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Source Localisation in Wireless Sensor Networks Based on Optimised Maximum Likelihood

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Abstract — **Maximum Likelihood (ML) is a popular and effective estimator for a wide range of diverse applications and currently affords the most accurate estimation for source localisation in wireless sensor networks (WSN). ML however has two major shortcomings namely, that it is a biased estimator and is also highly sensitive to parameter perturbations. An *Optimisation to ML* (OML) algorithm was introduced that minimises the sum-of-squares bias and exhibits superior performance to ML in statistical estimation, particularly with finite datasets. This paper proposes a new model for acoustic source localisation in WSN, based upon the OML estimation process. In addition to the performance analysis using real world field experimental data for the tracking of moving military vehicles, simulations have been performed upon the more complex source localisation and tracking problem, to verify the potential of the new OML-based model.**

Keywords-Maximum likelihood; estimation; source localisation; wireless sensor networks.

I. Introduction

The emergence of Wireless Sensor Networks (WSN) has provided considerable impetus to research in this area, because of the wide range of potential applications, from environmental monitoring and manipulation in physical world scenarios to pervasive computing [6], [7], [8], [1]. Typically, WSN comprise many disparate, small and usually inexpensive types of network nodes, such as control and sensing nodes, all of which are characterized by having limited sensing, communication and computational capability. Some control nodes possessing higher computational and communication capacity are also available in WSN for data aggregation and summarisation.

Estimating the source locations within a region covered by a WSN is a very challenging task. Source localisation can be performed based on readings taken from different sensors, such as acoustic, seismic or infra-red. Seismic signal propagation for instance, is very sensitive to the medium and also there is no standard model to estimate the propagation speed while in contrast, acoustic signal propagation has a well established theoretical basis for estimating source location and direction [16]. This provided the motivation in this particular paper to focus upon localisation using acoustic sensors.

There are many real world applications of source localisation employing acoustic sensors, including underwater acoustic localisation with hydrophone arrays in sonar [4], microphone arrays in room environments for speaker head location estimation and tracking [2], [14], [21] and vehicle location estimation in open-field sensor networks [15], [5].

Localisation methods typically depend on three types of physical variables being either measured or derived from actual sensor readings, namely; *i*) the Direction Of Arrival (DOA), *ii*) Time Delay Of Arrival (TDOA) and *iii*) Received Signal Strength (RSS). DOA [13] can be estimated by measuring the phase difference at different sensors and is applicable to coherent, narrowband sources [17]. A drawback of such measurements is that they require costly antenna arrays at each node. Conversely, TDOA [11] is suitable for broadband sources and is more sensitive to accurate time delay measurements [15], while there are well-established acoustic energy decay models [23] that can be applied to measure the RSS and thereby a locus of source position from various sensor readings. A straightforward approach to localisation using RSS measurements is the *Closest Point of Approach* (CPA) [23] which assumes the source location being that of the nearest sensor, i.e., that which measures the largest RSS reading. More sophisticated strategies are based upon *Maximum Likelihood* (ML) methods [23], [24], which exhibit superior estimation accuracy as well as flexibility in handling multiple sources for localisation compared with other energy-based source localisation methods [12]. ML however, has two major drawbacks in that it is a biased estimator and also highly sensitive to parameter perturbations [20].

An *Optimisation to Maximum Likelihood* (OML) algorithm [19] has been developed that minimises the sum-of-squares bias, and it has been shown to consistently provide superior estimation performance compared with ML for some reference statistical datasets. It has also been proven in an asymptotic sense, that OML and ML are equivalent. To exploit its gain over ML in the finite sampling domain, this paper formulates an acoustic energy-based source localisation model as an OML estimation problem. Source localisation has many influencing factors in the decision making process and optimisation therefore affords many performance benefits by minimizing the bias.

The remainder of the paper is organized as follows: Section II presents a review of classical ML estimation theory and discusses an acoustic energy attenuation model for source location. Section III firstly presents a brief overview of the OML estimator before theoretically formulating a source localisation model, with the experimental performance of the proposed technique being numerically evaluated in Section IV. Section V provides some conclusions.

