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Creativity from Two Perspectives: Prospective Mathematics Teachers and Mathematician

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Abstract: Although creativity plays a critical role in mathematics, it remains underestimated in the context of a mathematics classroom. This study aims to explore the views and differences creativity displays in prospective teachers and one of their lecturers with respect to the characteristics and practices of creative teachers and the characteristics of creative students. We collected data through interviews with four prospective mathematics teachers and one mathematics lecturer. The study results revealed that their perspectives on creativity varied greatly and were mostly influenced by the characteristics of their diverse backgrounds and teaching practices. The views of the prospective mathematics teachers with respect to creativity were related to classroom activities prepared by teachers and students' approaches to problem solving. The lecturer appreciated focusing on the process rather than the product itself. The lecturer did not consider the nature of being creative to be an absolute necessity for a prospective mathematics teacher, and consequently, the lecturer's preferences regarding classroom materials affected the prospective teachers' views regarding creativity.

Key Words: Creativity, Mathematical creativity, Prospective mathematics teachers, Teacher training

Introduction

The importance of creative thinking has come into prominence in response to our rapidly changing world to adapt to ongoing technological and scientific improvements (Leikin, 2013). Creativity plays a significant role in mathematics. Creative thinking is considered to be the essence of mathematics (Mann, 2006). Sriraman (2004) reported that "Mathematical creativity ensures the growth of the field of mathematics as a whole" (p.19). Creativity arises not only from mathematical knowledge but also from the ability to think outside typical mindsets to envision potential possibilities and apply various aspects of mathematical knowledge for realizing opportunities (Bolden, Harries, & Newton, 2010). It is not surprising that both mathematicians and mathematics educators appreciate the role of creativity in doing mathematics (Levenson, 2013; Sriraman, 2004). However, they focus on the different aspects of creativity. For instance, mathematics educators focus on the teaching practices in the classroom, whereas mathematicians focus on mathematical creativity, which concentrates more on the nature of mathematics.

Conceptual learning and teaching are not acquired via the rule-based algorithms that dominate most mathematics classrooms (Mann, 2006); whereas creativity that provides "deep, vast and, thorough" knowledge of concepts is a fundamental element of conceptual learning (Ma, 1999). Thus, we need creativity if we aim to go beyond the algorithms to enrich mathematical thought at a conceptual level. Creative aspects in teaching and learning can result in "meaningful understanding of the mathematical concepts underlying the algorithm or when to apply it" (Beghetto & Kaufman, 2009, p.301). Consequently, creativity has become a key component in both learning and teaching mathematics (Lev-Zamir & Leikin, 2011; Panaoura & Panaoura, 2014). From Vygotskian perspectives, "creativity ('imagination' in Vygotsky's terms) is one of the basic mechanisms that allow development of conceptual knowledge" (Lev-Zamir & Leikin, 2011, p.17). Creativity is regarded as an inherent part of mathematics education programs. Flexible thinking, an aspect of creativity, is probably the most important ability in successful problem solving (Pehkonen, 1997). The main goal of teachers is to enable students to develop mathematical ability, which will help them in solving problems they experience in both school and everyday lives (Vale, Pimentel, Cabrita, Barbosa, & Fonseca, 2012). Students must be able to understand and represent problems, discern patterns, make generalizations, draw analogies and connections, see an old problem from different perspectives, find personal solutions to an existing problem, and evaluate and reflect solutions. Some of these activities, which are closely related to mathematical creativity, have been made part of the educational curriculum for all students (Levenson, 2013). Thus, the promotion of creativity has the potential to elevate teaching and learning activities (Lev-Zamir & Leikin, 2011). Teacher awareness and knowledge of creativity also has a profound impact in fostering student creativity (Runco & Johnson, 2002). Yet there is insufficient teacher awareness of the tasks that may require mathematical creativity (Levenson, 2013, 2015). A mathematical task may be inherently creative of itself, but its creative aspect may not be realized if the teacher is unable to facilitate a creative approach in the classroom (Panaoura & Panaoura, 2014). In this respect, while teachers have a critical role in stimulating creativity in their classrooms (Levenson, 2013), they usually attribute the greatest importance to logic in the learning of mathematics, and ignore the power of creativity (Pehkonen, 1997). Many researchers (Bolden et al., 2010; Shriki, 2010) have confirmed that teachers' overall knowledge of creativity is quite limited and narrow. Prospective teachers' views on creativity appear to focus more on the concept of "teaching creatively" rather than "teaching for creativity" (Bolden et al., 2010).

