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Using Semiotic Resources to Build Images when Teaching the Part-Whole Model of Fractions

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This paper reports an exploration into the use of a combination of semiotic resources when teaching the part- whole model of fractions. The study involved a single case study of one class teacher and six students in an Australian primary classroom. Using video as the predominate research tool it was possible to describe how gesture and language were combined with two and three dimensional representations of folding paper, fraction walls and number lines to build images that appeared to enhance students understanding of the part-whole model of fractions. I conjecture that the variety semiotic resources including gesture should have more prominence in teacher's planning documents.

Traditionally, classroom-based research about effective teaching and learning has tended to focus on the analysis of audio recordings (Sfard, 2008). With the emergence of advanced digital video recording and analysis technologies, the subtleties and complexities of the learning process can more easily be captured, recorded, observed, and analysed - thus facilitating a deeper understanding of how learning takes place (O'Halloran, 2005). It was of interest therefore to the researcher, who lecturers in primary mathematics education at a tertiary level, to use advanced digital technology and a fine grained lens to investigate, on a micro level, how different semiotic resources such as gesture, two and three dimensional representations, and language were combined in order to build images of the part-whole model of fractions in the insitu setting of the classroom. In this article the author is reporting on a case study which explored how one teacher used a number of semiotic resources including gesture, language, and representations. These semiotic resources were at times planned by the teacher and at other times used spontaneously.

Conceptual Framework and Theoretical perspective

This study was underpinned by sociocultural theory, which emphasises the social setting of the classroom and the concept of a community of learners (Rogoff, 1995). Whereas in transmissive approaches, the role of the teacher is to transmit information, sociocultural theory emphasises how students engage in social activity before and during individualising knowledge for themselves (Jaworski, 1996). Overall, sociocultural theory makes the assumption that all mental actions "are inevitably situated in cultural, historical, and social settings" (Mortimer & Scott, 2003, p. 120). Within sociocultural theory, Sfard's (2008) theory of 'commognition' was central to framing this research study (2008); Commognition unifies thinking and communicating, perceiving learning as a participationist process in which realisations support the gradual deep understanding of the abstract mathematics.

Combining Semiotic Resources

Sociocultural theory emphasises that learning is mediated through tools such as concepts, language and artefacts, and as students master their use of tools their learning increases (Vygotsky, 1933). Palincsar (1994, p. 353) contends that "These semiotic means

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are both the tools that facilitate the co-construction of knowledge and the means that are internalized to aid future independent problem solving". Spoken language is the major semiotic resource in learning (Griffiths, 1994), and because this is connected with thinking and reasoning it lies at the heart of our understanding of the world. It is now emerging that gesture is also a component of thinking and explaining (Hwang & Roth, 2011; Radford, 2009). Gesture is very important as "physical gestures appear to be essential to teacher's ability to conduct their practice" (Alibali & Nathan, 2012, p. 20). A useful framework that delineates types of gestures, including pointing and metaphoric gestures, has been outlined by Alibali and Nathan (2012); this categorisation is pertinent when analysing the semiotic resources at play in the teaching – learning of mathematical concepts. An important point made by Lemke is the connection between drawings and gesture. Lemke (1990, p. 11) asserts that "Pictures are in some sense originally traces of gestures (in the sand, on the paper, on the bedroom wall with crayon) but with elaborate typological conventions". Whereas our understanding of the value of using semiotic resources is generally well established (O'Halloran, 2005), more research is necessary to further understand the role that gesture plays in combination with other thinking tools (Radford, 2009).

Teaching Fractions

Fractions are difficult to teach and challenging for students to learn (Harvey, 2012; Lemke, 1990). Different representations may be used in the classroom to co-construct the meaning of fractions (Ball, 1993). English and Halford (1995) assert that the representation of the fraction needs to be selected carefully in order to facilitate students' understanding because some models and constructs have limited valued in developing appropriate images in students' minds. Fazio and Sieglar (2011) suggest that when teaching the conceptual understanding of fractions procedures, using symbols, should be introduced alongside manipulatives. Whereas in the past mathematics educators recommended students learn mathematical concepts through one set of manipulatives in a prescribed way (Bruner, 1967), mathematics educators now assert that students should be introduced to a variety of models, creating a richer understanding. Sfard (2008) outlines how a rich understanding of a mathematical concept may be constructed through students engaging with different tasks or deeds that highlight different constructs of the mathematical concept. Clarke and Roche (2010) outlined how the rectangular area model is an effective representation to teach equivalent fractions. Mitchell (2012) cautions that the part- whole model of fraction should not be the only model of fractions that is introduced in the mathematics classroom.

