2013

Effectiveness of teacher professional learning : enhancing the teaching of fractions in primary schools

Derek Hurrell
Edith Cowan University

USE OF THESIS

The Use of Thesis statement is not included in this version of the thesis.

EFFECTIVENESS OF TEACHER PROFESSIONAL LEARNING: ENHANCING THE TEACHING OF FRACTIONS IN PRIMARY SCHOOLS

Derek P. Hurrell

M. Ed. (Edith Cowan University)

Submitted in fulfilment of the requirements of the Doctor in Philosophy

School of Education
Faculty of Education and Arts

Edith Cowan University

March 2013

Abstract

This study was motivated by the need to develop professional learning for primary school teachers that would support them to more effectively teach the mathematics topic of fractions. What seemed evident, was that previous professional learning attended by teachers had not adequately met their needs.

The aim of this study was to investigate whether professional learning, with a focus on subject content knowledge, pedagogical knowledge and reflective practice could enhance primary school teachers' PCK for teaching fractions and make them more confident teachers of fractions. Demonstrating this to be the case would have wide implications for the development of professional learning opportunities for in-service teachers and would also be highly beneficial in informing teacher education.

This study brought together teachers from a variety of backgrounds and experiences. These experiences comprised not only what they had encountered in their teaching of mathematics, but also what they had encountered in their learning of mathematics. Therefore a study of the affective elements of attitudes, beliefs and self-efficacy were not only warranted, but pivotal.

The professional learning was conducted over an extended period of time and the teachers were involved in workshops where clear links were explored between the required content and what the current research considered to be the most efficacious pedagogy. They were then required to take at least one of the activities from the workshops and use it in their classroom. After they had taught the lesson, they were asked to reflect upon the lesson and bring those reflections to the next session to share with the group. This cycle was repeated.

This research showed that the professional learning amplified both Pedagogical Knowledge (PK) and Subject Matter Knowledge (SMK), which in turn provided pathways to increased PCK. The results also indicated that well-structured professional learning can have a positive effect on the beliefs and attitudes of teachers towards teaching the difficult mathematical topic of fractions. This improvement in attitudes and beliefs is important, as the impact of efficacy on the teaching and learning of mathematics cannot be underestimated.

COPYRIGHT AND ACCESS DECLARATION

I certify that this thesis does not, to the best of my knowledge and belief:

(i) incorporate without acknowledgement any material previously submitted for a degree or diploma in any institution of higher education;

(ii) contain any material previously published or written by another person except where due reference is made in the text; or

(iii) contain any defamatory material.

I also grant permission for the library at Edith Cowan University to make duplicate copies of my thesis as desired.

Signed……………………………………..

Date……………………………………….

ACKNOWLEDGEMENTS

Anyone who takes on a commitment such as writing a doctoral thesis knows that it requires a team of people to make it happen even if it’s only one person who gets to put their name to the finished product.

First and foremost my thanks and love go out to my family. Without the support of Sandra and my children Chris and Sarah and their forbearance at me disappearing into my room to do my writing, this endeavour would have been impossible.

I cannot start to express my thanks to Dr Christine Ormond, Professor Mark Hackling and Dr Paul Swan who have all played a part in helping me craft what I hope will be a useful addition to the ever growing knowledge of something of which I am very passionate, teaching and learning. If there is anything of worth in this document they have probably had a big hand in it, if there are things which are of lesser worth it’s probably because I wasn’t clever enough to properly heed their advice.

To my colleagues over the span of the life of this thesis many thanks for being a sympathetic ear or a critical eye, your advice and friendship has always been greatly appreciated. A special thanks to the Dean of Education Michael O’Neil who has always provided the support and the culture that valued this work.

Finally, a big thank you to all of those wonderful teachers who joined me on this journey. It is always a privilege to be able to work with some of our sensationally good teachers who invest their lives in trying to provide the very best of education to our children.

Table of Contents

List of Tables

List of Figures

CHAPTER 1

1 Background and Introduction to the Study

1.1 *Background*

This study arose from many conversations with teachers at various professional learning (PL) sessions. When asked about areas of need, the teachers often raised the content area of fractions as being problematic. More precisely they wanted well researched guidance about how they could improve their own and their students' knowledge about fractions.

From the perspective of providing the support to teachers it became quite clear that the professional learning which had previously been made available by a number of agencies, both pre-service and in-service, had not adequately served these teachers' needs. The teachers had two fundamental questions about the teaching of fractions, "What do I teach and how do I teach it?" It would seem that many teachers were lacking critical aspects of Pedagogical Content Knowledge (PCK). Careful consideration needs to be paid to how the content and pedagogy could be provided for the teachers in order that PCK on this difficult topic might be developed.

This study aims to investigate whether PL, with a focus on subject content knowledge, pedagogical knowledge and reflective practice can enhance primary school teachers PCK for teaching fractions and make them more confident teachers of fractions. Demonstrating this to be the case would have wide implications for the development of professional learning opportunities for in-service teachers and would also be highly beneficial in informing teacher education.

Shulman's (1986) seminal work on PCK offered thee domains which needed to intersect for PCK to be evident. These were content, pedagogy and context. It is therefore important to consider the prevailing educational context in which this study was executed.

Education in Western Australia, Australia and the world is constantly under review. At the outset of this study Western Australia's education climate was much affected by the employment of Outcomes Based Education (OBE) and a Federal Government push towards a national curriculum. These two major curriculum reforms necessitated a searching look at the curriculum and the expectations for education that such reforms bring.

In Western Australia Outcomes Based Education (OBE) was articulated in a document called the Curriculum Framework (Curriculum Council, 1998) which stated as its purpose:

> The Curriculum Framework sets out what all students should know, understand, value and be able to do as a result of the programs they undertake in schools in Western Australia, from kindergarten through to Year 12. Its fundamental purpose is to provide a structure around which schools can build educational programs that ensure students achieve good outcomes.
>
> It is neither a curriculum nor a syllabus, but a framework identifying common learning outcomes for all students… (p. 6)

The Curriculum Framework articulated seven strands in the learning area of Mathematics (Appreciating Mathematics, Working Mathematically, Number, Measurement, Chance and Data, Space and Pre-Algebra/Algebra). The topic of fractions was placed in the Number strand.

Because OBE did not have an accompanying syllabus document, but instead produced Curriculum Guides which outlined what the students should learn in relation to their own development rather than against their age, many parents, teachers and the community at large were concerned that the system might be 'dumbing down' education. The concern was that students in general were not receiving an education that was academically rigorous and no learning area received more attention than did mathematics. The whole notion of the lack of rigour was heavily supported by the print, television and radio media. For some quite vocal sections of the community Donnelly, 2007; Swan, 2.13), the only way to assure the rigour was to have students providing 'pen and paper' records of achievement where the correct answer was paramount and where instruction was given in quite formal ways, usually supported by a text book. In this manner, OBE, which was about the students

developing and applying understanding, at their own developmental rate, was in conflict with the more traditional and populist view.

Further, a good deal of competitiveness between schools was developing with regards to school results. There was a growing trend for the Year 12 results to be aggregated, and for 'league tables' to be made available to the community for the sake of 'transparency.' One of the key indicators of a school's 'success' was the number of students who scored above a benchmark in the different learning areas, and mathematics was no exception. There were also regular mandatory benchmark assessments for all students. In Western Australia these were the Western Australian Literacy and Numeracy Assessments (WALNA) in Year 3, Year 5 and Year 7 and the Monitoring Standards in Education (MSE) assessment in Year 9. These were state wide benchmarked assessments and schools received a comprehensive statement as to how they were faring.

Both WALNA and MSE had a great deal of currency in the community and therefore schools were very aware of their performance in them. A school's reputation could be greatly enhanced through a strong performance. Alternately a poor performance could negatively affect the enrolments that a school enjoyed, as parents were less likely to want to send their children to a school which did not feature positively in the 'league tables'. Hence there was a divide between the OBE pedagogy and philosophy and the 'political' reality to which schools were forced to accede.

Shortly after, due to pressure from a variety of sources, syllabus documents were introduced into the Western Australian education system for all schools (DETWA, 2007). These documents were supposed to be an adjunct to the mandated Curriculum Framework but were not in themselves mandated. These documents spoke of what teachers needed to teach in each year level of compulsory schooling through to Year 10. With the 2008 introduction of the national NAPLAN (National Assessment Program - Literacy and Numeracy tests) taking the place of the WALNA testing, increased accountability pressures on schools for student achievement in mathematics.

Towards the conclusion of this study the Australian Curriculum (AC) was introduced through the Australian Curriculum, Assessment and Reporting Authority (ACARA). The goal of this curriculum was to provide a consistent curriculum across all of the states and territories of Australia. In the a newspaper article from Barry McGaw (2010) the Chair of ACARA

> The Australian curriculum sets out students' learning entitlements – the knowledge, understanding and skills that all students should have the opportunity to acquire. It does not prescribe how teachers should organise their students' learning but it offers suggestions in 'content elaborations' of ways in which teachers might develop ideas.
>
> (McGaw, 2010)

Once again the curriculum spelled out what content was required but not how to teach it. The documents themselves (ACARA, 2012) have a very heavy emphasis on understanding and a review of the rationale reveals that a preponderance of the key words was centred on understanding and the skills with algorithmic aspects demanding far less prominence. However, there was and is a tension that comes from the ever increasing importance placed upon students obtaining strong results in NAPLAN. Therefore, any move to take students and teachers away from the more formal and traditional 'pen and paper' instruction requires a good deal of strong evidence. This evidence must support that any alternate methods of teaching and learning provides students with better understandings.

1.2 *Research Problem*

This research was based around the need to improve the teaching and learning of the important topic of fractions. As professional learning (PL) is the predominant way in which teachers add to their knowledge and understanding, it is therefore imperative that the provided PL be effective. In order to be effective the PL should be action research based to encourage the teachers to be curious about, collect data regarding, and then create alternative pathways to improve their practice, (Teddlie & Tashakkori, 2009) it should also be predicated on some well-established general principles of PL (Clarke, 2003; Cohen & Hill, 2000; Loucks-Horsley, Hewson, Love & Stile, 1998; Supovitz & Turner, 2000) and result in an improvement of teachers' capacity to teach this demanding topic.

The challenge is developing a PL design that effectively improves teachers' PCK and capacity for teaching fractions. Can this capacity be improved through a focus on developing content and pedagogical knowledge under a lens of reflective practice and thereby linking content and pedagogy to develop PCK? Therefore, a PL design needed to be established which developed content and pedagogy, both based around evidence based best practice. This needed to be done in such a way as to develop the confidence of the teachers in the efficacy of the PL and consequently develop confidence in their ability to teach the topic.

1.3 *Rationale and Significance*

Fractions are an important mathematical topic (Booth & Newton, 2012; Brown & Quinn, 2007; Chinnappan, 2005; Wu, 2001) with applications in other areas of mathematics and in contributing towards being a numerate person. When this Researcher asked teachers about the more problematic areas of mathematics in both teaching and learning, understanding of fractions was often mentioned. It is probably no coincidence then, that when teachers were asked to explain why they teach fractions and what fractions are, the answers varied markedly, and the question was answered in a rather cursory manner.

One of the questions pursued at the start of this study was to ascertain what the research tells teachers about why it is necessary to teach fractions. This information was remarkably difficult to find, considering the seemingly unanimous voice regarding their difficulty to teach and learn. Whether it is assumed by most authors that the reader comes to the text with an innate understanding of why we teach fractions, or that the writer wants the reader to come to their own conclusion, is not clear. However, what is clear, is that to find information on the reasons as to why we teach fractions, and more importantly why students should learn fractions, would be a challenge to most time-poor teachers. If due to a lack of information and knowledge teachers see fractions of little or no importance, then they may be reluctant to afford them the time and effort required to develop the conceptual understanding for themselves and for their students.

Whilst many teachers are themselves able to recite ‘rules’ for dealing with operating with fractions, for example, “to divide fractions we invert and multiply,” very few would understand the specialised mathematical knowledge of fractions which makes it necessary to apply such a rule. Their understanding about fractions is predominantly operational without a clear conceptual understanding to underpin it. As Nunes and Bryant (2009) state

> ... studies show that students can learn procedures without understanding their conceptual significance. Studies with adults show that knowledge of procedures can remain isolated from understanding for a long time: some adults who are able to implement the procedure they learned for dividing one fraction by another admit that they have no idea why the numerator and the denominator exchange places in this procedure.
>
> (p. 5)

It is important that when attending PL on the topic of fractions, the participants are engaged in learning which acknowledges fractions as being worthy of attention and encourages them to engage with content and pedagogy which develops conceptual understandings. Such engagement should then help the teachers to answer the question, “What do I teach and how do I teach it?” questions which are fundamental in the application of Pedagogical Content Knowledge (PCK). (PCK will be discussed in detail in sections 2.15, 2.16 and 2.17.)

This study aimed to determine whether the PCK of primary school teachers of mathematics could be enhanced by applying reflective practice to professional learning focussed on content and pedagogy. A synthesis of the literature established the importance on: PCK (Shulman, 1986; Hill et al., 2008) for teachers; the importance of subject content knowledge (Ambrose, 2004; Ball, Thames & Phelps, 2008; Charalambous et al., 2012; Cobb & Jackson, 2011; Hill et al., 2008; Toluk-Uçar, 2009); pedagogical knowledge (Ball, Thames & Phelps, 2008; Hill et al., 2008; Park & Oliver, 2008; Shulman, 1986) and reflection on practice (Park & Oliver, 2008; Barkatsas & Malone, 2005). This study aims to make some significant contributions to research about the teaching of fractions.

An area of significance in this study can be drawn from the empowerment of teachers through the production of effective PL experiences. PL experiences which have a focus on content and pedagogy, and the PCK which is exercised when these two domains complement each other.

At present there is a body of research that supports the development of PCK as an effective way in which to improve teaching and learning. However, the majority of this work has been completed in the disciplines of science education with a little in the social sciences. The work linking PCK and mathematics teaching and learning is embryonic and mostly theoretical in nature. Further, as Hill, Blunk, Charalambos, Lewis, Phelps, Sleep and Ball (2008b) state, two major shortcomings of research into the field of mathematical knowledge for teaching have been the focus on one mathematical topic in the context of one lesson and the analysis of this through the 'lens' of one teacher. This study was concerned with many contributing elements, over an extended number of lessons with a group of teachers from a variety of backgrounds and bringing with them a variety of experience.

Further, the implications of demonstrating the effectiveness of this PL design is that it could also be beneficial in informing teacher education and perhaps stimulating further research to see if a similar design would benefit these prospective teachers. The implications are also wide-reaching for teachers of mathematics, if this approach is proven useful in one of the more problematic teaching areas in the mathematics syllabus. Through electing to work with the teachers on the specific concept of fractions, a topic which the research has established as being particularly conceptually difficult, it is expected that success in this project could lead to similar success in other primary or lower secondary mathematics topic. At the very least, this study aims to give impetus to further studies on the efficacy of such an approach.

1.4 ***Purpose and Research Questions***

The purpose of this study is to investigate whether PL, with a focus on content knowledge, pedagogical knowledge and reflective practice can enhance teachers PCK for teaching fractions and make them more confident teachers of fractions. More specifically the study attended to the following research questions:

1. What is the current status of teaching fractions in middle and upper primary school classrooms in Western Australia?

2. What impacts will well-structured, action research based, professional learning opportunities and reflective practice have on primary school teachers' content knowledge of fractions?
3. What impacts will well-structured, action research based, professional learning opportunities and reflective practice have on primary school teachers' pedagogical knowledge of teaching fractions?
4. What impacts will well-structured, action research based, professional learning opportunities and reflective practice have on primary school teachers' beliefs and attitudes with regards to teaching mathematics in general and fractions in particular?

1.5 *Definition of Terms*

For the purpose of this study the following definitions will be adopted when discussing fractions.

Table 1.1

Definition of terms

Term	Definition
Real Numbers	All numbers on the number line. This includes but is not limited to positive and negative numbers, integers and rational numbers, square roots, cube roots, etc.
Rational Numbers	A number which can be expressed as a fraction or a ratio of integers. e.g. ⅔ ⅝. Each rational number can be written in infinitely many ways, for example 3 / 6 = 2 / 4 = 1 / 2.
Whole Numbers	Zero and all counting numbers. e.g. {0, 1, 2, 3, 4, 5, 6 …}. Non-negative integers.
Natural Numbers	Counting numbers, either positive integers {1, 2, 3, 4…} or non-negative integers {0, 1, 2, 3, 4…}
Irrational Numbers	Numbers which cannot be written as integers or ratios. e.g. π or $\sqrt{2}$.
Integers	Positive or negative whole numbers including zero. e.g. {-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5…}.
Unit Fractions	A unit fraction is a fraction whose numerator equals 1 and its denominator is greater than 1.
Common/Integer/Simple/ Vulgar Fraction	A fraction $\frac{n}{d}$ where $n>1$ and $n<d$.

1.6 *Summary*

This study arose from the Researcher's contact with practising teachers, who expressed a desire to improve their teaching of fractions. Many of these teachers were skilled practitioners who had experience, previous professional learning and access to a variety of curriculum documents. Regardless, they felt their classrooms were perhaps not proving to be the most effective learning environment for this topic.

This study therefore had the intention of investigating whether PL, with a focus on content knowledge, pedagogical knowledge and reflective practice can enhance teachers' PCK for teaching fractions and make them more confident teachers of fractions. Demonstrating this to be the case would have wide implications for the development of professional learning opportunities for in-service teachers and would also be highly beneficial in informing teacher education.

The following chapter reviews the available literature to inform the study and provide background on themes and topics such as: fractions, what they are and why we should teach them; the causes of difficulties in teaching and learning fractions; the roles of teachers, texts and representations in the teaching of fractions; professional development; what knowledge teachers required to teach mathematics; and the roles of beliefs, attitudes and self-efficacy.

CHAPTER 2

2 Literature Review

The literature review provides a context to the study and informs the conceptualisation and design of the research. The literature review considers: fractions how they are taught and learned; difficulties in learning fractions; teachers and the different resources available to them to teach fractions; pedagogical content knowledge and its domains; the affective domain and how it impacts on the teaching and learning; and, professional learning. A conceptual framework which emerges from the literature is then described.

2.1 *What is a Fraction?*

This study will concentrate on dealing with the rational numbers thought of as fractions, and only those fractions which are represented as common/integer fractions. Decimal fractions and ratios will not be the focus of this study, although reference will be made to them.

Zevenbergen, Dole and Wright (2004) express a connection between whole and rational numbers (Figure 1.1). They assert that whole number understanding provides the foundation for the understanding of rational numbers.

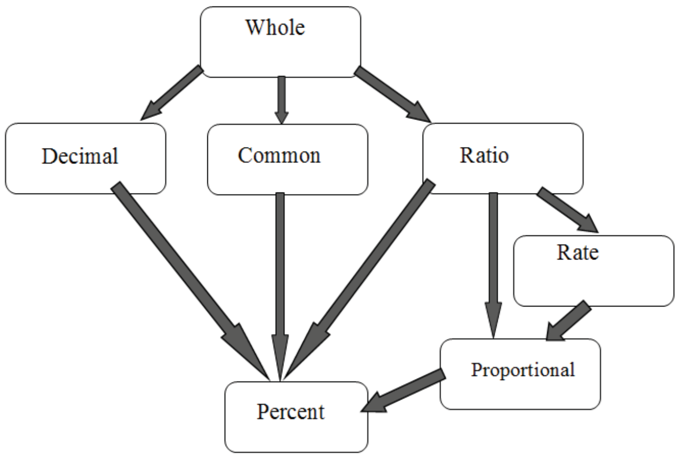


Figure 1.1 Zevenbergen, Dole and Wright (2004) whole number and rational number connections

Rational numbers take the form of a /b where a and b are natural numbers and where b is not zero and can be thought of in terms of fractions (both decimal and common), ratios and proportions. It is therefore useful to determine the difference between the three terms. Because of the similarity in the symbolic representation of fractions, ratios and proportions, it is advantageous to illustrate the differences that the context of a situation can bring to the reading of the symbols.

Smith (2002) uses the term 'quotient' to name any numeral which is ambiguous because the context has not been set. Where the context has been set and the quotient obviously refers to a quantity that has been divided into some number of equal sized parts, this can be called a fraction (e.g. ¾ = three parts out of a total of four equal parts). In this format it is generally referred to as a common fraction. A decimal fraction is a fraction where the denominator is a power of 10. Decimal fractions are commonly expressed without a denominator, the decimal separator being inserted into the numerator (with leading zeros added if needed), at the position from the right corresponding to the power of 10 of the denominator. For example, 8/10 is expressed as 0.8, 73/100 as 0.73, 64/1000 as 0.064 and so forth.

When a quotient refers to the multiplicative relationship between two quantities (e.g. a ratio of 1 pencil to 3 pens, implies there are three times as many pens as pencils) it is then a ratio; and when we have an equation with a ratio on each side (a/b = c/d or 3/4 = 6/8) then we refer to this as a proportion.

2.2 ***Why Teach Fractions?***

So why are factions so important for students to learn? Siemon writes;

> It is no longer acceptable that students leave school without the foundation knowledge, skills and dispositions they need to be able function effectively in modern society. This includes the ability to read, interpret and act upon a much larger range of texts than those encountered by previous generations. In an analysis of commonly encountered texts, that is, texts that at least one member of a household might need to, want to, or have to deal with on a daily, weekly, monthly

> or annual basis approximately 90% were identified as requiring some degree of quantitative and/or spatial reasoning. Of these texts, the mathematical knowledge most commonly required was some understanding of rational number and proportional reasoning, that is, fractions, decimals, percent, ratio and proportion. An ability to deal with a wide range of texts requires more than literacy - it requires a genuine understanding of key underpinning ideas and a capacity to read, interpret and use a variety of symbolic, spatial and quantitative texts. This capacity is a core component of what it means to be numerate.
>
> (2003, p. 16).

Where it is relatively easy to think of significant applications for proportional reasoning and ratios (missing value problems, linear equations, rates, scaling, co-ordinate graphs etc.) which have both 'real life' and mathematical import, examples of the significant use of common fractions are a little less obvious. Certainly there are applications in the areas of equivalent fractions, long division, percentage, place value, measurement conversion and algebra which occur in late primary and early secondary mathematics classrooms. It can be quite a challenge to convince most teachers and students about the importance of this area of mathematical understanding. For instance, the reading of analogue clocks, in particular the key times of a quarter past, a half past and a quarter to, all hold a bit less importance with the advent and uptake of digital time pieces. It is indeed difficult to think of many situations where other than the use of perhaps unit fractions and a few key common fractions (perhaps ¼, ½, ⅓, ¾ and ⅔), using decimal fractions or percentages would not appear to be more illustrative.

Nunes and Bryant (2009) claim that in primary schools there are only two types of situations where fractions are employed, these being measurement and division. When we measure, we often have to describe the object being measured in whole units and fractions to represent parts of the unit. In the division situation we use fractions to represent a quantity when the dividend is smaller than the divisor. This is a fact that when stated seems obvious, but the connection may not always be made in the mind of the teacher and therefore is less likely to be made in the mind of the students.

In 1983 when writing about rational numbers for the seminal *Rational Number Project* (RNP), Behr, Lesh, Post, and Silver wrote:

> Rational-number concepts are among the most complex and important mathematical ideas children encounter during their pre-secondary school years. Their importance may be seen from a variety of perspectives: *(a)* from a practical perspective, the ability to deal effectively with these concepts vastly improves one's ability to understand and handle situations and problems in the real world; *(b)* from a psychological perspective, rational numbers provide a rich arena within which children can develop and expand the mental structures necessary for continued intellectual development; and *(c)* from a mathematical perspective, rational-number understandings provide the foundation upon which elementary algebraic operations can later be based.
>
> (p. 1)

Whilst this is a neat summary there is little in the remainder of the research paper to justify their assertions regarding these important mathematical ideas; neither practical 'real world' situations nor school based situations are expounded upon. Therefore, for many teachers it is almost a leap of faith that they teach fractions not necessarily for immediate application and understandings, but rather as a valuable basis for further learning and as scaffolding for important conceptual frameworks.

Mathematics curricula from around the world address the issue of fractions, which is a topic which has long been documented to cause students difficulties (Anthony & Ding, 2011; Anthony & Walshaw, 2003; Capraro, 2005; Carpenter, Corbitt, Kepner, Lindquist & Reys, 1981; Cramer, Behr, Post & Lesh, 1997; Mack, 1995; Nunes & Bryant, 2009; Usiskin, 2007; Watanabe, 2002; Wu, 2005). Smith (2002, p. 3) asserts; "No area of school mathematics is as mathematically rich, cognitively complicated, and difficult to teach as fractions, ratios and proportion." The National Assessment of Educational Progress Report, which was published in the United States of America in 2001, declares that fractions are "exceedingly difficult for children to master" (Braswell et al., p. 5). Not only do the studies show that fractions are difficult to learn but also that students are frequently unable to remember prior experiences about fractions covered in previous years (Groff, 1996).

Research tells us that students enter school with an already developing concept of fractions (Empson, 2002; Meagher, 2004; Nunes & Bryant, 2009; Sharp, Garafolo & Adams, 2002), gathered from his or her 'real life' circumstantial knowledge (Mack, 1995). Smith (2002) expresses it as such

> …the need for fractions and the development of action sequences to generate them arise quite early in children's social activities with physical objects. Often objects (like cookies) are desirable and scarce and therefore must be divided up and shared. 'Fair' sharing quickly leads to the necessity of parts of equal size.
>
> (p. 5)

This understanding should be consistently further developed in schools and indeed in many cases is being addressed. Yet research by Chapin and Johnson (2000) concluded that "…this complex topic causes more trouble for elementary and middle school students than any other area of mathematics" (p. 73). Indeed Baba (2002) found that some university students could not understand fractions. This sort of fractional understanding is part of a wider understanding of rational numbers.

Chinnappan (2005) stated that "fractions provide teachers with an insight into developments in children's understanding of numbers and relations between numbers", and, that they "provide important prerequisite conceptual foundations for the growth and understanding of other number types and algebraic thinking" (p. 241). Apart from these number based aspects of mathematical understanding, the topic of fractions also supports students to make critical conceptual links in such strands as space and measurement (Pitkethly & Hunting 1996).

Booth and Newton (2012), Brown and Quinn (2007) and Wu (2001) strongly link a robust knowledge of fractions to success in algebra. Wu (2001) states:

> I will argue in this paper that no matter how much "algebraic thinking" is introduced in the early grades and no matter how worthwhile such exercises might be, the failure rate in algebra will continue to be high unless we radically revamp the teaching of fractions and decimals.
>
> The proper study of fractions provides a ramp that leads students gently from arithmetic up to algebra.
>
> (p. 1)

Further Brown and Quinn (2007) espouse that fractions and algebra are closely linked and that much of the basis of thinking algebraically is based upon a clear understanding of, and ability to, manipulate fractions. Indeed the National Mathematics Curriculum Framing paper (ACARA, 2008) uses fractions to highlight the need to identify the more important topics to teach.

2.3 ***Student Understanding of Fractions***

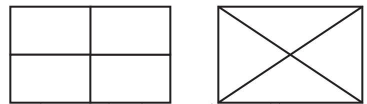
Students arrive for their formal schooling with a developing concept of fractions, ratios and proportionality (Mamede, Nunes & Bryant, 2005; Smith, 2002) and this is in part due to the development of the important concept of sharing (Sharp, 1998). This is sometimes called systematic dealing (Davis & Pitkethly 1990), and because of the importance of being able to share items with others, students are often very familiar with the fractions ½ and ¼ (and a little less so with $^1/_3$ and $^1/_5$).

It seems more than reasonable to suppose that if students come to school with certain concepts already in place there is also the possibility that the concepts they have developed are only partially developed or indeed misconceived. Martinie (2005) writes that "Research shows that students have misconceptions that stem from their previous knowledge that interferes with their understanding of rational numbers" (p. 6). Indeed, many concepts which are related to working with whole numbers actually interfere with how children think about fractions (Post & Cramer, 1987).

First Steps in Mathematics (Willis, Devlin, Jacob, Powell, Tomazos & Treacy, 1994) is a series designed to enhance teacher's professional judgements about mathematics teaching and learning which originated in Western Australia. It has since been adopted in many states of Australia and areas of Great Britain, Canada, New Zealand and the United States of America. This document argues that developmentally there are seven key understandings regarding fractions in order to achieve the Western Australian Curriculum Framework outcome of "Read, write and understand the meaning, order and relative magnitudes of numbers, moving flexibly between equivalent forms" (Curriculum Council, 2005, p. 36). These key understandings are outlined in Table 2.1.

Table 2.1

First Steps in Mathematics (Number), Key Understandings – Understand Fractional Numbers

Key Understanding	Description	Priority for students at age:
1	When we split something into two equal-sized parts, we say we have halved it and that each part is half the original thing.	pre 8 years old
2	We can partition objects and collections into two or more equal- sized parts and the partitioning can be done in different ways. 	8 year old to 10 year old
3	We use fraction words and symbols to describe parts of a whole. The whole can be an object, a collection or a quantity.	10 year old to 14 years old
4.	The same fractional quantity can be represented with a lot of different fractions. We say fractions are equivalent when they represent the same number or quantity	10 year old to 14 years old
5	We can compare and order fractional numbers and place them on a number line.	10 year old to 14 years old
6	A fractional number can be written as a division or as a decimal.	10 year old to 14 years old
7	A fraction symbol may show a ratio relationship between two quantities. Percentages are a special kind of ratio we use to make comparisons easier.	10 year old to 14 years old

Willis et al., 2004, p. 87

Indeed there was no suggestion from the *First Steps in Mathematics* materials that students deal with fractions in anything but concrete ways in the early years before they are 8 years old, and that the use of symbols should be treated at a later stage. This view was further supported by Bezuk and Cramer (1989) and Cramer, Post and Del Mas (2002). In 1988, Kieren asserted that there is a gradual expansion of children's knowledge and thinking about fractions through them building it up from personal environments. So by moving from their knowledge about ½ and ¼, students can then be provided with the environment to do more equal sharing through invented strategies (Empson, 2002; Hunting, 1991).

Specifically once this invented understanding of ½ has been developed then repeated halvings into ¼ and ⅛ are possible for students by about third (3rd) grade (according to United States school grade levels). Further strategies for equal sharing can then be developed for numbers such as three and six (Empson, 2002). Empson (2002) described how equal sharing strategies develop according to how the child "co-ordinates the number of shared items with the number of sharers to solve the problem" (p. 31). Further, Empson (2002) has proposed a developmental continuum for the equal sharing strategies as see in Table 2.2, Table 2.3 and Table 2.4.

Table 2.2

Empson's (2002) Development of children's equal- sharing strategies - Early

Early strategies	**Description**
a) Repeated halving	Child repeatedly halves each unit, regardless of number of sharers. Little or no coordination with number of sharers
b) Trial and error	Child tries various partitions with little or no coordination with the number of sharers. Some children may go through a list of fractions (e.g., halves, thirds, fourths) until they find one that yields the right number of pieces to deal out.

Table 2.3

Empson's (2002) Development of children's equal- sharing strategies - Intermediate

Intermediate Strategies	**Description**
c) Give out halves	Child starts by giving out halves, if possible. The rest of the partition is coordinated with the sharers in some way.
d) Coordinating sharers with single units	Child partitions each shared unit into enough pieces for all sharers. (This is a useful, all-purpose strategy, within the zone of understanding of many first and second graders.)
e) Coordinating sharers with multiple units 1) coordinates total sharers with every two units	Child partitions every 2 units into enough pieces for all sharers. There may be a leftover unit to partition.
2) coordinates total sharers with every 3, 4, 5 or more units	Child partitions every 3, 4, 5, or more units into enough pieces for all sharers. There may be leftover units to partition.

Table 2.4

Empson's (2002) Development of children's equal- sharing strategies - Later

Later Strategies	Description
f) Coordinating sharers with all units 1) creates same number of pieces as sharers	Sometimes children try to create partitions that give each sharer exactly one piece. This means they have to use multiplication, division, or trial-and-error skip counting to figure out how many pieces to partition each unit into. Some children think of this as creating big pieces. This strategy is related to the idea of reducing to a unit fraction. It does not work in all equal-sharing situations.
2) creates a number of pieces that is a multiple of the number of sharers:	This sophisticated strategy is used mainly by children who are fluent with multiplication. The child's goal is to create a number of pieces greater than the number of sharers that can be equally distributed among the sharers.

(Empson, 2002, pp. 32-34)

The research seems to suggest that students should or can acquire procedures (ability to operate with fractions) and conceptual (understanding) knowledge independently (Hallett, Nunes & Bryant, 2010; Martinie 2005). However it has also been argued that "students that do not make the connection between the rules/procedures and an understanding of the concept that drives the rules and procedures may suffer serious consequences in their learning of mathematics" (Martinie, 2005 p. 5). Further, understanding fractions should precede asking students to perform operations with them, although this is not always the case (Cramer, Behr, Post & Lesh, 1997). All this is known, and yet Kouba, Zawojewski and Struchens (1997) asserted that students are generally reasonably proficient with fraction computations but lack an understanding of what fractions mean. This statement leads to the conjecture that understanding may be sacrificed in the classroom for the sake of teaching procedures.

Siemon (2003) suggested that there are 11 steps in formalising fraction ideas.

1. Review initial fraction language and ideas by discussing 'real-world', every-day examples involving continuous and discrete fractions.

2. Practice naming and recording (not symbols) every-day fractions using oral and written language, distinguishing between the count (how many) and the part (how much) and including mixed as well as proper fractions.

3. Use practical examples and non-examples to ensure foundation ideas are in place, that is
    - recognition of the necessity for equal parts or fair shares and an appreciation of part-whole relationships (e.g., half of this whole may be different to half of that whole) - fractions are essentially about proportion;
    - recognition of the relationship between the number of equal parts and the name of the parts (denominator idea), particularly the use of ordinal number names; and
    - an understanding of how equal parts are counted or enumerated (numerator idea).

4. Introduce the 'missing link' - partitioning (the ability to physically divide continuous and discrete wholes into equal parts and generalise that experience to create own fraction diagrams and representations on a number line) - to support the making and naming of simple common fractions and an awareness that the larger the number of parts, the smaller they are.

5. Introduce (or revisit) the fraction symbol in terms of the 'out of' idea for proper fractions:

6. Introduce tenths via fraction diagrams and number line representations. Make and name ones and tenths using the fifthing and halving partitioning strategies (keeping in mind that zero ones is just one example of ones and tenths).

7. Extend partitioning techniques to develop understanding that thirds by fourths produce twelfths, tenths by tenths give hundredths and so on.

8. Extend decimal fraction knowledge to hundredths using diagrams (tenths by tenths), number line representations and metric relationships (money and MAB can lead to misconceptions), introduce percentage as another way of writing hundredths.

9. Explore fraction renaming (equivalent fractions) using paper-folding, diagrams, and games.

10. Introduce thousandths in terms of metric relationships. Rename measures (grams to kilograms etc.). Use partitioning strategies to show where decimals live. In particular, emphasise the relationship, 1 tenth of these is 1 of those.

11. Introduce addition and subtraction of decimals and simple fractions to support place-value ideas, extend to multiplication and division by a whole number.

(pp. 6 -11)

Siemon (2003) asserts that taking these steps will enable students to develop a number of required generalisations for fractions such as; partitioning, the capacity to develop their own fraction diagrams and representations and the understanding behind the role of factors in determining equivalent fractions. It should also be noted that operating with fractions in an algorithmic manner is not mentioned until the final dot point, suggesting support for the idea of allowing the conceptual understanding to develop properly before the introduction of the use of procedures.

Johanning (2008) describes the difference between procedural and conceptual knowledge. Procedural knowledge comes in two forms, the first being a familiarity with symbols and the syntactic familiarity with the configuration of those symbols. The second form concerns the rules and procedures employed when solving problems and this often consists of sequences of procedures and is quite linear in nature. Conceptual knowledge differs from procedural knowledge in that it is networked, connected and relationships rich. This study will be concentrating on developing conceptual understanding of fractions, rather than the manipulation of them.

2.4 *Learning Theory*

It is illustrative at this point to proceed with a short exploration of the predominant learning theories of behaviourism and constructivism, as the adoption of either of these two theories will position an educator differently as to the manner in which they approach the whole area of misconceptions and error.

Behaviourism was founded on the work on such people as Pavlov, Thorndike and Skinner and is based upon the premise that:

> pupils' learn what they are taught, or at least some of what they are taught, because it is assumed that knowledge can be transferred intact from one person to another. The pupil is viewed as a passive recipient of knowledge, an "empty vessel" to be filled...
>
> (Olivier, 1989 p. 2)

Behaviourists speak of stimulus–induced response (Thorndike, 1922) and learning through an accumulation or stock piling of ideas (Bouvier, 1987). They also conclude that errors and misconceptions can be analogously compared to faulty data in a computer, that is, if what is there is incorrect, it can be erased or written over, by giving the student the correct information (Strike, 1983).

Bartlett (1932) pioneered what became the constructivist approach (Good & Brophy, 1990). Constructivists believe that learners construct their own reality or at least interpret it based upon their perceptions of experiences, so an individual's knowledge is a function of their prior experiences, mental structures, and beliefs that are used to interpret objects and events, in other words, "…that a person's ability to learn from and *what* he learns from an experience depends on the quality of the ideas that he is able to bring to that experience." (Olivier, 1989, p. 2)

The two learning theories are therefore not compatible. One might simplistically describe the difference in the two learning theories as the difference between training and learning.

2.5 *Errors and Misconceptions*

In order to use the terms precisely it is necessary to determine the difference between the terms error and misconception. According to Hawker and Cowley (1998) "errors are mistakes or a condition of being wrong" (p. 163) and are typically associated with performance that is evaluated after instruction. Therefore a student making a mathematical error is making an error after instruction that has a systematic basis, unlike slips which are wrong answers due to processing and are sporadic and careless (Olivier, 1989). This differs

again from a misconception which according to Bell (1984, p. 58) is "…the implicit belief held by a pupil, which governs the errors that pupil makes" and can be a concept which the student carries within themselves before instruction has begun. This Olivier (1989) explains as being erroneous thinking that students consistently apply, and as Fong (1995) asserts, frequently makes sense from the student's point of view.

Steinle (2004) reports on a review of literature on misconceptions carried out by Confrey (1990) in which misconceptions were referred to in a variety of ways: "alternative conceptions, student conceptions, pre-conceptions, conceptual primitives, private concepts, alternative frameworks, systematic errors, critical barriers to learning, and naive theories" (p. 460).

As Confrey (1990) writes:

> …in learning certain key concepts in the curriculum, students were transforming in an active way what was told to them and those transformations often led to serious misconceptions. Misconceptions were documented to be surprising, pervasive, and resilient. Connections between misconceptions, language, and informal knowledge were proposed (p. 19).

Tripp (1993) incisively states "Students do misunderstand, but it is seldom because they cannot understand, most often it is because they understand something else" (p. 88). According to Mestre (1989) misconceptions are a problem for two reasons. They interfere with subsequent understandings if the student attempts to use them as the basis for further learning, and they have been actively constructed by the student and therefore have emotional and intellectual attachment for that student, and consequently are only relinquished by the student with great reluctance.

Because there are a number of ways in which the literature refers to misconceptions and because errors can indeed be exhibited by students as a result of misconceptions, it is easy to understand how the two came to be used erroneously as interchangeable terms (Ashlock, 2002). But if as Confrey (1990) asserts that "…misconceptions were documented to be surprising, pervasive, and resilient" (p. 19), can they be addressed through a concerted interaction or are students 'doomed' to live with the misconceptions until they are

developmentally able to somehow grow out of them through a process of natural maturation of understanding, or through self-induced cognitive conflict which requires them to alter their understandings? Helme and Stacey (2000) report on a successful intervention applied to alleviate a misconception. This has support through Graeber and Johnson (1991) as reported in Steinle (2004) when they commented:

> It is helpful for teachers to know that misconceptions and buggy errors do exist, that errors resulting from misconceptions or systematic errors do not signal recalcitrance, ignorance, or the inability to learn; how such errors and misconceptions and the faulty reasoning they frequently signal can be exposed; that simple telling does not eradicate students' misconceptions or "bugs" and that there are instructional techniques that seem promising in helping students overcome or control the influence of misconceptions and systematic errors. (pp. 1-2)

Given that learning is not a linear and diagonal process, that learning does not proceed like a line of best fit in a correlation graph, but rather it is a series of understandings and misunderstandings, Bell (1984) asks teachers to embrace misconceptions as an important and necessary stage of the learning process and not something which is intrinsically negative. This study will therefore concentrate on the misconceptions students and teachers have regarding fractions and work with the definition as constructed by Bell (1984).

2.6 *Causes of Difficulties in Learning Fractions*

A number of studies on learning difficulties and misconceptions of fractions have been carried out in the past (Pitkethly & Hunting, 1996; Taber, 1999; Tirosh, 2000) and various reasons have been attributed for this difficulty. Research by Baroody and Hume (1991), Streefland (1991), and D'Ambrosio and Mewborn (1994) as reported by Newstead and Murray (1998) and Hanson (2001) consider the following as possible causes:

- The way and the sequence in which the content is initially presented to the students, in particular exposure to a limited variety of fractions (only halves and quarters), and the use of pre-partitioned manipulatives.

- A classroom environment in which, through lack of opportunity, incorrect intuitions and informal (everyday) conceptions of fractions are not monitored or resolved.
- The inappropriate application of whole number schemes, based on the interpretation of the digits of a fraction at face value or seeing the numerator and denominator as separate whole numbers.

Other researchers declare that fractions are for many students too abstract to understand (Saenz-Ludlow, 1995). Tirosh, Fischbein, Graeber and Wilson (1998), also conclude that children do not have the same everyday experiences with rational numbers that they do with natural numbers. Mamede et al. (2005) also stress that fraction knowledge is not a simple extension of whole number understanding.

One way teachers try to convey the meaning of fractions is through language which uses definitions, examples or models. However, the language teachers often use is "influenced by cultural factors, including the characteristics of the language used in the mathematical domain" (Muira, 2001, p. 53). In some east Asian languages the concept of fractional parts is embedded in the mathematics terms used for fractions. However, this is not the case in English, the predominant language of instruction in Australia and in many other countries around the world. For instance, Muira (2001) gives the example:

> In Japanese, one third is spoken as san bun no ichi, which is literally translated as "of three parts, one." Thus, unlike the English word third, the Japanese term, san bun (three parts), directly supports the concept of the whole divided into three parts (p. 55).

The English language offers no such support, often a fraction is expressed as three over four, which gives no clue to the uninformed as to what actions to take with the numbers. Even using the language of three divided by four can set up a notion which is algorithmic rather than conceptual.

Certainly the language used to describe fractions can be problematic for some students. A number can have many names: one half is also five tenths, zero point five (0.5) and fifty percent (50%); as well as two quarters, three sixths, four eighths and so on. Different words

are also used when fractions are expressed in different ways. There are improper fractions and mixed numbers as well as common fractions that may have common, low or lowest denominators and be equivalent or irrational. This may suggest why some students and teachers may find fractions difficult and confusing (Kaur, 2004).

If one considers the manner in which students in certain English speaking countries say their unit and common fractions there is the problem that a some of the fractions have 'special' names (1/2 is said as one half not one twoth, ¼ is said as one quarter not one fourth and ¾ is said as three quarters not three fourths) whilst all of the others are said using their ordinal name. Using an ordinal name in itself may cause some students, perhaps those with a language background other than English, some confusion, as ordinal numbers are usually reserved to indicate position, whereas cardinal numbers are usually used to indicate size. Even if a student can accept the use of ordinal numbers as an indicator of size, in the place of cardinal numbers, they then must wrestle with the understanding that usually when cardinal numbers are used, a larger positive number (one further away from zero than another) refers to an increase in quantity for example 8 is greater than 2, whereas with unit fractions the larger the number the smaller the quantity, for example ⅛ is smaller than ½.

Baroody and Hume (1991) suggest that student errors in fractions may be caused by poor understanding of underlying concepts, as well as by an inability to recognise accurate visual representations. Hanson (2001) has suggested that students' desire to memorise formulas or algorithms instead of understanding the concepts was a major reason why they continued to experience difficulties with fractions.

2.7 ***Teachers and the Teaching of Fractions***

Carnine, Jitendra and Silbert (1997) cite research (Stevenson, 1992; Stevenson, Chuansheng & Lee, 1993; Stevenson & Stigler, 1992) that shows that differences in student mathematical performance may be a function of the amount and kind of exposure to mathematical instruction rather than in real differences in abilities. Once again this supports the idea that the quality of the teaching is paramount in the learning of mathematics, a view supported by Wenglinsky (2002) who proffers that teacher education and content

background and the decisions teachers make about classroom practices can have a great effect on student learning. Zientek and Thompson (2008) also state that teacher quality impacts on students' success in mathematics.

The overwhelming evidence indicates that students continue to have considerable difficulty with fractions raises the question as to why teachers are not addressing this area well. If, as the literature suggests, (Capraro, 2004) the teaching of fractions is problematic and frustrating for many teachers, it would also seem likely that their understanding in this area is not well developed.

Content knowledge of mathematics is crucial for improving the quality of instruction (Ambrose, 2004; Borko, 2004; Hill et al., 2008b; Hill & Ball, 2004; Lamb, Cooper & Warren, 2007) and it is well documented that teachers often lack a deep conceptual understanding of mathematics (Ball, 1990; Ma, 1999; Tirosh, Fischbein, Graeber & Wilson, 1998). Many exhibit weaknesses in mathematics, misapply rules, and are generally not prepared to teach the mathematical content entrusted to them (Hungerford, 1994; Tsao, 2005). As Zhou et al. (2006) state, unfortunately teachers who do not acquire mathematical competency during their school years are unlikely to have another opportunity to do so.

Tsao (2005) reported on a study conducted by Johnson (1998) which identified some common misconceptions held by prospective elementary school teachers which included, the beliefs that:

- the fraction having a larger denominator is always larger;
- two fractions that are almost equal are equivalent; and,
- area models must be rectangular or regular in order to find a fractional portion.

More positively, Hill et al. (2008b) report on a 1998 study by Sowder, Phillip, Armstrong and Schappelle, where they concluded that a two year professional development program showed gains in teacher knowledge of rational number which lead to teachers covering more content with their students. The teachers also had instructional goals which featured conceptual understandings as well as the development of skills, moreover, they asked for explanations from their students and investigated for understandings.

Naiser and Wright (2004) argue that students need to be more strongly engaged in learning fractions. They state:

> One area of improvement could be on how teachers engage students. Students should be shown how fractions apply to their personal lives. This could make the lesson more motivating and successful for the students. Many teachers did not make the connections from the content to real life applications. Many of the fraction lessons were not engaging nor did they actively involve the students. Instead, they consisted of the teacher providing examples and the students practicing.
> (p. 182)

One can readily assume that many of the examples alluded to above would be taken from worksheets and/or text books. Yet most teachers would support Naiser and Wright (2004) when they claim that worksheets and seatwork do not seem to actively engage students. There seems to be a disparity here between the understanding of what constitutes a good mathematics lesson and the actual practice of teachers when teaching fractions. This is not to suggest that teachers do not have an adequate grasp of constructivist practice and its efficacy – which is argued extensively in the literature to be a highly successful pedagogical approach - but they can often fall back to more algorithmic and less pedagogically sound methods when teaching fractions.

Anecdotal evidence and research (Carnine, Jitendra & Silbert, 1997) suggests that the text book, with drawn analogues, is the preferred method with which to address this area of mathematics learning. The *Everybody Counts* Report (Mathematical Sciences Education Board, 1989) states that “teachers tend to teach only what is in the textbook and students learn only what is in the text” (p. 45) an opinion which will be further explored in section 2.8. This begs the question of the level of teacher confidence and attitude and their capacity to divert from the text book as a major method of instruction. One must also consider Charalambous’s (2010) proposition that “...strong mathematical knowledge for teaching (MKT) supports teachers in using representations to attach meaning to mathematical procedures...” (p. 273). He further asserts that “...strong MKT supports teachers in giving and co-constructing explanations that illuminate the meaning of mathematical procedures.” (p. 274). If these propositions are correct then the reverse may also be true. Weak mathematical knowledge for teaching would impede teachers in using representations to

attach meaning to mathematical procedures, and impede teachers in giving and co-constructing explanations that illuminate the meaning of mathematical procedures. This would in turn lead some teachers to the use of text books.

There have been a number of studies completed on teacher confidence/attitude (Baxter, 1983; Becker, 1986; Ernest, 1988; Steffe, 1990; Tirosh, 1990) and content knowledge (Kajander, 2005). The Department of Education Employment and Training (1989) states that primary teachers often hold negative attitudes towards mathematics and this negative attitude may be reflected in poor teaching of mathematics. This is supported by Askew (2008) who states that; "…one thing is clear from the research evidence: many prospective and practicing primary teachers have, or express, a lack of confidence in their mathematical knowledge" (p. 16). This is a worrying finding, as a positive attitude towards the teaching of mathematics has a direct influence on the levels of success that teachers can help students to achieve (Kulm, 1980; Sullivan, 1987).

A good deal of research has been conducted as to the type of mathematical knowledge that teachers require in order to teach well (Ambrose, 2004; Hill & Ball, 2004; Ma, 1999). Two important and revealing findings were that: a number of teachers in elementary classes lack a conceptual understanding of mathematics (Ball, 1990; Even, 1990; Ma, 1999); and, that teaching quality (as determined by performance on standard tests) related to whether the teachers' knowledge of mathematics was procedural or conceptual (Hill & Ball, 2004). Given the previously stated difficulty that teachers have with the topic of rational numbers (Moseley, Okamoto & Ishida, 2007; Moss & Case, 1999; Tirosh, 2000; Zhou et. al, 2006) and the fact that many teachers do not have a deep conceptual understanding of mathematics in general, it is probable that their conceptual understanding of fractions would also be somewhat lacking. Indeed, Tirosh et al. (1998) concluded that the main goal of teacher education should be to develop prospective teachers' mathematical and pedagogical content knowledge of rational numbers.

Van de Walle and Folk (2005) assert that the topic of fractions may be one of the most common area in mathematics in which students are often presented with rules without justification. A further stating of the belief, that many students are able to operate with

fractions, (that is use them in calculations) without necessarily understanding them comes from Moss and Case (1999) who write;

> The domain of rational numbers has traditionally been a difficult one for middle school students to master. Although most students eventually learn the specific algorithms they are taught, their general conceptual knowledge often remains remarkably deficient
>
> (p. 122)

Hiebert (1999) explored the difficulty in building conceptual understanding after a procedural approach has been espoused. A position supported by Ambrose (2004) who stated that:

> …pre-service teachers may be faced with a great challenge in improving their own conceptual understanding of fractions, especially if their experiences of learning mathematics in school consisted mostly of memorizing procedures, which may have been the case.
>
> (p. 93)

2.8 *The Role of Texts in Teaching Fractions*

Research conducted in Australia has shown that mathematics teachers are dependent on a variety of commercially produced materials, particularly the student workbook (Shield, 1991; Watt, 2004). International research by Johansson (2006) and Reys, Reys and Chaves-Lopez (2004) suggests that up to 90% of mathematics lessons employ a textbook to form content, sequencing and instructional activities and ideas for lessons. International studies have shown that textbooks influence what teachers teach by delineating what topics are covered and how these topics are presented (Stein, Remillard, & Smith, 2007), how they teach it, and what homework or activities they assign to their students (Alajmi, 2009; Weiss, Pasley, Smith, Sanilower, & Heck, 2003). However, there is a lack of conclusive evidence to support the efficacy of textbooks (Zevenbergen, Dole & Wright, 2004) and as contemporary schooling has a heavy dependence on textbooks (Jamieson-Proctor & Byrne, 2008) such unquestioning reliance could be quite problematic. This reliance can lead to what Haberman (2010) calls the "pedagogy of poverty" (p. 45) where teachers are overly didactic, dole out information and assign problems. As Vincent and Stacey (2008) assert about explanations contained in text books:

> …in general, very short with essential aspects of the reasoning unstated. Hence they are unlikely to stand alone, so students must rely on teachers to elaborate on the explanation provided. It is unlikely that all teachers can present these elaborations from the material provided …This highlights the need for teachers' deep mathematical pedagogical content knowledge
> (p. 480).

Stein et al. (2007) assert that how topics are presented in text books is important as it sets the type of pedagogical approaches employed and therefore the different opportunities for students to learn. This being said, research indicates that the primary instructional approach for teaching fractions which receives almost all the attention in textbooks is the part to whole model (Martinie, 2005) and the teaching of mainly procedural skills (Behr, Post & Lesh, 1984; Moss & Case, 1999). Other interpretations which are important to developing rational number skills and concepts such as fractions as measures, quotients, partitioning, equivalence, ordering and so on are barely approached (Empson, 2002). It is also worth noting that the main method of illustration of concepts in books is the use of diagrams and illustrations, but as Smith (2002) points out, drawings do not speak for themselves. Constructivism asserts that learning occurs through individuals constructing ideas, processes and understandings for themselves, rather than through the transmission of performed knowledge from teacher to learner (Goldin, 2002). Therefore, rather than simply accepting new information, students interpret what they see, hear, or do in relation to what they already know (Carpenter, 2003, p. 29). Further, constructivism is based upon the following premises:

- Knowledge is actively created or invented, not passively received
- New ways of knowing are built through reflection on physical and mental actions
- Learning is a social process requiring engagement in dialogue, discussion, argumentation and negotiation of meaning

(Booker, Bond, Sparrow & Swan, 2004, p. 11).

Constructivism is the seemingly preferred epistemology for mathematics teaching in Australian schools. If one accepts this theory of knowledge and its acquisition as at least a working hypothesis then the use of text books cannot fit comfortably as the primary teaching tool within this pedagogy. Textbooks by their inherent physical restrictions cannot accommodate constructivist theories on learning nor provide concrete representations to help in the development of the required concepts. Further, Vincent and Stacey (2008)

asserted that only one in four text books from the United States emphasised multiple representations in worked examples and that few connections were made in the books between the concepts practised. This paucity of multiple representations may be related to the restriction on the number of pages in a text, even though this restriction in the number of representations could be deleterious to the learning of fractions (Cramer & Henry, 2002).

Naiser et al. (2004) restate the assertion of Burrill (1997) that good teaching is not about making learning easy, but about making it active and engaging for the students. Textbooks provide a passive form of learning which too often rely on symbolic representations to carry understanding, rather than some form of learning with activity at its core.

Further, texts rarely acknowledge that students learn in a variety of ways (Fotoples, 2000) and that students can arrive at the same answer using many different pathways. Texts by their very nature preclude many of the kinaesthetic and aural learners from engaging with the content in the manner in which best suits their learning styles.

Research by Tzur (2000) on the learning of fractions explains how students may be prompted to use their informal knowledge to construct meanings about fractions for themselves, if a realistic situation is applied. Naiser et al. (2004, p. 181) claim that "Manipulatives combined with real life problems can interest students in solving problems, which in turn, helps them construct meaning." No such assertion is made for the use of textbooks in the teaching of fractions.

Hill et al. (2008b) offer some sobering and quite provocative countervailing arguments to the objections to the use of text books:

> Teachers are flooded with messages *not* to use their textbooks, starting with scholarly work (Ben-Perez, 1990) and continuing on to the materials thrust upon them in professional development and ending with district curriculum documents that piece together units from disparate resources. This may have been appropriate in an era when most textbooks were similar in their mathematical drabness; however, the quality of available materials has sharply improved, yet this ethos persists. And we argue that solid mathematical tasks and representations that come from a drab textbook are preferable to teacher-created math lessons in the hands of teachers with little mathematical knowledge for teaching. Without the

> ballast of mathematical knowledge, teachers' implementation of supplementary materials is chancy at best.
>
> (p. 499)

However, much this statement may seem to be at odds with prevailing opinion, it is a position which should probably be considered and this Researcher would suggest that for many teachers it is a position that would resonate.

2.9 *Interpretation of Fractions and the Curriculum*

Researchers and educators (Behr, Harel, Post & Lesh, 1992; Usiskin, 1979; Wu, 2005) declare that rational numbers can be interpreted in at least six ways: as a part to whole comparison, a ratio, an indicated division (quotient), an operator, a measure or a decimal.

The part-whole construct of rational numbers relies on the ability to partition either a continuous quantity or a set of discrete objects into equal sized parts or sets. This is when an object is equally divided into d parts then c/d denotes c of those d parts.

Continuous quantity usually refers to length, area, or volume. In this case, the whole, of which a fraction is a part, is made up of one single object. When the whole consists of more than one object (for example a dozen eggs or 8 biscuits) then this whole is referred to as being discrete. The part-whole notion of rational numbers is fundamental to all other interpretations. This interpretation is usually introduced very early in the formal schooling of students in informal ways, and then refined in a substantial and systematic fashion later in school.

Geometric regions, sets of discrete objects, and the number line are the models most commonly used to represent fractions in the primary and middle school mathematics class (Watanabe, 2002, p. 5). For example, ½ could be represented with a geometric region as in Figure 2.1, with a discrete set as in Figure 2.2, or with a number line as in Figure 2.3. Interpretation of geometric regions apparently also involves an understanding of the notion of area.

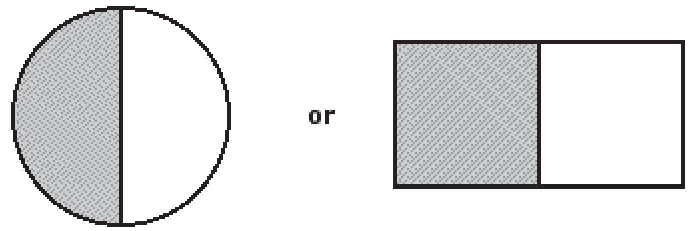


Figure 2.1 Geometric region models

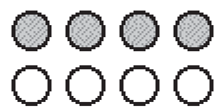


Figure 2.2 Discrete objects model

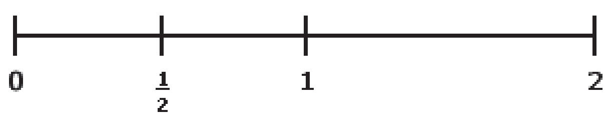


Figure 2.3 Number line model

The ratio interpretation of rational numbers conveys the idea of relative magnitude. The use of proportions is an important problem-solving tool in a variety of physical situations that requires comparisons of magnitudes for example: A large box of cereal contains 750 grams, and a small box contains 330 grams. What is the ratio of the mass of the small box to the large box?

A quotient of the integer *c* divided by *d* can also be used to refer to the operation of division. That is, a/b is sometimes used as a short way of writing $a \div b$. This is the indicated quotient (or indicated division) interpretation of rational numbers. Rational numbers can be used as operators, or as an instruction to carry out a process such as ⅔ of 21 which positions the user to multiply 21 by 2 (the numerator) and then divided by three (the denominator) to arrive at an answer.

The measure interpretation is usually reflected in the use of the number line as a physical model. The reason for the phrase, measure interpretation, is that rational numbers are defined as a measure. When one thinks of measure, the notion of a unit of measure and of subunits of that unit of measure comes to mind. On a number line the unit of measure is the distance on the line from zero to one (Figure 2.4). In some cases this distance is a centimetre, in others a millimetre or in others a kilometre, but the basic notion of the distance between zero and one defining the unit remains intact. Multiples of this unit distance are generated on the number line by iterating the distance from zero to one along the line (Figure 2.5).

Figure 2.4 Number line model – unit of measure 0 - 1

Figure 2.5 Number line model – multiples of unit of measure 0 - 1

Frequently teachers suggest to children that each rational number represents a point on the number line, but it actually represents a distance on the number line. We can think of ⅝ as being associated with a point on the number line, provided we take the distance starting at zero and iterate five subunits of ⅛ in the direction of one.

Decimals are yet another important interpretation of rational number and are very useful in a wide variety of settings, such as measurement using the metric system, percentage, and money. Decimals can be particularly problematic for some students as they have characteristics similar to both whole numbers and fractions. Decimals, however, are different from each of these in the way that they are conceptualised and in the way they are manipulated.

This variety of interpretations alone can confuse students. Compound this confusion with erroneously trying to deal with fractions using counting and matching (Mitchell, 2005) or that equivalence and ordering of fractions often defy students' intuition from natural numbers (Smith, 2002), and fractions become a very challenging proposition.

Behr, Lesh, Post and Silver (1983) working on the long running *Rational Number Project* (RNP) devised a theoretical model which linked the different interpretations of fractions to the basic operations of fractions and to problem solving. This is illustrated in Figure 2.6. The solid arrows suggest established relationships and the dotted arrows hypothesised relationships.

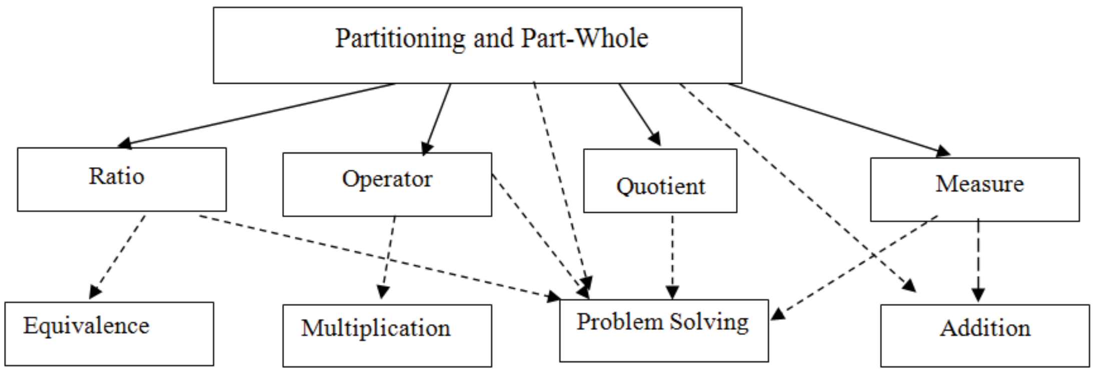


Figure 2.6 Sub-constructs relationships for fractions (Behr, et al., 1983)

Behr, Lesh, Post and Silver (1983) considered which of the sub-constructs (part to whole comparison, a ratio, an indicated division [quotient], an operator and as a measure) might best serve to develop in students a basic fraction concept. Charalambous and Pitta-Pantazi, (2005) state that the RNP found it most plausible that the part-whole construct represents the primary construct for rational number development and is fundamental in developing understanding of the subordinate constructs. This assumption in part explains the preponderance of this construct across curricula as the most commonly employed point of entry for teaching fractions. Charalambous and Pitta-Pantazi (2005) provided empirical support for the fundamental role of the part-whole sub-construct, thereby justifying this traditional instructional approach. They claim that their study supports part-whole as being the dominant interpretation of fractions but add "they [the findings] also underline the need for emphasising the other sub-constructs of fractions, and especially those that are not so highly related to the foregoing notion" (p. 239).

Ellerbruch and Payne (1978) identified four stages for the successful teaching of fractions:

- using concrete materials to make equal size partitions;
- recognizing and using the oral names for the various parts;
- drawing diagrams of the concrete objects and attaching oral names to the parts; and,
- using the concrete objects and diagrams together with the oral names to write the fraction symbols.

Cramer and Henry (2002) wrote about a curriculum which was created and then revised on the basis of what was learned through several long-term teaching experiments conducted in the United States as part of the *Rational Number Project*. This project reflected the following four beliefs:

- children's learning about fractions can be optimised through active involvement with multiple concrete models;
- most children need to use concrete models over extended periods of time to develop mental images needed to think conceptually about fractions;
- children benefit from opportunities to talk to one another and with their teacher about fraction ideas as they construct their own understandings of fraction as a number; and,
- teaching materials for fractions should focus on the development of conceptual knowledge prior to formal work with symbols and algorithms.

(Cramer et al., 1997).

The fact that there are multiple interpretations of fractions (Behr, Harel, Post & Lesh, 1992; Wu, 2005) may prove for some to be challenging in both the teaching and learning of fractions. However, Charalambous and Pitta-Pantazi (2005) strongly recommend that there is a need to explore all of the different constructs. Further, the research gives direction regarding how teachers might approach the successful teaching of fractions (Ellerbruch & Payne, 1978; Cramer et al., 1997).

2.10 ***Methods of Representing Fractions***

According to Behr, Lesh, Post and Silver (1983) there is an interactive relationship between the different representational models for fractions (Figure 2.7). They also assert that students learn best when the different relationships between the models are exploited. The model also suggests that mathematical problems are frequently solved by: translating from the 'real' situation to one or more of the representation/s; operating with the representation/s to produce a result or a hypothesis; and then, translating the result back to the real situation. It also suggests that many problems are solved through a combination of partial mappings from a number of representations.

