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Joint Optimization in Capacity Design of Networks with p-Cycle Using the Fundamental Cycle Set

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***Abstract*— We propose a joint optimization model for capacity design of networks with p-cycles. The model is based on a modified definition of network fundamental cycles and the available straddling links. Concepts about visible and hidden straddling links, which are essential components of our model are also introduced. This is the first ILP model for joint optimization of p-cycle network that can be solved without enumerating p-cycle candidates, and has the ability to achieve optimum solutions. In addition, the complexity of our proposed model is much smaller than any conventional models, particularly when applying to a planar network. This model is suitable large size networks and for shared risk link group networks or backbone networks protected by p-cycle schemes.**

I. INTRODUCTION AND MOTIVATION

Protection of communication networks against failures is an essential step in network design and development. This requires not only high efficiency in resource allocation, but also fast recovery time after the failure of network components, particularly in optical WDM (Wavelength Division Multiplexing) networks with data rates up to 10 Tb/second. Restoration time is a significant limitation of the mesh networks compared to the ring networks. The discovery of the p-cycle [1] has opened a new approach in the protection of mesh networks, and allows the networks to achieve both, ring-like fast recovery speed and mesh-like resource efficiency. However, the design of the p-cycle protection faces a complexity problem when the optimization models are formulated in either link form as in [2] or cycles enumeration as in [3], [4], where the number of variables is increasing exponentially with the size of the network. In both of the above cases, a heuristic algorithm approach is usually the appropriate choice to solve the problem. Obtaining the optimal solution is particularly important in studying network topology designs, where the precision of data is essential for analytical purposes. In addition, when combining and formulating multiple quality of protection service classes (MQoS) in one network, this usually involves a very large number of constraints and variables, and the implementation is a very complicated process. Thus much time and efforts can be saved if the complexity of the model is significantly reduced. These problems lead to the development of new approach for formulating a joint capacity allocation of working path and p-cycles placement.

II. BACKGROUND

A. Related works

Since the introduction of p-cycles by D. W. Grover [1], many articles have been published concerning various aspects of p-cycles from finding the candidates, designing survivable network with shared p-cycle, placement of wavelength converters, p-cycles in multi-failure scenarios, and particularly, a large percentage of them searching for new ways to obtain the minimum set of p-cycle candidates but still can provide the highest resource efficiency. The first IP (Integer Programming) model for placement of p-cycles was introduced by W. D. Grover *et al.* [1]. The objective of this model was to determine the set of p-cycles from the given p-cycle candidates which minimizes the total cost of spare capacity subject to a constraint of 100% restorability. Because the number of candidate cycles grows exponentially with network size. This problem has led to the development of many algorithms targeting the selection of suitable p-cycle candidates as in [4], where a cycle generation algorithm that can find good candidate cycles for use by IP or the work in [5] involves the identification of primary p-cycles using straddling link algorithm (SLA), followed by a search for better cycles using a number of search algorithms to produce the final set of candidates with highest efficiency. The maximum deviation compared to pure ILP model can be up to 14% and varies with network topology. The complexity of this model is greatly reduced compared to the pure model, e.g. 270/7321 for USA network. Dominic A. Schupke [2] introduces a different approach to formulate a non-joint optimization without enumeration of candidates before optimization. However, the proposed model is very complex, and the author suggests a four-step heuristic models which makes the calculation tractable and achieves near-optimal solution. To our knowledge, up to now, there is no published p-cycle formulation that can be solved with ILP other than the models proposed in [1].

B. Preliminary theory

A network physical topology is usually represented by an undirected graph $G(V, E)$, where V is a set of network nodes and E is a set of network spans.

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$V = \{v_1, v_2, \ldots, v_N\}$, where N is the number of network nodes. $E = \{e_1, e_2, \ldots, e_M\}$, where M is the number of network spans.

Please note that in this paper, the term 'span'implies the direct physical connection between two end nodes and 'link' is referred as the direct logical connection between two end nodes.

Definition 1: A cycle of a given undirected graph is called a fundamental cycle if it contains no straddling link, and any sub-graph $H(V', E') \in G(V, E)$ can be built from the fundamental cycle set of G. Fig. 1 shows an arbitrary network with 9 nodes and 14 spans, and has the following set of fundamental cycles: c_1={1 2 3}, c_2={3 4 8}, c_3={5 6 7}, c_4={5 7 9}, c_5={2 3 8 6}, c_6={4 5 6 8}, c_7={2 3 4 5 6} (c_7 contains no straddling link, thus it is valid). Please note that our definition of the fundamental cycle is different from what has been defined in the literature [6], [7], [8], where the fundamental cycles are a set of unique cycles found from the spanning tree(s) T and edges of the graph that are not in T. Clearly, unlike in our model, these cycles may contain straddling links.

