# A comparison of 2D and 3D Delaunay triangulations for fingerprint authentication

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# A COMPARISON OF 2D AND 3D DELAUNAY TRIANGULATIONS FOR FINGERPRINT AUTHENTICATION

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## Abstract

*The two-dimensional (2D) Delaunay triangulation-based structure, i.e., Delaunay triangle, has been widely used in fingerprint authentication. However, we also notice the existence of three-dimensional (3D) Delaunay triangulation, which has not been extensively explored. Inspired by this, in this paper, the features of both 2D and 3D Delaunay triangulation-based structures are investigated and the findings show that a 3D Delaunay structure, e.g., Delaunay tetrahedron, can provide more feature types and a larger number of elements than a 2D Delaunay structure, which was expected to provide a higher discriminative capability. However, higher discrimination does not necessarily lead to better performance, especially in biometric applications, when biometric uncertainty is unavoidable. Experimental results show that the biometric uncertainty such as missing or spurious minutiae causes more negative influence on the 3D Delaunay triangulation than that on the 2D Delaunay triangulation in three out of four experimental data sets.*

**Keywords:** Biometrics, Fingerprint, Delaunay triangulation, Delaunay tetrahedron

## INTRODUCTION

In computer security, personal authentication properties are derived from knowledge, ownership of an object or some physical property of the self, commonly expressed as something you know (e.g., a password), something you have (e.g., a smart card) or something you are (e.g., a fingerprint). It is the last of these properties that is of interest here. Biometrics, derived from the Greek for life (βιο) and to measure (μετρικός) is especially attractive as the other methods of authentication tend to be less secure. People choose poor passwords or misplace access cards. Fortunately, people do not need to remember their fingerprint and cannot lose it (except under somewhat extreme circumstances).

The fingerprint is a complex set of segments formed by ridges and valleys that are unique to each person. Fingerprint matching is a process used to classify and match the fingerprint based on a group of features that are extracted from fingerprint images and used during the comparison process to minimise the effects of biometric uncertainty or error. Biometric uncertainty, which is generated during the fingerprint image acquisition process by image distortion, translation, and rotation, is undesirable in fingerprint matching due to the fact that it can generate unreliable results as noted by Yang et al. (2015) and Wang et al. (2017). In other words, two fingerprint images never look exactly the same even if they are scanned one after another which can affect the reliability of the fingerprint extraction and comparison process.

To mitigate the negative influence of biometric uncertainty, especially non-linear distortion, many local structures have been explored. Examples include tree-based structures (Moayer and Fu, 1986), n-nearest minutiae based structures (Liu et al., 2011) and minutia-pair based structures (Ahmad et al., 2011). From these examples, Delaunay triangulation has shown to be an effective structure which has been subsequently widely applied in fingerprint-based authentication systems, as it is able to provide some favorable local and global features (Yang et al., 2014). For example, in a Delaunay triangulation, each minutia will form a stable structure with its neighbors. Even if there is a certain degree of non-linear distortion, the local structure is still invariant. Moreover, missing or spurious minutiae only influence the local triangles that contain those missing or spurious minutiae, thus the error is localised.

There is considerable literature using Delaunay triangulation to achieve promising matching performance and security. For instance, in Parziale and Niel (2004), a Delaunay triangulation net is formed by a set of minutia points. Several invariant features such as rotation and translation are extracted from each Delaunay triangle and utilised to conduct matching. In the method of Deng and Huo (2005), some edge pairs are considered as best-matching pairs in a first round of matching. Then a second round is conducted, with matching based on global features and a matching score is computed under the guidance of the best-matching edge pairs obtained in the first round. In Yin et al. (2005), some similar minutiae pairs are chosen from both template and query fingerprint images and considered as references for alignment. Then the aligned feature sets based on the references are used for fingerprint matching. There are other Delaunay triangulation-based fingerprint matching methods which can be found in Liang et al., (2006), Wang and Gavrilova (2006), Liang et al. (2007), Zhang and Yan (2007), Yang et al. (2012), Yang et al. (2014), and Yang et al. (2013).

