

2000

## **M-GARCH Hedge Ratios And Hedging Effectiveness In Australian Futures Markets**

Wenling Yang  
*Edith Cowan University*

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**M-GARCH HEDGE RATIOS AND  
HEDGING EFFECTIVENESS IN AUSTRALIAN  
FUTURES MARKETS**



**WENLING YANG**

**Submitted in partial fulfillment of the requirements for the  
degree of master of business in finance  
Edith Cowan University  
May 2000**

## ***Declaration***

This thesis contains no material that has been accepted for the award of any other degree or diploma in any tertiary institution. To the best of my knowledge and belief, this thesis contains no material previously published or written by any other person, except when due reference is made in the text of this thesis.

Wenling Yang



## ***ABSTRACT***

This study deals with the estimation of the optimal hedge ratios using various econometric models. Most of the recent papers have demonstrated that the conventional ordinary least squares (OLS) method of estimating constant hedge ratios is inappropriate, other more complicated models however seem to produce no more efficient hedge ratios. Using daily AOIs and SPI futures on the Australian market, optimal hedge ratios are calculated from four different models: the OLS regression model, the bivariate vector autoregressive model (BVAR), the error-correction model (ECM) and the multivariate diagonal Vec GARCH Model. The performance of each hedge ratio is then compared.

The hedging effectiveness is measured in terms of ex-post and ex-ante risk-return trade-off at various forecasting horizons. It is generally found that the GARCH time varying hedge ratios provide the greatest portfolio risk reduction, particularly for longer hedging horizons, but they do not generate the highest portfolio return.

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# Chapter 1

## General Introduction

### *Introduction*

There is a large literature on hedging with futures contracts that has emerged since the 1950s. Those traditional methods of calculating hedge ratios are subject to increasing criticism and are generally viewed as inappropriate. On the other hand, other more complicated measures using recent time series techniques don't seem to be overwhelmingly superior. This contradiction has spurred a wide range of literature on futures hedging in the last two decades.

In this study, various time series econometric models are applied to the All Ordinaries Stock Index (AOIs) and corresponding Share Price Index futures (SPI) on the Australian Stock Exchange (ASX) to calculate optimal hedge ratios. The performance of the hedge ratios derived is compared to assess whether the more advanced time varying hedge ratios calculated from Bollerslev, Engle and Wooldridge's (1988) Multivariate- GARCH model provide more efficiency than other constant hedge ratios from the regression model, the Bivariate VAR model and the Error-Correction Model.

## ***Purpose of the Study***

As noted in most of the literature on futures trading, hedging is viewed as a major function and reason for the existence of futures markets. In order for hedgers to set up an efficient portfolio combining cash assets and futures contracts, they encounter the question of how many futures contracts should be held for each unit of cash asset, that is, how should the appropriate hedge ratio be calculated?

Although the earliest literature on hedging theory goes back to the 1950s, a prevailing theory today is Ederington's (1979) portfolio and hedging theory extended from Johnson (1960) and Stein (1961). Portfolio and hedging theory postulate that the objective of hedging is to minimize the variance of cash portfolio held by the investor. Therefore, the hedge ratio that generates the minimum portfolio variance should be the optimal hedge ratio, which is also known as minimum variance hedge ratio<sup>1</sup>.

Although Ederington's (1979) approach to calculation of the hedge ratio was highly recognised, at the same time it has also brought about questions and critiques. Many authors criticized it by drawing attention to the inefficiency of the residuals in the OLS method used to estimate the optimal hedge ratio. Examples encompass Herbst, Kare and Marshall (1989),

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<sup>1</sup> This study has presented the procedure for deducting Ederington's (1979) minimum variance hedge ratio in Chapter 3.

Park and Bera (1987) and Thompson (1989, 1996). Herbst, Kare and Marshall (1989) claim that the estimation of the minimum variance hedge ratio suffers from problem of serial correlation in the OLS residuals. Park and Bera (1987) point out that the simple regression model is inappropriate to estimate hedge ratios because it ignores the heteroscedasticity often encountered in cash and futures price series. Myers and Thompson (1989, 1996) argue that the hedge ratio should be adjusted continuously based on conditional information and thus calculated from conditional variance and covariance.

In this paper these two problems are dealt with using time series econometric techniques. In order to account for the serial correlation in the residuals, spot and futures prices are modelled under a bivariate-VAR framework in the presence of a cointegrating relationship. The appropriate lag length of the VAR model is chosen where the autocorrelation in residuals is eliminated from the system equations. At the second stage of catering for heteroscedasticity, Bollerslev, Engle and Wooldridge's (1988) multivariate-GARCH approach was applied allowing for time varying covariability in variables to the residuals obtained from VAR model (with an error-correction term). The time varying hedge ratios are thus calculated from the conditional variance of the futures prices and the conditional covariance between spot and futures prices.

Previous research on the time varying hedge ratio estimation has largely used the multivariate-GARCH modeling. To avoid the computational complexity involved, most of these studies have restricted the model to a pure GARCH process that displays only



conditional variance dynamics. In this case, the mean equation of the GARCH model is reduced to a simple naïve equation with a drift term ( $\alpha$ ) only. That is,

$$Y_t = \alpha + \varepsilon_t$$

Examples of using this equation can be found in Myers (1991), Baillie and Myers (1991), Sephton (1993) and Park and Switzer (1995a, 1995b). Although this simplification partly diminishes the complexity in the calculations, the over-evaluation of variance dynamics gives rise to the concern that the serial correlation problem commonly existing in price series can possibly be ignored. In this study, with the help of the econometric software packages, a Bivariate-VAR model (with an error-correction term) can be estimated and then treated as the mean equation, and the conditional variance ( $h_s, h_f$ ) of the residuals from the VAR model are utilised for M-GARCH modelling. The “clean” hedge ratios are thus obtained.

As noted by a number of authors including Ghosh (1993a, 1993b) and Lien (1996), the cointegration between spot and futures prices can play an important role in determining optimal hedge ratios. The error-correction term (ECT) is thus incorporated in the VAR model, given that there is evidence of cointegrating relationship in prices.

A second issue that is addressed in this study is the performance comparison of hedge ratios derived from various models. An ample literature has criticized Ederington's (1979) assumption that the hedgers are risk minimizers only, rather than expected utility

maximizers<sup>2</sup>. In this paper hedge ratios are generated from four econometric models. They are the traditional regression model based on changes in spot prices and changes in futures prices (model I); the bivariate -VAR framework (model II); the error-correction model (VAR including an error-correction term) (model III) and the multivariate GARCH model (model IV). In comparing hedging performance, it is assumed that hedgers seek to maximise their expected utility based on risk-return trade-off in choosing appropriate hedging techniques.

The objective of the study is to estimate constant and time varying hedge ratios from various econometric models using stock price index and index futures series on the Australian futures market and to examine the hedging performance of time varying dynamic hedge ratios obtained from conditional variances and covariances relative to static minimum variance hedge ratios derived from the other models.

### ***Plan of the Study***

The remainder of the study is as follows: Chapter two provides a general background and description of the purpose of futures markets with particular emphasis on the Sydney Futures Exchange, the market in which this study is conducted. The characteristics of futures contracts are described and each type of futures contract traded in the Sydney

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<sup>2</sup> See Howard and D'Antonio (1984, 1987), Chang and Shanker (1986), Lindahl (1991), Cecchetti, Cumby and Jiglewski (1988) for critiques on the ground.

Futures Exchange is briefly highlighted. Chapter three reviews the large amount of literature written on futures hedging and various measures of hedge ratios that currently exist. Chapter four provides an overview of the econometric tests and methods that are employed in the course of empirical estimation and analysis. Chapter five presents the implementation of methodology, estimates the parameters of models and calculates all types of hedge ratios. The empirical results generated are reported and explained. Chapter 6 concludes the paper and discusses the limitations and further research that can be explored from the results.

# Chapter 2

## The Futures Market in Australia – Background

### **2.1 Introduction**

What are futures contracts? The Sydney Futures Exchange (SFE) provides the following definition:

“Futures contracts are legally binding agreements to buy or sell commodities (such as wool or wheat), or financial instruments (such as government bonds or shares), at a fixed time in the future at a price agreed upon today.”

The organized futures markets began in Chicago with the opening of the Chicago Board of Trade (CBOT) in 1848 to standardize the quantities and qualities of grains. Today, the CBOT is the world’s largest futures exchange. Futures markets were originally concerned with commodity products, such as agricultural goods, livestock and metals. However, today most futures trading occurs in financial futures including equity, interest rate and foreign exchange rate instruments.

In Australia, all futures and options on futures contracts are traded at the Sydney Futures Exchange (SFE). The Sydney Futures Exchange was formed in 1960 primarily to provide a hedging facility for the wool industry. In the 1970s the SFE diversified its

product base by listing a trade steels futures contract (1975), a gold futures contract (1978), financial futures contract (1979) and finally the currency futures contract (1980). The Share Price Index (SPI) futures contract was listed in 1983 followed two years later by the listing of options on the SPI futures contract. In 1995, the SFE was ranked thirteenth in the world in terms of size.

## ***2.2 The Characteristics of Futures Contract***

Generally, futures contracts possess five characteristics that differentiate them from other financial instruments traded in financial markets. I outline these characteristics briefly in the following subsections as for readers to better understand futures contracts and futures trading.

### **2.2.1. Futures trade on organized Exchange**

Futures contracts always trade on an organized exchange. Exchange members have a right to trade on the exchange and to have a voice in the exchange's operation. Trading may take place during official trading hours in a pit, a physical location on the floor of the exchange. In contrast to the specialist system used on stock exchanges, futures contracts trade by a system of open outcry. Members of the exchange who trade in the pits are typically speculators, who enter the futures market in pursuit of profit by bearing risk. To speculators, many traders are hedgers, who trade futures to reduce some pre-existing risk exposure.

### **2.2.2. Futures Contracts Have Standardized Contract Terms**

Futures contracts are highly uniform and well-specified commitments for a carefully described good to be delivered at a certain time and in a certain manner. Generally, the futures contract specifies the quantity and quality of the good that can be delivered to fulfill the futures contract. The contract also specifies the delivery date and method for closing the contract, and the permissible minimum and maximum price fluctuations permitted in trading.

### **2.2.3. The Functions of Clearinghouses**

To ensure that futures contracts are traded in a smoothly functioning market, each futures exchange has an associated clearinghouse. The clearinghouse guarantees that all of the traders in the futures market will honor their obligations. The clearinghouse serves this role by adopting the position of buyer to every seller and seller to every buyer. This means that every trader in the futures markets has obligations only to the clearinghouse and has expectations that the clearinghouse will maintain its side of the bargain as well. The clearinghouse substitutes its own credibility for the promise of each trader in the market.

### **2.2.4. Futures Trading Requires Margin Payments and Daily Settlement**

The main purpose of margin payments and daily settlement is to ensure that traders will perform on their contract obligations. Normally, there are three types of margin: the initial margin, the maintenance margin and the variation margin. For most futures contracts, the initial margin may be 5 percent or less of the underlying commodity's

value. The maintenance margin is generally about 75 percent of the amount of the initial margin. The futures contracts' prices are settled on a daily basis. Once the cumulated loss of the day brings the deposit of the trader from initial margin to a level lower than the maintenance margin, he or she will get margin call from the clearinghouse to replenish the margin account to the level of initial margin. This amount of deposit is the variation margin.

### **2.2.5. Futures Positions Can Be Closed Easily**

Three ways of closing a futures contract are delivery or cash settlement, offset and exchange-for-physicals (EFP). Most futures contracts are written to call for completion of the futures contract through the physical delivery of a particular good. In recent years, exchanges have introduced futures contracts that allow completion through cash settlement. In cash settlement, traders make payments at the expiration of the contract to settle any gains or losses, instead of physical delivery. By far, most futures contracts are however completed through offset or via a reversing trade. To complete a futures contract obligation through offset or reversing trade, the trader transacts in the futures market by selling (buying) the same contract that was bought (sold) originally to bring his or her net position in a particular futures contract back to zero. A trader can also complete his obligation to a futures contract by engaging in an EFP. In an EFP, two traders agree to simultaneous exchange of a cash commodity and futures contracts based on that cash commodity.

## **2.3 Purposes of Futures Markets**

Duffie (1989) has described two key services provided by futures contracts as price discovery and insurance, or hedging:

“Futures contracts serve many purposes. Beyond their obvious role of facilitating the exchange of commodities and financial instruments, futures contracts are essentially insurance contracts, providing protection against uncertain terms of trade on spot markets at the futures date of delivery....Futures contracts are useful even to those who do not trade in them, since futures prices are publicly available indicators of futures demand and supply conditions”

Traditionally, futures markets have been recognized as meeting the needs of three groups of futures market users: those who wish to discover information about future prices of commodities, those who wish to speculate (speculators), and those who wish to hedge (hedgers). Therefore, futures markets exist mainly for the purpose of price discovery, hedging and trading (speculating).

### **2.3.1 Price Discovery**

Price discovery is the revealing of information about future cash market prices through the futures market. By buying or selling a futures contract, a trader agrees to receive or deliver a given commodity at a certain time in the future for a price that is determined now. In this way a relationship between the futures price and the price that people expect to prevail for the commodity at the delivery date is formed. By using the information



contained in futures prices today, market observers can form estimates of the price of a given commodity at a certain time in the future. It is found that for some commodities, the future prices that can be drawn from the futures market compare in accuracy quite favorably with other types of forecasts. Futures markets serve a social purpose by helping people make better estimates of future prices, so that they can make their consumption and investment decisions more wisely. For example, the futures price of the silver listed today may guide a mine operator who is trying to decide whether to reopen a marginally profitable silver mine or not. The financial wisdom of operating the mine will depend on the price the miner can obtain for the silver after it is mined and refined. In this situation, the miner can choose to use the futures market as a vehicle of price discovery. Farmers, lumber producers, and other economic agents can use futures markets the same way. They all use futures market estimates of futures cash prices to guide their production or consumption decisions.

### **2.3.2. Hedging**

Perhaps the most widely accepted view on the purpose of futures market is to hedge against price variability. As described by the SFE, the fundamental reason organizations use futures is for risk management -- hedging. Hedging involves the act that reduces the price risk of an existing or anticipated position in the cash market. Futures contracts enable risk to be transferred from those exposed to risk, to those seeking to benefit from the risk. That is, futures enable participants to hedge or protect the value of their assets (shares and bonds etc.) against the risk of price fluctuations in a commodity or financial instrument, by taking a position in the futures market opposite to the physical position they hold to benefit from the adverse price movement. By using futures, participants are

able to lock in a fixed buying or selling price and any profits or losses that arise from their futures position will offset any losses and gains in the physical market. For example, funds managers with share portfolios can hedge against a fall in the stock market by selling Share Price Index (SPI) futures.

However, there are still risks in using the futures markets for hedging, one of the most important risks is the basis risk. 'Basis' is the word used to describe the difference between the price of the commodity in the physical market and the price of the futures contract relating to that commodity.

$$\text{Basis} = \text{Cash Price} - \text{Futures Price}$$

As pointed out by Working as early as 1953, the fact that futures and spot (cash market) prices of assets tend to vary, that is, the basis tends to vary, most often unpredictably, makes it virtually impossible to guarantee that futures contracts can be used as a perfect hedging instrument. Essentially, a hedger exchanges price risk for basis risk. An unhedged investor faces price risk – the risk that the price of the cash asset will change. A hedged investor faces basis risk – the risk that the basis will change. Because of the fact that on the delivery date the futures price converges to the cash price, that is, the basis becomes zero, some of the change in basis is predictable.

If the basis were guaranteed to always remain unchanged, or if it were perfectly predictable at the end of the hedging horizon, then the investor would have a perfect hedge. Nevertheless, there is almost always risk that the basis will randomly change, so perfect hedges are rare. To minimize basis risk, the hedger hopes that price changes of the cash asset and the futures price will be highly correlated. The higher the correlation

between the price changes of the cash asset and those of the futures contract, the less basis risk will exist. A hedger must also decide how many futures contracts to trade in order to hedge the cash asset and reduce their risk exposure as much as possible. If he or she believes that the basis will change over the period of the contract, however, dynamic hedging with a changing hedge ratio according to the conditional covariability of the cash and futures prices will be more appropriate for him to profit than a constant hedging strategy. (See Park and Bera (1987) and Myers and Thompson (1989)).

### **2.3.3 Speculating**

The third purpose of using futures is for speculating, or for trading, where unlike hedging, no physical commodities or financial instruments are held and nor are traders seeking to reduce risk, but rather take risks in futures trading purely to profit from favorable price movements. The speculators are important to the efficient operation of the futures market. Speculators perform the useful role of often being the counter-party to a position that a hedger wants to take, effectively taking on the risk that hedgers seek to avoid.

## **2.4 The Sydney Futures Exchange**

Most developed countries that have a stock exchange also have a futures exchange. In Australia, it is the Sydney Futures Exchange (SFE) that provides a market place for the exchange of standardized futures contracts. The Sydney Futures Exchange (SFE) commenced trading in 1960 as the Sydney Greasy Wool Futures Exchange and by 1964 had become the world's leading wool futures market. Today the SFE is the largest

financial futures exchange in the Asia Pacific region with annual turnover approaching 30 million contracts. The SFE is also the first exchange outside the USA to list a financial futures contract (90-Day Commonwealth Bank Bills) in 1979 and a futures contract based on a stock index (All Ordinaries Share Price Index) in 1983.

There are mainly five types of futures contracts traded on the Sydney Futures Exchange in terms of the instruments underlying the futures contracts. They are bill futures contracts, bond futures contracts, commodity futures contracts, individual share futures contracts and share index futures contracts.

#### **2.4.1. Bill and Bond Futures Contracts**

By definition, Bill futures and bond futures are futures contracts over a short-term bill and coupon-paying bond, respectively. The maturity of the underlying bond is longer than a bill, though both are contracts over an interest-rate sensitive instrument whose value is determined by market yields.

The bill futures contract on the SFE is known as the 90-day BAB (bank-accepted bill) contract. The underlying contract unit is a BAB yet to be issued with a face value of AUD\$1,000,000 and maturity of 90 days. The BAB futures contracts expire every three months on a March, June September and December cycle and can be traded out to five years. The settlement of 90-day BAB futures contracts involves physical delivery of BABs or bank negotiable certificates of deposit.

The SFE has two bond futures contracts – the 3-year and 10-year Commonwealth Treasury bond futures contracts. As with bill futures contracts, these two bond futures contracts have an expiry cycle of March, June, September and December, they however only trade out to two quarters ahead. The underlying contract unit is a Commonwealth government Treasury bond with a nominal face value of \$100,000 and a coupon rate of 12% p.a. Unlike the bill futures contracts which are settled with physical delivery, both the 3-year and 10-year bond futures are cash settled. Therefore, the profit or loss on the investor's position is determined at settlement and this amount becomes payable in cash.

#### **2.4.2. Commodity Futures Contracts**

Commodity futures were among the first futures contracts ever developed. The most heavily traded contracts are crude oil, soybeans, corn and the base metals. In Australia, the SFE has listed a number of commodity futures contracts. Except for the greasy wool contracts, with which the SFE began life, the other commodity contracts listed on the SFE include live cattle, trade steers, boneless beef, fatty lambs, gold and silver.

However, the interest and activity in commodity futures tends to vary considerably over time. Recently, their relative popularity has declined, particularly in Australia, and financial futures tend to be the most heavily traded type of futures contract.

Nevertheless, commodity futures contracts continue to be listed and traded in significant volumes around the world.

### **2.4.3. Individual Share Futures Contracts**

Individual share futures are futures contracts on individual shares. The first contracts listed on the SFE were futures contracts on three of the largest stocks listed on the ASX being Broken Hill Proprietary Company Ltd (BHP), National Australia Bank Ltd (NAB) and The News Corporation Ltd (NCP) in May 1994. Today, the SFE has a total of ten stocks<sup>1</sup> on which individual share futures trade. These companies represent about 40% of the total capitalization of the Australian stock market. On the SFE, one individual share futures contract represents 1,000 shares of the underlying stock. The contracts are available on a three-month expiry cycle with only the two near-dated contracts listed for trading at any time. Although originally settled in cash, the contracts are now settled by physical delivery of the underlying shares.

### **2.4.4. Share Index Futures Contracts**

Share index futures are futures contracts over a stock market index. Probably the most well known and actively traded futures contract on a share index is the S&P 500 Contract on the Chicago Mercantile Exchange. In Australia, the SFE introduced in 1983 the All-Ordinaries Share Price Index (SPI) futures contract that tracks the movement of the All-Ordinaries Index (AOI). The All Ordinaries Index has long been the benchmark by which Australia's professional money managers measure portfolio performance. It is a value-weighted index that currently consists of a portfolio of approximately 270 large capitalization stocks traded on the ASX. The combined market worth of these companies represents over 95% of the market value of all Australian shares.

The underlying contract unit for the SPI is A\$25 times the AOI level. For instance, if the SPI contract is quoted at 2020, the value of one SPI contract is A\$50,500 ( $2020 \times \text{A\$}25$ ). If someone were to purchase (or sell) a SPI futures contract and the index rose 50 points to 2070, the contract would then be worth A\$5,750, representing a gain (or loss) of A\$1,250 ( $\text{A\$}25 \times 50$  points). The index price of the SPI contract at any particular time will reflect the underlying market plus the market's expectations of futures movements in the AOIs. Table 0 at the end of this chapter has given a specification of the share price index futures contract traded on the Sydney Futures Exchange.

SPI futures contracts provide equity market users with an easy and effective way to trade the share market. Since the AOI is not a tradable instrument in its own right, the SPI has the added advantage in that it allows a position to be taken in a market index that is not available on the ASX. Moreover, trading the SPI is equivalent to trading a balanced share portfolio that tracks the AOI, providing instant exposure to the overall share market with no company specific risk. Thus investors are able to protect their portfolio against adverse changes in the share market, or profit by taking a position based on a view of the performance of the AOI. Table 0 at the end of this chapter provides the contract specification for the SPI futures traded in the SFE.