Definition 2: A straddling link is called a *visible straddling link* if it can be created by joining two different fundamental cycles, and is the only common link that exists between them. Fig. 1 shows a typical *visible straddling link* e_3, which is the common link between two fundamental cycles c_1, c_5.

Definition 3: A straddling link is called a *hidden straddling link* if it can be created by adding two or more fundamental cycles together, but it is not part of any of those cycles. Note that, the number of fundamental cycles that are used to generate the *hidden straddling links* yields the trade off between resource efficiency, and minimum restoration time in the case of failure. Longer recovery path may result if the hidden straddling link is formed by many fundamental cycles. Fig. 1 shows a typical *hidden straddling link* e_{12}, which is formed by the two fundamental cycles c_4 and c_6.

Definition 4: A non-shareable set Λ contains groups of straddling links. Each group in the set consists of straddling links that have at least one identical fundamental cycle component. However, if joining these fundamental cycles into a subgraph causes the straddling links to disappear, this group is said to be non-shareable. For example, in Fig. 1, straddling link e_6 is formed by cycles c_2 and c_5, straddling link e_8 is formed by cycles c_2 and c_6, and straddling link e_{13} is formed by cycles c_5 and c_6. When the cycle c_2 is shared between straddling links, the relevant straddling links will vanish, i.e. the sharing of cycles will create a new subgraph {2 3 4 5 6 8} with no straddling links. Therefore, this group is non-shareable.

III. The new ILP model

In this section, we assume that all the fundamental cycles and available straddling links of the network under consider have been pre-processed or given. In addition, the network is assumed to have enough wavelength channels or wavelength converters to support the routing of connection demands. Thus wavelength continuity is not an issue in this context.

<u>Notation:</u>

$E = \{e_1, e_2, \ldots, e_M\}$	where M is the number of network spans
$V = \{v_1, v_2, \ldots, v_N\}$	where N is the number of network nodes
$\mathbf{C} = \{c_1, c_2, ..., c_C\}$	Set of network fundamental cycles, and C is the number of fundamental cycles
$\mathbf{S} = \{s_1, s_2, ..., s_S\}$	Set of visible straddling links, and S is the number of visible straddling links
$\mathbf{I} = \{i_1, i_2, ..., i_I\}$	Set of hidden straddling links, and I is the number of hidden straddling links
$\Lambda = \{\lambda_1, \lambda_2, ...\lambda_n\}$	Set of non-sharable straddling links
$\mathbf{D} = \{d_1, d_2, \ldots, d_D\}$	Set of demands, and D is the number of demands
$\mathbf{P} = \{p_1^i, p_2^j, \ldots, p_P^k\}$	Set of path candidates between end nodes of demands; $i, j, k \in \mathbf{D}$

<u>Constants:</u>

$$\delta_{x,j} = \begin{cases} 1, & \text{if cycle } x \text{ includes span } j; \\ & j \in E; x \in \mathbf{C}; \\ 0, & \text{otherwise.} \end{cases}$$

$$\xi_{s,c}^{j} = \begin{cases} 1, & \text{if } c = \{x, y | x \bigcap y = j; x, y \in \mathbf{C}\}; \\ & j \in E; c \subset \mathbf{C}; s \in \mathbf{S}; \\ 0, & \text{otherwise.} \\ & s \text{ is the straddling link formed by} \\ & \text{cycles } x, y \end{cases}$$

$$\pi_{i,\theta}^{j} = \begin{cases} 1, & \text{if } s \bigcap E(\theta) = \emptyset \text{ and} \\ & V(s) \subseteq V(\theta); \\ & j \in E; \theta \subseteq \mathbf{C}; i \in \mathbf{I}; \\ 0, & \text{otherwise.} \\ & i \text{ is the straddling link formed by} \\ & \text{set of cycles } \theta \end{cases}$$

$$\tau_{i,j}^{d} = \begin{cases} 1, & \text{if candidate path } i^{th} \text{ of demand d} \\ & \text{cross span } j,\ d \in \mathbf{D}; \\ 0, & \text{otherwise.} \end{cases}$$

