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An empirical comparison using both the term structure of interest rates and alternative models in pricing options on 90-day BAB futures

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USE OF THESIS

The Use of Thesis statement is not included in this version of the thesis.

AN EMPIRICAL COMPARISION USING BOTH THE TERM STRUCTURE OF INTEREST RATES AND ALTERNATIVE MODELS IN PRICING OPTIONS ON 90-DAY BAB FUTURES

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ABSTRACT

The use of the term structure of interest rates to price options is relatively new in the literature. It describes the relationship between interest rates and the maturities of bonds. The first model that described the interest rate process was the Vasicek (1977) model. There have been many studies on the formulation of theoretical pricing models. Yet limited empirical research has been done in the area of actually testing the models. In this thesis we report the results of a set of tests of the models indicated below.

This paper involves analysis of the pricing errors of the Black model (1976), Asay model (1982), Extended-Vasicek model (1990) and Heath-Jarrow-Morton model (HJM) (1992) as applied to call options on 90-day Bank Accepted Bill (BAB) futures. Monthly yield curves are generated from cash, futures, swap and interest rate cap data.

A number of different methods of analysis are used. These include the use of inferential statistics, non-parametric sign tests and Ordinary Least Square Regressions. The Wilcoxon non-parametric sign test assists the interpretation of whether the pricing errors are from the same distribution. Ordinary Least Square Regressions are used to assess the significance of factors affecting pricing errors. In addition, data are plotted against different variables in order to show any systematic patterns in how pricing errors are affected by the changes in the chosen variables.

Monthly options data on BAB futures in the year 1996 suggest that the term structure models have significantly lower pricing errors than the Black and the Asay model. The

Heath-Jarrow-Morton model (1992) is overall the better model to use. For the term structure models, pricing errors show a decreasing trend as moneyness increases. The Extended-Vasicek model and the HJM model have significantly lower errors for deep in-the-money and out-of-the-money options. Higher mean absolute errors are observed for at-the-money options for both term structure models. The HJM model overprices at-the-money options but underprices in and out-of-the-money options while the Extended-Vasicek model underprices deep-in-the-money options but overprices options of other categories.

The mean and absolute errors for both the Black model and the Asay model rise as time to maturity and volatility increases. The two models overprice in, at and out-of-the-money options and the mean pricing error is lowest for in-the-money options.

The results suggest that the factor time to maturity is significant at the 0.05 level to the mean pricing error for all four models. Moneyness is the only insignificant factor when the Asay model is used. It is also negatively correlated to mean pricing error for the Black model, the Asay model, the Extended-Vasicek model and the HJM model. The R-square for the Extended-Vasicek model was found to be the lowest. Overall, the HJM model gives the lowest pricing error when pricing options on 90-Day Bank Accepted Bill Futures.

"I certify that this thesis does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any institution of higher education; and that to the best of my knowledge and belief it does not contain any material previously published or written by another person except where due references is made in the text."

Signature:

Date: 30/3/99

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Chapter 1: INTRODUCTION

1.1 BACKGROUND TO THE STUDY

Empirical analysis of option prices has concentrated on two distinct questions. The first is concerned with discriminating between alternative pricing models. The second area of concern deals with the accuracy with which market participants estimate the parameters needed to implement the option pricing formulas. The present study is within the first category, and is aimed specifically at testing the efficiency of alternative models in pricing options on bond futures. The purpose of the study is to apply two term structure models: the Extended-Vasicek model and the Heath-Jarrow-Morton model, to the pricing of call options on 90-day bank accepted bill futures options. Theoretical prices will be used to compare with actual settlement prices to determine the accuracy of the models. Any systematic discrepancies are analyzed. The prices obtained from the two term structure models are also compared to the Black (1976) model and the Asay (1982) Model in order to determine the effect on pricing.

The most widely used model in option pricing is the Black-Scholes Option Pricing model (1973). Black and Scholes were the first to derive an analytic solution for the price of a European option on a non-dividend paying stock. The value of the option was determined by arbitrage considerations rather than by an investor's risk preferences about the future performance of the stock. The value of an option depends on (1) the stock price, (2) the exercise price of the option, (3) the volatility of the stock's return, (4) the time to maturity, and (5) the continuously compounded short-term interest rate for borrowing and lending. They assumed that the stock's returns followed a normal distribution. The model assumes that the distribution of security prices is skewed, so that higher prices are more likely to occur than lower prices.

The model assumes stock price behavior corresponds to a lognormal model and the short-term risk-free rate of interest is constant. Black (1976) extended the Black-Scholes model to cover European options on commodities and futures contracts:

$$c = e^{-rT}[FN(d_1) - XN(d_2)]$$
$$p = e^{-rT} [XN(-d_2) - FN(-d_1)]$$

where

$$d_1 = [\ \ln(F/X) + \sigma^2 T/2]/ (\sigma\sqrt{T})$$
$$d_2 = [\ \ln(F/X) - \sigma^2 T/2]/ (\sigma\sqrt{T}) = d_1 - \sigma\sqrt{T}$$

N(.) is the cumulative probability function for a standardized normal variable, c and p are European call and put prices, F is the futures price, X is the strike price, r is the risk-free interest rate, T is the time to expiration, and σ is the volatility of the bond price.

1.1.1 Term Structure of Interest Rates

The term structure theory is relatively new. Economists have held an interest in the subject, as an understanding of the term structure has always been important to an understanding of the impact of monetary policies. The term structure of interest rates has been of primary interest to economists. The term structure is the relationship between interest rates and the maturities of bonds. The focus in this area is useful for risk management i.e. the pricing of fixed income securities and interest rate options. This approach has already been employed on Wall Street to price and hedge numerous types of fixed-income securities and interest rate options. The main purpose of this study is to investigate the effect of term structure models in pricing options. It provides a more valid guide to the pricing of interest rate-contingent claims in an Australian context. To price interest rate derivatives, the evolution of the entire yield curve must be modelled.

The uses of the term structure are as follows:

(1) Analyze the returns for asset commitments of different terms. Portfolios can be varied according to the quality, coupon level, time to maturity and the type of issuer. The term structure allows investors to make judgements about the short-term rewards of different maturity strategies as interest rates change.

(2) Assess the expectation of future interest rates. Analysis of the term structure allows the interpretation of the expectation of future interest rates.

(3) Price bonds and other fixed-payment contracts. The yield curves give an expectation of the alternative yields for coupon-bearing bonds. Some bonds and contracts are priced in such a way that the yield would be equal to the yield at the same maturity on the yield curve with the adjustment for credit quality or other important factors. The pricing errors can be minimized for zero coupon bonds and other financial instruments with non-traditional cash-flow patterns. The separation of pricing of cash flows with different term structures can increase the accuracy of pricing.

(4) Pricing contingent claims on fixed income securities. The use of the term structure of interest rates to price options is relatively new in the literature. It describes the relationship between interest rates and the maturities of bonds. Yield curve models describe the probabilistic behaviour of the yield curve over time. They deal with movements in a whole yield curve - not changes to a single variable. As time passes, the individual interest rates in the term structure change causing the shape of the curve to change.

The Black model does not include any of the underlying term structure information when applied to options on interest rate instruments. Asay (1982) modifies the Black Model and allows for the application of the marking-to-market position for options on futures as is the case in the Australian market.

It has been suggested that there are four approaches in the literature to the valuation of interest rate options. The first follows Black and Scholes and uses the price of the underlying bond as an exogenous variable. The second models the endogenous term structure of interest rates in a no-arbitrage framework. These include the Vasicek (1977) model and Brennan and Schwartz (1979, 1982). The term structure model developed for pricing in Cox, Ingersoll and Ross (1985) represents an equilibrium specification that is completely consistent with stochastic production and with changing investment opportunities. The third approach, pioneered by Ho and Lee (1986) and Heath, Jarrow, and Morton (1990, 1992) begins with the evolution of the entire zero coupon price curve. The fourth approach, as suggested by Black, Derman, and Toy (1990) and Hull and White (1990, 1993), specifies the spot rate process and determines the model in such a way that the model is consistent with the current term structure. There have been many studies on the formulation of theoretical pricing models but limited empirical research has been done in this area particularly as applied to the no-arbitrage models. Buhler, Uhrig, Walter and Weber (1995) provide one of the few empirical comparisons of some of these models.

1.2 THESIS OUTLINE

This dissertation is divided into six sections. The first chapter is the introduction; it justifies the research and it gives a brief summary of the purpose and scope of the study.

The contribution of the term structure of interest rates in the financial services industries is also discussed. Chapter two provides a literature review in terms of general and specific literature in the area of term structure of interest rates, assumptions and an overview of the different pricing models. Some definitions of specific terms are also given. Chapter three presents the research methodology. Results are presented in chapter four which examines how the pricing errors are affected by the factors like time to maturity, moneyness and volatility. Comparisons are made between the Black model, the Asay model, the Extended Vasicek model and the Heath-Jarrow-Morton model. Chapter five summarizes the findings. The significance of the study can be assessed. This is followed by suggestions for future research.

Chapter 2: REVIEW OF THE LITERATURE

An outline of the Black model was given in chapter one. This chapter elaborates on the different tests and hypothesis being put forward. How the term structure of interest rates was introduced in application to option pricing will be discussed. The chapter forms a basis of understanding for the alternative models used for pricing in this study.

2.1 Black Model

After the introduction of the Black model, it was challenged by different researchers. They looked into the possibility of alternative distributional hypotheses.

Galai (1983) summarizes the empirical approaches to validating option-pricing models. There are a few methodologies within which the models can be tested. The first approach is by means of simulations and quasi-simulations of deviations from the basic assumptions of the models. The sensitivity of the model prices to empirical deviations from the assumptions is tested. Bhattacharya (1980) tests the actual distribution of stock prices rather than the assumed lognormal distribution.

The other approaches in testing the models involve comparisons of the model prices to actual prices. The estimated parameters of the model and the actual observations of stock prices are placed in the pricing model to generate expected option prices. The model prices are compared to the actual, realized option prices. The tests have the ability to show whether model prices are unbiased estimators of actual prices and whether there are consistent deviations that can be exploited for better prediction or for making above-normal profits. The third approach in testing the models involves imputing the standard deviations from actual option prices by using a pricing model. It assumes that all parameters are known and that markets are efficient and synchronous. The standard deviation can be imputed as the only unknown in the equation when equating the actual price to the model price. The last approach is based on creating neutral hedge positions and testing the behavior of the returns from the investment. This should create a riskless position for options and their underlying stock when the model is correct. In this case, the problem of risk-adjustment for investment in options is eliminated and returns on the hedge position should equal the risk-free rate.

In his summary of the tests put forward by different studies, Galai (1983) concludes that the Black-Scholes model performs relatively well, especially for at-the-money options. No alternative model consistently offers better prediction of market prices than the Black-Scholes model. Also, for short time periods, the Black-Scholes model gives good predictions of market prices for options that are undervalued and overvalued. The Black-Scholes model assumes constant variance. There has been a great deal of work examining alternative diffusion processes. There is some evidence in favour of the constant elasticity of variance model, but this is inconclusive. Nonstationarity of the risk estimator of the underlying stock is a major problem that affects the performance of the Black-Scholes model. The evidence does not seem to support the null hypothesis of market synchronization. The tests of the boundary conditions suggest that trading synchronization or data synchronization are important considerations.

The other approach in testing the consistency between options and time series is by estimating model parameters implicit in option prices and testing the distributional predictions for the underlying time series. This is usually done by two procedures: (1) the parameters inferred from option prices are assumed known with certainty. (2) their informational content is tested using time series data. In order to test the time-series distributions, option prices must satisfy certain no-arbitrage constraints. Firstly, option prices relative to the synchronous underlying asset price cannot be below intrinsic value and European call and put prices of common strike price must satisfy put-call parity. Also, American and European option prices, with respect to strike price, must be equivalent to the risk-neutral distribution function being non-decreasing. The risk-neutral probability must be non-negative. Evnine and Rudd (1985) and Bhattacharya (1983) find that option prices tend to violate lower bound constraints. Bhattacharya (1983) examines CBOE American options on 58 stocks over the period 1976-1977 to find that 1.3% of the options tested violate the immediate-exercise lower bound and 2.38% violate the European intrinsic value lower bound.

Ogden and Tucker (1987) examined pound, Deutschemark and Swiss franc call and put options and found only 0.8% violate intrinsic-value bounds. Consistent with Ogden and Tucker (1987), Bates found around 1% of the Deutschemark call and put transaction prices over 1984-1991 violate intrinsic value bounds computed from futures prices. Violations which are generally less than estimated transaction costs suggests that the violations may originate in imperfect synchronization between the options market and the underlying asset market or in bid-ask spreads.

The literature on ARCH and GARCH models addresses the issue of estimating conditional variance when volatility is time-varying. Bates (1996) provides a brief survey of work in this area. Melino and Turnbull (1990) find that the stochastic volatility model does reduce pricing errors. Further confirmatory evidence is provided by Cao (1992), and Myers and Hanson (1993).

Besides the Black-Scholes Option Pricing model, other models that were proposed in pricing interest rate contingent claims include the jump diffusion model and the constant elasticity of variance models. Merton (1976) introduced a jump-diffusion model under the assumption of diversifiable jump risk and independent lognormally distributed jumps. He suggested that distributions with fatter tails than the lognormal model might explain the tendency for deep-in-the-money, deep-out-of-the money, and short-maturity options to sell for more than their Black-Scholes value, and the tendency of near-the-money and longer-maturity options to sell for less. Cox and Ross (1976b) proposed pricing models for European options under absolute diffusion, pure jump, and square root constant elasticity of variance models of the return on the underlying asset. Option pricing models under stochastic volatility were put forward by Hull and White (1987). The main issue of concern is whether option prices are consistent with the time series properties of the underlying asset price. Hypotheses tested include cross-sectional tests of whether high-volatility stocks tend to have high priced options. Bates (1996) summarizes the various theoretical implications behind different models tested. Other tests examine whether volatility inferred from option prices using the Black-Scholes model is an unbiased and efficient predictor of future volatility of the underlying asset price. The other important problem relates to non-constant variance which is the focus of the previously mentioned ARCH/GARCH time series estimation procedures. This questions whether the term structure of volatilities inferred from options of different maturities is consistent with predictable changes in volatilities.

2.2 Asay Model

The Asay model is similar to the Black model except that it is used to price options that are marked-to-market. Asay (1982) and Lieu (1990) derive a pricing model for European call and put options which takes into account this margining in circumstances where the short-term interest rate is assumed to be known with certainty.

The call premium is given by:

$c = FN(d_1) - XN(d_2)$

where

$d_1 = [\ln(F/X) + \sigma^2 T/2]/(\sigma\sqrt{T})$
$d_2 = [\ln(F/X) - \sigma^2 T/2]/(\sigma\sqrt{T}) = d_1 - \sigma\sqrt{T}$

c is the European call price

N(.) is the cumulative probability function for a standardized normal variable,

F is the futures price,

X is the strike price,

T is the time to expiration, and

σ is the volatility of the bond price.

The put premium (p) is given by:

$p = XN(-d_2) - FN(-d_1)$

The model assumes that variance and interest rates are non-stochastic. If interest rates are stochastic, the pricing equations will be a function of the covariance between the forward futures price and short-term interest rates over the life of the option. The pricing error when ignoring the stochastic nature of interest rates is about 5 to 7 percent (Ramaswamy and Sundareson (1985)).

Brace and Hodgson (1991) use the Asay model to compare different estimates of historical volatility. Actual standard deviation was regressed on both historical and implied standard deviations. It was concluded that no one measure of historical volatility was consistent with observed market price. The price sensitivity in the Black and Asay model pricing equations are highly affected by volatility estimates. Empirical analysis has tended to focus on issues related to volatility (Brown and Shevlin (1983), Hull and White (1987)).

Options on futures in the Sydney Futures Exchanges have futures style margining. The contracts are marked to market at the end of each day. Brown and Taylor (1997) examined the Asay model on transaction prices from the SPI futures option market for the period from 1 June 1993 through to 30 June 1994. They found that the errors are related to the degree of moneyness and the maturity of the option. The model generates significant pricing errors which are consistent with an observed 'smile' in the implied volatilities.

2.3 SPECIFIC TERMS IN TERM STRUCTURE

Spot Rates

A spot interest rate is the rate applying to money borrowed now, to be repaid at some future date. Money can be borrowed for various lengths of time and therefore there will be a range of spot rates at any moment in time, each rate relating to the period of the borrowing. The relationship between these interest rates and the term of the borrowing is known as the term structure of interest rates. If P(0,1) is the price of a one-year zero coupon bond and P(0,2) is the price of a two-year zero-coupon bond. Using the term structure of interest rates, the price of a one-year bond with one dollar face value will be

P(0,1) = 1/(1 + r(0,1)). The two-year bond price will be P(0,2) = 1/(1 + r(0,2)) etc. Therefore, spot rates r(0,1), r(0,2), ..., r(0,T) make up the term structure going out T periods. They represent transactions as bonds undertaken on the spot.

Forward Rate

The forward rate is a rate in which one can contract at time 0 to borrow or lend at a future date. If F(0,1,2) is the rate that can be locked in today for a bond that would be issued in one year and matures in 2 years. The bond would have a one-year maturity and its price can be specified as F(0,1,2). By the end of two years, for every dollar invested, the total amount of [1/P(0,1)][1/F(0,1,2)] can be gained. This should be equal to the return per dollar from buying a two-year bond today and holding it for two years i.e. 1/P(0,2). If F(0,1,2) ≠ P(0,2)/P(0,1), an arbitrage profit can be earned. Therefore, any forward price F(0,i,j) = P(0,j)/P(0,i).

Since F(0,1,2) = P(0,2)/P(0,1) and that F(0,2,3) = P(0,3)/P(0,2); combining the two relationships, F(0,2,3) = P(0,3)/[P(0,1) F(0,1,2)] or P(0,1) F(0,1,2) F(0,2,3) = F(0,3). Thus, the price of a three-period bond today is the product of the price of a one-period bond and the forward price of a one-period bond starting at time 1 and another one-period bond starting at time 2.

Example : The yield on the two year bond (7.53% per annum) can be replicated by buying a one year bond now (yielding 7.24%) and a one-year-to-maturity bond in one year's time.

$(1 + R_2)^2 = (1 + R_1)(1 + {}_1R_1)$

$(1.0753)^2 = (1.0724)(1 + {}_1R_1)$

$(1 + {}_1R_1) \quad = (1.0753)^2 / (1.0724)$

$= 1.0782\ (7.82\%)$

${}_1R_1$ is known as the implied forward rate - the interest rate implied by the current term structure for one-year borrowing in one year's time. Similar calculations can indicate the implied forward rate for a bond of any term at any future point in time covered by the term structure.

A one-year rate in two years' time (${}_2R_1$) is given by:

$(1 + R_3)^3 \quad = (1 + R_2)^2\ (1 + {}_2R_1)$

$(1.0765)^3 \quad = (1.0753)^2 (1 + {}_2R_1)$

$(1 + {}_2R_1) \quad = (1.0765)^3 / (1.0753)^2$

$= 1.0789$

${}_2R_1 \quad = 0.0789\ (7.89\%)$

The yield on the three one-year borrowing equals the current yield on a three-year bond:

$(1 + R_3)^3 \quad = [(1 + R_1)\ (1 + {}_1R_1)\ (1 + {}_2R_1)]^{1/3}$

$= [1.0724 \times 1.0782 \times 1.0789]^{1/3}$

$= 1.0765$ (7.65% as per the term structure data)

Overall, the series will be raised to the power of 1/N where N is the number of years involved.

Arbitrage Vs No-arbitrage Models

No-arbitrage models take the term structure as an input whilst arbitrage models produce the term structure as an output. Since market prices do not conform to these model prices, this creates the possibility of arbitrage even when the volatility parameter of the model used is fairly accurate. No-arbitrage models take the current price of the asset as given and derive a model that relates to the evolution of the term structure.

Some examples of these approaches are shown below:

Table 1 Examples of Arbitrage and No-arbitrage Models

Arbitrage Models	No-arbitrage Models
Vasicek (1977)	Ho and Lee (1985)
Cox, Ingersoll and Ross (1985)	Extended-Vasicek (1989)
	Heath, Jarrow and Morton (1992)

The arbitrage-free binomial model is based on a lattice of interest rates. The yields on the lattice represent a series of possible future short-term interest rates formed to satisfy conditions preventing changes in the yield curve that allow arbitrage opportunities. Cox and Ross (1976a) explained that the no-arbitrage constraints reflect the fundamental properties of the risk-neutral distribution implicit in options prices. The no-arbitrage constraints are respectively : (1) call and put option prices relative to the synchronous underlying asset price cannot be below intrinsic value and American option prices cannot be below European prices. (2) American and European option prices must be monotone and convex functions of the underlying strike price. (3) synchronous European call and put prices of common strike price and maturity must satisfy the put-call parity.

If the no-arbitrage constraints are violated, there is no distributional hypothesis consistent with observed option prices. Studies that use more carefully synchronized transactions data have found that substantial proportions of option prices violate lower bound constraints (Bhattacharya (1983), Evnine and Rudd (1985)). Violations of intrinsic value constraints are observed for short-maturity, in-the-money and deep-in-the-money options, as outlined in Section 2.1. Interest rate based derivative securities have structures that are much more complicated compared with those of derivatives on stocks. This makes it difficult to value the contracts analytically. The evolution of the entire yield curve has to be known. The price of interest rate derivatives is the value of the expected discounted future cash flow, with the assumption of risk-neutral expectations. This is similar to the Black and Scholes model for stock option prices. However, when contingent claims based on interest rate sensitive securities are being priced, interest rates change over time. The discount rate is usually correlated with the cash-flow of the interest rate contingent claim which further complicates the issue.

2.4 THE EXPECTATIONS HYPOTHESIS

The expectations theory holds in a world of certainty or risk neutral borrowers and lenders. Investors are not assumed to be risk neutral but rather when hedging a derivative with the underlying asset, arbitrage possibilities can and will be exploited by all investors regardless of which way prices go because of the assumption of full information.

According to the pure expectations theory, forward rates exclusively represent the expected future rates. Therefore, the entire term structure at a given time reflects the market's current expectation of the future short rates. To clearly explain the forward rates, assume a discount bond that matures in period four. r1, r2, r3 and r4 are short-term interest rates in periods one, two, three and four.

If the short-term rate moves as follows:

A $1 face value discount bond should then be priced at:

$$P = \frac{1}{(1+r1)(1+r2)(1+r3)(1+r4)}$$

The pure expectation theory hypothesis implies that the price should be a simple expectation of this quantity.

$$P = E\ \frac{1}{(1+r1)(1+r2)(1+r3)(1+r4)}$$

For example, if P(0,1) and P(0,2) are the prices of the one and two year zero-coupon bonds today. The forward rate is P(0,2)/P(0,1). The forward rate is the price that could be contracted today for a bond that is issued in the future. The forward rate is the rate of return implicit in the forward price. In a world of certainty, forward rates will equal future spot rates. That is P(0,2)/P(0,1) which is the forward price, would equal the known future spot price of a one-year zero coupon bond issued at time 1, P(1,2).

If the expectations theory holds, the shape of the term structure provides a prediction of the direction of future interest rates. For example, a downward sloping term structure will exist if longer-term rates are lower than shorter-term rates and suggest that short rates will decline.

2.4.1 RISK NEUTRAL PRICING

Cox, Ross and Rubinstein (1979) explain clearly how the process of risk neutral pricing works. Using a binomial economy, if the economy has two future values, Su or Sd with probabilities of p and 1-p respectively. The expected value of future prices is:

$$E(S) = p\,Su + (1 - p)Sd$$

Since the expected return carries an uncertainty risk, it should be higher than the risk free rate.

$$k = E(S)/S > R,$$

where $R = (1 + r_f)$

k is greater than $(1 + r_f)$ since a risk averse person will ask for compensation for taking risks. A risk neutral person is insensitive to risks.

A risk neutral person however has the following view of probabilities:

$$S = E(S)/R = \frac{pSu + (1 - p)Sd}{R}$$

Since the risk free return is expected, the probabilities can be found by equating the expectation of the stock price to $S \times R$. Risk-neutral probabilities can be solved for u and d. If there exists a riskier asset in the economy, for example, an option; the expected value of the option from the view point of the risk adverse person will be:

$E(C) = p\,Cu + (1 - p)Cd$

This requires a higher return to compensate for the increase in risk

$K = E(C)/C > k$

The expected option value will be

$E(C) = p'\,Cu + (1 - p')Cd = R \times C$

The probability p' is the 'risk-neutral' probability. Once the probability p' becomes known, the expected value of any asset can be calculated because the expected value can be discounted at the risk free rate. Risk-neutralization is the difference between the real probability p and the risk-neutral probability p'. In order for p' to be solved, there has to be the assumption of complete markets. If the stock market did not exist, there would be no current stock price for us to calculate p and the risk-neutral pricing methodology would not work.

The expected value is equal to $R \times C$ since only the risk free return is required regardless of the risk of the asset. Since p' can be solved from the stock market, it can be applied to the option market in order to calculate the option price. When p' is known, the expected value of any asset can be derived and the value can be found by discounting at the risk free rate.

In order to price under a term structure in a continuous framework, a utility function is assumed to obtain risk aversion. The risk premium between the actual probability and the risk-neutral probability can be found. For example, the face value of a $1 bond in a continuous time setting has the following pricing formula:

$P(t,T) = E_t [\exp(-\int_t^T r(x)dx)]$

t and T are the current time and the maturity time of the bond. The expectation is taken at time t.

If the random movements of future interest rates over time are assumed to follow known distributions, the bond prices can be computed by using the expected value risk neutral formula.

2.5 THE TERM STRUCTURE THEORY

2.5.1 Assumptions

The assumptions underlying the use of the term structure in pricing is explained in Jarrow (1996) and other earlier studies such as Ho and Lee (1986). The economy is assumed to be frictionless and competitive. The frictionless market's assumptions are justified since the activities of large institutional traders approximate frictionless markets as their transaction costs are minimal. All securities are assumed to be infinitely divisible and the market for any financial security is perfectly liquid.

When markets are liquid, arbitrage profits can be made. For pure arbitrage to exist, enough related assets are required in order to form a 'complete market'. Pricing models using the term structure use arbitrage in a continuous time sense. In continuous time, markets are complete if there are two different non-redundant assets (Black and Scholes (1973)). The risks exist over a discrete time period as no one can trade continuously in reality. When assets cannot be priced by arbitrage, a utility-based formula can be used.

If $P(t, T)$ is the price at time t of a discount bond maturing at $T(t < T)$, the instantaneous return on the bond is given by the ratio $dP(t, T)/ P(t, T)$

Suppose this return is given by:

$$dP(t, T) / P(t, T) = \mu(t, T)dt + \sigma(t, T)dW(t)$$

where

μ and σ are fixed constants,

$W(t)$ is a standard Wiener process.

The total return is the sum of the expected return and the random part of the return. The equation assumes that the randomness is generated by a diffusion process.

Two bonds of different maturities T_1 and T_2 can form a portfolio in which the return is instantaneously riskless. If a proportion w_1 of the total value is invested in bonds of maturity T_1 and a proportion $1- w_1$ is invested in bonds of maturity T_2, the return on the portfolio can be given as:

$$dV/V = (w_1 \mu(t, T_1) + (1- w_1)\mu(t, T_2))\, dt + (w_1 \sigma(t, T_1) + (1 - w_1)\, \sigma (t, T_2))\, dW(t)$$

The instantaneous return on the portfolio is riskless when w_1 is chosen to eliminate the second term on the right hand side.

This instantaneous return has to be equal to the short-term interest rate:

$\{\mu(t, T_1) - r\} / \sigma(t, T_1) = \{\mu(t, T_2) - r\} / \sigma(t, T_2)$

The equation says that the expected return in excess of risk-free rate associated with holding a bond divided by the standard deviation of the return (excess return per unit risk) is independent of the maturity of the bond. Let $\lambda(r, t) = (\mu(t, T) - r)/\sigma(r,t)$. $\lambda(r, t)$ is the market price of risk.

The return on the bond maturing at T can be shown to be:

$dP(t, T) / P(t, T) = (r(t) + \sigma(t, T)\lambda(r, t))\, dt + \sigma(t, T)dW(t)$

The bond price can be obtained as the solution to the boundary condition $P(T,T) = 1$, in which the price at maturity is equal to 1. The models described in the following sections demonstrate the different approaches used to solve the bond price process.

2.5.2 VALUING INTEREST RATE DERIVATIVE SECURITIES

The stochastic behavior of interest rates is very difficult to model. The various risk-free interest rates available in the economy can be represented by the term structure. This is the interest rate earned on a default-free discount bond until its time to maturity. Interest rates also appear to follow mean-reverting processes. This refers to the drift which pulls the interest rate back to the long-run average level. Forward interest rates can also be deduced from the term structure. Early studies assume all the underlying assets' distributions be lognormal with known parameters.

Models of the short-term interest rate assume the short rate follows a diffusion process and the price of the discount bond depends only on the short-term rate over its term (Attari 1997). The general form of the evolution of the short-term interest rate is normally assumed to be:

$$dr = \alpha(r, t)\,dt + \rho(r, t)\,dW(t)$$

where $\alpha(r, t)$ is the 'drift', the instantaneous expected change in the short-term interest rate; and $\rho(r, t)$ is the volatility or the random change in the short-term interest rate. The drift and the volatility can both be functions of the current level of interest rates.

When the short-term interest rate is assumed to be the only source of uncertainty in the model, Ito's Lemma applied to the bond price gives:

$$dP = P_t dt + P_r dr + 0.5\, P_{rr}(dr)^2$$

P is used in place of P(t,T) and the subscripted variables denote partial derivatives.

P_t is the partial derivative of the bond price with respect to current time.

Substituting for dr from the general evolution of short-term interest rate equation and comparing with the return on the bond equation yields:

$$P_t + \alpha(r, t) + \rho(r, t)\lambda(r, t))P_r + 0.5\rho(r, t)^2 P_{rr} - rP_r = 0$$

This can be solved for P(t, T), the price of the discount bond using the appropriate boundary equation. The above equation is referred to as the fundamental partial differential equation for the bond price.

The different types of short-term interest rate models depend on how the market price of risk $\lambda(r, t)$ is specified. $\lambda(r, t)$ can be treated as a function of short-term interest rate r and current time t. If $\lambda(r, t)$ is chosen to make models analytically tractable, it is important that economic equilibrium arguments are also considered.

The evolution of short-term interest rate models can generally be summarized in the following equation:

$dr = \alpha(\mu - r)dt + \sigma^{\gamma}\, dW(t)$

The short-term interest rate process should allow for mean reversion. The volatility of interest rates should also depend on the level of short-term interest rates.

The following section is a summary of the different types of models that incorporate the term structure of interest rates. Chen (1996) provides the following classifications.

2.6 REVIEW OF MODELS

2.6.1 ONE FACTOR MODELS

Discrete single factor models are models with one source of uncertainty in which only one of two possibilities can happen (movements in interest rates up or down) at each node in the tree. In a continuous framework, one factor is solely responsible for the evolution of interest rates. A model that provides great insights on how the term structure of interest rate could be modelled is the Vasicek Model.

2.6.1.1 Vasicek Model (1977)

Vasicek (1977) modelled the interest rate as a continuous time process. The interest rate process was:

$dr = \alpha(\mu - r)dt + \sigma dW(t)$

where α, μ and σ are fixed constants and W(t) is a standard Wiener process,

dr is the change in the spot rate r,

dW can be viewed as normal variate with mean 0 and variance dt,

$\alpha(\mu - r)dt$ is the instantaneous expected change in the short-term interest rate. This is consistent with mean reversion of interest rates.

σ is the volatility or the random change in the short-term interest rate.

Vasicek (1977) obtained the price of bonds of all maturities using a constant market price of risk ($\lambda(r, t) = \lambda$). He shows that when r is smaller than $R(\infty) - \frac{1}{4}\sigma^2/\alpha$, increasing yield curves are obtained. When r is larger than the above but less than $R(\infty) + \frac{1}{4}\sigma^2/\alpha$, the yield curves are humped; and the yield curves can be decreasing when the values for r above is $R(\infty) + \frac{1}{4}\sigma^2/\alpha$.

The model is captured by assuming that the market price of interest rate risk, $(\mu - r)/\sigma = \lambda$, is constant across the term structure. This is the same assumption as the no arbitrage/equivalent martingale assumption.

At a given time, the distribution for r is normal with the following attributes:

$$E[r(s)] = r(t)e^{-\alpha(s-t)} + \mu(1 - e^{-\alpha(s-t)})$$

$$Cov[r(\mu), r(s)] = \sigma^2/2\alpha \times e^{\alpha(s+\mu-2t)}(e^{2\alpha(\mu-t)} - 1) \qquad \text{for } \mu < s$$

Conditional variance is:

$$var[r(s)] = cov[r(s), r(s)] = \sigma^2(1 - e^{-2\alpha(s-t)})/2\alpha$$

where s is the timing of the cash flows of the underlying spot bond that come after the expiration of either the option or the futures and the interest rate is mean reverting to μ. To find the price of a pure discount bond, it is necessary to compute the expectation. If the distribution of r in $R = \int_t^T r(\mu)du$ is normally distributed, it follows that R is also normally distributed. Once a risk-neutral process (p') can be identified, assets can be priced using the risk-free return regardless of their actual risks. The risk-neutral mean and variance are:

$$E'_t[R] = \int_t^T E'_t\,[r(s)]ds = r(t)(1-e^{-\alpha(T-t)}/\,\alpha) + (\mu - (q\sigma)/\alpha)\,[T - t - (1-e^{-\alpha(T-t)}/\,\alpha)]$$

and $$V'_t\,[R] = V_t[R]$$

The risk-neutral mean is changed by the risk parameter q which is fixed under log utility. The risk-neutral variance remains unchanged since r is normally distributed.

Given the parameters α, μ , σ and q, bond prices for a given maturity can be calculated:

$$P(t,T) = e^{-E't(R) + V't[R]/2} = e^{-r(t)F(t,T) - G(t,T)}$$

where

$$F(t,T) = (1- e^{-\alpha(T-t)})/\alpha$$

$$G(t,T) = (\mu - (q\sigma)/\alpha - \sigma^2/2\sigma^2)[T - t - F(t,T)] + [\sigma^2F^2(t,T)]/4\alpha$$

Since interest rates are normally distributed, it is possible for the interest rate to become negative. Taking the limit of the expected rate and variance when $T \rightarrow \infty$ shows that as long as $\alpha > 0$, the expectation will converge to b and the variance will converge to $\sigma^2/2\alpha$.

While the Vasicek (1977) model is arbitrage-free in the sense that no bond or options prices produced by the model will permit arbitrage, it is not arbitrage-free in the context of actual market prices. This is due to the fact that the model produces a term structure as an output but does not accept the term structure as an input. Another limitation of the Vasicek (1977) model is that it cannot capture the more complex term structure shifts that occur since it is only a single factor model. Moreover, all rates have the same volatility.

There is no known solution for American options so the Vasicek model must be laid out in a binomial or trinomial tree. Hull and White (1989) modify the Vasicek model by using the trinomial tree to solve the problem of fitting the current term structure.

Dothan (1978) models the interest rate process as an exponential random walk with no drift:

$dr = r\sigma dt$

This is obtained by setting $\alpha = 0$ and $\gamma = 1$. In this case the short-term interest rates cannot become negative.

2.6.1.2 Cox-Ingersoll-Ross Model (CIR) (1985)

Cox, Ingersoll, and Ross (1985) develop a one factor model and propose an economy driven by a number of processes that affect the rate of return to assets including technological change or an inflation factor. The short-term interest rate process in Cox, Ingersoll, and Ross (1985) is assumed to be:

$dr = \alpha(\mu - r)dt + \sigma\sqrt{r}\, dt$

where α, μ and σ are fixed constants,

dr is the change in the spot rate r,

$\alpha(\mu - r)dt$ is the instantaneous expected change in the short-term interest rates,

This is consistent with the mean reversion of interest rates.

σ is the volatility or the random change in the short-term interest rate. A square root process for the evolution of interest rate is used.

They show the bond price solution to be:

$P_t + \alpha(\mu - r)P_r - \lambda r P_r + \frac{1}{2}\sigma^2 r P_{rr} - r P = 0$

This is a model similar to Vasicek's but overcomes the problem of negative interest rates. If the interest rate can be negative, bond prices can exceed one. When the current rate moves to zero, the square root of zero causes the volatility to go to zero and the rate will be pulled up by its drift. To ensure that the short-term rate does not become 0, CIR assume that $2\alpha\mu \geq \sigma^2$.

The boundary behaviour of the short-rate process does not need to be specified when the process does not allow the short rate to reach infinity. This model assumes the diffusion process has a square root of r. All future interest rates are non-negative.

The analytical solution for the term structure in the CIR model is:

$P_t(\tau) = A_1(\tau)\exp(-B_1(\tau)r_t)$ (Cox et al. (p. 393))

where

$A_1(\tau) = [2\delta\exp((\delta + \gamma)\tau/2) / (\delta + \gamma)(\exp(\delta\tau) - 1) + 2\delta]^{2\alpha/\sigma 2}$

$B_1(\tau) = [2(\exp(\delta\gamma) - 1) / (\delta + \gamma)(\exp(\delta\tau) - 1) + 2\delta]$ and $\delta = (\gamma + 2\sigma^2)^{1/2}$

Converting to a yield

$r_t(\tau) = \{-\log(A_1(\tau)) + B_1(\tau)r_t\} / \tau$

The level of the term structure depends on the value of r_t at any point in time while the slope of the curve depends upon the variables of the diffusion equation and the market price of risk. One deficiency of their model, however, is that it will never exactly reproduce an observed yield curve.

Arbitrage models such as Vasicek (1977) and Cox-Ingersoll-Ross (1985) value all interest rate derivatives on a common basis. Nevertheless, the model's term structure does not correctly price actual bonds. These models have too few parameters to be adjusted and they do not take the initial term structure into account. They may allow negative interest rates (Vasicek (1977)) or assume perfect correlation between volatility and the short rate (Cox-Ingersoll-Ross (1985)). The short rate is not sufficient to explain the future yield curve changes.

2.6.1.3 Empirical Research on the use of One-Factor Models

Brown and Dybvig (1986) test the parameters of the CIR model and compute the residuals defined by the gap between the observed bond prices and the predictions of the model. Residuals show specification errors present in the model.

From the evidence obtained, it seems unreasonable to assume that the entire money market is given by only one explanatory variable. Moreover, it is hard to obtain a realistic volatility structure for the forward rates without introducing a very complicated short rate model. These considerations have led authors to propose models that use more than one state variable.

2.6.2 MODEL PROCESSES

2.6.2.1 The State Space Process

This is the extension of the binomial model previously put forward. Jarrow (1996) explains the process. Between time 0 and 1, the three possible outcomes will be: up(u), middle(m) and down (d); i.e. $s_1 \in [u,m,d]$

Over the time interval [t,t+1], the new state s_{t+1} is generated according to the expression:

$$s_{t+1} = \begin{cases} s_t\,u & \text{with probability } q_t^u(s_t) > 0 \\ s_t\,m & \text{with probability } q_t^m(s_t) > 0 \\ s_t\,d & \text{with probability } 1 - q_t^u(s_t) - q_t^m(s_t) > 0 \end{cases}$$

At time $t \in \{1, 2, \ldots, \tau\}$, the generic initial state is labeled s_t {all possible t sequences of u's, m's and d's}.

2.6.2.2 The Bond Price Process

The evolution of the zero-coupon bond price curve is described by the expressions:

$$P(t+1, T; s_{t+1}) = \begin{cases} u(t,T; s_t)P(t,T; s_t) & \text{if } s_{t+1} = s_t\,u & \text{with probability } q_t^u(s_t) > 0 \\ m(t,T; s_t)P(t,T; s_t) & \text{if } s_{t+1} = s_t\,m & \text{with probability } q_t^m(s_t) > 0 \\ d(t,T; s_t)P(t,T; s_t) & \text{if } s_{t+1} = s_t\,d & \text{with probability } 1 - q_t^u(s_t) - q_t^m(s_t) > 0 \end{cases}$$

for all $t \leq T\text{-}1 \;\; \leq \tau - 1$ and s

2.6.2.3 The Forward Rate Process

The rate of change of the forward rate will be:

$$\alpha(t, T; s_t) \equiv f(t+1,T; s_t u) \,/\, f(t,T; s_t) \qquad \text{for } t+1 \leq T \leq \tau - 1$$
$$\gamma(t, T; s_t) \equiv f(t+1,T; s_t m) \,/\, f(t,T; s_t) \qquad \text{for } t+1 \leq T \leq \tau - 1$$
$$\beta(t, T; s_t) \equiv f(t+1,T; s_t d) \,/\, f(t,T; s_t) \qquad \text{for } t+1 \leq T \leq \tau - 1$$

The evolution of the forward rate curve can be described by:

$$f(t, T; s_{t+1}) = \begin{cases} \alpha(t, T; s_t)\, f(t,T; s_t) & \text{if } s_{t+1} = s_t\,u & \text{with probability } q_t^u(s_t) > 0 \\ \gamma(t, T; s_t)\, f(t,T; s_t) & \text{if } s_{t+1} = s_t\,m & \text{with probability } q_t^m(s_t) > 0 \\ \beta(t, T; s_t)\, f(t,T; s_t) & \text{if } s_{t+1} = s_t\,d & \text{with probability } 1 - q_t^u(s_t) - q_t^m(s_t) > 0 \end{cases}$$

for all $t \leq T\text{-}1 \;\; \leq \tau - 1$ and s_t

The spot rate process evolution is described as:

$$r(t+1, s_{t+1}) = \begin{cases} u(t+1, t+2; s_t\ u) & \text{with probability } q_t^u(s_t) > 0 \\ m(t+1, t+2; s_t\ m) & \text{with probability } q_t^m(s_t) > 0 \\ d(t+1, t+2; s_t\ d) & \text{with probability } 1 - q_t^u(s_t) - q_t^m(s_t) > 0 \end{cases}$$

Longstaff (1989) develops a 'double square-root' process which makes the term structure fit more accurately compared to the CIR model. The interest rate process is:

$$dr = \alpha(\theta - \sqrt{r})dt + \sigma\sqrt{r}\, dt$$

His model allows the short-term rate to be zero in which $2\alpha\mu < \sigma^2$ is possible. The possibility of the short-term rate being zero allows the model to fit the current term-structure better.

