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Abstract: The present study aimed to analyse the potential difficulties in the problems posed by pre-service teachers about first degree equations with one unknown and equation pairs with two unknowns. It was carried out with 20 pre-service teachers studying in the Department of Elementary Mathematics Educations at a university in Eastern Turkey. The problem Posing Test (PPT) including five items concerning the equation types was used as the data collection instrument.

Furthermore, semi-structured interviews were made with each pre-service teacher. It was found that the pre-service teachers had difficulties in seven categories of problem posing. These difficulties were centered on incorrect translation of mathematical notations into problem statements, unrealistic values assigned to unknowns, and posing problems by changing the equation structure. Moreover, the pre-service teachers were found to have greater difficulty in posing problems about equations pairs.

Introduction

Algebra is a language of patterns, rules and symbols (Dede & Peker, 2007). In this respect, it assumes the role of a language and a bridge between the elements of mathematics and those of other scientific disciplines. Some researchers argue that algebra has a key role in understanding mathematical concepts (Choike, 2000; Kieran, 1992; Maccini & Hughes, 2000; Van Dooren, Verschaffel & Onghena, 2002). Kaput (1999) notes that algebra has made significant contributions to today's technology and modern life. Due to its significance, it is advised that algebra learning should be started in the early years of school life (Carraher, Schliemann, Brizuela & Earnest, 2006; National Council of Teachers of Mathematics [NCTM], 2000). In elementary education, learning arithmetical operations and the basic characteristics of such operations (for instance, commutativity and associativity) is deemed to be important. During further stages, the focus shifts toward students' skills of thinking in algebraic terms and problem solving (Nathan & Koedinger, 2000; Van Dooren, Verschaffel & Onghena, 2002). In this process, equations are formed by representing unknowns with symbols and problems are solved through the operations performed on these equations. However, it is not as easy as it seems to acquire such skills.

Translation Verbal Expressions to Symbols

The difficulties experienced in translating the verbal expressions into algebraic expressions were emphasized in studies about algebra. Rosnick (1981) noted that the transition from verbal expressions into the algebraic expressions is difficult for students of every age. MacGregor and Stacey (1994) conducted a study researching the difficulties students in the 7-10 age group experienced in learning algebra. In the study, they mentioned that the three main obstacles against forming the algebraic notations are *the meanings of letters, the belief that an expression containing an operation sign should be simplified to a single "answer" without an operation sign, and lack of awareness of the need for parentheses.* In the study, the students were asked to write the algebraic expression corresponding to the verbal expression "Add 5 to an unknown number x, then multiply the result by 3". Although the rate of the success in giving the correct answer increases according to age, it was determined to be generally low. The researchers stated that the students do not remember to use parentheses and they do not know how the use of parentheses affects the explanations. Furthermore, the students were also asked to form an algebraic expression for the relation between x and y variables in the table. It was determined that the rate of success in expressing this relationship algebraically was low in all of the age groups.

Another difficulty encountered in the process of writing algebraic expressions corresponding to verbal expressions is to concentrate on the unknown. Herscovics and Kieran (1980) noted that students fail to comprehend the use of letters in place of the unknown. Ergöz (2000) mentioned that students in the 12-13 age group considered the letters that represent the unknown in the problem as the labels of objects. It is a common misconception to think of algebraic symbols as abbreviations or labels of objects (for example, P means “professor” or represents a professor) (Capraro & Joffrion, 2006; Macgregor & Stacey, 1994). Letters represent different meanings in different contexts. When letters are present in algebraic entities, this seems difficult for students. Kieran, et al. (1990) explained an example. In arithmetic, 12m can mean 12 meters, that is, 12 times 1 meter. However in algebra, 12m can mean 12 times some unknown number of meters. Therefore, the letter carries two different meanings depending on the context. MacGregor and Stacey (1994) posed the problem the students in the 7-10 age group: “David is 10 cm taller than Con. Con is h cm tall. Use algebra to write David's height". Many students, gave arbitrary numerical answers, assumed alphabetical coding, or used letters to stand for words, and progressed to writing algebraic expressions that, although not always correct, used letters to signify unknown numbers. This misconception can pose an obstacle between the students and understanding the significance of the equal sign, as well as the structure of the equations.

Clement (1982) stated that 99% of freshman engineering students correctly solved the equation “$5x = 50$”. Nevertheless, only 27% of them could correctly formulate the algebraic expression that corresponds to the following verbal expression: "At Mindy's restaurant, for every four people who ordered cheesecake, there are five people who ordered strudel”. The author found that the students assigned false meanings to the variables in the algebraic expression corresponding to the verbal expression. (For example: let an equation corresponding to any verbal expression be in the form of 6x=y. Students write it as 6y=x by assigning false meanings to the unknowns in the verbal expression, which is seen in the “Student and Professor problem” and in many other similar types of problems. The original problem was: Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this university." Use S for the number of students and P for the number of professors (Clement, 1982)).

Clement stated that these types of mistakes encountered arise from syntactic translations. In the syntactic translations, the equations are formed by writing the mathematical symbols from left to the right in order in place of the words in the verbal expressions. MacGregor and Stacey (1993) researched the cognitive models of the students in the 8-10 age group in forming the equations corresponding to the verbal expressions. In their study, they posed a problem as "s and t are numbers. The number s is eight more than t. Write an equation showing the relation between s and t." They determined that the students made three major categories of errors while they formed the equations corresponding to the given verbal expression. The three major categories of errors were: (1) writing expressions, usually products or totals, instead of equations, such as $s(8 - t)$ or $8s\ x\ t$, (2) writing inequalities, such as $s8 > t$ and (3) writing reversed equations, such as $t = s + 8$. Researchers stated that the verbal expressions consist of syntactic and semantic processes as well as other processes, so the errors made cannot be explained merely by syntactic translation.

