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## Innovative transition matrix techniques for measuring extreme risk: an Australian and U.S. comparison

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**Abstract:** Comparing Australia and the U.S. both prior to and during the Global Financial Crisis (GFC), using a dataset which includes more than six hundred companies, this paper modifies traditional transition matrix credit risk modelling to address two important issues.

Firstly, extreme credit risk can have a devastating impact on financial institutions, economies and markets as highlighted by the GFC. It is therefore essential that extreme credit risk is accurately measured and understood. Transition matrix methodology, which measures the probability of a borrower transitioning from one credit rating to another, is traditionally used to measure Value at Risk (VaR), a measure of risk below a specified threshold. An alternate measure to VaR is Conditional Value at Risk (CVaR), which was initially developed in the insurance industry and has been gaining popularity as a measure of extreme market risk. CVaR measures those risks beyond VaR. We incorporate CVaR into transition matrix methodology to measure extreme credit risk. We find significant differences in the VaR and CVaR measurements in both the US and Australian markets, as CVaR captures those extreme risks that are ignored by VaR. We also find a greater differential between VaR and CVaR for the US as compared to Australia, reflecting the more extreme credit risk that was experienced in the US during the GFC.

The second issue is that relative industry risk does not stay static over time, as highlighted by the problems experienced by financial sector during the GFC. Traditional transition matrix methodology assumes that all borrowers of the same credit rating transition equally, whereas we incorporate an adjustment based on industry share price fluctuations to allow for unequal transition among industries. The existing CreditPortfolioView model applies industry adjustment factors to credit transition based on macroeconomic variables. The financial sector regulator in Australia, APRA, has found that banks do not favour such credit modelling based on macroeconomic variables due to modelling complexity and forecasting inaccuracy. We use our own *i*Transition model, which incorporates industry factors derived from equity prices, a much simpler approach than macroeconomic modelling. The *i*Transition model shows a greater change between Pre-GFC and GFC total credit risk than the traditional model. This means that those industries that were riskiest during the GFC are not the same industries that were riskiest Pre-GFC. The *i*Transition model also finds that the Australian portfolio, which has a much higher weighting towards financial stocks than the US portfolio, transitions very differently to the more balanced industry-weighted US portfolio. These results highlight the importance of including industry analysis into credit risk modelling.

To ensure a thorough analysis of the topic we use various approaches to measuring CVaR. This includes an analytical approach which is based on actual credit ratings as well as a Monte Carlo simulation approach which generates twenty thousand observations for each entity in the data set. We also incorporate historical default probabilities into the model in two different ways, one method using an average historical default rate over time, and the other method using annual default probabilities which vary from year to year. Overall, this comprehensive analysis finds that innovative modelling techniques are better able to account for the impact of extreme risk circumstances and industry composition than traditional transition matrix techniques.

**Keywords:** *credit models; credit value at risk; probability of default*

## 1. INTRODUCTION AND BACKGROUND

The GFC provided overwhelming evidence of the problems caused by poor credit risk management. The losses and increase in problem loans experienced by banks during the GFC is staggering. Over the two years to March 2009, impaired assets (problem loans) in the US more than trebled from 2.4% of total assets to 8.8%, an increase of USD 480 billion. The 5 largest US banks showed losses of nearly \$50 billion over the 2008 year. Australian bank impaired assets increased fivefold (although off a smaller base) from 0.2% to 0.95%, an increase of USD 24 billion (Reserve Bank of Australia, 2009). The US Government needed to provide a \$700bn Troubled Asset Relief Programme (TARP) to support banks. Australia introduced government deposit guarantees to shore up confidence in banks. In short, the subject of credit risk in extreme circumstances has become an intensely scrutinised topic in finance today. Yet, to date it has been poorly modelled and understood. According to all the widely used credit models, the GFC was not supposed to happen, because none of these models predicted the problem. In fact, the credit ratings of all major Australian banks were increased just before the GFC from AA- to AA on the basis of their sound credit risk outlook. Yet over the next two years, in addition to the five-fold increase in impaired assets, Australian banks' market capitalisation plunged by 58%.

