1-1-1997

Year 7 students' understanding of the relationship between area and perimeter

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USE OF THESIS

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YEAR 7 STUDENTS' UNDERSTANDING OF THE RELATIONSHIP BETWEEN AREA AND PERIMETER

BY

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A Thesis Submitted in Partial Fulfilment of the
Requirements for the Award of

Master of Education

at the Faculty of Education, Edith Cowan University

Date of Submission: 20 February 1997

ABSTRACT

The aim of this study was to determine Year 7 students' understanding of the relationship between area and perimeter. This is an important part of the measurement strand of mathematics. Two methods of collecting data were used: a multiple-choice pencil-and-paper test item; and clinical interviews with a class of Year 7 students. Two Perth metropolitan government primary schools allowed access for the research to take place: one was used for the trial of the test item with eleven students; the students at the other school were given the validated test item followed, one week later, by clinical interviews.

Analysis of the data suggested that students of this year level have a sound understanding of the concept of perimeter, but that their understanding of the area concept was not as well developed. There also did not appear to be a widespread understanding of the relationship between area and perimeter. Several categories of understandings and misunderstandings were identified, as were other areas of concern.

The research highlights some interesting implications for teachers. A better understanding of their students' beliefs about the concepts of area, perimeter, and the relationship between the two, may influence teachers' decisions when planning for the teaching of these attributes of measurement.

DECLARATION

I certify that this thesis does not, to the best of my knowledge and belief:

(i)   incorporate without acknowledgement any material
        previously submitted for a degree or diploma in any
        institution of higher education;

(ii)   contain any material previously published or written by
        another person except where due reference is made in
        the text; or

(iii)   contain any defamatory material.

Signature

Date   _20 February 1997_

ACKNOWLEDGEMENTS

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CHAPTER 1

INTRODUCTION

There is not a constant relationship between the area of a shape and its perimeter. Two shapes may have equal areas but different perimeters, for example, a rectangle of 2 metres by 8 metres, and a quare with sides of 4 metres have perimeters of 20 metres and 16 metres respectively, yet both enclose an area of 16 square metres. Conversely, two shapes may have equal perimeters but different areas, for example, a triangle with sides of 3 metres, 4 metres and 5 metres and a rectangle of 5 metres by 1 metre both have perimeters of 12 metres, but have areas of 6 square metres and 5 square metres respectively. The shape with the largest area may not always have the largest perimeter and vice versa. Understandably, this has the potential to cause a great deal of confusion.

Chapter 2, which reviews the existing literature discusses the following pertinent areas of study: Measurement, length, area, the relationship between area and perimeter, Western Australian and Australian expectations, the range of misconceptions about the relationship between area and perimeter, and multiple-choice pencil-and-paper test items.

The need for measurement is explored in *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991), which describes measurement as, "The quantification of some feature of objects, people or events and underlies many of the descriptive statements we make". As well as an important area of study on its own, measurement provides a link to other areas of mathematics, in particular, number and space (Booker, Briggs, Davey & Nisbet, 1992; Cruikshank & Sheffield, 1988; Reys, Suydam & Linquist, 1984; Osborne, 1980).

There is a generally accepted sequence of teaching that students encounter when learning about measurement concepts. These include: identifying the attribute; direct comparison and indirect comparison; arbitrary (non-standard) units and standard units. (Cruikshank & Sheffield, 1988; Booker, et al., 1992). Two fundamental concepts of measurement are length and area.

The concept of length is one that is better considered in practical situations. The issue of students' understanding of conservation of length is important, as is the notion of perimeter. It is important for students to have a sound understanding of the concept of area (Hirstein, Lamb & Osborne, 1978). There is also a need to ensure that students are not rushed into working with area formulas, instead being provided with better foundations on the concept of area in their early years of schooling (Booker et al., 1992; Batista, 1982; Latham & Truelove, 1980; Williams & Shuard, 1982; Dickson, 1989; Osborne, 1980).

On the concept of the relationship between area and perimeter, several authors describe the confusion that frequently arises for students, such as confusing perimeter with area (Doig, Cheeseman, & Lindsay, 1995; Reys, et al., 1984; Nunes, Light, Mason & Allerton, 1994). They attribute the causes of these misunderstandings to various factors: a lack of understanding of the underlying concepts; inability to distinguish area and perimeter; a belief that if areas of regions remain constant, so must the perimeters, and if the perimeters of regions remain constant, so also must their areas; and the early introduction of formulas.

State and National expectations on the relationship between area and perimeter are examined in Chapter 2, in the light of *The WA Learning Mathematics Syllabus K-7* (Ministry of Education, 1989), *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991), *The WA Mathematics Student Outcome Statements Draft Edition* (Education Department of Western Australia, 1994) and *Making the Links: A First Steps in Mathematics Interim Document* (Education Department, 1995). These documents, which inform mathematics teaching in Western Australian primary schools, provide information and activities that aim to help teachers enable their students to gain a sound understanding of this relationship.

The above documentation gives guidelines for the progress of understanding of these concepts, but misconceptions still occur. In order to identify causes of

confusion, a number of authors have employed different methods, which will be described in Chapter 2.

Identifying Areas of Misunderstanding

Woodward and Byrd (1983) used a multiple choice paper-and-pencil test item that they administered to three groups of students in the USA. Another method of collecting relevant data consists of students following a structured interview protocol such as the Newman Error Analysis Guideline [NEAG] (See Appendix 1) (Newman, (b) 1983, p. 125). This technique was to be used in this research as a basis for determining if students understood the Woodward and Byrd test item.

There were four research questions posed in relation to this study. The first was concerned with testing the validity of the Woodward and Byrd (1983) test item. The second question looked at the proportion of one Year 7 class of students who appeared to understand the relationship between area and perimeter, using the test item, and how this compared with the original 1983 data. Thirdly, the range of understanding of the students on the relationship between area and perimeter was to be determined using clinical interviews. Finally, a comparison was to be made between the results of the test item and the understandings demonstrated by the students in the interviews.

The first phase of this research was to check the suitability of the Woodward and Byrd test item in the context of WA primary schools. Newman interviews were conducted to determine the validity of the test item. The interviews were audio-taped. From an analysis of the data collected, a decision was made that the students appeared to understand what the question was asking them to do, and, on this basis, the test item was accepted as a valid measure of students' understanding of the relationship between area and perimeter.

The next phase of the research was to survey the extent of area/perimeter misconceptions by presenting the test item to a class of 21 Year 7 students at a different metropolitan government primary school. One week later, these students were each interviewed, using the clinical interview method described by Ginsburg (1981). These interviews were also audio-taped.

From the test item, a comparison was made with the data from the USA study. It was found that the Perth students fared considerably better than the USA students in their results, and appeared to have a clearer understanding of the relationship between area and perimeter.

Range of Understanding on the Relationship Between Area and Perimeter

The students' range of understandings about the relationship between area and perimeter was explored by means of the clinical interviews, an approach that was broader than the Newman interviews. Questions that examined their

understandings of the concepts of area and perimeter themselves were also included. There appeared every reason to believe that the students of Year 7 in this research had a clear understanding of the concept of perimeter. The same could not be said, however, for the concept of area. There appeared to be a high number of students who had confusion about some aspects of the concept of area. It was found that some students seemed to believe that there was a direct relationship between the two aspects of measurement, that is, for shapes with the same areas, the perimeters must be the same, and conversely, for shapes with the same perimeters, the areas must be the same. Other students appeared to believe, correctly, that no relationship existed. There seemed to be a significant suggestion that area and perimeter dealt with straight-sided figures, and there was considerable confusion as to the correct units of measure for area and perimeter. There also existed a strong desire to use the 'Length time Width' formula when determining how to calculate areas of shapes, whether rectangular or not.

Comparison of Test Item Results and Interview Results

Finally, this research looked at a comparison of the results of the test item and the responses from the clinical interviews with the same students. This was examined in the light of research by Ellerton and Clements (1995), who tested 115 Year 8 students in NSW, on a range of mathematical questions considered 'fair' by the teachers at the schools concerned. Their finding was that over one-third of the responses in their research could be classified as either: those who

gave correct answers but did not have a sound understanding of the concept being tested; or those who gave incorrect answers but who had partial or full understanding of the concept.

In this research, it was found that most of the students who answered the test item correctly seemed to have a sound understanding of the relationship between area and perimeter, although one student had only partial understanding, and one other seemed to display limited understanding. However, two-thirds of those who 'failed' the test item had partial or sound understanding, as indicated by the interviews.

CHAPTER 2

REVIEW OF THE LITERATURE

The purpose of this chapter is to review the existing literature related to the following pertinent areas of study: Measurement, length, area, the relationship between area and perimeter, state and national expectation, the range of misconceptions about the relationship between area and perimeter, and multiple-choice pencil-and-paper test items. The research questions will then be described.

Measurement

Measurement is an important part of mathematics, one that is used in everyday life. "It provides quantitative information about certain familiar aspects of our environment" (Cruikshank & Sheffield, 1988, p. 291). *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) describes measurement as, "The quantification of some feature of objects, people or events and underlies many of the descriptive statements we make", and later states that

> The fundamental idea which underlies measurement is the
> comparison of one thing with another according to some
> specified feature. ... The process of measuring involves
> choosing a unit (e.g. handspan), repeating the unit until it
> 'matches' the thing to be measured according to the attribute

of interest (e.g. length), and counting how many of the units it takes to make the match. (p.136)

It goes on to describe the practical use of measurement in everyday life, as well as the need for knowledge of the skills involved in undertaking these measurements.

Booker et al. (1992), Cruikshank and Sheffield (1988), Reys et al. (1984) and Osborne (1980) all make the point that measurement provides a link to other concepts in mathematics, particularly number and space. It provides a context for manipulation of numbers and shapes. It also lends itself to the integration of mathematics with other subject areas, such as science and social studies.

Wilson and Rowlands (1993) describe what teachers' goals are when teaching measurement. "We want students to understand the attributes to be measured, to choose appropriate units, to estimate, to develop useful processes, and to use instruments and student-created formulas to facilitate their work" (p. 181). In its introduction to basic mathematical skills, *The Mathematics Framework P-10* (Ministry of Education, Victoria, 1988), the need for students to be able to make "reasonable accurate estimates of physical quantities" is emphasised. It goes on to say, "Measurement involves a choice about precision. Students should be able to decide how precise a measurement needs to be and either estimate or use an appropriate instrument" (p. 13).

In developing measurement concepts, Cruikshank and Sheffield (1988) suggest that there are four stages of activities that students should engage in. These are: direct comparison, indirect comparison, arbitrary (non-standard) units and standard units (p. 291). Direct comparison means that students compare two objects by holding them side-by-side or one on top of the other, whereas indirect comparison requires the comparison to be made without the objects being moved. The use of arbitrary units enables students to measure the attributes of an object with materials such as counters, straws, shells, blocks, pencils, etc. The use of standard units, in Australia, means students becoming familiar with the SI system of metric measures.

Booker et al. (1992) suggest the following sequence: identifying the attribute, comparing and ordering, non-standard units, standard units and applications. They maintain that some of the attributes to be measured "may not yet form part of the child's experience" (p. 274). Hence students will need to be exposed to activities which enable them to experience the attribute by manipulation and discussion. Once the students have an awareness of the particular attribute, for example length, then they can compare two objects, and later seriate three or more objects, according to that attribute. The use of non-standard, or arbitrary units, followed by standard units are the next two stages in this model. The final stage is the application of their measurement skills, involving generalisations and the use of formulas.

Nitabach and Lehrer (1996), when looking at the foundations of measure, describe six important ideas of which students need to be aware. The first is that "units of measure should be adapted to the objects of measure" (p. 473). By this, they suggest that to measure length, a unit that has length is needed; to measure area, a unit that has area should be used. The other foundations of measure are: that units of measure should be identical; that measurement involves iteration, that a scale has a zero point; that measurement is characterised by additivity; and that measuring area is based on space filling (p. 473-474).

Length

Length is a measure with which students are familiar before starting school, and the one that they find easiest, as they can clearly see what is being measured. It is "one of the most perceptual attributes of an object" (Reys et al., 1984, p. 143). It is the "one-dimensional concept [and] is related to the geometric concepts of direction and line" (Booker et al., 1992, p. 276). Measurement of length needs to be undertaken in practical situations.

Schwartz (1995) describes five understandings that he feels are necessary for students to be able to use linear measurement effectively, that is, "a developmental sequence that children follow as they construct understandings and build skills in linear measurement" (p. 413). The first is that length is "an attribute that can be specifically described and that serves as useful information under particular conditions" (p. 413). This is concerned with valuing the

usefulness of measuring length in various contexts. The second is that "the distance between the end points of an object define the length of that object" (p. 413). Thirdly, the "length of an object can be described by using (a) another object in direct comparison, (b) non-standard units, or (c) standard units" (p. 413). The fourth understanding is that "the difference between non-standard units and standard units is important when talking about length to someone who is not in the same place as the object or the tool of measurement" (p. 414), and the fifth is that "some tools are easier to use, others are more efficient, and still others are more accurate" (p. 414).

Misconceptions about length are sometimes associated with a student's inability to conserve length. This is characterised by a student believing that a piece of string is shorter when rolled up than when it is pulled straight. Cruikshank and Sheffield (1988) argue two points of view in regard to students' ability in conserving length: The first is from Piaget's work, where there is a suggestion that "until certain stages of intellectual development have occurred, children will have difficulty measuring successfully, ... [and that] measuring length should be held off until the child is able to conserve length" (p. 292). The second point of view Cruikshank and Sheffield (1988) cite is from Heibert (1984) who "found that the absence of conservation did not seem to limit children learning about measurement concepts" (p. 292).

When looking at the concept of perimeter, and making the connection between length and perimeter, Shaw (1983) suggests that "teachers can use several activities to help students see that perimeter is a linear measure involving the measure of distance around a polygon" (p. 4). Her article goes on to offer a series of activities which use centimetre grid paper to experiment with perimeter of straight-sided shapes. Another description for students learning about perimeter in upper primary school, is that they "will devise their own short cuts for finding perimeters: rather than measuring each side of a regular pentagon and adding all sides they will measure one and multiply by five; similarly they will measure two adjacent sides of a rectangle, add them and double and they will explain why 'it must work'" (Education Department, 1995, p. 21).

Area

The concept of area is one that causes some difficulty for children in primary school. It is "an attribute of plane regions that can be compared by sight if the differences are large enough and the shapes similar enough" (Reys et al., 1984, p. 143). Reynolds and Wheatley (1996) describe understanding of area in the following terms:

> The area of a region is determined (i.e. assigned a number) by comparing that region to another region, usually a square unit. One makes four assumptions when comparing regions. These assumptions are: (a) a suitable two-dimensional region is chosen as a unit; (b) congruent regions have equal areas; (c) regions are disjoint (no overlapping); (d) the area of the union of these disjoint regions is

the sum of their areas. Thus determining area can be thought of essentially as tiling of the plane with congruent regions that become units of measure. (p. 567)

The understanding of conservation of area comes when a student recognises, for example, that if a shape is cut and rearranged without overlaps, the area has not changed. A frequent Piagetian test item for determining an understanding of conservation of area is where students are shown diagrams of two squares of tin with the same number of equal-sized holes punched out in different patterns. They are asked if there is the same amount of tin in each square. As part of a series of tests with a small group of 9 and 10 year olds in Britain, Dickson (1989) found that "all pupils readily conserved area within the context of the item involving two squares of tin, ... But for David this was temporary as in a parallel item ... he did not conserve". Hart (1984), when testing students aged 12 - 14 years, found that 72% of their sample of 986 could successfully solve the problems involving conservation of area. Interestingly, 70% of those who could not conserve length could conserve area, and 70% of those who could not conserve area could conserve length. "Thus, it seems that one ability is not [necessarily] a pre-requisite for the other" (p. 27).