Purpose and Research Question

In this research the author aims to inform mathematics educators about the nature of a teacher's use of particular representations when teaching fractions in a naturalistic setting of the classroom, and how certain combinations of semiotic resources allowed students to create a robust image of the part-whole model of fractions. The following research questions were constructed to guide the research:

What were the different combinations of semiotic resources used by the teacher to convey mathematical images?

How did these semiotic resources affect students' understanding of fractions?

Methodology

A case study methodology was adopted because it suited collecting multiple sources of data in a rich context (Yin, 2009). A case study is a bounded system which allows the researcher to gain an understanding of people, situations, events or programs involved (Yin, 2009). Within this approach there were elements of action research (McNiff & Whitehead, 2005). Video was the main data collection tool due to its the ability to capture object manipulation and gesture (Flewitt, 2005).The subjects of this research were a teacher and a small group of six students from a low-fee independent school in Metro Perth, Western Australia. The six students in Year six were withdrawn from their mainstream class for intensive mathematics tuition because they had experienced some difficulties in learning mathematics. The teacher was a female, experienced teacher who is responsible for supporting students who have numeracy difficulties. The teacher was selected due to her belief in the value of developing number sense and her interest and willingness to take part in the study.

Data Collection

Data were collected from multiple sources over two terms. Initially the teacher was interviewed about her long term planning that she perceived was appropriate to underpin this process. The headings for these were content areas to be covered, the main activity (key content and concrete representations), homework and assessment. Kathy then provided short-term plans with the headings of focus, resources, language, activities, key questions, and reflections.

The teaching of the three selected lessons from Year six was videoed; work samples were collected; and the researcher conducted semi-structured focus group interviews with the students about how they perceived the learning experiences and how the language, representations and drawings helped them to understand the various concepts. The three lessons chosen for the in-depth analysis were a sequenced set of fraction lessons that focussed on the part-whole model of fractions. The research focused on the images used by the teacher to illuminate a part-whole fraction. As the focus for the research is how teachers develop students conceptual understanding, gestures that were clearly identifiable as imitating a process of creating parts of a whole such as, folding, marking, and grouping were focussed on rather than other gestures such as pointing at symbols (Alibali & Nathan, 2012) .

Data Analysis

The data collected from the Year 6 classroom which captured the combination of representations, drawings, language, and gesture to teach fractions was analysed using an analytical approach taken and adapted from the Erikson's model (2006). It used computer software NVIIVO 9 to analyse the transcripts and Studiocode to support the video analysis. Initially using Studiocode software, themes or codes of gestures, representations, and drawings were the foci for this fine-grained analysis. NViivo 9 was then utilised to further analyse the discourse and work samples. At this stage the analysis progressed to focus on how these different semiotic resources were used simultaneously to support each other to create one image or metaphor i.e., cutting. Using the findings from the Studiocode and Nviivo analysis a matrix was created to facilitate this process. Finally the students' response to these learning experiences was analysed. To gain different perspectives the author presented these findings to the teacher for her comments and reflections.

Results

In this section the author presents some key findings that emerged from the analysis about the different combinations of semiotic resources used by the teacher to create mathematical images in the minds of the students.

A Rich Combination of Semiotic Resources

The semiotic resources identified in the delivery of these images were gesture, pre-designed paper representations, counters, drawings created before the lesson and drawings created during the lesson, language, and symbolic notation. One example of how various semiotic resources were combined in an holistic way to create a supported image of a single mathematical metaphor of a fraction was noted in the first lesson on fractions. In this section of the lesson the teacher provided a combination of language, gesture, drawings and folded paper which all contributed towards building an image of an area divided into equal portions. Initially the students created their own wall by using the semiotic resource of folding paper rectangles into different equal size pieces such as halves, quarters and eighths. The teacher then had the students play a pre-prepared fraction game that also used imagery of a wall with bricks, using the different area of the drawn bricks to represent the proportions of the fractions. This game is recommended by Clarke, Roche and Mitchell (2008) as useful for developing an understanding of fractions and equivalent fractions. When the students were playing the game it was noted that the teacher supported the creation of this wall image with spatial language: "*Can you see that space ? …. If I put a line on the quarter, can you see that it is equivalent to two eighths?"* (7/05/2012). As these verbal and concrete semiotic resources were being used, the teacher made the gestures of repeatedly moving her hands to show how she compared the two sections of the wall (7/5/2012). When asked about this activity the students were able to discuss the intended learning outcome of considering the physical relationships of fractions. As an example of the focus on the physical relationship, when asked about fractions Peter (a pseudonym) stated:

> We learned about folding fractions and we learned how a half could be three sixths. Ann[another student] stated how we can fold fractions , so when we roll the dice and roll one fourth, but you have already coloured them all in you can colour in two eighths (student gesturing at the stage the colouring in process of looking for an area that was equivalent) (10/9/2012)