While fostering creativity is recognized as a matter of great importance in education, there are a limited number of published studies examining teaching perspectives regarding the use of creativity in the teaching environment (Levenson, 2015), and there are few studies of teacher awareness regarding creativity (Leikin, Subotnik, Pitta-Pantazi, Singer, & Pelczer, 2013). Nor does mathematical creativity have the significance it merits in related literature (Leikin, 2009). Mathematical creativity in high school education is not precisely emphasized in Turkey's curriculum of. For example, problem posing, one of the activities that has a central role in fostering mathematical creativity, has no recognized role in mathematical creativity (MoNE, 2013). Bolden et al. (2010) discussed the need to conduct studies of teacher education programs regarding the stimulation of mathematical creativity in students. Teaching education programs are one of the main sources for implementing teacher knowledge (Kleickmann et al., 2013). It is significant for the experts to transfer their interpretive perspectives to the prospective teachers, who are considered novices (Davis & Renet, 2013). Similarly, teachers as experts are significant actors in activating creativity in the classroom. Therefore, raising awareness in teachers around creativity and the quality of knowledge is crucial. One of the fundamental variables affecting teachers' knowledge is undergraduate education. Thus, in order to obtain deeper meanings regarding the teaching practices of prospective teachers, it is vital to focus on the teaching practices of the lecturer. Examining the influences of lecturer on the prospective teachers can provide us with significant insight. In this study, we investigated the ideas about mathematical creativity of a lecturer responsible for educating prospective mathematics teachers, and the resulting reflection of those ideas (if any) by prospective teachers. The lecturer and prospective teachers possibly have different perspectives with regard to the role of creativity in teaching and learning mathematics based on their backgrounds. Teaching approaches applied by the lecturer and prospective teachers also vary because their students have different learning needs. This study has the potential to contribute to and reveal the differences between the lecturers' views and prospective teachers' needs. It is valuable to analyze the different views of the lecturer and prospective teachers on the characteristics and practices of creative teachers and students. Essentially, we investigate the views and differences observed in the prospective mathematics teachers and their lecturer regarding the characteristics and practices of creative teachers and students in the Turkish context.

Literature Review

Creativity is a complicated concept. It has been defined in various ways, with some definitions focusing on the process and others stressing the creative product (Haylock, 1987; Kattou, Kontoyianni, & Christou, 2009; Leikin et al., 2013). Producing something authentic or original via a personal activity is a common basis of these definitions (Bolden et al., 2010; Sriraman, 2004). Guilford (1950) made a distinction between convergent thinking, which provides only one correct solution to a problem, and divergent thinking, which involves creatively generating multiple solutions to a problem. Highlighting and distinguishing between these two views has been the source of inspiration to the authors of various related literature (Beghetto & Kaufman, 2009). Divergent or productive thinking requires factors such as fluency, flexibility, originality, and elaboration (Yuan & Sriraman, 2010). To measure creativity, Torrance (1974) used these four factors and Guilford (1950)'s definition of creativity to develop the Torrance Tests of Creative Thinking [TTCT]. Fluency relates to the continuity of ideas, flow of associations, and use of basic and universal knowledge. Flexibility is associated with changing ideas, approaching a problem in multiple ways, and producing different solutions. Originality is defined as a unique way of thinking, and producing unique outcomes through a mental activity (Lev-Zamir & Leikin, 2011). Elaboration refers to the abilities of extension and improvement (Mann, 2006).

Scholars have presented various definitions, approaches, and interpretations regarding both mathematical and general creativity (Leikin et al., 2013; Shriki, 2010). Hadamard (1945) and Poincaré (1948) were inspired by the model of the creative process (Wallas, 1926) in their attempt to explain the creative thinking process of mathematicians. The model, which is called the Gestalt model, involves stages of preparation, incubation, illumination, and verification. Sriraman (2004) conducted a qualitative study with five creative mathematicians and found the model applicable. He attempted to add to the model the roles of imagery, intuition, and social interaction, as well as the use of heuristics, and the necessity of using proof (Sriraman, 2004). Ervynck (1991) considered mathematical creativity to be an essential component of advanced mathematical thinking. He also described a three-stage model of mathematical creativity: the preliminary technical, algorithmic activity, and creative activity stages. While the preliminary technical stage involves the use of procedures and rules without any awareness of doing so, the algorithmic stage consists of the utilization of mathematical techniques. The real action for mathematical creativity, however, occurs in the last stage, which involves creative activity and non-algorithmic decision making (Ervynck, 1991). One of the definitions of mathematical creativity at the professional level is that by Liljedahl and Sriraman (2006): the ability to produce an original study, which may substantially expand the whole body of knowledge (including the synthesis and expansion of known ideas), or which may ask new questions of mathematicians.