All of the abovementioned Delaunay triangulation-based methods use only the 2D pattern of a Delaunay triangulation (as shown in Figure 1.a). However, the 3D Delaunay Triangulation (as shown in Figure 1.b) also exists as shown by Maur (2002), but it has not been extensively explored, suggesting that it might be a fruitful avenue to traverse as a research challenge and solution for application to real-world problems, e.g., fingerprint matching. Each unit, e.g., tetrahedron (a tetrahedron is called a 3D triangle in this paper), of 3D Delaunay triangulation, would be expected to have better discriminative ability than a unit, e.g., Delaunay triangle, of a 2D Delaunay triangulation, as the former have more properties. A tetrahedron can provide more feature elements than a triangle. For example, a 3D triangle has six edges, whilst a Delaunay triangle has only three edges (Frey et al., 1998) as shown in Figure 2 and Figure 3, respectively.

In this paper, the feature differences between 2D and 3D Delaunay triangulation-based structures are investigated. Also, through experiments, the effect of spurious and missing minutiae on both 2D and 3D Delaunay triangulation is studied, which lays a basis for further application of 3D Delaunay triangulation in fingerprint matching.

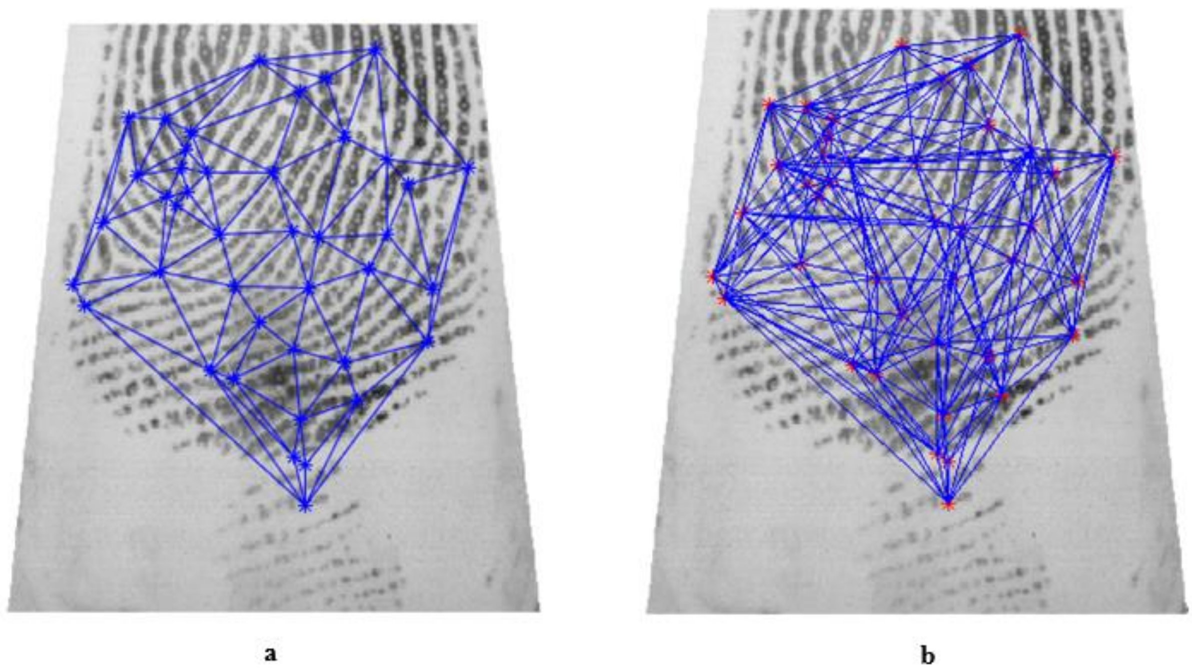

*Figure 1. An example of (a) a 2D Delaunay triangulation, and (b) a 3D Delaunay triangulation*

## FEATURE DIFFERENCES BETWEEN A 2D TRIANGLE AND A 3D TRIANGLE

### Definition of a 2D triangle and its features

In mathematics, a triangle is defined as a three-sided polygon which has a sum of 180 degrees for all its internal angles, as shown in Figure 2. A triangle is the simplest form of a polygon and has been studied and applied in areas such as construction due to its strength and precision. Any triangle belongs to one of three types, Equilateral, Isosceles or Scalene. Specifically, the Equilateral triangle has three sides of equal length and three equal angles, the Isosceles triangle has two sides of equal length and two equal angles, and the Scalene has no equal sides of equal length and no equal angles. Trigonometry studies the relationship between angles and sides of these shapes and there are many features that can be extracted from them as listed in Table 1.