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<sup>1</sup> As well as those already mentioned, these are Australia and New Zealand Banking Group Ltd, CRA Limited, Foster's Brewing Group Ltd, MIM Holdings Ltd, Pacific Dunlop Ltd, Westpac Banking Cooperation Ltd and WMC Ltd.

## 2.5 Conclusions

This chapter has provided a general outline and background of the futures market with particular reference to the Sydney Futures Exchange. Although the organized futures markets originated from the opening of CBOT in Chicago 1848, the futures trading in Australia did not begin until 1960 when the SFE was formed. The major five types of futures contract listed in the SFE today are: bill futures contracts, bond futures contracts, commodity futures contracts, individual share futures contracts and share index futures contracts. With its well-marked characteristics and rules, the futures market exists for the purpose of price discovery, hedging and speculating.

**Table 2.1 Share Price Index Futures Contract Specifications**

	<b>Specifications</b>	<b>Explanation</b>
<b>Trading Unit</b>	A\$25 × the All Ordinaries Index (AOI)	An index level of 2800 represents an equivalent share exposure of A\$70,000 i.e. $2800 \times A\$25 = A\$70,000$
<b>Tick Size</b>	1.0point = A\$25	One point movement in the SPI is worth A\$25
<b>Contract Months</b>	March, June, September and December, up to six contract months ahead.	If it is now Jun 1999, futures expiring on a quarterly basis would be available until Dec 2000.
<b>Last Trading Day</b>	On futures expiry at 4:15pm on the last business day of the contract month.	A Sep 1999 future will expire on 30 September 1999.
<b>Cash Settlement Price</b>	All futures positions still open at expiry are settled in cash. The final settlement price is taken from the closing quotation for the AOI on the last day of trading calculated to one decimal place, as adjusted and provided by the ASX to the SFECH at 12:00 noon on the business day following the last day of trading.	Futures expire automatically and all profits and losses are credited or debited in cash.
<b>Settlement Day</b>	The second business day following the last permitted day of trading.	
<b>Trading Hours</b>	09:50-12:30 14:00 – 16:15	Total trading coverage of 19 hours per day.

Source: Sydney Futures Exchange (March 2000)



# Chapter 3

## Hedging Effectiveness and Hedge Ratios— Literature Review

### ***3.1 Introduction***

Since hedging is a major reason for the existence of futures exchanges, the number of futures contracts that are required to hedge an un-hedged asset position is the key question in the hedging decision. The literature on futures hedging theory and optimal hedge ratio estimation has emerged since the 1950s, while the portfolio and hedging theory that prevails today is extended by Ederington (1979) from the hedging theory of Johnson (1960) and Stein (1961). Ederington (1979) derived the well known “minimum-variance hedge ratio” using traditional regression procedure that is presented later in this chapter.

However, with the development of time series econometrics, the conventional minimum-variance hedge ratio is found to bear many inadequacies. This chapter is devoted to reviewing the current wide literature which covers the drawbacks of the traditional regression method of Ederington (1979) and then provides alternative methods of estimating hedge ratios using time series techniques.

### ***3.2 Traditional Hedging Measurements***

As organized futures markets are traditionally thought of as vehicles for hedging and minimizing risk, the use of hedging strategies with futures instruments has generated an

extensive literature and such strategies have been widely adopted in a number of practical settings. The estimation of optimal hedge ratios (the number of futures contract a hedger should hold for each unit of spot commodity or financial instrument) arises from work as far back as Working (1953), Johnson (1960) and Stein (1961).

Considerable work has been devoted to the question of the proper volume of futures contracts needed to protect or enhance the value of a given anticipated volume of a physical commodity until the time of the commodity's final disposal in the cash market. Three major theories of determining hedge ratios and hedging effectiveness have been the traditional theory, the theories of Holbrook Working, and portfolio theory.

Traditional hedging theory emphasizes the risk avoidance potential of futures markets. It argues that spot and futures generally move together and hedgers only have to take futures market positions equal in magnitude but of opposite sign to their position in the cash market. If the cash prices at times  $t_1$  and  $t_2$  are  $P_s^1$  and  $P_s^2$  respectively, the gain or loss on an unhedged position,  $U$ , of  $X$  units is

$$X (P_s^2 - P_s^1)$$

But the gain or loss on a hedged position,  $H$ , is

$$X [(P_s^2 - P_s^1) - (P_f^2 - P_f^1)]$$

where the  $f$  subscript denotes the futures price. Since the traditional theory believes that spot and futures prices generally move together so that the absolute value of  $H$  is less than  $U$ :

$$\text{Var} ( H ) < \text{Var} ( U )$$

This question is often discussed in terms of the change in the cash price versus “the change in the basis,” defined as

$$[(P_f^2 - P_s^2) - (P_f^1 - P_s^1)]$$

A hedge is viewed as perfect if the change in the basis is zero. Since it is clear that basis changes over time, traditional hedges are not perfect.

Working (1953b) criticized the traditional approach in terms of its naïve tests of hedging effectiveness associated with routine hedging. He suggested that, realistically, merchants' or dealers' hedging is determined by inventory levels and expected basis changes. In contrast to much "optimal" hedge ratio research published in recent years, Working was concerned with "discretionary hedging" based on predictable basis changes. Discretionary hedging reduces cash speculation and lowers inventory if unfavorable basis changes are expected. The essence of Working's view of hedging is that hedging is a form of arbitrage between cash and futures prices. It is undertaken to profit from predictable changes in the relationship between cash and futures prices and not specifically to reduce risk. A hedger is not a risk averter, as tradition would have it, but rather a risk selector. A hedger prefers to profit from a skillful prediction of changes in the basis rather than from predictions of price levels. In so doing, the hedger becomes exposed to "basis risk." Hence, the view of hedging is "speculating on the basis."

The view of hedging that appears to prevail today is the one that draws from portfolio theory. It originates from Johnson (1960) and Stein (1961). Extensions or applications of this theory are seen in Heifner (1972), Ederington (1979), Anderson and Danthine (1980), Beninga, Eldor, and Zilcha (1984), Brown (1985), Adler and Detemple (1988), Briys, Crouhy, and Schlesinger (1990) and others. The rationale underlying the theory is that hedgers are risk-averse utility of wealth maximizers. Given the hedger's degree of risk

aversion, the hedger chooses to hedge partially or fully in an attempt to trade-off risk against return.

Johnson (1960) formulated the hedger's problem using traditional theory and derived the optimal variance-minimizing hedge. The optimal hedge ratio formula in the portfolio model resembles the Ordinary Least Squares (OLS) estimator of the slope in a simple regression framework. Hence, a convenient and usual method of estimating the hedge ratio is through OLS regression where the spot price is regressed against the futures price, and the estimated slope coefficient is taken to be the hedge ratio.

The study of Ederington (1979) is an extension of Johnson (1960) and Stein (1961), in which he investigated the government National Mortgage Association (GNMA) and T - Bill futures contracts within the context of Markowitz Portfolio Theory (MPT). He concluded that hedging decisions in the futures market are no different from any other investment decision and that traditional hedging theory and Working's (1960) theory are special cases of the broader portfolio theory. In contrast to the simple regression price level or price change ratios and Working's change in basis model, Ederington's (1979) model includes price change expectations and thus yields a hedge ratio designed to maximize profit subject to some risk averse weighted variance<sup>1</sup>. However, absent are some means of reliably forecasting changes in price over the hedge duration, the portfolio model is reduced to providing a price risk minimization hedge ratio (Heifner (1972), Ederington (1979)).

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<sup>1</sup> Also see Peck (195) and Tumblin (1982) for examples.

Ederington (1979) assumes that the spot market holdings,  $X_s$ , are viewed as fixed and the decision is how much of this stock to hedge. Again letting  $U$  represent the return on an unhedged position, we have the return and variance equations for the unhedged cash portfolio,

$$E(U) = X_s E(P_s^2 - P_s^1)$$

$$\text{Var}(U) = X_s^2 \sigma_s^2$$

Let  $R$  represent the return on a portfolio which includes both spot market holdings,  $X_s$ , and futures market holdings,  $X_f$ , then the return and variance equations for the hedged portfolio are

$$E(R) = X_s E(P_s^2 - P_s^1) + X_f E(P_f^2 - P_f^1) - K(X_f)$$

$$\text{Var}(R) = X_s^2 \sigma_s^2 + X_f^2 \sigma_f^2 + 2 X_f X_s \sigma_{s,f} \quad (3.1)$$

Where  $X_s$  and  $X_f$  represent spot and futures market holdings.  $K(X_f)$  are brokerage and other costs of engaging in futures transactions including the cost of providing a margin.  $\sigma_s^2$ ,  $\sigma_f^2$  and  $\sigma_{s,f}$  represent the subjective variances and the covariance of the possible price changes from time 1 to time 2. Ederington (1979) lets

$$b = -X_f / X_s$$

where  $b$  represents the proportion of the spot position which is hedged. Since in a hedge  $X_s$  and  $X_f$  have opposite signs,  $b$  is usually positive.

To obtain the minimum variance hedge ratio  $b^*$ , we take the first derivative of equation (3.1) with respect to  $b^*$  and set it to zero.

$$\frac{\partial E(R)}{\partial b} = X_s^2 (2b\sigma_f^2 - 2\sigma_{s,f}) = 0$$

$$b^* = \frac{\sigma_{s,f}}{\sigma_f^2}$$

This is the minimum variance hedge ratio derived in Ederington (1979).  $b^*$  is also known as the portfolio model's optimal hedge ratio. Theoretically,  $b^*$  is the proportion of futures contracts needed to hedge an established cash position when the hedger's goal is to maximize risk reduction. Therefore, a frequently recommended solution to the determination of optimal hedge ratio for a risk-averse hedger is to set the hedge ratio equal to the ratio of the covariance between spot and futures prices to the variance of the futures price (Benninga, Eldor, and Zilcha (1984), Kahl (1983)). Equivalently,  $b^*$  can be defined as the coefficient of the independent variable in a regression of spot price changes on futures price changes. Ederington (1979) showed that the optimal hedge ratio in most cases is significantly different from the traditional one-to-one ratio of futures to spot position holdings. He suggested that even pure risk minimizers should not hedge their entire spot portfolio but only a portion of it because minimum risk is achieved with a ratio of less than one-to-one.

Carter and Loyns (1985) regress the spot price changes on futures price changes to remedy the problem of non-stationarity in the price levels. Brown (1985) has also regressed spot market returns on futures market returns, where returns are defined as the proportional price change from period to period. The question of whether levels, changes, or returns should be used in the simple regression approach to optimal hedge ratio estimation has become controversial (Bond, Thompson and Lee (1987), Witt, Schneeweis and Hayenga (1987)). However, Hill and Schneeweis (1982) discussed the

merits of levels versus differences in the context of foreign currency hedging and claimed that a commonly used alternative is first differences.

Brown (1985) suggested that the portfolio approach for hedging is not supported because he found hedge ratios of approximately one when regressing cash and futures returns over the duration of the hedge and ratios not equal to one regressing price levels at the close of the hedge. Shafer (1993) noted that the “full” portfolio model is generally not appropriate because hedging as practiced is essentially either (1) an attempt at arbitrage via Working’s anticipated change in relative prices (basis change) and/or (2) a zero return process where the objective is a minimum risk bearing price relative to some break-even price, probably in an anticipatory hedge. Hartzmark (1988) has suggested that even large commercial firms are generally risk minimizers rather than “nimble footed speculators” reacting to expected price changes.

The methodology applied by Ederington (1979) has experienced other criticism. From the econometric side, Herbst, Kare and Caples (1989) claim that Ederington’s estimate of the minimum variance hedge ratio suffers from the problem of serial correlation in the OLS residuals. To remedy this problem, Herbst, Kare and Caples (1989) recommend a Box-Jenkins autoregressive, integrated moving average (ARIMA) technique. Their results of hedge ratio estimation show significant improvement over results from OLS regression. Moreover, the optimal hedge ratios yielded by ARIMA methodology are lower, which implies a corresponding reduction in the margin deposit and transaction costs. Another drawback of Ederington's portfolio approach described by Herbst, Kare and Marshall (1993) is that this approach implicitly assumes a constant basis. In reality,

in a direct hedge the basis must decline over the life the futures contract and vanish at the contract maturity.

The information inefficiency problem with the traditional OLS regression was studied early on by Bell and Krasker (1986). They showed that if the expected futures price change depends on the information set, then the traditional regression methods would yield a biased estimate of the hedge ratio. Hilliard (1984) proposed to correct for the estimation bias by regressing unexpected change in the spot price on the unexpected change in the spot price. Myers and Thompson (19989) also point out that the optimal hedge ratio should take account of relevant conditioning information associated with spot and futures prices. They propose a single equation framework to replace the multiple equation estimation required by Hilliard's method. Castelino (1990a, 1990b) argues that the regression procedure is inappropriate as spot and futures prices are expected to converge at maturity. He suggested the possibility that inclusion of structural relationships, such as the time to maturity effects arising from arbitrage, may improve hedge ratio performance.

Viswanath (1993) considers a modification of Myers and Thompson (1987) and Castelino (1990a, 1990b) procedure taking into account the possibility of spot-futures convergence and the dependence of the hedge ratio on the hedge duration and the time left to maturity. His model consists of regressing the changes of spot price on the changes of futures price and the current basis<sup>2</sup>. Hedge ratios are estimated under both the traditional method and this basis-corrected method. The findings indicate that the basis-



correction methodology produces significantly smaller hedged portfolio return variances in most of the cases, even though the improvement is not similar across the board.

Lindahl (1992) re-estimated the minimum-variance hedge ratio by adding in the expiration effect and duration effect following the previous studies of Hill and Schneeweis (1982); Marmer (1986); and Chen, Sears, and Tzang (1987). The results of the study show that the minimum-variance hedge ratios for MMI and S&P 500 stock index futures contracts increase significantly as hedge duration increases from one to four weeks. Moreover, when the sample is subdivided by weeks to expiration, both simple and multiple OLS confirm that the minimum-variance hedge ratio increase as hedges approach the contract expiration dates (consistent with Castelino (1992)). The author concludes that when hedging an established cash position, hedging with futures should be viewed as a dynamic process – being adjusted as the futures hedges increase in duration and approach the contract expiration date. This is consistent with the findings of Lo and MacKinlay (1988) and Malliaris and Urrutia (1991).

Lo and MacKinlay (1988) run a variance-ratio test which shows empirically the random walk hypothesis for two stock index futures: standard and Poor's 500 and New York Stock Exchange; and for four foreign currency futures: British Pound, German Mark, Japanese Yen and Swiss Franc. The findings imply that hedgers cannot consistently place optimal hedges and that dynamic hedging techniques must be considered.

Malliaris and Urrutia (1991) explored the non-stationarity of the hedge ratio and the

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<sup>2</sup> This method is motivated by Fama and French's (1987) argument that the current basis should have power to predict changes in the spot and futures price.

measure of the hedging effectiveness. Their empirical tests confirm the hypothesis that the hedge ratios and the measures of hedging effectiveness follow a random walk.

Myers and Thompson (1996) point out the inappropriateness of these simple regression approaches. The covariance between the dependent and explanatory variable and variance of the explanatory variable in the optimal hedging rule should be conditional moments that depend on information available at the time the hedging decision is made. They develop a generalized approach allowing for a more flexible specification of equilibrium pricing models where the conventional simple regression approaches to optimal hedge ratio estimation are special cases under particular sets of restrictions on equilibrium spot and futures price determination.

### ***3.3 Measure of Hedging Effectiveness***

In studies of futures markets much attention has been paid to the hedging effectiveness criteria of futures contracts because it is an important determinant in measuring the success of futures contracts. Several studies express the usefulness of trading a futures contract after comparing the results of a combined cash-futures portfolio and the cash position only. They can be roughly classified into these three categories. Ederington (1979) defines hedging effectiveness as the percentage reduction in variance of the hedged portfolio relative to the unhedged portfolio.

$$E = [ \text{Var} ( U ) - \text{Var} ( H ) ] / \text{Var} ( U )$$

where  $E$  is the measure of hedging effectiveness,  $\text{Var}(U)$  and  $\text{Var}(R)$  are the variance of unhedged and hedged portfolio respectively. The objective of a hedge is to minimise the risk of a given position. This risk is presented by the variance of returns.

Howard and D'Antonio (1984, 1987) and Chang and Shanker (1986) define hedging effectiveness as the ratio of the excess return per unit of risk of the optimal portfolio of the spot commodity and the futures instrument to the excess return per unit of risk of the portfolio containing the spot position alone. The objective of hedging here is to maximise a hedger's risk-return trade-off. Numerically it is expressed as:

$$E = \frac{\overline{R}_p - i}{\sigma_p} \bigg/ \frac{\overline{r}_s - i}{\sigma_s}$$

Where  $\overline{R}_p$  and  $\sigma_p$  are the expected return (percent) and the standard deviation of return (percent) respectively for the spot, futures portfolio,  $i$  is the risk-free rate of return (percent),  $\overline{r}_s$  and  $\sigma_s$  are the expected return (percentage) and the standard deviation of return (percentage) respectively for the spot portfolio.

Lindahl (1991) defined a joint return-risk hedging effectiveness measure as the mean and standard deviation of the fully hedged portfolio return less the corresponding T-bill rate for the same time period and duration.

$$M_r = R_{pt} - R_{ft}$$

$$\sigma_r = \sigma_{pt} + \sigma_{ft}$$

where  $M_r$  and  $\sigma_r$  are two parts of hedging effectiveness measure;

$R_{pt}$  represents the return of the fully hedged portfolio in time period  $t$ ;

$R_{ft}$  represent the risk-free T-bill rate for time period  $t$ .

Chang and Fang (1990) however criticised these one-period measures of hedging effectiveness in that they ignore the effect of stochastic interest rates on mark-to-market which can lead to significant biases. Chang and Fang derived optimal hedge ratios and a measure of hedging effectiveness in an intertemporal framework that contains previous one-period ratios and measures as special cases. They concluded that with stochastic interest rates, the optimal hedge ratios and the measure of hedging effectiveness are determined not only by the correlation between a futures contract and its underlying cash price changes, and the relative pay-off of a futures to its underlying cash position per unit of risk, as in one-period models; but also by the correlation between a cash price and the interest rate changes, the correlation between a future price and the interest rate changes, and the relative pay-off of a default-free bond to the cash position per unit of risk.

Pennings and Meulenberg (1995) argue that all of the previous models are estimated by implicitly assuming that the futures contract is perfect and thus introduces no risks. In fact, futures contracts do introduce risks that will have an impact on the variance of the hedger's returns. In their study a new measure of hedging efficiency focused on the hedging service of the futures contract is introduced. This measure expresses the distance between the hedging service provided by the exchange and the perfect hedge. This distance is divided into a systematic part, which can be managed by the futures

exchange, and a random part, which is dependent on factors that are beyond the influence of the futures exchange. The measure also takes commission costs into account, for the estimation of optimal hedge ratio in the environment of hedging cost<sup>3</sup>. The above mentioned research in the area of hedge ratio estimation and hedging effectiveness imply that the already complex relationships among hedge ratios, measures of hedging effectiveness, volume of trade and open interest require further empirical research.

### ***3.4 Hedge Ratios with Cointegration***

Since the introduction of the concept of cointegration by Granger (1981) and then Engle and Granger (1987), researchers have tested for cointegrating relationships between numerous economic time series. Most empirical results support cointegration between spot and futures markets. For example, Chowdhury (1991) finds cointegration between 3-month futures prices and spot prices at the maturity date for copper, lead, tin and zinc trade at the London Metal Exchange; Lai and Lai (1991) also demonstrate that, in five major foreign exchange markets, 1-month forward rates and spot rates at the maturity date are cointegrated. Krehbiel and Adkins (1993) show that spot prices of the silver, copper, gold and platinum traded in the metal markets are cointegrated with their corresponding three-month futures prices traded at the New York Mercantile Exchange for the period of 1964-1992. Therefore, the use of error-correction models which takes specific account of non-stationary time series' cointegrating relationship to calculate

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<sup>3</sup>Also see Howard and D'Antonio (1994) for discussion of this issue.

optimal hedge ratios has appeared in the futures literature as a means of calculating optimal hedge ratios.

This technique is used in analysis of stock index futures in Ghosh (1993a, 1993b), Wahab and Lashgari (1993) and Lien and Luo(1993)<sup>4</sup>. Ghosh (1993a, 1993b) finds that minimum variance hedge ratio estimates are biased downwards due to mis-specification if spot and futures are cointegrated and the error-correction term is not included in the regression. Lien and Luo (1994) point out that, although GARCH (Generalized Autoregressive Conditional Heteroscedasticity) may characterize the price behavior, the cointegration relationship is the only truly indispensable component when comparing ex post performance of various hedge strategies. Lien (1996) provided theoretical support for the importance of the cointegrating relationship and pointed out that:

“A hedger who omits the cointegration relationship will adopt a smaller than optimal futures position, which results in a relatively poor hedging performance.”