Creativity in the mathematics education context differs markedly from the creativity of professional mathematicians (Sriraman, 2005). Taking the past experiences of students into account, creativity can be considered in a specific group context, which includes those students' peers or classmates who share a common history (Leikin, 2009; Leikin & Pitta-Pantazi, 2013). Students become creative in mathematics classes when they construct the meanings of symbols, signs, and operations; when they understand mathematical problems; and when they plan a solution and determine various ways to check and justify that solution; that is, when they produce something new on their own (Haylock, 1987). Liljedahl and Sriraman (2006) define mathematical creativity at the school level as a process that results in novel, insightful solutions to a problem, and/or the formulation of new questions and/or possibilities, which allow the viewing of an old problem from different perspectives. Given that there are various definitions of mathematical creativity for specific contexts, in our study we considered the definition of mathematical creativity by Liljedahl and Sriraman (2006), both at the professional and school levels.

In recent years, creativity has become a more multifaceted concept with respect to teaching and learning mathematics (Levenson, 2013). We also consider creativity to have a dynamic feature, whereby students can develop if teachers provide them with appropriate learning environments (Leikin, 2009). Several studies have investigated the role of teachers, who may facilitate creativity. For example, Lev-Zamir and Leikin (2011) used a model to characterize teachers' conceptions of mathematical creativity, which consists of teacher-directed and student-directed conceptions. Teacher-directed creativity relates to teacher actions within the context of creative teaching, such as generating original tasks and using different models and manipulatives. Student-directed creativity relates to the opportunities given that foster student creativity. Bolden et al. (2010) found that prospective elementary teachers understand two types of creativity in mathematics lessons: creative teaching and creative learning. Creative teaching involves the creative use by teachers of resources and the application of mathematics to everyday examples. Creative learning consists of students undertaking practical activities and investigations as well as developing computational flexibility. Similarly, in this study, we considered the characteristics and practices of creative teachers, as well as the characteristics of creative students.

Although creativity plays a significant role in the thinking process, many teachers seem to lack the knowledge and skills required to foster student creativity in mathematics classrooms. Shriki (2010) argued that teachers have insufficient knowledge regarding creativity. Bolden et al. (2010) analyzed the written questionnaires and semi-structured interviews of prospective elementary school teachers regarding their conceptions of creativity, and revealed that these conceptions were narrow and related only to their own actions. In another study, teachers described a creative environment as one in which there are open-ended activities and non-routine problems, and that such environments provide students with the freedom to apply imaginative ideas and discover novel methods (Shriki, 2008). Sheffield (2006) argued that a task that enables further questioning can foster mathematical creativity. Further, teachers who enhanced the creative environment observed that student creativity is activated when they cooperate with their classmates (Fleith, 2000). To summarize, teachers must consider several factors when working to promote creativity in their classrooms (Levenson, 2013). We subscribe to the idea of Lev-Zamir and Leikin (2011) that "teaching is a complex activity that must be creative and directed at students' creativity." (p.18). In this study, we focused on the views of prospective teachers and their mathematician lecturer, regarding creativity in teaching and learning mathematics. Related literature reveals that teacher conceptions and expertise can promote creativity in the school context. As such, our study contributes to existing teacher training literature.

Methodology

The participants of this study were four prospective teachers in their final year of a secondary mathematics education program in Turkey. Our aim was to reflect the views of the prospective teachers who received no general course on creativity or on creativity within the context of their education and training program.

The prospective teachers were given experience in lesson and activities planning in their course titled “Teaching methods on mathematics education,” which they studied for two semesters. They also made observations and carried out various high school classroom activities within the scope of their “School experience” course. One of these activities involved implementing an activity they had planned earlier in the classroom. Briefly, these included the prospective teachers’ experiences in education, lesson, and activity planning, and the activities they carried out within the scope of the “School experience” course.

We collected data from interviews with the four participants, referred to as Hayal, Eda, Esra, and Bilge (pseudonyms), who had created a common lesson plan. We chose these prospective teachers as participants because they excelled in relation to their peers (other prospective teachers) in terms of their lesson plans, eagerness, and critical views of teaching. Each of the selected participants provided us with in-depth information, and we employed the intensity sampling strategy (Patton, 2002) for this qualitative research, which enabled us to obtain more detailed information regarding our research problem. The prospective teachers were asked four open-ended questions (Table 1) in video-recorded interviews that each lasted approximately 15 minutes. When necessary, prompts were employed during the interview to encourage the prospective teachers to elaborate upon their answers with concrete examples.