II. Maximum Likelihood and Its Application to Source Localisation

This section firstly presents an overview of the underlying principles of ML estimation before examining an acoustic energy decay model that uses ML for both single and multiple source localisation.

A. Classical Maximum Likelihood Estimator

ML estimation has been extensively employed because of its flexibility and simplicity of derivation. Given a set of observations $Z=(z_1, z_2, z_3,\ldots,z_n)$, represented by n random variables and parameter values to be estimated θ, the likelihood $p(Z|\theta)$ of these observations is defined as:-

$$p(Z \mid \theta)=\prod_{i=1}^{n} p(z_i \mid \theta) \tag{1}$$

Parameter values θ is estimated as the one which will maximise the likelihood function $p(Z|\theta)$. The log-likelihood function is defined as:-

$$l(\theta)=\log p(Z \mid \theta)=\sum_{i=1}^{n}\log p(z_i \mid \theta) \tag{2}$$

The fact the log-likelihood function can be used instead of the likelihood function in ML estimation highlights the unique invariance property of ML though in general, ML estimators by themselves are not a sufficient statistic to fully describe a distribution [22].

B. Acoustic Energy Based Source Localisation Model

Let N and M be the number of sensors and acoustic sources in a WSN field respectively. The emission of acoustic signal energy can be modelled as omni-directional signal starting from a point sound source and attenuating at a rate inversely proportional to the square of the distance from the source [16]. It is assumed that the acoustic energy received by the i^{th} sensor will be the linear summation of the attenuated energies without any interference between them. A sensor in a WSN is modelled by $[\mathbf{p}_i, \lambda_i]$ where $\mathbf{p}_i$ denotes a p dimensional position vector and λ_i is the gain factor for the i^{th} stationary sensor, with the location of each sensor node being known *a priori*. The acoustic energy received at the i^{th} sensor during time interval t is expressed as [23]:

$$z_i(t)=\lambda_i\sum_{m=1}^{M}\frac{E_m(t)}{\left\|\mathbf{p}_i-\rho_m(t)\right\|^2}+\varepsilon_i(t) \tag{3}$$

where $\varepsilon_i(t)$ is a perturbation term that summarizes the net effect of background additive noise and parameter modelling error. $E_m(t)$ and $\rho_m(t)$ are the energy emitted by the m^{th} source (measured 1 meter from the source) and its location during the t^{th} time interval respectively. The distribution of $\varepsilon_i(t)$ has been shown [24] to be an independent and identically distributed (*iid)* Gaussian random variable, whenever the time period T for averaging the energy is sufficiently large, i.e., $T > 40/f_s$ where f_s is the sampling frequency. The mean $\mu_i(t)$ and variance $\sigma_i^2(t)$ of each $\varepsilon_i(t)$ are empirically estimated using a constant false alarm (CFAR) detector [24] and the proposed energy attenuation model has previously been validated [9].

The estimation of a moving source, such as a vehicle is made at each time instance by a set of energy readings from different individual sensors. The acoustic energy model for a specific sampling interval t, is represented in a concise matrix notation, with the time index t omitted, adopting the same convention proposed by [23].

$$\begin{aligned}
&\mathbf{Z}=[(z_1-\mu_1)/\sigma_1 \quad \ldots \quad (z_N-\mu_N)/\sigma_N]^T\\
&\boldsymbol{\lambda}=diag[\lambda_1/\sigma_1 \quad \lambda_2/\sigma_2 \quad \ldots \quad \lambda_N/\sigma_N]\\
&\mathbf{D}=\begin{bmatrix}1/d_{11}^2 & 1/d_{12}^2 & \ldots & 1/d_{1M}^2\\ 1/d_{21}^2 & 1/d_{22}^2 & \ldots & 1/d_{2M}^2\\ \vdots & \vdots & \ddots & \vdots\\ 1/d_{N1}^2 & 1/d_{N2}^2 & \ldots & 1/d_{NM}^2\end{bmatrix}\\
&d_{im}^2=\left\|\mathbf{p}_i-\rho_m\right\|^2\\
&\mathbf{E}=[E_1 \quad E_2 \quad \ldots \quad E_M]^T\\
&\mathbf{K}=\boldsymbol{\lambda}\mathbf{D}\\
&\xi=[\xi_1 \quad \xi_2 \quad \ldots \quad \xi_N]^T
\end{aligned} \tag{4}$$