According to Engle and Granger (1987), the general statistical model for spot and futures prices in the presence of cointegration relationship is:

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<sup>4</sup> This technique is also applied to foreign exchange futures prices in Lien and Luo (1993), interest rates futures in Fung and Leung (1993) and Fung and Lo (1993), oil futures in Crowder and Hamed (1993) and metal futures in Krehbiel and Adkins (1993).

$$\Delta S_t = c_s + \sum_{i=1}^n \beta_{si} \Delta S_{t-i} + \sum_{i=1}^n \beta_{fi} \Delta F_{t-i} + \gamma_s Z_{t-1} + \varepsilon_{st}$$

$$\Delta F_t = c_f + \sum_{i=1}^n \beta_{si} \Delta S_{t-i} + \sum_{i=1}^n \beta_{fi} \Delta F_{t-i} - \gamma_f Z_{t-1} + \varepsilon_{ft}$$

where  $Z_{t-1}$  is the error-correction term which accounts for the long-run relationship equilibrium. (This Vector Autoregression Representation (VAR) model with error-correction model is further explained in Chapter 5.) Lien (1996) shows that a hedger proceeding with the correctly specified model chooses the minimum risk hedge ratio as follows:

$$h^* = \text{cov}(\Delta f_t, \Delta p_t \mid \Delta f_{t-j}, \Delta p_{t-i}, Z_{t-1}) / \text{Var}(\Delta f_t \mid \Delta f_{t-j}, \Delta p_{t-i}, Z_{t-1})$$

$$= \text{Cov}(\varepsilon_{1t}, \varepsilon_{2t}) / \text{Var}(\varepsilon_{2t}) = \rho(\sigma_1 / \sigma_2)$$

In contrast, an errant hedger ignorant of the cointegration relationship concludes with the following minimum-risk hedge ratio:

$$h' = \text{cov}(\Delta f_t, \Delta p_t \mid \Delta f_{t-j}, \Delta p_{t-i}) / \text{Var}(\Delta f_t \mid \Delta f_{t-j}, \Delta p_{t-i})$$

$$= \rho(\sigma_1 \sigma_2 - \gamma_1 \gamma_2 \text{Var}(\Delta Z_{t-1} \mid \Delta f_{t-j}, \Delta p_{t-i})) / (\sigma_2^2 + \gamma_2^2 \text{Var}(\Delta Z_{t-1} \mid \Delta f_{t-j}, \Delta p_{t-i}))$$

The difference between the two hedge ratios is:

$$h' - h^* = -\gamma_2 \sigma_Z^2 (\gamma_1 + \gamma_2 h^*) (\gamma_2^2 \sigma_Z^2 + \sigma_2^2)^{-1}$$

When  $\gamma_2 = 0$ ,  $h' = h^*$ . That is, if the futures price does not adjust to the deviation from the long-run equilibrium relationship, the errant hedger obtains the correct minimum risk hedge ratio in spite of model mis-specification. On the other hand, if  $\gamma_2 > 0$ , then  $h' < h^*$ . If  $\gamma_1 = 0$ , then  $h' < h^*$  whenever  $\gamma_2 \neq 0$ . Thus, the errant hedger under-hedges due to omitting the effect of the cointegrating variable in spot price behavior.

Ghosh and Clayton (1996) compare the performance of hedge ratios derived from the traditional price change OLS regression and the error correction model in the presence of cointegration. The hedge ratios are estimated using daily spot prices and nearby stock index futures contracts of France (CAC 40), the United Kingdom (FTSE 100), Germany (DAX) and Japan (NIKKEI) for the sample period 1990-1992. They conclude that the optimal hedge ratio derived from the error correction model (generally greater in magnitude) is superior to that obtained from the traditional price change model in that: (1) it is more effective in controlling and/or reducing the risk of the cash portfolio, (2) it significantly reduces the root mean squared error of the spot portfolio for each country, (3) hedging using the error correction model results in higher mean holding-period returns and lower standard deviation for portfolios based on the spot and futures markets for each country. Chou, Denis and Lee (1996) find the similar results by estimating a bivariate VAR model of spot and futures prices incorporating the error-correction model.

Interestingly, this single cointegrating vector relationship was re-examined by Heaney (1998), drawing attention to the cost of carry terms that constrain the linkage between spot and futures prices prior to maturity. Heaney (1998) tested the cost of carry



relationship using the lead contracts on the London Metal Exchange. He found the existence of one cointegrating vector that consists of spot, futures price, interest rate, stock level and a constant term. This implies that estimating a single cointegrating vector between spot and futures prices but omitting cost-of-carry terms may lead to misspecification.

Following Heaney (1998), Ferguson and Leistikow (1999) empirically compared the hedge ratios calculated from a cost-of-carry-adjusted regression method and error-correction model (ECM) using COMEX gold futures and spot price on a weekly basis. The modified regression model (MRM) they consider is

$$(S_{t+1} - S_t - C_{st}) = \alpha + \beta (F_{Tt+1} - F_{Tt}) + \varepsilon_{t+1}$$

where  $S_{t+1} - S_t$ ,  $C_{st}$  and  $F_{Tt+1} - F_{Tt}$  denote the change in spot price, the spot asset cost of carry and the change in the maturity-date- $T$  futures contract price, respectively, from time  $t$  to  $t+1$ . Their findings indicate that MRM hedge ratios are slightly smaller and more variable than ECM hedge ratios. It is also found that the hedging performance of the two approaches do not appear to be economically, nor statistically, significantly different.

Using data from the Sydney Futures Exchange, Allen, McDonald and Walsh (1999) employed a panel-based technique to test the stability of constant hedge ratio derived from VAR model across different futures contracts. The finding indicates that the individual hedge ratio estimates differ significantly between contracts and the estimated

parameters show considerably stronger evidence of stability when the data are manipulated on a contract by contract basis, with each individual contract considered as a panel of data.

### ***3.5 Time-Varying Hedge Ratios***

The strategies suggested thus far to derive the hedge ratios have restricted the hedge ratio to be constant over time. However, it is argued that if the joint distribution of stock index and futures prices is changing through time, estimating a constant hedge ratio may not be appropriate, and the estimation of optimal or minimum risk hedges with futures contracts should therefore use a time-dependent conditional variance model.

Since the development of the autoregressive conditional heteroskedastic (ARCH) model of Engle (1982) and the generalized ARCH model of Bollerslev (1986) to formulate the second moments (variance) of a time series, a considerable amount of literature has focused on testing the validity of constant hedge ratios obtained under the assumption of unconditional information and constant covariability between the cash and futures price series<sup>5</sup>. Park and Bera (1987) first claimed that all of the previous methods were inappropriate to estimate hedge ratio because they ignore the heteroskedasticity often encountered in price series and the non-linearity between cash and futures prices. Myers and Thompson (1989) argue that the hedge ratios should be adjusted continuously based on conditional information.

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<sup>5</sup> See Anderson (1985) and Fackler (1986) for discussions on this issue.

Cecchetti, Cumby and Figlewski (1988) argue that using the conventional regression model to estimate optimal hedge ratio suffers from two major shortcomings. First, it is assumed that the objective is to minimize risk, not to maximize expected utility, which also depends on expected return. Second, when joint distribution of cash and futures price changes, therefore so does the hedge ratio, but there is no adjustment for the fact that hedge ratio varies substantially over time. In their study, the estimates of the optimal hedge ratios from bivariate ARCH (3) specification of joint distributions are used to obtain the series of expected utility maximizing hedge ratios. They demonstrate that an investor with log utility would have preferred the utility maximizing hedge to a variance-minimizing hedge by a certainty-equivalent rate of return.

Baillie and Myers (1991) adopt a bivariate GARCH (1,1) model with vech parameterization to estimate the optimal hedges ratios for six different commodities allowing for time-varying correlation relationship between cash and futures returns. In line with other studies, they found strong support for the hypothesis that the estimated optimal hedge ratios are time varying and indeed non-stationary. Baillie and Myers (1991) have also compared the performance of dynamic hedge ratios with that from conventional hedging model and find the former is remarkably superior.

Bera, Bubnys and Park (1993) challenge the validity of the conventional OLS model in that the significant heteroskedasticity and non-normality are found in the disturbance term of OLS regression. They provide alternative dynamic hedge ratios based on the ARCH model of Engle (1982) and compare its performance with OLS regression

estimates. It is found that the dynamic hedging model that takes into account the ARCH effects (heteroskedasticity) in the course of estimation indicates markedly improved efficiency of the hedge ratio estimates. But they also noted that the estimation of the ARCH model is computationally more demanding than running an OLS regression. Sephton (1993) applied this technique to Canadian data and obtained a similar conclusion.

Park and Switzer (1995a, 1995b) estimate the time-varying minimum hedge ratios with stock index futures in the presence of transaction costs. The percentage variance reduction of bivariate GARCH (1,1) model over the naïve hedging model, the traditional OLS and OLS model incorporating error-correction term shows noticeable improvement over the hedging effectiveness through GARCH for both Standard and Poor 500 and Toronto 35 Index futures.

Park and Switzer (1995b) consider the utility function of the spot-futures hedged portfolio and conclude that GARCH hedge is even more economically useful in improving the utility function of investors with a mean-variance expected utility function. They assume an investor faces the mean-variance expected utility function as

$$EU(x) = E(x) - \lambda Var(x)$$

Where  $x$  is the return from the spot-futures hedged portfolio,  $E(x)$  and  $Var(x)$  represent the return and variance of the hedged portfolio, respectively.  $\lambda$  is the degree of risk aversion.

If it is assumed that an investor compares the utility function with costly rebalancing against that without rebalancing, he will rebalance his portfolio only when the potential gains in utility from the reduced variance offset the transaction costs that must be incurred. The sum of weekly utility functions from March of 1990 to December of 1991 indicates that the utility increase is evident with the GARCH method for both futures contracts. Therefore, they concluded that the GARCH method gives an improved hedging strategy even after accounting for transaction costs.

Tong (1996) compares GARCH-modelled dynamic hedging strategies with the traditional OLS model for the spot and futures Tokyo Stock Index. He finds that the dynamic hedging reduces risk more than static hedging, but only slightly. This finding is consistent with some previous findings<sup>6</sup> that more complex hedging methods may not improve the performance much. Tong (1989) analyses the reason as the tight relationship between the spot and futures price in a direct hedging. The GARCH model tries to exploit the nature of time-varying variances and covariances of asset returns, while the non-arbitrage condition guarantees that the spot and futures prices have a relatively stable variance-covariance structure. Therefore, Tong (1989) claims that “GARCH cannot gain much mileage under such a situation.” However, if there is no means for direct hedging and other correlated instruments have to be used to hedge indirectly, then a dynamic strategy would show more power. Myers (1991) produces similar results demonstrating that a multivariate GARCH specification performs only

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<sup>6</sup>McNew and Fackler (1994) use an interactive generalized least squares procedure that produces maximum likelihood estimates and is simpler to apply than GARCH procedures. Moreover, the results show the time-varying hedge ratios estimated using GARCH model is not significantly better than those from simpler model.

marginally better than regression techniques in the application of commodity price series.

Lien and Tse (1998) examined the performance of hedge ratios estimated from various econometric models including the newly introduced fractionally cointegrated error-correction model. The other three models estimated are a GARCH model, error-correction model and VAR model. They find that firstly, the error correction model strategy for more than ten days incorporating conditional heteroskedasticity is the dominant strategy. Secondly, incorporating the fractional cointegration relationship does not improve the hedging performance over the error correction model. Finally, the conventional regression method provides the worst hedging outcomes of all.

### ***3.6 Conclusions***

This chapter has provided a review of vast research on hedge ratio estimation and hedging effectiveness. The recent evidence leads us to believe that the hedging position should be adjusted conditional on new information. Therefore, the hedge ratio estimation should be based on the conditional variance and covariance of spot and futures price series. Nevertheless, there has not been explicit superiority of this dynamic time varying hedge ratio estimation technique over the conventional methodology in empirical studies. In the field of measuring hedging performance, there is an on-going debate in terms of whether the hedging effectiveness should be measured only by the extent of minimizing portfolio risk, or by the degree of risk-return trade-off in the hedged portfolio.

However, most of the previous work was done on the US markets. In this paper, we particularly focus on Australian futures markets by applying various hedging techniques to testify the superiority of the time varying hedge ratios comparing to the traditional constant hedge ratio. Also, the more comprehensive risk-return trade-off of the hedged portfolios using different hedge ratio is going to be compared in measuring hedging performance.

# Chapter 4

## Econometric Methodology

### **4.1 Introduction**

As pointed out by Engle and Granger (1987), the time series should be checked for the characteristics of stationarity (unit roots) and cointegration before they are modelled. If there is cointegrating relationship present, an error-correction term should be incorporated into the model to account for this long-run equilibrium relationship between the series. The first section of this chapter introduces the concepts and procedures of two of the most commonly used methods to test the unit root of the time series: the augmented Dickey-Fuller tests (ADF) and the Kwiatkowski, Phillips, Schmidt and Shin (1992) tests (KPSS). The second and third sections in turn explain the meaning of cointegration, and define the features of the error correction term. In addition, Engle and Granger's (1987) two-step cointegration test and Johansen's (1988) cointegration test are described and compared.

In the last section, the concept of Autoregressive Conditional Heteroscedasticity (ARCH) type modeling and its wide range of derivations are introduced. Since Engle's (1982) development of ARCH to explicitly model the variance of time series, his original ARCH(1) model has been tremendously generalized, extended and supplemented to fit economic time series of different characteristics. In the application of hedging with futures, researchers have found the multivariate-generalized ARCH



model to be the most suitable in capturing the factor of conditional information entering the market during the life of the hedge. The idea of a bivariate-GARCH model to estimate hedge ratios is explained in this chapter, and the estimates are presented in Chapter 5.

## 4.2 Unit Root Tests

A crucial property of any economic variable influencing the behaviour of statistics in econometric model is the extent to which that variable is stationary. If the autoregressive description such as:

$$y_t = \alpha + \gamma y_{t-1} + \varepsilon_t$$

has a root on the unit circle, that is, the autoregressive coefficient generates a random walk ( $\gamma = 1$ ), then this equation is said to have a unit root and stationarity is violated.

McAleer (1999) pointed out that: “the single topic in the 1980s that attracted the most attention and to which most econometricians have devoted their energies is that of testing for unit roots.” It has also been found that most empirical applications of unit root testing have been in the field of economics and finance. One reason why economists became interested in the existence of unit roots is that, for economic time series, nonstationary behaviour is often the most dominant characteristic. If that is the case, then those models that are estimated using OLS are mis-specified in that the error term  $e_t$  violates the three basic conditions (unbiased, consistent and efficient) required for a valid estimate. In consequence, economists have been intrigued by the prospect of re-testing these models in the theories. In the first modern attempt to do so using a unit root

test, Nelson and Plosser (1982) analysed the logarithms of fourteen historical macroeconomic time series for the United States by the ADF test and found empirical evidence to support a unit root in thirteen of them. Since then, these series have been re-tested hundreds of times with other methods, and thousands of other series have been examined in the literature for the existence of a unit root.

#### 4.2.1 ADF Test for Unit Roots

The Augmented Dickey-Fuller test is a modification of Dickey and Fuller (1981) Dickey-Fuller test to deal with the situation that error term  $\varepsilon_t$  is not white noise. Formally, it provides three additional F-statistics (called  $\Phi_1$ ,  $\Phi_2$ , and  $\Phi_3$ ) to test joint hypotheses on the coefficients. Three different regression equations are used to test for the presence of a unit root.

$$\Delta y_t = \beta y_{t-1} + \sum_{i=2}^p \delta_i \Delta y_{t-i} + \varepsilon_t \quad (a)$$

$$\Delta y_t = \alpha_0 + \beta y_{t-1} + \sum_{i=2}^p \delta_i \Delta y_{t-i} + \varepsilon_t \quad (b)$$

$$\Delta y_t = \alpha_0 + \beta y_{t-1} + t\alpha_2 + \sum_{i=2}^p \delta_i \Delta y_{t-i} + \varepsilon_t \quad (c)$$

The difference between the three regressions concerns the presence of the deterministic elements  $\alpha_0$  and  $\alpha_2$ . The first one is like a pure random walk model<sup>1</sup>, the second one

<sup>1</sup>Cochrane (1997) demonstrates that random walks have a number of properties:

1. The impulse-response function of a random walk is one at all horizons.
2. The forecast variance of the random walk grows linearly with the forecast horizon  
 $\text{VAR}(y_{t+k} | y_t) = \text{VAR}(y_{t+k} - y_t) = k\sigma\varepsilon^2$
3. The autocovariances of a random walk aren't defined, strictly speaking
4. The variance of a random walk is primarily due to low-frequency components. The signature of a random walk is its tendency to wander around at low frequencies.

adds in an intercept or drift term and the third one includes both a drift and a linear time trend. In all cases, the parameter of interest in the regression equations is  $\beta$ ; if  $\beta = 0$ , the  $\{y_t\}$  sequence contains a unit root. With equation (a), the null hypothesis of  $\beta = 0$  is tested by looking at the significance of the  $t$  ratio of  $\beta$ . If the null hypothesis is rejected, the series  $\{y_t\}$  is stationary, otherwise, equation (b) is tested for the null hypothesis  $\beta = \alpha_0 = 0$  using the  $\Phi_1$  statistic. If the null hypothesis is rejected, then the series  $\{y_t\}$  is stationary with drift, otherwise, model (c) including a time trend is estimated-- the joint hypothesis hypothesis  $\beta = \alpha_0 = \alpha_2 = 0$  is tested using the  $\Phi_2$  statistic and the joint hypothesis  $\beta = \alpha_2 = 0$  is tested using the  $\Phi_3$  statistic. If the null hypothesis is rejected during the course of testing, we concluded that the series  $\{y_t\}$  is stationary with a time trend, otherwise, the series considered is non-stationary.

The  $\Phi_1$ ,  $\Phi_2$ , and  $\Phi_3$  statistics are constructed in the  $F$ -test as:

$$\Phi_i = \frac{[rss(\text{restricted}) - rss(\text{unrestricted})]/r}{rss(\text{unrestricted})/(T - k)}$$

where  $rss(\text{restricted})$  and  $rss(\text{unrestricted})$  are the sums of the squared residuals from the restricted and unrestricted models;  $r$  is the number of restrictions;  $T$  is the number of useable observations; and  $k$  = number of parameters estimated in the unrestricted model. Hence,  $(T - k)$  is the degree of freedom in the unrestricted model.

#### 4.2.2 KPSS Test for Unit Roots

Recent studies have found that for many economic time series the standard unit root tests such as ADF and Phillip-Perron (1988) tests have consistently shown that the null

hypothesis of a unit root (that is  $\gamma = 1, \beta = 0$ ) cannot be rejected. This led some researchers as Schwert (1987) and DeJong et al. (1992) to question the power of the standard unit roots tests in that these tests often tend to accept the null too frequently against a stationary alternative. For example, they do not reject the null hypothesis in unit root tests  $H_0: \gamma = 1$ , but they do not reject the hypothesis  $\gamma = 0.95$  either. This result has been attributed to the fact that the tests have very low power against relevant TS (trend stationary) alternatives.

Actually, in the theory of testing of hypothesis, the null hypothesis and the alternative are not in the same footing. The null hypothesis is on a pedestal and it is rejected only when there is over whelming evidence against it. If, on the other hand, the null hypothesis and alternative were to be

$$H_0: y_t \text{ is stationary} \quad \text{and} \quad H_1: y_t \text{ is non-stationary}$$

Therefore, tests for unit roots with the null hypothesis being stationarity (no unit root) are performed using the method proposed by Kwiatkowski, Phillips, Schmidt and Shin (1992). In the KPSS tests, the null hypothesis is that a series is stationary around a deterministic trend (TS) and the alternative hypothesis is that the series is difference stationary (DS). The series is expressed as the sum of deterministic trend, random walk, and stationary error as:

$$y_t = \xi t + r_t + \varepsilon_t$$

where  $r_t = r_{t-1} + u_t$ , and  $u_t$  is i.i.d.(0,  $\sigma_u^2$ ).

The test is the LM test of the hypothesis that  $r_t$  has zero variance, that is,  $\sigma_u^2 = 0$ . If  $\sigma_u^2 = 0$ , the random walk part of the above equation,  $r_t$ , becomes a constant and thus the series

$\{y_t\}$  is trend stationary. The asymptotic distribution of the statistic is derived under the null and under the alternative hypothesis. The test is based on the statistic:

$$\eta(u) = (1/T^2) \sum_{t=1}^T S_t^2 / \sigma_k^2 \quad \text{where} \quad S_t = \sum_{i=1}^t v_i, t = 1, \dots, T$$

with  $v_t$  being the residual term from a regression of series  $y_t$  on a intercept, and  $\sigma^2$  is a consistent long-run variance estimate of  $y_t$ , and  $T$  represents the sample size.

Kwiatkowski et al (1992) shows that the statistic  $\eta(u)$  has a non-standard distribution and critical values have been provided therein. If the calculated value of  $\eta(u)$  is large, then the null of stationarity for the KPSS test is rejected.

In the case of the ETA ( $\mu$ ) statistic, the null hypothesis is that the series  $\{y(t)\}$  is stationary around a level, while in the case of the ETA( $\tau$ ) statistic, the null hypothesis accepts that  $\{y(t)\}$  is trend stationary (TS). These tests with the no unit root as null are used in many papers as a complement to standard unit root tests. By testing both the unit root hypothesis and the stationary hypothesis, we can distinguish series that appear to be stationary, series that appear to have a unit root, and series for which the data (or the tests) are not sufficiently informative to be sure whether they are stationary or integrated.

### **4.3 Cointegration and Error Correction**

As previously mentioned, a large number of time series show evidence of non-stationarity. However, the fact that any two series, spot and futures prices ( $S$  and  $F$ ) say, are I(1) process does not always imply that the error  $u_t$  generated from their linear combination is also I(1). If there is some linear combination of them,  $S_t = \alpha F_t + u_t$ , that makes  $u_t$  stationary,  $S_t$  and  $F_t$  are said to be cointegrated. Thus there is a long-run

equilibrium relationship between them. This means that the spot prices ( $S_t$ ) and the futures prices ( $F_t$ ) do not drift too much apart from each other over time.

The theory of cointegration was first developed by Granger (1981), and elaborated in Engle and Granger (1987). Their formal analysis begins by considering a set of economic variables in long-run equilibrium when:

$$\beta_1 x_{1t} + \beta_2 x_{2t} + \dots + \beta_n x_{nt} = 0$$

Let  $\beta$  and  $x_t$  denote the vectors  $(\beta_1, \beta_2, \dots, \beta_n)$  and  $(x_{1t}, x_{2t}, \dots, x_{nt})$ , the system is in long-run equilibrium when

$$\beta x_t = 0$$

The deviation from long-run equilibrium is called the equilibrium error,  $e_t$ , so that:

$$e_t = \beta x_t$$

If the equilibrium is meaningful, it must be the case that the equilibrium error process is stationary. Engle and Granger (1987) provide the following definition of cointegration:

The components of the vector  $x_t = (x_{1t}, x_{2t}, \dots, x_{nt})'$  are said to be cointegrated of order  $d$ ,  $b$ , denoted by  $x_t \sim CI(d, b)$  if:

1. All components of  $x_t$  are integrated of order  $d$
2. There exists a vector  $\beta = (\beta_1, \beta_2, \dots, \beta_n)$  such that linear combination

$$\beta x_t = \beta_1 x_{1t} + \beta_2 x_{2t} + \dots + \beta_n x_{nt}$$

is integrated of order  $(d - b)$ , where  $b > 0$ . The vector  $\beta$  is called the cointegrating vector.