1. How necessary and important is being mathematically creative for prospective mathematics teachers?
2. What kind of activities should a teacher select in order for you to be mathematically creative?
3. What should a student do in the classroom in order for you to consider his/her actions to be a mathematically creative behavior/action?
4. Do your activities serve to activate mathematical creativity of students in the classroom? If so, could you give some examples of your activities?

Table 1: Interview questions

During the interviews with the prospective teachers, they constantly referred to Dr. Ali (pseudonym) when providing examples of their creative activities. For instance, Esra mentioned a creative activity introduced by Dr. Ali in the classroom as follows: “One of our lecturers [Dr. Ali] drew a star in the class and asked if it was a polygon. It was an outside-the-box demonstration for us, all of these can be examples of mathematical creativity.” Following our interviews with prospective teachers, we were prompted to interview Dr. Ali, since he seemed to be a hidden participant with rich data on the characteristics of mathematics teachers and students. We interviewed Dr. Ali, who teaches in the department of secondary mathematics education and is a professor of mathematics. As such, in line with the objectives of our study, we were able to compare the views of the prospective teachers with those of their lecturer, whom they considered to have a wealth of creative activities and ideas.

In choosing Dr. Ali as a study participant, we employed the snowball sampling strategy (Patton, 2002). We asked Dr. Ali the same interview questions as we had asked the prospective teachers. This interview, which was also video-recorded, took approximately 20 minutes.

We analyzed the views of the prospective mathematics teachers and the mathematician through qualitative content analysis (Patton, 2002). First, we read only the raw data by taking into consideration the views of teachers as discussed in the literature (e.g. Bolden et al., 2010; Kattou et al., 2009; Lev-Zamir & Leikin, 2011, 2013). This literature guided determination of patterns that signify the characteristics and practices of creative teachers and students in the mathematics classroom context. Next, we categorized these patterns. For example, when prospective teachers made statements such as "I provide real life problems for [teaching] difficult concepts such as derivatives" and "For one manipulative, for example, I used a geometry board but I will not need it for every problem, maybe just for some," we coded these responses as "using various resources," which is one of the characteristics and practices of creative teachers.

Findings

We organized our findings according to (i) the characteristics and practices of creative teachers, and (ii) the characteristics of creative students. Each section includes the views of both the prospective mathematics teachers and their lecturer. As such, the reader is able to consider different views.

Characteristics and Practices of Creative Teachers

The prospective mathematics teachers described creativity through the activities implemented by teachers. The lecturer, however, emphasized the thinking process with respect to creativity (Table 2).

According to the Prospective Teachers	According to the Lecturer
- Use various resources (e.g., real life problems, open-ended problems, manipulatives, and modeling) - Make lessons interesting for students (e.g. drama, puzzles) - Assign group projects to increase interaction among students - Create plan Bs for successful students - Assign research projects to students - Teach students to question ideas - Nature of the teacher (e.g., attention-grabbing, effective tone of voice) - Have good communication skills	- Lead creative students - Enable students to think divergently - Compel students to think creatively

Table 2: Characteristics and Practices of Creative Teachers

Characteristics and Practices of Creative Teachers According to the Prospective Teachers

The prospective teachers defined the nature of a creative teacher as "attention-grabbing, being able to use his/her tone of voice effectively, and being attentive while approaching students." Thus, they highlighted the behaviors displayed by a teacher as a characteristic of a creative teacher. It could be inferred that they appreciated the good behavior of a teacher specifically while communicating with students.

Hayal emphasized the importance of the teacher paying attention to the ideas of students in a classroom setting with respect to activating student creativity: “Teachers must respect their students and attach importance to their ideas within the context of creativity.” Hayal stated that having good communication skills in classroom settings could contribute to their development of creativity. In the mathematics discourse community, it has been recognized that paying equal importance to the ideas presented by every student in the classroom helps to balance the environment (Sfard, 2008). Alternatively, Hayal may have meant that classrooms in which creativity for both the teachers and students is encouraged have a more comfortable atmosphere and are more democratic (Fleith, 2000; Leveson, 2011).

Esra proposed to encourage creativity by creating and using plan Bs for students who are a step ahead and/or more eager than their peers with respect to classroom activities. She explained her ideas by saying

Esra: An activity could be prepared and used as a plan B, and a few extra activities could be added for those who are a step ahead or more eager than their peers.

Researcher: What would these extra activities be for?

Esra: For those who are creative and think faster. For instance, we might implement activities containing puzzles or proofs. We can encourage their creativity by implementing activities that would compel them to think independently and to find their own solutions.

