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***Abstract*—In wireless OFDM applications, since the radio channel is frequency selective and time-varying, a dynamic estimation of channel must be achieved before the demodulation of the transmitted OFDM signals. As an effective approach for solving the channel estimation problems, the pilot-assisted channel estimation technique has received considerable attention in recent years. In this paper, we will investigate the channel estimation error in the existed pilot-assisted channel estimation approaches in detail, a new effective channel estimation approach with lower estimation error is proposed as well.**

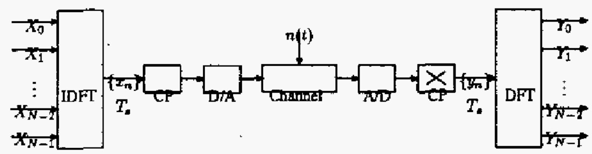

Fig. 1. A baseband model of OFDM system

I. Introduction

In high speed wireless OFDM systems, the channel estimation can be achieved by the pilot-assisted scheme, this technique has been studied for several years [1][2]. In this scheme, the channel is usually assumed to have a finite-length impulse response, a cyclic prefix which is longer than the channel impulse response is put between consecutive symbol blocks for avoiding inter-channel interference (ICI) and preserving the orthogonality of the subcarriers. The pilot symbols are inserted at selected two-dimensional time-frequency pilot locations, the channel estimation is achieved by estimating the channel attenuations in frequency domain using least square (LS) method or linear minimum mean square error (LMMSE) method [3].

In this paper, we will present an analysis of the estimation error in the existed pilot-assisted channel estimation approaches for multipath Rayleigh fading channels. The paper is organized as follows. In Section 2, the OFDM systems and channel models are represented. The analysis of channel estimation is provided in Section 3, an improved channel estimation algorithm is also proposed in this section. The simulation results are presented in Section 4.

II. System Model and Channel Estimation

A. OFDM System Model

A baseband OFDM system model is shown in Fig. 1, where $\{X_n\}$ is the transmitted parallel complex symbol sequence with symbol rate $f_s = 1/NT_s$. The serial time-domain sequence $\{x_n\}$ with data rate $1/T_s$ is obtained by taking IDFT on $\{X_n\}$. The cyclic prefix (CP) of time length T_g is used to eliminate inter-channel interference and preserve the orthogonality of the subcarriers, the D/A converter which contains ideal low-pass filter with bandwidth $W_s = 1/T_s$ converts the digital signal $\{x_n\}$ into the transmitted analog signal $x(t)$.

At receiver, the input analog signal $y(t)$ is discretized with sampling rate $1/T_s$ to obtain the digital data sequence $\{y_n\}$. Removing the cyclic prefix from $\{y_n\}$ and then performing DFT on it, we obtain the received symbol sequence $\{Y_n\}$.

B. Channel Model

The channel model considered throughout the paper is a multipath Rayleigh fading channel with time-varying finite impulse response [1][3], which can be written as

$$h(t,\tau) = \sum_{m=0}^{M-1} h_m(t)\delta(\tau - \tau_m(t)) \qquad (1)$$

where M is the total number of the propagation paths of channel, $h_m(t)$ and $\tau_m(t)$ are the attenuation coefficient and excess delay, respectively, of the mth path component at time t. The excess delay $\tau_m(t)$ is required to satisfy the condition $0 \le \tau_m(t) \le T_g$, which ensures the entire impulse response lies inside the guard interval. The corresponding frequency response of channel is

$$H(t,f) = \sum_{m=0}^{M-1} h_m(t) e^{-j2\pi\tau_m(t)f} \qquad (2)$$

Furthermore, the channel $h(t,\tau)$ is also assumed to be a wide-sense stationary uncorrelated scattering and slow fading channel [4][5], in which case the channel attenuation coefficient $h_m(t)$ and the excess delay $\tau_m(t)$ can remain constant during one symbol duration. Thus, the channel can be treated as a set of parallel independent stationary complex-valued Gaussian processes. The attenuation on the nth subcarrier with carrier frequency $f_n = \frac{n}{NT_s}$ during the pth symbol can be written as