Chen (1996b) describes in detail the use of Brennan-Schwartz (1979), Richard model (1978) and Longstaff-Schwartz (1993) model.

2.6.3 TWO-FACTOR MODELS

2.6.3.1 Brennan-Schwartz Model (1979)

Brennan and Schwartz (1979) use short and long rates as factors, which are the two ends of the yield curve. The short and long rate follows a jump diffusion lognormal process and the short rate displays mean reversion to the long rate.

$$d \ln r = a(\ln l - \ln r)dt + b_1 dW_1$$

$$d\, l = l\, a(r, l, b_2)dt + b_2\, l\, dW_2$$

where

$$E[dW_1 dW_2] = \rho dt$$

Since there is no closed form solution, the partial equation has to be derived through the standard arbitrage argument. The finite difference method was used to solve the problem.

$$\tfrac{1}{2} P_{11} b_1^2 r^2 + P_{12} \rho\, b_1 r\, b_2\, l + \tfrac{1}{2} P_{22}\, b_2^2\, l^2 + P_1(\ln l - \ln r - \lambda_1 b_1 r) + P_2(l\, a_2 - \lambda_2 b_2\, l) - P_t$$

$$= rP$$

With the 'no-arbitrage condition' in place, they conclude that the price of the instantaneous risk associated with the long rate can be eliminated, and the two factors can become the instantaneous rate and the yield spread between the short rate and the long rate. The need to linearize to find a solution since the elimination of the price of risk for the long rate makes the model non-linear. Another solution to this problem is to allow for stochastic volatility to be involved in the analysis.

2.6.3.2 Chen and Scott (1992)

Chen and Scott (1992) assume the instantaneous rate is the sum of two factors:

$$r_t = x_{1t} + x_{2t}$$

where

$$d\, x_{1t} = (\alpha_1 - \beta_1 x_{1t})dt + \sigma_1 x_{1t}^{1/2}\, dn_t$$

$$d\, x_{2t} = (\alpha_2 - \beta_2 x_{2t})dt + \sigma_2 x_{2t}^{1/2}\, dn_{2t}$$

where dn_{it} are independent, the solution for the bond price is

$$f_t(\tau) = A_1(\tau) A_2(\tau) \exp\{-B_1(\tau)x_{1t} - B_2(\tau)x_{2t}\}$$

A_2 and B_2 are defined analogously to A_1 and B_1. Where A_1 and B_1 are the same as those presented in the CIR model.

$$f_t(\tau) = A_1(\tau) \exp(-B_1(\tau) r_t) \qquad \text{(Cox et al. (p. 393))}$$

where

$$A_1(\tau) = [2\delta \exp((\delta + \gamma)\tau/2) / (\delta + \gamma)(\exp(\delta\tau) - 1) + 2\delta]^{2\alpha/\sigma 2}$$

$B_1(\tau) = [2(\exp(\delta\gamma) - 1) / (\delta + \gamma)(\exp(\delta\tau) - 1) + 2\delta]$ and $\delta = (\gamma + 2\sigma^2)^{1/2}$

The equation allows the inclusion of any number of factors as long as they are assumed to be independent.

2.6.3.3 Longstaff-Schwartz Model (1993)

In Longstaff-Schwartz (1993) model, two factors are also used. They are the short-term interest rate and the volatility of the short-term rate. They retain the rest of the assumptions of the CIR model. Factors are observable and parameters can be directly estimated from the data. Maximum likelihood estimation is possible since the process assumption is imposed directly on factors. Longstaff and Schwartz write the two state variables as:

$d\,y_1 = (a - b\,y_1)dt + c\sqrt{y_1}dW_1$

$d\,y_2 = (k - e\,y_2)dt + f\sqrt{y_1}dW_2$ where $dW_1\,dW_2 = 0$

The equilibrium rate of interest and its volatility are:

$r = \alpha\,y_1 + \beta\,y_2$

$V = \alpha^2\,y_1 + \beta^2\,y_2$

The two factors are related to the underlying rate of return process rather than directly to the instantaneous rate as in Chen and Scott (1992). The second factor they use affects only the conditional variance of the rate of return process but both factors affect the conditional mean. $P(0, 0, t, T) < 1^4$ implies that the forward rates are strictly positive. As the short-term rate increases, the price of the bond can either increase or decrease, for small values of T-t bond values decrease but for larger values of T-t, bond values may either decrease or increase. This is due to the fact that an increase in the short-term interest rate, while keeping the volatility constant, implies a lower production uncertainty and a lower λ. This is an important factor that makes this model differ from the other models considered. Changes in volatility and the interest rate constant will have an effect on the shape and the slope of the term structure. No evidence is found in support of the rejection of the Longstaff and Schwartz two-factor model, whilst similar tests reject the one-factor CIR model.

In Chen and Scott (1992), two factors are regarded as driving the short-term rate and its conditional volatility. The nominal instantaneous interest rate is the sum of the two components. They both affect the mean and variance. However, more research has to be done to know how well the models can replicate the unconditional standard deviations of yield changes.

2.6.3.4 Chan, Karolyi, Longstaff and Sanders (1992)

Chan, Karoyl, Longstaff and Sanders (1992) compared the various short-term riskless rate using the Generalized Method of Moments. They find that the most successful models in capturing the dynamics of the short-term interest rates are those that allow the volatility of interest rate changes to be highly sensitive to the level of the riskless rate. The equation that represents the interest rate process is:

$$dr = (\alpha + \beta r)dt + \sigma r^{\gamma} dz$$

To estimate the parameters of the above equation, the discrete time specification equation is given as:

$$r_{t+1} - r_t = \alpha + \beta r_t + \varepsilon_{t+1} \qquad E(\varepsilon_{t+1}) = 0$$

$$E(\varepsilon^2_{t+1}) = \sigma^2 r^{2\gamma}$$

Various short-term rate specifications with different parameter restrictions are then evaluated against each other. Only weak evidence of mean reversion (β is not significantly different from 0) is found. The models explain 1-3 per cent of the variation in r and they explain up to 20 per cent of the variation in volatility.

Of the most frequently used models the Vasicek (1977) model performs poorly relative to the less known models (Dothan (1978) and Cox, Ingersoll and Ross (1985)). It was commonly known that interest rate volatility is important in valuing contingent claims and hedging interest rate risk. However, these models fail to capture the dependence of the term structure's volatility on the level of interest rate.

2.6.3.5 Empirical Research on the use of Two-Factor Models

Single factor models are useful for clarifying the mathematical concepts involved but are not useful for applications. They imply that zero-coupon bonds of different maturities are perfectly correlated, which is not true. Therefore, more factors can be added to the term structure models in order to improve pricing. The two-factor model has been used in a framework which either assumes arbitrage-free conditions or is based on utility equations (Richard (1978), Brennan and Schwartz (1979), Langetieg (1980), Cox, Ingersoll, and Ross (1985) and Longstaff and Schwartz (1992)). Cox, Ingersoll, and Ross (1985) find that the instantaneous rate can be expressed by separate factors in equilibrium. One method of modelling is to decompose the instantaneous rate into two factors following two stochastic processes. The other way is to view the volatility of the instantaneous interest rate as a function of two factors. The process then follows a single stochastic volatility model. Chen and Scott (1993) find the parameters of the model by maximum likelihood estimation and provide some evidence that at least two factors are required to capture the term structure adequately. Other studies using the generalized method of moments (Heston (1989), Gibbons and Ramaswamy (1993)) and factor analysis also found this to be true. This suggests the need for an increased number of factors in the models.

2.6.4 MULTIFACTOR MODELS

The extension from a one-factor model to a two-factor model corresponds to adding an additional branch on every node in the appropriate tree. The procedure for extending the model to a multifactor framework is similar in the process of evolution.

The need for fitting the yield curve suggested tests for multiple factors. Factors used are to be arbitrarily specified. Recent studies use short and long interest rates and other interest rates as factors.

2.6.4.1 Chen (1996)

Chen (1996) incorporated that both the short mean and volatility of short interest rates are stochastic. The pricing of interest rate derivatives based on his model is able to reflect more factors. The no-arbitrage approach has been accommodated into his three-factor model. In the model the future short rate depends on the current short rate, the short-term mean of the short rate and the current volatility of the short rate. The system of stochastic differential equations that determines the interest rate dynamics is given by:

$dr = k\,(\theta - r)dt + \sqrt{\sigma}\,\sqrt{r}\,dz_1$

$d\theta = a\,(\theta' - \theta)dt + b\,\sqrt{\theta}\,dz_2$

$d\sigma = c\,(\sigma' - \sigma)dt + e\,\sqrt{\sigma}\,dz_3$

where

dr is the change in the future short rate r,

dθ is the change in the short-term mean of the short rate,

dσ is the change in the volatility of the short rate,

k, a and c are constants and are the reversion rates of the short rate, short-term mean of the short rate and the volatility of the short rate.

Despite authors like Brennan and Schwartz (1979), Schaefer and Schwartz (1984), incorporating more factors in their models; multifactor models are disadvantaged in that they may not fit perfectly a given term structure.

2.6.5 FORWARD RATE MODELS

Another approach is to take the current term structure as given and model term structure fluctuations. These models take the initial term structure as input. Thus, the model prices of bonds of all maturities match the observed market prices. Therefore, these models are comparatively more suitable for applications in pricing derivative securities. Perturbation functions were used on the forward price in Ho and Lee (1986). Black, Derman and Toy (1990) assumed the distribution function of the short rate to be lognormal. It is similar to the Ho-Lee model but it also fits the volatility curve. Heath, Jarrow and Morton (1992) and Hull and White (1990) developed continuous time models by letting the parameters in the stochastic processes of the instantaneous rate be deterministic functions of time.

There are many variants of equations put forward by different studies, but the common use of the process is:

$$F_t(\tau - 1) - F_{t-1}(\tau) = c_{t,\tau-1} + \sigma_{t,\tau-1}\varepsilon_{t,\tau-1}$$

where

$F_t(\tau - 1)$ and $F_{t-1}(\tau)$ are the forward rates at different points in time and the other parameters are constants.

The different models make different assumptions made about volatilities $\sigma_{t, \tau, 1}$. The model could have a constant volatility or a proportional volatility assumption. $c_{t, \tau, 1}$ is the current price of the asset as given and it reflects the no-arbitrage assumption.

The equation used by HJM model for the evolution of the forward rate incorporates spreads and changes in yields.

$F_t(\tau - 1) - F_{t-1}(\tau) = [1/\tau(\tau - 1)]sp_t(\tau) - [(\tau + 1)/\tau](r_t(\tau + 1) - r_t(\tau)) + [(\tau + 1)/\tau] \Delta r_t(\tau + 1) - 1/\tau \Delta r_t(1)$

For small τ, constant volatility models with martingale difference errors could not describe the data. The rank of the covariance matrix of the errors $\varepsilon_{t, \tau, 1}$ are generally found to be two or three so the assumption of a single error to drive all forward spreads proves to be unreliable.

2.6.5.1 Ho and Lee Model (1986)

Ho and Lee (1986) find that although the multi-factor models can improve the fitting of the yield curve, they still do not perform well enough. Ho and Lee adopt the approach of taking bond prices as given instead of pricing bonds. Therefore, their model cannot be used to find bond prices. The model is mainly used for pricing interest rate contingent claims.

A series of forward prices are calculated from the observed term structure. The term structure of pure discount bonds is first defined.

$P(0,1), P(0,2), \ldots, P(0,n)$.

Under certainty, the one-year bond price one-year from now should equal today's one-year forward price on a one-year bond.

$P(1,2) = P(0,2) / P(0,1) = \varphi\,(0,1,2)$

A binomial tree is created by adding perturbations for up and down states in order to incorporate uncertainty.

$P(1,1,2) = P(0,2) / P(0,1)\ u(1)$ state up

$P(0,1,2) = P(0,2) / P(0,1)\ d(1)$ state down

The two yield curves in the next period can be created as:

$P(1,1,i) = P(0,\ i) / P(0,1)\ u(i-1)$

$P(0,1,i) = P(0,\ i) / P(0,1)\ d(i-1)$

The three yield curves two periods from now can be derived from the two yield curves one period from now:

$P(2,2,i) = P(1,1,i) / P(1,1,2)\ u(i\text{-}2)$

$P(1,2,i) = P(0,1,i) / p(0,1,2)\ u(i\text{-}2) = P(1,1,i) / P(1,1,2)\ d(i\text{-}2)$

$P(0,2,i) = P(0,1,i) / p(0,1,2)\ d(i\text{-}2)$

The closed form solution for u and d are as follows:

$u(k) = 1/\ [p' + (1\text{-}p')\ \delta^k]$

$d(k) = \delta^k / [p' + (1\text{-}p')\ \delta^k]$

δ = a constant measuring the magnitude of the interest rate volatility

p' = the risk neutral probability

The higher the δ, the higher the volatility; p and δ^k are constants between 0 and 1.

The two parameters, p and δ, must be estimated from the prices of traded options. This is done by first calculating the values for a set of traded options based on estimated parameters. The calculated values can be compared with market values. After, this is done, values of parameters are adjusted. The process is carried out until no further improvement is possible.

The underlying process for the short-term interest rate r is given by:

$dr = \theta(t)dt + \sigma dW$

The drift θ(t) is a function of time and is chosen to make the process consistent with the term structure. The volatility factor is constant. It has the disadvantage that it involves no risk reversion and leads to a flat term structure of interest rate volatilities. Moreover, interest rates can become negative. The price of the discount bond using a similar risk premium to the Vasicek model and the assumption of risk-neutral expectation is:

$P(t,T) = E'_t [\exp(-\int_t^T r(u)du = \exp[(-r(t) - \int_t^T \int_t^s \theta(u)duds + (\sigma^2(T-t)^3/6)]$

$= D(t,T)e^{-r(t)(T-t) + (\sigma 2(T-t)3/6)}$

D depends on the time-dependent parameter θ

The time-dependent parameter θ needs to generate D for every point on the yield curve in order to price the bond correctly.

The option formula for Ho and Lee (1986) is:

$var[\ln P(T,s)] = var[r(T)] = \sigma^2(T-t)(s-T)^2$

The spot rate is normally distributed, the difference is variable volatility in the equation.

The Ho and Lee model has a number of disadvantages. The model describes the whole volatility structure by a single parameter and it does not incorporate mean reversion.

One problem with the Ho and Lee model is that it allows negative interest rates. This issue was pinpointed by Ritchken and Boenawan (1990). The major difference between the Ho and Lee model and other models is that Ho and Lee model the bond price process while the others model the interest rate processes.

2.6.5.2 Hull and White Model (1990)

The model by Hull and White (1990) overcomes the defects of Ho and Lee. They discuss how the one-factor models of Vasicek (1977) and CIR (1985) can be extended so that they are consistent with both the current term structure of interest rates and the current volatility of all spot rates or the current volatility of all forward rates. The underlying distribution process is normal.

It is based on the equation:

$$dr = \alpha(t)[u(t) - r]dt + \sigma(t)dW$$

$\sigma(t)$ is a constant but also time dependent. The time varying parameters allow a better fit of the model.

Hull and White (1990b) employ a trinomial method that allows for a different branching procedure. The method permits the user to solve for different probabilities at each node, which uphold the constraint that the probabilities must sum up to one and that they must guarantee that the interest rate will be normally distributed with mean and variance correctly defined.

Hull and White (1990a) have proposed a modification to the model to incorporate the current term structure. The extended Vasicek model is normally distributed and parameters are time dependent. The market price of risk can be time dependent if the parameters are time dependent. Hull and White obtain their solution by solving a partial differential equation. The risk neutral process is as follows:

$dr = [\alpha(t)u(t) - \sigma q(t) - \alpha(t)r]dt + \sigma dW$

where q(t) is the market price of risk

The stochastic linear equation has the solution of :

$r(s) = \phi(s)[r(t) + \int_t^s \phi(u)^{-1}[\alpha(u)\mu(u) - \sigma q(u)]du + \int_t^s \phi(u)^{-1}\sigma dW_u$

where $\phi(s) = \exp(-\int_t^s \alpha(u)du)$

The mean and variance of the state variables are:

$E_t[r(s)] = \phi(s)[r(t) + \int_t^s \phi^{-1}(u)q(u)du)]$

$cov_t [r(s),r(u)] = \phi(s)[\int_t^{min(u,s)} (\phi^{-1}(w) \sigma)^2 dw] \phi(u)$

The term structure model will be:

$P(t,T) = E_t [\exp(-\int_t^T r(s)ds)] = e^{-m(t,T)+[V(t,T)/2]}$

where

$m(t,T) = \int_t^T E'[r(s)]ds = \int_t^T \{\phi(s)[(r(t) + \int_t^s \phi(u)^{-1} [\alpha(u)\mu(u) - \sigma q(u)]du]ds$

$V(t, T) = \int_t^T \int_t^s 2K_t [r(s),r(u)]duds = \int_t^T \int_t^s 2 \phi(s) [\int_t^u (\phi^{-1}(w)\sigma)^2 dw] \phi(u)duds$

$\alpha(t)$ and $\mu(t)$ have to be in closed form for the bond price

The pricing formula can generate any bond prices to match the observed ones traded in the marketplace. The option formula of the model is based on a log normal distribution.

The option pricing formula is:

$C(t, T_c, T) = P(t,T)N(d) - P(t, T_c)KN(d - \sqrt{V_p})$

where

$d = \{\ln(p(t, T)/KP(t,T_c) + V_p/2\}/ \sqrt{V_p}$

$V_p = var(\ln P(T_c , T)) = F(T_c , T)^2 \, var(r(T_c)) = F(T_c , T)^2 \, \phi(T_c)^2 \sigma^2 \int_t^{Tc} \phi(w)^{-2}dw$

When σ is known, the flexibility of $\mu(t)$ and $\alpha(t)$ can be used to fit the yield curve. The equation becomes a one-time dependent parameter model if the model is not required to fit the yield curve.

The bond and option formula become:

$P(t,T) = E_t [\exp(-\int_t^T r(s)\, ds)] = e^{-m(t,T) + [V(t,T)]/2}$

where

$m(t,T) = r(t)F(t,T) + \int_t^T [e^{-\alpha(s-t)} (\int_t^s e^{\alpha(u-t)} [\alpha\mu(u) - \sigma q(u)]du)]ds = r(t)F(t,T) + D(t,T)$

$V(t,T) = \sigma^2 /\alpha^2 [T-t-(e^{-2\alpha(T-t)} /2\alpha) + [2(e^{-\alpha(T-t)} /\alpha)] - (3/2\alpha)$

In order to fit all bond prices, D is used to provide flexibility. The yield curve can be fitted without changing the option prices (but changing D) since the time dependent parameters (μ and q) are not part of the equation.

Hull and White (1993) propose another variation of the model with α being constant. Their model fits the current term structure to the model and updates the parameters as they step through time. A disadvantage in this model is it recalibrates with no real time-dependent structure. However, except for the Extended-Vasicek model, Hull and White's (1990) approach provides no closed form solution and has to rely on numerical methods.

2.6.5.3 Heath-Jarrow-Morton (HJM) (1992)

Heath, Jarrow and Morton (1992) drop the path-independence condition of the Ho-Lee model. Forward rates are used as the primitive variables and they model the evolution of an infinite set of forward rates.

The Ho-Lee, Vasicek and the Hull-White extension of the Vasicek model modelled the short or the forward rates as Gaussian processes. Any model within the Heath-Jarrow-Morton framework possessing a deterministic volatility will also give rise to a Gaussian forward rate curve. The main defect of the model is that negative interest rates may be generated with positive probability. If the existence of cash is an assumption, negative interest rates would lead to theoretical arbitrage opportunities.

The HJM model is a framework under which all arbitrage-free term structure models can be derived. Instead of using multi-factors as the state variables, their model takes the entire forward rate curve as their state variable.

Attari (1997) describes how the HJM model is generated. First, the initial forward rate curve and the volatility functions are used to specify the arbitrage-free movements in the forward rates. This allows the computation of unique martingale probabilities which can be used to price contingent claims. The price of a bond is:

$$P(t, T) = \exp \{ - \Sigma f(t, j\Delta)\Delta\}$$

where the forward rate $f(t, T)$ at t for T to T + Δ is assumed to satisfy the stochastic process

$$f(t, T) = f(0, T) + \Sigma a_j [u_i (j\Delta, T) - v_i (j\Delta, T)] + \Sigma v_i (j\Delta, T)]$$

The a_j takes on values 0 or 1 with an objective probability given by $q_a(j)$. The u's and v's are random functions that at each time t can depend on any information prior to time t. The forward rate process is an arbitrage-free process.

Under a continuous time framework, only the forward rate volatility needs to be specified. The forward rate at time t from time T to time T + dT can be given by $f(t, T)$. The prices of discount bonds can then be specified in terms of the forward rates:

$P(t, T) \exp\{-\int_t^T f(t, v)dv\}$

The forward rate $f(t, T)$ at time t can be given in terms of the initial forward rate $f(0, T)$ as:

$f(t, T) = f(0, T) + \int_0^t \alpha(v, T, w)dv + \sum \int_0^t \sigma_i(v, T, w)\, dW_i(v)$

This is under the objective probabilities. The spot rate at time t is $f(t, t)$ and is given as

$r(t) = f(0, T) + \int_0^t \alpha(v, T, w)dv + \sum \int_0^t \sigma_i(v, T, w)\, dW_i(v)$

Using the forward rate and the spot rate the relative price of the bond is

$Z(t, T) = P(t,T) / B(t)$ where $B(t) = \exp(\int_0^t r(y)\, dy)$

The forward rate process under the risk-neutral measure is given as:

$f(t, T) = f(0, T) + \sum \int_0^t \sigma_i(v, T) \int_v^t \sigma_i(v, y)\, dy\, dv + \sum \int_0^t \sigma_i(v, T)\, dW_i(v)$

By specifying $f(0, T)$ and σ_i, $f(t, T)$ can be obtained.

The forward rate can be expressed as:

$df(t, T) = \alpha(t,T)dt + \sigma(t,T)dW$

Bond price and the forward rate volatility need to be a function of time to maturity.

Discount bonds can be priced when $f(t, T)$ is found and contingent claims C(t) can be priced using:

$C(t) = B(t) E_t^* (C(T) / B(T))$

where E_t^* is the expectation computed using the risk-neutral probabilities

One drawback of the Heath-Jarrow-Morton model is that the interest rate models resulting from their approach are usually non-Markov. Because the HJM model is a generalization of the Ho and Lee model that is obtained by dropping the path-independence condition, the forward rates are path dependent and the resulting tree does not recombine. The distribution of interest rates in the next period depends not only on the current rate but also on the rates in the earlier periods. Moreover, there are only a small number of known forward rate volatility functions that give rise to the Markov models (Chen (1996a)). It is difficult to derive closed-form solutions for the values of bond and interest rate derivatives. However, if the path dependence can be represented by a single statistic, the evolution of the term structure can then be characterized by just the spot rate. This reduces the computational complexity of the model.

Abken (1993) fitted HJM models to forward rates by the generalized method of moments whilst Amin and Morton (1994) used option prices to recover implied volatilities whose evolution was compared to those of the most popular variants of the HJM model. Abken demonstrates that the latter tends to follow a constant volatility formulation and the former tends to follow a proportional one. It is important to examine the evidence regarding volatility together with model specifications.

Amin and Morton (1994) tested six models in the HJM class. The models tested have the implied volatility function of:

$\sigma(.) = [\sigma_0 + \sigma_1 T - t] \exp[-\lambda(T-t)] f(t, T)^\gamma$

Data on Eurodollar futures and options contracts are tested. Two parameter models fit observed option prices better than single parameter models but the results are inconsistent. Their results support those of Flesakar (1993) who uses data on Eurodollar futures options.

All the time-dependent parameter models are similar in that the parameters are adjusted to fit the observed yield and volatility curves. When a normal distribution is assumed, all models can be translated to one another. When pricing American contracts, the Hull and White model is the most suitable for the lattice framework. It provides the simplest way to build the model when numerical methods are required to price the options. Only the short rate is used in pricing compared to the HJM model which requires a set of points on the forward rate or the short rate curves. The various approaches and model specifications are summarised in Table 2 below.

Table 2 Developments in Term Structure Modeling (Taken from Chen (1996a))

Authors	Model Specifications	
Merton (1973)	$dr = \theta dt + \sigma dz$	θ, σ are constants
Vasicek (1977)	$dr = k(\theta - r)dt + \sigma dz$	k, θ, σ are constants
Brennan-Schwartz (1979)	$dr = \theta_r dt + \sigma_{r1}\, dz_1 + \sigma_{r2}\, dz_2$	θ_r, θ_l, σ_{r1}, σ_{r2}, σ_{l1}, σ_{l2}
	$dl = \theta_l dt + \sigma_{l1}\, dz_1 + \sigma_{l2}\, dz_2$	are constant
Dothan (1978)	$dr = \sigma r dz$	σ is constant
Schaefer-Schwartz (1984)	$ds = m(\mu - s)dt + \eta\, dz_1$	m, μ, η, σ are constant
	$dl = m(\sigma^2 - ls)dt + \sigma\sqrt{l}\, dz_2$	
Cox-Ingersoll-Ross (1985)	$dr = k(\theta - r)dt + \sigma\sqrt{r} dz$	k, θ, σ are constants
Ho-Lee (1986)	$dr = \theta dt + \sigma dz$	θ time varying, σ constant
Black-Derman-Toy (1990)	$d \ln r = [\theta - \sigma'(t)/\sigma(t) \ln r]dt + \sigma(t)dz$	θ is time varying
Hull-White (1990)	$dr = k(\theta - r)dt + \sigma\sqrt{r} dz$	θ, σ are time varying
Heath-Jarrow-Morton (1992)	$df = \alpha(t)dt + \sigma(t)dW$	f is the forward rate
Longstaff-Schwartz (1992)	$dy_1 = (a - b\, y_1)dt + c\sqrt{y_1}dW_1$	a, b, k, e are constant
	$dy_2 = (k - e\, y_2)dt + f\sqrt{y_1}dW_2$	
Chen (1994)	$dr = k\,(\theta - r)dt + \sqrt{\sigma}\,\sqrt{r}\, dz_1$	k, a, θ', b, c, σ', e are constant
	$d\theta = a\,(\theta' - \theta)dt + b\,\sqrt{\theta}\, dz_2$	
	$d\sigma = c\,(\sigma' - \sigma)dt + e\,\sqrt{\sigma}\, dz_3$	

Eurodollar futures and futures options are tested using the HJM approach using the Generalized Method of Moments with three years of daily data (Flesaker (1993)). He concludes that the approach is not compatible with the data for most periods. Backus, Foresi and Zin (1994) demonstrate that the Black, Derman and Toy model is likely to overprice call options on long bonds when interest rates exhibit mean reversion. They find that the no-arbitrage term structure model can lead to systematic arbitrage opportunities as a result of its mispricing of some assets. Mispricing can also occur when no other traders offer the mispriced assets. They conclude that the empirical results are disappointing.

The problem with the no-arbitrage models of Ho and Lee (1986), Heath-Jarrow-Morton (1992), Hull and White (1990, 1993), Black, Derman and Toy (1990) is that on any day a function for the term structure of interest rates needs to be estimated and there is no guarantee that the estimated function will be consistent with the previously estimated one. Although this approach has the ability to fit the initial term structure, its empirical performance is not sound.

2.7 NEW DIRECTIONS

Point Process Models

Researchers introduced jumps due to the empirical evidence that interest rate path movements cannot be represented by diffusion processes. Instead, they behave more like pure jump processes. This is why point process models are introduced and this theory has been actively discussed currently in the literature.

Daz (1995) uses a jump-diffusion process for the short-term interest rate. A jump-diffusion process is suitable as the changes in short-term interest rate have too many outlying values to be generated by one diffusion process alone. Daz obtains expressions for discount bond prices in the presence of jumps. Option prices can also be computed in an equilibrium model where the short-term interest rate follows a jump-diffusion process (Attari 1996).

Shirakawa (1991), Bjork (1995) and Jarrow-Madan (1995) consider interest rate models driven by a finite number of counting processes. Jarrow and Madan construct a model for asset prices driven by semimartingales. Bjork, Kabanov and Runggaldier (1995) modeled the forward rates as:

$$df(t, T) = \alpha(t, T)dt + \sigma(t, T)dW_t + \int_E \delta(t, x, T)\mu(dt, dx)$$

where:

μ is a multivariate point process.

It covers the case of a finite number of driving counting processes as well as the case of an infinite mark space E. The inclusion of the infinite mark space into the model will introduce an infinite number of random sources. The measure-valued portfolios which at each point in time contain bonds with a continuum of maturities have to be used in order to produce any effect of hedging. Suppose the number of bonds held at time t is denoted by h(t, dT) with maturities in the interval [T, T + dT] and g(t) is the number of units of the risk free asset B. The definition of the value process becomes:

$$V(t) = g(t) B(t) + \int_t^\infty p(t, T)\, h(t, dT)$$

The formal generalization of the standard self-financing condition can be summarized as:

$$dV(t) = g(t)dB(t) + \int_t^\infty h(t, dT)dp(t,T)$$

Bjork, Kabanov and Runggaldier (1995) find that the standard portfolio concept can be extended to include measure valued portfolios. The no-arbitrage condition on the interest rate dynamics can provide an extension of the HJM condition. They conclude that as opposed to the standard models with only a finite set of assets, market completeness is no longer equivalent to uniqueness of the martingale measure. It is shown that the uniqueness of the martingale measure is equivalent to 'approximate completeness' of the market. In this case, claims belonging to a dense subspace of the space of all claims can only be hedged.

2.8 IMPLICATIONS

In summary, the literature has been concentrated in empirical research on single and multifactor models. The analysis in the use of time-dependent models is very limited. This is true especially in the Australia context. There are many questions required to be justified including how pricing errors are affected by time to maturity, the degree in which options are in and out-of-the-money and volatility? What are the deviations of pricing errors when different models are used? How do they differ? This study addresses these issues by using call options on 90-day Bank Accepted Bill futures as taken from the *Sydney Futures Exchange* in order to investigate the accuracy of different pricing methods.

Chapter 3: METHODOLOGY

3.1 DESIGN

Monthly data is used taken from the year 1996 to price call options on 90 Day Bank Accepted Bill Futures from the period 1 January 1996 to 31 December 1996. The futures and options data are closing prices taken from the last Wednesday of the month to ensure consistency. Despite the fact that options are priced on individual days, interest rates and reversion rates are generated on a continuous basis using the yield curve, volatilities and reversion rates generated each month at that point in time from interest rate caps and market data at that point in time. (This is better explained in Part 3.3 Parameter Estimation).

Both the option and the futures data were collected from the *Sydney Futures Exchange* (SFE) web site directly from the internet. Each futures trade and option is matched according to the futures prices at the end of a particular day since prices are quoted on the end-of-the-day basis. (Thus, there may be a problem with lack of data synchronicity as the end of day prices may not represent trades made at identical times).

The bank bill market is the largest short-term interest rate market in Australia. Bank Accepted Bills are negotiable short-term securities used to affect short-term borrowing and lending for periods typically between 30-180 days. A Bank Bill represents a promise to pay the full face value of the bill at maturity with its credit risk based on the debt rating of the bank which guaranteed its payment. Bank bills are quoted on a yield per cent per annum basis and are priced according to a yield formula that discounts the face value to earn the appropriate interest cost.

Options on Bank Bill futures are American style options. Future options are marked to market on a daily basis, profits and losses are withdrawn and paid as they accrue. When the position is marked to market at the end of the day, the holder of a call option can withdraw any excess of the futures price over the exercise price. The holder of the put option can withdraw any excess of exercise price over futures price. The futures contract is left with no value when it is closed out. The use of appropriate models (i.e., reflecting margining) are needed to price these options. Both the Extended-Vasicek and HJM Model, as applied in this work, incorporate adjustments for marking-to-market futures style. The Asay Model, as discussed before, is derived principally to price futures or options that are subject to futures style margining. Despite the fact that the Black model is similar to the Asay Model and the Asay model is derived from the former, the Black model calculates the present value of the exercise price which is inappropriate for futures style margining. In this study, however, the Black model is also used for comparison to the other models to investigate how prices change when the marking-to-market property is not including in pricing.

The face value of the 90-Day Bank Accepted Bill futures contract is A$500,000. The delivery months are the spot month (nearest) and the next consecutive months. March, June, September and December quarters are then traded up to two years ahead. Eight quarter months are available for trading with strike prices at 0.25% intervals. As the 90-Day Bank Bill contract involves physical delivery, options expire five days prior to futures expiry so as to close or adjust positions. The market convention is to quote the option premium as yield per cent per annum x 100 (basis points). The dollar value of an option premium is calculated by comparing its contract value at the strike price and the value at the same price minus 0.01%. Quotes are expressed as an index (I) equals to

(100 - yield). For example, a futures price of 88.25 corresponds to a yield of 11.75 percent. The dollar value (f) of futures contracts at any index value (I) can be derived by the equation:

$f = 365(500{,}000)/(365+90-0.9I)$ since the contract unit is A$500,000

The minimum price movement is 0.01 index points, which at a yield of 11.85 percent corresponds to around $11.64.

Contract information on the options is down-loaded via the web site of the SFE on the internet. The SFE provides information on the date, strike price and the maturity date together with the future contract's settlement prices. The total number of transactions obtained were 2,097 for the year 1996. Data on interest rate caps, market rates for cash, futures and swaps were retrieved from the Australian Financial Review. (These were required to calibrate the term structure models).

Table 3 Number of Call Options on 90-Day Bank Accepted Bills Traded at the last Wednesday of the month in 1996

Date	No. of contracts with settlement prices
31 Jan 96	174
28 Feb 96	170
27 Mar 96	169
24 April 96	168
29 May 96	165
26 Jun 96	157
31 July 96	170
28 Aug 96	182
25 Sep 96	183
30 Oct 96	180
27 Nov 96	191
18 Dec 96	188

3.2 INSTRUMENT

The requisite data was entered in a spreadsheet in *Microsoft Excel* Version 7.0 for the purposes of calculating prices using both the Black model and Asay model. Prices are calculated based on the relevant formulas entered. Prices for the other models were calculated using the *Optimum: Fixed Income Monis Software* which takes data required to calibrate the yield curve and calculates the theoretical price of an interest rate option using the calibrations.

The pricing of the options on futures is based on the Heath, Jarrow, Morton (1992) and the Extended-Vasicek (1989) models. The actual valuation formulae and tree implementation are built in the software and are shown below:

- The Extended-Vasicek Model

$dP(t,T) = \mu_a(t, T)P(t, T)dt + \sigma_a(t, T)P(t, T)dZ_t$

where:

T is the bond maturity time

Z_t is a standard Brownian motion

σ_a is a deterministic function given by: $\sigma_a(t, T) = (\sigma_a/\alpha)(1\text{-}\exp(-\alpha(T\text{-}t)))$

for positive constants α and σ_a

The associated short rate satisfies the stochastic differential equation

$dr(t) = \alpha(m(t) - r(t))dt + \sigma_a dZ_t$

The function m(t) is the mean level of the short rate. The parameter α is the reversion rate, and σ_a is the Vasicek volatility.

- The HJM model

Uses two factors with deterministic volatility functions

The bond price is given by:

$dP(t, T) = \mu_b(t, T)P(t, T)dt + \sigma_{b1}(t, T)P(t, T)dW_t + \sigma_{b2}(t, T)P(t, T)dZ_t$

where W and Z are independent standard Brownian motions

$\sigma_{b2}(t, T)$ is as in the Extended-Vasicek model

$\sigma_{b1}(t, T)$ is given by $\sigma_{b1}(t, T) = \sigma_{b1}(T - t)$

The short rate is not a Markov process, so has a less convenient stochastic differential equation. The forward rate has the form of:

$df(t, T) = v(t, T)dt + \sigma_{b1} dW_t + \sigma_{b2} \exp(-\alpha(T-t))dZ_t$

The software permits the taking into account of the futures style margining feature of trading on the SFE in order to make the analysis consistent when the results from the above models are compared to the Asay model's.

The advanced statistical software package *SPSS for Windows, Release 7.0* was used to assist the analysis of data. The *Shazam* econometrics software package was used to run the Ordinary Least Square Regressions.

3.3 PARAMETER ESTIMATION

3.3.1 BLACK MODEL

3.3.1.1 Time to Maturity

Call options become more valuable as the time to expiration increases. Options on 90 day Bank Accepted Bill futures mature every three months. The time to maturity is obtained by dividing the number of days-to-maturity by the number of days in a year (365).

3.3.1.2 Volatility

Volatility measures the uncertainty of future stock price movements. The holder of a call benefits from a price increase; whereas the owner of a put benefits from a price decrease. The value of call prices increase as volatility increases. Implied volatility is used to monitor the market's opinion about the volatility of a particular option.

Latane and Rendleman (1976), Chiras and Manaster (1978) and Brown and Shevlin (1983) found that implied volatility is a more efficient predictor of options prices than historical volatility measures. Implied volatilities are taken from SFE accompanied by each of the option contracts. The fact that implied volatilities were taken from the exchange, should if anything, favour the Black and Asay models in the empirical tests. It may also lead to subtle biases in that the Vega of a European call option on a non-dividend-paying stock is given by:

$$Vega = S(T-t)^{1/2} N'(d_1)$$

where

S represents the value of the underlying security,

T-t is the time to maturity of the option,

N is the cumulative normal distribution

d_1 is as defined below.

The above suggest that estimates of the implied volatility will be sensitive to the level of the stock price and the time to maturity. This may affect the degree of pricing errors across different option series and maturities when the Black and Asay models are applied.

The Black (1976) model used to price the 90-day Bank bill options can be described by the following formula:

$c = e^{-rT}[FN(d_1) - XN(d_2)]$

where

$d_1 = [\ \ln(F/X) + \sigma^2 T/2]/\ (\sigma\sqrt{T})$
$d_2 = [\ \ln(F/X) - \sigma^2 T/2]/\ (\sigma\sqrt{T}) = d_1 - \sigma\sqrt{T}$

N(.) is the cumulative probability function for a standardized normal variable,

c and p are European call and put prices,

F is the futures price,

X is the strike price, r is the risk-free interest rate,

T is the time to expiration, and σ is the volatility of the bond price.

The following assumptions are made when using the model proposed by Black.

1) The futures price F is a continuous-time stochastic process that can be represented by the stochastic differential equation:

$$dF/F = \mu dt + \sigma dz$$

where dz is a Wiener process.

2) There are no transaction costs.

By invoking the condition that no risk-free arbitrage opportunities exist in an efficient market.

3.3.2 ASAY MODEL

The SFE trade options on futures where the options have futures-style margining. Option contracts are marked to market at the end of the day like futures contract. Thus, another model in this study is used to compare the theoretical prices of the Black model. The Asay model (1982) has prices similar to the Black model except that the use of discounting by the risk-free rate is not required.

The formula used to price options under the Asay Model (1982) is:

$c = FN(d_1) - XN(d_2)$

where

$d_1 = [\ln(F/X) + \sigma^2 T/2]/(\sigma\sqrt{T})$
$d_2 = [\ln(F/X) - \sigma^2 T/2]/(\sigma\sqrt{T}) = d_1 - \sigma\sqrt{T}$

N(.) is the cumulative probability function for a standardized normal variable,

c and p are European call and put prices,

F is the futures price,

X is the strike price,

T is the time to expiration, and

σ is the volatility of the bond price.

It should be noted that the pricing equations for call options are derived for European options, whereas the options traded are American options. An American option can be expected to have a value in excess of that predicted for a European option. However, Lieu (1990) discusses that it is never optimal to exercise a call or put option early under futures-style margining. The fact that the SFE uses the Asay model to calculate implied volatilities may create some biases when these are input into the model. However, the volatilities are used since these provide the best approximation to the underlying volatilities for the options and are readily available to market traders.

3.3.3 THE TERM STRUCTURE MODELS

3.3.3.1 The Yield Curve, the Extended Vasicek volatility, the HJM volatility and the Reversion Rates

The yield curve is generated from market data. This is done by a standard bootstrapping method. Market Rates for the last Wednesday of the month are gathered. Cash market information used includes both the offer and bid prices for overnight, one week, one month, two months, three months, six months and up to nine months maturity interest rates. Prices of futures with maturity from three months up to three years are included. Swap market prices with expiry from one year's time up to 10 year's time are also entered for bootstrapping. The cash, futures, and swap market prices collected at the last Wednesday of the months from January to December 1996 are shown in Tables 4-9.

The cash market is used as far as the futures market, which takes priority over everything. The swap market is used for the longer dated points, with intermediate swap rates found by interpolation if they are missing. There is an adjustment on the futures market data to allow for the volatility of interest rates for the more advanced models. The true futures' price was used rather than forward prices. Interest rate caps with maturity one, three and fives years for each month are also added.

The method of Cubic Splines is used as the interpolation method. Data is converted into discount factors at each known time point, and these are connected with cubic splines, which preserve continuity of the first derivative of the discount factors at each time point.

3.4 DATA PROCESSING PROCEDURE

3.4.1 Black Model and Asay Model

After the parameters for volatility and risk-free rates have been estimated, they are plugged into a spreadsheet containing the pricing formula for call options for Black Model. The time to maturity is on a 365 day basis for a year.

3.4.2 The Term Structure Models

The current term structure is formulated by entering the current market values of market rates for cash, futures, and swaps in the 'Market Rates' worksheet. The software calculates the yield curve from the input data by the bootstrapping method. The start date and number of points are automatically displayed. The yield curves generated for each of the months in 1996 are shown in Tables 10-11.