Esra proposed activities that she labeled as plan Bs, for students who are more creative and who think faster than other students participating in the activities. She offered examples for possible plan B activities, such as puzzles and proofs, to enable successful students to think more deeply and find their own solutions.

The prospective teachers proposed a number of activities for making lessons interesting in order to ensure creativity in teaching. In addition, they suggested giving lessons not only to present accurate mathematical concepts but also to use different teaching approaches, which Esra called “creative tools.” She explained as follows:

Even if I wanted to explain concepts such as a triangle or a point creatively, they have a certain basic description. However, the way I present it, i.e., using inverted sentences, explaining it in another way, or using materials or explaining by drama—these are all creative tools I can use.

Esra also stated that she could teach the lesson using drama to activate student creativity. She also mentioned that she could use puzzles to “direct attention to the lesson.”

The prospective teachers evaluated creativity by the teacher’s use of various resources, including real life problems, open-ended problems, manipulatives, and modeling within the context of in-class activities. For example, Eda mentioned that modeling problems for use in classroom settings could improve students’ creativity by stating that “Using modeling problems can be considered as a separate approach. From my own experience, I realized that my creativity had really improved while I was taking a modeling course. When necessary, creativity can be improved even through those activities.” Eda shared her own experience as an example of one way her creativity had been activated. She then incorporated this experience as a student to her perspective as a prospective teacher. She stated that this approach could also be used for high school students to contribute to student creativity. Bilge also considered activities prepared using various resources, such as real-life problems and manipulatives, to be creative activities. She elaborated as follows:

I use different books, create my own problems, and associate them with real life, and that is how I use these problems. For one manipulative, for example, I used a geometry board but I will not need it for every problem, maybe just for some.

Bilge also mentioned using different books and creating her own problems in the classroom, which could be an example of mathematical originality in teachers' conceptions of creativity in mathematics teaching (Lev-Zamir & Leikin, 2013).

The prospective teachers also included assigning research projects to the students to be completed outside the classroom as falling within the scope of creative education, and gave examples such as organizing seminars on the history of mathematics or assigning research projects about mathematicians. Bilge suggested "pi day" as an example of a research project, saying "It is possible to organize activities related to famous mathematicians or specific concepts such as pi, and celebrate pi on the 14th of March. I could ask students to make out-of-class research projects that week." Hayal suggested a "photo-math" project as an example within the context of the "Teaching methods on mathematics education" course, which was taught by Dr. Ali. She explained her ideas as follows:

> *For example, if we are on the subject of similarity, we can discuss it not only in the classroom but also outside the classroom. In the photo-math project last year, I used the picture of a statue at a university campus, which resembled an arch for the subject of similarity. I used something that looked like three interlaced arches. For instance, we could assign students such a project and leave them with the problem.*

Hayal suggested a "photo-math" project as an example of a research project. The "photo-math" project involves taking a photo of a structure or natural object that is located in the city where students live. Students must then describe the photo as defining a mathematical concept, proving a theorem, or solving a mathematical problem.

The prospective teachers also stated that activating interaction among students via group work is an important part of in-class activities, and that group work could be used to activate creativity in students. For instance, Bilge discussed the advantages of group projects as follows:

> *First, group work could be used to activate creativity. We could group students into twos or threes, so that they could produce better ideas, because sometimes one student's ideas can trigger those of others. Group work can produce more fruitful results than the students working on their own. Therefore, group work can be more beneficial than individual work.*

Esra gave another creative activity example for group work—the manipulative—which was developed by the prospective teachers, and is displayed in Figure 1.

Figure 1: Example provided by Esra as a creative manipulative

The manipulative shown in the figure uses ping pong balls to relate the powers of imaginary unit (i) numbers. The green cards represent the congruence classes of i, -1, $-i$, and 1. The ping-pong balls can be put on and taken off from the sticks, and this manipulative can hold a maximum of 16 ping-pong balls. In the figure, 15 ping-pong balls represent the imaginary number i^{15}. Students can determine that i^{15} is congruent with $-i$ by referring to the green cards.

The prospective teachers considered having students question a specific subject or problem to be a technique characteristic of creative teachers in the classroom context. Eda explained her ideas as follows:

> *I will really care about the questioning part when I become a teacher. What is important is not giving a definition nor making the student provide a definition. I believe that a student's ability to question is important in order to improve her creativity.*

Eda mentioned that students who can question even a simple definition tend to improve their creativity. When the prospective teachers provided examples of mathematical activities that activate creativity, they most often mentioned explorative or investigative activities that cause students to question. For instance, Esra made the following comment regarding a problem that made students question: "One of our lecturers [Dr. Ali] drew a star in the class and asked if it was a polygon (see Figure 2). It was an outside-the-box demonstration for us, all of these can be examples of mathematical creativity."