$$H_{p,n} = H\left(pT, \frac{n}{NT_s}\right) = \sum_{m=0}^{M-1} h_{p,m} e^{-j\frac{2\pi\tau_{p,m} n}{NT_s}} \quad (3)$$

where T is the symbol duration with $T = NT_s + T_g$, and $h_{p,m} = h_m(pT)$, $\tau_{p,m} = \tau_m(pT)$. For simplifying the discussion, we consider the case that only one symbol is transmitted in each sub-channel, thus we can drop the time (OFDM symbol) index p from the (3) to obtain

$$H_n = \sum_{m=0}^{M-1} h_m e^{-j\frac{2\pi\tau_m n}{NT_s}} \quad (4)$$

Consequently, the corresponding observed channel impulse response $\{\hat{h}_n\}$ is the IDFT of $\{H_n\}$.

III. Channel Estimation

A. Analysis of Channel Estimation Error

The pilot-assisted channel estimation problem is usually formulated to estimate the channel attenuations $\{H_n\}$ from the received data $\{Y_n\}$ and the transmitted data $\{X_n\}$ [2][3]. Since $Y_n = X_n H_n + N_n$, where $\{N_n\}$ is the additive Gaussian white noise of the channel, which is independent and uncorrelated with the channel, the channel attenuation estimate $\{\widetilde{H}_n\}$ can then be written as

$$\widetilde{H}_n = \frac{Y_n}{X_n} = H_n + \frac{N_n}{X_n} \quad (5)$$

Thus, maximum likelihood (ML) method or LMMSE method can be adopted to estimate the channel attenuations. For holding the equation $Y_n = X_n H_n + N_n$, the channel excess delay τ_m is usually assumed to be equal to mT_s. However, this assumption is seldom satisfied in practice [6]. In that case, it will be demonstrated next that even for an ideal channel without noise, the proposed approaches based on (5) can not provide a good estimate of the channel attenuations.

Without loss of generality, consider a multipath fading channel whose excess delay τ_m equals $\frac{mT_s}{K}$, where K is an integer, then the channel impulse response can be represented by

$$h(\tau) = \sum_{m=0}^{M-1} h_m \delta\left(\tau - \frac{mT_s}{K}\right) \quad (6)$$

Notice that the transmitted signal can be written as

$$x(t) = \sum_{n=0}^{N-1} x_n \delta(t - nT_s) \quad (7)$$

If we upsample the data sequence $\{x_n\}$ with the upsampling rate K, then we can obtain a new date sequence $\{\hat{x}_k\}$ with $\hat{x}_k = x_n$ if and only if $k = nK$ or $\hat{x}_k = 0$ elsewhere. Thus, $x(t)$ can be also expressed as

$$x(t) = \sum_{k=0}^{KN-1} \hat{x}_k \delta\left(t - \frac{kT_s}{K}\right) \quad (8)$$

At receiver, discretizing the received signal $y(t)$ with the sampling rate $\frac{K}{T_s}$, then the obtained data sequence $\{\hat{y}_k\}$ can be written as $\hat{y}_k = \hat{x}_k \otimes h_m$, where $\otimes$ denotes the circular convolution. Let $\{\widehat{X}_k\}$, $\{\widehat{H}_k\}$ and $\{\widehat{Y}_k\}$ be the DFTs of the data sequences $\{\hat{x}_k\}$, $\{h_m\}$ and $\{\hat{y}_k\}$, respectively, with the length of KN, then we have $\widehat{Y}_k = \widehat{X}_k \widehat{H}_k$.