On the way to successfully completing this mathematical task, the students physically compared the wall areas to find out which fractions were equivalents. This type of activity aligns itself with Sfard's definition of a deed (2008), in which students complete the mathematics within the context of the task only. The students went through the reasoning process that areas could be folded into different equal sized pieces and then that these areas could be compared to see that they actually are part of the same area. This suggests that using the game combined with language and gesture, was effective for creating one realisation (a procedure that links the abstract mathematics with the concrete task) of an area partitioned into an equal number of parts that can be compared with the same area portioned into a different number of equal parts. Even students who appeared to be nervous in the class were keen to discuss this process of comparing the equal areas. For example Mary, a student in the class, stated in the reflection of the lesson:

> When you colour all different rows and you haven't any row left you can colour in other row to make the same one (Mary gesturing the colouring action). (10/09/2012)

Generally the students in this research continually referred back to the physical act of comparing the two different areas. The teacher concurred with this observation in the data analysis. She also stated "the students are not multiplicative thinkers". It appears that the students did not progress to discussing this multiplicative relationship and the abstract mathematics.

In the second lesson (21/05/2012) the teacher began requesting that the students play the fraction wall game again. When discussing this game the teacher used a chopping gesture to support the image of the delineated fraction wall. The teacher then moved the lesson forward through discussion with the students concerning the difference between improper and proper fractions. At this stage the teacher started to incorporate linear images in her gestures. This is noteworthy as the teacher had not used this gesture in the previous lesson. When conducting a member check the teacher confirmed that it had been subconscious and it not in her planning (1/11/2012). The teacher used linear gesturing as she explained to the students the improper fractions are "greater than one"[she moves her hand along an imaginary line]. She repeated, using the linear gesturing, to explain "when we talk about mixed number fractions it's got to be greater than one because it's a mixture of whole and fractions so I can have one (gestures a fraction on a line and then moved her hands forward] and a half " (21/05/2012).

Whilst not a planned semiotic resource, this linear gesturing then acted as an advance organiser for the students' thinking when they later explored the task where they had to use a number line. The teacher then used the actual paper number line and used her language, gestures to create an image of a line divided into equal parts.

> **Teacher:** You show me what, so where's the whole one, where's one whole? (Teacher gesturing and pointing to the paper model of the number line)
> **Jack:** Like one quarter and one quarter.
> **Teacher:** No. That's one whole. That's one whole there (pointing), remember I said to you the thick line, if you want to write the number 1 up there you can for 1 whole, 2 wholes, 3 wholes. Where's one quarter?
> [Josh points to one quarter on the number line]
> **Teacher:** If you add another quarter to it how many quarters do you have?
> **Josh:** 2.

The Use of Unplanned Semiotic Resources in the Fraction Lesson

In the first lesson when the teacher was revising and recapping the students' general understanding of part-whole, it appeared that the teacher spontaneously decided to create a visual representation to support her verbal explanation, namely a drawing on the white board of equal parts in a square. This model was used to reflect the teacher's own thinking. The teacher used her gestures to revise the process of creating a representation of a whole and then dividing it equally into pieces. In her reflection of the lesson, the teacher stated that she perceived it as important for the students to see the process of her actually splitting the shape up into equal pieces. The flexibility of this semiotic resource allowed it to perfectly match the teacher's verbal explanation. Despite the teacher modelling this, the students struggled to represent 4/3 using these two dimensional drawings. They appeared to not understand that each square represented the "whole". It may be the case that the children had not had a lot of experience drawing these types of images themselves and this may be the reason why they struggle to complete the drawing correctly. The teacher reflected in the member check that she had noticed that the students did not have enough experience with this type of drawing and has purchased some small magnetic white board to facilitate this (1/11/2012).

Discussion and Conclusion

In this research it was found that when teaching about the part-whole concept of fractions, images were successfully created when the teacher used a combination of semiotic resources including gestures, concrete models and language to create images of folding and cutting paper, or marking and comparing areas or lines equally. This finding supports the notion that mathematics teaching and learning is an holistic and social process and that all of these resources are important. The research findings presented above concurs with more recent research findings (Thomas, 2008) which suggests that it is important to highlight the complex use of a number of different semiotic resources such as gesture and language as well as concrete materials that are used concurrently in the classroom by teachers to scaffold children's creation of an image or realisation (Sfard 2008). An implication of these research findings is that as primary school teachers plan learning experiences they need to identify how the use of different semiotic resources including concrete materials, drawings, language and gesture may be interwoven so that the mathematical images created for students are as rich as possible.

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