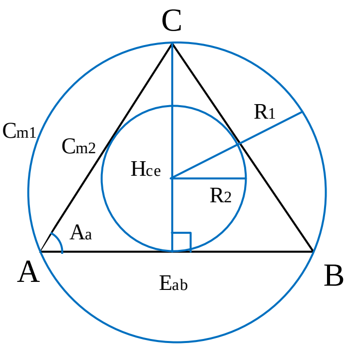


*Figure 2. An example of a 2D triangle*

*Table 1. Features that can be extracted from a 2D triangle*

| Feature type | Definition | # Elements |
|---|---|---|
| Edge | A line connects two points, e.g., $E_{ab}$ | 3 |
| Angle | The curvature of a combination of two lines with a common endpoint, e.g., $A_a$ | 3 |
| Radius of the inscribed circle | A straight line from the centre of the inscribed circle to the circumference, e.g., $R_2$ | 1 |
| Radius of the circumscribed circle | A straight line from the centre of the circumscribed circle to the circumference, e.g., $R_1$ | 1 |
| Altitude (Height) | The length of a line segment through a vertex and perpendicular to the opposite edge, e.g., $H_{ce}$ | 3 |

**Definition of a 3D triangle and its features**

A 3D triangle defined in this paper is a tetrahedron as shown in Figure 3. It is formed by the four triangular faces which will form a group of four vertices and six edges. Table 2 shows features that can be extracted from a 3D triangle.

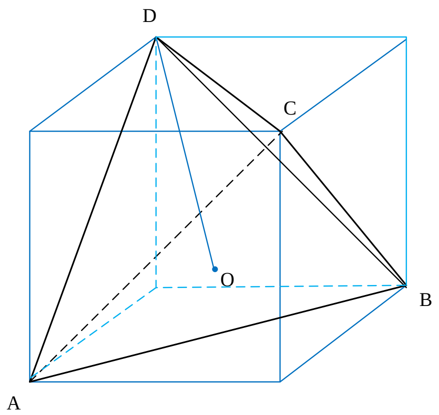


*Figure 3. An example of a 3D triangle*

*Table 2. Features that can be extracted from a 3D triangle*

| Feature type | Definition | # Elements |
|---|---|---|
| Edge | As per a 2D triangle in Table 1 | 6 |
| Angle | As per a 2D triangle in Table 1 | 12 |
| Radius of the inscribed circle | As per a 2D triangle in Table 1 | 1 |
| Radius of the circumscribed circle | As per a 2D triangle in Table 1 | 1 |
| Altitude (Height) of a face | As per a 2D triangle in Table 1 | 12 |
| Faces | A triangular shape, e.g. $\Delta_{ABC}$ | 4 |
| Altitude (Height) | The length of a line segment through a vertex and the perpendicular to a line containing the base shape, e.g., $H_{DO}$ | 4 |

From a comparison of tables 1 and 2, it can be seen that a 3D triangle can provide more features than a 2D triangle, in terms of feature type and number of elements, which means the former may be able to provide higher discrimination and lead to potential better matching performance in a 3D triangle based fingerprint authentication system than a system that uses 2D triangles.

## EFFECT OF BIOMETRIC UNCERTAINTY IN 2D AND 3D DELAUNAY TRIANGULATION

The aim of this section is to demonstrate how missing and spurious minutiae can affect the Delaunay triangulation, a well-known issue of fingerprint matching. The number of missing or spurious minutiae depends on the quality of the fingerprint image that is used during the minutiae extraction process and this factor will likely influence the matching performance.

In experiments, the fingerprint image "1_1.tif" from the public database FVC2002 DB2 was chosen as the experimental subject and a set of minutiae $M_p$ were extracted from it using the software Verifinger 4.0 SDK from Neurotechnology (Verfinger, 2010). A specific percentage of minutiae based on $M_p$ are randomly removed or added to simulate biometric uncertainty.