$\upsilon_{s,j} = 2$	the number of useful paths provided by hidden straddling link s to restore span $j \in E; s \in \mathbf{S}$
α_j	the cost per channel on span $j \in E$;
h_d	volume of demand d; $d \in \mathbf{D}$
ϕ_j	maximum capacity provided by span $j \in E$

<u>Variables:</u>

y_j	the capacity on span j that can support the cycle that crossing it; $j \in E$.
n_x	the number of unit capacity copies of the cycle x in the design; $x \in \mathbf{C}$.
$m_{i,x}$	the number of unit capacity copies of the hidden straddling i using cycle x in the design; $i \in \mathbf{I}; x \in \mathbf{C}$

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$u_{i,x}$ the number of unit capacity copies of the visible straddling i using cycle x in the design; $i \in \mathbf{S}; x \in \mathbf{C}$

w_j the working capacity on span j to support the routing of working paths; $j \in E$

p_i^d the number of unit capacity copy of the i^{th} path candidate chosen to serve demand d; $i \in \mathbf{P}; d \in \mathbf{D}$

ILP model

- Objective

$$\text{Minimize}: \quad \sum_{j \in E} \alpha_j (y_j + w_j) - \sum_{i \in S} \sum_{j \in E} 2\xi_{i,c}^j u_{i,x}, \quad x \in c; c \subset \mathbf{C} \tag{1}$$

- Constraints

1) Capacity on each span is sufficient to support all cycles that cross it.

$$\sum_{x \in \mathbf{C}} \delta_{x,j} n_x = y_j, \qquad \forall j \in E \tag{2}$$

2) Number of cycles must be sufficient to support the chosen straddling links.

$$\sum_{i \in \mathbf{S}} \xi_{i,c}^k u_{i,x} - n_x \leq 0, \tag{3}$$

$$\sum_{j \in \mathbf{I}} \pi_{j,c}^k m_{j,x} - n_x \leq 0, \tag{4}$$

$$\forall c \subset \mathbf{C}; x \in c; k \in E$$

3) Working capacity allocated on each span.

$$\sum_{i \in \mathbf{P}} \sum_{d \in \mathbf{D}} p_i^d \tau_{i,j}^d = w_j, \quad \forall j \in E \tag{5}$$

4) Spare capacity allocated on links sufficient to support 100% restorability.

$$\sum_{i \in I} v_{i,j} m_{i,x} + y_j \geq w_j, \quad \forall j \in E; x \in \mathbf{C} \tag{6}$$

5) The total number of useful capacity provided by the i^{th} visible straddling link at span j must be less than or equal to the total number of spare capacities of that span form by the corresponding cycles.

$$\sum_{i \in \mathbf{S}} 2\xi_{i,c}^j u_{i,x} - y_j \leq 0, \quad \forall j \in E; x \in c; c \subset \mathbf{C} \tag{7}$$

6) The constraint for non-shareable straddling links.

$$\left(\sum_{i \in \mathbf{S}} \xi_{i,c}^m u_{i,x} + \sum_{j \in \mathbf{I}} \pi_{j,c}^n m_{j,x} \right) - n_x \leq 0, \quad \forall \{\xi_{i,c}^m \pi_{k,c}\} \subseteq \Lambda; \quad \forall x \in c; c \subset \mathbf{C}; m, n \in E \tag{8}$$

7) Path selection constraint: each connection of demand requires to be assigned to one candidate path.

$$\sum_{i \in \mathbf{P}} p_i^d = h_d, \quad \forall d \in \mathbf{D} \tag{9}$$

8) Total capacity assigned for each span (working plus spare capacity) must be less than or equal to the maximum capacity that can be provided by the corresponding span.

$$\sum_{i \in \mathbf{S}} -2\xi_{i,c}^j u_{i,x} + y_j + w_j \leq \phi_j, \quad \forall j \in E; x \in c; c \subset \mathbf{C} \tag{10}$$

The objective have a constant number 2 (the part after the minus sign): This is because as in the model, for example, when merging 2 cycles to create an straddling link (the visible straddling link) , the total number of links is equal to the sum of the number of links on these cycles. There is no working capacity requires for the straddling link, thus we subtract 2 out of the total for each straddling link that is selected by the model.

The number of variables and constraints that are introduced in the model are shown in Table I, where $\bar{v}$ is the average of number of cycles forming a straddling link.