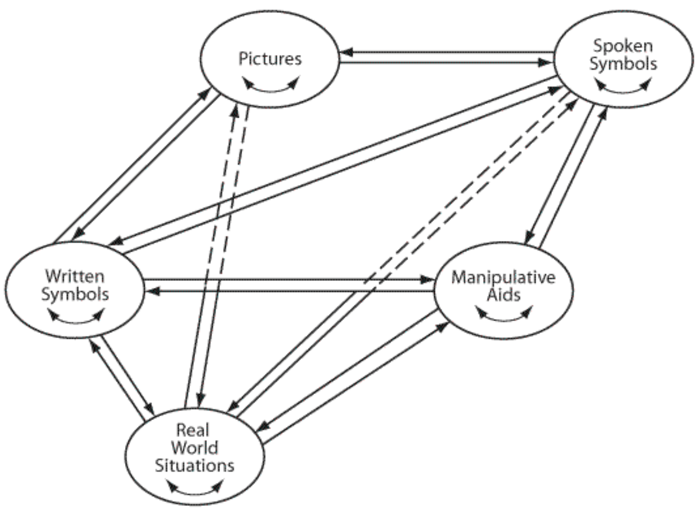


Figure 2.7 Behr, et al.'s (1983) Interactive model for using representational systems

According to Watanabe (2002), although other fraction models are used, there are three common fraction models found in typical mathematics textbooks; linear models, area models and discrete models. In addition to these static drawings found in textbooks teachers and students often represent fractions through the use of a variety of concrete objects, for example Cuisenaire rods, counters, linking cubes, fraction 'cakes', and pattern blocks.

For each of these models Watanabe (2002) declares that there are two distinct methods for representing fractions: the part-whole method (Figure 2.8) and the comparison method

(Figure 2.9). In the part-whole method the fractional part is embedded in the whole and in the comparison method the whole and the fractional part are constructed separately.

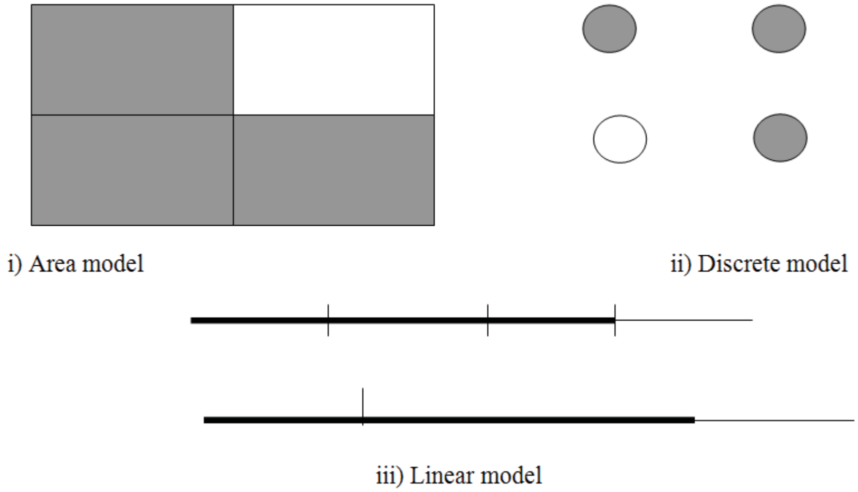


Figure 2.8 Part – whole models

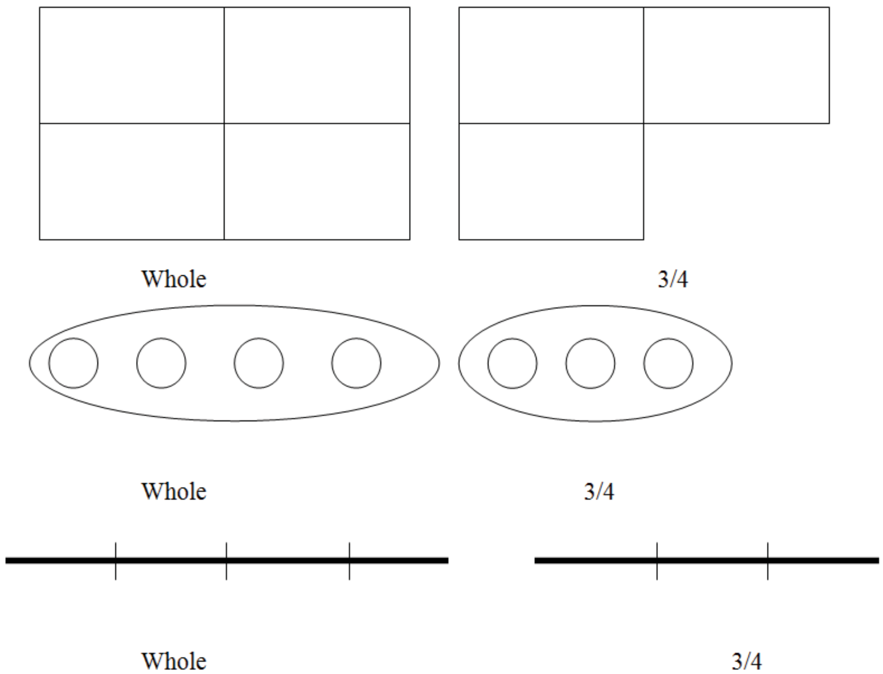


Figure 2.9 Comparison method

2.11 ***Analogues***

An analogy is defined by the Collins Dictionary of English Language (Hanks, 1986, p. 52) as "a form of reasoning in which a similarity between two or more things is inferred from a

known similarity between them in other respects." In order for good analogical matching to occur, overlap in relations is necessary for any strong perception of similarity between two domains (Gentner, 1983). In mathematics, analogies assign a concrete or pictorial model (analogues) as the source and the concept to be constructed as the target. If there is to be no ambiguity in the mapping of the source to the target then the learner must understand the structure of the source and recognise the correspondence between the source and the target (English & Sharry, 1996; Kurtz, Miao & Gentner, 2001).

English and Halford (1995) also identify two categories of mathematical analogues commonly used in the teaching of fractions: unstructured and structured. Ball (undated p. 6) asserts that one of the many decisions teachers must make in teaching fractions concerns the relative advantages of providing students with structured representational materials, as opposed to having students refine existing models, thereby developing their own representational media.

Collections of counters or other discrete objects are examples of unstructured analogues as are set models. In other words, the counters themselves lack inherent structure, that is, the structure needs to be created by the manner in which the counters are manipulated or arranged (English & Halford, 1995). Region models and length models form the basis of structured mathematical analogues used in the teaching of fractions (Charles, Nason & Cooper, 1999). Charles, Nason and Cooper (1999, p. 2) state that structured analogues are more appropriate for the initial learning of fractions than unstructured analogues due to the "complex mappings and high information processing loads associated with unstructured mathematical analogues."

English and Halford (1995) stress the need for caution in the application of structured analogues for the teaching of fractions. They declare that: analogues alone cannot impart meaning; children may not make the appropriate mappings from concrete to abstract; and, the accompanying mapping language and processes must be appropriate.
They therefore contend that for an analogue to be effective it needs to follow the principles espoused by Gentner (1988) and English and Sharry (1996):

- the clarity of source principle – the source is clearly articulated and understood by the child;
- the clarity of mapping principle – the analogue should, from the concrete model to the target fraction concept, facilitate an unambiguous mapping;
- the principle of conceptual coherence – there should be a cohesive structure in the relations mapped from the source (concrete model) to the target (concept to be constructed); and,
- the principle of scope – any analogy must be transferable.

Charles et al. (1999) conducted a study in which they used seven different analogues: pizzas and apple pies (based on large circular region models); pancakes (based on medium circular region models); pikelets (based on small circular region models); ice cream bars (based on long and narrow rectangular region models); rectangular cakes (based on short and wide rectangular region models); and, licorice straps (based on length models). They evaluated them in terms of their effectiveness and how this might be limited by:

1. ecological validity (How realistic was the sharing concept engendered by the analogue?);
2. abstraction ability (How well they facilitated abstraction from the source to the target?); and,
3. ease of partitioning (How easy was the concrete model to physically partition?).

Their findings indicated that:

- circular regions are difficult to partition into thirds, fifths and sevenths;
- the educational efficacy of the ice cream bars and the licorice straps was compromised by their lack of ecological validity; As long, narrow rectangle and length models provided high levels of abstraction ability and ease of partitioning it is suggested that teachers should try to devise analogs other than licorice straps and so on which have greater ecological validity for their students.

(Charles et al,. 1999)

Watanabe (2002) asserts that Cuisenaire rods, simple counters and pattern blocks are typically used for modeling fractions in the mathematics classroom. He also insightfully declares that "representations are meaningful to the one who created them, whether that creator is the teacher or the student" (p. 461). Research in science education is demonstrating the positive learning outcomes that arise when students are challenged to construct their own representations and explain them (Hubber, Tytler & Haslam, 2010). Clearly then the use of manipulative materials is more than just a case of providing the students with hands-on materials. Deliberation is required if learning is to be maximized.

2.12 *Manipulative Materials*

Greenes (1996) stated that in general terms, manipulative materials have been central to the call by several mathematics professional organisations to reform mathematics teaching. More specifically, Ellerbruch and Payne (1978) and Cramer and Henry (2002) expressly call for the use of manipulative materials early in the development of the concept of fractions. This is generally predicated on the belief that developmentally it is advantageous for students to be allowed to move from the concrete to the abstract.

Behr, Wachsmuth and Post (1988, p. 1) stated that "the research concerning learning via manipulative materials is somewhat equivocal. The question of what characteristics of manipulative aids best facilitate learning is unanswered." This has since been refuted by Cramer and Henry (2002) who claim that most children need to use concrete models over extended periods of time to develop mental images needed to think conceptually about fractions is the most important

Research by Berry et al. (2009), Daniels, Hyde and Zemelman (1993), Marsh and Cooke (1996), Mastropieri, Scruggs and Shiah (1991), Reimer and Moyer (2005) and Sowell (1989), all indicate that manipulatives do have a positive effect and can promote positive classroom behaviours. These positive effects were noted in the areas of: acquisition of basic mathematics concepts; as a tool for developing competence in identifying the correct operation to employ when solving mathematics story problems; and, as an aid to students becoming more knowledgeable and confident in the mathematics area.

Driscoll (1983), Sowell (1989) and Suydam (1986) all report that students who use manipulatives in their mathematics classes usually outperform those who do not; a benefit which holds across all year levels, ability levels and topics. The important proviso they make in this assertion is that this applies when the manipulative is appropriate to the topic. This means it must be carefully selected and must stimulate students' thinking (Clements & McMillen, 1996).

Millsaps and Reed (1998) assert that students retain fraction concepts for longer and at a higher level, after using manipulative materials. Boulet (1998) adds to this view saying, that in order to understand the unit fraction conceptually, students must first develop an understanding of the underlying physical concepts before then moving on to the emerging mathematical concepts. This is a view shared by McGuire (2004).

Capraro (2004) states that the use of manipulatives is a strategy by which teachers can make a lesson more engaging by providing a hands-on experience. It also allows the students an effective way in which to represent their thinking in a manner which the teacher then can explore further with the student. It enables the teacher to determine if there are any misconceptions in the student's understanding of fractions.

What then are manipulatives? Perry and Howard (1997, P. 26) defined manipulatives as including "all materials, both inside and beyond the mathematics classroom, which can be experienced through senses of sight, touch and/or sound." Manipulative materials are used to create an external representation, a representation outside of the mind that stands for a mathematical idea (Puchner et. al, 2008), in order to eventually develop an internal representation. Cramer and Wyberg (2009) reaffirm this by stating that manipulative materials are tools to support meaningful learning, where students construct insights and create physical and then mental representations for those ideas. This later supports more abstract and symbolic work. The manipulative materials act as the analogous source to reach the 'target.'

Swan and Sparrow explain that manipulatives also come in various forms:

- unstructured (such as buttons, pop sticks, match sticks and so on);
- sructured (such as Attribute Blocks, Multi-base Arithmetic Blocks [MABs] and so on); and

- virtual (computer based simulations of concrete or physical manipulatives).

(2004, p. 518)

Yet not all manipulative materials are as efficacious as others and each one needs to be judged against the ‘target’ it is chosen to represent. For instance, research (Cramer & Wyberg, 2009) on the different manipulative models for developing the part-whole model suggests that continuous models (for example area and measurement models) are more effective than discrete models (counters and so forth).

In summary, it seems that children’s learning about fractions can be optimised through active involvement with multiple concrete models (Cramer, 2002; Cramer & 2009; Sebasta & Martin, 2004) and that an effective method of teaching fractions and promoting understanding is to move students in a sequence from a concrete to a semi-concrete and then to an abstract conception (Jordan, Miller & Mercer, 1998). If so, this then raises the issue of what manipulative materials are best suited to teach fractions.

Some may believe that children will automatically understand fraction concepts simply as a result of using the various representations or manipulatives. This is not necessarily the case (English & Halford, 1995; Millsaps & Reed, 1998; Puchner et al., 2008; Thompson & Lambdin, 1994, Viadero, 2007). For instance, some students define a fraction as "a piece of pie to eat," because they have only seen fractions represented using circle diagrams (Niemi, 1996). Clements (1999) noted:

> Their (the manipulatives) physicality does not carry the meaning of the mathematical idea. They can be used in a rote manner…They need teachers who can reflect on their students’ representations for mathematical ideas and help them develop increasing sophisticated and mathematical representations.
>
> (p. 47)

Clements (1999) writes that “… even if children begin to make connections between manipulatives and nascent ideas, physical actions with certain manipulatives may suggest different mental actions than those we wish students to learn,” and that “…although manipulatives have an important place in learning, their physicality does not carry the meaning of the mathematical idea” (p. 47). These findings are clearly compatible with the findings of the work of Holt (1992).

Clements and McMillen (1996) are even more forthright in their views when they warn that manipulatives are not sufficient to guarantee meaningful learning and require proper use. They further caution against manipulatives being used in a rote manner, whereby the students learn how to perform the correct steps in the use of that material, for a particular concept, without understanding how that manipulative illustrates the concept or how other materials could be employed to reach the same result.

Further, Boulet (1998) writes that concrete activities which are without context and purpose can actually be harmful to students developing fraction concepts. Therefore the power of each of the analogues chosen as physical representations of fractions is important.

Clements and McMillen (1996, pp. 276 - 277) offer the following guidelines for selecting and using manipulatives:

1. select manipulatives primarily for students' use, not for teacher demonstration;
2. select manipulatives that allow students to use their informal methods for finding solutions or making sense of mathematical ideas;
3. use caution in selecting manipulatives which are 'pre-structured' or in which the mathematics is 'built in' such as Base 10 materials;
4. select manipulatives that can serve many purposes.
5. choose particular representations of mathematical ideas with care;
6. use a single manipulative to introduce a topic rather that multiple representations; and,
7. after the initial use of a single representation to introduce a topic, employ different representations so that other students who find a different representation meaningful, are catered for.

Puchner et al. (2008) ask the question: if the evidence seems quite compelling for the use of manipulatives and that teaching with hands on materials tends to be appealing to teachers, why is it then that teachers are not employing them more? Naiser and Wright (2004) described how manipulatives, although widely regarded by, and available to teachers, had a low usage rate primarily because teachers did not feel competent in their use. This position

is supported by Capraro (2004) and Hatfield (1994) in their assertion that teachers may not be using manipulatives as much as possible because of a lack of training.

2.13 ***Virtual Manipulatives***

One of the burgeoning areas in mathematics education is the use of learning technologies. The proliferation of computers and internet access has brought the use of virtual manipulatives into the majority of classrooms in the developed world.

The British Educational Communications and Technology Agency (BECTA) (2003) indicate that:

> …high quality, interactive learning resources are more likely to be related to higher learning gains for pupils than other resources. The reviews point to substantial evidence of the impact of specific uses, for example, using simulations and modeling in Science and Mathematics. However, impact is dependent on teachers' use and quality of implementation (p. 19).

Freebody (2006) further concludes that there is a reasonably well established body of research regarding the efficacy of information and communication technology use in classrooms, and that much of the research shows improved learning outcomes through its use. Research by Hoyles, Healy and Sutherland (1991), Clements and McMillen (1996) and Papert (1980) also indicates that computers focus attention and increase motivation. However, there has been something of a debate as to whether materials rendered on a screen can be classed as being manipulatives. As recently as 2001 Moyer, Bolyard and Spikell asserted that "No accepted standard meaning or definition for the phrase 'virtual manipulative' currently exists (p. 184)." By 2002 the same researchers had defined a virtual manipulative as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (p. 373). Spicer (2000) further states that there are two types of representations which are being called virtual manipulatives. These are static and dynamic representations of concrete manipulatives.

According to Moyer et al. (2002) it is important to define these two forms of virtual manipulatives. They assert that static visual representations are essentially pictures, which although they appear to resemble concrete manipulatives, cannot be flipped, slid or turned

by the user as concrete manipulatives can. In contrast, dynamic representations of concrete manipulatives are essentially visual images which have the capacity for students to manipulate them just as they would the concrete materials, but through the use of a computer mouse. These are increasingly the more common form of virtual manipulatives available to students.

Clements (1999) warns that even if we agree that 'concrete' cannot simply be equated with physical manipulatives, we might have difficulty accepting objects on the computer screen as valid manipulatives. However, computers might provide representations that are just as personally meaningful to students as physical objects. Paradoxically, research indicates that computer representations may even be more manageable, 'clean,' flexible, and extensible than their physical counterparts (Clements & McMillen, 1996).

Clements and McMillen (1996, p. 6) offer a further positive aspect of the use of manipulatives on computers: "They avoid distractions often present when students use physical manipulatives. They can also mirror the desired mental actions more closely." They also quote Hoyles, Healy and Sutherland's (1991) research which claims that the computer "…somehow draws the attention of the pupils and becomes a focus for discussion", thus resulting in very little off-task talk.

Moyer-Packenham, Salkind and Bolyard (2008) write that dynamic virtual manipulatives are unique in that they offer a visual image like a pictorial model but can be manipulated like a physical model. They further report on their belief that due to the fact that some virtual manipulatives contain links among enactive, iconic and symbolic notations their potential for increased mathematically meaningful action for users is increased. In their study of a single Year 3 class they reported a statistically significant improvement in student post-test scores (for a unit of work on fractions) and attributed this at least in part to the use of virtual manipulatives. They claim that an important element of this success was the capacity for differentiation which was inherent in the virtual manipulatives and stated

> These instances of individual feedback, multiple representations for support, and a variable pace for completing tasks may have been an important aspect in the differentiation of instruction during these lessons… (Moyer-Packenham, et al., 2008, p. 21)

Not only do virtual manipulatives have a role as a learning tool in themselves but they can also act as a bridge in the progression from physical objects to representational forms (Reimer & Moyer, 2005). There is, however, the warning codicil which comes with manipulative materials, whether they be physical or virtual that the mere manipulative in no way guarantees learning will happen. Teachers carefully helping students make connections between what is hoped to be learned and the materials is essential (Reimer & Moyer, 2005).

2.14 *Professional Development and Professional Learning*

Professional development (PD) around the world is a multi-billion dollar concern: in 2004-2005 in the USA alone, US$1.5 billion was spent on teacher development (Wayne et al., 2008). According to Kelleher (2003), the National Staff Development Council in the USA recommended devoting 10% of school budgets and 25% of teacher time to professional development. With any investment, and particularly one of this magnitude, the question of effectiveness is vital. If as research suggests, much professional development does not take into account how adults learn, is intellectually superficial (Borko, 2004) and is fragmented or sporadic and disconnected (Feinman-Nemser, 2001), it is unlikely to be effective. Therefore a pertinent question is, how can professional development be designed to have a positive impact on teachers' professional learning, and subsequently on student achievement?

According to Penuel, Fishman, Yamaguchi and Gallagher (2007), this focus on what makes PD effective has developed from a time when little attention was directed to the outcomes achieved through PD as at the time the focus was more upon the teacher satisfaction gained from the PD experience. Whilst teacher satisfaction is no doubt important in engaging the teachers, the more important question that needed to be asked was how PD affected change in classroom practice (Penuel et al., 2007).

It is impossible to be sanguine about believing that merely instituting PD will make a difference. As Cohen, Raudenbush and Ball (2003) assert, the mere addition of resources to a school, in this case professional learning experiences, will not necessarily improve instruction. However, Zambo and Zambo (2008) cite a number of studies which support the

premise that "professional development for teachers has been shown to have potential positive effects for both changing teachers' beliefs about mathematics instruction and the instruction they provide" (p. 159). Their own study also asserts that increased competence for teaching mathematics was indicated by teachers with high levels of participation in PD (Zambo & Zambo, 2008). Penuel et al. (2007) report that there have been large scale survey studies which indicate that PD can influence teachers' knowledge and practice. Yoon, Duncan, Lee, Scarloss and Shapley (2007) summarised the recent literature and posited that carefully considered PD can have an effect on student achievement. This is a position supported in the work of Hill, Ball and Schilling (2008a) and of Desimone, Porter, Garet, Yoon and Birman (2002).

In 1986 Guskey constructed a model for teacher change which was predominantly linear in nature. This model was then adapted by Rogers (2007) who proposed an alternative which was more cyclical in nature while retaining the same elements.

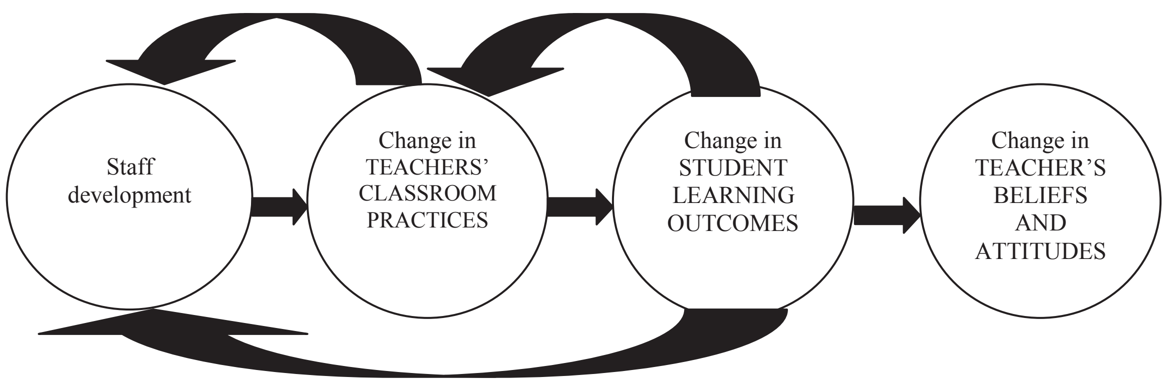


Figure 2.10 Cyclic model of the process of teacher change (Rogers, 2007)

In a fundamental sense (and perhaps almost self-evidently, regardless of the appellations in the circles) the whole process was driven by the desire for change to occur. Yet this Researcher believes there is an implied, though not articulated, connection between the change in teacher's beliefs and attitudes and a cycling back to staff development. Another significant limitation of Rogers' (2007) model is that there is no reference to impacts on teachers' knowledge.

Other than the benefit of the intuitive connection between PD and increased student achievement, showing the gains is difficult. Yoon et al. (2008) created a logic map (Figure 2.11) to illustrate the link between PD and student achievement and how it always runs through a contextual filter. The filter containing teacher knowledge, skills and motivation in itself raises questions which speak to the complexity of such a model. Questions such as; what knowledge? Knowledge to what depth and breadth? What skills?

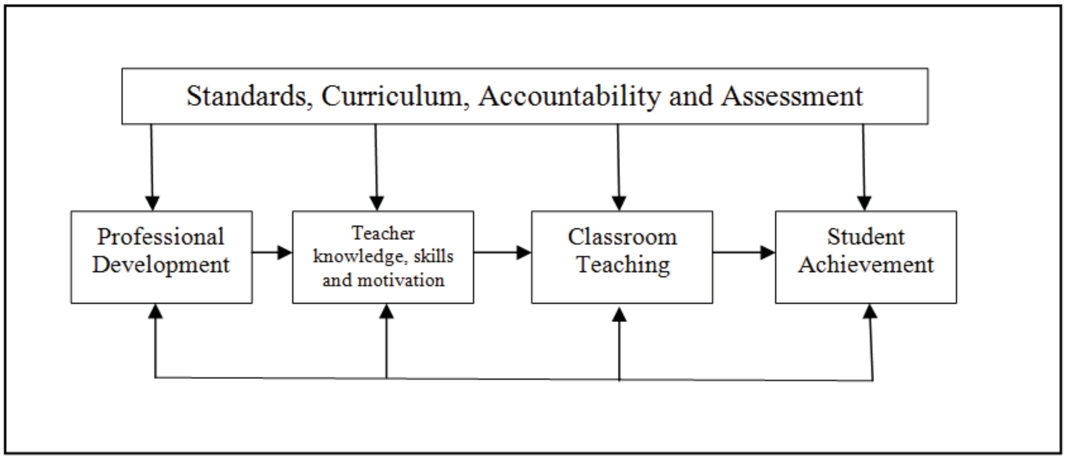


Figure 2.11 Yoon et al. (2008) logic model of the impact of professional development on student achievement

A developing body of literature has emerged as to what constitutes effective PD for teachers (Clarke, 2003; Cohen & Hill, 2000; Loucks-Horsley, Hewson, Love & Stile, 1998; Supovitz & Turner, 2000; Supovitz, Mayer & Kahle, 2000; Zigarmi, Betz & Jensens, 1977). However, in spite of the amount of literature, relatively little research seems to have been conducted concerning the effects of different PD programs upon improving teaching or improving student outcomes (Hiebert & Grouws, 2007; Kennedy, 1998; Loucks-Horsley & Matsumoto, 1999; Supovitz, 2001; Watson, Beswick & Brown, 2012). The research indicates some consensus of thought on what constitutes effective PD (Loucks–Horsley et al., 2003), but there appears to be little written about the extent to which these characteristics are related to better teaching and learning (Yoon, Garet, Birman & Jacobson, 2006). It is important to emphasise the word some, as none of the lists are identical, even though there are significant overlaps (Guskey, 2003).

A summary of the research is presented in Appendix 1. This uses as a basis (but not exclusively) three extensive studies by Clarke (2003), In- Praxis Group Inc. (2006) and Guskey (2003). This synthesis indicates that effective professional learning opportunities should contain at least some of the elements listed in Appendix 1.

For this study only the characteristics of effective PD that were mentioned in four or more of the studies (labelled in Table 2.5 as M for Medium frequency) and those mentioned six or more times, labelled H, (High frequency) are considered to be sufficiently universal as to warrant attention. Those labelled L for Low frequency are of interest but perhaps these characteristics did not resonate with enough of the researchers.

The one exception to this will be the inclusion of reflection as a key element. This is a surprising omission from many lists, but nevertheless it deserves attention due to the body of research (for example Barkatsas & Malone, 2005; Clarke, 1994; Griffin, 2003; Grouws & Schultz, 1996; Yost & Sentner, 2000) which has emphasised its efficacy to teachers and to teaching. Indeed, Johnson, Hodgen and Adhami (2007) declare that "Reflection is integral to all professional development components" (p. 209). All of the identified characteristics have been further grouped in Table 2.5 under the headings of; *Content, Process* and *Context*, as defined by the National Staff Development Council (2001). In recognition of the research, the professional learning experience which was provided for the participants in this study was based on those identified characteristics of effective professional development which are indicated with an asterisk (*) in Table 2.5.

Table 2.5

Characteristics of effective Professional Development

Characteristics of effective professional development - Content	**Level of support from the literature**
*Focuses on increasing knowledge and skills to bring about change in teaching practice.	**H**
*Recognises the ways adults learn, and the impact of constructivist learning theory	**H**
*Accommodates diversity and promotes equity in schools	**H**
Involves families and other stakeholders in the process	**M**

Characteristics of effective professional development – Context	**Level of support from the literature**
*Promotes the development of leadership capacity	**H**
Is centred in the school community and based on teachers' identified needs	**H**
*Involves the formation of learning communities	**H**
Recognises and explores the impact of initiatives on school culture	**H**
Recognises the impact of change on school improvement processes	**M**
*Is centred on the development and maintenance of collaborative environments	**H**
*Addresses issues and concerns and interest which are largely (but not exclusively) identified by the teachers	**H**
Aligns with other reform initiatives	**H**
Is school or site based	**M**
*Focuses on individual and organisational improvement	**M**
*Includes follow up and support	**M**
*Is ongoing and job embedded	**M**
*Allows time and opportunity for planning, reflection and feedback	**L**

Characteristics of effective professional development – Process	**Level of support from the literature**
*Is centred on the improvement of student achievement and growth	**M**
*Emphasises, and makes choices informed by, the link between teacher quality and student success	**H**
*Recognises multiple contexts, formats and factors	**H**
*Increases teacher knowledge and understanding	**H**
*It is purposeful, sustained and sustainable over time	**H**
*Is based on the best available research evidence	**H**
*Is driven by analyses of student learning data	**H**
*Assesses the impact of initiatives and decisions on student outcomes	**H**

Support
L = Low frequency - 1 – 3 researchers
M = Medium frequency - 4 – 6 researchers
H = High frequency – more than 6 researchers

It is intended that all of the elements which are a focus in this study are what are extensively referred to as a teacher's zone of enactment, a term drawn from the work of Spillane (1999). Spillane characterised the zone of enactment as "that space where reform initiatives are encountered by the world of the practitioners" (p. 144).

An important but unstated nuance of what is offered to the teachers who undertake professional development is its reframing to a focus on learning. As Webster-Wright (2009) asserts, simply using the term professional development may set a discourse which does not reflect the need to focus on learning over development, as using the word development may be seen as coming from a deficit view point. Therefore the first aspect of reframing the professional learning should be to call it just that, professional learning (PL). Further, Webster-Wright (2009) stresses that merely applying the nomenclature of professional development implies a transmission model which can set an epistemological expectation.

In order to frame the content of professional learning (PL), the knowledge which the participants in a professional learning situation might be seeking must be considered. Some courses focus on pedagogy whilst others focus on content. White et al. (2004) advocate a blending of both, a position supported by other researchers (Darling-Hammond & Richardson, 2009; Higgins & Parsons, 2009; Loucks-Horsley et al., 2003).
Further, in considering the structure of PL, Darling-Hammond and Richardson (2009) posit that research supports professional learning (learning is the Researcher's own word here as Darling-Hammond and Richardson use the word development) which:

- Deepens teachers' knowledge of content and how to teach it to students.
- Helps teachers understand how students learn specific content.
- Provides opportunities for active, hands-on learning.
- Enables teachers to acquire new knowledge, apply it to practice, and reflect on the results with colleagues.
- Is part of a school reform effort that links curriculum, assessment, and standards to professional learning.
- Is collaborative and collegial.
- Is intensive and sustained over time.

(p. 49)

Point four of Darling-Hammond and Richardson's (2009) list speaks to the need to be cognisant of praxis in constructing the PL. In order to encourage teachers to make fitting judgments, then the PL should create an environment in which McCotter's (2001) guidelines for professional development (experiential, ongoing, collaborative, empowering, contextual and related to practice and theory) are adhered to. MacIntyre-Latta and Kim (2010) argue that "...creating, adapting, and discerning are fundamental to the nature of

human beings and that the costs for teachers and learners of thwarting the agentic ways of being and acting in classrooms are vastly underestimated” (p. 137).

Thomas (2007) argues that too often PL can be criticised for little or no attention being paid as to just how the participants will apply their newly acquired knowledge, understandings or skills. If this is to be avoided then it can be argued that one measure of effectiveness of professional learning should be transfer. Hager and Hodkinson (2009) define transfer as a metaphor for “trying to understand what happens when people learn something new and/or move into new and different situations” (p. 620). They consider the word transfer and its definition to be simplistic and misleading as there is no external, reified entity that is learning. They suggest a shift in the metaphor, towards thinking not of transfer, but of learning as becoming a transitional process of boundary crossing. Smith (2006) defines boundary crossing as the collective formation of new concepts that requires moving into unfamiliar domains and can be said to involve reconstructing knowledge rather than simply transferring it.

Regardless of the term employed there seems to be little doubt that one important measure of successful PL is if transfer is achieved in terms of altering the learning environment. Goldman and Schmalz (2005) claim that no more than 20% of training investment actually results in transfer to the job, although this was data quoted in an article in relation to health workers there seems no reason to believe this would not be applicable to other situations. If transfer is a measure of effectiveness then a return of 20% on the PL investment may not seem adequate.

It is intended that the strong reflective element of this action learning/action research study will act as a vehicle for transfer of the learning. The learning which comes from the workshops will be reflected upon, and the teachers will be asked to enact them in the classroom. These classroom experiences will then be reflected upon in the following PL session to further integrate content and pedagogical knowledge, and strengthen the transfer of learning into classroom practice.

2.15 *What is Pedagogical Content Knowledge (PCK)?*

In his seminal work Shulman (1986) enumerated seven categories of the professional knowledge base required for teaching. These categories included:

1. content knowledge;
2. general pedagogical knowledge;
3. curricular knowledge;
4. knowledge of learners (their characteristics, cognition, motivation and development);
5. knowledge of educational contexts;
6. knowledge of educational aims, goals and purposes; and,
7. pedagogical content knowledge (PCK).

According to Abd-El-Khalick (2006) and Ball, Thames and Phelps (2008), the category that gained the greatest attention from researchers was PCK. Shulman (1986) argued that PCK builds upon, but is not the same as, subject matter knowledge or knowledge of general principles of pedagogy. Rather, the epistemological concept of PCK could be described as a link between the knowledge bases of content (both the substantive and syntactic structures of a discipline) and of pedagogy. A third necessary domain is knowledge of context. One could characterise PCK as a practical knowledge of teaching and learning guided through a contextualised knowledge of a particular classroom setting. The conceptual framework below (Figure 2.12) is intended to express in diagrammatic form these three domains, and their interaction in developing PCK.

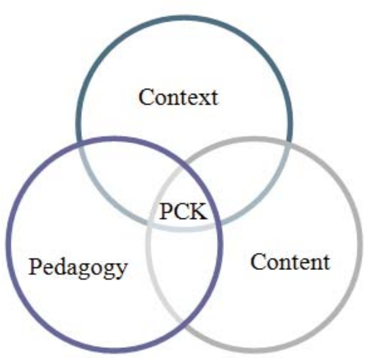


Figure 2.12 Shulman’s (1986) pedagogical content knowledge domains

Shulman (1986) stated that PCK helps teachers to guide students to understanding. PCK includes useful forms of representations, analogies, illustrations, examples, explanations and demonstrations. In order for teachers to have these forms of understanding, they need to be underpinned by content knowledge. In fact, Veal and MaKinster (1999) in their hierarchy of PCK stated that a strong content background is essential to its development. Park and Oliver (2008) reviewed and analysed literature available on defining PCK and arrived at what they consider to be a comprehensive working definition:

> PCK is teachers' understanding and enactment of how to help a group of students understand specific subject matter using multiple instructional strategies, representations, and assessments while working within the contextual, cultural, and social limitations in the learning environment. (p. 264)

Wilson, Shulman and Richert (1987) suggested that experience is a major influence on the shaping and development of PCK, a view which was supported by Gess-Newsome and Lederman (1995). They found evidence that novice teachers with up to five years of teaching experience did not recognise and put into practice the necessary connections to fulfil what Shulman would determine as PCK. Further, Wilson, Floden and Ferrini-Mundy (2002) state that beginning teachers possess a limited repertoire of PCK. This being said, Gess-Newsome and Lederman (1995) concluded that teaching experience, although an important factor in the development of PCK, is not as significant in contributing to PCK as a teacher's opportunity and disposition towards reflection on content knowledge. This proposition was also supported by the work of Hoz, Tomer and Tamir (1990).

In 1999 Gess-Newsome proposed two models for PCK, integrative and transformative. The integrative model supposes that the relevant knowledge bases used in teaching (subject matter, pedagogy and context) are developed separately and that the act of teaching provides the opportunity for their integration. Therefore "an expert teacher, then, is one with organised knowledge bases that can be quickly and easily drawn upon while being engaged in the act of teaching" (Silverman & Thompson, 2008, p. 501). A potential danger in this model is that teachers may not see the equal importance of content, pedagogy and context. This may occur if they are struggling with the problems of the domains in particular or, if as Nilsson (2008) states, that the teacher never sees the importance of such

integration. Nilsson (2008) also proposes that the advantage of the integrative model is that the constituent parts can be developed independently and integrated at a later time.

The transformative model is a model which recognises the value of synthesised knowledge, the fundamental transformation of knowledge and the creation of new knowledge. According to Silverman and Thompson (2008) the transformative model requires purposefully integrated experiences that allow teachers the opportunity to not only extend their mathematical and pedagogical understandings but to also create connections to create a new knowledge. The transformative model also recognises that whilst the knowledge bases of content, pedagogy and context exist they are only useful when transformed into PCK, which by extrapolation cannot occur in a professional learning situation or in pre-service training, but only in the classroom. This then raises questions regarding the efficacy of the current manner in which systems and sectors try to enhance the PCK of teachers and teaching neophytes. This Researcher believes that the truth probably sits between the two positions and that teachers need to be prepared, through purposeful development opportunities, to reflect upon their teaching. This is necessary in order to become more responsive when the opportunity for development of PCK in the teaching work place presents itself. For pre-service teachers and neophyte teachers this may take the form of having opportunities provided to them to observe, analyse and reflect upon other teachers' teaching (Nilsson 2008). Furthermore, these other teachers, whether they are classroom based or in a university setting, may in the words of Nilsson (2008) "…need to portray and explicate aspects of their own PCK…" (p. 1296).

2.16 ***Factors Leading to Strengthened Pedagogical Content Knowledge***

Regardless of its pre-eminence, Shulman's notion of PCK has been challenged (Graeber & Tirosh, 2008) and the concept has been expanded and modified by a number of authors. Ball and Bass (2000) for example, regard the ability to unpack the mathematics from constructs, concepts, analogies, metaphors and images as being an important aspect of PCK.

Ball, Thames and Phelps (2008) argue that although the term PCK is very widely used, it actually lacks clarity of definition, and its potential has not been fully realised. Their refinements of the concept of PCK and its attempt to reframe the study of teaching knowledge are predicated around placing the emphasis on the use of "knowledge in and for teaching rather than on teachers themselves." (p. 394). In elaborating on Shulman's construct of PCK, Ball, Thames and Phelps (2008) use a construct (Figure 2.13) (also used in Hill et al. 2008a) which maps their domains of content knowledge for teaching onto two of Shulman's (1986) initial categories for PCK, those of subject matter knowledge and pedagogical content knowledge.

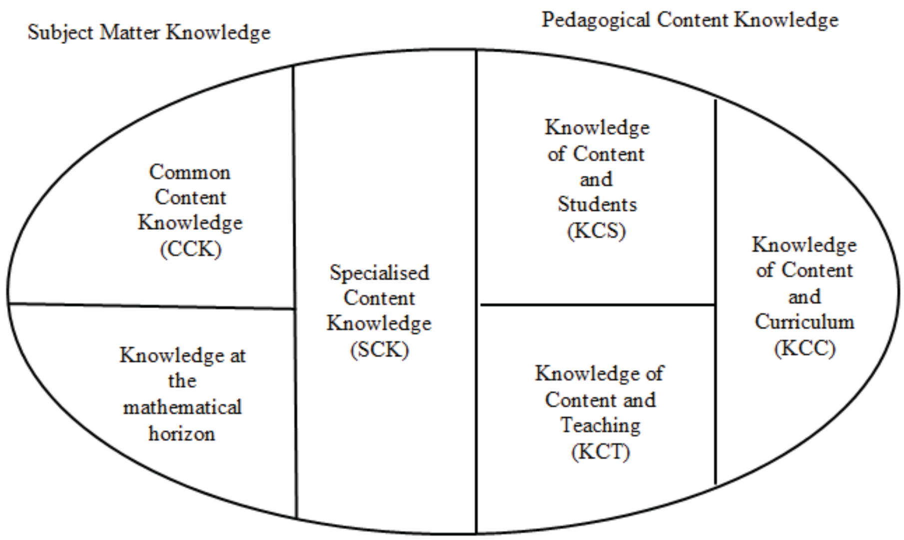


Figure 2.13 Mathematical knowledge for teaching (Hill et al., 2008a, p. 377)

Common Content Knowledge (CCK) is mathematical knowledge and skill used in general settings, settings not necessarily unique to teaching. Specialised Content Knowledge (SCK) is the mathematical skills and knowledge particular to teaching. Knowledge of Content and Students (KCS) is that knowledge which is a combination of knowing about students and about mathematics, and Knowledge of Content and Teaching (KCT) is a combination of knowing about teaching and about mathematics. The final two domains Knowledge at the mathematical horizons and Knowledge of Content and Curriculum (KCC) are both considered by the authors as interim placements, still in need of revision and refinement, as both may run across several categories or be categories on their own (Ball, Thames & Phelps, 2008).

It is the contention of this study that in order to strengthen PCK, teachers must develop the domains of Subject Matter Knowledge (SMK) and pedagogical knowledge which in turn will have a positive effect upon teachers' self-efficacy. Self-efficacy is defined as "…people's beliefs about their capabilities to produce designated levels of performance that exercise influence over events that affect their lives" (Bandura, 1994, p. 72). It is further contended that this can be achieved when these elements are passed through a process of reflective practice (Figure 2.14).

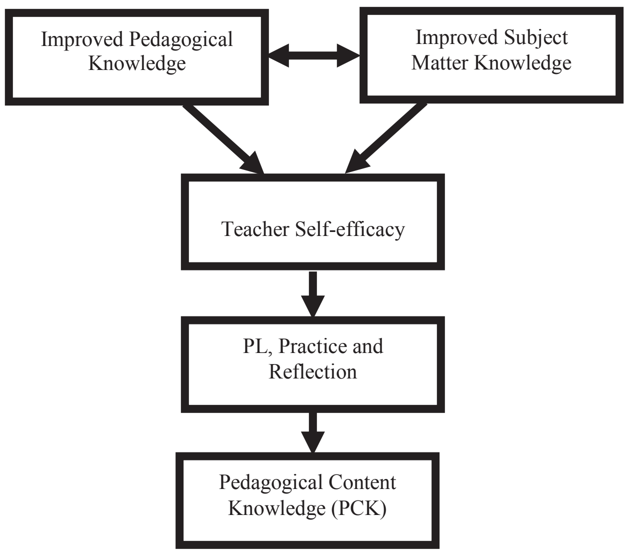


Figure 2.14 Factors leading to strengthened PCK

Park and Oliver (2008) determined through their research five salient features which they claim add to or complement current understanding about PCK. These are:

- PCK is developed through reflection.
- Teacher efficacy is an affective affiliate of PCK.
- Teachers' understanding of student misconceptions are vital in that they shape PCK in planning, instruction and assessment.
- Students influence the manner in which PCK is developed, organised and validated.
- PCK is idiosyncratic in that it is particular to the individual.

This Researcher, whilst recognising all of the features of PCK as stated by Park and Oliver (2008), finds the first three points to be compelling and therefore they will be discussed

further. Dot point four takes the research into the sphere of context, an area that this research, is not pursuing. Dot point five speaks of PCK being idiosyncratic, which would suggest that there is no single structure or design which will allow PCK to be learned. This, the Researcher believes to be axiomatic, as in all learning there are basic precepts which can be employed but the enactment is always individual. Therefore, as stated, the focus will be on the first three points.

What is of further interest from the work of Park and Oliver (2008) is that they produced a heuristic model of PCK for science which incorporates many of the aspects as defined in the Mathematical Knowledge for Teaching model (Hill et al., 2008a). However, it also uses a more dynamic representation in an attempt to show how the components influence each other and therefore, and in turn, understanding and enactment. The Researcher contends that there is nothing in this representation that suggests that it is not wholly applicable to mathematics teaching and learning.

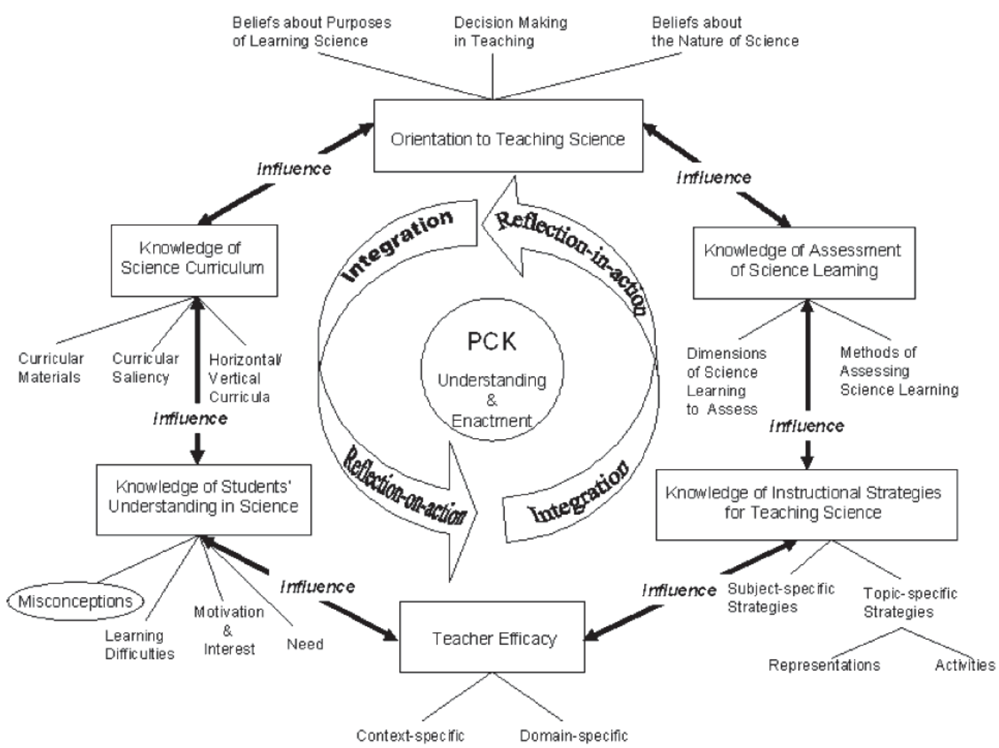


Figure 2.15 Park and Oliver’s (2008) hexagon model of pedagogical content knowledge for science teaching

Although the model speaks of knowledge, understandings and orientation to the teaching of science it would lose no power or truth from having, in all cases, the word science replaced by the term mathematics.

2.17 ***Subject Matter Knowledge (SMK)***

In section 2.15 the assertion was made that in order to strengthen Pedagogical Content Knowledge (PCK), teachers must develop the domains that comprise Subject Matter Knowledge (SMK). Hattie (2003) looked at what made the difference in achievement in schools. In this study he determined that there were six major sources of variance, with the second biggest influence that affected student achievement in the classroom being the teacher. Hattie (2005) wrote about the difference between the 'expert' teacher and the 'experienced' teacher and identified five major dimensions of the 'expert' teacher that differentiates him or her from the 'experienced' teacher. He or she can:

1. identify essential representations of their subject;
2. guide learning through classroom interactions;
3. monitor learning and provide feedback;
4. attend to affective attributes; and,
5. can influence student outcomes.

It can be asserted that the first three dimensions articulate areas which are underpinned by and are heavily reliant on the teacher's mathematical knowledge element of PCK. The importance of mathematical knowledge in differentiating between an experienced teacher and an expert teacher is further supported by Nathan and Petrosino (2003), Ericsson and Lehmann (1996) and Shulman (1986).

In research focussed on mathematics (Charalambous, Hill & Mitchell, 2012; Hill et al, 2008b; Hill, Rowan & Ball, 2005), the ability to do the following were seen as indicators of teachers who could provide quality instruction; they:

- provide better mathematical explanations;
- choose and/or construct more appropriate representations;
- are responsive to student mathematical ideas;

- have a clearer understanding of the structures and connections in mathematics; and,
- avoid mathematical imprecision and error.

There is much that is similar when one compares Hattie's (2005) 'expert' teachers and Hill, Rowan and Ball's (2005) 'knowledgable' teachers (Table 2.6).

Table 2.6

Comparison of Hattie (2005) and Hill, Rowan and Ball (2005) research on 'Expert' and 'Knowledgable' teachers.

Hattie	Hill, Rowan and Ball
• can identify essential representations of their subject;	• provide better mathematical explanations • **choose and/or construct more appropriate representations** • have a clearer understanding of the structures and connections in mathematics;
• can guide learning through classroom interactions;	• provide better mathematical explanations • choose and/or construct more appropriate representations • **better 'hear' student explanations** • have a clearer understanding of the structures and connections in mathematics • avoid mathematical imprecision and error
• can monitor learning and provide feedback;	• provide better mathematical explanations • **better 'hear' student explanations** • have a clearer understanding of the structures and connections in mathematics
• can attend to affective attributes	
• can influence student outcomes.	

The overwhelming evidence that indicates that students continue to have considerable difficulty with fractions raises the question as to why teachers are not addressing this area well. If, as the literature states, the teaching of fractions is problematic and frustrating for

many teachers (Capraro, 2004) it would also seem likely that their own understanding in this area is not well developed. Although it is not unequivocal (Hill, Rowan & Ball, 2005), research generally supports that mathematical knowledge is crucial for improving the quality of instruction in classrooms (Ambrose, 2004; Ball, Thames & Phelps, 2008; Charalambous et al., 2012; Cobb & Jackson, 2011; Hill, et al., 2005; Hill et al., 2008b; Toluk-Uçar, 2009) and it is well documented that teachers often lack a deep conceptual understanding of mathematics (Ball, 1990; Ball, Hill & Bass, 2005; Ma, 1999; Tirosh, Fischbein, Graeber & Wilson, 1996). Many exhibit weaknesses in mathematics, may misapply rules, and are generally not prepared to teach the mathematical content entrusted to them (Hill et al., 2008b; Hungerford, 1994; Tsao, 2005).

When looking into mathematical knowledge for teaching, Hill et al. (2008b) used several key studies where measures of teacher knowledge and classroom instruction were taken simultaneously. They concluded that there were approaches, deficit and affordance.

> In the former the authors draw linkages between a teacher's lack of mathematical understanding and patterns in her mathematics instruction; in the latter (affordance), the authors highlight the affordances strong mathematical (and related) understandings create for classroom culture and instruction. (p. 433)

A good deal of research has been conducted as to the type of mathematical knowledge that teachers require in order to teach well (Ambrose, 2004; Hill & Ball, 2004; Ma, 1999). However according to Hill et al. (2005):

> ..educational production function literature...reveals that researchers... have typically measured teachers' knowledge using proxy variables, such as courses taken, degrees attained, or results of basic skills tests.
> (p. 372)

They further assert that as mathematical knowledge for teaching goes beyond that which can be captured and measured regarding mathematics competencies these measures are not adequate. These are assertions, which the work of Graeber and Tirosh (2008), Ball and Bass (2000), Shulman (1986), and others, would support in reference to pedagogical content knowledge (PCK).

An important and revealing finding was that, in the case of the many teachers in elementary classes who lack a conceptual understanding of mathematics (Ball, 1990; Ball, Hill & Bass, 2005; Even, 1990; Ma, 1999), teaching quality (as determined by performance on standard tests) related to whether the teachers' knowledge of mathematics was procedural or conceptual (Hill & Ball, 2004). As Silverman and Thompson (2008) relate, if the mathematical knowledge of a teacher is comprised of disconnected facts and procedures, they are unlikely to be able to compose a structural web of mathematical ideas for their students to emulate. Such structural webs are something their students' need in order to develop high quality understanding. This is a position supported by the research of Meaney and Lange (2010) and Wilson, Cooney and Stinson (2005). The importance of conceptual and procedural knowledge continues to be a point of discussion for researchers (Forrester & Chinnappan, 2010) and the tensions and balances between them are still topical.

Given the already discussed difficulty that teachers experience with the topic of rational numbers (Moss & Case, 1999; Tirosh, 2000) and the fact that many teachers do not have a deep conceptual understanding of mathematics in general, it can be expected that their conceptual understanding of fractions would also be somewhat lacking. As stated earlier, Van de Walle and Folk (2005) asserted that the topic of fractions may be one of the most common areas in mathematics in which students are often presented with rules without justification in schools.

The research supports the idea that efforts to improve teachers' mathematical knowledge through content focussed professional development will improve both teacher self-efficacy (Swackhamer et al., 2009; Swars et al., 2007) and student performance (Ball et al., 2005; Hill et al., 2005). Therefore it would seem self-evident that improved specific content knowledge in the topic of fractions would be beneficial firstly in helping the confidence and understanding of teachers in this problematic area and then secondly, in the improvement in student attainment.

2.18 *Self-Efficacy, Beliefs and Attitudes*

As stated in the previous section teacher efficacy is an affective affiliate of PCK (Park & Olive, 2008). The impact of self-efficacy on the teaching and learning of mathematics cannot be underrated. Tschannen-Moran and Hoy (2007) call self-efficacy "...a little idea with big impact" (p. 954). It is an important influence on teaching practices and student learning (Barkatsas & Malone, 2005; Philipp, 2007; Smith, 1996); it has a high correlation between it and classroom instruction practices and also the willingness to embrace innovations (Gabriele & Joram, 2007; Gresham, 2008; Swars, Daane & Giesen, 2006); highly efficacious teachers are more likely to use a variety of instructional strategies such as inquiry based learning, student centred teaching strategies (Czerniak & Schriver, 1994; Swackhamer, Koellner, Basile & Kimbrough, 2009) and the use of manipulative materials (Gresham, 2006); it is highly associated with teacher motivation (Bandura, 1993) and reducing stress amongst teachers (Tschannen-Moran, Woolfolk-Hoy & Hoy, 1998), and efficacy influences teacher' expectations, attributions and goals (Zambo & Zambo, 2008). Clearly self-efficacy is important to the successful teaching of mathematics.

Bandura (1994) stated that self-efficacy is about people's beliefs about their capacity to exercise influence over events that affect their life and as such the assertion that self-efficacy is about confidence, attitudes and beliefs is a logical contention. Each of these is an affective process, that is, a process which regulates emotional states and the drawing out of emotional reactions and are closely associated.

To quote Bandura (1994):

> …self-efficacy is defined as people's beliefs about their capabilities to produce designated levels of performance that exercise influence over events that affect their lives. Self-efficacy beliefs determine how people feel, think, motivate themselves and behave. Such beliefs produce these diverse effects through four major processes. They include cognitive, motivational, affective and selection processes. (p. 72)

Teacher self-efficacy may be positively enhanced (Bandura, 1994) by providing professional learning (PL) opportunities which concentrate on subject content knowledge

and sound pedagogical practices (Lampert & Ball, 1998). Tschannen-Moran and Hoy (2007) define a vicarious experience as being one where the experience is modelled by someone else, and this is the prevailing model for most PL. They caution about the impact of the vicarious experience that PL affords upon the observer's efficacy beliefs. If the participant closely identifies with the modelled experience then self-efficacy is advanced. If the identification with the modelling is not achieved then even a very competent performance in the modelling may not enhance self-efficacy. Further the PL must make sure that the vicarious experience must lead to the participant being able to take it back to his or her classroom and achieve what Bandura (1997) calls mastery experiences, which comes from teaching success. Tschannen-Moran and Hoy (2007) assert that of the four major influences upon teachers' self-efficacy (mastery experiences, verbal persuasion, vicarious experiences and physiological arousal), mastery experiences are the most powerful. It is therefore essential that if self-efficacy is to be enhanced, PL providers must be cognisant of the effect of the vicarious experiences to be offered and that the experience is able to be successfully replicated in the classroom.

Bandura (1994) asserted that reducing stress reactions and altering negative emotional proclivities (towards mathematics) can have a major positive effect on self-efficacy which in turn can be used to promote reflective practice and lead to improved PCK (Gess-Newsome & Lederman, 1995; Hoz, Tomer & Tamir, 1990).

According to Tschannen-Moran and Hoy (2007) there has been an accumulation of compelling evidence to support the link between teachers' beliefs about their ability to enhance student motivation and student achievement. Beliefs are usually defined as being understandings, premises or propositions that are held true, for example empirical truth, validity, or applicability (Goldin, 2002; Philipp, 2007), and beliefs, unlike knowledge, maybe held with various degrees of conviction (Philipp, 2007). Beliefs can be seen as the responses that people (and in the case of this study, a particular set of people, teachers) provide in reaction to open "I believe" questions; in a manner that the participants believe to be true at that time and in that context (Ajzen & Fishbein, 1980), and as personal philosophies (Aguirre, 2009). Although much has been written about the inconsistency between teachers' beliefs and teacher practice (Beswick, 2006), teacher beliefs should not be discounted as being unauthentic but rather as representations of intended practice

(Liljadhar 2009). Alternatively, perhaps the implication of the belief has been misunderstood, or another belief has taken precedence over it in that situation (Leatham, 2006). Beliefs and action develop together and influence each other, and so can be logically argued to be related (Beswick, 2005; Grootenboer, 2008). Furinghetti and Moselli (2009) deemed that "...beliefs are behind reasons for teachers' decisions and in this role, relies the importance of beliefs in relation to practice" (p. 61). Raymond (1997) proposed a model (Figure 2.16) to represent the relationship between teachers' mathematical beliefs and their teaching practice. It can be seen through this model that the relationship is quite complex and this Researcher would suggests that the model only hints at the relationships gestalt.

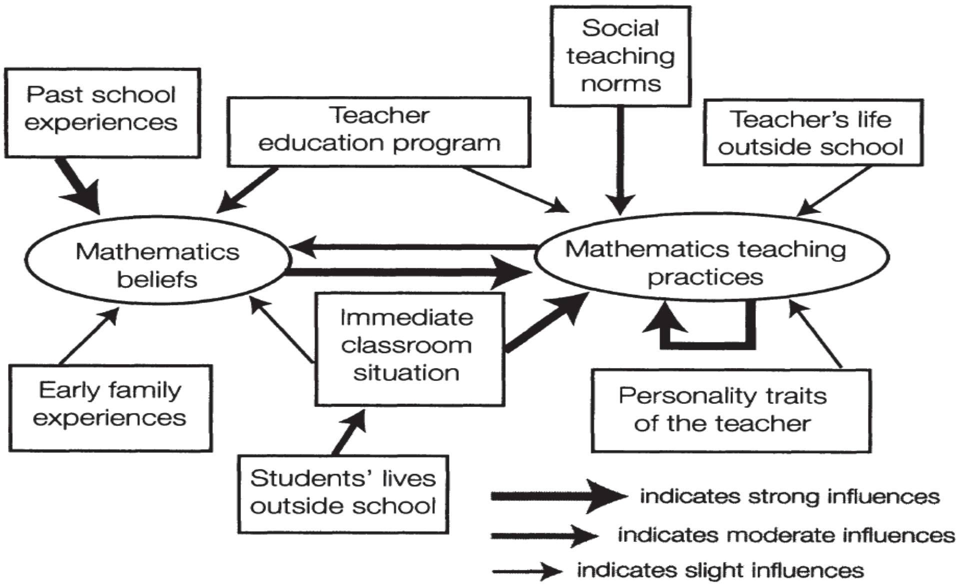


Figure 2.16 Raymond's (1997) revised model of the relationships between mathematical beliefs and practice

Goulding et al. (2002) suggest that beliefs are inextricably tied to subject matter knowledge as they determine how a teacher may approach a mathematical situation. If the teacher has a strong belief in the rote learning of routines, for instance, then this will probably have an impact on their teaching. Therefore, teacher beliefs are central to this study.

Attitude is an ambiguous construct (Hannula, 2002) and there is also a diversity of definitions regarding attitudes. Constructs are often seen to conceptualise attitudes as having emotional elements that places them nearer the affective than the cognitive end of the spectrum; as having an impact on intention and hence behaviour (Ajzen & Fishbein,

1980); and as depending upon experience (McLeod, 1992) and beliefs (Ajzen & Fishbein, 1980). Cognition and emotion are seen as being two complementary aspects of mind and the interaction between the two is so entwined and intense that neither can be separated from each other (Hannula, 2002).

Attitudes are learned and they predispose a person to some degree of consistency and can be evaluated as positive or negative (Fishbein & Ajzen, 1975; Hannula, 2002). They are linked to beliefs, as each person has a corresponding attitude to each belief. This attitude manifests itself in the performance of a behaviour or task and the manner in which it was performed. The positive relationship which exists between attitudes and achievement is widely documented (Wilkins, 2008).

There have been a number of studies completed on teacher confidence and attitude (Baxter, 1983; Becker, 1986; Bobis & Cusworth, 1994; Ernest, 1988; Steffe, 1990; Tirosh, 1990) which generally paint a fairly bleak picture. Research indicates that many pre-service primary school teachers are fearful, anxious, pessimistic and resentful of mathematics as a subject (Ambrose, 2004; Biddulph, 1999; Bobis & Cusworth, 1994; Szydlik, Szydlik & Benson, 2003) and hold beliefs and attitudes about mathematics which can be narrow and debilitating (Ball, 1990; Szydlik et al., 2003). These beliefs have been found to be firm, tenacious and notoriously resistant to change (Stuart & Thurlow, 2000; Swars, Hart, Smith, Smith & Tolar, 2007). Goldin et al. (2009) assert that beliefs are unlikely to be replaced unless they are challenged and proven to be unsatisfactory. These tenacious and unchallenged beliefs often prevent teachers from teaching mathematics in ways that empower students (Schuck & Grootenboer, 2004) and can affect the level of cognitive complexity in instruction (Charalambous, 2010). Gess-Newsome (2003) argued that the beliefs held by teachers would ultimately make the difference between failure and success in classroom reform. Aguirre (2009) supported this by claiming that beliefs are critical to the successful implementation of mathematical education reform. If, as Ajzen and Fishbein (1980) report, beliefs and attitudes influence behaviour, and as Wilkins (2008) asserts, knowledge, beliefs and attitudes all influence instructional practice, then negative attitudes and beliefs about mathematics will probably manifest in actions contrary to increased mathematical learning taking place. This could be demonstrated through decreased

instruction time, over-reliance upon text books, lack of subject content knowledge and an inability to provide conceptual explanations.

Research also suggests that the negative beliefs and attitudes possessed by pre-service teachers can be ameliorated and challenged through tertiary courses in mathematics (Aldridge & Bobis, 2001; Grootenboer, 2003; Grootenboer, 2008; Swars et al. 2007). If this is the case for pre-service teachers, it is not unreasonable to suppose that in-service teachers who possess negative beliefs and attitudes towards mathematics (Bobis & Cusworth, 1994) may also be positively assisted through professional learning opportunities. This is a position supported by Swackhamer et al. (2009) who state that "... professional development or further education that impacts a teacher's understanding of their craft can affect the teacher's perceived ability level and therefore self-efficacy." (p. 64)

Although many attempts have been made, there has been very little consensus as to how beliefs can be differentiated from attitudes (Furinghetti & Pehkonen, 2002) and therefore how they relate as elements of self-efficacy. In summary and for the purposes of this study the following definitions (Table 2.7) will be applied:

Table 2.7

Definition of terms used in this study for confidence, self-efficacy, beliefs and attitudes.

Term	Definition
Confidence	Faith or belief that one will act in a right, proper, or effective way (Merriam-Webster Online Dictionary, 2008)
Self-efficacy	People's beliefs about their capabilities to produce designated levels of performance that exercise influence over events that affect their lives. (Bandura, 1994)
Beliefs	The responses that people provide in reaction to open "I believe" questions, in a manner that the participants believe to be true at that time and in that context (Ajzen & Fishbein, 1980)
Attitudes	Attitudes are generally regarded as having been learnt. They predispose an individual to action that has some degree of consistency and can be evaluated as either negative or positive (Fishbein & Ajzen, 1975).

2.19 ***Conceptual Framework***

The conceptual framework outlined in Figure 2.17 has emerged from the literature reviewed in this chapter. The conceptual framework highlights proposed relationships between key variables related to teacher professional learning.