Hirstein et al. (1978) emphasise that an understanding of the concept of area is important for two reasons. The first is that it is important in its own right in everyday life. The second is that it is the "base of many of the models used by

teachers and textbooks to explain numbers and operations with numbers" (p. 10). They give an example of the area model for displaying fraction concepts.

When teaching area, there are 'steps' which teachers generally follow leading up to the introduction of the use of formulas.

> Investigations of the amount of surface of plane regions help identify the attribute which can then be compared, measured using non-standard and standard units, and then investigated for regular shapes by the informal development of area formulas. The use of formulas to calculate areas of common shapes is the appropriate final stage of the learning sequence, and not the beginning stage as has often happened in schools in the past. (Booker et al., 1992, p. 276)

Outhred and Mitchelmore (1991) interviewed 37 young students from Years 2 to 5, to find out about their understandings about the area of rectangles. They were asked how many squares were in a rectangle which was drawn on grid paper, and the majority of students counted each square rather than looking at the squares as representing an array. The students were then asked to cover a rectangular shape with square tiles, and had little trouble with this task. However, when given a rectangle showing only the centimetre marks around the perimeter, and asked to show how many squares would fit into the rectangle (a 1 cm square was shown next to the rectangle), there were wide discrepancies in the students' abilities to draw in the squares. The use of these activities may well provide a link for students who experience difficulties in understanding the formula for working out the area of a rectangle.

Studies have found that there appears to be a common idea amongst children that area is all about length times width. Batista (1982, p. 362) found that when students were asked to compare the areas of two irregularly shaped figures, many of them attempted to determine the perimeter. Of those who tried to use a formula, few understood *why* the areas of certain regions could be determined by taking linear measurements and using formulas, that is, they had little or no understanding of the development of these formulas.

Many students are taught formulas for working out the areas of shapes before they really have a sound grasp of what 'area' actually means. Latham and Truelove (1980), Williams and Shuard (1982), Dickson (1989) and Osborne (1980) all discuss this rush to formulas, and describe the need for better foundations to be made in the early years of schooling. Students need to have had much experience in comparing areas of different regions, both regular and irregular. They need to understand about using firstly arbitrary units to cover surfaces, and later standard units of measure. The use of square paper to enable students to count squares to determine area is an important step in later understanding of formulas. Students in upper primary school should then know "that although they could determine the area of a rectangle directly by covering it with unit squares and counting the number of squares, they can work out the area of a rectangle composed of squares by thinking of it as an array and multiplying the number of squares high by the number of squares wide (that is, the number of

rows by how many in each row)" (Education Department, 1995, p. 28). This use of the language which emphasises the number of *squares* within a rectangular region rather than just a numerical figure, has been suggested as an important factor which may help overcome some students' confusion with the use of the formula (Latham & Truelove, 1980, p.88).

Dickson (1989) found that, when looking at the concept of the area of rectangles, many students were confused about the appropriate unit to use. The students had been instructed in the use of centimetre grid paper for helping to work out area. Dickson found that when using the grids, there was more likelihood of the students using 'square centimetres' for linear measure, but when the grid was not used, then both lengths and areas were more likely to be given in centimetres (p. 111).

The Relationship Between Area and Perimeter

Many children, and indeed some adults, are confused as to whether a relationship does exist between area and perimeter. When working with children in upper primary school, as well as with tertiary students, it becomes apparent that this is a mathematical concept that is not always clearly understood. Doig, Cheeseman and Lindsay (1995) stated that "confusing perimeter with area is a well-known problem with young children" (p.232), while Reys et al. (1984) found that there was often confusion between perimeter and area. They attributed this partly to a lack of basic understanding of area, and partly due to the premature introduction

of formulas. Nunes, et al. (1994) noted that "the concept of area is prone to misconceptions, is difficult to teach, and remains unclear to many students even in the upper-primary school age range" (p. 156), and argued that the most common misconception concerned the relationship between area and perimeter. They cited Vinh Bang and Lunzer (1965) who first documented these misconceptions in a task in which children had to decide whether the area of a shape remained the same when the shape itself changed but the perimeter remained the same. A loop of string was arranged into a shape that was fixed at the corners, and then the tacks were moved, thus changing the area of the shape. Children tended to believe that the area remained constant even when the shape changed. Another French study cited by Nunes et al. was that of Douady and Glorian (1989), who found that "children treated perimeter and area as interchangeable 'measures' of a surface" (Nunes et al., 1994, p. 156).

In a practical example, described by Hart (1981, p. 15), students were directed to look at two shapes, one a square and the other a trapezium clearly constructed from the square (Figure 1). The instructions made it obvious that the square had been cut into three pieces and rearranged into the trapezium shape. Students were asked to tick the statement that they believed was true from a choice of four. The choices were:

1. A has the bigger area.
2. B has the bigger area.
3. A and B have equal area.

4. You cannot tell if one area is bigger or not.

Hart found that 80% of 12-year-olds, 85% of 13-year-olds and 84.5% of 14-year-olds were able to successfully answer the question. This question was accompanied by further questions as to the nature of the perimeters of the altered figures. "The number of children who then went on to say the perimeters of the two figures in [the] question were the same (presumably because the areas were the same) was 36 per cent (12); 29 per cent (13); 20 per cent (14)" (p. 15). She also made the point that "there is a powerful incentive to say the perimeter has not changed because the area has not changed" (p. 10).

A   B
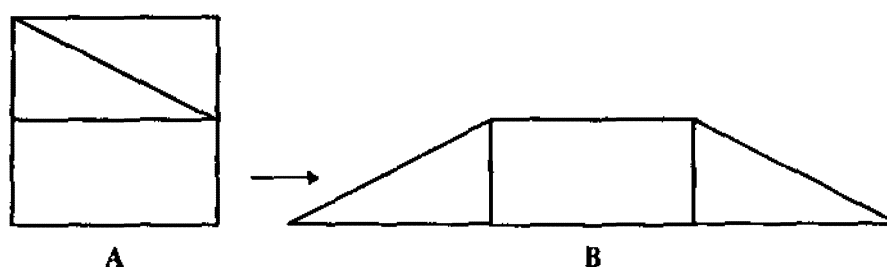
Figure 1

Furthermore, understanding is not present simply where students learn a formula. Indeed, Jamski (1978) warned that superficial manipulation of formulas should not be equated with understanding of the area concept.

Woodward and Byrd (1983) described their research in which American eighth grade students were given a question about rectangles of different dimensions, different areas, but equal perimeters. The students were asked to identify which

shape(s) had the largest area. Labelled diagrams of these different rectangles, drawn to scale, were provided for the students' use. Only 23% of the 129 students tested answered the question correctly, while 59% said that all of the rectangles were the same size. The same test item was administered to eighth grade students at another junior high school, where only 25 (19%) answered the question correctly while 81 students, or 63%, said they were all the same size. Later these researchers presented the same test item to tertiary students and similar results were obtained.

These examples point to an apparent lack of understanding about the areas of rectangles with equal perimeters, that is the relationship between area and perimeter.

A further example of this apparent lack of understanding, or confusion when children have been introduced to the Length times Width (L x W) formula for area of a rectangle, was evident in the work of a class of Year 4 and 5 children at a Perth primary school in 1995. Groups of children were given outlines of dinosaurs and, when asked to find the perimeter or size of the outline, one group put string along each side of the shape, forming a rectangle around it with the dinosaur contained within, and measured the length of the string, perhaps believing that perimeter requires rectangles (Wayne Hawkins, personal communication, March 4, 1996).

Approaches to the teaching of measurement, and more specifically, the concepts of area, perimeter and the relationship between the two, are documented in state and national curricula.

State and National Expectations

The WA Learning Mathematics Syllabus K-7 (Ministry of Education, 1989), for organisational reasons, breaks mathematics into three strands: Space, measurement and number. Each of these strands is further divided into parts, with the measurement strand having five parts: Length, area, volume and capacity, mass and time. The concepts of length and area are introduced in the pre-primary grades. At this level, the teachers' entries (which include instructional activities) consist of freely selected activities, with some direct comparisons of length. In Stage 1, students are measuring length by carrying out directed activities such as sorting, matching and seriating according to length. They are also carrying out directed activities in area, such as sorting and matching and they use arbitrary units of measure in both length and area activities.

By Stage 2, length activities include the use of the 10 centimetre rod as a measuring unit, and finding their own height in centimetres with the teachers' assistance. Area activities include direct comparison of surfaces. For the first time, at this stage there is an entry in the length strand where the students relate

measurement of length to other measures. In the final area entry for this stage, they relate activities with area to arbitrary measures of other attributes.

In Stage 3 of the syllabus, the entries on length formally introduce the units of centimetres and metres. The area entries now include manipulation of two-dimensional shapes.

It is in Stage 4 that the term 'perimeter' is first introduced. This is in relation to perimeters of a variety of polygons, regular shapes and other shapes. They also carry out activities that lead to the recognition of the relationship between metres and centimetres. In the area entries, the use of geoboards is suggested, where students use these to make figures from connected squares.

Stage 5 has the introduction of measuring in millimetres, and also practical experiences of one kilometre. Direct measurement of perimeters of circles is also introduced. This is also the stage where students are required to find areas of regular and irregular shapes by counting squares, and investigate the surface areas of three-dimensional shapes.

By Stage 6, entries are included where students measure and compare the perimeters of polygons, determine the perimeters of squares and rectangles, and measure and compare diameters and circumferences of circles. Tessellations are introduced in the area strand, as is the informal measurement of areas of various

regions and surfaces. For the first time, students are expected to determine areas beyond the limits of concrete experience.

At Stage 7, the measuring and calculating of perimeters of polygons are carried out in practical situation, as is the measuring of diameters and circumference of circles. Area entries include the informal measurement of areas of various regions and surfaces, including parallelograms and other polygons as well as surface areas of three-dimensional shapes. This is extended to the informal measurement of areas of various triangular regions and surfaces. They carry out area calculations for squares and other rectangles, and investigate the relationship between areas of triangles and areas of rectangles. Hectares are also introduced as units of measure.

In *The WA Learning Mathematics Syllabus K-7* (Ministry of Education, 1989), there are two entries where the relationship between perimeter and area are specifically mentioned in Stage 7. These entries are both concerned with relating one type of measure to another. The first is M7:P1:5 *Relate measurement of length to other measures.* The background section of this entry states that "it is not necessarily the case that, as the perimeter of a shape is increased or decreased, the area of the shape is increased or decreased" (p. 11). The second relevant entry in the syllabus is M7:P2:6 *Relate measurement of area to other measures.*

A National Statement on Mathematics for Australian Schools (Australian

Education Council, 1991) introduces the concept of the relationship between area

and perimeter in Band B which is intended to relate to children in upper primary

grades. The introduction to the measurement strand states that children "should

also realise that, for figures with different shapes, perimeter and area are not

necessarily related; that is, one rectangle may have a larger area than a second

rectangle but have a smaller perimeter" (p. 144).

The specific entry is:

> B6: *Investigate relationships between measures for different attributes*
> *and apply to solve problems.*
>
> • Investigate relationships between perimeter and area (e.g. sketch
>   and interpret tables and graphs showing the areas of different
>   rectangles with the same perimeter and the perimeters of different
>   rectangles of the same area). (p. 147)

Similarly, *The WA Mathematics Student Outcome Statements Draft Edition*

(Education Department of Western Australia, 1994) has a relevant entry in the

measurement strand, in the sub-strand 'Use Formulas' (Understanding and using

generalisations about length, angle, area and volume) at Level 4, which

encompasses children in upper primary and lower secondary school. This is the

first mention of the perimeter/area relationship in the document:

> 4.15 *Understands and uses relationships involving perimeters of*
> *polygons and areas of regions based on squares.*

- understand that shapes with the same area may have different perimeters and those with the same perimeter may have different areas (find which rectangle has the least perimeter for a fixed area). (p. 39)

A follow-up document to *The WA Mathematics Student Outcome Statements Draft Edition* (Education Department of Western Australia, 1994) is *Making the Links: A First Steps in Mathematics Interim Document* (Education Department, 1995) which seeks to provide further clarification of the Student Outcome Statements. In the introduction to the 'Use Formulas' strand, it explains that students who have attained Level 4, "realise that, for figures with different shapes, perimeter and area are not necessarily related; that is, one rectangle may have a larger area than a second rectangle but have a smaller perimeter" (p. 28).

The above documents, which inform mathematics teaching in Western Australian primary schools, provide information and activities that aim to help teachers enable their students to gain an understanding of the relationship between area and perimeter. This research intended to find out if this understanding has developed in the setting of one Year 7 classroom.

Range of Misconception about the Relationship between Area and Perimeter

There appears to have been little research into the different types of misconceptions that exist in children's understanding of the relationship between area and perimeter. One study by Hirstein (1981) looked at children's ideas

about right-angle triangles. This study found five categories related to right triangles. He described an exercise in which two groups of students aged 13 and 17 were asked to work out the area of a right triangle given the lengths of the three sides. Unit squares were not included on the figure. He stated that:

> the following detectable errors were noted in the students'
> open-ended responses: (a) add the lengths of the three sides,
> that is, find the perimeter, (b) give the length of one side,
> (c) multiply the length of the three sides, (d) find the product
> of the lengths of the two legs, (e) multiply the length of one
> leg by the length of the hypotenuse. (p. 704)

Kouba, Brown, Carpenter, Lindquist, Silver and Swafford (1988) found that the confusion between area and perimeter lessened by eleventh grade, but was not eliminated entirely, and that the most common error made in items about area involved working out the perimeter and vice versa. They went on to say that:

> evidence suggests that the confusion between perimeter and area is
> not the only misconception students have about area. About one-
> fourth of the seventh-grade students indicated that the area of a
> rectangle could not be determined after the rectangle had been
> separated into parts, even though the students were given the
> dimensions of the original rectangle. One could argue that the students
> were not able to conserve area, but a more plausible explanation is
> that they lacked a conceptual understanding of area. (p. 704)

In order to determine the range of misunderstandings that may exist for Year 7 children about the relationship between area and perimeter, the current research used interviews that were carried out with a group of Year 7 students. These

clinical interviews were intended to give an insight into the range of understanding that students have about this relationship, and to assist in finding if there were specific categories that can be described for these areas of misunderstandings.

How Widespread are the Difficulties?

A study by Woodward and Byrd (1983) shows that there is widespread confusion about the relationship between area and perimeter. In their study, a multiple-choice pencil-and paper test item was used (See Appendix 2). It consists of an explanation of a problem about Mr Young who wishes to fence his garden with a length of fence wire 60 feet long. Labelled diagrams of five different shaped rectangles that can be produced with that set length of fencing (perimeter) were provided. The students were then asked to tick which of six statements about the areas of the gardens was most appropriate.

Only 23% of the 129 students tested in the first sample of the USA study answered the question correctly, while 59% said that all of the rectangles were the same size. Two students did not complete the question. The same test item was administered, again to eighth grade students, at a junior high school in Clarksville, Tennessee, where only 25 (19%) answered the question correctly while 81 students, or 63%, said they were all the same size. Later these researchers presented the same test item to tertiary students where similar results were obtained.

