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A comparison of mental strategies used by skilled and unskilled mental calculators

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ABSTRACT

The purpose of this study was to investigate the various strategies used by year seven students when carrying out division computations mentally. A comparison was made between the strategies used by high and low performing mental calculators.

A number of high and low performing mental calculators were chosen as a result of their performances on twelve interview items. Both groups of students were given a set of division problems to complete mentally. After solving each problem the students were asked on a one-to-one basis to reflect on the strategy or method they used to solve the problem. The interviews were audio-taped, transcribed and coded. Non verbal behaviour was recorded on a separate sheet during the interview.

The data were analysed to determine what differences existed between high and low performing mental calculators in relation to the strategies they used to solve division computations mentally. The diversity and range of strategies used by each group were compared. Commonly used strategies were noted together with those which hindered the mental solution of problems.

It is hoped that the results of this investigation can be used to aid teachers to improve the teaching of mental calculation in ordinary classrooms. The results may also be helpful to those working in remedial mathematics. Further it is hoped that a follow up study may be carried out to determine the best way of improving the performance of both skilled and unskilled mental calculators.

"I certify that this thesis does not incorporate, without acknowledgement, any material previously submitted for a degree or diploma in any institution of higher education and, that, to the best of my knowledge and belief, it does not contain any material previously published or written by another person except where due reference is made in the text."

Paul Swan

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CHAPTER 1: INTRODUCTION

A widely accepted purpose of mathematics education is that of preparing students to solve problems they will encounter in the real world. In many classrooms written calculations are used to fulfil this aim. It is clearly evident, however, that adults carry out very few pencil and paper calculations compared to the number of mental calculations performed. The question that must therefore be raised is "Are students being provided with the skills they will use in the real world?"

The research suggests (Cockcroft, 1982; Maier, 1977; Wandt & Brown, 1957) that most calculations carried out by adults are done mentally or with the aid of a calculator. The teaching of children to calculate mentally therefore meets an important practical need.

Mental arithmetic was once part of the routine of every mathematics teacher but it has lost some prominence over the years. There are many reasons cited for the decline. Some suggest that the self esteem of less able mental calculators suffers under the typical ten-quick-questions mental session, where a range of unrelated exercises given out of context are called out to students in rapid succession. These sessions tend to suit the more able students but do little to teach the less able student how to solve mental problems.

Others think it is too difficult to find mental questions suited to the range of abilities of students in the class. The decline of mental arithmetic skills "represents a failure to recognise the central place which 'working in the head', occupies throughout mathematics" (Cockcroft, 1982, p. 75).

In recent years, especially after the introduction of calculators in the mathematics curriculum, the role of mental mathematics has received greater attention. Chronometric research has been used to measure student reaction times to mental questions. Others have focused on the role that memory plays in mental calculation. Further research has been carried out to ascertain what makes one student more proficient at mental calculation than another.

Researchers such as Reys, (1984) and Menchinskaya and Moro (1975) believe that mental calculation provides a vehicle through which number sense may be developed. The term number sense refers to an understanding of the relationship between numbers and their properties. Hope (1986) suggests that, "the study of arithmetic should help children to develop some measure of quantitative thinking about, and reasoning with, numbers" (p. 49). This statement was made in the context of mental arithmetic.

The notion of helping children to develop an understanding of the principles and ideas that underpin arithmetic is not a new one. In the mid nineteen thirties Brownell (as cited by Reys, 1984) urged a move away from the mere mechanical teaching of basic number facts to one that developed understanding on the part of students.

It has also been established that those who are mathematically effective in daily life seldom make use of the standard written methods taught in the classroom, but either adapt them in a personal way or make use of methods which are highly idiosyncratic. Maier (1977) felt that the methods used by adults to deal with the mathematical problems they face were so different from those taught in school that he coined the phrase 'folk math', to distinguish 'real mathematics' from school mathematics.

Carraher, Carraher and Schliemann (1985) went slightly further, suggesting that children learn to operate in two different systems. When at school they use the methods taught by the teacher and when 'out on the streets' they adopt their own methods of computation. Carraher et al. found that the children in their study were able to solve mental calculations when posed in the natural context but were unable to perform the same calculation when in the classroom. They concluded that in many cases attempts to follow the routines learned at school only served to interfere with the solving of the problem.

Hope (1986) documented cases of what he termed 'calculative monomania' to support the argument that schooling was producing a generation that believed there was only one way to perform a calculation. 'Calculative monomania' is described by Hope as "the tendency to ignore number relationships useful for calculation and, instead, resort to more cumbersome and inappropriate techniques" (pp. 50 – 51). The cases cited as examples by Hope are all too familiar to mathematics educators. A child who employs a written algorithm approach to multiply a whole number by 100 or to perform a subtraction where the difference between the two numbers is one, is suffering from 'calculative monomania'.

Unfortunately the situation does not seem to improve as children mature. It has been found that as children become adults they do not simply grow out of these slow and inept ways of calculating. Many students leaving school still have trouble carrying out the most elementary calculations mentally. Hope and Sherrill (1987) referred to The National Assessment of Educational Progress which monitors standards in the United States and noted that nearly half of the 17 year-olds sampled could not multiply 90 by 70 mentally.

Hope (1986) believes that the term calculative monomania aptly describes the unskilled mental calculators from his 1984 study which compared skilled and unskilled mental calculators. He believes unskilled mental calculators rely far too heavily on the written algorithm taught in school as a means of performing mental calculations. Hope found that children who he described as 'skilled' were less tied to these cumbersome methods and were able to make use of number relationships and patterns in solving mental calculations. These idiosyncratic methods as described by Maier (1977) were developed by the children themselves. They chose not follow any prescribed method.

Further research has shown that even though children tend to develop these idiosyncratic methods independent of the teacher there is a remarkable similarity between the methods used by different children. Researchers have been able to categorise these methods and in this way certain methods or 'strategies' have been identified as being used by skilled or unskilled mental calculators. Even though the picture is incomplete the identification of many of the strategies used by children when performing a mental calculation has enabled researchers to speculate on what makes one student more able at mental computation than another.

Rathmell (1978) found that children used a variety of different strategies when performing the same mental calculation. For example, to calculate 8 + 7 mentally some students count on from eight until they reach fifteen. Others take two from seven and add this to eight to make ten and then add the ten and the five. Still others double eight and subtract one to obtain the answer.

Hope (1986) identified some of the characteristics of skilled and unskilled mental calculators performing multiplication. He observed that skilled mental calculators tended to work in a left to right fashion rather than the usual right to left fashion. He inferred that by using this approach the children were able to reduce the load on short-term working memory. The unskilled children who tended to use the written algorithm approach were placing more demands on their short-term working memory. This raises the question of the role that memory plays in performing a mental calculation.

In the same study Hope found that there was little difference between the memory capacity of the skilled and unskilled calculators. A similar result was found by McIntosh (1991). "There is no indication here that short term memory is a decisive factor in the superiority of more competent calculators" (p. 4).

Researchers such as Hitch (1978), Howe and Ceci (1979) and Hunter (1977, 1978), believe that skilled and unskilled mental calculators make different use of their long and short-term working memory when performing mental calculations. Much of this work is based on studies of exceptionally talented mental calculators.

Hunter (1977) suggests that expert mental calculators devise a 'calculative plan' of tackling a mental calculation based on the need to reduce the load on short-term working memory. He even goes so far as to suggest the mental calculation performance of unskilled mental calculators could be improved if they developed techniques that helped to reduce the load on short-term working memory. There is a growing body of literature which discusses the link between memory and mental arithmetic performance.

The way skilled mental calculators approach a problem has been described in the following terms, "Skilled mental calculation demands that the user 'search for meaning' by scanning the problem for salient number properties and relationships" (Hope, 1986, p. 52). Profiles of skilled and unskilled mental calculators are beginning to emerge. Thus far these profiles are only sketchy because of the limited nature of the research in this area.

A reasonably comprehensive group of strategies for addition and subtraction involving basic number facts has been found, but further work is being carried out to determine the full range of multiplication and division strategies, used when dealing with numbers beyond the basic number facts. Basic facts are defined as $0 + 0$ to $9 + 9$ for addition and their associated subtraction relations; and 0×0 to 9×9 for multiplication and the associated division relations.

Knowledge of these basic fact strategies has caused educators to re-think the way mental mathematics is carried out in the classroom. There is some thought that the various strategies should be taught to students as one would teach any skill. Another school of thought suggests that children should be aided to discover these strategies.

This research has been designed with all the foregoing in mind. The research questions that follow have been framed with the goal of adding to the body of knowledge about how children perform mental computation and what makes one child more able at mental computation than another.

RESEARCH QUESTIONS

The aim of this research is to note the differences in the strategies used by skilled and unskilled mental calculators when dealing with mental computation beyond the basic facts. The main focus will be upon the division operation as this is the operation which has received the least attention in research studies. This research will focus only on division problems without remainders. The consideration of division with remainders would significantly broaden the parameters of the research. Restricting the research to division problems without remainders allows for a more manageable focus to be adopted. The main question to be explored in this study is:

- What differences are there between the strategies used by skilled and unskilled year seven students when solving division problems mentally?

Further to this the following subsidiary questions will be explored:

- Are there differences between the skilled and unskilled groups in:

  (i) their use of particular strategies;

  (ii) their success or lack of success in the use of particular strategies;

  (iii) their reliance on multiplication to solve division problems, and

  (iv) their use of known facts to solve problems?

LITERATURE REVIEW

This literature review will focus on:

- What is mental arithmetic?
- The history of mental arithmetic.
- The place of mental arithmetic.
- The role of memory.
- The nature and use of mental calculation strategies.
- Classification of mental strategies.

What is Mental Arithmetic?

It has been argued that in a sense all arithmetic is carried out mentally. When a written algorithm is performed the student becomes engaged in a series of mental computations momentarily interrupted by jottings on paper.

Hall (1954) recognised that confusion among educators about the precise meaning of mental arithmetic hindered the acceptance and usefulness of the practice. To clarify the term mental arithmetic he surveyed the usage of the term in textbooks, by teachers, and by authorities and compared the usage with then current definitions. Hall's conclusion is stated below:

> The writer believes, therefore that the expression "mental arithmetic" should be used exclusively and should have the following meanings: (1) arithmetic problems which arise (a) in an oral manner (b) in a written form, or (c) "in the head" of the person who needs to solve the problem; (2) problems in which pencil and paper and other mechanical devices, such as calculators, are not used to record the intermediate steps between the statement of the

> problem and its answer; (3) problems in which pencil and paper are used; and problems in which they are not used to record the answer. (Hall, 1954, pp. 352-353)

Hall's definition is slightly dated and somewhat lengthy. Atweh (1982) provides a more up-to-date and succinct definition. "Mental arithmetic is a method of thinking through a problem, performing an operation, or obtaining a result, as opposed to using paper and pencil or some other concrete aid" (p.51).

Reys (1986) concurs with Atweh. She defines mental computation as "The process of producing an exact answer to a computational problem without any external computational aid" (p. 22). The definition put forth by Reys will be adopted in this research because it distinguishes between estimation and exact mental calculation and precludes the use of pencil and paper to calculate any portion of the answer.

The History of Mental Arithmetic

The role of mental arithmetic has changed considerably over the past hundred years. This change has been governed by many factors. The prevailing learning theories, aims of teaching, and the advent of calculators are the three most common factors tending to affect the role of mental arithmetic.

During the late nineteenth century the theory of mental discipline prevailed. "This theory viewed mental computation as a perfect technique for developing the faculties of the mind" (Reys, 1984, p.549). Early in the twentieth century a strong reaction against the theory of mental discipline meant that the approach toward mental arithmetic was changed. Thorndike showed that the theory of mental discipline was based on false argument. His research led to the development of the stimulus-response (S-R) bond theory (Bana & Bourgeois, 1976).

Thorndike's theory had considerable influence over the teaching of mental arithmetic. Bana and Bourgeois (1976) explain:

> In the case of arithmetic the content had to be analysed into a multitude of discrete elements of knowledge and skill. Each element was to be learnt by internalising an S–R bond, and this bond or connection could be strengthened by repetition and drill. This theory did not consider meaning to be of any importance. (pp. 12-13)

This theory came to be known as 'drill theory'. Speed and accuracy were stressed through mechanical drill and practice. No attempt was made to develop an understanding of number relationships such as 12 - 4 = 8, 8 + 4 = 12 , 4 + 8 = 12 and 12 - 8 = 4. "The emphasis was on speed and accuracy of computation and not meaning" (Bana & Bourgeois, 1976, p. 14).

Throughout the 1920s drill theory gained popularity. By the late 1930s and early 1940s, a new theory which advocated that understanding should precede drill began to gain acceptance. This theory came to be known as the 'meaning theory' and was developed by William Brownell. Brownell as cited by Reys (1984) suggested that "[meaningful learning helps] make arithmetic less a challenge to the pupil's memory and more a challenge to his intelligence."(p. 549).

It was during this period that the social utility of mental arithmetic came to the fore. Mental arithmetic was seen as a means of preparing students to enter the real world. A number of surveys were carried out to determine the type of arithmetic people used. Questions were couched in context in an effort to promote the social utility of mental arithmetic. Often the real life context was simply cosmetic, using a broad context or a non-descript farmer to promote the multiplication of two numbers.

During the 1960s 'new mathematics' came into vogue, suggesting that 'old mathematics' was no longer useful. Under the 'new mathematics' regime mental arithmetic was de-emphasised. The emphasis was placed on the structural properties of mathematics. Reys (1984) notes that it was ironic that mental mathematics was played down because mental computation calls for understanding, number sense and the use of structural relationships.

Bana and Bourgeois (1976) point out that a wide variety of teaching aids were introduced into the teaching of mathematics during the 1960s and 70s. This was partly due to the work of Piaget, who maintained that concepts develop from working with concrete materials. The emphasis was on teaching for understanding. Number relationships were taught using a variety of concrete aids. Cockcroft (1982) also noted that mental arithmetic had declined over the sixties and seventies.

Several factors combined to promote mental mathematics during the 1980s. The increasing availability of calculators and the de-emphasis on written algorithms combined to highlight the role of mental mathematics. Further, the 'back to the basics' movement also advocated a return to mental arithmetic. Unfortunately many educators associate mental arithmetic with a daily routine of testing children's recall of the basic number facts.

A change in emphasis was required. Rather than simply return to the days of mechanical drill and practice a different approach was advocated. McIntosh (1980) summed up the situation this way:

> We need to do more mental mathematics. But I do not believe children enjoy or learn from the traditional mental arithmetic lessons in which they write answers to a large number of unrelated brief questions, as a result of which a few feel superior and the rest feel varying degrees of discomfort. (p. 14)

French (1987) suggested that the emphasis of mental arithmetic lessons should be on discussion of the methods used to solve various calculations. He concurs with McIntosh stating the following:

> Undoubtedly one of the reasons for the lack of interest is the association that mental arithmetic has with the daily mental tests once used almost universally in schools, with their emphasis on recall of facts and speed . . . . The variety of methods that children and adults use in doing mental calculations is very great and discussion of these in the classroom is very valuable, not to produce a "best method", but to encourage a flexible approach and make explicit the advantages and insights that come from considering alternatives. (p. 39)

French well describes the approach currently being espoused as the most suitable way of developing mental arithmetic.

What about the future? The current "West Australian Primary School Mathematics Syllabus, Handbook: Pre-Primary to Stage 7 Mathematics Syllabus," (Ministry of Education, 1989) provides a glimpse into the future. This document advocates a reduction in time spent on written algorithms, plus a subsequent increase in time spent on mental calculation, estimation and the use of calculators and computers.

It is almost ironic that educators are beginning to take heed of his words well over a hundred and fifty years after Colburn (1830) made the following statement:

> Most persons, when such a question is proposed [George had five cents, and his father gave him three more, how many had he then?] do not observe the process going on in the child's mind; but because he does not answer immediately, they think that he does not understand it, and they begin to assist him . . . Many teachers seem not to know that there is more than one way to do a thing or think of a thing; and if they find a scholar pursuing a method different from their own, they suppose of course that he must be wrong, and they check him at once, and endeavor to force him into their way, whether he understands it or not. If such teachers would have patience to listen to their scholars and examine their operations they would frequently discover very good ways that had never occurred to them before. (p.31)

The Place of Mental Arithmetic

One method of evaluating a curriculum is to examine the social utility of the content. The relevance of the curriculum to the 'real world' is brought into question. According to this form of evaluation a good curriculum is one which provides students with the skills to solve problems encountered in the real world.

To determine which forms of calculation were most commonly used by people for everyday purposes, Wandt and Brown (1957) carried out a survey in which participants were asked to note what types of calculations they used, except for those carried out in the workplace, over a twenty-four hour period. Close to 75% of the calculations reported were either mental calculations or approximations, whereas only 25% were written methods of calculating.

Although this study is somewhat dated and does not consider the impact of calculators it does serve to highlight the disparity between what is taught in school and what people use in society. Bastow (1988) suggests that most of the instructional time used in mathematics is taken up by the teaching of written algorithms, when quite clearly mental calculation methods are favoured over written calculation in real life. Even though his conclusion was based on somewhat flimsy evidence, he does highlight a possible anomaly in mathematics education.

Jones (1988) also questions whether the time spent teaching written algorithms is well spent. When using a written method children are not encouraged to think but simply to apply a set of rules in a particular order. Little thought is given to the structure and properties of number.

Plunkett (1979) was more forthright in his criticism of the inordinate amount of time spent teaching and practicing written algorithms. With the advent of calculators he wrote, "We can abandon the standard written algorithms, of general applicability and limited intelligibility, in favour of methods more suited to the minds and purposes of the users" (p. 5). He proposed that much of the time spent on written algorithms could be more wisely spent on improving the ability of children to calculate mentally.

Maier (1977) went a step further, claiming that adults use methods different from those taught in school to tackle problems encountered in real life. He felt the differences were so great that he referred to these untaught procedures as "folk math." He wrote:

> Some of the differences between school math and folk math are clear. One is that school math is largely paper and pencil mathematics. Folk mathematicians rely more on mental computations and estimations and on algorithms that lend themselves to mental use. When computations become too difficult or tedious to do mentally, more and more folk mathematicians are turning to calculators and computers. In folk math, paper and pencil are a last resort. Yet, they are the mainstay of school math. (p. 86)

The role of mental arithmetic has also been recognised and promoted in "An Agenda for Action" (N.C.T.M, 1980) and the report, "Mathematics Counts" (Cockcroft, 1982). In both cases an increase in the quality and quantity of instruction given to mental calculation and estimation is endorsed.

It is hoped, however, that a balanced education would provide a person with more than simply the skills to solve everyday problems. Hope (1986) writes: "the study of arithmetic should help children to develop some measure of quantitative thinking, namely, a way of thinking about, and reasoning with, numbers" (p. 49). Hope (1986) further cites Brownell to show that as early as 1935 he urged that meaning and understanding should be promoted in mathematics education:

> The "meaning" theory conceives of arithmetic as a closely knit system of understandable ideas, principles and processes. According to this theory, the test of learning is not mere mechanical facility in "figuring." The true test is an intelligent grasp upon number relations and the ability to deal with arithmetical situations with proper comprehension of their mathematical as well as their practical significance. (p. 49)

Reys (1984) lists five benefits of teaching mental computation and links mental computation with the development of a number of skills:

> Five widely accepted reasons for teaching mental computation are:
>
> (1) it is a prerequisite for successful development of all written algorithms;
>
> (2) it promotes greater understanding of the structure of numbers and their properties;
>
> (3) it promotes creative and independent thinking and encourages students to create ingenious ways of handling numbers;
>
> (4) it contributes to the development of better problem-solving skills; and
>
> (5) it is a basis for developing computational estimation skills. (p. 549)

The list is most comprehensive and provides a basis for the study and teaching of mental arithmetic.

The Role of Memory

A number of researchers (Hitch, 1978; Hope 1986, 1987; Howe & Ceci, 1979; Hunter, 1977, 1978) working in the area of mental arithmetic and memory suggest that the capacity of the memory to temporarily store information plays a significant role in the ability to calculate mentally. There is little doubt that memory plays a role in mental computation, but the exact nature of that role is still unclear.