Translation Symbols to Verbal Expressions

Capraro and Joffrion (2006) researched the middle school students' skills to form the algebraic expressions corresponding to the verbal expressions. They posed a question as "Tachi is exactly one year older than Bill. Let T stand for Tachi's age and B stand for Bill's age. Write an equation to compare Tachi's age to Bill's age" to the students. Meaningful conceptual knowledge may not be necessary to answer this question. Translating this statement one word at a time would result in, "Tachi (T) is (=) exactly one (1) year older than (+) Bill (B)" or the algebraic sentence T = 1 + B. However, only less than half of the students were able to answer this question correctly. Furthermore, it was also determined that the students assign various values to the T and B variables such as 7-8 or 49-50. The fact that the students write the equities by using numerical values instead of writing algebraic equations indicates that they cannot understand the meanings of the letters representing the variables in the equation. In a similar way, Kieran (1992) noted that students correctly solved the operations relating to the given algebraic equation, while they had difficulty forming the equations to be obtained from the relations in verbal expressions.

It is believed that middle school students need to develop representational techniques for a profound understanding of and fluency with linear equations (Silver, 2000). However, the transition between representations is not limited with forming the algebraic equation corresponding to the verbal expression. Moreover, the transition from the algebraic expressions to the verbal expressions or to the problems can be considered as another dimension of transition between the representations. One way of making the process of learning algebra meaningful and effective for middle grade students is to use multiple representations. McCoy, Thomas, and Little (1996) state that equation solving and algebraic expressions establish relationships that are slightly related with daily life, which is unsatisfactory. This requires students to learn about different forms of representation of algebraic expressions and equations and about the shifts between them, and further requires teachers to carry out activities in this regard. For Dede (2005), when teaching the equation concept, students should be asked to write stories or create scenarios concerning equations. Thus, they could be prevented from performing meaningless and insensible operations on given equations. Laughbaum (2003) indicated that the traditional equation solving method was purely practical, and students could not establish a connection between daily life situations and the equation concept. NCTM (2000) mentions the need to form problems that are appropriate for the algebraic expressions given in the form of equations, along with forming the equations corresponding to verbal problems.

Problem Posing

Problem posing is a process that necessitates performing a series of mental activities. Problem posing involves generating questions to be analyzed or discovered with regard to a given situation (Akay, 2006). According to Leung (1993), posing problems is the rearrangement of a given problem, while NCTM (2000) believes that it is posing different problems on the basis of a given situation or experience. In general, problem posing is defined as generating new problems or reformulating a given problem (Cai & Hwang, 2002; English, 2003; Silver, 1994; Ticha & Hospesova, 2009). English (1998) noted that identifying mathematical expressions and relating them to the situations of daily life skills of the students can be evaluated and improved by posing problems. According to Dickerson (1999), students find the opportunity to use their own language, grammar structure, syntax and subject story in the problem sentences they write. He further mentions that the teaching environment that encompasses problem posing, as well, offers an alternative to the traditional problem-solving approach. When they pose their own problems, students can develop a mathematical language, gain insight into the symbolic representations in problems, and establish links between the steps that are necessary for solution (Rudnitsky, Etheredge, Freeman & Gilbert, 1995).

According to Hedden and Langbauer (2003), rather than teach specific algebraic skills and then progress to story problems that use those skills, we start from interesting contexts and pose problems about those contexts. Işık and Kar (2012), ascertained in a qualitative study they conducted with elementary school mathematics teachers that the teachers had a tendency to include, at the most, posing problems about equations. Teachers who took part in the research noted that posing problems about equations contributed to establishing relationship with daily life and solving verbal problems involving equations. Akkan, Çakıroğlu and Ünal (2009), on the other hand, determined that the students, ranging in age from 12 to 15, were less successful in posing problems about equations than they were in solving problems. In addition, they further mentioned that students proved to be more successful in posing problems corresponding to arithmetical equations than in posing problems corresponding to algebraic equations.

English (1998) held activities of problem posing about arithmetical operations in his experimental study he conducted with students in the 3rd grade. At the end of the study, he determined that the students could also pose different problems about different meanings of addition and subtraction operations. He mentioned that this situation improved the students' conceptual understanding of addition and subtraction operations. Moreover, Silver and Cai (1996), presented an open-ended story to middle school students of sixth and seventh grades as follows; "Jerome, Elliot, and Arturo took turns driving home from a trip. Arturo drove 80 miles more than Elliot. Elliot drove twice as many miles as Jerome. Jerome drove 50 miles." Students were asked to pose three different problems that will require using the information given in the story while solving the problem. The posing problems were analyzed first in terms of solvability and then in terms of mathematical complexity. Researchers found that 30% of the posed problems were not mathematical problems at all and that the majority of solvable problems were like; "How many miles did Elliot drive?", "How many miles did Arturo drive?" Researchers stated that such problems are the weakest type of problems with respect to complexity. Additionally, they used eight open-ended tasks to measure the students' mathematical problem solving performance. They found that students' problem solving performance was highly correlated with their problem posing performance. Compared to less successful problem solvers, good problem solvers generated (posed) more and more complex, mathematical problems.

Crespo and Sinclair (2008) presented different problem posing activities for pre-service mathematics teachers, in addition to the above mentioned activity. A majority of the problems posing by pre-service teachers proved to be weak, in terms of complexity. Furthermore, they indicated that the candidates had limited understanding about the content of a good mathematical problem and the problems posed by them were in aesthetics. However, as a result of the meetings held to discuss the problems, the pre-service teachers developed richer understandings of how a good problem should be and the problems posed by the same teachers improved in terms of aesthetics. Stickles (2006) conducted a study researching different problem posing abilities of teachers and pre-service teachers. She used two different problem posing strategies. These strategies involve the posing different problems based on the problems and posing problems from semi-structured situations. The problems posed by teachers and pre-service teachers were analyzed in three categories, which are as follows; problem, not a problem, and exercise. It was concluded from the study that the problem posing performance of the teachers and pre-service teachers was low and that exercise-type questions were preferred more often.