There has been no data found for Australian students on this relationship. This research used the test item from Woodward and Byrd (1983), with the length measure of 'feet' changed to 'metres'. A comparison of the Perth data, taken in 1996, and the USA data, taken in 1983, was then made to determine whether similar results were experienced.

Concerns about Multiple-choice Pencil-and-paper Test Items

The main source of comparison between the Woodward and Byrd (1983) study and this one, was a multiple-choice pencil-and paper test item. (See Appendix 2)

Ellerton and Clements (1995) researched students' understanding of mathematical concepts when using multiple-choice paper-and-pencil test items. They administered 16 pencil-and-paper questions (8 short-answer and 8 multiple-choice questions covering corresponding concepts) to 115 students in six Year 8 classes in NSW. The questions were shown to the teachers beforehand, and were regarded as 'fair', in that the teachers believed that they concerned topics that had been covered and that the language used was appropriate. Fifty of the students were also interviewed. They found that, in these multiple-choice paper-and-pencil tests, over one-third of the responses could be classified into one of two categories: those who gave correct answers but did not have a sound understanding of the concept being tested; and those who gave incorrect answers but who had partial or full understanding of the concept. For this reason, all of

the students in the Year 7 group tested in this study were to be interviewed regardless of their "success" with the question. From this interview a clearer picture of the students' understandings of the relationship between area and perimeter was to be determined.

RESEARCH QUESTIONS

This research began by replicating the test item used by Woodward and Byrd (1983) who claimed that students' performance on this item could be used to determine the percentages of children with understanding of the concept of the relationship between area and perimeter. To check the validity of the wording of the item, and to ensure that it made sense to students in Perth, a trial of the test item was undertaken. Next the reviewed test item was presented to another class of Year 7 students, followed by clinical interviews with all of the students in that class.

The four research questions posed in relation to this study are:

1. **How valid is this test item in determining the relationship between area and perimeter of rectangles?**

    This was determined by analysing the test item results and conducting Newman Error Analysis interviews with a trial group of eleven Year 7 students from a metropolitan primary school. If needed, changes would have been made to the test item based on these interviews, and further trialing undertaken.

2. **What proportion of one Year 7 class of students appear to understand the relationship between area and perimeter? How do these results compare with the original 1983 data?**

    This was based on the revised test item.

3. **What is the range of understanding of Year 7 students on the relationship between area and perimeter? What are the areas of (mis)understandings?**

    This was determined using clinical interviews with all of the students from the Year 7 class.

4. **How do the results from the revised test item compare with the understandings demonstrated by these children in the interviews?**

    This was determined by comparison of the test item data and the results of the clinical interviews.

CHAPTER 3

METHODOLOGY

Introduction

The aim of this research was to look at Year 7 students' understandings about the relationship between area and perimeter. This was undertaken in several stages. The first stage was to determine the validity of the test item being used. (See Appendix 2) This item was taken from a study by Woodward and Byrd (1983) who claim that students' performance on this item can be used to determine the percentages of children with understanding of the concept of the relationship between area and perimeter. This question was to be trialed with a sample group of eleven Year 7 students.

The second stage was the application of the validated test item to a class of Year 7 students from a different school. This was to enable a comparison to be made between the USA data and the data obtained from this research.

The third stage was to interview each of the students who had undertaken the test item, with two main purposes. The first was to look at the range of the students' understandings or misunderstandings that may become apparent, in terms of the concepts of area, perimeter, and the relationship between them. The second purpose was to compare the results of the interviews with the results of the test item, to determine any significant differences.

Research Question 1.

How valid is this test item in determining the relationship between area and perimeter of rectangles?

Test Item Validity

The first part of this research was to determine the validity of the test item to be used in the research. A trial of the question to be replicated was undertaken with Year 7 students at a Perth metropolitan government primary school, in a suburb of average socio-economic status. Agreement was obtained from the principal and teacher concerned, and permission notes (see Appendix 3), with a detailed explanation of the research, were sent home to the parents and/or guardians of the children in the class. These letters explained the processes to be followed as well as the fact that the interviews were to be audio-taped. As only ten students were needed for the trial, it was decided by the teacher that, as soon as she had received ten permission slips, the trial would take place that day. Eleven permission slips were returned after two days, and these became the students, 4 boys and 7 girls, who undertook the trialing of the test item, and were presented with the test item during interviews on a one-to-one basis in class time.

The test item (shown below) was posed by Woodward and Byrd (1983, p. 344) to two different groups of students in eighth grade at junior high schools in the USA. This approximates mid-Year 8 - 9 students in Western Australia, which is

about 18 months older than the students tested for this research. They also administered the test item to a group of tertiary students for further comparison of their results, although this research did not attempt to make this comparison.

The Test Item

Mr Young had 60 feet of fencing available to enclose a garden. He wanted the garden to be rectangular in shape. Also, he wanted to have the largest possible garden area. He drew a picture of several possibilities for the garden, each with a perimeter of 60 feet. These drawings are pictured below:

Garden I

8 ft.

22 ft.

Garden II

10 ft.

20 ft.

Garden III

15 ft.

15 ft.

Garden IV

5 ft.

25 ft.

Garden V

2 ft.

28 ft.
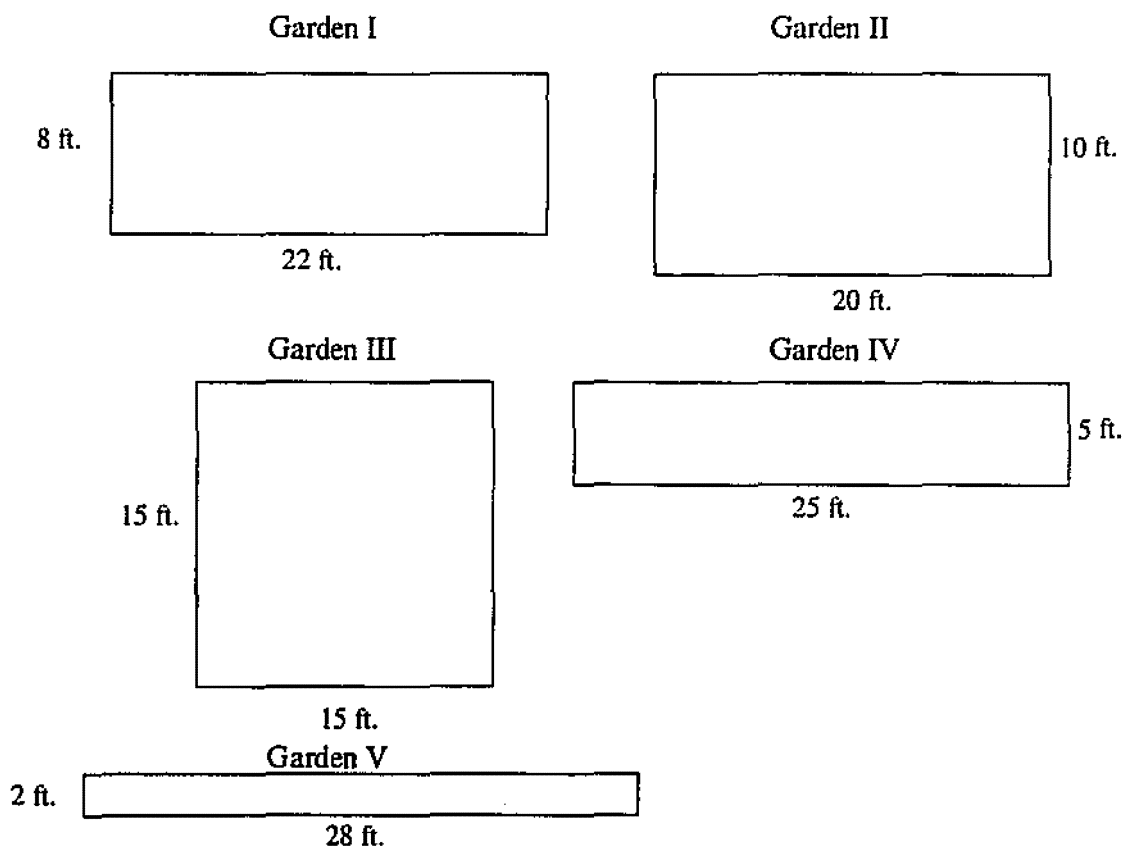
Consider Mr Young's drawings of the garden plots. Check the statement below that he found to be true.

_____1.   Garden I is the biggest garden.
_____2.   Garden II is the biggest garden.
_____3.   Garden III is the biggest garden.
_____4.   Garden IV is the biggest garden.
_____5.   Garden V is the biggest garden.
_____6.   The gardens are all the same size.

Prior to administration, two minor alterations to the original question were made: changing 'feet' to 'metres', and 'check' to 'tick'. (See Appendix 2)

Multiple-choice Pencil-and-paper Test Items

Multiple-choice paper-and-pencil test items are frequently used in mathematics to ascertain children's abilities on a particular concept or concepts. Questions are often set up with foils to enable the marker to decide if students are able to distinguish between two similar or commonly-confused concepts, such as area and perimeter, addition and subtraction, or volume and mass. Hart (1984) describes a research project that looked at measurement skills of students aged 11 to 16 years. The questions were all concerned with the concepts of length, area and volume. The test items were multiple-choice paper-and-pencil test items. Later, Hart, Brown, Kerslake, Kuchemann and Ruddock (1985) wrote about the Chelsea Diagnostic Mathematics Test. Many of the test items were not of the multiple-choice paper-and-pencil type, but the majority of those which dealt with the concepts of area and perimeter, and conservation of area were of this type.

Another project which makes use of multiple-choice paper-and-pencil test items is the National Assessment of Educational Progress (NAEP), which has been reporting on the "status and progress of US Educational achievement in a variety of subject areas, including mathematics for over 20 years" (Silver & Kenney, 1993, p. 159). In 1986, Kouba et al. (1988) described the result of the Fourth NAEP assessment. Their article is concerned with the aspects of the tests in measurement, geometry, data interpretation and attitudes. In these test items,

students were also given, as one of their choices, the option of an 'I don't know' box.

In the 1990 mathematics assessment, the Fifth NAEP (National Assessment in Education Progress) program, the latest reported on, there were 137 mathematical test items. Of these, three-quarters were multiple-choice, and one-quarter were open-response items (Silver & Kenney, 1993, p. 160).

While multiple-choice paper-and-pencil test items are commonly used, Ellerton and Clements (1995) have expressed doubts about their validity. Their finding was that over one-third of the responses could be classified as either: those who gave correct answers but did not have a sound understanding of the concept being tested; or those who gave incorrect answers but who had partial or full understanding of the concept.

The possible lack of validity may be due, for example, to a lack of understanding of key terms. In order to check the validity of the test item, the Newman Error Analysis Guideline was used.

The eleven students were presented with the test item during their interviews, using an adaptation of the Newman Error Analysis Guideline [NEAG] (See Appendix 1) (Newman, (b) 1983, p 125). Newman stated that, "with most items children's difficulties in solving the problems cannot be analysed accurately from the written responses alone. That is why it is advisable to talk to the children about the difficulties which they are having" (Newman, (a), 1983, p. 4). She went on to suggest that random questioning of children on a small number of items could easily lead to some of their problems being overlooked, and so makes the point that the interviews need to be structured in a such a way that will lead to the reason why the children are having difficulty. Questions to be put to the students followed an exact format, with as little variation as practical (see Table 1, over).

The Newman Error Analysis Guideline uses codes to be completed by the interviewer during the questioning. The symbol "E" is circled if the student makes an error in his/her response to a particular strategy or to the complete test item. The symbol "C" signifies that the student responded correctly. The right hand column is used by the interviewer during the interview to record words or symbols that cause the student difficulty.

Table 1

Interview Sheet - Adapted Newman Error Analysis Guideline (NEAG)

Test Item Newman Error Analysis Guideline (NEAG)		Name: Answer: E C
Strategies	**Expected Response**	**Pupil Response**
1 **Reading Recognition** *Please read the question to me. If you don't know a word or number leave it out.*	Mr Young had 60 metres of fencing available to enclose a garden. He wanted the garden to be rectangular in shape. Also, he wanted to have the largest possible garden area. He drew a picture of several possibilities for the garden, each with a perimeter of 60 metres. These drawings are pictured below: Consider Mr Young's drawings of the garden plots. Tick the statement below that he found to be true.	Words E C
2 **Comprehension** (a) *What does this word mean?* Point to the word in the item.	*What do they mean by "area"?* *What do they mean by "largest possible garden area"?* *What do they mean by "perimeter"?*	Symbols E C
	(b) *Tell me what the question is asking you to do.*	"Put the question in your own words."	General E C
3 **Transformation** *Tell or show me how you start finding an answer to this question*		E C
4 **Process Skills** *Show me how you work the answer out for this question. Tell me what you are doing as you work.*	Numerical Spatial Logic	C Random Resp E Wrong Op E Faulty Algor E Faulty Comput E No Resp E Other E
5 **Encoding ability** The pupil verbalises the correct answer to the task at Strategy 4, but writes the answer incorrectly.	..	Words E C Symbols E C
6 **Carelessness** Task performed correctly during interview. Carelessness possible cause of error.		E
7 **Motivation** Task performed correctly during interview. Pupil's attitude possible cause of error		E
8 **Task Form** Form of task appears to have brought about the pupil's error.		E
Correct answer: 3. Garden III is the biggest garden. (G I = 172 cm^2; G II = 200 cm^2; G III = 225 cm^2; G IV = 125 cm^2; G V = 56 cm^2)		

In strategy 4, process skills can be categorised according to whether the incorrect response is one of number, space or logic, and whether the error is a random response, a wrong operation, a faulty algorithm, a faulty computation, no response offered or an incorrect response that cannot be categorised into one of the other strategies.

As part of the Newman interview technique, a means of checking the clarity of the test item is to ask each child to put the question into his or her own words. Each student was asked to explain what the question was asking him or her to do, and how to actually go about doing the problem, and where he or she would start. He or she then continued on to complete the question.

The interviews were audio-taped to enable the researcher to transcribe all of the responses for further detailed analysis at a later date. These interviews with the eleven students were designed to find out if the test item itself was clear and easily interpreted by students of this age group.

From the results of this validity check, a decision was to be made to determine whether the original version of the test item or a revised one was to be used in the next phase. The item would have been modified if necessary, and trialed further. It was decided that this was not necessary, as the students appeared to fully

understand what the question was asking, even if they were not all able to answer it correctly.

The use of the qualitative method of the Newman Error Analysis Guideline, as well as the clinical interviews employed in determining research questions 3 and 4, later, were considered appropriate for the type of questions being asked.

Why Use Qualitative Methods?

This research employed mainly qualitative methods. Miles and Huberman (1984) make the point that "qualitative data are attractive. They are a source of well-grounded, rich description and explanation of processes occurring in local contexts ... Finally, qualitative findings have a certain undeniability that is often far more convincing to a reader than pages of numbers" (p. 21-22).