In discussing the role of memory in the mental calculation process it is important to distinguish between long-term and short-term working memory, each having a separate function. Long-term memory may simply be described as a store of knowledge. Howe and Ceci (1979) state:

> The contents of short-term memory roughly correspond to "what is remembered" by a person at a given time, and form a type of "working memory" that temporarily retains both newly perceived environmental information and information retrieved from long-term memory while the information from both these sources is being used by the individual to cope with the demands of the task. Short-term memory provides a holding mechanism that stores data at the interface or working area where items that the individual has just perceived and information that he already possesses are brought together to deal with cognitive tasks. (p. 63)

People can and do store vast amounts of information in long-term memory, but individuals only have a limited capacity to keep items in their mind for short periods. Most research in the area of memory and mental arithmetic has concentrated on the role of short-term working memory on mental calculation.

According to Hunter (1978) there are three kinds of demands made on memory during a mental calculation. The first, a memory for calculative method, may be considered as the steps that a person must remember in order to carry out the calculation. Secondly, a memory for numerical equivalents is needed. For an average child the numerical equivalents roughly correspond to the basic number facts. These numerical equivalents are of the type 6 x 8 = 48 and 5 - 3 = 2. A gifted mental calculator may, however, remember far more than the basic facts and hence the term numerical equivalents may be applied to number facts outside the basic number facts such as 15 x 15. Memory for numerical equivalents can be likened to a store of basic facts. Finally, memory for interrupted working is called for if the problem is to be tackled successfully, because at several points in a calculation a part of the calculation is stored while another part is worked on. The first part must be retrieved later to complete the calculation.

The first two recall demands are met by long-term memory, whereas the third demand is filled by using a form of temporary storage. In written mathematics this would equate to the use of pen and paper to record interim parts of the calculation. When it comes to mental calculation this temporary storage role is fulfilled by the short-term working memory. The more complex the calculation, the greater the strain that is placed on short-term working memory.

It is noteworthy that researchers using a chronometric approach (where the reaction times of students' answers to mental arithmetic calculations are recorded) have found that reaction times slow considerably as the problem size increases. Various reasons are given for this slow down. At some point, which differs for each individual, the efficiency of the mental calculation decreases to such a degree that an alternative to mental calculation must be used to solve the problem.

Hitch (1978) points out that long-term memory may act as a store or library of strategies such as doubling or halving, removing zeros and the like which can be applied to different problems. Number facts are also stored in long-term memory for use as the need arises. These two roles are similar to the first two demands suggested by Hunter (1978).

Hitch (1978), Howe and Ceci (1979) and Hunter (1978), believe that skilled mental calculators make different uses of long-term memory and short-term working memory from their unskilled counterparts. It appears that most breakdowns occur in short-term working memory. Hope (1987) suggests that skilled mental calculators shift the burden of mental calculation from short-term working memory to long-term memory.

Svenson and Sjoberg (1983) claim that a shift in mental computation strategies occurs as children grow older. Young children tend to rely on primitive, less demanding strategies such as counting on their fingers. Finger counting serves as an external memory aid thereby reducing the load on short-term working memory. As students mature they shift toward a 'reconstructive' memory process by which answers are derived using short-term working memory. The final stage in the development of memory strategies involves a reproductive or retrieval process. The answer is stored in long-term memory and retrieved when the need arises.

The terms 'procedural knowledge' and 'declarative knowledge' have been used by Baroody (1983) to distinguish between the two main elements that must be present to perform a mental calculation. Procedural knowledge may be thought of as heuristics or strategies used to construct answers to problems. Declarative knowledge is simply another name for a stored body of facts from which retrieval can take place.

A third element sometimes referred to as pathways or connections ties these two bodies of knowledge together. Not only do these pathways form a link between strategies and number facts but they also join strategies to other strategies and tie number facts to other number facts. This combination of pathways is sometimes referred to as a network. The strength and number of these connections plays an important role in bringing together a person's knowledge to solve a mental computation.

It appears very likely that different individuals use procedural and declarative knowledge to different degrees. The type of question asked will also have a bearing on the degree to which each type of knowledge is used. It also appears likely that as students mature a shift from procedural to declarative knowledge occurs, but to what extent this occurs often depends on the individual and the strength and number of connections that have been formed.

The role of memory in mental computation, is acknowledged but it is outside the scope of this research project to study the degree to which memory differs between skilled and unskilled mental calculators.

The Nature and Use of Mental Strategies

It is often difficult to separate the role of memory and the use of strategies when solving problems mentally. As noted earlier, procedural knowledge and declarative knowledge are stored in long-term memory, and, together with short-term working memory, form a partnership to solve mental computation problems. The strength of this partnership is dependent on the number and calibre of the pathways connecting long and short-term working memory.

The relative use of heuristics and strategies or procedural knowledge as opposed to the use of a bank of stored facts or declarative knowledge in the solution of basic number facts is still being debated.

Hope and Sherrill (1987) argue that individual differences in mental calculation ability may reflect differences in the choice of strategy used. The word 'choice' in this context implies that students have several strategies at their disposal from which a selection can be made. The question of whether skilled mental calculators possess a wider variety of strategies compared to their unskilled counterparts or whether they simply use a more sophisticated range of strategies is one requiring further investigation.

Vakali (1985) notes that although simple problems have been studied "the processing of complex problems with multi-digit numbers have received less attention" (p.107).

In this section the most common strategies observed by previous investigators will be discussed. When comparing strategies documented by other researchers the problem of different researchers calling similar strategies by different names arises. Often different researchers describe the same strategy using a completely different term.

A second problem occurs because of the type of previous research undertaken. Most of the research in this area has been limited to the basic number facts and more specifically the operations of addition and subtraction. The strategies observed in these settings in some cases do not relate to the types of strategies used by children mentally computing the answer to division items beyond the range of the basic number facts. The discussion of strategies observed by other researchers will therefore be limited to those who have studied mental computation applied to problems beyond the range of the basic facts or to those who have studied the division operation.

The results of Vakali's study of more complex addition and subtraction problems show that children from year three onward tend to invent their own strategies or heuristics to solve problems. Mulligan (1990) also found similar results in her study of multiplication and division word problems. She noted that "75% of the children were able to solve the problems using a wide variety of strategies even though they had not received formal instruction in multiplication or division " (p.1). These findings might appear to conflict with Ashcraft's (1982) suggestion that children move toward declarative knowledge or answers stored in long-term memory, beginning around the year three level. This is possibly true when dealing with the basic facts, but Vakali's findings show this is not the case when dealing with more complex computations. Procedural knowledge comes to the fore in this case and if this knowledge is not available many students adapt strategies or invent their own to solve the problem.

Vakali (1985) further adds, "as the complexity of a problem increases, the mental effort and nature of solution strategies also become more complex" (p.112). Vakali was not surprised to find that some invented strategies appear more often than others. Whether the strategies are shared among students through discussion or whether students develop their own strategy independently is not known.

Ginsburg, Posner and Russell (1981) compared the development of mental addition in schooled and unschooled children. They noted that five main strategies were used to solve these problems:

1. Number fact: The subject was able to recall the answer without performing a mental calculation.
2. Counting.
3. Regrouping: When calculating the answer to 27 + 58 the tens would be added and the units would be added and the results combined. i.e. (20 + 50) + (7 + 8).
4. Algorithm: The subject calculated the answer using the written algorithm mentally.
5. Other. (p. 171)

Carraher and Schliemann (1985) found that the students in their study used similar strategies to those in the research cited above. They list four main strategies:

1. Counting;
2. Using the written algorithm taught in school;
3. Breaking the numbers into tens and units and in some cases, fives and then working out the solution; and
4. Using previous results to deduce a new one. (p. 40)

The strategies described above appear in most studies dealing with mental computation beyond the range of the basic facts.

Several researchers (Ashfield, 1989; Hope & Sherrill, 1987; McIntosh, 1990; Rathmell, 1978) have documented many examples of the strategies most commonly used by students. The use of a known fact is a good example of a strategy used by children to solve a mental computation problem. Thus, six times eight may be solved by using a known fact such as five times eight and then adding on another eight to reach the answer of forty-eight.

It has become clear from work carried out by McIntosh (1990) that students might use a strategy without fully understanding how it works. An example of this is the removal of zeros to simplify the solution of a problem like 70 x 90. Often, students will multiply seven by nine and then "add two zeros". Less cognitive processing is involved, demands on short-term working memory are decreased and therefore fewer errors should occur. Unfortunately this is not always the case. If the student has little understanding of place value then a 'remove zeros' strategy may cause the student to make an error when the zeros have to be replaced.

Several different strategies may be used to solve any one problem. Ashfield (1989) uses the terms "variable" and "flexible" to describe the strategies used by students tackling mental computation problems. It also appears that the same student tackling two similar problems may use completely different strategies to solve the problems. One possible reason for this apparent instability is that a student may be in the process of adopting a new strategy in place of an older less efficient strategy. Rathmell (1978) also found that an individual child may use different strategies to calculate answers to similar mental computation problems. Some strategies appear more dominant than others when specific question types are analysed. For example in the question 90 x 40 it is very likely that a remove zeros strategy will be used by the majority of students.

Ashfield (1989) suggests that strategies may be classed as efficient or inefficient on the grounds of speed and accuracy. Rathmell (1978) classifies strategies as either mature or immature and efficient or inefficient according to the speed and amount of cognitive processing involved. This classification appears reasonable on the basis of the literature that has been reviewed on the role of memory. It appears that a strategy which reduces cognitive processing more than another strategy and also reduces solution time will provide less opportunity for errors.

Counting strategies may be used to illustrate the notion of efficiency. Young children will often solve the problem six plus seven by counting in ones from six to thirteen. After some time a child may then realise it is more efficient to start from the bigger number and then count on. Later a more appropriate strategy like bridging tens may be used (6 + 4 + 3) or a doubling strategy (2 x 6 + 1) may be adopted. Memorising the number bond '6 + 7' in order to give an immediate response may be considered the most efficient method of all. Unfortunately, the results of many calculations beyond the basic facts cannot be memorised and hence, strategies play an important role in these computations.

The problem with such reasoning is that before a strategy may be classified as efficient or inefficient one needs to consider the computation and the individual performing the computation. For example a child who counts on from six to nine in ones may be classified as using an inefficient counting on strategy. If, however, the child were five years-old this strategy may not necessarily be classified as inefficient. Likewise a number of strategies may be equally efficient for calculating the sum of two numbers. When adding seven and eight it would be extremely difficult to classify doubling seven and adding one as being more efficient than bridging the ten by adding three to the seven to make ten and then adding the remaining five to make 15. It is for this reason that strategies will not be classified as being efficient or inefficient in this research.

Interviews carried out with exceptionally gifted mental calculators such as Aitken show that while they store many more numerical equivalents in long-term memory, they still make use of strategies which draw on their knowledge of stored number facts (Hunter, 1977). This finding suggests that the strategies adopted by students will depend on the number facts that they have stored in long-term memory. Hunter (1977) suggests that individuals build up through their own numerical experience a distinct calculative system. Hunter concluded that an "increase in ability concerns the development of techniques which enable the person to make more effective and economic use of his basically limited capacities for handling information" (p.40).

A number of researchers working in the field use interviews as their means of gathering data. One can never be a hundred percent certain that the explanation given by an interviewee of how a calculation was performed was the one that was actually used but it is one of the few ways to find out the type of information being sought. Rather than ask the child to provide a running commentary while solving a mental computation most researchers prefer to wait until after a problem is completed to ask the child to explain how they did it. In this way the explanation process does not interfere with what is going on in short-term working memory.

A major problem that concerns many researchers using interviews as their prime data gathering technique is the sheer volume of material that is collected. When the scientific method is used raw data are condensed by use of statistics into manageable pieces from which conjectures may be made. A similar approach must be applied to the data gathered by interviews. Many researchers who have gathered data on strategies used in mental computation have developed coding systems to help them condense the data into manageable pieces.

Most research has been confined just to basic facts and even more specifically addition and subtraction. A comprehensive list of strategies can be drawn from the literature. McIntosh (1990) has developed a list of strategies and coding which may be applied to all operations and which is not confined just to the basic number facts. The use of codes to classify mental strategies will be discussed in the next section.

Classification of Mental Strategies

Several researchers (Ashcraft, 1985; Ginsburg, Posner & Russell, 1981; Hitch, 1978; Hope, 1985; Vakali, 1985) have found it necessary to code strategies according to various characteristics. In each case the coding systems used were broad. Most coding systems were confined to single operations within the basic facts.

Codes are usually supported with information about the question asked and the response given. It is important that the code used is given in the context of the question that was asked. The sample verbalization clarifies the code being used and alerts the reader to the subtle differences between the various codes.

The names or codes given to the various strategies differ from one research article to the next. For example, the mental strategy of working from the left of a problem to the right for addition and subtraction problems might be coded as a "ten's column" strategy by researchers working with two digit numbers, while other researchers use the code "LR" to signify a similar strategy. The "LR" strategy appears to be quite a common one despite the fact that the written addition and subtraction algorithms are generally carried out from right to left.

While it is recognised that codes can become confusing they are the only means of condensing the data to a manageable form. McIntosh (1990) has developed a coding system to classify the set of strategies uncovered by his research. This coding system stems from his own research and from the early work of others in the field. The system devised by McIntosh is the most comprehensive of those to be found because strategies covering computations beyond the basic facts are listed. This list which is adapted to suit the needs of this research is shown in Appendix 6.

It is difficult, however, to cover all the possible strategies that might be used by a person because many strategies are highly idiosyncratic. Many researchers have found it necessary to record interesting or unique strategies in word-for-word fashion. Strategies and codes may need to be added to this set and others will not be used because of the focus on division that has been adopted for this research.

Unfortunately, when data are condensed in this fashion some elements are lost. To avoid this, the subject's transcribed explanations are often provided.

CHAPTER 2: METHOD

Forty students were chosen from a population of 300 year seven students drawn from seven inner Perth metropolitan primary schools. The 40 were chosen on the basis of their performance on a screening test. Nineteen high and twenty-one low performing mental calculators were chosen as a result of their performances on the twelve interview items. A more detailed discussion of the methodology follows.

The main research question and subsidiary questions dictated that a qualitative approach be adopted for this research. Data was gathered through the use of interviews. Cohen and Manion (1980) define the research interview as a "two-person conversation initiated by the interviewer for the specific purpose of obtaining research-relevant information" (p. 241).

Twelve division items formed the basis of the interview. After solving each problem the students were asked on a one-to-one basis to reflect on the strategy or method they used to solve the item. The interviews were audio-taped, transcribed and coded. Non verbal behaviour was recorded on a separate sheet during the interview.

DESIGN OF STUDY

As the aim of this research was to gain an understanding of how skilled and unskilled mental calculators carry out mental computations a qualitative approach was adopted. Hunting (1983) noted that little work had been done to explore the mental mechanisms that children possess or might have the potential to develop in relation to mathematics. He pointed out that a shift in research paradigm is called for in order to investigate these mechanisms. Currently the trend in mathematics education research is moving away from experimental research to qualitative methods such as those used by Piaget. Piaget made use of a special type of interview technique known as the 'clinical method'.

Hunting (1983) describes the clinical method as follows:

> The clinical method usually takes the form of a dialogue or conversation held in an interview session between an adult, the interviewer, and a child, the subject of the study. Usually the discussion is centred upon a task or problem which has been carefully chosen to give the child every opportunity to display behaviour from which mental mechanisms used in thinking about that task or solving that problem can be inferred. It is typical in this methodology, for the investigator to pose a verbal question to which the child makes some type of response, the investigator then asks another question, poses a variation of the problem, or in some way sets up a new stimulus situation. (p. 48)

Central to this research was the need to gather data relating to the stages or processing that a subject works through in order to arrive at an answer to a mental problem. Ginsburg (1981) suggests that if a researcher is interested in the stages or steps taken in solving a problem, then verbal reports are a valuable source of information. The clinical interview method provides a framework by which the question "How do you get at thinking if everyone thinks differently?" may be answered.

There are some problems inherent in the clinical interview method. Central to the method is the reliance on the verbal reflections of the subjects. The flexibility and variability of questions asked during interviews lead to questions of reliability and validity being raised against any research employing such a methodology.

Most weaknesses associated with the use of the clinical interview method as a means of gathering data stem from the dependence on the verbal reflections of the subject and the ingenuity of the interviewer.

The most common weaknesses associated with the clinical interview method according to Hunting (1983) are outlined below:

(1) The lack of a set of standardised procedures.

(2) The inability to precisely replicate the research.

(3) The reliance on the skills of the interviewer.

(4) The questionable reliability of one-off interviews.

A number of methods were employed to reduce the threats to reliability and validity of the research. These are discussed in the next section.

Issues of Reliability and Validity

With any research issues of reliability and validity need to be addressed. When considering reliability a researcher is principally concerned with the consistency of the measurements taken. The question of whether using the same instrument would produce similar results over a number of trials is one that must be answered by a researcher seeking to ensure reliability.

Validity refers to what the instrument measures and how well it does so (Anastasi, 1982). While reliability and validity are often linked it does not necessarily follow that because an instrument is reliable it is also valid.

One condition of validity, however, is that an instrument be reliable. A number of measures may be taken to prove and attain reliability but as Lecompte and Goetz (1982) acknowledge:

> Attaining absolute validity and reliability is an impossible goal for any research model. Nevertheless investigators may approach these objectives by conscientious balancing of the various factors enhancing credibility within the context of their particular research problems and goals. (p. 55)

The terms reliability and validity need to be defined in the context of this research because as Hammersley (1987) states, "when one looks at discussions of reliability and validity one finds not a clear set of definitions but a confusing diversity of ideas" (p. 73).

Reliabilty and Validity Issues Relating to the Use of the Clinical Interview

"Reliability is the extent to which a procedure produces similar results under constant conditions on all occasions" (Bell, 1987, p. 51). When interviews are to be used as the prime source of data collection Bell (1987) suggests that a researcher needs to ask, "Would two interviewers using the schedule or procedure get similar results? Would an interviewer obtain a similar picture using the procedures on different occasions?" (p. 51).

Validity refers to the accuracy of the data collected. "Validity tells us whether an item measures or describes what it is supposed to measure or describe" (Bell, 1987, p.51). The whole issue of validity is rather complex. The relationship between reliability and validity is such that reliability does not necessarily ensure validity, but items which are unreliable will also be invalid. An interview carried out on a number of occasions may elicit the same responses, but still not measure or describe what it is supposed to measure.

A variety of strategies were adopted throughout this research to reduce threats to validity and reliability.

Cohen and Manion (1980) point out that one of the major causes of invalidity in research employing the interview as the main data gathering instrument is bias. One way of establishing validity is to compare the data gathered with other data which has already been shown to be valid. The results of this research were compared with the findings of other researchers. The results of this research were found to be in agreement with most of the findings of the previous research in the field.

A delicate balance, however, exists between reliability and validity in the interview situation. Cohen and Manion (1980) cite Kitwood:

> In proportion to the extent to which 'reliability' is enhanced by rationalisation, 'validity' would decrease. In other words, the distinctively human element in the interview is necessary to its 'validity'. The more the interviewer becomes rational, calculating and detached, the less likely the interview is to be perceived as a friendly transaction, and the more calculated the response is likely to be. (pp. 252-253)

Best (1981) states, "The key to effective interviewing is the extent to which the interviewer can establish rapport" (p.166). If respondents feel threatened by some aspect of the interview they will tend to tell the interviewer what they think the interviewer wants to hear, or they will hold back information which they consider may reflect poorly on them.

Every attempt was made to help the child feel at ease during the interview without compromising the reliability of the data collected. The children were aware that the results of the screening test had not been leaked to their teachers and were pleased that a breach of promise had not occurred. The presence of the audio-tape recorder did cause a number of children to feel ill at ease. Most of these relaxed after the first two items and responded in a more open manner. The first two questions were simple and designed to relax the students.