The volatilities and the reversion rates are generated by entering the data for three interest rate caps as quoted in the market on the same day. Interest rate caps are derivative securities which restrict the rate of interest that can apply to floating-rate loans. It is a contract where the seller of the contract promises to pay a certain amount of cash to the holder of the contract if the interest rate exceeds a certain predetermined level (the 'cap rate') at some future date. If a loan is taken at a floating rate of interest, the investor may buy a cap from the bank in order to ensure that he/she will never have to pay more than the cap rate. In the same way, the seller of a floor contract promises to pay cash if some future interest rate falls below a certain predetermined level.

Table 4 MARKET DATA (JAN & FEB 1996)

31 JANUARY 1996

CASH MARKET	Offer	Bid
1 WEEK	5.87	5.75
1 MONTH	5.87	5.75
2 MONTH	5.90	5.78
3 MONTH	5.88	5.69
6 MONTH	5.84	5.71
9 MONTH	6.06	5.81
12 MONTH	6.19	5.94

FUTURES	Offer	Bid
Sep-96	94.27	94.26
Dec-96	94.21	94.20
Mar-97	94.00	93.98
Jun-97	93.65	93.64
Sep-97	92.90	93.25
Dec-97	92.63	92.89
Mar-98	92.43	92.61
Jun-98	92.29	92.42
Sep-98	92.29	92.27
Dec-98	92.18	92.17
Mar-99	92.07	92.04
Jun-99	91.97	91.94

SWAP	Offer	Bid
1YEAR	6.19	5.94
2 YEAR	6.53	6.49
3 YEAR	6.95	6.91
4 YEAR	7.24	7.20
5 YEAR	7.44	7.40
7 YEAR	7.73	7.69
10 YEAR	8.04	8.00

27 FEBRUARY 1996

CASH MARKET	Offer	Bid
1 WEEK	7.52	7.52
1 MONTH	7.50	7.50
2 MONTH	7.50	7.50
3 MONTH	7.50	7.50
6 MONTH	7.65	7.65

FUTURES	Offer	Bid
Mar-97	92.50	92.49
Jun-97	92.29	92.23
Sep-97	92.19	92.07
Dec-97	92.09	91.98
Mar-98	92.02	91.98
Jun-98	92.90	91.90
Sep-98	91.90	91.89
Dec-98	91.83	91.81

SWAP	Offer	Bid
1YEAR	7.82	7.82
3 YEAR	8.20	8.20
5 YEAR	8.43	8.43

Table 5 MARKET DATA (MAR & APRIL 1996)

27 MARCH 1996

CASH MARKET	Offer	Bid
OVERNIGHT	7.55	7.55
1 MONTH	7.52	7.52
2 MONTH	7.53	7.53
3 MONTH	7.56	7.56
6 MONTH	7.66	7.66

FUTURES	Offer	Bid
Jun-96	92.37	92.35
Sep-96	92.08	92.04
Dec-96	91.82	91.77
Mar-97	91.63	91.59
Jun-97	91.51	91.46
Sep-97	91.43	91.38
Dec-97	91.38	91.34
Mar-98	91.32	91.28
Jun-98	91.26	91.26

SWAP	Offer	Bid
1 YEAR	7.90	7.90
3 YEAR	8.39	8.39
5 YEAR	8.63	8.63

23 APRIL 1996

CASH MARKET	Offer	Bid
OVERNIGHT	7.55	7.55
1 MONTH	7.52	7.52
2 MONTH	7.54	7.54
3 MONTH	7.55	7.55
6 MONTH	7.67	7.67

FUTURES	Offer	Bid
Jun-96	92.42	92.37
Sep-96	92.24	92.09
Dec-96	91.93	91.72
Mar-97	91.65	91.42
Jun-97	91.48	91.27
Sep-97	91.29	91.17
Dec-97	91.24	91.11
Mar-98	91.10	91.07
Jun-98	91.18	91.06
Sep-98	91.04	91.03
Dec-98	91.01	91.00

SWAP	Offer	Bid
1YEAR	7.85	7.85
3 YEAR	8.37	8.37
5 YEAR	8.57	8.57

Table 6 MARKET DATA (MAY & JUNE 1996)

29 MAY 1996

CASH MARKET	Offer	Bid
OVERNIGHT	7.50	7.50
1 MONTH	7.52	7.52
2 MONTH	7.53	7.53
3 MONTH	7.55	7.55
6 MONTH	7.63	7.63

FUTURES	Offer	Bid
Jun-96	92.47	92.46
Sep-96	92.43	92.32
Dec-96	92.09	92.05
Mar-97	91.80	91.75
Jun-97	91.56	91.54
Sep-97	91.41	91.39
Dec-97	91.28	91.27
Mar-98	91.23	91.22
Jun-98	91.18	91.17
Sep-98	91.14	91.12
Dec-98	91.10	91.08
Mar-99	91.06	91.03

SWAP	Offer	Bid
1YEAR	6.19	5.94
3 YEAR	6.95	6.91
5 YEAR	7.44	7.40

26 JUNE 1996

CASH MARKET	Offer	Bid
OVERNIGHT	7.35	7.35
1 MONTH	7.52	7.52
2 MONTH	7.55	7.55
3 MONTH	7.60	7.60
6 MONTH	7.70	7.70

FUTURES	Offer	Bid
Sep-96	92.34	92.32
Dec-96	92.10	92.06
Mar-97	91.81	91.77
Jun-97	91.58	91.53
Sep-97	91.43	91.38
Dec-97	91.31	91.29
Mar-98	91.25	91.25
Jun-98	91.21	91.19
Sep-98	91.17	91.16
Dec-98	91.13	91.12
Mar-99	91.08	91.08
Jun-99	91.03	91.03

SWAP	Offer	Bid
1YEAR	7.91	7.91
3 YEAR	8.42	8.42
5 YEAR	8.63	8.63

Table 7 MARKET DATA (JULY & AUG 1996)

31 JULY 1996

CASH MARKET	Offer	Bid
OVERNIGHT	7.05	7.05
1 MONTH	7.05	7.05
2 MONTH	7.05	7.05
3 MONTH	7.05	7.05
6 MONTH	7.04	7.04

FUTURES	Offer	Bid
Sep-96	92.71	92.60
Dec-96	92.75	92.62
Mar-97	92.58	92.46
Jun-97	92.33	92.20
Sep-97	92.07	91.95
Dec-97	91.88	91.79
Mar-98	91.70	91.70
Jun-98	91.64	91.59
Sep-98	91.53	91.51
Dec-98	91.47	91.45
Mar-99	91.41	91.41

SWAP	Offer	Bid
1YEAR	7.04	7.04
3 YEAR	7.65	7.65
5 YEAR	7.99	7.99

28 AUGUST 1996

CASH MARKET	Offer	Bid
OVERNIGHT	7.00	7.00
1 MONTH	6.98	6.98
2 MONTH	6.92	6.91
3 MONTH	6.90	6.90
6 MONTH	6.81	6.81

FUTURES	Offer	Bid
Sep-96	93.09	93.06
Dec-96	93.24	93.16
Mar-97	93.15	93.06
Jun-97	92.94	92.86
Sep-97	92.67	92.60
Dec-97	92.43	92.36
Mar-98	92.27	92.22
Jun-98	93.13	92.05
Sep-98	92.06	92.06
Dec-98	91.98	91.97
Mar-99	91.90	91.90
Jun-99	91.82	91.79

SWAP	Offer	Bid
1YEAR	6.90	6.90
3 YEAR	7.41	7.41
5 YEAR	7.79	7.79

Table 8 MARKET DATA (SEP & OCT 1996)

25 SEPTEMBER 1996

CASH MARKET	Offer	Bid
OVERNIGHT	7.00	7.00
1 MONTH	6.98	6.98
2 MONTH	6.91	6.91
3 MONTH	6.90	6.90
6 MONTH	6.81	6.81

FUTURES	Offer	Bid
Sep-96	93.09	93.06
Dec-96	93.24	93.16
Mar-97	93.15	93.06
Jun-97	92.94	92.86
Sep-97	92.67	92.60
Dec-97	92.43	92.36
Mar-98	92.27	92.22
Jun-98	93.13	92.05
Sep-98	92.06	92.06
Dec-98	91.98	91.97
Mar-99	91.90	91.90
Jun-99	91.82	91.79

SWAP	Offer	Bid
1YEAR	6.90	6.90
3 YEAR	7.41	7.41
5 YEAR	7.79	7.79

30 OCTOBER 1996

CASH MARKET	Offer	Bid
OVERNIGHT	7.05	7.05
1 MONTH	6.77	6.77
2 MONTH	6.67	6.67
3 MONTH	6.59	6.59
6 MONTH	6.55	6.55

FUTURES	Offer	Bid
Dec-96	93.54	93.50
Mar-97	93.67	93.64
Jun-97	93.63	93.59
Sep-97	93.46	93.44
Dec-97	93.28	93.24
Mar-98	93.07	93.05
Jun-98	92.94	92.92
Sep-98	92.84	92.83
Dec-98	92.75	92.75

SWAP	Offer	Bid
1YEAR	6.47	6.47
3 YEAR	6.93	6.93
5 YEAR	7.25	7.25

Table 9 MARKET DATA (NOV & DEC 1996)

27 NOVEMBER 1996

CASH MARKET	Offer	Bid
OVERNIGHT	6.50	6.50
1 MONTH	6.49	6.49
2 MONTH	6.45	6.45
3 MONTH	6.41	6.41
6 MONTH	6.40	6.40

FUTURES	Offer	Bid
Dec-96	93.60	93.58
Mar-97	93.84	93.75
Jun-97	93.87	93.80
Sep-97	93.76	93.68
Dec-97	93.56	93.53
Mar-98	93.42	93.39
Jun-98	93.31	93.27
Sep-98	93.20	93.17
Dec-98	93.10	93.09
Mar-99	93.00	92.99
Jun-99	92.90	92.89
Sep-99	92.80	92.80

SWAP	Offer	Bid
1YEAR	6.33	6.33
3 YEAR	6.70	6.70
5 YEAR	7.03	7.03

18 DECEMBER 1996

CASH MARKET	Offer	Bid
OVERNIGHT	6.00	6.00
1 MONTH	6.01	6.01
2 MONTH	6.00	6.00
3 MONTH	5.99	5.99
6 MONTH	5.94	5.94

FUTURES	Offer	Bid
Mar-97	94.12	94.09
Jun-97	94.07	94.04
Sep-97	93.82	93.78
Dec-97	93.53	93.47
Mar-98	93.26	93.21
Jun-98	93.03	92.99
Sep-98	92.85	92.79
Dec-98	92.73	92.69
Mar-99	92.57	92.57

SWAP	Offer	Bid
1YEAR	6.06	6.06
3 YEAR	6.78	6.78
5 YEAR	7.23	7.23

Table 10 : Yield Curves- Jan 96 - Jun 96
(Interest rates shown are compounded annually)

Start: 31-Jan-96

Dates	Days	Rate (%)
7-Feb-96	7	7.8360
29-Feb-96	29	7.7620
20-Mar-96	49	7.7360
19-Jun-96	140	7.6780
18-Sep-96	231	7.6070
18-Dec-96	322	7.5620
19-Mar-97	413	7.5480
18-Jun-97	504	7.5580
17-Sep-97	595	7.5800
17-Dec-97	686	7.6080
18-Mar-98	777	7.6380
17-Jun-98	868	7.6660
29-Jan-99	1094	7.3814
31-Jan-01	1827	7.7048

Start: 28-Feb-96

Dates	Days	Rate (%)
29-Feb-96	1	7.8090
20-Mar-96	21	7.7710
19-Jun-96	112	7.7250
18-Sep-96	203	7.8310
18-Dec-96	294	7.9140
19-Mar-97	385	7.9800
18-Jun-97	476	8.0260
17-Sep-97	567	7.9870
17-Dec-97	658	8.0320
26-Feb-99	1094	8.2100
28-Feb-01	1827	8.4690

Start: 27-Mar-96

Dates	Days	Rate (%)
28/03/96	1	7.8410
29/04/96	33	7.7830
27/05/96	61	7.7700
19/06/96	84	7.7680
18/09/96	175	7.8160
18/12/96	266	7.9380
19/03/97	357	8.0680
18/06/97	448	8.1830
17/09/97	539	8.2800
17/12/97	630	8.3590
18/03/98	721	8.4240
17/06/98	812	8.4790
29/03/99	1097	8.3920
27/03/01	1826	8.6674

Start: 24-Apr-96

Dates	Days	Rate (%)
25-Apr-96	1	7.8410
24-May-96	30	7.7830
19-Jun-96	56	7.7710
18-Sep-96	147	7.8060
18-Dec-96	238	7.9050
19-Mar-97	329	8.0480
18-Jun-97	420	8.1940
17-Sep-97	511	8.3170
17-Dec-97	602	8.4250
18-Mar-98	693	8.5120
17-Jun-98	784	8.5880
16-Sep-98	875	8.6440
16-Dec-98	966	8.6955
26-Apr-99	1097	8.3630
24-Apr-01	1826	8.5955

Start: 29-May-96

Dates	Days	Rate (%)
30/05/96	1	7.7880
19/06/96	21	7.7770
18/09/96	112	7.7540
18/12/96	203	7.7930
19/03/97	294	7.9080
18/06/97	385	8.0390
17/09/97	476	8.1650
17/12/97	567	8.2740
18/03/98	658	8.3700
17/06/98	749	8.4480
16/09/98	840	8.5120
16/12/98	931	8.5680
31/05/99	1097	8.5679
29/05/01	1826	8.7917

Start: 26-Jun-97

Dates	Days	Rate (%)
27-Jun-96	1	7.6260
26-Jul-96	30	7.7850
26-Aug-96	61	7.7910
18-Sep-96	84	7.7940
18-Feb-96	175	7.8530
19-Mar-97	266	7.9570
18-Jun-97	357	8.0830
17-Sep-97	448	8.2080
17-Dec-97	539	8.3170
18-Mar-98	630	8.4090
17-Jun-98	721	8.4840
16-Sep-98	812	8.5460
16-Dec-98	903	8.5986
17-Mar-99	994	8.6507
28-Jun-99	1097	8.4206
26-Jun-01	1826	8.6621

Table 11: Yield Curves- July 96 - Dec 96
(Interest rates shown are compounded annually)

Start: **31-Jul-96**

Dates	Days	Rate (%)
01-Aug-96	1	7.3040
30-Aug-96	30	7.2830
18-Sep-96	49	7.3140
18-Dec-96	140	7.4480
19-Mar-97	231	7.4770
18-Jun-97	322	7.5370
17-Sep-97	413	7.6300
17-Dec-97	504	7.7380
18-Mar-98	595	7.8410
17-Jun-98	686	7.9360
16-Sep-98	777	8.0180
16-Dec-98	868	8.0940
17-Mar-99	959	8.1645
30-Jul-99	1094	7.6313
31-Jul-01	1826	8.0246

Start: **28-Aug-96**

Dates	Days	Rate (%)
29-Aug-96	1	7.2500
18-Sep-96	21	7.2260
18-Dec-96	112	7.1240
19-Mar-97	203	7.0570
18-Jun-97	294	7.0610
17-Sep-97	385	7.1140
17-Dec-97	476	7.1990
18-Mar-98	567	7.2980
17-Jun-98	658	7.3900
16-Sep-98	749	7.4140
19-Dec-98	840	7.4930
17-Mar-99	931	7.5660
30-Aug-99	1097	7.4138
28-Aug-01	1826	7.8438

Start: **25-Sep-96**

Dates	Days	Rate (%)
26-Sep-96	1	7.2500
25-Oct-96	30	7.1870
25-Nov-96	61	7.1260
18-Dec-96	84	7.0790
19-Mar-97	175	6.9350
18-Jun-97	266	6.8840
17-Sep-97	357	6.8860
17-Dec-97	448	6.9250
18-Mar-98	539	6.9810
17-Jun-98	630	7.0450
27-Sep-99	1097	7.1750
25-Sep-01	1826	7.6100

Start: **30-Oct-96**

Dates	Days	Rate (%)
31-Oct-96	1	7.3040
29-Nov-96	30	6.9840
18-Dec-96	49	6.8990
19-Mar-97	140	6.7250
18-Jun-97	231	6.6350
17-Sep-97	322	6.6080
17-Dec-97	413	6.6300
18-Mar-98	504	6.6800
17-Jun-98	595	6.7450
16-Sep-98	686	6.8110
16-Dec-98	777	6.8730
29-Oct-99	1094	6.9380
30-Oct-01	1826	7.2947

Start: **27-Nov-96**

Dates	Days	Rate (%)
28-Nov-96	1	6.7150
18-Dec-96	21	6.6940
19-Mar-97	112	6.5870
18-Jun-97	203	6.4820
17-Sep-97	294	6.4270
17-Dec-97	385	6.4270
18-Mar-98	476	6.4620
17-Jun-98	567	6.5090
16-Sep-98	658	6.5590
16-Dec-98	749	6.6100
17-Mar-99	840	6.6600
16-Jun-99	931	6.7100
15-Sep-99	1022	6.7601
29-Nov-99	1097	6.8025
27-Nov-01	1826	7.0702

Start: **18-Dec-96**

Dates	Days	Rate (%)
19-Dec-96	1	6.1830
20-Jan-97	33	6.1770
18-Feb-97	62	6.1490
18-Mar-97	90	6.1240
19-Mar-97	91	6.1240
18-Jun-97	182	6.0740
17-Sep-97	273	6.0740
17-Dec-97	364	6.1400
18-Mar-98	455	6.2430
17-Jun-98	546	6.3560
16-Sep-98	637	6.4700
16-Dec-98	728	6.5800
17-Mar-99	819	6.6774
20-Dec-99	1097	6.8001
08-Dec-01	1826	7.2993

In Australia, interest rate caps have a quarterly interest reset pattern (frequency at which new caplets are started). The notional amount of the contract is \$1,000,000. Payment of each contract occurs at the start of the caplet. The interest rate caps used are summarized in Table 12.

Table 12 Interest Rate Caps for Swap Contracts Quoted in the Australian Financial Review

	[illegible]		[illegible]		[illegible]	
	Value	Strike	Value	Strike	Value	Strike
31 Jan 96	3,260	7.25	1,849	7.40	38,690	7.68
27 Feb 96	3,850	7.82	20,820	8.20	41,960	8.43
27 Mar 96	3,400	7.90	21,500	8.39	42,800	8.69
23 April 96	3,400	7.90	21,500	8.39	42,800	8.69
29 May 96	3,250	7.93	20,850	8.37	40,700	8.57
26 Jun 96	3,300	7.91	20,300	8.42	40,400	8.63
31 July 96	2,700	7.04	18,300	7.65	36,900	7.99
28 Aug 96	2,860	6.90	17,750	7.41	36,650	7.79
25 Sep 96	2,920	6.74	16,500	7.16	34,800	7.55
30 Oct 96	2,800	6.47	16,500	7.16	34,800	7.55
27 Nov 96	2,280	6.33	14,660	6.70	30,110	7.03
18 Dec 96	2,880	6.06	19,200	6.78	39,990	7.23

The generation procedure is call the 'calibration process'. Given the yield curve, the model parameters are extracted from the cap market prices. Volatility parameters and reversion rates are generated based on a simple optimisation. This is done by minimizing the sum of squares of the percentage errors in the theoretical price and the market price entered. The algorithm starts with a set of parameters and makes intelligent choices as to the next set to try. Details of the option contract are entered into the futures worksheet and theoretical prices based on the observed term structure can be calculated. The marking-to-market feature of Australian futures contract is also taken into account by choosing the appropriate pricing method in the software. According to the equations for the Extended-Vasicek Model and the HJM Model, the parameters for the two models are calculated from the market data and are shown in Table 13.

- The Extended-Vasicek Model

$$dP(t,T) = \mu_a(t, T)P(t, T)dt + \sigma_a(t, T)P(t, T)dZ_1$$

- The HJM Model

It uses two factors with deterministic volatility functions

The bond price is given by:

$$dP(t, T) = \mu_b(t, T)P(t, T)dt + \sigma_{b1}(t, T)P(t, T)dW_t + \sigma_{b2}(t, T)P(t, T)dZ_t$$

The volatility and reversion rates generated by the software using the market data are as follows:

Table 13 Volatilities and reversionrates generated by the *Optimum: Fixed Income Monis Software*

	Extended-Vasicek Model		HJM Model		
	μ_a	σ_a	μ_b	σ_{b1}	σ_{b2}
Jan	0.043578	0.019525	0.456679	0.012127	0.002979
Feb	0.047616	0.219440	1.273052	0.015611	0.004585
Mar	0.057830	0.022756	0.068429	0.009814	0.014066
April	0.090240	0.021682	0.504750	0.013545	0.003310
May	0.035287	0.021363	0.089875	0.014125	0.008242
June	0.066042	0.021137	0.569794	0.013768	0.003367
July	0.146819	0.017734	0.413811	0.005566	0.001381
Aug	0.053007	0.018244	0.565565	0.010401	0.002543
Sep	0.046683	0.017333	0.456168	0.009851	0.002420
Oct	0.115609	0.014850	0.449618	0.009076	0.002232
Nov	0.002084	0.013966	0.417195	0.006913	0.001714
Dec	0.005579	0.017607	0.590749	0.010586	0.002592

3.5 DATA ANALYSIS

The pricing errors (E_m) for the different models are calculated by:

E_m = Model price - Market price

Absolute pricing errors are the absolute values of the above.

Mean pricing errors can be found by finding the average of the pricing errors grouped under different categories of moneyness, volatility, time to maturity etc.

3.5.1 SPSS

A spreadsheet was set up in SPSS for Windows, Release 7.0. The information on each of the options was coded and entered as variables. The data was analyzed by the applications of split files, transform and compute functions. The descriptive statistics used to summarized the data were the mean, standard deviation and standard error. Maximum and minimum were also included to assist data analysis. The parametric statistics are best represented by the mean, which shows the mathematical average of the data.

3.5.2 Graphical Analysis

After the data has been grouped using SPSS, mean pricing errors and mean absolute errors are plotted over time; against time to maturity, the degree to which options are in or out-of-the-money.

3.5.3 Wilcoxon Signed Ranks Test

The Wilcoxon signed ranks test explains the sign of the difference between any pair and rank the differences in order of absolute size. The null hypothesis is that the two paired samples are from populations with the same medians and the same continuous distribution. It makes no assumptions about the shapes of the distributions of the two variables. The test takes into account information about the magnitude of differences within pairs and gives more weight to pairs that show large difference than to pairs that show small differences. Therefore, if the ranks having plus signs and negative signs were summed respectively, the two sums should be about equal when H_0 is true. But if the sum of the positive ranks is very much different from the sum of the negative ranks, it is inferred that the treatment of the two groups differs, and thus H_0 is rejected.

3.5.4 OLS Regression and White's Corrections

To shed more light on the nature of the model's pattern of mispricing, a set of Ordinary Least Square (OLS) regressions of the pricing errors on a constant, time to maturity, and the degree to which the options were in- or out-of-the-money were performed. The regressions were run for all the call futures options. This test examines whether the error variance is affected by any of the regressors, their squares or their cross-products. It tests specifically whether or not any heteroskedasticity present causes the variance-covariance matrix of the OLS estimator to differ from its usual formula.

Four regression models were used:

$R_a = \alpha_a + \mathbf{TTM}\gamma_1 + \mathbf{MON}\gamma_2 + \varepsilon_a$

$R_b = \alpha_b + \mathbf{TTM}\gamma_1 + \mathbf{MON}\gamma_2 + \varepsilon_b$

$R_c = \alpha_c + \mathbf{TTM}\gamma_1 + \mathbf{MON}\gamma_2 + \varepsilon_c$

$R_d = \alpha_d + \mathbf{TTM}\gamma_1 + \mathbf{MON}\gamma_2 + \varepsilon_d$

where

R_a, R_b, R_c, R_d represents the pricing errors of the Black model, the Asay model, the Extended-Vasicek model and the HJM model respectively. **TTM** is time to maturity; **MON** is moneyness.

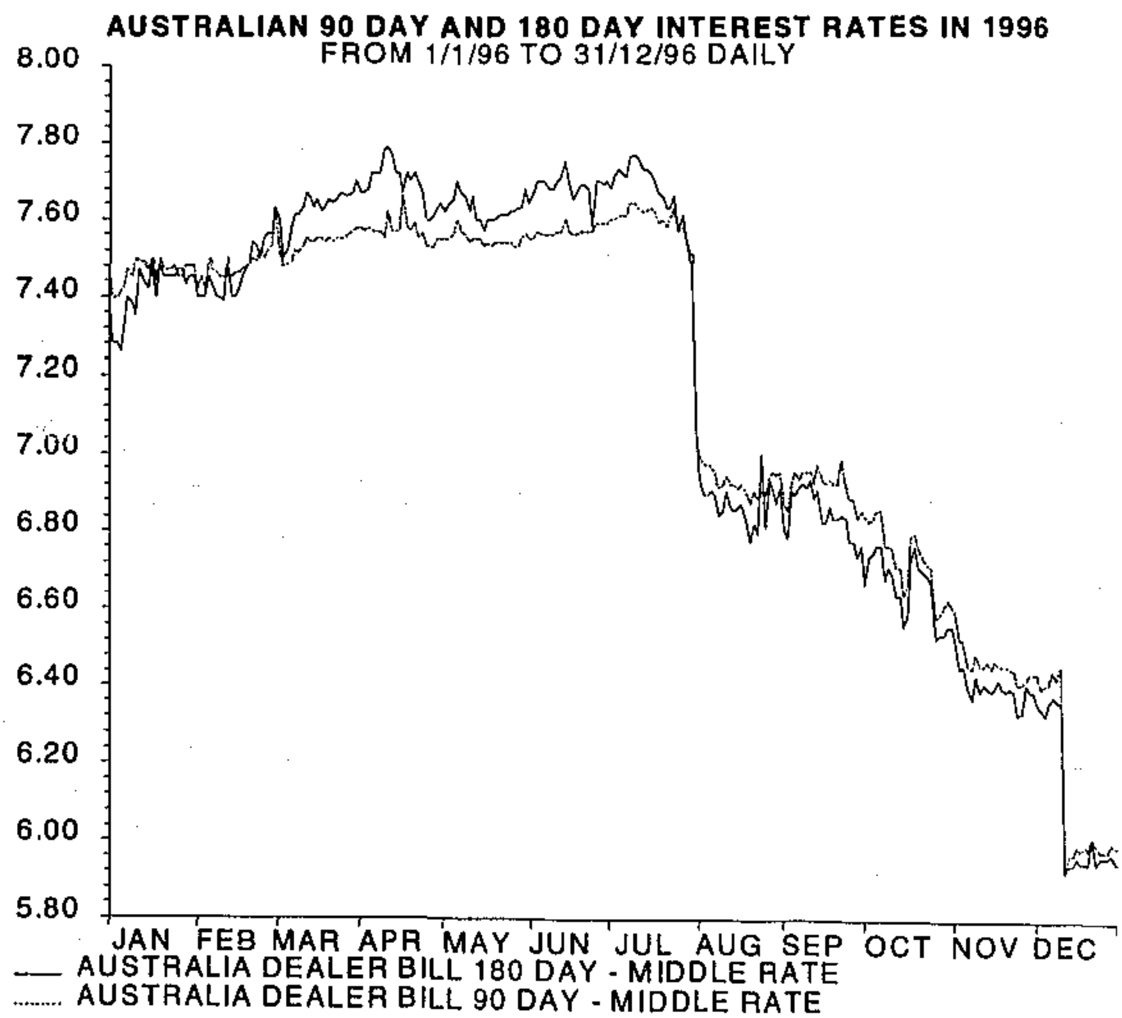
I did consider including a factor representing the option delta, which is the rate of change of the option premium with respect to movements in the underlying futures price. This is provided by the SFE based on the Asay Model but there seemed to be an element of circularity involving in applying this, given that both the Black and Asay model have similar constructions. The SFE also provides an estimate of volatility, which again is an implied volatility from the Asay model. Similar concerns lead me to reject its' use also.

Chapter 4: RESULTS

The following graph represents the 90-day and 180-day interest rates fluctuations in 1996. This provides a guideline as to how volatilities and reversion rates generated for the 90-day Bank Accepted Bills move relative to the interest rates in the short-term market. Both the interest rates for 90-day and 180-day short-term market show similar trends. As shown in the graph, interest rates moved up from 7.22% to 7.68% in January to July in 1996. In July, there was major drop from 7.60% to 6.82%. Subsequently, interest rates continued to decrease from July to November. The interest rates also drop considerably in the middle of November. Overall, in the year 1996, interest rates increased for the first half of the year but decreased thereafter.

Source : Datastream

GRAPH 1



4.1 GRAPHICAL ANALYSIS

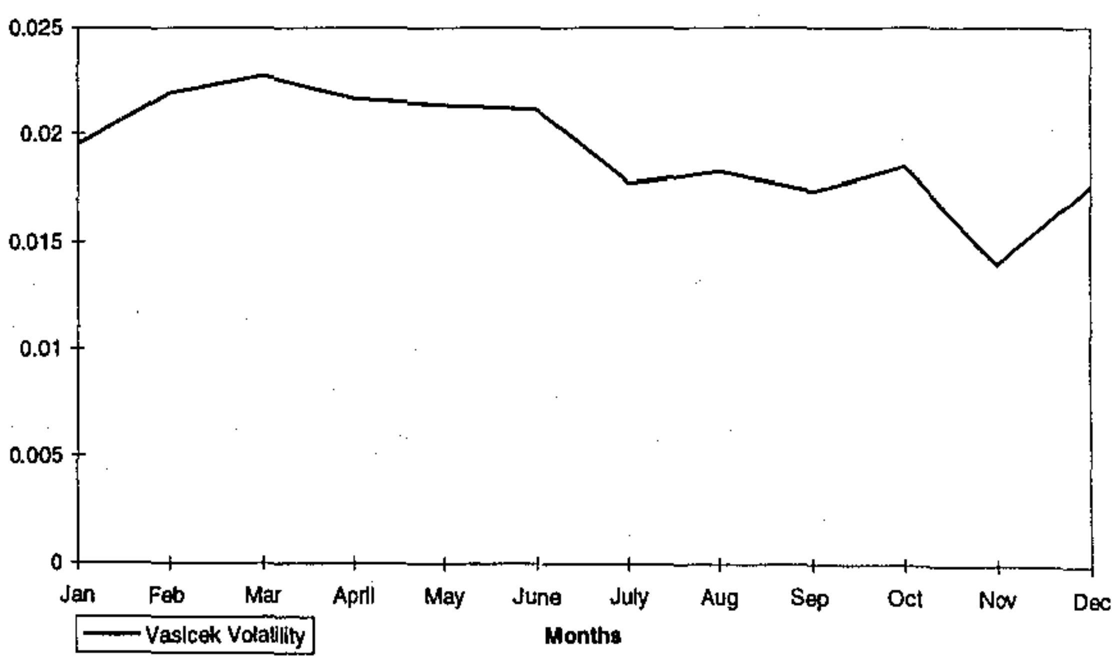
4.1.1 VOLATILITY

Extended-Vasicek Model

$dP(t,T) = \mu_a(t, T)P(t, T)dt + \sigma_a(t, T)P(t, T)dZ_1$

The volatility value (σ_a) ranges from 0.013 to 0.023 over the year with a comparatively lower value in November (Graph 2). The volatility rises from January to March and it shows a decreasing trend with slight fluctuations from March onwards. The lowest is in November which reaches around 0.013, the value rises again to above 0.015 in December. In general, the volatility for the Extended-Vasicek model shows a similar trend as the 90-day and 180-day short-term interest rate market. Interest rates decrease in the second half of the year from June onwards.

Graph 2
Extended-Vasicek Model: Changes in Volatility Over Time

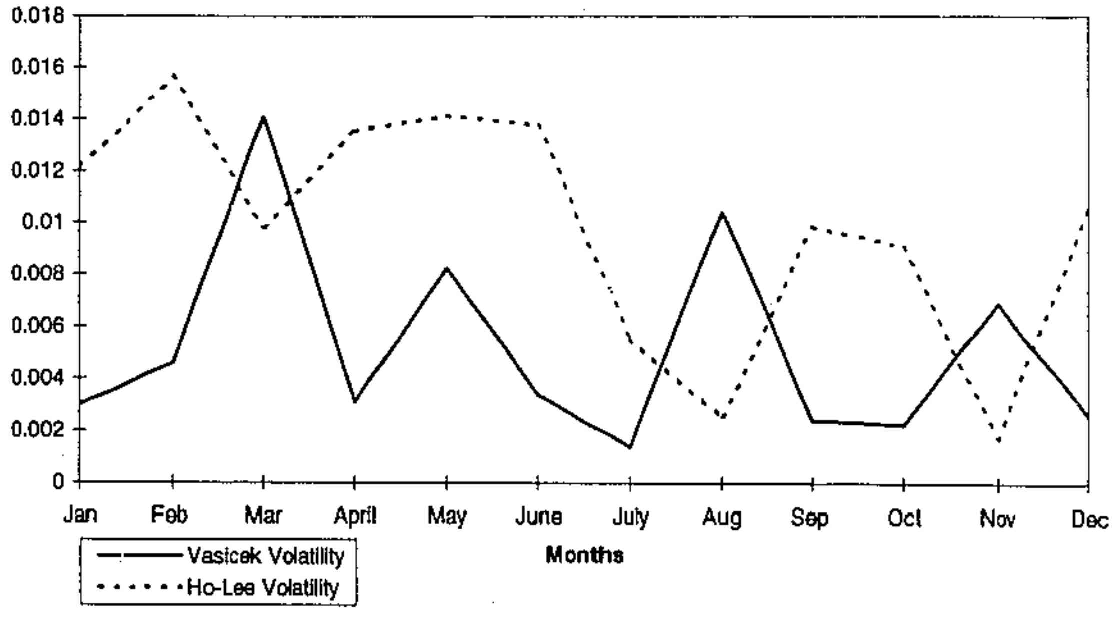


HJM Model

$dP(t, T) = \mu_b(t, T)P(t, T)dt + \sigma_{b1}(t, T)P(t, T)dW_t + \sigma_{b2}(t, T)P(t, T)dZ_t$

As shown in Graph 3, the Vasicek volatility (σ_{b1}) ranges from 0.001 to 0.014 from January to December in 1996. The values fluctuate significantly with higher rates obtained in March, May, August and November. Conversely, the Ho-Lee volatility (σ_{b2}) is much lower in March, August and November compared to other times of the year.

Graph 3
HJM Model: Changes in Volatility Over Time



4.1.2 REVERSION RATE

Reversion Rate for the Extended-Vasicek Model (μ_a)

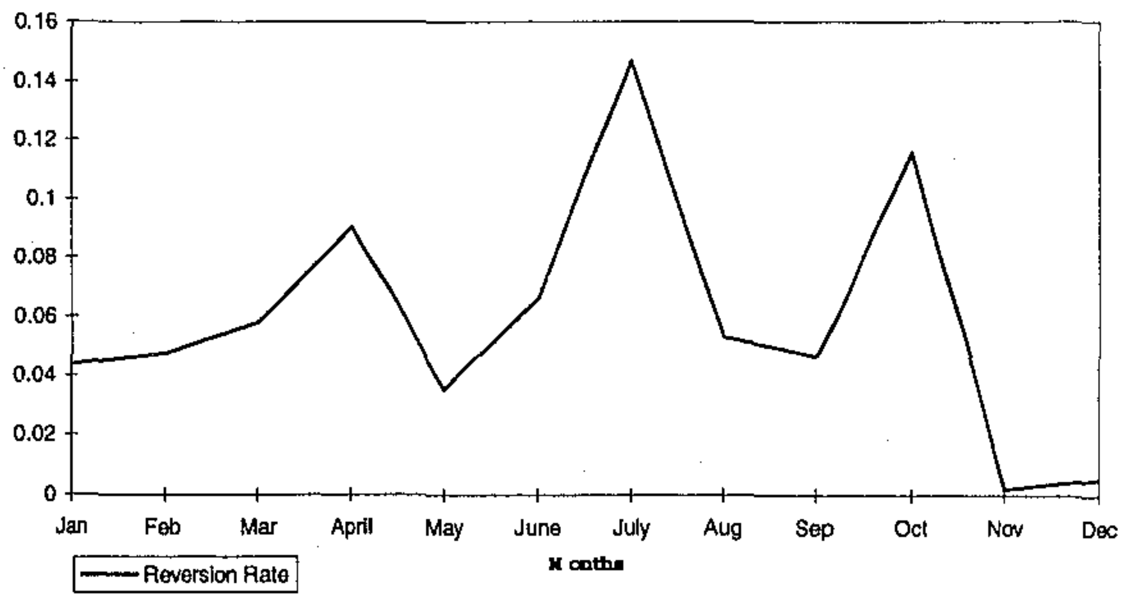
The reversion rate (μ_a) fluctuates significantly through the months over the year with the three highest rates occurring in April, July and October (Graph 4). The highest value reaches up to around 0.15. There is a considerable drop from 0.12 to less than 0.002

from October to November. The rate rises slightly in December. This is consistent with the increase in the Vasicek volatility in that month (Graph 2).

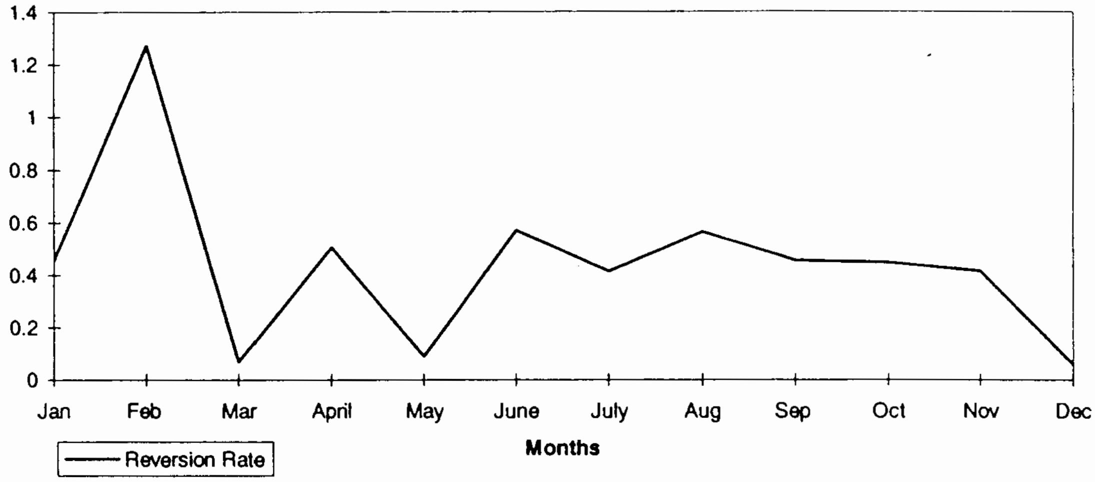
Reversion Rate for the HJM Model (μ_b)

The lowest reversion rates (μ_b) occur in months March, May and December (Graph 5). The biggest change is from January to March where the reversion rate rises from 0.45 to 1.27 and drops back to around 0.15 in March. The value is quite consistent from June to November.

Graph 4
Extended-Vasicek Model: Changes in Reversion Rate Over Time



Graph 5
HJM Model: Changes in Reversion Rate Over Time



4.2 PRICING DEVIATIONS

4.2.1 MONEYNESS

Black Model and Asay Model

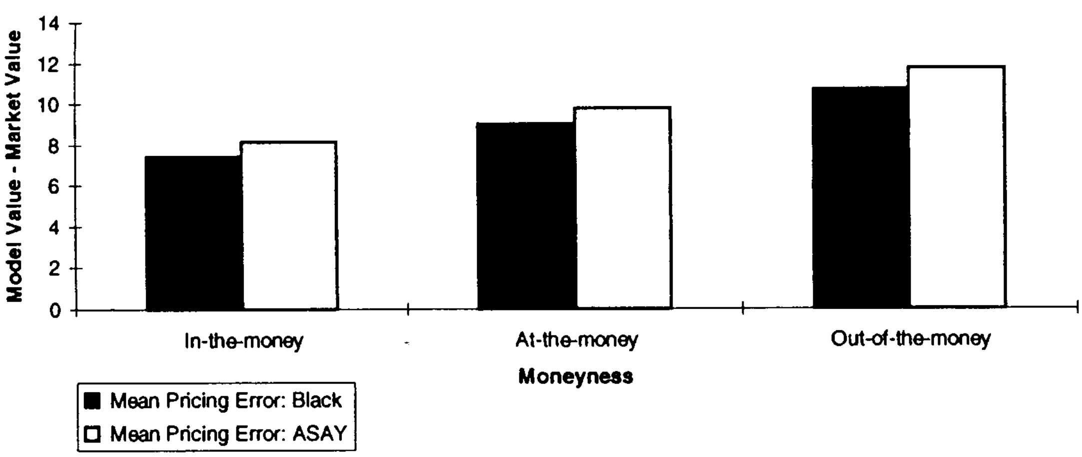
The manner in which pricing error changes for the four models when the options are grouped into in-the-money, at-the-money and out-of-the-money[1] is illustrated in Graphs 6 & 7. The graphs are plotted using data shown in Table A.1 & A.2 in the Appendices. All mean pricing errors for the Black and Asay models are positive. As shown in Graph 6, the mean pricing error for both models increases as the value of the 'futures price - strike price' increases. However, the Asay model shows a higher mean pricing error.

[1]The range of pricing error for different option categories:
In-the-money options - (Futures Price - Strike Price) > 0.02
At-the-money options - -0.02 ≤ (Futures Price - Strike Price) ≤ 0.02
Out-of-the-money options: - (Futures Price - Strike Price) < 0.02
The categories are chosen to make sure the at-the-money options have a small of difference between the futures price and the strike price.