where as alluded above, $\xi_i=(\varepsilon_i-\mu_i)/\sigma_i\sim\mathcal{N}(0,1)$ are *iid* Gaussian random variables. Using this notation, (3) can be expressed as

$$\mathbf{Z}=\boldsymbol{\lambda}\,\mathbf{D}\mathbf{E}+\boldsymbol{\xi}=\mathbf{K}\mathbf{E}+\boldsymbol{\xi} \tag{5}$$

With the probability density function of $\mathbf{Z}$ given by:-

$$f(\mathbf{Z}\mid\theta)=(2\pi)^{-n/2}e^{-\frac{1}{2}(\mathbf{Z}-\mathbf{KE})^T(\mathbf{Z}-\mathbf{KE})} \tag{6}$$

where $\theta=[\rho_1^T\ \rho_2^T\ldots\rho_M^T\ E_1\,E_2\ldots\,E_M]^T$ is a vector of unknown parameters, with ρ_M and E_M being the M^{th} source location and M^{th} source energy respectively. From [24], the parameter of the ML estimator θ_{ML} is then calculated from the following equations:-

$$\theta_{ML}=\arg\min_{\theta}\left\{(\mathbf{Z}-\mathbf{KK}^{\dagger}\mathbf{Z})^T(\mathbf{Z}-\mathbf{KK}^{\dagger}\mathbf{Z})\right\} \tag{7}$$

where $\mathbf{K}^{\dagger}$ is the *pseudo-inverse* of matrix $\mathbf{K}$ and $\mathbf{E}=\mathbf{K}^{\dagger}\mathbf{Z}$ following the maximisation of (6) with respect to $\mathbf{E}$.

In the next section, the *optimisation of maximum likelihood* (OML) estimator model is formulated for multiple source location estimation utilising the aforementioned acoustic energy decay model in (3).

III. OPTIMISATION OF MAXIMUM LIKELIHOOD AND ITS FORMULATION TO SOURCE LOCALISATION

The rationale for the *optimisation of maximum likelihood* (OML) estimator is to improve the accuracy of acoustic energy-based source localisation by explicitly minimising the sum-of-squares estimation bias of the ML estimator identified in Section I. The optimisation for ML is discussed in [19] and so is only briefly outlined in this section to aid understanding of the underlying theory. A source localisation strategy based on OML for ad hoc WSN is then subsequently presented.

A. Optimisation of Maximum Likelihood Estimation

In an asymptotic sense, the ML estimation is equivalent to maximum entropy method and in [19] it has been shown that:

$$\theta_{ML} = \arg\max_{\theta}\left\{-E_f\left[\log p(Z\mid\theta)\right]\right\} \text{ for } n \to \infty \tag{8}$$

where E_f [log $p(Z|\theta)$] is the population mean of log likelihood i.e., the expected value of log $p(Z|\theta)$, and subscript f denotes the expectation is based on true probability distribution of the data.

From (8), it is evident that ML is asymptotically efficient, since as the number of observations becomes large it tends towards the expected log-likelihood value with respect to the true distribution [22]. An alternative interpretation is that ML will always generate an optimal estimation provided the true distribution f is known and E_f [log $p(Z|\theta)$] is accordingly estimated. In such circumstances, (8) is the optimal ML estimation θ_{OML}.

The essential tenet underpinning OML is that while it is not feasible to find the true distribution from finite sampling, it can however be better approximated with respect to a particular parameter value θ by the uniform Gaussian Mixture Distribution of the likelihood functions through a sum-of-squares bias minimisation strategy (Lemma 2 of [19]).

Let f_{OML} be the optimal approximation of the true distribution. Following (8)

$$\theta_{OML} = \mathbf{arg}\max_{\theta}\left\{-E_{f_{OML}}\left[\mathbf{log}\, p(Z\mid\theta)\right]\right\}$$

simplifying,

$$\theta_{OML} = \arg\min_{\theta}\left\{\left(\frac{1}{n}\sum_{i=1}^{n} p\ (z_i\mid\theta)\right)\left(\log p(Z\mid\theta)\right)\right\} \tag{9}$$

which corresponds to the OML estimation of parameter θ. Note, that since there is only one additional term in (9), OML will have exactly the same order of computational complexity as ML.