A protocol was used to guide each interview. The subjects were asked the same questions. When the explanations given by the subjects as to how they carried out the problem were unclear a series of probes (Appendix 5) were used to try to elicit further information. Swanson, Schwartz, Ginsburg and Kossan (1981) warn that the aim must be to avoid putting words into the subject's mouth. Every attempt was made to avoid biasing the subject's response. The use of pre-determined probes aided in counteracting any such tendencies.

When the roles of task developer, interviewer and investigator coincide, as is the case with this study, problems relating to the skills of the interviewer tend to decrease. The skills of the interviewer may improve because of the familiarity of the interviewer with the task. The use of a single interviewer meant that consistency was maintained over the forty interviews.

Although it is argued that replication is difficult in this type of research it is not impossible, because given the appropriate documentation a researcher could undertake a similar study.

The problems associated with the clinical method need to be considered in relation to the purpose for which the method is adopted. Swanson et al. (1981), while recognising the limitations of clinical interviews, are quick to defend their use when it comes to securing understanding of a person's mathematical knowledge and reasoning. They go on to state:

> Indeed, with many of our more abstract or complex mathematical thoughts . . ., it would be difficult to make sense of the claim that a subject had such knowledge independent of the accompanying ability to articulate it in language or other symbol system. So there is good reason to believe that the clinical interview can be a useful tool for securing information about the facts and principles subjects may use in their mathematical reasoning. (p.32)

Hunting (1983) tempers the argument by suggesting that the problems associated with the clinical method need to be considered in the light of the purpose for which the method is adopted. Ginsburg (1981) concurs:

> Research into mathematical thinking has three basic aims: the discovery of cognitive processes; the identification of cognitive processes; and the evaluation of competence. Theoretical analysis shows that the clinical interview is the most appropriate method for accomplishing these aims. (p. 10)

Ginsburg does, however, clarify his statement by noting that the clinical method is far from foolproof and that other methods of collecting data such as naturalistic observation and standardised testing also have their uses.

The clinical interview was employed in this study to discover cognitive processes and to identify or specify cognitive processes. The choice of the clinical interview therefore, according to Ginsburg was most appropriate. Data of this type may also be collected using a 'talk through' approach, whereby the subjects verbalise the processes they are using to solve a problem as they are working toward a solution. This method of data gathering was dismissed because it was felt that it might interfere with the solution process. Ginsburg (1981) compares a number of methods that could be used to gather data but he concludes that "the clinical interview is the most appropriate" (p. 10).

SUBJECTS

The subjects were drawn from the population of year seven pupils who attended seven inner Perth metropolitan primary schools. Approximately 300 students were given a screening test. A random sample of 40 students was drawn from the top and bottom 27% of the three hundred students tested. Twenty students were then classified as 'skilled mental calculators' and 20 as 'unskilled'. Later the members of these two groups were redistributed according to their results on the twelve interview items. The students were spread across all of the seven schools.

The students were redistributed into two categories, 'high performers' and 'low performers'. A high performer was defined as a student who achieved a result of 10 out of 12 on the interview items. A student who achieved nine or less on the interview questions was classified as a low performer. The redistribution was necessary because the original screening test contained questions from all four operations. The interview questions focussed entirely on the division operation and therefore it was thought would provide a better indicator of performance of division computation carried out in a mental fashion. The redistribution only affected a few students, thus showing that the original screening test had provided a fair indication of performance on the twelve interview items. Details of this redistribution are given in Chapter Three.

INSTRUMENTS

Very few tests of mental calculation ability exist. Hope, Reys and Reys (1987) produced some tests which they claim assess the mental calculation ability of students. Unfortunately, no data regarding the reliability or validity of these tests were given.

A screening test was therefore developed, using the above-mentioned tests as a guide. Every attempt was made to design a test with items that closely followed the "Western Australian Mathematics Syllabus: Learning Mathematics Pre-Primary to Stage 7" (Ministry of Education, 1989).

Reys (1985) gives several suggestions for preparing mental-computation tests. He suggests that the test should be kept short (between 10 and 20 questions). Starting with a narrow focus (one operation), with specific numbers (whole numbers, decimals, or fractions) is also recommended.

The mental nature of the test should be emphasised and to this end Reys recommends that the students only be supplied with a small answer sheet. A small answer sheet discourages writing any working on the paper and reminds the students of the importance of mental computation. Reys also encourages the use of a variety of testing formats such as reading the problems aloud or displaying them on an overhead projector. Finally he suggests that 'nested questions', or problems of a similar nature be placed into the test so that patterns are easily recognised.

The suggestions made by Reys were taken into account when designing the screening test to be used in this study. The screening test consisted of 15 questions, most of which were division. A copy of the screening test is included as Appendix 1. A small number of addition and subtraction questions were placed at the beginning to give the students a measure of confidence. A few multiplication problems were also given because of the strong links between multiplication and division. The main emphasis, however, was on division.

The answer sheet (Appendix 2) provided the students with only enough room to write down their answer. A dual testing format was used to administer the screening test. The problems were read aloud twice and shown on the overhead projector at the same time.

The screening test was trialled to determine the length of time to be given to students to complete each calculation. The time taken to answer different questions varied according to the complexity of the problem. Members of the trial group were asked to comment on the difficulty of the questions, timing and the manner in which the questions were asked. In response to the comments made by those in the trial group some adjustments were made in the timing of the questions and a few questions were altered.

A panel consisting of three independent judges all working in the field of mathematics education considered the content validity of the screening test. The test was also slightly modified in accordance with the suggestions of the panel.

A time of 20 seconds was allocated for the answering of each question. The 20 seconds was measured from the time the question was asked to the beginning of the next question. The children were also simultaneously shown the question on the overhead projector. A standard time of 20 seconds was chosen because it was too difficult to administer a test where the timings for each question fluctuated.

The screening test was administered by the same person and the same instructions were given to the participants. Care was taken to note whether any students wrote down interim calculations. The children were told that the test would not contribute to their school marks and that it was important for them to try and work the questions out in their head and not to write things down on the desk or the back of their hand. One or two children preferred not to participate and one parent in response to the letter sent home regarding the research requested that her child not participate (Appendix 7).

The screening tests were scored and the children were ranked. The top and bottom 27% were separated and 20 students from each group were randomly chosen to form the basis of the more and less competent groups.

A set of twelve items formed the basis of the second instrument. These twelve items were used as a basis for a clinical interview with each of the 40 students chosen as a result of their performance on the screening test.

A panel consisting of three independent judges all working in the field of mathematics education also considered the content validity of the interview items. Changes were made to these interview items in response to the suggestions of the panel. A trial was carried out using the division items. As a result three items were removed from the original set of 15 items. The final twelve items used as the basis of the interview are given in Appendix 3.

The interview consisted of 12 division items, each without remainders, to be solved mentally. The 12 items were chosen in such a manner so as not to force students into using particular strategies. Many of the questions were similar to those given in the screening test. A number of different divisors were chosen and most consisted of a single digit.

The same instructions were given to each student to achieve standardisation across all the interviews. An attempt was made to reduce the anxiety of the participants by explaining that their answers would remain confidential, and that the results would not be supplied to their teacher nor be used to grade them. Two very simple questions were placed at the beginning of the interview to provide the participants with a feeling of confidence and to ease them into the style of interview to be conducted. After trying each question the student was asked to explain how he/she arrived at the solution.

A standard set of probes was used to probe the students for any extra information required to clarify unclear answers (Appendix 5). The interviews were audio-taped and student explanations coded. The coding system developed by McIntosh (1990) was used as the basis for the coding of student interviews (Appendix 6).

A third instrument, the interview recording sheet (Appendix 4) was used to record any observations made at the time of the interview. Non verbal behaviour, especially the use of fingers was noted on this sheet. Notations were kept to a minimum to avoid the student feeling threatened by the process. The recording sheet was used along with transcripts of the interviews to help code student responses to the interview items.

A sample of the transcripts and recording sheets was given to an independent analyst to code. The results from the two independent codings were used to determine the validity of the coding.

PROCEDURE

Two non-government and five government schools were contacted and asked to participate in the research. The two non-government schools were large and both had two classes of year seven students. Three of the government schools also had two classes of year seven students. The fourth government school only had a single year seven class while the fifth school only had a mixed year six/seven group consisting of 18 year seven students. The year seven teachers were questioned as to the type of mental arithmetic programme they used. None of the teachers had a programme running where mental computation strategies were highlighted. A copy of the letter sent to these schools is included as Appendix 8. Arrangements were made to send a letter to the parents of children in year seven at these schools seeking permission to test and possibly interview their children (Appendix 7).

The screening test was administered in the fortnight preceding the July 1990 school holidays. Student responses to the 15 questions were scored and the results were entered onto a spreadsheet. The data were sorted and the top and bottom 27% separated. Twenty students were randomly chosen from the top 27% and another 20 students from the bottom 27%.

These students were spread across all seven schools that participated in the screening test.

Interviews were arranged in the fortnight after the July 1990 holiday break. All schools were most co-operative arranging for rooms where the interviews could be conducted and audio-taping could take place. The interviews were conducted in the morning and generally in the time allocated to mathematics to avoid disrupting the school programme.

The interview began with a short chat to put the child at ease. The children were not told how they performed on the screening test. Most children were keen to co-operate and did not mind being audio-taped. One or two were hesitant but were put at ease when told that the interview was confidential and their teacher would not hear the tape. The time taken for individual children to complete the interview varied considerably. Generally 15 to 20 minutes was sufficient to complete the interview. The few students who took longer than 20 minutes to complete the interview began to show signs of fatigue.

Non verbal behaviour was noted on the recording sheet (Appendix 4). A few children showed signs of concern whenever recordings were made on this sheet. Many were concerned about being caught using their fingers. Often children would try to peer over the file to see what was being written about them. To avoid making the children anxious written observations were kept to a minimum. A form of short-hand was developed to streamline the process.

Each audio-tape was transcribed as soon as possible after the interview. Coding did not take place until all the interviewing was completed and all the tapes were transcribed. The transcriptions and non verbal recording sheets were used in conjunction when coding. A sample of these were coded on a trial basis. Copies were distributed to two other researchers familiar with the coding system. There was a high level of agreement about the codes applied to the transcripts. No formal analysis of this agreement was undertaken because all of the transcripts were coded by the same person. Discrepancies between the coding of interviews were discussed and some adaptation to the coding system developed. The final coding system will be discussed in Chapter Three.

The 40 interviews were coded and details entered onto a database. These students were then classified as high or low performers on the basis of their results in the division items. A student scoring 10 or more on the 12 interview items was classified as a high performer. Students scoring below this were classified as low performers. A designation of 'HP' was applied to high performers and 'LP' to low performers. Data were analysed according to this classification.

DATA ANALYSES

Miles and Huberman (1984) identify three components of qualitative data analysis: data reduction, data display, and conclusion-drawing and verification. Data reduction constantly occurs throughout a qualitatively oriented research project. Sampling decisions along with data coding and summaries are all examples of data reduction in one form or another. Miles and Huberman go on to state: "Data reduction is not separate from analysis. It is a part of analysis that sharpens, sorts, focuses, discards, and organizes data so final conclusions can be drawn and verified" (p.24).

The transcribed interviews and the recording sheets were examined to identify primary or dominant strategies used by each student to solve each mental division task. The frequencies of strategies used by high and low performing students were then tabulated. This allowed for a comparison to be made between the strategies adopted by high and low performers.

Raw data often provides an insight that cannot be gained from consolidated data and therefore segments from various interviews have been reported verbatim. The analysis focused on the main research question and subsidiary questions in order to determine the differences and similarities of the two groups under study.

Miles and Huberman (1984) link conclusion-drawing and verification. From the beginning of data collection a qualitative analyst starts to draw conclusions. These conclusions may simply be in the form of patterns or regularities that are noted. "The competent researcher holds these conclusions lightly, maintaining openness and scepticism, but the conclusions are still there . . . "(p. 26). Verification may take the form of reflections in the mind of the researcher; a return to raw data or to the subject; or an attempt at replication.

A detailed analysis of the strategies used by both groups follows in Chapter Three.

CHAPTER 3: DATA ANALYSIS

The purpose of this chapter is to analyse the data in relation to the original research question:

> "What differences are there between the strategies used by skilled (high performing) and unskilled (low performing) year seven students when solving division problems mentally?"

Associated with the main research question are the subsidiary questions outlined in the opening chapter of this thesis. Essentially the subsidiary questions focus on the use of particular strategies by high performing and low performing students and their success or lack of success in the use of particular strategies.

First, a review of the original more competent and less competent groupings will be undertaken on the basis of performance on the twelve division items tested during the interview. Second, an overview of performance on each item will be presented. Third, the strategies used will be defined and examples of each strategy will be given. An outline of how particular strategies were used in each item will then be provided.

The frequency of strategy usage will then be analysed. Strategy groupings will also be considered in the discussion. Some items will be grouped according to type to facilitate the analysis of broad strategy patterns. Items involving the use of place value will be included as one type of grouping. Those items just outside the range of the basic number facts will also be considered as another grouping. Differences between high and low performers will then be discussed in relation to the use of strategies. Finally, a few items that discriminated well will be discussed in more depth.

PERFORMANCE GROUPINGS

Students were chosen from the target population of year seven students based on performance by students in a fifteen-question screening test. Students were ranked and 20 were randomly chosen from the top 27% and called 'more competent' (MC). Another 20 were randomly chosen from the bottom 27% and called 'less competent' (LC).

The original screening was carried out to ensure that the interview sample would contain an appropriate split of high and low achievers. However the split was carried out on the basis of a screening test made up of mental computation questions covering all four operations. As the original research question focussed purely on division, it was more appropriate to group the subjects according to performance on the twelve interview items, all of which involved division.

The aim was to split the subjects into two equal groups, one called 'high performers' (HP) and the other, 'low performers' (LP). A split of 21 low performers and 19 high performers was achieved by using a cut-off point of 10 correct out of 12 items. Subjects who scored 10 or more on the twelve interview items were placed into the category of 'high performers'. Subjects who scored nine or less were classed as 'low performers'. A comparison between the original 'more competent' and 'less competent' groupings and the 'high performer' and 'low performer' groupings was made and is shown in Table 1.

Table 1 shows the correlation between the 'more competent'/'less competent' grouping and the 'high performer'/'low performer' grouping. The relationship between these groupings is indicated by the discrimination index 'ϕ'. In this case the relationship between the two groups was reasonably high given the small sample. Totals for each group are provided to indicate the numbers in each group.

Table 1

<u>A comparison of screening test results with performance on interview items</u>

	MC	LC	TOTALS
HP	16	3	19
LP	4	17	21
TOTALS	20	20	40
			$\phi = 0{\cdot}65$

The results show that 16 of the 'more competent' group were 'high performers'. Four of the same group became 'low performers' based on their performance on the twelve interview items. Three of the 'less competent' group performed well and therefore were classified as 'high performers'. The other 17 from the 'less competent' group were classified as 'low performers' on the basis of their performance on the twelve interview items.

The correlation between the 'more competent'/'less competent' and 'high performer'/'low performer' groupings is indicated by the discrimination index 'ϕ'. The discrimination index was calculated and found to be 0·65. This figure indicates that a reasonably strong correlation exists between the performance of members of each group on the screening test and the twelve interview items. This result suggests that the original screening test provided a good indication of how the subjects would perform on the twelve division items asked during the interview.

From this point on, the terms 'HP' and 'LP' will be used to describe 'high performers' and 'low performers'. All analysis will make use of these terms because these groupings should provide a truer indication of performance on mental division problems.

PERFORMANCE ON INDIVIDUAL ITEMS

The performance of the two groups on each of the twelve items asked during the interview will now be examined. Those items which best discriminated between the two groups will be identified and discussed in more detail during this part of the analysis. The twelve interview items are shown in Appendix 3.

Items 1 and 2. 20 ÷ 5 and 140 ÷ 10

The first two items, 20 ÷ 5 and 140 ÷ 10 were designed to put the children at ease. It was not surprising therefore, that everyone in both groups answered the first item correctly. In order to produce data in a succinct fashion the following symbols have been used to streamline the tables contained in this section. A correct answer is depicted by the tick symbol, '✔', an incorrect answer by the use of a cross, '✖' and the symbol 'ϕ' refers to the discrimination index, or the extent to which the item discriminated between the high and low performing groups.

Table 2

<u>A comparison of performance on Item 1</u>

Item 1. 20 ÷ 5	HP		LP		
	✔	✖	✔	✖	
	19	0	21	0	$\phi = 0$

The result for Item one as shown in Table 2 does not show any difference whatsoever between the 'high' and 'low performing' groups. The item is well within the realm of the basic number facts. No differences were found due to the absence of errors.

The question of how members of each group arrived at the correct answer will be considered when the strategies used by each group are examined in more detail later in the chapter.

Table 3 below shows how subjects from both groups performed on item two. This question was also relatively simple although it could not be classified as a basic number fact. Sometimes it may be taken for granted that children can perform simple multiplication and division problems involving tens but Table 3 indicates that this is not necessarily the case. Five of the 'low performing' children failed to answer the item, '140 ÷ 10' correctly.

Table 3

<u>A comparison of performance on Item 2</u>

Item 2. 140 ÷ 10	HP		LP		
	✔	✖	✔	✖	
	19	0	16	5	$\phi = 0{\cdot}36$

The relatively low discrimination index of 0·36 indicates that there was little difference between the performance of each group on the item. This was not surprising because the item was designed to put the subjects at ease. It was also designed to find out the strategies children use when confronted with calculations involving tens. These strategies will be discussed later in the chapter.

Item 3. 34 ÷ 2

The third item produced an interesting result as indicated by Table 4. Just over half of the 'low performing' group failed to correctly answer this question. Members of the 'high performing' group did not experience any difficulty obtaining the correct answer. The discrimination index, 'ϕ' indicates that there was a marked difference between the results obtained by both groups.

Table 4

A comparison of performance on Item 3

Item 3. 34 ÷ 2	HP		LP		
	✔	✖	✔	✖	
	19	0	10	11	$\phi = 0{\cdot}59$

Possibly the way a subject perceives the problem may have a bearing on whether the correct answer is attained. For example a subject might view 'thirty four divided by two' as 'half of thirty four' or 'two times what is thirty four?' or 'how many twos are there in thirty four?' The strategies used by the subjects should provide an insight into how they performed the calculation and hence how they viewed the question.

Item 4. 45 ÷ 15

Table 5 indicates that this item did not cause any significant difficulties to members of either the 'high' or 'low performing' group, with only two errors in the LP group. This is also confirmed by the low discrimination index of 0·21 recorded for this question.

Table 5

<u>A comparison of performance on Item 4</u>

Item 4. 45 ÷ 15	HP		LP		
	✔	✖	✔	✖	
	19	0	19	2	$\phi = 0{\cdot}21$

The similar performance of the two groups raises the question, "does it matter whether different approaches to a problem are used by high and low performers as long as the correct answer is achieved?" Some might argue that the time taken to produce an answer should also be considered as well as the accuracy of the answer. For the purposes of this research only the accuracy of the answer was considered. Response times were noted when transcribing the audio-tapes. These times, however, were only used on a few occasions to verify a student's response. For example one would expect an extremely short response time from a student responding that they knew the answer. A longer response time would be expected if the child used a strategy to determine the answer to an item.

Item 5. 78 ÷ 6

The performance of both groups on the fifth item is indicated by Table 6 below. Relatively few of the LP group and none of the HP group answered incorrectly.

Table 6

A comparison of performance on Item 5

Item 5. 78 ÷ 6	HP		LP		
	✔	✖	✔	✖	
	19	0	16	5	$\phi = 0{\cdot}31$

This item was included as an example of a calculation just outside the range of the basic number facts. Studying the strategies applied to this calculation may provide some useful information about the way children approach problems of this nature.

Item 6. 75 ÷ 3

The sixth question revealed a very marked difference in performance between the two groups. Table 7 indicates that most of the LP group answered incorrectly while the majority of the HP group answered correctly. The relatively high discrimination index of 0·75 reflects this large difference between the two groups.

