

1986

Exhibition of the mathematical 'reversal error' by final-year secondary-teacher trainees

Martin Cooper

University of New South Wales

Recommended Citation

Cooper, M. (1986). Exhibition of the mathematical 'reversal error' by final-year secondary-teacher trainees. *Australian Journal of Teacher Education*, 11(1).

<http://dx.doi.org/10.14221/ajte.1986v11n1.6>

This Journal Article is posted at Research Online.

<http://ro.ecu.edu.au/ajte/vol11/iss1/6>

Exhibition of the mathematical 'reversal error' by final-year secondary-teacher trainees

Martin Cooper
University of New South Wales

Abstract

A mathematical error which involves the reversal of natural referents for symbols is displayed to a substantial degree by high school teachers and university staff among others. It has been shown, moreover, that the phenomenon is difficult to correct by teaching methods.

In the present study, a sample of final-year pre-service Education students destined for secondary-school teaching was asked to write a short verbal statement giving the same information as a simple algebraic equation and to construct an equation corresponding to a simple statement printed in plain language. A substantial proportion of students displayed reversal in both equation-interpretation and equation-formation modes. Students specialising in some academic fields appeared to be more prone to the error than those with other specialisations.

Introduction

In recent years, there has been interest in the "reversal error", a phenomenon in which many people reverse natural referents for symbols. Rosnick (1980), for example, found that many calculus graduates write the equation $6S = P$ when asked to produce an algebraic expression for the statement that there are six times as many students as professors in a particular university. Lochhead (1980) collected from university faculty members and high school teachers responses to a request to write an English sentence giving the same information as the equation $A = 7S$ where A is the number of assemblers in a factory and S in the number of solderers. Of 202 faculty members, 35% gave an incorrect response, the proportion being over 50% for those in faculties other than Physical, Natural, Behavioural and Social Sciences. Of the 148 high-school teachers, the proportion incorrect was 47%; for teachers of the physical sciences it was 28%, while for teachers of Natural, Behavioural and Social Sciences it was 67%. As a result of tutoring interviews, moreover, carried out with undergraduates, Rosnick and Clement (1980) concluded that "the reversal misconception is a resilient one which is not easily taught away".

In a study of over a hundred graduate business-administration students, Cooper (1984) found about the same proportion of reversal as Lochhead did with high-school teachers. Moreover, an even larger proportion was found to reverse when writing an equation corresponding to a written verbal statement.

The study reported in this paper focusses on final-year students undertaking education for high school teaching.

Method

Fifty-one final-year secondary-teacher trainees responded to two questions, as follows:

Question 1

In the equation $R = 4G$, the letter R stands for the number of Russians in a particular community while G stands for the number of Germans. Write a simple sentence that conveys the same information:

Are there more Russians or more Germans?

Question 2

For every worker using a Packard machine in a particular office, four workers use Canon machines.

Using the letter P to represent the number of Packard machines used in the office and C to represent the number of Canon machines, write a simple equation corresponding to the above statement:

Equation _____

The above questions were used in order to provide respondents with opposite modes of operation: the first involves interpreting an equation, the second requiring the writing of an equation to represent a situation. We refer to the explanation of an equation in words as the equation-interpretation mode; writing an equation corresponding to a particular situation we call the equation-formation mode. In addition to the possibility that the reversal phenomenon might be stronger in one mode than the other, the use of both modes allows the examination of tendencies among those who exhibit reversal in one mode but not the other.

As well as being asked to respond to the questions, each participant was asked to state the subjects which he or she intended to teach in high schools.

Results

The results, summarised in Table 1, show that in the equation-interpretation mode, secondary school teacher trainees exhibit reversal to a slightly smaller, although substantial, extent than high-school teachers and MBA students. In the equation-formation mode, however, the proportion exhibiting reversal is greater than that shown by MBA students.

		Question 2 (equation-formation mode)		
		reversal	correct	
Question 1 (equation-interpretation mode)	correct	15 (30%)	16 (31%)	31 (61%)
	reversal	20 (39%)	0 (0%)	20 (39%)
		35 (69%)	16 (31%)	51

Table 1 : Frequencies and percentages giving correct responses and exhibiting reversal

One marked difference found between teacher-trainees and graduate business students, however, was that all the Education students who reversed in only one mode did so when forming an equation; of Business students who reversed in only one mode, about one third reversed in the equation-interpretation mode.

That the reversal phenomenon is not confined to non-mathematicians and non-scientists was demonstrated by Lochhead (1980), who found that a substantial proportion reversed when presented with his equation-interpretation problem. Cooper (1984) showed that of MBA students who were graduates in areas of science and technology, 31% reversed on an equation-interpretation task and 47% reversed in the equation-formation mode. For graduates in other areas, the proportions were 63% and 69%, respectively. When the responses of the teacher trainees examined in the present study were partitioned by "teaching subject area", the proportions reversing were as shown in Table 2.

Teaching-subject area	Equation-interpretation mode	Equation-formation mode	Number in subject area
Mathematics	3 (37%)	6 (75%)	8
Sciences	3 (21%)	6 (43%)	14
Humanities	14 (48%)	23 (79%)	29

Table 2 : Numbers and percentages exhibiting reversal, by teaching-subject area

These results indicate that of final-year Education students destined to teach secondary school mathematics and sciences, just over one-quarter reverse in equation interpretation and just over one-half in equation formation. The result for mathematics-sciences teacher trainees involved in equation interpretation is about the same as that found by Lochhead for high school teachers of "physical sciences". In the case of the equation-formation mode, the teacher trainees in humanities exhibit reversal to a markedly lesser extent than Lochhead's high-school teachers of natural, behavioural and social sciences.

In view of Rosnick and Clement's claim that the reversal misconception is resilient, some may experience disquiet at the degree to which it appears to be present in teachers and teachers-in-training. Cooper (1984) demonstrated, however, that the incidence of the phenomenon can be substantially reduced, even with low-key teaching. There may not, therefore, be cause for undue concern provided that the potential widespread existence of the problem is acknowledged.

References

- Kaput, J. and J. Clement, Letter to the Editor, *Journal of Children's Mathematical Behaviour*, 1979, 2.
- Lochhead, J., "Faculty interpretations of simple algebraic statements : the professor's side of the equation", *Journal of Mathematical Behaviour*, 1980, 3, 29-37.
- Rosnick, P. and J. Clement, "Learning without understanding: the effect of touring strategies on algebra misconceptions", *Journal of Mathematical Behavior*, 1980, 3, 3-27.
- Cooper, M., "The mathematical reversal error and attempts to correct it", paper present ED at the eighth annual conference of the International Group for the Psychology of Mathematics Education (PME), Sydney, 1984.