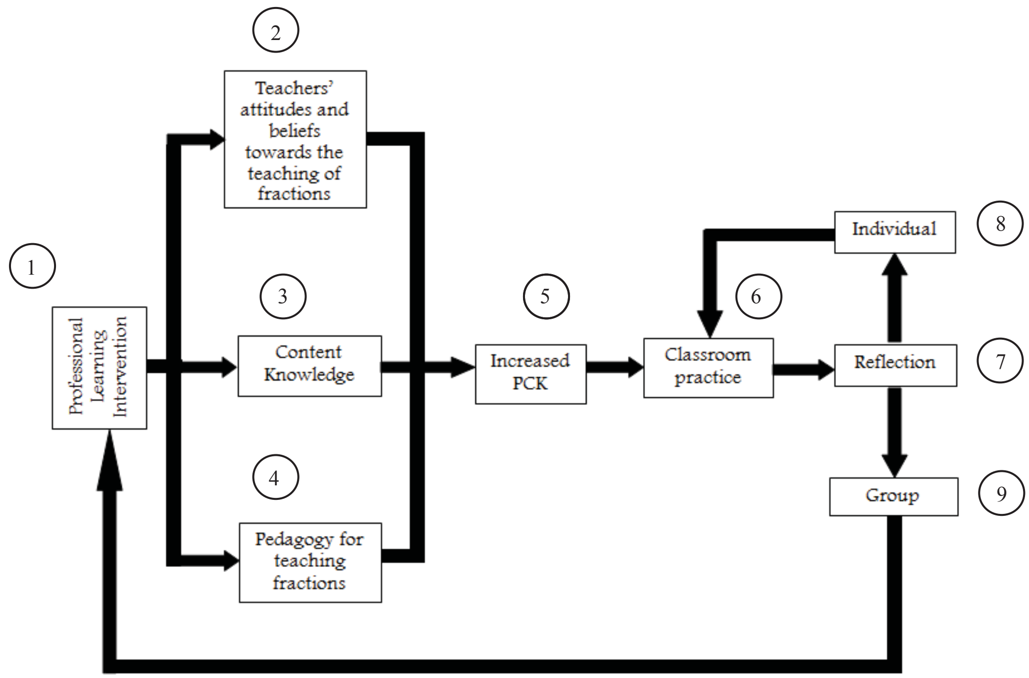


Figure 2.17 Conceptual framework for the study

This notion of multiple PL sessions came from the literature and particularly the work of Rogers (2007), Zambo and Zambo (2008) and Penuel et al. (2007) in effecting teacher change. The structure of the PL was arrived at through a synthesis of the research regarding effective PL (Clarke, 2003; In- Praxis Group Inc., 2006; Guskey, 2003).

The aim of the PL was to make positive changes in the beliefs and attitudes, content knowledge and pedagogy of the participating teachers. Content knowledge and pedagogy were chosen as foci as identified domains in the literature for the development of PCK (Shulman, 1986; Ball, Thames & Phelps, 2008). Working with these two domains at the same time and in a blended way was supported through the work of amongst others, Darling-Hammond and Richardson (2009) and Higgins and Parsons (2009).

Following each PL session the teachers were asked to conduct a lesson or lessons in their classroom, so that they may see how they could directly apply what has been learned (Thomas, 2007) and reflect on those lessons in light of the information gathered in the PL session. Those reflections had a dual purpose in that the teachers could refine the approach to the next lesson taken on the topic of fractions by that teacher, and be used as the basis of the group discussion in the next PL session. The opportunity to reflect in a variety of ways was seen as vital to the PL design and process and the efficacy of doing so was supported by researchers such as Johnson, Hodgen and Adhami (2007), Barkatsas and Malone (2005) and Yost and Sentner (2000).

2.20 *Summary*

This chapter provides a comprehensive review of the pertinent literature. The literature was reviewed and organised into topics to act as background to the field of inquiry: what has previously been written; some commentary by particularly seminal or pre-eminent authors; what questions are being asked; and hypotheses drawn. It was also a forum for the Researcher to begin to establish his perspectives on the field through the lines of inquiry taken and these are represented in the conceptual framework that emerged from the literature.

The following chapter discusses the methodology used in conducting the study. It describes the research approach and explains why a mixed methods research study was adopted. It then gives details regarding the participants in the study. Further, details will be provided about the structure of the professional learning experience which was adopted, and the instruments that were employed to collect the data.

CHAPTER 3

3 Method

3.1 *Introduction*

This study focussed on the manner in which professional learning could be structured to improve the teaching of fractions. The design of the professional learning (PL) was based on the available research with regards to developing content and pedagogical knowledge and engaging teachers in reflection to enhance PCK, positive attitudes and beliefs regarding the teaching of fractions.

The research addressed the following research questions:

1. What is the current status of teaching fractions in middle and upper primary school classrooms in Western Australia?
2. What impacts will well-structured, action research based, professional learning opportunities and reflective practice have on primary school teachers' content knowledge of fractions?
3. What impacts will well-structured, action research based, professional learning opportunities and reflective practice have on primary school teachers' pedagogical knowledge for teaching fractions?
4. What impacts will well-structured, action research based, professional learning opportunities and reflective practice have on primary school teachers' beliefs and attitudes with regards to teaching mathematics in general and fractions in particular?

3.2 *Research Approach*

It was determined that a case study methodology using mixed methods best suited this Researcher's epistemological position (Yin, 2009). Case study was chosen, as it can contribute to knowledge of individual, group, organisational, social, political and related phenomena (Yin, 2009). Selection of this methodology allowed this Researcher to "retain the holistic and meaningful characteristics of real life events" (Yin, 2009, p. 4) and to explore the how and why questions of the phenomena. This methodology was chosen as its fundamental nature is one of trying to "…illuminate a decision or a set of decisions, why

they were taken, how they were implemented and with what results" (Schramm, 1971, p. 6).

This is summed up by Yin (2009) who writes:

> A case study is an empirical inquiry that investigates a contemporary phenomenon within its real-life context; when the boundaries between phenomenon and context are not clearly evident. The case study enquiry copes with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result relies on multiple sources of evidence, with data needing to converge in a triangulating fashion, and as another result benefits from the prior development of theoretical propositions to guide data collection and analysis. (p. 18)

The Researcher wished to gain an understanding of the participating teachers' depth of pedagogical and content knowledge and how these were affected by a professional learning program. Klingner and Boardman (2011) state that "Mixed method designs are better suited to unraveling educational phenomena of enormous complexity" (p. 208) and that education systems are "…heterogeneous and the nature of teaching and learning so complex" (p. 209). For these reasons and others articulated later, it was therefore decided that a case study using mixed methods was the best vehicle through which to collect rich data to recognise and try to deal with this complexity.

The decision to choose a mixed methods approach is supported by the work of Bergman (2010):

> Mixed method research is eminently suited for exploring variations in the construction of meaning of concepts in relation to how respondents, for instance, make sense of their experiences or report on attitudes in interviews and questionnaires, respectively. An (sic) systematic inquiry into the variations of social constructions of meaning among interview and survey respondents may not only help in validating research instruments and scales, but may go further in that they could produce complementary subsets of results, which would enrich overall findings.
> (p. 172)

Employing qualitative data allows for rich description and explanation of processes which can be viewed in the local context (Miles & Huberman, 1984), a vital feature when considering the domains of MKT. Miles et al. (1984) also claim that it is also possible to preserve chronological flow, assess local causality and work towards deriving explanations.

The collection of quantitative data enables statistics to be calculated to further strengthen any interpretations made by the Researcher.

The data were collected through a combination of: participant constructed concept maps and the application of either the Fraction Knowledge Assessment Tool (Appendix 3) or the Rational Number Interview to determine content knowledge; Likert scales to determine attitudes (Appendix 4) and beliefs (Appendix 5); an assessment tool to determine PCK (Appendix 6); an exit questionnaire regarding the characteristics of effective professional learning (Appendix 7); audio-recorded individual semi-structured interviews (Appendix 8); the participants' diary logs; and, researcher field notes.

The use of the semi-structured interviews allowed the participants to express their understandings of the experiences and interactions (Gorman & Clayton, 1997). Verbatim transcripts of the complete interviews were produced by a second party adept at this work. This Researcher used verbatim transcription as it is central to "reliability, validity, and veracity of qualitative data collection" (Halcomb & Davison, 2005, p. 40). These transcripts were then read by the Researcher whilst listening to the recording. The purpose of this was to determine the accuracy of the transcription; make certain of the placement of the punctuation in the text was correct so that the intended meaning of statements was clear and so as not to change the intent or emphasis of the interviewee's response or comment; hear the nuances surrounding the spoken word, the intonation of the respondents and the non-verbal sounds (sighs, laughs and so forth); including the silences and the extended pauses, all of which may give further clue to meaning (Halcomb & Davison, 2005; McLennan, MacQueen & Neidig, 2003). Further, this process allowed the Researcher to determine the few sections which had been concluded to be inaudible by the transcriber could indeed reveal anything of importance through further, more focused interrogation.

After determining that the transcripts were true representations of the interviews, further listening accompanied by reading the transcripts allowed the Researcher to hear the nuances surrounding the spoken words and to ponder in depth the responses made. These transcripts and the audio-recordings also allowed the Researcher to repeatedly revisit the data to make some preliminary observations, highlight significant statements, sentences or quotes, to help identify themes, check the validity of conclusions drawn and to triangulate with other data sources (Creswell, Hanson, Plano-Clark & Morales, 2007).

3.3 *Mixed Methods*

For the purposes of this research, a mixed method study will be defined as “research in which the investigator collects and analyses data, integrates the findings and draws inferences using both qualitative and quantitative approaches or methods in a single study or program of inquiry” (Tashakkori & Cresswell, 2007, p. 4). In the past there has been a good deal of debate about the usefulness of combining qualitative and quantitative research methodologies in the same study (Creswell, 2003; Tashakkori & Teddlie, 1998). However, Hart, Smith, Swars and Smith (2009) state that many researchers believe that a more pragmatic view of research necessitates a wide variety of methods.

Mixed methods investigations may be used to: better understand the problems in research by forming a junction between the numeric information from quantitative data and the specific details that can be elicited from qualitative data (Punch, 1998); identify variables/constructs that may be measured subsequently through the use of existing instruments or the development of new ones (Punch, 1998); obtain statistical data from a sample of a population and use them to identify individuals who can be called upon to further illuminate this data through a qualitative approach (Punch, 1998); sharpen our understanding of the research findings (Frechtling & Sharp, 1997); convey the needs of individuals or groups of individuals who are marginalised or underrepresented (Punch, 1998); and, allow researchers to simultaneously generalise results from a sample to a population and to gain a deeper understanding of the phenomena of interest (Hanson, Creswell, Plano-Clark, Petska, & Creswell, 2005).

The disadvantages of mixed methods have also been investigated, and it has been found that: the mixed-method researcher has to be knowledgeable in both qualitative and quantitative designs and this generally requires increased time and effort on the part of the researcher (Creswell, 2003); for mixed-method research, the confidentiality of human research subjects can be problematic (Leahey, 2007); in quantitative research the larger numbers of subjects increases the reliability of the findings, but it is not always feasible to work with large numbers of subjects when taking a qualitative approach (Collins, Onwuegbuzie & Jiao, 2006); and, quantitative and qualitative approaches can pose philosophical differences (Pagnucci, 2004). A position, which pragmatism rejects, as the

compatibility thesis supports the idea that the combination of the two is good, and denies that the wedding is epistemologically incoherent (Teddlie & Tashakkori, 2009).

Steps were taken to limit the deleterious effects of the identified disadvantages by: being cognisant of the data which was required to be collected and the manner in which it was collected; making sure that protocols to protect anonymity were strictly adhered to; and, working to the strengths of each approach to marry it with the type of data to be collected. This Researcher believes that the advantages of employing mixed methods outweigh any of the disadvantages.

3.4 *Participants*

Participation in the professional development and associated research was sought from teachers who taught in Independent Schools in the metropolitan area of Perth, Western Australia. This was a convenience sample. No attempt was made to stratify the sample by year level taught or years teaching. The convenience sample resulted in two distinct groups being created.

Group One (G1) comprised of teachers from a Kindergarten to Year 10 school. The nine teachers in G1 worked in the Kindergarten to Year 7 section of the school and came from Years 4, 5, 6 and 7. All of the teachers from these year levels were required to attend. There was concern that the requirement to attend the PL would make the teachers feel obliged to participate in the research component through completion of the questionnaires and interviews. The teachers were informed that they were under no obligation to do any more than attend the PL and that they could decline the offer to be part of the data gathering, or after completion of the measures, withdraw their permission to use their responses at any point. One teacher chose the option of not completing all of the post-intervention measures. The students they taught ranged from nine to 12 years old. At the time of this study the school was less than two years old and the teachers varied in teaching experience from new graduates to teachers of many years' experience. The intervention came about through a request from the deputy principal of the school. This was prompted by an internal assessment which showed that fractions, decimal fractions and percentages were not a strong feature in the curriculum of the school and that testing of students showed weaknesses in these areas.

Group 2 (G2) was constituted by offering professional learning opportunities to all schools registered as members with the Association of Independent Schools of Western Australia (AISWA). It was offered as a four day program, with two days in March, one in June and one in August. The stipulations on attendance, phrased as questions on the flyer that was sent to schools to advertise this opportunity were:

- Are you a teacher of students between Years 3 and 9?
- Are you looking for something different in a professional learning experience?
- Would you like to strengthen your capacity to teach fractions in particular and mathematics in general?
- Would you like to have the chance to gain recognised prior learning credit to a higher degree at no financial cost to you?
- Are you willing to be part of a research study?

Group 2 (G2) comprised 17 teachers who accepted the invitation to join the professional learning sessions, ranging from teachers of Year 3 to teachers of Year 10 and from a total of 13 different schools. These schools ranged from small Islamic Primary schools, to small primary schools from low socio-economic areas, to large schools considered to be mid-fee paying schools covering Kindergarten to Year 12 (the last formal year of non-tertiary schooling in Western Australia), to representation from one of the highest fee paying schools. All attendees were offered substantial assistance towards paying for teacher release, so as to allow the teachers to attend the professional learning sessions, and so as not to discriminate against schools with less money for teacher replacement. No selection criteria were leveraged against this group therefore everyone who applied for attendance was accepted. When asked privately, no teacher claimed they were directed to attend, but attended on their own volition.

There were differences in the way the PL was delivered to the two groups. To accommodate that the meetings for G1 would be delivered at the school the sessions were of a three hour duration rather than the six hour duration of the professional learning for G2. A further difference between the groups was that the Researcher spent time in classrooms working alongside participants from Group 1 in the days between the

professional learning sessions, in order to build a relationship with the teachers and to embed the suggested pedagogical practices.

All participants indicated their willingness to be involved in the following facets of the investigation and signed the appropriate consent forms. They agreed to:

a. have their attitudes and beliefs towards teaching mathematics in general and fractions in particular, examined;
b. have their ability to answer questions regarding understanding of fractions examined;
c. follow a ‘guide’ on the most effective way in which to teach fractions;
d. introduce manipulative materials into their pedagogy for teaching fractions;
e. complete a ‘log’ recording their thoughts; and,
f. participate in a semi-structured audio-taped interview.

The study was conducted over one year from the time of the pilot study to the time of the post-intervention data collection (Table 3.1). The intervention phases were six months in duration, January to June for Group 1 and March to August for Group 2. The difference in the dates of delivery was necessitated by logistical reasons and the availability of the teachers participating in the study. The phases of the research study are summarised in Table 3.1.

Table 3.1
Overview of the phases of the study

Phase	Months	Group	Activity
1. Pilot Study	September 2008	PS1 & PS2	Application of concept maps and FKAT
2. Recruitment	January 2009	G1	See section 3.4
	February 2009	G2	
3. Pre-intervention data collection	January 2009	G1	See Table 3.2
	March 2009	G2	
4. Professional Learning	January – June 2009	G1	See section 3.5
	March – August 2009	G2	
5. Post-intervention data collection	June 2009	G1	See Table 3.3
	August 2009	G2	
6. Data analysis	Commenced September 2009		See section 3.6.10

3.5 *Design of the Professional Learning Program*

The professional learning sessions were constructed around three major tenets. Those tenets were to provide subject content knowledge, some alternative pedagogies for the teaching of fractions, and the opportunity and tools to reflect on both the content and the pedagogy.

After the first professional learning session the participants were asked to take the content and pedagogies from the PL session and to incorporate them into some lessons on fractions. This was accompanied by the invitation to share their experiences through feedback to the group in the following session. They were also invited to complete a reflective journal. The journal could take the form of writing in a self-carboning exercise book provided to all participants, or as an e-mail. These reflections were then used to enhance the content and pedagogies of the following sessions and therefore the capacity of the participants to "reflect-on-action" and "reflect-in-action" (Schon, 1987). They also provided information

to the Researcher for shaping the professional learning sessions and the direction of the research. The Researcher found these reflections to be of limited value, as will be discussed later. The following figure is an attempt to capture the flow of the opportunities for reflection created by this study.

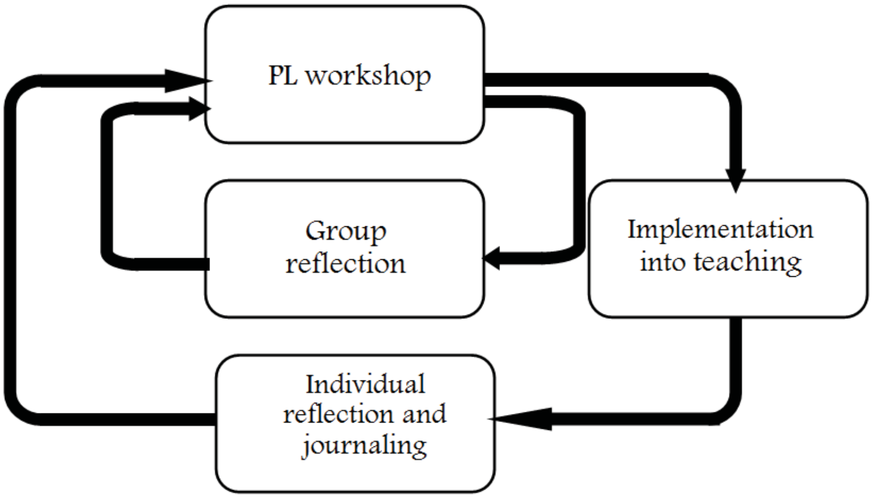


Figure 3.1 Flow of the opportunities for reflection created by this study

The above process was repeated in sessions approximately three weeks apart as one, one full-day and then six three-hour sessions for G1; and four full-day, spaced sessions, approximately one month apart, for G2. Data were collected at a variety of times (Table 3.2). Predominantly the pre-intervention data was collected at the very start of the professional learning sessions. It was deemed important by the Researcher that there was no intervention prior to the data gathering so as not to compromise the reliability of the assessment of the teachers' initial beliefs, attitudes and content and pedagogical knowledge.

Table 3.2
Pre-intervention data collection points

Tool	Source	Group 1	Group 2
Concepts and understanding of fractions (CM)		Session 1	Session 1
Fraction Knowledge Assessment tool (FKAT)		Session 1	Not collected
Attitudes (TAM)	White, Way, Perry and Southwell, 2006	Session 1	Session 1
Beliefs (TBM)	White, Way, Perry and Southwell, 2006; Yates, 2006	Session 1	Session 1
Pedagogical Content Knowledge Situations (PCKS)	Clarke and Mitchell, 2008	Session 2	Session 1
Rational Number Interview (RNI)	Clarke, Mitchell and Roche 2005	Not collected	Between Sessions 1 and 2

Apart from the semi-structured interviews conducted with G1, the post-intervention data was collected during the final sessions for both groups, although the reflections were accepted by the Researcher whenever the participants felt the desire to submit them. The FKAT instrument was not collected for G2 as its efficacy (as discussed in section 3.6.1) was brought into question. Similarly, the RNI was not applied to G1, as discussed in section 3.6.2.

Table 3.3
Post-intervention data collection points

Tool	Source	Group 1	Group 2
Concepts and understanding of fractions (CM)		Last session	Last session
Attitudes (TAM)	White, Way, Perry and Southwell, 2006	Last session	Last session
Beliefs (TBM)	White, Way, Perry and Southwell, 2006; Yates, 2006	Last session	Last session
Exit Questions – Effective PL		Last session	Last session
Semi-structured interview		After last session	Last session
Reflections		At the participants' pleasure	At the participants' pleasure

3.6 *Instruments*

This section outlines the instruments employed, along with a brief explanation as to why each instrument was chosen. The Researcher acknowledges that whilst the purpose of a case study is to provide a rich and deep understanding of complex environments, such as classroom teaching, the information gathered should be absolutely pertinent to the study. No request to the participants must be made for information which, while it may prove to be interesting, would lend marginal strength to the study. 'Trawling' for information and hoping for some serendipitous responses from respondents runs the risk of answers becoming facile and ill considered. Therefore, each instrument was carefully considered before being employed.

Two of the instruments, namely the Fraction Knowledge and Assessment Tool (FKAT) and concept maps (CM) were first trialed with pilot study groups to determine their efficacy. The FKAT was subsequently modified after feedback and initial analysis, for use in the

main study. A total of nine instruments were used (Table 3.4) with G1 and G2. These are further explored in the following sections.

Table 3.4

Instruments used for data collection with G1 and G2

		Data Source		
		Measurement	Written response	Verbal response
1.	Concepts and understanding of fractions(CM)		Concept map	
2.	Fraction Knowledge Assessment tool (FKAT)		Fraction knowledge instrument – 36 questions	
3.	Attitudes (TAM)	20 statements on an eight point Likert scale	Questionnaire	
4.	Beliefs (TBM)	16 statements on a three point Likert scale	Questionnaire	
5.	Pedagogical Content Knowledge Situations (PCKS)	5 situations	Multiple choice and then short answer	
6.	Clarke, Mitchell & Roche – rational number constructs (RNI)		Fraction knowledge instrument	
7.	Exit questions – effective PL	19 statements on a three point Likert scale	Questionnaire	
8.	Exit questions			Semi-structured interview
9.	Reflective journals		Written as desired by respondents – some in the narrative genre, some in the report genre, some as a mixture of the two.	Some incidental conversation

3.6.1 Fraction Knowledge Assessment Tool (FKAT)

The intent of the Fraction Knowledge Assessment Tool (FKAT) was to determine the respondents' understanding of common fractions. An initial test of 59 questions was devised (Appendix 2) by adapting items and elements from a number of text books. This version of the instrument was trialed with Pilot Study Group 1 (PS1).

After attending to understandings gained through the trial with PS1 (see section 4.1) and engaging in discussion with an expert review group comprised of mathematics educators, a revised version of the FKAT was then used with G1. The intention was to see if there was any immediate difference in the way that G1 answered the questions to that of PS1, and if the information collected was at all instructive. A decision was made to reduce the number of items from 59 items used with PS1to 36 items. Duplicate items were eliminated, as were items which did not directly pertain to answering the research questions. This served to limit question fatigue for the respondents and allow them to concentrate more on the items which had been problematic for the PS1 respondents. The revised 36-item version of the FKAT was administered to Group 1. However, preliminary analysis of the data indicated that the revised instrument was still not ideal, as the ability to score well in this assessment did not necessarily correspond to other measures which showed some weakness in a conceptual understanding of fractions.

3.6.2 Clarke, Mitchell and Roche – Rational Number Constructs (RNI)

Given the limitations of the FKAT, there was a need to identify another instrument that could be used to determine the content knowledge of the teachers in G2. The instrument selected was the Rational Number Interview (RNI), which was constructed by Clarke, Mitchell and Roche (2005) and was originally used as an assessment tool with Victorian students in Years 6 (12-year-olds). As a tool for use with students it has strong research reliability, having been trialed with 323 students. It provides an assessment interview which when:

> …embedded within an extensive and appropriate in-service or pre-service program, can be a powerful tool for teacher professional learning, enhancing teachers' knowledge of how mathematics learning develops,

> knowledge of individual mathematical understanding as well as content knowledge and pedagogical content knowledge.
>
> (Clarke & Roche., 2009, p. 129)

The RNI contains questions relating to common fractions, decimals and percentages. It was decided that the respondents would be asked to answer all of the questions, but as common fraction understanding is the basis of the study only the questions relating to common fractions would be analysed. As stated previously the RNI was originally used with 12 year old students, but it was considered by this Researcher also to be a valuable tool for use with the study's participants. This was due to its ease of use and the challenging nature of some of the questions. These questions appeared to have a high degree of probability of revealing some valuable findings.

It was never intended that the RNI would be used as a pre- and post-test. The tool was only used to try to gain some understanding of the level of competence and content understanding with common fractions, and not to try to determine the growth of subject content knowledge. It was felt that obtaining this information would allow the Researcher to prepare PL activities that met the needs of the participants.
Due to the constraint of time, the RNI was conducted between the first and the second professional development sessions at the workplace of the teachers and at a time of their choosing.

This assessment tool focuses on

> …the rational number constructs of part-whole, measure, division, and operator, and the "big ideas" of the unit, using discrete and continuous models, partitioning, and the relative size of fractions, a range of around 50 assessment tasks was established, drawing upon tasks that had been reported in the literature, and supplemented with tasks that the research team developed.
>
> (Clarke & Roche, 2009, p. 130)

The important notion of a fraction as a quantity was scrutinised in a section of the assessment (questions 14 a –h) where the respondents were asked to compare pairs of fractions. The pairs of fractions were shown to the teachers, one pair at a time and in order of increasing difficulty. They were asked to determine which of each pair was larger in magnitude and articulate why they thought this. The reasoning behind the responses were analysed and recorded on a rubric. This rubric had been constructed by Clarke et al. (2005)

to delineate the appropriateness of the articulated strategies. The more appropriate strategies were placed above a dividing line and the less appropriate below that line. For example, the choices given for the pair 3/4 and 7/9 (considered the most difficult comparison) are shown in Figure 3.2:

Strategy	Category
• Residual with equivalent (2/8 > 2/9) • Residual thinking (1/4 > 2/9) with proof • Converts to decimals • Converts to common denominator	More appropriate
• Higher or larger numbers • Gap thinking • More area (sometimes related to an image) • Other ………………………….	Less appropriate

Figure 3.2 RNI Question 14h – Matrix for assessment

3.6.3 Concept maps (CM)

According to Novak and Cañas (2008) concept maps are “graphical tools for organising and representing knowledge” (p. 1). Further, they define a concept as being “a perceived regularity in events or objects, or records of events or objects, designated by a label” (p. 1). In a concept map, concepts are usually enclosed and lines linking the concepts indicate relationships between them (Novak & Cañas, 2008).

As instruments for data collection and for capturing the inter-relatedness of concepts within a domain, the concept map is considered to provide a direct and powerful approach (Ruiz-Primo & Shavelson, 1996). This sits well with the position that mathematics is about the interconnectedness of the big (and small) ideas about mathematics and about how teachers can develop this for themselves and their students. Ma (1999) referred to this as longitudinal coherence.

The case has been made that concept maps are devices which: are flexible (Boujaoude & Attieh, 2008, Hough et al., 2007) and have been widely used in education (McClure, Sonak & Suen, 1999, Boujaoude & Attieh, 2008); provide a measure of structural knowledge (Markham, Mintzes & Jones, 1994); are successful pedagogical tools (Ruiz-Primo &

Shavelson, 1996); are a respectful (Hough et al. 2007) and successful (Herl, Baker & Niemi, 1996) approach to assessing teacher growth; have been used by mathematics educators as a strategy for teaching subject matter and as means of identifying misconceptions (Hough et al., 2007); and have been successfully used for identifying pre-instruction and post-instruction knowledge and identifying the difference between the two (Hough et al., 2007).

There is a metric available for scoring concept maps (Herl, Baker & Niemi, 1996), which measures semantic content, organisational structure, the number of links used in the map and comparison with expert teacher maps. This study however did not require such a level of complexity. It concentrated only on the number of links used and the number of nodes developed, to see whether or not that painted an unambiguous picture of growth.

The Researcher chose to use concept maps rather than a mind map because concept maps are often more structured and include multiple concepts, whereas mind maps are usually less formal and generally focus on one concept at a time (Wheeldon, 2010). The facility to explore multiple concepts was seen as advantageous and that the concept map's structure leaned towards deeper exploration of a topic, was seen as being more effective than the mind map.

Following a trial of the concept map with Pilot Study Group 2, research participants were asked to create a concept map, with the instructions: "Tell me everything you know about two thirds (2/3) and the different ways we can show it." The concept maps were then collected to analyse the participants' ability to make connections and the degree of complexity with which they could do this.

3.6.4 Teachers' Beliefs about Mathematics Questionnaire (TBM).

The Teachers' Beliefs about Mathematics questionnaire (TBM) (Appendix 5) was constructed by White, Way, Perry and Southwell (2006), as a three-point Likert scale (the responses to select from were; disagree, undecided and agree). The respondents were asked to indicate their beliefs about 16 statements regarding mathematics. It should be noted that the beliefs were of a generic nature about mathematics and not directly about fractions. Some of the statements contained negative beliefs about mathematics (for example "Right

answers are much more important in mathematics than the ways in which you get them") whereas others were considered to be positive in nature (for example, "Mathematics is the dynamic searching for order and pattern in the learner's environment"). The statements were concerned with constructivist beliefs about mathematics and beliefs about its meaningfulness.

3.6.5 Teachers' Attitudes towards Mathematics Questionnaire (TAM).

The Teachers' Attitudes towards Mathematics questionnaire (TAM) (Appendix 4) was constructed by White, Way, Perry and Southwell (2006) as an eight-point Likert scale (the responses to select from were: 'definitely false', 'false', 'mostly false', 'more false than true', 'more true than false', 'mostly true', 'true', 'definitely true' and 'not applicable to me'). The respondents were asked to indicate their attitudes concerning 20 statements regarding mathematics. The statements were once more of a broad nature about mathematics and not directly about fractions. Some of the statements displayed negative attitudes towards mathematics (for example, "Mathematics makes me feel inadequate") where others were considered to be positive in nature (for example, "Generally I feel secure about the idea of teaching mathematics").

3.6.6 Pedagogical Content Knowledge Situations (PCKS).

Pedagogical Content Knowledge Situations (PCKS) offered five scenarios for which the participants had to provide solutions and (in most cases) give reasons for their answers, Figure 3.3 as an example of the situations (and Appendix 6 includes the complete set of questions). This was based upon some early work by Clarke and Mitchell (2008) and by Roche and Clarke (2009), and was used as a tool for trying to make judgments with regards to the pedagogical content knowledge of the participating teachers.

1. Which numbers should be used as initial examples to illustrate the place of partitioning in understanding fractions?

a) $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$

b) $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{6}$ $\frac{1}{8}$

c) $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$

d) $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ $\frac{4}{4}$ $\frac{5}{4}$

e) All above work equally as well

Figure 3.3 Example of Pedagogical Content Knowledge Situations (PCKS) - Situation 1

Roche and Clarke (2009) assert that there are five components underpinning the development of PCK items:

> ***Pathways:*** Understanding possible pathways or learning trajectories within or across mathematical domains, including identifying key ideas in a particular mathematical domain.
> ***Selecting:*** Planning or selecting appropriate teaching/learning materials, examples or methods for representing particular mathematical ideas including evaluating the instructional advantages and disadvantages of representations or definitions used to teach a particular topic, concept or skill.
> ***Interpreting:*** Interpreting, evaluating and anticipating students' mathematical solutions, arguments or representations (verbal or written, novel or typical), including misconceptions.
> ***Demand:*** Understanding the relative cognitive demand of tasks/activities.
> ***Adapting:*** Adapting a task for different student needs or to enable its use with a wider range of students.
>
> (p. 469)

They further assert that; “If teachers can demonstrate capacities in the areas of Pathways, Selecting, Interpreting, Demand and Adapting as we have defined them, it can be argued reasonably that they are likely to have the PCK to teach mathematics well (Roche & Clarke, 2009, p. 473)."

In this study there were a total of five situations given. Situations 1 and 2 addressed Selecting, situations 3 and 5 concerned Interpreting, and situation 4 was based around Pathways. The Researcher made the judgment that more questions which looked into the components of Demand and Adapting were not as germane to this study as the other chosen questions. This should not be interpreted as the Researcher not seeing value in these categories, but had rather decided that the risk of fatigue was quite strong. Application of just the five questions and the response they received vindicated this decision (section 11.2.7).

When scoring the five situations, the respondents were awarded a higher score for responses which were judged by the Researcher to display higher levels of PCK. Situation 1 was scored on a 3 point scale, with increments of 0.5 marks to reward detail and depth of thought. It was scored as follows:

- if the teacher chose response ‘c’ and used partitioning as the reason behind this decision they received 3 points;
- if the teacher chose any other than response ‘c’ and arrived at that position through a well-reasoned argument they received 2 – 2.5 points;
- if the teacher chose any of the responses without giving an explanation for their choice they received 1 to 1.5 points; and,
- although no teacher chose not to answer the question, if they had done so, they would have received a score of zero.

Situation 2 had a value of 5 marks and was scored on increments of 1. It was scored as follows:

- if the respondent chose to tick response 7 or all of the first five boxes they received 5 points ;

- if the respondent chose not to tick response 6 or 7 they received a mark for each of the first five they did mark; and,
- if the respondent chose to tick response 6 they would have received a mark of zero.

Situation 3 had a value of 5 marks and was scored on increments of 0.5. The respondent earned 0.5 marks for each possible misconception they posited and a further 0.5 marks for a plausible reason as to why a student might arrive at that solution. Situation 4 had a value of 3 marks and was scored on increments of 0.5. These marks were awarded on the strength of the argument used to support the respondent's position. Situation 5 had a value of 5 marks and was scored on increments of 0.5. These marks were awarded on the strength of the respondent's perceived capacity to interpret, evaluate and anticipate the students' mathematical solutions through the persuasiveness of their reasoning.

3.6.7 Exit questions – Effective professional learning

At the conclusion of the Professional Leaning (PL) sessions the participants were asked to respond to 19 statements on a three point Likert Scale (Yes, Undecided and No) regarding the question: "Do you believe the PL addressed the following characteristics of effective professional development?" (Appendix 7). Questions asked included, "Do you believe the PL focused on increasing knowledge and skills to bring about change in my teaching practice?" and "Do you believe the PL addressed issues and concerns and interest identified by the teachers?" These statements of characteristics were chiefly, but not exclusively, informed by the work of three extensive studies by Clarke (2003), Guskey (2003) and In-Praxis Group Incorporated (2006).

3.6.8 Exit questions – Semi-structured interview

The semi-structured audio taped interview comprised 17 questions which were grouped under the headings of the original research questions. The three interviewers involved were supplied with the list of these 17 questions (Appendix 8) and were extensively briefed about the protocols for asking the questions (Appendix 9).

Questions one to five addressed Research question 1 and 11 of the remaining 12 questions addressed Research questions 2, 3 and 4. The final question was a general one, "What were you hoping to gain from attendance in this PL?"

The semi-structured audio-taped interviews for Group 1 participants were conducted at a negotiated time in their school environment, but not in their classrooms. The semi-structured audio-taped interviews for Group 2 participants were conducted at the professional learning venue, but in a separate room from the main group. At the time of the interviews seven respondents had either not completed the course or were unavailable for interview. Nineteen interviews were conducted. It should be indicated at this point that due to the semi-structured nature of the interview and the direction taken in some of the interviews, some questions were not responded to by the participants or were responded to in a manner that defied coding, due to their ambiguous nature.

3.6.9 Participant diary logs and Researcher's field notes

As part of the structure of the professional learning experience the participants were given the opportunity for reflection through the completion of a log, through informal conversations, and by e-mail communication. The participants were supplied with a self-carboning book which facilitated a copy being made as notes were taken. This allowed the original to stay with the participant and a copy to be submitted to the Researcher if the participant felt so inclined.

At no time were the reflections structured in a manner which required a reflection before and a reflection after a particular event, as all inputs from the participants were

discretionary and engaged in purely at the participants' pleasure. The Researcher did not want the act of reflection to seem as if it were a data collection point, but rather wished to encourage and enable the participants to "reflect-on-action" and "reflect-in-action' (Schon, 1987), and to promote the idea that this could be, for some, a more integral part of their planning. The reflections which were submitted by the participants were used to illuminate or support findings from other data collection tools and to add further richness to the analysis of those findings.

At the conclusion of the first PL session the participants were asked to reflect on the session and were given prompting questions which they could choose to answer.

a. Some aspects that are of interest/new to me about teaching fractions from today are…?
b. Have any of my conceptions about teaching fractions been:
    i. reinforced today?
    ii. challenged so that I might consider changing them?
c. What part of my practice in the teaching of fractions will I make a focus in the next classroom lesson?

Before the commencement of the second PL session the participants were e-mailed a set of prompts to guide their thinking. These formed the basis of a discussion which was held at the start of the session.

a. What activity did you do with your students? Particularly if it isn't one that was presented in the last session please be prepared to describe it to the group (perhaps work samples or something similar might be illustrative here).
b. What understanding about fractions were you trying to promote?
c. What were the strengths and weaknesses of the lesson?
d. Did you find out anything that surprised you either positively or not so positively about your students' understanding?
e. How did you adapt the activity to meet the needs of your particular students?
f. What would you change given the opportunity to do the lesson again?

During the second session the participants were shown one type of tool for reflection, (Appendix 10). Towards the end of the session they were then asked to apply that model to one of the activities that had been conducted that day. Between sessions 2 and 3, and again between sessions 3 and 4, the participants were asked to reflect upon at least one fraction lesson that they had taught in their classrooms.

3.6.10 Methods of data analysis

Two major forms of data analysis were pursued in this mixed method research. The quantitative data which was collected through the administration of the questionnaires and the assessments for the different types of pedagogical and content knowledge was collected and scored. Where simple descriptive statistics were required, the Microsoft Excel program was employed. Where statistical significance needed to be ascertained, the statistical package SPSS was used to conduct Wilcoxon Signed Ranks Tests, and on some occasions, the paired-sample t-test was also utilised.

The qualitative data were collected through semi-structured interviews, field notes and invited reflections from the participating teachers. In the case of the semi-structured interviews the conversations were recorded as audio-recordings and then transcribed. The field notes were informal written records of conversations and overheard comments, and the invited reflections from the teachers were also in writing.

The Researcher used the collected data to engage in concurrent data triangulation (Creswell, 2009; Yin, 2009) where the quantitative and qualitative data were compared to see where there was convergence, inconsistency or contradiction (Burke Johnson, Onwuegbuzie & Turner, 2007). This was determined to be useful in validating the data to explain any variance. As Bouchard elegantly states (1976), the convergence "…enhances our belief that the results are valid and not a methodological artefact" (p. 268). Prevailing literature suggests that no measures are perfect (Johnson, Onwuegbuzie & Turner, 2007; Creswell, 2009) and that multiple measures in a study share theoretically relevant and have different patterns of irrelevant components (Johnson et al., 2007). This reduces the uncertainty of the interpretation.

All of these data collection were then carefully scrutinised so as to identify and make sense of themes that emerged. The initial analysis of the data generated a number of findings which were interpreted, drawing on the literature to generate assertions. The assertions formed the basis of the conclusions which answered the research questions.

3.6.11 Ethical considerations

An application to undertake this research was first approved by the Human Research Committee at Edith Cowan University in 2008. Informed written consent was obtained from: the members of the trial groups of pre-service teachers (Appendix 11); the principals from the schools of the teachers participating from G1 and G2 (Appendix 12); and the members of G1 and G2 (Appendices 13 and 14). As supporting the teachers involved classroom visits, a letter outlining the limited contact that the Researcher would have with students of the teachers involved, was sent to the parents, via the school principal (Appendix 15).

In the initial invitation to attend the PL the principals and teachers were given information with regards to the purposes of the PL and it was clearly indicated that they would be part of a research study. At the commencement of the first session of the PL the teachers were then informed of the exact nature and the processes to be adopted for the PL and the research. They were informed of the expectations required of them and the resultant time commitments. All participants were given assurances regarding the confidentiality of the research data and the anonymity of the collection and reporting of the data, as all participants were referred to by coding (for example G1-R4 for the fourth coded teacher from Group 1). The teachers were also informed that they were free to withdraw at any stage and that if at any point they wished the data already collected to be removed from the study, this would be done. The collected data will be securely stored for at least five years post the publication of any papers.

3.7 *Summary*

This chapter has concerned itself with the design and research methods in this study. It has made links to the literature to explain the conceptual design, and outlined the methods for the study with attention to why a mixed methods research study was adopted. The participants in the study and the instruments used to collect the data are described. The choice of research design was shaped by the Researcher's acknowledgement of the number and complexity of the elements of the craft of successful teaching, and therefore also of the need to provide a comprehensive set of measures in trying to appraise those elements.

In the next chapter, more details regarding the pilot studies are discussed. The two groups, Pilot Study group 1 (PS1) and Pilot Study group 2 (PS2), were considered important in the trialing and refinement of some of the instruments which were to be employed with the groups to be used in the main study. Both pilot study groups were composed of pre-service teachers.

CHAPTER 4

4 Pilot Study

Prior to the commencement of this study, two pilot studies were conducted to trial some of the research instruments. A convenience sample of pre-service teachers participated in the pilot study.

4.1 *Pre-Service Teachers – Pilot Study Group 1 (PS1).*

After an initial hour long lecture on fractions to an audience of approximately 230 pre-service teachers in their second year of training an offer was extended to all students to attend a session where the major themes and ideas touched on in the lecture would be further expanded.

This offer was accepted by 12 students and they participated in a three hour tutorial in which one of the assessment tools was trialed and they were offered the chance to explore the topic of common fractions more closely. The fraction knowledge instrument, *Fraction Knowledge Assessment Tool* (FKAT) was trialed with Pilot Study Group 1 (PS1) (Appendix 2).

In conversations conducted during the tutorial with the students in PS1, all but one of the students expressed a level of anxiety regarding mathematics and a lack of understanding regarding fractions. This is perhaps not surprising as this was a self-selecting group. One student went so far as to state that she had what she termed "massive difficulty and worries" about the thought of having to teach fractions as she "was an absolute failure" with fractions at school and "never understood what all the numbers meant."

4.2 ***Fraction Knowledge Assessment Tool (FKAT).***

On the application of the *Fraction Knowledge Assessment Tool* (FKAT) it was found that the students performed quite well. Of the total of 59 items that required an answer only 23 of the items showed any errors (Table 4.1). In Table 4.1, 'Item number' refers to the number given to each individual item in the FKAT. Items which did not attract any errors were not recorded in the table.

Table 4.1
Items and number of errors noted for PS1

(n=12) Item	3	6	10a	10d	10e	10f	11	15	19	20	21	25
No. of errors	7	1	1	4	1	1	2	2	1	1	2	2
% error	58	8	8	33	8	8	17	17	8	8	17	17
Item	26	27	28	29	30	31	32	33	37a	37b	49d	
No. of errors	2	3	2	3	2	3	4	5	2	3	3	
% error	17	25	17	25	17	25	33	42	17	25	25	

The items which proved to be somewhat problematic for at least four members of PS 1 were: 3, 10d, 32 and 33. These items have been exemplified to see where the difficulties may lay.

The errors encountered in Question 3 where the respondents were asked to identify the representations of ½ could be put down more to a visual misinterpretation rather than fractional misunderstanding. This is of particular concern for pre-service teachers, as it is important that teachers have an understanding of a range of ways of presenting mathematics ideas, including different visual representations of fractions. On further investigation, it was found that for two of the participants who were incorrect on this item, this was the case.

Figure 4.1 Question 3 - Region model of ½

Question 10d displayed a region model where the fractional parts were not contiguous (Figure 4.2), and was problematic. Four of the participants offered an incorrect response.

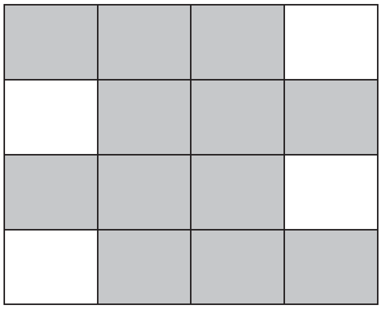


Figure 4.2 Question 10d - Region model where the fractional parts are not contiguous

Question 32 had the distracter of asking for two-fifths with a pre-divided rectangular shape divided into 10 pieces (Figure 4.3). Four respondents found looking for this equivalent fraction a challenge.

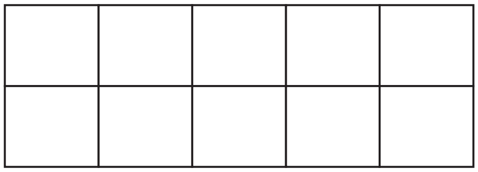


Figure 4.3 Question 32 - Shade two-fifths on a rectangle pre-divided into 10 pieces

Question 33 (Figure 4.4), colouring 2/5 on a pre-divided area representation where the parts were not all represented in the same manner, was problematic for five of respondents. All of the other items had a 75% rate of correctness or better.

Figure 4.4 Question 33 - shade two fifths on an area model

This raised concerns as to whether the application of such a test would be at all instructive. It was decided, based on the doubts raised through this pilot study, that in future, an abridged version of this assessment would be used and its efficacy re-assessed. This abridged version was employed with *Group 1* (G1).

4.3 *Pre-Service Teachers – Pilot Study Group 2 (PS2).*

To trial the use of concept maps (CM) as a tool to determine knowledge of representations of fractions, Pilot Study Group 2 (PS2) was instituted. This was a different group of 25 pre-service teachers, in their sixth week of lectures and tutorials of their second year of training. They were asked to develop a concept map to show what they knew about fractions. These were then collected in order to analyse the complexity and connections regarding fractions that the pre-service teachers might demonstrate.

4.4 *Concept Maps (CM).*

Used in this study, are the 13 different written representations of fractions commonly used in schools and curriculum documents, as identified by this Researcher (see Table 4.2). These are not necessarily constructs (part-whole, quotient, measurement, operation or ratio) as described by other researchers (Kieren 1980), but rather are representations of those constructs.

Table 4.2
Written representations of fractions

	Region model	All regions are the same shape and area
	Area model	All regions are the same area but not necessarily the same shape
 ¼ ½ ¾ 4/4	Length model	
	Set (discrete) model	
	Fractions as division	$1 \div 4 = {}^1/_4$
	Three dimensional	
 ¼	Number line	
Algorithm		${}^3/_4 - {}^2/_4 = {}^1/_4$
Percentage	Conversion to a percentage	25%
Decimal fraction	Conversion to a decimal fraction	0.25
Symbolic		¼
As operators		${}^1/_4$ of 24
Equivalence		¼ = 2/8 = 3/12
As words		One quarter
Ratio		1:3 – part to part ratio 1:4 – part to whole ratio

As could be expected these concept maps displayed a marked difference in the understandings shown by pre-service teachers. One pre-service teacher made 17 links (links being lines of connection from the central ‘main idea’ to the subordinate headings and ideas) while another made six. These connections did not all show different constructs (representations) but often were about other issues related to fractions.

Some pre-service teachers wrote about general mathematical knowledge that they had developed such as the part/whole concept and subitising, whereas others illustrated some specialised mathematical knowledge they had developed, for example, “Equivalent fractions are fractions which equal the same, however are constructed by different parts.”

Some mention of pedagogical issues was made, such as, “Fractions can be taught through games and manipulatives” and “Don’t start with a pizza model” as well as references to materials such as pattern blocks which could be employed in the teaching of fractions. There were some allusions to the variety of representations needed to teach fractions. Interestingly though, the most complete set given was one which articulated only the linear models, area models, sets, decimals, percentages and ratios and number lines out of the 13 possibilities. Some pre-service teachers constructed lists of specialised vocabulary particular to fractions, such as denominator and numerator. A few of the pre-service teachers made reference to knowledge of teaching and learning and referred to sequencing the learning, developmental issues and the introduction of strategies. It should be noted that a number of the statements made which were intended to show understanding of fractions (such as “the bigger the denominator the smaller the fraction”), actually illustrated incomplete understandings or at best an imprecision of definition (as this is something that only holds true in certain circumstances, but not all, for example $5/10 > 1/8$).

Generally speaking these concept diagrams showed a variation in the understandings and connections explored by the pre-service teachers and in general were not comprehensive in displaying either a depth or breadth of knowledge. That is to say that the interpretation of what was required from this activity seems to be confused. As such it suggested that in order to gain a more consistent view of fractional understanding, the concept map should be constructed around a fractional number such as two-fifths and a question such as “Tell me everything you know about this number and its different representations” should be asked.

It was felt that this might better guide the respondents to showing their knowledge in a more diverse way, rather than the more ‘open’ word of fractions, which seemed to obfuscate what was required. It was therefore decided to use the concept maps in the study, paying careful attention to the initial instructions given to the participants for its completion.

4.5 *Summary*

The pilot study allowed the Researcher the opportunity to make some necessary refinements to the instruments. Due to its length, and the fact that some of the questions did not seem to be instructive, the efficacy of the fraction knowledge instrument (FKAT) was brought into question. Likewise, concerns were raised about the wording used to ask the respondents to produce a concept map for the topic of fractions. The pilot study also highlighted to the Researcher the level of anxiety that the topic of fractions can cause some people. This heightened awareness of some people’s reaction to fractions was taken into the main study, and informed the kind of rhetoric and discussion regarding fractions that was entered into.

Undertaking the main study with two groups, operating under slightly different circumstances, required careful consideration as to whether the two could be combined and their results considered together. The following chapter will explain the rationale for combining the groups.

CHAPTER 5

5 Rationale for Combining Group Results

Undertaking the main study with two groups, operating under slightly different circumstances, required careful consideration as to whether the two could be combined and their results considered together. In this chapter the rationale for combining the groups will be explored.

5.1 *Comparison Between Group 1 and Group 2*

In dealing with both G1 and the G2 the following observations about the similarities in the groups were made. Both groups:

- were from Western Australia;
- were teaching in the Independent schools' sector;
- had group members who completed their teacher training in a variety of locations (Western Australia, other states of Australia, overseas);
- worked within the constraints of the Western Australian Curriculum Framework;
- had the same recourse to professional learning experiences offered by AISWA;
- were given professional development which dealt with the same content and was of the same duration;
- had the same facilitator(s) for the sessions; and,
- consisted of members who varied in teaching experience, age, gender, year levels taught, confidence for teaching mathematics, confidence for teaching fractions and Mathematical content knowledge.

The primary differences between the two groups were:

- one group was constituted from teachers at the same school (G1) and one group was constituted by teachers from a variety of schools (G2);
- G1's attendance was due to a perceived whole school need to improve in the area of fractions;

- G1 was afforded the opportunity to have the Researcher work in their classrooms in between PL sessions, while this was not available to G2; and
- G1's PL was conducted initially over one complete day and then in 3 hour sessions, G2 completed their PL in whole day sessions.

The similarities lead the Researcher to consider amalgamating the groups and treating their responses to the post-intervention assessments as one group rather than as two. A comparison of the data collected from the pre-intervention assessment tools used which were common to both groups (that is, concept maps (CM), Beliefs questionnaire (TBM) and Attitudes questionnaire (TAM)) was used to consider the possibility of doing so.

5.2 *Concept Maps*

In completing the concept map the teachers were awarded a point for each different representation they mentioned from the 13 representations identified by the Researcher (Table 4.2). This was summed for the group and represented as a percentage. When G1 and G2 were compared, G1 had a mean of the means of 37.30% across all the different references to representations for fractions, with G2 having a mean of the means of 33.61%. Using a Kruskal-Wallis test the group means were not significantly different (p<.05) (Figure 5.1).

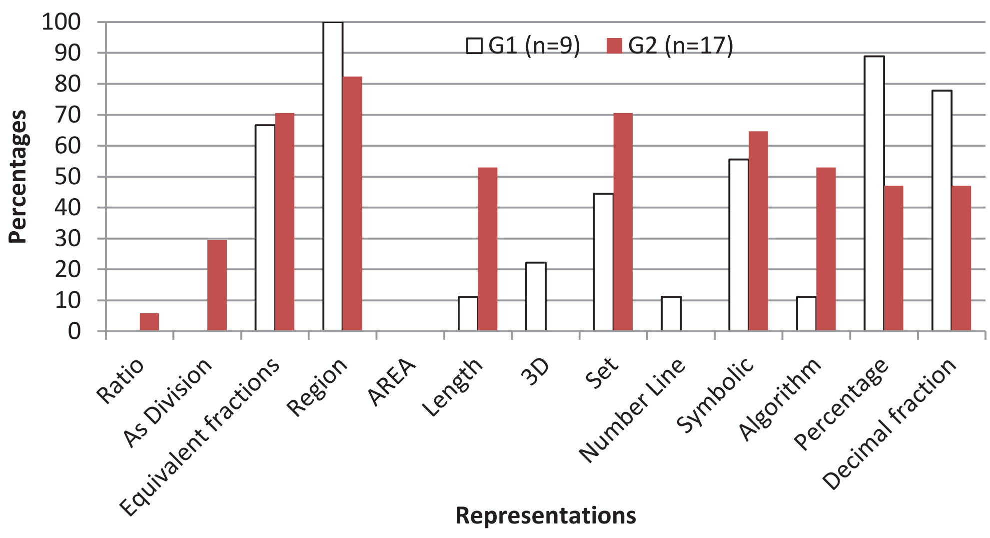


Figure 5.1 Percentage of respondents who referred to the representation in their concept map

As groups, G1 used 10 out of the 13 possible representations and G2 used 10 out of the possible 13, but not the same 10. The area model was employed by neither group. Ratio, fractions as division, length, 3D, Set, number line and algorithms were all used by less than 50% of the respondents in G1 and ratio, fractions as division, 3D, number line, percentage and decimal fractions were all used by less than 50% of the respondents in G2. Between G1 and G2 there was a statistically significant difference in the use of the length, 3D, algorithms and percentages as representations (Figure 5.2).

Table 5.1
Kruskal-Wallis Test of statistical significance of difference between G1 and G2 in total use of representations.

	Composite of representations
Chi-Square	.592
df	1
Asymp. Sig.	.442

a. Kruskal-Wallis Test

b. Grouping Variable: Group

Table 5.2
Kruskal-Wallis Test of statistical significance of difference between G1 and G2 use of particular representations.

	Ratio	As Division	Equiv. Fractions	Region	Area	Length	3D
Chi-Square	.529	3.151	.565	.186	.000	4.183	3.935
df	1	1	1	1	1	1	1
Asymp. Sig.	.467	.076	.452	.667	1.000	.041	.047

	Set	No. line	Symbol	Algorithm	%	Dec. Fraction
Chi-Square	1.634	1.889	.200	4.183	4.183	2.188
df	1	1	1	1	1	1
Asymp. Sig.	.201	.169	.655	.041	.041	.139

a. Kruskal-Wallis Test

5.3 ***Teachers' Beliefs About Mathematics Questionnaire (TBM).***

The Teachers' beliefs about mathematics questionnaire (TBM) (Appendix 5) was constructed using a three point Likert scale and the respondents were asked to indicate their beliefs about 16 statements regarding mathematics.

	Disagree	Undecided	Agree
Statement			

Figure 5.2 Likert scale used to assess beliefs towards mathematics of the respondents

G2 started in this study holding slightly more positive beliefs regarding mathematics. When employing the Kruskal-Wallis test, the mean per cent positive response for G2 (91.18) was significantly different from G1 (87.73) ($p < .05$). However, for all but three of these items (1, 10 and 14) the differences between G1 and G2 responses were 6.1% or less.

Table 5.3
Kruskal-Wallis Test of statistical significance of difference between G1 and G2 beliefs

	Total Beliefs
Chi-Square	5.024
df	1
Asymp. Sig.	.025

a. Kruskal-Wallis Test
b. Grouping Variable: Group

Statement 10, "Being able to memorise facts is critical in mathematics learning" was the statement with the biggest disparity with a 30.28% difference, followed by Statement 14, "Teachers or the textbook – not the student –are the authorities for what is right or wrong", with 18.3%. The third largest difference of 13.94% was seen in Statement 1, "Mathematics is computation". All other statements were below 6% difference. Of these, Statement 10 and 14 proved to be statistically significantly different (Table 5.4) but Statement 1 did not.

Table 5.4
Kruskal-Wallis Test of statistical significance of difference between G1 and G2 particular beliefs

	Belief 1	Belief 2	Belief 3	Belief 4	Belief 5	Belief 6	Belief 7	Belief 8
Chi-Square	.869	.653	.618	.009	.529	.595	1.390	1.101
df	1	1	1	1	1	1	1	1
Asymp. Sig.	.351	.419	.432	.923	.467	.441	.238	.294

	Belief 9	Belief 10	Belief 11	Belief 12	Belief 13	Belief 14	Belief 15	Belief 16
Chi-Square	.562	7.063	.529	.000	.154	5.779	.238	1.889
df	1	1	1	1	1	1	1	1
Asymp. Sig.	.454	.008	.467	1.000	.694	.016	.626	.169

a. Kruskal-Wallis Test

b. Grouping Variable: Group

5.4 *Attitudes Towards Mathematics Questionnaires (TAM).*

The Teachers' Attitude towards Mathematics (TAM) questionnaire was constructed so as to provide 20 statements to which the participants could respond. The responses ranged from 'Definitely false' to 'Definitely true' and allowed the respondents eight possible responses (Figure 5.3). A ninth possibility was 'Not applicable to me' which none of the respondents chose to employ.

	Definitely False	False	Mostly False	More False Than True	More True Than False	Mostly True	True	Definitely True	Not applicable to me
Statement									

Figure 5.3 Likert Scale used to assess attitudes towards mathematics of the respondents

Ten of the statements (Statements 1, 2, 5, 7, 8, 10, 13, 14, 15 and 17 – see Appendix 4) were considered to be positive by this Researcher as they showed attitudes which were efficacious in mathematics teaching (for example “Statement 1: I generally feel secure about the teaching of fractions”). For these items a ‘definitely true’ response was given a score of 8 and a ‘Definitely false’ response was given a score of 1. That is, the higher score reflected a more positive response.

Ten of the statements (Statements 3, 4, 6, 9, 11, 12, 16, 18, 19 and 20 – see Appendix 4) were considered to be negative by this Researcher as they showed attitudes which were not efficacious in mathematics teaching (for example “Statement 3: Mathematics makes me feel inadequate”). For these items a ‘definitely true’ response was given a score of 1 and a ‘Definitely false’ response was given a score of 8. As before, the higher score reflected a more positive response.

G2 started this study holding slightly more positive attitudes towards mathematics. This difference between the mean scores for G1 (5.83) and G2 (5.63) did not produce a statistical difference ($p<.05$) using a Kruskal-Wallis test.

Table 5.5
Kruskal-Wallis Test of statistical significance of difference between G1 and G2 attitudes

	Total Attitudes
Chi-Square	.059
df	1
Asymp. Sig.	.808

a. Kruskal-Wallis Test
b. Grouping Variable: Group

The Kruskal-Wallis test showed no evidence of a significant difference between the paired means for the item numbers (Table 5.6).

Table 5.6

Kruskal-Wallis Test of statistical significance of difference between G1 and G2 particular attitudes

	Att. 1	Att. 2	Att. 3	Att. 4	Att. 5	Att. 6	Att. 7	Att. 8	Att. 9	Att. 10
Chi-Square	.079	.709	.000	.508	.241	.691	.597	.467	.252	.450
df	1	1	1	1	1	1	1	1	1	1
Asymp. Sig.	.779	.400	1.000	.476	.624	.406	.440	.494	.615	.502

	Att. 11	Att. 12	Att. 13	Att. 14	Att. 15	Att. 16	Att. 17	Att. 18	Att. 19	Att. 20
Chi-Square	.549	.342	1.022	.061	1.425	.028	.128	.112	.019	.247
df	1	1	1	1	1	1	1	1	1	1
Asymp. Sig.	.459	.559	.312	.805	.233	.867	.721	.738	.889	.619

a. Kruskal-Wallis Test

b. Grouping Variable: Group

5.5 ***Pedagogical Content Knowledge Situations (PCKS).***

Table 5.7 displays the percentages of what this Researcher defined as positive responses to the Pedagogical Content Knowledge Situations (PCKS). Responses were considered to be positive if they described the link between the knowledge bases of content (both the substantive and syntactic structures of a discipline) and of pedagogy. When applying the Kruskal-Wallis test the overall mean scores for G1 (60.74) and G2 (63.06) were not statistically different ($p<.05$).

Table 5.7
Comparison between G1 and G2 of percentages of positive responses to Pedagogical Content Knowledge Situations (PCKS)

	G1 *(n=9)*	G2 *(n=17)*
PCKS	%	%
1	59.26	61.76
2	75.56	68.24
3	54.44	56.47
4	55.56	61.76
5	58.89	67.06
Mean	60.74	63.06

Table 5.8
Kruskal-Wallis Test of statistical significance of positive responses to Pedagogical Content Knowledge Situations (PCKS)

	PCKS1	PCKS2	PCKS3	PCKS4	PCKS5
Chi-Square	.019	1.351	.064	.498	3.027
df	1	1	1	1	1
Asymp. Sig.	.889	.245	.800	.480	.082

a. Kruskal-Wallis Test

b. Grouping Variable: Group

5.6 ***Summary***

There were demographic similarities between the two groups that participated in the professional learning. Analysis of the data from both G1 and the G2, revealed there was no statistically significant difference with regards to the representations provided through a concept maps (CM), their responses to the attitudes questionnaire (TAM), or the responses to the Pedagogical Content Knowledge Situations (PCKS). There was a significant, but still relatively small statistical difference in the response to the beliefs scale (TBM). Therefore, there did not seem to be sufficient reason to prohibit amalgamating the groups, and then treating their responses to the pre-intervention and post-intervention assessments as one group rather than two. It was, however important to remain cognisant of the possible

differences between the two groups which might be of interest given the differences in the ways in which the PL was implemented in the groups.

In searching the available literature, there was an almost unequivocal support for the notion that fractions were an important topic in mathematics which deserves focussed attention (Booth & Newton, 2012; Brown & Quinn, 2007; Chinnappan, 2005; Nunes & Bryant, 2009; Wu, 2001). Of interest to the Researcher was if teachers' regard for the importance of the teaching and learning of fractions matched that of the literature. Further, because the teachers were not autonomous, they work within parameters set by their schools and the curriculum documents, the teachers were asked their perceptions regarding the status of fractions as expressed by their schools and the prevailing curriculum documents. This is explored in the next chapter.

CHAPTER 6

6 The Current Status of Teaching Fractions in Middle and Upper Primary School Classrooms in Western Australia.

This study was predicated on the supposition that any teachers who attended the professional learning would be reasonably representative of the general teaching population in Western Australia. When compared with research from other studies, none of the demographic data, nor any of the consequent results from the use of the instruments employed with the participants, suggested that this was not a reasonable assumption.

It was deemed valuable by the Researcher to try to ascertain the status of fractions in the classrooms and schools of the participants. This was approached by studying the participants' confidence and the perceived ability for teaching fractions and their belief in the importance of fractions in the curriculum at a personal, school and curriculum level. Further, the content knowledge displayed by the respondents was also assessed. It was assumed that if a topic such as fractions had little status in comparison to other areas of mathematics, then the nature of a crowded curriculum would see it garner less time and attention and therefore be relegated to a minor and unimportant part of the curriculum. Further, by asking for information which invited the participants to compare their position at the commencement of the professional learning (PL) and its ending, some conclusions might be made as to whether or not the PL had any positive effect. Combined with information regarding the content knowledge of the teachers in the study, this might then allow for subsequent generalisations which could be employed for the wider school population.

As it seemed very suitable for the purpose of ascertaining the status of fractions, a semi-structured interview was employed. It should be noted that due to the semi-structured nature of the interview and the direction that some of the interviews took, some questions were not responded to by the participants, or were responded to in a manner that defied coding, due to ambiguity. This means that the results from the statistics do not always refer to exactly the same sample size.

In order to collect data, the three interviewers were supplied with a list of 17 questions and briefed about the protocols for asking the questions. The questions themselves were grouped under the headings of the original research questions. Questions one to five were intended to answer Research question 1: “What is the current status of teaching fractions in middle and upper primary school classrooms in Western Australia?”

6.1 *Respondents’ Confidence in Teaching Fractions*

The teachers were asked to rate their confidence for teaching fractions and give a rating on a 10-point scale. Some of the respondents did not offer a numerical score. In recording the responses for a participants’ level of confidence, the responses that were non-numerical in nature were given a conservative numerical value (minimal = 2, average and fair = 5, pretty good = 7 and higher and better are one more than the pre-intervention score).

Using the Wilcoxon Signed Ranks Test, the two sets of data were compared to determine if there was any statistically significant difference between them. For the pre-intervention results ($p = .593$) and for the post-intervention results ($p = .138$) there was no significant statistical difference between those who gave a numerical score and those for whom a score was ascribed. Therefore this Researcher felt comfortable in ascribing scores to non-numerical responses to questions so as to ensure all respondents were represented in the data.

Using the data which included all participants, application of the Wilcoxon Signed Ranks Test displayed a highly significant difference between the pre and post-intervention scores in the measures of confidence ($p = .000$).

When the participants were asked the questions “At the outset of this PL how would you have rated your confidence in teaching fractions and how would you now rate your confidence to teach fractions?” (Semi-structured interview Question 1), apart from G1-R2 who offered no response to this question, the respondents unanimously reported that they thought that their confidence in teaching fractions had been raised. Although the intention

was to have the participants quantify their statement on a one to 10 scale, four respondents (G1-R1, G1-R4, G1-R6, G1-R9) chose not to apply a number ranking, instead using personal phrases to describe their pre-intervention confidence in teaching fractions (for example, fair, minimal, average and pretty good). Similarly in their response to rating the improvement in their confidence, they employed words such as better and higher.