Table 7

A comparison of performance on Item 6

Item 6. 75 ÷ 3	HP		LP		
	✔	✖	✔	✖	
	17	2	3	18	$\phi = 0{\cdot}75$

Even though this calculation falls well outside the range of the basic number facts one might assume that most children would know the first four multiples of twenty-five. It appears that student knowledge of number facts beyond the basic number facts may be a limiting factor in performing calculations of this nature. An examination of the strategies used by each group should help reveal why such a vast difference in performance occurred.

Item 7. 424 ÷ 4

The item "four hundred and twenty four divided by four" also showed up a marked diversity in performance between the two groups. The high discrimination index of 0·76 reflects the situation outlined in Table 8 below. Almost all of the LP group failed to answer the question correctly, whereas most of the HP group gave a correct response.

Table 8

<u>A comparison of performance on Item 7</u>

Item 7. 424 ÷ 4	HP		LP		
	✔	✖	✔	✖	
	15	4	1	20	$\phi = 0{\cdot}76$

This question was chosen to test the subject's ability to cope with the problem of a zero in the middle of the quotient. The types of errors made by the LP group will be examined later, along with the strategies used, to try and determine the cause of the wide difference in results between the groups.

Item 8. 320 ÷ 8

"Three hundred and twenty divided by eight" was an item designed to test whether children associate 320 with 32 and how they cope with this idea. The data show that the HP group had no trouble with this item, while over half of the LP group failed to furnish a correct answer.

Table 9

<u>A comparison of performance on Item 8</u>

Item 8. 320 + 8	HP		LP		
	✔	✖	✔	✖	
	19	0	9	12	$\phi = 0{\cdot}62$

It appears that not all children can make use of their knowledge of place value in solving calculations of this nature. Perhaps a lack of knowledge of place value is one cause of the low performer's problems. Further examination of the strategies used for this item may help determine the factors that differentiated between low and high performers.

Item 9. 290 + 5

Table 10 provides a summary of how children from both groups performed on Item 9. The disparity between both groups is most evident. One might expect that an item involving a divisor of five would not pose much of a problem. Clearly this was not the case.

Table 10

<u>A comparison of performance on Item 9</u>

Item 9. 290 ÷ 5	HP		LP		
	✔	✖	✔	✖	
	15	4	5	16	$\phi = 0{\cdot}60$

It appears from the results of children calculating the answer to "two hundred and ninety divided by five" that the LP group had difficulty applying their knowledge of the multiples of five beyond the basic number facts. An examination of the strategies applied by the LP group should help to indicate where this breakdown might be occurring.

Item 10. 144 ÷ 9

The results of Item 10 were very similar to the previous question, although twice as many LP children gave an incorrect answer as gave the correct answer. Only two HP children gave an incorrect response, hence the discrimination index was moderately high.

Table 11

A comparison of performance on Item 10

Item 10. 144 ÷ 9	HP		LP		
	✔	✖	✔	✖	
	17	2	7	14	$\phi = 0{\cdot}57$

Possibly the size of the divisor may have some bearing on the strategies used to solve the problem. The multiples of nine, for example, produce a pattern which some children may be aware of. Perhaps children may make use of this pattern as a strategy to solve a question of this type.

Item 11. 180 ÷ 30

Item 11 caused more difficulty for the HP group than any other item in the interview. The item, "one hundred and eighty divided by thirty" was chosen to further explore the children's understanding of place value. Table 12 below shows that what might at first seem like a rather simple item can cause problems to both high and low performers. The relatively low discrimination index of 0·27 suggests there was only a small difference in performance between the two groups. This item involving multiples of ten caused problems to members of both the HP and LP groups.

Table 12

A comparison of performance on Item 11

Item 11. 180 ÷ 30	HP		LP		
	✔	✖	✔	✖	
	14	5	10	11	$\phi = 0{\cdot}27$

An analysis of the strategies applied and the errors produced should aid in gaining a better understanding of the problems children face when carrying out a mental division problem of this nature. Items two, eight and nine also drew on children's understanding of place value so these will be grouped at the end of the strategy analysis section to see if any common threads appear.

Item 12. 161 ÷ 7

Table 13 outlines the performance of both groups on this item. It is quite evident that the LP group experienced a great deal of difficulty with this question while the HP group experienced very few problems. The high discrimination index of 0·76 also bears this out.

Table 13

A comparison of performance on Item 12

Item 12. 161 ÷ 7	HP		LP		
	✔	✖	✔	✖	
	18	1	4	17	$\phi = 0{\cdot}76$

A more detailed analysis of the strategies used by low performers may provide further information to explain the poor performance on this item.

CLASSIFICATION OF COMPUTATION STRATEGIES

In order to appreciate much of what is to follow in terms of the analysis of strategies used by various students to perform division calculations mentally, a clear understanding of what constitutes a particular strategy must be developed. In this section each strategy will be discussed and an example of each strategy in use will be provided to clarify subtle differences between certain strategies. The codes used to represent strategies will also be provided. The specific use of strategies in particular questions will be discussed in the following section.

The review of the literature indicated that little is known about the strategies used by children to perform mental calculations. What is known is confined to the basic number facts and then almost always to addition and subtraction. As this research focused on division outside the range of the basic number facts it was accepted that existing coding systems would need to be modified to suit the data being collected. This modification process could only take place once the data had been collected and analysed. The coding system devised by McIntosh (1990) was used but some alterations were necessary.

The system devised by McIntosh covered the four operations and included calculations both within and beyond the basic number facts. As this research dealt with division only and focused on calculations outside the range of the basic number facts, many of the strategies found by

McIntosh did not apply to this research. Some of his strategies were therefore discarded.

A second problem arose due to the relatively small sample chosen. Some strategies were only used by a very small number of the subjects and therefore a number of similar strategies needed to be collapsed into broader groupings to allow meaningful analysis to take place. Most notable were the strategies that had basic number facts as their base.

Table 14 below provides a summary of the strategies used by children when attempting to solve the twelve division items. The table outlines the name of the strategy, the code given to it and a simplified example of the strategy in use, as shown below.

The identification and classification of strategies to solve particular items in this research at times became rather complex. Mental calculation methods are often highly idiosyncratic and hence no coding system will adequately describe the way every person will approach every problem. In this study if more than one code was used to describe a calculation then the codes were listed in order of their use.

The strategies: 'basic number facts', 'repeated addition' and 'recited tables', were collapsed under the category 'basic number facts' for the purposes of statistical analysis. These strategies are delineated by the double lines in Table 14. They were coded separately, however, so a more accurate picture of how a child attempted to solve an item was maintained. Appendix 9 provides a summary of strategy use by group and item. The code 'T' is used to refer to the single entity of basic number facts rather than the combined group of three strategies. The code 'T' was used because most children referred to a specific multiplication table fact.

Table 14

Summary of strategies and codes

STRATEGY DESCRIPTION	CODE	EXAMPLE
Used mental form of written algorithm.	WA	Child gives a verbal description of the written algorithm. Makes use of terms such as "put down" and "carry the"
Changed division to multiplication.	DM	Item 20 ÷ 4. 5 x 4 = 20.
Used tens and/or hundreds.	UTH	Item 144 ÷ 9. 10 x 9 = 90 plus 6 x 9 so it's 16.
Split calculation into parts.	SP	Item 34 ÷ 2. "2 into 30 is 15 and then 2 into 4."
Removed zero(s).	RZ	Item 180 ÷ 30. "Take off the zeros; 3 goes into 18 six times."
Used doubling/halving.	DH	Item 161 ÷ 7. "7 tens are 70 and then I doubled it . . . "
Used fingers to aid in calculation.	F	Non-verbal behaviour. Noted on recording sheet as child performed calculation.
Related calculation to a known fact.	RK	Item 78 ÷ 6. "12 sixes are 72 and so it must be 13."
Multiples.	MU	Item 180 ÷ 30. "I just went 30, 30, 60 , 90, 120, 150, 180."
Knew or recalled the answer.	K	Child responded automatically to question. Child stated "I just know it."
BASIC NUMBER FACTS		
Basic number fact.	BNF	Child stated that he/she knew 'a table' that answered the question.
Repeated addition.	RA	Item 45 ÷ 15. "15 add 15 is 30 and another 15 is 45."
Recited 'Tables'.	RT	Item 78 ÷ 6. "6 sixes are 36, 7 sixes are 42, 8 sixes are 48, 9 sixes are 54, 10 sixes are 60, 11 sixes are 66, 12 sixes are 72 . . . "
Worked from the right.	WR	Child began with the units.
Mental picture.	MP	Child referred to a mental picture such as an array.
Counted on.	CO	Item 78 ÷ 6. 'Cause there's 10 in 60, 11 in 66, 12 in 78."
Couldn't do.	CD	Child responded "Can't do it."
See script.	SS	Unusual or interesting responses.

It should be noted from Table 14 that in a number of cases there was only a subtle difference between some of the strategies. When the line between one strategy and the next became blurred, the method of computation was classified according to the general 'approach' taken by the student.

The term 'approach' simply refers to a combination of strategies. The 'approach' was then classified, according to which strategy appeared to be the dominant one or which strategy underpinned the calculation and the method was classified in this manner. This method of classification often needed to be adopted when the 'split calculation into parts' (SP) strategy was used. A calculation was often split in order to 'relate calculations to a known fact' (RK), which often entailed the 'use of tens and hundreds' (UTH). A decision was made as to which strategy was the dominant one. A number of approaches were noted and a consistent recording system was used to code these approaches.

A pattern was noted in the order in which strategies were used. As mentioned earlier, a calculation was coded in the order in which a student approached it. Certain strategies continually showed up as being the first in a chain of strategies. These beginning or 'initial strategies' as McIntosh (1990) describes them need not be the dominant strategies. Initial strategies are those that might be used by children to transform the calculation into one with which they are more comfortable. In other words, when first faced with a calculation what does a child do? These strategies will be considered first of all.

Changing the calculation from one involving division to one involving multiplication (DM) was a commonly used strategy. This result was not altogether surprising because Fielker (1986) notes that:

> Division is traditionally done by multiplication, as we can clearly see by vocalising mental or written algorithms for it. One says "How many twos in eight?" or "Two into eight" rather than "eight divided by two", and computation is based on the multiplication tables rather than a memory of the division bonds. (p.35)

Even though Fielker spent much of his time studying how children deal with 'doubles' he concluded that children tend to avoid doing division if they can find other ways to carry out the problem. The results of this study are in harmony with the findings of Fielker's research. The 'division-to-multiplication strategy' was one of the most widely used strategies found in this research.

To better illustrate how the various strategies were applied to items in the research a number of verbatim accounts of children's responses will be provided. In each case the item will be identified first. The 'I' indicates when the interviewer was speaking. The first initial of the child's name was used to identify when he/she was speaking.

Note how the following student 'M' applied the 'division-to-multiplication' (DM) strategy to the item $75 \div 3$:

Item 6. $75 \div 3$

M (pause for 5 seconds) I don't know.

I Where could you start on a problem like that do you think?

M I don't know. I'm not . . . I don't . . . really know how to do divides. I just work 'em out by timesing.

The DM strategy was characterised by the subject restating the division problem in terms of multiplication by using phrases such as 'How many x's in y?' or 'x times y gives z.' Further examples of this strategy's use are given below:

Item 1. 20 ÷ 5

N 4.

I Can you explain how you get an answer of 4?

N I um, I said 5 x what = 4, I mean 20.

Item 1. 20 ÷ 5

K 4.

I Right, and how do you know that there is 4?

K Oh, um I remember 5 x 4 is 20.

Item 1. 20 ÷ 5

R Um 5, ah 4.

I Alright, and how did you come about solving that?

R Well, um I think of my tables and I go 5 fours.

In each case above the student used the 'division-to-multiplication' strategy in conjunction with a basic number fact.

Another example of what might be termed an initial strategy is 'removing zeros' (RZ). The students appeared much happier working with a problem like "eighteen divided by three", than with "one hundred and eighty divided by thirty". Apparently children found working with smaller numbers less daunting.

Unlike many other strategies this one appears not to have been self taught. In many cases after being questioned as to their use of the RZ strategy the students revealed that a teacher or parent had taught them how to use it. Further questioning revealed a lack of understanding on the part of many students as to why it worked. The use of this strategy often caused low performers to err. Note the use of the RZ strategy by the same student in both the following problems.

Item 7. 424 ÷ 4

G (pause for 10 seconds) 60.

I How did you get that as your answer?

G I went 4 into 24 goes 6 and added a zero.

I Alright, and why did you add the zero?

G Because . . . it was three numbers in the um 424.

I Oh, because it was 4 hundred and 24. I see. Fine.

Item 9. 290 ÷ 5

G (pause for 32 seconds) 40.

I And how did you solve that one?

G 5 into 20 goes 4 and add a zero.

I Why do you add the zero?

G Because there's three numbers in 290.

In both examples the student has applied a rule in an invalid fashion to try to solve the problem. A more successful use of this strategy is illustrated below:

Item 8. 320 ÷ 8

M (pause for 15 seconds) 40.

I 40, and how did you work that out?

M I went, um . . . um . . . 320, no 32 divide by 8 is 4 . . . then I just added a zero.

I So you added zero.

M Mm. And I got 40.

Item 11. 180 ÷ 30

S Um, chop the zeros off so it's 3 into 18. 3 goes into 18 six times.

Many students chose to approach the mental calculation in a similar fashion to the way it would be done on paper. The code WA was used to signify the 'written algorithm approach'. This code was only applied when students verbalised the steps of the written algorithm. Terms such as 'carry', 'borrow' and 'bring down' were commonly used in the descriptions given by children employing this method. Note the use of these terms in the following student's explanation of how she calculated the answer to item 7:

Item 7. 424 ÷ 4

R Um, 106.

I And would you explain how you solved that?

R 4 into 4 goes once. 4 into 2 goes 0. Carry the 2. 4 into 24 goes 6.

This method of calculation proved to be the most popular. This may be due in part to the types of questions asked. The division operation perhaps more than any other suits the use of a written algorithm approach when performing a mental calculation. The division algorithm is the only written algorithm to work in a left to right fashion. Working from the left to right is often used as a mental strategy in other operations. For example when adding two digit numbers children often begin with the tens. A lack of experience in dealing with division problems of this nature may also have caused the students to fall back on methods they knew or felt comfortable using.

A large number of students chose to split a calculation into manageable parts, find the answer to each part, and then add them together to produce the final answer. This strategy was recorded as 'split into parts' and coded SP. Even though the students reported splitting a calculation into manageable parts as the first step, in most cases the choice of split was dependent on one of two strategies. The split was often dependent on a 'known fact' (K) or on the 'use of tens and hundreds' (UTH). An example of each is given below. Note in the first example that the split was based on a multiple of ten whereas in the second example the split was based on a known fact:

Item 6. 75 ÷ 3

E (pause for 7 seconds) 25.

I And how do you get that answer?

E Um, I broke it up into 30 and 30, so it's 10 threes are 30 and 20 so it's 60 and 15. Five threes are 15. 25.

Item 10. 144 ÷ 9

G (pause for 53 seconds) 16.

I How did you solve that?

G Ah, a hundred and 12 nines is 108 and I just kept adding nines on from that.

In some cases it was dif. cult to determine whether the dog was wagging the tail or the tail was v agging the dog. The results clearly show that many students try to use tens and hundreds wherever possible and so in order to accommodate their use of tens and hundreds they are forced to split the problem into two or more parts.

The following example shows how one student split a calculation so as to make use of tens and a related table fact to find an answer:

Item 10. 144 ÷ 9

S (heavy sigh, pause for 13 seconds) How many in it?

I How many 9s in 144?

S Oh 100 let's see. (pause for 22 seconds) 15.

I How did you get 15?

S Oh. 10 nines are 90, so that's 10 and another 9, that's 11, and then it's 99 and then add it 45. 5 nines are 45. Add the other one is 46.

I So what was your final answer?

S 16.

The 'use tens and hundreds' (UTH) strategy was favoured by a large number of students. Essentially a student using this strategy would endeavour to make use of a multiple of ten in a mental computation so the intermediate calculations leading up to the solution involved trailing zeros. This may have the effect of easing the burden on short-term working memory. Another factor to keep in mind is that most children find it easy to recall the multiples of ten. They might therefore use a multiple of ten because it is the largest number fact at their disposal. In this case the strategy could more aptly be described as 'relating the calculation to a known fact' (RK). In many cases students chose to use the largest known number fact at their disposal, often a multiple of ten, as the basis of a split.

The following excerpt is a good example of a child making use of multiples of ten:

Item 2. 140 ÷ 10

L Um, 14.

I And how did you get that answer?

L Um there's 10 . . . there's um . . . well there's 10 x 10 is 100 and then 10 into 40 is 10.

Table 15 outlines the five most common strategies used by the children in the the twelve items. The five strategies were:

- written algorithm (WA);
- division to multiplication (DM);
- using tens and hundreds (UTH);
- splitting into parts (SP); and
- remove zeros (RZ).

These were all used as 'initial strategies' and some were also used later in the mental computation.

Table 15

Five most common strategies

STRATEGY USAGE					
Type	WA	DM	UTH	SP	RZ
Percentage	18%	15%	13·5%	10%	8%
Number	133	114	101	77	64

Altogether 753 strategies were used. One might expect 480 strategies considering that 40 children were asked to answer twelve division items but many children used more than a single strategy in answering each item.

Table 15 represents the most common strategies overall. Apart from these most common strategies several other strategies were used. There were some items where these strategies were not used to the same extent as shown in Table 15. For example in Item 1 the most common strategy was using a known fact, which does not appear among the most common strategies shown in Table 15.

In its simplest form division may be thought of as repeated subtraction and yet no student chose to use this strategy. A few students, however, chose to change the division problem into one involving multiplication and then performed the computation using repeated addition. The repeated addition strategy was coded RA. It should be noted that while this practice was limited, it was mainly used by low performers and often resulted in errors. Note the use of the word 'plus' rather than 'and' in the following example. This example also illustrates the use of DM as the initial strategy followed by RA:

Item 4. 45 ÷ 15

J 3.

I That was quick. How did you work that out?

J Um, 15 plus 15 is 30 plus another 15 is 45.

The use of fingers was a strategy which was noted and recorded on the interview sheet (Appendix 4). In some cases the student would state how they had used their fingers in the particular problem. In most cases children tried to conceal the fact that they were using their fingers by trying to hide their hands under the desk.

The use of fingers often served as an external memory aid. It appeared that some children felt restricted by not being able to write intermediate steps down for a mental computation as they would in a written calculation and tended to make use of their fingers as an interim recording device. The use of fingers to record interim steps may relieve the strain on short-term working memory. The use of fingers became very noticeable when children chose to use the 'written algorithm' strategy. Twice as many students used 'fingers' in conjunction with the 'written algorithm' strategy as used them with any other strategy.

Members from both the high and low performer groups made use of their fingers when carrying out mental computations. It is debatable as to whether the use of fingers is efficient or inefficient. It may depend on the nature of the calculation.

It appears that children often change a division computation to one involving multiplication so they can make use of a particular basic number fact. While it may appear from the data that this strategy was not widely used it can be misleading because this strategy lends itself to use in questions within the realm of the basic number facts. The use of a basic number fact to solve a question outside the basic facts such as in the case of seventy eight divided by six would be recorded as RK, 'relating to a known fact'. A child using the basic number fact 6 x 10 as the basis of a solution to this question would not be recorded as having used 'tables' (T) but rather as 'splitting the question into parts' (SP) and 'relating one part to a known basic number fact' (RK). The use of a basic number fact in conjunction with 'division to multiplication' (DM) can be seen below. A coding of DM, RK was applied to this explanation:

Item 1. $20 \div 5$

K 4.

I Right, and how do you know that there's 4?

K Oh, um I remember 5 x 4 is 20.