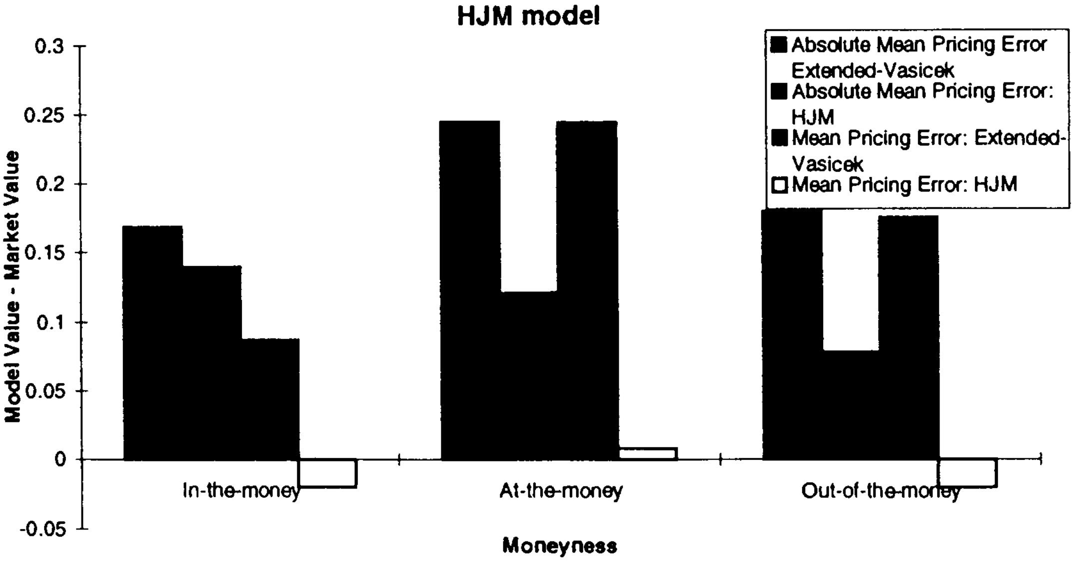
Term Structure Models

The comparison of the mean absolute pricing error between the Extended-Vasicek model and the HJM model shows that the absolute error is considerably smaller for the HJM model (Graph 7). This supports the fact that the HJM model is a relatively better model to use. As illustrated in Graph 7, the HJM model overprices at-the-money options but underprices both in-the-money and out-of-the-money options. Contrarily, the Extended-Vasicek model overprices options in all categories.

Graph 6
Mean Pricing Error by Moneyness: Black Model and ASAY Model



Graph 7
Comparison of Pricing Errors for Extended-Vasicek model and HJM model

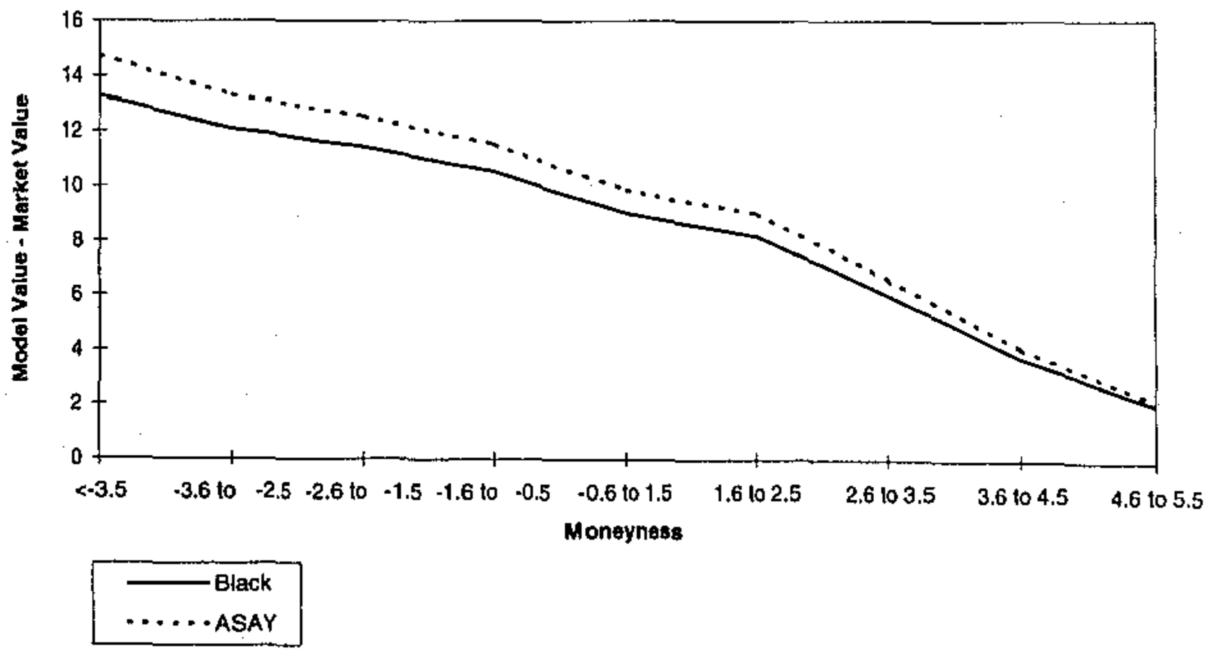


Graphs 8, 9 & 10 show how the mean pricing error and the mean absolute pricing error change against the degree which the options are in or out-of-the-money for the four models. The degree to which the options are in or out-of-the-money is calculated using the futures price minus the strike price. This is categorized into groups for analyzing the pricing error.

Black Model and Asay Model

Graph 8 shows how the mean pricing error changes for the Black model and the Asay model with different degrees of moneyness. Both models overprice the options. Overall, the pricing error decreases as moneyness increases. The Asay model creates a higher pricing error but the difference in error between the two models also decreases when moneyness increases. The pricing errors decrease with an increasing rate from category 1.6 to 2.5 onwards.

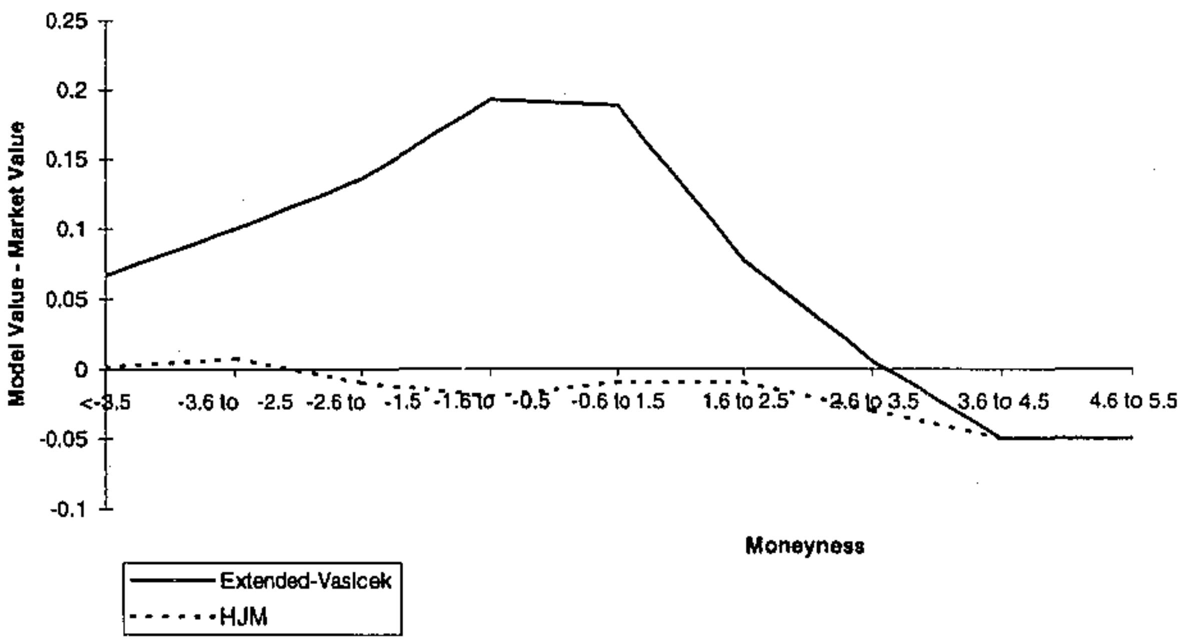
Graph 8
Mean Pricing Error: Black Model and Asay Model



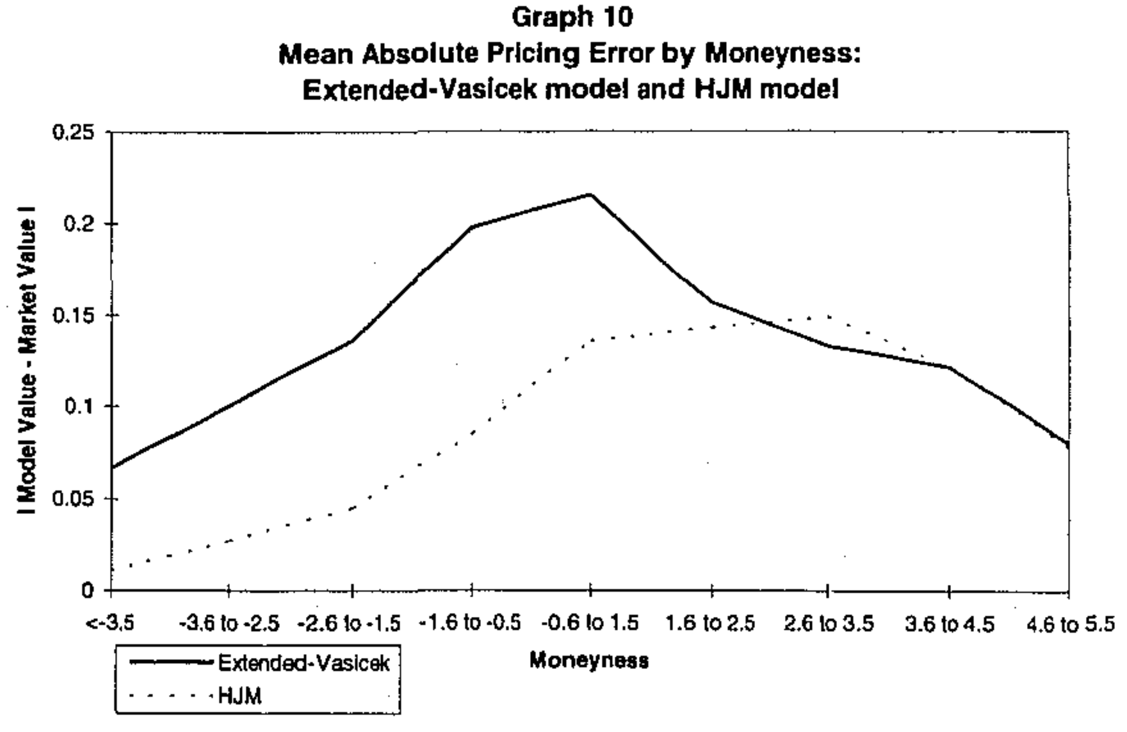
Term Structure Models

From Graph 9, it can be seen that the Extended-Vasicek model underprices deep in-the-money options with moneyness 2.6 to 5.5 but overprices the other categories. Mean pricing error increases from category <-3.5 to ranges -1.6 to -0.5 then decreases again from -0.6 to 2.5 onwards. The HJM model overprices deep out-of-the-money options but underprices most of the options in other ranges of moneyness. Mean pricing error decrease significantly for deep in-the-money options (from ranges 1.6 to 2.5 onwards).

Graph 9
Mean Pricing Error by Moneyness: Extended-Vasicek model and HJM model



Looking at the mean absolute pricing error in Graph 10, it shows that the mean absolute pricing error decreases significantly when using the Extended-Vasicek model for in-the-money options from category -0.6 to 1.5 onwards. There is not much difference in error between the two models for deep in-the-money options. Only in the ranges 2.6 to 3.5 does the HJM model have a higher mean absolute pricing error. Moreover, there is a considerable decrease in mean absolute pricing error using the HJM model for at-the-money and out-of-the-money options. Overall, there is a general increase in trend for mean absolute pricing errors as moneyness increases up to category -0.6 to 1.5.

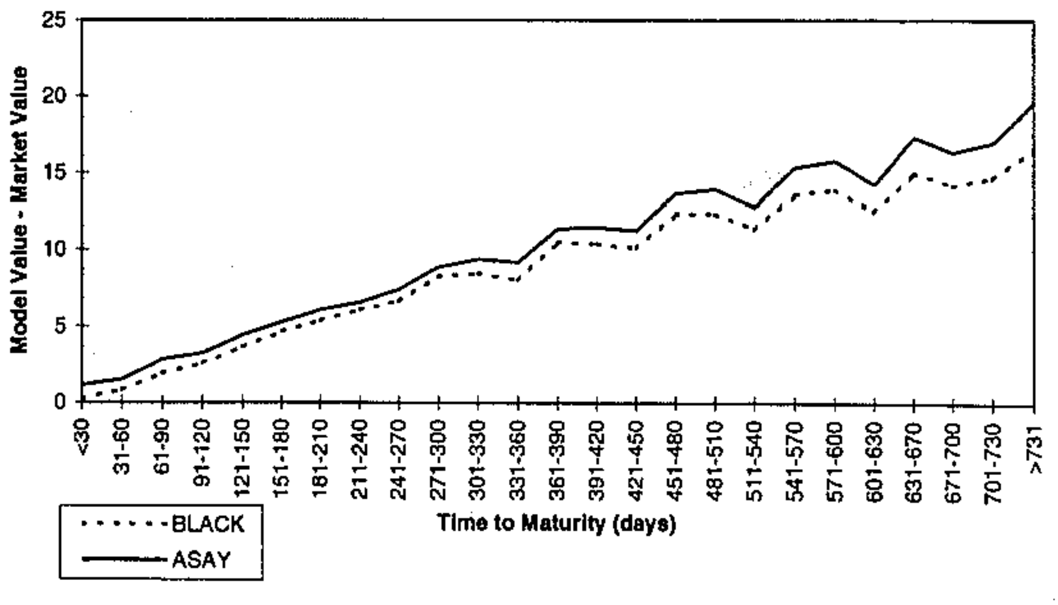


4.2.2 TIME TO MATURITY

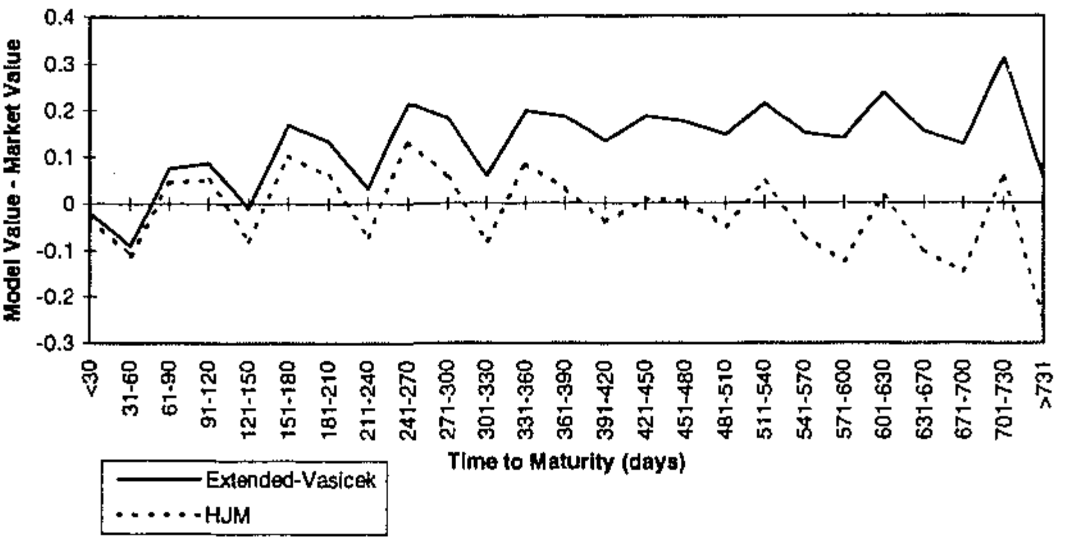
Graphs 11 - 19 describe the change in mean pricing error and mean absolute pricing error as time to maturity increases for in-the-money, at-the-money and out-of-the-money options for the four models. Information on data involved are presented the Table A.3 in the Appendices. Graph 11 & 12 are the mean pricing error for the four models when all options are grouped together. Graph 11 demonstrates that as time to maturity increases, the mean pricing error and mean absolute pricing error increases for both the Black model and the Asay model. Graph 12 shows that as time to maturity increases, the mean pricing error for the Extended-Vasicek model on average increase while that for the HJM model decreases. The graphs also show that the pricing error starts to fluctuate as time to maturity reaches to around 9 months (270 days). Graphs 13-15 provide the same information with options groupings according to their moneyness.

In Graphs 13 & 14, the mean pricing error appears to fluctuate but there seems to be a general decreasing trend for in and at-the-money options for the HJM model. For out-of-the-money options (Graph 15), mean pricing error decreases for options with time to maturity of 331 days (13 months) or higher for the HJM model. The mean pricing error also decreases significantly for options with time to maturity of 701 days (24 months) or higher. In general, the mean pricing error fluctuates from negative figures to positive figures every three months as time to maturity increases.

Graph 11
Mean Pricing Error by Time to Maturity:
Black Model and Asay Model - All options

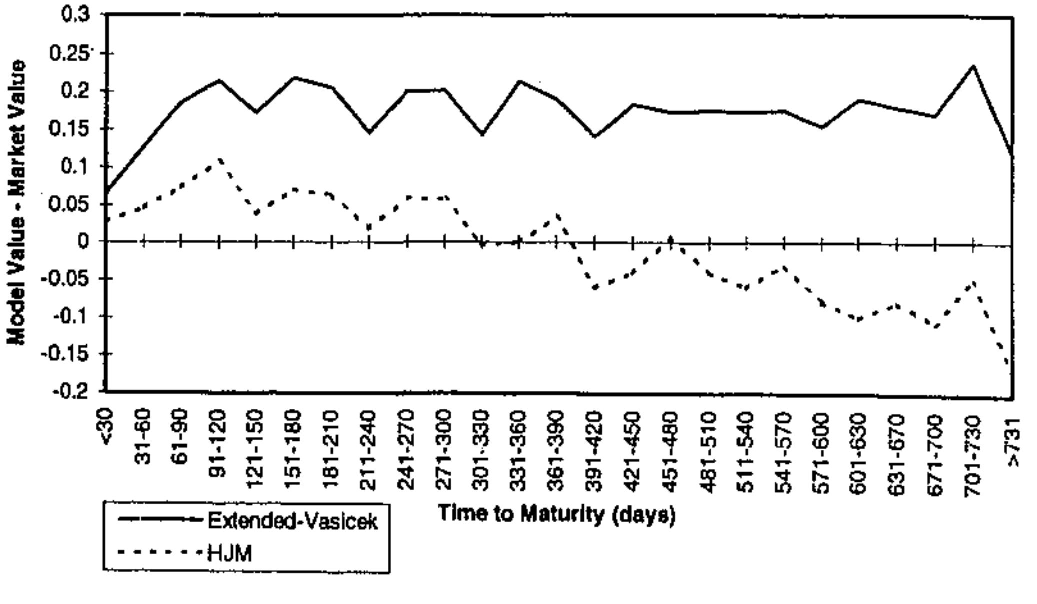


Graph 12
Mean Pricing Error by Time to Maturity: Extended-Vasicek Model and HJM Model - All options

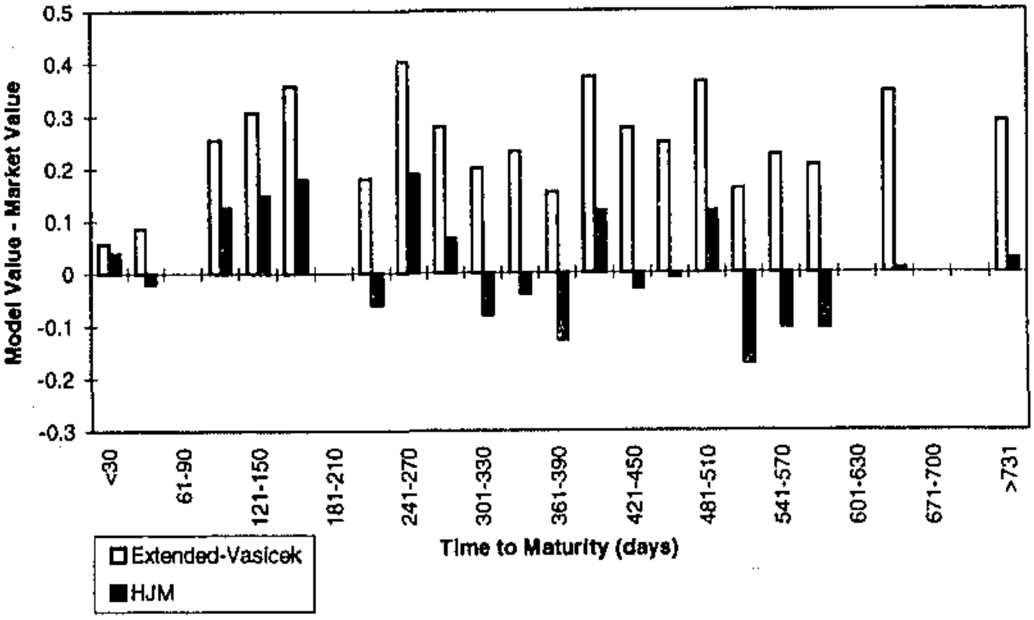


The mean absolute pricing errors for the four models have similar trends to those of the mean pricing errors (Graphs 16-19). They clearly show the overall magnitude of the fluctuation. Graph 16 demonstrate that there is a general increasing trend for the mean absolute pricing error for the term structure models as time to maturity increases.

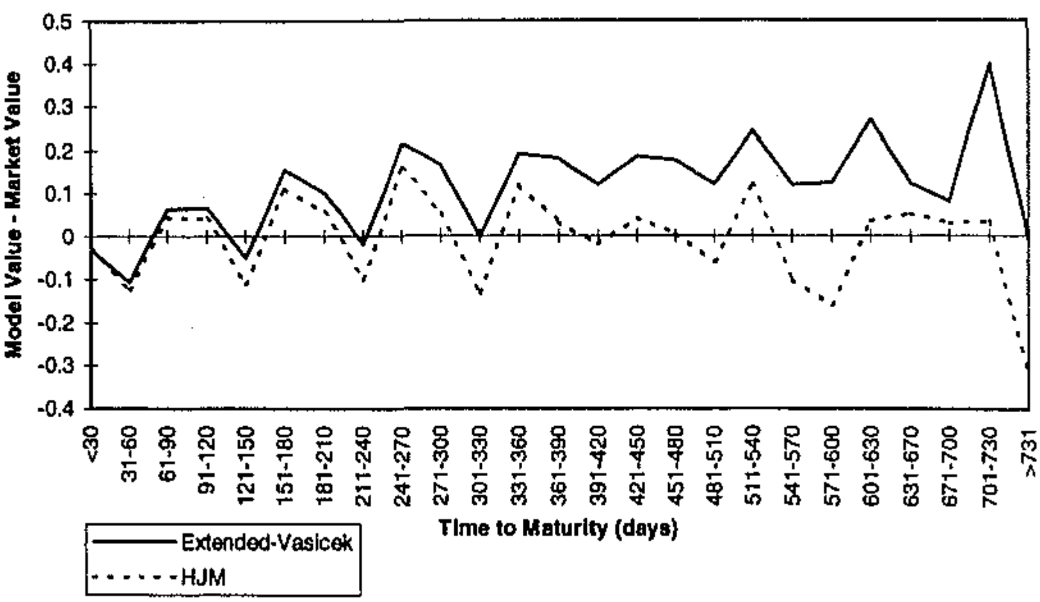
Graph 13
Mean Pricing Error by Time to Maturity:
Extended-Vasicek Model and HJM Model - In-the-money options



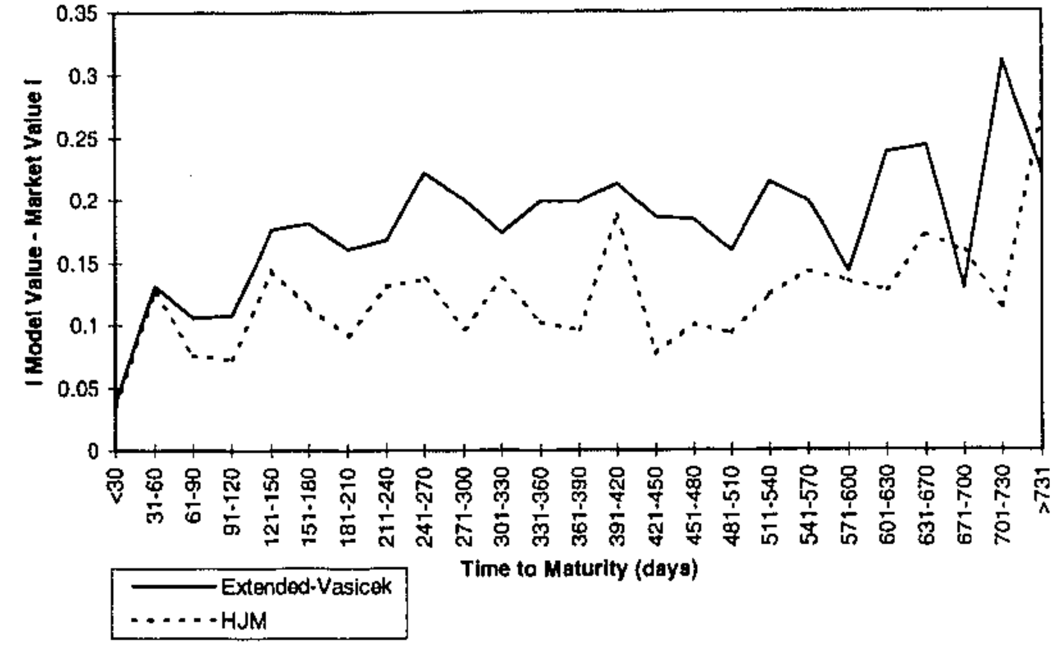
Graph 14
Mean Pricing Error by Time to Maturity:
Extended-Vasicek Model and HJM Model- At-the-money options



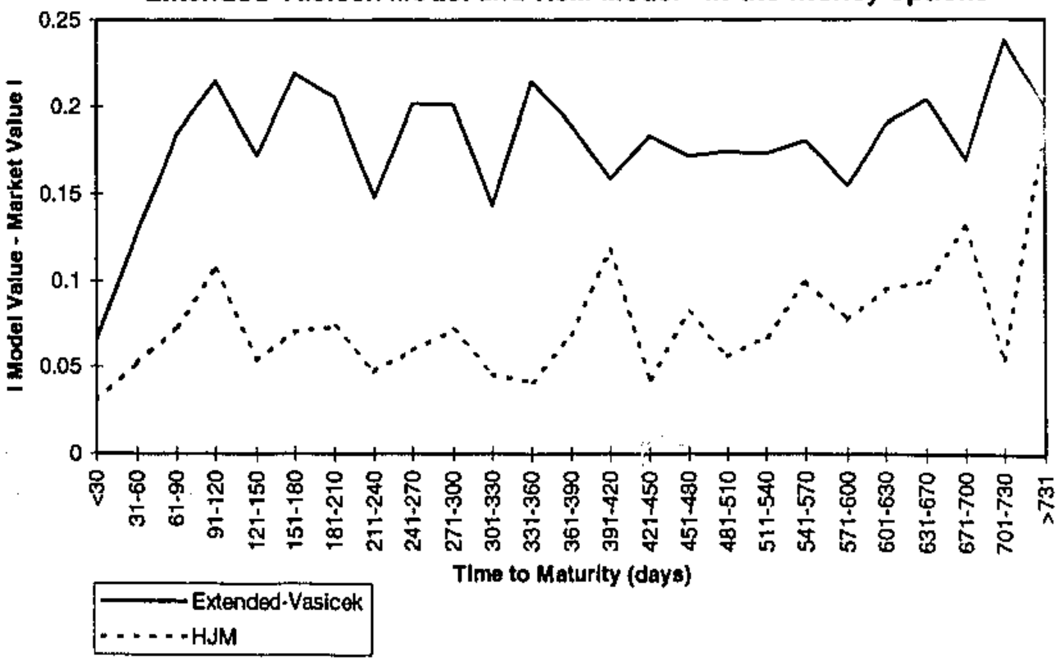
Graph 15
Mean Pricing Error by Time to Maturity:
Extended-Vasicek Model and HJM Model - Out-of-the-money options



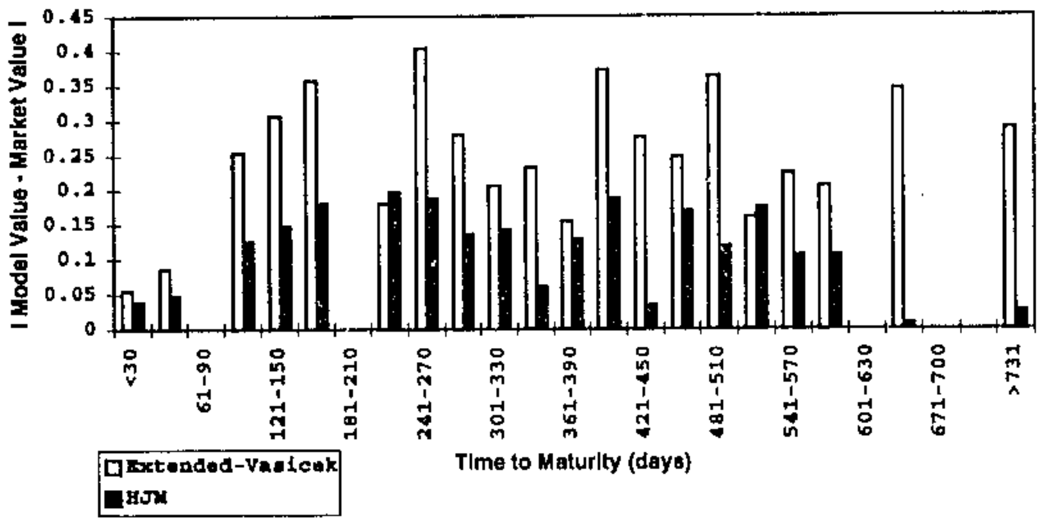
Graph 16
Mean Absolute Pricing Error by Time to Maturity:
Extended-Vasicek Model and HJM Model - All options



Graph 17
Mean Absolute Pricing Error by Time to Maturity:
Extended-Vasicek Model and HJM Model - In-the-money options



Graph 18
Mean Absolute pricing error by Time to Maturity: Extended-Vasicek Model and HJM Model - At the money options



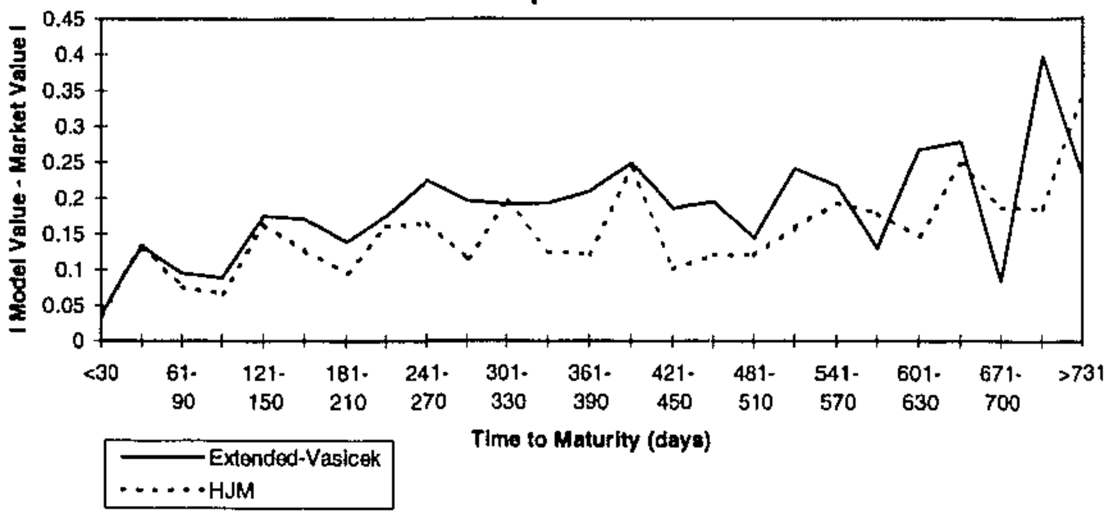
As shown in Graphs 17 & 18, the HJM model leads to a lower pricing error for in- and at-the-money options. For in-the-money options (Graph 17), mean absolute pricing error increases as time to maturity increases. The error tends to be lower for short-term (<90 days) and long-term (>600 days) at-the-money options when priced using the HJM model (Graph 18). Graph 19 shows a similar trend to Graph 16 for out-of-the-money options.

4.2.3 VOLATILITY

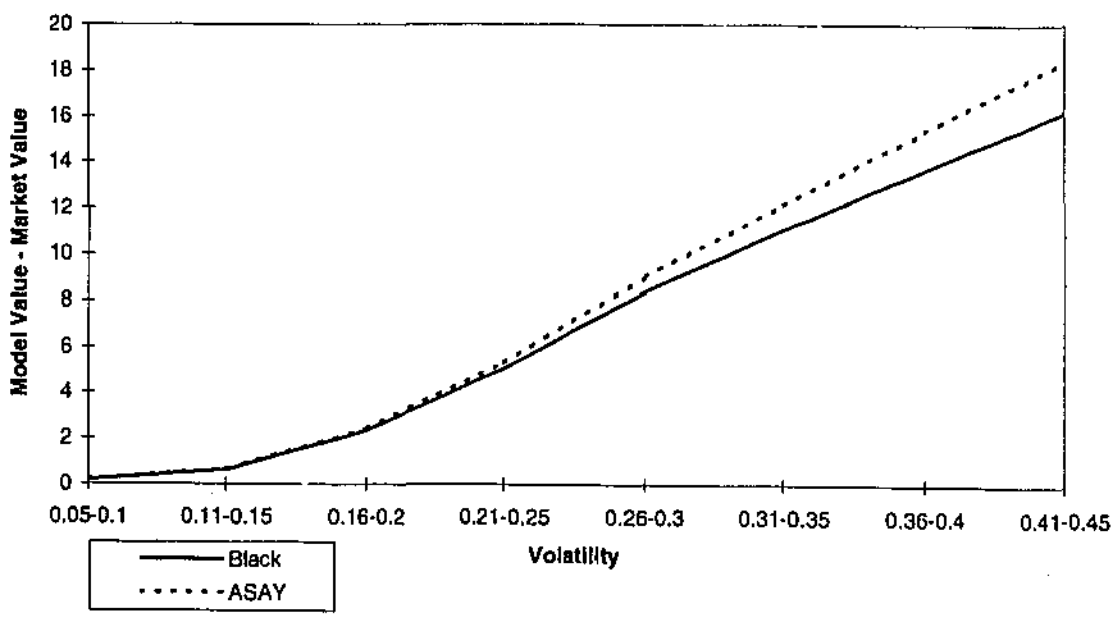
Black Model and Asay Model

The relationship between mean pricing error changes and different volatility ranges for the Black model and the Asay model are presented in Graph 20. Mean pricing error increases as volatility increases. The mean pricing error for the Asay model also becomes higher compared to the Black model as volatility increases.

Graph 19
Mean Absolute Pricing Error by Time to Maturity:
Extended-Vasicek Model and HJM Model - Out-of-the-money options



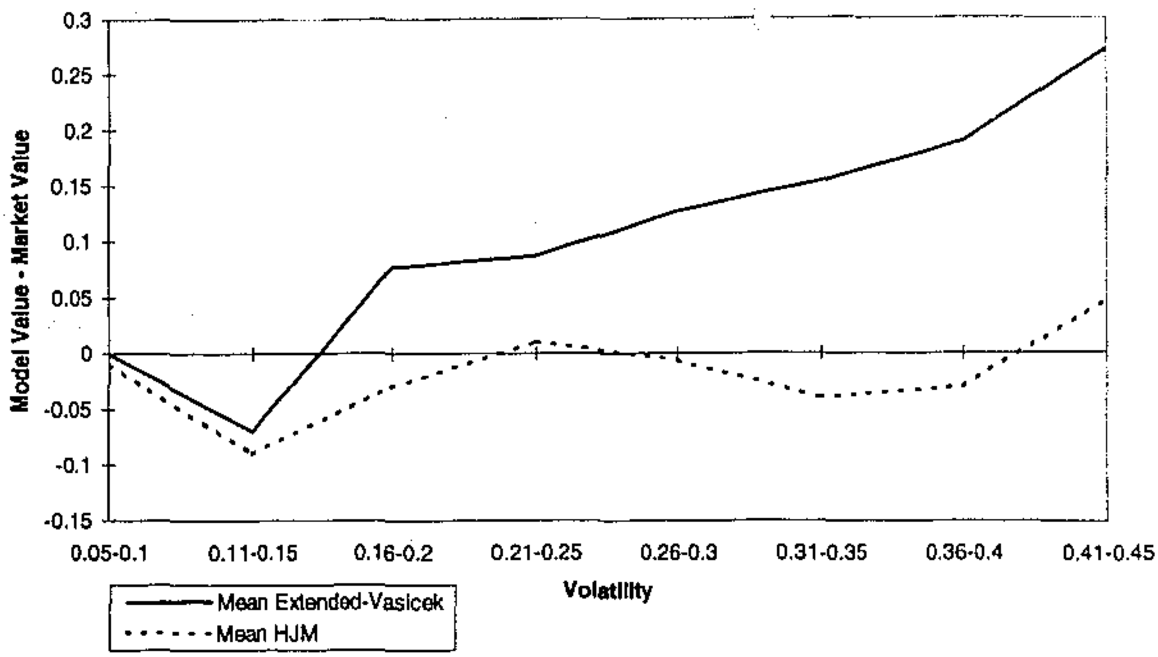
Graph 20
Mean Pricing Error by Volatility: Black Model and Asay Model



Term Structure Models

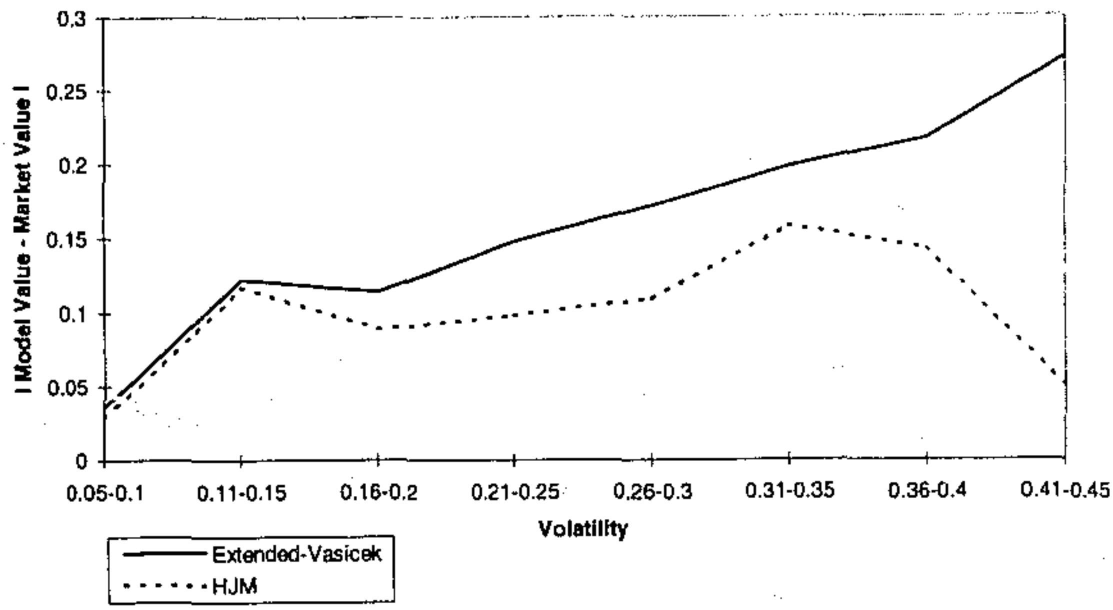
The mean pricing error for at-the-money options shows a completely different trend for the two term structure models. The Extended-Vasicek model shows that pricing error is negative for options with volatility values of less than 0.16 (Graph 21). Above 0.16, pricing error tends to be positive. The model overprices the options above this point and pricing error increases as volatility increases. The HJM model only overprices options in volatility ranges 0.21-0.25 and 0.41-0.45. The model underprices options in other categories.

Graph 21
Mean Pricing Error by Volatility: Extended-Vasicek Model and HJM Model



The absolute pricing error in Graph 22 shows that the error is lower especially for options in categories 0.05-0.1 and 0.41-0.45 (deep in and out-of-the-money options). The mean absolute pricing error is lower when the HJM model is used. Both term structure models show approximately the same trend for volatility ranges from 0.05 to 0.35. With volatility values of 0.35 or above, mean absolute pricing error for the HJM model decreases while that for the Extended-Vasicek model increases. The difference in error between the two models then becomes larger.