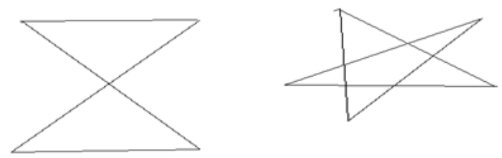

Figure 2: Activity Dr. Ali uses in his classes

Characteristics and Practices of Creative Teachers according to the Lecturer

Dr. Ali stated that it is not directly necessary for a prospective mathematics teacher to be mathematically creative. Rather, he stated that it was necessary for mathematics teachers to be cognizant of flexibility and fluency, which are components of creativity, because they may be useful for some students in their future work. When we asked the lecturer to clarify what he referred to by "some students," he responded with the following:

> *Let's say you are studying a problem and the student proposes a different solution and this happens more than once. That means he is already giving signs of being a creative student. In that case, you should work to improve the abilities of that student.*

He stated that he recommended that prospective teachers prepare different activities and lead creative students. With respect to how a mathematically creative teacher could use such activities during the lesson preparation process, the lecturer made the following statement:

> *You can make a problem open-ended in an activity in which you work with normal students (I do not know if it would be correct to call them normal though) or make them do some research outside the classroom. If there are such students, I believe teachers can guide the students in that way and lead them in doing so. It is not necessary to aim at being Olympiads, but it is good to have knowledge about mathematical creativity so as to consciously guide the students.*

In his statement, the lecturer also expressed that it was possible to make open-ended the problems contained in an activity by rearranging them for the students who are ahead of the classroom level. He also highlighted leading creative students via the use of extracurricular activities.

Dr. Ali also expressed ideas regarding how a creative mathematics teacher could enable a student to think divergently while solving a problem. He highlighted the importance of not directly giving students the answer to a problem, saying: “If I give them the solution, everybody will go in that direction.” In addition, he suggested that prospective teachers should make a habit out of thinking divergently, in order to guide their own students in the future. In another remark, Dr. Ali recommended that prospective teachers reflect on solving a problem: “This may be how it is solved, but can I use a more divergent method or an advanced method or a more basic method?” Dr. Ali stated that his intent to not directly give students answers to problems is one of the reasons that he strives to enable students to deal with problems independently. He gave the following example:

> *The other day one of our students sent me an e-mail asking me “how do we solve a problem we saw in the class?” I told her “I do not know, tell me tomorrow morning when you find out.” They stayed up all night until they found the solution. She told me “I did not let my friends in the dorm even sleep.” That’s what I call “solve it yourself instead of asking others.” Okay, you can get hints, but what’s essential is solving it yourself. And they were happy when they realized they were able to solve it.*

In the above statement, Dr. Ali pointed out that he prefers to guide students in his courses, but not provide them with the correct answer to a problem, to enable them to reflect on the problem themselves. Dr. Ali encourages his students to focus and think deeply, so that they can find their own way without imitating a solution, which they did not come up with by themselves. One of his suggestions is to make a habit out of “finding out and reflecting in their own way” while solving a problem.

Dr. Ali poses divergent problems based on situations that are not included in the standard lesson content, in order to compel his students to think creatively in a mathematical sense. Figure 2 shows an example of a divergent problem that Dr. Ali used in one of his courses. He also reported that he asked various problems to activate student creativity. Another problem he has used in the classroom is “Prove that there do not exist positive integers x, y, z such that $x^n + y^n = z^n$ for all integers $n \geq 3$” (Erickson & Flowers, 1999, p.30). He explained that what he expects from students in solving this kind of problems is as follows:

> *First, I tell them “stick to only one thing and do not let it go.” Then I ask them “can you produce five different ideas?” and I do not get five ideas very often. I mean for them to think of why, it is not easy, okay “this and that might not give you the solution but let me look at it from those perspectives.” It is not too important to have too many ideas, I mean, “can I develop another method that is convenient for this? It does not have to be five, six, or too many;” even two methods are enough. The first one may not work but then the second one does.*

Thus, we see that Dr. Ali encourages his students to focus on an idea, and then compels them to produce several different ideas. Dr. Ali allows students to view problems from a wide perspective in order to identify different ways to solve the problem. However, his main goal in having the students find different ways to solve a problem is for them to experience the process, not to produce the outcome.

Characteristics of Creative Students

The prospective mathematics teachers explained the characteristics of creative students primarily as a matter of problem solving, while the lecturer emphasized the generation of novel ideas as characterizing creative students (Table 3).