Closely allied with the use of basic number facts was the reciting of basic number facts, in almost a chanting fashion, as a means of solving a question. There is a marked difference between a child who says "seven times eight is fifty six" and a child who recites "one times eight is eight, two times eight is sixteen, . . . , seven times eight is fifty six" to arrive at an answer. The first child has developed automatic recall, the second has not.

While relatively few children used this strategy, it was still considered worth noting. Most children have developed automatic recall of the basic facts by the time they reach year seven and therefore it was surprising to still find some children reciting basic number facts to reach a particular basic number fact.

In the following extract note how the child relates the calculation to a known fact and then continues to recite the 'six times table' from that point:

Item 5. $78 \div 6$

S (pause for 13 seconds) What is it ? 78.

I Yes. How many sixes in 78?

S (11 seconds) 14.

I Right, how did you work out 14 as your answer?

S Well, um I started from 6 sixes and went up to 12 sixes and then I added another 2 to 78.

I Why did you start at 6 sixes?

S 'Cause I knew that 6 sixes were 36.

I Right, and then you went straight to 12 sixes.

S Yeh.

I How did you do that?

S Um, 6 sixes are 36, 7 sixes are 42, 8 sixes are 48, 9 sixes 54, 10 sixes are 60, 11 sixes are 66, 12 sixes are 72.

I You seem to know those tables pretty good, but you start at 6 anyway?

S Yeh.

I Alright, you didn't go straight to 10 or 11?

S No.

A further strategy with strong links to basic number facts and basic number fact recitation is the use of 'multiples' (MU). A child using multiples to solve a question such as "one hundred and eighty divided by thirty" would first change the division into a multiplication and then count in thirties until the desired target, in this case until one hundred and eighty was reached:

Item 11. 180 ÷ 30

G (pause for 23 seconds) 6.

I Right, how did you get 6 as your answer?

G I just went 30, 30, 60 , 90, hundred and, . . . 120, 150, 180.

A number of children used their fingers to keep track of the number of multiples used to reach the target. An example of this is given below:

Item 11. 180 ÷ 30

A How many 30s in 180? (pause for 36 seconds) 6.

I How did you solve that?

A Add the 30s together.

I Can you tell me how you did it?

A Oh, 30 add 30 is 60, then 90, 120, 150 and then 180.

I How did you keep track of them? What you were doing?

A Oh, just counted them with the fingers you know.

Note below how one child used multiplication to check his answer. The child demonstrated an understanding of a number of different strategies and used them to good effect. The MU strategy was not widely used except in Item 11:

Item 11. 180 ÷ 30

A 180 ÷ 30. (pause for 25 seconds) 6 times.

I How did you work out 6?

A Because I went 30, 60, 90, and so on to 180 and then I ...to make sure if it went 6 times I went 6 x 30 so it's 180.

A strategy which has come to light in many research reports on mental computation strategies is 'doubling and halving' and the use of near doubles. Some researchers treat 'doubling and halving' as a special case. While these strategies tend to be used a lot in addition and multiplication problems the use of 'doubling and halving' was not as popular in this research on division. The strategy, while appearing very powerful, is restricted to questions that lend themselves to the use of doubles and halves. The code DH was used to represent the doubling and/or halving strategy in use.

In the first example doubling is combined with the use of a known fact, a multiple of ten, in an attempt to find the answer:

Item 12. 161 ÷ 7

M (pause for 10 seconds) 20.

I 20.

M Yeh.

I So how did you work out how many 7s in 161 then?

M Oh, sorry. 161. Um. (pause for 8 seconds). Sorry. 23.

I 23 okay, how did you work that out?

M Um, times table. Um 7 tens are 70 and then I doubled it and 3 sevens are 21.

Note the combination of the SP and DH strategies in the next example:

Item 5. 78 ÷ 6

C (pause for 9 seconds) 12.

I Alright, and how did you work that out?

C I just said 6 sixes are 36 and doubled it.

Some children make use of 'mental pictures' (MP) to help them perform a mental computation. For example, when carrying out a simple addition students might imagine a number line or ruler to help them perform the addition. The use of this strategy was found to be very limited in this research, possibly due to the nature of the questions. This may be a reflection on the practice that many educators have of using concrete and diagrammatic aids in dealing with the basic facts but abandoning them as complexity increases.

Although the student in the following example referred to a mental picture it is doubtful whether it assisted him in finding a solution to the question. It is more likely that the use of tens and hundreds was the key strategy in solving this problem:

Item 5. 78 ÷ 6

M (pause) 12.

I Right, now how did you solve that one?

M I had a picture in my mind of one of those times tables sheets that we have in the classroom.

I And why did you go for 12?

M Oh, because 60 is 10 times and 2 more is 72. That's the question was it , 72?

I No, 78.

M Oh, and that's 13 then.

Two further codes were used to describe student behaviour. Neither refers to a strategy, although some might argue that the first shows common sense on the part of the student. The code CD was applied to any students who replied that they 'couldn't do' a particular computation mentally. A code of CD was not recorded unless a number of probes such as "well, where might you start?" (Appendix 5) had confirmed that the student had no idea of how or where to start the problem. A typical response to the probe was "no idea", or "wouldn't have a clue". Rather than record an error the code CD was used to show that a child did not even attempt the problem. If a student attempted a problem but did not get very far with it then the attempted strategy was coded.

The ability to determine whether and when a problem is beyond one's grasp could be considered in itself a strategy. The knowledge of when to carry out a problem mentally, on paper or with a calculator is most important. Perhaps children apply a number of tests to determine whether or not a problem is within their grasp.

A final notation of 'see script' SS was used when a very unusual, ingenious or particularly interesting approach was employed to answer a question. It was a means of referring to the verbatim transcript of a particular student's approach to a question. A number of unusual responses are included as Appendix 10. The usage of all the various strategies is shown in Table 16 below.

Table 16

Summary of strategy usage

WA	DM	UTH	SP	RZ	DH	BNF	F	RK	K	MU	WR	CO	MP
18%	15%	13·5%	10%	8·5%	7·5%	6·5%	6%	5%	5%	3%	1%	0·5%	0·5%
133	114	101	77	64	56	48	47	38	34	23	10	4	4

Note approximate percentages only

While the foregoing has only been a brief description of each strategy it should provide enough background to illustrate the use of these strategies in particular questions. Where the use of a particular strategy in a specific question appears to be obscure or where the strategy is consistently used by a number of children, the discussion will include a verbatim example of how the strategy was used. In the next section the strategies used by high and low performers in relation to particular questions will be discussed.

STRATEGIES USED BY HIGH AND LOW PERFORMERS

The strategies used by members of the HP and LP groups will be examined in relation to each question. Items with a common element, such as those involving the use of place value will be combined so that trends might be examined. Strategy usage and strategy grouping will also be considered. Children's levels of success when using particular strategies will also be noted.

Item 1. 20 ÷ 5

As described previously the performance of both groups on Item 1 was the same. An examination of the strategies revealed only a minor variation in the strategies used. Members from both the HP and LP groups claimed either to know the answer (that is automatically recall that twenty divided by five is four) or they changed the 'division to a multiplication' and used a basic number fact to solve the problem. Table 17 shows the most common strategies used by high and low performers when attempting Item one. The category 'others' was formed by pooling all those strategies together that individually were used by less than 20% of the children. This cut-off point was used because in most items it was found that three or four strategies tended to dominate.

Table 17

Item 1: 20 ÷ 5. Most common strategies

	K (25)		DM (12)		BNF (10)		Others (6)	
	HP	LP	HP	LP	HP	LP	HP	LP
✔	11	14	6	6	4	6	4	2
✖	0	0	0	0	0	0	0	0

It should be pointed out that many children used more than one strategy when calculating an answer to a particular division item. Overall 753 strategies were used. If each child had only used one strategy to answer each question 480 strategies would have been used.

Six members of each group chose to change the division problem into one involving multiplication (DM). In every case the members of the LP group then made use of a basic number fact, either 4 x 5 = 20 or 5 x 4 = 20 to complete the solution, whereas only half of the HP group chose to follow the 'division to multiplication' strategy with use of a basic number fact (BNF). High performers tended to use a slightly broader range of strategies than their LP counterparts.

Item 2. 140 ÷ 10

The second item "one hundred and forty divided by ten" was designed to test the way members of both groups handled the place value aspect of the question. The most common strategy was to remove the zeros as illustrated by the following excerpt:

Item 2. 140 ÷ 10

C 14.

I Alright, and can you explain how you did that one?

C I just take off the zero, 'cause 140 ÷ 10; Ten has a zero and so you just take off the zero.

I Alright, and how does that help you get the answer?

C Like ten has a zero on the end and 140 has a zero on the end so, so, you take off the zero on both of them and ones into 14.

Table 18 below indicates that twice as many high performers were likely to use this strategy as low performers. It should be noted that in this case every child who used this strategy arrived at the correct answer. At first glance it might appear that the remove zeros strategy (RZ) is ideal to use in this situation.

Table 18

Item 2: 140 ÷ 10. Most common strategies

	RZ (18)		UTH (9)		DM (9)		Others (19)	
	HP	LP	HP	LP	HP	LP	HP	LP
✔	12	6	2	5	1	5	8	9
✖	0	0	0	2	0	3	0	2

Three questions in this research involved the use of place value. The use of the RZ strategy along with the success rate will be monitored and reported on later in this chapter. A table indicating the strategy use and success rate of members of both groups for all twelve division items is contained in Appendix 9.

Item 3. 34 ÷ 2

No single strategy stood out in item three, "thirty four divided by two", but rather the use of strategies was almost evenly spread among five of the strategy types. This is evidenced by Table 19 given below.

Table 19

Item 3: 34 ÷ 2. Most common strategies

	SP (13)		DM (12)		UTH (11)		DH (10)		WA (9)		Oth (15)	
	HP	LP	HP	LP	HP	LP	HP	LP	HP	LP	HP	LP
✔	7	4	2	4	5	4	5	4	8	0	5	4
✖	0	2	0	6	0	2	0	1	0	1	0	6

A wide range of strategies was pooled together to form the category 'Others'. This question did not cause high performers any difficulty but over half of the low performers answered incorrectly. Much of the analysis of this question will concentrate on those strategies used by low performers and which produced incorrect answers.

Six of the ten low performing children using the DM strategy gave an incorrect answer. The mistakes appeared to occur when the low performing student followed the DM strategy with a further strategy. In some cases the choice of secondary strategy was inappropriate and made the problem more difficult by increasing the number of steps involved, thereby increasing the strain on short term working memory. This can be seen by considering the following example of a student who used DM followed by the use of a basic number fact or multiples of two:

Item 3. 34 ÷ 2

C (pause 20 seconds) 16.

I And what went through your mind when you were solving that?

C Say your two times table.

I And how do you say your two times table?

C 2, 4, 6, 8, and so on.

It was surprising to note the number of high performers who chose to use a written algorithm approach in their head. Not one low performing child used this strategy in this question. In each case the high performing child who used a written algorithm approach answered correctly. While one might imagine that this approach is somewhat clumsy it would be hard to criticise this approach based on the results of this question. It does raise the question, however, of whether the high performing children use the most efficient mental strategy (if one can make a distinction) or whether they simply use the one they have the most confidence will produce the correct answer.

The fact that only one low performing child used the WA strategy and then failed to answer correctly shows quite a marked difference between the two groups. This raises some questions about the carry over of written algorithms to mental computation.

Menchinskaya and Moro (1975) note that "the Russian school has always been distinguished by its great attention to mental calculation" (p. 73). They found that Russian students are encouraged to develop competency at mental computation before developing written calculation.

Why did children choose to apply a written algorithm method to the mental computation of a relatively simple problem? Perhaps children, especially high performers value accuracy over speed. For many this may be the only method at their disposal. Other studies have shown (Carraher, Carraher & Schliemann, 1985, 1987) that when children are given problems in the school environment they tend to use school-taught methods of solution but when outside of school they prefer to use their own methods. Thus if the question had been raised outside the classroom the method used may have differed.

There is quite possibly a strong link between high performers' ability at written mathematics and their mental computation ability and this may in turn affect the methods applied to mental computations. Perhaps the children are taught to use the written algorithm approach to such an extent that they believe it to be the most appropriate method to use all the time.

The use of the 'doubling and halving' strategy was fairly limited on this problem as double seventeen does not appear to be a commonly known double. A number of students chose to split the problem into parts (generally two) and then apply their knowledge of doubles to those parts. In most cases the children using this approach chose to split the problem so that a multiple of ten was formed thus making use of tens and hundreds. Note the use of this approach in the following example:

Item 3. 34 ÷ 2

K (pause for 5 seconds) 6, 17.

I Alright, and why do you say 17?

K Because I halved it and I said . . . First I halved 30 which is 15 and then I had 4 left over so I halved that which is 2 and then I add that on to 15.

The following student used a knowledge of double seven as the basis for solving the problem. Once again the use of tens is evident:

Item 3. 34 ÷ 2

A (pause for 23 seconds) Nup, can't work that one.

I There's no time limit on this. Where would you start?

A Um . . . 17.

I Right, you think the answer is 17?

A Yep.

I Okay, now how did you do that?

A Just double the number into what into 17 and everything.

I So, what . . . You tried a number of doubles or did you . . . ?

A Yeh , just double it.

I Which one did you start with?

A 17. I didn't think it would work out but . . .

I Why did you pick 17? Any reason?

A cause 7 and 7 is 14 so just add the 2 tens and it's 37 . . . 34.

In the following example the child has chosen a double which makes use of tens and then uses a type of 'counting on' approach with 'doubles' to arrive at the correct answer:

Item 3. It's 34 ÷ 2

J Oh. (pause for 22 seconds) It'd be 17.

I Alright, and how did you work 17 out?

J Well, 15 and 15 is 30 and so 16 and 16 is 32 and 17 add 17 is 34.

The use of the 'split into parts' strategy (SP) was most noticeable in this question. The ability to split a question into manageable parts may be limited by the number of different strategies a person has at his/her disposal. The ability to split a problem into manageable parts may also be a factor which differentiates between high and low performers. A high performer, because of his/her ability to break a problem into a series of simpler parts may be able to 'see' a method of solution. There are two possible routes that might be followed. Firstly, students might 'see' a method of solution and split the problem accordingly or they may split the problem into parts first and then endeavour to find a method of solution.

A low performer, for one or both of the above reasons, may not be able to apply the SP strategy, or once they use the strategy may not be able to apply other strategies successfully to the component parts. A further obstacle which may stand in the way of a correct solution is the need to remember the answer to each component so they might be combined to form the final answer. The load on short-term working memory may be too great. The error might simply occur at the final stage when the two parts are combined.

An examination of the two students who failed to answer the question correctly after having applied the SP strategy did not reveal any significant findings.

The questions raised above will be considered in further detail at the end of the individual analysis of strategies used in each question when the use of strategies and groups of strategies is examined over the whole twelve questions.

Item 4. 45 ÷ 15

Item four, "forty five divided by fifteen", while not producing any significant difference in performance between the two groups did show a reasonably consistent pattern of strategies that were used by members of both groups. The most common strategies are shown in table 20 below.

Table 20

Item 4: 45 ÷ 15. Strategy usage.

	DH (23)		SP (14)		DM (9)		Others (19)	
	HP	LP	HP	LP	HP	LP	HP	LP
✔	11	11	6	8	4	5	10	9
✖	0	1	0	0	0	0	0	0

The most common strategy grouping, or approach of DH and SP is given below. The two strategies DH and SP as shown in the extracts were combined to produce the solution to this question:

Item 4. 45 ÷ 15

R (pause - 4 seconds) 3.

I How did you work that out?

R Well there's 2 fifteens in 30 and another 15 is 45.

Item 4. 45 ÷ 15

M (pause for 11 seconds) 3.

I Right and how did you work that out?

M Um, 2 fifteens are 30 so an extra 15 has to be 45.

It should be pointed out that while the high and low performing groups did not differ to any large extent in their use of initial strategies such as SP and DM, what they did from this point on reveals some differences. Members of the low performing group made much more use of repeated addition than their high performing counterparts. This approach as used by a low performer is outlined below. All of the low performing children who applied this strategy gave the correct answer to the question:

Item 4. 45 ÷ 15

J 3.

I That was quick. How did you work that out?

J Um, 15 plus 15 is 30 plus another 15 is 45.

An example of particular interest given below shows how one student used a doubling approach to isolate the answer to this problem:

Item 4. 45 ÷ 15

K 3.

I That was quick. How did you solve that?

K There's 2 in 30 and there's 4 in 60 and I know that there's 3 in 45.

Item 5. 78 ÷ 6

The fifth item, "seventy eight divided by six", falls just outside the range of the basic number facts. While the difference in performance between both groups was almost insignificant it is interesting to note the approaches adopted by members of each group. The variety of strategies used may be seen by referring to Table 21.

Table 21

Item 5: 78 ÷ 6. Most common strategies

	UTH (15)		DM (15)		WA (12)		SP (9)		Oth (21)	
	HP	LP	HP	LP	HP	LP	HP	LP	HP	LP
✔	5	5	6	5	8	3	4	4	6	10
✖	1	4	0	4	1	0	0	1	2	3

The strong use of the WA strategy by high performers is most noticeable. Of the nine high performers using this strategy eight answered correctly. Note how this strategy is used to solve this question:

Item 5. 78 ÷ 6

C Is 12.

I Right, now how did you get 12 as your answer?

C I did the same as the other one. I did a division in my head.

I When you say you did a division, how does that look in your head?

C I just have the 78, and with the 6 in front of it and just do a normal division.

I And then can you go a step further? Can you give me the steps you do in that division?

C Well I put the six into the 7 which is one and then I carried the 1 over to the eight and then put the 6 into 18 which is 2.

Consider a second more successful use of the WA strategy illustrated below:

Item 5. 78 ÷ 6

C 13

I Good, now how did you do that one?

C Did a division sum, like a division sum.

I Can you run through the steps please?

C Yeh. I put 6 into 7 goes once and carried the one on and then I did 6 into 18 goes 3 times.

I Right, how did you work out the 6 into 18 part?

C Um, 'cause it goes 6, 12, 18

The two examples show that the successful employment of the WA strategy relies on the student's competence with the basic number facts, in this case six times three. High performers displayed less tendency to make the type of simple errors shown in the first example above. Many low performers showed they could also apply the WA strategy but often made simple mistakes of the type depicted in the first example above. These simple errors may in part be attributed to the strain placed on short-term working memory when using the WA strategy to perform a computation.

By their very nature written algorithms are designed to be performed with pencil and paper so that intermediate steps may be recorded. Once the pencil and paper are removed these intermediate steps have to be stored in memory and retrieved at various points in the computation. High performers may possess a better memory for this type of work and therefore perform better when applying this strategy. Perhaps high performers also perform well on written computations and have developed a high level of skill, therefore prompting the use of this strategy as an automatic choice. These observations will be pursued when the use of the WA strategy is considered for all questions.

A factor which may have had a bearing on strategy use in this type of problem is the student's prior knowledge of the basic number facts. Some students knew the multiples of twelve and made use of this when solving this question. Students who only have a knowledge of number facts up to the multiples of ten were therefore limited in the choice of strategy. The two examples below indicate how the recall of certain number facts may have a bearing on the method of solution:

Item 5. 78 ÷ 6

R Ah, 78 ÷ 6. Um, I'd go 6 times 13 is 78 which goes 13 times.

I Right, so you'd turn that round to a multiplication to work that out.

R Yeh.

I And how did you work out it was 6 x 13 to go for?

R Oh, well 6 x 12 is 72 and add another 6 is 78.

Item 5. 78 ÷ 6

M (pause for 10 seconds) 13.

I Alright, and would you explain how you worked that one out?

M Um, well, ten sixes are 60 and then 11 sixes are 66 and 72 and then . . . and then um I just got there from there.

I Right, so you started at the 10 sixes.

Both children were able to calculate the correct answer based on a particular number fact. It should not be implied, however, that the second child did not know the number fact 'six times twelve'. All that can be determined from the account is that she did not use it. What can be said is that without a knowledge of the multiples of twelve the first child could not have used that particular method.