The intercept term can also be included by setting all realizations of one  $\{x_i\}$  sequence equal to unity.

An important issue in econometrics is the need to integrate short-run dynamics with long-run equilibrium. The analysis of short-run dynamics is often done by first eliminating trends in the variables, usually by differencing. This procedure, as being pointed out by Engle and Granger (1987), throws away potential valuable information about long-run relations about which economic theories have a lot to say. Under the condition that two series are cointegrated, Engle and Granger (1987) propose the error correction model (ECM) that integrates short-run dynamics with long-run equilibrium. In the error-correction model, the short-term term dynamics of the variables in the system are influenced by the deviation from long-run equilibrium.

In the context that spot prices and futures prices are  $I(1)^2$  processes, an autoregressive model incorporated in the error-correction representation that could be applied to  $S_t$  and  $F_t$  is:

$$\begin{aligned}\Delta S_t &= c_s + \sum_{i=1}^n \beta_{si} \Delta S_{t-i} + \sum_{i=1}^n \beta_{fi} \Delta F_{t-i} + \gamma_s Z_{t-1} + \varepsilon_{st} \\ \Delta F_t &= c_f + \sum_{i=1}^n \beta_{fi} \Delta S_{t-i} + \sum_{i=1}^n \beta_{fi} \Delta F_{t-i} - \gamma_f Z_{t-1} + \varepsilon_{ft}\end{aligned}\tag{4.1}$$

where  $c$  is the intercept, the two terms represented by  $\varepsilon_{st}$  and  $\varepsilon_{ft}$  are white-noise disturbance terms and  $\beta_s$ ,  $\beta_f$ ,  $\gamma_s$  and  $\gamma_f$  are positive parameters.  $Z_{t-1}$  is the error-correct term, which measures how the dependent variable adjusts to the previous period's deviation from long-run equilibrium.

$$Z_{t-1} = S_{t-1} - \alpha F_{t-1}$$

Where  $\alpha$  is, what we have defined above, the cointegrating vector. This two-variable error-correction model expressed in equation (1) is a bivariate VAR (n) model in first

differences augmented by the error-correction term  $\gamma_s Z_{t-1}$  and  $-\gamma_f Z_{t-1}$ . The coefficients  $\gamma_s$  and  $\gamma_f$  have the interpretation of speed of adjustment parameters. The larger  $\gamma_s$  is, the greater the response of  $S_t$  to the previous period's deviation from long-run equilibrium.

#### **4.4 Test for the cointegration**

There are two most common used ways to test for cointegration. The Engle-Granger methodology seeks to determine whether the residuals of the equilibrium relationship are stationary, while the Johansen (1988), Johansen and Juselius (1990) and Stock-Watson (1988) methodologies determine the rank of matrix consisting of the cointegrating vectors in the error-correction model.

##### **4.4.1 Engle & Granger Approach**

Engle and Granger (1987) propose a straightforward two-step test whether two I (1) variables are cointegrated of order CI(1,1).

Step 1: Consider two time-series  $\{x_t\}$  and  $\{y_t\}$ , given that they are both I (1) processes, then it is appropriate to estimate the long-run equilibrium relation in the form:

$$Y_t = \beta_0 + \beta_1 x_t + e_t \quad (4.2)$$

In order to determine if the variables are actually cointegrated, denote the residual sequence from equation (4.2) by  $\{\hat{e}\}$ . Thus  $\{\hat{e}\}$  is the series of the estimated residuals of the long-run relationship. If these deviations from long-run equilibrium are found to be stationary, the  $\{x_t\}$  and  $\{y_t\}$  sequences are cointegrated of order (1,1).

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<sup>2</sup> I(1) process means the series is stationary after first difference.



Step 2: They apply Dickey-Fuller test on  $\{\hat{e}\}$  series to determine its order of integration<sup>3</sup>:

$$\Delta \hat{e}_t = \alpha_1 \hat{e}_{t-1} + \varepsilon_t$$

If the residuals in this regression equation,  $\varepsilon_t$ , do not appear to be white-noise, than the Augmented Dickey-Fuller test can be used instead:

$$\Delta \hat{e}_t = \alpha_1 \hat{e}_{t-1} + \sum_{i=1}^n \alpha_{i+1} \Delta \hat{e}_{t-i} + \varepsilon_t \quad (4.3)$$

If the null hypothesis  $\alpha_1 = 0$  cannot be rejected, then it is concluded that the residual series contains a unit root. Hence, the  $\{x_t\}$  and  $\{y_t\}$  sequences are not cointegrated. Otherwise, the two series are cointegrated and the error-correct term should be appended to the VAR model.

Although the Engle and Granger (1987) procedure is easily implemented, Enders (1995) claims that it does have several important limitations. First, the estimation of the long-run equilibrium regression requires that the researcher place one variable on the left-hand side and use the others as regressors. In practice, it is possible to find one regression that indicates the variables are cointegrated whereas reversing the order indicates no cointegration. This is a very undesirable feature of the procedure since the test for cointegration should be invariant to the choice of the variable selected for normalization. Second, this procedure lies on a two-step regression estimator. The coefficient  $\alpha_1$  for ADF test in equation (4.3) is obtained by estimating regression using the residuals from another regression. Hence, any error introduced by the researcher in Step 1 is carried into Step 2. Third, Enders (1995) points out that the method has no systematic procedure for the separate estimation of the multiple cointegrating vectors in

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<sup>3</sup> Since the  $\{\hat{e}\}$  sequence is a residual from a regression equation, there is no need to include an intercept term.

tests using three or more variables, where there may be more than one cointegrating vector.

#### 4.4.2. Johansen's Cointegration Tests

Fortunately, the Johansen (1988) and Stock and Watson (1988) maximum likelihood estimators circumvent the use of two-step estimators and can estimate and test for the presence of multiple cointegrating vectors. Moreover, the tests allow the researcher to test restricted versions of the cointegrating vector(s) and the speed of adjustment parameters. Both the Johansen (1988) and Stock and Watson (1988) procedures rely heavily on relationship between the rank of a matrix and its characteristic roots. It can be seen as a multivariate generalization of the Dickey-Fuller test. In the univariate case, it is possible to view the stationary of  $\{y_t\}$  as being dependent on the magnitude  $(\alpha-1)$ , that is,

$$y_t = \alpha_1 y_{t-1} + \varepsilon_t$$

or

$$\Delta y_t = (\alpha_1 - 1) y_{t-1} + \varepsilon_t$$

If  $(\alpha_1 - 1) = 0$ , the  $\{y_t\}$  process has a unit root. Now consider the simple generalization to  $n$  variables, let

$$x_t = A_1 x_{t-1} + \varepsilon_t$$

so that

$$\Delta x_t = (A_1 - I) x_{t-1} + \varepsilon_t$$

$$\Delta x_t = \pi x_{t-1} + \varepsilon_t \quad (4.4)$$

where  $x_t$  and  $\varepsilon_t$  are  $(n \times 1)$  vectors

$A_I = \text{an } (n \times n) \text{ matrix of parameters}$

$I = \text{an } (n \times n) \text{ identity matrix and } \pi \text{ is defined to be } (A_I - I)$

The rank of  $(A_I - I)$  equals the number of cointegrating vectors. By analogy to the univariate case, if  $(A_I - I)$  consists of all zeros, so that  $\text{rank}(\pi) = 0$ , all the  $\{\Delta x_{it}\}$  sequences are unit root processes. Since there is no linear combination of the  $\{x_{it}\}$  processes that are stationary, the variables are not cointegrated. If we rule out characteristic roots that are greater than unity, if  $\text{rank}(\pi) = n$ , equation (4.4) represents a convergent system of difference equations, so that all variables are stationary. It follows that all the  $\Delta x_{it}$  in equation (4.4) can be written as:

$$\Delta x_{1t} = \pi_{11}x_{1t-1} + \pi_{12}x_{2t-1} + \dots + \pi_{1n}x_{nt-1} + \alpha_{10} + \varepsilon_{1t}$$

$$\Delta x_{2t} = \pi_{21}x_{1t-1} + \pi_{22}x_{2t-1} + \dots + \pi_{2n}x_{nt-1} + \alpha_{20} + \varepsilon_{2t}$$

...

$$\Delta x_{nt} = \pi_{n1}x_{1t-1} + \pi_{n2}x_{2t-1} + \dots + \pi_{nn}x_{nt-1} + \alpha_{n0} + \varepsilon_{nt}$$

#### **4.5 ARCH Modeling in Finance**

A feature common to much of the early work in finance has been its focus on modeling the first moment of the data. Any temporal dependencies in the higher order moments were treated as a nuisance and generally ignored or, occasionally, some informal procedure was adopted to take account of this changing variance, see Mandelbrot (1963a) and Klien (1977) for examples. The increased role played by risk and uncertainty in models of financial decision making and the finding that common measures of risk and volatility exhibit strong variation over time lead to the development

of new time series techniques for modeling time-variation in second moments<sup>3</sup>.

However, it was not until 1980s that parsimonious modeling of volatility was developed with one of the most prominent techniques known as the Autoregressive Conditional Heteroscedasticity (ARCH) model by Engle (1982). Since that time, the ARCH-type models have gone through enormous extensions and variations to feature different type of time-series data in financial markets and economics.

An ARCH process can be defined in a variety of contexts. Following Engle (1982), the standard approach of heteroscedasticity is to introduce an exogenous variable  $x_t$  which predicts the variance of the variable of interest,  $y_t$ . With a known zero mean, the model might be:

$$y_t = \varepsilon_t x_{t-1}$$

Where  $Var(\varepsilon) = \sigma^2$ . The variance of  $y_t$  is simply  $\sigma^2 x_{t-1}^2$  and, therefore, the forecast interval depends upon the evolution of an exogenous variable,  $x_t$ . If the magnitude  $(x_t)^2$  is large (small), the variance of  $y_{t+1}$  will be large (small) as well. In this way, the introduction of the  $\{x_t\}$  sequence can explain periods of volatility in the  $\{y_t\}$  sequence. However, in practice this seems unsatisfactory, as it requires a specification of the causes of the changing variance of  $y_t$ . Often, we may not have a firm theoretical reason for selecting one candidate for the  $\{x_t\}$  sequence over other reasonable choices. Instead of using ad hoc variable choices for  $x_t$ , Engle (1982) shows that it is possible to simultaneously model the mean and variance of a series. To elaborate, suppose we estimate the stationary ARMA model:

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<sup>3</sup> For examples see Mandelbrot (1963a, 1963b, 1967), Fama (1965), Pagan (1996) and Bollerslev, Engle and Nelson (1994). They emphasize that the variance of speculative price series typically changes through

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \varepsilon_t, \quad \varepsilon \sim \text{i. d. d.}(0, \sigma^2) \quad (4.5)$$

The conditional forecast of  $y_{t+1}$  is  $E$

$$E y_{t+1} = \alpha_0 + \alpha_1 y_t$$

And the forecast error variance is

$$E_t [(y_{t+1} - \alpha_0 - \alpha_1 y_t)^2] = E_t \varepsilon_{t+1}^2 = \sigma^2$$

Similarly, if the variance of  $\{\varepsilon\}$  is not constant, we can estimate any tendency for sustained movements in the variance using an ARMA model. In this case the conditional variance of  $\varepsilon_t$  from equation (4.5) becomes:

$$\begin{aligned} \text{Var}(y_{t+1} | y_t) &= E_t [(y_{t+1} - \alpha_0 - \alpha_1 y_t)^2] \\ &= E_t \varepsilon_{t+1}^2 \end{aligned}$$

Thus far, we have set  $E_t \varepsilon_{t+1}^2 = \sigma^2$ . Now suppose that the conditional variance is not constant. One simple strategy is to model the conditional variance as an AR (Q) process using the square of the estimated residuals:

$$\hat{\varepsilon}_t^2 = \alpha_0 + \alpha_1 \hat{\varepsilon}_{t-1} + \alpha_2 \hat{\varepsilon}_{t-2} + \dots + \alpha_q \hat{\varepsilon}_{t-q} + v_t \quad (4.6)$$

where  $v_t$  is a white-noise process

If the values of  $\alpha_0, \alpha_1, \dots, \alpha_n$  all equal zero, the estimated variance is simply the constant  $\alpha_0$ . Otherwise, the conditional variance of  $y_t$  evolves according to the autoregressive process given by equation (4.6). For this reason, an equation like equation (4.6) is called

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time (heteroscedasticity) and distribution of speculative prices series exhibits a disproportionately high

an Autoregressive Conditional Heteroscedastic (ARCH) model. In actuality, in order to facilitate the simultaneous estimation of the model for  $\{y_t\}$  and the conditional variance using maximum likelihood techniques, researchers find it more tractable to specify  $v_t$  as a multiplicative disturbance. The simplest example from the class of multiplicative conditionally heteroscedastic models proposed by Engle (1982) is:

$$\varepsilon_t = v_t \sqrt{\alpha_0 + \alpha_1 \varepsilon_{t-1}^2} \quad (4.7)$$

where  $v_t$  = white-noise process such that  $\varepsilon_{t-1}$  has mean of zero and are independent of each other, and to ensure that the conditional variance is never negative, it is necessary to assume that both  $\alpha_0$  and  $\alpha_1$  are positive constants. Moreover, to ensure the stability of the autoregressive process, it is also necessary to restrict  $\alpha_1$  such that  $0 < \alpha_1 < 1$ .

If the conditional Variance of  $\varepsilon_t$  is defined as  $h_t$ , substituting  $h_t$  into equation (4.7), we obtain most common appearance of the ARCH (1) model:

$$\begin{aligned} \varepsilon_t &= v_t h_t^{1/2} \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \end{aligned}$$

As noted before, in an ARCH model, the error structure is such that the conditional and unconditional means are equal to zero. Moreover, the  $\{\varepsilon_t\}$  sequence is serially uncorrelated since for all  $j \neq 0$ ,  $\text{cov}(\varepsilon_t, \varepsilon_{t-j}) = 0$ . The key is that the errors are not independent since they are related through their second moment in a linear relationship. The conditional variance itself is an autoregressive process resulting in conditionally heteroscedastic errors.

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quantity of vary large and small changes compared to that of a normal distribution (leptokurtosis).

The ARCH (1) process can be extended to an ARCH ( $q$ ) processes:

$$\varepsilon_t = v_t \sqrt{\alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2}$$

We can see that in the ARCH model, the variance of the error  $\varepsilon_t$ , conditional on the realised values of the lagged errors  $\varepsilon_{t-i}$ ,  $i = 1, 2, \dots, q$ , is an increasing function of the magnitude of the lagged errors, irrespective of their signs. Hence large (small) errors of either sign tend to be followed by a large (small) error of either sign. The order of the lag  $q$  determines the length of time for which a shock persists in conditioning the variance of subsequent errors. The larger the value of  $q$ , the longer the episodes of volatility will tend to be.

Not only in the first empirical applications of ARCH to the relationship between the level and the volatility of inflation by Engle (1982, 1983), in many of other applications with the linear ARCH ( $q$ ) model a long lag length  $q$  is called for in the conditional variance function. Bollerslev (1986) extended Engle's original work by developing a technique that allows the conditional variance to be an ARMA process. A GARCH ( $p, q$ ) process can be expressed as

$$\begin{aligned} \varepsilon_t &= v_t \sqrt{h_t} \\ h_t^2 &= \omega + \sum_{i=1}^q \alpha_i \cdot \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \cdot h_{t-i}^2 \end{aligned} \quad (4.8)$$

Again, to ensure a well-defined process all the parameters in the infinite-order AR representation must be non-negative. For example, in a GARCH (1,1) model, this

amounts to ensuring that both  $\alpha_l$  and  $\beta_l$  are non-negative. It follows also that  $\varepsilon_t$  is covariance stationary if and only if  $\alpha(1) + \beta(1) < 1$ . GARCH ( $p, q$ ) process is an infinite order ARCH process with a rational lag structure imposed in the coefficients. The generalization of ARCH to GARCH is similar to the generalization of an MA process to an ARMA process. The intention is that GARCH can parsimoniously represent a high order ARCH process. The GARCH model also has the appealing features that the time series lacks serial correlation in  $\varepsilon_t$  while there is dependence in the second moments. Moreover, the nature of the unconditional density of an ARCH process can be analyzed by the higher order moments, which indicate further properties of the ARCH process. According to Engle's (1982) expression for the fourth moment (kurtosis) of an ARCH (1) process, it is revealed that its kurtosis coefficient is greater than the kurtosis coefficient of the normal distribution (the coefficient of the normal distribution is 3). Nelson (1990b) demonstrates that under suitable conditions, as the time interval goes to zero, a GARCH (1,1) process approaches a continuous time process whose stationary unconditional distribution is a Student's  $t$ . These results indicate why heavy tailed distributions are so prevalent with high frequency financial data which have leptokurtosis as a common characteristic, see (Pagan (1996), Bollerslev, Engle and Nelson (1994)).

In applications with high frequency financial data it is also found that the estimate for

$$\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j$$

turns out to be very close to unity. This provides an empirical motivation

for the so-called integrated GARCH ( $p, q$ ) or IGARCH ( $p, q$ ) model introduced by Engle and Bollerslev (1986). In the IGARCH class of models the autoregressive polynomial in



GARCH ( $p, q$ ) equation (12) has a unit root, and consequently a shock to the conditional variance is persistent in the sense that it remains important for future forecasts of all horizons.

It is important to notice that for the above models positive and negative past values have symmetric effects on conditional variance  $\varepsilon^2$ . Many financial series however are strongly asymmetric. Negative equity returns are followed by larger increases in volatility than equally large positive returns. This phenomenon was first interpreted by Black (1976) as the “leverage effect”, which could be partly explained by the fixed costs of a firm such as financial and operating leverage. A firm with debt and equity outstanding typically becomes more highly leveraged when the value of the firm falls. This raises equity returns volatility if the returns to the firm as a whole are constant<sup>4</sup>. GARCH models are thus modified to allow for this asymmetric phenomenon in the equity market, among which are exponential GARCH (EGARCH) of Nelson (1991), the quadratic GARCH (QGARCH) model of Sentana (1991), the threshold GARCH (TGARCH) of Zakoian (1994) and the non-linear ARCH model (NARCH) of Higgins and Bera (1992). Also, Harvey, Ruiz, and Sentana (1992) have recently proposed an unobserved components time series ARCH model (STARARCH) in which the innovation is composed of several sources of error where each of the error sources has heteroskedastic specifications of the ARCH form. A good illustration and comparison of these different parametric models presented here is given in Bollerslev, Chou and Kroner (1992) and Pagan and Schwert (1990).

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<sup>4</sup> Black (1976), however, argued that the response of stock volatility to the direction of returns is too large to be explained by leverage alone. This conclusion is also supported by the empirical work of Christie (1982) and Schwert (1989b).

The models discussed above are all univariate specifications. However, in much of the empirical studies in economics and finance, it is the interaction between two or more variables that is meaningful. In this context, the estimation of a number of financial ‘coefficients’ such as the hedge ratio requires sample values of covariances between the share price index as well as the related futures prices on the index. This is motivated from the fact that these two variables react to the same market information and hence, should have nonzero covariances conditional on the information set. A multivariate ARCH ( $q$ ) model is first given in Kraft and Engle (1983) and subsequently generalized to the multivariate linear GARCH ( $p, q$ ) model in Bollerslev, Engle, and Wooldridge (1988). Let  $\varepsilon_t$  in equation (12) of univariate GARCH ( $p, q$ ) model denote an  $n \times 1$  vector stochastic process then any process that permits the representation

$$\varepsilon_t = v_t \Omega_t^{1/2}$$

$$v_t, i. d. d., E(v_t) = I$$

where the time-varying  $n \times n$  covariance matrix  $\Omega_t$  is given by a linear function of the contemporaneous cross-products in the past squared errors and

$$vech(\Omega_t) = \omega + \sum_{i=1}^q A_i vech(\varepsilon_{t-1}; \varepsilon_{t-1}') + \sum_{i=1}^p B_i vech(\Omega_{t-i})$$

where  $vech(\cdot)$  denotes the operator that stacks the lower portion of an  $n \times n$  matrix as an  $(n(n+1)/2) \times 1$  vector.  $\omega$  denotes an  $(n(n+1)/2) \times 1$  vector, and  $A_i$  and  $B_i$  are  $(n(n+1)/2) \times (n(n+1)/2)$  matrices. Several properties of this model, including sufficient conditions for this parameterization to ensure that  $\Omega_t$  are positive definite, have been derived in Bera, Engle, Kraft and Kroner (1991). In this paper a simple

bivariate GARCH (1,1) model is applied to the All Ordinaries Index ( $S_t$ ) and the corresponding futures prices on the AOI ( $F_t$ ):

$$\begin{bmatrix} h_{ss,t} \\ h_{sf,t} \\ h_{ff,t} \end{bmatrix} = \begin{bmatrix} c_{ss,t} \\ c_{sf,t} \\ c_{ff,t} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} \varepsilon_{s,t-1}^2 \\ \varepsilon_{s,t-1}, \varepsilon_{f,t-1} \\ \varepsilon_{f,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \times \begin{bmatrix} h_{ss,t-1} \\ h_{sf,t-1} \\ h_{ff,t-1} \end{bmatrix}$$

where  $h_{ss}$ ,  $h_{ff}$  are the conditional variance of the errors ( $\varepsilon_{s,t}$ ,  $\varepsilon_{f,t}$ ) from the estimated within VAR framework as the mean equations of spot price and futures price respectively, and  $h_{sf}$  represent the conditional covariance. One problem with this general specification of  $H_t$  is that the number of parameters to be estimated is prohibitively high. According to Pagan (1996), there are 21 parameters to be estimated in the simple bivariate GARCH (1, 1) model. One way of simplifying this computational complexity, as proposed by Bollerslev (1990), is to assume that matrix  $A_i$  and  $B_i$  are diagonal and the correlation between the conditional variances are to be constant. However, Bera and Rog (1991) conducted a test for the constant correlation hypothesis and found that for many financial time series, the hypothesis can be rejected. Bollerslev, Engle and Wooldridge (1988) propose a parameterization of the conditional variance equation in the multivariate-GARCH model termed the Diagonal Vec (DVEC) model. Like the constant correlation model, the off-diagonal in the matrices  $A_i$  and  $B_i$  are set to zero, i.e. the conditional variance depends only on its own lagged squared residuals and lagged values. Thus, the diagonal representation of the variances elements  $h_{ss}$  and  $h_{ff}$  and the covariance element  $h_{sf}$  can be expressed as:

$$h_{s,s,t} = c_{ss} + \alpha_{ss} \varepsilon_{s,t-1}^2 + \beta_{ss} h_{ss,t-1}$$

$$h_{s,f,t} = c_{sf} + \alpha_{sf} \varepsilon_{s,t-1} \varepsilon_{f,t-1} + \beta_{sf} h_{s,f,t-1}$$

$$h_{f,f,t} = c_{ff} + \alpha_{ff} \varepsilon_{f,t-1}^2 + \beta_{ff} h_{f,f,t-1}$$

Unlike the constant correlation model, the DVEC model to be applied in this study explicitly incorporates a time varying conditional correlation coefficient between the spot and futures prices and hence generates more realistic time-varying hedge ratios.