According to the Prospective Mathematics Teachers	According to the Lecturer
- Associate the subject with other subjects - Use different approaches to solve a problem - Question mathematical arguments	- Solve problems using different methods - Maintain continuity in generating novel ideas

Table 3: Characteristics of Creative Students

Characteristics of Creative Students according to the Prospective Teachers

The prospective teachers described students who propose the use of different approaches to a problem as being mathematically creative, thereby explaining the characteristics of creative students based on the student approaches to problem solving. For instance, Bilge expressed her appreciation for receiving suggestions from students about using different approaches while solving a problem as follows: “Solving the problems by using different approaches, different ways. There are at least 30 students in the class and it is nice to have two or three different solutions, but of course that depends on the problem or the subject.” She mentioned the importance of using different approaches to address a problem in the context of a classroom, and having the students discuss their solutions with each other. Similarly, Eda highlighted the importance of using different methods to solve a problem by saying

> *When a student only thinks of different methods, it is a sign of creativity for me. I mean, just proposing a new idea, I do not mean it has to be correct, it is about being able to think of different things, to look at a subject from a different perspective, and as I said before, it does not have to be correct.*

Eda considers the students’ abilities to consider different ideas and perspectives in in-class activities to be signs of creativity. She also added that the important aspect is not the accuracy of a student’s idea, but rather the fact that the student himself/herself came up with and shared a different idea.

As stated above, the prospective teachers proposed that the ability to associate one subject with other subjects is an indicator of mathematical creativity. For instance, Esra mentioned a student’s statement about the line y = x being the first bisector line as follows: “While I teach students about the line y = x, one of them called it the first bisector line. She might have used mathematical creativity to solve it by associating that subject with others.” Esra considered that relating one mathematical object to another mathematical context is characteristic of creative students.

Furthermore, the prospective teachers noted the importance of questioning mathematical arguments as being a characteristic and practice of creative students. Hayal mentioned that a student who questions mathematical arguments is thinking creatively: “When we give students a formula, they may directly apply it. On the other hand, if the formula goes against that student’s logic, which makes the student question it, then the student could be creatively thinking in a mathematical sense.” Bilge also stated that students who ask questions about the nature of mathematics are students who think creatively: “The students ask questions such as “Where did it come from? Why it is formed in this way? Why do we need to learn derivatives?” These questions are about the nature of mathematics and knowledge itself." As such, Bilge considers that questions about the necessity of mathematical arguments are indicators of creativity.

Characteristics of Creative Students according to the Lecturer

Dr. Ali expressed that students whom he considers to be mathematically creative in the classroom setting solve problems using different methods. He profiled these students as follows:

> *Working in a student-oriented manner helps. When I ask my students to solve a problem, I recognize the ones who produce different ideas in the classroom. However, it is even more visible in the exam papers. For example, when I evaluate exam papers, I can see solutions that are quite systematic even when they give a wrong answer.*

According to Dr. Ali, conducting his mathematical teaching and pure mathematics lessons in a student-oriented manner allows him to directly communicate with his students. By doing so, he finds that he is able to recognize the students who display mathematically creative behaviors and who generate different ideas.

Dr. Ali stated that maintaining continuity in generating novel ideas is a basic requirement for him to consider a student as being mathematically creative. He also added that he further identified such students while reading exam papers: "It is important to produce different ideas, but it does not happen all the time. I generally look at how often it occurs, and if this happens continuously then it is really valuable. If producing different ideas happens once, it is not as remarkable." Producing different ideas still involves recognition of creative thinking; however, Dr. Ali interprets producing different ideas only once as not remarkable. This could be due to Dr. Ali's expectations from his students. Dr. Ali underlined not only the significance of generating different ideas, but also the ability to maintain continuity in generating new ideas. He valued the repetition of the mathematical thinking process that results in novel ideas.

Discussion and Conclusion

The aim of this study was to investigate the views and differences in prospective secondary mathematics teachers and one of their lecturers regarding the characteristics and practices of creative teachers and students. We analyzed individual interviews conducted with these five participants and identified categories of creative characteristics. We make no claim that our findings represent a comprehensive overview of the views of prospective teachers and their lecturers. The number of participants, who were purposefully selected, is small, thus it makes no sense to make any general inferences regarding creativity in teaching and learning mathematics. Despite this limitation, this study reveals a dichotomy between the views of prospective teachers and their lecturer regarding mathematical creativity within the classroom context. Moreover, even within the small scope of our participants, the study provides some insight for relating mathematical creativity to teacher training.