For the upsampling transceiver system described above, the data sequences $\{y_n\}$, which is obtained by discretizing the signal $y(t)$ with sampling rate $\frac{1}{T_s}$, equals the sequence downsampled from $\{\hat{y}_k\}$ with downsampling rate K, i.e. $y_n = \hat{y}_k$ if and only if $n = \frac{k}{K}$, and $y_n = x_n \otimes h_m^{(\downarrow K)}$ where $\{h_m^{(\downarrow K)}\}$ is the sequence downsampled from $\{h_m\}$ with downsampling rate K. Let

$$Y(f) = \mathscr{F}(y_n), \quad \hat{Y}(f) = \mathscr{F}(\hat{y}_n) \quad (9)$$

where $\mathscr{F}(\cdot)$ denotes discrete time Fourier transform, then from [7], we have

$$Y(f) = \frac{1}{K}\sum_{k=0}^{K-1} \hat{Y}\left(\frac{f+k}{K}\right) \quad (10)$$

Discretizing the frequency at $f = \frac{n}{N}$, then we can obtain

$$Y_n = \frac{1}{K}\sum_{k=0}^{K-1} \hat{Y}\left(\frac{n+kN}{KN}\right) = \frac{1}{K}\sum_{k=0}^{K-1} \hat{Y}_{n+kN} \quad (11)$$

Similarly, it can also be proved that

$$X_n = \widehat{X}_n, \quad H_n^{(\downarrow K)} = \frac{1}{K}\sum_{k=0}^{K-1} \widehat{H}_{n+kN} \quad (12)$$

where $\{H_n^{(\downarrow K)}\}$ is the N-point DFT of $\{h_m^{(\downarrow K)}\}$.

Thus, for the system model presented in Section 2, it can be verified that in the case of ideal channel without noise, the channel attenuation estimate obtained in (5) is

$$\widetilde{H}_n = \frac{Y_n}{X_n} = H_n^{(\downarrow K)} \quad (13)$$

On the other hand, substituting $\tau_m = \frac{mT_s}{K}$ into (4), we can see that the estimated channel attenuations should be

$$H_n = \sum_{m=0}^{M-1} h_m e^{-j\frac{2\pi mn}{KN}} = \widehat{H}_n = \frac{\hat{Y}_n}{X_n} \quad (14)$$

From (12), we know that $\widehat{H}_n \neq H_n^{(\downarrow K)}$ for $0 \leq n \leq N-1$ in general, it demonstrates that even for the case of ideal channel, the channel estimation approaches based on (5) can not give us an accurate estimate of the channel attenuation.

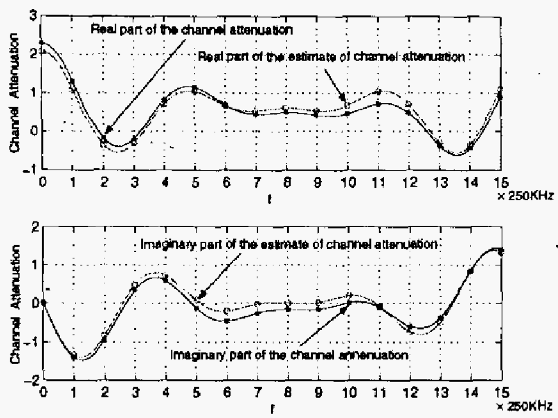

Fig. 2. Comparison between the channel attenuation $\{H_n\}$ and the estimate $\{\widehat{H}_n\}$ obtained from the existed algorithm

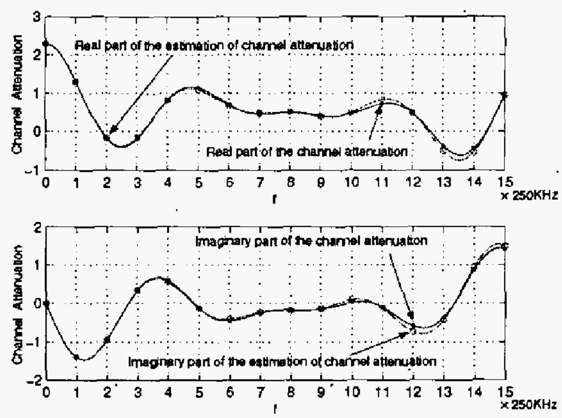

Fig. 3. Comparison between the channel attenuation $\{H_n\}$ and the estimate $\{\widehat{H}_n\}$ obtained from the improved algorithm

B. Improved Channel Estimate

In the previous section, it has been shown that the channel estimation algorithms based on (5) may have estimation error even for ideal channels. In this section, we will investigate the methods to reduce this type of estimation error.