#### **4.6 Conclusion**

This chapter has discussed econometric concepts and procedures that are necessary in identifying the characteristics of the data (unit roots and cointegration), in deriving a constant hedge ratio with and without an error-correction term incorporated (VAR model and Error-Correction model), and in estimating time varying hedge ratios (ARCH modeling). Recall that in total four types of models are employed to estimate hedge ratios, the fourth one is the conventional regression model<sup>5</sup>. The estimated hedge ratios from each of the four models are reported in the next chapter.

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<sup>5</sup> The equation of the basic regression model, as may be familiar with the reader, is thus not included in this chapter, but is presented in the next chapter with the estimated results.

# Chapter 5

## Empirical Results

### **5.1 Introduction**

This chapter implements the tests and presents the estimation results. Sections 5.3 –5.5 illustrate the properties of the data using ADF tests, KPSS tests and Johansen's cointegration test. Sections 5.6-5.7 provide the estimates of hedge ratios from the Vector Autoregressive Representation (VAR) model, the Error-Correction model and the simple regression model. Section 5.8 in turn presents the results of time varying hedge ratios estimated using a bivariate-GARCH model. The last section 5.9 compares the performance of these hedge ratios from all of the above models: the regression model, the VAR model, the Error-Correction model and the bivariate GARCH model in terms of the degree of maximizing the portfolio's return and minimizing portfolio risk.

### **5.2 The Data**

The data used in this study is downloaded from the Datastream database. It encompasses the All Ordinaries Share Price Index (AOI) and the corresponding share price index (SPI) futures prices on a daily basis for the period of January 1<sup>th</sup>, 1988 – December 12<sup>th</sup>, 2000 summing up to totally 3139 observations. Only the first 2987 observations are used in the empirical tests, leaving the last 269 observations starting from 1<sup>st</sup> January 1999 for an ex-ante hedge ratio performance comparison.

There are four delivery months per annum for the futures on the stock price index: March, June, September and December. The three-month futures contracts are adopted and the contracts in the delivery month are rolled over to the next three-month contracts on the first day of the delivery month. For example, the March contract is renewed to the June contract on the 1<sup>st</sup> of June and hence the settlement prices of the June contracts are used in June, July and August; similarly, the September contracts are used in September, October and November.

### 5.3 Descriptive Statistics

Table 1 below presents the descriptive statistics of the logarithm of All Ordinaries Index series and the futures price series. Figures 1 – 2 depict the plots of logarithm of the AOIs, futures and the basis, which is calculated as the difference between the spot prices and the corresponding futures prices.

**Table 1. Descriptive Statistics of the Data**

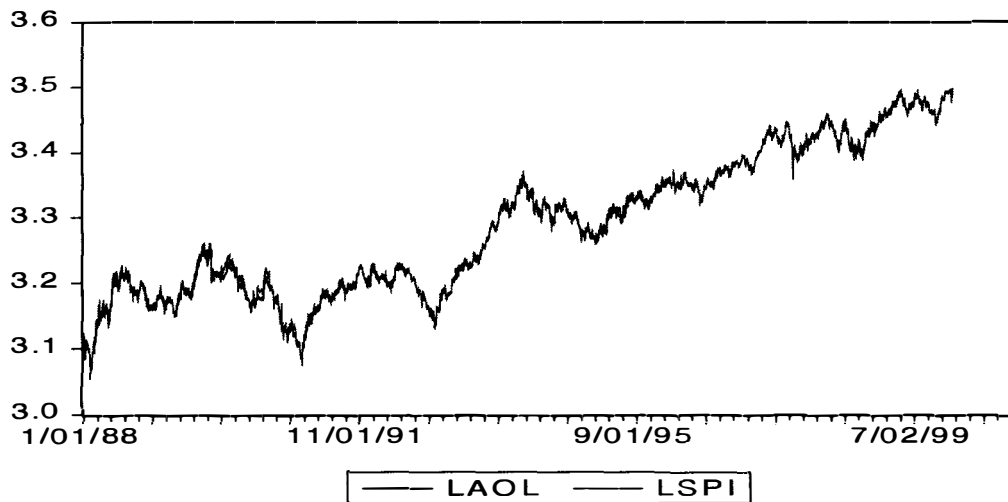
	Mean	Max	Min	Sd. D.	Skewness	Kurtosis	Jarque-Bera Test	Probability
<b>LAOI</b>	3.29	3.50	3.07	0.11	0.23	1.83	208.15	0.00
<b>LSPI</b>	3.29	3.50	3.06	0.11	0.21	1.84	201.20	0.00

Notes: This table reports the descriptive statistics of the data sets. Under the hypothesis of normal distribution, the Skewness (SK) takes value of 0 and the Kurtosis (EK) takes value of 3. Jarq-Bera is a test for normality, due to Bera and Jarque (1982):  $B - J = \left[ \frac{T}{6} SK^2 + \frac{T}{24} (EK - 3)^2 \right]$  where T is the number of observations, and under the null that the error term is normally distributed this will be distributed as  $\chi^2(2)$ .

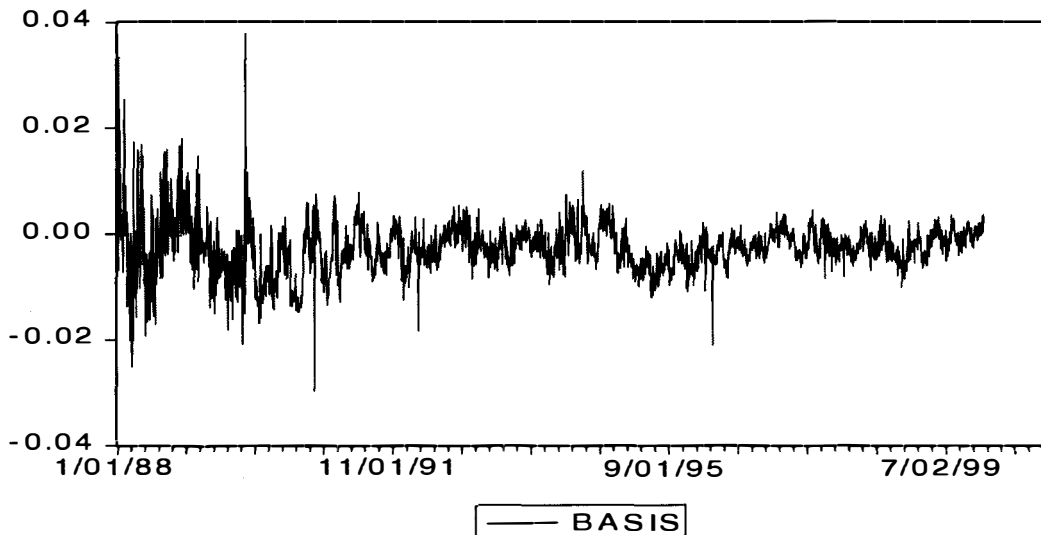
According to the values shown in table 1, the large test statistics of the Jarque-Bera normality test indicate that we have to reject the null hypothesis that the two series

follow normal distributions. Therefore, it can be concluded that both the logged AOIs and futures prices are not normally distributed. Another noticeable fact is that the statistics for the kurtosis are markedly different from 3. This tells us that the problem of non-normality may come from the heavy tails in the distribution of the series, or what is known as leptokurtosis. This feature in economic time series has been found and studied by Fama (1965), Bollerslev et al. (1994), Pagan (1996) and others.

**Figure 1. The Logarithm of AOI and SPI Series**



As is clearly evident in figure 1, the AOI price index and the futures contracts are closely correlated. From the indication of obvious time trend in the diagram it is suspected that both series are characterized by non-stationarity in levels. This will be tested formally in the next section.

**Figure 2. The Basis**

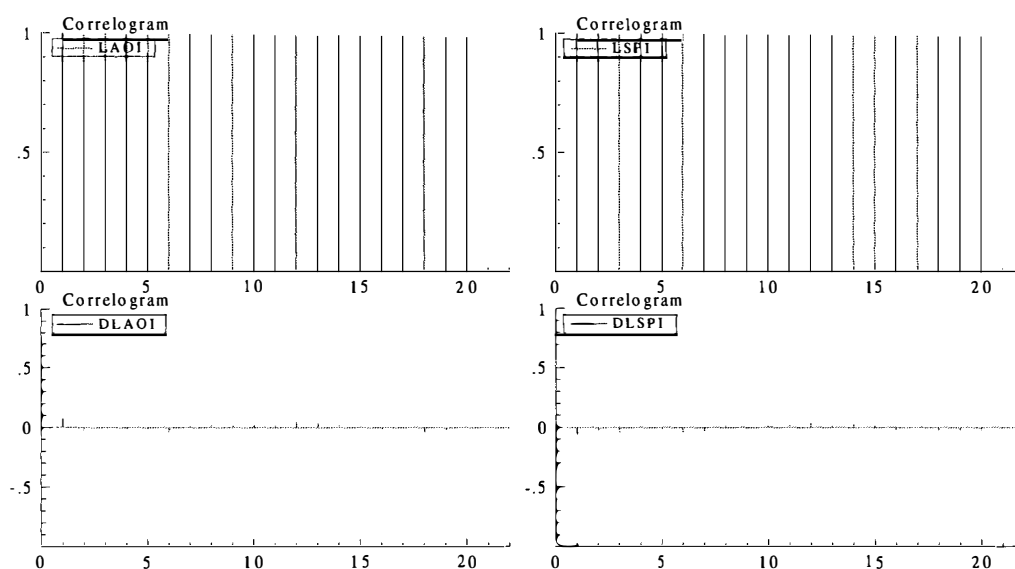
It is shown in figure 2 that the basis is indicative of extreme volatility in the late 1980s relative to other parts of the sample period. This reflects the fluctuations in the spot and futures prices as an impact of the October 1987 crash of the Australian financial markets. From the early 1990s, the basis has become less volatile and we can roughly tell that the basis steadily decreases as the spot and futures collapse towards each other as expiry approaches. On the other hand, the continuous fluctuation in the basis implies a non-linear relationship between the spot and futures price. Park and Bera (1987) pointed out that in this case a conventional regression which represent the linear relationship between spot and futures price as linear may be mis-specified and the hedge ratio estimated is misleading.



### 5.4 Tests of Unit Roots

The autocorrelation function (ACF) is a simple and commonly used method to examine the presence of non-stationarity in the series. It gives us a rough indicator of whether trend is present in a series. According to Pindyck and Rubinfeld (1991), if the series is stationary, the autocorrelation function (ACF) should converge to zero geometrically.

**Figure 3. Correlograms**



In figure 3, the correlograms of the logged function in levels (as shown in the top two correlograms) have autocorrelation coefficients close to one for each lag of the AOIs and SPI futures in levels even after twenty lags showing no inclination of convergence to zero. This suggests that these series are non-stationary. On the contrary, the autocorrelation coefficients of the differenced terms (as shown in the bottom two correlograms) are all significantly close to zero suggesting stationarity in the first differences.

Although the properties of a sample correlogram are useful for detecting the possible presence of unit root, the method is necessarily imprecise. The problem arises in that near unit root process will have the same shaped ACF as a unit root process. Hence, the Augmented Dickey Fuller (ADF) test with the null hypothesis that the series contains a unit root is employed in this study. Considering the shortcomings of the ADF tests as discussed in the last chapter, the KPSS tests is also applied with the null hypothesis that the series is either stationary around a certain level (ETA ( $\mu$ ) statistic) or is trend stationary (ETA ( $\tau$ ) statistic).

The results of the ADF and the KPSS tests for spot and futures prices in levels and changes are reported in table 2. Both series are evidenced of non-stationary in their levels, as the ADF  $t$ -statistic is insignificant and conversely the ETA( $\mu$ ) and ETA( $\tau$ ) statistics significant. After being difference, they all become stationary, that is, the ADF  $t$ -statistic becomes significant and both the ETA( $\mu$ ) and ETA( $\tau$ ) statistics turn insignificant. Therefore, it can be concluded that spot and futures prices are I (1) processes. This finding is consistent with evidence from the correlograms.

This feature of the data forms an important precondition for the tests of a cointegrating relationship, which requires that each of the variables of concerned should be integrated to the same order great than zero (Enders (1995)). The next step is to therefore test for conegration between these variables.

**Table 2. Tests for Unit Roots**

	<b>ADF Tests:</b>	<b>KPSS Tests:</b>	
	t-statistic	ETA (mu)	ETA (tau)
<b>LAOI</b>	-0.7746	***56.9338	***5.58186
<b>LSPI</b>	-1.0764	***56.73143	***5.37582
<b>DLAOI</b>	***-52.3981	0.04104	0.02813
<b>DLSPI</b>	***-59.4324	0.02495	0.02111
<b><u>Critical Values:</u></b>			
<b>Level</b>	1%	5%	10%
<b>ADF</b>	-3.43	-2.86	-2.57
<b>ETA (mu)</b>	0.739	0.463	0.347
<b>ETA (tau)</b>	0.21	0.146	0.119

Notes: for the ADF tests, \*\*\*means that the series is stationary at 99% confidence level. For the KPSS tests, \*\*\* represents that the series is non-stationary at 99% confidence level. The ETA (mu) statistic tests whether the series is stationary around a certain level, whereas the ETA (tau) statistic tests whether the series is trend stationary. The critical values are sourced from Doan, T.A. (1996).

### **5.5 The Cointegration Tests**

There are several tests for cointegration, including those developed by Engle and Granger (1987) and then extended to a multivariate version by Engle and Yoo (1987). These two-step cointegration tests have been criticized on various grounds as discussed in Chapter 4. Therefore, in this application the more advanced Johansen and Juselius (1990) cointegration tests are employed<sup>1</sup>. The model selection criteria method is also used as a supplement to cointegration tests in finding the rank of the cointegrating vector.

<sup>1</sup> For the procedure of this method, see Chapter 4.

**Table 3. Tests for Cointegration**

<b>A. Johansen's Cointegration Test</b>							
<b>Ho:</b>	<b>H<sub>1</sub>:</b>	<b><u>Eigenvalue Test</u></b>			<b><u>Trace Test</u></b>		
			95%	90%		95%	90%
<b>r = 0</b>	<b>r &lt; 1</b>	**165.7941	14.88	12.98	**166.3389	17.86	15.75
<b>r = 1</b>	<b>r &lt; 2</b>	0.54473	8.07	6.5	0.54473	8.07	6.5

Notes: Cointegration LR Test Based on Maximal Eigenvalue of the Stochastic Matrix and Trace of the Stochastic Matrix.  $r$  represents the number of linearly independent cointegrating vectors. Trace statistic =  $-\sum_{i=r+1}^n T \ln(1 - \lambda_i)$ ;  $\lambda_{\max} = -T \ln(1 - \lambda_1)$ , where  $T$  is the number of observations,  $n$  is the dimension of  $x$ , and  $\lambda_i$  is the  $i$ th smallest squared canonical correlations in Johansen (1988, 1991) or Johansen and Juselius (1990, 1992). \* and \*\* represent the significance level of 90% and 95% respectively.

**B. Choice of the Number of Cointegrating Relations Using Model Selection Criteria**

<b>Rank</b>	<b>AIC</b>	<b>SBC</b>	<b>HQC</b>
<b>r = 0</b>	25575.7	25533.6	25560.6
<b>r = 1</b>	*25655.6	*25604.5	*25637.2
<b>r = 2</b>	25654.9	25600.8	25635.4

Notes: AIC = Akaike Information Criterion, SBC = Schwarz Bayesian Criterion, HQC = Hannan-Quinn Criterion. \* marks the largest statistic value for a certain criterion.

The results of Johansen's cointegration test are presented in panel A of table 3. Two tests, one designed to test for the presence of  $r$  cointegrating vectors (the 'trace' test) and one designed to test the hypothesis of  $r$  cointegrating vectors in  $r+1$  cointegrating vectors (the maximum eigenvalue test) are undertaken on the logarithm of spot and futures price series. Under the null hypothesis that there is no cointegrating vector existing, both eigenvalue and trace statistics strongly reject the null. When the null hypothesis is that there exists a single cointegrating vector, both statistics tend not to reject it. Therefore, there is an indication of a cointegrating relationship between the variables.

Panel B in table 3 shows an alternative way of selecting the number of cointegrating relationships. The values from three model selection criteria (AIC, SBC, HQC) give the same information that the rank of cointegrating vector is one in that the statistic of each criterion its reaches the largest value when the rank of cointegrating rank is one. Ghosh (1993) and Lien (1996) suggest that in the presence of a cointegration relationship, the time series model used to estimate the minimum variance hedge ratio should account for this relationship by incorporating the error correction term (ECT). To examine whether the cointegrating relationship would also have a significant impact on Australian market, a VAR model (no error-correction term), a Error Correction Model (with error-correction term) and the conventional OLS model that regresses the changes in spot prices on the changes in futures prices have been estimated and resulting hedge ratios are compared. In addition, the error-correction model is combined with multivariate GARCH modeling that acts to eliminate the conditional heteroskedasticity in the residuals thus producing time-varying hedge ratios conditional on the information at each period of time. The hedge ratios from all of the above four models are compared in terms of in-sample and out-of-sample performance.

### ***5.6 Selecting the Lag Length of VAR***

In selecting the lag length of the bivariate VAR, a number of criteria were employed: Akaike Information Criterion (AIC), Schwarz Bayesian Criterion (SBC), and Log-likelihood ratio statistics (LL). Considering daily data used in this study, a maximum order of ten-lag is chosen to start with. The values of each criterion for each lag are obtained by using the Microfit econometrics program, where all the eleven unrestricted

VAR ( $p$ ) ( $p = 0, 1, \dots, 10$ ) models are estimated over the same sample period to find out the appropriate lag length at which the autocorrelation is eliminated.

**Table 4. Test Statistics and Choice Criteria for Lag-Length of VAR**

Order of Lag	AIC	SBC	LR test
10	25674.2	25548.1	-----
9	25675.9	25561.8	CHSQ( 4)= 4.6256[.328]
8	*25679.1	25577	CHSQ( 8)= 6.3032[.613]
7	25676.8	25586.8	CHSQ( 12)= 18.7487[.095]
6	25662.5	25584.4	CHSQ( 16)= 55.4801[.000]
5	25662.6	25596.6	CHSQ( 20)= 63.2339[.000]
4	25660.2	*25606.1	CHSQ( 24)= 76.0987[.000]
3	25645.3	25603.3	CHSQ( 28)= 113.8136[.000]
2	25604.3	25574.3	CHSQ( 32)= 203.8430[.000]
1	25396.5	25378.5	CHSQ( 36)= 627.3943[.000]
0	14357.8	14351.8	CHSQ( 40)= 22712.9[.000]

Notes: The lag of ten was used as the maximum to start with. The appropriate lag length is chosen using LR test, AIC, SBC criteria, where AIC = Akaike Information Criterion, SBC = Schwarz Bayesian Criterion. \* in the table marks the greatest value for a certain criterion and thus helps find out the appropriate lag. Note that LR test rejected the lag of eight but not the lag of four.

It is found in table 4 that the Akaike (AIC) and the Schwarz (SBC) have the most significant value in order eight and four, respectively. The log-likelihood ratio statistics strongly reject order 8, but do not reject a VAR of order 4. Therefore, the bivariate VAR (4) model is chosen<sup>2</sup>.

Empirical regularity can be described as a situation in which financial variables display a persistent pattern over time or a persistent relationship with each other (See Dimson (1988)). With the daily data being examined in this context, two types of empirical regularities should be considered. One is the weekend effect, which relates to the

observation that returns are not independent of the day-of-the-week<sup>3</sup>. The other is the holiday effect, which is based on the observation that abnormally high returns are commonly observed following the incidence of public holidays compared to other trading days which do not occur near a public holiday. For this reason, a VAR model with dummy variables representing the holiday and the weekend effects are estimated. A public holiday variable takes a value of unity on the day following an Australia public holiday and zero at all other times. A weekend dummy variable is generated in a similar way. This variable takes on a value of unity on the particular day-of-the-week (say, Monday) to which it is assigned and a value of zero for all other days of the week. The *t*-statistics of the coefficient of each dummy variable are assessed at a 5% level of significance. It is found that the coefficients of all of the eleven dummy variables (5 day-of-the-week dummies and 6 holiday dummies) are not significantly different from zero in the sample period. Hence, it is concluded the restriction of adding extra dummy variables is not binding and the VAR model should be modeled in its initial form. In the interest of saving space, the results are not presented here<sup>4</sup>.