It might be argued that high performers have a vast store of facts at their disposal that provides them with more strategy alternatives. There is nothing to suggest that this would improve performance and it is beyond this research to suggest a link between performance and range of known facts.

Apart from the use of the WA strategy, the choice of strategy between high and low performers did not vary greatly. What is noteworthy, however, is that four of the nine low performers choosing to use the DM strategy answered incorrectly. In each case the initial use of the DM strategy was carried out successfully, but the follow up strategy caused problems. An examination of the responses of the four children showed that each had given an answer of twelve, one away from the actual answer. No common thread appeared when the follow up strategies were examined.

Similarly four out of nine low performing students using the UTH strategy failed to answer the question correctly. In this case the incorrect respondents all had different answers. No common trend was found when the groups of strategies used by these children were examined.

Item 6. $75 \div 3$

Item six, 'seventy-five divided by three', showed quite a marked difference in performance between both groups. The most commonly applied strategies can be seen by examining Table 22 which shows that high performers and low performers differed considerably in their use of three strategies, WA, DM and DH. Once again students from the high performing group made much more use of the WA strategy than any other. This strategy was the most popular for members of the HP group with ten of the nineteen students opting to use it.

Table 22

Item 6: 75 ÷ 3. Most common strategies.

	UTH (16)		WA (14)		DM (11)		DH (8)		Oth (19)	
	HP	LP	HP	LP	HP	LP	HP	LP	HP	LP
✔	6	2	9	1	2	1	2	2	7	0
✖	1	7	1	3	1	7	0	4	0	12

Table 22 clearly indicates that on this question low performers preferred to avoid division by converting the division into a multiplication problem, hence the large number of LP students making use of the DM strategy. The 'use tens and hundreds' (UTH) strategy proved to be the most common strategy used overall.

Consider how the WA strategy and the DM strategy were employed in the following examples:

Item 6. 75 ÷ 3

R 15

I And can you explain how you get 15 as your answer?

R Oh, no hang on 25 not 15.

I So.

R I just did a division sum in my head.

I Alright, would you run through the steps of that for me please.

R Well, 3 into 7 goes 2. There's 1 remainder so I put that and it makes 15. 3 into 15 is 5.

I How do you know 3 into 15 is 5?

R From my times tables.

Item 6. 75 ÷ 3

M 25

I What steps did you go through to get 25 as your answer?

M I went um I knew that 3 x 10 is 30 and then I added another 30. That was 60 and then I added 15.

The example above illustrates how the child proceeded after applying the 'DM' strategy. Note how the calculation was split into parts based on a multiple of ten and the subtle use of doubles. The extract above is a good example of a child using an 'approach' rather than a single strategy.

The most popular method was to make use of tens and hundreds when calculating the answer to this question. The coding UTH can be somewhat deceptive as it covers quite a range of methods which rely on tens or hundreds as their base. The following examples outline a number of ways in which tens and hundreds were used in this question:

Item 6. 75 ÷ 3

M (pause) 25.

I Alright, and how did you solve that question?

M Well there's 3 tens in 30 , 30, 60, and 15 is another 5 , 25.

I Right, so then you can jump to 60 and then the next 15.

Item 6. 75 ÷ 3

E (pause for 7 seconds) 25.

I And how do you get that answer?

E Um, I broke it up into 30 and 30, so it's 10 threes are 30 and 20 so it's 60 and 15. Five 3s are 15. 25.

In the first example the child has shown signs of following the UTH strategy with a doubling of thirty to make sixty. In this case, even though the child has split the problem into parts it appears that the driving force was to use tens which in turn caused a split to occur. The second child uses the phrase "I broke it up" which tends to indicate a conscious thought of splitting the problem up into manageable parts. These two examples also indicate the subjective nature of a coding system.

In the following example the statement, "I know that there's four twenty-fives in 100" suggests that the strategy RK was being used but this method might also be construed as a use of tens and hundreds. The use of tens and hundreds can be thought of as a subset of the RK strategy because multiplications involving tens and in most cases hundreds invoke an automatic response. A child who therefore makes use of tens and/or hundreds to solve a question is relating the question to a known fact. In the cases where the number fact involved the 'use of tens and hundreds' the strategy was coded as UTH because this gives a clearer picture of how the child performed the computation.

Item 6. 75 ÷ 3

K 25.

I Alright, didn't take long to think about that. How did you solve that one?

K There's um because I know that there's four twenty-fives in 100 and then there's three twenty-fives in 75.

I Just take one off to get the three did you?

K Mm.

Table 22 also shows the success rate of children from both groups using these strategies. Two features stand out. First the number of low performing children using the UTH strategy who failed to answer the question correctly and secondly the lack of success by members of the same group who used DM as part of their method of solution.

There appeared to be no common patterns among the low performing students who answered incorrectly. However many of the students made use of doubling and halving along with UTH to solve the question.

The second area of concern relates to the poor performance of low performers using the DM strategy. Once again the transition from 'division to multiplication' appears to have been carried out successfully. The use of another strategy following the application of the DM strategy appears to have caused a problem in most cases.

Three low performers tried to relate the question to a known fact but the fact was too far away from the answer to be of any real help. It appears that when the method of solution is not readily discernible to the children, they tend to choose the largest basic number fact that relates to the question and try to work from that point. In most cases, such as the one in this question, the difference between the basic number fact and the answer is so great that the known fact is of little use.

This strategy of using the largest known basic fact is very useful when dealing with problems that are just outside the realm of the basic facts but hopelessly inadequate when dealing with computations of the nature of item six. When applying this method the low performers would try to count on from the known basic number fact or use multiples to progress toward the answer. In many cases they lost track of how many they counted and found it difficult to keep all the parts of the calculation stored in memory. In each case it appears that the number of steps used by the low performing students caused them to become confused, which in turn contributed to them making silly errors. One student even commented that he had "forgotten the number". One student became hopelessly lost in the calculation and gave up trying to complete the computation. Once again the WA strategy proved to be popular and most successful for HP students but not for LP students.

Item 7. 424 ÷ 4

The seventh item, 'four hundred and twenty four divided by four' discriminated well between the two groups with only one low performer answering the question correctly. Table 23 shows the dominance of two strategies, WA and UTH.

Table 23

Item 7: 424 ÷ 4. Most common strategies

	WA (16)		UTH (16)		SP (10)		Oth (19)	
	HP	LP	HP	LP	HP	LP	HP	LP
✔	8	0	6	0	6	0	4	2
✖	3	5	1	9	1	3	1	12

Children applying the WA strategy to this question often left out the zero in the ten's place, thus giving an answer of 16 rather than 106. This mistake is fairly common among children performing the written algorithm on paper so this finding is not altogether surprising. All five LP children applying the WA strategy to this question failed to answer it correctly. The following example shows how one student successfully applied the WA strategy. Note the combination of the WA strategy and the RK strategy:

Item 7. 424 ÷ 4

C 106

I A little bit harder I thought, but you did it fairly quickly. How did you do that one?

C 4 goes into 4 once. 4 into 2 doesn't go. Carry the 2, then 4 into 24 goes 6.

I How did you do 4 into 24?

C I just divided it. Like I knew there was 4 fives are 20, and 4 sixes are 24.

I Knowing 4 fives helps you work . . .

C Yeh.

I That's an easy one to remember is it?

C Yeh.

I Fine, now you had 106. Where does the 0 come from?

C The 4 into the 2.

I Right that . . .

C 'Cause that doesn't go.

I Then you . . .

C Carry the two.

The following example indicates how the UTH strategy was applied in this question:

Item 7. 424 ÷ 4

R (pause for 24 seconds) hundred and, hundred and, . . .6.

I How did you work that out?

R Well there's 25 fours in 100 and there's 400 so um so that's 100 and then there's 6 fours in 24.

In many cases the SP and UTH strategies were closely allied, while in some other cases one of these strategies tended to dominate. Note the use of these strategies in the following example.

Item 7. 424 ÷ 4

M 16, no hang on 106.

I You changed your mind. What was going on there?

M I don't know. I just mucked it up. I was thinking it was 24. I thought of 100 and instead of 100 I thought of 10.

I Right, now how did you do that question?

M I did the 100s first and then I was left with the 24 and I divided that by 4.

I Right, so it was 4 went into 424 so what did you do first?

M I divided 400 by 4 and got 100 and then I divided 24 by 4 and got 6.

I I see . . .

M And then I put 16 and then I remembered it should be 106.

Most of the answers given by the low performers using the UTH strategy were not even close to the correct answer. There were no strategy groupings based on the UTH strategy which were commonly used by low performing students attempting this question.

Examining the way low performing students used the UTH strategy provides an insight into the cause of their problems. Low performing students used one of two approaches involving tens and hundreds to solve this problem. The first approach involved using ten times four as the basis of the solution. Students adopting this approach would then use repeated addition or multiples to slowly progress toward four hundred, often losing track of how many groups of forty they had added. Exhausted from this effort, a number of students then failed to progress any further. The second approach, somewhat akin to working from the left and the written algorithm, involved starting with the known fact, 'twenty five fours are one hundred'. Even though students were considerably closer to the final answer most still failed to calculate the correct answer. One student was clearly confused and answered "four hundred and six" instead of "one hundred and six".

A consideration of the strategies listed under the heading 'Others', showed that a wide range of strategies was used. Only one strategy WR, worked from the right, was used by any more than two low performers. Only one of the five students applying the WR strategy answered correctly.

Item 7. 424 ÷ 4

M (pause for 42 seconds) 106.

I That's pretty good. Can you explain how you arrived at that answer?

M Well I went 24 divided by 4 and went 4 divided by 4.

I Right so when you did the 400 bit, you thought of it as a 4?

M Yeh.

Item 8. 320 ÷ 8

Item 8 was another problem which probed children's understanding of place value. When confronted with this problem 18 students applied the 'written algorithm' (WA) strategy. Twelve students removed the zero (RZ) effectively, breaking the problem down to 32 ÷ 8. From this point on a variety of strategies were applied. A common approach involved children changing the problem from 'division to multiplication' (DM). Table 24 below indicates that these three strategies proved to be the most common. Their use relative to each other and across the high and low performer groupings may also be seen from examining Table 24.

Table 24

Item 8: 320 ÷ 8. Most common strategies

	WA (18)		RZ (12)		DM (8)		Others (21)	
	HP	LP	HP	LP	HP	LP	HP	LP
✔	11	5	6	4	3	1	7	2
✖	0	2	0	2	0	4	0	12

Once again it should be noted that high performers applied the WA strategy more often than their low performing counterparts. Students employing the WA strategy did not encounter any significant problems because of the nature of the item. As the following example shows, the use of the WA strategy only really involves one calculation. A student following this approach is also less likely to forget to add a zero to make the answer forty.

Item 8. 320 ÷ 8

S Um (pause) 8 goes into 3 , zero times. 8 goes into 32 um 4 times and it doesn't go into 0 at all. So it's 40.

The 'remove zero' strategy did not seem to cause any significant problems possibly because only one zero had to be removed to carry out the computation and only one zero needed to be added to complete the answer. In many cases the removal of the zero was almost automatic as the following example indicates. The removal of the zero created a simpler problem which the student was able to solve.

Item 8. 320 ÷ 8

M (pause for 15 seconds) 40.

I Forty, and how did you work that out?

M I went, um . . . um..320 no 32 divide by 8 is 4 . . . then I just added a zero.

Low performing students who changed the problem from 'division to multiplication' (DM) often failed to produce a correct answer. In each case the conversion from 'division to multiplication' was carried out without problems. Mistakes occurred when follow-up strategies were applied in an effort to complete the solution.

'Using tens and hundreds' as a means of solving the problem also proved to be unsuccessful. All three of the low performers who applied the UTH strategy to this question failed to answer correctly.

Item 9. 290 ÷ 5

This item was well handled by the high performers but only five low performers answered the question correctly. The most common strategies used by members of both groups are outlined in Table 25 below

Table 25

Item 9: 290 ÷ 5. Most common strategies

	WA (18)		UTH (9)		Others (33)	
	HP	LP	HP	LP	HP	LP
✔	11	4	1	0	9	4
✖	2	1	2	6	4	16

Once again the 'written algorithm' strategy proved to be most popular. High performing students accounted for the bulk of those using this strategy. Most of the students applying this strategy, both high and low performers successfully tackled this problem.

This was in stark contrast to those students 'using tens and hundreds' (UTH) to solve the question. Every low performing student who applied this strategy gave an incorrect response. Only one of the three high performing students who applied this strategy gave a correct response. The remaining students from both groups applied a wide range of strategies to try and solve the question.

Item 10. 144 ÷ 9

This item caused relatively few problems to the high performing students. Solving this question, however, caused a number of difficulties to low performing children. Two thirds of the responses given by low performers were incorrect. Table 26 indicates that three main strategies were used to solve this question.

Table 26

Item 10: 144 ÷ 9. Most common strategies

	WA (21)		UTH (11)		DM (8)		Others (18)	
	HP	LP	HP	LP	HP	LP	HP	LP
✔	14	2	2	3	1	3	5	5
✖	1	4	1	5	0	4	1	7

The table clearly shows the dominance of the 'written algorithm' strategy. The bulk of those using this strategy were high performers. It appears that many high performers automatically revert to using this strategy when no obvious alternate strategy is available.

Low performing students tended to try other strategies such as changing the question from 'division to multiplication' or 'using tens and hundreds' as a first step. Once the initial strategy had been applied then the question was re-appraised. If a path to solution became more obvious then a further strategy or a number of strategies was applied to find a solution.

A breakdown tended to occur at one of two points. The first occurred straight after the use of an initial strategy, when the child was confronted with an equally complex problem. For example in this question a child who changed the problem from division to multiplication was required to solve '9 x ? = 144' rather than '144 ÷ 9 = ?'. The application of the 'division to multiplication' strategy did not achieve the desired result because the new question was not any easier to solve than the previous question. The child either gave up, made a guess or tried another strategy. It was during the application of a secondary strategy that further problems began to surface. A child who reached the point 9 x ? = 144 might then 'split the problem into parts', generally so as to produce a ten and then work toward the solution. The child would use 9 x 10 = 90 combined with another strategy such as 'counting on' to complete the solution. This procedure places a strain on short-term working memory. It is not difficult, therefore to understand why these children often failed to mentally solve items of this nature.

High performing children in many cases have a better grasp of the written algorithm than low performers and therefore the application of the 'written algorithm' strategy to a problem of this nature is probably most reasonable. When the written algorithm is used as it was intended, with paper and pencil, all the intermediate steps are carried out mentally and the paper and pencil only serve as an external memory aid to record the results of the various intermediate steps. The only difference between using the 'written algorithm' strategy mentally and with paper and pencil is that in the former case the results of any intermediate steps need to be stored in short-term working memory.

High performers may tend to have a better short term working memory, able to cope with this need for intermediate storage, whereas this might be beyond the ability of a low performer. Possibly this might explain why high performers adopt this strategy much more often than low performers. Working from the left to the right also tends to reduce the burden on short-term working memory. One can only speculate on why high performers use the 'written algorithm' strategy much more than their low performing counterparts. What is significant is that high performers used this strategy more often than low performers. High performers using the 'written algorithm' strategy generally gave a correct response and the frequency with which they applied the strategy increased as the items became more difficult.

Item 11. 180 ÷ 30

This item caused more difficulties than anticipated, but in doing so provided some rich data. The 'written algorithm' strategy was abandoned completely in favour of a variety of other strategies. This tends to indicate that rather than simply applying the same strategy to all questions encountered children apply different strategies depending on the type of question. In this particular item there were a number of strategies that could be applied to the solution of the problem in preference to the written algorithm strategy. Table 27 shows that a wide variety of strategies were applied.

Table 27

Item 11: 180 ÷ 30. Most common strategies

	RZ (26)		DM (20)		F (10)		MU (9)		Oth (12)	
	HP	LP	HP	LP	HP	LP	HP	LP	HP	LP
✔	10	4	5	9	4	4	4	5	3	2
✖	5	7	1	5	0	2	0	0	0	7

An understanding of place value would aid in solving a question of the nature of 180 ÷ 30. The most common strategy was to 'remove zeros' to produce a simpler problem, 18 ÷ 3. Often the 'division was changed to multiplication' and therefore became 3 x ? = 18. This approach well illustrates the grouping of two strategies to produce a solution. One might imagine that solving a question of this nature would have been a relatively simple task for year seven students. Table 27 also shows that almost half of the students applying the RZ strategy answered incorrectly.

When examined in detail the cause of the difficulties was an unclear understanding of place value. Many students felt that because they had removed one or two zeros they should add them on at the end of the calculation. It was not unusual to find children giving answers of 60 rather than six to the question "180 ÷ 30". When probed as to how they got an answer of 60 a number of children gave an explanation in terms of a rule. The following example illustrates how one child applied the rule without understanding why it worked:

Item 11. 180 ÷ 30

L (pause for 32 seconds) 60.

I Right, and how did you solve that one?

L I took off both the zeros and went 3 sixes are 18.

I It's easy when you can take those zeros off

L Yeh.

I How come you took two zeros off and you only put one back on?

L Um, I don't know. I forgot

Quite a few asked to change their answer from 60 to six after reflecting on their solution. This seems to indicate that the children were using a 'remove zeros rule' without thinking about the question.

Children also used 'multiples' to solve this question. Firstly they would change the problem from 'division to multiplication' and then count in multiples of 30 until reaching 180. Children following this approach tended to make use of their fingers as a means of keeping track of how many thirties they had counted.

Item 12. 161 ÷ 7

This question discriminated very well between the two groups with the low performing children experiencing considerable difficulty answering the question. It was not surprising therefore to find that the 'written algorithm' strategy was dominant. Few other strategies were so consistently used. The only other strategy that was used to any relative degree was 'splitting the problem into parts'. Six students adopted this strategy. Five applied the 'SP' strategy successfully. A wide variety of strategies were used in an attempt to solve the problem, most of which were unsuccessful. The overall dominance of the 'written algorithm' strategy can be seen in Table 28.

Table 28

Item 12: 161 ÷ 7. Most common strategies

	WA (23)		UTH (7)		SP (6)		Oth (19)	
	HP	LP	HP	LP	HP	LP	HP	LP
✔	14	3	3	1	4	1	3	1
✖	1	5	0	3	0	1	1	14

A large number of students chose to use a strategy other than the 'written algorithm' strategy but no particular strategies stood out beside the UTH and SP. The 'written algorithm' strategy proved to be a most successful strategy when used by high performers and even three low performers using this strategy answered correctly. This question appears to be of the type where the solution path was not obvious and hence the 'written algorithm' approach was adopted.

The few students who chose to split the problem into parts chose a split based on tens. For example they may have used seventy or one hundred and forty as a base for the split and then carried on from there.

PREFERRED STRATEGIES FOR HIGH AND LOW PERFORMERS

The five most common strategies will be considered first because these were chosen by the majority of children. Table 29 indicates the numbers from each group using each of the five most common strategies. Separating the data in this manner gives a clearer picture of which strategies were favoured by particular groups.

Table 29

Use of most common strategies by high and low performers

WA		DM		UTH		SP		RZ	
HP	LP	HP	LP	HP	LP	HP	LP	HP	LP
94	39	33	81	38	63	40	37	35	29

Table 29 clearly shows how common the 'written algorithm' strategy was among the high performers. The 'written algorithm' strategy proved to be the most popular overall but it may clearly be seen that this popularity was mainly due to the large number of high performers who adopted this strategy.

The reliance of low performers on changing the problem from 'division to multiplication' may also be seen by examining Table 29. The 'division to multiplication' strategy was by far the most popular strategy used by low performers. Out of the five most common strategies it was the least favoured by high performers.

'Using tens and hundreds' also proved to be more popular with members of the low performing group than their high performing counterparts. It was the second most common strategy chosen by low performers. There was little variation in the use of the 'split into parts' strategy and the 'remove zeros' strategy between the two groups.

Further differences showed up when a number of strategies were collapsed under the category of 'basic number facts'. These are reflected in Table 30. Table 30 also indicates the difference between high and low performers using the 'doubling and halving' strategy and children who responded 'can't do'.