According to prospective teachers interviewed, a creative teacher uses various resources such as real-life and open-ended problems, as well as manipulatives to verify findings (Fleith, 2000; Kattou et al., 2009). Furthermore, the prospective teachers in this study believed that assigning group projects to students and creating plan Bs for more creative students will contribute to the activation of creativity in mathematics classrooms. Therefore, we can deduce that the prospective teachers consider that using different pedagogical approaches can make a difference in the creativity of teachers. The prospective teachers considered traits such as making students question and listening to the students to be characteristic of creative teachers. Therefore, we can assume that the prospective teachers interviewed associate such characteristics with creative environments. The lecturer thinks that leading creative students, enabling students to think divergently, and compelling students to think creatively could constitute a set of techniques for establishing a creative classroom context. The lecturer also stated that studying outside of classroom activities can contribute to fostering student creativity. In addition, he stressed the importance of teaching students to think mathematically. We can conclude that the lecturer believes that teachers who can enable students to think critically about mathematical content are, in fact, creative.

The prospective mathematics teachers described the characteristics of creative students as being able to associate one subject with other subjects, the use of different approaches in solving a problem, and questioning mathematical arguments. We conclude that the prospective mathematics teachers define the characteristics of creative students based on their approaches to problem solving and their thinking styles. The lecturer, on the other hand, emphasizes the significance of solving problems by different methods, as well as maintaining continuity in generating novel ideas. We conclude that the lecturer considers the ability to retain continuity when producing novel ideas to be a sign of creativity.

We discovered that prospective teachers and lecturers have different perspectives of creativity in their teaching practices. Based on our analysis, we can say that, with respect to mathematical creativity, the prospective teachers do not focus on the mathematical content itself, but rather the teachers and their practices (Kattou et al., 2009). In contrast, the lecturer associates creativity with the student's ability to obtain a mathematical result that is independent from the presented school mathematics—a view that parallels statements made in the literature by mathematicians regarding creativity (Liljedahl & Sriraman, 2006). Of course, creativity in classroom mathematics varies from that of mathematicians (Sriraman, 2005). The audience of the lecturer (mathematician) and the audience of the prospective teachers were different and had different learning needs. Creative thinking by prospective teachers is triggered by the lecturer's creative thinking and teaching practices. As clearly suggested by the results of this study, the views on creativity that are held by a mathematician differ from those held by prospective mathematics teachers. As Ma (1999) stated, it is vital to be an expert in mathematics (as a lecturer) and understand and know how to communicate with the prospective teachers in the classroom. This statement could clarify the dichotomy between the different perspectives on mathematical creativity within the context of our study. The lecturer, who is an expert of mathematics, does not focus on how to communicate with the prospective teachers as novices, who are in need of being led in terms of embedding creativity into their teaching practices.

The lecturer stated that it is not necessary for a prospective teacher to be mathematically creative while teaching mathematics. The prospective teachers' views on creativity, however, are limited to concerns about the activities that teachers prepare and their students' approaches to problem solving. The reason that prospective teachers identified the most effective determinant for their views on creativity is due to cultural and contextual factors (Emre-Akdoğan & Yazgan-Sağ, 2015). In Turkey today, teachers primarily give direct instructions. As such, we consider that the prospective teachers consider as being creative those teachers who employ activities and tasks without giving direct instructions. The prospective teachers interviewed also focused on the importance of providing various classroom activities in their descriptions of the characteristics of creative teachers (Mann, 2006; Sheffield, 2006). We found that the views of the prospective teachers were inspired by their lecturer's mode of instruction. However, the lecturer perceived the design of such in-class activities as creativity promoting, but not creative in and of themselves (Shriki, 2008).

Consequently, prospective teachers' approaches on creative teaching materials and learning activities are diverse and rigorous. However, the lecturer's views on mathematical creativity, in contrast to the views of the prospective teachers, are similar to the definition of mathematical creativity, as found in the literature (Ervynck, 1991; Liljedahl & Sriraman, 2006; Sriraman, 2006). One reason for this might be that they had not received any training on creativity in their teacher education curriculum. It has been reported that what students primarily need in order to activate their creativity is having teachers equipped with pedagogical knowledge related to creativity (Shriki, 2008). Hence, we assume that this is made possible through the training teachers receive throughout their education program. Teacher education programs should be designed to support prospective teachers to acquire knowledge and teaching skills related to the development of student creativity (Hosseini & Watt, 2010; Levenson, 2013). Future research might focus on designing courses on creativity for teacher education programs, determining course content and implementation in the classroom, and analyzing the impact these courses on prospective mathematics teachers.

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