B. OML Source Location Estimation

The optimal approximation of the true distribution using the uniform Gaussian mixture distribution of the likelihood and the acoustic energy decay model from (3), (4) and (5) can be written as:

$$\begin{aligned} f_{OML}(\mathbf{Z}\mid\theta) &= \frac{1}{n}\sum_{i=1}^{n} p\ (z_i\mid\theta) \\ &= \frac{1}{n\sqrt{2\pi}}\, tr\left(e^{-\frac{1}{2}(\mathbf{Z}-\mathbf{KE})(\mathbf{Z}-\mathbf{KE})^T}\right) \end{aligned} \tag{10}$$

From (6), (9) and (10), θ_{OML} can thus be expressed as:-

$$\begin{aligned} \theta_{OML} &= \mathbf{arg}\ \mathbf{min}_{\theta}\left\{\left(\frac{1}{n\sqrt{2\pi}} tr\left(e^{-\frac{1}{2}(\mathbf{Z}-\mathbf{KE})(\mathbf{Z}-\mathbf{KE})^T}\right)\right)\times \left(-\frac{n}{2}\mathbf{log}(2\pi) - \frac{1}{2}(\mathbf{Z}-\mathbf{KE})^T(\mathbf{Z}-\mathbf{KE})\right)\right\} \\ &= \arg\max_{\theta}\left\{tr\left(e^{-\frac{1}{2}(\mathbf{Z}-\mathbf{KE})(\mathbf{Z}-\mathbf{KE})^T}\right)(\mathbf{Z}-\mathbf{KE})^T(\mathbf{Z}-\mathbf{KE})\right\} \end{aligned} \tag{11}$$

The minimisation process of (9) with respect to **E** and setting $\frac{\partial\left\{tr\left(e^{-\frac{1}{2}(\mathbf{Z}-\mathbf{KE})(\mathbf{Z}-\mathbf{KE})^T}\right)(\mathbf{Z}-\mathbf{KE})^T(\mathbf{Z}-\mathbf{KE})\right\}}{\partial E} = 0$ gives:

$$\mathbf{E} = \mathbf{K}^{\dagger}\mathbf{Z} \tag{12}$$

where $\mathbf{K}^{\dagger}$ is the *pseudo-inverse* of matrix **K**. Substituting the value of **E** into **(9)** gives :-

$$\theta_{OML} = \mathbf{arg}\ \mathbf{max}_{\theta}\left\{tr\left(e^{-\frac{1}{2}(\mathbf{Z}-\mathbf{KK}^{\dagger}\mathbf{Z})(\mathbf{Z}-\mathbf{KK}^{\dagger}\mathbf{Z})^T}\right)\times (\mathbf{Z}-\mathbf{KK}^{\dagger}\mathbf{Z})^T(\mathbf{Z}-\mathbf{KK}^{\dagger}\mathbf{Z})\right\} \tag{13}$$

Since all the terms of (13) are known for a specific parameter θ, (13) can be used to find the parameter set θ that is most suitable under the OML criterion.

IV. EXPERIMENTAL RESULTS

The proposed OML source localisation model and other popular source localisation techniques including ML and CPA were implemented in Matlab 6.5.1. Firstly, a simulation for source localisation was performed in part A, and finally its application to a set of practical data obtained by a DARPA funded field experiment (ITO SensIT [18]) was conducted to evaluate the effectiveness of OML for the practical detection and tracking of moving vehicles within a WSN region.

A. Simulation of Source Localisation Methods

The formulation (3) is utilised for generation of acoustic energy readings for a 2-D (p= 2) sensor field of size 100 by 100 m^2 . The source and sensor locations were uniformly distributed over the WSN field in each sample, with the source energy set at E = 5000 intensity and background noise level modelled as $N(\sigma_i^2, \sigma_i^2/\Delta)$ with Δ = 100.