Table 30

Use of less common strategies by high and low performers

BNF		DH		CD	
HP	LP	HP	LP	HP	LP
20	51	21	35	0	12

The use of basic number facts by low performers may be related in part to their use of the 'division to multiplication' strategy. Low performing students often chose to use a 'basic number fact' after applying the 'division to multiplication' strategy. Often this 'approach' was unsuccessful because the basic number fact was too far away from the desired result.

A reliance on 'doubling and halving' on the part of low performing children may also be noted from Table 30. Low performing children often made use of doubles as a means of progressing toward an answer after applying the largest basic number fact they knew for the problem. For example in item seven, 424 ÷ 4, a number of children used 4 x 10 as a starting point and doubled 40 to make 80 and then doubled 80 to make 160 and so on. Unfortunately many became confused after reaching 320 and failed to answer the question.

The 'couldn't do' category may not be significant but it was noticeable that a number of the low performers were able to discern when a problem was beyond their reach mentally. Perhaps these children apply some form of strategy in order to determine whether a problem is within their ability to calculate mentally. The ability to decide whether to calculate mentally or with the aid of a pencil and paper or perhaps calculator is in itself most important.

SUCCESS RATE FOR HIGH AND LOW PERFORMERS

Clearly it is one thing to use a strategy, and it is another to use a strategy and achieve the correct answer. A consideration of the five most common strategies revealed some interesting findings. Table 31 indicates how successful members of each group were after having chosen to apply a particular strategy.

Table 31

Success rate for each each of the most common strategies

	WA		DM		UTH		SP		RZ	
	HP	LP	HP	LP	HP	LP	HP	LP	HP	LP
✔	85	18	30	40	32	22	38	24	30	15
✖	9	21	3	41	6	41	2	13	5	14

Most high performing students who chose to apply the 'written algorithm' strategy were successful, whereas low performing students using the same strategy had an almost 50% chance of giving an incorrect response. One can only speculate on whether the results would have been any different if the low performers were allowed to use a pencil and paper.

Low performing students also experienced trouble in applying the 'division to multiplication' strategy and the 'remove zeros' strategy. Almost 50% of the low performing students using these strategies failed to correctly answer the question.

Low performing children tended to prefer to change the division to a multiplication and then reappraise the situation from the multiplication perspective. From this vantage point often they would choose the largest known basic number fact (often a multiple of ten) as the basis of a 'split'.

The 'remove zeros' strategy was limited to just a few items which lent themselves to the use of this particular strategy. A number of children from both groups tended to simply apply a 'rule' which they had been taught but did not necessarily understand. The results indicate that most high performers were able to successfully apply this strategy. Many of these children, however, originally gave an incorrect response but corrected themselves when asked to explain how they arrived at their answer.

The least successful strategy employed by low performers was the 'using tens and hundreds' strategy. Two thirds of the low performing students 'using tens and hundreds' failed to answer the question correctly. One reason for this occurrence was that low performing students often resorted to using the largest known basic number fact when groping for a solution. Invariably, the largest known basic number fact was a multiple of ten and hence the code UTH was given to this approach. Low performers often failed to progress past this point.

The most successful strategy employed by low performers was the 'split into parts' strategy. The low performing students using the split into parts strategy got the correct answer on two out of three occasions. Possibly splitting the question into smaller manageable parts helped to relieve the strain on short-term working memory. There were, however, some difficulties that low performing children experienced when applying this strategy. Firstly many low performers could not discern how a problem might be split into smaller, more manageable parts. Secondly, many of those low performers who were capable of breaking the problem up into manageable parts were then unable to store the results of all the interim computations in short-term working memory in order to arrive at an answer.

What is of interest is the reliance of high performing children on one particular strategy. It should be pointed out, however, that high performing children did not simply continually apply the written algorithm strategy without considering the question. When individual items were taken into account high performing students tended to apply a number of different strategies. When the item was such that no path toward the solution became apparent then the high performing students tended to rely on the 'written algorithm' strategy. When faced with a similar situation low performing students chose to apply the 'division to multiplication' strategy.

SUMMARY OF RESULTS

The results clearly showed that both high and low performers relied on seven main strategies when dealing with division problems beyond the range of the basic number facts. The strategies listed in order of use are shown in Table 32 which also outlines the frequency of use of particular strategies by the HP and LP group. The level of success achieved by the HP and LP groups when utilizing particular strategies can also be seen by examining Table 32. A number of lesser used strategies such as 'repeated addition', 'basic number facts' and 'recited tables' were combined under the heading of basic number facts. The category 'others' was used to describe a number of strategies which individually were not used to any large degree. Details of individual strategy use in particular items by each of the groups are given in Appendix 9.

Table 32 indicates the dominance of particular strategies such as the 'written algorithm' strategy and the 'division to multiplication' strategy. It should be reiterated at this stage that many children chose to use more than one strategy when performing a mental calculation. The 'written algorithm' strategy was used exclusively on its own, whereas the 'division to multiplication' strategy was nearly always used in conjunction with another strategy. This was also the case with a number of the other strategies shown in Table 32. When this fact is taken into account the overall dominance of the 'written algorithm' strategy comes sharply into focus.

Table 32

Summary of strategy use

Strategy	% of total strategy use	Frequency of use	HP	% HP	% HP ✔	LP	% LP	% LP ✔
W A	18	133	94	71	90	39	29	46
DM	15	114	33	29	91	81	71	49
UTH	13·5	101	38	38	84	63	62	35
SP	10	77	40	52	95	37	48	65
RZ	8·5	64	35	55	86	29	45	52
DH	7·5	56	21	38	100	35	62	57
BNF (RA, BNF, RT)	6·5	48	12	25	100	36	75	55
Others (WR, F, MU, CO, MP, RK, K)	21	160	69	43	90	91	57	49
Total	100	753	342			411		

A major difference between the high and low performing groups was in their respective utilization of the 'written algorithm' strategy and the 'division to multiplication' strategy. Of the students choosing to apply the written algorithm strategy 71% were from the HP group. Likewise 71% of those choosing to apply the 'division to multiplication' strategy were low performers. There was quite a marked difference between the two groups in terms of the most common strategy they applied overall to the twelve division items.

Apart from the use of the written algorithm strategy no particular strategy stood out for high performers in comparison to their low performing counterparts. Low performers, however, tended to make more use of the strategies involving division to multiplication, tens and hundreds, basic number facts, and doubling and halving. Many of these strategies were used in conjunction with the 'division to multiplication' strategy. High performers using the written algorithm strategy had little need for back-up strategies except perhaps the use of basic number facts on some occasions. Both groups used the same range of strategies but high performers tended to focus on a single strategy whereas low performers were more inclined to use a number of strategies.

No single strategy stood out as being more or less successful than another when used by a high performer. This was not the case for low performers. In most cases their success rate when using a particular strategy hovered around the 50% mark. However, when applying the 'use tens and hundreds' strategy the low performing students performed very poorly. The possible reasons for this occurrence have been outlined earlier in the discussion. The strategy which produced the best results for low performers was the 'split into parts' strategy. The LP group also experienced a measure of success using basic number facts.

Few strategies caused the high performers any trouble although the 'used tens and hundreds' strategy and the 'removed zeros' strategy were the only two strategies where as many as 14-16 % of the high performers gave an incorrect response.

The lack of success experienced by low performers applying the 'use tens and hundreds' strategy may be attributed in part to the approach used by low performers when they encountered a difficult problem. Typically they would change the problem from division to multiplication and then use the largest known number fact at their disposal, which in most cases was a multiple of ten as a starting point toward solving the problem. Generally this was as far as the low performers reached.

High performers as one might expect experienced little difficulty in obtaining a correct solution regardless of the strategy used. In some cases, on individual items, high performers did experience a little trouble in correctly applying particular strategies. This was particularly noticeable on Item 11, '180 ÷ 30', when a number of high performers failed to apply the 'remove zeros' strategy correctly.

The results presented above must be considered in the context of this study. By their very nature it was expected that high performers would, on the whole successfully apply a chosen strategy and that low performers would experience difficulty in obtaining the correct answer. Some possible causes of these results will be discussed in Chapter 4.

CHAPTER 4: DISCUSSION AND IMPLICATIONS

The purpose of this chapter is to discuss the findings in relation to the research questions posed for the study. A brief overview of the results will be presented prior to discussing the results in relation to these original research questions. Limitations of the research will also be considered. The relationship of this research to other research in the field will then be discussed followed by the implications of the findings for the classroom and for further research.

DISCUSSION OF RESULTS

The findings of this study indicate that the high and low performing children differed mainly in their use of two strategies, WA and DM. The HP group as one might expect were able to achieve good results using a wide range of strategies. Only when applying the UTH and RZ strategies did their success rate fall below 90% In most cases the success rate of the LP group stayed near 50%. The DH and BNF strategies proved to be slightly more successful. The most successful strategy used by the LP group was SP. Members of the LP group who applied the UTH strategy were only able to achieve a 35% success rate.

There was no difference in the range of strategies used by both groups but low performers showed more reliance on the DM, UTH, DH and BNF strategies whereas the high performers tended to rely mainly on the WA strategy. The DM strategy, favoured by the LP group was often used in conjunction with another strategy.

The original research question was as follows:

- What differences are there between the strategies used by skilled and unskilled year seven students when solving division problems mentally?

The subsidiary research questions were as below:

- Are there differences between the skilled and unskilled groups in:

  (i) their use of particular strategies;

  (ii) their success or lack of success in the use of particular strategies;

  (iii) their reliance on multiplication to solve division problems?

The findings of the study indicate that the high and low performing children mainly differed in their use of two strategies, WA and DM. One possible explanation for this difference is that the HP group may have experienced success in using the written algorithm with pen and paper and therefore naturally adopted this successful method for mental computation. High performers did not simply apply the written algorithm strategy to every item so it appears that the written algorithm was used whenever an alternative strategy could not be easily used.

The left to right progression employed when using a written algorithm strategy may also contribute to the popularity of this method among high performers. All of the items used in the interview could be solved without the use of remainders and therefore students using the WA strategy only needed to perform two or three calculations in their head before combining the intermediate results to produce an answer. The need to store intermediate results when using the WA strategy may also explain why the LP group did not use the WA strategy or failed to answer correctly when applying this strategy. The low performers may not have the same short-term working memory capacity as their high performing counterparts. Essentially the written algorithm was designed to be used with pen and paper, not mentally. Interim results can be recorded on paper when the written algorithm is used for the purpose it was designed. The strain of storing interim results may have been too great for the LP group and therefore they would choose not to use the WA strategy or if they did they would fail to answer the question.

When using the written algorithm for division the student has to break the problem into a series of multiplications and combine these using their knowledge of place value. The low performers may not have as good a grasp of the basic number facts or place value as high performers. They would therefore encounter more difficulties in applying the WA strategy than a person who possessed a sound knowledge of place value and the basic number facts.

The items given to the children to solve may also have influenced the choice of strategy. Many children when faced with problems involving 'big numbers' automatically assume that the only way to solve them is with the use of the written algorithm. Generally children are not encouraged to pursue alternative methods at school and are often chastised for not showing their working. It is possible, therefore, that children are given the impression that there is only one way of solving problems with 'big numbers'.

It is quite possible that the children felt that the interviewer was hoping they would use the written algorithm because they spend so much time in school learning how to use it. This may partially account for the high number of children who stated that they used a mental form of the written algorithm to perform a division calculation mentally. However, it is doubtful whether many children adopted this stance and certainly it would not fully account for the large number of students applying the 'written algorithm' strategy.

The finding that the LP group tended to rely on the DM strategy may be due to the manner in which teachers present division problems. Often when a child does not understand a division problem the teacher will rephrase the division problem in terms of a multiplication. Often the phrase "how many . . . ?" is used to describe a division problem. Many of the LP group may have come to associate division problems, especially more difficult ones, with multiplication.

The LP group may also have tried to work from the known to the unknown. Applying the DM strategy would then enable the LP group to use other strategies such as multiples with which they were more familiar. The LP group possibly applied the DM strategy when they could not think of any other suitable strategy. Having applied the strategy they may have, from this multiplication perspective, found it easier to 'see' a path to a solution.

The difference between the use of the WA and DM strategies by high and low performers partially answers the research questions shown above. Further differences arise when the levels of success in applying particular strategies are considered.

The 'split into parts' strategy proved to be the most successful for the LP group. Children opting to use this strategy would break the calculation into manageable parts and then work on each part adding the results together as they went. This would involve the need to store interim calculations as in the case of the WA strategy but in this case often the interim results were often based on multiples of ten. The flexibility of the SP strategy possibly allows children enough freedom to use number facts or multiples of ten which they find easy to work with. This procedure may have had the effect of reducing the strain on short-term working memory.

It should be pointed out that the UTH strategy proved to be the least successful for both the LP and HP groups. The LP group may have relied on this strategy when all else failed. As a last resort they would make use of any multiple of ten at their disposal to try and get close to the solution. Children may prefer to work with numbers that have trailing zeros. Often children are taught rules for multiplying by 10 and 100 such as 'add on a zero when multiplying by ten' and perhaps when under pressure these students fall back on the rules they have learned. As in the case of the RZ strategy many of these children do not understand why the rule works and therefore make mistakes when using tens and hundreds in multiplication and division problems.

The low performing children also used the 'doubling and halving' and 'basic number fact' strategies much more often than high performers. Doubling and halving is a common strategy. Most children can apply this strategy without any difficulty and often experience success using doubling and halving. It was not surprising to find the LP children using this strategy. Most children are reasonably proficient in using the basic number facts by year seven and therefore it was understandable that this strategy was favoured by the LP group. The HP group were able to apply more appropriate strategies to the situation and therefore tended not to use the DH and BNF strategies as often as their LP counterparts.

LIMITATIONS

The foregoing results need to be viewed in the light of the limitations of the research.

The issues relating to the reliability and validity of the clinical interview data gathering technique have previously been discussed. Although measures were taken to reduce the threats to reliability and validity some aberrations may have occurred. The results may include examples of children saying what they felt the interviewer wanted to hear.

The relatively small sample of children drawn from a sample of 300 year seven students from a number of schools in the metropolitan area also makes it difficult to generalise the results to any large extent. The trends indicated from the results do, however, add to the growing body of research in this area and in most cases concurs with what other researchers have found.

A close examination of Appendix 5 reveals that some of the probes may have influenced the responses of the children. Two probes in particular may have lead children into using particular strategies. The probe "could you break the question into simpler parts to help you solve it?" may have caused some students to adopt the SP strategy when they possibly may not have thought of applying this strategy. The second probe which may have influenced the children was, "did any pictures come to your mind when trying to work this question out?". This probe may have caused the children to use a mental picture when they had no intention of using one. It should be pointed out, however, that both these probes were only used on a limited number of occasions and therefore did not influence the results to a large extent.

The particular division items used during the interviews may also have influenced the results. Every effort was made to provide a blend of division problem types. However, some questions lend themselves to the use of particular strategies such as RZ. A decision was made to only use items that produced a whole number answer. Some differences in results may have been found if items with remainders had been included.

RELATIONSHIP TO OTHER RESEARCH

The outcome of this research serves to confirm what many other studies have concluded.

- When calculating in their heads children employ a variety of methods or strategies.
- Children invent their own methods to try and solve mental computations.
- Children generally understand the strategy they employ. Although this was not the case when using the 'remove zeros' strategy.
- Number sense is related to mental computation.
- Memory plays a role in mental computation.
- Children changed or altered a problem to produce one which was easier to manipulate mentally.

A brief outline of how this research confirmed previous findings follows.

Carraher and Schliemann (1987) found that children tended to change or alter a problem to produce one which was easier to manipulate mentally. This was also the case in this research. The manipulation was most evident when children applied strategies such as 'changing the problem from division to multiplication', 'splitting the problem into parts' and 'using tens and hundreds'.

When using the 'split into parts' strategy the children tried to split the problem in order to make use of a basic number fact. In order to do so they changed the problem from one involving division to one requiring multiplication. As Fielker (1986) noted children feel more comfortable multiplying than they do dividing.

The use of tens and hundreds was also a common strategy, possibly because it had the effect of reducing the strain on short-term working memory. The use of tens and hundreds also reduced the burden of having to 'carry'. Hope and Sherrill ((1987) noted that the burden of carrying numbers in the short-term working memory can become so excessive that performance eventually suffers. The children also seemed to feel more at ease working with tens and hundreds.

Closely related to 'using tens and hundreds' was the 'removal of zeros'. Zeros were removed in an attempt to reduce the mental processing required to solve the question. Many of the children in this research had apparently been taught how to use this strategy rather than having developed the strategy for themselves. It was clear from the interviews that many children did not fully understand why or how this strategy worked.

A number of researchers (e.g., Hope, 1986) have suggested that a child only tends to use strategies which he/she understands (p. 53). The results of this research seem to indicate that this was not necessarily the case especially in regard to the 'remove zeros' strategy. The implications of this particular finding may have some bearing on the argument of whether strategies should be taught or whether perhaps they should be nurtured by discussion and other means.

The findings of Hope and Sherrill (1987) appear to conflict with those of this study in relation to the use of the written algorithm strategy to tackle a question. Hope and Sherrill studied the 'characteristics of skilled and unskilled mental calculators performing multiplication calculations' (p. 104) and found that unskilled mental calculators relied on the written algorithm strategy. They also concluded that this strategy proved to be inefficient because the written algorithm strategy increased the burden on memory and therefore children often forgot interim calculations and failed to achieve the correct result. The difference appears to lie in the operation being researched.

When the division operation is considered, the written algorithm strategy which utilizes a left to right approach may be a most efficient method of performing a division calculation mentally. High performers relied heavily on this strategy. Most experienced a great deal of success applying this strategy to the more complex division items contained in the interview. The tendency to work from the left to the right, which is the basis behind the written algorithm approach to division, was also noted by Hope and Sherrill (1987). They suggested that working from the left is less demanding on short term working memory.

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It therefore appears that the written algorithm approach to solve division problems mentally may not be inefficient because a left to right approach is utilised, in contrast to the other operations where the written algorithm operates from right to left.

It has been suggested that a child's number sense and mental computation performances are closely allied. This appears to have held true in this research. The role of number sense was most apparent when children used the 'doubling and halving', 'split into parts' and 'removed zeros' strategies. The problem of not adding enough or adding too many zeros back on after using the 'removed zeros' strategy was one that affected both high and low performing children. It was interesting to observe, however, that once asked to explain how they arrived at their answer high performers often corrected their mistake. When asked to elaborate on why one answer was dismissed in favour of another, high performers often commented that their original response 'did not make sense'.

The children often manipulated calculations using the 'split into parts' strategy so they could make use of a known fact or use tens or perhaps doubling and halving to answer the question. This manipulation of a calculation often depended on the child's number sense and facility with numbers. Obviously the child's knowledge of basic number facts had a bearing on the manipulation, but the ability to alter a problem in order to accommodate what the child already knows so as to provide a path to solution gives evidence of number sense coming into play.

One final observation made during this research which appears to be in harmony with other research is that children not only use individual strategies but they group these strategies to form an 'approach' to solving a mental computation. Hunter (1977) uses the term 'a calculative plan' to describe the method by which exceptionally talented mental calculators perform computations. Hope (1986) used the term 'path' to describe a method of solution rather than the use of a particular strategy. This 'calculative plan' or 'path' appears somewhat similar to the 'approach' applied by children when answering items posed in this research. A number of common 'approaches' used by children tackling the same computation were noted in this research.

Hope (1986) suggests that "A good mental calculator is able to travel many more paths than the poor mental calculator."(p. 53, 54). The results from this research tend to suggest that there was little difference between high and low performers in terms of the range of strategies used by each group. If anything the high performers tended to use a narrower range of strategies than their low performing counterparts. The low performers may have used a wider range of strategies in their attempts to grope for a solution whereas the high performers had more definite ideas as to which strategies should be applied to particular types of questions. If the various 'approaches' used by members of each group had been examined in more detail then a difference between the number of 'paths' used by members of each group may have become more apparent.