Three respondents (G2-R11, G2-R12 and G2-R13) gave a numerical value to their pre-intervention confidence in teaching fractions and then gave a non-numerical (worded) answer such as better or higher in response to their confidence post-intervention. When non-numerical responses were given a conservative numerical response a mean rating improvement of 2.25 was determined (Figure 6.1).

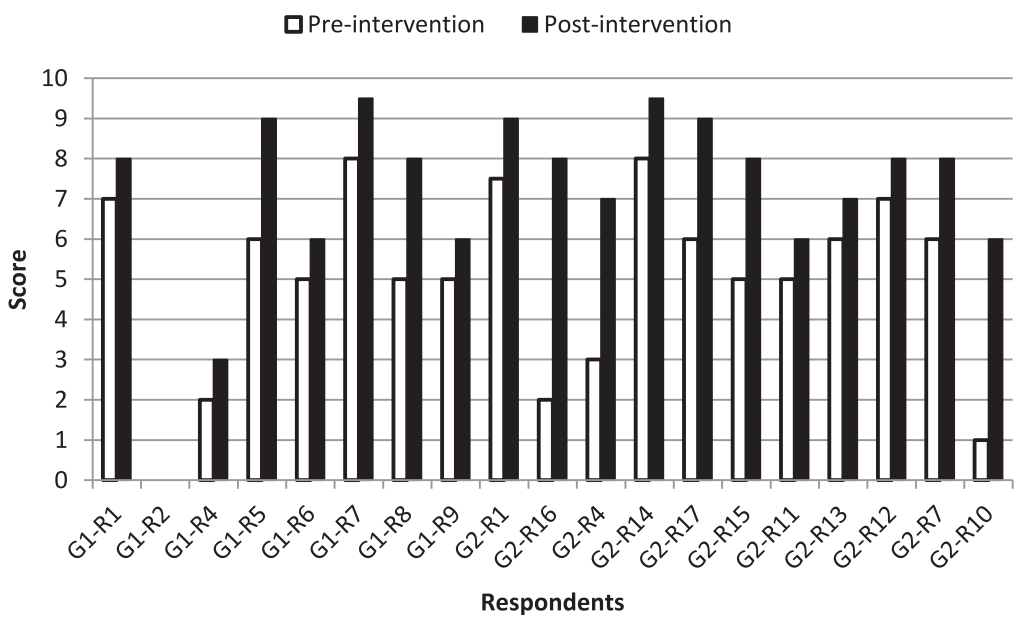


Figure 6.1 Confidence in teaching fractions pre and post-intervention responses with numerical values ascribed for all respondents

One comment from G1-R6 was:

> I would say that at the start it was probably, confidence was kind of average. I know how to do fractions, I've done it in school, I've done it in maths at university. I would say after the PD sessions my confidence is actually quite a lot better because I know different ways to teach fractions and there are different strategies that I can now try to incorporate and put into the kids to try to get them to work out how to work out fractions.

This suggests that from a purely pre-intervention knowledge of content viewpoint ("I know how to do fractions, I've done it in school, I've done it in maths at university") this respondent was at the post-intervention phase more focused on the pedagogy and had developed his/her PCK (Pedagogical Content Knowledge). This development of PCK was echoed in the statement by G1-R8 who reported:

> Not very confident, but after doing this professional development, just all the examples and the way they use concrete materials has helped me explain the different concepts of fractions to children.

In speaking about her confidence respondent G2-R11 was reflective about her practice:

> I think it's a bit of up and down for me because I feel like I am going a step forward but also like I am going like a step back because I am thinking oh I haven't covered this properly and I am thinking I think I should go back and revisit that again because I don't think I taught that correctly the first way and after coming here it's like ooooh.

Key Finding 6.1: There was a statistically significant positive difference in pre-intervention and post-intervention means of the participants in regards to their confidence in teaching fractions.

6.2 *Perceived Ability to Teach Fractions*

The teachers were asked to rate their ability to teach fraction on a 10-point scale. Some respondents did not offer a numerical score. In recording a participant's perceived ability the responses that were non-numerical in nature were given a conservative numerical value (average = 5, good = 6, pretty good = 7 and higher and better being one more than the pre-intervention score). Using the Wilcoxon Signed Ranks Test, the two sets of data were compared to determine if there was any statistically significant difference between them. For the pre-intervention results ($p = .878$) and for the post-intervention results ($p = .339$) there was no significant statistical difference between those who gave a numerical score and those for whom the score was ascribed. Therefore this Researcher felt comfortable in ascribing scores to non-numerical responses to questions to ensure all respondents were represented in the data.

Using the data which included all participants, application of the Wilcoxon Signed Ranks Test, again displayed a highly significant difference between the pre and post-intervention scores in the measures of perceived ability ($p = .000$).

When the participants were asked to respond to the question, "At the outset of this PL how would you have rated your ability to teach fractions and how would you now rate your ability to teach fractions?" (Semi-structured interview Question 2) the unanimous response was that there had been an improvement in their ability to teach fractions since the commencement of the PL. This was consistent with the response in the previous question regarding confidence. Again, although asked to quantify their statement on a one to 10 scale, three respondents (G1-R1, G1-R6, G1-R9) chose not to apply a number but used phrases to describe their pre-intervention ability to teach fractions. They employed words and phrases such as, pretty good, good and able. These respondents also used words and phrases such as better and higher in response to how they assessed their ability to teach fraction after the professional learning.

Four respondents (G1-R4, G2-R11, G2-R12, G2-R13) gave a numerical value to their pre-intervention ability to teach fractions and then gave a non-numerical answer of better or

higher in response to their ability post-intervention. When non-numerical responses were given a conservative numerical response a mean rating improvement of 2.37 was determined (Figure 6.2).

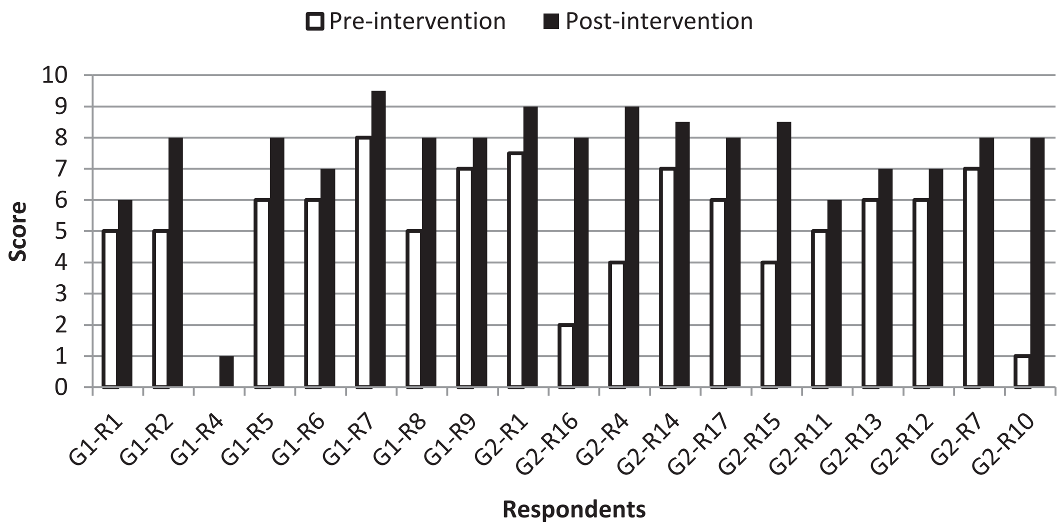


Figure 6.2 Perceived ability in teaching fractions pre and post-intervention responses with numerical values ascribed for all respondents

G1-R2 suggests that the ability to teach is closely linked with the resources available; that is, that a teacher needs the resources and then can construct the learning rather than constructing the learning and then applying resources to enhance that learning.

> I think we are all able to do things, it's just whether or not we find good resources, so I think the resources we've got has made us more able and the resources, the different activities we've done has given us the ability to teach fractions in different ways that will target different learning styles so that's probably…

This dependence on resources could be seen as concerning. The research states that teachers are reliant on textbooks as their primary resource (Shield, 1991; Watt 2002; Johansson, 2006) and yet there is doubt regarding the efficacy of textbooks (Zevenbergen, Dole & Wright, 2004). This then, could contribute to what Haberman (2010) calls a pedagogy of poverty.

G2-R16 stated:

> Knowing what I know now, my ability would have been very low because I didn't even know that I didn't know stuff so that means it would have been very low, 2 again.

This respondent raises an important consideration which speaks directly to the beliefs and attitudes questionnaires that the respondents answered at both the commencement and the conclusion of the intervention. Asking people about their beliefs and attitudes prior to any input may result in skewed results, as they can only answer from a position of what they perceive their understanding to be at that time. One of the advantages/disadvantages (depending on a point of view and the purpose) of professional learning can be that it can illustrate understandings not before considered. Therefore a participant can leave a PL feeling less confident, or at least questioning their beliefs and attitudes more than when they entered it.

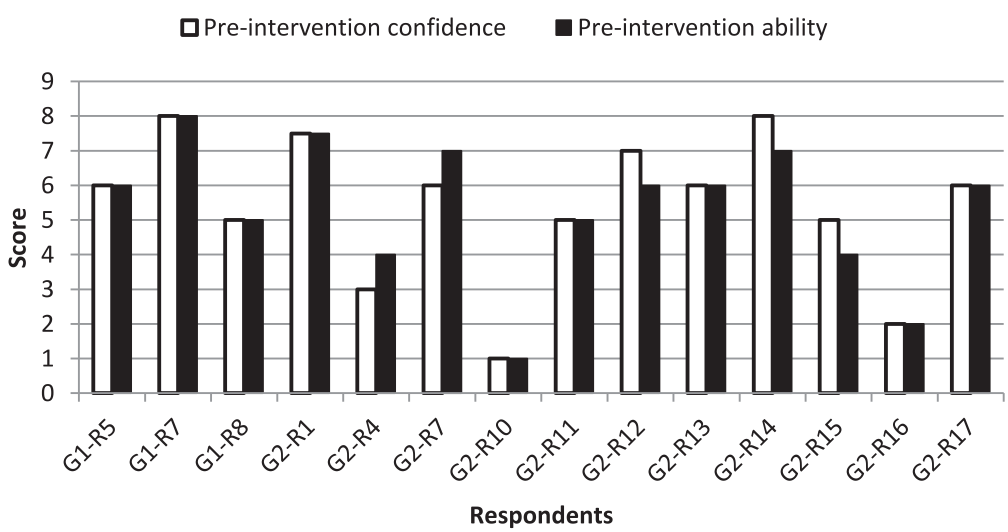


Figure 6.3 Comparison of respondents offering numerical values only, pre-intervention scores for confidence and perceived ability in teaching fractions

If one loosely equates the confidence and ability scores, it is interesting to note that Figure 6.3 reveals that two of the respondents (G2-R4 and G2-R7) indicated their ability to teach fractions was greater than their confidence in doing so, and three respondents (G2-R12, G2-R14 and G2-R15) indicated that their confidence for teaching fractions was greater than their perceived ability. This Researcher would have thought that questions of confidence in teaching fractions and then the ability to do so would be so inextricably linked as to generate evenly scored responses, as was the case with the other nine teachers. There are no supporting statements made in the audio-taped, semi-structured interviews to explain the disparity in the scores.

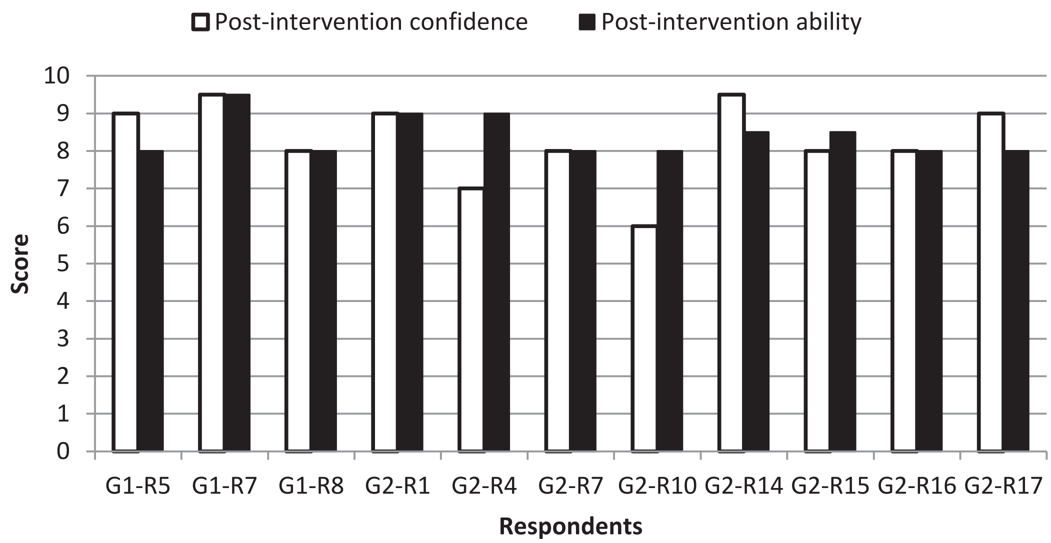


Figure 6.4 Comparison of respondents offering numerical values only, post-intervention scores for confidence and perceived ability in teaching fractions

Figure 6.4 reveals that three of the respondents (G2-R4, G2-R10 and G2-R15) indicated that post-intervention their ability to teach fractions was greater than their confidence in teaching fractions and three respondents (G1-R5, G2-R14 and G2-R17) indicated that their confidence for teaching fractions was greater than their perceived ability. As previously asserted, this Researcher finds this intriguing. G2-R4 and G2-R14 are the only participants that have this inequality in answering both pre and post the intervention. This disparity would have made an interesting line of enquiry.

Key Finding 6.2: The responses from the teachers in this study show that in general terms they perceived themselves to have limited ability to teach fractions at the outset of this PL. At the conclusion of this PL there was a statistically significant rise in the teachers' perception of their ability to do so.

6.3 *Personal Perception on the Importance of Fractions as a Topic in Mathematics*

In collating the data for the answers to the related questions of "In terms of its importance as a topic in mathematics how did you rate fractions before the start of this professional development? Has that rating changed and if so, why?" the Researcher decided to group the responses using the headings of; Numerical (where responses were given as numbers), Low, Medium, Quite High, High and Higher, and NR where no response was forthcoming. The numerical scores were then adapted to fit with the other responses. A numerical score of 10 was coded as Higher, a numerical score of eight or nine was coded as High, a score of seven was coded as Quite High and a score of six was coded as Medium.

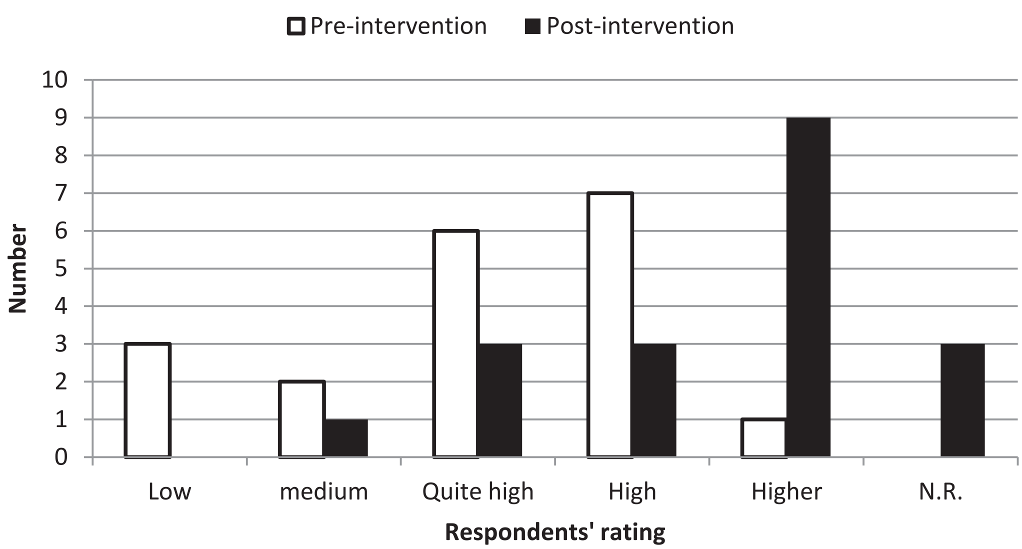


Figure 6.5 Comparison of importance of the topic of fractions to the respondents, pre and post-intervention

Figure 6.5 illustrates that there is a skewing of the data from the lower to the higher ratings when comparing the pre-intervention and the post-intervention responses. Although not apparent in Figure 6.5, the individual data revealed that none of the respondents rated

fractions as a topic lower in importance after the intervention than they did before its commencement.

All three of the respondents below have made advances into what Askew, Brown, Rhodes, Wiliam and Johnson (1997), calls being a connectionist teacher. According to Askew et al. (1997), a connectionist teacher is one, who among other things, has an emphasis on establishing connections within mathematics, something that they consider to be an important element of becoming an effective teacher of mathematics. These statements also point towards these respondents being reflective regarding their practice.

G1-R2 commented:

> Yeah, we've definitely given it (fractions) more emphasis especially as a whole school…the good thing about fractions is that you can bring it into your measurement, you can bring it into other topics.

G2-R15 reported:

> Yeah, it's probably increased to more than what I'd put into it, or what I originally thought, just because I understand more how it can be linked to so many other areas of maths. So yeah, it has improved.

And further G2-R17 stated:

> Yes, but I didn't realise how widely it touched into other areas of number.

Although it has previously been established that for the purposes of data analysis the groups (G1 and G2) could be considered one group, it was considered of interest to see if the two groups showed differences in the perceived level of importance they ascribed to the topic of fractions. If there was to be a difference it may stem from the fact that one of the groups (G1) was a group of teachers from a single school who were instructed to attend the PL due to a perceived weakness in the topic at the school. Conversely, G2 was constituted of participants from a variety of schools with a variety of reasons for attendance.

In both groups, the compulsory attendance and the self-selecting, the pattern was similar, in that after the intervention, the status of fractions was seen to be raised. In the second group, however, the original judgment about the status of fractions was initially somewhat higher.

<u>Key Finding 6.3:</u> The data collected suggests that prior to the PL, the topic of fractions held high status in the teachers' classrooms, but that the status of fractions was raised at the conclusion of the PL.

6.4 *Respondents' Perception of the Status or Importance the Topic of Fractions Holds in their School*

When the data was combined from both groups in answer to the question, "What kind of status or importance do you think the topic of fractions holds in your school?" it was of interest to note that 42.11% of the respondents perceived the status in their school to be low (Figure 6.6).

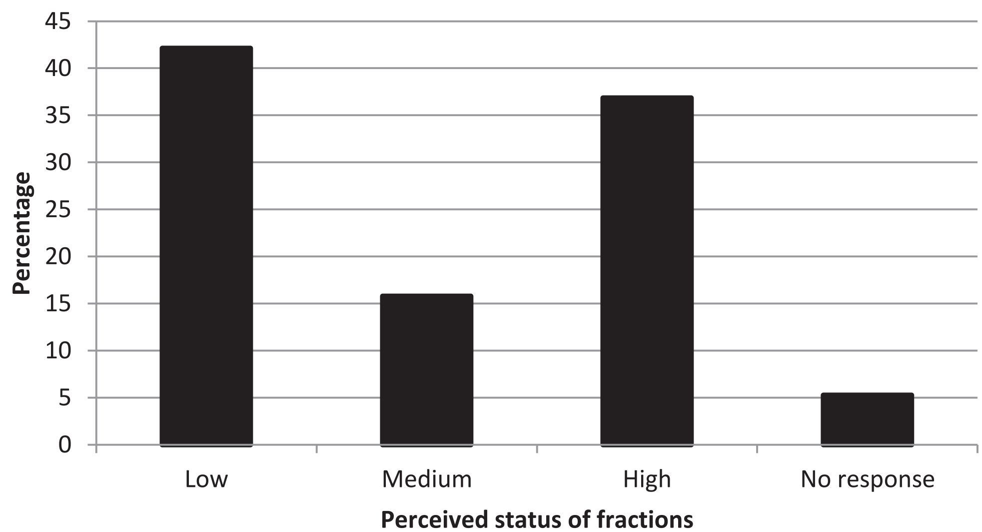


Figure 6.6 Perceived status of importance of fractions in schools for all respondents

When the data were interrogated to see if the two groups (G1 and G2) responded differently to the question, it was noted that they were considerably different (Figure 6.7). It is possible that the participants in G2 saw their attendance at the PL as exposure to mathematics

content and pedagogy in general rather that fractions in particular. Whereas, the attendance of G1, was borne out of an identified school priority regarding the topic of fractions.

G2-R12 expressed the opinion, echoed by others, that fractions only seem to gain any sense of importance in the middle to upper primary classrooms. G2-R16 stated:

> It's (the importance of fractions in her school) very low and it's to do with the same reason I wanted to have a PD, is you rate it low because you don't understand it and you are scared of it and it's overwhelming so you rate it low and fill your timetable with something else.

G2-R4 also cited a lack of perceived support for the importance of fractions, but when challenged on her low rating and asked to reflect on the actuality that the school had supported her attendance at the PL stated:

> I mean, they wanted me to do the PD, so yeah, I guess the Principal thought it would be worthwhile…but there is no follow up in regards to what we have learned or how we are teaching it or anything like that.

At the other end of the spectrum was a comment from G2-R12; "Yes, it's planned for specifically, planned for in the curriculum and taught as a topic."

In comparing the post-intervention status that teachers hold for fractions and the status they perceive that their schools hold, there is quite a disparity (Figure 6.5). In both the case of G1 and G2 the respondents personally rated the status of fractions higher than they perceived their schools to do so. For the respondent's personal judgment of the status of fractions, responses which were categorised under the words; quite high, high or higher were grouped for comparison with the perception of the high status held on fractions by schools.

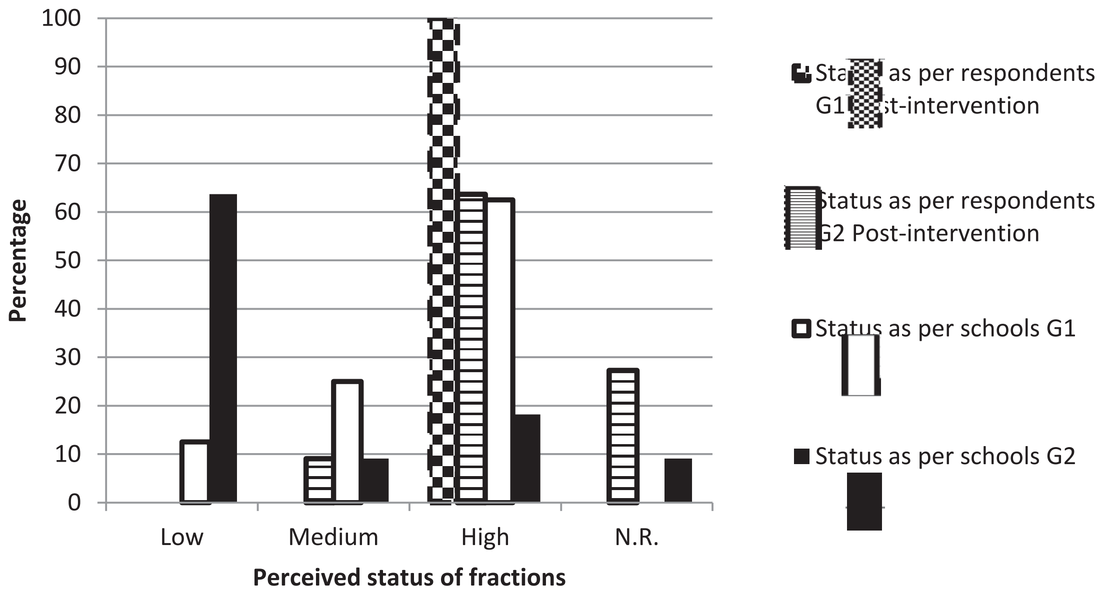


Figure 6.7 Comparison of respondent's personal judgment of status of the importance of fractions and their school's perceived status of importance

<u>Key Finding 6.4:</u> The data collected suggests that 42.11% of teachers even after having been given permission to attend a PL on the content topic of fractions still perceived the topic to be of low importance in their school.

6.5 *Respondents' Perception of the Status or Importance the Topic of Fractions Holds in the Western Australian Curriculum*

When asked the question "What kind of status or importance do you think the topic of fractions holds in the WA curriculum?" nearly half of the respondents (Table 6.1) were unsure as to the importance fractions held in the Western Australian Curriculum (as expressed in the Curriculum Framework documents, the principal and only mandated document in Western Australia). Eight out of the nine respondents came from G2 which could suggest that the whole school approach taken to engage the members of G1 had made them more aware of the place of fractions in the relevant documentation, and consequently its perceived importance as part of the curriculum. It needs to be acknowledged that it is possible that relevant school-delivered work on the curriculum was carried out without report from the teachers.

Table 6.1
Perceived status of importance of fractions in Western Australian curriculum

(n=19)	Total	%
Low	3	15.79
Medium	3	15.79
High	4	21.05
Unsure	9	47.37

In one exchange the following was said:

> **G2-R7**: I don't think people spend time on fractions as they do on other areas.
>
> **Interviewer**: And do you think if you read the Curriculum Framework you would get from it that it's important or not, what do you think about that?
>
> **G2-R7**: Oh yeah, I mean it certainly seems important but it's the relativity of that, you know, how important as compared when you are trying to do everything else.
>
> **Interviewer**: And from reading the written documentation what would you think of that?
>
> **G2-R7**: I don't think it makes it clear.

This respondent, a teacher of many years experience, and one who had previously defended the open nature of the Curriculum Framework documents, seemed to have some issue with the lack of clarity on the teaching of fractions. Another teacher of many years experience G2-R16 supported this view when she declared:

> Yeah, well in the Curriculum Framework, I wouldn't suggest that you're asked to put more time into studying it, it's just there, as a matter of fact. I believe that the 1977 curriculum specified in this year you need to make sure that your children know halves and quarters…so a lot of the older teachers are saying look, look at my precious bible called the 1977 maths book, this is what they asked us to do in Year 5, now we've got this six month older student so they really should know that…we'll say have a look at this, this will give you guidelines because outside that they still

> think the Curriculum Framework is too broad, they want to get specific so they are not disadvantaging a child.

As did G2-R1 who related:

> It just seems to be a little bit of a blip in the number section really, I mean when I think of the curriculum framework which is what we are currently using it is kind of mentioned but it's drowned by the amount of other stuff that you have to teach.

All three of these respondents speak to the fact that although they perceive fractions to be important, they did not feel that the curriculum documents from which teachers are supposed to be working, necessarily have the required emphasis placed upon them. G1-R8, a teacher of limited experience, indicates in her response that she perceived fractions to hold limited importance in the curriculum, as in her words she was "…just going with what I've been told to do (by the syllabus documents)":

> I thought fractions were really important but just according to what I've been told in my syllabus it doesn't look like it's that important any more, at year 4 level I'm not sure if at the higher year levels it may be more important.

<u>Key Finding 6.5:</u> The data collected suggests that teachers are either unsure about the status of fractions as determined by the prevailing curriculum documents or feel that the documents give them low or medium status.

6.6 *Status of the Respondents' Content Knowledge of Fractions*

In order to try to ascertain the status of the content knowledge of fractions with the respondents, and to further explore its usefulness as a tool, it was decided to apply the Fraction Knowledge Assessment Tool (FKAT) with G1, the tool which had been trialed with Pilot Study group 1. The focus was to see if there was any immediate difference in the way that G1 answered the questions to the PS1 group and whether the information collected was at all instructive. It was decided to reduce the number of items to 36 of the 59 items used with PS1. The items which did not seem to be illustrative were dropped so as to

eliminate some duplication, to speed up the time required to do the assessment and to allow the respondents to concentrate more on the items which had been problematic to the PS1 respondents.

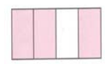
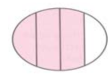
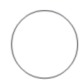
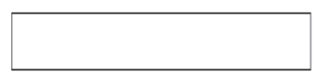
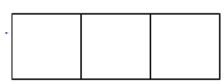
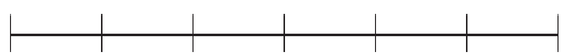
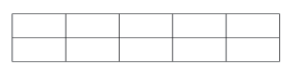
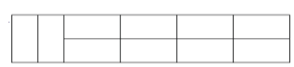
Table 6.2
Items and percentage of errors noted for FKAT

Item	3(3) Rectangular region	6(6) Rectangular area representation	10a(10a) Oval Representation of 3/4	10d(10d) Non-contiguous representation of	10e(10e) Non-contiguous representation of 3/4	10f(10f) Incorrect representation of	11(11) 3D representation	25(12) Circular region	26(13) Pentagonal region representation	27(14) "Free" region representation
% error (G1) *(n=9)*	22	11	0	0	11	0	0	0	0	11
% error (PS1) *(n=12)*	58	8	8	33	8	8	17	17	17	25

Item	28(15) Rectangular region representation	29(16) Number line representation	30(17) Blank number line representation	31(18) Unhelpfully marked number line representation	32(19) Rectangular region	33(20) Rectangular area representation	37a(24a) Comparing fraction size	37b(24b) Comparing fraction size	49d(25d) Fractions as division
% error (G1)	22	0	0	22	11	11	33	0	22
% error (PS1)	17	25	17	25	33	42	17	25	25

In Table 6.2, 'Item' refers to the number given to each individual item in the FKAT and a brief description of its purpose. Examples of these items can be found in Table 6.3. These appeared as they were presented to PS1. Items which did not attract any errors were not recorded in the table. In the truncated version given to G1 the items were renumbered and this is shown by the item number in brackets. Percentages of errors are then displayed for G1, with a comparative line of percentage of errors for PS1.

Table 6.3
Examples of items which drew incorrect responses found in FKAT

Item number and brief description	Example
3(3)Rectangular region representation	
6(6)Rectangular area representation	
10a(10a)Oval Representations of 3/4	
10d(10d) Non-contiguous representation of 3/4	
10e(10e) Non-contiguous representation of 3/4	
10f(10f) Incorrect representation of 3/4	
11(11) 3D representation	11. Sarah and Chris shared the drink from a bottle. They had half each. Put a tick on the picture that shows their shares of the drink? 
25(12) Circular region representation	
26(13) Pentagonal region representation	
27(14) "Free" region representation	
28(15) Rectangular region representation	
29(16) Number line representation	
30(17) Blank number line representation	
31(18) Unhelpfully marked number line representation	
32(19) Rectangular region representation	
33(20) Rectangular area representation	
37a(24a) Comparing fraction size	Which of these fractions is greater, ½ or ¾?
37b(24b) Comparing fraction size	Which of these fractions is greater, 3/8 or 5/20?
49d(25d) Fractions as division	Maria, Carlos and Terry want to share 4 medium, square pizzas. Each person gets an equal amount. How many pizzas will Carlos get?

Across the items the teachers in G1 showed slightly better understanding than the pre-service teachers from PS1, with the exception of item 24a. Use of the statistics program SPSS to examine the number of errors made in each question, across PS1 and G1, shows a highly significant difference (Tables 6.4) between the two groups in favour of G1. This is a result which one would expect as this group is comprised of practising teachers as opposed to PS1 who were pre-service teachers.

Table 6.4
Paired-sample t-test of difference between the number of errors of PS1 and G1.

	Paired Differences							
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	Lower	Upper	*t*	*df*	Sig. (2-tailed)
PS1-G1	13.105	14.169	3.251	6.276	19.935	4.032	18	.001

Using SPSS to look at the non-parametric Wilcoxon Signed-Ranks Test applied to the same data supported the assertion of significance ($p = .002$).

The capacity to score well in a written assessment on fractions did not necessarily correspond with either a positive attitude towards fractions or a powerful understanding of them. G1-R4 (Respondent 4 from Group 1) commented:

> I never understood fractions at school; I didn’t know they weren't one number on top of another number so when I added I never got the right answer.

The same teacher went on to say;

> I get the lowest common denominator because it’s multiplication and I know those numbers need to be the same but I don’t know why I’m doing it.

Another teacher, G1-R8, acknowledged;

> I know how to add and subtract fractions but I can’t multiply or divide with them…I don’t know why.

Key Finding 6.6: In G1, a group of nine teachers, two acknowledged they have serious Specialised Content Knowledge (SCK) deficits in fractions.

Due to the perceived shortcomings of the FKAT data collection tool and this Researcher's reluctance to employ it, a different tool was introduced to gain an insight into the knowledge of the participants in G2. The tool selected was the Rational Number Interview (RNI), the tool previously described in section 3.6.1.

It was noted that none of the participants in the study scored every item correct in the interview. In fact five of the respondents scored more than 10% of errors on an assessment tool designed to test the understanding of rational number constructs in Year 6 students. One respondent scored more than 25% of all the questions incorrect.

Table 6.5
Number of incorrect responses to RNI by respondent

(n=15)	Respondent	1	2	4	6	7	8	9	10
No. of incorrect responses		1	3	5	7	2	1	2	3
%. of incorrect responses		2.63	7.89	13.16	18.42	5.26	2.63	5.26	7.89

	Respondent	11	12	13	14	15	16	17
No. of incorrect responses		10	8	3	2	4	2	1
%. of incorrect responses		26.32	21.05	7.89	5.26	10.53	5.26	2.63

There were18 questions which elicited incorrect responses (Table 6.5) and nine of those questions (2f, 4d, 6, 8, 10b, 10d, 13b, 14g and 14h) drew incorrect responses from 20% or more of the respondents.

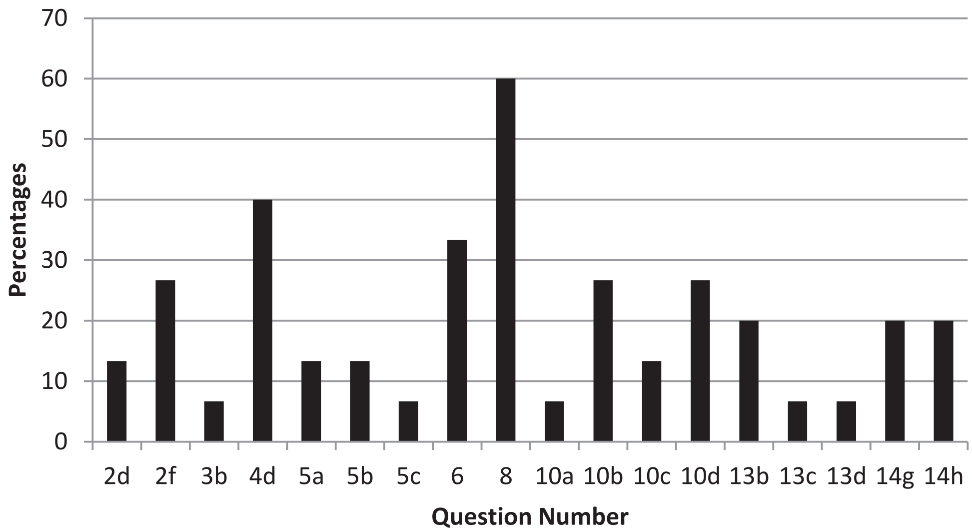


Figure 6.8 RNI questions which elicited an incorrect response by item number

The item which proved to be the most difficult for the respondents was Question 8, a task designed to ascertain students' understanding of the size of fractions. Nine (60%) of the respondents failed to articulate an answer within 0.1, the limit determined by Clarke, Mitchell and Roche (2005) as being acceptably close to the optimum answer, of 1/7 plus 5/6. It is worth noting that none of the teachers arrived at this optimal solution.

When the items are viewed in terms of the understandings (as Clarke et al., 2005 name them, 'Big Ideas') that they are trying to assess, errors were made by the respondents in five areas (Figure 6.9); part-whole (Questions two and three), operations (Questions four, eight and 14), relative size (Questions five and 13), fractions as division (Question six) and fractions as numbers (Question 10). Due to the structure of the tool, each question varied in the number of answers to be supplied by the respondents. Therefore although operations had the most number of errors, in percentage terms (due to the number of items which were addressed) (Figure 6.9), respondents performed least successfully with the big idea of fractions as division (Figure 6.10).

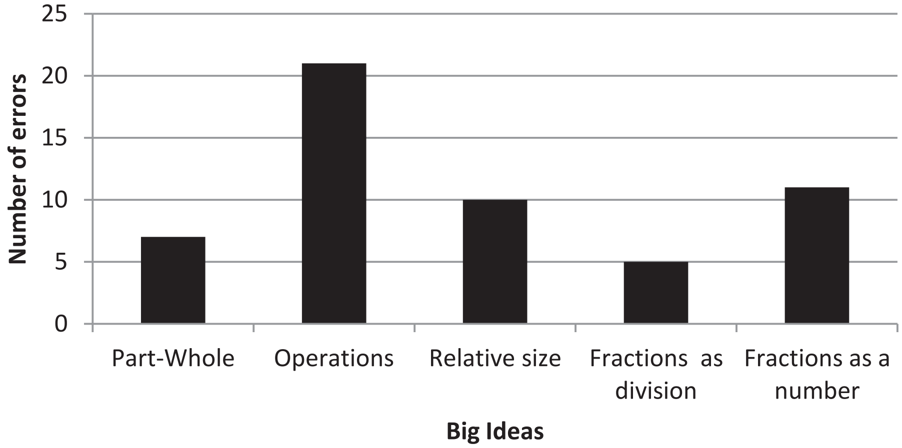


Figure 6.9 Number of RNI questions which elicited an incorrect response grouped as "Big Ideas"

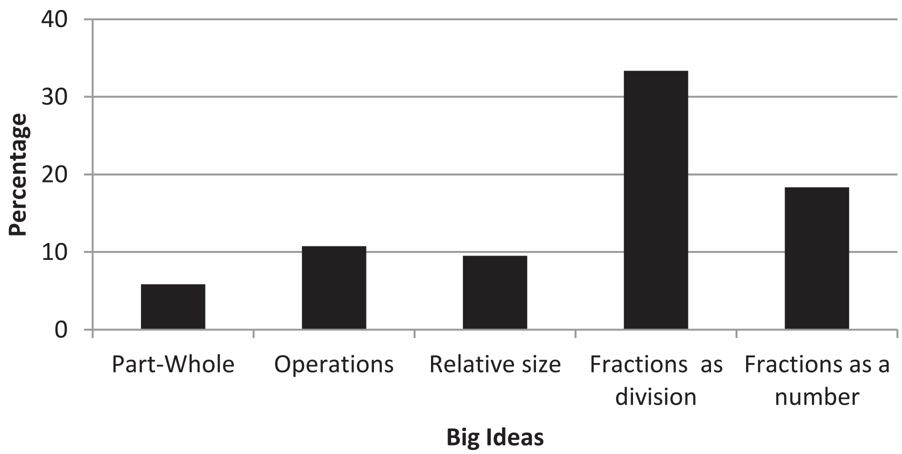


Figure 6.10 Percentage of RNI questions which elicited an incorrect response grouped as "Big Ideas"

One of the questions which particularly interested this Researcher was Question 14, the fraction pairs question. This was a question which allowed for some detailed analysis of the respondents' thinking concerning fractions. The respondents were asked to identify the bigger of two fractions and express the strategy they used to reach their decision.

An error was recorded if the respondent chose the incorrect item as being larger in value. One error (6.67%), was made on 2/4 and 4/2. Four errors (26.67%) were made with 5/6 and 7/8 and three errors (20.00%) with 3/4 and 7/9. No errors were recorded with the other five questions. These results compare favourably with the Clarke, Mitchell and Roche (2005) group of Year 6 students (n=323) of which 49.5% made an error with 2/4 and 4/2, 85.1% made an error with 5/6 and 7/8 and 89.2% made an error with 3/4 and 7/9. However, the fact that any errors at all were made on this assessment by the teachers is perhaps worthy of concern.

Analysis of the responses to the fraction pairs was achieved through the use of rubrics supplied by Clarke et al., (2005). Each of the correct answers to the items can be delineated as being either; the most preferred strategy, a preferred strategy or a least appropriate strategy. Analysis of the responses to the fraction pair 3/8 and 7/8 (considered by Clarke, Mitchell and Roche, 2005 as the easiest of the questions) revealed that all respondents described using more appropriate strategies (those above the line) though only 67% used the most preferred strategies.

Considering the responses to the fraction pair 1/2 and 5/8, all but one respondent preferred using more appropriate strategies to obtain the correct answer, however only 53% used the most preferred strategy of benchmarking to 1/2. Respondent G2-R10 could not give an explanation about how she arrived at the correct answer. For the fraction pair 4/7 and 4/5 five respondents used the most preferred method and a further 46% used other preferred methods. One person (G2-R16) used gap thinking, looking solely at the gap between the numerator and denominator; one respondent (G2-R6) said that "the bottom number is smaller" and one (G2-R14) converted the numbers into percentages, "80% and about 50%." This solution is a variation of benchmarking to ½ using a conversion to percentages.

In making a response to the fraction pair 2/4 and 4/8, all but one respondent preferred using the most appropriate strategy. One person, G2-R14, used an algorithmic cancelling down before making a decision.

In responding to the fraction pair 2/4 and 4/2, 12 respondents used the most preferred method. One person (G2-R17) made an error by stating that the fractions were equivalent. G2-R14 stated the “one is two wholes, and the other is not.” G2-R16 had difficulty in expressing why they gave the correct answer.

For the fraction pair 3/7 and 5/8 eight respondents (53%) used the most preferred method and a further 27% used one of the other more preferred methods. Respondent G2-R4 claimed that they obtained the correct answer through a guess, G2-R14 benchmarked to one rather than ½, and G2-R15 gained the correct answer due to the fact that it “feels bigger.”

Responding to the fraction pair 5/6 and 7/8, forty percent (six) of the respondents described using residual thinking, the most preferred strategy. (Residual thinking is useful for comparing fractions that are one away from the whole. For example, 6/7 is one seventh away from the whole and 7/8 is one eighth away from the whole. As one eighth is smaller, 7/8 is closer to the whole). Four others (27%) used the other of the more preferred strategies of converting to common denominators. G2-R6 vacillated between believing the 5/6 to be bigger and that the two fractions had the same value, respondents G2-R8 and G2-R14 converted the fractions into decimals to get the correct answer, G2 –R10 acquired the correct answer by using ‘intuition’ and expressing that “7/8 is larger due to the size of the denominator”.

Considering the fraction pair 3/4 and 7/9, 33% (five) of the respondents described using the most preferred strategies of residual thinking or converting to decimals. Four others (27%) used the other of the more preferred strategy of converting to common denominators.

G2-R4 and G2-R5 provided incorrect answers with the reasoning that ¾ looked bigger. G2-R10 proffered the right answer with the reason that in 7/9 the denominator has more parts. G2-R11 gave an incorrect response and simply stated that they did not know how to work

out the answer. G2-R16 and G2-R17 answered correctly but could not articulate how they processed the answer.

When determining the larger of two fractions, most respondents were utilising methods which fall into the most preferred or preferred categories (Figure 6.11). The use of less appropriate strategies which lead to the correct or incorrect solutions constituted between zero and 40%, with a mean of the scores for employing a less preferred or appropriate method being 18.33%.

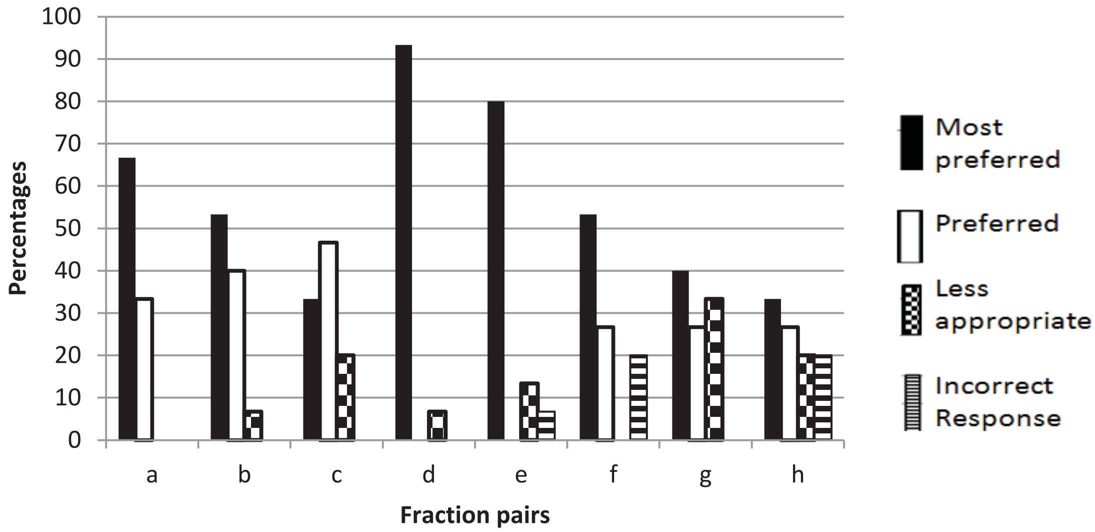


Figure 6.11 Strategies employed to solve fraction pair problems using criteria from the Rational Number Interview

Key Finding 6.7: It was noted that none of the participants scored every item correct in the Rational Number Interview. In fact five of the respondents scored more than 10% incorrect responses on an assessment tool designed to test the understanding of rational number constructs of 12 year old students. One respondent got more than 25% of all the questions incorrect.

6.7 ***Summary***

Having collected data regarding the status of fractions, it showed that the teachers felt that their belief about the status of fractions was not congruent with their perception of the status, that their school, or the prevailing curriculum documents, afford the topic. The data also showed that the teachers felt more confident and able to teach fractions at the conclusion of the PL experience than they did prior to it. Further, the data showed some of the teachers to have deficits with Specialised Content Knowledge for fractions and that when asked to complete an assessment (RNI) constructed to test the knowledge of 12 year old students on fractions, five of the teachers displayed errors. All of these issues will be more deeply explored in Chapter 11.

Having determined the status of fractions, the next area for consideration was the building of Pedagogical Content Knowledge (PCK) (as determined by Shulman, 1986) through the development of the two domains of PCK, content and pedagogy. The following chapter will explore the data collected to determine if well-structured PL can improve the content knowledge of teachers in the topic of fractions.

CHAPTER 7

7 The Impact that Well-Structured, Action Research Based, Professional Learning Opportunities and Reflective Practice have on Primary School Teachers' Content Knowledge of Fractions.

7.1 *Concept Maps (CM)*

This Researcher identified 13 different written representations of fractions commonly used in schools and curriculum documents (Table 4.2). These are not necessarily constructs (part-whole, quotient, measurement, operation or ratio) as described by other researchers (Kieren, 1980) but rather representations of those constructs (for example, region models, area models, number lines).

At the commencement of the professional learning sessions the teachers were asked to create a concept map with the instructions, "Tell me everything you know about two-thirds (2/3) and the different ways we can show it." These were then collected, to see what complexity and connections regarding fractions were apparent to these teachers.

The numbers of connections made by the teachers (Figure 7.1) ranged between two and nine (out of a potential set of 13). There was a mean score of 9.07 representations and a median score of 10 representations.

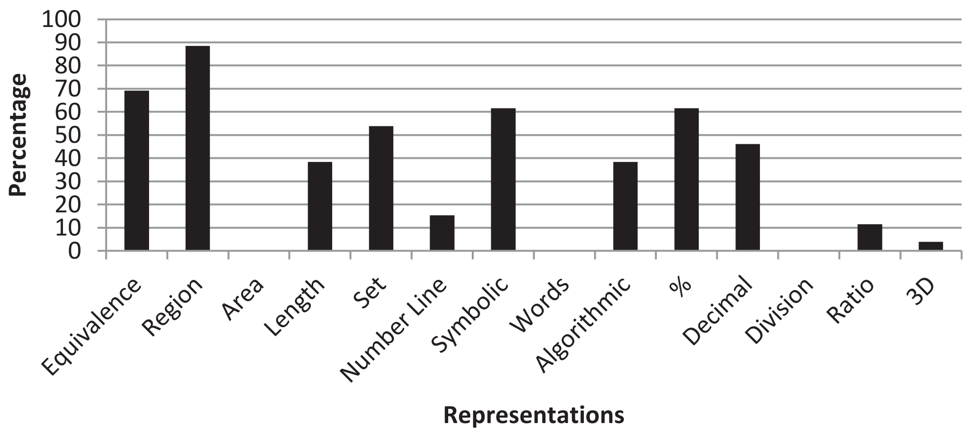


Figure 7.1 Fractional representation and the percentage of respondents who referred to them in their pre-intervention concept map

No teacher thought to represent 2/3 as written in words, as an area representation (where the pieces may not look the same but have the same area) or as a problem using the division symbol. The most commonly referred to representations were the region model with 23 (88%), 18 (69%) for equivalent fractions and 16 (61%) for symbolic and percentage representations. Of the remaining, only the set or discrete representation (where if there is a whole collection of four objects, then one object is ¼ of the whole) at 53% (14) was used by more than half of the respondents.

The concept map (CM) was the first of the post-intervention assessment tools that the participants were asked to undertake. They were asked to write the number 2/5 in the middle of a blank piece of paper and then to record everything they knew about that number.

The number of representations (constructs) each of the teachers produced ranged between six and 11. All 13 representations were illustrated at least once. Symbolic equivalence, be it as equivalent fractions, decimal fractions or percentages, were utilised by all respondents.

There had been a number of respondent withdrawals (eight in total) between the pre-intervention and post-intervention assessments, and the data collected from these participants was taken out of the analysis. Therefore all further data sets illustrate only the pre-intervention scores of those who participated in the post-intervention testing.

There was a growth in the mean number of representations from the pre-intervention assessment (5.72) to the post-intervention assessment (8.61). When testing the statistical significance of this difference the Wilcoxon Signed Ranks Test of median of differences showed it to be significantly different ($Z = -3.400$, $p = .001$).

Post-intervention, the representations of fractions as division, equivalent fractions, region models, length models, set models, number lines, symbols, algorithms, percentages and decimal fractions were used by more than 50% of the respondents (Figure 7.2). Overall it can be stated that there was an increase in the number of representations post-intervention to pre-intervention.

Figure 7.2 shows the number of respondents, pre-intervention and post-intervention, who used particular representations on their content map (CM). In the pre-intervention drawings only equivalent fractions, region models, set models, symbolic representation, percentages and decimal fractions were mentioned by more than 50% of the participants.

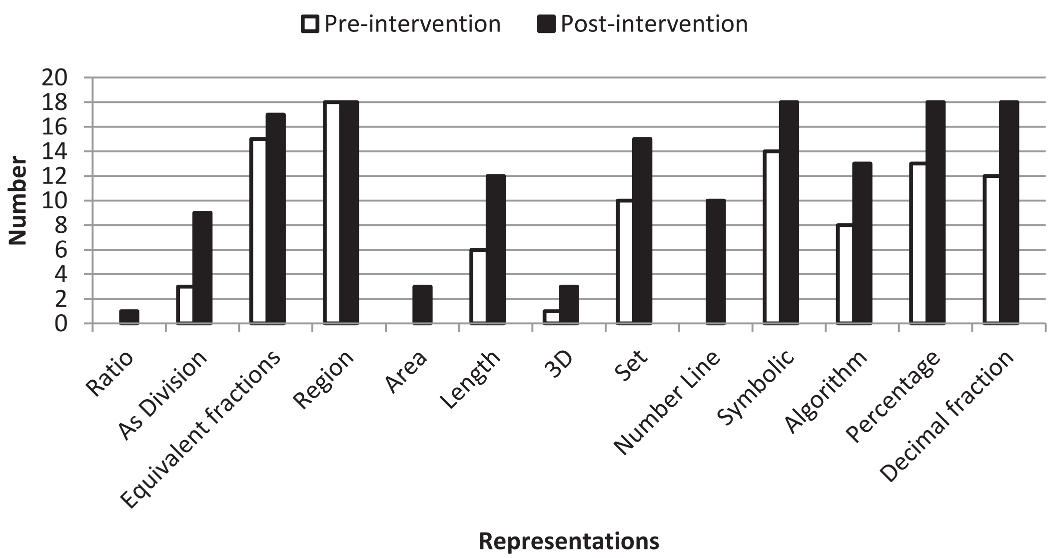


Figure 7.2 Number of respondents who referred to particular representation in their pre and post-intervention concept map

To determine if the increase of use of any of the particular representations was statistically significantly the Wilcoxon Signed Ranks Test (Table 7.1) was employed. Statistically significant differences were found in five of the representations (as division, number line, symbolic, percentage and decimal fractions).

Table 7.1
Wilcoxon Signed Ranks Test for statistically significant differences between pre and post-intervention number of representations in CM

Related-Samples Wilcoxon Signed Ranks Test	Representation	Median of differences	Statistically significant? [a]
	Ratio	.317	No
	As Division	**.034**	**Yes**
	Equivalent fractions	.317	No
	Region	1.000	No
	Area	1.000	No
	Length	.058	No
	3D	.317	No
	Set	.059	No
	Number Line	**.002**	**Yes**
	Symbolic	**.046**	**Yes**
	Algorithm	.096	No
	Percentage	**.025**	**Yes**
	Decimal fraction	**.014**	**Yes**

Note. [a] $p<.05$

The number of different representations used by the respondents rose from 10 in the pre-intervention assessment to 13 in the post-intervention assessment (Figure 7.2). In the post-intervention assessment, seven of the representations were used by at least 15 out of the 18 respondents. Three of the representations were employed by between nine and 14 of the respondents and three of the representations were used by three or less of the respondents.

Even though the respondents had been asked to consider all of the representations during the professional learning sessions, fractions as ratios, fractions as area (as previously mentioned, for area representations, regions are the same area but not necessarily the same shape) and fractions as 3D representations *escaped* being captured on the post-intervention concept maps by respectively 94%, 83% and 83% of the respondents.

7.2 ***Semi-Structured Interview: Capacity to teach fractions***

When asked in the semi-structured interview if they thought they were better equipped to promote student learning in fractions, because of attendance at the PL (Interview Question 7), the response to this question was universally positive. More than one-third of the respondents used the superlative of definitely, to express their positive position (Table 7.2). Many of the conversations focused on the Specialised Content Knowledge (SCK) (as defined by Hill et. al., 2008a), that the teachers had acquired (for example, unitising, partitioning, and so on) and pedagogical concerns such as the use of manipulative materials.

Table 7.2
Perception of being better equipped to promote student learning in fractions, through attendance at this PL

(n=19)	Number of responses	% of responses
No	0	0.00
Yes	11	57.89
Definitely	7	36.84
No response	1	5.26

Some of the comments made to support the answering of this question were about self-growth and the development of the understanding of constructivist principles and developmental learning.

G1-R4 commented:

> …that's why I personally have enjoyed the PD as being our school together and all the teachers together because some people learn things quickly, some people learn things slowly, some of the teachers here are brilliant at mathematics and some people like me take longer to understand concepts. So when I ask questions I'm actually acting like a student …and I find as much as it makes me feel stressed, because I'm opening myself up for my weaknesses, it's also a strength, because these guys, the teachers, get to see how people learn differently, including teachers.

This development of the further understanding of constructivist principles and developmental learning might appear to be fundamental pedagogy in teaching, but it raises a question about whether this theory is actually practised.

G1-R2 also commented:

> …I was just on to it very quick, waiting, and then people were like getting it wrong., and I was like, how could you even get that wrong like it's just, it's so obvious and then Geoff kind of said to me, yeah, yeah, you've got it but do your kids get it? If you're that far ahead, you know like, that's probably true, I've probably left kids behind quite a few times…

One interviewee, G2-R14, was generally not forthcoming with answers and his answers were terse and not well elaborated. On this question he replied with his most detailed answer:

> I think before I came here fractions was just sort of on the whiteboard doing calculations, you know, adding, multiplying and working out the system of learning I think which I was used to when I was at school but now I have gone back to the very basics maybe year two, year three which I can still teach with my year fives, going from the beginning, getting activities, learning the basics and getting them to do things, physically, going to the board, holding things, moving things which I didn't really have when I was at school so I sort of taught that way as well but this is a lot better with this (sic) interactive activities.

This response by G2-R14 supports the response to the earlier interview question where he was asked about his ability to teach fractions. He indicated a pre-intervention score of seven and a post-intervention score of 8.5.

7.3 *Semi-Structured Interview: Fractions content knowledge*

A second, two part question, asked during the semi-structured interview (Question 10) was "How has your content knowledge about fractions changed through this course? Could you articulate a couple of the things you have learned?" Comments were made by the respondents who indicated that they felt they had improved their content knowledge, their pedagogy and therefore, their ability to teach fractions.

Table 7.3
Respondents' perception as to if their content knowledge about fractions had changed

(*n=19*)	Number of responses	% of responses
Improved	12	63.16
Not improved	5	26.32
Not answered	2	10.53

There was an implicit and intended meaning attached to the word changed in this question. The implication was that changed would mean that the Subject Matter Knowledge (SMK) and particularly the participants Specialised Content Knowledge (SCK) would have improved. The word improved was not used in the question so as not to influence the respondents, perhaps biasing them towards thinking they necessarily needed to provide a positive response.

The pressure of responding to questions 'on the spot' was highlighted again in the response to this question by G1-R8 who said:

> Oh, this is quite difficult because I haven't…I wasn't prepared for this.

Respondent G1-R8 did however continue by saying:

> I've never been really good at maths so this really helped me by using all of the concrete materials, helping me with my content knowledge or being able to take it over and explaining to the children

G2 - R1 expressed the view of quite a few respondents:

> I knew a lot of the content, but it's looking at different ways to present it, I think it's more than just knowing the content it's actually how to make it interesting.

Again there was a desire to illustrate the symbiotic relationship between SMK and PCK.

Respondent G2-R10 did not itemise areas of content growth but instead gave a more 'global' but revealing reflection:

> **G2-R10:** My content knowledge was pretty poor because I would never go near it as a choice and in fact the only thing that stops me from teaching year 7 is having to teach them the higher maths, because of my confidence and ability. But having said that now I am much more confident and have a better ability to start thinking in ways that would enhance my teaching of fractions. The reason I came is not because I love maths or fractions it's because I wanted to learn to do it properly and I feel that I am more than half way to doing it really, really well.
>
> **Interviewer:** So how has your teaching changed then, so you are saying you are not avoiding it anymore but you're…
>
> **G2-R10:** Because I can get up there in front of them and say let's go step by step because I can see now that it's not a quick thing to learn and so going from place value or whole numbers to the number lines to show where the decimals fit and when we talk about fractions we also talk about decimals concurrently so they get it that they mean the same thing yeah, I don't shy away from teaching it now.

Key Finding 7.1:
The results from the concept map and Questions 7 and 10 from the semi-structured interview indicate that the teachers' Specialised Content Knowledge was increased through professional learning. There is also the suggestion that the manipulative materials allowed some respondents to improve their Subject Matter Knowledge (SMK) as well as their Pedagogical Knowledge (PK).

7.4 *Summary*

The data collected from concept maps and interviews indicated that the PL intervention enhanced the teachers' Specialised Content Knowledge of fractions. When this outcome is considered in the light of the available literature, this is extremely important for the development of effective teachers of this difficult topic. This will be more deeply explored in Chapter 11.

Along with the development of content knowledge, the second domain to consider in the building of Pedagogical Content Knowledge (PCK) is the development of pedagogical knowledge. Chapter 8 will look at the collected data with regards to the impact that well-structured PL has on primary school teachers' pedagogical knowledge of teaching fractions.

CHAPTER 8

8 *The Impact that Well-Structured, Action Research Based, Professional Learning Opportunities and Reflective Practice have on Primary School Teachers' Pedagogical Knowledge of Teaching Fractions.*

8.1 ***Pedagogical Content Knowledge Situations (PCKS)***

All respondents were asked to complete an initial Pedagogical Content Knowledge Situations (PCKS) questionnaire (Appendix 6) which offered scenarios for which they had to provide solutions and (in most cases) give reasons for their answers. This instrument was based upon some early work by Clarke and Mitchell (2008). A total of five situations were provided. When the teachers' responses were scored, Situation 1 had a value of three marks, Situation 2, five marks, Situation 3, five marks, Situation 4, three marks and Situation 5, five marks. The higher (and therefore more desirable) the level of PCK as determined by the Researcher, the higher the score that was achieved by the respondent.

8.1.1 Situation 1

Situation 1 (Figure 8.1) asked the respondents to consider the concept of partitioning in understanding fractions and they were given five choices of number sets to use and then asked to explain their reasoning. Whatever set of numbers the respondent chose they then had to justify this decision. Indeed the justification was the most important part of the answer as it was needed to articulate the specialised knowledge of the respondent.

1. Which numbers should be used as initial examples to illustrate the place of partitioning in understanding fractions?

a) $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$

b) $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{6}$ $\frac{1}{8}$

c) $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$

d) $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ $\frac{4}{4}$ $\frac{5}{4}$

e) All above work equally as well

Figure 8.1 Situation 1 Question

Seventeen (65.4%) of the respondents chose ‘c’ with fairly strong reasoning such as "…because there is a progression of numbers and all are even numbers" and "…it allows us to see equivalence easily." Other less well expressed (perhaps misconceived) reasons were "Simple activities that show how the large denominator actually means a smaller part" and "You can teach that the whole part may change, but your (sic) still talking about one part of." Two people (7.6%) thought that ‘a’ was the most favourable number set, four (15.3%) ‘b’ and three (11.5%) ‘d’. Interestingly, one person who chose ‘d’ (1/4, 1/2, 3/4, 4/4 and 5/4) did so to work from the premise of building understanding through using the same size denominator. Of the respondents who answered b, c and d, six of them justified their answer using the word equivalence, and argued that that combination would be best to illustrate equivalence. Two of the respondents, both who answered ‘c’, also suggested that the partitioning of the fractions might be more easily accomplished using that number set.

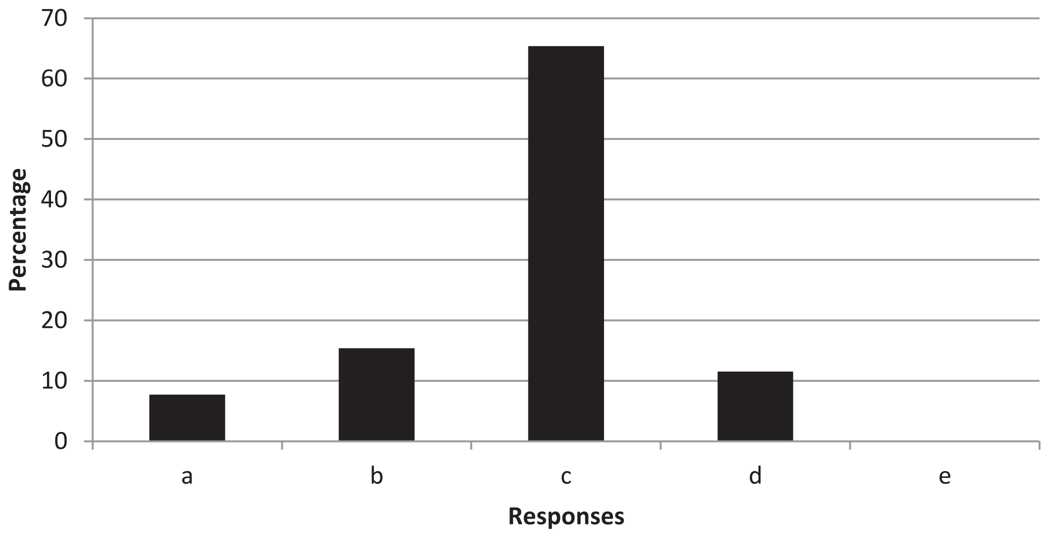


Figure 8.2 Percentage responses to Situation 1

8.1.2 Situation 2

Situation 2, (Figure 8.3) was primarily aimed at seeing if the participants understood the need for multiple representations of fractions. The respondents were advised that they could use as many ticks to indicate their choices as they thought necessary.

2. Which representations of fractions are necessary to foster student understanding of unit fractions i.e. ¼, ½, ⅓ etc. Please indicate with a tick.

- Symbols ☐
- Area/region models ☐
- 3 Dimensional models ☐
- Number Lines ☐
- Set models ☐
- None of the above ☐
- All of the above ☐

Figure 8.3 Situation 2 - Question

Thirteen (50%) of the teachers thought that all of the representations were required (symbols, area/region model, three dimensional models, number lines and set models) and ticked the corresponding box or all of the available boxes (Figure 8.4). Nine more (34.6%) of the respondents picked multiple representations, but not all representations, and four of the respondents (15.3%) chose the area model as their only representation.