The mean and covariance matrices of single source location estimation error for all dimensions (*horizontal and vertical axes*) are listed in Table 1.

Table 1: Mean and Covariance Matrices of Single Source Location Estimation Error

	5 Sensors	10 Sensors	20 Sensors
CPA	$\begin{bmatrix}-2.44 & 1.21\end{bmatrix}$ $\begin{bmatrix}631.85 & 11.77 \\ 11.77 & 588.34\end{bmatrix}$	$\begin{bmatrix}-2.23 & -2.76\end{bmatrix}$ $\begin{bmatrix}663.53 & -23.68 \\ -23.68 & 625.00\end{bmatrix}$	$\begin{bmatrix}1.41 & 0.46\end{bmatrix}$ $\begin{bmatrix}842.62 & -40.32 \\ -40.32 & 646.71\end{bmatrix}$
ML	$\begin{bmatrix}2.31 & -0.92\end{bmatrix}$ $\begin{bmatrix}153.83 & 5.00 \\ 5.00 & 180.29\end{bmatrix}$	$\begin{bmatrix}0.94 & 1.64\end{bmatrix}$ $\begin{bmatrix}45.65 & 11.70 \\ 11.70 & 65.16\end{bmatrix}$	$\begin{bmatrix}0.66 & 1.65\end{bmatrix}$ $\begin{bmatrix}31.74 & -3.61 \\ -3.61 & 29.83\end{bmatrix}$
OML	$\begin{bmatrix}1.01 & 0.22\end{bmatrix}$ $\begin{bmatrix}192.69 & 9.21 \\ 9.21 & 206.10\end{bmatrix}$	$\begin{bmatrix}0.76 & 0.47\end{bmatrix}$ $\begin{bmatrix}34.66 & 9.15 \\ 9.15 & 45.15\end{bmatrix}$	$\begin{bmatrix}-0.94 & -0.56\end{bmatrix}$ $\begin{bmatrix}45.23 & 34.53 \\ 34.53 & 35.77\end{bmatrix}$

From simulation results, we observe that the mean error value in case of OML estimation is less than both ML and CPA. The covariance matrices show that errors in both dimensions are uncorrelated too. OML method outperforms ML consistently at low sensor density as well as higher densities.

Another representation of the results showing the distribution of the magnitude of location estimation error is presented in Figure 1.

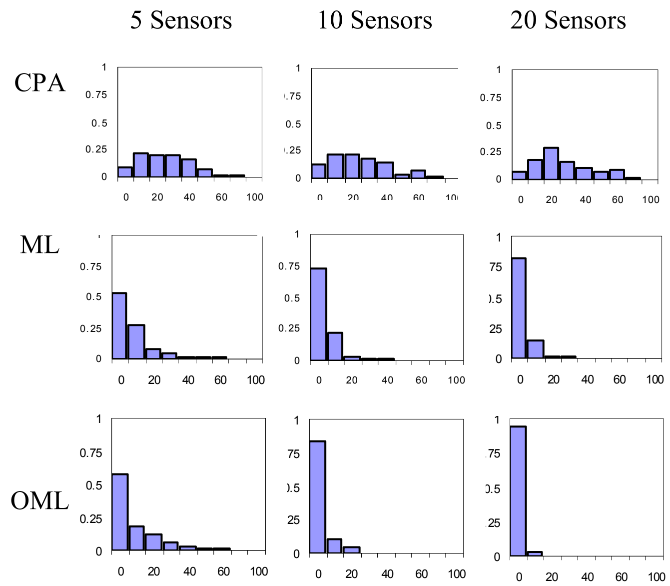

Figure 1. Distribution of errors of the three localisation algorithms for single source localisation.

The histograms of the errors are plotted with a bin size of 10 metres. The histograms can be considered as a rough distribution of the errors and clearly OML distributions are better than CPA and ML as they generate most of the estimations within the low error ranges.