Prevailing credit models were generally designed to predict credit risk on the basis of 'average' credit risks over time, or, in the case of Value at Risk (VaR) models on the basis of risks falling below a pre-determined threshold at a selected level of confidence, such as 95% or 99%. The problem with these models is that they are not designed to measure the most extreme losses, that is, those in the tail of the credit loss distribution. It is precisely during these extreme circumstances when firms are most likely to fail.

Transition matrix methodology, as used most notably by CreditMetrics (Gupton, Finger, & Bhatia, 1997), measures the probability of transitioning from one credit rating to another, and uses these probabilities to calculate VaR. Some examples of other studies in respect of ratings based modelling or transition matrices that can be referred to include bond pricing aspects (Jarrow, Lando, & Turnbull, 1997; Lando, 2004; Thomas, Allen, & Morkel-Kingsbury, 2002), time or business cycle sensitivity (Altman & Kao, 1992; Cowan, 2001; Trück, 2010), discussion of the transition modelling approach (Crouhy, Galai, & Mark, 2000; Saunders & Allen, 2002), and fixed interest credit spreads in Australia (Carrett, 2004).

We extend CreditMetrics methodology to calculate Conditional Value at Risk (CVaR) which measures extreme risk beyond VaR, thus addressing VaR's shortcoming in its lack of tail risk measurement. CVaR was traditionally used in the insurance industry but has gained popularity as a measure of market and credit risk. For examples of CVaR studies see Artzner, Delbaen, Eber & Heath, (1997; 1999), Bucay & Rosen (1999) and Andersson, Mausser, Uryasev & Rosen (2000).

CreditPortfolioView (Wilson, 1998) extends CreditMetrics methodology to incorporate industry risk, using macroeconomic factors, thus recognising not all industries transition equally. This phenomenon was illustrated over the GFC where, for example, the financial industry experienced more severe problems than other industries. However, macroeconomic approaches to industry risk measurement are not always popular as noted by the Australian Prudential Regulation Authority (APRA, 1999, p.4) in their statement "Currently none of the Australian banks favours a credit risk modelling approach conditioned on the state of the economy. Apart from the additional modelling complexity involved, the banks express concern that errors in forecasting economic turning points could lead, in particular, to a shortfall in desired capital coverage just as the economy turns sharply downwards". Based on the premise that all risks inherent in an industry should already be captured in market prices and VaR, we use a transition matrix (which we call *i*Transition) incorporating industry risk factors derived from equity price fluctuations, without the need for macroeconomic analysis.

The innovative techniques discussed in this paper were first introduced by us prior to the GFC to compare credit risk among Australian sectors (Allen & Powell, 2009). This study now extends their application to compare Australia and the US over both Pre-GFC and GFC periods. The key question explored is whether there are differences in credit risk measurements between traditional transition matrix techniques (which we call the undiversified transition model) and our innovative techniques (which include CVaR and *i*Transition) and whether these outcomes are consistent across Australia and the US.

## 2. DATA AND METHODOLOGY

### 2.1. Data

Data is divided into two periods: Pre-GFC and GFC. Our Pre-GFC period includes the 7 years from January 2000 to December 2006. This 7 year period aligns with Basel Accord advanced model credit risk requirements. Our GFC period includes January 2007 to June 2009. We use all Australian entities (less exclusions as noted later in this paragraph) with a Standard and Poor's or Moody's rating. For the US, there are a lot more rated entities so we limit it to two sources which ensure a mix of investment and speculative entities. The US data includes entities from the S&P500 index as well as Moody's Speculative Grade Liquidity Ratings list (Moody's Investor Services, 2010). We exclude entities with insufficient data. For example, we exclude those entities which do not have ratings in both Pre-GFC and GFC periods, and to ensure a reasonable number of companies are included in each industry for calculation of *i*Transition factors, we exclude any industries with less than 5 companies. This yields approximately 250 Australian entities and 370 US entities. We recognise that the differences in sources of data for the US and Australia mean the outcomes are not directly comparable. In addition, the Australian portfolio is more heavily weighted towards financial entities than the US portfolio which has a greater spread among industries. However, this has the advantage of allowing us to assess the outcomes of the modelling for portfolios with different characteristics. For standardisation of ratings, all Moody's ratings are mapped to S&P, using mapping provided by the Bank for International Settlements (2011).