### ***5.7 Bivariate VAR Model and an Error-Correction Model***

The Vector Autoregression (VAR) method is used by Ghosh (1993), Chou, Denis, and Lee (1996), Lien and Tse (1998) in that it incorporates the history of both spot and futures prices as the conditional information. In this context, it is assumed that the

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<sup>2</sup> Pasaran (1994) notes that it is quite usual for the SBC to select a lower order VAR than the AIC.

<sup>3</sup> A number of research on this issue suggests that negative returns are more common to Mondays, whereas positive returns are more common to the latter part of the business week. See French (1980), Ball and Bowers (1988), Faff (1992) for evidence.

<sup>4</sup> However, the results are ready to supply on request.

previous four lags of movements in the spot and futures markets affect the current price movements. The following VAR model incorporates these considerations

$$\begin{aligned}\Delta S_t &= c_s + \sum_{i=1}^4 \beta_{si} \Delta S_{t-i} + \sum_{i=1}^4 \beta_{fi} \Delta F_{t-i} + \varepsilon_{st} \\ \Delta F_t &= c_f + \sum_{i=1}^4 \beta_{fi} \Delta S_{t-i} + \sum_{i=1}^4 \beta_{si} \Delta F_{t-i} + \varepsilon_{ft}\end{aligned}\quad (5.1)$$

**Table 5. Estimates of A Bivariate VAR (4) Model**

	DLAOI			DLSPI		
	Coefficient	Stand. D.	t-Ratio	Coefficient	Stand. D.	t-Ratio
<b>DLAOI(-1)</b>	-0.3252	-0.0265	*-12.2565	0.1673	-0.0387	*-4.3218
<b>DLAOI(-2)</b>	-0.1959	-0.0283	*-6.9279	0.0935	-0.0413	*-2.2661
<b>DLAOI(-3)</b>	-0.0958	-0.0278	*-3.4412	0.0567	-0.0406	*-1.3972
<b>DLAOI(-4)</b>	-0.0526	-0.0235	*-2.242	-0.0157	-0.0343	-0.4578
<b>DLSPI(-1)</b>	0.3698	-0.0182	*-20.3156	-0.1471	-0.0266	*-5.5379
<b>DLSPI(-2)</b>	0.1816	-0.0214	*-8.4852	-0.1127	-0.0312	*-3.6065
<b>DLSPI(-3)</b>	0.1023	-0.0215	*-4.7492	-0.0904	-0.0314	*-2.8767
<b>DLSPI(-4)</b>	0.0674	-0.0192	*-3.5026	-0.0121	-0.0281	-0.4303
<b>Constant</b>	0.0001	-0.0001	-1.8597	0.0001	-0.0001	-1.4217

Notes: The results are the estimates of Equation (5.1), a bivariate VAR(4) model. The DLAOI(.) and DLSPI(.) represent the coefficients of each lag: 1, 2, 3, and 4 for the differenced logarithm of spot and futures prices, respectively. The standard errors and t-ratios are presented beside the corresponding coefficients to show each coefficient's relative significance at 95% level. The statistically significant coefficients are marked with \*'s .

Where  $c$  is the intercept, and  $\beta_s, \beta_f, \gamma_s$  and  $\gamma_f$  are positive parameters.  $\varepsilon_{st}, \varepsilon_{ft}$  are independently identically distributed (i.i.d) random vectors. Let  $\text{var}(\varepsilon_{st}) = \sigma_{ss}$ ,  $\text{var}(\varepsilon_{ft}) = \sigma_{ff}$ , and  $\text{cov}(\varepsilon_{st}, \varepsilon_{ft}) = \sigma_{sf}$ . The minimum variance one-day hedge ratio is  $\sigma_{sf} / \sigma_{ff}$ , which will be called the VAR hedge ratio. The coefficients of Equation (5.1) are presented in table 5.



Following Ghosh (1993b), Lien and Luo (1994) and Lien (1996), if the two price series are found to be cointegrated, a VAR model should be estimated along with the error-correction term which accounts for the long-run equilibrium between spot and futures price movements. Thus equation 5.1 is modified as:

$$\begin{aligned}\Delta S_t &= c_s + \sum_{i=1}^4 \beta_{si} \Delta S_{t-i} + \sum_{i=1}^4 \beta_{fi} \Delta F_{t-i} + \gamma_s Z_{t-1} + \varepsilon_{st} \\ \Delta F_t &= c_f + \sum_{i=1}^4 \beta_{fi} \Delta S_{t-i} + \sum_{i=1}^4 \beta_{si} \Delta F_{t-i} - \gamma_f Z_{t-1} + \varepsilon_{ft}\end{aligned}\tag{5.2}$$

Where  $Z_{t-1}$  is the error-correct term, which measures how the dependent variable adjusts to the previous period's deviation from long-run equilibrium.

$$Z_{t-1} = c + S_{t-1} - \alpha F_{t-1}\tag{5.3}$$

Where  $c$  is the intercept term,  $\alpha$  is the cointegrating vector. The two-variable error-correction model expressed in equation (5.2) is a bivariate VAR (4) model in first differences augmented by the error-correction term  $\gamma_s Z_{t-1}$  and  $-\gamma_f Z_{t-1}$  in (5.2). The several properties of coefficients  $\gamma_s$  and  $\gamma_f$  and the econometric sufficiency of incorporating the error-correction term  $Z_{t-1}$  have been discussed in the previous chapter. In this application of spot price and its corresponding futures contract, Lien and Tse (1998) provide an alternative explanation to the inclusion of the Error-Correction Term. They suggest that due to an arbitrage-free relationship and the convergence of spot and futures price at delivery, the basis may help determine the price movements. Therefore the error-correction term can be considered as a version of the basis and thus should be

incorporated into the VAR model. Table 6 presents the results of the error-correction model.

**Table 6. Estimates of Error Correction Model**

	D(LAOI)			D(LSPI)		
	Coefficient	Stand. D	t-Ratio	Coefficient	Stand. D	t-Ratio
<b>Cointegrating Equation (<math>Z_{t-1}</math>)</b>	-0.0684 ( $\gamma_s$ )	-0.0159	<b>*-4.3054</b>	0.1014 ( $\gamma_f$ )	-0.0232	<b>*-4.3768</b>
D(LAOL(-1))	-0.2813	-0.0283	*-9.9362	0.1041	-0.0413	*-2.5200
D(LAOL(-2))	-0.1660	-0.0290	*-5.7253	0.0510	-0.0423	*-1.2048
D(LAOL(-3))	-0.0778	-0.0281	*-2.7728	0.0319	-0.0410	*-0.7781
D(LAOL(-4))	-0.0505	-0.0234	*-2.1568	0.0173	-0.0342	-0.5060
D(LSPI(-1))	0.3179	-0.0218	*-14.6070	0.0708	-0.0318	*-2.2293
D(LSPI(-2))	0.1439	-0.0231	*-6.2346	0.0560	-0.0337	-1.6640
D(LSPI(-3))	0.0751	-0.0224	*-3.3535	0.0497	-0.0327	-1.5227
D(LSPI(-4))	0.0526	-0.0195	*-2.6965	0.0104	-0.0285	-0.3651
<b>Cointegrating Equation</b>						
	Coefficient	Stand. D	t-Ratio			
$S_{t-1}$	1.000	-	-			
$F_{t-1}$	-1.0047	-0.0029	351.6024			
C	0.0178	-0.0094	-1.8927			

Notes: The upper part of the results are the estimates of Equation (5.2), the error-correction model, or a bivariate VAR(4) model incorporated in an error-correction term. The coefficients of cointegration equation are  $\gamma_s$  and  $\gamma_f$  in Equation (5.2). The DLAOI(.) and DLSPI(.) represent the coefficients of each lag: 1, 2, 3, and 4 for the differenced logarithm of spot and futures prices, respectively. The standard errors and t-ratios are presented beside the corresponding coefficients to show each coefficient's relative significance at 95% level. The statistically significant coefficients are marked with \*'s. The bottom part of the table presents the results estimated from the cointegration equation of spot and futures prices in levels,  $Z_{t-1} = c + S_{t-1} - \alpha F_{t-1}$  in Equation 5.3.

From table 6, it is shown that for both equations of changes in spot prices and changes in futures prices, the coefficients of the error-correction term (as shown in bold characters) are significant, as indicated by the large values of the  $t$ -ratios. It is noticed that  $\gamma_s = 0.069$ , while  $\gamma_f = 0.1$ . This implies that the futures price series  $F_t$  have a greater speed of

adjustment to the previous period's deviation from long-run equilibrium than the spot price series. This finding is consistent with the fact that on the delivery date of each contract the futures price has to adjust itself to the prevailing spot price.

For the sake of curiosity, the conventional OLS regression method is also applied to estimating the constant minimum variance hedge ratio. The linear regression model is given as:

$$\Delta S_t = c + h\Delta F_t + \varepsilon_t \quad (5.4)$$

Table 7 reports the results of the parameters of the model. It is shown that the minimum variance hedge ratio is 0.48, smaller than that obtained from the VAR model (0.51) and the Error-correction model (0.52) as calculated in table 8.

**Table 7. The Estimation Results of Regression Model**

Variable	Coefficient	Std. Error	t-Statistic	Prob.
<i>h</i>	<b>0.477719</b>	0.00978	48.8587	0.0000
<i>c</i>	6.08E-05	4.86E-05	1.25165	0.2108
<b>R<sup>2</sup></b>	43.21%			

Notes: this table reports the estimates from the conventional regression model using ordinary least square (OLS) method. The coefficient of the independent variable (the changes in the logged futures prices) is taken as the optimal hedge ratio.

Then the hedge ratios calculated from the VAR and the Error-Correction Model are presented in table 8 for the purpose of illustration.

**Table 8. Covariance Matrix and Hedge Ratios**

	VAR Model		Error-Correction Model	
	$\sigma_{ss}$	$\sigma_{ff}$	$\sigma_{ss}$	$\sigma_{ff}$
$\sigma_{ss}$	1.14E-05	1.24E-05	1.14E-05	1.25E-05
$\sigma_{ff}$	1.24E-05	2.43E-05	1.25E-05	2.42E-05
$\sigma_{sf} / \sigma_{ff}$	<b>0.5083</b>		<b>0.5165</b>	

Notes: the unconditional variances of the spot prices ( $\sigma_{ss}$ ), futures prices ( $\sigma_{ff}$ ) and the covariance ( $\sigma_{sf}$ ) of the two are calculated from the residuals of the VAR model (equation 5.1) and the error-correction model (equation 5.2), respectively. The optimal hedge ratios are thus calculated from  $h^* = \sigma_{sf} / \sigma_{ff}$ .

As expected and in line with most of the previous studies by Ghosh (1993b) and others, the hedge ratio estimated by the error-correction model is greater than that obtained from other models. The hedger ignorant of the cointegrating relationship between futures and spot prices is likely to take a smaller than optimal futures position.

### **5.8 Dynamic Hedge Ratios using B-GARCH Model**

In this paper, Lien's (1996) study is extended to examine the efficiency of the error-correction model by further investigating the features of the residual series. The autocorrelation functions of the two streams of residuals from Equation 5.2 are presented in table 9. For daily data in this application, the lag of 20 is chosen to correspond to a period of approximately one calendar month, and the actual residual values are plotted in figure 4.

It is indicated clearly in table 9 that the autocorrelation coefficients for all 20 lags are close to zero, with probabilities well greater than 5% significance level. This leads us to believe that the estimated mean equation, that is, the bivariate VAR model incorporated

in the error-correction term, has adequately accounted for the serial correlation in the logarithm of spot and futures price series.

**Table 9. The Autocorrelation Function of the Residuals**

**(a) Residuals of All Ordinaries Share Price Index**

	Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
1			-0.003	-0.003	0.0239	0.877
2			-0.002	-0.003	0.0434	0.979
3			0.003	0.003	0.0650	0.996
4			-0.005	-0.005	0.1529	0.997
5			-0.007	-0.007	0.3224	0.997
6			-0.030	-0.030	3.1228	0.793
7			-0.015	-0.016	3.8707	0.795
8			0.022	0.022	5.4221	0.712
9			0.001	0.001	5.4254	0.796
10			0.015	0.015	6.1198	0.805
11			0.025	0.024	8.0163	0.712
12			0.039	0.038	12.685	0.392
13			0.016	0.016	13.518	0.409
14			0.027	0.029	15.814	0.325
15			0.016	0.018	16.636	0.341
16			-0.013	-0.012	17.182	0.374
17			0.008	0.010	17.376	0.429
18			-0.023	-0.021	19.084	0.387
19			-0.022	-0.021	20.599	0.359
20			-0.020	-0.020	21.838	0.349

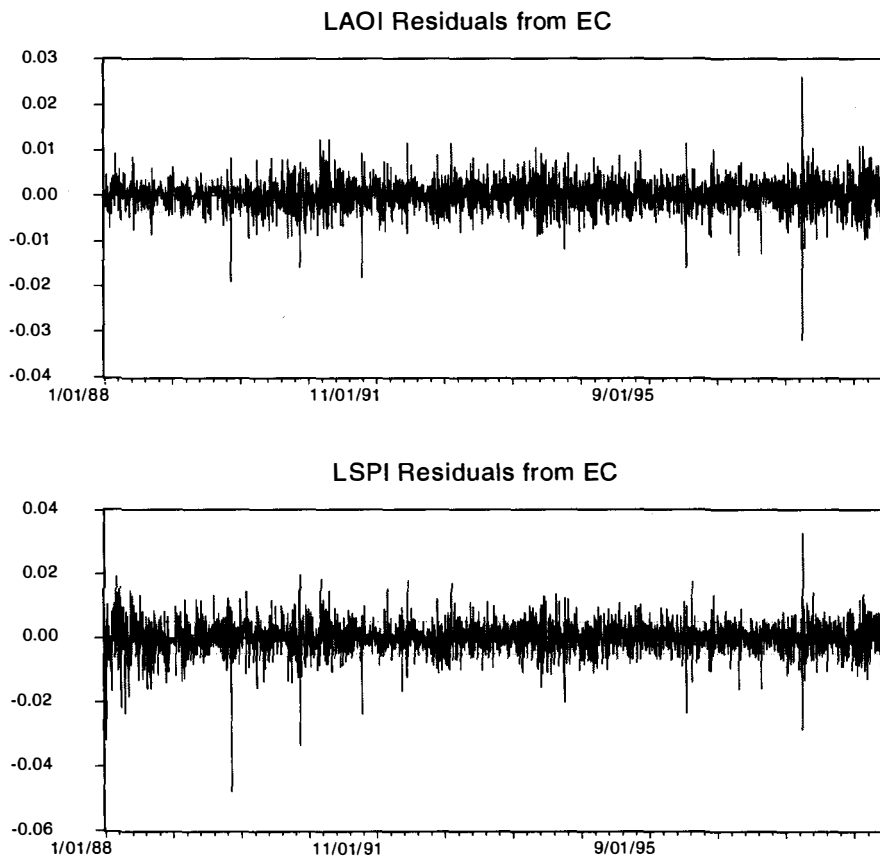
**(b) Residuals of SPI Futures**

Lag	Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
1			-0.002	-0.002	0.0106	0.918
2			-0.002	-0.002	0.0180	0.991
3			0.004	0.004	0.0563	0.997
4			-0.005	-0.005	0.1194	0.998
5			-0.008	-0.008	0.3166	0.997
6			-0.035	-0.035	3.8109	0.702
7			-0.012	-0.013	4.2545	0.750
8			0.026	0.026	6.2190	0.623
9			0.005	0.005	6.2794	0.712
10			0.012	0.012	6.7241	0.751
11			0.027	0.026	8.8561	0.635
12			0.034	0.033	12.113	0.437
13			0.017	0.017	12.950	0.452
14			0.029	0.031	15.418	0.350
15			0.014	0.015	15.962	0.385
16			-0.014	-0.013	16.539	0.416
17			0.014	0.016	17.105	0.447
18			-0.022	-0.019	18.515	0.422
19			-0.022	-0.021	19.921	0.399
20			-0.019	-0.019	20.976	0.399

Notes: This table presents the plots and values of autocorrelation function (AC) and partial autocorrelation correlation function (PAC) of the residuals from Equation 5.2. The last two columns are Q-statistics for high order autocorrelation and the corresponding probability. The null hypothesis is that there is no autocorrelation at a certain order. The probabilities tell us that we have to accept the null of no autocorrelation up to 20 lags.

However, the plots of the actual values of the residuals in figure 4 exhibit volatility clustering even though the mean seems constant. The variance of the series is changing through time and large (small) changes tend to be followed by large (small) changes of either sign. This characteristic has been commonly found in most economic time series by Mandelbrot (1963, 1967), Klien (1977), Engle (1982) etc. and it is indicative of the presence of an autoregressive conditional heteroskedastic (ARCH) effect.

**Figure 4. The Plot of Residuals**











































Another way to test for the presence of ARCH effects has been suggested by McLeod and Li (1983). According to McLeod and Li, a casual examination of the sample

autocorrelation function of the mean equation squared residuals for a significant Q-statistic at a given lag can be used to infer the presence of ARCH effects. The (Ljung-Box) Q-statistic at lag  $k$  is a test statistic for the null hypothesis that there is no autocorrelation up to order  $k$ . Table 10 presents the Q-statistic for squared residuals ( $\varepsilon_t^2$ ) generated from equation 5.2. They are all highly significant confirming the presence of ARCH effects. Therefore, a bivariate GARCH method is necessary to explicitly model the variance of the residuals of the error-correction model.

**Table 10. Autocorrelation Function of the Squared Residuals**

**(c) Squared Residuals of All Ordinaries Share Price Index**

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.381	0.381	456.28	0.000
		2	0.124	-0.025	504.45	0.000
		3	0.112	0.086	543.85	0.000
		4	0.092	0.027	570.42	0.000
		5	0.046	-0.004	577.00	0.000
		6	0.015	-0.010	577.68	0.000
		7	0.044	0.041	583.71	0.000
		8	0.050	0.018	591.49	0.000
		9	0.013	-0.017	592.04	0.000
		10	0.021	0.020	593.43	0.000
		11	0.026	0.006	595.57	0.000
		12	0.012	-0.005	596.01	0.000
		13	0.042	0.045	601.68	0.000
		14	0.022	-0.015	603.22	0.000
		15	0.016	0.008	604.02	0.000
		16	0.012	-0.002	604.46	0.000
		17	0.022	0.016	605.94	0.000
		18	0.027	0.012	608.32	0.000
		19	0.000	-0.018	608.32	0.000
		20	0.020	0.025	609.56	0.000

## (d) Squared Residuals of SPI Futures

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.158	0.158	78.430	0.000
		2	0.066	0.043	92.283	0.000
		3	0.039	0.023	97.029	0.000
		4	0.039	0.028	101.90	0.000
		5	0.030	0.018	104.77	0.000
		6	0.049	0.039	112.28	0.000
		7	0.049	0.033	119.83	0.000
		8	0.013	-0.006	120.35	0.000
		9	0.047	0.039	127.16	0.000
		10	0.022	0.005	128.73	0.000
		11	0.013	0.001	129.28	0.000
		12	0.022	0.014	130.75	0.000
		13	0.046	0.035	137.36	0.000
		14	0.048	0.032	144.67	0.000
		15	0.011	-0.009	145.05	0.000
		16	0.034	0.024	148.67	0.000
		17	0.019	0.006	149.85	0.000
		18	0.040	0.029	155.01	0.000
		19	0.014	-0.005	155.60	0.000
		20	0.025	0.013	157.57	0.000

Notes: This table presents the plots and values of autocorrelation function (AC) and partial autocorrelation correlation function (PAC) of the residuals from Equation 5.2. The last two columns are Q-statistics for high order autocorrelation and the corresponding probability. The null hypothesis is that there is no autocorrelation at a certain order. The probabilities tell us that we have to accept the null of no autocorrelation up to 20 lags.

Following Bollerslev, Engle and Wooldridge (1988), a diagonal-vec MGARCH model

(DVEC MGARCH) is defined as:

$$\begin{aligned}
 h_{s,s,t} &= c_{ss} + \beta_{ss} h_{ss,t-1} + \alpha_{ss} \varepsilon_{s,t-1}^2 \\
 h_{s,f,t} &= c_{sf} + \beta_{sf} h_{sf,t-1} + \alpha_{sf} \varepsilon_{s,t-1} \varepsilon_{f,t-1} \\
 h_{ff,t} &= c_{ff} + \beta_{ff} h_{ff,t-1} + \alpha_{ff} \varepsilon_{f,t-1}^2
 \end{aligned} \tag{5.5}$$

Where  $h_{ss}$  and  $h_{ff}$  are the variance elements and the covariance elements of spot and futures prices and  $h_{sf}$  represents the covariability of both price series. Like the constant correlation model of Bollerslev (1987), the off-diagonal in the matrices  $A_i$  and  $B_i$  are set



to zero, i.e. the conditional variance depends only on its own lagged squared residuals and lagged values. However, the diagonal-vec model is superior to the constant correlation model in that the DVEC model explicitly incorporates a time varying conditional correlation coefficient between the spot and futures prices and hence generates more realistic time-varying hedge ratios<sup>5</sup>.

For a bivariate MGARCH model in the study, I use the BHHH (Berndt, Hall, Hall and Hausman) optimization method and the Simplex Algorithm optimization method to estimate all the coefficients  $c_{ij}$ ,  $\alpha_{ij}$  and  $\beta_{ij}$  simultaneously. The Simplex method is a search procedure that requires only function evaluations, not derivatives; while the other method BHHH required twice-differentiable formulas. The use of a combination of the two methods is suggested by Doan (1996), who states that the Simplex used in the program is to refine initial estimates before applying BHHH. The latter method is more sensitive to the choice of initial estimates. However, a disadvantage of the Simplex method is that it cannot provide standard errors for the estimated parameters. The program automatically selects parameter values that maximize the log likelihood function of the model. The results are presented in Table 11. The parameter estimates are all positive definite and highly significant. Furthermore, the sum of the coefficients for each equation is close to unit, (for example:  $c_{ff} + \alpha_{ff} + \beta_{ff} = 0.988$ ), suggesting the persistence of ARCH effects in the data sets. This implies that current information remains important for forecasts of the conditional variance at all horizons.