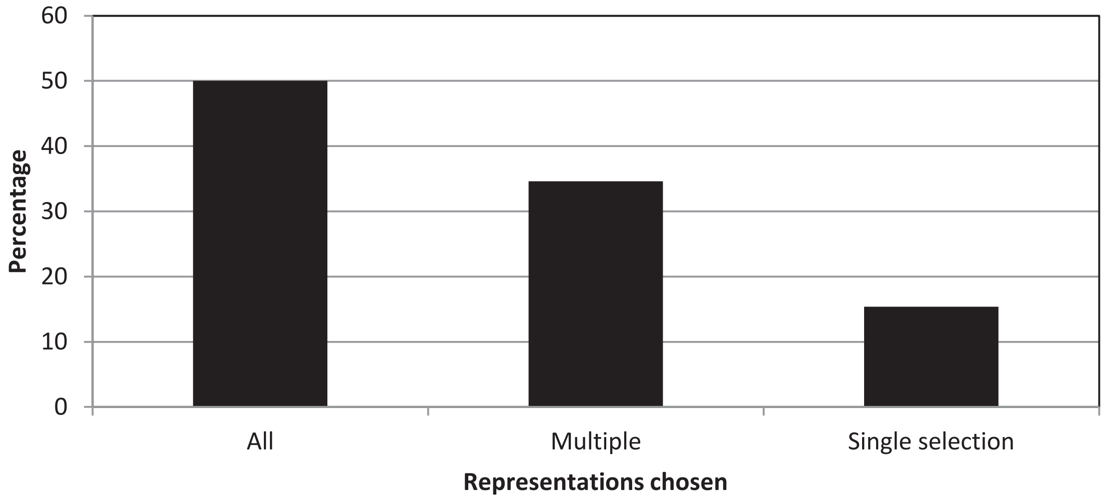


Figure 8.4 Responses to which representations of fractions are necessary to foster student understanding of unit fractions

Whilst at this stage, the recognising all of the representations might have been a desirable, if overly ambitious expectation, the fact that four respondents chose only one representation was quite disturbing.

8.1.3 Situation 3

Situation 3 required the participants to respond to the following statement:

> "When given the sum 3/7 + 2/9 the student gave an incorrect answer. Give 5 different answers that the student might have given and give reasons for the misconceived answer."

Figure 8.5 - Situation 3 Statement and question

Participants were asked to propose five misconceptions that the student might display in giving this incorrect answer (Table 8.1). Misconception ‘e’ (reasoning with no obvious understanding of misconceptions) was utilised by the Researcher in instances where the teachers wrote an explanation which did not to this Researcher seem plausible, but might have been possible (for example “Multiplying three and nine then adding two and seven.”) Although asked to provide five possible explanations for the incorrect answer, three teachers provided only one possible explanation, 12 teachers provided two explanations, five teachers provided three explanations, three provided four and three provided five reasonable and therefore possible explanations.

Table 8.1
Number of respondents who described possible misconceptions for Situation 3

	Misconception*(n=17)*	Number
a	Whole number reasoning – adding the numbers for an answer of 5/16	26
b	Use of either denominator without finding a common denominator – for answers of 5/7 or 5/9	9
c	Incorrect generalisation and multiplying denominators but adding numerators – 5/63	12
d	Variations on previous answers to misconception a, b and c	15
e	Reasoning with no obvious understanding of misconceptions	14
f	Adding all numbers together with no concept of fractions	5

8.1.4 Situation 4

Situation 4 posed the following situation:

> "The Year 4 teacher told her class on the first day of working with fractions; "Nothing is more important in understanding fractions than finding out what the unit is!" Is she correct? Explain your reasoning."

Figure 8.6 Situation 4 Statement and question

Situation 4 of this assessment (Figure 8.6) posed a question which required the teachers to express what they considered a key understanding with regards to fractions. Seventeen respondents agreed with the statement, seven respondents disagreed with the statement and declared that they believed that the understanding that fractions are about equal parts was pivotal and two failed to respond to the question. Of the teachers who agreed with the statement, five were unable to articulate why they so responded.

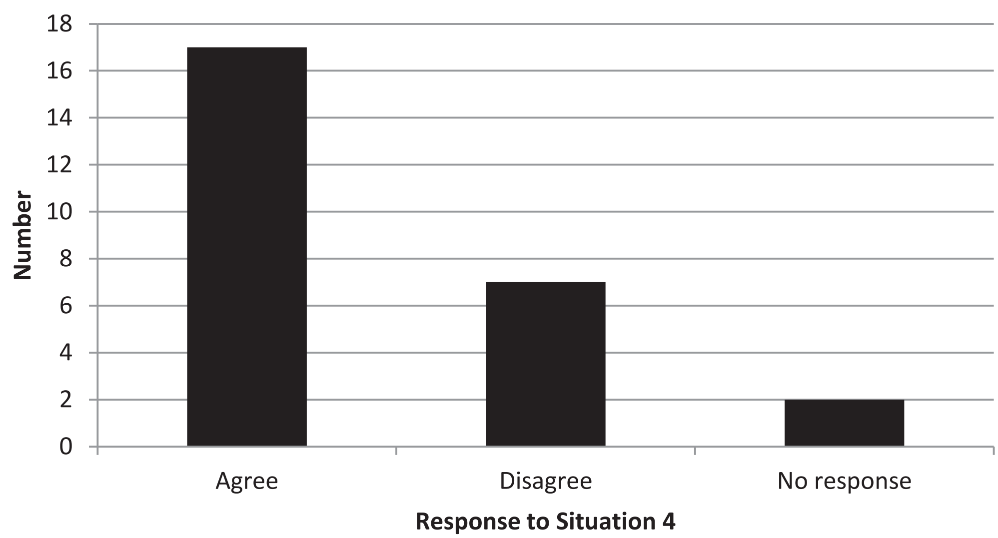


Figure 8.7 Situation 4 – Responses to the statement

If there was an area of concern here, it was less about whether or not the teachers agreed and more about the fact some could not articulate a reason for their answer.

8.1.5 Situation 5

The final situation (Figure 8.8) was intended to reveal the participant's own knowledge of fractions and then to ask them to identify some possible misconceptions that others might have. The respondents were given five responses, one of which was correct, asked to identify the correct answer and then asked to articulate how an answer which was incorrect might have been thought correct.

5. The students in a Year 7 class gave the following answers to the following problem. Determine and indicate which the correct answer is and then how they might have achieved their misconceived answer.

What fraction of the circle is part D?

a. $\frac{1}{3}$

b. $\frac{1}{6}$

c. $\frac{1}{5}$

d. $\frac{1}{4}$

e. $\frac{1}{8}$

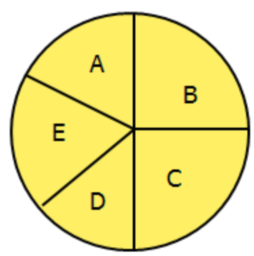


Figure 8.8 Situation 5 Statement and questions

The correct answer of 1/6 was identified by 23 of the teachers, while one selected option c (1/5) and one selected option e (1/8). One respondent did not indicate which choice was correct. Eight respondents posited four possible explanations, 11 of the respondents posited three plausible explanations for the wrong answer and eight suggested two explanations. Ten of the respondents (58%) couldn't identify the correct answer nor then give plausible reasons why the other answers might have been offered. Answers supplied for this part of the assessment covered possible solution strategies such as those shown in Table 8.2.

Table 8.2
Number of occasions a particular answer was supplied for Situation 5

Answer supplied	Number of times considered
Poor estimation	9
Viewing only left hand side of shape	23
Not determining need for equal parts	27
Using knowledge of familiar but incorrect fractions	16
Guessing	8

The Pedagogical Content Knowledge Situations (PCKS) questionnaire (Appendix 6) was administered for a second time to the respondents during the last professional learning session, after the intervention. As there were a number of participant withdrawals before the post-intervention questionnaire was given, all subsequent data sets illustrate only the pre-intervention input of those who participated in the post-intervention assessment. Situation 1 had a value of three marks, Situation 2, five marks, Situation 3, five marks, Situation 4, three marks and Situation 5, five marks. The more PCK that was identified by the Researcher, the higher the numerical score it was given. Therefore, a score of one out of five would indicate little PCK shown, and a score of four out of five, a good deal of PCK shown.

Table 8.3
Pre and post-intervention responses to PCKS questionnaire

(n=17) Situation	Pre-intervention	Post-intervention	Diff.[a]
1	30.0	45.5	15.5
2	65.0	78.0	13.0
3	51.5	52.5	1.0
4	31.0	41.5	10.5
5	55.5	67.5	12.0
Mean	46.6	56.6	10.0

[a] $p<.01$ Wilcoxon Signed Ranks

A simple survey of the difference between the pre and post interview results suggested that there was growth in PCK, and the statistical package SPSS was employed to judge the significance of that growth. When the difference in the mean scores (Table 8.3) was compared using the Wilcoxon Signed Ranks Test a highly significant statistical difference was noted (p = .002).

Table 8.4
Wilcoxon Signed Ranks Test for Pre (PCKS) and post (PCKSA) intervention statistical significance of differences between individual situations

Related-Samples Wilcoxon Signed Ranks Test	Representation	Median of differences
	Question 1 pre and post-intervention	.004*****
	Question 2 pre and post -intervention	.106
	Question 3 pre and post-intervention	.918
	Question 4 pre and post-intervention	010*****
	Question 5 pre and post-intervention	.007*****

*****p<.05 Wilcoxon Signed Ranks

Key Finding 8.1:
The teachers made statistically significant gains in PCK for teaching fractions in response to the PL intervention.

8.2 ***Semi - Structured Interview: Approach to teaching fractions***

When asked in the semi-structured interview the question, "Has this PL changed the manner in which you will teach fractions? If so, why and how?" (Question 8) all respondents indicated that the PL had changed the manner in which they would teach fractions. Information as to how their practice would change was collated, in order to see if themes had started to appear (Table 8.5). Nearly half of the participants (48.15%) stated that the use of manipulative materials would become more prevalent in their teaching of fractions.

Table 8.5
Indicated changed manner in teaching of fractions

(n=27)	Number of responses	% of responses
More use of manipulative materials	13	48.15
Start from basics	4	14.81
Moved away from early teaching of algorithms	2	7.41
Use of more and/or different representations	2	7.41
Integration into other mathematics topics	1	3.70
Strengthened and supported pedagogical beliefs	1	3.70
A more systematic approach to teaching fractions	1	3.70
Constant revision of concepts	1	3.70
More time on fractions	1	3.70
No response	1	3.70

What is of interest here is not the assertion that change will take place but rather how the respondents indicated that they will change their practice. This change in pedagogical practice is seen by the Researcher as a positive recognition of the use of manipulative materials, which in turn goes some way to providing the platform for reflection upon why one would utilise these materials.

Using the domains and definitions of Hill et al. (2008a), all of the responses fell into the category of Pedagogical Content Knowledge (PCK), with 96.30% of the responses being able to be categorised as Knowledge of Content and Teaching (KCT). If this indicated alteration of practice is carried into the classroom then the PL will have affected the PCK of the respondents.

G1-R1 speaks of using manipulative materials:

> Yeah, again with the hands-on activities. I probably wouldn't have done as many as what I do now, and I suppose, yeah, just building from certain things and how to move forward using those hands on activities. I guess it's one thing to do a once off lesson with them but to bring them back in again and how to build on that.

This was a positive shift in thinking in this respondent, as in a conversation that was held prior to any professional learning taking place, she strongly indicated that she felt quite confident and able to teach fractions and thought that text books and the teaching of algorithms lay at the heart of understanding fractions. It should also be stated that when completing the post-intervention interview questions regarding her confidence and ability to teach fractions at the commencement of the PL she only rated herself as pretty good in relation to her confidence and average in relation to her ability. This again was in contrast to her verbal and unrecorded pre-intervention perceptions.

G1-R7 spoke of strengthening and supporting her pedagogical beliefs:

> It adds value rather than changes the way, because I was hands on anyway.

This focus on PCK was further explored through the interview question, "If you had to advise on your top five tips for teaching fractions what would they be and why? Don't worry about the order and if you need more than five that's fine." Using the domains and definitions of Hill et al. (2008a), 37 of the 42 responses (88.1%) related to PCK and in particular Knowledge of Content and Teaching (KCT). The remaining five responses (11.9%) came under the banner of Subject Matter Knowledge (SMK), highlighting the

particular dimension of Specialised Content Knowledge (SCK, indicated below by #). (Table 8.6)

The responses in Table 8.6 highlighted by asterisks (*) were considered by the Researcher to be of great importance and their importance was expressed in the PL but never explicitly expressed. Not all of the respondents provided five answers for this question.

Table 8.6
Respondents' top five tips for the teaching of fractions

	(n=42)	Number of responses	% of responses
KCT	* Use manipulative materials	14	33.33
	Start with the basics	7	16.67
	Proceed slowly	4	9.52
	* Be precise with language	3	7.14
	* Choose representations carefully	3	7.14
	* Use multiple representations	1	4.76
	Apply principles of constructivism	2	4.76
	Link to the 'real' world (contextualise)	2	4.76
SCK	# Make explicit links to decimal fractions and percentages	2	4.76
	# Make explicit links to place value	2	4.76
	*# Stress knowledge of the 'unit'	1	2.38

The *raison d'être* for this question was to try to ascertain what elements of the PL the teachers perceived most valuable, and which were therefore pivotal to the teaching and learning of fractions. The responses seem to indicate that carrying the message of good pedagogy was perhaps seen as more important than improving or increasing the SCK of teachers.

G1-R7 talked about having an 'aha' moment for herself through using manipulative materials over an abstract explanation.

> Oh, I just had this aha moment because one of the teachers asked me, she had done ½ x $^1/_5$ or something, no, ½ divided by $^1/_5$ and I was, she didn't understand it and I even went to the high school, the HOD (Head of Department) and said can you show me this in pictures? And he basically said oh, no, no, no, that's too hard to show in pictures, you accept it and you invert the other one and you times it. Everyone knows how to do that but I wanted to know how it worked. I took the…Pattern Blocks, I was determined; I ended up doing it with a picture rather than the Pattern Blocks. I was very excited.

G2-R14 responded in a different manner in that he did not look at answering the question through his experience from the elements of the PL but rather the PL experience itself:

> You've got to go to a PD and go and surround yourself with people who know what they are talking about who have experienced all different activities and lessons in classrooms, go and steal their ideas and take them, you've got to be creative yourself and adapt it to your own classroom and school, if you don't have the equipment you've got to make it which we did, which is a lesson in itself, probably got to find where the level of your kids are at, so you've got to start from somewhere and probably start from below that even just to make sure the grounding is good and then go from there that's what I would say.

Another person G2-R15 could only find hands-on activities as being important but qualified this by saying:

> …that was probably the really major thing for me, and other things that…, I would really need to reflect…that's terrible, it's been a very rewarding experience and I've really got a lot out of it, but on the spot with a little…

This lack of opportunity to reflect, is perhaps one of the drawbacks of an interview situation, particularly one that the respondents know is being recorded. Perhaps

unsurprisingly, given the proximity of the asking of the questions, the two most popular responses to this question were the same as to the previous question concerning how the respondents would change their manner of teaching fractions.

A further question asked during the semi-structured interview that elicited more support for manipulative materials was “What do you think you have learned through attending this PL and how will it change your practice from now on?” Of the 19 responses to this question (Table 8.7), 11 (57.89%) espoused the increased use of manipulative materials. One other response concerned the use of more paper folding activities. Although this could be seen as a use of manipulative materials (as it is), this was placed in a separate category due to the fact that this reply was explicitly about paper folding and the different representations required to ensure that a understanding was generalisable across a variety of different sized and shaped regions. Three of the respondents (15.79%) did not offer a response to this question.

Using the domains and definitions of Hill et al. (2008a), three of the groupings (more use of manipulative materials, more care about lesson sequence and lots of paper folding) representing 68.21% (13) of the responses sitting under the heading of PCK and the sub-element of Knowledge of Content and Teaching (KCT). Of the remaining, three of the groupings (contextualising fractions, articulating links to other mathematics and spending more time on fractions) represented 15.8% (three respondents). These also relate to PCK but abide under the sub-heading of Knowledge of Content and Curriculum (KCC).

Table 8.7
How the teaching practice of the respondents be changed post-intervention

	(n=19)	Number of responses	% of responses
	More use of manipulative materials	11	57.89
KCT	More care about lesson sequence	1	5.26
	Lots of paper folding	1	5.26
	Articulating links to other maths	1	5.26
KCC	Spending more time on fractions	1	5.26
	Contextualising fractions	1	5.26
	No response	3	15.79

One particularly revealing comment came from G1-R6 who said:

> I think I've learned that fractions are a lot harder to teach than you initially think, its…I would probably rate it as one of the hardest things to teach in maths because there is so much stuff you've got to understand. You've got to make sure kids have the understanding and knowledge before they actually start moving into the harder concepts and the harder fractions. So yeah, I think it has really opened my eyes up as to how difficult it actually is and how much as adults we actually take for granted, because we know about fractions. We are familiar with them but there was even some stuff that was done in the PD where I was sitting there going, ooh, I don't fully understand this, so, if I'm having trouble with it and I've gone through primary school and high school and done it all, I can't imagine what kids are feeling and thinking…

This respondent had previously stated in an unrecorded conversation that he had completed mathematics units at university.

Key Finding 8.2:
Post-intervention semi-structured interviews revealed strong growth in the teachers' PCK for teaching fractions.

8.3 *Semi-Structured Interview – Generalising the knowledge to other domains of mathematics*

The final question from the semi-structured interview which probed the development of PCK was, "What have you learned from this PL about fractions that you can generalise into wider teaching about mathematics?" (Table 8.8). Using the domains and definitions of Hill et al. (2008a) this Researcher determined that seven of the 10 categories of responses (86.38%) could be categorised as PCK. Three reside in Knowledge of Content and Teaching [KCT]; a more "hands on pedagogical approach", "more careful use of language" and "a concentration on process over solutions". Three reside in Knowledge of Content and Students [KCS], "making connections to other learning", "aiming for depth of understanding" and "a need to move more slowly into symbolic representations." One resides in Knowledge of Content and Curriculum [KCC], "the need to differentiate"). Two of the 10 categories (9.1%) belonged in Subject Matter Knowledge [SMK], one as Common Content Knowledge [CCK] ("the importance of place value") and one as Specialised Content Knowledge [SCK], "part - whole understanding". Some respondents provided more than one answer to this question and one respondent did not offer a response.

Table 8.8
Learning from this PL which can be generalised into wider teaching about mathematics

		(n=22)	Number of responses	% of responses
PCK	KCT	Pedagogical approach - more hands on	8	36.36
		Careful use of language	1	4.55
		Concentrate on process over solutions	1	4.55
	KCC	Need to differentiate	3	13.64
	KCS	Making connections to other learning	4	18.18
		Aim for depth of understanding	1	4.55
		Need to move more slowly into symbolic	1	4.55
SMK	SCK	Part - whole understanding	1	4.55
	CCK	Importance of place value	1	4.55
		No response	1	4.55

One major focus for this research was determining if PL could be structured in such a manner, that attendance would provide participants with the opportunity to improve their PCK. An assertion made in this regard, was that if this could happen whilst tackling a topic such as fractions that has been proven to be difficult to teach and learn, then the structure might be successfully applied to other content areas in mathematics.

G1-R4 commented about using manipulative materials:

> For me it's a way of thinking, it's changed my way of thinking and that has got to be brilliant. Because if you've changed your teacher's understanding and way of thinking, it filters through, not just to maths, it filters through everything because then I'll pick up in a different area…to change a teacher's thinking and deepen their understanding I think for me is one of the things that has been really good.

Key Finding 8.3: The teachers repeated in post-intervention interviews that they had developed knowledge that could be applied to teaching mathematics more broadly than just fractions.

Some of the data collected in the questionnaire on beliefs were centred on pedagogical beliefs which would be used to speak to pedagogical content knowledge. As can be seen in Figure 8.9, of the five items relating to beliefs, three of the items showed a decrease in the score attained (Item 1, "Mathematics is computation", Item 2, "Mathematics problems given students should be quickly solvable in a few steps" and Item 5, "Right answers are much more important in mathematics than the ways in which you get them") and two remained static (Item 13, "Teachers should provide instructional activities which result in problematic situations for learners" and Item 15, "The role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge". Of the responses the only one that showed significant statistical difference in pre and post-intervention measures was Item 1.

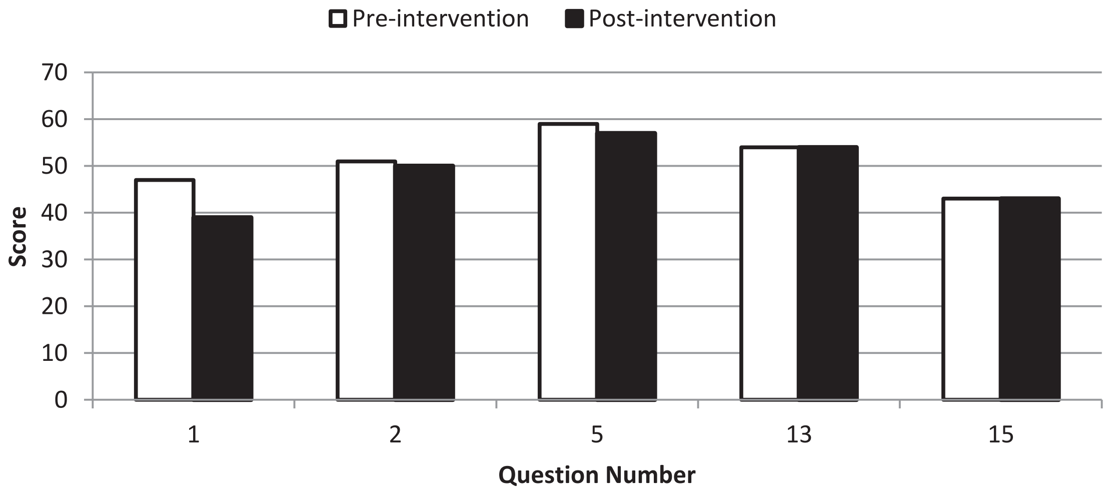


Figure 8.9 Responses from pre-intervention and post-intervention questionnaires to items related to pedagogy

8.4 ***Summary***

The data collected from pre and post-intervention Pedagogical Content Knowledge Situations (PCKS) demonstrated that the teachers made statistically significant gains in PCK in response to the PL intervention. Corroborating evidence also gathered from post-intervention interviews.

Literature supports the thesis that the beliefs and attitudes that teachers hold in relation to the teaching of mathematics are vital (Barkatsas & Malone, 2005; Philipp, 2007; Gabriele & Joram, 2007; Gresham, 2008; Swackhamer et al., 2009; Swars, et al., 2006; Tschannen-Moran & Hoy; 2007; Zambo & Zambo, 2008). Further, the literature is unified on the fact that many teachers, both neophyte and experienced, hold negative beliefs and attitudes towards this subject (Askew, 2008; Baxter, 1983; Becker, 1986; Ernest, 1988; Steffe, 1990; Tirosh, 1990). The following chapter will explore the impact of well-structured PL on primary school teachers' beliefs and attitudes with regards to teaching mathematics in general and fractions in particular.

CHAPTER 9

9 The Impact that Well-Structured, Action Research Based, Professional Learning Opportunities and Reflective Practice have on Primary School Teachers' Beliefs and Attitudes with Regards to Teaching Fractions.

9.1 *Teachers' Beliefs About Mathematics Questionnaire (TBM)*

9.1.1 Pre-intervention

Prior to the intervention the participants answered the Teachers' Beliefs about Mathematics questionnaire (TBM). This questionnaire (TBM) (Appendix 5) was constructed as a three point Likert scale (agree, undecided and disagree) and the respondents were asked to indicate their beliefs about 16 statements regarding mathematics. A positive response (agree) was awarded a score of three, the "undecided" response was given a score of two and the "disagree" was given a score of one. Thus, a score of 48 (the maximum score for the set of 16 statements) would be seen as 100% agreement of the statements.

Seven of the statements in the questionnaire (Statements 1, 2, 5, 7, 10, 14 and 15) were considered by the Researcher to be supportive of beliefs which suggested a very narrow view of mathematics and its applicability to different situations. Each of these was scored with one if the respondent supported these negative positions (agree), two for undecided and three if they rejected them (disagree).

Answering this questionnaire revealed that the belief that "mathematics is computation" (Statement 1) was relatively strong (Table 9.2) as six out of the 26 respondents agreed with this statement. Likewise, the narrow view and therefore negative proposition that "being able to memorise facts is critical to mathematics learning" (Statement 10), had nine (34.6%) respondents who were undecided about this statement, and eight (30.7%) who declared that they agreed that this statement was true. Statement 15, "The role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge" saw seven of the respondents (26.9%) support this negative statement and a further nine (34.6%) undecided.

The four most positively supported propositions all enjoyed 97% support or better. Statement 12, “Mathematics learning is enhanced by challenge within a supportive environment” was the most positive with 100%, followed by Statement 16, “Teachers should recognise that what seem like errors and confusions from an adult point of view are students’ expressions of their current understanding” with 98%. Statement 5, “Right answers are much more important in mathematics than the way you get them” saw most respondents answering with “disagree” (which meant this was coded as being positive) and also scored 98%. One could perhaps see a small amount of dislocation between the response to Statement 5 and the leaning towards the statements that mathematics is about computation and memorising facts. Statement 11, “Mathematics learning is enhanced by activities which build upon students’ experiences” also gained 97% support. It is prudent to be aware however, that a respondent’s beliefs about mathematics may be inconsistent, but as Liljadhar (2009) asserts, they should not be discounted as being unauthentic but rather representations of intended practice.

This result prompted the Researcher to consult with the respondents and to question several teachers as to the reason why they scored Statement 10 (“Being able to memorise facts is critical to mathematics learning”) as they did. It became obvious that they reasoned that without memorised facts then deeper mathematical understanding was difficult. They did not appear to interpret the question in the manner in which the Researcher had intended. A better question would probably have been, “Being able to memorise facts is the *most* critical aspect in mathematics learning.” This would then have spoken more to the balance between understanding and memorisation, which was really the intent of the question.

9.1.2 Post-intervention

The Teachers’ Beliefs about Mathematics questionnaire (TBM) (Appendix 5) was re-administered in the last professional learning session, after the intervention. There were a number of withdrawals between the pre-intervention and post-intervention questionnaires, which precipitated taking the data collected from these participants out of the analysis. Therefore all further data sets illustrate only the pre-intervention input of those who participated in the post-intervention assessment. The results were coded in the same way as the pre-intervention questionnaire.

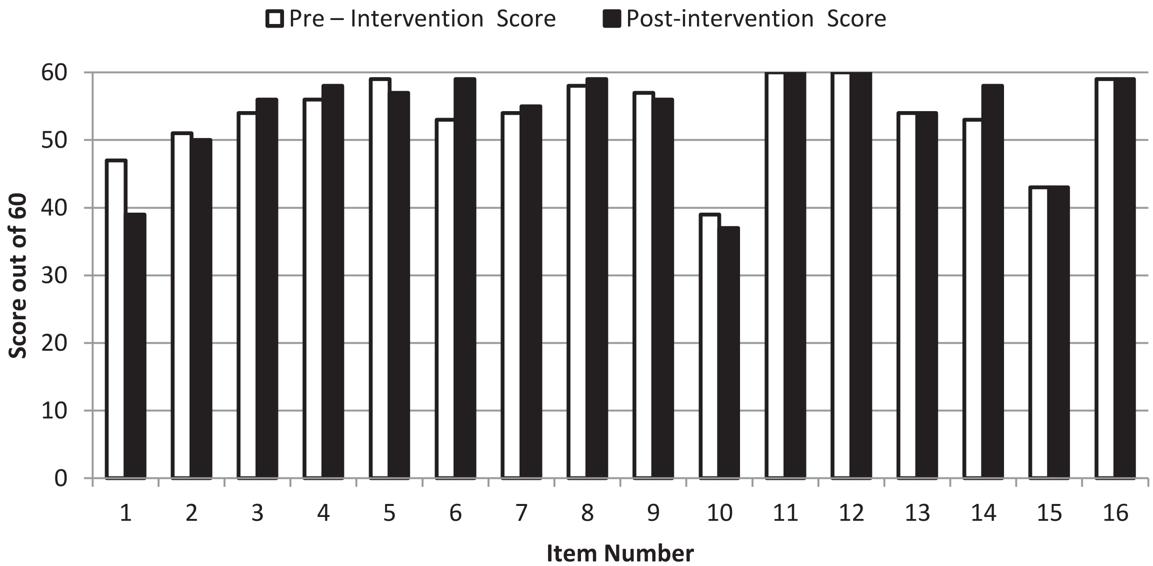


Figure 9.1 Pre and post-intervention positive responses to statements on beliefs (n=20)

When the difference in the means for the scores for the pre and post-intervention responses to questions about beliefs was compared (53.56 for pre-intervention and 53.75 for the post-intervention) there was not a significant statistical difference found Table 9.1). The Wilcoxon Signed Ranks Test was again employed.

Table 9.1
Wilcoxon Signed Ranks Test of statistical significance of differences between pre and post-intervention means of beliefs

	Total Belief 2 – Total Belief
Z	-.156[a]
Asymp. Sig. (2-tailed)	.876

a. Based on positive ranks.
b. Wilcoxon Signed Ranks Test

Use of the Wilcoxon Signed Ranks Test (Table 9.2) to determine if there were any significant differences between the pre- and post-intervention scores of each individual item testing the beliefs of the respondents, showed that only two beliefs were significantly different. These were, Belief 1, “Mathematics is computation,” and Belief 6, “Mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences.”

Table 9.2

Wilcoxon Signed Ranks Test for pre and post-intervention statistical significance of differences between individual beliefs

		$\overline{X}$ agreement Pre	$\overline{X}$ agreement Post	Change	p	Statistically significant?
6	Mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences.	2.65	2.95	0.30	.014	Y
1	Mathematics is computation	2.35	1.95	-0.40	.033	Y
14	Teachers or the textbook – not the student – are the authorities for what is right or wrong.	2.65	2.90	0.25	.096	N
3	Mathematics is the dynamic searching for order and pattern in the learner's environment.	2.70	2.80	0.10	.317	N
4	Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking.	2.80	2.90	0.10	.317	N
5	Right answers are much more important in mathematics than the ways in which you get them.	2.95	2.85	-0.10	.317	N
9	Young students are capable of much higher levels of mathematical thought than has been suggested traditionally.	2.85	2.80	-0.05	.564	N
10	Being able to memorise facts is critical in mathematics learning.	1.95	1.85	-0.10	.564	N
8	Periods of uncertainty, conflict, confusion, surprise are a significant part of the mathematics learning process.	2.90	2.95	0.05	.655	N
7	Mathematics learning is being able to get the right answers quickly.	2.70	2.75	0.05	.739	N
2	Mathematics problems given students should be quickly solvable in a few steps.	2.55	2.50	-0.05	.755	N
13	Teachers should provide instructional activities which result in problematic situations for learners.	2.70	2.70	0.00	.931	N
11	Mathematics learning is enhanced by activities which build upon and respect students' experiences.	3.00	3.00	0.00	1.000	N
12	Mathematics learning is enhanced by challenge within a supportive environment.	3.00	3.00	0.00	1.000	N
15	The role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge.	2.15	2.15	0.00	1.000	N
16	Teachers should recognise that what seem like errors and confusions from an adult point of view are students' expressions of their current understanding.	2.95	2.95	0.00	1.000	N

Key Finding 9.1:
Pre-intervention and post-intervention measures of teachers' beliefs about mathematics indicated that the intervention had little impact on beliefs as measured by the Teachers' beliefs about Mathematics instrument. There was no statistically significant difference in the beliefs of the respondents. It seems that the professional learning did not have an effect.

9.2 *Teachers' Attitudes Towards Mathematics Questionnaire (TAM).*

9.2.1 Pre-intervention

At the commencement of the intervention program a Teachers' Attitudes towards Mathematics questionnaire (TAM) (Appendix 4) was constructed as an nine point Likert scale (the responses to select from were: definitely false, false, mostly false, more false than true, more true than false, mostly true, true, definitely true and not applicable to me). The respondents were asked to indicate their attitude relating to 20 statements regarding mathematics. The responses were then coded with a number between one and eight and the scores summed to generate a total scale score (/160). Any negatively stated items were scored in reverse, meaning that the more the teachers disagreed with the negative statement the higher the score they received. It was intended by the Researcher that a response of "not applicable to me" was to be investigated through further conversation with the respondents. As it was, none of the respondents chose this option. In answering the Teachers' Attitudes towards Mathematics questionnaire (TAM), eight of the teachers scored in excess of 10 points below the mean score for the cohort of teachers. Of these teachers, two of them (G1-R4 and G1-R8) proved in later informal discussions, to be very unsure about their capacity to teach mathematics in general, and fractions in particular.

The least positively answered statement by the teachers was, "At school, my friends always came to me for help in mathematics" (Statement 14). Two possible explanations for this negative perception may have been that these respondents were not seen by their peers as being either mathematically successful or being able to explain their understanding. Neither of these perceptions are ideal for people who are to be teachers. The most positive response

of true was from only three participants (no one opted for definitely true). Seventeen of the respondents felt that this statement was untrue with six claiming that it was definitely false, four claiming it was false, two claiming it was mostly false and five claiming it was more false than true. The second least positive answer was given in response to the proposal, “I generally have done better in mathematics courses than other courses” (Statement 8) which had a mean item score of 4.5.

The third least positive response came in answer to the item, “I have always done well in mathematics classes” (Statement 5). This related to the respondents’ perception of their own school experience. Ten of the respondents (38.4%) responded to this statement in the negative.

On the positive side of the ledger there was almost contradictory buoyancy regarding the following statements, all of which were strongly denied:

- “I have hesitated to take course that involve mathematics” (Statement 11),
- “I do not enjoy teaching mathematics” (Statement 6),
- “I have trouble understanding anything that is based upon mathematics” (Statement 16),
- “I’m not the type of person who could teach mathematics very well” (Statement 4),
- “If I taught in a team or with a teaching partner, I’d like to have another teacher teaching the mathematics” (Statement 20).

The top three positively phrased statements which were supported were:

- “Time passes quickly when I am teaching mathematics” (Statement 10),
- “I find mathematical problems interesting and challenging” (Statement 2),
- “Generally I feel secure about the idea of teaching mathematics” (Statement 1).

All of these items had a mean score between 6.08 and 6.73 (Table 9.4).

9.2.2 Post-Intervention

The Teachers' Attitudes towards Mathematics questionnaire (TAM) (Appendix 4) was again applied in the last professional learning session, after the intervention. The application of the questionnaire and the collection of data were carried out in the same fashion as the pre-intervention questionnaire. In collecting the data it was noted that no-one responded with "Not applicable to me."

There were a number of withdrawals between the pre-intervention and post-intervention questionnaires being given, which precipitated taking the data collected from those participants out of the analysis. Therefore all further data sets illustrate only the pre-intervention input of those who participated in the post-intervention assessment. Similarly to the pre-intervention, the responses to the post-intervention questionnaire attracted a score of between one and eight, with eight being for the most positive of responses.

When the difference in the means scale scores for the pre- and post-intervention questionnaires was compared through the use of the Wilcoxon Signed Ranks Test a significant statistical difference was not found (Table 9.3).

Table 9.3
Wilcoxon Signed Ranks Test for pre and post-intervention statistical significance of differences between means regarding attitudes

	Total Attitudes2 – Total Attitude
Z	-.672[a]
Asymp. Sig. (2-tailed)	.501

a. Based on negative ranks.
b. Wilcoxon Signed Ranks Test

Use of the Wilcoxon Signed Ranks Test (Table 9.4) between the pre- and post-intervention scores of each individual item regarding the attitudes of the respondents, showed no statistically significant differences.

Table 9.4

Wilcoxon Signed Ranks Test for pre and post-intervention statistical significance of differences between individual attitudes

		$\bar{X}$ agreement Pre	Post	Change	p	Statistically significant?
2	I find many mathematical problems interesting and challenging.	6.38	6.90	0.52	.143	No
14	At school, my friends always came to me for help in mathematics.	5.54	5.65	0.11	.199	No
1	Generally I feel secure about the idea of teaching mathematics.	6.38	6.65	0.27	.206	No
17	It wouldn't bother me to teach a lot of mathematics at school.	5.92	5.70	-0.22	.257	No
20	If I taught in a team or with a teaching partner, I'd like to have another teacher teaching the mathematics.	6.08	5.75	-0.33	.285	No
19	Of all the subjects, mathematics is the one I worry about most in teaching.	5.96	5.50	-0.46	.329	No
12	I would get a sinking feeling if I came across a hard problem while teaching mathematics.	5.08	4.93	-0.15	.357	No
10	Time passes quickly when I'm teaching mathematics.	6.46	6.35	-0.11	.360	No
3	Mathematics makes me feel inadequate.	5.46	5.70	0.24	.497	No
13	Teaching mathematics doesn't scare me at all.	5.46	5.35	-0.11	.501	No
11	I have hesitated to take courses that involve mathematics.	6.75	6.50	-0.25	.608	No
5	I have always done well in mathematics classes.	4.62	4.85	0.23	.629	No
16	I have trouble understanding anything that is based upon mathematics.	6.55	6.55	0.00	.668	No
18	I never do well on tests that require mathematical reasoning.	5.75	5.50	-0.25	.689	No
4	I'm not the type of person who could teach mathematics very well.	6.23	6.30	0.07	.710	No
9	I'm not sure about what to do when I'm teaching mathematics.	5.81	5.80	-0.01	.755	No
6	I do not enjoy having to teach mathematics.	6.38	6.70	0.32	.763	No
7	I am quite good at mathematics.	5.46	5.60	0.14	.809	No
15	I am confident about the methods of teaching mathematics.	5.46	5.45	-0.01	.854	No
8	I have generally done better in mathematics courses than other courses.	4.50	4.40	-0.10	.929	No

Key Finding 9.2:

In comparing pre and post-intervention mean scale scores on the Teachers' Attitudes towards Mathematics questionnaire, there was no statistically significant difference in the attitudes of the respondents. It seems that the professional learning did not have a measureable effect on teachers' attitudes towards mathematics.

Several of the semi-structured interview questions (Questions 1, 2, 7, 15 and 16) alluded to self-efficacy through key words or statements such as confidence, ability, being equipped or taking leadership. Both questions one and two (discussed respectively in sections 6.1 and 6.2 as evidence of the current status of the teaching of fractions) supported the fact that in these instances participants perceived that their confidence (as seen in Question 1) and ability (as discussed in Question 2) had improved.

9.3 *Semi-Structured Interview: Capacity to promote student learning of fractions*

When asked in the semi-structured interview if they thought they were better equipped to promote student learning in fractions, because of attendance at the PL (Interview Question 7), the response was universally positive (with more than one third of the respondents using the superlative of definitely to express their positive position). The answers to this question certainly supported the idea that teachers understood the complexity of the teaching 'craft' and that for effective instruction the domains are very closely connected. On quite a few occasions the respondents talked about being better equipped to promote student learning by referring to a change in pedagogy which often requires: an improved Knowledge of Content and Curriculum (KCC) to know when to teach something; improved Specialised Content Knowledge (SCK) so as to know what to teach; Knowledge of Content and Students (KCS) to determine why a student might arrive at a particular answer; and Knowledge of Content and Teaching (KCT) to know which representations would be of greatest advantage to the learner.

G2-R12 alluded to Knowledge of Content and Curriculum (KCC):

> We've probably all just recognised that it's not an area of maths that you can just glaze over and knock it on the head in 3-4 weeks you know, it's something that you have to constantly revisit

G1-R5 reacted in a similar way:

> Yeah just because I've had, we've sort of talked about the basics of fractions and what they mean and then I'm equipped with the activities … we've all done the same sort of activities in our classrooms with different varying outcomes so you know Year 4s have had it one way and Year 7s have been able to take it further and that's been really interesting.

Knowledge of Content and Students (KCS) and Knowledge of Content and Curriculum (KCC) was referred to by G2-R13:

> …another part that I found very valuable was as we were doing an activity you were giving examples of where the kids would have trouble, they might possible have an answer like this, they might possibly have an answer like that, which I found really helpful because you can give an activity and they come back with an answer and I will just look at it thinking how on earth did you get there, so it's really good for when you're giving these examples I am able to say oh ok they've done this or they've done that or whatever else so you know where their difficulty is, what to hone in on, where to work on with them. (KCS)

G1-R8 and G2-R15 provided answers which indicated Knowledge of Content and Teaching (KCT) and Specialised Content Knowledge (SCK):

> G1-R8: Yes I definitely think I am better at promoting learning in fractions because of all the activities that we have been through and it just makes it easier for me to explain to the children by using the activities and the concrete materials.
>
> G2-R15: I think probably personally the pedagogy has been really good that whole, like the big ideas and things yeah make a good connection for me, I think content wise I mean as I said before I kind of get fractions at a level that I think is acceptable but still need, you know, there were still probably some holes there, or maybe even some more kind of reminders that I needed you sort of you know get out of practice with some things.

On the other hand, G2-R10 suggested Knowledge of Content and Curriculum (KCC) and Specialised Content Knowledge (SCK):

> Oh most definitely, equipped with more knowledge, equipped with more ways of teaching so that I know that I can attend to the needs of all the children not just the ones who can pick it up straight away. I guess the more hands on and the more reinforcing the basics and the place value and the language as well so that the children in my Year Four class know that my specific language means something very specific so when they can answer me they know that if they use that same language I get that they understand what they are talking about.

G2-R7 mentioned a combination of Knowledge of Content and Curriculum (KCC) and Knowledge of Content and Teaching (KCT):

> I think probably the pedagogy, because the activities have always been out there but its knowing why you are doing them and when you should do them and in what order you should do them, I think primarily being more aware of the sequence in which you would present the concepts and lots of extra activities that I can now link to that sequence.

In order to enact G2-R13's conversation, a mixture of Knowledge of Content and Curriculum (KCC), Specialised Content Knowledge (SCK) and Knowledge of Content and Students (KCS) would be required:

> For me I think it was probably the pedagogy, just learning I suppose what to teach and the order in which to teach it to make sure that I am not going too far ahead and then coming a step back but it's all taught in a sequential order …I was picturing in my class and I would say OK these 15 kids I know will have no problems, these ones I could see having a bit of trouble…how for these three kids how can I modify it to make it a bit easier for them and these ones will find it really easy...

9.4 *Semi-Structured Interview: Perceptions of problematic areas in teaching fractions*

This question, “If any, what aspect/s do you find most problematic in teaching fractions?” had as its origin in trying to further explore the area of confidence and ability in teaching fractions but with an attempt to draw from the participants whether any issues that might have, stemmed from Subject Matter Knowledge (SMK) or Pedagogical Content Knowledge (PCK).

Table 9.5
Respondents perceived problematic areas in teaching fractions

	(n=19)	Number of responses	% of responses
SCK	Content knowledge	5	26.32
	Algorithms	3	15.79
KCT	Explaining using correct language	3	15.79
	Being too algorithmic in pedagogy	1	5.26
KCS	Assessment	1	5.26
KCC	Justifying importance in curriculum	2	10.53
	Nothing problematic	4	21.05

Of the problematic areas (Table 9.5), 36.8% were situated in Pedagogical Content Knowledge (21.05% in Knowledge of Content and Teaching (KCT), 5.26% in Knowledge of Content and Students (KCS) and 10.53% in Knowledge of Content and Curriculum (KCC)) and 42.11% in Subject Matter Knowledge (SMK) with a particular emphasis on Specialised Content Knowledge (SCK). It should be noted that due to the response given it is not possible to determine if the word ‘assessment’ was used in a generic manner (that is, how we assess) or as a more specialised understanding (that is, what we assess). The Researcher has opted for the latter definition. The split between the elements of PCK and SMK was reasonably equal, with the remaining 21.05% of respondents claiming not to have any problematic aspects in the teaching of fractions.

When asked to respond to the question about any problematic areas G2-R17 alluded to her own Specialised Content Knowledge (SCK), and then moved onto considering Knowledge of Content and Students (KCS):

> Your more irregular fractions, in the sense of your odd numbers, bigger numbers as in numerator and denominator for me to get my head round … we have students in year 5 who are at that year 7 level and it's been good just to refresh your memory of how far you can go on to develop the concept.

G1-R1 responded by talking about the challenge of change in her pedagogy:

> I really tell them about the formulas and you know … well this is how you do it, rather than showing them more hands on. So I've just got to retrain myself in just showing them, you know, how we convert the mixed number into an improper fraction rather than it just being a formula.

Respondent G2-R10 discussed the pedagogical concern of Knowledge of Content and Teaching (KCT) in selecting the most appropriate language to describe the representations:

> Making sure my language is at their level, because I enjoy teaching the upper grades I have to catch myself and stop myself from speaking in a way that the upper grades would realise what I mean when I have got the younger ones, so trying to turn what's an obvious thing to me into something that's much more simple for them and to find a few different ways to explain the same thing.
>
> I try to relate everyday language to the fractional language so I think the language is probably the most problematic because it's what we use and assume that they know it so we spent a long time on that and the kids are now starting to use that language, talking about the denominator, not just the number on the bottom, number on the top, so that's been quite powerful.

Respondent G2-R12 was quite forthcoming when speaking about a lack of Specialised Content Knowledge (SCK):

> Multiplying fractions, like generally we do the adding and subtracting but there is always that one kid that says what if we have to multiply, and you go, “Oh why did you say that?” That’s when my brain just deflates and I just go “Oh we don’t really have to do that?” and I kind of bite the bullet and go all right, well I am not 100% sure on how to do it and that’s because I’ve never really had to in a sense but I want to be prepared…the concept is quite daunting for me.

Respondent G2-R13 delved into the heart of PCK, that being, how do we take the knowledge we have as teachers and transform it in such a way that will facilitate students’ learning?

> I think my problem is if I had a fractions problem that I needed to solve I am the sort of person who will just sit there and do all sorts of things and play around and work out an answer but then my problem is yes I’ve got an answer how do I teach the kids how to do that, I don’t know the process properly that I went through to get the answer …

Of the four respondents (G1-R9, G2-R7, G2-R14 and G2-R15) who perceived they did not have any areas they found problematic, three applied ratings to themselves between eight and 9.5 out of 10 for confidence and eight and 8.5 out of 10 for ability. The fourth person rated themselves as “better than “fairly confident” and “better than pretty good” regarding their ability to teach fractions (Table 9.6). One could argue that these ratings do not reflect people who are totally convinced that they possess no areas of weakness. One could equally well argue that this is an illustration of the reticence of people to over-estimate their capabilities.

Table 9.6

Respondents who reported no problematic areas in teaching fractions and their confidence (Q1) and perceived ability (Q2) in teaching fractions

	Q1		Q2	
Respondent	Pre-intervention	Post-intervention	Pre-intervention	Post-intervention
G1-R9	fairly	better	pretty good	better
G2-R14	8	9.5	7	8.5
G2-R15	5	8	4	8.5
G2-R7	6	8	7	8

Key Finding 9.3:

Problematic areas in the teaching of fractions stem from issues with Pedagogical Content Knowledge and Specialised Content Knowledge. Therefore PL must be constructed which recognises the need to have *both* of these elements addressed.

9.5 *Semi-Structured Interview: Perceptions of capacity to take leadership in the teaching of fractions*

Semi-structured interview Question 16 involved the respondents answering the question, "If you were now asked to take a 'leadership' role in the teaching of fractions in your school do you feel more confident now, as opposed to when you began this PL, that you could do so? If so, what has lead to this increased confidence?" Seventeen of the 19 (89.47%) responses indicated that at the conclusion of the PL, the teachers felt more confident to take on a leadership role in their schools.

Table 9.7

Respondents perceived ability to take on a leadership role in the teaching of fractions

(n=19)	Number of responses	% of responses
No	2	10.53
Yes	17	89.47

This question was an attempt to try and ascertain what the triggers for any increased confidence might be. The question was constructed around taking a leadership role in school, to position the respondents to make their decision about confidence in regards to a situation which many teachers would find stressful or which might place them at their most vulnerable.

G1-R5 was asked what had increased her confidence, “Just having success in the classroom has lead to that. Having personal success in PD, so knowing what you are doing and you know, having it just make sense and knowing that you know the content well.” G2-R17 talked about the need for the professional learning to provide activities which can translate straight back into the classroom and therefore gives confidence that the teacher has something worthwhile to share with other staff members:

> And I think as a teacher you know that what you want to walk away with from a PD is something that you can take into your class the next day and use and each time … I think I have got so many games and resources that I wasn’t aware of before that I could go and give to others now and I think they would be appreciative because quite often you get a PD in the afternoon and you stand and listen to one of your colleagues talk for an hour when you really want a cup of coffee, and you’ve got a stack of marking but I really think this is something, its hands on you can use it in the classroom it’s not just something to numb your brain.

Having a set of activities to take back to a school also featured in the conversations with G1-R1, G1-R4, G1- R7, G1-R9, G2 R1, G2-R4 and G2-R11.

When asked if she felt more confident to take on a leadership role G2-R15 replied with a comprehensive answer which talked about the qualities required of PL:

> I do get approached by staff on how to teach those *(fractions)* and actually I have referred them to some of the things I have done on the fraction course, I also feel confident to help people plan to teach fractions and I think that the big part of that is down to having more content and content that I trust …when you come to a PD that is run by AISWA *(the association who organised the PL)*, run by people that you’ve heard of who are well thought

> of it gives you the confidence to say, "Right well I will do that" and anything that you do that comes out of that you feel has credibility because it's based on good pedagogy so I would feel more confident after this PD. It's the content knowledge and the fact that I trust it the fact that I value what we are doing here because it's based on sound research it's presented by people well regarded in the field.

G2-R12 attributed a confidence in taking a leadership role through having been provided with appropriate pedagogical tools and representations and the opportunity to reflect on those activities through engaging in them:

> The amount of hands on stuff that we have done, I have never been to a PD in my life where I have had to do that much hands on, getting on the kids level and playing the games and finding out all the little bugs and strategies and that I have never had that sort of, to me that opens your eyes to what else is out there.

G2-R7 concentrated on Knowledge of Content and Students (KCS) in explaining her confidence to lead. She said: "Well I think just greater understanding of how children learn about fractions, about how they develop understanding."

The two teachers (G1-R8 and G2-R10) who claimed not to be confident in taking on a leadership role, both cited the reason of being inexperienced in terms of length of service as the impediment. Neither suggested they were ill-equipped for the responsibility.

9.6 *Semi-Structured Interview: Benefits of attendance at PL*

Somewhat surprisingly, although semi-structured interview Question 17 asked the participants to reflect upon the question, "What were you hoping to gain from attendance in this PL and was that gain achieved?", the reply furnished by respondent G2-R10 saw her answer this question through reflecting on her own growing level of confidence:

> Did it exceed my expectations, definitely! Because the minute I walked into the door and sort of saw what a small number of people were here I thought oh! I felt like walking out. Because this is very confronting because I am aware of my own lack of ability and I know that there are people in there who are very mathematically minded, whereas I am not a mathematical person. I don't think mathematically at all and so to sit in there in a small group on the first day made me feel like I'd better keep my mouth closed and not show how foolish I am with working things out. But at the end of the day it has been a good thing because I have had questions answered and my bad ideas are shown for what they are and you sort of have to put your pride in your pocket and realise that you are here to learn. You know, not to show what you know, but to get in, and so in that respect I am very thankful to have come, yeah… I am rapt really.

This was echoed in a much more succinct manner by G1-R4 who said: "Just more confidence in it really I think, yeah, so yeah."

Key Finding 9.4: Although the attitudes and beliefs questionnaires did not show a statistically significant impact of the PL on the teachers' beliefs and attitudes towards mathematics, the comments collected during the semi-structured interview would support that the PL has had some positive impacts on teachers' confidence in teaching mathematics.

9.7 ***Summary***

Although the data collected through the attitudes and beliefs questionnaires did not show a statistically significant impact of the PL, the comments collected during the semi-structured interview would support the idea that the PL has had some positive impacts on the teachers' confidence for teaching mathematics. When considered in the light of the available literature, a positive attitude and belief are extremely important, for the development of effective teachers of this difficult topic. This will be more deeply explored in Chapter 11.

The following chapter contains data collected from questions from the semi-structured interviews that did not fit neatly under any of the research questions, but nevertheless informed all of them. Also contained within this chapter are the results that were gained through the application of an exit questionnaire regarding the participants' perception as to whether the PL displayed elements of effective PL. The contention here, borne out by the literature, is that effective PL can make a positive difference (Desimone et al., 2002; Hill et al., 2008a; Penuel et al., 2007; Yoon et al., 2007; Zambo & Zambo, 2008) to teaching and learning and so the perceptions of the worth of the PL are important.

CHAPTER 10

10 Teachers' Perceptions of the Effectiveness of the Professional Learning Program

The following questions from the semi-structured interviews were worded in such a way that they did not fit neatly under any of the research questions, but nevertheless it was felt that they informed all of them. Also contained within this chapter are the results that were gained through the application of an exit questionnaire regarding the participants' perception as to whether the PL displayed elements of effective PL. There is also reference to some other invited reflections.

10.1 *Semi-Structured Interview: The relative importance of pedagogy, content and reflective practice*

This study was constructed by the Researcher around three major elements of the PL: pedagogy, content and reflective practice. Question six asked the participants to consider which of these had been the most important to them in their development as mathematics teachers. They were then asked to expand upon their answers.

It was never explicitly expressed in one encompassing statement that all three elements of content, pedagogy and reflection were seen as integral and of equal importance. At varying times during the PL each element's importance was highlighted. It was assumed, erroneously as it transpired, that each element would be seen as equally crucial by the participants.

When the results were collated content and pedagogy were both mentioned 12 times (Table 10.1) but the reflective element was only mentioned three times. Three teachers named all three components as being important, 14 named one component.

The wording of the statement to open the discussion may have lead some of the respondents to think that only one, the most important *one*, was required. The wording of, "There are 3 major elements in this PL, which *has* been the most important to your development as a mathematics teacher and why?" perhaps should have been, "There are 3 major elements in this PL, which *have* been the most important to your development as a mathematics teacher and why?"

A few of the teachers who expressed pedagogy as their only choice inadvertently referred to content as illustrated by comments recorded for G2-R13:

> For me it was probably the pedagogy, just learning I suppose what to teach *(content)* and the order in which to teach it to make sure that I'm not going too far ahead and then coming a step back, but it's all taught in a sequential order.

It is not unwelcome to the Researcher, that this respondent and others see the two elements as irrevocably tied.

Another G2-R16 did not use the word pedagogy but gave a defining example:

> …helped me realise these kids can get it right on paper because they've done the exercises, they've coloured this, they've matched that, they've ruled a line from this to this on the paper for their assessment. They look like they understand but they don't…I can have my class all doing an activity that is hands-on learning and I and I can pull one child away for two minutes and I can talk to them, and it's in the talking that I realise not only don't you get it, but I can see at what point you don't get it and that's what I have found really important, and I wouldn't have done so much hands on if I hadn't come to this PD.

10.2 ***Semi-Structured Interview: Revealing PL activities***

The respondents were asked to consider the activities from the PL which had been most revealing to them and to indicate why this was so (Table 10.1). There was always an element of ambiguity in using the word revealing in this question, something that this

Researcher wanted to exploit. Rather than use the word challenging or interesting or difficult it was thought that by using the word revealing the participants would not be lead by the Researcher, and therefore give an almost unconscious reflection of the their thinking and thinking processes.

Table 10.1
Most revealing activity engaged in during professional learning (n=27)

	Number of responses	% of responses
Fraction bridge	7	25.93
Fraction strips	5	18.52
Paper folding	3	11.11
Pattern Blocks	2	7.41
Chocolate frogs	2	7.41
Number lines	2	7.41
Tangrams	1	3.70
Cuisenaire fractions	1	3.70
Fraction walls	1	3.70
Concept maps	1	3.70
Hundreds squares	1	3.70
All activities	1	3.70

Whilst it is to be expected that there would be a spread of activities which were considered revealing, the fact that the Fraction Bridge was mentioned seven times is perhaps the least surprising to the Researcher. During its completion this was considered by many to be the most challenging of the activities, the one where more clarification was required as the activity unfolded and the one which created the most conversation about its application to the classroom setting.

G1-R4 expressed that:

> …I just couldn't work out how to fold it, and so everyone was done and it just makes you feel really bad, and it's just like students in the classroom really. It's quite frustrating for them as well, which is why I like teaching maths and why I like learning…

This particular activity allowed the respondent to empathise with the struggles that some students have with learning.

G2-R17 commented on how the activity forced them into reflecting upon learning, their own and others:

> I really like the suspension bridge, I learnt a lot about what I could do myself and again my expectations of what others could do and that feeling of I can't do this.

Another respondent G2-R1 found that the activity exercised the students' knowledge of the size of fractions:

> I think the suspension bridge *(fraction bridge)* one because that was really powerful because...when we came to the 7ths we actually had to look at alternative suggestions and again I sort of lead the kids a little bit. Just looking at…well…a 6^{th} and an 8^{th}, where is the 7^{th} going to fit and they said, "It's smaller than a 6^{th} but it will be bigger than the 8^{th}."…because they understand fractions are an equal part but they had to judge how equal they could make it, so that was really powerful…

Question 13 asked the participants to consider which activity from the PL was their favourite and then explain why this was the case. Unlike Question 11 where the respondents were given a question regarding the most revealing activity, purposely intended to be somewhat ambiguous, this question was specifically directed to the most positively received activities, by using the word favourite.

Table 10.2
Favourite activity for teaching fractions (n=23)

Activity	Number of responses	% of responses
Pattern Blocks	5	21.74
Fraction Bridge	4	17.39
Fraction strips	2	8.70
Fraction Dominoes	2	8.70
Number Lines	2	8.70
Chocolate frogs	2	8.70
Fraction wall	2	8.70
Paper folding	2	8.70
Decimal straws	1	4.35
All activities	1	4.35

One respondent (G2-R10) was impressed by all of the activities:

> I think I am just as amazed with each of the activities, because even the simplest ones can reinforce concepts in a really powerful way and I don't think they have to be elaborate. I think that is the other thing, that when you look at the commercial things available to teach anything, the simplest of things can be used to give a powerful representation and get the message across. So I don't think there is any one thing that I have gone wow, because I feel that way about all of it.

It was of interest to this Researcher to determine whether the favourite activities had any sort of relationship with the revealing activities as articulated in Question 11. Eight of the respondents (G1-R1, G1-R5, G1-R7, G2-R1, G2-R4, G2-R11, G2-R13 and G2-R17) chose the most revealing activity as being their favourite activity (Figure 10.1). When talking about their favourite activity all five respondents who chose Pattern Blocks did so because of their availability and the ease of use amongst their students.

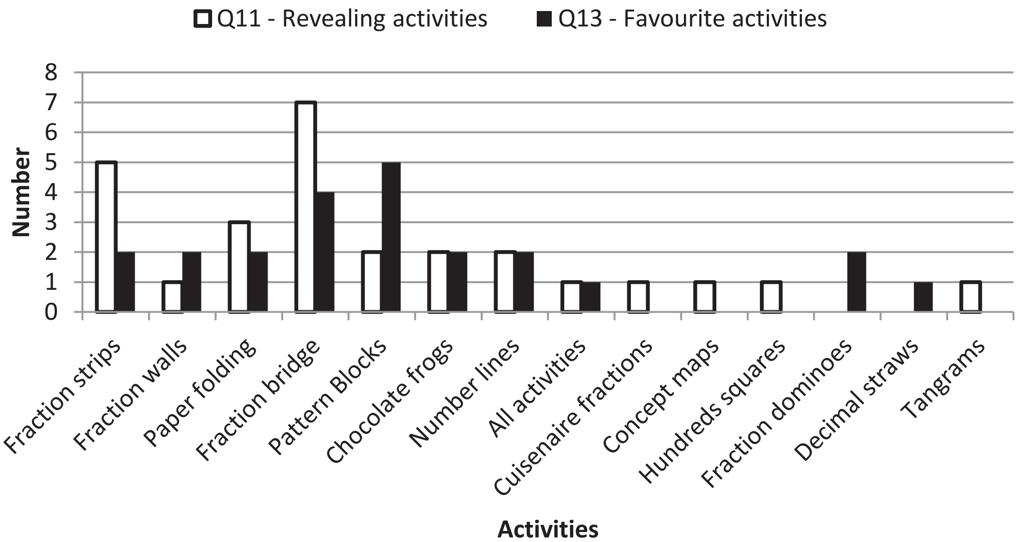


Figure 10.1 A comparison of respondents' perception of revealing activities (Q11) and favourite activities (Q13)

A cursory viewing of these data suggests that activities considered by the teachers as being revealing are not necessarily the ones they selected as being favourites. Conversely, the activities considered as favourites may not indicate that the activity was also considered to be revealing.

10.3 ***Semi-Structured Interview: Teachers' goals for the PL***

Semi-structured interview Question 17 asked the participants to reflect upon the question, "What were you hoping to gain from attendance in this PL and was that gain achieved?" On indicating what their expectations were from attendance at the PL each person gave one response and three people offered a second. Six (30%) of the respondents gave answers which displayed an increase in Content Knowledge and Teaching (KCT) through stressing that that pedagogical issues were key (Table 10.7). Four of the responses (20%) indicated that the participants had no particular expectations of the PL.

Table 10.3
Respondents' expectations of attending PL (n=20)

Expectations	Number of responses	% of responses
Improvement of pedagogy	6	30.00
No particular expectations	4	20.00
To 'teach' fractions better	4	20.00
Affirmation of practice	2	10.00
Whole school improvement	2	10.00
More activities	1	5.00
Improved content knowledge	1	5.00

G2-R1 is one example of the respondents who focused on the shift in pedagogy (KCT) in adopting activities which on the whole employed manipulative materials and in a variety of representations:

> I was really hoping to get practical ideas to put into play. Because again, it's the practical stuff and again I can't say enough about the suspension bridge activity, because I would never ever in a million years have thought of it. But it's just been fantastic and the kids enjoyed it and they were engaged and it took us a couple of sessions but I reckon that that cut out maybe 6 sessions of just ploughing through, "What does this mean, what does that mean?", so it's definitely the content has been huge and also pedagogy, those are my main things that I have really valued.

Some of the responses indicated that expectations had been met, but the expectations were not clearly articulated and statements such as "teach fractions better or effectively" (G2-R11) and "expand my teaching repertoire" (G1-R5) were employed. G2-R7, who also mentioned teaching fractions better, clarified that statement somewhat by adding "…so how to teach it so that more children felt confident about it because I don't think it's an area where a lot of kids in general feel particularly confident."

One of the participants G1-R7 attended the PL with a particular intention in mind, in that she had arranged the PL for that group. This respondent's expectation was a broader

expectation for the staff at her school, and some capacity building of her own to act in a support role at the conclusion of the PL:

> My aim was to equip the eight teachers to be able to teach, to have the knowledge themselves because they can't teach what they don't know; you can't go where you haven't been yourself. So that was my objective of the whole thing, my objective *(of)* being there was to support them, so that then I can encourage and follow up afterwards and yes I think it's made a big difference. I don't know about all the teachers because I haven't heard their feedback but certainly for some teachers it's been significant to their own learning but also their ability to then teach and to watch students really, really learn well.

The second part of the question regarding whether the expectations were in fact met saw all of the respondents answer in the affirmative. The implications of this response are further explored in the discussion chapter.

10.4 ***Exit Questionnaire: Effective professional learning***

At the conclusion of the Professional Leaning sessions the participants were asked to respond to 19 statements on a three point Likert Scale (Yes, Undecided, No) regarding the question: "Do you believe the PL addressed the following characteristics of effective professional development?" (Table 10.4).

Table 10.4

Answers to the question stem: Do you believe the PL addressed the following characteristics of effective professional development?

Numb.		**No** (Response)	**Undecided** (Response)	**Yes** (Response)	**Mean score**
1	Focused on increasing knowledge and skills to bring about change in my teaching practice	0	0	19	2.00
2	Recognised the ways adults learn, and the impact of constructivist learning theory	0	0	19	2.00
6	Was centred on the improvement of student achievement and growth	0	0	19	2.00
9	Included follow up and support	0	0	19	2.00
15	Increased teacher knowledge and understanding	0	0	19	2.00
16	Was purposeful, sustained and sustainable over time	0	0	19	2.00
10	Was ongoing and job embedded	0	1	18	1.95
12	Was centred on the development and maintenance of collaborative environments	0	1	18	1.95
14	Recognised multiple contexts, formats and factors	0	1	18	1.95
3	Accommodated diversity and promoted equity in schools	0	2	17	1.89
5	Involved the formation of learning communities	0	2	17	1.89
8	Focused on individual and organisational improvement	0	2	17	1.89
7	Addressed issues and concerns and interest identified by the teachers	0	3	16	1.84
13	Emphasised, and made choices informed by, the link between teacher quality and student success	0	3	16	1.84
17	Was based on the best available research evidence	0	3	16	1.84
18	Was driven by analyses of student learning data	0	3	16	1.84
11	Allowed time and opportunity for planning, reflection and feedback	1	3	15	1.74
19	Assessed the impact of initiatives and decisions on student outcomes	0	4	15	1.79
4	Promoted the development of leadership capacity	0	11	8	1.42

All of the statements enjoyed in excess of 80% support, that the characteristic of effective PL were addressed, with only one below 90% (Statement 4, six between 90 and 95% (Statements 7, 11, 13, 17, 18 and 19) and the remaining 12 in excess of 95%. Six of the Statements (1, 2, 9, 12, 15 and 16) drew the response of yes from all of the respondents.