Gonzales (1998) defines problem posing as the fifth one of Polya's problem solving steps. Dickerson (1999) applied a kind of problem solving teaching based on problem posing in learning numbers on seventh grade students. The students who were trained to cover various problem posing activities proved to be more successful when compared to those who did not receive such training. Cai and Hwang (2002) concluded from the study, within the scope of which they focused on problem posing and solving skills of sixth grade students from the USA and China, that the posed problems were classified into extension problems, non-extension problems, or others. A problem is considered an extension problem if it asks about the pattern beyond the first several given figures or terms. A non-extension problem restricts itself solely to the first several given figures or terms in a pattern. Findings of the study showed that the students preferred non-extension problems more than others. In addition, researchers have found that the students who were successful in posing problems preferred abstract strategies while solving problems.

Reality was another condition underlined in the studies that were based on problem posing and solving. Cooper and Harries (2002), stated that students, ranging in age from 11 to 12, focused on arithmetical operations for verbal problems that require realistic answers, yet, they failed in interpreting the result in daily life. Carpenter, Lindquist, Matthews and Silver (1983) asked students of age 13 the following question; *A military bus can carry 30 soldiers. Accordingly, how many buses are needed to carry 1128 soldiers?* Seventy percent of the same students answered as *37 buses, with 18 as a remainder, or as 37.6 buses*. Verschaffel, Greer and De Corte, (2000) mentioned that students could not realize in such problems that the answer needed to be a natural number, not a fractional one. Likewise, Chen, Dooren, Chen and Verschaffel (2011) ascertained in their studies focusing on the skills of Chinese teachers and pre-service teachers in posing and solving problems corresponding to division operations with remainder that the teachers acted more realistically in solving problems than posing them. They also highlighted that there was congruity between the problem posing and solving performances of teachers and the performance of students in evaluating their unrealistic answers.

Işık and Kar (2012), investigated an error analysis in the problems posed by pre-service elementary mathematics teachers about fractional division operation. In the research, seven types of errors were identified in the problems posed by the pre-service teachers about fractional division. In addition, they founded that the pre-service teachers overlooked the conceptual aspect of division in the problems they posed about fractional division. Similarly, Luo (2009) investigated the ability of pre-service elementary teachers to write word problems that represent symbolic expressions of fraction multiplication. Results indicated that a significant percentage of the pre-service teachers were unable to construct appropriate word problems for the given symbolic expressions of fraction multiplication. In addition, some pre-service teachers included expressions like "Four children have each got $1\frac{2}{3}$ of a sheet cake in the problems they generated corresponding to the $1\frac{2}{3} \times 4 =?$ operation. The researcher noted that it was not possible to have more than a whole out of a sheet cake and, thus, the expression of "$1\frac{2}{3}$ of a sheet cake" was not correct in logical terms. Such situations point that the realistic aspect of a problem posed by pre-service teachers was ignored.

In the studies carried out with regard to equations and problem posing, students, pre-service teachers and teachers have been observed to have difficulty at different levels. Difficulties experienced in equations seem to focus on the use of language in transitions between representations and mathematical mistakes made about the components (unknowns, equalities and operational symbols) building the equation. In the literature, on the other hand, problems generated for symbolic expressions and open-ended situations are indicated to be simple, unsatisfactorily structured and at the exercise level. Furthermore, it is mentioned that the realism aspect of problems posed for symbolic expressions is ignored. No comprehensive and qualitative analysis from different aspects has been found about the difficulties undergone while posing problems about equations and their causes. In this sense, the possible difficulties of problems posed by elementary school pre-service mathematics teachers, in relation to the daily life conditions for the first-degree equations with one unknown and equation pairs with two unknowns, have been aimed to be analyzed.

Methodology

Participants

The elementary education in Turkey involves a continuous eight-year period starting from the age of seven. This elementary period is carried out in two stages as 1-5 and 6-8. While elementary teachers are in charge of education within the first stage, subject teachers replace them in the second stage. In elementary school mathematics is taught in the fields of numbers, geometry, algebra, probability-statistics and measurement. Depending on the topic of the study, learning domains of algebra consist of subjects such as patterns and relations, algebraic expressions, equity and equality, inequalities.

The mathematic teachers who are educated to teach mathematics to elementary grades 6-8 graduate from the faculty of college after completing a four year program. Selection and placement of the students to be admitted to such programs is done on the basis the results of a national university entrance exam, which aims to make a general assessment of the students' twelve-year education. As a result of the examination, students with nearly the same achievement levels are admitted to a university's elementary mathematics education program. This program focuses on how to teach mathematics to students in the 12-15 age groups. The elementary mathematics education program involves courses on general knowledge, pedagogy and mathematical content knowledge. Along with field courses, the program also offers the pre-service teachers two courses about mathematical teaching methods during their third year. These courses involve the basic elementary concepts and theoretical and practical activities to teach them. In these courses, the problems and what these problems are, the significance of solving problems, the purposes of teaching problem solving, the process of problem solving, problem posing and activities of problem posing are taught. Moreover, during the final year of the Elementary Mathematics Education program, pre-service teachers participate in educational activities at schools. In this way, they find the chance to observe and implement in-classroom teaching activities.

The present study was carried out with 20 pre-service teachers who were about to graduate from the Department of Elementary Mathematics Education at a university in eastern Turkey during the spring semester of 2009-2010. Since the pre-service teachers were in the graduation stage, they completed all the courses with success and participated in teaching activities in elementary schools. The pre-service teachers to participate in the study were selected among a total of 42 pre-service teachers to whom the researcher lectured. Since an intimate environment was desired for the interviews, the participants were to be chosen among the students to whom the researcher lectured. The pre-service teachers were informed about the study procedure prior to the study. At the end of the instruction process, 20 pre-service teachers agreed to participate in the study voluntarily. Twelve of the pre-service teachers who participated in the study were female and 8 of them were male. Their average age is approximately 22. Each participant was assigned by the researcher with codes such as PT1, PT2,…, PT20 to symbolize the concept of pre-service teacher.

Data Collection

The Problem Posing Test (PPT) was employed to identify the pre-service teachers' difficulties in posing problems about different equations. Additionally, a semi-structured interview was conducted with the pre-service teachers who were observed to experience difficulties in answering the PPT.