Graph 22
Mean Absolute Pricing Error by Volatility:
Extended-Vasicek Model and HJM Model

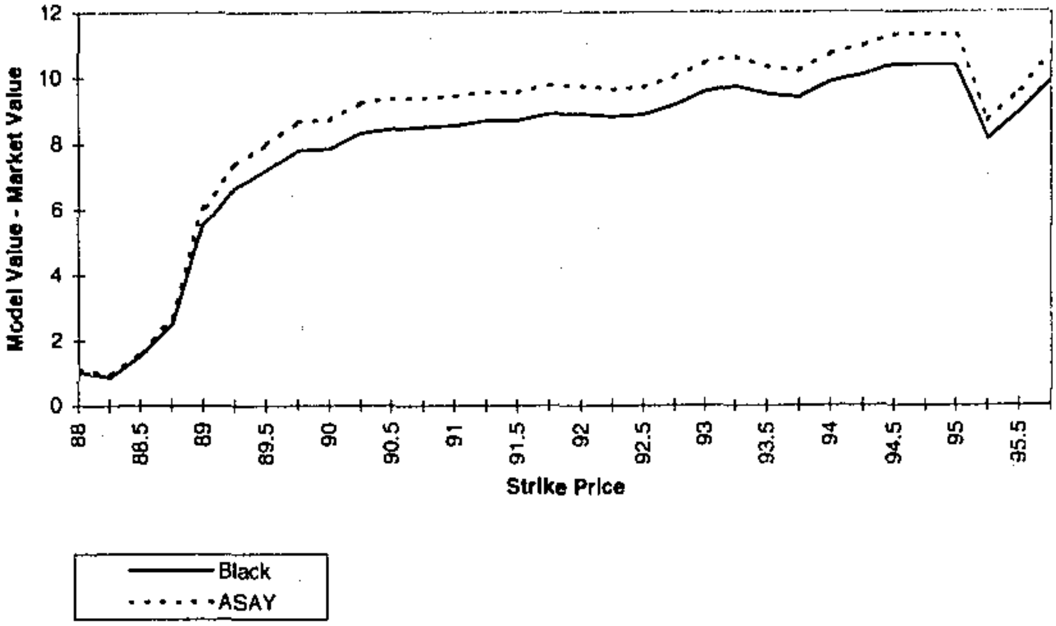


4.2.4 STRIKE PRICE

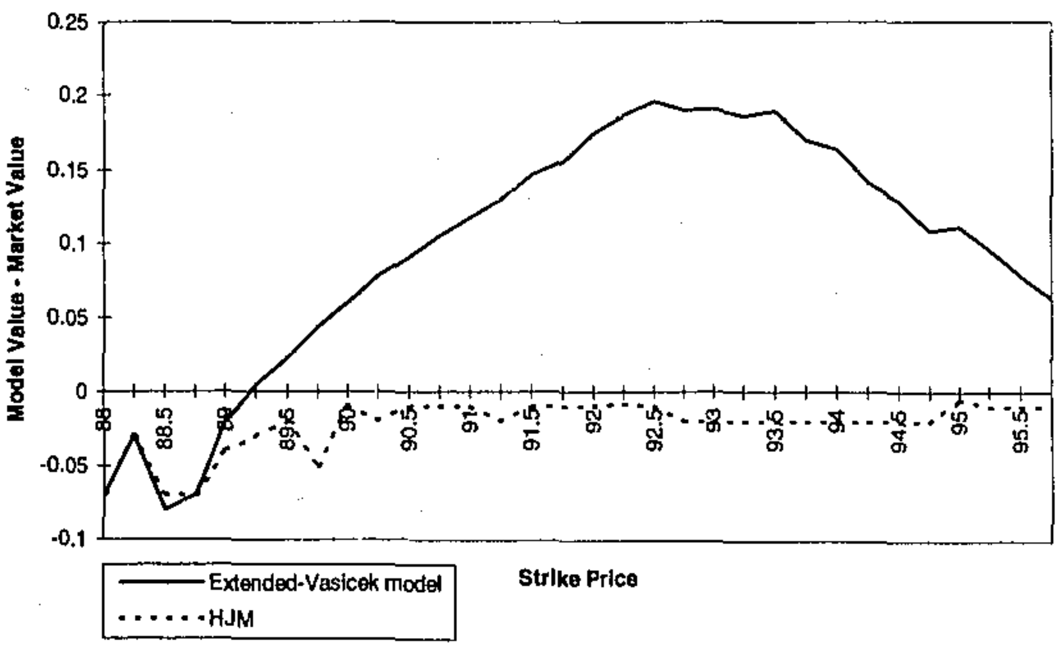
Black Model and Asay Model

Data in Table A.5 in the Appendices are plotted and shown in Graphs 23-25. The change in pattern for mean pricing error and mean absolute pricing error for the Black Model and Asay Model is the same. On the whole, the error for the Asay model is higher compared to the Black model's. Error increases at a very high rate from a strike price of 88 to 90. Except for a strike price of 95.25, the rate of increase in mean pricing error at strike price above 90 is relatively less in comparison to that below 89.5.

Graph 23
Mean Pricing Error: Black Model and Asay Model



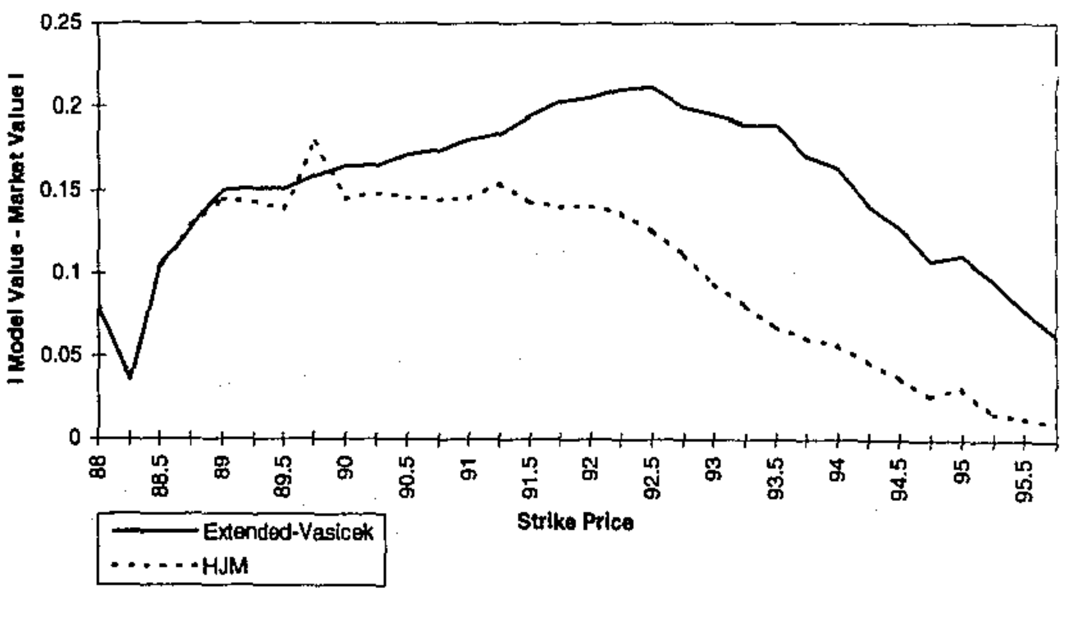
Graph 24
Mean Pricing Error by Strike Price: Extended-Vasicek Model and HJM Model



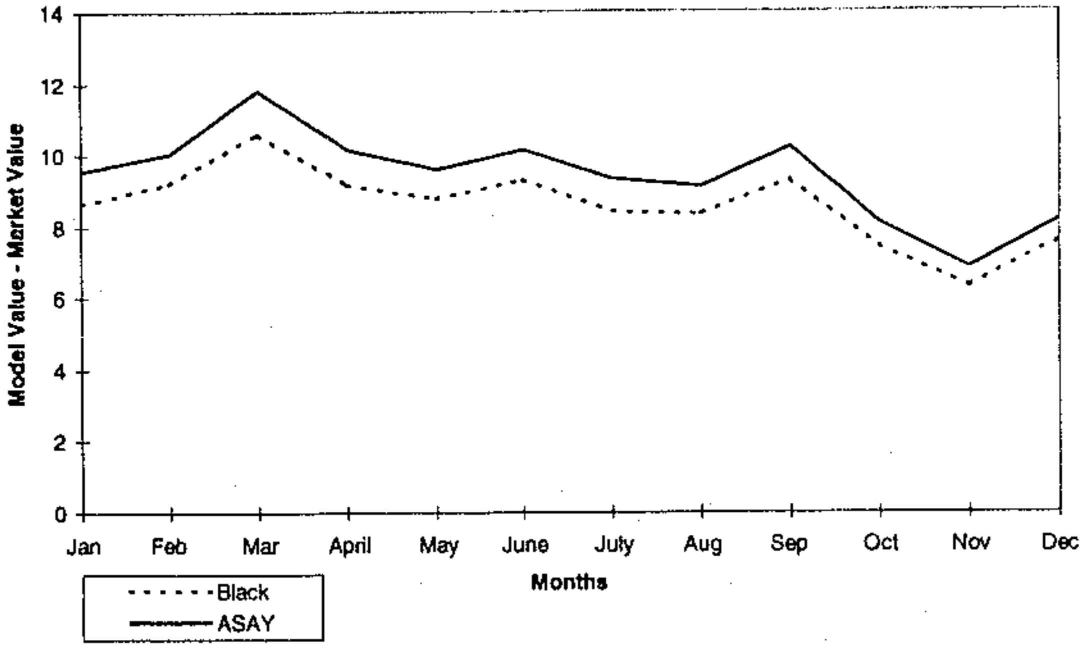
Term Structure Models

Graphs 24 & 25 show how mean pricing error and mean absolute pricing error change as strike price increases. In Graph 24, it shows that the HJM model tends to underprice all options with different strike prices. Mean pricing error fluctuates between 0 to -0.1 with error between 0 to -0.05 above strike price 89.25. The Extended-Vasicek model overprices options with strike price of above 89.25 but underprices options with strike prices below that point. Mean pricing error increases as strike price increases up to 92.5. Thereafter, error tends to drop subsequently as strike price increases. This shows that the HJM model gives a better approximation for pricing since the spread of the error is of a lesser extent. As seen in Graph 25 for mean absolute pricing error, the HJM model provides an overall lower absolute error in pricing. Moreover, as strike price increases, the HJM model gives a substantially lower error compared to the Extended-Vasicek model.

Graph 25
Mean Absolute Pricing Error by strike price: Extended-Vasicek Model and HJM Model



Graph 26
Mean Pricing Error Over Time: Black Model and Asay Model



4.2.5 CHANGE IN PRICING ERROR OVER TIME

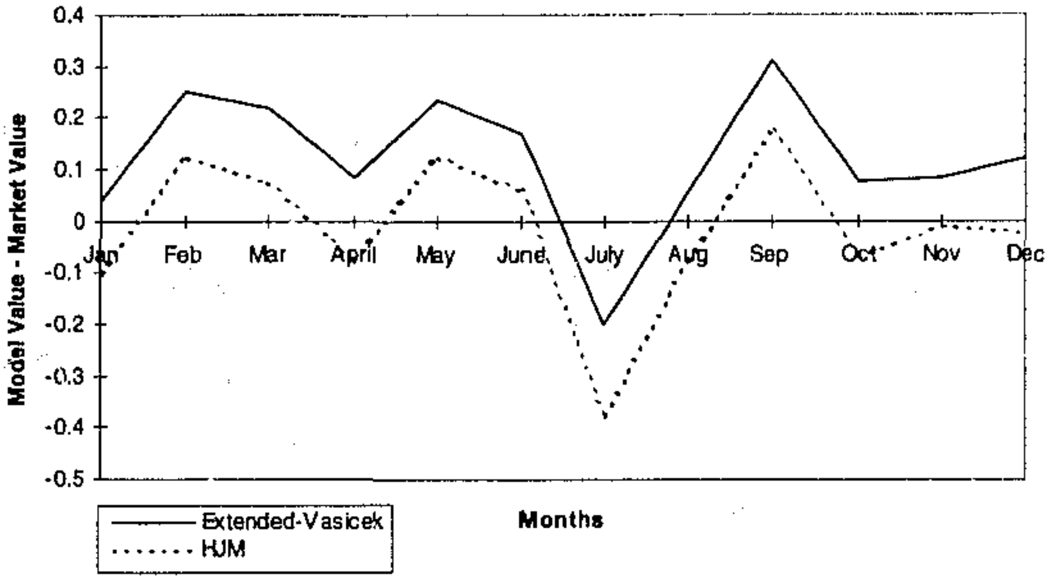
Black Model and Asay Model

Graphs 26-28 are generated from data presented in Table A.6 in the Appendices. Mean pricing error and mean absolute pricing error for Black model and Asay model have the same change in pattern as illustrated in Graph 26. The errors range between 6 to 12 and they tend to decrease over the months. Again, the Black model provides lower errors compared to the Asay model.

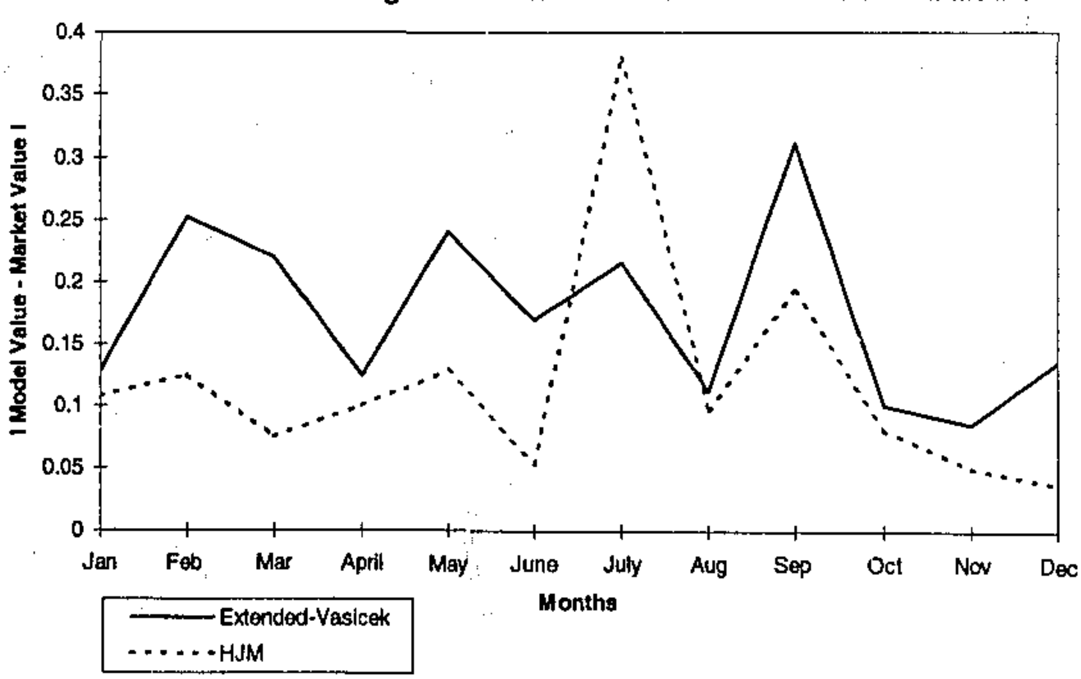
Term Structure Models

Graphs 27 & 28 demonstrate how pricing errors change over time for both term structure models. With the exception for the month of July, the Extended-Vasicek model appears to overprice the options most of the time (Graph 27). Mean pricing error for the HJM model fluctuates above and below zero over the months in 1996. The mean absolute pricing error for HJM model is lower for all months except for July when the Extended-Vasicek model underprices the options (Graph 28).

Graph 27
Mean Pricing Error Over Time: Extended-Vasicek Model and HJM Model



Graph 28
Mean Absolute Pricing Error: Extended-Vasicek Model and HJM Model



4.3 WILCOXON SIGNED RANKS TESTS

Results from the use of the Wilcoxon Signed Ranks Test in testing pricing errors and absolute pricing errors for the three models are given in Tables 14, 15, 16 &17. The sign tests report the mean rank, the sum of the ranks and the test statistics for mean and absolute pricing errors. The test statistics are based on the ranks of the absolute value difference between the two variables. The pricing errors and the absolute pricing errors generated by different models are grouped by pairs in order to test for their significance. 2-tailed tests are used to compare the data. In Tables 14, 15, 16 &17, the mean and mean absolute pricing errors of the Black Model, the Extended-Vasicek Model and the HJM Model are shown to be significantly different from each other at the 0.05 level. Therefore, the null hypothesis that the pricing errors and the absolute pricing errors of the four models have the same distribution can be rejected.

Table 14 & 15 analyze the pricing error for the four models, they show that the number of the negative ranks is greater than the number of positive ranks when: 1) Black Error - Asay Error; 2)Vasicek Error - Black Error; 3) HJM Error - Vasicek Error.

This demonstrates that:

1) Black Error < Asay Error;

2)Vasicek Error < Black Error

3) HJM Error < Vasicek Error.

Similarly, by looking at Tables 16 & 17, they prove that:

1) Absolute Black Error < Absolute Asay Error;

2) Absolute Vasicek Error < Absolute Black Error

3) Absolute HJM Error < Absolute Vasicek Error.

Table 14

Tables 14 & 15 show the nonparametric statistics for the pricing error for the Black model, Asay model, Extended-Vasicek model and HJM model for call options on 90-Day Bank Accepted Bill Futures.

Ranks

		N	Mean Rank	Sum of Ranks
Black Error - Asay Error	Negative Ranks	2097[a]	1049.00	2199753
	Positive Ranks	0[b]	.00	.00
	Ties	0[c]		
	Total	2097		
Vasicek Error - Black Error	Negative Ranks	2075[d]	1059.95	2199389
	Positive Ranks	22[e]	16.55	364.00
	Ties	0[f]		
	Total	2097		

a. Black Error < Asay Error

b. Black Error > Asay Error

c. Asay Error = Black Error

d. Vasicek Error < Black Error

e. Vasicek Error > Black Error

f. Black Error = Vasicek Error

Test Statistics[a]

	Black Error - Asay Error	Vasicek Error - Black Error
Z	-39.663[b]	-39.650[b]
Asymp. Sig. (2-tailed)	.000	.000

a. Wilcoxon Signed Ranks Test

b. Based on positive ranks.

Note: Pricing Error = Model Price - Actual Price
Absolute Pricing Error = I Model Price - Actual Price I
Black Error = Pricing Error for the Black Model
Asay Error = Pricing Error for the Asay Model
Vasicek Error = Pricing Error for the Extended-Vasicek Model
HJM Error = Pricing Error for the HJM Model
Abs Black Error = Absolute Pricing Error for the Black Model
Abs Asay Error = Absolute Pricing Error for the Asay Model
Abs Vasicek Error = Absolute Pricing Error for the Extended-Vasicek Model
Abs HJM Error = Absolute Pricing Error for the HJM Model

Table 15

Ranks

		N	Mean Rank	Sum of Ranks
Vasicek Error - HJM Error	Negative Ranks	340[a]	229.07	77883.50
	Positive Ranks	1753[b]	1205.64	2113488
	Ties	4[c]		
	Total	2097		

a. Vasicek Error < HJM Error

b. Vasicek Error > HJM Error

c. HJM Error = Vasicek Error

Test Statistics [a]

	Vasicek Error - HJM Error
Z	-36.808[b]
Asymp. Sig. (2-tailed)	.000

a. Wilcoxon Signed Ranks Test

b. Based on negative ranks.

Note: Pricing Error = Model Price - Actual Price
Absolute Pricing Error = | Model Price - Actual Price |
Black Error = Pricing Error for the Black Model
Asay Error = Pricing Error for the Asay Model
Vasicek Error = Pricing Error for the Extended-Vasicek Model
HJM Error = Pricing Error for the HJM Model
Abs Black Error = Absolute Pricing Error for the Black Model
Abs Asay Error = Absolute Pricing Error for the Asay Model
Abs Vasicek Error = Absolute Pricing Error for the Extended-Vasicek Model
Abs HJM Error = Absolute Pricing Error for the HJM Model

Table 16

Tables 16 & 17 show the nonparametric statistics for the pricing error for the Black model, Asay model, Extended-Vasicek model and HJM model for call options on 90-Day Bank Accepted Bill Futures.

Ranks

		N	Mean Rank	Sum of Ranks
Abs Black Error - Abs Asay Error	Negative Ranks	2070[a]	1062.02	2198385
	Positive Ranks	27[b]	50.67	1368.00
	Ties	0[c]		
	Total	2097		
Abs Vasicek Error - Abs Black Error	Negative Ranks	2078[d]	1058.20	2198931
	Positive Ranks	19[e]	43.26	822.00
	Ties	0[f]		
	Total	2097		

a. Abs Black Error < Abs Asay Error

b. Abs Black Error > Abs Asay Error

c. Abs Asay Error = Abs Black Error

d. Abs Vasicek Error < Abs Black Error

e. Abs Vasicek Error > Abs Black Error

f. Abs Black Error = Abs Vasicek Error

Test Statistics[a]

	Abs Black Error - Abs Asay Error	Abs Vasicek Error - Abs Black Error
Z	-39.613[b]	-39.633[b]
Asymp. Sig. (2-tailed)	.000	.000

a. Wilcoxon Signed Ranks Test

b. Based on positive ranks.

Note: Pricing Error = Model Price - Actual Price
Absolute Pricing Error = I Model Price - Actual Price I
Black Error = Pricing Error for the Black Model
Asay Error = Pricing Error for the Asay Model
Vasicek Error = Pricing Error for the Extended-Vasicek Model
HJM Error = Pricing Error for the HJM Model
Abs Black Error = Absolute Pricing Error for the Black Model
Abs Asay Error = Absolute Pricing Error for the Asay Model
Abs Vasicek Error = Absolute Pricing Error for the Extended-Vasicek Model
Abs HJM Error = Absolute Pricing Error for the HJM Model

Table 17

Ranks

		N	Mean Rank	Sum of Ranks
Abs Vasicek Error - Abs HJM Error	Negative Ranks	543[a]	856.06	464840
	Positive Ranks	1549[b]	1113.26	1724438
	Ties	5[c]		
	Total	2097		

a. Abs Vasicek Error < Abs HJM Error

b. Abs Vasicek Error > Abs HJm Error

c. Abs HJM Error = Abs Vasicek Error

Test Statistics [a]

	Abs Vasicek Error - Abs HJM Error
Z	-22.793[b]
Asymp. Sig. (2-tailed)	.000

a. Wilcoxon Signed Ranks Test

b. Based on negative ranks.

Note: Pricing Error = Model Price - Actual Price
Absolute Pricing Error = I Model Price - Actual Price I
Black Error = Pricing Error for the Black Model
Asay Error = Pricing Error for the Asay Model
Vasicek Error = Pricing Error for the Extended-Vasicek Model
HJM Error = Pricing Error for the HJM Model
Abs Black Error = Absolute Pricing Error for the Black Model
Abs Asay Error = Absolute Pricing Error for the Asay Model
Abs Vasicek Error = Absolute Pricing Error for the Extended-Vasicek Model
Abs HJM Error = Absolute Pricing Error for the HJM Model

Overall results can be summarized by their direction of dominance:

- Mean Pricing Error

Asay Model Error > Black Model Error > Extended-Vasicek Model Error > HJM Model Error

- Mean Absolute Pricing Error

Asay Model Error > Black Model Error > Extended-Vasicek Model Error > HJM Model Error

The Wilcoxon Signed Ranks tests reveal that the HJM model is the best pricing model to use. Evidence shows that the term structure models have considerably lower mean and mean absolute pricing error. However, as discussed previously, data on options and the underlying stock may not be taken at the same time and the market can appear to be nonsynchronous. This can affect the results generated.

4.4 THE OLS REGRESSIONS AND WHITE'S ADJUSTMENTS

Tables 18-21 show the Ordinary Least Square (OLS) Regressions for mean pricing error for the Black Model, the Asay Model, the Extended-Vasicek Model and the HJM Model. Mean pricing error is chosen as the dependent variable as the direction of error can be involved in the analysis. The pricing error was regressed on time to maturity and the degree to which options were in and out-of-the-money. The regressions were run for all the call options throughout the period. R-squares indicate the percentage of error variation explained by the independent variables. The correlation matrices of different variables are also given in the tables. They give an idea of how much the different variables relate to one another.

Black Model

The OLS Regression for the Black Model is shown in Table 18. Both the variables time-to-maturity and moneyness are statistically significant at the 0.05 level and are able to explain the error change. The coefficients in the regression have a t-value of

41.32 and -4.0840 respectively. The R-squares for the regression is demonstrated to be high (0.5780) suggesting that the variables are highly related to the pricing error for the Black Model.

The standard error of the estimate measures variation around the regression line. Moneyness (0.0391) has a higher standard deviation compared to time to maturity (0.0004) for the OLS regression. The factor time to maturity provides a more reliable prediction. Moneyness is negatively correlated to pricing error but time to maturity is shown to be positively correlated. Results show that the correlation between time to maturity (TTM) and moneyness (MON) is low (-0.3710) and negative. Since the correlation between the two variables is quite low, the regression is less likely to suffer from problems of multicollinearity.

Table 18 BLACK MODEL

THE OLS REGRESSION AND WHITE'S ADJUSTMENTS USING HETEROSKEDASTICITY-CONSISTENT COVARIANCE MATRIX
Call Options on 90-Day Bank Accepted Bill Futures

R_a measures the pricing error of the Black model on each option traded on the last Wednesday of the month in year 1996. Time to maturity of the options are based on 365 days. Moneyness is the difference between the future price and the strike price. The α and γ terms represent the intercept and coefficient of determination. R^2 is the explained variance of the regression. The total number of observations are 2098.

$$R_a = \alpha_a + TTM\gamma_1 + MON\gamma_2 + \varepsilon_a$$

where **TTM** = Time to maturity; **MON** = Moneyness

	α_a	TTM	MON
Estimated Coefficient	1.5836	0.0181(γ_1)	-0.1599(γ_2)
Standard Error	0.1372	0.0004	0.0391
T-Ratio	11.5500	41.3200	-4.0840
p-value	0.0000	0.0000	0.0000
Partial Correlation	0.2450	0.6700	-0.0890
Standardized Coefficient	0.0000	0.7352	-0.0619
Elasticity At Mean	0.2058	0.8117	-0.2058

R-SQUARE = 0.5780 R-SQUARE ADJUSTED = 0.5776
VARIANCE OF THE ESTIMATE-SIGMA **2 = 9.8112
STANDARD ERROR OF THE ESTIMATE-SIGMA= 3.1323
SUM OF SQUARED ERRORS-SSE = 20555.00

MEAN OF DEPENDENT VARIABLE = 7.6964
LOG OF THE LIKELIHOOD FUNCTION = -5370.85

Note: The values are based on a 5% significance level for a two-tailed test.
Pricing Error = Model price - Actual price

CORRELATION MATRIX OF VARIABLES

TTM	1.0000	
MON	-0.3710	1.0000
R_a	0.7578	-0.3347
	TTM	**MON**

Asay Model

The Asay Model is similar to the Black Model except the risk-free rate is not taken into account since discounting of both the futures price and the strike price is not required. In Table 19, the R-square has a value of 0.0535. Compared to the Black Model (R square = 0.5780), variables in the Asay Model are less able to explain the change in mean pricing error.

Table 19 ASAY MODEL

THE OLS REGRESSION AND WHITE'S ADJUSTMENTS
USING HETEROSKEDASTICITY-CONSISTENT COVARIANCE MATRIX
Call Options on 90-Day Bank Accepted Bill Futures

R_a measures the pricing error of the Asay model on each option traded on the last Wednesday of the month in year 1996. Time to maturity of the options are based on 365 days. Moneyness is the difference between the future price and the strike price. The α and γ terms represent the intercept and coefficient of determination. R^2 is the explained variance of the regression. The total number of observations are 2098.

$$R_a = \alpha_a + TTM\gamma_1 + MON\gamma_2 + \varepsilon_a$$

where **TTM** = Time to maturity; **MON** = Moneyness

	α_a	TTM	MON
Estimated Coefficient	0.1275	0.0001(γ_1)	-0.0021(γ_2)
Standard Error	0.0057	0.0000	0.0016
T-Ratio	22.5300	9.9890	-1.3760
p-value	0.0000	0.0000	0.1690
Partial Correlation	0.2450	0.2130	-0.0300
Standardized Coefficient	0.0000	0.2175	-0.0322
Elasticity At Mean	0.2058	0.2745	-0.0104

R-SQUARE = 0.0535 R-SQUARE ADJUSTED = 0.0526
VARIANCE OF THE ESTIMATE-SIGMA **2 = 0.0015
STANDARD ERROR OF THE ESTIMATE-SIGMA= 0.1207
SUM OF SQUARED ERRORS-SSE = 30.5210
MEAN OF DEPENDENT VARIABLE = 0.1733
LOG OF THE LIKELIHOOD FUNCTION = 1460.66

Note: The values are based on a 5% significance level for a two-tailed test.
Pricing Error = Model price - Actual price

CORRELATION MATRIX OF VARIABLES

TTM	1.0000	
MON	-0.3710	1.0000
R_a	0.7578	-0.3347
	TTM	**MON**

All variables are statistically significant at the 0.05 level. T-value for time to maturity is 9.9890 while that for moneyness is -1.3760. The standard error for time to maturity is again lower compared to moneyness. Similar to results presented in the Black Model, moneyness is negative correlated to the pricing error. The figures for both the estimated (-0.0021) and standardized coefficient (-0.0322) for moneyness are negative. Therefore, the higher the degree of moneyness, the lower the pricing error. This is also consistent with the result in Graph 8.

The variable time to maturity is positively correlated to pricing error. Partial correlation also suggests that time to maturity can better explain pricing error (0.2130 for TTM and -0.0300 for MON).

The Extended Vasicek Model

Table 20 presents the OLS regression with the mean pricing error of the Extended-Vasicek Model as the dependent variable. The results indicate that the values for the T-ratio for all the variables are high and significant at the 0.05 level. The T-ratio for time to maturity is very high (167.2). This indicates that time to maturity is a crucial factor that drives the mean pricing error. The T-Ratio for moneyness is also high (-20.11), although not as significant as that for time to maturity. The variables also have a very high correlation (shown by the R-square of 0.9429) with the mean pricing error. This suggests that the factors successfully account for the change in pricing errors.

Time to maturity has a higher standardized coefficient for the pricing error. The partial correlation for pricing error is lower and negative for moneyness. As with both the Black Model and the Asay Model, time to maturity is positively correlated while moneyness is negatively correlated with the mean pricing error.

The HJM Model

The OLS Regressions for the HJM Model are shown in Table 21. The variables have R-square values of 0.1535 which is lower than that of the Extended-Vasicek Model. The regression in this case is less able to predict the change in pricing error. Both the variables time to maturity and moneyness are significantly related to mean pricing error. As opposed to the regressions for the other models, the variable moneyness is more significant as shown with a T-Ratio of -13.84. The standardized coefficients for moneyness is larger than time to maturity in the OLS Regression. Similar to the other regressions, moneyness is still shown to be negatively correlated to the pricing error.

In this case, the point estimation for moneyness also has a higher standard deviation around the regression line (with a standard error of 0.0021 compared to 0 for time to maturity). Time to maturity provides a more reliable prediction for the pricing error.

Table 20 EXTENDED-VASICEK MODEL

THE OLS REGRESSION AND WHITE'S ADJUSTMENTS USING HETEROSKEDASTICITY-CONSISTENT COVARIANCE MATRIX
Call Options on 90-Day Bank Accepted Bill Futures

R_a measures the pricing error of the Extended-Vasicek model on each option traded on the last Wednesday of the month in year 1996. Time to maturity of the options are based on 365 days. Moneyness is the difference between the future price and the strike price. The α and γ terms represent the intercept and coefficient of determination. R^2 is the explained variance of the regression. The total number of observations are 2098.

$$R_a = \alpha_a + TTM\gamma_1 + MON\gamma_2 + \varepsilon_a$$

where **TTM** = Time to maturity; **MON** = Moneyness

	α_a	TTM	MON
Estimated Coefficient	1.1620	0.0245(γ_1)	-0.2965(γ_2)
Standard Error	0.0524	0.0001	0.0147
T-Ratio	22.1600	167.2000	-20.1100
p-value	0.0000	0.0000	0.0000
Partial Correlation	0.4360	0.9650	-0.4020
Standardized Coefficient	0.0000	0.9263	-0.1069
Elasticity At Mean	0.1240	0.9026	-0.0266

R-SQUARE = 0.9429 R-SQUARE ADJUSTED = 0.9428
VARIANCE OF THE ESTIMATE-SIGMA **2 = 1.5322
STANDARD ERROR OF THE ESTIMATE-SIGMA= 1.2378
SUM OF SQUARED ERRORS-SSE = 3210.00
MEAN OF DEPENDENT VARIABLE = 9.3669
LOG OF THE LIKELIHOOD FUNCTION = -3423.06

Note: The values are based on a 5% significance level for a two-tailed test.
Pricing Error = Model price - Actual price

CORRELATION MATRIX OF VARIABLES

TTM	1.0000	
MON	-0.3710	1.0000
R_a	0.7578	-0.3347
	TTM	**MON**

From the OLS regression and White's Adjustments, the variables time to maturity and moneyness best account for the mean pricing error in the Extended-Vasicek Model (R-square of 0.9429). This is followed by the Black Model (0.5780), the HJM Model (0.1535) and the Asay Model (0.0535). Knowledge of how the factors relate to pricing error enables conclusions to be made as to which model can provide the best estimate given the factors. In the next chapter, the results are presented with an overall analysis.

Table 21 HJM MODEL

THE OLS REGRESSION AND WHITE'S ADJUSTMENTS USING HETEROSKEDASTICITY-CONSISTENT COVARIANCE MATRIX

Call Options on 90-Day Bank Accepted Bill Futures

R_a measures the pricing error of the HJM model on each option traded on the last Wednesday of the month in year 1996. Time to maturity of the options are based on 365 days. Moneyness is the difference between the future price and the strike price. The α and γ terms represent the intercept and coefficient of determination. R^2 is the explained variance of the regression. The total number of observations are 2098.

$$R_a = \alpha_a + TTM\gamma_1 + MON\gamma_2 + \varepsilon_a$$

where **TTM** = Time to maturity; **MON** = Moneyness

	α_a	TTM	MON
Estimated Coefficient	0.0932	0.0002(γ_1)	-0.0286(γ_2)
Standard Error	0.0077	0.0000	0.0021
T-Ratio	12.1200	7.3020	-13.8400
p-value	0.0000	0.0000	0.0000
Partial Correlation	0.2560	0.1580	-0.2890
Standardized Coefficient	0.0000	0.1609	-0.3025
Elasticity At Mean	0.7817	0.4205	-0.2021

R-SQUARE = 0.1535 R-SQUARE ADJUSTED = 0.1527

VARIANCE OF THE ESTIMATE-SIGMA **2 = 0.0264

STANDARD ERROR OF THE ESTIMATE-SIGMA= 0.1626

SUM OF SQUARED ERRORS-SSE = 55.3730

MEAN OF DEPENDENT VARIABLE = 0.1192

LOG OF THE LIKELIHOOD FUNCTION = 835.822

Note: The values are based on a 5% significance level for a two-tailed test.
Pricing Error = Model price - Actual price

CORRELATION MATRIX OF VARIABLES

TTM	1.0000	
MON	-0.3710	1.0000
R_a	0.7578	-0.3347
	TTM	**MON**

Chapter 5: FINDINGS

5.1 CONCLUSION

From the analysis of the graphs, OLS regressions, inferential and nonparametric statistics, the following conclusions can be made:

Black Model and Asay Model

1) The Black model and Asay model significantly overprice in, at, and out-of-the money options (Graph 6).
2) Mean pricing error is lowest for in-the-money options. The pricing error for at-the-money options is greater, but error for out-of-the-money options is the greatest (Graph 6).
3) As moneyness increases, mean pricing error for both the Black model and the Asay model decreases (Graph 8).
4) As time-to-maturity increases, mean pricing error for the Black model and the Asay model increases. Both the Black model and Asay model prices options correctly with less than 30 days of maturity (with mean pricing error < 1) (Graph 11).
5) As volatility increases, mean pricing error for Black model and Asay model increases. The difference between the error of the two models increases as volatility increases with the Asay model having a higher mean pricing error (Graph 20).
6) The mean pricing error for Black model and Asay model increases at an increasing rate up to the point when strike price reaches 89.25. Error increases at a lesser extent when strike prices increase beyond that point (Graph 23).

Extended-Vasicek Model and HJM Model

1) The HJM model has lower mean absolute pricing error compared to the Extended-Vasicek model (Graph 7).
2) The Extended-Vasicek model underprices deep-in-the-money options but overprice other categories (Graph 9).
3) The HJM model overprices at-the-money options but underprices in and out-of-the-money options (Graph 7).
4) Mean absolute pricing error decreases for both term structure models as moneyness increases for in-the-money options (Graph 10).
5) The Extended-Vasicek model and the HJM model have significantly lower errors for deep in-the-money (4.6-5.5) and out-of-the-money options (<-3.5) (Graph 10).
6) Higher mean absolute pricing errors are observed for at-the-money options for both term structure models (Graph 10).
7) For in-the-money options, the HJM model overprices options with short time to maturity (up to 390 days/3months), but underprices options with longer time to maturity (Graph 13).
8) The mean absolute pricing error for both models fluctuates but there seems to be a general increasing trend as time to maturity increases (Graph 16).
9) The mean absolute error increases for the Extended-Vasicek model as volatility increases but the error for the HJM model tends to fluctuate more randomly (Graph 22).

Overall, the results show that mean pricing error of the term structure models are much lower than those of the Black model and the Asay model. The Wilcoxon Signed Ranks Tests demonstrate that the pricing errors and the absolute pricing errors of the three models have significantly different distributions. The mean and absolute errors are both the lowest for the HJM model.

The OLS Regression results can be summarized in Table 22 below. The OLS regression demonstrates that when using the Asay Model, moneyness fails to explain the pricing errors. Moneyness is significant at the 0.05 level for the Black Model, the Extended-Vasicek model and the HJM model. It is negatively correlated to mean pricing error in the four models. Time to maturity is positively correlated and significant to the mean pricing error of all models.

Table 22 OLS Regression results

(Mean pricing errors for the four models are regressed against the independent variables time to maturity and moneyness. The table summarized the results for the models.)

	Positively Correlated	Negatively Correlated	Insignificant Variables	Variables in the [illegible] regression
Black Model				
	Time to maturity	Moneyness,	Nil	Time to maturity
Asay Model				
	Time to maturity	Moneyness	Moneyness	Time to maturity
Extended-Vasicek Model				
	Time to maturity	Moneyness	Nil	Time to maturity
HJM Model				
	Time to maturity	Moneyness	Nil	Time to maturity

The Black model has an R-square of 0.5780. However, the model does not incorporate the futures-style margining of the Australian market. Since the R-square of the Extended-Vasicek model (0.9429) and the HJM model (0.1535) is higher than that of the Asay model (0.0535), this implies that the term structure of interest rates does provide important information in pricing options on 90-day Bank Accepted Bill Futures. The fact that the R-square for the HJM model is lower than that of the Extended-Vasicek's suggests the inclusion of the mean reversion of the forward rates may cause the variables to be less correlated.

The mean pricing error for the Black model is relatively high compared to the other term structure models. This implies that although the factors: time to maturity and moneyness, are highly correlated to the mean pricing error, the pricing method is not very accurate compared to other models. The HJM model has an R-square of 0.1535. From the Wilcoxon Signed Ranks tests and evidence from the graphs, the HJM model was found to have the lowest pricing error.

5.2 COMPARISON WITH SIMILAR RESEARCH

The Asay model was tested by Brace and Hodgson (1991) using different estimates of historical volatility. They compared the theoretical prices of call options using the Asay model with observed market prices. Actual standard deviation was regressed on both implied and historical volatility. They find that very low explanatory power is provided by implied volatilities.

Results in this study might be affected by market synchronization. Markets which are synchronous have trading in assets that take place simultaneously. For markets to be synchronous, there has to be parallel trading in two related securities. Moreover, data recording must also be synchronized. This is important as the data can accurately present the timing of the transaction and the time the information is made available to market participants.

The study by Brace and Hodgson (1991) uses transaction data with option and futures trades matched to within one minute, this reduces the problem of measurement errors. In this study, end-of-the-day data are taken from the SFE from the *Internet*, the above process done by Brace and Hodgson has not been followed. Moreover, there may not be enough liquidity in longer dated contracts to enable this to be done.

Brown and Taylor (1997) also examine the Asay model using options on the SPI futures contract. Their results show that the model significantly overprices call options, which is supported by this study. They find that for call options, out-of-the-money options are overpriced and in-the-money options are underpriced while at-the-money options are not significantly mispriced. On the other hand, it is shown in this study that mean pricing error increases in the order of:

in-the-money options < at-the-money options < out-of-the-money options

Brown and Taylor conclude that the model accurately prices short-term options for both calls and puts. Medium and long-term calls are overpriced. Their results are supported here as it is shown that the mean pricing error for the Asay model with short-term to maturity (<30 days) is much lower compared to options with longer-term to maturity.

They prove that the overall mispricing is largely driven by the in and out-of-the-money medium and long-term options. However, in this study, mean pricing error increases as time to maturity increases for all options (in, at and out-of-the-money options). The Asay model and the Black model tend to price in-the-money options accurately comparatively to at-the-money and out-of-the-money options. While the OLS regression demonstrate that time to maturity, and moneyness are significantly related to mean pricing error for the Black model; the magnitude of mispricing is high compared to the term structure models. This suggests the need to incorporate the effect of stochastic short and long-term interest rates. The comparison of the Black model and the Asay model in this study proves that the Black model is still the better alternative for pricing between the two.

Buhler, Uhrig, Walter and Weber (1995) value options on futures and interest rate warrants in the German market using the one-factor and two-factor inversion models. One and two-factor inversion models of Hull and White type and one- and two-factor Heath, Jarrow, Morton models are considered. They found that the one-factor inversion model underpriced out-of-the-money and at-the-money calls but the two-factor HJM model overpriced in-the-money calls. The deep-in-the-money calls were underpriced by all the models. However, in many cases, the absolute pricing error decreased when moneyness increased.

In this study, the Extended-Vasicek (one-factor inversion model) underpriced deep in-the-money options but overpriced the at- and out-of-the-money options. Consistent with the above study, the deep in-the-money calls were underpriced by the two term structure models. In addition, this study found that mean pricing errors tend to increase as volatility increases.

Buhler, Uhrig, Walter and Weber (1995) demonstrate that the average pricing errors and average absolute pricing errors decrease as time to maturity increase. On the contrary, this study reports a general increasing trend in mean absolute error for all the models investigated as time to maturity increases for call futures options in the Australian context. The difference may be caused by the futures-style margining characteristic in the SFE.

Mean pricing errors for the term structure models were found to fluctuate over time. No consistent pattern could be found. For the Black and Asay model, mean pricing error decreases over time.