### Effect of missing minutiae to 2D and 3D triangle Delaunay triangulations

In this test, a certain number of points (10% and 20% of $M_p$) are randomly chosen and removed from the minutiae set $M_p$, the new minutiae sets are used to generate 2D and 3D Delaunay triangulations. Thus, four cases are generated:

**Case 1**: 10% of points are removed from minutiae set $M_p$ and the new minutia set is used to generate 2D Delaunay triangulation, $DT_{2D\text{-}M10}$.

**Case 2**: 20% of points are removed from minutiae set $M_p$ and the new minutia set is used to generate 2D Delaunay triangulation, $DT_{2D\text{-}M20}$.

**Case 3**: 10% of points are removed from minutiae set $M_p$ and the new minutia set is used to generate 3D Delaunay triangulation, $DT_{3D\text{-}M10}$.

**Case 4**: 20% of points are removed from minutiae set $M_p$ and the new minutia set is used to generate 3D Delaunay triangulation, $DT_{3D\text{-}M20}$.

The generated 2D Delaunay triangulations, e.g., $DT_{2D-M10}$ and $DT_{2D-M20}$, and 3D Delaunay triangulations, e.g., $DT_{3D-M10}$ and $DT_{3D-M20}$, in Cases 1 to 4, are shown in Figure 4, where the small yellow circles represent the points (10% of $M_p$) that are removed from the original set $M_p$ and the green ones represent the points (20% of $M_p$) that are removed from the original set $M_p$.