The PPT included two items for first-degree equations with one unknown and three items for first-degree equation pairs with two unknowns. During the process of determining which equations were going to be involved in the PPT, the equations in the national mathematics program and course books were examined. In this process, it was determined that various equations differing in terms of whether the unknown is on one side or on both sides of the equation, the use of parentheses, the values assigned to the coefficients and unknowns were used. Since an extensive test to be prepared by taking these differing situations into consideration would complicate the analysis of the study even if it would extend the study, the researchers resorted to limiting the items of the test. In the national mathematics program, equations with one unknown on only one side of the equation are taught initially. Later on, equations with one unknown on both sides of the equation are taught. The same applies for first-degree equation pairs with two unknowns. From this perspective, it was determined that the PPT's would be prepared by taking into consideration whether the unknown is on only one side or on the both sides of the equality. For this reason, it was determined that in the PPT, there would be two items about equations with one unknown, whereas the number of the items about equation pairs would be three. Although there are various cases of equations, the test was limited with five types of equations in total. Three equations of each type were formed. According to the opinions of five elementary school mathematics teachers, the equations in the Table 1 were agreed to be used in the study. The teachers stated that the types of the equations chosen were used in the education process and they frequently encounter such equations while solving problems within the course books.

Equations	Equation Characteristics
$5(x+4)=4(x+8)$	Equation with one unknown on both sides of the equal sign
$5(x-4)=35$	Equation with one unknown on only one side of the equal sign
$x+20=\frac{2y}{3}$ $x-30=\frac{y}{2}$	An equation pair with two unknowns on both sides of the equal sign
$x+y=840$ $4x=3y$	An equation pair with unknowns on both sides of only one of the equal sign
$x+y=16$ $5x+10y=100$	An equation pair with unknowns on a single side of the equal sign

Table 1. The Types and Characteristics of the Equations in the PPT

The PPT was performed in one course hour (approximately 50 minutes). In the application directions of the test, the following explanation was provided: *For a single moment, imagine that you are a teacher. In respect to teaching equations, you have discovered that some of your students have difficulties in understanding the meanings of equal signs, unknowns, brackets and operations, which are components of equations. In order to eliminate such difficulties, you have decided to pose problems related to daily life situations that will reflect the components of an equation. Given such circumstances, pose a problem related to daily life situations that will reflect the equations.* Answers, given by the pre-service teachers to each item mentioned in the PPT, were analyzed. In order to ascertain the difficulties observed in the answers and elaborately defining the reasons of these difficulties, semi-structured interviews were made with the 20 pre-service teachers. The aim of these interviews was to reveal their ways of thinking during the process of posing problems about equations. During the interview process, alternative questions were asked in order to obtain their opinions about the acceptability of the problem posed for the given equation. The interviews were made in a silent and spacious environment and took about 10-15 minutes each. They were recorded under the express permission of the students. The recordings were then transcribed and these data were employed in the analysis.

Data Analysis

The problem statements regarding the equation in the PPT were analyzed by two researchers. Two researchers analyzed the answers of the pre-service teachers independently from each other and simultaneously. The answers were analyzed using the categorical analysis technique, which is a type of content analysis. In this process, each researcher formed categories according to the problems posed. Next, their analyses were compared and a consistency of 92% was achieved in the classification of the difficulty types identified. In the process of comparing the analyses, a researcher divided the difficulties experienced in translating the operations and parentheses into verbal expressions under two categories.

However, the difficulties in these two categories were agreed to be presented combined under the same category, since the operation symbols and parentheses are mathematical notations. The researchers named this category *incorrect translation of mathematical notations (operations and parentheses) into problem statements* [Difficulty 1(D1)]. In the last meeting, a consensus was arrived at differing classifications of the given answers. In addition, the researchers hesitated on the issue of whether counting the use of symbolic representations while posing problems as difficulties or not. The performed interviews indicated that the pre-service teachers prefer such usage in order to overcome the difficulties they experience while translating into verbal expressions. For this reason, this case was agreed to be presented as a category of difficulty. This category was named "use of symbolic representations in the problems posed" (D4).

An analysis of the pre-service teachers' responses to the PPT demonstrated that they had difficulty in seven categories. Explanations of such difficulties have been presented in the results section. The data obtained from the interviews are presented in the results section using descriptive analysis method. Descriptive analysis makes frequent use of direct quotations in order to strikingly show the opinions of the interviewed or observed individuals (Mcmillan & Schumacher, 2010). The distribution of the difficulty categories observed in the problems statements written for each equation type in the PPT is shown by using frequency value.

Results

An analysis of the pre-service teachers' responses to the PPT demonstrated that they had difficulty in seven categories concerning the equations, which are as follows: (a) *incorrect translation of mathematical notations (operations and parentheses) into problem statements* (D1), (b) *unrealistic values assigned to the unknowns in the problems posed* (D2), (c) *posing problems by changing the equation structure* (D3), (d) *the use of symbolic representations in the problems posed* (D4), (e) *the failure to establish a part-whole relationship* (D5), (f) *posing separate problems for each equation in an equation pair* (D6), and (g) *the failure to establish a relation between the variables* (D7). Explanations of the specified difficulties over equation types are as follows:

The results regarding the difficulties experienced in posing problems for first-degree equations with one unknown

Table 2 presents the results regarding the categories of difficulties experienced by the pre-service teachers in posing problems for first-degree equations with one unknown.

Equations	D1	D2	D3	D4	D5	Total
$5(x-4)=35$	6	1	1	0	0	8
$5(x+4)=4(x+8)$	2	0	5	2	1	10

Table 2. Distribution of the difficulty categories for equations with one unknown

The pre-service teachers experienced a total of 18 difficulties in posing problems about first-degree equations with only one unknown. Eight of these difficulties were experienced while posing problems about the equation of $5(x-4)=35$, whereas the other

10 of them were experienced while posing problems about the equation $5(x+4)=4(x+8)$. The explanations related to the difficulty categories are:

Incorrect translation of mathematical notations (operations and parentheses) into problem statements (D1): This type of difficulty involves cases in which the operations and parentheses in an equation are incorrectly translated in problem statements. (For example, for the equation $5(x-4)=35$, posing a problem as follows: "*Ali has a certain amount of marbles. He gave 4 of his marbles to his brother and equally shared the rest of the marbles with 4 of his friends. Since Ali has 35 marbles in the end, how many marbles did have in the beginning?").* In the problem statement, the expression *"equally shared the rest of the marbles with 4 of his friends"* does not correspond to the multiplication operation in the equation.