Of the statements, the characteristic of effective PL which fewest participants felt was addressed was Statement four (analysed by assigning a numerical value of two to each response of yes, one to each response of undecided and zero marks for no). The statement, “Promoted the development of leadership capacity” scored 27 points from a possible 38 (80.7%), and was followed by Statement 11, “Allowed time and opportunity for planning, reflection and feedback” with 52 (91.12%).

The least convinced participants regarding the effectiveness of the PL in covering the characteristics (Figure 10.2) were G1-R2 and G1-R8, who both gave scores of 34 out of a possible of 38 (89.47%). Three of the respondents (G1-R1, G2-R1 and G2-R7) indicated that they perceived that the PL had accomplished all of the characteristics of effective PL.

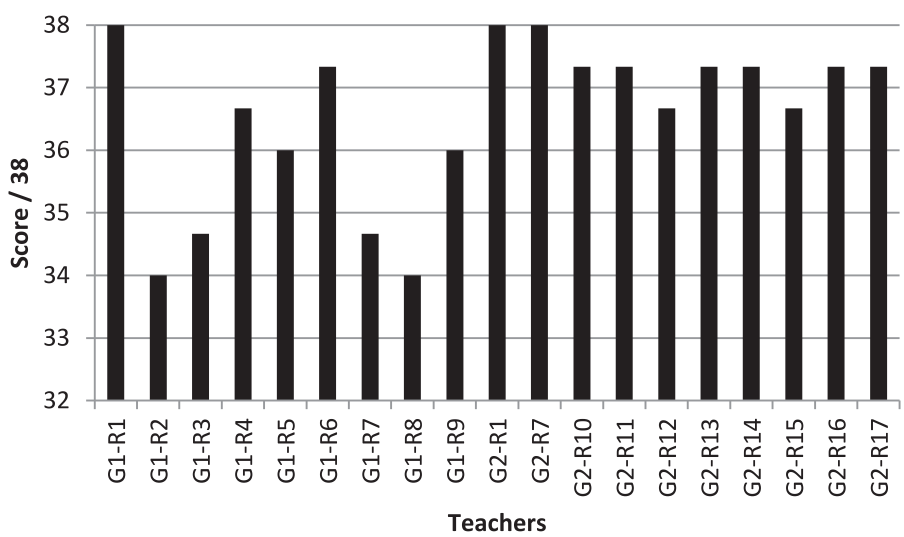


Figure 10.2 Scores for responses to Exit Questionnaire by respondent

Key Finding 10.1:
The teachers gave very positive responses to the exit questionnaire that indicated that the PL addressed 19 characteristics of effective PL.

10.5 ***Other Invited Reflections.***

Seven of the 20 participants (35%) chose not to supply the Researcher with any reflection other that the semi-structured reflection completed during the first professional learning session. Ten participants took one opportunity to submit their reflections (65%) and none of the participants provided more than one reflection. Although the invited reflections may have helped to encourage respondents to believe that there was a further line of communication open to them, few of them took up this opportunity. Of all of the data collection methods applied in this study, the reflection was seen by this Researcher as being the least informative or useful, although a few of the comments were confirmatory in nature.

G2-R4 wrote a fairly lengthy reflection which centred around the development of Specialised Content Knowledge (SCK) and Knowledge of Content and Teaching (KCT) and also took the opportunity to write about her own increased confidence in teaching fractions. This respondent displayed her Specialised Content Knowledge (SCK) through being able to articulate some key fraction concepts. "They clearly were working on a doubling and halving concept...understood the multiplicative structure of fractions…During this process we were constantly emphasising equal parts…"

G2-R4 also made comments with regards to Knowledge of Content and Teaching (KCT) with particular reference to changes in pedagogy:

> This PL has expanded my awareness of the range of activities I can do with the children in teaching fractions, which gives me greater confidence that I can meet the learning needs of students… I realise that I need to work on making my teaching more concrete, create more 'problems' for students to solve in a kinaesthetic way…using the concrete materials really helped them visualise the fractions as part of a whole.

They further expanded on the growth of their own confidence with statements such as:

> I have always dreaded teaching fractions as I always considered them to be difficult. These activities make it easier for me to explain the different fractions and how they relate to each other" and "I feel as if my students are already more successful in working with fractions than previous students.

G2-R1 raised the important issue of employing multiple representations; "It has been exciting to have such concrete and visual examples, focusing on forming and reinforcing concepts. I like that there's been an emphasis on looking at the same idea in a number of ways."

G2-R3 revealed a growing understanding, stemming from experiences gained in the PL, of the need to use manipulative materials not just for demonstration but as a tool for students to explore the mathematics:

> I had been introduced to the *(fraction)* strips before but it was already made and was placed on the board as a visual for us to see and observe and identify equivalent fractions. As a result it did not help in my understanding. Today by actually cutting out the strips, folding them into equal parts etc., etc. I have found it to be more interesting, engaging and truly understand it.

They went on to comment about their level of confidence with teaching fractions by writing, " Definitely my conceptions about teaching fractions has (sic) been challenged and I feel more optimistic already about imparting my knowledge with the kids."

G2-R16 actually employed the word fear to describe her relationship with fractions and explained how this fear can be ameliorated through well planned PL: "My fear of fractions is being addressed because I can implement meaningful activities and allay any parent concerns by showing the depth of thinking that is taking place."

10.6 ***Summary***

This chapter explored the teachers' perceptions of the important elements of the PL; which activities they found the most revealing and which were their favourites; what aspects of the teaching of fractions they felt most challenging; and what they had hoped to gain through attendance at the PL and if that had been attained. Not all PL can be run with a timeframe as generous as that available for this study; therefore it was important to determine the activities to make the focus should the timeframe be more restricted.

Also contained within this chapter were the results of the exit questionnaire. This questionnaire was constructed around the literature as to what constitutes effective PD for teachers (Clarke, 2003; Cohen & Hill, 2000; Loucks-Horsley, Hewson, Love & Stile, 1998; Supovitz & Turner, 2000; Supovitz, Mayer & Kahle, 2000; Zigarmi, Betz and Jensens, 1977). This was deemed important, as, if teachers perceive worth in the PL it will be more likely to affect their practice.

The conclusions drawn from the data displayed in this chapter will be discussed in greater detail in the next chapter, the Discussion chapter.

CHAPTER 11

11 Discussion

This discussion chapter is constructed around the key findings (K.F.) deriving from analysis of the reported data in Chapters 6 to 10. These key findings have been grouped according to the themes of: current status of teaching fractions; teachers' mathematical knowledge for teaching fractions; pedagogical content knowledge, teachers' self-efficacy, beliefs, attitudes and confidence; and attributes of effective professional learning. Although the key findings have been grouped thus, the inter-connectedness of all of the themes is worthy of comment. On many occasions the teachers' responses made reference to more than one theme.

11.1 *The Teachers' Perceptions Regarding the Status of Teaching Fractions*

The research (Bailey, Hoard, Nugent & Geary, 2012; Brown & Quinn, 2007; Chinnappan, 2005; Fazio & Siegler, 2011; National Curriculum Board, 2009; Nunes & Bryant, 2009; Siegler et al., 2012) is unequivocal regarding the importance of the teaching and learning of fractions. It is important as fractions: are a core component of being numerate (Siemon, 2003); are an integral part of understanding division (Nunes & Bryant, 2009); provide an insight into children's' understanding of numbers and number operations (Chinnappan, 2005); impact upon the strands of measurement and space (Pitkethley & Hunting, 1996); and are necessary for achieving success in algebra (Brown & Quinn, 2007). As such, teachers and schools should ensure that fractions receive the attention they deserve. Teachers are constantly making decisions on how to best utilise the limited teaching time available to them and consequently seek guidance from the school setting and the curriculum documents about what needs priority.

11.1.1 The Teachers' perception of the importance of fractions

The semi-structured interviews showed that the teachers' perception of the importance of fractions at the start of the PL was quite high (K.F. 6.3a) and at the conclusion of the PL it was deemed by them as even more important (K.F. 6.3b). As the teachers developed a deeper knowledge of fractions they may have recognised the implications with regards to learning other concepts.

Although teachers can be fairly autonomous within their own classrooms, school plans and curriculum priorities can shape the amount of time and effort that goes towards the teaching and learning of any curriculum area. Therefore it is interesting to compare and contrast the importance of the topic of fractions from the teachers' perspective and their perception of the status of fractions in the curriculum at their schools. One could assume that attendance at a professional learning program which was advertised as being about fractions, indicated that the schools had recognised the importance of the topic, and that their teachers would benefit from attendance. It could, however, be the case that the teachers' attendance at the PL was supported by the schools because the sessions were based on the generic subject of mathematics rather than the more specific topic of fractions.

11.1.2 The Teachers' perception of the status of fractions in their schools and the curriculum

When interviewed at the conclusion of the PL about their perceptions of the importance of fractions in their school, the teachers found difficulty in categorically stating that fractions enjoy a high priority (K.F. 6.4). In some cases (eight in total) the responses were answered with qualifying words and phrases such as "I *guess* (emphasis added) it's important." This raises the interesting issue as to why, if schools *do* (emphasis added) feel fractions are important, and wish to affect whole school practice, this importance is not recognised, articulated and supported back in the school setting?

The responses from a group of teachers from the same school were ambiguous. One teacher from the school used the qualifying word pretty, twice in one sentence:

> I think it holds a *pretty* important one because I think they've realised that fractions is weak across the board with most students, they have trouble grasping fractions and being able to understand what fractions are, so I think the school is trying to put more of our focus onto fractions and fractions work so I think it is *pretty* important for the school.

Some teachers spoke of their belief that a topic such as fractions may have more importance in some phases of learning in a school than in others. There was a prevailing belief that fractions are more important in the middle and later years of primary school than the earlier years. Anecdotal evidence suggested that text books and the NAPLAN testing required the students to operate with fractions in these phases drove this belief.

Amongst the participants, one group of teachers was required to attend the PL (G1) and the other teachers (G2) were volunteers or self-selecting. The pattern of responses was similar in both groups, in that after the intervention, the status of fractions was seen to be raised. However, the original judgment about the status of fractions was initially somewhat higher in the latter group. It is likely to assume that the volunteer group attended the PL because they realized the importance of enhancing their teaching of this topic.

As stated earlier, the data suggest that being involved in well-structured PL about fractions increases the perception of the status of fractions (K.F. 6.3b). Given that there is a clear link between what the curriculum documents' focus on as being important, and consequently, the focus required of the teacher, there appears to be a dislocation in the level of focus between the two. The teachers perceived a lower level of status in the documents (K.F. 6.5) than they personally accorded it. When asked the question, "What kind of status or importance do you think the topic of fractions holds in the WA (Western Australian) curriculum?" nearly half of the teachers were surprisingly unsure as to the importance fractions held as expressed in the Curriculum Framework documents, the only mandated document in Western Australia at the time of the study.

Eight out of the nine teachers who were unsure came from G2, and this could suggest that the whole school approach taken to engage the members of G1 had made them more aware of the place of fractions in the Curriculum Framework, and consequently of its perceived importance as part of the curriculum. This resonates with Coburn's (2003) assertion that any classroom is inextricably linked to the broader school and education system and that a teacher's personal view is likely to be broadened by systemic concerns and priorities.

When asked about the topic of fractions held in the WA curriculum, G2-R10 expressed a view which was repeated in various forms by other teachers. The comments all contained elements of vagueness, which suggested that either the teachers were not overly familiar with the documents, or that the manner in which the documents alluded to fractions was unclear:

> Yeah, it's not specific enough in the Curriculum Framework I don't think, it certainly has its place in the Framework I am not saying it's not there, I don't know that I can really answer that question though.

One teacher expressed this vagueness slightly differently, as well as alluding to both the syllabus and the curriculum documents, documents which in the Western Australian context had markedly different purposes. Regardless, a knowledgeable reference to either of these documents would have been considered advantageous. G1-R2 stated:

> Oh, erm, the WA curriculum, yeah, it's not something I really know a lot about because it's something that keeps coming in and out, we're using it, we're not using it and now we're using syllabus and it's like, but yeah it's definitely, it's part of our syllabus and it holds an importance, if it wasn't important it wouldn't be there, but maybe it's not taught as much as other concepts are taught, it could kind of be integrated into like other concepts as well like I was saying before.

G2-R2 vacillated as to its importance with the sentence; "Probably quite a lot, I mean that's my opinion I guess, I don't know."

If familiarity with the topic of fractions, as represented in the curriculum documentation, can be used as a measure of the importance of a particular mathematics topic, from these teachers' point of view at least, there seems to be dislocation. This dislocation appears to be

between their perceptions of the status fractions has in the documentation as opposed to the importance that the system places upon it. There are two possible explanations for this dislocation: either the teachers are unfamiliar with the documents or the documents do not provide adequate guidance as to the importance of fractions.

If the teachers in this study are representative, then there does not seem to be any cognisance paid to a considerable body of research which promotes fractions as being important to teach and learn (Brown & Quinn, 2007; Chinnappan, 2005; Fazio & Siegler, 2011; National Curriculum Board, 2009; Nunes & Bryant, 2009; Siegler et al., 2012). This particular research has prompted the responsible agencies to include fractions as being worthy of a place in the Australia-wide test in numeracy, the National Assessment Program – Literacy and Numeracy (NAPLAN). NAPLAN, in the interest of manageability of the assessment tool, is very selective about the concepts it includes, featuring only those that it considers to be of greatest importance.

These data were collected when the teachers were using the Western Australian Curriculum documents. The Researcher would posit that the new Australian Curriculum, which has been produced to guide the delivery of the content and implicitly the pedagogy, offers no more support to teachers than did the previous state based documents in Western Australia. Consequently there is no reason to assume that in future, the perception of teachers will in any way be different.

Assertion 1: Teachers hold the teaching and learning of fractions as being important (K.F. 6.3a) but this status can be further raised through attendance at well-structured PL (K.F. 6.3b). They perceive that the importance of fractions has not been made explicit by their schools (K.F. 6.4). This perception further extends to the education system in which they teach, through the lack of explicit guidance in the curriculum documents (K.F. 6.5).

11.2 ***Mathematical Knowledge for Teaching (MKT)***

A second theme to emerge from the collected data was that of Mathematical Knowledge for Teaching (MKT). As stated in Chapter 2, the research argues that MKT is essential for effective teaching and learning of mathematics and teachers who lack subject specific knowledge will be less effective teachers (Ball et al., 2008; Cobb & Jackson, 2011; Hill, Rowan & Ball, 2005; Hill et al., 2008b; Toluk-Uçar, 2009). Although no research question was proposed to directly ascertain the growth of MKT, it is a natural extension from asking questions about the promotion of pedagogy and content knowledge through well-structured PL.

11.2.1 Improving MKT to increase student attainment in fractions

In a prior section the argument was made that a good understanding of fractions is critical for students' general mathematical education (Brown & Quinn, 2007; Chinnappan, 2005; Fazio & Siegler, 2011; National Curriculum Board, 2009; Nunes & Bryant, 2009; Siegler et al., 2012). It was further argued that teachers are constantly making decisions on how best to utilise the limited teaching time available to them, and that if fractions are not seen as being important, then in a mathematics curriculum that is already viewed as crowded fractions will not be afforded the time they require. Albeit in the science learning area, Hackling, Peers and Prain (2007) offer a pathway (Figure 11.1) to increased teaching time and consequently increased achievement through improving teacher knowledge.

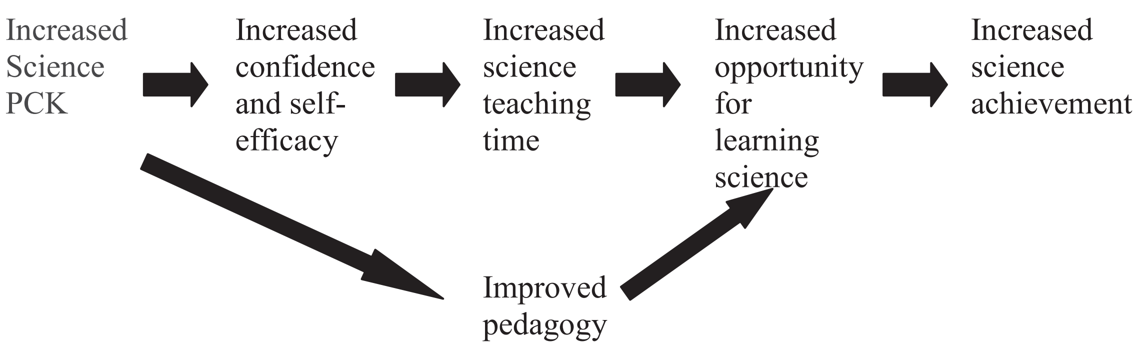


Figure 11.1 Pathway to increased achievement (Hackling, Peers & Prain, 2007)

11.2.2 Teachers' fraction knowledge prior to the PL

This research acknowledges that mathematical knowledge for teaching goes beyond that which can be captured and measured by examining mathematics competencies, as these measures are inadequate (Hill, Rowan & Ball, 2005). Further, a balance is needed between conceptual and procedural knowledge, and these need to be integrated so that teachers are able to create a framework of mathematical ideas for their students to follow (Forrester & Chinnappan, 2010; Meaney & Lange, 2010; Silverman & Thompson, 2008).

Whilst acknowledging this and agreeing that using competency measures are not adequate, this Researcher believes them to be a useful and convenient tool if applied with other measures. Therefore, prior to the PL intervention, to ascertain the teachers' fraction content knowledge, it was decided to administer the Fraction Knowledge Assessment Tool (FKAT) (Appendix 3) with Group 1 (G1), the tool which in a different version had been trialed with Pilot Study Group1 (PS1) (see section 1.5.1).

Across the items in the assessment the teachers in G1 showed slightly better understanding than the pre-service teachers from PS1. A Wilcoxon Signed-Ranks Test showed a significant difference between the two groups in favour of G1. This is a result which one would expect as this group is comprised of practising teachers, as opposed to PS1 who were pre-service teachers. However, the interview and the Rational Number Interview (RNI) showed that the capacity to score well in a written assessment on fractions did not necessarily correspond with either a positive attitude towards fractions or a perceived capacity to teach them (K.F. 6.2).

When the FKAT was completed by the teachers, two of them anecdotally acknowledged that they felt they had serious deficits in their content knowledge when it came to teaching fractions (K.F. 6.6). It should be noted that these teachers were not the same teachers described earlier in this chapter, those who expressed concerns about their Specialised Content Knowledge (SCK) of fractions. It is not the intent of the Researcher to assert from this that the remainder of the teaching population would have the same concern with their fractions knowledge, but it does demand some pause, particularly as this is a second

instance where content knowledge has been shown or perceived to be lacking at a required level. The content knowledge that was required to complete the FKAT assessment only involved conceptual understandings of common fractions and did not ask the teachers to also engage in operating with fractions. None of the knowledge necessary to complete the assessment was, according to currently available curriculum documents (ACARA, 2012) outside that required in a primary school setting.

The FKAT was perceived by this Researcher to have shortcomings in that even in its abbreviated form, the tool had too many repetitive questions which were not really insightful and tended to test the patience of the teachers rather than their conceptual content knowledge of fractions. Therefore, at the commencement of the PL for Group 2 (G2), the Rational Number Interview (RNI) was employed to ascertain their conceptual knowledge of fractions. The RNI was conducted between the first and the second professional development sessions at the workplace of the teachers and at a time of their choosing. It was never intended that the RNI would be used as a pre- and post-test. The interview was only used to try to gain some understanding of the level of the teachers' competence and content understanding with common fractions, and not to try to determine the growth of subject content knowledge.

Without looking at the reasons behind the incorrect responses, it was noted that none of the teachers scored every item correct in the interview (K.F. 6.7). One teacher answered more than one quarter of all the questions incorrectly and five of the teachers had more than 10% of errors.

As first reported in section 6.6, the item which proved most difficult for the teachers was Question 8, a task designed to ascertain understanding of the size of fractions. The teachers were given cards with the numerals 1, 3, 4, 5, 6 and 7 on them. They were also given an empty fraction sum template. They were then given the instruction to choose from the numbers to form two fractions that when added together were close to one, but not equal to one (Figure 11.2). There was no time limit for the completion of this task.

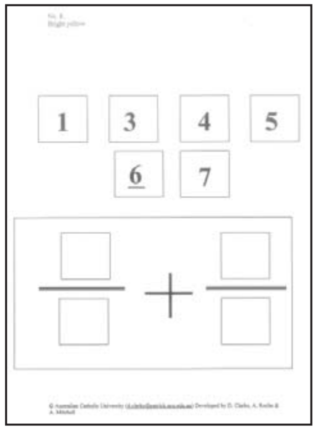


Figure 11.2 RNI Question 8

Nine (60%) of the teachers failed to answer within ± 0.1 of the best response, the limit determined by Clarke and Roche (2009) as being acceptably close to the optimum answer. None of the teachers arrived at the optimal solution ($^1/_7 + ^5/_6$). It should be noted that some of the anecdotal comments offered by the teachers regarding this task were centred on not taking the time needed to exhaust all possibilities. But, rather the teachers adopting an attitude whereby, once an answer was achieved that was *relatively* close, no further efforts were made. The necessary question that could be asked here is if the attitude of being close enough was more about a lack of application or about an inability to complete the task properly.

In completing the RNI errors were made by the teachers in five areas of what Clarke and Roche (2009) call "Big Ideas" (p. 130), part-whole, operations, relative size, fractions as division, and fractions as numbers. One of the questions which particularly interested this Researcher was Question 14. This was a question which allowed for some detailed analysis of the teachers' thinking about fractions. The teachers were asked to identify the larger fraction in eight sets of two fractions and to explain the strategy they used to reach their decision. No incorrect responses were recorded for five of the eight questions and in order to determine the larger of two fractions, most teachers were utilising methods which fall into the most preferred or preferred categories as defined by Clarke and Roche (2009).

It is worth keeping in mind that this assessment was constructed to test the understanding of primary school students (K.F. 6.7). Therefore, it seems wise to question whether *any* number of incorrect responses is acceptable, and further, if these responses are due to slips,

errors or misconceptions. If they are slips (as defined by Olivier, 1989) then this is not too concerning. If they are misconceptions (as defined by Bell, 1984) then PL based around the further development of both content and pedagogy should be efficacious in addressing these misconceptions and provide a meaningful platform to explore them. If, however, they are errors (as defined by Fong, 1995) then perhaps more of the balance of time of PL for fractions needs to be devoted to Specialised Content Knowledge (SCK) rather than pedagogy. It does indeed seem that there is a case for data-driven professional learning. It may prove to be an unpopular (and unrealistic) desire, but to gain the greatest benefit from PL it may then be necessary at the commencement of each PL to assess teachers on the content knowledge they require for that PL, and then to determine through analysis of that assessment if the teachers have areas of misconception or error and thus shape the PL accordingly.

11.2.3 Development of MKT

As previously stated, research claims that mathematical knowledge is crucial for improving the quality of instruction in classrooms (Ball, Hoover-Thames & Phelps, 2008; Cobb & Jackson, 2011; Hill et al., 2008a, 2008b). Further, the research supports the argument that efforts to improve teachers' mathematical knowledge through content focused professional development will improve both teacher self-efficacy (Swackhamer et al., 2009) and student performance (Ball, Hill & Bass, 2005; Hill, Rowan & Ball, 2005). This study was in part predicated on the aforementioned research, and consequently the PL was designed to support the development of the teachers' content and pedagogical knowledge for teaching fractions.

Given that the contention is that MKT requires attention, the question is raised as to whether PL is an efficacious tool in fulfilling that requirement. Results obtained in this study such as those provided by the concept maps and the interview, indicate that the Specialised Content Knowledge (as defined by Hill et al., 2008a) can be increased through professional learning (K.F. 8.2). Responses from the teachers also suggested that manipulative materials allowed some teachers to improve their Subject Matter Knowledge (SMK) as well as their Pedagogical Content Knowledge (PCK) (K.F. 7.1, K.F. 10.1). In

more than one instance there was a desire by the teachers answering the questions to illustrate the close association between SMK and PCK.

During, and after the PL, when addressing the question of their own perceptions of the importance of fractions, some teachers commented on an increase in their content knowledge and how it would improve their capacity to teach fractions (Knowledge of Content and Teaching (KCT)). Comments were made, both incidentally and in the interview, that suggested that an increase in KCT had raised the confidence (K.F. 6.1) of the teachers to make fractions a more prominent topic in the future (K.F. 6.3b). These are comments which are in concordance with Hackling et al.'s (2007) pathway (Figure 11.1). This increased prominence or status should in the future result in more time, thought and effort being expended on this challenging topic.

Some teachers attempted to articulate the connections between fractions and other areas of the mathematics curriculum and were consequently displaying elements of being teachers with a connectionist orientation (Askew et al., 1997). According to Askew et al. (1997), a teacher with a connectionist orientation is one, who among other things, places an emphasis on establishing conceptual links within mathematics, something that they consider to be intrinsic in becoming an effective teacher of mathematics. This view is supported by others (ACER & Stephens, 2009; Singer, 2007; Wilson et al., 2005).

Research supports the position that mathematical knowledge is crucial for improving the quality of instruction in classrooms, and that efforts to improve teachers' mathematical knowledge through content focused professional development will improve teacher self-efficacy (Swackhamer et al., 2009). Although these positions are well supported they are not unequivocal (Hill, Rowan & Ball, 2005). Hill and Ball (2009) state:

> These studies suggest that, though it would be foolish to say that mathematical knowledge is not important to teaching mathematics, conventional content knowledge seems to be insufficient for skilfully handling the mathematical tasks of teaching.
>
> (p. 69)

Commentators such as Barton (2009) would agree; they insist that with the advances in teaching practice that more mathematical understanding and content knowledge is demanded. Clearly PL cannot hope to address all areas of Subject Matter Knowledge (SMK), and therefore serious consideration need to be as to the Specialised Content Knowledge (SCK) which is most important in each topic.

Assertion 2: Prior to this PL intervention, a significant number of the teachers in this study showed conceptual shortcomings with fractions (K.F. 6.6 and K.F. 6.7) which would have limited their capacity to effectively teach this important topic. Following this PL the teachers cited an increase in their confidence (K.F. 6.1) and many attributed this to improved content knowledge (K.F. 7.1). Evidence collected shows that this form of PL, built around the platforms of addressing Pedagogical Knowledge and Specialised Content Knowledge (K.F. 10.1) has resulted in the teachers expressing a belief in the growth of content knowledge for teaching fractions (K.F. 8.2).

11.2.4 The Importance of multiple representations

One particular part of Specialised Content Knowledge (SCK) which is crucially important for the successful teaching and learning of fractions is that of multiple representations (Pape & Tchoshanov, 2001). It can be claimed that if one is not aware of the various representations of fractions then the likelihood of teaching those representations is very small. This is significant if we want to develop strong mathematical knowledge for teaching to support teachers in using representations to attach meaning to mathematical procedures (Charalambous, 2010). One measure of an expert teacher is a teacher who can identify essential representations of their subject (Hattie, 2003). Professional learning programs should therefore assist teachers to provide quality instruction through their choice and/or construction of appropriate representations (Hill et al., 2008b; Hill, Rowan & Ball, 2005).

Concept maps were used to collect data from the participants so that they might show their understanding of the different written representations of fractions commonly used in schools and curriculum documents. As previously stated in section 7.1, it should be noted

that the representations are not necessarily constructs themselves (part-whole, quotient, measurement, operation or ratio) as described by other researchers (Kieren, 1980), but rather representations of those constructs (See Table 4.2).

The teachers were asked to construct a concept map at the commencement of the PL and then at its conclusion. Comparing the post-intervention with the pre-intervention results, statistically significant differences were found in five of the representations: as division, number line, symbolic, percentage and decimal fractions. This is considered by the Researcher to be a significant improvement in the use of extra representations.

This increase in the number of representations familiar to the teachers is important. This combined with the move away from a more algorithmic instruction of fractions, which uses only a couple of representations (as related by a number of the teachers), represents an important addition to SCK (K.F. 7.1).

11.2.5 Participants' perception of the effect of PL on content knowledge

A two part question, asked during the semi-structured interview (Question 10) was, "How has your content knowledge about fractions changed through this course? Could you articulate a couple of the things you have learned?" Comments were made by the teachers that indicated that they felt they had improved their content knowledge (K.F. 7.1), their pedagogy (K.F. 8.1) and therefore, their ability to teach fractions (K.F. 6.2).

The use of the word changed, implies that the Subject Matter Knowledge (SMK), and particularly the domain of Specialised Content Knowledge (SCK), would be improved. Through the activities they chose to illustrate their growth, many of the teachers suggested a growth in Specialised Content Knowledge (SCK) through reference to Knowledge of Content and Curriculum (KCC).

One such example (fully articulated in Chapter 9) was seen where a teacher spoke at length about how she turned the abstract concept of solving division of fractions through the rule of invert and multiply into a pictorial representation. Coincidentally this is the very same

mathematical activity that Borko et al. (1992) used when discussing a highly trained teacher who lacked suitable representations to successfully make the content comprehensible to students, and therefore failed to display PCK. Through participation in this study, this teacher has developed this particular piece of PCK.

There are few structures or processes in place across education systems and sectors which require teachers (other than perhaps a moral and professional obligation) to explore the depth of their own Specialised Content Knowledge (SCK). This PL placed the teachers in a position where they were required to do so. In incidental conversations and in the completion of activities many of the teachers indicated through explicit words or implicit expression (nodding of head, a smile, and so on) that they had experienced a moment of improved SCK (K.F. 7.1).

Assertion 3: This PL increased the number of representations of fractions available to teachers (SCK), a vital piece of SMK (K.F. 7.1) a domain of Mathematical Knowledge for Teaching (MKT). This improvement in MKT is further evidenced through the careful consideration shown by teachers of Knowledge of Content and Curriculum (KCC) in the activities chosen to articulate the required fractions content (K.F. 8.1). This increased KCC then strengthened the teachers' perception of their ability to teach fractions (K.F. 6.2).

11.2.6 Pedagogical content knowledge

Another theme to emerge from the collected data was that of developing Pedagogical Knowledge (PK) as a vital element of PCK. With the understanding firmly established that the development of PCK is profoundly linked to the development of content knowledge, the focus throughout this theme was upon seeing where the content and pedagogy were brought together to enact each other. It is contended here, that where explicit connections have been drawn between the domains of knowledge and suitable pedagogical practices, PCK is being displayed.

As discussed throughout this dissertation, Shulman (1986) argued the existence of a particular form of teachers' professional knowledge, calling it Pedagogical Content Knowledge (PCK), and that this form of knowledge builds upon, but is not the same as, subject matter knowledge or knowledge of general principles of pedagogy. Rather, the epistemological concept of PCK could be described as a link between the knowledge bases of content and of pedagogy. Shulman (1986) stated that PCK included those special teacher attributes that helped him or her guide students to understanding. PCK is a way of making the complexity of teaching transparent.

Research (Gess-Newsome & Lederman, 1995; Hoz, Tomer & Tamir, 1990) shows that teaching experience, although an important factor in the development of PCK, is not as significant in developing PCK as is a teacher's opportunity and disposition towards reflection on their content knowledge. In order to reflect, time and opportunity needs to be available. One manner in which this opportunity can be afforded is during PL experiences. PL affords the opportunity, but how can PL be structured to be as effective as possible in developing PCK?

A central claim is that, if it is to be efficacious in developing PCK, PL must be constructed in a manner which recognises the need to address both Pedagogical Knowledge (PK) and Subject Matter Knowledge (SMK) (K.F. 10.1). This is in concordance with Higgins and Parsons' (2011) research which talks about the need to provide PL which improves both pedagogical skills and knowledge to develop good teaching, and therefore improves student outcomes.

This study was constructed around a core of developing pedagogy, content and reflective practice. At the conclusion of the intervention a question was asked "There are three major elements in this PL, which has been the most important to your development as a mathematics teacher and why?

- The chance to see some alternative pedagogy
- Content - the variety of activities
- Taking everything back into your context and then given a chance to reflect".

Although it was never explicitly expressed that all three elements of content, pedagogy and reflection were seen as integral and of equal importance, comments about the importance of each were made during the course of the PL. Only three of the teachers claimed all three elements were important, two claimed two to be important and 14 chose one. When the results were collated, content and pedagogy were both mentioned an equal number of times, but the reflective element gained only a quarter of the number of responses of the other two. However, a claim could be made that the PL achieved all three of its aims as although reflection, was not always explicitly mentioned, meta-cognition was demonstrated on numerous occasions through talk of practice, the effect of this current practice and subsequent change of that practice. To quote Parsons and Stephenson (2006), "The notion of meta-cognition, of being aware of and able to monitor the development of one's own learning and the application of that learning to their practice, would therefore seem to be a feature of the reflective practitioner" (p. 97).

To further explore the claim that to be efficacious in developing PCK and that PL must be constructed which recognises the need to address both PK and SMK (K.F. 10.1) the teachers were asked to respond to the question, "What activity or activities introduced in this PL have been most revealing to you and why?" It should be remembered that there was a calculated degree of ambiguity in the use of the word revealing, so as to encourage the participants to reflect upon the different elements which may have led to such a revelation. This proved fruitful, as it prompted conversation which displayed the juxtaposition of; the content the activity showed (Common Content Knowledge (CCK) and Specialised Content Knowledge (SCK)), the pedagogical changes the activity suggested (Knowledge of Content and Teaching (KCT) and Knowledge of Content and Students (KCS)) or the confidence of the teacher in order to implement it. This was evidence that the teachers were developing or refining their understanding the connectedness of PK and SMK (K.F. 10.1)

Whilst it is to be expected that there would be a spread of activities which were considered revealing, the Fraction Bridge activity, was mentioned most often. During its completion, this activity evoked the most conversation and required the most clarification. Although some clarification about the task was to do with its execution (it seems that no matter how comfortable the PL setting is, teachers seem very reluctant to take risks and perhaps make

mistakes in carrying out a task) most clarification was needed around the thinking that it promoted.

The activity which was mentioned second most often was the Fraction Strip activity. One teacher saw this as most revealing through the experience of using this task in her classroom. On using the task she was struck by the understanding that her students achieved through completion of the activity in moving from an algorithmic understanding to a more conceptual understanding. She referred to this as having the ability to visualise the fractions. Some other teachers commented how this activity allowed them to engage in an informal assessment of the level of understanding of their students. This was best expressed by a teacher who said, "… and just to hear their language when they are speaking, you think OK, they actually don't understand what we are trying to do, so you go back a bit and you try and start off at the basics again." This resonates well with the work of van Es and Sherin (2008) who talk of one type of teaching expertise being the ability "…to notice and interpret key features of classroom interaction" (p. 156). One teacher actually found all of the activities to be revealing as instead of highlighting one particular activity, they focused on the necessity of what she referred to as powerful representations to promote understanding.

The spread of the activities which were thought to be most revealing does demonstrate that no matter how powerful a single activity might seem to be in illustrating a concept, a range of activities using a range of representations is required in order to uncover meaning for learners (Cramer & Wyberg, 2009; Park & Oliver, 2008; Sebasta & Martin, 2009). If this is the case with teachers, should it not also be the case with students?

Another question from the post-PL interview asked the participants to consider, "What is your favourite activity when teaching fractions? What makes it your favourite?" Unlike Question 11 where the teachers were posed a question regarding the most revealing activity, where the word revealing was ambiguous, this question was purposely focusing on the most positively received activities by using the word favourite. It was of interest to this Researcher to determine if the favourite activities had any sort of relationship with the revealing activities as articulated in Question 11. Eight of the teachers chose the most

revealing activity as also being their favourite activity. This may be a good tip to teachers, that to make mathematics more fun for students, they need to make it a challenge!

The activity that was most favoured was one which employed the use of Pattern Blocks as the representation to carry the understanding of equivalence and proportional reasoning, amongst other concepts. One teacher used Pattern Blocks and Specialised Content Knowledge (SCK) to develop an understanding of unitisation, which is one of the basic precepts of proportional reasoning.

Teachers mentioned that an important consideration in using manipulative materials was their availability. As schools work under the constraints of a limited budget, purchases are generally rationalised to materials with multiple applications. Therefore PL constructed around materials which are particular to only one task or which cover only one mathematical concept may not be appropriate. Rather, on the proviso they are effective, materials which have multiple applications need to be the focus of well-structured PL. Pattern Blocks are manipulative materials which have the distinction of having multiple uses, something of which was articulated by a teacher who said; "…so I reckon any school that's got the Pattern Blocks, you know there are so many activities that you can do".

The focus on PCK was further explored through the interview question, "If you had to advise on your top five tips for teaching fractions what would they be and why? Don't worry about the order and if you need more than five that's fine". Using the domains and definitions of Hill et al. (2008a), 37 of the 42 responses (88.1%) related to PK and in particular Knowledge of Content and Teaching (KCT). The remaining five responses (11.9%) related to Subject Matter Knowledge (SMK) and particularly Specialised Content Knowledge (SCK) (K.F. 7.1). Once again the boundary problem as acknowledged by Ball et al. (2008) is very much in evidence when trying to perceive into which domain any knowledge mentioned by the participants is positioned. It is also obvious how different participants responded to the question in referencing and connecting different domains (K.F. 7.1). The combinations and permutations seem to be as individual as the teachers. Therefore it might be reasonable to suggest that any PL which has a desire to improve PCK

must accept that no particular domain can be excluded from consideration, as any domain might be an appropriate pathway in developing PCK (K.F. 7.1).

Overall the responses seem to indicate that carrying the message of good pedagogy was seen as more important to the teachers than perhaps improving or increasing their Specialised Content Knowledge (SCK). However, responses to questions often had some specific SCK (for example; multiplicative reasoning, the relationship between common fractions, decimal fractions and percentages, and references to unitisation) which were couched in relation to Knowledge of Content and Teaching (KCT). One such example is where teachers articulated the need to develop physical representations and the language to describe them (KCT) through the use of unit fractions (SCK).

11.2.7 Responses to the Pedagogical Content Knowledge Situations (PCKS) questionnaire

All teachers were asked to complete a pre-intervention and a post-intervention Pedagogical Content Knowledge Situations, (PCKS) questionnaire (Appendix 6) which offered scenarios for which they were asked to provide solutions and (in most cases) give reasons for their answers. This instrument was based upon some early work by Clarke and Mitchell (2008). A total of five situations were provided. A simple survey of the difference between the pre-intervention and post-intervention results suggested that there was growth in PCK and this growth proved to be statistically significant (K.F. 8.1).

What emerged in the responses to these statements and questions was not that the teachers did not necessarily arrive at the answer or response that the Researcher thought most appropriate, but that in many cases the responses were poorly articulated. As an example, when looking at the responses to Situation 4 (discussed in more detail in section 8.1.4), “The Year 4 teacher told her class on the first day of working with fractions: ‘Nothing is more important in understanding fractions than finding out what the unit is!’ Is she correct? Explain your reasoning.” One of the better responses was supplied by a teacher who responded with:

> A fraction is always related to the whole you are dealing with. When using the same size whole, ½ is bigger than $^1/_3$, is bigger than ¼ etc. However, comparing a fraction from a larger whole may show that ½ from a smaller whole is smaller than ¼ of a larger whole. Therefore it is essential to find out what size the whole/unit you are referring to is.

This teacher's clear and concise answer contrasted with those of other teachers, who provided responses, which this Researcher feels, would only serve to obfuscate the understanding for students. For example:

> Yes. She means what is the label or value we are attaching to something? For example if 2 counters are ½ what is the whole? It doesn't have to be a whole number; both the items and value/label can be more than one or less than one.

The completion of the PCKS assessment was not universally welcomed. One of the teachers claimed that such an assessment was difficult as it was "…stuff you do in the classroom without thinking". Another teacher asked, "How can we know what the student is thinking without actually asking them?" When asked if every student who supplies an incorrect answer in her classroom had the opportunity to explain their thinking, she replied that this did not happen and then would not enter into further conversation. A response such as this suggests that the teacher is not engaging in what Ball et al. (2008) refer to as the need to "hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language (p. 401)." This discussion ended without a resolution as far as the teacher was concerned.

The discomfort expressed by these two participants may be symptomatic of a larger problem in trying to capture and portray PCK (Loughran, Mulhall & Berry, 2004). It might have been that the negative responses about the PCKS were expressed by two people who were perhaps being defensive about their own performance. It however must raise the issue of the legitimacy of this assessment tool. If the teachers could not see the potential of the tool to deliver relevant and trustworthy information, then their application to finishing it to the best of their ability might be compromised. However, the responses received from the

other teachers meant that, even with this reservation noted, the assessment tool was still considered valuable.

One teacher discussed the use of appropriate language and the use of manipulative materials (KCT). At the same time she displayed Knowledge of Content and Students (KCS) by saying that the teacher needs to listen to their students and find out what they have discovered through exploration.

Other teachers provided responses which spoke of KCS in finding prior knowledge and spoke of KCT in the use of manipulative materials and a variety of representations. What was apparent was the large degree of uniformity of the answers amongst the participants. Reference to correct language use, teaching from the point of the students' prior experience, the use of manipulative materials and using multiple and carefully selected representations all featured strongly.

A further question asked during the interview that elicited more support for manipulative materials was, "What do you think you have learned through attending this PL and how will it change your practice from now on?" Of the responses to this question, nearly two-thirds espoused the increased use of manipulative materials of one form or another.

One teacher said "Well like I've said before just using, starting really basic, using concrete materials and Pattern Blocks as a start before actually going on to the real teaching of fractions". This Researcher finds the use of the statement about using manipulative materials "…before actually going on to the real teaching of fractions" quite disturbing. The implication here is that manipulative materials do not provide the opportunity for learning, but rather act as a novelty or motivation before the learning commences!

When the responses to the question were grouped, three of the groupings related to KCT (more use of manipulative materials, more care about lesson sequence and lots of paper folding). A further three (contextualising fractions, articulating links to other mathematics and spending more time on fractions) related to Knowledge of Content and Curriculum (KCC), and these together constituted 86% of the responses. The data displayed that the

teachers clearly believed that learning, in this case a growth in PCK (K.F. 8.2) had occurred through the completion of this PL (K.F. 10.2).

11.2.8 Developing generalised PCK through well-structured PL

A further claim made in this small study is that it may be possible to develop PCK through well-structured PL regarding other mathematical topics. The teachers were asked to respond to the final question from the semi-structured interview which probed the development of PCK, "What have you learned from this PL about fractions that you can generalise into wider teaching about mathematics?" Using the domains and definitions of Hill et al. (2008a) this Researcher determined that seven of the 10 categories of responses could be characterised as PCK, and two of the 10 categories belonged in Subject Matter Knowledge (SMK) (K.F. 8.3).

One major focus for this research was to determine if PL could be structured in such a manner that participation would provide teachers with the opportunity to improve their PCK. It was claimed that if this could occur whilst tackling a topic, as difficult to teach and learn as is fractions, the structure might then be successfully applied to other content areas.

Some of the teachers alluded to the inter-connectedness of mathematics (Askew, 2008). For example, one teacher talked about particular connections between fractions and measurement which refers to one of Kieren's (1988) constructs of fractions, that of measurement. Some also raised the connection between fractions and Measurement and included a connection to the strand of Space (Geometry); others concentrated on the value of using manipulative materials to facilitate learning; whilst others still, talked about both the use of manipulative materials and exploiting the interconnectedness of mathematical concepts and ideas. Many of the teachers not only talked of the curriculum connections (KCC) but also displayed Knowledge of Content and Teaching (KCT). There were many instances which indicated in them a developing Pedagogical Content Knowledge through the Knowledge of Content and Teaching (KCT) and Knowledge of Content and Curriculum (KCC) (K.F. 8.2). The Researcher would also argue that the difficulty of the topic of fractions did not complicate this development.

Assertion 4: This PL enhanced the PCK of teachers (K.F. 8.1 and K.F. 8.2) in the difficult topic of fractions, and the PCK appeared to be generalised into other mathematical topics (K.F. 8.3). The PL amplified both Pedagogical Knowledge (PK) and Subject Matter Knowledge (SMK) which provided pathways to increased PCK (K.F. 10.1).

11.3 *Self-Efficacy, Confidence, Beliefs and Attitudes*

For the purposes of this study (and as argued in section 2.1.7) an acknowledgement has been made concerning the differences in meaning between the terms, attitudes, beliefs, confidence and self-efficacy. Notwithstanding these differences, they all sit firmly in the affective domain. If we adopt Bandura's (1994) thinking regarding self-efficacy, then we see it as people's beliefs about their capabilities to produce designated levels of performance that exercise influence over events affecting their lives. Therefore, although they may need different tools and perhaps different metrics, they all attempt to measure a positive disposition to learning.

A rise in confidence is a significant feature in addressing the issue of self-efficacy. As stated in Chapter 2 the research supports the belief that poor self-efficacy is a major obstacle in effective teaching and learning and that the implications of this are significant (Gresham, 2008; Park & Oliver, 2008; Zambo & Zambo, 2008). Therefore if teacher efficacy can be defined as the extent to which teachers believe they have the capacity to affect student performance (Berman, McLaughlin, Bass, Pauly & Zelman, 1977), then a teacher's confidence for and positive disposition towards teaching a subject will enhance their capacity to teach that subject.

11.3.1 Confidence

This study, in concordance with the findings of Hackling et al. (2007), argues that through the delivery of quality professional learning experiences the confidence of teachers for teaching fractions was significantly improved (K.F. 6.1). This is despite the fact that fractions is a topic that is obviously problematic for many teachers to teach and many students to learn (Capraro, 2004; Nunes & Bryant, 2009; Wu, 2005).

Elements of the qualitative data confirmed the finding that confidence was raised (K.F. 6.1). This qualitative data from a semi-structured interview identified the particular factors which helped to facilitate this rise in confidence. Teachers made comments related to Specialised Content Knowledge (SCK) and Knowledge of Content and Teaching (KCT), both elements of Mathematical Knowledge for Teaching as described by Hill et al. (2008a). Teachers commented upon the strategies they had developed, their increased knowledge of the misconceptions which students might carry and the instructive properties of manipulative materials. One teacher spoke of how attendance at the PL had created a sense of optimism towards the topic of fractions:

> … I am more looking forward to next year and my whole session on fractions I can see doing completely differently and I know how to teach the concepts properly that the kids will be able to understand and be able to consolidate everything well enough for them to have a grasp on it.

There may be a beneficial iterative relationship between the development of SCK, pedagogy and confidence. The results show that attainment of SCK and pedagogical knowledge (PK) develops confidence, and research indicates that an increase in confidence may encourage teachers to be innovative with their teaching (Gabriele & Joram, 2007; Gresham, 2008; Swackhamer, Koellner, Basile & Kimbrough, 2009). This increase in confidence may then encourage these teachers to further explore and develop SCK and their pedagogy, and consequently to further develop PCK.

In speaking about their confidence, teachers described their learning trajectory regarding fractions as not being linear, and used phrases such as “two steps forward and one step

back" a process which challenged some teachers' level of confidence. Some teachers, in one way or another asserted that "we don't know what we don't know." This leads to questioning if this lack of pre-intervention knowledge may not have lead these teachers to have inflated the ratings on the pre-intervention questionnaire. They recognised that from a learner's uninformed position they may have held a conceptual understanding which after some illumination in a PL experience, may in fact not hold true. This is in congruence with the work of Hackling, Smith and Murcia (2011). Although expressed by several teachers, one teacher succinctly summed this up by asserting "Yeah I thought I was pretty confident but then I suppose when we started I realised that I needed to know a lot more than what I did."

Two questions intended to determine an element of confidence were, "At the outset of this PL how would you have rated your ability to teach fractions? How would you now rate your ability to teach fractions?" The unanimous answer to these was that there had been an improvement in the teachers' ability to teach fractions since completing the pre-intervention questionnaire (K.F. 6.1 and K.F. 6.2). This further strengthens the claim that confidence was raised, which in turn increases the possibility for improved teaching and learning.

One teacher raised the issue of having no baseline against which to judge their teaching, which created difficulty in making an assessment if the efficacy of that teaching. This perspective alone does much to justify the provision of professional learning experiences for teachers.

> At the outset I felt pretty good, yeah, but I guess I didn't realise, I could've been doing it better, you know I felt pretty good about it but looking back I probably could've done it better.

Another question, related to the issue of confidence, asked teachers if they felt they were better equipped to promote student learning in fractions, because of attendance at the PL. The response to this question was universally positive. This measure suggested an improved level of confidence (K.F. 6.1 and K.F. 6.2). On quite a few occasions the teachers

talked about that they were better equipped to promote student learning by referring to a change in pedagogy which often requires:

- an improved Knowledge of Content and Curriculum (KCC) to know when to teach something;
- improved Specialised Content Knowledge (SCK) so as to know what to teach;
- Knowledge of Content and Students (KCS) to determine why a student might arrive at a particular answer; and ,
- Knowledge of Content and Teaching (KCT) to know which representations would be of greatest advantage to the learner (K.F. 7.1).

As can be seen above, in explaining the reason behind their increased confidence some teachers responded by indicating that the specialised knowledge developed through attendance at the PL meant that they felt better equipped to promote learning of fractions. Others, referred to a change in pedagogical practices by describing a move to using manipulative materials one would assume, from textbooks and worksheets. Their willingness to depart from the textbooks indicates a growing confidence. Others talked about how they were developing and employing a more specialised and precise language with which to discuss and illustrate fraction concepts, which they felt allowed them to raise the standard of the negotiated meanings around fractions, and therefore the quality of the discourse with the students. This is a view which is in concordance both with the work of Cobb (1994) and of Boaler & Greeno (2001).

As noted in responses to previous questions, often the answers given to this question regarding the teachers' capacity to promote the learning of fractions did not cause the teachers to refer to a single dominant domain of MKT, but rather combinations of the domains. This further supports the claim that PL must be constructed which recognises the need to address both PK (which in turn, through practice, is synthesised into PCK) and SCK (K.F. 10.1).

The teachers were asked, "If any, what aspect/s do you find most problematic in teaching fractions?" This question had its origin in an explicit attempt to further explore the area of confidence and ability in teaching fractions and an implicit attempt to inquire of the

participants whether any issues may have arisen concerning either PCK or SMK. Of the problematic areas, 37% were situated in Pedagogical Content Knowledge and 42% in Subject Matter Knowledge with the remaining 21% of teachers claiming not to have any problematic areas in the teaching of fractions. Somewhat contradictory was that four teachers who perceived themselves as not having any problems did not score themselves particularly highly on related questions regarding confidence and ability with regards to fractions.

It should also be noted that when problematic issues were raised by the teachers, the reporting of these issues was on the whole quite brief. This could be due to the fact that the teachers did not feel secure in expressing what could be perceived as short-comings. However, responses to other questions where the participants were quite forthcoming in expressing their doubts - and thus perhaps exposing what could be perceived as weaknesses - would not support this. This brevity of explanation may also be seen in some instances as belying the scores that the teachers gave themselves in answering the related interview questions of confidence and ability. However, as Phillip (2007) states, beliefs unlike knowledge may be held with various degrees of conviction. These seemingly contradictory results may perhaps be an illustration of that variation in that.

The questions were asked, “If you were now asked to take a leadership role in the teaching of fractions in your school do you feel more confident now, as opposed to when you began this PL, that you could do so? If so, what has lead to this increased confidence?” These questions were constructed around taking a leadership role in school, a circumstance which many teachers would find stressful. Seventeen of the 19 (89.47%) responses indicated that the participants in the PL felt more confident about taking on a leadership role in the teaching of fractions in their school on the completion of the course, as compared to before entering the course.

This question about taking on a leadership role elicited results that could be seen as somewhat less convincing if viewed with the response to the statement in the Exit Questionnaire (see section 10.5). Here the teachers were asked to score how effective they felt that the PL was at promoting leadership capacity, and they showed a good deal of equivocation. It may be that like beliefs, where a person can hold beliefs which are

contradictory without being aware of the contradiction (Beswick, 2006), someone may hold levels of confidence which are contradictory without being aware of the contradiction. There is also the possibility that although the PL had achieved its goal of making the teachers more confident to take a leadership role in the teaching of fractions, the teachers recognised that at no time did the PL ever explicitly try to develop the generic skills of leadership. Therefore, although both questions are regarding leadership, the teachers may have perceived that they were addressing two distinctly different notions.

Assertion 5: In concordance with Hackling et al. (2007) findings, this research suggested that through the delivery of a quality PL experience, the confidence of teachers for teaching fractions can be significantly improved (K.F. 6.1). The evidence strongly suggests that attendance to Pedagogical Knowledge and to Specialised Content Knowledge (K.F. 7.1) influenced this rise in confidence. It is not unreasonable to speculate that the implementation of SCK and PK in practice should then lead to further developing PCK (K.F. 8.2).

11.3.2 Beliefs

Comments collected during the interview show that the PL had an impact in promoting some positive beliefs about teaching and learning (K.F. 9.3). The claim that the beliefs regarding the effectiveness of manipulative materials to bring about conceptual understanding of fractions were legitimate, was supported by the research of Berry et al., (2009); Cramer and Wyberg (2009); Reimer and Moyer (2005) and Sebasta and Martin, (2004), amongst others.

This study also used the Teachers' Beliefs about Mathematics (TBM) questionnaire to explore the impact of well-structured professional learning opportunities and reflective practice on primary school teachers' beliefs with regards to teaching mathematics in general and fractions in particular. This study found little empirical evidence of significant changes to these beliefs as measured by this instrument (K.F. 9.1).

Only two individual items showed significant differences between pre-intervention and post-intervention assessments. These were Belief 1, “Mathematics is computation” and Belief 6, “Mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences.” It is the view of this Researcher that the movement away from the belief that mathematics is computation requires that teachers employ a broader definition of what mathematics is. This broadening in definition in turn creates the need to reflect on practice and pedagogy. The positive skewing of the responses for Belief 6 regarding the focus of the learning being gained from experiences is again viewed as a positive movement, as it suggests a reflection on pedagogy, perhaps towards the increased use of more manipulative materials.

The overall results should be viewed in the light that the beliefs of the teachers started from a reasonably positive base. It should also be noted that the three point beliefs scale adopted from the work of White, Way, Perry and Southwell (2006) was perhaps not sensitive enough to detect small changes.

A further point for discussion is that at no time were the belief statements directly and explicitly explored or challenged in the professional learning sessions. Therefore the capacity to affect any movement from what Beswick (2006) calls non-evidential beliefs to evidential beliefs, and therefore beliefs susceptible to change, was not thoroughly tested. Another opportunity to consider in any future studies would be to use the belief statements and to encourage the teachers to pursue active reflection regarding these. This would be in accord with Wilkins’ (2008) assertion that an important element of any professional learning should be the opportunity for teachers to develop an understanding of their own teaching practice through an examination of their beliefs.

Assertion 6: Although not indicated by all measures (K.F. 9.1), well-structured PL can improve the beliefs (K.F. 9.3) of teachers about teaching the difficult mathematical topic of fractions. Where the measures failed to discern growth in beliefs, three explanations are plausible: a lack of sensitivity in the tool employed; the lack of direct examination of the teachers’ beliefs during the PL; and the fact that the teachers started from a fairly positive base meaning growth was always going to be minimal and therefore difficult to discern.

11.3.3 Attitudes

Comments collected during the interview show that the PL had a positive impact in promoting some positive attitudes, with regards to teaching mathematics in general and fractions in particular (K.F. 9.3). However, a constructed questionnaire (Teachers' Attitudes towards Mathematics [TAM]), found no empirical evidence of any significant difference made by attendance at this PL (K.F. 9.2) The Teachers' Attitudes towards Mathematics questionnaire (TAM) was applied in the first session of the PL and then in the last professional learning session, after the intervention. When the difference in the mean responses was examined through application of a Wilcoxon Signed Ranks Test, no significant statistical difference was found. Further, no significant statistical differences were found between the pre-intervention and the post-intervention scores of each individual item testing the attitudes of the teachers.

A question to be raised here is whether or not the relative brevity of this study (February through to June) had an effect on the possible attitudinal gains that might be made from adopting the PL model that was used. That is, if the study had been conducted over a number of years instead of a number of months, there may have been more of an opportunity to see change not obviously apparent over this short time frame. It is also pertinent to note that of the four factors that Hew and Brush (2006) identify as being required to facilitate change - teachers' knowledge and skills, subject culture, assessment, and institutional support - only the first was directly addressed through this professional learning. The other three were addressed only incidentally.

Further, the questionnaire which was adopted from the work of White et al. (2006) did not specifically target the topic of fractions: the questions were phrased around the much more generic term of mathematics. The questions therefore captured data which might inform the first part of the research question regarding the impacts of PL on teachers' attitudes with regards to teaching mathematics in general, but not on the second part, which enquired about the teachers' attitudes to teaching fractions in particular. Further still, the White et al. (2006) questionnaire was constructed around 11 questions regarding attitudes towards teaching mathematics and nine towards the self as a mathematics student. Of the items, 13

were not addressed in any overt manner during the PL sessions, so there was no expectation of measuring effect.

Assertion 7: Although not indicated by all measures (K.F. 9.2), well-structured PL can improve the attitudes (K.F. 9.3) of teachers towards teaching the difficult mathematical topic of fractions. Where the measures failed to discern growth in attitudes, three explanations are plausible: the relative brevity of the study did not allow for the change to become apparent; many of the questions asked alluded to elements which were not overtly covered in the PL; and only one of Hew and Brush's (2006) four factors required to facilitate change was pursued, which may have inhibited the rate of change.

11.4 *Perceived Effectiveness of Professional Learning (PL)*

The final theme to emerge from the collected data concerned the effectiveness of the Professional Learning (PL). Although PL effectiveness was not an overt concern of any of the research questions, the provision of effective PL is central in answering these questions. As stated in Chapter 2, PL is an important and costly concern (Wayne et al., 2008) and it is unwise to assume that merely instituting PL will make a significant difference (Cohen, Raudenbush & Ball, 2003). Research also indicates that PL does have positive effects on teaching and learning and the efficacy of teachers (Penuel et al., 2007; Yoon, Duncan, Lee, Scarloss & Shapley, 2007; Zambo & Zambo, 2008). Clearly then, the provision of effective PL is a significant factor in the teaching and learning of mathematics in general and of fractions in particular.

The findings from this study suggest that it is feasible to construct PL which employs the majority of the researched characteristics of effective PL (K.F. 10.2) and also leaves participants with the perception that their needs have been met. It can be further contended that it is also feasible to deliver this PL in a manner which makes those characteristics explicit to the participants, thereby possibly enhancing that perception.

One of the semi-structured interview questions asked the participants, "What were you hoping to gain from attendance in this PL and was that gain achieved?" Surprisingly, one

fifth of the participants had no particular expectations when beginning the PL. It is probably not unreasonable to suggest that many participants in PL, attend with a vague idea that the PL will beneficial but are unsure how this might occur. This notion is certainly supported in this study, and was illustrated by a teacher who said “Actually, I came in with little expectation and actually, yeah, came out with a whole lot of stuff that I can use” and another who offered; “Well, it’s my first PD so I didn’t have a lot of expectations but it’s definitely exceeded, I’m really happy, yeah…any reflection I guess in teaching is good, yeah so I didn’t have a lot of expectations but it’s definitely been good so I’m glad I did it.”

The second part of the question regarding whether the expectations were met saw 100% of the teachers answering in the affirmative. It could be seen as unusual that, although a number of teachers came to the PL with no expectations, they were still comfortable in providing an affirmative response when asked if their expectations had been met. This suggests that even though people may attend PL with few clear or well-reasoned expectations they may well feel that they have gained from the PL and that it has been worthwhile.

In responding to this question, one teacher used the word simpler in a rather confusing way to describe the pedagogical shift from reliance on texts to the use of manipulative materials: “Oh just to learn how to teach fractions to the children in simpler ways than text book.” Although further questions were not raised to clarify this statement, it could be assumed by use of the word simpler, that this teacher was referring to the use of concrete materials as being more fundamental than the abstract representations from text books.

At the conclusion of the Professional Leaning sessions the participants were asked to respond to 19 statements on a Likert Scale regarding their belief that the characteristics of effective PL had been addressed. These statements on the characteristics of effective PL were informed mainly, but not exclusively, by the work of three extensive studies (Clarke, 2003; Guskey, 2003; In- Praxis Group Inc., 2006). At least 80% of the teachers indicated that the 19 elements of effective PL had been exercised.

This Researcher strongly contends that being able to provide PL participants with research which presents the elements of effective PL, and then allowing them to be able to identify if those elements have been met, should add to the perception of the worth and credibility of the PL. This in turn should lead to increased confidence in using what has been learned when back in the classroom. This contention is supported by the teacher who stated:

> … but when you come to a PD that is run by AISWA *(the association through which the PL was organised)*, run by people that you've heard of who are well thought of, it gives you the confidence to say, "Right well I will do that" and anything that you do that comes out of that, you feel has credibility because it's based on good pedagogy so I would feel more confident after this PD.

Assertion 8: It is feasible to construct PL which not only leaves participants feeling that their needs have been met, but that also employs the majority of the researched characteristics of effective PL (K.F. 10.2) and thereby enhancing their perception of the credibility of the PL. It can be further contended that it is also feasible to deliver this PL in a manner which makes those characteristics explicit to the participants.

11.5 ***Summary***

There is an ancient Indian parable that has a corollary with the work carried out in this project. In various versions of the tale, a group of blind men touch an elephant to learn what it is like. Each one feels a different part, but only one part, such as the trunk or the tusk or the leg. They then compare notes and learn that they cannot agree as to what they are presented with. At times, delivery of PL has been something like this. We present participants in PL with only part of the 'beast' to explore and they therefore fail to capture it in its entirety. Presenting professional learning in a problematic topic such as fractions may find that the many participants leave the PL with a less than adequate understanding of all that is required to teach this properly. To be effective there needs to be a clear pathway or process which ensures that participants are exposed to all of the parts. This is necessary for them to be able to construct a more complete understanding. They need to be given the

tools and the time to reflect upon the pedagogy and the content, and how they work together in context to develop Pedagogical Content Knowledge.

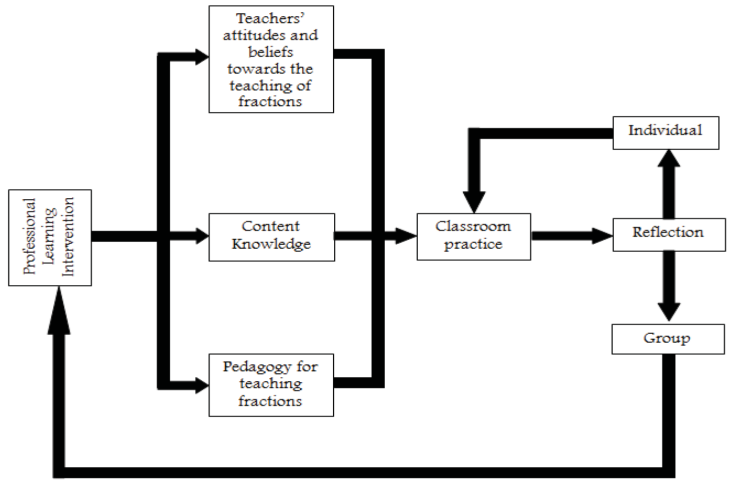


Figure 11.3 Original conceptual framework for the study

As this study progressed, the original conceptual framework (Figure 11.3) was reconceptualised so as to more sensitively illustrate the changing emphasis of the study. The original basis of the study centred around the teaching of fractions and the manner in which PL could be structured to improve the teaching of this topic and perhaps others in the area of mathematics. As the study progressed it became clearer that to achieve change in the teachers' capacity to teach fractions (or for that matter any area of mathematics) that the change needed to grow through the development of the teachers' PCK, through focussing on content, pedagogy and reflective practices. Therefore, PCK was also a fundamental part of the framework (Figure 11.4).