Flesaker (1993) finds that the HJM model tends to overvalue short-term options relative to long-term options. The model also has a tendency to undervalue options on days when the interest rate level is below the average for the subperiod. It indicates that the interest rate volatility is positively correlated with the interest rate level. Flesaker also concludes that the model fails to provide a reasonably good approximation for maturities of less than a year. The possible explanation for the finding is that the study has ignored all credit risks as well as credit risk premia. Consistent with Flesaker's study, this study finds that the mean pricing errors tend to be high for comparatively short-term options (with time to maturity <3 months) when using both the Extended-Vasicek and the HJM model. However, his results, which state that the HJM model fails to provide a reasonably good approximation for maturities of less than a year, cannot be supported.

Buhler, Uhrig, Walter and Weber (1995) also tested an identical set of bond warrant data in order to highlight the differentiation between the models. Not only do they assess the model's ability to predict observed option prices, but the following points are also taken into account: Difficulty of the estimation of the input data, problems in fitting the volatility structures of interest rates, and numerical problems in solving the valuation model.

When estimating forward rate curves, the historical estimation of parameters using forward rate changes for the HJM models is very sensitive to small changes. A factor analysis must be carried out for the two-factor model parameter estimation. Volatility parameters can be reasonable only if the forward rate curves were smoothed by splines with a small number of nodes. In their study, the mean reverting parameter κ was obtained by a standard maximum likelihood estimation, therefore it is biased. κ has a strong influence on the volatility of long-term interest rates within the model. Buhler, Uhrig, Walter and Weber (1995) use an implicit estimate for κ that results in a fitting value for the volatility of the long-term rate instead of the maximum likelihood estimate of κ. There is difficulty in parameter estimation of the two-factor inversion model with stochastic interest volatility as the state variable volatility is not observable directly. The short-term rate has to be estimated first before the other parameters of the model can be obtained by solving a number of equations.

Fitting problems are created for the inversion models. Buhler, Uhrig, Walter and Weber (1995) use a numerical method for both the one-factor inversion model and the two-factor inversion rate model with stochastic interest rate volatility. Besides determining the time-dependent market price of risk required for the one-factor inversion model, the parameter κ has to be determined. This causes a fitting problem for the model. They found that the current structure of interest rates has to be smooth enough in order to avoid the strong variations of the time-dependent functions.

HJM models are non-markovian models in terms of volatility functions. Buhler, Uhrig, Walter and Weber (1995) conclude that within the two-factor inversion models, the valuation problem is more difficult for the two-factor model with long-rate and spread than for the two-factor model with stochastic volatility. The choice of an Ornstein-Uhlenbeck causes problems in the treatment of the boundaries.

The models with linear absolute volatility show the best performance. The one-factor HJM models perform better when compared to the two-factor HJM model. For the inversion models, the two-factor inversion models perform better than the one-factor inversion model. This shows that there are difficulties for the implementation of the HJM approaches and the one-factor inversion model. The two-factor inversion model with long rate and spread was found to be the best model due to its easy applicability.

The use of the *Optimum: Fixed Income Monis Software* helps to negate most of the valuation, estimation and fitting problems. Market data required is input into the software to calibrate the yield curve. The software calculates the theoretical prices of the options using the calibrations. Overall, the HJM model was found to be a better model for estimation. The following table compares previous research to this study.

Table 23 Comparision of this study with previous literature

Previous Study	Findings	This Study
Brown & Taylor (1997) (Tested the Asay Model)	1) Asay model overprices call options 2) For call options, -out-of-the-money options are overpriced -in-the-money options are underpriced -at-the-money options are not significantly mispriced 3) Asay model accurately prices short-term options 4) Medium and long-term calls are overpriced. 5) Mispricing largely due to in and out-of-the-money medium and long-term options.	1) Results Supported 2) Mean Pricing Error increases in the order of: in-the-money < at-the-money < out-of-the-money 3) Results Supported 4) Results Supported 5) Mean pricing error increases as time to maturity increases.
Buhler, Uhrig, Walter Weber (1995) (Tested theHull and White (1990) HJM model (1990)	1)One factor inversion model underpriced out-of -the money and at-the-money options 2) Two factor HJM model overpriced in-the-money options. 3) Deep in-the-money options underpriced by the models. 4) Mean pricing and absolute mean pricing error decrease as time to maturity increases.	1 & 2)Extended-Vasicek (one factor and inversion model) underpriced deep in-the-money options but overpriced the at- and out-of-the-money options 3) Results Supported 4) Increasing trend in mean absolute error as time to maturity increases or all models.
Flesaker (1993) (Tested the HJM model (1990))	1) HJM model overvalue short-term options 2) HJM model cannot provide a good approximation for options with maturity less than one year	1) Mean pricing errors are high for short -term options for the Extended-Vasicek model and HJM model. 2) No evidence

Chapter 6: CHAPTER REVIEW AND CONCLUSION

6.1 OVERVIEW

This chapter integrates the results of the previous chapter. It specifies the implications of the study and recommends the direction of future research.

The term structure of interest rates has been of primary interest to economists. The focus in this area is useful for risk management i.e. the pricing of fixed income securities and interest rate options. This study provides a guide to competing pricing models of interest-rate contingent claims in an Australian context. The results suggest that term structure models price options much better than the Black model and the Asay model. However, the pricing of these two models may be affected by estimation of their parameters. The volatility data extracted from the SFE is calculated using the Asay model. The accuracy of the volatility parameter may affect pricing when volatility is input into the Black and Asay formula. However, since this is the best estimate of volatility for each option and is most reliable for market participants, this was assumed to be representative of the drift in the option prices. Volatilities and reversion rates used for the term structure models were calculated by the estimate of the best fit using yield curves generated from the market data during the period, this provides a good approximation to the parameters. As the movements of both short-term and long-term interest rates are incorporated when using the term structure models, they are theoretically more sound. This study proves this to be true empirically.

The mean and mean absolute pricing errors were lower for the term structure models. For the Black model and Asay model, both the mean and mean absolute error increase steadily as time to maturity increases. Whereas the pricing errors for both the Extended-Vasicek and the HJM model fluctuate as time to maturity increases. In general, the mean absolute error shows a decreasing trend, especially for in-the-money options.

The Heath-Jarrow-Morton model (1992) is a better model to use. This suggests that the inclusion of the stochastic process of not only the short-term but also the long-term forward rates in pricing interest rate contingent claims is important.

For the two term structure models, there is evidence of a general increasing trend for pricing errors as moneyness increases. Both term structure models underprice in-the-money and at-the-money options but overprice the out-of-the-money options. Mean pricing error decreases for deep-in-the-money and out-of-the-money options. Moreover, the HJM model better prices at-the-money and out-of-the-money options. Mean and absolute pricing error increased as volatility increased.

The null hypothesis that the pricing errors of the four models have the same distribution was rejected. The OLS regressions show moneyness fails to explain the pricing error for the Asay model. Time to maturity is statistically significant at the 0.05 level and is able to explain the error change in both the Black model and the Asay model.

Regressions of the pricing errors for the Extended-Vasicek and the HJM model indicate that the values for the T-ratio for all the variables are high and significant at the 0.05 level. Both factors time to maturity and moneyness are able to relate to the pricing error for both models. The R-square for the mean pricing errors for the Extended-Vasicek model is very high (0.9429), this suggests a high correlation between the variables and the errors. The HJM model gives lower mean and absolute mean pricing errors, but the variables time to maturity and moneyness have lower correlation to the errors.

Nonsynchronous Markets

For markets to be synchronous, there has to be parallel trading in two related securities. Moreover, data recording must also be synchronized. This is important as that data can accurately present the timing of the transaction and the time the information is made available to market participants. In this study, data are taken from the SFE, data on options and the underlying stock are not screened. This may in turn affect the results of the study. As the R-square coefficient for the HJM model is lower than that of the Extended-Vasicek's, this indicates the long-term interest rate fluctuation may affect the degree to which the variables correlate to the mean pricing error. The Black model and the Asay model better price options with short-term maturities. However, no major synchronized data problems can be identified.

6.2 FUTURE RESEARCH

Since empirical research in the use of term structure models is limited, the application of the models in different instruments is encouraged. The models can be used to calculate the value of a coupon-bearing bond, a futures contract written on a coupon-bearing bond, interest rate futures, swaptions and caplets. One area of interest is to use Bond options obtained using the Hull and White trinomial tree and Extended-Vasicek model as the interest rate process. This can be compared to prices attained from Black's Bond Option Model. However, adjustments have to be made for accrued interest in this case. The pricing error differences of the call and put options can be analyzed based on factors like time-to-maturity and moneyness. The valuation of different types of derivative securities allow the analysis of portfolios.

Another area of interest is the use of the control variate technique. The price of American option (T_1) and European option (T_2) can be calculated from the Hull and White trinomial tree using the Extended-Vasicek interest rate process. When the Black's Bond Option Model is used to obtain the price of the European option (B), the improved estimate of the price of the American option can be:

$(T_1) + (T_2) - B$

Pricing errors can be determined to find out whether this method increases the accuracy of pricing.

The analysis of the hedging parameters can be an important contribution in this area of research. Delta, Gamma, Theta, Vega and Rho can be applied to hedge some of the interest rate contingent claims. This determines whether they are effective hedging elements.

As discussed, point process models are currently being introduced. They view the interest rate process as jumps instead of being represented by diffusion. More empirical work is needed to be done in this area in order to support their existence.

6.3 CLOSING COMMENT

Given the results found in this study, it was shown that the two-factor HJM model gives the best approximation when used price call options on 90-day Bank Accepted Bill futures. This is consistent with the theory that pricing models have a better performance when the term structure of both the short-term and long-term rates are included. The results documented motivate empirical studies of term structure models, where the volatility is allowed to vary across the maturity of forward rates, as well as across time. Challenging econometric problems are waiting along this route.

APPENDICES

Table A.1

Call Options on 90-day Bank Accepted Bill Futures: Descriptive Statistics for In-the-money, At-the-money and Out-of-the-money categories

		N	Minimum	Maximum	Mean		Std. Deviation
In/Out		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
M<-0.02	Black Error	708	.5486	17.1472	10.6689	.1307	3.4765
	Asay Error	708	.5496	19.9244	11.6934	.1541	4.1011
	Vasicek Error	708	-.1518	.6416	.1744	.0036	.0971
	HJM Error	708	-.5986	.3230	-.0169	.0043	.1148
	Abs Black Error	708	.5486	17.1472	10.6689	.1307	3.4765
	Abs Asay Error	708	.5496	19.9244	11.6934	.1541	4.1011
	Abs Vasicek Error	708	.0010	.6416	.1790	.0033	.0882
	Abs HJM Error	708	.0001	.5986	.0773	.0033	.0865
	Valid N (listwise)	708					
0.02<M<0.02	Black Error	37	.3194	17.0578	8.9888	.6994	4.2545
	Asay Error	37	.3196	19.8819	9.7640	.8048	4.8952
	Vasicek Error	37	-.0146	.5407	.2436	.0196	.1191
	HJM Error	37	-.3620	.3049	.0083	.0250	.1521
	Abs Black Error	37	.3194	17.0578	8.9888	.6994	4.2545
	Abs Asay Error	37	.3196	19.8819	9.7640	.8048	4.8952
	Abs Vasicek Error	37	.0146	.5407	.2444	.0193	.1174
	Abs HJM Error	37	.0045	.3620	.1205	.0150	.0911
	Valid N (listwise)	37					
M>0.02	Black Error	1352	-.0661	17.0014	7.4173	.1256	4.6171
	Asay Error	1352	-.0587	19.8386	8.1447	.1433	5.2679
	Vasicek Error	1352	-.5158	.6935	.0870	.0054	.2001
	HJM Error	1352	-3.2400	.6256	-.0224	.0058	.2126
	Abs Black Error	1352	.0012	17.0014	7.4185	.1255	4.6151
	Abs Asay Error	1352	.0021	19.8386	8.1458	.1432	5.2663
	Abs Vasicek Error	1352	.0001	.6935	.1684	.0038	.1387
	Abs HJM Error	1352	.0001	3.2400	.1395	.0044	.1620
	Valid N (listwise)	1352					

Note: M = Futures Price - Strike Price
Pricing Error = Model Price - Actual Price
Absolute Pricing Error = I Model Price - Actual Price I
Black Error = Pricing Error for the Black Model
Asay Error = Pricing Error for the Asay Model
Vasicek Error = Pricing Error for the Extended-Vasicek Model
HJM Error = Pricing Error for the HJM Model
Abs Black Error = Absolute Pricing Error for the Black Model
Abs Asay Error = Absolute Pricing Error for the Asay Model
Abs Vasicek Error = Absolute Pricing Error for the Extended-Vasicek Model
Abs HJM Error = Absolute Pricing Error for the HJM Model

Table A.2

Descriptive Statistics for Call options on 90-day Bank Accepted Bill Futures with Different degree of Moneyness

		N	Minimum	Maximum	Mean		Std. Deviation
M		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
< -3.5	Black Error	7	11.0994	15.4957	13.2539	.5475	1.4487
	Asay Error	7	11.9458	17.6099	14.7094	.7045	1.8640
	Vasicek Error	7	.0316	.0933	.0672	.0084	.0223
	HJM Error	7	-.0143	.0164	.0016	.0049	.0131
	Abs Black Error	7	11.0994	15.4957	13.2539	.5475	1.4487
	Abs Asay Error	7	11.9458	17.6099	14.7094	.7045	1.8640
	Abs Vasicek Error	7	.0316	.0933	.0672	.0084	.0223
	Abs HJM Error	7	.0029	.0164	.0114	.0018	.0047
	Valid N (listwise)	7					
-3.6 to -2.5	Black Error	57	8.4555	15.7622	12.0686	.2606	1.9672
	Asay Error	57	9.1165	17.9208	13.3006	.3192	2.4099
	Vasicek Error	57	-.0053	.6416	.1002	.0113	.0850
	HJM Error	57	-.0500	.3230	.0075	.0066	.0500
	Abs Black Error	57	8.4555	15.7622	12.0686	.2606	1.9672
	Abs Asay Error	57	9.1165	17.9208	13.3006	.3192	2.4099
	Abs Vasicek Error	57	.0045	.6416	.1004	.0112	.0848
	Abs HJM Error	57	.0002	.3230	.0272	.0056	.0424
	Valid N (listwise)	57					
-2.6 to -1.5	Black Error	184	5.9957	17.1133	11.4149	.1887	2.5591
	Asay Error	184	6.2462	19.8473	12.5214	.2284	3.0986
	Vasicek Error	184	-.0010	.3011	.1363	.0044	.0596
	HJM Error	184	-.2853	.1208	-.0143	.0042	.0566
	Abs Black Error	184	5.9957	17.1133	11.4149	.1887	2.5591
	Abs Asay Error	184	6.2462	19.8473	12.5214	.2284	3.0986
	Abs Vasicek Error	184	.0010	.3011	.1363	.0044	.0595
	Abs HJM Error	184	.0003	.2853	.0449	.0027	.0372
	Valid N (listwise)	184					
-1.6 to -0.5	Black Error	301	2.7768	17.1472	10.5161	.2014	3.4937
	Asay Error	301	2.8311	19.9244	11.5101	.2391	4.1475
	Vasicek Error	301	-.1065	.4386	.1923	.0052	.0894
	HJM Error	301	-.5156	.3113	-.0242	.0068	.1179
	Abs Black Error	301	2.7768	17.1472	10.5161	.2014	3.4937
	Abs Asay Error	301	2.8311	19.9244	11.5101	.2391	4.1475
	Abs Vasicek Error	301	.0083	.4386	.1972	.0045	.0780
	Abs HJM Error	301	.0001	.5156	.0853	.0049	.0848
	Valid N (listwise)	301					

Table A.2 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
M		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
-0.6 to 1.5	Black Error	751	.0012	17.1217	8.9965	.1636	4.4823
	Asay Error	751	.0021	19.9198	9.8550	.1890	5.1797
	Vasicek Error	751	-.3961	.6682	.1877	.0057	.1574
	HJM Error	751	-.8540	.4881	-.0149	.0069	.1888
	Abs Black Error	751	.0012	17.1217	8.9965	.1636	4.4823
	Abs Asay Error	751	.0021	19.9198	9.8550	.1890	5.1797
	Abs Vasicek Error	751	.0010	.6682	.2153	.0043	.1167
	Abs HJM Error	751	.0001	.8540	.1356	.0048	.1321
	Valid N (listwise)	751					
1.6 to 2.5	Black Error	360	-.0212	16.6153	8.1508	.2363	4.4828
	Asay Error	360	-.0197	19.5191	8.9926	.2725	5.1700
	Vasicek Error	360	-.4337	.6935	.0775	.0104	.1974
	HJM Error	360	-.5680	.5813	-.0128	.0104	.1973
	Abs Black Error	360	.0056	16.6153	8.1511	.2362	4.4823
	Abs Asay Error	360	.0046	19.5191	8.9928	.2725	5.1696
	Abs Vasicek Error	360	.0001	.6935	.1564	.0075	.1430
	Abs HJM Error	360	.0003	.5813	.1430	.0072	.1364
	Valid N (listwise)	360					
2.6 to 3.5	Black Error	260	-.0325	15.8385	6.0009	.2493	4.0204
	Asay Error	260	-.0300	18.1053	6.5864	.2829	4.5610
	Vasicek Error	260	-.4984	.6925	.0050	.0123	.1983
	HJM Error	260	-3.2400	.6256	-.0311	.0175	.2819
	Abs Black Error	260	.0019	15.8385	6.0025	.2492	4.0180
	Abs Asay Error	260	.0040	18.1053	6.5877	.2827	4.5590
	Abs Vasicek Error	260	.0014	.6925	.1326	.0091	.1473
	Abs HJM Error	260	.0009	3.2400	.1486	.0150	.2414
	Valid N (listwise)	260					
3.6 to 4.5	Black Error	152	-.0574	10.7947	3.7235	.2353	2.9006
	Asay Error	152	-.0497	12.0654	4.0541	.2606	3.2124
	Vasicek Error	152	-.5158	.3717	-.0527	.0140	.1725
	HJM Error	152	-.5078	.3799	-.0493	.0141	.1734
	Abs Black Error	152	.0091	10.7947	3.7293	.2347	2.8930
	Abs Asay Error	152	.0026	12.0654	4.0589	.2601	3.2064
	Abs Vasicek Error	152	.0011	.5158	.1211	.0108	.1334
	Abs HJM Error	152	.0011	.5078	.1207	.0108	.1335
	Valid N (listwise)	152					

Table A.2 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
M		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
4.6 to 5.5	Black Error	25	-.0661	4.8898	2.0425	.3100	1.5500
	Asay Error	25	-.0587	5.2927	2.2065	.3346	1.6731
	Vasicek Error	25	-.3706	.2499	-.0486	.0213	.1064
	HJM Error	25	-.3696	.2543	-.0452	.0214	.1068
	Abs Black Error	25	.0207	4.8898	2.0538	.3069	1.5343
	Abs Asay Error	25	.0131	5.2927	2.2161	.3320	1.6598
	Abs Vasicek Error	25	.0010	.3706	.0797	.0169	.0846
	Abs HJM Error	25	.0032	.3696	.0780	.0170	.0849
	Valid N (listwise)	25					

Table A.3

Descriptive Statistics for call options on 90-Day Bank Accepted Bill Futures with different time to maturity (days)

		N	Minimum	Maximum	Mean		Std. Deviation
Time to Maturity (Days)		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
3	Black Error	17	-.0425	.3194	.0365	.0249	.1026
	Asay Error	17	-.0400	.3196	.0378	.0247	.1020
	Vasicek Error	17	-.0288	.0562	.0006	.0047	.0194
	HJM Error	17	-.0287	.0390	-.0005	.0040	.0164
	Abs Black Error	17	.0012	.3194	.0634	.0212	.0876
	Abs Asay Error	17	.0021	.3196	.0626	.0214	.0881
	Abs Vasicek Error	17	.0012	.0562	.0137	.0032	.0133
	Abs HJM Error	17	.0013	.0390	.0125	.0024	.0101
	Valid N (listwise)	17					
9	Black Error	60	-.0661	1.3664	.2232	.0406	.3142
	Asay Error	60	-.0587	1.3685	.2273	.0404	.3128
	Vasicek Error	60	-.0892	.1008	-.0183	.0048	.0369
	HJM Error	60	-.0890	.0750	-.0220	.0041	.0316
	Abs Black Error	60	.0019	1.3664	.2430	.0386	.2989
	Abs Asay Error	60	.0026	1.3685	.2435	.0387	.3002
	Abs Vasicek Error	60	.0018	.1008	.0321	.0033	.0255
	Abs HJM Error	60	.0020	.0890	.0294	.0032	.0248
	Valid N (listwise)	60					
30	Black Error	18	.1159	1.6974	.6584	.1055	.4475
	Asay Error	18	.1413	1.7077	.6743	.1043	.4425
	Vasicek Error	18	-.1121	.1105	-.0548	.0163	.0694
	HJM Error	18	-.1114	.0359	-.0701	.0100	.0422
	Abs Black Error	18	.1159	1.6974	.6584	.1055	.4475
	Abs Asay Error	18	.1413	1.7077	.6743	.1043	.4425
	Abs Vasicek Error	18	.0219	.1121	.0831	.0062	.0261
	Abs HJM Error	18	.0294	.1114	.0775	.0059	.0251
	Valid N (listwise)	18					
37	Black Error	41	.0754	2.1416	.9824	.0936	.5996
	Asay Error	41	.1075	2.1561	1.0038	.0930	.5953
	Vasicek Error	41	-.3706	.1417	-.1248	.0277	.1771
	HJM Error	41	-.3696	.0290	-.1466	.0262	.1675
	Abs Black Error	41	.0754	2.1416	.9824	.0936	.5996
	Abs Asay Error	41	.1075	2.1561	1.0038	.0930	.5953
	Abs Vasicek Error	41	.0034	.3706	.1560	.0234	.1497
	Abs HJM Error	41	.0025	.3696	.1554	.0248	.1591
	Valid N (listwise)	41					

Table A.3 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Time to Maturity (Days)		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
44	Black Error	17	-.0193	1.0882	.4269	.0933	.3847
	Asay Error	17	.0200	1.0980	.4502	.0910	.3752
	Vasicek Error	17	-.0830	.2321	-.0018	.0228	.0942
	HJM Error	17	-.0810	.1294	-.0252	.0139	.0572
	Abs Black Error	17	.0091	1.0882	.4292	.0926	.3820
	Abs Asay Error	17	.0200	1.0980	.4502	.0910	.3752
	Abs Vasicek Error	17	.0109	.2321	.0716	.0142	.0586
	Abs HJM Error	17	.0096	.1294	.0541	.0070	.0290
	Valid N (listwise)	17					
72	Black Error	40	.3711	3.9662	1.9653	.1582	1.0005
	Asay Error	40	.4326	4.0297	2.0182	.1585	1.0023
	Vasicek Error	40	-.0348	.2945	.1218	.0146	.0923
	HJM Error	40	-.0329	.1746	.0886	.0116	.0735
	Abs Black Error	40	.3711	3.9662	1.9653	.1582	1.0005
	Abs Asay Error	40	.4326	4.0297	2.0182	.1585	1.0023
	Abs Vasicek Error	40	.0001	.2945	.1283	.0131	.0828
	Abs HJM Error	40	.0015	.1746	.0950	.0103	.0649
	Valid N (listwise)	40					
79	Black Error	24	.8104	2.7901	1.8206	.1364	.6683
	Asay Error	24	.8895	2.8274	1.8759	.1337	.6548
	Vasicek Error	24	-.0897	.1838	-.0006	.0176	.0863
	HJM Error	24	-.0880	.0589	-.0270	.0086	.0423
	Abs Black Error	24	.8104	2.7901	1.8206	.1364	.6683
	Abs Asay Error	24	.8895	2.8274	1.8759	.1337	.6548
	Abs Vasicek Error	24	.0014	.1838	.0687	.0103	.0503
	Abs HJM Error	24	.0079	.0880	.0445	.0045	.0222
	Valid N (listwise)	24					
93	Black Error	20	1.0762	3.5110	2.0532	.1580	.7068
	Asay Error	20	1.1827	3.5882	2.1298	.1554	.6950
	Vasicek Error	20	-.0332	.3020	.0739	.0252	.1127
	HJM Error	20	-.0315	.2050	.0437	.0166	.0744
	Abs Black Error	20	1.0762	3.5110	2.0532	.1580	.7068
	Abs Asay Error	20	1.1827	3.5882	2.1298	.1554	.6950
	Abs Vasicek Error	20	.0016	.3020	.0852	.0233	.1040
	Abs HJM Error	20	.0015	.2050	.0539	.0150	.0670
	Valid N (listwise)	20					

Table A.3 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Time to Maturity (Days)		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
100	Black Error	83	1.1690	4.5525	2.6987	.0883	.8041
	Asay Error	83	1.2757	4.6546	2.7866	.0878	.7997
	Vasicek Error	83	-.1017	.3923	.0896	.0130	.1181
	HJM Error	83	-.0992	.2903	.0510	.0099	.0902
	Abs Black Error	83	1.1690	4.5525	2.6987	.0883	.8041
	Abs Asay Error	83	1.2757	4.6546	2.7866	.0878	.7997
	Abs Vasicek Error	83	.0033	.3923	.1130	.0105	.0957
	Abs HJM Error	83	.0001	.2903	.0777	.0075	.0683
	Valid N (listwise)	83					
121	Black Error	19	1.9398	4.7285	3.1705	.1823	.7945
	Asay Error	19	2.1058	4.8642	3.2980	.1780	.7759
	Vasicek Error	19	-.2239	.1868	-.0642	.0355	.1548
	HJM Error	19	-.2204	.0403	-.1203	.0216	.0941
	Abs Black Error	19	1.9398	4.7285	3.1705	.1823	.7945
	Abs Asay Error	19	2.1058	4.8642	3.2980	.1780	.7759
	Abs Vasicek Error	19	.0124	.2239	.1544	.0128	.0559
	Abs HJM Error	19	.0112	.2204	.1325	.0172	.0749
	Valid N (listwise)	19					
128	Black Error	45	1.4936	4.3798	2.9105	.1164	.7809
	Asay Error	45	1.6364	4.4945	3.0261	.1145	.7679
	Vasicek Error	45	-.4738	.2297	-.1085	.0340	.2283
	HJM Error	45	-.4696	.0285	-.1765	.0298	.2002
	Abs Black Error	45	1.4936	4.3798	2.9105	.1164	.7809
	Abs Asay Error	45	1.6364	4.4945	3.0261	.1145	.7679
	Abs Vasicek Error	45	.0028	.4738	.1866	.0252	.1690
	Abs HJM Error	45	.0004	.4696	.1812	.0292	.1958
	Valid N (listwise)	45					
135	Black Error	19	1.9331	5.3972	3.1675	.2229	.9714
	Asay Error	19	2.0906	5.5802	3.2991	.2232	.9730
	Vasicek Error	19	-.1567	.2970	.0269	.0386	.1681
	HJM Error	19	-.1532	.1280	-.0519	.0231	.1007
	Abs Black Error	19	1.9331	5.3972	3.1675	.2229	.9714
	Abs Asay Error	19	2.0906	5.5802	3.2991	.2232	.9730
	Abs Vasicek Error	19	.0025	.2970	.1449	.0190	.0829
	Abs HJM Error	19	.0142	.1532	.1043	.0089	.0386
	Valid N (listwise)	19					

Table A.3 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Time to Maturity (Days)		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
142	Black Error	22	4.8961	6.5213	5.8596	.1021	.4791
	Asay Error	22	5.1386	6.7230	6.0646	.0984	.4615
	Vasicek Error	22	.0691	.3322	.2016	.0195	.0913
	HJM Error	22	.0348	.1698	.1123	.0088	.0412
	Abs Black Error	22	4.8961	6.5213	5.8596	.1021	.4791
	Abs Asay Error	22	5.1386	6.7230	6.0646	.0984	.4615
	Abs Vasicek Error	22	.0691	.3322	.2016	.0195	.0913
	Abs HJM Error	22	.0348	.1698	.1123	.0088	.0412
	Valid N (listwise)	22					
163	Black Error	43	3.3258	6.4872	4.8105	.1309	.8584
	Asay Error	43	3.5468	6.7242	5.0122	.1304	.8551
	Vasicek Error	43	.0192	.3778	.2376	.0159	.1041
	HJM Error	43	.0056	.2945	.1649	.0158	.1037
	Abs Black Error	43	3.3258	6.4872	4.8105	.1309	.8584
	Abs Asay Error	43	3.5468	6.7242	5.0122	.1304	.8551
	Abs Vasicek Error	43	.0192	.3778	.2376	.0159	.1041
	Abs HJM Error	43	.0056	.2945	.1649	.0158	.1037
	Valid N (listwise)	43					
170	Black Error	26	3.0266	5.6335	4.3850	.1555	.7927
	Asay Error	26	3.2505	5.8181	4.5637	.1508	.7690
	Vasicek Error	26	-.0631	.2398	.0542	.0209	.1064
	HJM Error	26	-.0584	.0739	-.0010	.0075	.0382
	Abs Black Error	26	3.0266	5.6335	4.3850	.1555	.7927
	Abs Asay Error	26	3.2505	5.8181	4.5637	.1508	.7690
	Abs Vasicek Error	26	.0005	.2398	.0878	.0156	.0798
	Abs HJM Error	26	.0035	.0739	.0319	.0039	.0199
	Valid N (listwise)	26					
184	Black Error	39	3.7224	7.0662	5.7300	.1609	1.0047
	Asay Error	39	3.9953	7.3523	5.9870	.1633	1.0196
	Vasicek Error	39	.0096	.3738	.1937	.0176	.1097
	HJM Error	39	.0099	.2871	.0933	.0110	.0685
	Abs Black Error	39	3.7224	7.0662	5.7300	.1609	1.0047
	Abs Asay Error	39	3.9953	7.3523	5.9870	.1633	1.0196
	Abs Vasicek Error	39	.0096	.3738	.1937	.0176	.1097
	Abs HJM Error	39	.0099	.2871	.0933	.0110	.0685
	Valid N (listwise)	39					

Table A.3 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Time to Maturity (Days)		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
191	Black Error	68	2.9045	6.9665	5.1264	.1233	1.0170
	Asay Error	68	3.1604	7.2663	5.3673	.1242	1.0244
	Vasicek Error	68	-.3961	.4151	.0978	.0180	.1488
	HJM Error	68	-.1288	.2753	.0392	.0138	.1137
	Abs Black Error	68	2.9045	6.9665	5.1264	.1233	1.0170
	Abs Asay Error	68	3.1604	7.2663	5.3673	.1242	1.0244
	Abs Vasicek Error	68	.0010	.4151	.1426	.0128	.1059
	Abs HJM Error	68	.0016	.2753	.0912	.0094	.0778
	Valid N (listwise)	68					
219	Black Error	72	3.7734	7.2831	5.4500	.1007	.8542
	Asay Error	72	4.1038	7.6419	5.7523	.1003	.8508
	Vasicek Error	72	-.5158	.2577	-.0666	.0257	.2180
	HJM Error	72	-.5078	.2410	-.1624	.0224	.1903
	Abs Black Error	72	3.7734	7.2831	5.4500	.1007	.8542
	Abs Asay Error	72	4.1038	7.6419	5.7523	.1003	.8508
	Abs Vasicek Error	72	.0010	.5158	.1724	.0174	.1479
	Abs HJM Error	72	.0008	.5078	.1704	.0216	.1831
	Valid N (listwise)	72					
226	Black Error	21	5.1050	7.1087	6.1168	.1100	.5040
	Asay Error	21	5.4938	7.5092	6.4535	.1064	.4877
	Vasicek Error	21	-.2163	.2523	.0522	.0363	.1662
	HJM Error	21	-.2278	.0503	-.0786	.0235	.1077
	Abs Black Error	21	5.1050	7.1087	6.1168	.1100	.5040
	Abs Asay Error	21	5.4938	7.5092	6.4535	.1064	.4877
	Abs Vasicek Error	21	.0311	.2523	.1569	.0148	.0680
	Abs HJM Error	21	.0054	.2278	.1039	.0179	.0821
	Valid N (listwise)	21					
233	Black Error	23	7.4996	9.0524	8.3724	.0860	.4123
	Asay Error	23	8.0071	9.5018	8.8200	.0821	.3937
	Vasicek Error	23	.1080	.3680	.2534	.0182	.0874
	HJM Error	23	.0267	.1869	.1296	.0101	.0482
	Abs Black Error	23	7.4996	9.0524	8.3724	.0860	.4123
	Abs Asay Error	23	8.0071	9.5018	8.8200	.0821	.3937
	Abs Vasicek Error	23	.1080	.3680	.2534	.0182	.0874
	Abs HJM Error	23	.0267	.1869	.1296	.0101	.0482
	Valid N (listwise)	23					

Table A.3 (Cont'd)

Time to Maturity (Days)		N	Minimum	Maximum	Mean		Std. Deviation
		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
240	Black Error	26	4.2784	6.3553	5.6098	.1373	.6999
	Asay Error	26	4.6536	6.6324	5.9202	.1304	.6649
	Vasicek Error	26	-.0444	.2012	.0858	.0129	.0657
	HJM Error	26	-.0482	.1976	.0253	.0098	.0498
	Abs Black Error	26	4.2784	6.3553	5.6098	.1373	.6999
	Abs Asay Error	26	4.6536	6.6324	5.9202	.1304	.6649
	Abs Vasicek Error	26	.0114	.2012	.0892	.0119	.0608
	Abs HJM Error	26	.0062	.1976	.0424	.0070	.0357
	Valid N (listwise)	26					
254	Black Error	25	5.7099	8.6012	7.1660	.1640	.8201
	Asay Error	25	6.1834	9.1046	7.5818	.1623	.8115
	Vasicek Error	25	.1171	.4361	.3423	.0168	.0840
	HJM Error	25	.0088	.3626	.2519	.0263	.1313
	Abs Black Error	25	5.7099	8.6012	7.1660	.1640	.8201
	Abs Asay Error	25	6.1834	9.1046	7.5818	.1623	.8115
	Abs Vasicek Error	25	.1171	.4361	.3423	.0168	.0840
	Abs HJM Error	25	.0088	.3626	.2519	.0263	.1313
	Valid N (listwise)	25					
261	Black Error	27	4.7641	7.2954	6.1891	.1455	.7558
	Asay Error	27	5.1726	7.6722	6.5309	.1392	.7233
	Vasicek Error	27	-.0439	.2750	.0975	.0205	.1065
	HJM Error	27	-.0353	.2588	.0222	.0101	.0525
	Abs Black Error	27	4.7641	7.2954	6.1891	.1455	.7558
	Abs Asay Error	27	5.1726	7.6722	6.5309	.1392	.7233
	Abs Vasicek Error	27	.0066	.2750	.1093	.0180	.0937
	Abs HJM Error	27	.0026	.2588	.0309	.0092	.0477
	Valid N (listwise)	27					
275	Black Error	44	7.0259	9.3429	8.4157	.0969	.6428
	Asay Error	44	7.5935	9.8967	8.9423	.0997	.6612
	Vasicek Error	44	-.0053	.3781	.2163	.0145	.0963
	HJM Error	44	-.0013	.2014	.0776	.0090	.0599
	Abs Black Error	44	7.0259	9.3429	8.4157	.0969	.6428
	Abs Asay Error	44	7.5935	9.8967	8.9423	.0997	.6612
	Abs Vasicek Error	44	.0053	.3781	.2165	.0144	.0958
	Abs HJM Error	44	.0013	.2014	.0776	.0090	.0598
	Valid N (listwise)	44					

Table A.3 (Cont'd)

Time to Maturity (Days)		N	Minimum	Maximum	Mean		Std. Deviation
		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
282	Black Error	45	6.4580	9.1420	8.1065	.0972	.6521
	Asay Error	45	7.0134	9.7209	8.6083	.0968	.6495
	Vasicek Error	45	-.1250	.3935	.1497	.0236	.1582
	HJM Error	45	-.1180	.2334	.0318	.0196	.1318
	Abs Black Error	45	6.4580	9.1420	8.1065	.0972	.6521
	Abs Asay Error	45	7.0134	9.7209	8.6083	.0968	.6495
	Abs Vasicek Error	45	.0146	.3935	.1836	.0173	.1161
	Abs HJM Error	45	.0054	.2334	.1162	.0101	.0679
	Valid N (listwise)	45					
310	Black Error	76	5.6886	9.4912	7.7700	.0948	.8266
	Asay Error	76	6.2588	10.2418	8.3214	.0963	.8397
	Vasicek Error	76	-.4912	.2467	-.0243	.0225	.1959
	HJM Error	76	-.5133	.0004	-.1574	.0202	.1763
	Abs Black Error	76	5.6886	9.4912	7.7700	.0948	.8266
	Abs Asay Error	76	6.2588	10.2418	8.3214	.0963	.8397
	Abs Vasicek Error	76	.0025	.4912	.1514	.0144	.1255
	Abs HJM Error	76	.0003	.5133	.1574	.0202	.1762
	Valid N (listwise)	76					
317	Black Error	21	7.8284	8.8190	8.4943	.0636	.2916
	Asay Error	21	8.5043	9.4167	9.0999	.0598	.2740
	Vasicek Error	21	-.1152	.2358	.1190	.0241	.1105
	HJM Error	21	-.1968	.1929	-.0427	.0219	.1001
	Abs Black Error	21	7.8284	8.8190	8.4943	.0636	.2916
	Abs Asay Error	21	8.5043	9.4167	9.0999	.0598	.2740
	Abs Vasicek Error	21	.0116	.2358	.1437	.0160	.0735
	Abs HJM Error	21	.0005	.1968	.0802	.0157	.0720
	Valid N (listwise)	21					
324	Black Error	24	10.1211	11.0647	10.7385	.0576	.2822
	Asay Error	24	10.9692	11.8272	11.5056	.0562	.2751
	Vasicek Error	24	.0737	.3788	.2700	.0194	.0950
	HJM Error	24	.0104	.1790	.1193	.0124	.0609
	Abs Black Error	24	10.1211	11.0647	10.7385	.0576	.2822
	Abs Asay Error	24	10.9692	11.8272	11.5056	.0562	.2751
	Abs Vasicek Error	24	.0737	.3788	.2700	.0194	.0950
	Abs HJM Error	24	.0104	.1790	.1193	.0124	.0609
	Valid N (listwise)	24					

Table A.3 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Time to Maturity (Days)		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
331	Black Error	25	5.9232	7.9676	7.2544	.1370	.6850
	Asay Error	25	6.5302	8.4468	7.7797	.1279	.6394
	Vasicek Error	25	.0011	.1846	.0947	.0124	.0619
	HJM Error	25	-.0992	.0431	-.0093	.0102	.0512
	Abs Black Error	25	5.9232	7.9676	7.2544	.1370	.6850
	Abs Asay Error	25	6.5302	8.4468	7.7797	.1279	.6394
	Abs Vasicek Error	25	.0011	.1846	.0947	.0124	.0619
	Abs HJM Error	25	.0126	.0992	.0447	.0050	.0252
	Valid N (listwise)	25					
345	Black Error	24	7.8324	9.6537	9.0233	.1186	.5809
	Asay Error	24	8.6069	10.3063	9.7122	.1099	.5385
	Vasicek Error	24	.1575	.4424	.3668	.0156	.0766
	HJM Error	24	-.0044	.3903	.2493	.0301	.1475
	Abs Black Error	24	7.8324	9.6537	9.0233	.1186	.5809
	Abs Asay Error	24	8.6069	10.3063	9.7122	.1099	.5385
	Abs Vasicek Error	24	.1575	.4424	.3668	.0156	.0766
	Abs HJM Error	24	.0044	.3903	.2497	.0300	.1469
	Valid N (listwise)	24					
352	Black Error	25	6.5076	8.5594	7.8493	.1374	.6871
	Asay Error	25	7.1448	9.0727	8.4062	.1284	.6422
	Vasicek Error	25	-.0202	.2869	.1403	.0215	.1075
	HJM Error	25	-.0089	.0333	.0174	.0023	.0116
	Abs Black Error	25	6.5076	8.5594	7.8493	.1374	.6871
	Abs Asay Error	25	7.1448	9.0727	8.4062	.1284	.6422
	Abs Vasicek Error	25	.0063	.2869	.1431	.0207	.1036
	Abs HJM Error	25	.0011	.0333	.0181	.0021	.0104
	Valid N (listwise)	25					
366	Black Error	46	9.3674	11.8417	10.7518	.1207	.8188
	Asay Error	46	10.0799	12.7788	11.6240	.1302	.8829
	Vasicek Error	46	.0316	.3755	.2109	.0140	.0948
	HJM Error	46	-.0436	.1745	.0474	.0111	.0755
	Abs Black Error	46	9.3674	11.8417	10.7518	.1207	.8188
	Abs Asay Error	46	10.0799	12.7788	11.6240	.1302	.8829
	Abs Vasicek Error	46	.0316	.3755	.2109	.0140	.0948
	Abs HJM Error	46	.0046	.1745	.0652	.0089	.0604
	Valid N (listwise)	46					

Table A.3 (Cont'd)