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<sup>5</sup> Bera and Roh (1991) proposed a test for the constant correlation hypothesis and found that for many financial time series, the hypothesis is rejected.

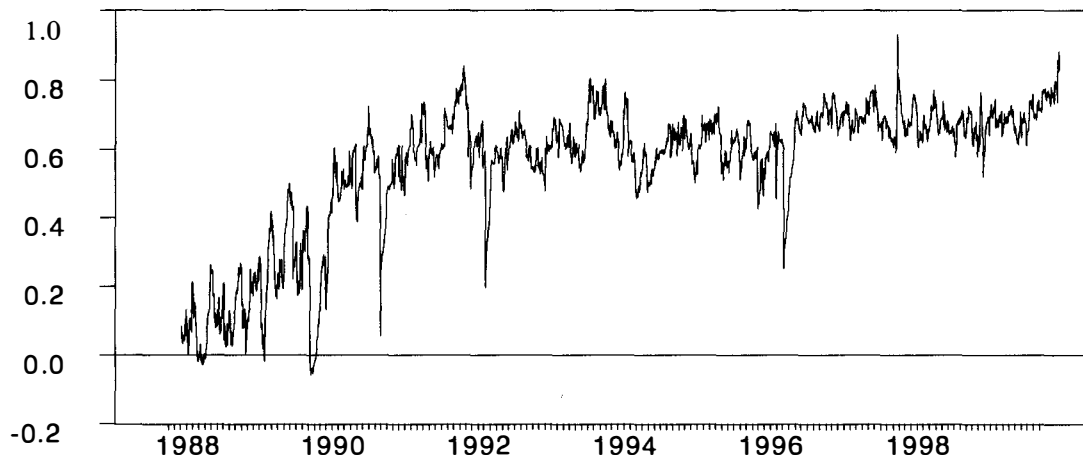
**Table 11. The Estimates of MGARCH Model**

<u>Variable</u>	<u>Coefficient</u>	<u>Std. Error</u>	<u>t-statistic</u>	<u>Probability</u>
$c_{ss}$	4.13E-07	4.56E-08	9.0512	0.0000
$c_{sf}$	3.84E-07	3.58E-08	10.7204	0.0000
$c_{ff}$	3.88E-07	4.57E-08	8.4900	0.0000
$\beta_{ss}$	0.07491	0.00338	22.1889	0.0000
$\beta_{sf}$	0.0716	0.00305	23.4462	0.0000
$\beta_{ff}$	0.07262	0.00278	26.1227	0.0000
$\alpha_{ss}$	0.89672	0.00571	157.1213	0.0000
$\alpha_{sf}$	0.90729	0.00428	212.2021	0.0000
$\alpha_{ff}$	0.91578	0.00382	239.9111	NA

Notes: This table report the results estimated from the MGARCH Model in Equation 5.5.  $c_{ss}$ ,  $c_{sf}$  and  $c_{ff}$  are constants.  $\beta_{ss}$ ,  $\beta_{sf}$  and  $\beta_{ff}$  are coefficients of the conditional variances and covariances, respectively.  $\alpha_{ss}$ ,  $\alpha_{sf}$  and  $\alpha_{ff}$  are coefficients of the squared error terms, respectively.

Figure 5 plots the dynamic hedge ratios obtained from the conditional variance and covariance between the spot and futures prices. Note that the hedge ratios display signs of extreme volatility and show continuous increase in the late 1980s, reflecting the irregular fluctuation in prices due to the impact of the October 1987 crash on both spot and futures markets. The hedge ratios are relatively more stable since 1991, except for 2 sharp drops in 1992 and 1996. Ranging from a minimum of  $-0.046$  to a maximum of  $0.92$ , the dynamic hedge ratio has a sample mean of  $0.59$ , which is well below  $1$ , but greater than the constant hedge ratios on average. This conclusion once again confirms the rejection of traditional 1: 1 hedging strategy. Moreover, the dynamic hedge ratio exhibits explicit random walk characteristics (non-stationarity) with its significant ADF statistic of  $2.12^6$ . This finding is consistent with that of Lo and MacKinlay (1988), Malliaris and Urrutia (1991), and Lindhal (1992) and others.

<sup>6</sup> The procedure of ADF test for the hedge ratios is not presented here, but can be provided on request.

**Figure 5. The time-Varying Hedge Ratios**

Notes: The diagram plots the estimates of the time varying hedge ratios from the MGARCH model. It is shown that they are highly non-stationary and are mostly positive.

### ***5.9 Hedging Effectiveness Comparison***

So far four hedging strategies have been used to derive optimal hedge ratios: a conventional regression model, a vector autoregression model, an error-correction model, and an MGARCH model. Each of these models is based on different econometric theories and involves different degrees of computational complexity. Previous studies testing the effectiveness of hedge ratios derived from various models of the US markets used in-sample and out-of-sample comparisons and yielded mixed results, though all found that the hedge ratio obtained from the regression model performs the worst. In this application, the ex post and ex ante forecasting method will be employed to compare the performance of these four types of hedge ratios and examine whether the similar results hold in a different market, or there are other striking results evident.

In order to compare the performances of each type of hedging strategy, the un-hedged portfolio is constructed, consisting of shares with the same proportion as the share price index held on the spot market. Also the hedged portfolio is constructed, consisting of a combination of the share price index held on both the spot and the futures markets. The number of futures contracts held is determined by the computed hedge ratios from each hedging strategy. The hedging performance is compared in terms of the risk-return trade-off, and the percentage variance reduction in the hedged portfolio relative to the un-hedged portfolio.

The mean and variance of the returns of the hedged portfolios, and the percentage reduction in the variance of the hedged portfolio relative to the un-hedged portfolio are calculated in each forecasting horizon. Following Baillie and Myers (1991) and Park and Bera (1987: appendix), the returns on the un-hedged and the hedged portfolios are simply expressed as:

$$r_u = S_{t+1} - S_t$$

$$r_h = (S_{t+1} - S_t) - h^* (F_{t+1} - F_t) \quad (5.6)$$

Where  $r_u$  and  $r_h$  are return on un-hedged portfolio and hedge portfolio, respectively.  $F_t$  and  $S_t$  are logged futures and spot prices at time period  $t$ , respectively, and  $h^*$  is optimal hedge ratio, and the return on the hedged portfolio is the difference between the return on holding the cash position and corresponding futures position.

Similarly, the variance of the un-hedged and the hedged portfolios are expressed as:

$$Var(U) = \sigma_s^2 \quad (5.7)$$

$$\text{Var}(H) = \sigma_s^2 + h^2 \sigma_f^2 - 2 h \sigma_{s,f} \quad (5.8)$$

Where  $\text{Var}(U)$  and  $\text{Var}(H)$  represent variance of un-hedged and hedged portfolios, respectively.  $\sigma_s$ ,  $\sigma_f$  are standard deviation of the spot and futures price, respectively, and  $\sigma_{s,f}$  represents the covariability of the spot and futures price. According to Ederington (1979), the effectiveness of hedging can be measured by the percentage reduction in variance of the hedged portfolio relative to the unhedged portfolio. The variance reduction can be calculated as:

$$\frac{\text{Var}(U) - \text{Var}(H)}{\text{Var}(U)} \quad (5.9)$$

Lien and Tse (1998) propose that the performance of the models may vary according to the hedge horizon, therefore, in this context hedging effectiveness of the four models will be considered over horizons of 1, 5, 10 and 20 days.

The more reliable measure of hedging effectiveness is the hedging performance for the post-sample periods. For each out-of-sample testing period, the same parameters estimated from M-GARCH are used to forecast the conditional variance and covariance for the following day. The forecasted hedge ratio will be the one-period forecast of the conditional covariance divided by the one-period forecast of the conditional variance. Such forecasts are conducted for each day for the following 20 observations from the 16<sup>th</sup> December 1999 to 12<sup>th</sup> January 2000. For the other three models which generate

constant hedge ratios, the estimated hedge ratios are used for the out-of-sample period.

The results for the in-sample and post-sample performance are presented in table 12.

The first section of Table 12 displays the within-sample comparisons. In the one-day hedge case, a trade-off between risk and return occurs. Although the M- GARCH model generates the greatest daily return of approximately 0.07%, it incurs a considerable risk greater than any other method. It is also the poorest one in terms of percentage reduction of the variance of the un-hedged portfolio. This is not the case for the longer hedging horizons. Taking the twenty-day hedge as an example, it is shown that the greatest return is generated from the conventional regression model, and so is the greatest risk. The GARCH method tremendously reduces the overall risk in the un-hedged portfolio to a degree of 80%, but the return yielded from the hedged portfolio is the smallest.

Therefore, if risk aversion is the major goal of an investor, the GARCH model hedging strategy performs the best in reducing the conditional variance of the hedged portfolio.

This is consistent with most of the previous studies undertaken by Myers (1991), Baillie and Myers (1991) and Park and Switzer (1995a, 1995b) on US commodities and financial markets. Another striking feature of the in-sample results is that the longer the hedge horizon, the greater the extent to which the GARCH hedge ratios reduce the risk of the hedged portfolio relative to other alternatives.

Table 12. Hedging Performances Comparison

**In Sample Comparison:**

<b>Forecast Horizons</b>	<b>Mean of the Hedge Ratio</b>	<b>Mean of the Return of the Hedged Portfolio</b>	<b>Variance of the Return of the Hedged Portfolio</b>	<b>Percentage in variance Reduction</b>
<b><u>One – Day</u></b>				
<b>OLS</b>	0.4778	0.069%	0.00009%	88.58%
<b>VAR</b>	0.5083	0.067%	0.00009%	89.01%
<b>Error-Corre.</b>	0.5147	0.067%	0.00009%	89.12%
<b>M-GARCH</b>	0.5922	0.071%	0.00010%	87.95%
<b><u>Five – Day</u></b>				
<b>OLS</b>	0.4778	0.127%	0.00026%	67.99%
<b>VAR</b>	0.5083	0.126%	0.00026%	68.97%
<b>Error-Corre.</b>	0.5147	0.125%	0.00025%	69.22%
<b>M-GARCH</b>	0.5922	0.123%	<b>0.00024%</b>	71.08%
<b><u>Ten – Day</u></b>				
<b>OLS</b>	0.4778	0.145%	0.00029%	64.63%
<b>VAR</b>	0.5083	0.138%	0.00027%	67.92%
<b>Error-Corre.</b>	0.5147	0.136%	0.00026%	68.76%
<b>M-GARCH</b>	0.5922	0.090%	<b>0.00014%</b>	83.56%
<b><u>Twenty – Day</u></b>				
<b>OLS</b>	0.4778	0.136%	0.00028%	66.20%
<b>VAR</b>	0.5083	0.130%	0.00026%	68.61%
<b>Error-Corre.</b>	0.5147	0.129%	0.00025%	69.22%
<b>M-GARCH</b>	0.5922	0.108%	<b>0.00017%</b>	79.73%

Table 12. Continued

**Post-Sample Comparisons:**

<b>Forecast Horizon</b>	<b>Mean of the Hedge Ratio</b>	<b>Mean of the Return of the Hedged Portfolio</b>	<b>Variance of the Return of the Hedged Portfolio</b>	<b>Percentage in variance reduction</b>
<b><u>One – Day</u></b>				
OLS	0.4778	0.031%	0.00002%	99.04%
VAR	0.5083	0.032%	0.00002%	99.03%
Error-Corre.	0.5147	0.032%	0.00002%	99.03%
M-GARCH	0.5922	0.050%	0.00005%	97.60%
<b><u>Five – Day</u></b>				
OLS	0.4778	0.133%	0.00037%	82.11%
VAR	0.5083	0.132%	0.00035%	82.99%
Error-Corre.	0.5147	0.132%	0.00035%	83.21%
M-GARCH	0.5922	0.136%	<b>0.00028%</b>	86.57%
<b><u>Ten – Day</u></b>				
OLS	0.4778	0.130%	0.00031%	84.88%
VAR	0.5083	0.129%	0.00030%	85.47%
Error-Corre.	0.5147	0.129%	0.00030%	85.62%
M-GRCH	0.5922	0.123%	<b>0.00024%</b>	88.23%
<b><u>Twenty – Day</u></b>				
OLS	0.4778	0.189%	0.00079%	61.48%
VAR	0.5083	0.182%	0.00074%	64.16%
Error-Corre.	0.5147	0.181%	0.00072%	64.85%
M-GARCH	0.5922	0.132%	<b>0.00035%</b>	83.19%

Notes: The return on the hedge portfolio is calculated using Equation 5.6 for each hedge horizon. The percentage of variance reduction is calculated by substituting the  $\text{Var}(H)$  and  $\text{Var}(U)$  in Equation 5.7, 5.8 to Equation 5.9.

However, if the return factor is taken into account, the M-GARCH hedging strategy does not seem too outperform the other alternatives. Although a number of previous studies of hedging effectiveness of hedging using M-GARCH optimal hedge ratios has found



either marginal or significant superiority to other alternative hedge ratios, it is based on the presumption that the hedging performance is measured in terms of the reduction in variance only. This study measures the hedging performance under a risk-return trade-off basis. It is found that the GARCH model is no longer the best choice. The results reflect a two-parameter approach in the theory of finance that was developed by Markowitz (1952): the higher the risk, the higher the return. The investor's degree of risk aversion, in this case, plays an important role in selecting the hedging method. For instance, a return oriented investor is likely to select the regression hedge ratio to form their hedged portfolio.

The post-sample comparison tells a similar story. A noticeable fact is that for one-day and five-day hedging, the dynamic hedge-ratios from the GARCH model yield both highest return and variance reduction. But as the hedging horizon increases, the return produced from this method recedes to be the poorest. It can be noted that in a twenty-day hedging strategy, the constant hedge ratios reduce the conditional variance by 64%, whereas the GARCH method reduces the variance by as much as 83%. This significant improvement seems to deserve the investor to consider a sacrifice of a part of his potential return. The GARCH out-performs the others in longer term hedging strategies. It is also worth noticing that the hedge ratios of all models are markedly helpful in reducing the total risk in the un-hedged portfolio ex-ante, since in a 20-day out-of-sample forecast, hedge ratios derived from all of the models can still reduce the total variance by more than 50%.

In sum, it appears that Australian financial market is a regularized market that facilitates the activities of investors of different purposes and degrees of risk aversion. Hedging and non-hedging make dramatic differences in terms of reducing potential risks. Futures contracts have proved to be a useful financial instrument to meet the hedging necessities in this market.

### **5.10 Conclusions**

A cointegrating relationship was found to exist between the two series. Therefore an Error-Correction term (ECT) is incorporated in the VAR representation to account for this long-run equilibrium relationship. The results indicate that a bivariate VAR (4) model is adequate to eliminate the autocorrelation in the series. In view of the strong ARCH effects present in the residuals, the multivariate VEC- GARCH model of Bollerslev, Engle and Wooldridge's (1988) was estimated to cater for the heteroscedastic error terms. Four types of hedge ratios in total are calculated. Of the three constant hedge ratios, the one derived from the error-correction model is the greatest in size. This finding conforms with Ghosh (1993) and Lien's (1996) demonstration that the hedge ratios derived from the other models which fail to recognize the cointegrating relationship between spot and futures prices would be biased downward. The time varying hedge ratios calculated from the M-GARCH model show a high degree of non-stationarity, though it seems to have moved upwards in the late 1980's due to the impact of the October 1987 crash on Australian cash and futures markets.

To compare the hedging effectiveness of these hedge ratios, the un-hedged and hedged portfolios are used for the in-sample and out-of-sample forecasting into various horizons. Regardless of potential hedged portfolio returns, the GARCH time varying hedge ratios provide the best results in variance reduction relative to un-hedged portfolio variance. This finding is in line with most of the recent studies on the usefulness of time varying hedge ratios. At the same time, this paper takes one more step to investigate the probable return on the hedged portfolio, for a rational investor would be likely to choose the optimal hedge ratio that maximize their expected utility, rather than minimize risk only<sup>7</sup>. When measuring hedging performance under a risk-return context, it is shown that the M-GARCH time varying hedge ratios are no longer the best. The greater risk reduction is obtained at the price of generating poorer expected return on the hedged portfolio. Therefore, the investor's degree of risk aversion plays an important role in deciding which hedging strategy is to be undertaken in this circumstance. The results also indicate that for longer term hedging, time varying hedge ratios reduce the total variance by an even greater percentage than constant hedge ratios.

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<sup>7</sup> Cecchetti, Cumby and Figlewski (1988) and Lien and Tse (1998) used the expected utility to measure the performance of time varying hedge ratios.

# Chapter 6

## General Conclusion

### **6.1 Conclusions**

The futures hedge ratios have been calculated in this study using various econometric time series models, and the performance of these hedge ratios have been compared in terms of a risk-return trade-offs in the ex-post and ex-ante forecasting horizons. Of the three constant hedge ratios derived from the regression model, the VAR model and the error-correction model, the error-correction model generates the hedge ratios that display the largest value in size. This finding agrees with Ghosh (1993) and Lien's (1996) demonstration that non-inclusion of a cointegration relationship leads to a hedge ratio that is biased downwards in size. The time varying hedge ratios calculated from conditional information set exhibit high degree of non-stationarity through time, though the excess volatility in the late 1980s may be due to the impact of October 1987 crash.

It is indicated that all of the hedge ratios calculated in this application are well below unity. This finding further rejects the traditional 1:1 hedging theory. It also implies that for a given amount of cash assets held, the amount of futures contracts needed to hedge against the spot exposure is less than that already held in the cash market. The hedge ratios in this context show that approximately half of the cash assets need to be hedged. This conclusion is consistent with the empirical work of Ederington (1979:p169) who reported that:

*“Contrary to traditional hedging, our empirical results indicate that even pure risk-minimizers may wish to hedge only a portion of their portfolios. In most cases the estimated  $b^*$  was less than one.”*

In the performance of these hedge ratios, the in-sample and out-of-sample forecasts tell the similar story. Firstly, the M-GARCH dynamic hedge ratios provide the greatest degree of variance reduction in most of the forecasting horizons, but also generate the smallest rate of return. On the other hand, the hedge ratio calculated from the conventional regression model performs the worst in terms reducing portfolio variance, but yields the highest rate of return. This finding implies that in selecting the most appropriate hedge ratio, the investor's degree of risk aversion plays an important role. In Park and Switzer (1995b), the investor's degree of risk aversion is actually included in the utility function to jointly compare the performance of different hedging methods<sup>1</sup>. The finding in this paper agrees with their methodology. It suggests that return and variance are not the only variables that determine the appropriate hedge ratio, the investor's preference for risk is also an essential factor. Secondly, it's found that in longer term hedging, the time varying hedge ratios out-perform the constant hedge ratios in terms of reducing portfolio variance. Thirdly, the hedge ratios of all models are markedly helpful in reducing the total risk ex-ante, since it is shown that still more than 50% of the total variance of the un-hedged portfolio can be reduced in a 20-day out-of-sample hedging horizon.

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<sup>1</sup> See Chapter 3 for discussion of this application.

## **6.2 Limitations and Further Research**

Due to the crash in October 1987, the results and the estimates of time varying hedge ratios are slightly affected, though a sample period was chosen beginning on January 1<sup>st</sup> 1988 in hoping to avoid the influence of crash. According to the results shown in this study, the influence of crash can be further reduced by re-selecting sample period starting from 1990, since the price movements as well as the hedge ratios had largely come back to normal after 1990. This research has drawn attention to the time varying hedge ratios' great power to hedge against spot market exposure even in the out-of-sample forecast. However, in the real world, frequently re-balancing of the position in the futures market would incur an increase in relative transaction costs. Therefore, given a certain degree of risk aversion, further research on the investor's trade-off between the cost of re-balancing the existing position and the potential utility of the newly-formed portfolio will be worth exploring.

## Bibliography

- Adler, M., & Detemple, J. (1988). Hedging Futures in an Intertemporal Portfolio Context. *The Journal of Futures Markets*. 8(3), 249-270
- Allen, D., Macdonald, G. A., & Walsh, K. (1999). Using Regression Techniques to Estimate futures Hedge Ratios, Some Results from Alternative Approaches Applied to Australian ten year Treasury Bond Futures. *Working Paper Series, Edith Cowan University, Australia, May 1999*.
- Anderson, R. W., & Danthine, J. P. (1980). Hedging and Joint Production. *The Journal of Finance*. 35(2), 478-498.
- Baillie, R. T., & Myers, R. (1991). Bivariate GARCH Estimation of The Optimal Commodity Futures Hedge , *Journal of Applied Econometrics*. 6, 109-124.
- Ball, R., & Bowers, J. (1988). Daily Seasonals in Equity and Fixed Interest Returns: Australian Evidence and Tests of Plausible Hypothesis. In *dimson, E., Stock Market Anomalies*, Cambridge University Press, Cambridge.
- Bell, D. E., & Krasker, W. S. (1986). Estimating Hedge Ratios, *Financial Management*. Summer, 34-39
- Beninga, S., Eldor, R., & Zilcha, I. (1984). The Optimal Hedge Ratio in Unbiased Futures Markets. *The Journal of Futures Markets*. 1, 155-159.
- Bera, a. K., & Higgins, M. L. (1993). ARCH Models: Properties, Estimation and Testing, *Journal of Economic Surveys*. April, 305-365.
- Bera, A. K., & Roh, J. S. (1991). A Moment Test of the Consistency of the Correlation in the Bivariate GARCH Model. *Mimeo, Department of Economics, University of Illinois at Urbana-Champaign*.
- Bera, A., Bubnys, E., & Park, H. Y. (1993). ARCH Effects and Efficient Estimation of Hedge Ratios for Stock Index Futures. *Advances in Futures And Options Research*, 6, 313-328.
- Bera, A., Philip, G., & Roh, J. S. (1997). Estimation of Time-Varying Hedge Ratios for Corn and Soybeans: BGARCH and Random Coefficient Approaches , *OFOR Paper*, No. 97-6.
- Bierens, H., (1990). Testing Stationarity Against the Unit Root Hypothesis. *Manuscript, University of Amsterdam*.