Borg and Gall (1989) maintain that, in qualitative research the "data collected are usually subjective and the main measurement tool for collecting data is the investigator him[her]self" (p. 380). When discussing some assumptions that underlie qualitative research, they suggest that each subject is different and should be studied individually, that the researcher and the research subject interact, thereby influencing each other, and therefore are inseparably connected, rather than functioning independently. This is the case in the current research where, in the interview situation, the researcher is interacting with each student as an individual. Borg and Gall (1989) also make the point that the aim of the

inquiry is "to develop a body of knowledge that is unique to the individual being studied, and that can be used to developed on-going hypotheses about the individual" (p. 384). Their final point is that the research is value-bound, "because inquiries are inevitably influenced by the values of the researcher, … the methodology employed, and the values inherent in the context of the inquiry" (p. 385).

Research Question 2

What proportion of one Year 7 class of students appear to understand the relationship between area and perimeter? How do these results compare with the original 1983 data?

The Test Item

For this second phase of the research, the principal and one Year 7 teacher from a different Perth metropolitan primary school agreed to their school's participation, on condition that a summary of the findings about the students be submitted. The school is in a suburb of average socio-economic status. Explanatory permission notes were sent home to the parents and/or guardians of the students in this class. (See Appendix 3) Of the class of 29 students, 21 of these notes were signed and returned, and these students were given the multiple-choice paper-and-pencil test item.

From the test item, comparison was to be made between this data and that of the Woodward and Byrd (1983) study, taking into account the fact that the data has come from different curricula, from different cultures and from a different decade. This was to be collated in table form as well as in graph form.

By using more than one means of collecting data, that is, the multiple-choice pencil-and-paper test item and the clinical interview, the benefits of triangulation were to be obtained. This triangulation was pertinent, not only to this question, but to research questions 3 and 4, later.

Triangulation

Borg and Gall (1989) define triangulation as the "strategy of using several different kinds of data-collection instruments, such as tests, ... interviews, ... to explore a single problem or issue" (p. 393). Mathison (1988) makes the point that triangulation is a part of good research practice, and that the use of multiple methods or data sources enhances the validity of the findings. She goes on to suggest that, although triangulation "as a strategy provides a rich and complex picture of some ... phenomenon being studied, ... rarely does it provide a clear path to a singular view of what is the case" (p. 15), and adds that she believes that it is the researcher who makes sense of the data, not the triangulation strategy itself.

Mathison (1988) discusses the value of triangulation and describes it thus:

We end up with data that occasionally converge, but frequently are inconsistent and even contradictory. And do we throw our hands up in despair because we cannot say anything about the phenomenon we have been studying. Rather, we attempt to make sense of what we find and that often requires embedding the empirical data at hand with a holistic understanding of the specific situation and general background knowledge about this class of ... phenomenon. This conception shifts the focus on triangulation away from a technological solution for ensuring validity and places the responsibility with the researcher for the construction of plausible explanations about the phenomena being studied. (p. 17)

Research Question 3

What is the range of understanding of Year 7 students on the relationship between area and perimeter? What are the areas of (mis)understandings?

The same 21 students were all given clinical interviews one week after the test item had been completed. The interviews were audio-taped to enable the researcher to transcribe all of the responses for further detailed analysis at a later date.

Clinical Interviews

The interviews took the form of a clinical interview, as described by Ginsburg (1981). He suggested that clinical interviews are intended to facilitate rich verbalisation that may give a deeper insight into a child's thinking processes than the simple checking of a response to a test item. They can also be used to clarify

any ambiguous statements, which in this instance should not arise from the test item itself, but may occur as a result of the initial explanation of the child's interpretation of the question being posed. Indeed this unstructured, open-ended procedure for questioning was first documented by Piaget as a means of giving children an opportunity to display their "natural inclinations". Ginsburg (1981) cited Piaget (1929) who described a practitioner allowing him/her self to be led as a result of a child's unanticipated responses. Ginsburg (1981) went on to consider how the clinical interview achieves its goals:

> When the aim is to identify structure by eliciting verbalisations,
> evaluating them, and checking alternative hypotheses, the
> clinical interview procedure (a) employs tasks which *channel*
> the subject's activity into particular areas; (b) it demands
> *reflection;* (c) the interviewer's questions are *contingent* on
> the child's response; (d) the interviewer employs basic features of
> the *experimental method*; and (e) some degree of *standardisation*
> may be possible. (p. 7)

Schoenfeld (1994) discusses the clinical interview and states that, "if we want to understand what goes on in people's heads when they solve problems (and I assume we do!), we have to watch them solving problems" (p. 702).

The clinical interviews with these 21 students were intended to further explore their understanding of the concepts of area, perimeter and the relationship between area and perimeter. In these interviews, a series of questions were used to elicit discussion on what the children's understandings were about these

concepts. Extra questions were also posed to probe, not only the students' understanding of any relationship, but also their concepts of "area", "perimeter" and "biggest" to attempt to determine what attribute they attended to when asked about which shape is "biggest".

Questions that involve areas and perimeters of regular and irregular, non-rectangular shapes were included. This was intended to help give a clearer understanding of the children's concepts about area and perimeter generally, as well as seeing if they relied on the formula for working out the area of a rectangle (length multiplied by width), as was the hypothesis resulting from the Woodward and Byrd (1983) study.

From these interviews, the researcher intended to define specific categories of misunderstanding.

The Interview Questions

The clinical interviews consisted of 12 questions (See Appendix 4) that were designed to give a picture of the students' understandings of the concepts of area, perimeter, and the relationship between area and perimeter. The order of the questions remained the same for all participants.

There were eight questions in the clinical interviews that were designed to explore the students' understandings of the concepts of perimeter and area. They were questions 1, 2, 3, 5, 6, 7 and 11.

The first two questions simply asked the students to define the terms 'perimeter' and 'area'. The third, follow-up question was for the students to "Draw a shape of your choice and show what you mean by area and perimeter". This question was designed to find out what shapes children identified with when considering these concepts.

Questions 5, 6 and 7 were related in that they all were concerned with the students finding out information about a 4 cm by 4 cm square. They were shown a labelled diagram of the square. Question 5 asked the students what the perimeter of the shape was, and question 6 what the area was. These questions were included to find out how well, and by what methods, students were able to undertake these calculations. Question 7 then asked "What can you say about the area and perimeter of the shape?". This was designed to find out if the students were able to distinguish between the 16 cm of the perimeter, and the 16 cm^2 of the area. Question 8 was, "Do the area and perimeter always measure the same?", and the intention of this question was to see if the students were confused by the 'special' set of numbers the dimensions of this square produced.

The final question on these concepts was Question 11. This asked the students, "How could I work out the area and perimeter of these shapes?". They were shown labelled diagrams of two shapes, a 6 m x 4 m rectangle and a trapezium with dimensions of 6 m, 9 m, 3 m and 3 m. The students did not necessarily have to work out the areas and perimeters of these shapes, but to explain how they would go about working them out. This question was included with the intention of finding out the various methods they would employ, firstly for the perimeters, and secondly for the areas of these shapes. There was particular interest in discovering if the students were able to realise that the most common method for calculating the area of a rectangle, that is the 'length times width' formula, would not be appropriate for working out the area of a trapezium.

There were five questions in the clinical interview that were designed specifically to find out more about students' understanding of the relationship between area and perimeter. These were questions 4, 8, 9, 10 and 12.

The first of these was posed as a problem. "A family has 2 islands for sale, both for the same price. A company which grows valuable trees wants to buy one of the islands, and plant as many trees as possible on it. Which island would be the best buy? Why?" The students were shown drawings of Island 1, which was almost circular in shape, and Island 2 which had many 'bays' and 'inlets', and an observably larger perimeter and smaller area. The intention with this problem was to give a context for needing to find out about either the area or perimeter,

and to determine which of the two attributes the students would select as the most appropriate.

Question 8 was, "Do the area and perimeter always measure the same?", and was a follow-up question to 5, 6 and 7, which dealt with a square with sides of 4 cm. The intention was to see if the students were confused by the 'special' set of numbers a square of these dimensions produced, and how, or even if, they were concerned about the different units of measure that result from the calculations (i.e. 16 cm and 16 cm^2).

Question 9 involved having a 4 cm x 3 cm rectangle set up with an elastic band on a geoboard. The students were asked, "What are the perimeter and area of this shape? Can you make another shape that has the same area, but a larger perimeter? If yes, show me". The first part of the question was designed to find out how well the students were able to work out the answer in this concrete representation. The second part was concerned with determining if they could use this information and take it another step to solve the problem. It forced the students to think about whether the problem could be solved, and how. They needed, perhaps, to be aware that the area could stay the same and yet the perimeter could be different.

The tenth question was, "I have a loop of string that is 40 cm in length. If I use it to make different shapes, what can you tell me about the area of each shape?". A 40 cm long, brightly coloured loop of string was available so the students could

actually manipulate it and look at the resulting shapes made. This was intended to determine whether having the material to handle would make any difference to the students' responses.

The twelfth and final question was, "I have two different shapes which both have the same area. Can I always say that the perimeters of them are the same?". Previously the students had been asked a similar question that involved them thinking about this situation of same areas, different perimeters. This last question was to check if the results would be the same without the concrete materials available to assist the students.

Students' answers to these questions were probed further, with questions contingent on the students' responses.

Research Question 4

How do the results from the revised test item compare with the understandings demonstrated by these children in the interviews?

After the interviews with the 21 Year 7 students had taken place, responses were analysed to determine which of the students were able to demonstrate a sound understanding of the relationship between area and perimeter, those who had partial understanding, and those who appeared to have limited or no understanding. The result of this comparison between the students' success or

failure in the written multiple-choice pencil-and paper test item, and apparent understanding of the relationship between area and perimeter, were collated.

This information was then used to determine whether the results from the multiple-choice pencil-and-paper test item were similar to the three categories of understanding on the relationship between area and perimeter, as demonstrated in the clinical interviews. This was to be compared with the conclusions drawn in the Ellerton and Clements (1995) study of multiple-choice pencil-and-paper test items.

CHAPTER 4

ANALYSIS AND RESULTS

Research Question 1.

How valid is this test item in determining the relationship between area and perimeter of rectangles?

Trial of Test Item

The trial of the original test item (see Appendix 2) was carried out with eleven Year 7 students from a metropolitan government primary school. Permission notes explaining details of the research were sent and received back from the parents or guardians of each of the students involved.

The first instruction in the interview, using the Newman Error Analysis Guideline (see Appendix 1), was for the students to read the test item to the interviewer, missing out any words or numbers that might cause difficulty. The only problems encountered were that one child stumbled on the word 'enclose' and another child had difficulty pronouncing the word 'perimeter', although, with follow-up questions, it was clear that he understood the term. Further questions related to understanding of terms.

What do they mean by 'area'?

All but one of the students referred to the 'inside' of the shape in some way. This student replied, "The whole block thing", probably implying some understanding of the concept. One student, alluding to the formula, went on to say, "... and you times that by what it says there", and pointed to the dimensions on the drawing. One other student mentioned one of the units used to measure area, as in, "Like how many square metres are inside the area", and another gave the unit incorrectly, saying, "The stuff that's inside the ... the centimetres that's inside the shape".

What do they mean by 'perimeter'?

Here, each of the students used one of the terms 'outside' or 'around'. Typical was the statement, "the around it, around the rectangle". Again two students specifically mentioned the units of measure, one with 'metres' (which was the unit used in the question in front of them), the other with 'centimetres'. All clearly demonstrated a sound understanding of the concept of perimeter.

What do they mean by 'the largest possible garden area'?

On this question, four students referred specifically to 'area', one of whom also mentioned the "biggest area and the biggest perimeter", whilst another three mentioned 'shape' or 'space'. One student stated that it needed to be "as wide as possible and as big, um, long as possible". Two students concentrated more on

the word 'largest', and rephrased this into "he wants to get as much as possible, the biggest garden he can get", and "the largest garden". The eleventh student pointed at the rectangles and said, "out of them".

Tell me what the question is asking you to do.

In a further attempt to discover if the students were able to understand the intent of the test item, they were asked to rephrase the question. One student put it succinctly by saying, "look for the largest garden area with the perimeter of 60 metres". With eight of the students, it was difficult to tell from their replies whether they fully understood the question, in that they rephrased it in simple terms similar to 'finding the biggest garden', without reference to area or perimeter.

Tell me what you'd do to start working it out.

One student immediately ruled out Garden 3, as it was a square, and said, "he only wants rectangles, so it can't be that one". One student went straight to, "I'd times 22 metres by 8, and I'd see, probably write it in the middle, and then do all of them", whilst another three mentioned 'timesing'. Another said that she would look at the dimensions. Only one student said that she would solely measure the perimeter, although two others said that they would need to work out the area and the perimeter. Finally, one student indicated that he would "just look at the pictures and try to figure out which one is the biggest".

Show me how you work the answer out for this question. Tell me what you are doing as you work.

The students were also asked to continue on to complete the task. Two of the students worked out only the perimeters, discovered they were all 60 metres, although that information was included in the initial part of the question, and ticked Statement 6, that the gardens were all the same size. Another three worked out the perimeters to be 60 metres, then went on to work out the areas and (correctly) ticked Statement 3 although one of these also ticked Statement 6 because she seemed to want to make sure she covered herself, saying, "I'll just put in number 6 too". A further student also worked out all of the perimeters as 60 metres, and knew that she needed to calculate the areas next, but said that she could not remember how to do this. However, she ticked Statement 3 because she felt that it looked the biggest. Two students only worked out the areas and ticked Statement 3. Two students went straight to ticking Statement 3 without seeming to do any calculations, I assume by looking at the comparative sizes, and the final student ticked Statement 6 without any obvious working out.

From the above data, a decision was made that, although not all of the children were able to complete the test item correctly, they did appear to understand what the question was asking them to do. The wording of the problem did not appear to cause any confusion, however, the strategies they used to answer it were problematic. For this reason, it was decided that the test item was valid, and the

test item would be used without further adjustment for the next phase of the research.

Research Question 2

What proportion of one Year 7 class of students appear to understand the relationship between area and perimeter? How do these results compare with the original 1983 data?

Children's Responses to the Test Item

The accepted question from the trial phase was posed to a class of 21 children in Year 7 at a different Perth metropolitan government primary school. Woodward and Byrd (1983) described their research in which American eighth graders were given the test item (See Appendix 2). The age of the students used in the study is about 18 months more than students in Year 7 in the Perth primary school system. Two groups of 129 eighth grade students were studied. In the first study, only 23% of the 129 students answered the question correctly, while 59% said that all of the rectangles were the same size, and two students did not complete the question. In the second study, only 25 (19%) answered the question correctly while 81 students, or 63%, said they were all the same size. Later the same test item was presented by these researchers to a group of tertiary students where similar results were obtained.

To compare the Perth and USA studies, the results of the Perth study were compared with the results of the first testing in the USA study, as this was the one described in most detail in the article by Woodward and Byrd (1983), and also as the results between the two USA studies were not vastly different to each other. The results, seen below in Table 2, are broken down into the number of responses for each category.

Table 2
Comparison of responses from USA and Perth samples

Responses	USA (n = 129) % of students with that response	Perth (n = 21) % of students with that response
1. Garden I is the largest garden	1	0
2. Garden II is the largest garden	12	29
* 3. Garden III is the largest garden	23	43
4. Garden IV is the largest garden	2	0
5. Garden V is the largest garden	1	0
6. The gardens are the same size	59	28
7. Did not answer question	2	0
TOTAL	**100**	**100**

NB.   *   Correct answer.