Six of the pre-service teachers experienced the D1 category of difficulty in posing problems about the equation of $5(x-4)=35$, while two of them experienced the same type of difficulty in posing problems about the equation of $5(x+4)=4(x+8)$. Most of the difficulties about the equation of $5(x-4)=35$ were under this category. They failed to translate the operation of subtracting 4 units from what x represents and multiplying the solution by 5 into a problem statement using appropriate language. One example of a problem statement written by PT3 is as follows:

> *I cannot afford a 35 lira dress I saw in a store and liked. The store owner said that I could buy it by paying 4 liras less than what I have in my pocket in 5 months. How much money do I have in my pocket?*

Below is an excerpt from the interview made with the pre-service teacher who wrote down the above problem.

Researcher: Can you tell me how you posed the problem?

PT3: In fact, I thought about it a lot. Finally, I found this. Actually, I said to myself: what should I multiply to obtain 35 in the equation $5.(x-4)=35$? I find 35 if I multiply 5 by 7. Since there is $(x-4)$ in parentheses, I thought about how I could arrive at 7.

Researcher: What did you mean by your following statement *"The store owner said that I could buy it by paying 4 liras less than what I have in my pocket in 5 months"*?

PT3: It seems as if the store owner knows the amount of money in my pocket. I really did not think about the details there. I just thought what I should do to arrive at the solution.

Researcher: What do you think about whether the sentences in the problem statement you posed correspond to the equation "$5.(x-4)=35$"?

PT3: Let me explain. The amount of money in my pocket is x. It is an unknown amount of money. Four liras less than that, which is normal. Paying 4 liras less than the amount of money in my pocket in 5 months means multiplication. The first month (x-4), the 2nd month (x-4), and so on until it makes 5 months. And I made this sum equal to 35.

The focus of the problem to be posed is the fact that 5 times the amount in parentheses should be equal to 35. If the problem contained a statement like "you could buy that dress in 5 months by paying each month a sum that is 4 liras less than the amount in your pocket", then "x-4" could be multiplied by 5. Nevertheless, the problem statement formulated by the pre-service teacher is understood to mean that "x-4" liras should be paid in 5 months, if the amount of money in her pocket is represented by "x". Pre-service teachers have been observed in the foregoing and other interviews, to have failed in conveying algebraic expressions in a language appropriate for verbal expressions.

Unrealistic values assigned to the unknowns in the problems posed (D2): This type of difficulty involves cases in which the numerical value for the unknown in a given equation is ignored during the problem posing process, from which an inconsistency results between the values assigned to the unknown and the problem's story. This difficulty category was experienced by only one pre-service teacher in posing problems about the equation of $5(x-4)=35$. The answer of this pre-service teacher is;

> *Ali owns a car gallery. He has 5 cars with identical properties and the same price. But to easily sell all of them during an economic crisis, he made 4 liras of discount on each car. If Ali sold all the cars he had for a total price of 35 liras, then what was the original price of the cars?*

In this problem, making a discount of 4 liras (about 3 dollars) on each car and the total price of five cars being 35 liras (about 24 dollars) are unrealistic cases.

Posing problems by changing the equation structure (D3): This type of difficulty involves cases in which one poses a problem for a new equation obtained by performing a series of operations on a given equation (For example, for the equation "$5.(x+4)=4.(x+8)$", posing a problem as follows: "*A group of trainee teachers visit a school. In the classroom, if they and the students sit as 5 people in each desk, 20 people have to stand up; and if they sit as 4 people in each desk, then 32 people have to stand up. Then, how many desks are there in the classroom?"*). The equation corresponding to this problem is "$5x+20=4x+32$" if the number of desks is taken as "x".

One pre-service teacher experienced difficulty category of D3 in posing problems about the equation of $5(x-4)=35$ while five of them experienced this difficulty in posing problems about the equation $5(x+4)=4(x+8)$. Most of the difficulties about the equation of $5(x+4)=4(x+8)$ were experienced under this category. As an example for the difficulty category of D3, the most common category observed in posing problems for the equation $5(x+4)=4(x+8)$, PT12 wrote down the following problem statement:

> *A group of trainee teachers visit a school. In the classroom, if they and the students sit as 5 people at each desk, 20 people have to stand up. If they sit as 4 people at each desk, then 32 people have to stand up. How many desks are there in the classroom?*

Below is an excerpt from the interview with PT12 who wrote this problem:

Researcher: Can you tell me how you posed the problem?
PT12: I posed it after thinking for a long time. But I do not exactly know how I posed it.
Researcher: Can you write down the mathematical equations required to solve this problem?
PT12: I wrote x for the number of desks. 20 people have to stand up when they sit as 5 people in a desk, so 5x+20, and 32 people have to stand up when they sit as 4 people in a desk, so 4x+32 and they are equal to each other.
Researcher: What do you say about the consistency between the statements in your problem and the equation for which you were asked to pose a problem?
PT12: (After looking at the problem once again) Not exactly consistent. No, it is not consistent.
Researcher: What points do you think make it inconsistent?
PT12: There is no difference between the solution sets. No difference as an equation, only its distributed form.

If the number of desks is represented by x in this problem, the equation to be written will be "$5x+20=4x+32$", which is the version of the original equation "$5.(x+4)=4.(x+8)$" without any parentheses. Although both equations have the same solution set, verbal expressions that will correspond to the algebraic expressions mentioned in the equation will be different from each other. This is because there is a bracketed expression in the equation asked to be used for generating a problem. Verbal expressions corresponding to $5x + 20$ and $5.(x + 4)$ will, therefore, differ. Similar conditions apply for the right side of the equations. Considering the results of other interviews, it is understood that language-related difficulties steer pre-service candidates to pose problems by changing the equation structure.