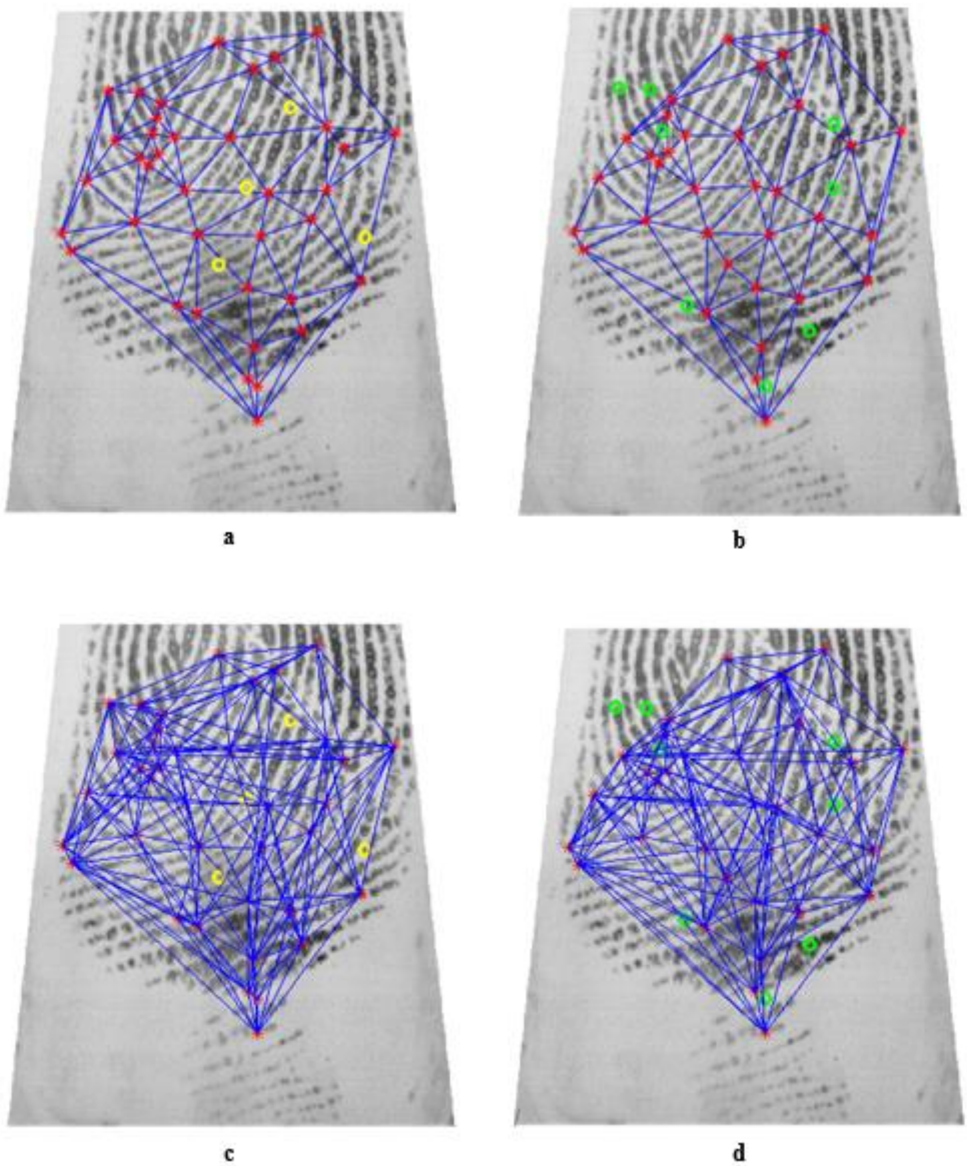


*Figure 4. The generated 2D and 3D Delaunay triangulations in Cases 1 to 4*

**Effect of spurious minutiae in 2D and 3D triangle Delaunay triangulations**

In this test, a certain number of points (10% and 20% of $M_p$) are randomly generated and added into the minutiae set $M_p$, the new minutia sets are used to generate 2D and 3D Delaunay triangulations. As before, there are four cases generated:

**Case 5**: 10% of points are added into minutiae set $M_p$ and the new minutia set is used to generate 2D Delaunay triangulation, $DT_{2D-S10}$.

**Case 6**: 20% of points are added into minutiae set $M_p$ and the new minutia set is used to generate 2D Delaunay triangulation, $DT_{2D\text{-}S20}$.

**Case 7**: 10% of points are added into minutiae set $M_p$ and the new minutia set is used to generate 3D Delaunay triangulation, $DT_{3D\text{-}S10}$.

**Case 8**: 20% of points are added into minutiae set $M_p$ and the new minutia set is used to generate 3D Delaunay triangulation, $DT_{3D\text{-}S20}$.

The generated 2D Delaunay triangulations, e.g., $DT_{2D\text{-}S10}$ and $DT_{2D\text{-}S20}$, and 3D Delaunay triangulations, e.g., $DT_{3D\text{-}S10}$ and $DT_{3D\text{-}S20}$, in Cases 5 to 8, are shown in Figure 5, where the small yellow circles represent the points (10% of $M_p$) that are added into the original set $M_p$ and the green ones represent the points (20% of $M_p$) that are added into the original set $M_p$.