B. *Performance Analysis through practical data*

In the DARPA funded ITO SensIT project, custom-made prototype sensor nodes were deployed along the roadside. Military amphibious assault vehicles (AAV) were driven past the sensors and the corresponding data sampled by different sensor types (acoustic, seismic and polarized infrared) at each node. The ground truth was obtained by interpolating an onboard Global Positioning System (GPS) recording, which sampled a position at every 15 sec. The acoustic signal was sampled at 4.96 KHz at 16-bit resolution. The sampled energy readings were collated from all sensor nodes within the WSN region within a 750ms time window. The data segments used were taken from the acoustic signatures of a single AAV travelling from east to west along the road during a time period of approximately 2 minutes.

Figure 2 shows the AAV ground truth and the localisation results based on the OML and ML estimations for run 6 of the practical data. For showing the tracking of the source within the field, the ground truth and locations estimated by OML and ML for each sampling interval are shown in different colours.

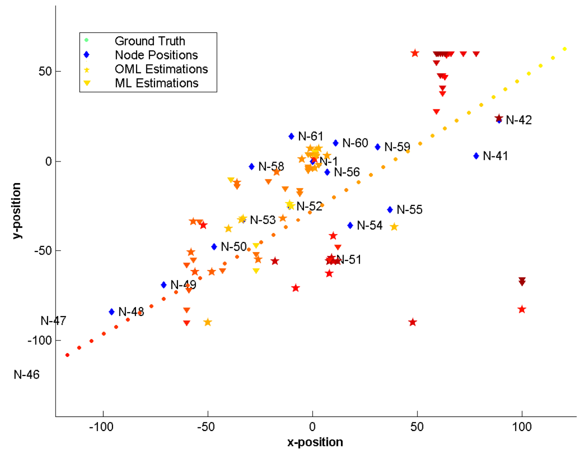

Figure 2: Estimation error histogram for AAV experiment data

If the estimated locations are compared with their corresponding ground truth, it is readily apparent that OML produced more accurate estimations in many more cases than ML. The middle portion of Figure 2 shows more accurate estimations for both OML and ML than the top region. Note the sampling process for this experiment occurred in a very noisy environment, with strong winds present which often blew directly into the microphone causing random energy transients. Also, many of the microphones were not properly calibrated [18], which is the reason for the presence of inaccurate estimates at the top of Figure 2.

The localisation errors in terms of the number of estimation points within a range (in metres) of estimation errors are summarized in the histogram in Figure 3. This reveals that OML generated approximately 10% estimation points within the small error range between 0—30m, whereas ML was able to produce only 1.6% within this low error range due to presence of noisy conditions. In addition, as the number of estimation points was fixed, more accurate estimations in the small error range reduces the probability for larger range errors. It is palpable that OML consistently provided more accurate estimations in comparison with ML, so corroborating the theory developed for the new OML estimator, in its ability to produce more accurate estimates than ML by reducing the error and thereby the bias.

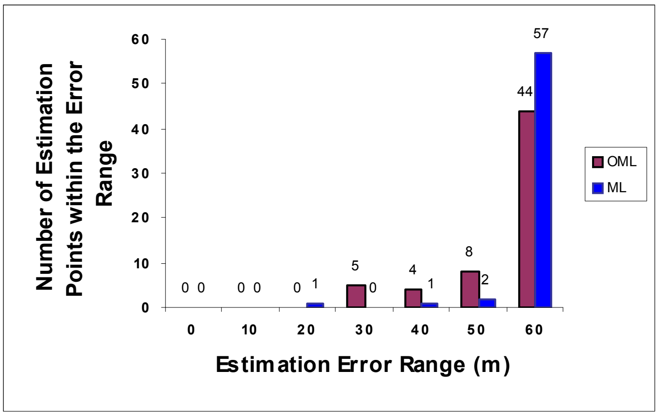

Figure 3: Estimation error histogram for AAV experiment data

V. CONCLUSIONS

In this paper the Optimisation of Maximum Likelihood (OML) estimation for a passive acoustic source localisation problem in WSN has been mathematically formulated and its performance analysed and compared with the most accurate existing source localisation estimation e.g. Maximum Likelihood based approach and also for Closest Point of Approach (CPA) method. The results confirm the fundamental hypothesis that OML provides a consistently lower estimation error compared with ML and CPA, for the same order of the computational complexity of ML. Results for both simulated and field experimental data also confirmed the effectiveness of the proposed technique.

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