The use of symbolic representations in the problems posed (D4): This type of difficulty involves cases in which the unknown is represented by parameters such as x and y in the problem statements posed (For example, for the equation "$5.(x+4)=4.(x+8)$", posing a problem as follows: *"A vehicle travelling from Erzurum to Istanbul had an initial speed of x km/h. If the vehicle speeds up by 4 km, it will reach Istanbul in 5 hours, and if it speeds up by 8 km, it will reach there in 4 hours. What is the vehicle's initial speed?"* This type of difficulty was not experienced in posing problems about the equation$5(x - 4) = 35$. However, it was experienced by two pre-service teachers in posing problems about the equation of $5(x + 4) = 4(x + 8)$.

The failure to establish a part-whole relationship (D5): This type of difficulty involves cases in which one assumes a greater amount than the value of the unknown and uses it in the problem statement (For example, for the equation "$5.(x+4)=4.(x+8)$", posing a problem as follows: "*Ali has a certain amount of money. He gives 4 times the amount of his money plus 8 liras to his younger sister. He gives 5 times the amount of his money plus 4 liras to his elder sister. If he gave equal amounts of money to his younger and elder sisters, how much money did Ali have in the beginning?*). In this problem posed, Ali gave out not a part of his money, but more than the amount to his elder and younger sisters, which does not make sense in terms of part-whole relation. This type of difficulty was not experienced in posing problems about the equation $5(x - 4) = 35$. However, it was experienced by one pre-service teacher in posing problems about the equation $5(x + 4) = 4(x + 8)$. In this regard, the pre-service teacher has overlooked reality in the posed problem.

The results regarding the difficulties experienced in posing problems for equation pairs

Table 3 presents the results regarding the categories of difficulties experienced by the pre-service teachers in posing problems for equation pairs.

Equations	D1	D2	D4	D6	D7	Total
$x+20=\frac{2y}{3}, \quad x-30=\frac{y}{2}$	1	3	3	1	4	12
$x+y=840, \quad 4x=3y$	2	7	1	0	0	10
$x+y=16, \quad 5x+10y=100$	0	3	3	1	0	7

Table 3. Distribution of the difficulty categories for equation pairs

The pre-service teachers experienced a total of 29 difficulties in posing problems about first-degree equation pairs with two unknowns. As for the first-degree equation pairs with two unknowns, five categories were identified including the following: (a) incorrect translation of mathematical notations (operations and parentheses) into problem statements(D1), (b) unrealistic values assigned to the unknowns in the problems posed (D2), (c) the use of symbolic representations in the problems posed (D4), (d) posing separate problems for each equation in an equation pair (D6), and (e) the failure to establish a relation between the variables (D7). The same difficulties (D1, D2 and D4) experienced while posing problems for first-degree equations with one unknown were similarly observed in the problems posed for equation pairs. Therefore, D1, D2 and D4 difficulty categories are represented with the same codes, while the two different difficulty categories identified are symbolized by D6 and D7. The explanations related to the difficulty categories of D6 and D7 are:

Posing separate problems for each equation in an equation pair (D6): This difficulty category involves problem statements written independently from each other for each equation that make up an equation pair (For example, writing two different problem statements by thinking each equation in item 3 of the PPT separately). This type of difficulty was observed only in the problems posed by two pre-service teachers about the equations of $x+20=\frac{2y}{3},\quad x-30=\frac{y}{2}$ and $x+y=16,\quad 5x+10y=100$. Posing problems that are appropriate for equation pairs has been emphasized for pre-service teachers in the test directive. Despite that, posing separate problems for each equation indicate that the pre-service teachers have deficiencies in conceptual terms regarding equation pairs.

The failure to establish a relation between the variables (D7): This difficulty category involves cases in which the change between the variables in equation pairs is not translated into problem statements using appropriate expressions. This type of difficulty was observed only in the problems posed by four pre-service teachers about the equation pair $x+20=\frac{2y}{3},\quad x-30=\frac{y}{2}$. PT7's answer about this difficulty type is;

> *Ali's apples plus 20 apples equals to two-thirds of Bahar's apples. If Ali gives 30 of his apples to Bahar, then Ali's apples are equal to half of Bahar's apples. Then, how many apples do Ali and Bahar have?*

The first sentence of this problem can be represented by the equation $x+20=\frac{2y}{3}$ in the equation pair. However, the second sentence cannot be represented by the equation $x-30=\frac{y}{2}$. Below is an excerpt from the interview with PT7 who wrote this problem;

Researcher: Can you tell me how you posed the problem?

PT7: The equations involve x and y. So I thought of comparing the apples of Ali and Bahar. Since the first equation is $x+20$, I said Ali's apples plus 20. Since it says $\frac{2y}{3}$ on the other side, I said two-thirds of Bahar's apples. I did the same for the second equation.

Researcher: Could you be more specific about what you thought for the second equation?

PT7: As I said. I did the same things for the second equation. Ali's giving 30 of his apples to Bahar corresponds to the expression x-30. Since it is $\frac{y}{2}$ on the other side, I wrote "half of Bahar's apples" for this side.

Researcher: What can you tell me if you compare the verbal expressions you wrote down for each equation in the equation pair?

PT7: After all, each equation has the expressions of +20 and -30 on the left side. Here, the statements "Ali's apples plus 20" and "if Ali gives 30 of his apples to Bahar" corresponds to these cases. As I said, they are similar in this way.

It is clear from this and other interviews that some of the pre-service teachers ignored the fact that a change on one side of an equation can affect the statements on the other side. In this problem, the statement "if Ali gives 30 of his apples to Bahar" can be written as "$x-30$", which corresponds to the left side of the equation "$x-30=\frac{y}{2}$". With this statement in place, the number of Bahar's apples will increase by 30 and will be represented by "$y+30$". So in the end, half of Bahar's apples will be "$\frac{y+30}{2}$". But this statement does not correspond to the right side of the equation "$x-30=\frac{y}{2}$". It would not be wrong to say that such answers indicate that the pre-service teachers have conceptual deficiencies.