Time to Maturity (Days)		N	Minimum	Maximum	Mean		Std. Deviation
		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
373	Black Error	47	8.6170	11.1812	10.2501	.1006	.6895
	Asay Error	47	9.4932	12.0466	11.0719	.1037	.7112
	Vasicek Error	47	-.1363	.3799	.1588	.0223	.1531
	HJM Error	47	-.1557	.2077	.0153	.0200	.1369
	Abs Black Error	47	8.6170	11.1812	10.2501	.1006	.6895
	Abs Asay Error	47	9.4932	12.0466	11.0719	.1037	.7112
	Abs Vasicek Error	47	.0087	.3799	.1868	.0170	.1165
	Abs HJM Error	47	.0160	.2077	.1241	.0083	.0569
	Valid N (listwise)	47					
394	Black Error	22	8.5888	10.2786	9.6326	.1059	.4965
	Asay Error	22	9.5710	11.1927	10.5216	.0966	.4530
	Vasicek Error	22	-.1514	.2108	.0644	.0257	.1205
	HJM Error	22	-.1721	-.0020	-.0983	.0126	.0592
	Abs Black Error	22	8.5888	10.2786	9.6326	.1059	.4965
	Abs Asay Error	22	9.5710	11.1927	10.5216	.0966	.4530
	Abs Vasicek Error	22	.0139	.2108	.1220	.0122	.0572
	Abs HJM Error	22	.0020	.1721	.0983	.0126	.0592
	Valid N (listwise)	22					
401	Black Error	49	7.6910	10.0058	9.2512	.0906	.6343
	Asay Error	49	8.5663	10.8138	10.0558	.0872	.6101
	Vasicek Error	49	-.4093	.2178	-.0295	.0253	.1771
	HJM Error	49	-.5332	.1974	-.1955	.0279	.1956
	Abs Black Error	49	7.6910	10.0058	9.2512	.0906	.6343
	Abs Asay Error	49	8.5663	10.8138	10.0558	.0872	.6101
	Abs Vasicek Error	49	.0035	.4093	.1358	.0166	.1159
	Abs HJM Error	49	.0200	.5332	.2036	.0267	.1870
	Valid N (listwise)	49					
408	Black Error	24	9.4132	11.1407	10.6758	.0952	.4666
	Asay Error	24	10.2127	12.1172	11.6467	.1026	.5025
	Vasicek Error	24	-.0688	.2474	.1277	.0178	.0873
	HJM Error	24	-.1709	.0159	-.0636	.0127	.0622
	Abs Black Error	24	9.4132	11.1407	10.6758	.0952	.4666
	Abs Asay Error	24	10.2127	12.1172	11.6467	.1026	.5025
	Abs Vasicek Error	24	.0056	.2474	.1373	.0144	.0705
	Abs HJM Error	24	.0002	.1709	.0663	.0121	.0592
	Valid N (listwise)	24					

Table A.3 (Cont'd)

Time to Maturity (Days)		N	Minimum	Maximum	Mean		Std. Deviation
		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
415	Black Error	25	11.6919	13.6200	13.1345	.1045	.5223
	Asay Error	25	12.7040	14.8403	14.3432	.1135	.5675
	Vasicek Error	25	.1453	.6935	.5156	.0370	.1851
	HJM Error	25	.0322	.6256	.3499	.0409	.2043
	Abs Black Error	25	11.6919	13.6200	13.1345	.1045	.5223
	Abs Asay Error	25	12.7040	14.8403	14.3432	.1135	.5675
	Abs Vasicek Error	25	.1453	.6935	.5156	.0370	.1851
	Abs HJM Error	25	.0322	.6256	.3499	.0409	.2043
	Valid N (listwise)	25					
422	Black Error	27	7.6912	9.5436	8.9625	.1120	.5817
	Asay Error	27	8.5723	10.2782	9.7376	.1020	.5301
	Vasicek Error	27	.0267	.1763	.1022	.0096	.0499
	HJM Error	27	-.1418	.0614	-.0242	.0135	.0702
	Abs Black Error	27	7.6912	9.5436	8.9625	.1120	.5817
	Abs Asay Error	27	8.5723	10.2782	9.7376	.1020	.5301
	Abs Vasicek Error	27	.0267	.1763	.1022	.0096	.0499
	Abs HJM Error	27	.0036	.1418	.0618	.0076	.0397
	Valid N (listwise)	27					
436	Black Error	25	9.6172	11.3520	10.8235	.1106	.5531
	Asay Error	25	10.7370	12.3250	11.8394	.0999	.4993
	Vasicek Error	25	.1254	.4198	.3411	.0176	.0879
	HJM Error	25	-.6328	.3766	.1561	.0454	.2271
	Abs Black Error	25	9.6172	11.3520	10.8235	.1106	.5531
	Abs Asay Error	25	10.7370	12.3250	11.8394	.0999	.4993
	Abs Vasicek Error	25	.1254	.4198	.3411	.0176	.0879
	Abs HJM Error	25	.0005	.6328	.2133	.0344	.1720
	Valid N (listwise)	25					
450	Black Error	72	8.4218	11.1666	10.2928	.0795	.6742
	Asay Error	72	9.3472	12.0327	11.1460	.0790	.6703
	Vasicek Error	72	-.0186	.2816	.1638	.0096	.0816
	HJM Error	72	-.1095	.1257	-.0308	.0046	.0388
	Abs Black Error	72	8.4218	11.1666	10.2928	.0795	.6742
	Abs Asay Error	72	9.3472	12.0327	11.1460	.0790	.6703
	Abs Vasicek Error	72	.0023	.2816	.1644	.0095	.0804
	Abs HJM Error	72	.0002	.1257	.0353	.0041	.0347
	Valid N (listwise)	72					

Table A.3 (Cont'd)

Time to Maturity (Days)		N	Minimum	Maximum	Mean		Std. Deviation
		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
457	Black Error	50	11.6839	13.2726	12.6764	.0629	.4449
	Asay Error	50	12.8040	14.5985	13.9614	.0681	.4813
	Vasicek Error	50	.0494	.3494	.1945	.0118	.0837
	HJM Error	50	-.1702	.1295	.0124	.0119	.0839
	Abs Black Error	50	11.6839	13.2726	12.6764	.0629	.4449
	Abs Asay Error	50	12.8040	14.5985	13.9614	.0681	.4813
	Abs Vasicek Error	50	.0494	.3494	.1945	.0118	.0837
	Abs HJM Error	50	.0057	.1702	.0715	.0063	.0445
	Valid N (listwise)	50					
464	Black Error	49	9.7807	12.7826	12.0499	.0911	.6376
	Asay Error	49	10.6981	14.0264	13.2363	.0966	.6763
	Vasicek Error	49	-.1087	.3730	.1561	.0199	.1395
	HJM Error	49	-.1879	.3113	-.0011	.0205	.1435
	Abs Black Error	49	9.7807	12.7826	12.0499	.0911	.6376
	Abs Asay Error	49	10.6981	14.0264	13.2363	.0966	.6763
	Abs Vasicek Error	49	.0016	.3730	.1727	.0168	.1179
	Abs HJM Error	49	.0163	.3113	.1295	.0084	.0589
	Valid N (listwise)	49					
492	Black Error	47	9.5223	12.5880	11.2365	.1106	.7584
	Asay Error	47	10.7280	14.0144	12.4632	.1183	.8108
	Vasicek Error	47	-.1174	.2150	.0940	.0142	.0973
	HJM Error	47	-.1498	-.0206	-.0958	.0061	.0416
	Abs Black Error	47	9.5223	12.5880	11.2365	.1106	.7584
	Abs Asay Error	47	10.7280	14.0144	12.4632	.1183	.8108
	Abs Vasicek Error	47	.0014	.2150	.1177	.0096	.0658
	Abs HJM Error	47	.0206	.1498	.0958	.0061	.0416
	Valid N (listwise)	47					
499	Black Error	24	10.1539	12.7748	12.3418	.1213	.5941
	Asay Error	24	11.2183	14.1599	13.7147	.1345	.6587
	Vasicek Error	24	-.0436	.1952	.1178	.0137	.0670
	HJM Error	24	-.1816	.0120	-.0866	.0128	.0628
	Abs Black Error	24	10.1539	12.7748	12.3418	.1213	.5941
	Abs Asay Error	24	11.2183	14.1599	13.7147	.1345	.6587
	Abs Vasicek Error	24	.0060	.1952	.1220	.0120	.0587
	Abs HJM Error	24	.0019	.1816	.0878	.0125	.0611
	Valid N (listwise)	24					

Table A.3 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Time to Maturity (Days)		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
506	Black Error	21	12.6346	15.8586	14.9666	.1664	.7627
	Asay Error	21	13.9798	17.7061	16.6328	.1900	.8707
	Vasicek Error	21	.1318	.3653	.2966	.0157	.0720
	HJM Error	21	.0100	.1929	.0945	.0123	.0563
	Abs Black Error	21	12.6346	15.8586	14.9666	.1664	.7627
	Abs Asay Error	21	13.9798	17.7061	16.6328	.1900	.8707
	Abs Vasicek Error	21	.1318	.3653	.2966	.0157	.0720
	Abs HJM Error	21	.0100	.1929	.0945	.0123	.0563
	Valid N (listwise)	21					
513	Black Error	25	9.2039	10.7220	10.2712	.0928	.4642
	Asay Error	25	10.3649	11.7387	11.3186	.0826	.4130
	Vasicek Error	25	.0384	.1646	.1066	.0087	.0437
	HJM Error	25	-.1741	.0476	-.0553	.0157	.0783
	Abs Black Error	25	9.2039	10.7220	10.2712	.0928	.4642
	Abs Asay Error	25	10.3649	11.7387	11.3186	.0826	.4130
	Abs Vasicek Error	25	.0384	.1646	.1066	.0087	.0437
	Abs HJM Error	25	.0038	.1741	.0778	.0110	.0549
	Valid N (listwise)	25					
527	Black Error	22	11.5101	13.0020	12.5546	.1032	.4843
	Asay Error	22	13.0226	14.3590	13.9622	.0911	.4272
	Vasicek Error	22	.1677	.3958	.3352	.0146	.0687
	HJM Error	22	-.0480	.3532	.1578	.0334	.1568
	Abs Black Error	22	11.5101	13.0020	12.5546	.1032	.4843
	Abs Asay Error	22	13.0226	14.3590	13.9622	.0911	.4272
	Abs Vasicek Error	22	.1677	.3958	.3352	.0146	.0687
	Abs HJM Error	22	.0073	.3532	.1760	.0288	.1350
	Valid N (listwise)	22					
541	Black Error	14	11.6174	12.0372	11.9202	.0346	.1295
	Asay Error	14	12.8170	13.1708	13.0622	.0285	.1066
	Vasicek Error	14	.1818	.2562	.2263	.0066	.0248
	HJM Error	14	-.1085	-.0648	-.0917	.0042	.0157
	Abs Black Error	14	11.6174	12.0372	11.9202	.0346	.1295
	Abs Asay Error	14	12.8170	13.1708	13.0622	.0285	.1066
	Abs Vasicek Error	14	.1818	.2562	.2263	.0066	.0248
	Abs HJM Error	14	.0648	.1085	.0917	.0042	.0157
	Valid N (listwise)	14					

Table A.3 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Time to Maturity (Days)		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
549	Black Error	39	12.5212	14.7394	14.2470	.0707	.4417
	Asay Error	39	13.9736	16.5111	15.9901	.0775	.4838
	Vasicek Error	39	.0990	.3041	.2167	.0086	.0535
	HJM Error	39	-.1017	.0727	.0020	.0095	.0596
	Abs Black Error	39	12.5212	14.7394	14.2470	.0707	.4417
	Abs Asay Error	39	13.9736	16.5111	15.9901	.0775	.4838
	Abs Vasicek Error	39	.0990	.3041	.2167	.0086	.0535
	Abs HJM Error	39	.0117	.1017	.0546	.0036	.0223
	Valid N (listwise)	39					
555	Black Error	64	12.3058	14.4815	13.7845	.0657	.5258
	Asay Error	64	13.9572	16.1520	15.4170	.0702	.5617
	Vasicek Error	64	-.0872	.3597	.2001	.0170	.1357
	HJM Error	64	-.2383	.1565	.0061	.0176	.1412
	Abs Black Error	64	12.3058	14.4815	13.7845	.0657	.5258
	Abs Asay Error	64	13.9572	16.1520	15.4170	.0702	.5617
	Abs Vasicek Error	64	.0076	.3597	.2091	.0151	.1211
	Abs HJM Error	64	.0160	.2383	.1277	.0073	.0583
	Valid N (listwise)	64					
569	Black Error	25	11.9402	13.3222	12.9270	.0820	.4100
	Asay Error	25	13.6410	14.8638	14.4985	.0712	.3559
	Vasicek Error	25	-.3428	.0075	-.1231	.0247	.1235
	HJM Error	25	-.5668	-.0300	-.3447	.0359	.1793
	Abs Black Error	25	11.9402	13.3222	12.9270	.0820	.4100
	Abs Asay Error	25	13.6410	14.8638	14.4985	.0712	.3559
	Abs Vasicek Error	25	.0010	.3428	.1253	.0242	.1211
	Abs HJM Error	25	.0300	.5668	.3447	.0359	.1793
	Valid N (listwise)	25					
583	Black Error	41	11.0454	13.9888	12.7141	.1277	.8175
	Asay Error	41	12.5838	15.8240	14.3277	.1481	.9483
	Vasicek Error	41	-.0432	.1869	.1094	.0112	.0720
	HJM Error	41	-3.2400	-.0501	-.1983	.0762	.4882
	Abs Black Error	41	11.0454	13.9888	12.7141	.1277	.8175
	Abs Asay Error	41	12.5838	15.8240	14.3277	.1481	.9483
	Abs Vasicek Error	41	.0019	.1869	.1153	.0097	.0618
	Abs HJM Error	41	.0501	3.2400	.1983	.0762	.4882
	Valid N (listwise)	41					

Table A.3 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Time to Maturity (Days)		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
590	Black Error	25	13.4995	14.3741	14.0995	.0501	.2503
	Asay Error	25	15.4946	16.2305	15.9581	.0473	.2365
	Vasicek Error	25	-.0085	.1741	.1118	.0105	.0523
	HJM Error	25	-.1718	-.0095	-.1036	.0118	.0588
	Abs Black Error	25	13.4995	14.3741	14.0995	.0501	.2503
	Abs Asay Error	25	15.4946	16.2305	15.9581	.0473	.2365
	Abs Vasicek Error	25	.0085	.1741	.1125	.0102	.0508
	Abs HJM Error	25	.0095	.1718	.1036	.0118	.0588
	Valid N (listwise)	25					
597	Black Error	17	15.7217	16.4631	16.2571	.0576	.2376
	Asay Error	17	18.0002	18.6137	18.4439	.0458	.1889
	Vasicek Error	17	.2013	.2834	.2518	.0063	.0260
	HJM Error	17	-.0055	.0548	.0237	.0054	.0221
	Abs Black Error	17	15.7217	16.4631	16.2571	.0576	.2376
	Abs Asay Error	17	18.0002	18.6137	18.4439	.0458	.1889
	Abs Vasicek Error	17	.2013	.2834	.2518	.0063	.0260
	Abs HJM Error	17	.0001	.0548	.0250	.0050	.0205
	Valid N (listwise)	17					
604	Black Error	20	10.2479	11.4175	11.0790	.0831	.3717
	Asay Error	20	11.6737	12.7025	12.4019	.0713	.3190
	Vasicek Error	20	.0723	.1650	.1272	.0070	.0311
	HJM Error	20	-.1947	.0539	-.0894	.0195	.0874
	Abs Black Error	20	10.2479	11.4175	11.0790	.0831	.3717
	Abs Asay Error	20	11.6737	12.7025	12.4019	.0713	.3190
	Abs Vasicek Error	20	.0723	.1650	.1272	.0070	.0311
	Abs HJM Error	20	.0102	.1947	.1064	.0144	.0643
	Valid N (listwise)	20					
618	Black Error	20	13.0877	14.2053	13.8851	.0796	.3559
	Asay Error	20	14.9894	15.9516	15.6803	.0671	.3002
	Vasicek Error	20	.1640	.6300	.3467	.0228	.1021
	HJM Error	20	-.0643	.3591	.1134	.0348	.1554
	Abs Black Error	20	13.0877	14.2053	13.8851	.0796	.3559
	Abs Asay Error	20	14.9894	15.9516	15.6803	.0671	.3002
	Abs Vasicek Error	20	.1640	.6300	.3467	.0228	.1021
	Abs HJM Error	20	.0024	.3591	.1470	.0273	.1223
	Valid N (listwise)	20					

Table A.3 (Cont'd)

Time to Maturity (Days)		N	Minimum	Maximum	Mean		Std. Deviation
		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
639	Black Error	35	14.6454	15.9897	15.4994	.0725	.4289
	Asay Error	35	16.8925	18.2356	17.7187	.0765	.4528
	Vasicek Error	35	.0933	.3055	.2297	.0092	.0542
	HJM Error	35	-.0742	.0599	.0035	.0084	.0498
	Abs Black Error	35	14.6454	15.9897	15.4994	.0725	.4289
	Abs Asay Error	35	16.8925	18.2356	17.7187	.0765	.4528
	Abs Vasicek Error	35	.0933	.3055	.2297	.0092	.0542
	Abs HJM Error	35	.0029	.0742	.0453	.0033	.0453
	Valid N (listwise)	35					
646	Black Error	20	13.4127	14.3714	14.1068	.0657	.2937
	Asay Error	20	15.4211	16.2298	16.0051	.0539	.2412
	Vasicek Error	20	.1524	.3800	.3122	.0166	.0743
	HJM Error	20	-.0653	.2808	.0603	.0284	.1272
	Abs Black Error	20	13.4127	14.3714	14.1068	.0657	.2937
	Abs Asay Error	20	15.4211	16.2298	16.0051	.0539	.2412
	Abs Vasicek Error	20	.1524	.3800	.3122	.0166	.0743
	Abs HJM Error	20	.0082	.2808	.1085	.0195	.0872
	Valid N (listwise)	20					
660	Black Error	17	14.1280	15.2867	14.9099	.0939	.3872
	Asay Error	17	16.3590	17.3510	17.0449	.0802	.3307
	Vasicek Error	17	-.3262	-.0581	-.1898	.0217	.0896
	HJM Error	17	-.6512	-.3049	-.5074	.0256	.1055
	Abs Black Error	17	14.1280	15.2867	14.9099	.0939	.3872
	Abs Asay Error	17	16.3590	17.3510	17.0449	.0802	.3307
	Abs Vasicek Error	17	.0581	.3262	.1898	.0217	.0896
	Abs HJM Error	17	.3049	.6512	.5074	.0256	.1055
	Valid N (listwise)	17					
674	Black Error	37	12.0723	14.4734	13.5887	.1255	.7634
	Asay Error	37	13.9211	16.6103	15.5679	.1552	.9441
	Vasicek Error	37	-.0034	.6416	.1466	.0165	.1002
	HJM Error	37	-.1947	.3230	-.1107	.0135	.0821
	Abs Black Error	37	12.0723	14.4734	13.5887	.1255	.7634
	Abs Asay Error	37	13.9211	16.6103	15.5679	.1552	.9441
	Abs Vasicek Error	37	.0034	.6416	.1468	.0164	.1000
	Abs HJM Error	37	.0574	.3230	.1281	.0082	.0496
	Valid N (listwise)	37					

Table A.3 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Time to Maturity (Days)		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
681	Black Error	17	14.7744	15.5305	15.3200	.0591	.2435
	Asay Error	17	17.2565	17.8671	17.6996	.0459	.1891
	Vasicek Error	17	-.0367	.1315	.0833	.0134	.0553
	HJM Error	17	-.8540	-.0416	-.2223	.0419	.1727
	Abs Black Error	17	14.7744	15.5305	15.3200	.0591	.2435
	Abs Asay Error	17	17.2565	17.8671	17.6996	.0459	.1891
	Abs Vasicek Error	17	.0100	.1315	.0888	.0110	.0453
	Abs HJM Error	17	.0416	.8540	.2223	.0419	.1727
	Valid N (listwise)	17					
709	Black Error	20	14.1026	14.9029	14.6870	.0531	.2376
	Asay Error	20	16.3712	17.0319	16.8380	.0427	.1911
	Vasicek Error	20	.1333	.4192	.3102	.0220	.0983
	HJM Error	20	-.0766	.3143	.0546	.0309	.1381
	Abs Black Error	20	14.1026	14.9029	14.6870	.0531	.2376
	Abs Asay Error	20	16.3712	17.0319	16.8380	.0427	.1911
	Abs Vasicek Error	20	.1333	.4192	.3102	.0220	.0983
	Abs HJM Error	20	.0090	.3143	.1131	.0209	.0935
	Valid N (listwise)	20					
730	Black Error	17	16.3977	17.1472	16.9354	.0590	.2431
	Asay Error	17	19.3285	19.9244	19.7602	.0451	.1859
	Vasicek Error	17	.2105	.2879	.2569	.0062	.0255
	HJM Error	17	.0047	.0342	.0227	.0022	.0090
	Abs Black Error	17	16.3977	17.1472	16.9354	.0590	.2431
	Abs Asay Error	17	19.3285	19.9244	19.7602	.0451	.1859
	Abs Vasicek Error	17	.2105	.2879	.2569	.0062	.0255
	Abs HJM Error	17	.0047	.0342	.0227	.0022	.0090
	Valid N (listwise)	17					
751	Black Error	15	15.6338	16.4536	16.1976	.0714	.2766
	Asay Error	15	18.3413	19.0117	18.8133	.0574	.2224
	Vasicek Error	15	-.2763	-.0750	-.1787	.0168	.0650
	HJM Error	15	-.6824	-.3340	-.5543	.0277	.1071
	Abs Black Error	15	15.6338	16.4536	16.1976	.0714	.2766
	Abs Asay Error	15	18.3413	19.0117	18.8133	.0574	.2224
	Abs Vasicek Error	15	.0750	.2763	.1787	.0168	.0650
	Abs HJM Error	15	.3340	.6824	.5543	.0277	.1071
	Valid N (listwise)	15					

Table A.4

Descriptive Statistics for Call options on 90-Day Bank Accepted Bill Futures with different ranges of volatility

		N	Minimum	Maximum	Mean		Std. Deviation
Ranges of volatility		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
Missing Values	Black Error	2	6.1736	11.0994	8.6365	2.4629	3.4831
	Asay Error	2	6.4327	11.9458	9.1892	2.7566	3.8984
	Vasicek Error	2	.0316	.1046	.0681	.0365	.0516
	HJM Error	2	-.0257	-.0143	-.0200	.0057	.0081
	Abs Black Error	2	6.1736	11.0994	8.6365	2.4629	3.4831
	Abs Asay Error	2	6.4327	11.9458	9.1892	2.7566	3.8984
	Abs Vasicek Error	2	.0316	.1046	.0681	.0365	.0516
	Abs HJM Error	2	.0143	.0257	.0200	.0057	.0081
	Valid N (listwise)	2					
0.05 to 0.10	Black Error	53	-.0574	1.0882	.1873	.0411	.2991
	Asay Error	53	-.0497	1.0980	.1966	.0413	.3006
	Vasicek Error	53	-.0830	.2321	-.0014	.0078	.0570
	HJM Error	53	-.0810	.1294	-.0103	.0052	.0382
	Abs Black Error	53	.0012	1.0882	.2069	.0392	.2857
	Abs Asay Error	53	.0021	1.0980	.2129	.0397	.2891
	Abs Vasicek Error	53	.0012	.2321	.0357	.0061	.0442
	Abs HJM Error	53	.0013	.1294	.0285	.0037	.0271
	Valid N (listwise)	53					
0.11 to 0.15	Black Error	75	-.0661	2.4926	.6185	.0651	.5642
	Asay Error	75	-.0587	2.5418	.6388	.0658	.5695
	Vasicek Error	75	-.3706	.3661	-.0698	.0192	.1659
	HJM Error	75	-.3696	.2635	-.0890	.0175	.1517
	Abs Black Error	75	.0019	2.4926	.6264	.0641	.5554
	Abs Asay Error	75	.0067	2.5418	.6455	.0649	.5619
	Abs Vasicek Error	75	.0001	.3706	.1223	.0152	.1315
	Abs HJM Error	75	.0015	.3696	.1171	.0151	.1309
	Valid N (listwise)	75					
0.16 to 0.20	Black Error	226	-.0207	4.9988	2.3394	.0774	1.1640
	Asay Error	226	-.0131	5.1722	2.4308	.0799	1.2018
	Vasicek Error	226	-.4738	.3923	.0077	.0107	.1603
	HJM Error	226	-.4696	.2903	-.0329	.0093	.1400
	Abs Black Error	226	.0099	4.9988	2.3397	.0774	1.1634
	Abs Asay Error	226	.0026	5.1722	2.4310	.0799	1.2015
	Abs Vasicek Error	226	.0010	.4738	1139	.0075	.1128
	Abs HJM Error	226	.0004	.4696	.0890	.0075	.1128
	Valid N (listwise)	226					

Table A.4 (Cont'd)

Descriptive Statistics for Call options on 90-Day Bank Accepted Bill Futures with different ranges of volatility

		N	Minimum	Maximum	Mean		Std. Deviation
Ranges of volatility		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
0.21 to0.25	Black Error	338	1.3664	7.9676	5.0389	.0814	1.4958
	Asay Error	338	1.3685	8.4468	5.3010	.0372	1.6025
	Vasicek Error	338	-.5158	.4151	.0872	.0090	.1652
	HJM Error	338	-.5078	.2945	.0103	.0082	.1502
	Abs Black Error	338	1.3664	7.9676	5.0389	.0814	1.4958
	Abs Asay Error	338	1.3685	8.4468	5.3010	.0872	1.6025
	Abs Vasicek Error	338	.0005	.5158	.1476	.0062	.1142
	Abs HJM Error	338	.0001	.5078	.0981	.0062	.1141
	Valid N (listwise)	338					
0.26 to 0.30	Black Error	514	3.9662	11.4175	8.3918	.0722	1.6376
	Asay Error	514	4.0297	12.7025	9.0339	.0832	1.8868
	Vasicek Error	514	-.4912	.4424	.1270	.0071	.1603
	HJM Error	514	-.5133	.3903	-.0072	.0068	.1543
	Abs Black Error	514	3.9662	11.4175	8.3918	.0722	1.6376
	Abs Asay Error	514	4.0297	12.7025	9.0339	.0832	1.8868
	Abs Vasicek Error	514	.0014	.4912	.1714	.0049	.1114
	Abs HJM Error	514	.0002	.5133	.1089	.0048	.1095
	Valid N (listwise)	514					
0.31 to 0.35	Black Error	385	7.4996	14.4815	11.0975	.0781	1.5320
	Asay Error	385	8.0071	16.1520	12.1858	.0957	1.8786
	Vasicek Error	385	-.4093	.6300	.1541	.0089	.1739
	HJM Error	385	-3.2400	.3766	-.0436	.0129	.2535
	Abs Black Error	385	7.4996	14.4815	11.0975	.0781	1.5320
	Abs Asay Error	385	8.0071	16.1520	12.1858	.0957	1.8786
	Abs Vasicek Error	385	.0010	.6300	.1983	.0062	.1208
	Abs HJM Error	385	.0002	3.2400	.1579	.0103	.2029
	Valid N (listwise)	385					
0.36 to 0.40	Black Error	454	10.1211	16.4536	13.6867	.0625	1.3321
	Asay Error	454	10.9692	19.0117	15.3683	.0828	1.7641
	Vasicek Error	454	-.3262	.6935	.1891	.0079	.1682
	HJM Error	454	-.8540	.6256	-.0280	.0097	.2058
	Abs Black Error	454	10.1211	16.4536	13.6867	.0625	1.3321
	Abs Asay Error	454	10.9692	19.0117	15.3683	.0828	1.7641
	Abs Vasicek Error	454	.0034	.6935	.2172	.0061	.1298
	Abs HJM Error	454	.0029	.8540	.1430	.0071	.1504
	Valid N (listwise)	454					

Table A.4 (Cont'd)

Descriptive Statistics for Call options on 90-Day Bank Accepted Bill Futures with different ranges of volatility

		N	Minimum	Maximum	Mean		Std. Deviation
Ranges of volatility		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
0.41 to 0.45	Black Error	50	14.8341	17.1472	16.1831	.1006	.7112
	Asay Error	50	16.6325	19.9244	18.4379	.1612	1.1398
	Vasicek Error	50	.1318	.3653	.2717	.0070	.0498
	HJM Error	50	-.0055	.1929	.0485	.0073	.0517
	Abs Black Error	50	14.8341	17.1472	16.1831	.1006	.7112
	Abs Asay Error	50	16.6325	19.9244	18.4379	.1612	1.1398
	Abs Vasicek Error	50	.1318	.3653	.2717	.0070	.0498
	Abs HJM Error	50	.0001	.1929	.0489	.0073	.0513
	Valid N (listwise)	50					

Table A.5

Descriptive Statistics for Call options on 90-Day Bank Accepted Bill Futures with different strike price

		N	Minimum	Maximum	Mean		Std. Deviation
Strike Price		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
88	Black Error	5	-.0574	2.2085	1.0295	.4745	1.0609
	Asay Error	5	-.0497	2.3384	1.1194	.5019	1.1223
	Vasicek Error	5	-.2239	.0139	-.0743	.0405	.0905
	HJM Error	5	-.2204	.0164	-.0723	.0402	.0899
	Abs Black Error	5	.0193	2.2085	1.0602	.4572	1.0224
	Abs Asay Error	5	.0200	2.3384	1.1393	.4906	1.0970
	Abs Vasicek Error	5	.0139	.2239	.0798	.0377	.0844
	Abs HJM Error	5	.0164	.2204	.0788	.0370	.0827
	Valid N (listwise)	5					
88.25	Black Error	4	-.0467	2.2992	.8571	.5562	1.1123
	Asay Error	4	-.0394	2.4260	.9248	.5825	1.1650
	Vasicek Error	4	-.0722	.0139	-.0292	.0177	.0355
	HJM Error	4	-.0710	.0164	-.0277	.0181	.0361
	Abs Black Error	4	.0091	2.2992	.8805	.5439	1.0877
	Abs Asay Error	4	.0394	2.4260	.9445	.5719	1.1437
	Abs Vasicek Error	4	.0139	.0722	.0361	.0128	.0256
	Abs HJM Error	4	.0164	.0710	.0359	.0123	.0246
	Valid N (listwise)	4					
88.50	Black Error	15	-.0457	4.8961	1.5342	.4546	1.7606
	Asay Error	15	-.0400	5.2270	1.6498	.4835	1.8724
	Vasicek Error	15	-.3706	.1177	-.0758	.0323	.1251
	HJM Error	15	-.3696	.1198	-.0741	.0321	.1245
	Abs Black Error	15	.0207	4.8961	1.5487	.4510	1.7469
	Abs Asay Error	15	.0131	5.2270	1.6620	.4804	1.8608
	Abs Vasicek Error	15	.0096	.3706	.1052	.0258	.0998
	Abs HJM Error	15	.0099	.3696	.1042	.0255	.0988
	Valid N (listwise)	15					
88.75	Black Error	32	-.0661	7.4996	2.5007	.4001	2.2635
	Asay Error	32	-.0587	8.0071	2.6886	.4303	2.4344
	Vasicek Error	32	-.5158	.2499	-.0691	.0296	.1672
	HJM Error	32	-.5078	.2543	-.0658	.0301	.1701
	Abs Black Error	32	.0099	7.4996	2.5096	.3983	2.2533
	Abs Asay Error	32	.0026	8.0071	2.6961	.4288	2.4258
	Abs Vasicek Error	32	.0126	.5158	.1282	.0223	.1261
	Abs HJM Error	32	.0102	.5078	.1290	.0225	.1274
	Valid N (listwise)	32					

Table A.5 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Strike Price		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
89	Black Error	62	-.0549	13.4995	5.5104	.5213	4.1049
	Asay Error	62	-.0479	15.4946	6.0649	.5857	4.6115
	Vasicek Error	62	-.5151	.6717	-.0195	.0266	.2098
	HJM Error	62	-.5078	.6256	-.0377	.0256	.2017
	Abs Black Error	62	.0133	13.4995	5.5143	.5207	4.0996
	Abs Asay Error	62	.0202	15.4946	6.0683	.5851	4.6070
	Abs Vasicek Error	62	.0010	.6717	.1503	.0186	.1464
	Abs HJM Error	62	.0032	.6256	.1449	.0183	.1443
	Valid N (listwise)	62					
89.25	Black Error	78	-.0533	16.3977	6.6711	.5030	4.4420
	Asay Error	78	-.0466	19.3285	7.3861	.5738	5.0675
	Vasicek Error	78	-.5039	.6840	.0046	.0234	.2064
	HJM Error	78	-.4978	.6243	-.0290	.0223	.1971
	Abs Black Error	78	.0294	16.3977	6.6741	.5025	4.4375
	Abs Asay Error	78	.0240	19.3285	7.3887	.5733	5.0636
	Abs Vasicek Error	78	.0029	.6840	.1513	.0158	.1394
	Abs HJM Error	78	.0011	.6243	.1429	.0156	.1378
	Valid N (listwise)	78					
89.50	Black Error	83	-.0410	16.5113	7.2302	.5141	4.6838
	Asay Error	83	-.0347	19.4286	8.0255	.5898	5.3733
	Vasicek Error	83	-.5018	.6808	.0228	.0225	.2053
	HJM Error	83	-.4978	.6059	-.0248	.0212	.1929
	Abs Black Error	83	.0152	16.5113	7.2321	.5138	4.6808
	Abs Asay Error	83	.0100	19.4286	8.0271	.5895	5.3710
	Abs Vasicek Error	83	.0023	.6808	.1512	.0153	.1397
	Abs HJM Error	83	.0012	.6059	.1389	.0148	.1353
	Valid N (listwise)	83					

Table A.5 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Strike Price		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
89.75	Black Error	91	-.0377	16.6153	7.8112	.5018	4.7871
	Asay Error	91	-.0318	19.5191	8.6878	.5779	5.5128
	Vasicek Error	91	-.4984	.6925	.0437	.0219	.2091
	HJM Error	91	-3.2400	.6013	-.0517	.0411	.3917
	Abs Black Error	91	.0087	16.6153	7.8127	.5016	4.7846
	Abs Asay Error	91	.0040	19.5191	8.6890	.5777	5.5108
	Abs Vasicek Error	91	.0016	.6925	.1588	.0149	.1419
	Abs HJM Error	91	.0010	3.2400	.1785	.0369	.3521
	Valid N (listwise)	91					
90	Black Error	90	-.0232	16.7097	7.8615	.5047	4.7883
	Asay Error	90	-.0197	19.6001	8.7153	.5798	5.5003
	Vasicek Error	90	-.4830	.6899	.0599	.0223	.2112
	HJM Error	90	-.4878	.5813	-.0134	.0211	.2001
	Abs Black Error	90	.0108	16.7097	7.8625	.5046	4.7866
	Abs Asay Error	90	.0151	19.6001	8.7161	.5796	5.4989
	Abs Vasicek Error	90	.0026	.6899	.1650	.0152	.1439
	Abs HJM Error	90	.0021	.5813	.1450	.0145	.1377
	Valid N (listwise)	90					
90.25	Black Error	93	-.0168	16.8046	8.3272	.4894	4.7197
	Asay Error	93	-.0117	19.6815	9.2395	.5642	5.4407
	Vasicek Error	93	-.4746	.6935	.0777	.0216	.2084
	HJM Error	93	-.6328	.5668	-.0210	.0217	.2093
	Abs Black Error	93	.0105	16.8046	8.3278	.4893	4.7187
	Abs Asay Error	93	.0091	19.6815	9.2399	.5641	5.4400
	Abs Vasicek Error	93	.0001	.6935	.1655	.0153	.1478
	Abs HJM Error	93	.0009	.6328	.1488	.0153	.1478
	Valid N (listwise)	93					
90.50	Black Error	94	-.0089	16.8798	8.4522	.4875	4.7267
	Asay Error	94	-.0077	19.7434	9.3533	.5613	5.4417
	Vasicek Error	94	-.4619	.6840	.0910	.0219	.2120
	HJM Error	94	-.5680	.5391	-.0121	.0208	.2017
	Abs Black Error	94	.0019	16.8798	8.4524	.4875	4.7264
	Abs Asay Error	94	.0067	19.7434	9.3535	.5612	5.4414
	Abs Vasicek Error	94	.0012	.6840	.1720	.0158	.1530
	Abs HJM Error	94	.0012	.5680	.1457	.0143	.1391
	Valid N (listwise)	94					

Table A.5 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Strike Price		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
90.75	Black Error	94	-.0056	16.9454	8.5030	.4894	4.7447
	Asay Error	94	-.0046	19.7958	9.3936	.5628	5.4562
	Vasicek Error	94	-.4337	.6821	.1049	.0210	.2034
	HJM Error	94	-.6144	.5191	-.0111	.0209	.2025
	Abs Black Error	94	.0056	16.9454	8.5031	.4894	4.7445
	Abs Asay Error	94	.0046	19.7958	9.3937	.5628	5.4561
	Abs Vasicek Error	94	.0012	.6821	.1743	.0152	.1477
	Abs HJM Error	94	.0003	.6144	.1450	.0145	.1410
	Valid N (listwise)	94					
91	Black Error	95	.0012	17.0014	8.5586	.4847	4.7246
	Asay Error	95	.0021	19.8386	9.4343	.5573	5.4323
	Vasicek Error	95	-.4126	.6682	.1174	.0206	.2006
	HJM Error	95	-.6544	.4881	-.0135	.0209	.2039
	Abs Black Error	95	.0012	17.0014	8.5586	.4847	4.7246
	Abs Asay Error	95	.0021	19.8386	9.4343	.5573	5.4323
	Abs Vasicek Error	95	.0012	.6682	.1807	.0149	.1455
	Abs HJM Error	95	.0013	.6544	.1453	.0147	.1428
	Valid N (listwise)	95					
91.25	Black Error	96	.0243	17.0578	8.6943	.4816	4.7186
	Asay Error	96	.0250	19.8819	9.5678	.5531	5.4191
	Vasicek Error	96	-.3967	.6530	.1297	.0199	.1946
	HJM Error	96	-.8540	.4571	-.0187	.0228	.2231
	Abs Black Error	96	.0243	17.0578	8.6943	.4816	4.7186
	Abs Asay Error	96	.0250	19.8819	9.5678	.5531	5.4191
	Abs Vasicek Error	96	.0018	.6530	.1841	.0147	.1436
	Abs HJM Error	96	.0020	.8540	.1549	.0164	.1609
	Valid N (listwise)	96					
91.50	Black Error	94	.0474	17.0945	8.6945	.4827	4.6799
	Asay Error	94	.0480	19.9056	9.5373	.5535	5.3667
	Vasicek Error	94	-.3716	.6369	.1468	.0195	.1893
	HJM Error	94	-.6824	.4273	-.0143	.0212	.2051
	Abs Black Error	94	.0474	17.0945	8.6945	.4827	4.6799
	Abs Asay Error	94	.0480	19.9056	9.5373	.5535	5.3667
	Abs Vasicek Error	94	.0018	.6369	.1947	.0143	.1390
	Abs HJM Error	94	.0020	.6824	.1431	.0152	.1469
	Valid N (listwise)	94					

Table A.5 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Strike Price		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
91.75	Black Error	93	.0855	17.1217	8.9101	.4834	4.6617
	Asay Error	93	.0860	19.9198	9.7712	.5553	5.3555
	Vasicek Error	93	-.3961	.6103	.1559	.0194	.1870
	HJM Error	93	-.6716	.3895	-.0135	.0210	.2022
	Abs Black Error	93	.0855	17.1217	8.9101	.4834	4.6617
	Abs Asay Error	93	.0860	19.9198	9.7712	.5553	5.3555
	Abs Vasicek Error	93	.0005	.6103	.2028	.0139	.1340
	Abs HJM Error	93	.0020	.6716	.1404	.0151	.1455
	Valid N (listwise)	93					
92	Black Error	95	.1439	17.1393	8.8966	.4746	4.6253
	Asay Error	95	.1443	19.9244	9.7395	.5452	5.3141
	Vasicek Error	95	-.2913	.5837	.1749	.0173	.1687
	HJM Error	95	-.6395	.3545	-.0136	.0203	.1977
	Abs Black Error	95	.1439	17.1393	8.8966	.4746	4.6253
	Abs Asay Error	95	.1443	19.9244	9.7395	.5452	5.3141
	Abs Vasicek Error	95	.0018	.5837	.2054	.0133	.1294
	Abs HJM Error	95	.0020	.6395	.1405	.0143	.1391
	Valid N (listwise)	95					
92.25	Black Error	93	.2257	17.1472	8.8194	.4848	4.6752
	Asay Error	93	.2277	19.9195	9.6450	.5564	5.3658
	Vasicek Error	93	-.2517	.5471	.1863	.0162	.1563
	HJM Error	93	-.5986	.3317	-.0082	.0199	.1916
	Abs Black Error	93	.2257	17.1472	8.8194	.4848	4.6752
	Abs Asay Error	93	.2277	19.9195	9.6450	.5564	5.3658
	Abs Vasicek Error	93	.0118	.5471	.2102	.0126	.1217
	Abs HJM Error	93	.0002	.5986	.1369	.0138	.1335
	Valid N (listwise)	93					
92.50	Black Error	96	.2887	17.1455	8.8796	.4706	4.6112
	Asay Error	96	.2907	19.9050	9.7107	.5409	5.2996
	Vasicek Error	96	-.1797	.5407	.1955	.0139	.1361
	HJM Error	96	-.5338	.3049	-.0118	.0176	.1720
	Abs Black Error	96	.2887	17.1455	8.8796	.4706	4.6112
	Abs Asay Error	96	.2907	19.9050	9.7107	.5409	5.2996
	Abs Vasicek Error	96	.0118	.5407	.2114	.0112	.1096
	Abs HJM Error	96	.0016	.5338	.1258	.0120	.1172
	Valid N (listwise)	96					