Looking at those who ticked Statement 6 as correct, that is those who (incorrectly) agreed that the perimeters are the same therefore the areas must be the same, demonstrates a distinct difference in the two samples, 59% in the USA study and 28% in the Perth study, and therefore in the apparent understanding. These differences are seen more clearly in Figure 2, over.
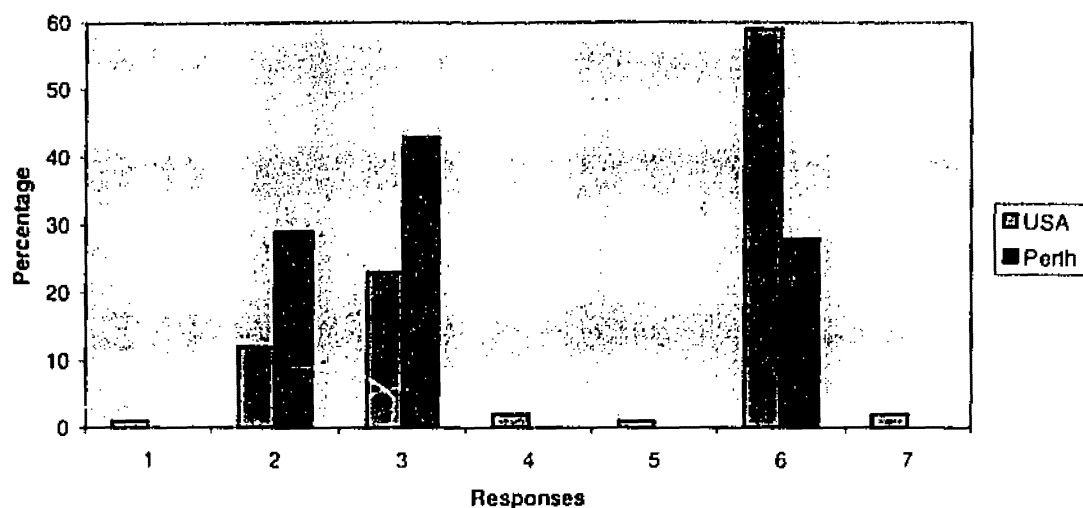
Figure 2. Comparison of USA and Perth Samples
(NB Response 3 is correct; Response 6 is incorrect)

It is evident that a higher percentage of students in the Perth study were able to identify Statement 3 as the correct response; that is, that the square gave the largest garden area, with 43% of the Perth students correct compared with 23% of the USA students. Also, there was a higher percentage of students in the USA study who ticked the incorrect response, Statement 6 (59%, compared to 28% in the Perth study) with the apparent belief that the perimeters were the same, therefore the areas had to be the same.

Research Question 3

What is the range of understanding of Year 7 students on the relationship between area and perimeter? What are the areas of (mis)understandings?

Clinical Interview Data

Children's Understanding of the concepts of Area and Perimeter

Before looking any deeper at students' understanding of the relationship between area and perimeter, it is worthwhile to look at their understanding of the concepts of area and perimeter. There were seven questions in the clinical interviews that were designed to explore these concepts. They were questions 1, 2, 3, 5, 6, 7, and 11. (See Appendix 4)

In the first question, the students were asked, "What is meant by the term 'area'?". Table 3, below, shows the overall responses to this question.

Table 3
Responses to the Question "What is meant by the term 'area'?"

	Satisfactory Description	Satisfactory description plus L x W	Only length times width	Total
No of children	17	3	1	21
Percentage	81	14	5	100

Seventeen of the 21 students (81%) were able to give a satisfactory general description of what the term 'area' means, referring to the 'space inside a shape'.

A further three students gave a general description, then went on to add that it meant length times width, as in one statement, "It's the sort of, there's only one word for it - area. You get the perimeter and it's the length times width, and that's the area, but it's the what you measure for the inside shape". One student saw area only in terms of "area, it's length times width".

The students were then asked to give their comments on what they thought the term 'perimeter' meant. All of the students were able to relate perimeter to distance, in terms of 'length of a border', 'outside of a shape' or 'boundary'.

Another question designed to check the students' understandings of the concepts of area and perimeter was Question 3, "Draw a shape of your choice and show what you mean by area and perimeter". The students were given a choice of four types of paper on which to draw their shapes: 1 cm square paper, 5 mm square paper, lined and blank paper. Table 4 below, shows the different shapes children chose to draw as part of their explanations.

Table 4
Shapes Children Chose to Draw to Help Explain Area and Perimeter

	Square	Rectangle	Other Polygon	Other	Total
No of children	7	10	2	2	21
Percentage	33	47	10	10	100

It is interesting to note that all but two students chose to draw a straight-sided figure, with the majority choosing a square or rectangle, seeming to equate the

concepts of area and perimeter with straight lines. One student's somewhat

tortuous description accompanying his drawing was, "Inside is the area of the

shape and the outside is the perimeter, sort of like the barrier that gives you the

area, it's the line, so that it's enclosed so that the perimeter gives you the area

inside". One of the two students who drew a free-form shape said, "The

perimeter is this, like what you actually see. The area is like the space inside",

and shaded the inside of the shape as she spoke.

Questions 5 and 6 were included to determine if the students were able to

calculate the area and perimeter of a square with sides of 4 cm. A labelled

diagram of the shape was provided, as was a calculator. Tables 5 below, show

the range of responses given to the question on perimeter.

Table 5
**Responses Given to Question 5, "What is the perimeter of this shape?" (4 cm x 4
cm square)**

	16	16 cm	16 cm after discussion	16 cm^2	12 cm	Total
No of children	3	13	3	1	1	21
Percentage	14	62	14	5	5	100
Total Percentage	90			5	5	100

All but one of the students were able to calculate the perimeter of the square,

although deciding which unit was appropriate provided a minor difficulty. 13 of

the 21 students (62%) were able to complete this question and give the

appropriate unit of measure without hesitation, and a further 3 students (12%)

gave the correct unit with some prompting. One student calculated the 16 correctly, but gave square centimetres as the units, and, when queried, still felt that it was the correct solution. The other student gave the perimeter as 12 centimetres, and did not explain how she arrived at this solution.

Table 6, below, shows the range of responses given to the question on area.

Table 6
Responses Given to Question 6, "What is the area of this shape?" (4 cm x 4 cm square)

	16	16 cm	16 square cm	16 cm squared	32	32 square cm	32 cm squared	Total
No of children	1	4	5	7	1	1	2	21
%	5	19	24	33	5	5	9	100
Total %	81				19			100

When it came to calculating the area of the same 4 cm by 4 cm square, only two numerical solutions were produced: 16 and 32. It is unclear how all of those who gave 32 as the answer obtained that solution, as only one offered an explanation. This was, "32, I was going to say that, but, it's that times that [points to two of the 4 cm sides] and times it by two, but I'm not sure if that's right". The issue of the appropriate unit of measure for this problem proved more difficult than for perimeter. Only 6 of the 21 students gave the unit of area correctly (i.e. 16 *square centimetres* or 32 *square centimetres*); 9 gave it as 16 *centimetres squared* or 32 *centimetres squared*; 4 as 16 *centimetres*; and 2 offered no units. There is obviously some confusion on this issue.

Question 9 involved asking students to determine the area and perimeter of a 4 cm by 3 cm rectangle set up with an elastic band on a geoboard. They were also asked to make another shape of the same area, but a larger perimeter. The students generally had some difficulty with this question. 11 of the 21 students (52%) were able to work out the perimeter, while 14 students (67%) could give the area. Most of the errors were caused by the students counting the 'pins' on the geoboard. It appeared that they had not had a lot of experience in using geoboards. The results of the students' attempts at making another shape with the same area but a larger perimeter will be discussed in a later section on the range of understanding on the relationship between area and perimeter.

In Question 11, the students were asked to explain *how* they would work out the area and perimeter of two shapes: a rectangle (6 m by 4 m) and a trapezium (sides of lengths 9 m, 6m, 3 m and 3 m). They were not required to actually calculate the solutions. Labelled diagrams were provided of each of the shapes, as well as 1 cm squared paper, 5 mm squared paper, lined and blank paper, a calculator and a pencil. Table 7, over, shows a summary of the students' suggestions as to how they would go about working out the area of the rectangle.

Table 7
Method for Working out the Area of the Rectangle

	L x W	Counting squares	P x 2	Confused A and P	Total
No of children	18	1	1	1	21
%	85	5	5	5	100
Total %	90		10		100

It can be seen that the majority of students knew the formula for working out the area of a rectangle, while one student had the strategy of copying the shape onto 1 cm square paper, and counting the squares. When it came to the perimeter of the rectangle, all but one of the 21 students knew to add the lengths of the sides, and that student confused area and perimeter. For the perimeter of the trapezium, 18 students knew to add the lengths of the sides. The other 3 students did not get to answer this particular part of the question, being more concerned with moving straight on to deciding how to work out its area. Over one-third of the students (8 of the 21; 38%) tried to manoeuvre the figures to attempt a 'length times width' solution for the area of the trapezium. Six of the students nominated tracing the trapezium onto 1 cm square paper and counting the squares and part-squares as their most appropriate strategy, three of them decided to work out the area of the internal rectangle and the areas of the two triangles on either side and add them together, one decided to multiply the perimeter by two; and three could not suggest a way to solve the problem. One student became totally confused with various concepts learned previously, and said,

Student (S):	Work out the exact middle of it, and ...
Interviewer (I):	The exact middle of which?

S: Half of the shape.

I: And you'd end up with something inside the middle?

S: Yes, so you knew what the radius was, no the diameter.

I: The diameter being from one side to the other side? [following the student's finger from top to bottom.]

S: Yes and work out the diameter across that way.

I: Across the long way?

S: Yes, and times the radius by the diameter to get the area.

Another convoluted description from a different student was offered thus:

S: You could make that 9 and the sides 1½ each, and then you times that by that ...

I: So you make the sides 1½ long, why do you do that?

S: Because if you take 3 cm and put it on the end which makes 9, but they're both parallel, and then you halve this one, so that ...

I: That gives you width does it?

S: Yes.

I: And what do you end up multiplying?

S: The length times width.

I: And what's that?

S: 1½ times 9.

This information shows that only 9 of the 21 students (43%) chose a strategy that would enable them to work out the area of the trapezium. The other 12 students (57%) either would have used an inappropriate strategy or did not really know what to do or where to start.

What is apparent from this data, is that students have a clear understanding of the concept of perimeter. However, when it comes to area, there is some confusion, and a tendency to fall back on the 'length times width' formula. The children involved in this research were all in Year 7, and *The WA Learning Mathematics Syllabus K-7* (Ministry of Education, 1989) recommendation for learning about the area of squares and other rectangles at this year level is that they "may be led to discover the formula A = l x w. However, formal application of this formula is not expected" (p. 19). Thus the findings about the students' understandings of the concept of area are interesting, and perhaps worrying.

Range of Understanding on the Relationship Between Area and Perimeter

There were five questions in the clinical interview that were designed specifically to find out more about students' understanding of the relationship between area and perimeter. These were questions 4, 8, 9, 10 and 12.

Question 4 of the interview was: "A family has 2 islands for sale, both for the same price. A company which grows valuable trees wants to buy one of the islands, and plant as many trees as possible on it. Which island would be the best buy? Why?" The students were provided with a piece of paper with two drawings of 'islands' on it: Island 1, which was close to a circular shape, and Island 2 which had many 'bays' and 'inlets', and an observably larger perimeter and smaller area. Table 8, below shows the students' reactions to this problem.

Table 8
Choice of Islands from Question 4

	Island 1			Island 2		Total
	Area	Area after discussion	Other explanation	Perimeter	Other explanation	
No of children	12	2	1	5	1	21
Percentage	57	10	5	23	5	100
Total Percentage	72			28		100

This table shows that over two-thirds of the students (72%) chose Island 1, which was the island with the largest area. Of these, 12 did not hesitate in choosing Island 1. Typical of these students was the comment, "Well, it sort of looks like it's got more space on it, and it would probably be an easier place to plant trees. Yeh, it's just looks like it's got more space". Two more students came to the conclusion that Island 1 would be the best after attempting to explain their choice. One such discussion was:

S: Probably this one [points to island 2].

I: Why do you think so?

S: Because it looks bigger, I don't know if it is, but it looks bigger.

I: So are you looking at the area or the perimeter?

S: Probably the perimeter.

I: So that would tell you which one you can fit the most trees on?

S: No because it might go in and out. Actually this one [points to Island 1].

I: Why do you think so?

S: Because the perimeter doesn't go into the island and back out, so that ...

I: We want to fit as many trees on the island as we can, so which one is going to be better?

S: This one [points to Island 1].

I: And the reason is...?

S: It's bigger, the area.

The other student's response started off with her putting her hands around Island 2, transferring her hand proportions to Island 1, and repeating the other way around. She then measured off the area of Island 2 in fingertip units.

I: What are you doing?

S: I'm figuring it out in fingers.

I: What are you figuring out in fingers?

S: The area of this one. It [Island 1] is about 43. [She went around Island 2 in fingertip units] I think that one looks bigger [Island 1] than that one [Island 2], but I'd buy that one [Island 1].

I: Why would you do that?

S: Because when I measured the islands in fingers, that one was 43, and that one was 41.

I: So there's not much difference in them?

S: No, but that one [Island 1] would probably be better to buy for agricultural reasons, because unless the trees could adapt to salt water, it would be much harder because there's more exposure to salt water.

Another student who identified Island 1 as the best island to plant lots of trees on, pointed to Island 1, and when asked why that would be the best island, she replied, "Island number two is very bumpy around the edges and it would be harder to plant anything". It is unclear whether she chose that island because of

its area or perimeter or for some entirely different reason, as appears to be the case.

Five of the 21 students (23%) chose Island 2, using the perimeter as their criterion. One of these students who chose Island 2, when asked if he had looked at the area or perimeter, said that he probably had looked at perimeter, and stated that, "even though it's like squiggled up, when you spread it out into a circle, it would probably, the whole thing would go round it, so like it would be a bigger circle, if you spread it all out". It appears that he was visualising the perimeter of each island being stretched into a circle, and then perhaps seeing a larger area for Island 2. The other student who also chose Island 2, but did not seem to use perimeter as a criterion, when asked why he chose Island 2 said, "because, I don't know ... because it's wider, that way [top to bottom], and it's longer that way [side to side]. It just looks like you'd grow more trees on it". In this case, he was perhaps looking at the overall width and breadth of Island 2, and saw that it reached further in each direction than Island 1.

Question 8 is a follow-up to questions 5, 6 and 7. In questions 5 and 6, the students were asked to work out the perimeter and area of a 4 cm by 4 cm square. Question 7 then asked them to make a statement about the two measures. Fifteen students said that the two were the same, while only 2 pointed out that the numbers were the same but the units were different. The other 4 students were not asked this question as they had previously miscalculated either the perimeter

or the area of the square, and therefore had different figures for the two measures.

Question 8 was then asked of the students, "Do area and perimeter always measure the same?". Questions 7 and 8 were, perhaps, simplistic questions, but were posed to get the students to consider the results they had obtained in working out the perimeter and area of the square. As both solutions were '16' the difference was in the units of measure; centimetres or square centimetres. All 17 of the students (81%) who had correctly calculated the area and perimeter of the square knew that the area and perimeter would not always be the same, and the other 4 were the ones, previously mentioned, who had different figures for the two measures.