One of the pre-service teachers experienced the difficulty category of D1 in posing problems about the equation pair $x+20=\frac{2y}{3}, \quad x-30=\frac{y}{2}$, while two of them experienced the same type of difficulty in posing problems about the equation couple of $x+y=840, \quad 4x=3y$. Three of the pre-service teachers experienced the difficulty category D2 in posing problems about the equation pair $x+20=\frac{2y}{3}, \quad x-30=\frac{y}{2}$, while seven of them experienced it in posing problems about the equation pair $x+y=840, \quad 4x=3y$ and three of them experienced it in posing problems about the equation pair $x+y=16, \quad 5x+10y=100$. To exemplify the difficulty category of D2, the most common category observed in posing problems for the equation pair "$x+y=840, \quad 4x=3y$", PT17 wrote down the following problem statement:

The total price for a pencil and an eraser is 840 liras. If 4 times the price of the eraser is equal to 3 times the price of the pencil, then how much do the pencil and eraser cost?

Below is an excerpt from the interview made with PT17 who wrote this problem:

Researcher: Can you tell me how you posed the problem?

PT17: I thought that the sum of two unknowns is 840 liras. I assigned names to the unknowns to ensure that students can better understand. Since the unknowns ($4x=3y$) are equal, I said 4 times the price of the pencil equals 3 times the price of the eraser.

Researcher: Did you take into account the values of x and y while you were posing a problem for the equation?

PT17: No, I did it without calculating. But if I had made a calculation, maybe I could have come up with a better problem.

Researcher: What can you say if you think your statement "The total price for a pencil and an eraser is 840 liras" in terms of realism?

PT17: When it comes to realism, 840 liras is not an appropriate sum. But I did not think that way then. It was impulsive. I never thought of the sum of 840 liras in terms of realism.

If we solve this problem formulated by the pre-service teacher, we will find that an eraser costs 360 liras (about \$240) and a pencil costs 480 liras (about \$320). Given that a pencil and an eraser cannot possibly cost that much in students' close surroundings, these values for the unknowns could be said to be unrealistic. In this and other interviews, some of the pre-service teachers noted that they selected their problems' stories from students' close surroundings so that they would be more interested. However, the numerical values assigned to the unknowns in the problems drew them away from real life.

Three of the pre-service teachers experienced difficulty in the D4 category in posing problems about the equation pair of $x+20=\frac{2y}{3},\quad x-30=\frac{y}{2}$, while only one of them experienced the same type of difficulty in posing problems about the equation pair of $x+y=840,\quad 4x=3y$ and three of them experienced it in posing problems for the equation pair of $x+y=16,\quad 5x+10y=100$. To exemplify the difficulty category of D4, the most common category observed in posing problems for the equation pair $x+y=16,\quad 5x+10y=100$, PT6 wrote down the following problem statement:

> *A group of 16 people including x students and y adults went to the theater. A students' ticket for the play costs 5 liras, while an adult ticket costs 10 liras. If the total sum paid for the tickets is 100 liras, then how many students are there in this group?*

Below is an excerpt from the interview made with PT6 who wrote this problem:

Researcher: Can you tell me how you posed the problem?

PT6: First of all, among the multiples in the equations, 16 made me think of the number of people and 5 and 10 made me think of the sum of money. And I took 16 as a group of people including students and adults and 5 and 10 would be ticket prices.

Researcher: Can you explain why you represented the number of people by x and y?

PT6: There are 16 people in the group. Some of them are students and the rest are adults. To correspond to the equation "$x + y = 16$", I wrote x for the number of students and y for the number of adults.

Researcher: What do you think about using the symbols x and y in the problem statement?

PT6: In fact, at that moment, I thought that it would be easier to pose the problem if I wrote x and y for the people in the group. But now, when I think again, I see that I could have put it without using x and y.

As revealed by this and other interviews, the pre-service teachers think of using symbolic representations like "x" and "y" in their problem statements as something that makes it easier to pose a problem. Yet, it could be argued that the use of symbolic representations like "x" and "y" in problem statements problem may orient students in the solution process into forming a mathematical representation of what is given and asked for.

Discussion

The results of the study showed that the pre-service teachers had difficulties in five categories (D1, D2, D3, D4, D5) when posing word problems for equations with one unknown and again in five categories (D1, D2, D4, D6, D7) when posing problems for equation pairs. The common difficulties observed in the problems posed for both types of equations include incorrect translation of mathematical notations (operations and parentheses) into problem statements (D1), unrealistic values assigned to the unknowns in the problems posed (D2), and the use of symbolic representations in the problems posed (D4). Posing problems by changing the equation structure (D3) and the failure to establish a part-whole relationship (D5) were only observed in the problems posed for equations with one unknown. On the other hand, posing separate problems for each equation in an equation pair (D6) and the failure to establish a relation between the variables (D7) were only observed in the problems posed for equation pairs. The difficulty categories of the posed problems seem to focus on language-related difficulties in transforming algebraic expressions into verbal expressions (D1, D3 and D4), ignorance of the realism of problems (D2 and D5) and lack of conceptual information on equations (D6 and D7).

The main reason behind the difficulties experienced in the category of D1 is the inability to translate the operations and the parentheses in the equation into verbal expression. In the interviews about this difficulty, the pre-service teachers were observed to calculate the numerical value of the expression with the bracket first of all. For instance, PT3's answer as *...what multiplied by what makes 35? 5 multiplied by 7 makes 35* supports this opinion. So, the pre-service teacher gave priority to calculating the expression of 5x7 first of all. Paying for the dress by installments means that this multiplication operation is considered as repeated addition. It is observed that the pre-service teacher tries to assign meaning to the expression of $(x-4)$ which corresponds to the multiplier value of 7 that represents the amount of installments of the pre-service teacher. As a result, in this type of difficulty the preference was given to forming the verbal sentences from the whole to the part (forming $5(x-4)$ from 5x7) on the left side of the equation. It can be said that this case causes difficulties in translating the parentheses and the operations into the verbal expressions. However, in the problem sentences where this type of difficulty was not observed, it was determined that there were answers such as *five times four units minus some amount is 35 units*. In other words, it was determined that they formed the verbal expressions from part to the whole on the left side of the equation (forming $5(x-4)$ through x).