Where the results differ from those found previously it does not necessarily indicate that these results conflict with those of other researchers. As stated at the outset very little research has been carried out in the area of mental division with calculations beyond the range of the basic number facts. Further study needs to be carried out in this area to find out more about the division operation and how it is used mentally.

IMPLICATIONS FOR THE CLASSROOM

A number of possible implications arise from from the results of this research. Firstly the data suggests that children performing mental computations involving problems beyond the range of the basic number facts mainly use a limited set of strategies and 'approaches' to solve these problems. This implies that it may be pos ible to make children aware of these strategies so at least the number of options they have at their disposal is increased. Whether a child's performance would improve if he/she had more strategies to choose from is a question that requires more investigation.

Possibly high performers applied the 'written algorithm' strategy when no other obvious alternate strategy was discernible. Perhaps educators should spend more time developing a child's 'number sense' by carrying out pattern searching activities and generally investigating numbers and their various properties. Discovering the various rules of divisibility comes to mind as an example of an activity which may enhance a child's ability to perform division calculations mentally. More study would need to be carried out to determine whether there was a transference of knowledge from such activities to mental arithmetic.

Low performers tended to produce better results using some strategies rather than others. This may not necessarily mean that low performers should avoid using particular strategies. This result may indicate that a number of low performing children were undergoing a transition from one strategy use to another. It is quite possible that low performers lag behind their high performing counterparts in terms of their adoption of particular strategies. If this were the case then the expectation would be that children who were just starting to apply new strategies may make a lot of errors.

Rathmell (1978), a proponent of using strategies as a means of improving mental arithmetic performance cites Brownell (1935) as pointing out that drill does nothing to develop new processes of solution. The terms strategies and 'approach' as described in this research may be substituted for the notion of processes of solution.

Current practice which often simply consists of drilling children in the basic facts is failing a number of students. Drill tends only to speed up the processes one already possesses rather than develop new or alternate ones. One way to improve the ability of low performers may be to change the way teachers deal with mental arithmetic.

It is probably true that very few teachers give children the opportunity to perform a division calculation mentally. For many teachers a mental arithmetic session consists of giving children a quick burst of miscellaneous questions. Rather than using this 'rapid fire' method as a means of developing mental arithmetic prowess a different approach involving the sharing of strategies amongst high and low performers might be encouraged. This is not to suggest that drill does not have a place but rather that drill is more appropriately used to increase the speed of a mental calculation rather than develop alternate strategies for performing the calculation.

The question of what is required to improve mental arithmetic performance has been considered by a number of researchers. From his memory perspective Hunter (1977) suggests that, "Increase in ability concerns the development of techniques which enable the person to make more effective and economic use of his basically limited capacities for handling information (p. 43).

It must be recognised that different people organise their knowledge in different ways and therefore one cannot prescribe a single method of developing mental arithmetic ability among children. What can be done, however, is to expose children to a variety of strategies which can be used to solve calculations mentally.

IMPLICATIONS FOR FURTHER RESEARCH

Throughout this chapter a number of questions alluding to possible further research have been raised. These questions are expanded below.

A replication of this study with students over a range of age groups could be carried out to determine whether a transition through strategy types occurs over time. Alternatively, students could be given a set of questions to calculate mentally and then at a later date asked to attempt the same set of questions. Similarities and differences in the strategies applied to corresponding questions could then be noted. In this way it may be determined whether children are consistent in the strategy they apply to different question types.

The issue of whether children should be taught to use certain strategies or simply be made aware of them is one that requires more research. Given that a body of knowledge is beginning to be built up about a number of strategies the question of what is the best way to impart this knowledge to children demands attention.

Further research also needs to be carried out to determine the relationship between mental arithmetic performance and written arithmetic performance. Many educators believe that too much time is spent dealing with written arithmetic. The time previously spent on written algorithms might then be used to develop mental arithmetic skills. Such a study could be used to determine whether overall computation performance changes as a result of increasing time spent on developing skills in mental arithmetic.

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APPENDIX 1: SCREENING TEST

1. 43 + 35
2. 28 + 55
3. 75 – 42
4. 80 – 24
5. 3 x 32
6. 4 x 23
7. 7 x 30
8. 90 ÷ 6
9. 80 ÷ 5
10. 72 ÷ 3
11. 56 ÷ 4
12. 150 ÷ 30
13. 74 ÷ 2
14. 128 ÷ 8
15. 189 ÷ 9

APPENDIX 2: SCREENING TEST ANSWER SHEET

ANSWER SHEET

Name:____________________

Age:________Sex: M / F

School:____________________

Main Language Spoken at Home

____

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

6. ____________

7. ____________

8. ____________

9. ____________

10. ______________

11. ______________

12. ______________

13. ______________

14. ______________

15. ______________

APPENDIX 3: INTERVIEW ITEMS MAIN STUDY

1. 20 ÷ 5
2. 140 ÷ 10
3. 34 ÷ 2
4. 45 ÷ 15
5. 78 ÷ 6
6. 75 ÷ 3
7. 424 ÷ 4
8. 320 ÷ 8
9. 290 ÷ 5
10. 144 ÷ 9
11. 180 ÷ 30
12. 161 ÷ 7

APPENDIX 4: INTERVIEW RECORDING SHEET

Name:______________________________Date________________

School______________________________

1. 20 ÷ 5 1st answer__________other answers________________

use of fingers____

comments____

2. 140 ÷ 10 1st answer__________other answers________________

use of fingers____

comments____

3. 34 ÷ 2 1st answer__________other answers________________

use of fingers____

comments____

4. 45 ÷ 15 1st answer__________other answers________________

use of fingers____

comments____

5. 78 ÷ 6 1st answer__________other answers________________

use of fingers____

comments____

6. 75 ÷ 3 1st answer__________other answers________________

use of fingers____

comments____

7. 424 ÷ 4 1st answer__________other answers__________

use of fingers__________

comments__________

8. 320 ÷ 8 1st answer__________other answers__________

use of fingers__________

comments__________

9. 290 ÷ 5 1st answer__________other answers__________

use of fingers__________

comments__________

10. 144 ÷ 9 1st answer__________other answers__________

use of fingers__________

comments__________

11. 180 ÷ 30 1st answer__________other answers__________

use of fingers__________

comments__________

12. 161 ÷ 7 1st answer__________other answers__________

use of fingers__________

comments__________

GENERAL COMMENTS__________

APPENDIX 5: INTERVIEW PROBES

ASK QUESTION

IF CHILD ANSWERS
Ask
How did you work that out (so quickly)?

IF NO ANSWER
Ask
Where might you begin?
What number might you start with?
Have you tried mental questions like this before?
Can you break the question into simpler parts to help you solve it?

IF THE RESPONSE IS SATISFACTORY
No further probes will be required

IF RESPONSE IS UNCLEAR
Tell the student that the explanation was a little difficult to follow and ask for a simpler/ slower explanation
or
Identify which part of the explanation was unclear and ask the student to explain that part in greater detail.
or
Ask the student to simplify his/ her explanation so that a younger student would understand.
or
Ask the student to describe each step s/he used to solve the problem and then ask her/ him to explain each step in as much detail as s/he can.

IF NO RESPONSE
Ask
What number did you start with?
What did you do first?
Did any pictures come to mind when trying to work this question out?
Have you tried questions like this before? What did you do?

IF CHILD GIVES INCORRECT REPONSE
If the student gives the wrong answer through mistaking the numbers in the question, this will be corrected immediately, but if the answer is wrong for any other reason it will not be pointed out. The student will simply be asked how s/he solved the problem and the appropriate probes shown above will be used.

APPENDIX 6: CODING SYSTEM AS DEVELOPED BY MCINTOSH

CD		Couldn't do the calculation
I		**Initial Strategy:**
	DM	Changed division to multiplication
	SA	Changed subtraction to addition
	CA	Used commutative law of addition
	CM	Used commutative law of multiplication
C_1		**Counting elementary**
	CO1	Counted on in ones
	CB1	Counted back in ones
	CBS1	Counted back to the second number in ones
C_2		**Counting in larger units**
	CO2/10	Counted on in twos/tens
	CB2/10	Counted back in twos/tens
	CBS2/10	Counted back to the second number in twos/tens
	RA	Repeated addition
	RS	Repeated subtraction
	MU	Multiples
	RT	Recited tables
P_1		**Used place value instrumentally**
	RZ	Removed zero
	WA	Used mental form of written algorithm
P_2		**Used place value relationally**
	ASP	Added/subtracted parts of second number
	B	Bridged tens/hundreds
	UTH	Used tens/hundreds
	WL	Worked from the left
	WR	Worked from the right
R		**Used other relational knowledge**
	DH	Used doubling/halving
	P	Used pattern
K		**Known fact**
	K	Knew (i.e. recalled) the answer
A		**Used aids**
	F	Used fingers
	MP	Used a mental picture
E		**Extra codings**
	G	Guessed
	SS	See script (1 - 5, SS5 being most significant)

APPENDIX 7: LETTER TO PARENTS

Dear Parent,

I am writing this letter to provide you with some information about a research project in which I am engaged and to ask if you would be willing to allow your child to take part.

The project is part of a Masters' Thesis that I am working on as part of my studies with the Western Australian College of Advanced Education. The main purpose of the project is to gain more detailed information about the development of mental arithmetic abilities of year seven students. It is hoped that this information will aid in the development of materials to improve the mental arithmetic abilities of children.

The project is being supervised by two well known academics, Dr Jack Bana and Mr Alistair McIntosh. Both are senior lecturers in mathematics education at W.A.C.A.E and have a deep interest in mental mathematics.

In the preliminary phase a short mental test of 15 minutes duration will be given to year seven students from a number of schools. Later on a few students will be selected for a follow up interview of approximately twenty minutes duration in which the students will be asked to explain how they go about solving some mental arithmetic questions.

All interviews will be audio-taped for further analysis. The identity of individual students and individual schools will not be used again once the data is collected. Thus complete confidentiality is assured.

If you have any concerns please feel free to contact me through your school.

Yours sincerely

Paul Swan
M.Ed Student

APPENDIX 8: LETTER TO SCHOOL

Dear Principal,

I am writing this letter to provide you with some information about a research project in which I am engaged and to ask if you would be willing for your school to be involved in the project.

The project is part of a Masters' Thesis that I am working on as part of my studies with the Western Australian College of Advanced Education. The main purpose of the project is to gain more detailed information about the development of mental arithmetic abilities of year seven students. It is hoped that this information will aid in the development of materials to improve the mental arithmetic abilities of children.

The project is being supervised by two well known academics, Dr Jack Bana and Mr Alistair McIntosh. Both are senior lecturers in mathematics education at W.A.C.A.E and have a deep interest in mental mathematics.

In the preliminary phase a short mental test of 15 minutes duration will be given to year seven students from a number of schools. Later, a few students will be selected for a follow up interview of approximately twenty minutes duration in which the students will be asked to explain how they go about solving some mental arithmetic questions.

All interviews will be audio-taped for further analysis. The identity of individual students and individual schools will not be used again once the data is collected. Thus complete confidentiality is assured.

Having taught in both the primary and secondary schools I realise that the demands placed on teachers are great. The data collection phase has therefore been designed to cause as little disruption as possible to the school and will not involve the relevant staff in any extra work load.

I would be very happy to discuss any matters with yourself and/or your staff prior to you making a decision if you wish. If possible I hope to commence. . .

Yours sincerely

Paul Swan
M.Ed Student

APPENDIX 9: SUMMARY OF STRATEGY USE BY GROUP AND ITEM

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1	Q	R/W GRP										STRATEGY										
2			HP	LP		SP		DM		RZ		RA		RK.		UTH		CO		F		T
3						HP	LP	HP	LP	HP	LP	HP	LP	HP	LP	HP	LP	HP	LP	HP	LP	HP
4	1	✓	19	21		0	0	6	6	0	0	1	0	0	1	0	0	0	0	2	0	3
5		×	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6																						
7	2	✓	19	16		2	4	1	5	12	6	0	0	0	0	2	5	0	0	0	0	1
8		×	0	5		0	1	0	3	0	0	0	0	0	0	0	2	0	0	0	0	0
9																						
10	3	✓	19	10		7	4	2	4	0	0	0	0	1	1	5	4	0	1	2	1	0
11		×	0	11		0	2	0	6	0	0	0	0	0	3	0	2	0	0	0	1	0
12																						
13	4	✓	19	19		6	8	4	5	0	0	1	5	2	1	0	1	0	0	1	1	0
14		×	0	2		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15																						
16	5	✓	17	13		4	4	6	5	0	0	0	1	4	3	5	5	0	2	0	1	1
17		×	2	8		0	1	0	4	0	0	0	0	1	0	1	4	0	0	0	1	0
18																						
19	6	✓	17	3		4	0	2	1	0	0	0	0	2	0	6	2	0	0	1	0	0
20		×	2	18		0	3	1	7	0	0	0	0	0	4	1	7	0	1	0	1	0
21																						
22	7	✓	15	1		6	0	0	0	0	0	0	0	0	1	6	0	0	0	2	0	1
23		×	4	20		1	3	0	2	0	1	0	2	0	0	1	9	0	0	1	2	0
24																						
25	8	✓	19	9		0	0	3	1	6	4	0	0	2	0	1	0	0	0	0	1	2
26		×	0	12		0	0	0	4	0	2	0	1	0	1	0	3	0	0	0	3	0
27																						
28	9	✓	15	5		3	1	0	1	2	1	0	0	2	0	1	0	0	0	1	0	1
29		×	4	16		1	1	1	3	0	3	0	1	1	0	2	6	0	0	1	3	0
30																						
31	10	✓	17	7		2	2	1	3	0	0	0	0	1	2	2	3	0	0	2	0	0
32		×	2	14		0	1	0	4	0	0	0	2	0	2	1	5	0	0	1	2	0
33																						
34	11	✓	14	10		0	0	5	9	10	4	0	0	1	0	1	1	0	0	4	4	0
35		×	5	11		0	0	1	5	5	7	0	3	0	1	0	0	0	0	0	2	0
36																						
37	12	✓	18	4		4	1	0	0	0	0	0	0	0	0	3	1	0	0	2	0	0
38		×	1	17		0	1	0	3	0	1	0	2	0	1	0	3	0	0	1	3	0
39																						
40		TOT				40	37	33	81	35	29	2	17	17	21	38	63	0	4	21	26	9
41																						
42		✓				38	24	30	40	30	15	2	6	15	9	32	22	0	3	17	8	9
43		×				2	13	3	41	5	14	0	11	2	12	6	41	0	1	4	18	0
44																						
45	GRAND TOT						77		114		64		19		38		101		4		47	
46							10%		15%		8%		3%		5%		13%		1%		6%	

	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM
1																	
2		K		RT		MP		WA		MU		WR		DH			TOT
3	LP	HP	LP	HP	LP	HP	LP	HP	LP	HP	LP	HP	LP	HP	LP		
4	6	11	14	0	0	1	0	1	0	0	0	0	0	0	1		53
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0
6																	
7	3	3	1	0	0	1	0	1	0	0	1	0	0	0	0		48
8	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0		7
9																	
10	0	1	0	0	0	0	0	8	0	0	1	1	0	5	4		52
11	0	0	0	0	0	0	0	0	1	0	2	0	0	0	1		18
12																	
13	0	3	1	1	0	0	0	0	0	2	0	0	0	11	11		64
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1		1
15																	
16	1	0	0	0	1	0	0	8	31	1	0	0	0	0	1		56
17	0	0	0	0	1	1	0	1	0	0	0	0	0	0	1		16
18																	
19	0	0	0	0	0	0	0	9	1	0	0	0	0	2	2		32
20	2	0	0	0	0	0	0	1	3	0	1	0	0	0	4		36
21																	
22	0	0	0	0	0	1	0	8	0	0	0	0	1	0	0		26
23	0	0	0	0	0	0	0	3	5	0	1	0	4	0	0		35
24																	
25	0	0	0	0	0	0	0	11	5	1	1	0	0	1	0		39
26	1	0	0	0	0	0	0	0	2	0	1	0	1	0	1		20
27																	
28	1	0	0	0	0	0	0	11	4	0	0	0	0	0	0		29
29	0	0	0	0	0	0	0	2	1	0	1	0	1	0	3		31
30																	
31	1	0	0	0	0	0	0	14	2	0	0	0	0	0	0		35
32	0	0	0	0	0	0	0	1	4	0	0	0	0	0	0		23
33																	
34	1	0	0	0	0	0	0	0	0	4	5	0	0	1	0		50
35	1	0	0	0	0	0	0	0	0	0	0	0	0	0	2		27
36																	
37	0	0	0	0	0	0	0	14	3	0	0	0	0	1	1		30
38	0	0	0	0	0	0	0	1	5	0	0	0	2	0	2		25
39																	
40	17	18	16	1	2	4	0	94	39	8	15	1	9	21	35		753
41																	
42	13	18	16	1	1	3	0	85	18	8	8	1	1	21	20		514
43	4	0	0	0	1	1	0	9	21	0	7	0	8	0	15		239
44																	
45	26		34		3		4		133		23		10		56		753
46	3%		5%		0%		1%		18%		3%		1%		7%		100%

APPENDIX 10: SOME UNUSUAL RESPONSES

The following response contains quite a wide variety of strategies, DM, MU, CO, F and RZ.

Item 11. 180 ÷ 30

M How many 30s? (pause for 50 seconds) 6.

I And how did you solve that?

M I started from 30 and then I said um 60 then 90 and then there was 120 , then 3 and 5 is 50 , then 50 and 30 is . . . um. 120 is 50 and then I and then 5 and 3 is 80.

I Right, I see. And how did you keep track of how many times you added..

M Oh, counted by my fingers again.

Note the reliance of the following students on the use of tens and hundreds.

Item 8. 320 ÷ 8

M 320? Um, oh I've got it . 40.

I Right, and how did you solve that one?

M 'Cause I know there's 50 in . . . I went to 8 only because that's the most 8s I know. I know there's a hundred there and I went to 50 and 400 and took away 80 which is 10 so it's 40.

I Right, sorry, you went . . .

M To 800. I know there's 100 eights in 800. I halved that so it's 50 eights in 400 and took away the 80 which is 10 eights so what I got was 40.

I So 50 eights in 400 , then you took away 10 eights.

Item 11. 180 ÷ 30

M (pause for 15 seconds) 5.

I How did you work out 5 thirties in 180?

M Well 30 into 100 goes 3 and you've got the 10 left over from the (inaudible) if you add to the um . Sorry, can I change the answer?

I Yes, for sure.

M 6.

I 6. You think it's six. Okay.

M Yeh, and you get the thing that makes . . . the 10 from the 100s to the 80s you add that on to make nine and there's 3 in that so 3 from the 100s and 3 from the 90s makes 3 and 3 together makes 6.

The following student makes use of tens and then applies a compensation procedure.

Item 3. 34 ÷ 2.

J You get , oh 6 , 6 , 17 or 16.

I And what goes through your head to solve that one?

J There's 10 in 20 and then you go to the next 20s , say 40, and then you take from that and then you've got your answer.

I Right, so you did 10 two's in 20 and then another 10 makes it 40..

J Yeh and then you take from that then 32 you take 8 and then you've got it so it 4, 16.

Note lack of understanding of what a remaider is on the part of the following student.

Item 5. 78 ÷6.

A (pause for 10seconds) 12.

I How do you get 12?

A (pause) I'm not exactly sure.

I That's alright.

A Um . (mutters) It's 11 remainder 4.

I Alright, and could you explain how you got . . .

A I remember 11 remainder 6.

I Alright, 11 remainder 6. So how do you do that ? 78 ÷ 6, how do you do that.

A 'Cause 12 sixes are 72 and 6 is 78 so 12 remainder 6.

I Alright, that's interesting. So you start with 12 sixes first.

A Yep.

I Is that 'cause you know your 12 times . . .

A Yeh, 12 sixes are 72, and add 6.

I So there's one more 6, so it would be a remainder 6.

A Yep.