Without loss of generality, we can divide the channel models into two classes. For the first type of system model, the channel excess delay τ_m is assumed to equal $\frac{mT_s}{K}$. For the second type of channel model, the channel excess delay τ_m is not a rational number. In this case, we can however always choose an integer K, such that $\left|\tau_m - \frac{k_m T_s}{K}\right| \le \varepsilon \ll 0$, where ε is an arbitrary constant. Thus we can use $\frac{k_m T_s}{K}$ as an approximation of τ_m and ignore the difference between τ_m and $\frac{k_m T_s}{K}$. In the previous section, it has been demonstrated that for the channel model with the channel excess delay $\tau_m = \frac{mT_s}{K}$, the channel attenuations $\{H_n\}$ are equal to the first N elements of $\{\widehat{H}_k\}$, so if we can get an accurate estimate of $\{\widehat{H}_k\}$, then the channel attenuation $\{H_n\}$ will be obtained.

Suppose that the system has additive Gaussian white noise, then we have

$$\widehat{Y}_k = \widehat{X}_k \widehat{H}_k + N_k \tag{15}$$

where $\{N_k\}$ is the channel Gaussian noise. Let

$$\widetilde{\mathbf{y}} = \begin{bmatrix} \widehat{Y}_0 \\ \widehat{Y}_1 \\ \vdots \\ \widehat{Y}_{KN-1} \end{bmatrix}, \ \widetilde{\mathbf{h}} = \begin{bmatrix} \widehat{H}_0 \\ \widehat{H}_1 \\ \vdots \\ \widehat{H}_{KN-1} \end{bmatrix}$$
$$\widehat{\mathbf{X}} = \begin{bmatrix} \widehat{X}_0 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \widehat{X}_{KN-1} \end{bmatrix}, \ \widetilde{\mathbf{n}} = \begin{bmatrix} N_0 \\ N_1 \\ \vdots \\ N_{KN-1} \end{bmatrix} \tag{16}$$

and

$$\mathbf{h} = \begin{bmatrix} h_0 & h_1 & \cdots & h_{M-1} \end{bmatrix}^T$$
$$\mathbf{W} = \begin{bmatrix} W_{KN}^{00} & \cdots & W_{KN}^{0(M-1)} \\ W_{KN}^{10} & \cdots & W_{KN}^{1(M-1)} \\ \vdots & \ddots & \vdots \\ W_{KN}^{(KN-1)0} & \cdots & W_{KN}^{(KN-1)(M-1)} \end{bmatrix} \tag{17}$$

where $W_{KN}^{kn} = e^{-j\frac{2\pi kn}{KN}}$. Because

$$\widetilde{\mathbf{h}} = \mathbf{W}\mathbf{h} \tag{18}$$

Equation (15) can then be rewritten in matrix notation as

$$\widetilde{\mathbf{y}} = \widehat{\mathbf{X}}\mathbf{W}\mathbf{h} + \widetilde{\mathbf{n}} \tag{19}$$

Based on the above equation, the channel estimation problem, in the sense of LS, can be formulated as the following optimization problem

$$\min_{\mathbf{h}} \left\{ (\widetilde{\mathbf{y}} - \widehat{\mathbf{X}}\mathbf{W}\mathbf{h})^T (\widetilde{\mathbf{y}} - \widehat{\mathbf{X}}\mathbf{W}\mathbf{h}) \right\} \tag{20}$$

Solving this LS problem using the algorithms in [8] and substituting the solution $\mathbf{h}$ into (18), we can obtain an estimate of the channel attenuation $\{H_n\}$.