Table A.5 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Strike Price		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
92.75	Black Error	92	.3635	17.1342	9.1684	.4597	4.4090
	Asay Error	92	.3654	19.8809	10.0105	.5303	5.0866
	Vasicek Error	92	-.1065	.4746	.1896	.0124	.1193
	HJM Error	92	-.4711	.2702	-.0154	.0157	.1502
	Abs Black Error	92	.3635	17.1342	9.1684	.4597	4.4090
	Abs Asay Error	92	.3654	19.8809	10.0105	.5303	5.0866
	Abs Vasicek Error	92	.0014	.4746	.1997	.0105	.1011
	Abs HJM Error	92	.0004	.4711	.1121	.0105	.1005
	Valid N (listwise)	92					
93	Black Error	87	.4512	17.1133	9.5557	.4488	4.1860
	Asay Error	87	.4528	19.8473	10.4358	.5203	4.8528
	Vasicek Error	87	-.0955	.4386	.1907	.0107	.0998
	HJM Error	87	-.4063	.2361	-.0245	.0134	.1250
	Abs Black Error	87	.4512	17.1133	9.5557	.4488	4.1860
	Abs Asay Error	87	.4528	19.8473	10.4358	.5203	4.8528
	Abs Vasicek Error	87	.0035	.4386	.1951	.0098	.0910
	Abs HJM Error	87	.0001	.4063	.0942	.0091	.0852
	Valid N (listwise)	87					
93.25	Black Error	85	.5524	17.0828	9.7139	.4479	4.1291
	Asay Error	85	.5538	19.8042	10.6073	.5196	4.7904
	Vasicek Error	85	-.0750	.4034	.1857	.0097	.0895
	HJM Error	85	-.3340	.1912	-.0241	.0114	.1051
	Abs Black Error	85	.5524	17.0828	9.7139	.4479	4.1291
	Abs Asay Error	85	.5538	19.8042	10.6073	.5196	4.7904
	Abs Vasicek Error	85	.0166	.4034	.1893	.0089	.0817
	Abs HJM Error	85	.0001	.3340	.0809	.0077	.0708
	Valid N (listwise)	85					
93.50	Black Error	74	.6576	16.3845	9.4873	.4330	3.7244
	Asay Error	74	.6587	18.4818	10.3083	.5008	4.3077
	Vasicek Error	74	-.0083	.4163	.1892	.0085	.0734
	HJM Error	74	-.1984	.2073	-.0182	.0098	.0847
	Abs Black Error	74	.6576	16.3845	9.4873	.4330	3.7244
	Abs Asay Error	74	.6587	18.4818	10.3083	.5008	4.3077
	Abs Vasicek Error	74	.0083	.4163	.1894	.0085	.0728
	Abs HJM Error	74	.0021	.2073	.0685	.0061	.0525
	Valid N (listwise)	74					

Table A.5 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Strike Price		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
93.75	Black Error	72	1.3664	15.8149	9.3867	.4308	3.6553
	Asay Error	72	1.3685	17.9847	10.1895	.4977	4.2232
	Vasicek Error	72	-.0444	.3693	.1698	.0091	.0773
	HJM Error	72	-.1882	.1976	-.0161	.0091	.0772
	Abs Black Error	72	1.3664	15.8149	9.3867	.4308	3.6553
	Abs Asay Error	72	1.3685	17.9847	10.1895	.4977	4.2232
	Abs Vasicek Error	72	.0010	.3693	.1710	.0088	.0744
	Abs HJM Error	72	.0005	.1976	.0610	.0058	.0495
	Valid N (listwise)	72					
94	Black Error	65	2.7895	15.7622	9.8572	.3888	3.1348
	Asay Error	65	2.8274	17.9208	10.7208	.4550	3.6686
	Vasicek Error	65	.0010	.3011	.1636	.0083	.0671
	HJM Error	65	-.1699	.2410	-.0167	.0089	.0715
	Abs Black Error	65	2.7895	15.7622	9.8572	.3888	3.1348
	Abs Asay Error	65	2.8274	17.9208	10.7208	.4550	3.6686
	Abs Vasicek Error	65	.0010	.3011	.1636	.0083	.0671
	Abs HJM Error	65	.0020	.2410	.0566	.0057	.0463
	Valid N (listwise)	65					
94.25	Black Error	63	2.7901	15.7000	10.0668	.3907	3.1010
	Asay Error	63	2.8264	17.8474	10.9550	.4577	3.6331
	Vasicek Error	63	-.0053	.2416	.1412	.0076	.0607
	HJM Error	63	-.1429	.2588	-.0187	.0078	.0618
	Abs Black Error	63	2.7901	15.7000	10.0668	.3907	3.1010
	Abs Asay Error	63	2.8264	17.8474	10.9550	.4577	3.6331
	Abs Vasicek Error	63	.0053	.2416	.1414	.0076	.0603
	Abs HJM Error	63	.0005	.2588	.0471	.0055	.0437
	Valid N (listwise)	63					
94.50	Black Error	58	2.7339	15.6381	10.3710	.4025	3.0657
	Asay Error	58	2.7687	17.7744	11.3046	.4728	3.6007
	Vasicek Error	58	.0046	.2298	.1284	.0067	.0511
	HJM Error	58	-.1211	.0622	-.0224	.0055	.0420
	Abs Black Error	58	2.7339	15.6381	10.3710	.4025	3.0657
	Abs Asay Error	58	2.7687	17.7744	11.3046	.4728	3.6007
	Abs Vasicek Error	58	.0046	.2298	.1284	.0067	.0511
	Abs HJM Error	58	.0002	.1211	.0366	.0040	.0302
	Valid N (listwise)	58					

Table A.5 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Strike Price		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
94.75	Black Error	46	4.7959	15.5667	10.3813	.4272	2.8974
	Asay Error	46	4.9548	17.6919	11.3074	.5054	3.4275
	Vasicek Error	46	.0075	.1957	.1081	.0068	.0460
	HJM Error	46	-.0986	.0442	-.0157	.0047	.0317
	Abs Black Error	46	4.7959	15.5667	10.3813	.4272	2.8974
	Abs Asay Error	46	4.9548	17.6919	11.3074	.5054	3.4275
	Abs Vasicek Error	46	.0075	.1957	.1081	.0068	.0460
	Abs HJM Error	46	.0008	.0986	.0259	.0035	.0239
	Valid N (listwise)	46					
95	Black Error	34	5.0452	15.4957	10.3521	.4714	2.7488
	Asay Error	34	5.1842	17.6099	11.2789	.5621	3.2774
	Vasicek Error	34	.0045	.6416	.1107	.0175	.1018
	HJM Error	34	-.0789	.3230	-.0047	.0108	.0630
	Abs Black Error	34	5.0452	15.4957	10.3521	.4714	2.7488
	Abs Asay Error	34	5.1842	17.6099	11.2789	.5621	3.2774
	Abs Vasicek Error	34	.0045	.6416	.1107	.0175	.1018
	Abs HJM Error	34	.0003	.3230	.0305	.0094	.0551
	Valid N (listwise)	34					
95.25	Black Error	8	4.9480	10.5417	8.1361	.7669	2.1690
	Asay Error	8	5.0841	11.3940	8.6677	.8708	2.4630
	Vasicek Error	8	.0567	.1270	.0958	.0104	.0296
	HJM Error	8	-.0457	.0144	-.0102	.0068	.0191
	Abs Black Error	8	4.9480	10.5417	8.1361	.7669	2.1690
	Abs Asay Error	8	5.0841	11.3940	8.6677	.8708	2.4630
	Abs Vasicek Error	8	.0567	.1270	.0958	.0104	.0296
	Abs HJM Error	8	.0037	.0457	.0159	.0050	.0141
	Valid N (listwise)	8					
95.50	Black Error	6	6.5418	10.4576	8.9357	.6738	1.6504
	Asay Error	6	6.8200	11.3122	9.5734	.7728	1.8929
	Vasicek Error	6	.0513	.1025	.0784	.0092	.0226
	HJM Error	6	-.0282	.0026	-.0118	.0044	.0109
	Abs Black Error	6	6.5418	10.4576	8.9357	.6738	1.6504
	Abs Asay Error	6	6.8200	11.3122	9.5734	.7728	1.8929
	Abs Vasicek Error	6	.0513	.1025	.0784	.0092	.0226
	Abs HJM Error	6	.0026	.0282	.0127	.0039	.0096
	Valid N (listwise)	6					

Table A.5 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Strike Price		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
95.75	Black Error	4	9.0678	10.3742	9.8625	.3010	.6020
	Asay Error	4	9.7438	11.2211	10.6405	.3456	.6912
	Vasicek Error	4	.0405	.0842	.0626	.0122	.0244
	HJM Error	4	-.0193	-.0016	-.0102	.0036	.0072
	Abs Black Error	4	9.0678	10.3742	9.8625	.3010	.6020
	Abs Asay Error	4	9.7438	11.2211	10.6405	.3456	.6912
	Abs Vasicek Error	4	.0405	.0842	.0626	.0122	.0244
	Abs HJM Error	4	.0016	.0193	.0102	.0036	.0072
	Valid N (listwise)	4					

Table A.6

Descriptive Statistics for Call options on 90-Day Bank Accepted Bill Futures for months January to December 1996

		N	Minimum	Maximum	Mean		Std. Deviation
Date		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
31/1/96	Black Error	174	.1159	14.4734	8.6262	.3312	4.3688
	Asay Error	174	.1413	16.6103	9.5564	.3827	5.0485
	Vasicek Error	174	-.2239	.6416	.0409	.0107	.1413
	HJM Error	174	-.2204	.3230	-.1014	.0053	.0701
	Abs Black Error	174	.1159	14.4734	8.6262	.3312	4.3688
	Abs Asay Error	174	.1413	16.6103	9.5564	.3827	5.0485
	Abs Vasicek Error	174	.0034	.6416	.1290	.0053	.0702
	Abs HJM Error	174	.0003	.3230	.1077	.0045	.0599
	Valid N (listwise)	174					
28/2/96	Black Error	170	-.0425	16.4631	9.2224	.3937	5.1327
	Asay Error	170	-.0400	18.6137	10.0329	.4448	5.7990
	Vasicek Error	170	-.0288	.6935	.2510	.0132	.1716
	HJM Error	170	-.0287	.6256	.1236	.0104	.1351
	Abs Black Error	170	.0012	16.4631	9.2251	.3933	5.1278
	Abs Asay Error	170	.0021	18.6137	10.0353	.4444	5.7947
	Abs Vasicek Error	170	.0012	.6935	.2523	.0130	.1697
	Abs HJM Error	170	.0001	.6256	.1251	.0103	.1338
	Valid N (listwise)	170					
27/3/96	Black Error	169	1.0762	17.1472	10.6013	.3745	4.8684
	Asay Error	169	1.1827	19.9244	11.8023	.4400	5.7203
	Vasicek Error	169	-.0332	.3781	.2183	.0081	.1052
	HJM Error	169	-.0315	.2295	.0739	.0046	.0596
	Abs Black Error	169	1.0762	17.1472	10.6013	.3745	4.8684
	Abs Asay Error	169	1.1827	19.9244	11.8023	.4400	5.7203
	Abs Vasicek Error	169	.0016	.3781	.2196	.0079	.1024
	Abs HJM Error	169	.0015	.2295	.0751	.0045	.0581
	Valid N (listwise)	169					
24/4/96	Black Error	168	-.0193	15.5305	9.1644	.3669	4.7560
	Asay Error	168	.0200	17.8671	10.1516	.4228	5.4803
	Vasicek Error	168	-.2163	.2970	.0844	.0088	.1139
	HJM Error	168	-.8540	.1929	-.0830	.0082	.1057
	Abs Black Error	168	.0091	15.5305	9.1647	.3669	4.7556
	Abs Asay Error	168	.0200	17.8671	10.1516	.4228	5.4803
	Abs Vasicek Error	168	.0025	.2970	.1240	.0053	.0685
	Abs HJM Error	168	.0002	.8540	.1002	.0069	.0894
	Valid N (listwise)	168					

Table A.6 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Date		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
29/5/96	Black Error	165	-.0574	14.4815	8.7721	.3841	4.9341
	Asay Error	165	-.0497	16.1520	9.5951	.4311	5.5381
	Vasicek Error	165	-.3961	.4151	.2327	.0103	.1328
	HJM Error	165	-.0449	.3113	.1261	.0062	.0794
	Abs Black Error	165	.0087	14.4815	8.7754	.3837	4.9282
	Abs Asay Error	165	.0040	16.1520	9.5978	.4308	5.5335
	Abs Vasicek Error	165	.0049	.4151	.2405	.0092	.1179
	Abs HJM Error	165	.0049	.3113	.1290	.0058	.0745
	Valid N (listwise)	165					
26/6/96	Black Error	157	.3711	15.1101	9.2829	.3523	4.4142
	Asay Error	157	.4326	17.2445	10.1365	.4016	5.0325
	Vasicek Error	157	-.0348	.3597	.1680	.0070	.0874
	HJM Error	157	-.1702	.2871	.0058	.0056	.0699
	Abs Black Error	157	.3711	15.1101	9.2829	.3523	4.4142
	Abs Asay Error	157	.4326	17.2445	10.1365	.4016	5.0325
	Abs Vasicek Error	157	.0001	.3597	.1697	.0067	.0840
	Abs HJM Error	157	.0013	.2871	.0539	.0036	.0447
	Valid N (listwise)	157					
31/7/96	Black Error	170	.0754	16.4536	8.4082	.3916	5.1064
	Asay Error	170	.1075	19.0117	9.3364	.4538	5.9174
	Vasicek Error	170	-.5158	.0856	-.2011	.0135	.1761
	HJM Error	170	-.6824	-.0099	-.3793	.0132	.1725
	Abs Black Error	170	.0754	16.4536	8.4082	.3916	5.1064
	Abs Asay Error	170	.1075	19.0117	9.3364	.4538	5.9174
	Abs Vasicek Error	170	.0010	.5158	.2155	.0121	.1581
	Abs HJM Error	170	.0099	.6824	.3793	.0132	.1725
	Valid N (listwise)	170					
28/8/96	Black Error	182	-.0207	14.3714	8.3306	.3326	4.4865
	Asay Error	182	-.0131	16.2298	9.1052	.3767	5.0818
	Vasicek Error	182	-.1363	.3800	.0542	.0096	.1298
	HJM Error	182	-.2383	.2808	-.0778	.0062	.0835
	Abs Black Error	182	.0099	14.3714	8.3310	.3325	4.4859
	Abs Asay Error	182	.0026	16.2298	9.1054	.3767	5.0815
	Abs Vasicek Error	182	.0016	.3800	.1114	.0063	.0855
	Abs HJM Error	182	.0001	.2808	.0971	.0044	.0599
	Valid N (listwise)	182					

Table A.6 (Cont'd)

		N	Minimum	Maximum	Mean		Std. Deviation
Date		Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
25/9/96	Black Error	183	1.5851	14.9029	9.2874	.2945	3.9832
	Asay Error	183	1.6672	17.0319	10.2099	.3435	4.6467
	Vasicek Error	183	.0953	.6300	.3118	.0073	.0983
	HJM Error	183	-.6328	.3903	.1724	.0116	.1569
	Abs Black Error	183	1.5851	14.9029	9.2874	.2945	3.9832
	Abs Asay Error	183	1.6672	17.0319	10.2099	.3435	4.6467
	Abs Vasicek Error	183	.0953	.6300	.3118	.0073	.0983
	Abs HJM Error	183	.0005	.6328	.1925	.0097	.1313
	Valid N (listwise)	183					
30/10/96	Black Error	180	.5086	13.1192	7.4051	.2834	3.8020
	Asay Error	180	.5430	14.8168	8.0921	.3226	4 3281
	Vasicek Error	180	-.0990	.2577	.0753	.0074	.0994
	HJM Error	180	-3.2400	.2410	-.0726	.0183	.2450
	Abs Black Error	180	.5086	13.1192	7.4051	.2834	3.8020
	Abs Asay Error	180	.5430	14.8168	8.0921	.3226	4.3281
	Abs Vasicek Error	180	.0010	.2577	.1008	.0055	.0733
	Abs HJM Error	180	.0004	3.2400	.0807	.0181	.2424
	Valid N (listwise)	180					
27/11/96	Black Error	191	-.0661	11.4175	6.2950	.2582	3.5683
	Asay Error	191	-.0587	12.7025	6.8267	.2874	3.9720
	Vasicek Error	191	-.0482	.2051	.0821	.0046	.0633
	HJM Error	191	-.1947	.1976	-.0122	.0048	.0667
	Abs Black Error	191	.0019	11.4175	6.2980	.2578	3.5628
	Abs Asay Error	191	.0067	12.7025	6.8293	.2871	3.9675
	Abs Vasicek Error	191	.0010	.2051	.0854	.0043	.0588
	Abs HJM Error	191	.0016	.1976	.0502	.0033	.0455
	Valid N (listwise)	191					
18/12/96	Black Error	188	.8104	12.0372	7.6011	.2381	3.2652
	Asay Error	188	.8895	13.1708	8.1678	.2621	3.5942
	Vasicek Error	188	-.0897	.2869	.1197	.0081	.1106
	HJM Error	188	-.1095	.2588	-.0167	.0035	.0482
	Abs Black Error	188	.8104	12.0372	7.6011	.2381	3.2652
	Abs Asay Error	188	.8895	13.1708	8.1678	.2621	3.5942
	Abs Vasicek Error	188	.0005	.2869	.1355	.0066	.0905
	Abs HJM Error	188	.0002	.2588	.0373	.0025	.0347
	Valid N (listwise)	188					

TABLE A.7

Summary table for mean pricing error for options with different time to maturity and moneyness

Black Model: Mean Pricing Error

Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	191-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-790	701-730	>731
M<-0.02	1.1457 (4)	1.4915 (3)	2.8021 (7)	3.1866 (10)	4.3336 (18)	5.1175 (15)	5.8457 (28)	6.5706 (38)	7.0983 (14)	9.4115 (37)	9.8267 (46)	8.6305 (19)	10.6003 (46)	10.8199 (51)	10.4637 (49)	12.4573 (54)	12.5954 (39)	11.6240 (18)	13.7650 (75)	13.9920 (34)	12.7545 (16)	15.2338 (35)	14.2837 (28)	14.8337 (11)	16.8614 (13)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	191-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
-0.02<M<0.02	0.3194 (1)	1.7422 (2)		3.4338 (4)	6.0922 (1)	8.2979 (2)		7.2625 (2)	7.8079 (1)	8.2721 (2)	9.0412 (4)	8.2546 (2)	10.2724 (1)	11.6951 (2)	10.5393 (2)	12.5253 (2)	14.3709 (1)	10.8994 (1)	13.1008 (2)	14.1888 (3)		14.3210 (1)			17.0578 (1)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
M>0.02	0.2329 (90)	0.7487 (53)	1.9018 (57)	2.4657 (89)	3.4443 (86)	4.5204 (54)	5.1402 (77)	5.6340 (102)	6.4667 (37)	6.1462 (50)	9.2315 (71)	7.8050 (53)	10.4011 (46)	10.2209 (67)	9.9610 (73)	12.2447 (43)	12.1738 (52)	11.1804 (29)	13.3511 (65)	13.7356 (46)	12.3004 (24)	14.7383 (36)	13.8723 (26)	14.5078 (9)	16.3676 (18)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	161-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-518	511-540	541-570	571-600	601-630	631-670	671-790	701-730	>731
All M	0.2723 (95)	0.8196 (58)	1.9110 (64)	2.5733 (103)	3.6220 (105)	4.6502 (69)	5.3464 (107)	6.0512 (142)	6.6567 (52)	9.2594 (89)	8.4845 (121)	8.0291 (74)	10.4992 (93)	10.4151 (120)	10.1101 (124)	12.3683 (99)	12.3783 (92)	11.3400 (47)	13.5768 (142)	13.6570 (83)	12.4820 (40)	14.9734 (72)	14.1336 (54)	14.6870 (20)	16.5895 (32)

Asay Model: Mean Pricing Error

Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
M<-0.02	1.1495 (4)	1.5023 (3)	2.8400 (7)	3.2471 (10)	4.4475 (18)	5.2729 (15)	9.0616 (29)	6.5706 (38)	7.4222 (14)	8.9857 (37)	9.3886 (46)	9.1636 (19)	11.4043 (46)	11.5002 (51)	11.2736 (49)	13.6458 (54)	13.9068 (39)	12.7630 (18)	15.3496 (75)	15.7274 (34)	14.2489 (16)	17.3082 (35)	16.3106 (28)	16.9297 (11)	19.5446 (13)
Time to maturity(days)	<30	31-60	61-90	81-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
-0.02<M<0.02	0.3196 (1)	1.7543 (2)		3.5010 (4)	8.2713 (1)	(2)	6.5448	7.8020 (2)	7.9859 (1)	9.7466 (2)	8.8282 (4)	9.7581 (2)	11.0340 (1)	12.7136 (2)	11.3635 (2)	13.7264 (2)	15.9381 (1)	11.7268 (1)	14.5474 (2)	15.9569 (3)		16.2126 (1)			19.8819 (1)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	161-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
M>0.02	0.2389 (90)	0.7687 (53)	1.8573 (57)	2.5552 (88)	3.5895 (86)	4.7239 (54)	5.3981 (77)	6.1913 (102)	6.8844 (37)	8.6815 (50)	8.9641 (71)	9.4173 (53)	11.2925 (46)	11.1996 (87)	10.7709 (73)	13.5424 (43)	13.5751 (52)	12.4525 (29)	15.0049 (85)	15.5941 (46)	13.9026 (24)	16.9896 (36)	16.1818 (26)	16.7259 (9)	19.1200 (18)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
All M	0.2871 (95)	0.8415 (58)	1.9648 (64)	2.6591 (103)	3.7613 (105)	4.8432 (69)	5.5932 (107)	8.3836 (142)	7.0381 (52)	8.7734 (89)	9.0881 (121)	8.6181 (74)	11.3450 (93)	11.3526 (120)	10.9791 (124)	13.6025 (99)	13.7414 (92)	12.5560 (47)	15.1805 (142)	15.8619 (83)	14.0411 (40)	17.0836 (72)	18.2390 (54)	18.8380 (20)	19.3164 (32)

Extended-Vasicek Model: Mean Pricing Error

Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	971-700	701-730	>731
M<-0.02	0.0660 (4)	0.1277 (3)	0.1847 (7)	0.2145 (10)	0.1724 (19)	0.2185 (15)	0.2053 (28)	0.1456 (38)	0.2015 (14)	0.2009 (37)	0.1432 (46)	0.2140 (19)	0.1896 (46)	0.1412 (51)	0.1837 (49)	0.1722 (54)	0.1749 (39)	0.1736 (18)	0.1753 (75)	0.1546 (34)	0.1901 (18)	0.1801 (35)	0.1698 (28)	0.2380 (11)	0.1172 (13)
Time to maturity(days)	<30	31-60	61-90	81-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
-0.02<M<0.02	0.0560 (1)	0.0960 (2)		0.2546 (4)	0.3064 (1)	0.3572 (2)		0.1803 (2)	0.4034 (1)	0.2790 (2)	0.1993 (4)	0.2321 (2)	0.1541 (1)	0.3716 (2)	0.2759 (2)	0.2490 (2)	0.3636 (1)	0.1597 (1)	0.2235 (2)	0.2048 (3)		0.3445 (1)			0.2879 (1)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	161-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
M>0.02	-0.0306 (90)	-0.1076 (53)	0.0630 (57)	0.0650 (89)	-0.0500 (86)	0.1546 (54)	0.1005 (77)	-0.0200 (102)	0.2153 (37)	0.1952 (50)	-0.0040 (71)	0.1915 (53)	0.1802 (46)	0.1192 (67)	0.1853 (73)	0.1762 (43)	0.1209 (52)	0.2413 (28)	0.1193 (65)	0.1237 (45)	0.2681 (24)	0.1224 (36)	0.0800 (26)	0.3985 (9)	-0.0070 (18)
Time to maturity(days)	<30	31-60	61-90	81-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	871-700	701-730	>731
All M	-0.02 (95)	-0.09 (58)	0.076 (64)	0.087 (103)	-0.01 (105)	0.1885 (89)	0.1328 (107)	0.031 (142)	0.2152 (52)	0.1626 (99)	0.058 (121)	0.1983 (74)	0.1846 (93)	0.1327 (120)	0.1661 (124)	0.1755 (99)	0.1484 (92)	0.2136 (47)	0.1503 (142)	0.1393 (63)	0.2369 (40)	0.1535 (72)	0.1267 (54)	0.3102 (20)	0.053 (32)

HJM Model: Mean Pricing Error

Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-790	701-730	>731
M<-0.02	0.027 (4)	0.046 (3)	0.073 (7)	0.1066 (10)	0.039 (19)	0.07 (15)	0.064 (29)	0.019 (38)	0.059 (14)	0.058 (37)	-0.004 (46)	0.00036 (19)	0.033 (46)	-0.06 (51)	-0.04 (49)	0.0079 (54)	-0.04 (39)	-0.06 (18)	-0.03 (75)	-0.08 (34)	-0.1 (16)	-0.08 (35)	-0.1077 (28)	-0.05 (11)	-0.1709 (13)
Time to maturity(days)	<30	31-60	61-90	81-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
-0.02<M<0.02	0.039 (1)	-0.02 (2)		0.125 (4)	0.1483 (1)	0.1805 (2)		-0.06 (2)	0.1979 (1)	0.066 (2)	-0.09 (4)	-0.04 (2)	-0.1278 (1)	0.1177 (2)	-0.03 (2)	-0.008 (2)	0.1162 (1)	-0.1741 (1)	-0.1048 (2)	-0.1054 (3)		0.0082 (1)			0.026 (1)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	461-480	481-510	611-640	541-570	571-600	601-630	631-670	871-700	701-730	>731
M>0.02	-0.03 (90)	-0.1234 (53)	0.042 (57)	0.04 (89)	-0.1115 (86)	0.1113 (54)	0.054 (77)	-0.1008 (102)	0.1591 (37)	0.051 (50)	-0.1337 (71)	0.118 (53)	0.033 (46)	-0.02 (67)	0.04 (73)	0.0037 (43)	-0.06 (52)	0.12 (28)	-0.1018 (85)	-0.1608 (46)	0.034 (24)	0.052 (36)	0.029 (26)	0.032 (9)	-0.3185 (18)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	191-218	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
All M	-0.03 (95)	-0.111 (58)	0.045 (64)	0.05 (103)	-0.08 (105)	0.1024 (69)	0.059 (107)	-0.07 (142)	0.1326 (52)	0.054 (89)	-0.09 (121)	0.084 (74)	0.031 (93)	-0.04 (120)	0.0083 (124)	0.0057 (99)	-0.05 (92)	0.044 (47)	-0.07 (142)	-0.1243 (83)	0.012 (40)	-0.1014 (72)	-0.1458 (54)	0.055 (20)	-0.2478 (32)

Note: The number of options within each category is shown in brackets

TABLE A.8

Summary table for mean absolute pricing error for options with different time to maturity and moneyness

Black Model: Mean Absolute Pricing Error

Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-219	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
M<-0.02	1.1457	1.4915	2.8021	3.1666	4.3336	5.1175	5.8457	9.5796	7.0963	8.4115	8.8267	6.6306	10.6003	10.6199	10.4637	12.4573	12.5954	11.6240	13.7650	13.9920	12.7545	15.2336	14.2837	14.8337	18.8614
	(4)	(3)	(7)	(10)	(18)	(15)	(28)	(38)	(14)	(37)	(46)	(19)	(48)	(51)	(49)	(54)	(39)	(18)	(75)	(34)	(18)	(35)	(28)	(11)	(13)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
-0.02<M<0.02	0.3194	1.7422		3.4338	8.0922	8.2979		7.2625	7.6078	8.2721	8.0412	8.2546	10.2724	11.6951	10.5393	12.5253	14.3709	10.6994	13.1008	14.1888		14.3210			17.0678
	(1)	(2)		(4)	(1)	(2)		(2)	(1)	(2)	(4)	(2)	(1)	(2)	(2)	(2)	(1)	(1)	(2)	(3)		(1)			(1)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
M>0.02	0.2512	0.7474	1.8016	2.4657	3.4443	4.5204	5.1402	5.8340	6.4667	8.1462	8.2316	7.8060	10.4011	10.2209	9.8610	12.2447	12.1736	11.1804	13.3511	13.7366	12.3004	14.7383	13.9723	14.5078	16.3876
	(90)	(53)	(57)	(89)	(86)	(54)	(77)	(102)	(37)	(50)	(71)	(53)	(46)	(87)	(73)	(43)	(52)	(28)	(65)	(46)	(24)	(36)	(26)	(9)	(18)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
All M	0.2896	0.8202	1.9110	2.5733	3.6220	4.6502	5.3464	6.0512	6.6587	8.2594	8.4845	8.0291	10.4962	10.4151	10.1101	12.3663	12.3763	11.3400	13.5768	13.8670	12.4820	14.9734	14.1336	14.6870	18.5896
	(95)	(58)	(64)	(103)	(105)	(69)	(107)	(142)	(52)	(89)	(121)	(74)	(93)	(120)	(124)	(99)	(92)	(47)	(142)	(83)	(40)	(72)	(54)	(20)	(32)

Asay Model: Mean Absolute Pricing Error

Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
M<-0.02	1.1496	1.5023	2.8400	3.2471	4.4475	5.2726	6.0618	8.5706	7.4222	8.8857	9.3668	9.1636	11.4043	11.5002	11.2736	13.8458	13.9068	12.7630	15.3496	15.7274	14.2489	17.3082	18.3106	18.9297	19.5448
	(4)	(3)	(7)	(10)	(18)	(15)	(28)	(38)	(14)	(37)	(46)	(19)	(48)	(51)	(49)	(54)	(39)	(18)	(75)	(34)	(18)	(35)	(28)	(11)	(13)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
-0.02<M<0.02	0.3196	1.7543		3.5010	8.2713		6.5448	7.6020	7.9859	8.7466	9.6282	8.7561	11.0340	12.7138	11.3636	13.7264	15.9361	11.7286	14.5474	15.9569		18.2126			19.8819
	(1)	(2)		(4)	(1)	(2)		(2)	(1)	(2)	(4)	(2)	(1)	(2)	(2)	(2)	(1)	(1)	(2)	(3)		(1)			(1)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
M>0.02	0.2544	0.7897	1.8573	2.5552	3.5886	4.7239	5.3981	6.1813	6.8644	8.6915	8.8641	9.4173	11.2925	11.1998	10.7709	13.5424	13.5751	12.4525	15.0049	15.5941	13.9026	16.8696	18.1819	19.7259	19.1200
	(90)	(53)	(57)	(89)	(86)	(54)	(77)	(102)	(37)	(50)	(71)	(53)	(46)	(87)	(73)	(43)	(52)	(28)	(65)	(46)	(24)	(36)	(26)	(9)	(18)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
All M	0.2927	0.8415	1.9648	2.6581	3.7813	4.8432	5.5932	6.3636	7.0361	8.7734	9.0881	8.6161	11.3450	11.3626	10.9791	13.8025	13.7414	12.5680	15.1806	15.8619	14.0411	17.0836	18.2390	18.8380	19.3164
	(95)	(58)	(64)	(103)	(105)	(69)	(107)	(142)	(52)	(89)	(121)	(74)	(93)	(120)	(124)	(99)	(92)	(47)	(142)	(83)	(40)	(72)	(54)	(20)	(32)

Extended-Vasicek Model: Mean Absolute Pricing Error

Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
M<-0.02	0.0660	0.1277	0.1847	0.2145	0.1724	0.2165	0.2063	0.1480	0.2015	0.2012	0.1432	0.2146	0.1896	0.1587	0.1837	0.1722	0.1749	0.1736	0.1806	0.1546	0.1901	0.2036	0.1696	0.2380	0.1990
	(4)	(3)	(7)	(10)	(18)	(15)	(28)	(38)	(14)	(37)	(46)	(19)	(48)	(51)	(49)	(54)	(39)	(18)	(75)	(34)	(16)	(35)	(28)	(11)	(13)
Time to maturity(days)	<30	31-60	61-90	61-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
-0.02<M<0.02	0.0660	0.0860		0.2546	0.3064	0.3672		0.1803	0.4034	0.2790	0.2066	0.2321	0.1541	0.3718	0.2756	0.2460	0.3636	0.1597	0.2236	0.2048		0.3445			0.2879
	(1)	(2)		(4)	(1)	(2)		(2)	(1)	(2)	(4)	(2)	(1)	(2)	(2)	(2)	(1)	(1)	(2)	(3)		(1)			(1)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	301-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
M>0.02	0.0370	0.1331	0.0960	0.0890	0.1757	0.1708	0.1401	0.1752	0.2239	0.1967	9.1915	0.1928	0.2086	0.2491	0.1859	0.1961	0.1443	0.2413	0.2178	9.1293	0.2681	0.2786	0.0840	0.3965	0.2318
	(90)	(53)	(57)	(89)	(86)	(54)	(77)	(102)	(37)	(50)	(71)	(53)	(46)	(67)	(73)	(43)	(52)	(28)	(65)	(46)	(24)	(36)	(26)	(9)	(18)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
All M	0.0360	0.1312	0.1059	0.1076	0.1764	0.1812	0.1612	0.1660	0.2213	0.1999	0.1736	0.1993	0.1967	0.2127	0.1665	0.1837	0.1596	0.2136	0.1961	0.1424	0.2369	0.2432	0.1265	0.3102	0.2202
	(95)	(58)	(64)	(103)	(105)	(69)	(107)	(142)	(52)	(89)	(121)	(74)	(93)	(120)	(124)	(99)	(92)	(47)	(142)	(83)	(40)	(72)	(54)	(20)	(32)

HJM: Mean Absolute Pricing Error

Time to maturity(days)	<30	31-60	61-90	81-120	121-150	151-180	181-210	211-240	241-279	271-300	301-330	331-360	361-396	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-736	>731
M<-0.02	0.0300	0.0520	0.0730	0.1066	0.0540	0.0700	0.0730	0.0470	0.0590	0.0710	0.0450	0.0410	0.0660	0.1174	0.0430	0.0810	0.0560	0.0870	0.0990	0.0770	0.0960	0.0990	0.1308	0.0540	0.1897
	(4)	(3)	(7)	(10)	(18)	(15)	(28)	(38)	(14)	(37)	(46)	(19)	(46)	(51)	(49)	(54)	(39)	(18)	(75)	(34)	(18)	(35)	(28)	(11)	(13)
Time to maturity(days)	<30	31-60	61-90	61-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-736	>731
-0.02<M<0.02	0.0390	0.0480		0.1250	0.1483	0.1605		0.1971	9.1878	0.1369	0.1415	0.0610	0.1278	0.1872	0.0340	0.1698	0.1162	0.1741	0.1048	0.1048		0.0082			0.0260
	(1)	(2)		(4)	(1)	(2)		(2)	(1)	(2)	(4)	(2)	(1)	(2)	(2)	(2)	(1)	(1)	(2)	(3)		(1)			(1)
Time to maturity(days)	<30	31-60	61-90	81-120	121-150	151-180	181-210	211-240	241-279	271-300	301-330	331-360	301-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	671-700	701-730	>731
M>0.02	0.0360	0.1328	0.0760	0.0670	0.1627	0.1271	0.0970	0.1603	0.1654	0.1161	0.1953	0.1256	0.1213	0.2405	0.1006	0.1212	0.1209	0.1585	0.1935	0.1786	0.1478	0.2475	0.1868	0.1654	0.3449
	(90)	(53)	(57)	(69)	(86)	(54)	(77)	(102)	(37)	(50)	(71)	(53)	(46)	(87)	(73)	(43)	(52)	(28)	(65)	(46)	(24)	(36)	(26)	(9)	(18)
Time to maturity(days)	<30	31-60	61-90	91-120	121-150	151-180	181-210	211-240	241-270	271-300	301-330	331-360	361-390	391-420	421-450	451-480	481-510	511-540	541-570	571-600	601-630	631-670	571-700	701-730	>731
All M	0.0350	0.1257	0.0760	0.0730	0.1440	0.1148	0.0920	0.1305	0.1371	0.0970	0.1364	0.1022	0.0950	0.1873	0.0770	0.1002	0.0930	0.1238	0.1423	0.1343	0.1287	0.1720	0.1578	0.1131	0.2719
	(95)	(58)	(64)	(103)	(105)	(69)	(107)	(142)	(52)	(89)	(121)	(74)	(93)	(120)	(124)	(99)	(92)	(47)	(142)	(63)	(40)	(72)	(54)	(20)	(32)

Note: The number of options within each category is shown in brackets

DEFINITION OF TERMS

Gaussian Distribution

The normal probability distribution. Its mathematical structure was developed by Carl Frederich Gauss (1777-1855) and the curve is often referred as the Gaussian distribution. The mathematical function that plots the normal curve has the following density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-[(x-\mu)/\sigma]^2/2}$$

where x is the value of the random variable, μ is its expected value and σ is the standard deviation. Any normally distributed random variable can be expressed as a standard normal random variable by subtracting its expected value and dividing by its standard deviation. The standardized normal variable has the following function:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

The standard normal variable has expected value of zero, a variance of 1 and is symmetric.

Mean Reversion

Interest rates do not typically drift upward or downward without encountering some resistance and a reversal of direction. The interest rate r responsible for the mean reversion can be seen as having an average drift or expected change, with volatility superimposed upon the drift. Mean reversion implies that the drift tends to pull interest back to some long-run average level. When the short-term interest rate is very high, r tends to have a negative drift; when the short-term interest rate is very low, r tends to have a positive drift. Mean reversion allows predictions about long-term interest rates. The volatility of a spot interest rate tends to be a decreasing function of its maturity. For example, the ten-year spot interest rate tends to have a lower volatility than the five-year spot interest rate; the five-year spot interest rate tends to have a lower volatility than the one-year spot interest rate etc.

Using the Vasicek model as an example, if the model assumes that the short rate follows the stochastic process of:

$$dr = \alpha(\mu - r)dt + \sigma dW$$

where dr is a standard d Wiener process and r is the current level of the interest rate. μ is the long run average interest rate. μ is positive and is the rate at which the interest rate is pulled. If the current rate is higher (lower) than the long run average $r > (<)\mu$, the factor $\alpha(\mu - r)dt$ induces negative (positive) expected change, which pulls the rate down (up) towards the average proportional to the factor dt, which is the length of time over which the change is observed.

σdW is a standard process for modelling uncertainty.

The coefficient μ is the speed of adjustment of the interest rate towards its long run normal level. With this feature, interest rates could not drift permanently upward the way stock prices do.

Stochastic Process
It is a sequence of observations from a probability distribution. Real world asset prices come from changing distributions though it is difficult to determine when a distribution has changed. Empirical analysis of past data can be useful in predicting when the numbers are coming out according to different bounds of probability.

Synchronous markets
Galai (1983) describes the attributes of synchronous markets. Synchronous markets are markets in which trading in related assets take place simultaneously and quoted prices reflect this simultaneity. Synchronization includes trading synchronization and data synchronization. Trading synchronization stands for parallel trading in two related securities. It is not sufficient for proving market synchronization since data recording may be nonsynchronized. The technology for registration of trades must be such that the data accurately present the timing of the transaction and the time the information is made available to market participants. If data on a class of options and the underlying stocks are used and the price quotes are not taken at the same time, based on parallel trading, the markets will appear to be nonsynchronous, though they may still be efficient.

White's Adjustments
It tests whether the error variance is affected by any of the regressors, their squares or their cross products. It examines whether or not any heteroskedasticity present causes the variance-covariance matrix of the OLS estimator to differ from its usual formula.

Steps must be taken to calculate β^{EGLS} once the presence of heteroskedasticity has been confirmed. The first step in the process is to determine the functional form of the relationship determining the variance. The relationship is then estimated and used to form an estimate of the variance-covariance matrix G of the disturbance term. Using the estimate (G'), the estimator β^{EGLS} can be calculated.

An alternative way of calculating β^{EGLS} can be used. This alternative way involves transforming the original equation to create an estimating relationship in transformed variables that has spherical disturbances. The OLS estimator can then be applied to the transformed data, producing the GLS estimator. In the case of heteroskedasticity, the appropriate transformation is obtained by dividing each observation by the square root of the estimated variance of the disturbance term.

Wiener Process
If ε(t) denotes a series of numbers coming out of a standard normal probability distribution and t denotes the point in time, the numbers are on average equal to zero and have a standard deviation of 1. The numbers are of the standard normal type. Taking any number and call it Z(t) at time. When moving ahead to time t+1 and call it ε(t+1), a transformation of the standard normal variable into the Z variable would be to add ε(t+1) to Z(t) to get Z(t+1). The difference between Z(t+1) and Z(t) is denoted as dZ(t) which can be defined as $dZ(t) = e(t) \sqrt{dt}$ and this is called a Wiener Process. When squaring the Wiener process, it becomes perfectly predictable. When multiplying the square root of the time interval dt by a

standard normal random variable $\varepsilon(t)$, the transformed value is unpredictable but the expected value and its variance are known. The expected value is zero. Using the rule that the variance of a constant times a random variable is the constant squared times the variance of the random variable, the variance is predicted to be dt. When the variable of interest is $dZ(t)^2$ and the value of $\varepsilon(t)$ is drawn. $\varepsilon(t)$ can be multiplied by the square root of dt and the entire expression is squared. This becomes $\varepsilon(t)^2$ dt. The variance of the expression can be found by squaring dt and multiplying it by the variance of $\varepsilon(t)^2$. By definition, all values of dt^k where $k > 1$ are zero. This results in that the length of the time interval becomes so short that squaring it makes it shorter and effectively zero. The expected value of $dZ(t)^2$ is the expected value of $\varepsilon(t)^2 dt$. This is dt times the expected value of $\varepsilon(t)$. Since the variance of any random variable x is defined as $E[x^2] - E[x]^2$ and when $\varepsilon(t)$ is a standard normal variable, $Var[\varepsilon(t)] = E[\varepsilon(t)^2] - E[\varepsilon(t)]^2 = 1$. When $E[\varepsilon(t)] = 0$, so $E[\varepsilon(t)]^2 = 1$. Since $Var[dZ(t)^2] = 0$ and $E[dZ(t)^2] = Z(t)^2 = dt$, therefore $E[dZ(t)^2] = 1* dt = dt$. This shows that any variable with zero variance can be expressed as its expected value i.e. $dZ(t)^2 = dt$

The process can be used to model stock price movements. Over the long run, stock prices go up or 'drift". The Wiener process does not drift but it is easy to make drift either upward or downward. The stock prices are random but with different volatilities. The Wiener process can be transformed to give different volatilities. It would be harder to forecast stock prices further into the future than nearby and stock prices are always positive.

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