Question 9 has previously been discussed in relation to the students' ability to work out the area and perimeter of a 3 cm by 4 cm rectangle made with an elastic band on a geoboard. A second part to this question required the students to make another shape on the geoboard, with the same area as the rectangle, but a larger perimeter. A summary of the results of the students' attempts at this is shown in Table 9 over.

Table 9
Making a shape on geoboard with same area, but larger perimeter

	Was able to make new shape			Was not able to make new shape		Total
	Rectangle	Other	Wrong area, otherwise correct	Rectangle	Other	
No of children	10	2	1	6	2	21
Total %	48	9.5	5	28	9.5	100

As can be seen, nearly one-half (10 of the 21 students) were able to make a rectangle that satisfied the request, two students made complicated polygons which also were correct and one made a polygon and gave the area as 12 and the perimeter as 14½, whereas in fact, the area was 12½ cm^2. He did not include any units of measure. The other 8 students tried, but were unable to make a new shape that satisfied the original request.

There were two main strategies for working out this part of the question. These were: trial and error, where the students made a shape, calculated the area and perimeter and systematically adjusted it to increase or decrease the area and/or perimeter; and making a calculation prior to using the elastic bands, where the students worked out that a rectangle of 12 cm by 1 cm, or 6 cm by 2 cm, would have the same area and a larger perimeter than the original rectangle, and then placed the elastic band appropriately onto the geoboard.

Question 10 was another that aimed to find out about the students' understanding of the relationship between area and perimeter. It was, "I have a loop of string that is 40 centimetres in length. If I use it to make different shapes, what can you

tell me about the area of each shape?". The students were given a 40 cm loop of brightly-coloured string to manipulate, to assist them in responding. Their replies are summarised in Table 10 below.

Table 10
Responses to Question 10, About Different Shapes Made with a 40 cm Loop of String

	Different Areas	Same areas	Total
No of children	18	3	21
Percentage	86	14	100

Clearly the students found this question easier to answer, perhaps because they could actually handle the piece of string and see what happened to the enclosed area as they moved it around. As one said, "the more you move it in, the less area you have. If you take it out here [like a circle], it's bigger than if you have it like that [long and thin]". Most of the students who gave the correct response appeared to know that a shape approaching a circle would give the largest area and a long, thin 'sausage' shape the smallest. One of the students who did not give the correct response said, "it doesn't change terribly". When queried about whether it changed at all firstly said, "yes", but then, when asked how it changed said, "no it's the same, it's just stretched a little differently". One stated, "they're probably the same, because it's the same length of string", and the other replied, "it's all going to be the same".

The final question of the interview was: "I have two different shapes which both have the same area. Can I always say that the perimeters of them are the same?". Reactions to this are summarised in Table 11 below.

Table 11
Results from Question 12, I have two different shapes with the same area, are the perimeters always the same?

	Perimeters Could be Different	Perimeters are always the Same	Total
No of children	20	1	21
Percentage	95	5	100

It is clear that the responses to this question were more positive than for any of the other questions posed. Certainly, some of the students appeared to have had changes of opinion in the course of the interviews, as their responses were probed, and they were forced to think more deeply on the topic. Apart from four students who asked for part of the question to be re-read, and one other, the students answered this question confidently. This one other student asked for a piece of paper to write on, and then asked, "do they have to have the same area?". When she was told that they did have to, she then drew a square with 4 cm sides and wrote 'P 16 A 16' inside it. Next to this she drew a 2 cm by 8 cm rectangle and wrote 'A 16 P 20' inside it. She then concluded by saying, "So they wouldn't have the same perimeter".

The student who decided that the perimeters would be the same did not appear to be altogether certain. He answered, "no, because, ... Is it the same size?". After

being reminded that they were different shapes but the same area, he replied, "yes [the perimeters would be the same]".

Whether the overall positive result on this question was because of the fact that this was the final question, and the children had been thinking about the relationship between area and perimeter, is uncertain. Compared to the responses to the other questions about the relationship between area and perimeter, a correct result of 95% seemed exceptional.

Areas of Misunderstanding

It was expected that certain categories of understanding or misunderstanding would become apparent from the interviews. What is apparent is that some students have more than one misconception, and that some misconceptions seemed to depend on the context of the problem. One student, responding to Question 7 that asked what she could say about the area and perimeter of a square with sides of 4 cm, had just finished answering the previous question about the area of the square. After Question 7 was posed, she referred back to the previous question and the conversation went:

S: I don't think it's right

I: Which one's not right?

S: I think it's the area that's not right.

I: OK, what do you think the area is?

S: Hang on, 4 plus 4 plus 4 plus 4, that's 16, um I don't think the area is right because *the area is meant to be more than the perimeter.*

I: Is it?

S: I think so.

I: So, do you know what the area is?

S: Um length times width equals, I think it's still 16 centimetres, because I can't figure out any other way.

I: OK so you're happy to have the area as 16?

S: Yes.

In general, these categories of understanding were apparent:

- there is no connection between area and perimeter;

- that if shapes have the same perimeter they have the same area;

- that if shapes have the same area they have the same perimeter;

- that the region enclosed is the area and that the boundary length is the perimeter;

- shows area of regions as only Length multiplied by Width;

- area and perimeter relate only to straight lines.

Research Question 4

How do the results from the revised test item compare with the understandings demonstrated by these children in the interviews?

Comparison of Test Item Results and Interview Results.

After the clinical interviews, a decision was made as to whether the students had demonstrated limited, partial or sound understanding of the relationship between area and perimeter. These decisions were based on the consistency of their

responses to the 12 interview questions. Students who had little or no difficulty in answering the questions and explaining their reasoning, were classified as having a 'sound understanding'. A student who, for some questions appeared to have a good understanding of the relationship, but in other questions was not as confident or clear, was classified as having only a 'partial understanding' of the concept. Those students who consistently gave inappropriate responses to the questions were deemed to have 'limited understanding' of the relationship between area and perimeter. These categories were then compared with the responses from the test item given the week before, and the results are summarised in Table 12 below.

Table 12
Perth Test Item Compared to Interview Results

Test	Interview		
	Limited Understanding	Partial Understanding	Sound Understanding
Pass	1	1	7
Fail	4	4	4

In the test item, 9 of the 21 students gave the correct response, that is they ticked Statement 3 in the multiple-choice list. Of these nine, 7 were later found to have indeed had a sound understanding of the relationship between area and perimeter. On the other hand, it was found that one of these students only had a partial understanding, that is, he was not always sure, and the other had a limited understanding of the relationship.

At the same time, the reverse was also true. Of the 12 who did not tick Statement 3 on the test item, that is they incorrectly answered the item, 4 demonstrated a sound understanding of the relationship during the interview.

Four others of the group who incorrectly answered the test item, were identified as having a partial understanding of the relationship between area and perimeter. They still had some misconceptions, but appeared to have a developing idea about the relationship.

Finally, there were 4 who demonstrated little understanding in either the test item or the interviews.

This information shows that two-thirds of those who failed the test item had partial or sound understanding as indicated by the interviews.

Summary

The first phase of the research was using the Newman interview technique to determine whether the test item was valid for Australian students. A series of questions was put to each of the eleven students who participated. From these interviews, it was apparent that the students understood what the test item required them to do, and the test item was accepted as valid.

The second phase was to present the test item to 21 Year 7 students from a different school. A comparison was then made with the data obtained by Woodward and Byrd (1983). 43% of the Perth were correctly able to solve the problem, whilst only 23% of the students in the USA study could do so. Conversely, only 28% of the Perth students answered the problem incorrectly, that is, said that the gardens were all the same size (as the perimeters were all the same), whilst 59% of the USA students gave that response.

Each of the students who completed the test item were interviewed a week later to find out if there were common areas of misconception. There were six main categories apparent: that there is no connection between area and perimeter; that if shapes have the same perimeter, they have the same area, and vice versa; that the region enclosed is the area, and the boundary length is the perimeter; that area means 'length times width'; and the area and perimeter relate to shapes with straight sides.

A comparison was then made with the results from the test item and the clinical interviews. Most of the students who correctly answered the test item, were found in the interviews to have a sound understanding of the relationship between area and perimeter. However, the reverse did not prove true. Of the 12 students who did not give the correct response to the test item, 4 were found to have a sound understanding of the relationship, 4 had partial understanding, and 4 had limited understanding of the relationship. This shows that two-thirds of

those who failed the test item had sound or partial understanding of the relationship between area and perimeter, compared to one-third in the Ellerton and Clements (1995) study.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

Research Question 1.

How valid is this test item in determining the relationship between area and perimeter of rectangles?

From the data presented in Chapter 4, a decision was made that, although not all

of the children were able to complete the test item correctly, they did appear to

understand what the question was asking them to do. The wording of the

problem did not appear to cause any confusion, however, the strategies they used

to answer it were not always appropriate. For this reason, it was decided that the

test item was valid, and the test item would be used without further adjustment

for the second phase of the research.

Research Question 2

What proportion of one Year 7 class of students appear to understand the relationship between area and perimeter? How do these results compare with the original 1983 data?

Based on the Woodward and Byrd (1983) test item which was designed to

explore students' understanding of the relationship between area and perimeter,

the students in the Year 7 Perth class studied fared considerably better than the USA study. 43% of the Perth students gave the correct response to the item as compared with 23% of the USA sample. (That is, they appeared to understand that the rectangles could have different areas even though they have the same perimeters.) There were six answers from which the students could choose. Looking at the final (incorrect) choice offered, that the gardens in question (all with a perimeter of 60 metres) were 'all the same size', comparisons can easily be made. On this solution, 28% of the Perth students (just over one-quarter) agreed with the option, whilst 59%, that is over one-half, of the USA students ticked this statement as being correct.

It must be taken into account that the USA study was undertaken a decade ago, in another culture and from different curricula. Therefore the results of this comparison must be viewed in that light.

Research Question 3

What is the range of understanding of Year 7 students on the relationship between area and perimeter? What are the areas of (mis)understandings?

Students' Understanding of the Concepts of Area and Perimeter

There appears every reason to believe that the students of Year 7 in this research had a clear understanding of the concept of perimeter. The same cannot be said,

however, for the concept of area. There appears to be a large number of students who have confusion about some aspects of the concept of area.

The vast majority (90%) of students appeared to equate the concept of area with straight-sided figures, particularly squares and rectangles.

When it came to calculating areas of non-rectangular shapes, only two strategies were apparent: tracing the shape onto square paper and counting the squares and half squares; and calculating the parts of the shape and adding them. Over one-third of the students tried to manoeuvre the figures to attempt a 'length times width' solution, even though this was inappropriate for the shapes being considered. This seems to confirm the implications made by Woodward and Byrd (1983) that students too frequently rely on the use of formulas, without necessarily knowing when to apply them.

There is a high degree of misunderstanding when it comes to the units of measure that should be used for determining area. Whilst one student was confused as to whether to use centimetres or square centimetres for perimeter, only six (29%) were certain about the use of the units of square centimetres for area. The remainder of the students used either centimetres or 'centimetres squared'. This confirms Dickson's (1989) finding that, when looking at the concept of the area of rectangles, many students were confused about the appropriate unit to use.

- there is no connection between area and perimeter;

- that if shapes have the same perimeter they have the same area;

- that if shapes have the same area they have the same perimeter;

- that the region enclosed is the area and that the boundary length is the perimeter;

- shows area of regions as only Length multiplied by Width;

- area and perimeter are associated only with straight lines.

Research Question 4

How do the results from the revised test item compare with the understandings demonstrated by these children in the interviews?

Comparison of Test Item Results and Interview Results.

Of the nine students who gave the correct response to the test item, seven had a sound understanding of the relationship between area and perimeter, one had a partial understanding and one had limited understanding. On the other hand, over two-thirds of those who failed the test item had partial or sound understanding, as indicated by the interviews. This figure is considerably higher than the one described by Ellerton and Clements (1995), that over one-third of the students tested on their multiple-choice pencil-and-paper test items could be classified into one of two categories: those who gave correct answers but did not

have a sound understanding of the concept being tested; and those who gave

incorrect answers but who had partial or full understanding of the concept.

A letter of appreciation, with a summary of the appropriate results considered

from the above data, was sent to the principal and Year 7 teacher from the two

schools that cooperated in this research. (See Appendix 5)

An Unexpected Finding

When the test item was given to the second group of students, an unexpected

aspect of mathematical misunderstanding arose. At the end of the test, one

student still had not handed in his paper. When asked if there was a problem, he

said that he knew that the square had the largest area, but as 'Mr Young' (the

owner of the garden) wanted a rectangular shaped garden, he wasn't sure which

statement to tick. He was told to tick the one he thought was most appropriate,

and he ticked Statement 2. After this final test had been handed in, another

student asked what the correct solution was. When asked what he thought, he

stated that he weren't sure which was the correct statement, 2 or 3. Several

students nodded in agreement. Again they were unsure because they were

concerned as to whether a square could be regarded as a rectangle. They clearly

understood what the question was asking them to do, knew *how* to calculate

areas, and knew to use area as the basis for comparison in this item. One other

student miscalculated the area of the square, giving it as 125 cm^2 instead of

225 cm^2, and therefore also ticked Statement 2, but again he clearly could be said to have understood what was required to complete the item. The responses obtained for the test item have been summarised in Table 13 below, highlighting these anomalies. The correct response was to tick Statement 3, that garden 3 was the largest garden.

Table 13
Perth responses to the Woodward and Byrd (1983) Test Item

	Ticked 3 Correct	Ticked 2 Square is not a rectangle	Ticked 2 Miscalculated area of square	Ticked 6 Worked out perimeter	Ticked 6 No working out shown	Total
No of children	9	5	1	2	4	21
Percentage	43	24	5	9	19	100
Total Percentage	72			28		100

Rather than 43% being correct as shown by the initial test results, the actual percentage of students who could be said to have a reasonable understanding of the relationship between area and perimeter could be as high as 72% of the Perth students tested, using this test item as a yardstick.

The USA data showed that 15 students (12%) of those tested gave the answer as Garden 2 being the largest garden, but no explanation was given for this, so a comparison cannot be made with the Perth data above. Perhaps these students also believed that a square is not a rectangle, or perhaps they also miscalculated the area of the square. This information was not available in the article.

In relation to the final research question, a comparison between the test item response and their responses in the interview, this spatial misunderstanding may also have had a bearing. Of the 12 students who did not give the correct solution to the test item, there were 3 of the 5 students who had ticked Statement 2 because of confusion over whether a square was a type of rectangle, and the one student who had incorrectly calculated the area of the square to be 125 cm^2 instead of 225 cm^2. As previously argued, these students could have been classified as having a reasonable understanding of the area/perimeter relationship, in that they knew how to calculate the areas, and knew to use area as the basis for comparison. Of the four from the group who incorrectly answered the test item and were identified as having a partial understanding of the relationship, there were the other two students who had ticked Statement 2 on the test item because of confusion about whether a square could be classified as a rectangle.

Another aspect of this spatial, rather than measurement misconception, is that only one student in the trial group seemed to hesitate on this point, but still went on to correctly tick Statement 3. Perhaps this is a misconception more peculiar to the second class being tested.

Limitations of Research

It is acknowledged that this research was conducted with a small sample of one class of 21 students. This particularly limits any comparison with the Woodward and Byrd (1983) study in the USA, where two groups of 129 students each were

studied. It is therefore not possible to generalise from these results to the whole population of Year 7 students in Perth.

Another factor that limits full comparison with the USA study is the fact that the USA data were collected in a different decade, from a different culture, from an older age group and from different curricula. The findings of this research need to be viewed bearing these factors in mind.