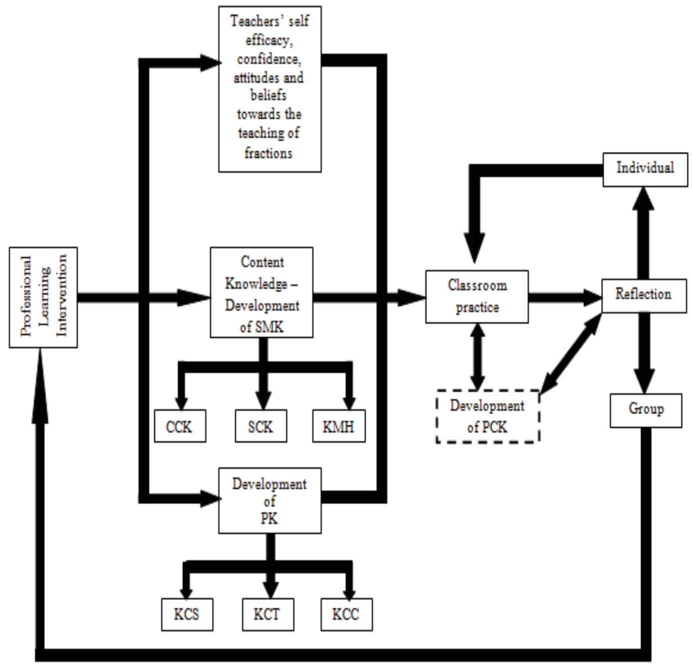


Figure 11.4 Revisited conceptual framework for the study

In this study the participants were taken through a PL process predicated upon research concerning its effective delivery. This PL was constructed around these, and each session was further informed by the need to include illustrations of best practice pedagogy and content knowledge which spanned the domains of Pedagogical Content Knowledge and Subject Matter Knowledge as described by Hill et al. (2008a). These domains were the structure which informed the whole 'beast' in this study. The participants were then asked to go back into their classrooms and use some of the activities presented to determine their effectiveness. Not only did this give them reason to trust what was being covered in the PL, it also allowed them to contextualise the activity. They were then asked to bring their reflections upon those activities to the next session. This was felt to be of vital importance to the teachers as they needed to become more aware of the power of reflection both *in* and *on* practice.

At each of the subsequent PL opportunities the participants were asked to use their growing understanding of the manipulative materials being utilised together with their developing understanding of pedagogical issues and increasing Specialised Content Knowledge. All this was considered in light of their classroom experiences. It was hoped that by increasing

this pedagogical knowledge and content knowledge, the self-efficacy, confidence, attitudes and beliefs of the participants would be affected in a positive manner.

Of all of the claims made through this small study, one of the major assertions is that PL can be a powerful agency for the nurturing of PCK. Certainly the classroom is a place where PCK can and should be developed, but the busy and diverse nature of the environment means that cultivating and refining PCK is not always easy. Indeed as previously stated, research informs us that more time in the classroom does not equate simply to more PCK development (Gess-Newsome & Lederman, 1995; Hoz, Tomer & Tamir, 1990). Primary school teachers are expected to cover the majority of learning areas and do not necessarily have the luxury of limiting their time and thinking to one particular learning area such as mathematics, let alone to one mathematical topic such as fractions. Therefore it is essential that a structure is developed with teachers, which allows them to develop knowledge and understanding across a range of topics.

In summary, this study shows that the effective teaching of fractions is important, but that the teachers do not generally believe that the school views this teaching area as having the same level of importance. Secondly, the collected data showed a rise in confidence through attendance at this PL, although no significant corresponding positive impact on beliefs and attitudes was measured using the instruments in this study. The failure to determine a corresponding positive impact on beliefs and attitudes may have been more to do with the appropriateness of the instruments employed and the timing of their use, rather than there being no changes in attitudes and beliefs. There is however, strong overall evidence to suggest that the experience was a positive one for the teachers, and that it was one which did much to alleviate some of the problems which they had experienced at the commencement of the course. It is also argued here that the PL resulted in some desirable reflection that in turn promoted an exploration of pedagogical content knowledge.

CHAPTER 12

12 Limitations, Conclusions and Implications

12.1 *Introduction*

This study arose from many conversations with teachers at various professional learning sessions. When asked about areas of need, the teachers often raised the content area of fractions as being problematic. More precisely they wanted well researched direction about how they could improve their own and their students' knowledge about fractions.

From the perspective of providing the support to teachers it became quite clear that the professional learning which had previously been made available by a number of agencies, both pre-service and in-service, had not adequately served these teachers' needs. Careful consideration needed to be paid to how the content and pedagogy would be provided for the participants in order that Pedagogical Content Knowledge (PCK) of this difficult topic might be developed.

The teachers had two fundamental questions about the teaching of fractions, "What do I teach and how do I teach it?" It would seem that many teachers were lacking critical aspects of PCK. This study therefore aimed at determining whether the PCK of primary school teachers of mathematics could be enhanced by applying reflective practice to professional learning, and during this PL, concentrating on subject content knowledge and sound pedagogy. This question has wide implications for the development of professional learning opportunities for in-service teachers and would also be highly beneficial in informing initial teacher education.

Further, as Hill et al. (2008b) have stated, a major shortcoming of research in the field of mathematical knowledge for teaching, is the limited scope of research often focusing on only one mathematical topic in the context of only one lesson, and the analysis of this through the focus of one teacher. This study was constructed on a topic with many

contributing elements, over an extended period with a group of teachers from a variety of backgrounds, bringing with them a variety of experiences.

Through electing to work with the teachers on the conceptually difficult topic of fractions, it was expected that success in this project could provide insights into approaches that might be successful with other mathematics topics. At the very least, this study aimed to give impetus to further studies on the efficacy of such an approach.

The research questions investigated were:

1. What is the current status of teaching fractions in middle and upper primary school classrooms in Western Australia?
2. What impacts will well-structured, action research based, professional learning opportunities and reflective practice have on primary school teachers' content knowledge of fractions?
3. What impacts will well-structured, action research based, professional learning opportunities and reflective practice have on primary school teachers' pedagogical knowledge for teaching fractions?
4. What impacts will well-structured, action research based, professional learning opportunities and reflective practice have on primary school teachers' beliefs and attitudes with regards to teaching mathematics in general and fractions in particular?

After the first professional learning session the participants were asked to take the content and pedagogies with which they engaged during the PL, into their classrooms, and to conduct some lessons. This was accompanied by the invitation to share their experiences through feedback to the group in the following session. This process was repeated in sessions approximately 3 weeks apart for Group 1 (G1) and approximately one month apart, for Group 2 (G2).

The teachers were asked questions in a semi-structured interview to ascertain their perceptions of the status of fractions. They were asked about their personal perceptions, their perceptions of fractions from their schools perspective and their perception of fractions from the school curriculum perspective. The teachers' content knowledge for

Group 1 was determined through completion of the Fraction Knowledge Assessment Tool (FKAT) and the construction of concept maps (CM). The teachers' content knowledge for Group 2 was determined through the construction of concept maps (CM) and the application of the Rational Number Interview (RNI). Pedagogical practices were assessed through applying the Pedagogical Content Knowledge Situations (PCKS) questionnaire and through particular questions posed in the semi-structured interview. Confidence, self-efficacy, attitudes and beliefs data was collected through the Teachers' Attitudes towards Mathematics (TAM) and Teachers' Beliefs about Mathematics (TBM) questionnaires and questions in the semi-structured interview.

These data were then analysed, interpreted and reported. Key findings were distilled from the data analysis. Interpretation of key findings drawing on the literature generated assertions and these assertions informed the development of the conclusions reported in this chapter.

12.2 ***Limitations***

This Researcher believes that, fundamentally, the study achieved what it set out to achieve. Nevertheless, there are a number of elements that if the study were to be replicated would probably need attention.

The small size of the cohort used in this study meant that some results may not always be able to be extrapolated to the whole of the teaching population. Although the teachers who were involved, worked in a broad spectrum of schools, taught different year levels, were of different gender and with differing levels of experience, they were perhaps not representative of the workforce. One major issue was that they chose, to a greater or lesser extent, to attend the PL. Experience indicates that many teachers who require the most help in mathematics may not volunteer to participate in PL and these are the individuals who conceivably would receive the most benefit.

Another issue regarding the size of the study was its duration. This study was completed over a period of about six months which meant that the teachers involved had about 20

weeks of school contact time in which to take the content and pedagogy attained in the PL and working with it in their classrooms. Within this contact time there was also the interruption to the normal classroom routine in the application of the WALNA test, which basically suspended the normal program for a period of at least one week. With the other expected curriculum pressures in the class, this probably affected the quantity and perhaps the quality of the exploration of topics which could be achieved by the teachers. This brevity would probably have affected the level of change that could be measured. An extended period would have been a much better time-frame for such a study. Keady (2007) states that it takes a "considerable time" (p. 248) for changes in critical and reflective practices, levels of confidence beliefs and classroom practice.

If this study were to be replicated, the choice of instruments used to measure the changes in beliefs and attitudes would need further consideration. Although the TAM and TBM instruments derived from well-credentialed sources, and proved to be reliable and valid in their original situations, neither was a comfortable fit with this study. The beliefs questionnaire with its three point scale proved not to be sufficiently sensitive to corroborate the changes which were indicated in the semi-structured interviews. Had there been more time for the study to run this may not have been such an issue, but the relative brevity of the study meant that a tool with more sensitivity needed to be employed. The questionnaire regarding attitudes was comprised of items that were stated a little too generally for the teachers to relate them to the specific context of the study.

A challenge for professional learning facilitators trying to promote positive change in the content knowledge of participants is the confronting nature of the task presented to them. For the teachers to reveal a lack of knowledge regarding something that they are supposed to be teaching competently requires a trusting relationship between them and the facilitator. They are offering up information which has negative consequences for perceptions of their own self-worth and potentially their employment. They need to be very comfortable that the collected information will be held in confidence and that they will not be judged by those data. This kind of trust requires time to develop; the brevity of this study may have inhibited the teachers from being as forthright as would have been desirable.

Considering the implications of some of these limitations - the small number of participants; the short time frame; the use of a case study conducted with a particular group of people, at a particular time and in a particular context limits the generalisability of the findings. The findings however, are compelling enough to make a case for a wider study to be conducted.

12.3 *Conclusions*

The status that a topic has amongst all of the topics of the mathematics curriculum will determine how much time and effort will be devoted to it. An increase in the amount of time committed to it and attention paid to its content and pedagogy are likely to make the teaching more effective and increase the opportunity for learning.

The evidence collected shows that the teachers in this study believe that the teaching and learning of fractions is important, and that its status can be further raised through attendance at well-structured PL. The teachers further indicated that the importance of fractions has not been made explicit by their schools. This perception of a lack of importance further extends to the education system in which they teach, through the lack of explicit guidance in the recently past and current curriculum documents (Assertion 1).

Knowledge of content is important for effective teaching. To be an effective teacher of fractions requires more than the common content knowledge which most people would hold (CCK). There is some specialised content knowledge that is beneficial to teachers so that they are in a position to help students develop a richer conceptual understanding of fractions (SCK).

Prior to this PL intervention, a significant number of the teachers showed conceptual shortcomings with fractions which would have limited their capacity to effectively teach this important topic. Following the PL, the teachers cited an increase in their confidence and many attributed this to improved content knowledge. Evidence also showed that the premise of basing the PL around the platforms of Pedagogical Knowledge and Specialised

Content Knowledge further supported the growth of their content knowledge for teaching fractions (Assertion 2).

Participation in the PL increased the number of representations of fractions available to teachers (SCK), which is a vital piece of Subject Matter Knowledge (SMK), a domain of Mathematical Knowledge for Teaching (MKT). This improvement in MKT is further evidenced through the careful consideration shown by teachers of Knowledge of Content and Curriculum (KCC) in the activities chosen to articulate the required fractions content. This increased KCC strengthened the teachers' perception of their ability to teach fractions (Assertion 3).

In addition to content knowledge, knowledge of pedagogy is required to develop effective teaching practice. A teacher must develop knowledge of their students and the pedagogy which is most beneficial for them (KCS), knowledge of the required curriculum and how the pedagogy might enact that curriculum (KCC), and knowledge about teaching and mathematics (KCT). An understanding of all of these, will enable the teachers to select pedagogy and representations which will best carry the content for the students (PCK).

Attendance at this PL enhanced the Pedagogical Content Knowledge of teachers in the difficult topic of fractions. The teachers also indicated the intention to generalise some of their new PCK into other areas of mathematics. Evidence was collected to show that the PL amplified both Pedagogical Knowledge (PK) and Subject Matter Knowledge (SMK), which in turn provided pathways to increased PCK (Assertion 4).

Research indicates that positive self-efficacy, beliefs and attitudes all play a vital role in developing effective teachers, and that the impact of self-efficacy on the teaching and learning of mathematics cannot be underestimated. There is a high correlation between teachers' beliefs and attitudes and: classroom instruction practices (Barkatsas & Malone, 2005; Philipp, 2007); the willingness to embrace innovations (Gabriele & Joram, 2007; Gresham, 2008); the use of a variety of instructional strategies (Czerniak & Schriver, 1994; Swackhamer, Koellner, Basile & Kimbrough, 2009); and the use of manipulative materials

(Gresham, 2006). Their beliefs and attitudes are also highly associated with teacher motivation (Bandura, 1993).

This research found that through participation in a quality PL experience, the confidence of teachers for teaching fractions could be significantly improved. The evidence strongly suggests that attending to Pedagogical Knowledge and to Specialised Content Knowledge influenced this rise in confidence. It is not unreasonable to speculate that if through attention to SCK and PK, confidence is raised, that this may result in a willingness to use of a variety of instructional strategies which improves the prospect of developing PCK (Assertion 5).

The results from this study indicate that well-structured PL can improve the beliefs (Assertion 6) and attitudes (Assertion 7) of teachers towards teaching the difficult mathematical topic of fractions. This improvement in attitudes and beliefs is important as it has an impact on the quantity of time devoted to this important topic.

It is feasible to construct PL which not only leaves participants feeling that their needs have been met, but also employs the majority of the researched characteristics of effective PL, and thereby enhances their perception of the credibility of the PL. It can be further contended that it is also feasible to deliver this PL in a manner which makes those characteristics explicit to the participants (Assertion 8).

12.4 *Implications*

12.4.1 Further Research

This results of this small study suggest that an extended study which takes into consideration the short-comings, as indicated in the limitations section of this chapter, would shed further light on the effectiveness of PL which is built around well researched precepts and provides an appropriate focus on the different forms of pedagogical knowledge and content knowledge. Should such larger scale replication studies generate

positive outcomes then the PL approach could be recommended for more widespread implementation.

12.4.2 Design of PL

PL is an important but costly enterprise which must not only be effective, but as this Researcher would argue, needs to be perceived by the participants to be effective. This is not to suggest that in the past PL presenters have been reticent to link PL to good practice. In fact organisations such as the Australian Association of Mathematics Teachers (AAMT) require the presenters at their conferences to indicate which elements of their standards they are addressing. One of the standards is based around Professional Knowledge which includes: knowledge of students, knowledge of mathematics; and knowledge of students' learning of mathematics (AAMT, 2006). Although these are advertised against the offerings for the conference there is no provision made that the participants consider the extent to which the standards have been addressed. The contention here, is that the PL's credibility would be enhanced by making it clear that it is based on best practice and addresses key professional standards, and then getting the participants to explicitly reflect on this.

12.4.3 Conceptualising PCK

If it is argued that transparency regarding the domains of PCK in both Pedagogical Knowledge and Content Knowledge is required when facilitating PL then some attention needs to be paid to the model which will be used to conceptualising these domains. Shulman's (1986) model for PCK, whilst perhaps simplistic and merely a good start, may not carry the complete message about what is required. Therefore, after some careful reflection, adoption of the Hill et al. (2008a) model of Mathematic Knowledge for Teaching (MKT) might be considered. This Researcher, whilst supporting the implementation of the Hill et al. (2008a) model, does have some reservations, as further explored in Chapter 2, regarding: the blurring of distinction between the Common Content Knowledge (CCK) and the Specialised Content Knowledge (SCK); the representation of

the domains in the diagram and the possibility of using the visual cue of region size to determine the importance of one domain of knowledge over another; the use of the term Pedagogical Content Knowledge (PCK) rather than the Pedagogical Knowledge (PK) to describe the domains regarding pedagogical concerns; and the fact that the model does not display the all of the possible interactions between the domains.

12.5 *A Final Note*

Every day teachers prove that they are adaptable people who can take the content knowledge they have and weave pedagogy around it to try to optimise learning. As part of the desire to do the work even better, they attend PL to equip or re-acquaint themselves with necessary content and alternative pedagogy. This PL has costs in terms of a commitment of money and time and the teachers' preparedness to subject themselves to scrutiny, which most find to be uncomfortable. If the teachers are making this commitment, then the PL facilitator is obliged to offer the most effective PL that he or she can. Therefore, the PL needs to give clear indications of what will be offered, deliver the offerings predicated on principles of effective PL, and then give the teachers confidence that what has been delivered is of value and therefore worthy of consideration in their classrooms. Anything short of this means the investment has not returned all possible rewards for the teachers.

If the teachers are to be confident that the PL has been delivered along best practice PL guidelines they need to be acquainted with those practices. They also need to be given the opportunity to become more informed about knowledge for teaching, and in particular, what constitutes mathematical knowledge for teaching. All of these measures will also add to the teachers' active involvement in the process of PL, and thus make it more effective.

CHAPTER 13

13 Bibliography

ACER, & Stephens, M. (2009, June). Numeracy in practice: teaching, learning and using mathematics. No.18, *NEALS*. Melbourne, Vic.: Department of Education and Early Childhood Development.

Australian Association of Mathematics Teachers. (2006). *Standards for excellence in teaching mathematics in Australian schools*. Retrieved from http://www.aamt.edu.au/Activities-and-projects/Standards.

Abd-El-Khalick, F. (2006). Preservice and experienced biology teachers' global and specific subject matter structures: Implications for conceptions of pedagogical content knowledge. *Eurasia Journal of Mathematics, Science and Technology Education 2*(1), 1-29.

Australian Curriculum, Assessment and Reporting Authority (ACARA) (2012). *Australian curriculum mathematics*. Retrieved from http://www.australiancurriculum.edu.au/Mathematics/Curriculum/F-10.

Aguirre, J. M. (2009). Teacher domain-specific beliefs and their impact on mathematics education reform. In J. Maab and W. Schloglmann (Eds.), *Beliefs and attitudes in mathematics education* (pp. 45-59). Rotterdam: Sense Publishers.

Ajzen, I., & Fishbein, M. (1980). *Understanding attitudes and social behavior*. Englewood Cliffs, NJ.: Prentice- Hall.

Alajmi, A. (2009). Addressing computational estimation in the Kuwaiti curriculum: Teachers' views. *Mathematics Teacher Education 12*(4), 263–283.

Aldridge, S., & J. Bobis (2001). Multiple learning contexts: A vehicle for changing preservice primary teachers' mathematical beliefs, knowledge, and practices. Numeracy and beyond. Proceedings of the 24th Annual Conference of MERGA, Sydney, MERGA. pp. 43-49

Aldridge, S., & Bobis, J. (2001). Multiple learning contexts: A vehicle for changing preservice teachers' mathematical beliefs, knowledge and practices. In J. Bobis, B. Perry & M. Mitchelmore (Eds.), *Numeracy and beyond*, *Proceedings of the Twenty-Fourth Annual Conference of the Mathematics Education Research Group of Australasia, Sydney* (pp. 43-49). Sydney: MERGA.

Ambrose, R. (2004). Initiating change in prospective elementary school teachers' orientations to Mathematics teaching by building on beliefs. *Journal of Mathematics Teacher Education 7*(2), 91-119.

Anthony, G., & L. Ding (2011). Teaching and learning fractions: Lessons from alternative example spaces. *Curriculum Matters, 7,* 159-174.

Anthony, G., & Walshaw, M. (2003). Pizza for dinner: "How much" or "how many"? *Mathematics Education Research: Innovation, Networking, Opportunity*, Sydney: MERGA.

Ashlock, R. B. (2002). *Error patterns in computation, using error patterns to improve instruction.* Columbus, Ohio: Merrill Prentice Hall.

Askew, M. (2008). Mathematical discipline knowledge requirements for prospective primary teachers, and the structure and teaching approaches of programs designed to develop that knowledge. In P. Sullivan and W. Terry (s.), *Knowledge and Beliefs in Mathematics Teaching and Teacher Development* (pp. 13-36). Rotterdam: Sense.

Askew, M., Brown, M., Rhodes, V., Johnson, D., & Wiliam, D. (1997). *Effective teachers of numeracy*. Report of a study carried out for the Teacher Training Agency 1995-96 by the School of Education King's College London.

Baba, I., & Iwasaki S. (2003). Teaching fractions: Pre-school children's understanding of fractions. *ICMI Regional Conference - The Third East Asia Regional Conference on Mathematics Education, Shanghai*. Retieved from www.math.ecnu.edu.cn/.../02_EARCOM3_Baba_Imiko_TSG1().doc.

Bailey, D. H., Hoard, M.K., Nugent, L, & Geary, D.C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology 113*(3), 447–455.

Ball, D. L. (1993). Halves, pieces, and twoths: Constructing representational contexts in teaching fractions. In T. P. Carpenter, E. Fennema & T. A. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 157-196). Hillsdale, NJ: Lawrence Erlbaum Associates.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83-104). Westport, CT: Ablex.

Ball, D. L., Hill, H.C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator 29* (1), 14-17.

Ball, D. L., Hoover-Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education 59*(5), 389-407.

Ball, D. T. (1990). Breaking with experience in learning to teach mathematics: The role of a preservice method course. *International Journal of Mathematics Education 10*(2), 10-16.

Bandura, A. (1993). Perceived self-efficacy in cognitive development and functioning. *Educational Psychologist 28*(2), 117-148.

Bandura, A. (1994). Self-efficacy. In V.R. Ramachaudran (Ed.), *Encyclopedia of human behaviour* (vol. 4, pp. 71-81). New York: Academic Press.

Bandura, A. (1997). *Self-efficacy: The exercise of control*. New York: Freeman.

Barkatsas, A. N., & Malone J. (2005). A typology of mathematics teachers' beliefs about teaching and learning mathematics and instructional practices. *Mathematics Education Research Journal 17*(2), 69-90.

Baroody, A., & Hume J. (1991). Meaningful mathematics instruction: The case of fractions. *Remedial and Special Education 12*(3), 54-68.

Barton, B. (2009). Being mathematical, holding mathematics: Further steps in mathematical knowledge for teaching. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing Divides*, *Proceedings of the 32nd Annual Conference of the Mathematics Education Research Group of Australasia*, (Vol 1, pp. 4-9). Palmerston North, NZ: MERGA.

Baxter, J. (1983). Status and trends in mathematics teacher education. In M. Zweng, T. Green, J. Kilpatrick, H. Pollak, & M. Suydam (Eds.), *Proceedings of the Fourth International Congress on Mathematics Education* (pp. 90-92). Berkeley, CA: Bikhauser.

Becker, J. R. (1986). Mathematics attitude of elementary education majors. *Arithmetic Teacher 33*(5), 50-52.

British Educational Communications and Technology Agency (Becta). (2005). *The Becta Review 2005. Evidence on the progress of ICT in education*. Retrieved November 12, 2011, from: http://dera.ioe.ac.uk/1428/1/becta_2005_bectareview_report.pdf

Behr, M., Harel, G., Post, T., & Lesh, R. (1992). Rational number, ratio, and proportion. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 296-333). New York: Macmillan.

Behr, M., Lesh, R., Post, T., & Silver E. (1983). Rational number concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes*, (pp. 91-125). New York: Academic Press.

Behr, M., Wachsmuth, I., & Post, T. (1988). Rational number learning aids: Transfer from continuous models to discrete models. *Focus on Learning Problems in Mathematics 10*(4), 1-17.

Bell, A. W. (1984). Short and long term learning - experiments in diagnostic teaching design. *Eighth International Conference for the Psychology of Mathematics Education*, Sydney.

Bergman, M. M. (2010). On concepts and paradigms in mixed method research. *Journal of Mixed Method Research 4*(3), 171.

Berman, P., McLaughlin, M. W., Bass, G., Pauly, E., & Zellman, G. (1977). *Federal programs supporting educational change: Vol. 7: Factors affecting implementation and continuation* (Report No. R-1589/7-HEW). Santa Monica, CA: Rand. (ERIC Document Reproduction Service No. ED 140 432)

Berry, R. Q., Bol, L., & McKinney, S.E. (2009). Addressing the principles for school mathematics: A case study of elementary teachers' pedagogy and practices in an urban high-poverty school. *International Electronic Journal of Mathematics Education 4*(1), 1-22.

Beswick, K. (2005). It depends on the students: Influencing teachers' beliefs about the ends and means of numeracy teaching. In P. C. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, U. and A. Roche (Eds.), *Building Connections: Research, Theory and Practice: Proceedings of the 28th Annual Conference of the Mathematics Education Research Group of Australasia* (pp.137-144). Sydney: MERGA.

Beswick, K. (2006). The Importance of mathematics teachers' beliefs. *Australian Mathematics Teacher 62*(4), 17-22.

Bezuk, N., & Cramer, K. (1989). Teaching about fractions: What, when, and how? In P. Trafton (Ed.), *National Council of Teachers of Mathematics 1989 Yearbook: New directions for Elementary School mathematics* (pp. 156-167). Reston, VA: National Council of Teachers of Mathematics.

Biddulph, F. (1999). The legacy of schooling: Student teachers' initial mathematical feelings and competence. *Mathematics Teacher Education and Development 1,* 64-71.

Boaler, J. & J. Greeno (2000). Identity, agency and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple Perspectives on Mathematics Teaching and Learning* (pp. 171-200). Westport: Ablex.

Bobis, J. & Cusworth, R. (1994) *Teacher education: An agent of change for attitudes towards mathematics and science/technology.* Paper presented at the Annual Conference of the Australian Association for Research in Australia.

Booker, G., Bond, D., Sparrow, L., & Swan, P. (2004). Teaching primary mathematics. (3rd ed.). Frenchs Forest, NSW.: Pearson Education.

Booth, J. L., & Newton, K.J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology 37*(4), 247–253.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33*(8), 3-15.

Borko, H., Eisenhart, M., Brown, C. A., Underhill R. G., Jones, D., & Agard P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education, 23*(3) 194-222.

Bouchard, T. J. (1976). Unobtrusive measures: An inventory of uses. *Sociological Methods and Research, 4*(3), 267-300.

Boujaoude, S., & Attieh, M. (2008). The effect of using concept maps as study tools on achievement in chemistry. *Eurasia Journal of Mathematics, Science & Technology Education 4*(3), 233-246.

Boulet, G. (1998). Didactical implications of children's difficulties in learning the fraction concept. *Focus on Learning Problems in Mathematics, 20*(4), 19-34.

Bouvier, A. (1987). The right to make mistakes. *Learning of Mathematics, 7*(3), 17-25.

Braswell, J. S., Lutkus, A.D., Grigg, W., Satapau, S., Tay-Lim, B., & Johnson, M. (2001). *The National Assessment of Educational Progress Report*, Institute of Education Sciences**:** 5.

Brown, G. & R. J. Quinn (2007). Fraction proficiency and success in algebra. *Australian Mathematics Teacher, 63*(3), 23-30.

Burrill, G. (1997). Choices and Challenges. *Teaching Children Mathematics, 4*(1), 58-63.

Capraro, R. M. (2005). The mathematics content knowledge role in developing preservice teachers' pedagogical content knowledge. *Journal of Research in Childhood Education, 20*(2), 102-118.

Carnine, D., Jitendra, A.K., & Silbert, J. (1997). A descriptive analysis of mathematics curricular materials from a pedagogical perspective: A case study of fractions. *Remedial and Special Education, 18*(2), 66-81.

Carpenter, S. (2003). Constructivism: A prospective teacher's perspective. *Australian Primary Mathematics Classroom, 8*(1), 29-32.

Carpenter, T., Corbitt, M, Kepner, H., Lindquist, M., & Reys, R. (1981). Decimals: Results and implications from the second NAEP mathematics assessment. *Arithmetic Teacher, 28*(8), 34-37.

Chapin, S. H., & Johnson, A. (2000). *Math matters: Understanding the math you teach*. Sausalito, CA: Math Solutions Publications.

Charalambous, C. Y. (2010). Mathematical knowledge for teaching and task unfolding: An exploratory study*. *The Elementary School Journal, 110*(3), 247-278.

Charalambous, C. Y., Hill, H.C., & Mitchell, R.N. (2012). Two negatives don't always make a positive: Exploring how limitations in teacher knowledge and the curriculum contribute to instructional quality. *Journal of Curriculum Studies, 44*(4), 489–513.

Charalambous, C. Y. & Pitta-Pantazi, D. (2005). Revisiting the theoretical model on fractions: Implications for teaching and research. *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 2, pp. 233-240, Melbourne: PME.

Charles, K., Nason, R.A., & Cooper, T. (1999). *Mathematical analogs and the learning of fractions*. Paper presented at the Australian Association for Research in Education Annual Conference (AARE99), Melbourne, December 1999.

Chinnappan, M. (2005). Children's mapping of part-whole construct of fractions. In *Building Connections: Research, Theory and Practice* - MERGA28 Melbourne. Retrieved from http://www.merga.net.au/documents/RP232005.pdf.

Clarke, D. (1994). Ten principles from research for the professional development of mathematics teachers. In D. B. Aichele, & A.F. Coxford (Eds.), *Professional development for teachers of mathematics: 1994 yearbook* (pp. 37-48). Reston, VA: National Council of Teachers of Mathematics.

Clarke, D.M. (2003). Effective Professional Development: What does research say? (ACU/CEO Research Monograph Series). Melbourne, Australia: Mathematics Teaching and Learning Centre, Australian Catholic University.

Clarke, D. M., & Mitchell, A. (2008). *Pedagogical Content Knowledge Situations (PCKS) questionnaire*. Melbourne: Australian Catholic University.

Clarke, D. M., Mitchell, A., & Roche, A. (2005). Student one-to-one assessment interviews in mathematics: A powerful tool for teachers. In J. Mousley, L. Bragg & C. Campbell (Eds.), *Mathematics: Celebrating achievement. Proceedings of the 100th Annual Conference of the Mathematical Association of Victoria* (pp. 66–80). Melbourne: MAV.

Clarke, D. M., & Roche, A. (2009). Students' fraction comparison strategies as a window into robust understanding and possible pointers for instruction. *Educational Studies in Mathematics, 72*(1), 127-138.

Clements, D., & McMillen, S. (1996). Rethinking concrete manipulatives. *Teaching Children Mathematics, 2*(5), 270-279.

Clements, D. H. (1999). Concrete manipulatives, concrete ideas. *Contemporary Issues in Early Childhood, 1*(1), 45-60.

Cobb, P. (1994). Individual and collective mathematical development: The case of statistical data analysis. *Mathematical Thinking and Learning, 1*(1), 5-43.

Cobb, P., & Jackson, K. (2011). Towards an empirically grounded theory of action for improving the quality of mathematics teaching at scale. *Mathematics Teacher Education and Development, 13*(1), 6-33.

Coburn, C. E. (2003). Rethinking scale: Moving beyond numbers to deep and lasting change. *Educational Researcher, 32*(6), 3-12.

Cohen, D. K. & Hill, H.C. (2000). Instructional quality and classroom performance: The mathematics reform in California. *Teachers College Record, 102*(2), 294-343.

Cohen, D. K., Raudenbush, S., & Ball, D. (2003). Resources, instruction and research. *Educational Evaluation and Policy Analysis, 25*(2), 119-142.

Collins (1986). *Collins Dictionary of the English Language*, Wm: Collins. P. Hanks (Ed.).

Collins, K. M. T., Onwuegbuzie, A. J., & Jiao, Q. G. (2006). Prevalence of mixed methods sampling designs in social science research. *Evaluation and Research in Education 19*(2), 83-101.

Confrey, J. (1990). A review of research on student conceptions in mathematics, science and programming. *Review of Research in Education 16,* 3-56.

Cramer, K., Behr, M., Post, T., & Lesh, R. (1997). *Rational number project: Fraction lessons for the middle grades*. Dubuque, Iowa: Kendall/Hunt Publishing.

Cramer, K., & Henry A. (2002). Using manipulative models to build number sense for addition of fractions. In *Making Sense of Fractions, Ratios and Proportions, 2002 Yearbook, National Council of Teachers of Mathematics,* 2002, pp. 41-48. Reston VA: National Council of Teachers of Mathematics.

Cramer, K., & Wyberg, T. (2009). Efficacy of different concrete models for teaching the part-whole construct for fractions. *Mathematical Thinking and Learning, 11*(4), 226-257.

Cramer, K. A., Post, T.R., & del Mas, R. (2002). Initial fraction learning by fourth- and fifth-grade students: A comparison of the effects of using commercial curricula with the effects of using the rational number project curriculum. *Journal for Research in Mathematics Education, 33*(2), 111–144.

Creswell, J. (2003). *Research design: Qualitative, quantitative, and mixed methods approaches* (2nd ed.). Thousand Oaks, CA: Sage Publications.

Creswell, J. W., Hanson, W.E., Plano Clark, V.L., & Morales, A. (2007). Qualitative research designs: Selection and implementation. *The Counseling Psychologist, 35*(2), 236-265.

Curriculum Council. (1998). *Curriculum framework*. Perth, Western Australia: Curriculum Council.

Curriculum Council. (2005). *Curriculum framework Progress Maps Mathematics*. Perth, Western Australia: Curriculum Council.

Czernaik, C. M., & Schriver, M. (1994). An examination of pre-service science teachers' beliefs and behaviours as related to self-efficacy. *Journal of Science Teacher Education, 5*(3), 77-86.

D'Ambrosio, B. S,. & Mewborn, D.S. (1994). Children's constructions of fractions and their implications for classroom instruction. *Journal of Research in Childhood Education, 8*(2), 150-161.

Daniels, H., Hyde, A., & Zemelman, S. (1993). *Best practice: New standards for teaching and learning in America's schools*. Portsmouth NJ: Heinemann.

Darling-Hammond, L., & Richardson N. (2009). Teacher learning: What matters? *Educational Leadership, 66*(5), 46-53.

Davis, G. E., & Pitkethly, A. (1990). Cognitive aspects of sharing. *Journal for Research in Mathematics Education, 21*(2), 145-153.

Department of Education and Training Western Australia. (2007). Mathematics Syllabus. Perth, Western Australia: DETWA.

Desimone, L. M., Porter, A. C., Garet, M.S., Yoon, K.S., & Birman, B.F. (2002). Effects of professional development on teachers' instruction: Results from a three-year longitudinal study. *Educational Evaluation and Policy Analysis, 24*(2), 81-112.

Donnelly, K. (2007). Australia's adoption of outcomes based education: A critique. *Issues in educational research*, 17(2), 183-206. http://www.iier.org.au/iier17/donnelly.html

Driscoll, M. J. (1983). *Research within reach: Elementary School mathematics and reading.* St Louis: CEMREL.

Ellerbruch L.W. & Payne J.N. (1978). A teaching sequence from initial fraction concepts through the addition of unlike fractions. In M.N. Suydam , & R.E. Rey (Eds.), *Developing computational skills* (pp. 129-147). Reston, VA: NCTM.

Empson, S. B. (2002). Organizing diversity in early fraction thinking. In B. Litwiller & G. Bright (Eds.), *Making Sense of Fractions, Ratios and Proportions* (pp. 29-40). Reston, VA: NCTM.

English, L. & Halford, G. (1995). Mathematics education: Models and processes. Hillsdale, New Jersey: Lawrence Erlbaum.

English, L. D. & Sharry, P.V. (1996). Analogical reasoning and the development of algebraic abstraction. *Educational Studies in Mathematics, 30*(2), 135-157.

Ericsson, K. A. & Lehmann, A.C. (1996). Expert and exceptional performance: Evidence of maximal adaptation to task constraints. *Annual Review of Psychology 47,* 273-305.

Ernest, P. (1988). The attitudes and practices of student teachers of primary school mathematics. *In Proceedings of 12th International Conference on the Psychology of Mathematics Education, Hungary*: PME.

Even, R. (1990). Subject matter knowledge for teaching and the case of functions. *Educational Studies in Mathematics, 21*(6), 521-544.

Fazio, L., & Siegler, R. (2011). *Educational Practices Series 22: Teaching Fractions*: UNESCO/IBE.

Feiman-Nemser, S. (2001). From preparation to practice: Designing a continuum to strengthen and sustain teaching. *Teachers College Record, 103*(6), 1013-1055.

Fishbein, M., & Ajzen, I. (1975). Belief, attitude, intention and behavior: An introduction to theory and research. Reading, MA: Addison-Wesley.

Fong, H. (1995). Schematic model for categorising children's errors in solving a ratio and proportion problem. *Hiroshima Journal of Mathematics Education, 3,* 15-29.

Forrester, T., & Chinnappan, M. (2010). The predominance of procedural knowledge in fractions. In *Shaping the future of mathematics education: Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, Fremantle*: MERGA.

Fotoples, R. M. (2000). Overcoming math anxiety. *Kappa Delta Pi Record, 36*(4), 149-151.

Frechtling, J., & Sharp, L. (1997). *User-friendly handbook for mixed method evaluations*. Retrieved from http://www.nsf.gov/pubs/1997/nsf97153/start.htm.

Freebody, P. (2006). *Early-stage use of The Le@rning Federation's learning objects in schools: Results of a field review*: The Le@rning Federation.

Furinghetti, F., & Morselli, F. (2009). Leading beliefs in the teaching of proof. In J. Maab & W. Schloglmann (Eds.), *Beliefs and Attitudes in Mathematics Education*. (pp. 59-75). Rotterdam: Sense Publishers.

Furinghetti, F., & Pehkonen, E. (2002). Rethinking characterizations of belief. In G. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp.39-57). Dordrecht: Kluwer.

Gabriele, A. & E. Joram (2007). Teachers' reflections on their reform-based teaching in mathematics: Implications for the development of teacher self-efficacy. *Action in Teacher Education 29*(3), 60-74.

Gentner, D. (1983). Structure mapping: A theoretical framework for analogy. *Cognitive Scientist,* 155-170.

Gentner, D. (1988). Metaphor as structure mapping: The relational shift. *Child Development, 59*(1), 47-59.

Gess- Newsome, J. (1999). Secondary teachers' knowledge and beliefs about subject matter and their impact on instruction. In J. Gess- Newsome & N. Lederman (Eds.), *Examining Pedagogical Content Knowledge* (pp. 51-94). Dordecht: Kluwer Academic Publishers.

Gess- Newsome, J. & N. Lederman (1995). Biology teachers' perception of subject matter structure and its relationship to classroom practice. *Journal of Research in Science Teaching, 32*(3), 301-325.

Gess-Newsome, J. (2003). Implications of the definitions of knowledge and beliefs on research and practice in science teacher education. Annual Meeting of the National Association for Research in Science Teaching. Philadelphia, Pennsylvania.

Goldin, G., Rosken, B., & Torner, G. (2009). Beliefs - no longer a hidden variable in mathematical teaching and learning processes. In J. Maab & W. Schloglmann (Eds.), *Beliefs and Attitudes in Mathematics Education* (pp.1-18). Rotterdam: Sense.

Goldman, K., & Schmalz, K. (2005). Yes, you can take it with you! Transfer of learning: From workshop to workplace. *Health Promotion and Practice, 6*(5), 5-7.

Good, T.L, & Brophy, J.E. (1990). *Educational Psychology: A realistic approach* (4th Ed.), White Plains, NY: Longman.

Gorman, G. E., & Clayton, P. (1997). *Qualitative research for the information professional - A practical handbook*. London: Library Association Publishing.

Goulding, M., Rowland, T., & Barber, P. (2002). Does it Matter? Primary teacher trainees' subject knowledge in mathematics. *British Educational Research Journal, 28*(5), 689-704.

Greenes, C. (1996). Investigations: Vehicles for learning and doing mathematics. *Journal of Education, 178*(2), 35-50.

Gresham, G. (2008). Mathematics anxiety and mathematics teacher efficacy in elementary pre-service teachers. *Teaching Education, 19*(3), 171-184.

Griffin, M. (2003). Using critical incidents to promote and assess reflective thinking in pre-service teachers. *Reflective Practice 4*(2), 486-499.

Groff, P. (1996). Is teaching fractions a waste of time? *Clearing House, 69*(3), 177-179.

Grootenboer, P. J. (2003). Facilitating affective change with pre-service primary teachers. *Annual Conference of the Mathematics Educational Research Group of Australia, Sydney*: MERGA.

Grootenboer, P. J. (2008). Mathematical belief change in prospective primary teachers. *Journal of Mathematics Teacher Education, 11*(6), 467-497.

Grouws, D., & Schultz, K. (1996). Mathematics teacher education. In J. Sikula (Ed.) *Handbook of Research on Teacher Education, (*2nd ed.): Macmillan.

Guskey, T. (2003). Analyzing lists of the characteristics of effective professional development to promote visionary leadership. *NASSP Bulletin, 87*(637), 4-18.

Haberman, M. (2010). 11 consequences of failing to address the 'Pedagogy of Poverty'. *Phi Delta Kappan, 92*(2), 45.

Hackling, M., Peers, S., & Prain, V. (2007). Primary connections: Reforming science teaching in Australian primary schools. *Teaching Science, 53*(3), 12-16.

Hackling, M., Smith, P., & Murcia, K. (2011). Enhancing classroom discourse in primary science: The puppets project. *Teaching Science, 57*(2), 18-25.

Hager, P., & Hodkinson, P. (2009). Moving beyond the metaphor of transfer of learning. *British Educational Research Journal, 35*(4), 619-638.

Halcomb, E. J., & Davidson, P.M. (2006). Is verbatim transcription of interview data always necessary? *Applied Nursing Research, 19*(1), 38-42.

Hallett, D., Nunes, T., & Bryant, P. (2010). Individual differences in conceptual and procedural knowledge when learning fractions. . *Journal of Educational Psychology, 102*(2), 395-406.

Hannula, M. S. (2002). Attitude towards mathematics: Emotions, expectations and values. *Educational Studies in Mathematics, 49*(1), 25-46.

Hanson, D. (2001). *Understanding Fractions (Grades 5 - 8)*: Saskatchewan Education.

Hanson, W. B., Creswell, J. W., Plano-Clark, Y. L., Petska, K. S., & Creswell, D. (2005). Mixed methods research designs in counseling psychology. *Journal of Counseling Psychology, 52*(2), 224-235.

Hart, L. C., Smith, S.Z., Swars, S.L., & Smith, W.E. (2009). An examination of research methods in mathematics education (1995-2005). *Journal of Mixed Method Research, 3*(1), 26-41.

Hatfield, M. M. (1994). Use of manipulative devices: Elementary school cooperating teachers selfreporting. *School Science and Mathematics, 94*(6), 303-309.

Hattie, J. A. (2003). *Teachers make a difference: What is the research evidence?* Retrieved from http://www.acer.edu.au/documents/RC2003_Hattie_TeachersMakeADifference.pdf.

Hattie, J. A. (2005). *What is the nature of evidence that makes a difference to learning? Using data to support learning*. Melbourne, Retrieved from http://www.acer.edu.au/documents/RC2005_Hattie.pdf.

Hawker, S., & Cowley, C. (Eds.), (1998). *The Oxford quick reference dictionary & thesaurus*.: Oxford University Press.

Helme, S., & Stacey, K. (2000). Can minimal support for teachers make a difference to students' understanding of decimals? *Mathematics Teacher Education and Development 2*, 105-120.

Herl, H. E., Baker, E.L., & Niemi, D. (1996). Construct validation of an approach to modeling cognitive structure of U.S. history knowledge. *The Journal of Educational Research, 89*(4), 206-218.

Hew, K. F., & Brush, T. (2006). Integrating technology into K-12 teaching and learning: Current knowledge gaps and recommendations for future research. *Educational Technology, Research and Development 55*, 223-252.

Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education, 30*(2), 122-147.

Higgins, J., & Parsons, R. (2009). A successful professional development model in mathematics. *Journal of Teacher Education, 60*(3), 231-242.

Higgins, J., & Parsons, R. (2011). Professional learning opportunities in the classroom: Implications for scaling up system-level professional development in mathematics. *Mathematics Teacher Education and Development, 13*(1), 54-76.

Hill, H., & Ball, D.L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education, 35*(5), 330-351.

Hill, H. C., & Ball, D.L. (2009). The curious and crucial case of Mathematical Knowledge for Teaching *Phi Delta Kappan 91*(2), 68-71.

Hill, H. C., Ball, D.L., & Schilling, S. (2008a). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372-400.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., et al. (2008b). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430–511.

Hill, H. C., Rowan, B., & Ball, D.L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Holt, J. (1982). *How children fail*. New York: Dell Publishing Co.

Hough, S., O'Rode, N.,Terman. N., & Weissglass, J. (2007). Using concept maps to assess change in teachers' understandings of algebra: A respectful approach. *Journal of Mathematics Teacher Education,* 10(1), 23-41.

Hoyles, C., Healy, L., & Sutherland, R. (1991). Patterns of discussion between pupil pairs in computer and non computer environments. *Journal of Computer Assisted Learning, 7*(4), 210-228.

Hoz, R., Tomer, Y., & Tamir, P. (1990). The relations between disciplinary and pedagogical knowledge and the length of teaching experience of biology and geography teachers. *Journal of Research in Science Teaching, 27*(10), 973-985.

Hubber, P,, Tytler, R., & Haslam, F. (2010). Teaching and learning about force with a representational focus: Pedagogy and teacher change. *Research in Science Education*, *40*(1), 5-28.

Hungerford, T. W. (1994). Future Elementary teachers: The neglected constituency. *The American Mathematical Monthly 10*(1), 15-21.

Hunting, R. P. (1991). The social origins of pre-fraction knowledge in three year olds. In R. P. Hunting & G. Davis (Eds.), *Early Fraction Learning* (pp.55-72). New York, Springer-Verlag.

InPraxis (2006). *Professional learning communities: An exploration*. Retrieved from http://education.alberta.ca/media/618576/prof_learng_communities_2006.pdf.

Jamieson-Proctor, R., & Byrne, C. (2008). *Primary teachers' beliefs about the use of mathematics textbooks*. Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia, Brisbane, MERGA.

Johanning, D. I. (2008). Learning to use fractions: Examining Middle School students' emerging fraction practices. *Journal for Research in Mathematics Education 39*(3), 281-310.

Johansson, M. (2006). *Teaching mathematics with textbooks. A classroom and curricular perspective*. Luleå, Luleå University of Technology. Retrieved from http://epubl.ltu.se/1402-1544/2006/23/LTU-DT-0623-SE.pdf.

Johnson, D. C., Hodge, J., & Adhami, M. (2007). Professional Development from a cognitive and social standpoint. In A. Millet, M. Brown & M. Askew (Eds.), *Primary Mathematics and the Developing Professional* (pp. 185-217). Dordrecht, Springer.

Johnson, R.B., Onwuegbuzie, A.J., & Turner, L. A. (2007). Toward a definition of mixed methods research. *Journal of Mixed Methods Research, 1*(2), 112-133.

Jordan, L., Miller, M.D., & Mercer, C.D. (1998). The effects of concrete to semi concrete to abstract instruction in the acquisition and retention of fraction concepts and skills. *Learning Disabilities: A Multidisciplinary Journal 9,* 115-122.

Kajander, A. (2005). Moving towards conceptual understanding in the preservice classroom: A study of learning fractions. Towards conceptual understanding in the preservice classroom: A study of evolving knowledge and values. 27th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Roanoke: University of Virginia.

Kaur, B. (2001). TIMSS and TIMSS-R - Performance of Grade 8 Singaporean Students. 2001 A Mathematical Odyssey. C. Vale, J. Horwood and J. Roumeliotis. Brunswick, *Mathematical Association of Victoria. 1***,** 132-144.

Keady, W. (2007). The impact of professional learning on science teachers' beliefs about assessment and their assessment practice. School of Education. Perth, Edith Cowan University. **Doctor of Philosophy:** 299.

Kelleher, J. (2003). A model for assessment driven Professional Development. *Phi Delta Kappan 84*(10), 751-756.

Kennedy, M. (1998). *Form and substance in in-service teacher education*. Madison, WI, National Institute for Science Education.

Kieren, T.E. (1980). The rational number construct—Its elements and mechanisms. In T.E.Kieren (Ed.), *Recent research on number learning* (pp. 125–149). Columbus, OH: ERIC/SMEAC. (ERIC Document Reproduction Service No. ED 212 463).

Kieren, T. E. (1988). Personal knowledge of rational numbers: Its intuitive and formal development. In J. Hiebert & M. Behr (Eds.), *Number Concepts and Operations in the Middle Grades* (162-181). Hillsdale NJ, Lawrence Erlbaum Associate.

Klingner, J. K., & Boardman, A.G. (2011). Addressing the "research gap" in Special Education through Mixed Methods. *Learning Disability Quarterly 34*(3), 208-218.

Kouba, V. L., Zawojewski, J.S., & Strutchens, M. E. (1997). What do students know about number and operations? Results from the sixth mathematics assessment of the National Assessment of Educational Progress. P. A. Kenney & E. A. Silver. Reston, VA**:** 97-140.

Kulm, G. (19800, Research on Mathematics Attitude, in R.J. Shumway (Ed.), *Research in mathematics education* (pp. 356-387). National Council of Teachers of Mathematics, Reston, VA,

Kurtz, K. J., Miao, C.H., & Gentner, D. (2001). Learning by analogical bootstrapping. *The Journal of the Learning Sciences 10*(4), 417-446.

Lamb, J., Cooper, T., & Warren, E. (2007). Combining teaching experiments and Professional Learning: Conflicts between research and teacher outcomes. *Mathematics Education Research Journal 19*(3), 73-92.

Lampert, M., & Ball, D.L. (1998). *Teaching, multimedia, and mathematics: Investigations of real practice*. New York, Teachers College Press.

Leahey, E. (2007). Convergence and confidentiality? Limits to the implementation of mixed methodology. *Social Science Research* 36(1): 149-158.

Leatham, K. R. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education 9,* 91-102.

Liljedahl, P. (2009). Teachers' insights into the relationship between beliefs and practice. In J. Maab & W. Schloglmann (Eds.), *Beliefs and Attitudes in Mathematics Education* (pp. 33-43). Rotterdam, The Netherlands, Sense Publishers.

Loucks-Horsley, Hewson, S.P.W., Love, N., & Stiles, K.E. (1998). *Designing Professional Development for teachers of Science and Mathematics.* Thousand Oaks, CA, Corwin Press.

Loucks-Horsley, Hewson, S.P.W., Love, N., & Stiles, K.E. (2003). *Designing Professional Development for Teachers of Science and Mathematics (2nd ed.).* Thousand Oaks, California, Corwin Press.

Loucks-Horsley, S., & Matsumoto, C. (1999). Research on Professional Development for teachers of Mathematics and Science: The state of the scene. *School Science and Mathematics 99*(5), 259-271.

Loughran, J., Mulhall, P., & Berry, A. (2004). In search of Pedagogical Content Knowledge in Science: Developing ways of articulating and documenting professional practice. *Journal of Research in Science Teaching 41*(4), 370-391.

Ma, L. (1999). *Knowing and Teaching Elementary School Mathematics*. Mahwah, NJ, Lawrence Erlbaum Associates.

MacIntyre-Latta, M., & Kim, J.-H. (2010). Narrative inquiry invites Professional Development: Educators claim the creative space of praxis. *The Journal of Educational Research 103,*137–148

Mack, N. K. (1995). Confounding whole number and fraction concepts when building on informal knowldge. *Journal for Research in Mathematics Education 26,* 422-441.

Mamede, E., Nunes, T., & Bryant, P. (2005). The equivalence and ordering of fractions in part-whole and quotient situations. In Chick, H. L. & Vincent, J. L. (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, pp. 281-288. Melbourne: PME.

Markham, K., Mintzes, J., & Jones, G. (1994). The Concept Map as a research and evaluation tool: Further evidence of validity. *Journal of Research in Science Teaching 31*(1), 91-101.

Marsh, L. G., & Cooke, N.L. (1996). The effects of using manipulatives in teaching math problem solving to students with Learning Disabilities. *Learning Disabilities Research and Practice 11*(2), 58-64.

Martinie, S. (2005). *Assessing conceptual understanding of rational numbers and constructing a model of the interrelated skills and concepts*. Retrieved November, 2011, from http://www.math.ksu.edu/math791/finalpaper/sherriterm2.pdf.

Mastropieri, M. A., Scruggs, T.E., & Shiah, S. (1991). Mathematics instruction for Learning Disabled students: A review of research. *Learning Disabilities Research and Practice 6,* 89-98.

McClure, J., Sonak, B., & Suen, H.K. (1999). Concept map assessment of classroom learning: reliability, validity, and logistical practicality *Journal of Research in Science Teaching 36*(4): 475-492

McCotter, S. S. (2001). Collaborative groups as Professional Development. *Teaching and Teacher Education 17*, 685-704.

McGaw, B. (2010). Blueprint sets high standard for education in the future *The Daily Telegraph.* Sydney.

McGuire, K. M. (2004). Exploring an interdisciplinary strategy for teaching fractions to second graders. *International Journal for Mathematics Teaching and Learning,* 1- 23.

McLellan, E., MacQueen, K.M., & Neidig, J.L. (2003). Beyond the qualitative Interview: Data preparation and transcription. *Field Methods 15*(1), 63-84.

McLeod, D. B. (1992). Research on affect in Mathematics Education: A reconceptualisation. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp.575-596). New York, McMillan Publishing Company.

Meagher, M. (2002) *Teaching fractions: New methods, new resources*. ERIC/CSMEE DOI: ED478711

Meaney, T., & Lange, T. (2010). Pre-service students' responses to being tested on their Primary School mathematical knowledge. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education: Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia.* Fremantle: MERGA.

Mestre, J. (1989). *Hispanic and Anglo students' misconceptions in Mathematics*. Retrieved 14th of June 2012, from http://www.eric.ed.gov/PDFS/ED313192.pdf

Miles, M. B,. & Huberman, A.M. (1984). Drawing valid meaning from qualitative data: Toward a shared craft. *Educational Researcher 13*(5): 20-30.

Millet, A., Brown, M., & Askew, M. (2007). *Primary mathematics and the developing professional*. Dordrecht, Springer.

Millsaps, G.M., & Reed, M.K. (1998). *Curricula for teaching fractions*. ERIC Digest. Information Analyses. Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education. (ERIC Document Reproduction Service No. ED433184)

Mitchell, A. (2005). Measuring fractions. In P. C. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Research, theory and practice* (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia, Melbourne, pp. 545- 552). Melbourne, Australia: MERGA.

Miura, I. T. (2001). The influence of Language on Mathematical Representations. In F. R. Curcio (Ed.), *The roles of representation in school mathematics: 2001 yearbook.* Reston: National Council of teachers of Mathematics.

Moseley, B., Okamoto, Y., & Ishida, J. (2007). Comparing US and Japanese elementary school teachers' facility for linking rational number representations. *International Journal of Science and Mathematics Education 5*, 165-185.

Moss, J., & Case, R. (1999). Developing children's understanding of the rational numbers: A new model and experimental curriculum. *Journal of Research in Mathematics Education 30*(2), 122-147.

Moyer, P. S., J. J. Bolyard, et al. (2001). Virtual Manipulatives in the K-12 Classroom. International Conference on New Ideas in Mathematics Education, Palm Cove.

Moyer, P. S., Bolyard, J.J., &Spikell, M.A. (2002). What are Virtual Manipulatives? *Teaching Children Mathematics (February)*, 372-377.

Moyer-Packenham, P. S., Salkind, G., & Bolyard, J.J. (2008). Virtual manipulatives used by K-8 teachers for mathematics instruction: Considering mathematical, cognitive, and pedagogical fidelity. *Contemporary Issues in Technology and Teacher Education 8*(3), 202-218.

Mathematical Sciences Education Board (1989). *Everybody Counts*. Washington, DC, National Academy Press, 1989

Naiser, E. A., Wright, W.E., & Capraro, R.M. (2004). Teaching fractions: Strategies used for teaching fractions to middle grades students. *Journal of Research in Childhood Education, 18*(3), 193-198.

Nathan, M. J., & Petrosino, A. (2003). Expert blind spots among pre-service Mathematics and Science teachers. *American Educational Research Journal 40*(4), 905-928.

National Curriculum Board, (2009). The shape of the Australian Curriculum. 1-16

Newstead, K., & Murray, H. (1998). *Young students' constructions of fractions*. Paper presented at the PME22, vol. 3, 295-302, Stellenbosch.

Niemi, D. (1996). A fraction is not a piece of pie: Assessing exceptional performance and deep understanding in Elementary School mathematics. *Gifted Children Quarterly 40*, 70-80.

Nilsson, P. (2008). Teaching for understanding: The complex nature of pedagogical content knowledge in preservice education. *International Journal of Science Education 30*(10), 1281-1299.

Novak, J. D., & Cañas, A.J. (2008). The theory underlying Concept Maps and how to construct and use them. Technical Report IHMC CmapTools 2010. Retrieved from http://cmap.ihmc.us/Publications/ResearchPapers/TheoryUnderlyingConceptMaps.pdf.

Nunes, T., & Bryant, P. (2009). *Paper 3: Understanding rational numbers and intensive quantities*. Key Understandings in Mathematics Learning. Retrieved 15/12/10, 2010, from http://www.nuffieldfoundation.org/sites/default/files/P3_amended_FB2.pdf.

Olivier, A. (1989). Handling pupils' misconceptions. Thirteenth National Convention on Mathematics, Physical Science and Biology Education, Pretoria.

Pagnucci, G. S. (2004). *Living the narrative life: Stories as a tool for meaning making*. Portsmouth, NH, Boynton/Cook Heinemann.

Pape, S. J., & Tchoshanov, M.A. (2001). The role of representation(s) in developing mathematical understanding. *Theory Into Practice 40*(2), 118-127.

Papert, S. (1980). Mindstorms: Children, computers and powerful ideas. New York, Basic Books.

Park, S., & Oliver, J.S. (2008). Revisiting the conceptualisation of Pedagogical Content Knowledge (PCK): PCK as a conceptual tool to understand teachers as professionals. *Research in Science Education 38*, 261-284.

Parsons, M., & Stephenson, M. (2005). Developing reflective practice in student teachers: collaboration and critical partnerships. *Teachers and Teaching: Theory and Practice 11*(1), 95-116.

Penuel, W. R., Fishman, B. J., Yamaguchi, R., & Gallagher, L. P. (2007). What makes professional development effective? Strategies that foster curriculum implementation. *American Educational Research Journal, 44*(4), 921–958.

Perry, B., & Howard, P. (1997). Manipulatives in primary mathematics: Implications for learning and teaching. *Australian Primary Mathematics Classroom 2*(2), 25-30.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257-315). United States: Information Age Publishing.

Pitkethly, A., & Hunting, R. (1996). A review of recent research in the area of initial fraction concepts. *Educational studies in Mathematics 30*, 5 - 38.

Post, T., & Cramer K. (1987). Research into practice: Children's strategies in ordering rational numbers. *Arithmetic Teacher 35*(2), 22-25.

Puchner, L., Taylor, A., O'Donnell, B., & Fick, K. (2008). Teacher learning and mathematics manipulatives: A collective Case Study about teacher use of manipulatives in Elementary and Middle School mathematics lessons. *School Science and Mathematics 108*(7): 313-325.

Punch, K. F. (1998). *Introduction to Social Research: Quantitative and qualitative approaches*. London, Sage.

Raymond, A. M. (1997). Inconsistency between a beginning Elementary School teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education 28*(5), 550-576.

Reimer, K., & Moyer, P.S. (2005). Third-graders learn about fractions using virtual manipulatives: A classroom study. *Journal of Computers in Mathematics and Science Teaching 24*(1), 5-25.

Reys, B., Reys, R., & Chávez, O. (2004). Why mathematics textbooks matter. *Educational Leadership 61*(5), 61-66

Roche, A., & Clarke, D. (2009). Making sense of partitive and quotitive division: A snapshot of teachers' Pedagogical Content Knowledge. Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, Palmerston North, New Zealand, MERGA.

Rogers, P. (2007). Teacher professional learning in mathematics: An example of a change process. Mathematics: Essential research, essential practice: 30th annual conference of the Mathematics Education Research Group of Australasia, Adelaide, MERGA.

Rogers, P. (2007). Teacher professional learning in mathematics: An example of a change process. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice. Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia*. (pp. 631-640). Adelaide: MERGA.

Ruiz-Primo, M. A., & Shavelson, R.J. (1996). Problems and issues in the use of concept maps in science assessment. *Journal of Research in Science Teaching 33,* 569-600

Saenz-Ludlow, A. (1995). Anne's fraction schemes. *Journal for Research in Mathematics Education 25*(1), 50-85.

Schon, D. (1987). *Education and the reflective practitioner*. San Fransisco, Jossey Bass.

Schramm, W. (1971), *Notes on Case Studies of Instructional Media Projects*, Academy for Educational Development, Washington, DC. Retrieved from http://pdf.usaid.gov/pdf_docs/PNAAD824.pdf

Schuck, S., & Grootenboer, P. J. (2004). Affective issues in mathematics education. In B. Perry, C. Diezmann, & G. Anthony (Eds.), *Review of mathematics education in Australasia 2000-2003* (pp. 53-74). Flaxton, QLD: Post Pressed.

Sebesta, L. M., & Martin, S.R.M. (2004). Fractions: Building a foundation with concrete manipulatives. *Illinois School Journal* (Spring 2004), 3 - 21.

Sharp, J. M. (1998). *Results of using writing tasks to enhance fourth grade children's acquisition of fraction knowledge*. Mid Western Educational Research Association, Chicago Illinois.

Sharp, J. M., Garofalo, J., & Adams, B. (2002). Children's development of meaningful fraction algorithms; A kid's cookies and a puppy's pill. In B. Litwiller & G. Bright (Eds.), *Making Sense of Fractions, Ratios and Proportions (pp.18-28)*. Reston, Virginia.

Shield, M. (1991). Mathematics textbooks: how are they used? *Australian journal of reading 14*(1), 60-68.

Shulman, L. S. (1986). Those who understand, knowledge growth in teaching. *Educational Researcher 15*(2), 4-14.

Siegler, R. S., Duncan, G.J., Davis-Kean, P.E., Duckworth, K., Claessens, A., Engel, M. et al. (2012). Early predictors of High School mathematics achievement. *Psychological Science 23*(6), 547-553.

Siemon, D. (2003). The missing link in building fraction knowledge and confidence. *Australian Mathematics Teacher 59*(2), 22-24.

Silverman, J. & Thompson, P.W. (2008). Towards a framework for the development of mathematical knowledge for teachers. *Journal of Mathematics Teacher Education 11*, 499-511.

Sims-Knight, J. E., Upchurch, R.L., Pendergrass, R., Meressi, T., Fortier, P. Tchimev, P., et al. (2004). *Using concept maps to assess Design Process Knowledge*. 34th ASEE/IEEE Frontiers in Education Conference, Savannah, GA. Retrieved from http://fie-conference.org/fie2004/papers/1167.pdf

Singer, F. M. (2007). Beyond conceptual change: Using representations to integrate domain-specific structural models in learning mathematics. *Mind, Brain, and Education 1*(2), 84-97.

Smith, J. (1996). Efficacy and the teaching of mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education 27*(4), 387-402.

Smith, J. P. (2002). The development of students' knowledge of fractions and ratios. In B. Litwiller and G. Bright (Eds.), *Making Sense of Fractions, Ratios, and Proportions* (pp. 3-17). Reston, VA.

Smith, L. (2006) Learning across communities of practice: How postgraduate students cope with returning to higher education in an international setting, in *Critical Visions, Proceedings of the 29th HERDSA Annual Conference, Western Australia, 10-12 July 2006: pp 340-346.*

Sowder, J. T., Phillip, R.A., Armstrong, B.E., & Schappelle, B.P. (1998). *Middle-grade teachers' mathematical knowledge and its relationship to instruction*. Albany, NY: State, University of New York Press.

Sowell, E. J. (1989). Effects of manipulative materials in mathematical instruction. *Journal for Research in Mathematics Education 20*(5), 498-505.

Spicer, J. (2000). Virtual manipulatives: A new tool for hands on Math. ENC Focus **7**(4), 14-15. (available at http://www.enc.org/focus/documents/0,1948,FOC-001754-index,00.shtm)

Spillane, J. P. (1999). External reform Initiatives and teachers' efforts to reconstruct their practice: The mediating role of teachers' Zones of Enactment. *Journal of Curriculum Studies 31*(2), 143-175.

Steffe, L. P. (1990). On the knowledge of Mathematics teachers. In R. B. Davis, M. C.A. & N. Noddings (Eds.), *Constructivist Views on Teaching and Learning Mathematics* (pp. 167-186) Reston VA.

Stein, M., Remillard, J., & Smith, M. (2007). How curriculum influences students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 557-628). Charlotte, Information Age.

Steinle, V. (2004). Detection and remediation of decimal misconceptions. In B. Tadich, S. Tobias, C. Brew, B.Beatty, & P. Sullivan (Eds.), *Towards Excellence in Mathematics* (pp. 460-478). Brunswick: The Mathematical Association of Victoria.