### 2.2. Undiversified Transition Model

This model is based upon obtaining the probability ( $\rho$ ) of a bank customer transitioning from one grade to another as shown for the following BBB example:

$$\text{BBB} \quad \rho_{AAA} \quad \rho_{AA} \quad \rho_A \quad \rho_{BBB} \quad \rho_{BB} \quad \rho_B \quad \rho_{CCC/C} \quad \rho_D$$

External raters such as Moody's and Standard & Poor's (S&P) provide transition probabilities for each grading and we use the S&P (2008) global transition probabilities. We exclude non-rated categories and adjust remaining categories on a pro-rata basis as is the practice of CreditMetrics (Gupton *et al.*, 1997). The sum of all probabilities must equal 1.

We follow CreditMetrics (Gupton *et al.*, 1997) methodology as described in the following paragraphs. The model obtains forward zero curves for each rating category (based on risk free rates) expected to exist in a year's time. Using the zero curves, the model calculates the market value ( $V$ ) of the loan, including the coupon, at the one year risk horizon. Effectively, this means estimating the change in credit spread that results from rating migration from one rating category to another, then calculating the present value of the loan at the new yield to estimate the new value. The following example values a 5 year loan, paying a coupon of 6%, where  $r$  = the risk free rate (the rate on government bonds) and  $s$  = the spread between a government bond and corporate bonds of a particular category, say AA:

$$V = 6 + \frac{6}{(1+r_1+s_1)} + \frac{6}{(1+r_2+s_2)^2} + \frac{6}{(1+r_2+s_2)^3} + \frac{106}{(1+r_2+s_2)^4} \quad (1)$$

The above is calculated for each rating category, with yields for government and corporate bonds obtained from central bank websites. Transition table probabilities (in this case obtained from the S&P global transition table) are multiplied by  $V$  for each rating category to obtain a weighted probability. Based on the revised probability table, VaR is obtained by calculating the probability weighted portfolio variance and standard deviation ( $\sigma$ ), and then using a normal distribution (for example  $1.645\sigma$  for a 95% confidence level) as per the example in Table 1. We extend CreditMetrics methodology to calculate CVaR (see also Allen & Powell, 2009) using the lowest 5% of ratings for each industry and call this 'Analytical CVaR'.

CreditMetrics use Monte Carlo modelling as an alternate approach to estimating VaR. Transition probabilities and a normal distribution assumption are used to calculate asset thresholds ( $Z$ ) for each rating category as follows:

$$\text{Pr(Default)} = \Phi(Z_{\text{Def}}/\sigma)$$

$$\text{Pr(CCC)} = \Phi(Z_{\text{CCC}}/\sigma) - \Phi(Z_{\text{Def}}/\sigma)$$

and so on, where  $\Phi$  denotes the cumulative normal distribution, and

$$Z_{\text{Def}} = \Phi^{-1}\sigma \quad (2)$$

The above process is illustrated by the example in Table 1 for a BB loan, derived from methodology provided by CreditMetrics (Gupton *et al.*, 1997).

Column A shows theoretical transition probabilities for a BB loan, which would be derived from historical observations of transition behaviour. The loan has 84% probability of staying BB within a year ( $\text{Pr}(\text{BB}) = 84\%$ ), 5.5% chance of improving to a BBB rating ( $\text{Pr}(\text{BBB}) = 80.53\%$ ), 1% chance of defaulting ( $\text{Pr}(\text{D}) = 1.00\%$ ) and so on. Column B shows theoretical new loan values derived using equation 1. VaR is then calculated from columns D, E and F as shown in the table.

To derive the asset thresholds per equation 2 (and column G of the table), normal distribution tables are used to obtain the cumulative normal distribution of the probabilities in column A. These thresholds are shown in column H. For example the cumulative normal distribution of the values shown in column A for a BB loan = 1.48 standard deviations. 20,000 random numbers are generated for each loan in the portfolio. If a random number falls between thresholds, it is allocated to the threshold above it, for example, if a random number for a BB loan falls between -1.34 and 1.48, then that random number would correspond to a BB loan (i.e. no change to the existing BB ranking). If the random number falls between -1.88 and -1.34, it would be assigned a new rating of B. Thus each of the 20,000 random numbers for every loan in the portfolio is assigned a new rating using this process. CVaR will then be calculated in a similar fashion to Table 1, but using the standard deviation of the worst 5% of the simulated ratings.