Recommendations

There are two distinct sections from the research with implications that need to be examined: understandings of the concepts of area and perimeter; and the concept of the relationship between area and perimeter.

Understandings of the concepts of area and perimeter

- Students need to experience finding out about area and perimeter of non-rectangular shapes as well as rectangular ones.

- Students need to experience using geoboards, as they are a useful concrete aid to helping them gain an understanding of these concepts.

- Students need to be shown suitable *strategies* for working out areas of shapes, such as copying or tracing the shape onto square paper and counting the squares and part-squares.

- Teachers need to be certain that their students have a sound understanding of the concept of area before encouraging them to explore the relationship between the length of sides of rectangles and their areas.

- The formal introduction of formulas does not take place in the primary school, but clear discussion is encouraged, where the students are led to discover the 'rules' for themselves.

- Teachers need to ensure that their students understand the units of standard measure to be used for area and perimeter. The use of the language which emphasises the number of *squares* within a rectangular region rather than just a numerical figure, has been suggested as an important factor which may also help overcome some students' confusion with the use of the formula. (Latham & Truelove, 1981, p.88). Part of the confusion may arise when writing the shorthand version of a measure (e.g. cm^2), where the order of writing is the reverse of the order of saying (i.e. we write the abbreviation for centimetre and then add the 'square' sign, whereas we say 'square centimetres).

- Students need to work with practical examples of shapes with the same areas and different perimeters, and shapes with the same perimeters and different areas.

- The use of concrete aids such as geoboards needs to be encouraged.

- Teachers need to make optimum use of discussion times, both with individual students when possible, and with the whole class. The students in this study appeared to improve in their responses as they were given the opportunity to think about and discuss each of the preceding questions. The results for the final question were significantly more accurate than for an earlier, similar question.

Implications for Further Research

There are several avenues apparent from this study with implications for further research. The first is that the study could be expanded to include a larger sample of students, in order to obtain a more general picture of the understandings of WA Year 7 students on the concept of the relationship between area and perimeter.

A second aspect of the findings would be to further explore students' confusion about the units of measure used in area measurement. It is clear that the majority of students who took part in this research were unsure about whether to use 'centimetres', 'square centimetres' or 'centimetres squared'.

Another implication for further research is that there seems to be a need to further explore students' understandings or misunderstandings about the spatial aspects of the test item, that is, how widespread is the confusion about whether a square is a rectangle or not? Certainly, the students who took part in the trial of the test item seemed to have little difficulty with this issue. However, in the second phase of the research, of the 21 students who participated, 6 of them (29%) appeared to have some confusion with the idea.

REFERENCES

Australian Education Council. (1991). *A national statement on mathematics for Australian schools.* Carlton, Victoria: Curriculum Corporation.

Batista, M. (1982). Understanding area and area formulas. *Mathematics Teacher, 75*(5), 362-368.

Booker, G., Briggs, J., Davey, G., & Nisbet, S. (1992). *Teaching primary mathematics.* Melbourne: Longman Cheshire.

Borg. W.R., & Gall, M.D. (1989). *Educational Research: An introduction: Fifth edition.* New York: Longman.

Cruikshank, D.E., & Sheffield, L.J. (1988). *Teaching mathematics to elementary school children: A foundation for the future.* Ohio: Merrill Publishing Company.

Dickson, L. (1989). Area of a rectangle. In K. Hart, D. Johnson, M. Brown, L. Dickson, & R. Clarkson (Eds), *Children's mathematical frameworks 8 - 13.* (pp. 89 - 125). Slough, England: NFER-Nelson.

Doig, B., Cheeseman, J., & Lindsey, J. (1995). The medium is the message: measuring area with different media. In B. Atweh, & S Flavel (Eds.), *Proceedings of the 18th MERGA Conference.* Darwin Mathematics Education Research Group of Australasia.

Education Department of Western Australia. (1994). *Mathematics student outcome statements (Working edition).* Perth: Curriculum Studies Branch.

Education Department of Western Australia. (1995). *Making the links: A first steps in mathematics interim document.* Perth: Curriculum Branch.

Ellerton, N.F., & Clements, M.A. (1995). Challenging the effectiveness of pencil and-paper tests for school mathematics. In J. Wakefield & L. Velardi (Eds.), *Celebrating mathematics learning* (pp. 464 - 469). Melbourne: Mathematical Association of Victoria.

Ginsburg, H. (1981). The clinical interview in psychological research on mathematical thinking: Aims, rationales, techniques. *For the Learning of Mathematics, 1*(3), 4-11.

Hart, K.M. (Ed.). (1981). *Children's understanding of mathematics: 11-16.* London: John Murray.

Hart, K. (1984). Which comes first - length, area or volume? *Arithmetic Teacher, 31*(9), 16-27.

Hart, K., Brown, M., Kerslake, D., Kuchemann, D., & Ruddock, G. (1985). *Chelsea diagnostic tests.* Windsor: NFER-Nelson.

Hirstein, J.J. (1981). The second national assessment in mathematics: Area and volume. *Mathematics Teacher, 74*(9), 704-708.

Hirstein, J.J., Lamb, C.E., & Osborne, A.. (1978). Student misconceptions about area measure. *Arithmetic Teacher, 25*(6), 10-16.

Jamski, W.D. (1978). So your students know about area? *The Arithmetic Teacher, 26*(4), 37.

Kouba, V.L., Brown, C.A., Carpenter, T.P., Lindquist, M.M., Silver, E.A., & Swafford, J.O. (1988). Results of the fourth NAEP assessment of mathematics: Measurement, geometry, data interpretation, attitudes and other topics. *Arithmetic Teacher, 35*(9), 10-16.

Latham, P., & Truelove, P. (1980). *Nuffield maths 3 teacher's handbook.* London: Longman.

Mathison, S. (1988). Why triangulate? *Educational Researcher, 17*(2), 13-17.

Miles, M.B., & Huberman, A.M. (1984). Drawing valid meaning from qualitative data: Towards a shared craft. *Educational Researcher, 13*(5), 20-30

Ministry of Education. (1989). *Learning mathematics pre-primary to stage 7 mathematics syllabus.* Perth: Curriculum Programmes Branch.

Ministry of Education (Schools Division). (1988). *The mathematics framework P-10.* Melbourne: Mathematics Centre of Curriculum Branch.

Newman, A. (1983 a). *Language and mathematics.* Sydney: Harcourt Brace Jovanovich Group (Australia) Pty Limited.

Newman, A. (1983 b). *Strategies for diagnosis and remediation.* Sydney: Harcourt Brace Jovanovich Group (Australia) Pty Limited.

Nitabach, E., & Lehrer, R. (1996). Developing spatial sense through area measurement. *Teaching Children Mathematics, 2*(8), 473-476.

Nunes, T., Light, P., Mason, J., & Allerton, M. (1994). The role of symbols in structuring reasoning: Studies about the concept of area. In L. Puig, & A.

Gutierrez (Eds.), *Proceedings of the eighteenth international conference for the PME.* vol IV, (pp.122-129). Lisbon: PME.

Osborne, A. (1980). Measurement: How much? In M.M. Lindquist (Ed.), *Selected issues in mathematics education* (pp. 54-68).Chicago: National Society for Study of Education.

Outhred, L., & Mitchelmore, M. (1991, July). *The transition from concrete to pictorial representation of area.* Paper presented at the 13th annual conference of the Mathematics Education Research Group of Australasia, University of Tasmania, Hobart.

Reynolds, A., & Wheatley, G. (1996). Elementary students construction and coordination of units in an area setting. *Journal for Research in Mathematics Education, 27*(5), 564-581.

Reys, R., Suydam, M., & Lindquist, M. (1984). *Helping children learn mathematics.* New Jersey: Prentice-Hall.

Schoenfeld, A.H. (1994). A discourse on methods. *Journal for Research in Mathematics Education, 25*(6), 697-710.

Schwartz, S.L. (1995). Developing power in linear measurement. *Teaching Children Mathematics, 1*(7), 412-416.

Shaw, J.M. (1983). Exploring perimeter and area using centimeter squared paper. *Arithmetic Teacher. 31*(4), 4-11.

Silver, E., & Kenney, P. (1993). An examination of relationships between the 1990 NAEP mathematics items for Grade 8 and selected themes from

NCTM Standards. *Journal for Research in Mathematics Education,* 24(2), 159-167.

Williams, E., & Shuard, H. (1982). *Primary mathematics today: Third edition: For the age of the calculator.* U.K.: Longman.

Wilson, P.S., & Rowland, R. (1993). Teaching measurement. In R. Jensen, (Ed.), *Research ideas for the classroom: Early childhood mathematics* (pp. 171-181). Reston, Virginia: The National Council of Teachers of Mathematics, Inc.

Woodward, E., & Byrd, F. (1983). Area: Included topic, neglected concept. *School Science and Mathematics. 83*(4), 343-347.
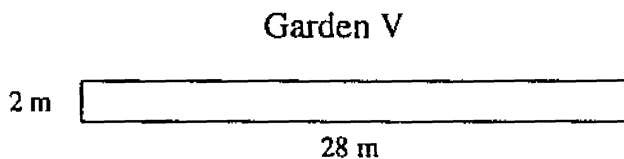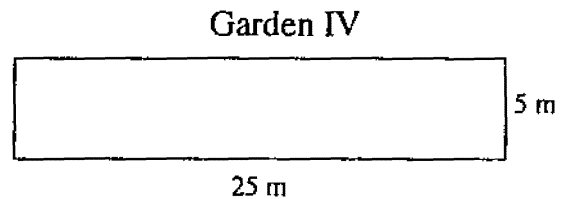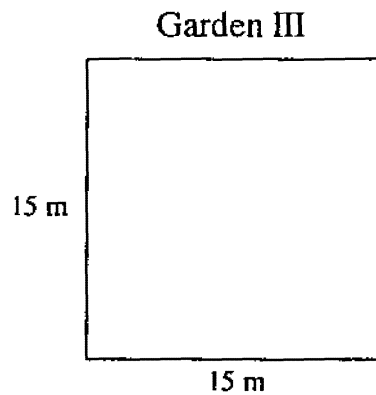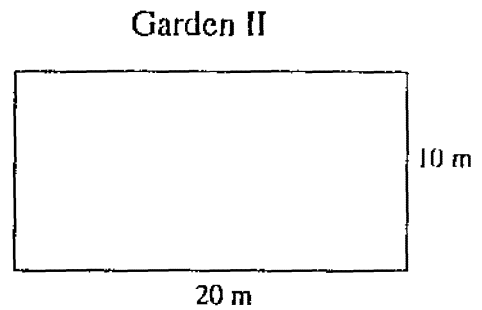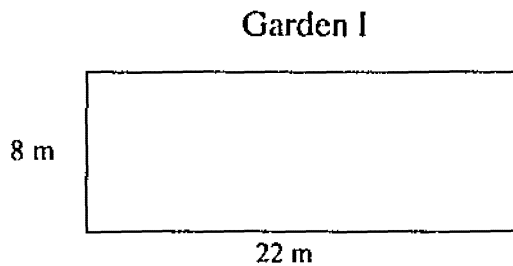
APPENDIX 1
Interview Sheet - Adapted Newman Error Analysis Guideline (NEAG)

Test Item Newman Error Analysis Guideline (NEAG)		Name: Answer: E C		
Strategies	**Expected Response**	**Pupil Response**		
1 **Reading Recognition** *Please read the question to me. If you don't know a word or number leave it out.*	Mr Young had 60 metres of fencing available to enclose a garden. He wanted the garden to be rectangular in shape. Also, he wanted to have the largest possible garden area. He drew a picture of several possibilities for the garden, each with a perimeter of 60 metres. These drawings are pictured below: Consider Mr Young's drawings of the garden plots. Tick the statement below that he found to be true.	Words	E	C
2 **Comprehension** (a) *What does this word mean?* Point to the word in the item.	*What do they mean by "area"?* *What do they mean by "largest possible garden area"?* *What do they mean by "perimeter"?*	Symbols	E	C
	(b) *Tell me what the question is asking you to do.*	"Put the question in your own words."	General	E	C
3 **Transformation** *Tell or show me how you start finding an answer to this question*		E C		
4 **Process Skills** *Show me how you work the answer out for this question. Tell me what you are doing as you work.*	Numerical Spatial Logic	C Random Resp E Wrong Op E Faulty Algor E Faulty Comput E No Resp E Other E		
5 **Encoding ability** The pupil verbalises the correct answer to the task at Strategy 4, but writes the answer incorrectly.		Words E C Symbols E C		
6 **Carelessness** Task performed correctly during interview. Carelessness possible cause of error.		E		
7 **Motivation** Task performed correctly during interview. Pupil's attitude possible cause of error		E		
8 **Task Form** Form of task appears to have brought about the pupil's error.		E		
Correct answer: **3. Garden III is the biggest garden.** (G I= 172 cm²; G II = 200 cm²; G III = 225 cm²; G IV = 125 cm²; G V = 56 cm²)				

APPENDIX 2 - Test Item

Mr Young had 60 metres of fencing available to enclose a garden. He wanted the garden to be rectangular in shape. Also, he wanted to have the largest possible garden area. He drew a picture of several possibilities for the garden, each with a perimeter of 60 metres. These drawings are pictured below:

Garden I

8 m

22 m

Garden II

10 m

20 m

Garden III

15 m

15 m

Garden IV

5 m

25 m

Garden V

2 m

28 m

Consider Mr Young's drawings of the garden plots. Tick the statement below that he found to be true.

_____ 1. Garden I is the biggest garden.
_____ 2. Garden II is the biggest garden.
_____ 3. Garden III is the biggest garden.
_____ 4. Garden IV is the biggest garden.
_____ 5. Garden V is the biggest garden.
_____ 6. The gardens are all the same size.

APPENDIX 3 - Permission Letters

Edith Cowan University
Mount Lawley Campus

Dear Parent/ Guardian

I am currently undertaking research into children's understandings of aspects of the mathematics curriculum as part of my Master of Education degree. Mathematics is an important area of education, with many applications to real-life situations.

I wish to ask one class of Year 7 children to complete a single test item, which will be a multi-choice problem. Following this test item, I propose to interview each child briefly about his or her understanding of the question. Several further questions will be posed to clarify the answers. I anticipate that these interviews will take no longer than 10 minutes per student. These interviews will be audio-taped to make it possible to transcribe the comments for analysis.

It is important for research purposes to obtain as many different students' comments as possible. This will help to give a clear picture of how children of this age think about aspects of mathematics.

General information about the children's understandings will be passed on to the classroom teacher, so that any misconceptions can be rectified during the course of the class's mathematics lessons. The benefits of the research will also be felt beyond this school, as recommendations pertinent to all Year 7 teachers will be made as a result of this work.

I wish to emphasise that all responses will remain confidential, as will the identity of the school.

Any questions concerning the project entitled Children's Understandings of Measurement can be directed to Linda Marshall of the Education Faculty on 351 7388.

If you give your consent for your child to be involved in the study, please complete the permission form attached and return it to the school as soon as possible.

Yours faithfully

Linda Marshall

I _____ have read the information attached and any questions I have asked have been answered to my satisfaction. I agree to allow my child to participate in this activity, realising that I may withdraw that permission at any time.

I agree that the research data gathered for this study may be published provided that neither the school nor my child will be identified.