TABLE I
NUMBER OF VARIABLES & CONSTRAINTS

Number of Variables	Number of Constraints
$C + S + I + 2M + kD$	$\bar{v}(2S + 2I) + C + D + 4M$

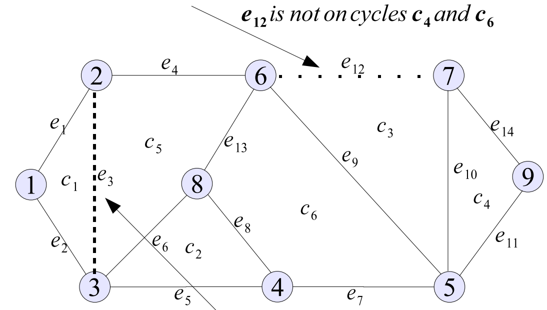

Fig. 1. Typical planar network 9N14S

IV. SIMULATION & DISCUSSION

In this section, we first present the performance of our proposed model on the 9n14s network shown in Fig. 1 with two different demand patterns as in Table III. Then, we compare the complexity of this model against the pure ILP formulation. The demands used in this simulation are randomly generated. The purpose for simulate these demands is just to prove the correctness of our model, which the optimum p-cycles required to protect the network can be obtained by using the network fundamental cycles.

- Simulation results of the of 9n14s network

The straddling links information which has been pre-processed is presented in Table II. Each visible straddling link is formed by two fundamental cycles only, whilst a hidden straddling link can be formed by more than two cycles. These data are essential to the formulation of the ILP as well as for generating the relevant p-cycles from the final results. Column one of this table gives the indices of the straddling links, column two gives information about how the straddling link is formed and the last column gives the physical link indices of the corresponding straddling links, for example, in Table II the visible straddling link with index 5 is constructed by cycles 3 and 4, and the link that becomes straddling has the physical index of 10.

Table III contains two demand patterns (a) and (b) used for the simulation. The first column of the table gives the index or connection ID of each source and destination pair. The second column gives the source and destination node pairs, and the last column gives the volume demand of the corresponding connection ID. Table IV shows the routing optimization results of the two demands patterns, where: (a) is the working channels allocated on each corresponding span that need to be protected, given in indexing order; (b) are the spare channels that are required for each span to guarantee 100% protection of the working channels, and (d) is the total of the network spare channels. (f), (g), (h) and (i) are the straddling links and their unit volumes needed to satisfy the objective of the model. The details of fundamental cycles that make the corresponding p-cycles are shown by (m) in the format [*cycles indices*]$\times volume$ (i.e., $[1\ 2\ 5] \times 3$ implies that there are 3 p-cycles, which are formed by fundamental cycles c_1, c_2, and c_5). The size of the model is shown in (k) with the format *no. constraints* $\times$ *no. variables*, for example, 139 constraints and 57 variables as shown in Result 1-(k).

- Model Comparison

The demand pattern (b) is also used for simulation on the pure ILP model. The performance of our model is comparable with the pure ILP model, and even better in some case as it has the ability to achieve the optimum solution by taking all the *extra straddling relationships with non-simple p-cycles* if available.

Before comparing the models complexity, we note that in our pre-processing of data, the *hidden straddling* links set are also limited by not allowing more than one pair of cycles to form a *hidden straddling* link. The complexity of the proposed model is much less then the pure ILP model, but this only happens when dealing with large size networks with average nodal degree greater than 3 or some network topologies that do not have many possible ways to create *hidden straddling* links. This is particularly true for small non-planar networks. The complexity of a non-planar network which uses the proposed model is usually higher than the pure ILP model because of the large number of hidden straddling links that can be constructed from the fundamental cycles, and the NFSNet is a typical example.

In Table V, the size of the first two networks are relatively small, thus there are not many candidates p-cycles exist in these networks. However, when the size of the network increase as in the last three cases, the number of candidate p-cycles increase significantly, but it is not applied to our model. The number of fundamental cycles only increase slightly with the size of the networks, event the depended on the structure of the given network the number of possible straddling links may vary differently between networks.

To the best of our knowledge, none of the ILP formulation can include the so called *extra straddling relationships with non-simple p-cycles*, which are the straddling links formed by two selected p-cycles [9] joined at a single node. Therefore, when compared to other conventional models, regardless of how the candidates are generated, our proposed model always outperforms them with the truly optimal solution as the formulation is built from the network's fundamental entities (the fundamental cycles and straddling link construction information). The main disadvantage of the proposed model is that when the network is a non-planar network, it requires a large number of constraints because of the need to constrain all the non-shareable cycles between straddling links. The number of constraints in our proposed model can be hundred times larger than the pure ILP model (the pure ILP model has about $\sim 4 \times M$ constraints). The number of constraints is relatively small and it is approximately the same as the pure model (eg. Table II: 9n14s network) if the network is planar as there are not as many relations between the fundamental cycles. This is the trade off for obtaining the optimum solution, but worthy when compared to the gain in minimizing the model complexity.