Summarizing the discussion above, we propose the following algorithm for solving the channel estimation problem:

Step 1: Choose suitable upsampling rate K and then discretize the received signal $y(t)$ with sampling rate $\frac{K}{T_s}$ to get the data sequence $\{\hat{y}_k\}$;

Step 2: Remove the cyclic prefix from $\{\hat{y}_k\}$ and then perform DFT on it to obtain $\{\widehat{Y}_k\}$;

Step 3: Solve the LS optimization problem (20) to obtain $\mathbf{h}$;

Step 4: Substitute $\mathbf{h}$ into (18) to obtain the estimate of channel attenuation $\{H_n\}$.

For the channel estimation approach based on the linear minimum mean square error criterion, the channel attenuation estimates $\{\widetilde{H}_n\}$ can also be obtained from (19). Suppose the channel is Gaussian channel and uncorrelated with the channel noise, then channel attenuation estimate is

$$\widetilde{\mathbf{h}} = \mathbf{C_{hh}} \mathbf{W}^H \widehat{\mathbf{X}}^H (\widehat{\mathbf{X}} \mathbf{W} \mathbf{C_{hh}} \mathbf{W}^H \widehat{\mathbf{X}}^H + \sigma_n^2 \mathbf{I}_N)^{-1} \widetilde{\mathbf{y}} \quad (21)$$

where $(\cdot)^H$ denotes Hermitian transposition, $\mathbf{C_{hh}}$ is the covariance matrix of $\mathbf{h}$, σ_n^2 is the channel noise variance.

IV. Simulation Results

In the simulation section, we consider a multipath time-variant Rayleigh fading channel. The number of propagation paths is $M = 5$, the channel impulse response and excess delay are $h_m = 0.5938, 0.2137, 0.3175, 0.7305, 0.4312$ and $\tau_m = 0, 0.167, 0.51, 0.751$ and $0.99\mu s$, respectively. The OFDM system is a 16-QAM OFDM system whose 4MHz bandwidth is divided into 16 subchannels, each subchannel has a bandwidth of 250KHz, the corresponding time-interval of the transmitted data sequence is $T_s = 0.25\mu s$, the guard interval T_g is $1\mu s$.

The channel attenuation $\{H_n\}$ and its estimate $\{\widetilde{H}_k\}$ at 16 subcarrier frequency points are shown in Fig. 2, where the mth subcarrier frequency f_m is $m \times 250$KHz. The estimate of channel attenuation $\{\widetilde{H}_n\}$ is obtained by solving the LS optimization problem based on (13), which implies that the data sequence $\{y_n\}$ is obtained by discretizing the analog signal $y(t)$ with sampling rate $\frac{1}{T_s}$, the channel is assumed to be ideal channel without noise. from the simulation results shown in Fig. 2, it can be seen that the difference between $\{H_n\}$ and $\{\widetilde{H}_k\}$ is obvious.

The channel attenuation estimate obtained by using the improved algorithm proposed in the paper is shown in Fig. 3, where the upsampling rate K is chosen as $K = 3$. The simulation result shows that the estimate is very close to the actual channel attenuation, the maximum error between them is about 0.147, the corresponding maximum error in Fig. 2 is about 0.322. Comparing the simulation results shown in Fig.2 and Fig. 3, it can be seen clearly that the proposed approach given in the paper has better performance than the existed methods.

V. Conclusion

In this paper, the channel estimation error based on the pilot-assisted approaches has been investigated. A new OFDM channel estimation scheme based on the filter bank theory is introduced. With the proposed scheme, a new effective channel estimation approach has also been provided. It is demonstrated that the proposed channel estimation method has lower estimation error than the existed approaches.

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