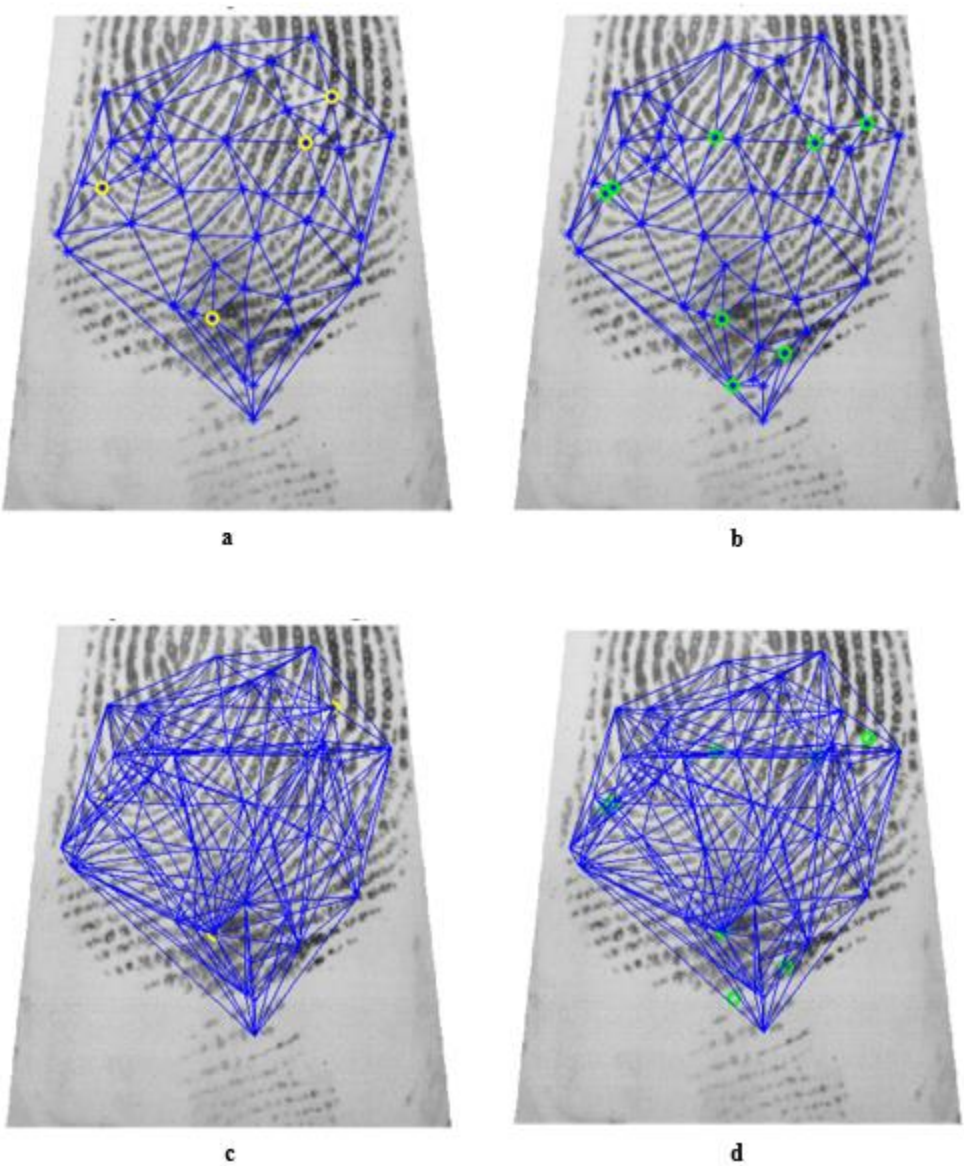


*Figure 5. The generated 2D and 3D Delaunay triangulations in Cases 5 to 8*

The effects of missing and spurious minutiae to 2D and 3D Delaunay triangulations are listed and compared in Table 3, from which it can be seen that when 10% of total minutiae are missing, compared with original number of units, 10.81% is reduced in 2D triangulation, while 13.37% is reduced in 3D triangulation, which means that missing minutiae have more negative influence on 3D triangulation than that on 2D triangulation. However, a contrary result is reported when 20% is missing. In the cases of spurious minutiae, it can be seen that spurious minutiae create more negative influence on 3D Delaunay triangulation with 11.05% and 24.42% of original number of units is increased (compared with only 10.81% and 21.62% in 2D Delaunay triangulation), when 10% and 20% spurious minutiae are added, respectively.

*Table 3- Comparison between 2D and 3D Delaunay triangulation under biometric uncertainty*

| Structure type | Units created with minutiae set $M_p$ | Units created with 10% points missing from $M_p$ | Units created with 20% points missing from $M_p$ | Units created with 10% points added into $M_p$ | Units created with 20% points added into $M_p$ |
|---|---|---|---|---|---|
| 2D triangles | 74<br>100% | 66<br>89.19%<br>(-10.81%) | 57<br>72.03%<br>(-27.97%) | 82<br>110.81%<br>(+10.81%) | 90<br>121.62%<br>(+21.62%) |
| 3D triangles | 172<br>100% | 149<br>86.63%<br>(-13.37%) | 133<br>77.33%<br>(-22.67%) | 191 3D triangles<br>111.05%<br>(+11.05%) | 214<br>124.42%<br>(+24.42%) |

## CONCLUSION

In this paper the feature differences between 2D and 3D Delaunay triangulation-based structures are investigated and it is demonstrated that a 3D triangle has more feature types and a larger number of elements, thus it may possess higher discriminative ability than a 2D triangle. However, the use of 3D Delaunay triangulation does not mean absolute better matching performance, especially under the presence of biometric uncertainty, e.g., missing or spurious minutiae. Biometric uncertainty is simulated by randomly adding or removing points from a generated minutiae set. The experimental results show that 3D Delaunay triangulation is more sensitive to the missing and spurious minutiae than a 2D Delaunay triangulation in three cases out of four. In future research, to explore and potentially reduce the negative influence of biometric uncertainty on the 3D Delaunay triangulation is needed.