TABLE II
VISIBLE STRADDLING LINK DETAILS

Index	Cycles	Network link-index
Visible straddling link details		
1	1 - 5	3
2	2 - 5	6
3	2 - 6	8
4	3 - 6	9
5	3 - 4	10
6	5 - 6	13
Hidden straddling link details		
1	4 - 6	12
2	5 - 6	5

V. CONCLUSION

In this paper, we proposed a novel joint ILP optimization for the p-cycle protection design using the set of network fundamental cycles and the straddling links formed by the fundamental cycles. The fundamental cycle of the network

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TABLE V

MODELS COMPARISON

Network	Average nodal degree	Complexity of the pure ILP model	Complexity of the proposed model	Complexity of the proposed model with limit *hidden straddling link*
9n14s	~ 3	$\sim 2^{27}$	$\sim 2^{67}$	$\sim 2^{65}$
NSFNet-14n21s	~ 3	$\sim 2^{139}$	$\sim 2^{409}$	$\sim 2^{302}$
USA-28n45s	~ 3.2	$\sim 2^{7321}$	$\sim 2^{270}$	$\sim 2^{234}$
EON-19n38s	~ 4	$\sim 2^{8857}$	$\sim 2^{298}$	$\sim 2^{257}$
COST239-11n26s	~ 4.73	$\sim 2^{3531}$	$\sim 2^{219}$	$\sim 2^{201}$

TABLE III

DEMAND

Connection ID	**Source - Destination**	**Volume**
a) Pattern 1		
1	1 - 6	3
2	2 - 3	4
3	1 - 4	2
4	3 - 4	3
5	3 - 8	3
6	4 - 6	1
7	4 - 8	1
b) Pattern 2		
1	1 - 6	3
2	2 - 3	6
3	1 - 4	2
4	3 - 4	3
5	3 - 8	6
6	4 - 6	4
7	4 - 8	4
8	6 - 7	3
9	2 - 9	2
10	6 - 8	4
11	5 - 7	4

was defined as a cycle that contains no straddling link, which is different from the literature. The proposed formulation can achieve the optimum solution by getting all the *extra straddling relationships with non-simple p-cycles* if available, which has never been formulated before. The complexity of our model is significantly smaller than the pure ILP model when dealing with large size networks. The drawback of the model is that it has more constraints compared to the pure ILP model if the network is a non-planar network. However, in the case of a planar network, the number of constraints is small due to a smaller number of relations between cycles, and thus a smaller number of hidden straddling links that can be formed by the fundamental cycles. This suggests that this model is highly suitable for shared risk link group p-cycles networks or backbone p-cycles networks survivability.

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TABLE IV

SIMULATED RESULTS

Result 1: Demand (a)	
a) Working capacity on each link	[3 2 4 3 4 4 0 3 0 0 0 0 1 0]
b) Spare capacity on each link	[3 3 0 3 4 1 0 4 0 0 0 0 3 0]
d) Total Spare capacity requested	21
e) Total Network capacity	45
f) Visible straddling link index	[1 2]
g) Number of unit vis. straddling link	[3 3]
h) Hidden straddling Index	[0 0]
i) Number unit hidden straddling link	[0 0]
k) Model size	[139 × 57]
l) Cycles used	[4 2 3 5 5 4 0]
m)*The network p-cycles are*:	[1 2 5] × 3
	[2]
Result 2: Demand (b)	
a) Working capacity on each link	[3 2 6 5 5 6 4 4 5 4 1 4 4 1]
b) Spare capacity on each link	[3 3 2 5 1 4 4 5 0 3 1 4 1 1]
d) Total Spare capacity requested	37
e) Total Network capacity	91
f) Visible straddling link index	[1 2 3 5 6]
g) Number of unit vis. straddling link	[3 1 4 4 1]
h) Hidden straddling Index	[2]
i) Number unit hidden straddling link	[4]
k) Model size	[147 × 65]
l) Cycles used	[3 1 4 1 5 4 0]
m)*The network p-cycles are*:	[1 2 5]
	[1 3 4 5 6]
	[1 3 5 6]
	[3 5 6]× 2

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