Stevenson, H. W. (1992). Learning from Asian schools. *Scientific America 267*, 70-76.

Stevenson, H. W., Chuansheng, C., & Lee, S. (1993). Mathematics achievement of Chinese, Japanese and American children: Ten years later. *Science 257*, 53-58.

Stevenson, H. W., & Stigler, J.W. (1992). *The Learning Gap*. New York, Summit.

Streefland, L. (1991). *Fractions in realistic Mathematics Education*. Dordrecht, Kluwer Academic.

Strike, K. A. (1983). Misconceptions and Conceptual Change: Philosophical Reflections on the Research Program. In H.Helm and J.Novak (Eds.), *Proceedings of the International Seminar on Misconceptions in Science and Mathematics* (pp 85-97). Ithaca, Cornell University.

Stuart, C., & Thurlow, D. (2000). Making it their own: Preservice teachers' experiences, beliefs and classroom practices. *Journal of Teacher Education 51*(2), 113-121.

Sullivan, P. (1987). The impact of a pre-service mathematics education course on beginning primary teachers. . *Research in Mathematics Education in Australia,* August, 1-9.

Supovitz, J. A. (2001). Translating teaching practice into improved student achievement. In S. Fuhrman (Ed.), *From the capital to the classroom: Standards-based reform in the States* (pp.81-98). Chicago, University of Chicago Press.

Supovitz, J. A., Mayer, D.P., & Kahle, J. B. (2000). Promoting inquiry based instructional practice: The longitudinal impact of Professional Development in the context of systemic reform. *Educational Policy 14*(3), 331-357.

Supovitz, J. A., & Turner, H.M. (2000). The effects of Professional Development on Science teaching practices and classroom culture. *Journal of Research in Science Teaching 37*(9), 963-980.

Suydam, M. N. (1986). Research Report: Manipulative materials and achievement. *Arithmetic Teacher 33*(10), 32.

Swackhamer, L. E., Koellner, K., Basile, C., & Kimbrough, D. (2009). Increasing the self-efficacy of inservice teachers through content knowledge. *Teacher Education Quarterly, 37*(3), 63-78.

Swan, J. (2013, February 22). Education about 'values' not money under Coalition: Pyne. The Sydney Morning Herald. Retrieved from http://www.smh.com.au/federal-politics/political-news/education-about-values-not-money-under-coalition-pyne-20130227-2f4tj.html#ixzz2aVIfY862

Swan, P., & Sparrow, L. (2004). Hands On-Heads On: Using manipulatives in the classroom. Towards Excellence in Mathematics, Victoria, MAV.

Swars, S., Daane, C., & Giesen, J. (2006). Mathematics anxiety and mathematics teacher efficacy: What is the relationship in Elementary pre-service teachers? *School Science and Mathematics 106*(7), 306-315.

Swars, S., Hart, L. C., Smith, S. Z., Smith, M. E., & Tolar, T. (2007). A longitudinal study of elementary pre-service teachers' mathematics beliefs and content knowledge. *School Science and Mathematics, 107*(9), 325-335.

Szydlik, J. E., Szydlik, S. D., & Benson, S. R. (2003). Exploring changes in pre-service elementary teachers' mathematical beliefs. *Journal of Mathematics Teacher Education, 6*, 253-279.

Taber, S. B. (1999). Understanding multiplication with fractions, an analysis of problem features and student strategies. *Focus on Learning Problems in Mathematics 21*(2), 1-25.

Tashakkori, A., & Teddlie, C. (1998). *Mixed methodology: Combining qualitative and quantitative approaches*. Thousand Oaks, CA, Sage.

Tashakkori, A., & Creswell, J. (2007). The New Era of Mixed Methods. *Journal of Mixed Method Research 1*, 3-7.

Teddlie, C., & Tashakkori, A. (2009). *Foundations of Mixed Methods Research integrating quantitative and qualitative approaches in the Social and Behavioral Sciences*. Thousand Oaks, CA, SAGE Publications, Inc.

Thomas, E. (2007). Thoughtful planning fosters learning transfer. *Adult Learning 18* (3-4), 4-8.

Thompson, P. W,. & Lambdin, D. (1994). Research into practice: Concrete materials and teaching for mathematical understanding. *Arithmetic Teacher 41*(9), 556-558.

Thorndike, E. (1922). *The Psychology of Arithmetic*. New York., Macmillan.

Tirosh, D. (1990). Inconsistencies in students' mathematical constructs. *Focus on Learning Problems in Mathematics 12*(3-4), 111-129.

Tirosh, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions. *Journal for Research in Mathematics Education 31*, 5-26.

Tirosh, D., Fischbein, E., Graeber, A.O., & Wilson, J.W. (1998) *Prospective Elementary teachers' conceptions of rational numbers*. Retrieved from http://jwilson.coe.uga.edu/Texts.Folder/Tirosh/Pros.El.Tchrs.html

Toluk-Uçar, Z. (2009). Developing pre-service teachers understanding of fractions through problem posing *Teaching and Teacher Education 25*(1), 166-175

Tripp, D. (1993). *Critical incidents in teaching: Developing professional judgement*. London, Routledge.

Tsao, Y. L. (2005). The number sense of preservice Elementary School teachers *College Student Journal 39*(4), 647-679.

Tschannen-Moran, M., & Woolfolk Hoy, A. (2007). The differential antecedents of self-efficacy beliefs of novice and experienced teachers. *Teaching and Teacher Education 23*, 944-956.

Tschannen-Moran, M., Woolfolk Hoy, A., & Hoy, K.A. (1998). Teacher efficacy: Its meaning and measure. *Review of Educational Research 68*(2), 202-248.

Tzur, R. (2000). An integrated research on children's construction of meaningful, symbolic, partitioning related conceptions, and the teacher's role in fostering that learning. *Journal of Mathematical Behavior 18*(2), 123-147.

Usiskin, Z. (1979). The future of fractions. *Arithmetic Teacher 27*, 18-20.

Usiskin, Z. P. (2007). The future of fractions. *Arithmetic Teacher 12*(7), 366-369.

van Es, E., & M. G. Sherin, M.G. (2010). The influence of video clubs on teachers' thinking and practice. *Journal of Mathematics Teacher Education 13*, 155-176.

Van de Walle, J., & Folk S. (2005). *Elementary and Middle School mathematics*. Toronto, Canada, Pearson Education.

Veal, W. R., & MaKinster J. G. (1999) Pedagogical Content Knowledge taxonomies. *Electronic Journal of Science Education 3*, Retrieved 28th of November 2011, from http://ejse.southwestern.edu/article/viewArticle/7615/5382

Viadero, D. (2007). Studies find that use of learning toys can backfire. *Education Week 26*(34), 12-13.

Vincent, J., & Stacey, K. (2008). Do mathematics textbooks cultivate shallow teaching? Applying the TIMSS Video Study criteria to Australian eighth-grade mathematics textbooks. *Mathematics Education Research Journal 20*(1), 81 - 106.

Watanabe, T. (2002). Representations in teaching and learning fractions. *Teaching Children Mathematics 8*(8), 457-463.

Watt, M. G. (2004). The role of curriculum resources in three countries: The impact of national curriculum reforms in the United Kingdom, the United States of America, and Australia. Canberra, University of Canberra. **Doctor of Philosophy**.

Wayne, A. J., Yoon, K. S., Zhu, P., Cronen, S., & Garet, M. S. (2008). Experimenting with teacher professional development: Motives and methods. *Educational Researcher*, *37*(8), 469-479.

Webster-Wright, A. (2009). Reframing Professional Development through understanding authentic Professional Learning. *Review of Educational Research 79*(2), 702-739.

Weiss, I.R., Pasley, J.D., Smith, P.S., Banilower, E.R., & Heck, D.J. (2003). *Looking inside the classroom: A study of K-12 mathematics and science education in the United States*. Chapel Hill, NC: Horizon Research.

Wenglinsky, H. (2002). How schools matter: The link between teacher classroom practices and student academic performance. *Education Policy Analysis Archives 10*(12), 1-30.

Wheeldon, J. (2010). Mapping Mixed Methods research: Methods, measures, and meaning. *Journal of Mixed Methods Research 4*(2), 87-102.

White, A. Way, J., Perry, B., & Southwell, B. (2006). Mathematical attitudes, beliefs and achievement In Primary pre-service Mathematics Teacher Education. *Mathematics Teacher Education and Development 7*, 33-52.

White, P., M. Mitchelmore, P., Branca, N., & Maxon, M. (2004). Professional Development: Mathematical content versus pedagogy. *Mathematics Teacher Education and Development 6*, 41-51.

Wilkins, J. L. M. (2008). The Relationship among Elementary teachers' content knowledge, attitudes, beliefs and practices. *Journal of Mathematics Teacher Education 11*, 139-164.

Willis, S., Devlin, W., Jacob, L., Powell, B., Tomazos, D., & Treacy, K. (2005). *First Steps in Mathematics: number: understand operations, calculate, reason about number patterns: book 2*. Port Melbourne, Victoria, Rigby/ Harcourt Education.

Wilson, P. S., Cooney, T. J., & Stinson, D.W. (2005). What constitutes good mathematics teaching and how it develops: Nine high school teachers' perspectives. *Journal of Mathematics Teacher Education 8*, 83-111.

Wilson, S., Floden, R., & Ferrini-Mundy, J. (2002) *Teacher Preparation Research: Current Knowledge, Gaps, and Recommendations*. Working Paper, February 2002, Center for the Study of Teaching and Policy, University of Washington. Retrieved 19th of December 2011, from http://depts.washington.edu/ctpmail/PDFs/TeacherPrep-WFFM-02-2001.pdf.

Wilson, S. M., Shulman, L. S., & Richert, A.E. (1987). 150 different ways of knowing: Representations of knowledge in teaching. In J. Calderhead (Ed.), *Exploring Teachers' Thinking* (pp.102-124). London, Caswell.

Wu, H. (2001). How to prepare students for Algebra. *American Educator 25*(2), 1-7.

Wu, H. (2005). *Some remarks on the teaching of fractions in Elementary school*, Department of Mathematics University of California, Berkeley. Retrieved from http://www.math.berkeley.edu/~wu/fractions2.pdf

Yates, S. M. (2006). Elementary teachers' mathematics beliefs and teaching practices after a curriculum reform. In Novotná, J., Moraová, H., Krátká, M. & Stehlíková, N. (Eds.), *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 5, pp. 433-440. Prague: PME. 5 - 433

Yin, R. K. (2008). *Case Study research: Design and methods*, SAGE Publications Inc.

Yoon, K. S., Duncan, T., Lee, S., & Shapley, K. (2008). *The effects of teachers' professional development on student achievement: Findings from a systematic review of evidence*. Paper presented at the Annual Meeting of the American Educational Research Association, March 24-28, 2008, New York. Retrieved from http://www.pdal.net/inc/docs/AERA%202008%20Paper_final_PD%20research%20review.pdf

Yoon, K. S., Duncan, T., Scarloss, B., & Shapley, K.L. (2007). *Reviewing the evidence on how teacher Professional Development affects student achievement*. Issues & Answers Report, REL 2007–No. 033. Washington, DC, U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Southwest.

Yost, D., & Sentner, S. (2000). An Examination of the construct of critical reflection: Implications for teacher education programming in the 21st Century. *Journal of Teacher Education 5,* 39-49.

Zambo, R., & Zambo, D. (2008). The impact of Professional Development in mathematics on teachers' individual and collective efficacy: The stigma of underperforming. *Teacher Education Quarterly 35*(1), 159-168.

Zevenbergen, R., Dole, S., & Wright, R. (2004). *Teaching mathematics in primary schools*. Crows Nest, NSW, Allen & Unwin.

Zhou, Z., Peverly, S.Y., & Xin, T. (2006). Knowing and teaching fractions: A cross-cultural study of American and Chinese mathematics teachers. *Contemporary Educational Psychology 31*(4), 438-457.

Zientek, L. R., & Thompson, B. (2008). Preparing high quality Mathematics and Science teachers: Are we meeting the challenge? *Research in the School 15*(2), 1-19.

Zigarmi, P., Betz, L., & Jennings, D. (1977). Teachers' preferences in and perceptions of inservice education. *Educational Leadership 34*(1), 545-551.

Appendices

14 Appendices

14.1 Appendix 1 – A Synthesis of What Makes for Effective PL.

Indicators of effective professional development	**Support from the literature - Source no.**
The purpose of professional development is centred on the improvement of student achievement and growth.	1, 3, 4, 5
Emphasises, and makes choices informed by, the link between teacher quality and student success.	6, 7, 8, 5, 9, 10, 11, 12, 13, 14, 15
Recognises multiple contexts, formats and factors. It uses combinations of different approaches, models and mediums, based on the needs of the school community.	3, 16, 17, 18, 19, 20, 21, 5, 23, 24, 33, 35
Increases teacher knowledge and understanding about their subject area and pedagogy.	25, 26, 16, 4, 27, 5, 22, 28, 14, 33, 46, 59, 60, 61, 62, 35, 63, 64, 65, 66, 67
Provides sufficient time and other resources. It is purposeful, sustained and sustainable over time.	16, 17, 20, 4, 5, 28, 30, 31, 46, 33, 60, 61, 62, 35, 63, 65, 66, 67, 58
Focuses on increasing knowledge and skills to bring about change in teaching practice. It models high-quality instruction. It recognizes the impact teachers have on students and honours their decision-making abilities.	16, 4, 32, 22, 28, 14, 33, 59, 35, 63, 65 67
Recognizes the ways adults learn, and the impact of constructivist learning theory on organizations and structures for professional development.	29, 7, 3, 32, 5, 22, 36, 12, 46, 58
Accommodates diversity and promotes equity in schools.	16, 38, 39, 40, 61, 66
Is based on the best available research evidence. It explores sources and methodologies of research before basing decisions on it.	16, 34, 41, 33, 60, 61, 67

Indicators of effective professional development	Support from the literature - Source no.
Is driven by analyses of student learning data.	16, 34, 58, 61, 63, 64, 65, 67
Involves families and other stakeholders in the professional development process.	16, 42, 32, 27, 67
Promotes the development of leadership capacity, including principals, administrators and teachers.	16, 8, 42, 43, 4, 44 45, 25, 1, 47, 14, 59, 60, 61, 63
Is centred in the school community and based on teachers' identified needs. It occurs within the context of the school community and involves people resources and models that include mentoring and community building.	16, 8 17, 7, 4, 20, 3, 32, 5, 22, 41
Involves the formation of learning communities.	48, 16, 49, 8, 1, 49, 4, 32, 5, 50, 28, 14
Recognizes and explores the impact of initiatives on school culture and is centred on a goal of organizational improvement. It makes connections between school culture, collaborative working teams, learning teams, communities of teacher researchers, collaborative exchanges and learning communities.	16, 51, 8, 17, 7, 42, 44, 27, 5, 53
Recognizes the impact of change on school improvement processes. Change is centred on those actions that are within a school's sphere of influence.	44, 54, 7, 5, 61
Is centred on the development and maintenance of collaborative environments.	49, 1, 5, 1, 55, 46, 58, 59, 60, 35, 63, 64, 65, 67
Assesses the impact of initiatives and decisions on student outcomes.	8, 16, 56, 57, 5, 12, 14, 20, 33, 58, 59, 60, 61, 63, 64, 67
The processes inherent in teacher evaluation and assessment, including collaborative approaches such as mentoring and coaching, are part of effective evaluation practices for professional development initiatives.	56, 7
Addresses issues and concerns and interest which are largely (but not exclusively) identified by the teachers themselves	30, 46, 33, 20, 61, 64, 67
Is driven by a well-defined image of effective classroom learning and teaching	14, 19, 59

Indicators of effective professional development	Support from the literature - Source no.
Uses or model with teachers the strategies teachers will use with their students;	14, 28
Provides links to other parts of the education system	14
Is continuously assessed and improved to ensure positive impact on teacher effectiveness, student learning, leadership, and the student community	28
Recognises and address many of the impediments to teacher growth at the individual, school and district level	28, 34, 37
Solicits a conscious commitment to be an active participant in the professional development	28
Allows time and opportunity for planning, reflection and feedback	28, 58
Recognises that change is a gradual, difficult and often painful process	28, 29, 33
Aligns with other reform initiatives	46, 58, 59, 60, 61, 35, 63, 64, 65, 67
Is school or site based	46, 58, 61, 62, 64, 65
Focuses on individual and organisational improvement	46, 33, 60, 61, 63
Includes follow up and support	46, 58, 61, 62
Is ongoing and job embedded	46, 60, 61, 65, 67
Provides opportunities for theoretical understanding	58, 64
Promotes continuous inquiry and reflection	63

Source number	Author	Date
1	Elmore	2001
2	Cohen & Hill	2000
3	Garet, Porter, Desimone, Birman and Yoon	2001
4	Reitzug	2002
5	Hawley and Valli	2000
6	Kent	2004
7	Danielson	2002
8	Marzano	2003
9	Killion	1999
10	Darling-Hammond	1998
11	Haycock	1998
12	Darling-Hammond and Loewenberg-Ball	1998
13	Fullan	1993
14	Loucks-Horsley, Hewson, Love & Stile	1998
15	Stoll and Fink	1996
16	Guskey	2004
17	Richardson	2003
18	Pritchard and Marshall	2002
19	Zigarmi, Betz & Jensen	1997
20	Sparks	2002
21	Lee	2001
22	Loucks-Horsley and Matsumoto	1999
23	Laferrière	1997
24	Lieberman	1995
25	Wenglinsky	2000
26	Porter, Garet, Desimone, Yoon and Birman	2000
27	Guskey and Sparks	2002
28	Clarke	2003
29	Gess Newsome	2001
30	Stupovitz & Turner	2000
31	Stupovitz, Mayer & Kahl	2000
32	Senge	2001
33	American Federation of Teachers	1996
34	Fullan	2001
35	Birman, B et. al.	2000
36	Alexander and Murphy	1998
37	Anderson & Helms	2001

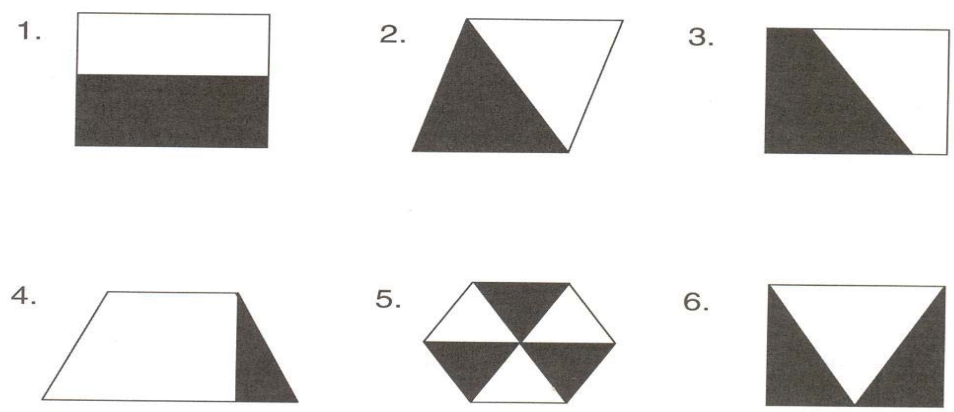
Source number	Author	Date
38	Johnson	2006
39	Malarkey	2003
40	Clair and Adger	1999
41	Little	1994
42	Sparks	2002
43	Dufour	2001
44	Fullan	2002
45	Clement and Vandenbergher	2001
46	Corcoran, T.B.	1995
47	Porter et al	2000
48	Kruse & Seashore	2005
49	Schmoker	2004
50	Hord	1997
51	Gamoran and Grodsky	2003
52	Morris, Chrispeels and Burke	2003
53	Busick, Hammond and Inos	1993
54	Clarke and Hollingsworth	2002
55	Fullan, Sparks and Guskey	2004
56	Kelleher	2003
57	Killion	2001
58	Wise, Spiegel & Bruning	1999
59	Loucks-Horsley, Stiles & Hewson	1996
60	U.S. Department of Education	1997
61	Educational Research Services	1996
62	Kennedy, M.M.	1998
63	Kent, K. & Lingman, C.	2000
64	National Partnership for Excellence and Accountability in Teaching.	2000
65	Terzian, M.	2000
66	Wenglinsky, H.	2002
67	National Staff Development Council	2001

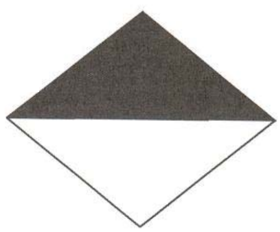
14.2 ***Appendix 2 – Fraction Knowledge Assessment Task (FKAT) Original Version***

Name ________________________________ Date ____________

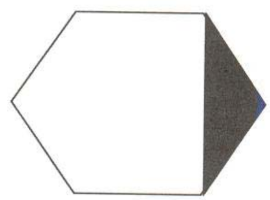
Year Level taught __________________

<u>Place a tick on the drawings that show ½.</u>

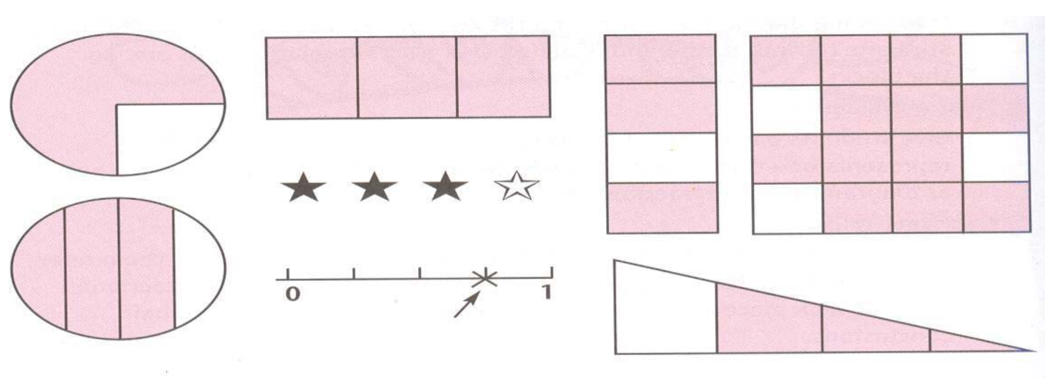


7. 

8. 

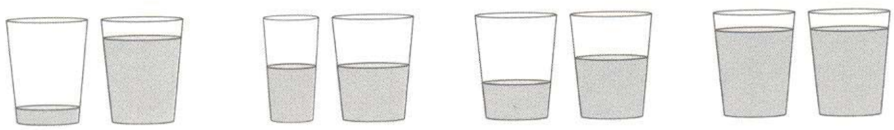
9. 

<u>10. Place a tick on the drawings that show ¾.</u>

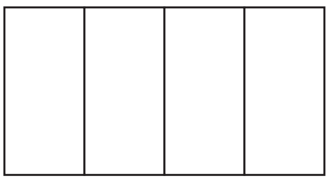


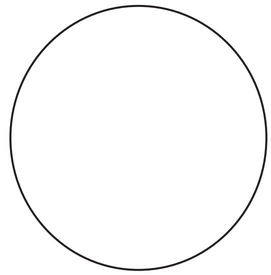
11. Sarah and Chris shared the drink from a bottle. They had half each.

Put a tick on the picture that shows their shares of the drink?

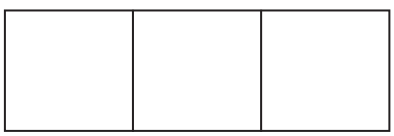


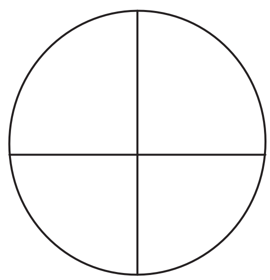
<u>Shade in one quarter on each of these shapes.</u>

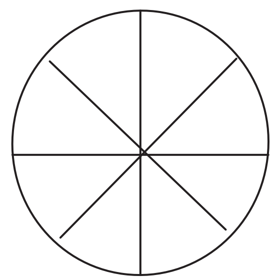
12. 

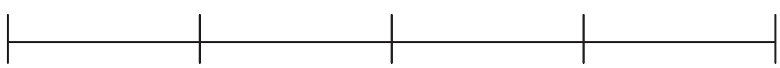
13. 

14. 

15. 

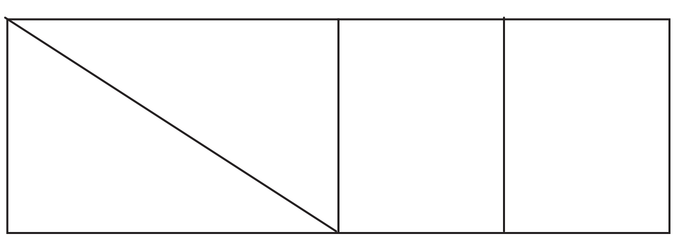
16. 

17. 

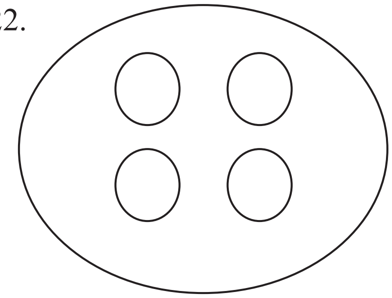
18. 

19. 

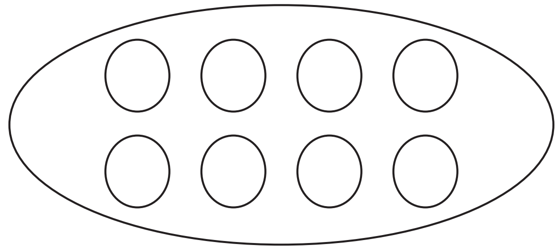
20. 

21. 

22.

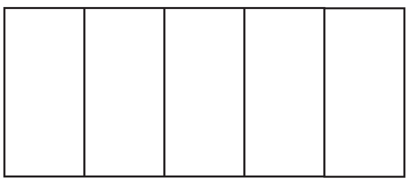


23.

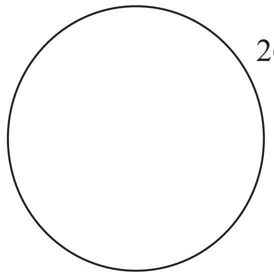


Shade in two fifths on each of these shapes

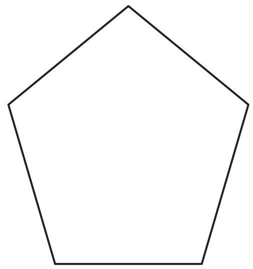
24.



25.

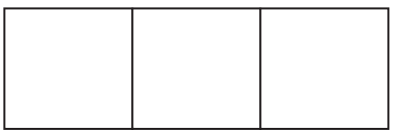


26.



27.

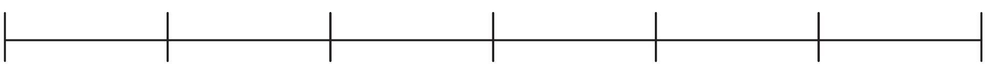
28.



29.

30.

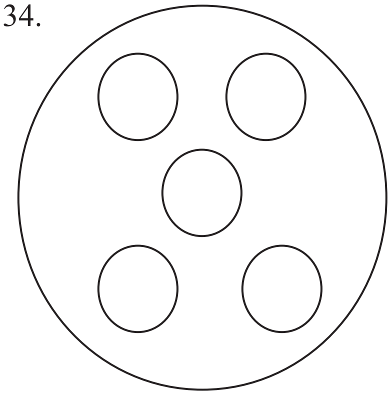
31.



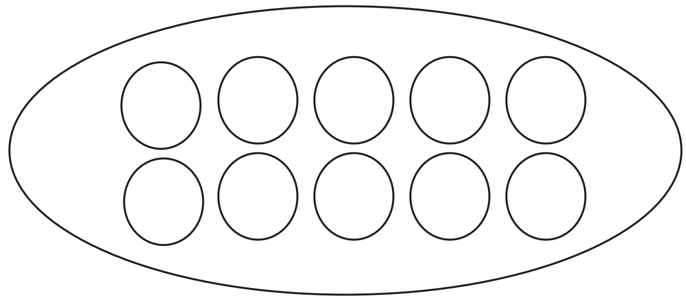
32.

33.

34.



35.



36. Sandra picked 24 apples, how many did she eat?

____

37. Which of these fractions is greater?

a) $\frac{1}{2}$ or $\frac{3}{4}$?

____

Why?

____

____

b) 3/8 or 5/20?

Why?

____

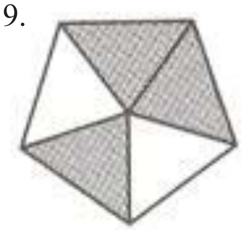
____

What are these shaded fractions?

38.

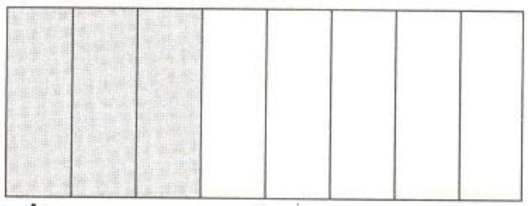
………………..

39.



………………..

40.

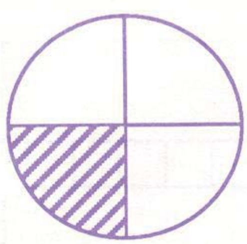


………………..

41.

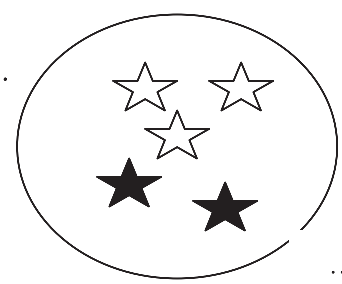
………………..

42.



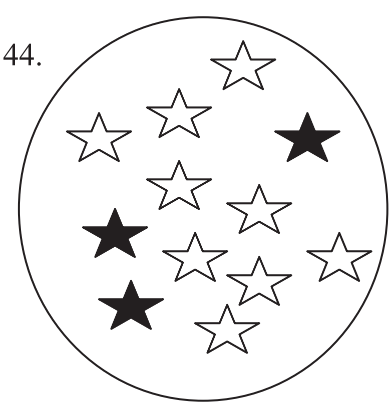
………………..

43.



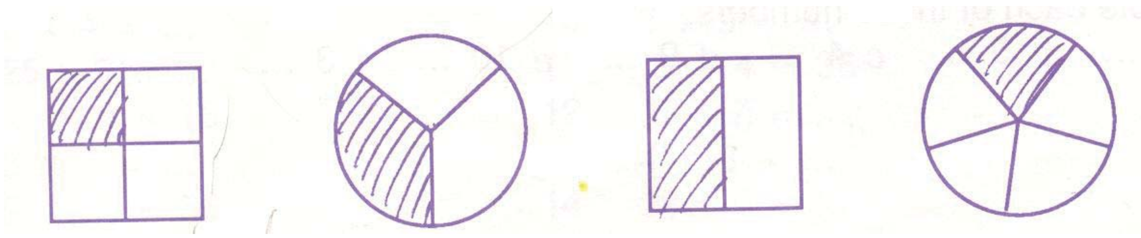
………………..

44.



………………..

45, 46. 47. 48.

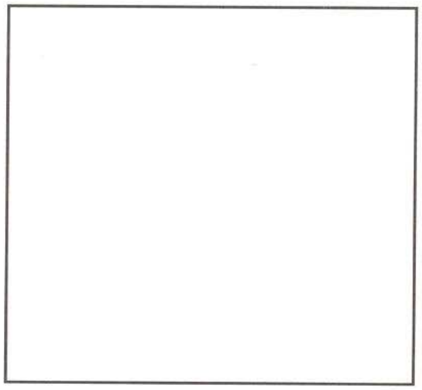


……………….. ……………….. ……………….. ………………..

Pizza

Luis, Elena, and Leslie plan to share 1 large, square pizza. Each person will get an equal amount.

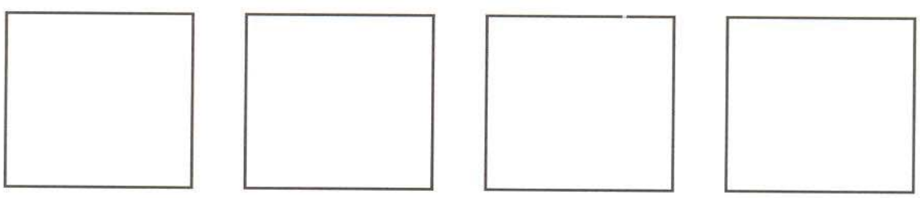
1. Show on the picture how much pizza Elena will get.



2. What fraction of the pizza will Elena get?

Maria, Carlos, and Terry want to share 4 medium, square pizzas. Each person will get an equal amount

3. Show on the picture how much pizza Carlos will get.

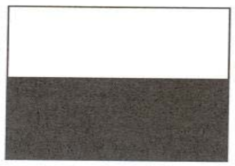


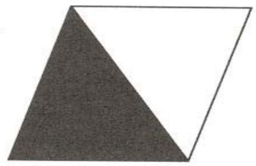
4. How many pizzas will Carlos get?

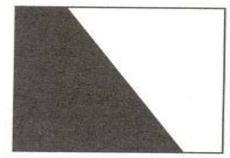
14.3 ***Appendix 3 – Fraction Knowledge Assessment Task (FKAT) - Abbreviated***

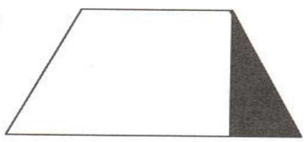
Name ______________________________ Date ____________________

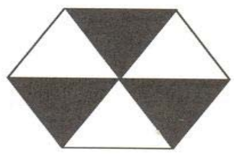
Place a tick on the drawings that show ½.

1. 

2. 

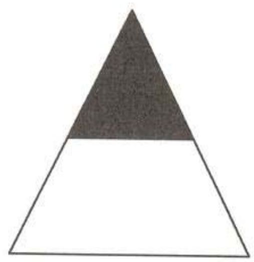
3. 

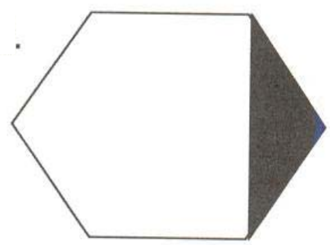
4. 

5. 

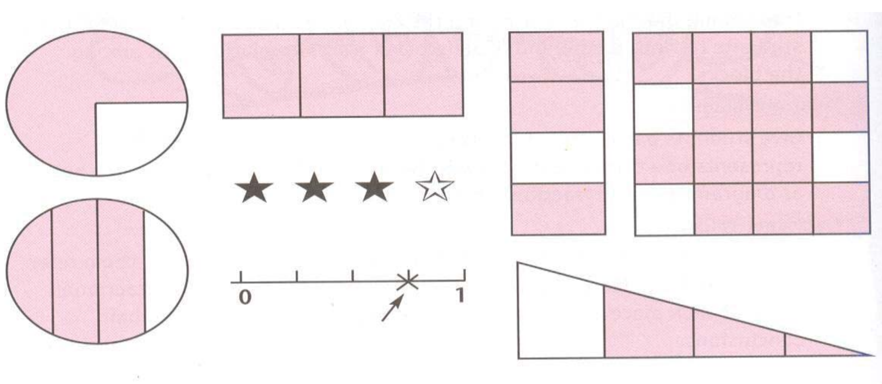
6. 

7. 

8. 

9. 

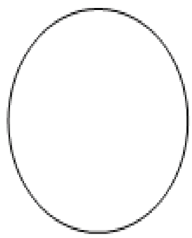
10. Place a tick on the drawings that show ¾.



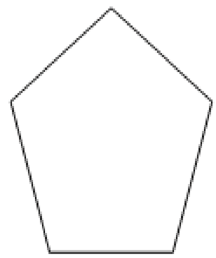
11. Sarah and Chris shared the drink from a bottle. They had half each.
Put a tick on the picture that shows their shares of the drink?

Shade in two fifths on each of these shapes

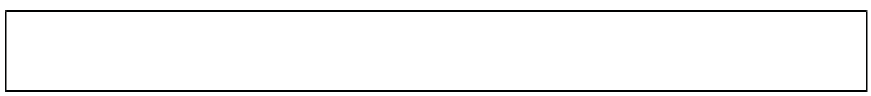
12.



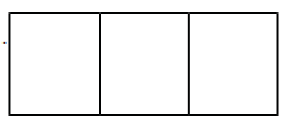
13.



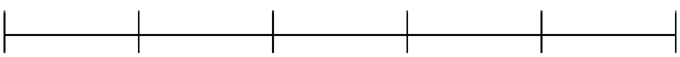
14.



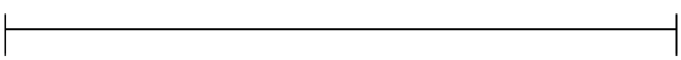
15.



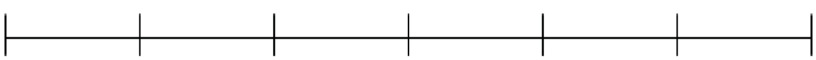
16.



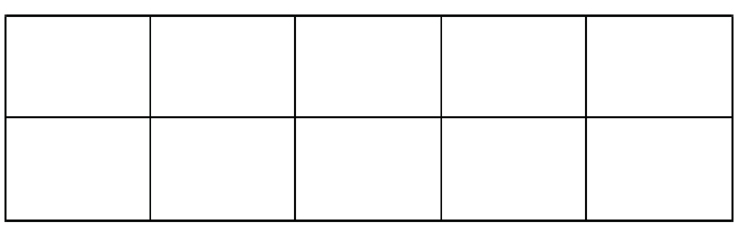
17.



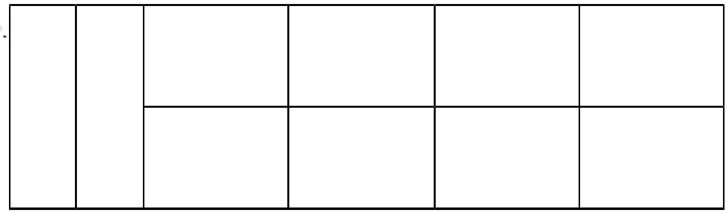
18.



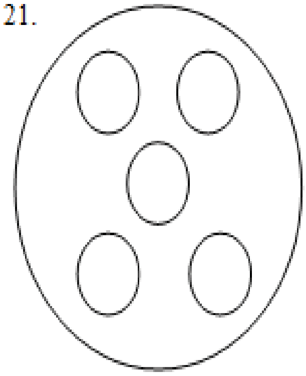
19.



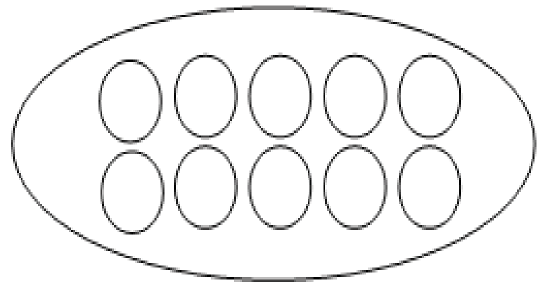
20.



21.



22.



23. Sandra picked 24 apples, how many did she eat if she ate ¼ of them?

____

24. Which of these fractions is greater?

a) ½ or ¾?

____

Why? ____

____

b) 3/8 or 5/20?

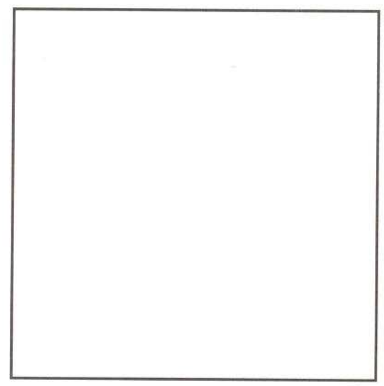
Why? ____

____

Pizza

Luis, Elena, and Leslie plan to share 1 large, square pizza. Each person will get an equal amount.

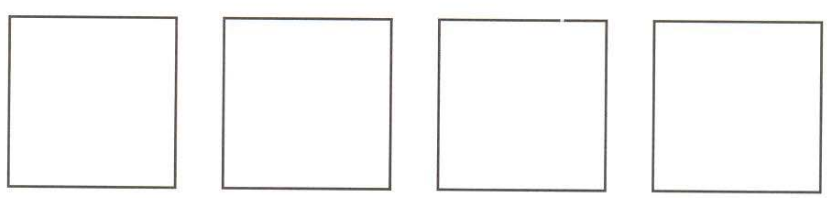
1. Show on the picture how much pizza Elena will get.



2. What fraction of the pizza will Elena get?

Maria, Carlos, and Terry want to share 4 medium, square pizzas. Each person will get an equal amount

3. Show on the picture how much pizza Carlos will get.



4. How many pizzas will Carlos get?

14.4 ***Appendix 4 – Teachers' Attitudes Towards Mathematics Questionnaire (TAM).***

Attitudes

		Definitely False	False	Mostly False	More False Than True	More True Than False	Mostly True	True	Definitely True	Not applicable to me
1.	Generally I feel secure about the idea of teaching mathematics.									
2.	I find many mathematical problems interesting and challenging.									
3.	Mathematics makes me feel inadequate.									
4.	I'm not the type of person who could teach mathematics very well.									
5.	I have always done well in mathematics classes.									
6.	I do not enjoy having to teach mathematics.									
7.	I am quite good at mathematics.									
8.	I have generally done better in mathematics courses than other courses.									
9.	I'm not sure about what to do when I'm teaching mathematics.									
10.	Time passes quickly when I'm teaching mathematics.									
11.	I have hesitated to take courses that involve mathematics.									
12.	I would get a sinking feeling if I came across a hard problem while teaching mathematics.									
13.	Teaching mathematics doesn't scare me at all.									
14.	At school, my friends always came to me for help in mathematics.									
15.	I am confident about the methods of teaching mathematics.									
16.	I have trouble understanding anything that is based upon mathematics.									
17.	It wouldn't bother me to teach a lot of mathematics at school.									
18.	I never do well on tests that require mathematical reasoning.									
19.	Of all the subjects, mathematics is the one I worry about most in teaching.									
20.	If I taught in a team or with a teaching partner, I'd like to have another teacher teaching the mathematics.									

14.5 *Appendix 5 - Teachers' Beliefs About Mathematics Questionnaire (TBM).*

Beliefs

		Disagree	Undecided	Agree
1.	Mathematics is computation			
2.	Mathematics problems given students should be quickly solvable in a few steps.			
3.	Mathematics is the dynamic searching for order and pattern in the learner's environment.			
4.	Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking.			
5.	Right answers are much more important in mathematics than the ways in which you get them.			
6.	Mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences.			
7.	Mathematics learning is being able to get the right answers quickly.			
8.	Periods of uncertainty, conflict, confusion, surprise are a significant part of the mathematics learning process.			
9.	Young students are capable of much higher levels of mathematical thought than has been suggested traditionally.			
10.	Being able to memorise facts is critical in mathematics learning.			
11.	Mathematics learning is enhanced by activities which build upon and respect students' experiences.			
12.	Mathematics learning is enhanced by challenge within a supportive environment.			
13.	Teachers should provide instructional activities which result in problematic situations for learners.			
14.	Teachers or the textbook – not the student – are the authorities for what is right or wrong.			
15.	The role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge.			
16.	Teachers should recognise that what seem like errors and confusions from an adult point of view are students' expressions of their current understanding.			

14.6 *Appendix 6 – Pedagogical Content Knowledge Situations (PCKS)*

Name ____ Date _____________

If more space is required to answer a question please use the back of the sheet and number your responses accordingly.

1. Which numbers should be used as initial examples to illustrate the place of partitioning in understanding fractions?

a) $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$

b) $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{6}$ $\frac{1}{8}$

c) $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$

d) $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ $\frac{4}{4}$ $\frac{5}{4}$

e) All above work equally as well

Please give reasons for your answer.

____

____

____

____

____

2. Which representations of fractions are necessary to foster student understanding of unit fractions i.e. ¼, ½, ⅓ etc. Please indicate with a tick.

Symbols ☐

Area/region models ☐

3 Dimensional models ☐

Number Lines ☐

Set models ☐

None of the above ☐

All of the above ☐

3. When given the sum 3/7 + 2/9 the student gave an incorrect answer. Give 5 different answers that the student might have given and give reasons for the misconceived answer.

____

____

____

____

____

____

____

____

____

4. The Year 4 teacher told her class on the first day of working with fractions; "Nothing is more important in understanding fractions than finding out what the unit is!" Is she correct? Explain your reasoning.

____

____

____

____

____

5. The students in a Year 7 class gave the following answers to the following problem. Determine and indicate which the correct answer is and then how they might have achieved their misconceived answer.

What faction of the circle is part D?

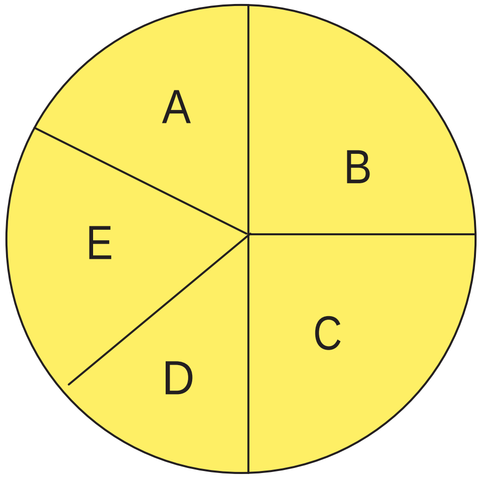
a. $^1/_3$

b. $^1/_6$

c. $^1/_5$

d. ¼

e. $^1/_8$



a. $^1/_3$____

b. $^1/_6$____

c. $^1/_5$____

d. ¼ ____

e. $^1/_8$____

14.7 *Appendix 7 – Exit Questionnaire*

Do you believe the PL addressed the following characteristics of effective professional development?

	Yes	Undecided	No
Focused on increasing knowledge and skills to bring about change in my teaching practice.			
Recognized the ways adults learn, and the impact of constructivist learning theory			
Accommodated diversity and promoted equity in schools			
Promoted the development of leadership capacity			
Involved the formation of learning communities			
Was centred on the development and maintenance of collaborative environments			
Addressed issues and concerns and interest identified by the teachers			
Focused on individual and organisational improvement			
Included follow up and support			
Was ongoing and job embedded			
Allowed time and opportunity for planning, reflection and feedback			
Was centred on the improvement of student achievement and growth			
Emphasised, and made choices informed by, the link between teacher quality and student success			
Recognised multiple contexts, formats and factors			
Increased teacher knowledge and understanding			
Was purposeful, sustained and sustainable over time			
Was based on the best available research evidence			
Was driven by analyses of student learning data			
Assessed the impact of initiatives and decisions on student outcomes			

14.8 ***Appendix 8 – Semi-Structured Interview Questions***

Research Question 1

What is the current status of teaching fractions in middle and upper primary school classrooms in Western Australia?

1. At the outset of this PL how would you have rated your confidence in teaching fractions? How would you now rate your confidence to teach fractions?
2. At the outset of this PL how would you have rated your ability to teach fractions? How would you now rate your ability to teach fractions?
3. In terms of its importance as a topic in mathematics how did you rate fractions before the start of this professional development? Has that rating changed and if so, why?
4. What kind of status or importance do you think the topic of fractions holds in your school?
5. What kind of status or importance do you think the topic of fractions holds in the WA curriculum?

Research Question 2

What impacts will well-structured, action research based, professional learning opportunities and reflective practice have on

- *primary school teachers' content knowledge of fractions?*
- *primary school teachers' pedagogical knowledge of teaching fractions?*
- *primary school teachers' beliefs and attitudes with regards to teaching fractions?*

6. There are 3 major elements in this PL, which has been the most important to your development as a mathematics teacher and why?
   - The chance to see some alternative pedagogy
   - Content - the variety of activities
   - Taking everything back into your context and then given a chance to reflect
7. Do you think you are now better equipped to promote student learning in fractions through attendance at this PL? Please elaborate on your answer.
8. Has this PL changed the manner in which you will teach fractions? If so, why and how?
9. If you had to advise on your top five tips for teaching fractions what would they be and why? Don't worry about the order and if you need more than five that's fine.

Primary school teachers' content knowledge of fractions?

10. How has your content knowledge about fractions changed through this course? Could you articulate a couple of the things you have learned?

Primary school teachers' pedagogical knowledge of teaching fractions?

11. What activity or activities introduced in this PL have been most "revealing" to you and why?
12. What do you think you have learned attending this PL and how will it change your practice from now on?
13. What is your favourite activity when teaching fractions? What makes it your favourite?
14. What have you learned from this PL about fractions that you can generalize into wider teaching about mathematics?

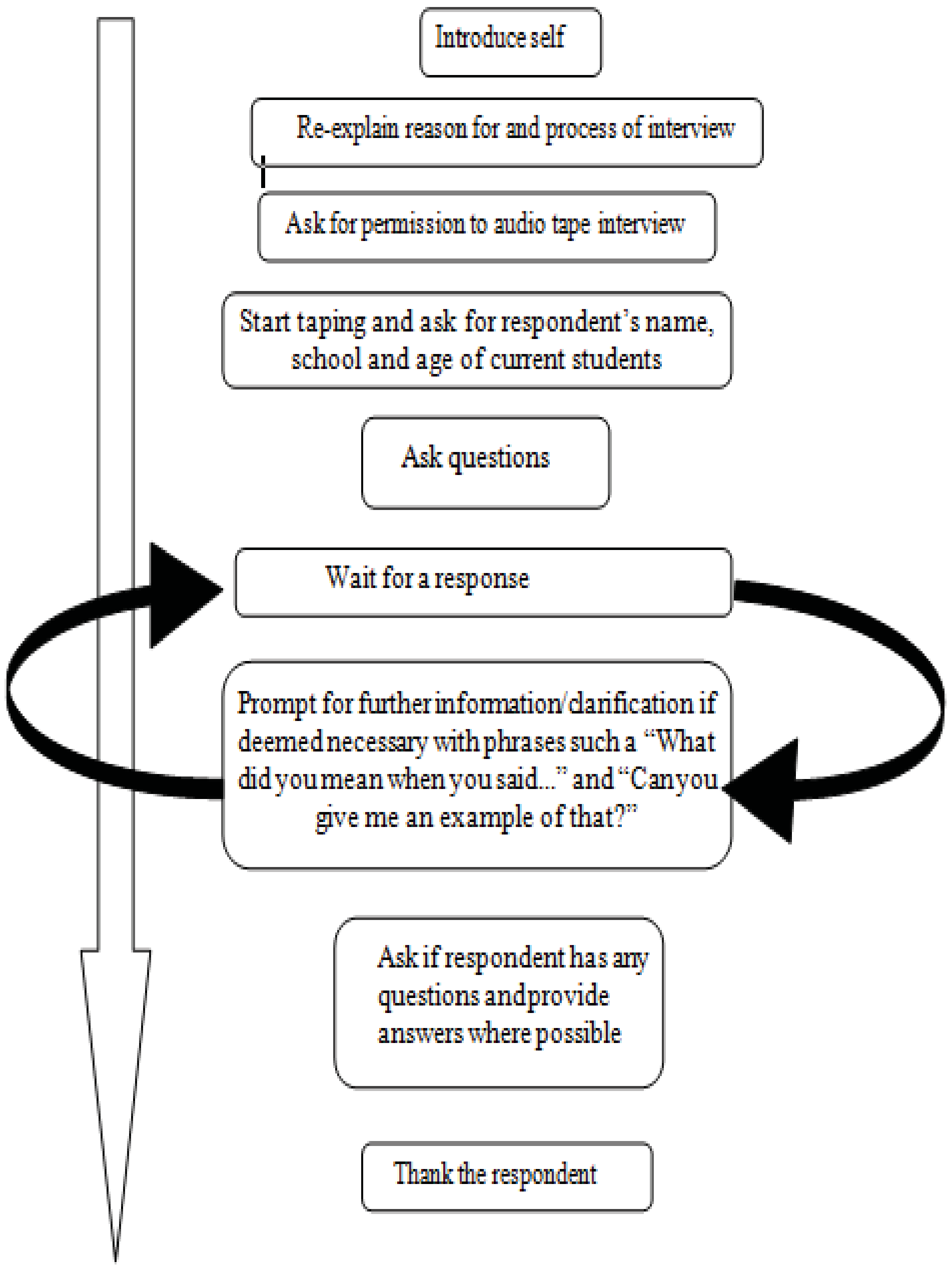
Primary school teachers' beliefs and attitudes with regards to teaching fractions?

15. If any, what aspect/s do you find most problematic in teaching fractions?
16. If you were now asked to take a "leadership" role in the teaching of fractions in your school do you feel more confident now, as opposed to when you began this PL, that you could do so? If so, what has lead to this increased confidence?

General question

17. What were you hoping to gain from attendance in this PL and was that gain achieved?

14.9 *Appendix 9 – Protocols for Semi-Structured Interviews*



14.10 ***Appendix 10 - Reflective Tools - Medical Lens Model by Charles Lovitt***

Using a medical metaphor, I liken it to placing the item on the 'medical examining table' and walking around it – 'pushing and prodding' to determine its educational health. And the more perspectives from which we scrutinize the more complete will be our examination.

i) to justify existing features and qualities
ii) to recognize missed opportunities for additional richness and balance

To make this process explicit, we can systematically pick up a series of lenses.

Some Lenses

Content: What mathematical content ideas and concepts are evident within the lesson?

Pedagogy: With what we know about teaching strategies, how much of this is exhibited or exploited within this lesson? Has the full potential of using concrete materials, co-operative group work, investigative or problem solving approaches, estimation, story-shells, social issues, personal involvement, etc been considered.

Policy: I am accountable to my system for certain objectives – how can I explain how this lesson 'fits' such policy?

Technology: With what we now know about how technology can contribute to learning, how much of this has been considered and visible within the lesson?

Equity: When I consider cultural, gender and ESL students, and the strategies that have been developed to provide access for all students, how many of these are evident?

Learning Theory: With what we know about how students learn, described by such terms as constructivism, meta-cognition, Blooms Taxonomy or multiple intelligence theory – how can I justify this lesson from those perspectives?

Context: There are many ways to seek making connections and providing a relevant context for learning – "does this content area have any vocational, recreational, preparatory, local, social issue, cultural, historical, scientific, technological, creative, artistic, aesthetic, macro (huge), micro (tiny), personal, humorous, current events or literary aspects?" – and have these opportunities been fully exploited?

Assessment: Have I employed a full rich array of strategies to help me monitor student learning within this lesson?

When picking up a particular lens, we can temporarily focus on just that perspective. For example if we pick up and peer through a *pedagogy lens* alone (as this I feel is the most important lens – the one on which the quality of the learning experience hinges) we can look at some "sub lenses."

Sub-lenses:

- Can I make it *concrete*?
- Can I make it a genuine *problem solving challenge*?

- Can I use the *outdoors*?
- Can I involve students *physically* (kinesthetic)?
- Can I inject an *estimation* component?
- Could I use a *simulation role play*?
- Can I make it *personal* for the students?
- Could I make it a *cooperative group challenge*?
- Can I make *links to other subject areas* e.g. language? And *other mathematics topics?*
- Can I exploit the *visual* aspects?
- Can I embed the learning within a *story-shell*?
- Can I provide choice and allow *student self-responsibility* and *ownership*?
- Can I make the task *open-ended and investigative*?
- Can I allow for *multiple entry and exit points*?
- Have I got an *interesting and meaningful context?*

14.11 *Appendix 11 – Consent for Participation from Pre-Service Teachers*

"Enhancing the pedagogical content knowledge of teachers of mathematics through reflective practices regarding content and pedagogical knowledge."

Student Informed Consent Form

I ______________________________ have read and understood the Research Information Letter, and have had the opportunity to ask any questions. Also I know that I may contact the research team if I have any further questions. I am happy to be asked some informal questions.

I agree that the research data gathered for this study may be published as part of this research only, provided that neither my name nor the schools are identified. I understand that I may withdraw at any stage if I wish to do so.

______________________________________ ____________________

(Signature of Student) (Date)

Thank you for your participation.

Derek Hurrell

Phone: 9441 1625

14.12 *Appendix 12 - Letter of Information to the Principals From the Schools of the Teachers Participating From G1 and G2*

16th of March 2009

ASSOCIATION OF INDEPENDENT SCHOOLS OF WESTERN AUSTRALIA (INC)

Dear Principal

Re: Participation in a research study

I am the Numeracy Consultant for the Association of Independent Schools of Western Australia (AISWA), working on a PhD thesis under the supervision of Dr. Paul Swan and Dr. Christine Ormond from Edith Cowan University.

I am currently offering professional development with classroom teachers in research which is focusing on the teaching of fractions, entitled "Enhancing the pedagogical content knowledge of teachers of mathematics through reflective practices regarding content and pedagogical knowledge." The ECU Human Research Ethics Committee has approved this research study.

I plan to work collaboratively with the teacher to investigate ways in which they can be aided in teaching the problematic area of fractions. The professional development will be based upon strengthening the content and pedagogical understandings required to teach fractions effectively and also in allowing the teachers opportunity to engage in reflective practices about the "craft" of teaching. It is highly desirable that the teachers will incorporate some of the content and pedagogical understandings gained into their classroom situation.

At no stage will I or any member of the research team have direct teaching contact with the students in the care of the participating teachers but it is likely that this study will have an impact on the teaching and learning program. Either myself or a research assistant may wish to attend the classroom of the participating teachers to audit what happens during lessons and therefore may have some informal contact with the students. This person will of course have the appropriate WACOT clearance to do so.

It should be understood that the pedagogical practices and the content to be trialled is grounded in research from across the globe and should prove highly beneficial to the students.

The research will take place over at least one academic year and the results will be published, as a thesis and parts may be used in papers delivered at conferences. In order to protect students' privacy, neither the students nor the school will be directly identified in any publication. Pseudonyms will be used to protect each student's identity. Tape-recorded sessions will be erased following transcription and all information will be securely stored.

Parents may withdraw their child from participation, at any stage in the process if they wish to do so.

Please do not hesitate to call me, Derek Hurrell on 9441 1625 or my principle supervisor Dr. Paul Swan, at Edith Cowan University on 6304 5224, if you have any queries. I would be happy to discuss with you any issues that you may have. I have enclosed copies of the parental information and informed consent form, for your perusal.

Yours faithfully,

Derek Hurrell

14.13 *Appendix 13 Letter of Information to the Members of G1 and G2*

16th of March 2009

Dear Teacher

I am the Numeracy Consultant for the Association of Independent Schools of Western Australia (AISWA), working on a PhD thesis under the supervision of Dr. Paul Swan and Dr. Christine Ormond from Edith Cowan University.

I am currently offering professional development with classroom teachers in research which is focusing on the teaching of fractions, entitled "Enhancing the pedagogical content knowledge of teachers of mathematics through reflective practices regarding content and pedagogical knowledge." The ECU Human Research Ethics Committee has approved this research study.

I have gained approval for the research study to be conducted in your school from the ECU Human Research Ethics Committee (which has also approved this research study) and will be seeking permission from your Principal for entry to your school and classroom.

I plan to work collaboratively with participating teachers to investigate ways in which they can be aided in teaching the problematic area of fractions. The professional development will be based upon strengthening the content and pedagogical understandings required to teach fractions effectively and also in allowing the participating teachers opportunity to engage in reflective practices about the "craft" of teaching. It is highly desirable that the participating teachers will incorporate some of the content and pedagogical understandings gained into their classroom situation.

At no stage will I or any member of the research team have direct teaching contact with the students in the care of the participating teachers but it is likely that this study will have an impact on the teaching and learning program. Either myself or a research assistant may wish to attend the classrooms of the participating teachers to audit what happens during lessons and therefore may have some informal contact with the students. This person will of course have the appropriate WACOT clearance to do so.

Participating teachers will also be required to bring to the sessions any de-identified work samples based on the ideas they will develop in the professional learning sessions. These work samples, although anonymous will be viewed and discussed by the teachers in the workshops.

It should be understood that the pedagogical practices and the content to be trialled is grounded in research from across the globe and should prove highly beneficial to the students.

The research will take place over at least one academic year and the results will be published, as a thesis and parts may be used in papers delivered at conferences. In order to protect students' privacy, neither the students nor the school will be directly identified in any publication. Pseudonyms will be used to protect each student's identity. Tape-recorded sessions will be erased following transcription and all information will be securely stored. Parents may withdraw their child from participation, at any stage in the process if they wish to do so.

The teachers used in the research will be gathered through an expression of interest circulated to all teachers in the Association of Independent Schools of Western Australia (AISWA). It is expected that no more than twenty (20) teachers will be needed for the professional learning sessions, with only five (5) required for the in-depth case studies. The Participants in this research will need to be willing:

g. to have their attitudes and beliefs towards teaching maths in general and fractions in particular, examined through a questionnaire
h. to have their ability to answer questions regarding understanding of fractions examined through a short assessment and interview
i. to follow a "guide" on the most effective way in which to teach fractions
j. to introduce manipulative materials into their pedagogy for teaching fractions
k. to complete a "log" recording their thoughts

If chosen to be part of the case study group the participants must also be willing;

l. to submit to audio-taped interviews about their attitudes and abilities in dealing with fractions.
m. to submit to audio-taping of discussions on content and pedagogical practices employed in their classroom.

Please do not hesitate to call me, Derek Hurrell on 9441 1625 or my principle supervisor Dr. Paul Swan, at Edith Cowan University on 6304 5224, or if you wish to speak to a person independent of the study, Kim Gifkins the Research Ethics Officer at Edith Cowan University on 6304 2170, if you have any queries. I would be happy to discuss with you any issues that you may have. I have enclosed copies of the parental information and informed consent form, for your perusal.

Yours faithfully,

Derek Hurrell

14.14 *Appendix 14 – Consent Form for Members of G1 and G2*

I, ____ (name of Teacher),

hereby consent to participate in research which is focusing on the teaching of fractions in the primary school setting, entitled "Enhancing the pedagogical content knowledge of teachers of mathematics through reflective practices regarding content and pedagogical knowledge." I have been provided with information about the study and have been given the opportunity to ask questions. I am aware that I may contact the researcher at any time. I understand that I will be involved in considering the teaching of fractions into my normal mathematics classroom. During this process my teaching will be observed and I will be interviewed. I agree that the research data gathered for this study may be published, provided that neither the students nor the schools are identified. I understand that I may withdraw at any stage if I wish to do so.

____ ______________________2009

(Signature of Teacher) (Date)

14.15 *Appendix 15 – Information Letter to Parents Outlining the Limited Contact that the Researcher Would Have with Students of the Teachers Involved*

March 2009

Dear Parent/s,

I am the Numeracy Consultant for the Association of Independent Schools of Western Australia (AISWA), working on a PhD thesis under the supervision of Dr. Paul Swan and Dr. Christine Ormond from Edith Cowan University. I am currently working with classroom teachers in research which is focusing on the teaching of fractions, entitled "Enhancing the pedagogical content knowledge of teachers of mathematics through reflective practices regarding content and pedagogical knowledge."

The Principal and your child's class teacher have both approved the conduct of this research. The Edith Cowan University Human Research Ethics Committee has also approved the research study. The study is planned to be incorporated into your child's normal mathematics class. The children will not be disrupted in any way, as the normal mathematics program will continue and should be enhanced.

At no stage will I or any member of the research team have direct teaching contact with the students in the care of the participating teachers but it is likely that this study will have an impact on the teaching and learning program. Either myself or a research assistant may wish to attend the classroom of the participating teachers to audit what happens during lessons and therefore may have some informal contact with the students. This person will of course have the appropriate Western Australian College Of Teachers clearance to do so.

Incidental student voices may appear when conducting audio taping of Teacher interviews but no student will be identified in any way what so ever.

Participating teachers will also be required to bring to the sessions any de-identified work samples based on the ideas they will develop in the professional learning sessions. These work samples, although anonymous will be viewed and discussed by the teachers in the workshops. It should be understood that the pedagogical practices and the content to be trialled is grounded in research from across the globe and should prove highly beneficial to the students.

The research will take place over at least one academic year and the results will be published, as a thesis and parts may be used in papers delivered at conferences. In order to protect your child's privacy, neither the students nor the school will be directly identified in any publication. Pseudonyms will be used to protect each student's identity. Tape-recorded sessions will be erased following transcription and all information will be securely stored.

Please do not hesitate to call me, Derek Hurrell on 9441 1625 or my principle supervisor Dr. Paul Swan, at Edith Cowan University on 6304 5224, or if you wish to speak to a person independent of the study, Kim Gifkins the Research Ethics Officer at Edith Cowan University on 6304 2170, if you have any queries. I would be happy to discuss with you any issues that you may have.

Yours faithfully,

Derek Hurrell