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## REFERENCES

Ahmad, T, Hu, J. and S. Wang (2011). Pair-polar coordinate-based cancelable fingerprint templates, *Pattern Recognition*, 44 pp. 2555-2564.

Deng H. and Huo Q. (2005) Minutiae Matching Based Fingerprint Verification Using Delaunay Triangulation and Aligned-Edge-Guided Triangle Matching. In: Kanade T., Jain A., Ratha N.K. (eds) Audio- and Video-Based Biometric Person Authentication. AVBPA 2005. Lecture Notes in Computer Science, vol 3546. Springer, Berlin, Heidelberg.

Frey, P.J., Borouchaki, H., and George, P-L., (1998). 3D Delaunay mesh generation coupled with an advancing-front approach, *Computer Methods in Applied Mechanics and Engineering*, 157 pp. 115-131.

Liang, X, Asano, T and Bishnu, A. (2006). Distorted fingerprint indexing using minutia detail and Delaunay triangle, IEEE 3rd International Symposium on Voronoi Diagrams in Science and Engineering, pp. 217-223.

Liang, X, Bishnu, A and Asano, T. (2007). A robust fingerprint indexing scheme using minutia neighborhood structure and low-order Delaunay triangles, *IEEE Transactions on Information Forensics and Security*, 2 pp. 721-733.

Liu, E., Zhao, H., Liang, J., Pang, L., Xie, M., Chen, H., Li, Y., Li, P., and Tian, J. (2011). A key binding system based on n-nearest minutiae structure of fingerprint, *Pattern Recognition Lett*., 32 pp. 666-675.

Maur, P. (2002). Delaunay triangulation in 3D, Technical Report DCSE/TR-2002-02, Department of Computer Science and Engineering, University of West Bohemia.

Moayer, B. and Fu, K-S (1986). A Tree System Approach for Fingerprint Pattern Recognition, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, PAMI-8 pp. 376-387.

Parziale G. and Niel A. (2004) A Fingerprint Matching Using Minutiae Triangulation. In: Zhang D., Jain A.K. (eds) *Biometric Authentication. Lecture Notes in Computer Science*, vol 3072. Springer, Berlin, Heidelberg.

VeriFinger. (2010). VeriFinger, S. D. K. Neuro Technology.

Wang, C. and Gavrilova, M.L. (2006). Delaunay Triangulation Algorithm for Fingerprint Matching, 3rd International Symposium on Voronoi Diagrams in Science and Engineering, pp. 208-216.

Wang, S., Yang, W., and Hu, J. (2017). Design of Alignment-Free Cancelable Fingerprint Templates with Zoned Minutia Pairs, *Pattern Recognition*, 66. pp. 295-301.

Yang, W., Hu, J., and Wang, S. (2012). A Delaunay Triangle-Based Fuzzy Extractor for Fingerprint Authentication, 11th International Conference on Trust, Security and Privacy in Computing and Communications (TrustCom), IEEE2012, pp. 66-70.

Yang, W., Hu, J., and Wang, S. (2013). A Delaunay triangle group based fuzzy vault with cancellability, 6th International Congress on Image and Signal Processing (CISP), 2013, pp. 1676-1681.

Yang, W., Hu, J., and Stojmenovic, M. (2014). NDTC: A novel topology-based fingerprint matching algorithm using N-layer Delaunay triangulation net check, 7th IEEE Conference on Industrial Electronics and Applications (ICIEA), IEEE2012, pp. 866-870.

Yang, W., Hu, J., and Wang, S. (2014). A Delaunay Quadrangle-Based Fingerprint Authentication System With Template Protection Using Topology Code for Local Registration and Security Enhancement, IEEE *Transactions on Information Forensics and Security*, 9 pp.1179-1192.

Yang, W., Hu, J, Wang, S., and Chen, C. (2015). Mutual dependency of features in multimodal biometric systems. *Electronics Letters*. 51. 234–235.

Yin, Y., and Zhang, H., and Yang, H.K. (2005). A method based on Delaunay triangulation for fingerprint matching, Proc. International Society for Optical Engineering, 5779 pp. 274-281.

Zhang, Q. and Yan, H. (2007). Fingerprint orientation field interpolation based on the constrained delaunay triangulation, *International Journal of Information and Systems Sciences*, 3 pp. 438-452.