Additionally, in the difficulty category of D3, the problem is posed on the expansion of the expression with the parentheses. The opinions of some of the pre-service teachers about such an approach eases problem posing and can be considered to support the difficulties experienced in translating the parentheses and operations into verbal expression. Different researchers have noted that the use of parentheses is ignored while forming equations that correspond to word problems (Booth, 1984; Kuchemann, 1981; Mcgregor & Stacey, 1994). Similar results were obtained in the present study with regard to posing word problems corresponding to equations. NCTM (2000) underlines the importance of translating algebraic expressions into verbal expressions, along with the importance of translating verbal expressions into algebraic ones. The difficulties of the pre-service teachers in formulating a verbal expression to correspond to an algebraic expression could be thought as one of the obstacles in the process of ensuring students to acquire the skill to switch between representations. Furthermore, in the interviews, some of the pre-service teachers mentioned that there are no differences between the equations with and without parentheses and their solution sets were similar. Such opinions indicate that the pre-service teachers ignored that the verbal expressions of the problems to be posed also differ when the structures of the equations are different.

In the difficulty experienced in problem posing in D4 , the pre-service teachers were observed to use symbolic representations such as x and y in the verbal expression of the problem. PT6 answered the question: *What do you think about using the symbols x and y in the problem statement?* with the response: *I thought that it would be easier to pose the problem if I wrote x and y for the people in the group*. This answer and the other answers in the interview such like this indicate that use of symbolic representations is considered a factor that eases forming verbal expressions. A problem statement creates a desire in students to solve the problem not by the symbolic representations of the unknowns involved, but by the story it tells to them. Rudnitsky, et al. (1995) note that the language of a problem helps students code mathematical values. A problem is not an encapsulated version of an equation.

On the contrary, the equation is an encapsulation of the problem. For this reason, using symbolic representations for unknowns in a problem statement will orient students in the problem solving process, which could be interpreted as an obstacle to the improvement of students' judgment and critical thinking skills.

The meanings assigned to the numerical values of x or y in the story of the problem statements posed in the category of D2 may seem to be unrealistic. The expressions of the pre-service teachers that they wrote as *a 4 lira discount on each car and the total price of five cars being 35 liras, an eraser costs 360 liras (about $240) and a pencil costs 480 liras (about $320)* in the problem statements indicates that the values assigned to the unknowns are not appropriate in terms of being realistic. The reason of this situation can be the fact that the pre-service teachers pose the problems without calculating the numerical values of the unknowns in the equations. In the interviews, it was determined that the pre-service teachers were unable to calculate the numerical values of x and y in the equation. So it can be said that the pre-service teachers were only trying to translate the given equation into verbal expression. The difficulty category of D5 was encountered in a problem about equations with only one unknown. In this problem sentence, giving more than all of the present money instead of a part of it is not meaningful in terms of part-whole relationship. In colloquial language, we can say that any times the numerical value that represents many is equal to lot many. Using a lot many, which is more than many we have (i.e. giving more money than the money present to the younger sister) is not reasonably meaningful. The expressions of the pre-service teacher in the problem he posed (He gives 4 times the amount of his money plus 8 liras to his younger sister, and he gives 5 times the amount of his money plus 4 liras to his elder sister.), indicate that he tries to cover the algebraic expressions on the both sides in only visual form. The process of bringing "the real world into mathematics" by starting from a student's everyday life experience is fundamental in school practice for the development of new mathematical knowledge (Bonotto, 2004). According to Palm (2009), the students should be presented authentic word problems more often so that they can relate school mathematics with the real world. Palm presented a theoretical framework of the appropriateness of the verbal problems or the activities held in school to the real life. Within this framework, he mentioned that the appropriateness of the questions in school activities to real-life situations is a pre-condition.

Another significant dimension is the content of the information presented in the activity. The realism of the values given in the school tasks is an aspect of importance in simulations of real-life situations. A mathematical situation may not be appropriate to real life in all aspects. However, when the structure of the equations in the study is taken into consideration, it can be said that the unknowns will form stories from real life that are appropriate to their numerical values. In this context, ignoring realism in the problems posed gives way to problems that are detached from daily life, which might adversely affect the development of students' problem solving skills during teachers' in-service period.

The D6 and D7 difficulties were observed only in the problems posed about the equation pairs. In the category of D6, problems were posed separately for each of the equations within the equation pair. In the category of D7, it was determined that the problems were posed by considering both sides of the equations as algebraic expressions independent from each other. An equation pair is a system that consists of two equations and two unknowns. Any change in one of the unknowns or either side of the equality will affect the equation system, which is otherwise balanced. In this context, it can be understood that the effect of such changes on the system is ignored in verbal sentences.

Translating algebraic expressions into other kinds of expressions and vice versa are cited in curricula among crucial skills to be acquired by students (Ministry of National Education [MONE], 2009; NCTM, 2000). Helping them acquire these skills, teachers should associate the types of equations with daily life situations. The difficulties experienced in problem posing for equations by pre-service teachers who will soon guide classroom activities may have adverse impact upon their activities in their teaching period. Various studies have also noted that problems posed by pre-service teachers fail to attract attention and are simply at the exercise level (Crespo, 2003; Crespo & Sinclair, 2008; Işık, 2011; Işık, Işık & Kar, 2011; Stickles, 2006). Nevertheless, the literature also contains studies demonstrating that pre-service teachers can pose suitable mathematical problems if they are given the chance to pose their own problems (Akay & Boz, 2010; Crespo & Sinclair, 2008; Dickerson, 1999).

In this study, the difficulties experienced in the process of posing problems about equations were studied through five equations. In these equations, the values of the unknowns are positive and the coefficients of the equations are generally whole numbers. Therefore, the problems to be posed about different equations will be seen different difficulties. In future studies, the difficulties experienced in posing problems about different equations can be determined. In the evaluation stage of the problems solved by using equations, the pre-service teachers can be asked to pose new problems about the equation used in the solution. These problems posed by the pre-service teachers can be discussed to make up the deficiencies. Furthermore, the incorrect problems the pre-service teachers or elementary students posed about the equations can be represented to the pre-service teachers. The pre-service teachers can be asked to evaluate these mistakes and correct them at the end of evaluations.

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