_____

Parent or Guardian   Date

_____

Investigator   Date

APPENDIX 4 - Interview Questions

1. What is meant by the term 'area'?

2. What is meant by the term 'perimeter'?

3. Draw a shape of your choice and show what you mean by area and perimeter.

4. A family has 2 islands for sale, both for the same price. A company which grows valuable trees wants to buy one of the islands, and plant as many trees as possible on it. Which island would be the best buy? Why?

5. What is the perimeter of this shape? (4 cm x 4 cm square)

6. What is the area of the shape?

7. What can you say about the area and perimeter of the shape?

8. Do area and perimeter always measure the same?

9. (Have a 4 cm x 3 cm rectangle set up on a geoboard) What are the perimeter and area of this shape? Can you make another shape which has the same area, but a larger perimeter? If yes, show me.

10. I have a loop of string that is 40 cm in length. If I use it to make different shapes, what can you tell me about the area of each shape?

11. How could I work out the area and perimeter of these shapes? (6 x 4 m rectangle & trapezium 6 m, 9 m, 3 m & 3 m). (The child does not necessarily have to work out the areas and perimeters)

12. I have two different shapes which both have the same area. Can I always say that the perimeters of them are the same?
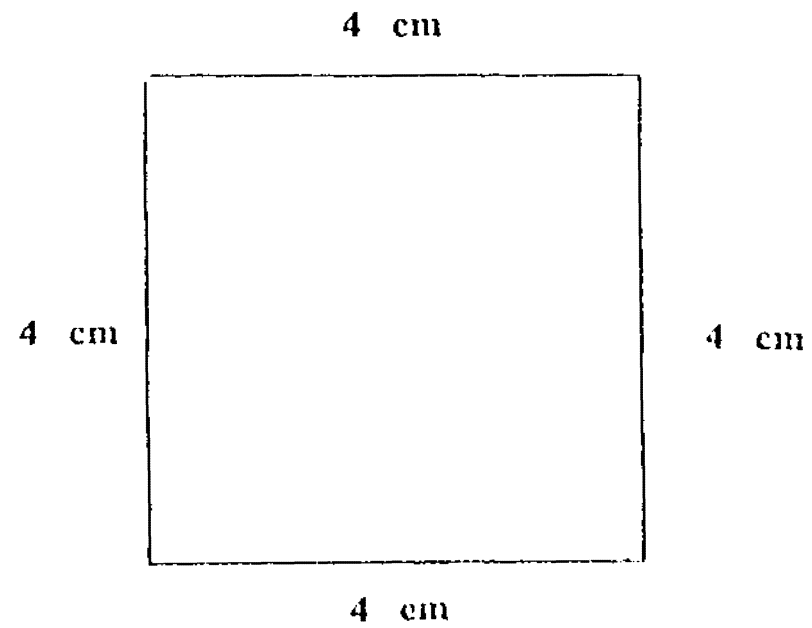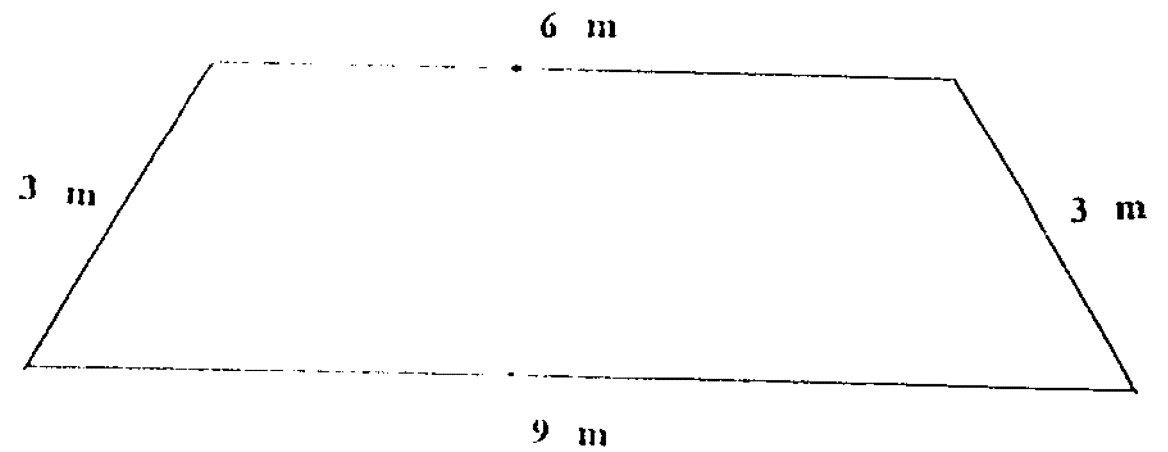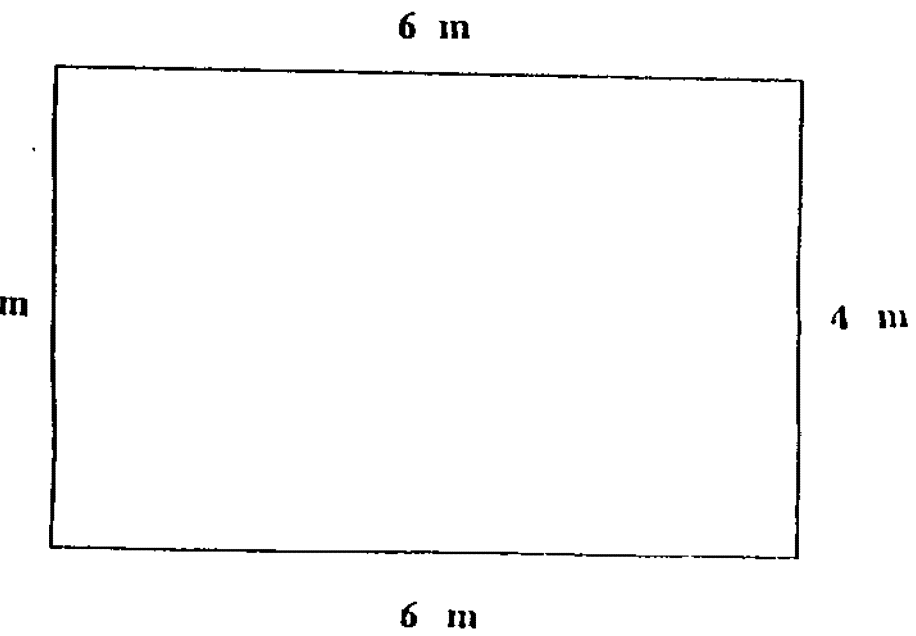
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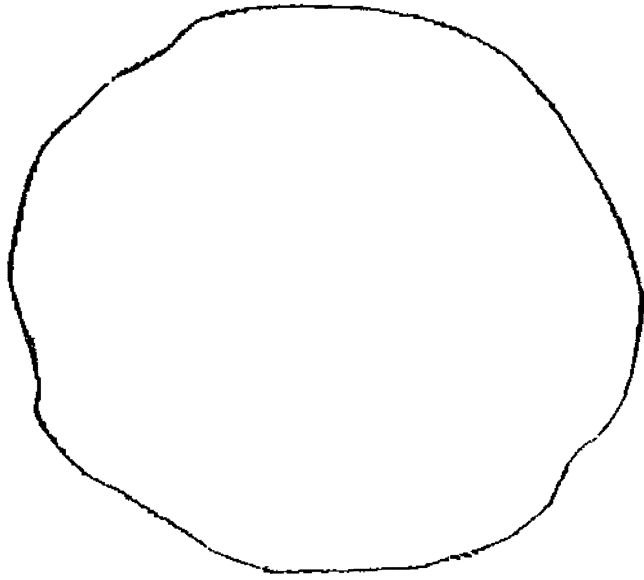
Island 1
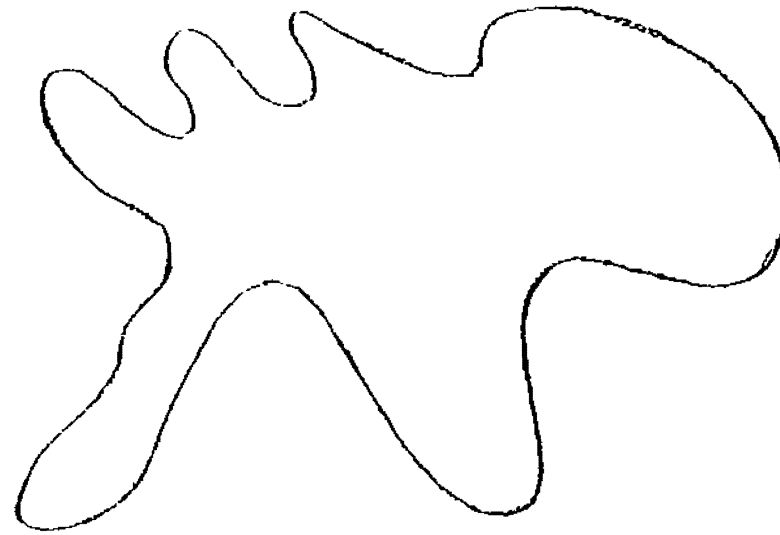
Island 2

4 cm

4 cm

4 cm

4 cm

Question 11

6 m

4 m

m

6 m

6 m

3 m

3 m

9 m

Edith Cowan University
Mt Lawley Campus

Mr AB

Dear A

Thank you for allowing me access to the children in Mrs D's Year 7 class, for gathering data for my Master in Education thesis. I was interested in finding out about children's understanding of the relationship between area and perimeter. I conducted a short interview with eleven of the children whose parents agreed to allow them to participate. In the interviews, the following points were evident:

- All of the children were able to give a reasonable definition of the term 'area'.
- All of the children were able to give a good definition of the term 'perimeter'.
- 7 students knew to use area as the basis for making a decision as to which of 5 different gardens would be the largest if they all had a perimeter of 60 metres. Three students appeared to believe that if the perimeters are constant, the areas must be the same; and one put a solution for both area and perimeter.
- 1 student gave the unit of area correctly (i.e. 225 square metres); 2 gave it as 225 metres; and 8 offered no units.

I trust that this information may be of benefit to Mrs D in any future lessons with her class on this concept. Thank you once again for the opportunity to work with the class. I appreciate the insight it has given me into the way children think about these concepts.

Yours sincerely

Linda Marshall

Edith Cowan University
Mt Lawley Campus

Ms CD

Dear C

Thank you for allowing me access to the children in your class, for gathering data for my Master in Education thesis. I was interested in finding out about children's understanding of the relationship between area and perimeter. I conducted a short interview with eleven of the children whose parents agreed to allow them to participate. In the interviews, the following points were evident:

- All of the children were able to give a reasonable definition of the term 'area'.
- All of the children were able to give a good definition of the term 'perimeter'.
- 7 students knew to use area as the basis for making a decision as to which of 5 different gardens would be the largest if they all had a perimeter of 60 metres. Three students appeared to believe that if the perimeters are constant, the areas must be the same; and one put a solution for both area and perimeter.
- 1 student gave the unit of area correctly (i.e. 225 square metres); 2 gave it as 225 metres; and 8 offered no units.

I trust that this information may be of benefit to you in any future lessons with your class on this concept. Thank you once again for the opportunity to work with the class. I appreciate the insight it has given me into the way children think about these concepts.

Yours sincerely

Linda Marshall

Edith Cowan University
Mt Lawley Campus

Mr EF

Dear E

Thank you for allowing me access to the children in Mrs H's Year 7 class, for gathering data for my Master in Education thesis. I initially gave each of the children in the class a problem to solve, testing their knowledge of the relationship between area and perimeter. I then followed this up with an interview with each of the children whose parents agreed to allow them to participate. The initial testing showed that 72% of the students (15 of the 21 students) appeared to understand the area/perimeter relationship, whilst 28% (6 students) did not appear to understand the concept. In the interviews, the following points were evident:

- All of the children were able to give a definition of the term 'area', although one child could only define it in terms of "Length times Width".
- All of the children were able to give a good definition of the term 'perimeter'.
- All but 2 of the students associated area with straight-sided shapes.
- 6 students did not know to use area as the basis for making a decision as to which of two islands would be better for planting the most trees.
- 19 of the students were able to correctly work out the perimeter of a 4 cm by 4 cm square; 1 gave the answer as 16 cm^2, and the other as 12 cm.
- 17 of the students were able to correctly work out the area of the square, with the other 4 giving the solution as 32.
- Only 6 students gave the unit of area correctly (i.e. 16 square centimetres); 9 gave it as 16 centimetres squared; 4 as 16 centimetres; and 2 offered no units.
- 15 of the students were unsure when using the geoboard to determine the area and perimeter of a rectangle, counting the 'nails' rather than the spaces.
- 18 students knew that with a loop of string of fixed length (40 cm) that the areas could vary, whilst the other 3 believed that the areas must be the same for any shape made with the string.
- When trying to determine the area of a trapezium, 6 chose to count squares; 3 decided to work out the area of the rectangle and the areas of the two triangles and add them together (both suitable strategies); 8 tried to manoeuvre the figures to attempt a Length times Width solution; 1 decided to multiply the perimeter by two; and 3 could not suggest a way to solve the problem.
- 5 students appeared to know that a square would offer the largest area from a choice of 5 four-sided shapes for a garden of fixed perimeter, but did not believe that the square could be chosen because the question asked for a rectangle.

I trust that this information may be of benefit to Mrs H in any future lessons with her class on this concept. Thank you once again for the opportunity to work with the class. I appreciate the insight it has given me into the way children think about these ideas.

Yours sincerely

Linda Marshall

Edith Cowan University
Mt Lawley Campus

Mrs GH

Dear G

Thank you for allowing me access to the children in your class, for gathering data for my Master in Education thesis. I initially gave each of the children in the class a problem to solve, testing their knowledge of the relationship between area and perimeter. I then followed this up with an interview with each of the children whose parents agreed to allow them to participate. The initial testing showed that 72% of the students (15 of the 21 students) appeared to understand the area/perimeter relationship, whilst 28% (6 students) did not appear to understand the concept. In the interviews, the following points were evident:

- All of the children were able to give a definition of the term 'area', although one child could only define it in terms of "Length times Width".
- All of the children were able to give a good definition of the term 'perimeter'.
- All but 2 of the students associated area with straight-sided shapes.
- 6 students did not know to use area as the basis for making a decision as to which of two islands would be better for planting the most trees.
- 19 of the students were able to correctly work out the perimeter of a 4 cm by 4 cm square; 1 gave the answer as 16 cm^2, and the other as 12 cm.
- 17 of the students were able to correctly work out the area of the square, with the other 4 giving the solution as 32.
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- 18 students knew that with a loop of string of fixed length (40 cm) that the areas could vary, whilst the other 3 believed that the areas must be the same for any shape made with the string.
- When trying to determine the area of a trapezium, 6 chose to count squares; 3 decided to work out the area of the rectangle and the areas of the two triangles and add them together (both suitable strategies); 8 tried to manoeuvre the figures to attempt a Length times Width solution; 1 decided to multiply the perimeter by two; and 3 could not suggest a way to solve the problem.
- 5 students appeared to know that a square would offer the largest area from a choice of 5 four-sided shapes for a garden of fixed perimeter, but did not believe that the square could be chosen because the question asked for a rectangle.

I trust that this information may be of benefit to you in any future lessons with your class on this concept. Thank you once again for the opportunity to work with the class. I appreciate the insight it has given me into the way children think about these ideas.

Yours sincerely

Linda Marshall