- Black, F. (1976). Studies in Stock Price Volatility Changes. Proceedings of the 1976 Business Meeting of the Business and Economic Statistics Section. *American Statistical Association*. 177-181.
- Bollerslev, T. (1986). A Generalized Autoregressive Conditional Heteroscedasticity , *Journal of Econometrics*. 31, 307-327.
- Bollerslev, T., Chou, R. Y., & Kroner, K. F. (1992). ARCH Modeling in Finance. *Journal of Econometrics*. 52, 5-59.
- Bollerslev, T., Engle, R. F., & Wooldridge, J. M. (1988). A Capital Asset Pricing Model with Time-Varying Covariances. *Econometrica*. 96, 116-131.
- Bollerslev, T., R. F. Engle, & Nelson, D. B. (1994). ARCH models , Northwestern University, *Working Paper*, prepared for the Handbook of Econometrics 4.
- Bollerslev, T. (1990). Modeling the coherence in Short-Run Nominal Exchange Rates: A Multivariate generalized ARCH Model. *The Review of Economics and Statistics*. 52, 5-59
- Bolleslev, T. (1987). A Conditionally Heteroscedastic time Series Model of Security Prices and Rates of Return. *Review of Economics and Statistics*. 59, 542-547.
- Bond, G. E., Thompson, S. R., & Lee, B. M. S. (1987). Application of a Simplified Hedging Rule. *Journal of Futures Markets*. 7, 65-72.
- Briys, E., Crouhy, M., & Schlesinger, H. (1990). Optimal Hedging under Intertemporally-Dependent Preferences. *Journal of Finance*. 45, 1315-1324.
- Brown, S. L. (1985), A Reofrmulation of the Portfolio Model of Hedging , *American Journal of Agricultural Economics*, 67, 508-512.
- Brown, S. L. (1986). A Reofrmulation of the Portfolio Model of Hedging: Reply , *American Journal of Agricultural Economics*. 68, 1010-1012.
- Carter, C. A., & Loyns, R. M. A. (1985). Hedging Feedlot Cattle: A Canadian Perspective. *American Journal of Agricultural Economics*. 67, 32-39.
- Castelino, M. G. (1992). Hedge Effectiveness: Basis Risk and Minimum-Variance Hedging. *The Journal of Futures Markets*. 12, 187-201.
- Castelino, M. G., (1990a). Futures Markets and Hedging: The Time Dimension. *The Journal of Portfolio Management*, Spring, 74-80.
- Castelino, M. G. (1990b). Minimum Variance Hedging with Futures Revisited. *Journal of Quantitative Economics*. 6, 271-287.



- Castelino, M. G. (1992). Hedge Effectiveness: Basis Risk and Minimum-Variance Hedging. *The Journal of Futures Markets*. 2(2), 187-201.
- Cecchetti, S. G., R. E. Cumby, & Figlewski, S. (1988). Estimation of the Optimal Futures Hedge. *The Review of Economic Statistics*. LXX(4), 623-630.
- Chang, J. S. K., & Fang, H. (1990). An Intertemporal Measure of Hedging Effectiveness. *Journal of Futures Markets*. 10(3), 307-321.
- Chang, J. S. K., & Shanker, L. (1986). Hedging Effectiveness of Currency Options and Currency Futures. *The Journal of Futures Markets*. 6 (2), 289-305.
- Chen, K., Sears, R., & Tzang, D. (1987). Oil Prices and Energy Futures. *The Journal of Futures Markets*. 7, 501-518.
- Chowdhury, A. (1991) Futures Market Efficiency: Evidence from Cointegration Tests. *The Journal of Futures Markets*. 11, 557-589.
- Christie, A. A. (1982). The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects. *Journal of Financial Economics*. 10, 407-432.
- Cochrane, F. H. (1997). Time Series for Macroeconomics and Finance, *Teaching Material*, University of Chicago.
- Crowder, W. J., & Hamed, A. (1993). A Cointegration Test for Oil Futures Market Efficiency. *The Journal of Futures Markets*. 13(8), 941-943.
- DeJong, D. N., & Whiteman, H. (1991). Redonsidering Trends and Random Walks in Macroeconomic Time Series. *Journal of Monetary Economics*. 28(2).
- Dickey, D. A., & Fuller, W. A. (1979). Distribution of the Estimators for Autoregressive time Series with a Unit Root. *Journal of the American Statistical Association*. 74, 427-431.
- Dickey, D. A., & Fuller, W. A. (1981). Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root. 49(4), 1057-1072.
- Dimson, E. 1988. *Stock Market Anomalies*. Cambridge University Press.
- Doan, T.A. (1996). *Rats Manual*. Estima, Evanston.
- Drost, F., & Nijman, T. (1993). Temporal Aggregation of GARCH Processes. 61, 909-927.
- Duffie, D., (1989), *Futures Markets*, Prentice Hall International Inc., *Econometrica*. Englewood Cliffs.

- Ederington, L. H. (1979) The Hedging Performance of the New Futures Markets. *The Journal of Finance*, XXXIV(1), 157-170.
- Enders, W. (1995). *Applied Econometric time Series*, John Wiley & Sons, Inc.
- Enders, W. (1995) *Rats: Handbook for Econometric Time Series*. John Wiley & Sons, Inc.
- Engle, R., & Kroner, K. F. (1993). Multivariate Simultaneous Generalized ARCH, *Discussion Paper*, 89-57R, University of California, San Diego.
- Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of Variance of U. K. Inflation. *Econometrica*. 50, 987-1008.
- Engle, R. F. (1983). Estimates of the Variance of U. S. Inflation Based upon ARCH Models. *Journal of Business and Economic Statistics*. 9, 345-359.
- Engle, R. F. (1987). Multivariate ARCH with Factor Structures – Cointegration in Variances. *Econometric Reviews*. 5, 1-87.
- Engle, R. F., & Bollerslev, T. (1986). Modeling the Persistence of Conditional Variances. *Econometric Reviews*. 5, 1-87.
- Engle, R. F., & Granger, C. W. J. (1987). Cointegration and Error Correction: Representation, Estimation and Testing. *Econometrica*. 5(2), 251-276.
- Engle, R. F., and C. W. J. Granger, (1991), *Long-run Economic Relationships: Readings in Cointegration*, Oxford University Press
- Evans, G., & Savin, N. E. (1981). Testing for Unit Roots: I. *Econometrica*. 49, 753-779.
- Evans, G., & Savin, N. E. (1981). Testing for Unit Roots: II. *Econometrica*. 52, 1241-1269.
- Faff, R. W. (1992). Capital Market Anomalies: A Survey of the Evidence. *Accounting Research Journal*. 5, 3-22.
- Fama, E. F. (1965). The Behavior of Stock Market Prices. *Journal of Business*. 38, 34-105.
- Fama, E. F., & French, K. R. (1987). Commodity Futures Prices: Some Evidence on forecast Power, Premiums, and the Theory of Storage. *Journal of Business*. 60, 55-73.
- Ferguson, R., & Leistikow, D. (1999). Futures Hedge Profit Measurement, Error-Correction Model vs. Regression Approach Hedge Ratios, and Data Error Effects. *Financial Management*. 28, 118-125.

- French, K. (1980). Stock Returns and the Weekend Effect. *Journal of Financial Economics*. 8, 55-70.
- Fung, H. G., & Lo, W. C. (1993). Memory in Interest Rate Futures. *The Journal of Futures Markets*. 13(8), 865-872.
- Fung, H. G., & Leung, W. K. (1993). The Pricing Relationship of Eurodollar Futures and Eurodollar Deposit Rates. *The Journal of Futures Markets*. 13(2), 115-126.
- Ghosh, A., & Clayton, R. (1996). Hedging with International Stock Index Futures: An Intertemporal Error Correction Model. *The Journal of Financial Research*. XIX(4), 477-491.
- Ghosh, A. (1993a). Hedging with Stock Index Futures: Estimation and Forecasting with Error Correction Model. *The Journal of Futures Markets*. 13(7), 743-752.
- Ghosh, A. (1993b). Cointegration and Error Correction Models: Intertemporal Causality Between Index and Futures Prices. *The Journal of Futures Markets*. 13(2), 193-198.
- Gjerde, M. (1987). Measuring Hedging Effectiveness in a Traditional One-Periodic Portfolio Framework. *The Journal of Futures Markets*. 7(1) 665-674.
- Granger, C. W. J. (1981). Some Properties of Time Series Data and Their Use in Econometric Model Specification. *Journal of Econometrics*. 16(1), 121-130.
- Grant, D., & Eaker, M. (1989). Complex Hedges: How Well do They Work?. *The Journal of Futures Markets*. 9, 15-27.
- Hamilton, J. D. (1994). *Time Series Analysis*, Princeton University Press, New Jersey.
- Hargreaves, C. P. (1994). *Non-stationary Time Series Analysis and Cointegration*. Oxford University Press
- Hartzmark, D. (1988). Is Risk Aversion a Theoretical diversion. *Review of Futures Markets*. 7(1), 1-25.
- Harvey, A., Ruiz, E., & Sentana, E. (1992). Unobserved Component Time Series Models with ARCH Disturbances. *Journal of Econometrics*. 52, 129-157.
- Heaney, R. H. (1994). Futures Contract Pricing and Hedging: A Review of Theory and Evidence. *Accounting Research Journal*. 48-65.
- Heaney, R. H. (1998). A Test of the cost-of-carry relationship Using the London Metal Exchange Lead Contract. *Journal of Futures Markets*. 18, 177-200
- Heifner, R. G. (1972). Optimal Hedging Levels and Hedging Effectiveness in Cattle Feeding. *Agricultural Economic Reviews*. 24, 25-36.

- Herbst, A. F., Kare, D. D., & Caples, S. C. (1989). Hedging Effectiveness and Minimum Risk Hedge Ratios in the Presence of Autocorrelation: Foreign Currency Futures. *The Journal of Futures Markets*. 9(3), 185-197.
- Herbst, A. F., Kare, D. D., & Marshall, J. F. (1993). A Time Varying, Convergence Adjusted, Minimum Risk Futures Hedge Ratio. *Advances in Futures and Options Research*. 6, 137-155.
- Higgins, M. L., & Bera, A. K. (1992). A Class of Non-linear ARCH Models, *International Economic Review*. 33, 137-158.
- Hill, J., & Schneeweis, T. (1982). The Hedging Effectiveness of Foreign Currency Futures. *The Journal of Financial Research*. 5(1), 95-104 (1982).
- Hilliard, J. E. (1984). Hedging Interest Rate Risk with Futures Portfolios under Term Structure Effects. *The Journal of Finance*. 39(5), 1547-1570.
- Hodgson, A., A. M. Masih, & Masih, R. (1998). Price Discovery Between Informationally Linked Markets During Different Trading Phases: Evidence From Australia , *Working Paper Series*, Edith Cowan University, Australia.
- Howard, C. T., & D'Antonio, L. J. (1984). A Risk-Return Measure of Hedging Effectiveness. *Journal of Financial and Quantitative Analysis*. 19(1), 101-112.
- Howard, C. T., & D'Antonio, L. J. (1987). A Risk-Return Measure of Hedging Effectiveness: A Reply. *Journal of Financial and Quantitative Analysis*. 22(3), 337-381.
- Howard, C. T., & D'Antonio, L. J. (1994). The Cost of Hedging and The Optimal Hedge Ratio. *The Journal of Futures Markets*. 14(2), 237-258.
- Johansen, S., & Juselius, K. (1990). Maximum Likelihood Estimation and Inference on Cointegration with application to the Demand for Money. *Journal of Econometrics*, 53, 211-244.
- Johansen, S. (1988) Statistical Analysis of Cointegration Vectors. *Journal of Economic Dynamics and Control*. 12, 231-254.
- Johnson, J. (1996). Hedging with the SPI: How Elusive is the Perfect Hedge Ratio? *Journal of Applied Finance and Investment*. 1, 36-47.
- Johnson, L. L. (1960). The Theory of Hedging and Speculation in Commodity Futures. *Review of Economic Studies*. 27, 139-151.
- Kahl, K. H. (1983). Determination of the Recommended Hedging Ratio. *American Journal of Agricultural Economics*. 65, 603-605.
- Kahl, K. H. (1986). A Reformation of the Portfolio Model of Hedging: Comment. *American Journal of Agricultural Economics*. 68, 1007-1009.

- Kahn, J. H., & Ogaki, V. (1990). A Chi-square Test for a Unit Root. *Economic Letters*. 34, 347-42.
- Kaminsky, G., & Peruga, R. (1990). Can a Time-Varying risk Premium Explain Excess Returns in the Forward Market for Foreign Exchange? *Journal of International Economics*. 28, 47-70.
- Klien, B. (1977). The Demand for Quality-Adjusted Cash Balances: Price Uncertainty in the U. S. Demand for Money Function. *Journal of Political Economy*. 85, 693-715.
- Kraft, D. F., & Engle, R. F. (1983). Autoregressive Conditional Heteroscedasticity in Multiple Time Series, *Unpublished Manuscript*, department of Economics, University of California, San Diego, U.S.
- Krehbiel, T., & Adkins, L. C. (1993). Cointegration Tests of the Unbiased Expectations Hypothesis in Metal Markets. *The Journal of Futures Markets*. 13, 753-763.
- Kwiatkowski, D; P. C. B. Phillips; P. Schmidt, & Shin, Y. (1992). Testing the Alternative of Stationary against the Alternative of a Unit Root: How Sure Are We That Economic Time Series Have a Unit Root? *Journal of Econometrics*. 54, 159-178.
- Lai, K., & Lai, M (1991). A Cointegration Test for Market Efficiency. *The Journal of Futures Markets*. 11, 567-576
- Lange, S. (1999). Modeling Asset Market Volatility in a Small Market: Accounting for Non-synchronous Trading Effects. *Journal of International financial Markets*. 9, 1-18.
- Lien, D., & Tse, T. K. (1999 ). Fractional Cointegration and Futures Hedging. *The Journal of Futures Hedging*. 19(4), 457-474.
- Lien, D., & Luo, X. (1993). Estimating Multi-period Hedge Ratios in Cointegrated Markets. *The Journal of Futures Markets*. 13(8), 909-920.
- Lien, D. D. (1996). The Effect of the Cointegrating Relationship on Futures Hedging: A Note. *The Journal of Futures Markets*. 16 (7), 773-780.
- Lien, D. H. D., & Luo, X. (1994). Multi-period Hedging in the Presence of Conditional Heteroscedasticity. *The Journal of Futures Markets*, 14(8), 927-955.
- Lindahl, M. (1991). Risk-Return Hedging Effectiveness Measures for Stock Index Futures. *The Journal of Futures Markets*. 11(4), 399-409.
- Lindahl, M. (1992). Minimum Variance Hedge Ratios for Stock Index Futures: Duration and Expiration Effects. *The Journal of Futures Markets*. 12(1), 33-53.

- Lo, A., & MacKinlay, A. C. (1988). Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test. *Review of Financial Studies*. 1, 41-66.
- Maddala, G. S. (1992). *Introduction to Econometrics*. 2<sup>nd</sup> edition. Macmillan Publishing Company, New York.
- Malliaris, A. G., & Urrutia, J. (1991). Tests of Random Walk of Hedge Ratios and Measures of hedging Effectiveness for Stock Indexes and Foreign Currencies. *The Journal of Futures Markets*. 11(1), 55-68.
- Mandelbrot, B. (1963a) The variation of Certain Speculative Prices. *Journal of Business*. 36, 394-419.
- Mandelbrot, B. (1963b). New Methods in Statistical Economics. *Journal of Political Economy*. 71, 421-440.
- Mandelbrot, B. (1967). The Variation of Some Other Speculative Prices. *Journal of Business*,. 40, 393-413.
- Mansur, I., & Elyasiani, E. (1998). Sensitivity of the Bank Stock Returns Distribution to Changes in the Level and Volatility of Interest Rate: A GARCH-M Model. *Journal of Banking & Finance*. 22, 535-563.
- Markwitz, H. M. (1952). Portfolio Selection. *The Journal of Finance*. 1(1), 77-91.
- Marmer, H. (1986). Portfolio Model Hedging with Canadian dollar Futures: A Framework for analysis. *The Journal of Futures Markets*. 6, 83-92.
- McAleer, M., & Oxley, L. (1999). *Practical Issues in Cointegration Analysis*. Blackwell.
- McKenzie, M. D., & Brooks, R. D. (1999). *Research Design Issues in Time-Series Modeling of Financial Market Volatility*. The McGraw-Hill Companies, Inc.
- McLeod, A. I., & Li, W. K. (1983). Diagnostic Checking of ARMA Times Series Models Using Squared Residual Autocorrelations. *Journal of Time Series Analysis*. 4, 269-273
- McNew, K. P., & Fackler, P. L. (1994). Nonconstant Optimal Hedge Ratio Estimation and Nested Hypotheses Tests. *The Journal of Futures Markets*. 14(5), 619-635.
- Myers, R. J. (1991). Estimating time-varying optimal hedge ratios on futures markets. *The Journal of Futures Markets*. 11, 139-153.
- Myers, R. J., & Thompson, S. R. (Nov, 1989). Generalized Optimal Hedge Ratio Estimation. *American Journal of Agricultural Economics*. 858-867.

- Nelson, C. R., & Plosser, C. I. (1982). Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications. *Journal of Monetary Economics*, 10, 139-162.
- Nelson, D. B. (1990b). ARCH Models as Diffusion Approximations. *Journal of Econometrics*. 45, 7-38.
- Nelson, D. B., & Cao, C. Q. (1992). Inequality Constraints in the Univariate GARCH Model. *Journal of Business & Economic Statistics*. 10, 229-235.
- Nelson, D. (1991). Conditional Heteroscedasticity in Asset Returns: a New Approach. *Econometrica*. 59, 347-370.
- Pagan, A. (1996). The Econometrics of Financial Markets. *Journal of Empirical finance*. 3, 15-102.
- Pagan, R. R., & Schwert, G. W. (1990) Alternative Models for Conditional Stock Volatility. *Journal of Econometrics*. 45, 267-290.
- Park, H. Y., & Bera, A. K. (1987). Interest Rate Volatility, Basis, and Heteroscedasticity in Hedging Mortgages. *The American Real Estate and Urban Economics Association*. 15, 79-97.
- Park, J. Y., & Choi, B. (1988). A New Approach to Testing for a Unit Root. *CAE Working Paper*. 88-123. Cornell University.
- Park, T. H., & Switzer, L. N. (1995). Bivariate GARCH Estimation of The Optimal Hedge Ratios For Stock Index Futures: A Note. *Journal of Futures Markets*. 15, 61-67.
- Park, T. H., & Switzer, L. N. (1995b). Time-Varying Distribution and the Optimal Hedge Ratios for Stock Index Futures. *Applied Financial Economics*. 5, 131-137.
- Pasaron, A. H. (1994). *Microfit Manual*, Cambridge University Press, London.
- Pennings, J. M. E., & Meulenberg, M. T. G. (1995). Hedging Efficiency: A Futures Exchange Management Approach. *The Journal of Futures Markets*. 17(5), 599-615.
- Phillips, P. C. B. (1987). Time Series Regression with a Unit Root. *Econometrica*. 55, 277-302.
- Phillips, P. C. B., & Perron, P. (1988). Testing for a Unit Root in Time Series Regression. *Biometrika*. 75, 335-346.
- Pindyck, R. S., & Rubinfeld, D. L. (1991). *Econometric Models and Economic Forecasts*. 3<sup>rd</sup> ed. McGraw Hill Inc, New York.

- Racine, M. D., & Ackert, L. F. (1998). Time-Varying Volatility in Canadian and U.S. Stock Index and Index Futures Markets: A Multivariate Analysis. *Working Paper 98-14*. Federal Reserve Bank of Atlanta.
- Said, S. E., & Dickey, D. A. (1980). Testing for Unit Roots in ARMA Models of Unknown Order. *Biometrika*. 71, 599-607.
- Schwert, G. (1987). Effects of Model Specification on Tests for Unit roots in Macroeconomic Data. *Journal of Monetary Economics*. 20, 73-103.
- Schwert, G. W. (1989b). Business Cycles, Financial Crises, and Stock Volatility. *Carnegie-Rochester Conference Series on Public Policy*. 39, 83-126.
- Sentana, E. (1991). Quadratic ARCH models: A Potential Re-interpretation of ARCH models , *Mimeo*, Department of Economics and Financial Markets Group, London School of Economics.
- Sephton, P. S. (1993). Optimal Hedge Ratios at the Winnipeg Commodity Exchange. *Canadian Journal of Economics*. XXVI(1), 175-193.
- Shafer, C. E. (1993). Hedge Ratios and Basis Behavior: An Intuitive Insight? *The Journal of Futures Markets*. 13(8), 837-847.
- Stein, J. L. (1961). The Simultaneous Determination of Spot and Futures Prices. *American Economic Review*. 51, 1012-25.
- Stock, J., & Watson, M. (1988). Testing for common Trends. *Journal of American Statistical Association*. 83, 1097-1107.
- Stoll, H. R., & Whaley, R. E. (1990). The Dynamics of Stock Index and Stock Index Futures Returns. *Journal of Financial and Quantitative Analysis*. December, 441-468.
- Tong, W. (1995). An Examination of Dynamic Hedging. *Journal of International Money and Finance*. 15(1), 19-35.
- Viswanath, P. V. (1993). Efficient Use of Information, Convergence Adjustment, and Regression Estimates of Hedge Ratios. *The Journal of Futures Markets*. 12(1), 3-53.
- Wahab, M., & Lashgari, M. (1993). Price Dynamics and Error Correcting Stock Index and Stock Index Futures Markets: A Cointegration Approach. *The Journal of Futures Markets*. 13, 711-742.
- Witt, H. J., Schnessweis, T. C., & Hayenga, M. L (1987). Comparison of Analytical Approaches for Estimating Hedge Ratios for Agricultural Commodities. *Journal of Futures Markets*. 7, 135-146.
- Zakoian, J-M. (1990). Threshold Heteroscedastic Model. *Mimeo*. INSEE, Paris.