Table 1. Transition Matrix VaR Methodology Example

	A	B	C	D	E	F	G	H
	Probability	New loan value plus coupon (\$)	Probability weighted value AxB	Distance from mean	Distance from mean <sup>2</sup> (D <sup>2</sup> )	Probability weighted difference <sup>2</sup> (AxE)	Threshold value calculation	Threshold value
AAA	0.10%	112	0.11	7.31	53.48	0.0535	$1 - \Phi(Z_{AA}/\sigma)$	
AA	0.40%	110	0.44	5.31	28.23	0.1129	$\Phi(Z_{AA}/\sigma) - \Phi(Z_A/\sigma)$	3.09
A	1.00%	108	1.08	3.31	10.98	0.1098	$\Phi(Z_A/\sigma) - \Phi(Z_{BBB}/\sigma)$	2.58
BBB	5.50%	107	5.89	2.31	5.35	0.2942	$\Phi(Z_{BBB}/\sigma) - \Phi(Z_{BB}/\sigma)$	2.17
BB	84.00%	106	89.04	1.31	1.72	1.4481	$\Phi(Z_{BB}/\sigma) - \Phi(Z_B/\sigma)$	1.48
B	6.00%	98	5.88	-6.69	44.72	2.6830	$\Phi(Z_B/\sigma) - \Phi(Z_{CCC}/\sigma)$	-1.34
CCC	2.00%	85	1.70	-19.69	387.58	7.7516	$\Phi(Z_{CCC}/\sigma) - \Phi(Z_{Def}/\sigma)$	-1.88
Default	1.00%	55	0.55	-49.69	2468.80	24.6880	$\Phi(Z_{Def}/\sigma)$	-2.33
<b>Mean (<math>\sum</math>Column C)</b>			104.69					
<b>Variance (<math>\sum</math>Column F)</b>			37.14					
<b><math>\sigma = \text{std deviation (sqrt variance)}</math></b>			\$6.09					
<b>95% VaR (1.645<math>\sigma</math>)</b>			\$10.03					

It has become common practice for modellers of transition matrices to use the average historical transition probabilities over the time period being modelled, as opposed to varying the probabilities year by year. This approach is effective in isolating how changes in ratings affect VaR over time. However, credit ratings change only periodically and, especially over periods like the GFC, this will not be effective in predicting actual changes in VaR as it ignores the impact of volatility in the default probabilities associated with the ratings, which changed dramatically over the GFC. Standard and Poor's provide annual probability matrices as well as historical averages over various extended time periods. We examine both approaches – one which uses fluctuating probabilities and one which uses an average for the 10 year period.

### 2.3. *i*Transition Model

CreditPortfolioView (Wilson, 1998) is a variation to the transition model which incorporates an adjustment to transition probabilities based on industry and country factors calculated from macroeconomic variables. As noted in Section 1, this model recognises that customers of equal credit rating may transition differently depending on their industry risk, but a study by APRA (1999) showed that banks did not favour using macroeconomic factors in their modelling due to complexities involved. Our own *i*Transition model (Allen, Kramadibrata, Powell, & Singh, 2011; Allen & Powell, 2009, 2011) uses the same framework as

CreditPortfolioView, but incorporates equity VaR instead of macroeconomic variables to derive industry adjustments. This is done by calculating market VaR for each industry, then calculating the relationship between market VaR and credit risk for each industry, using the Merton model to calculate the credit risk component. These factors are used to adjust the model as follows using a BBB rated loan example:

$$BBB \quad \rho_{AAi} \quad \rho_{AAi} \quad \rho_{Ai} \quad \rho_{BBBi} \quad \rho_{BBi} \quad \rho_{Bi} \quad \rho_{CCC/Ci} \quad \rho_{Di}$$

### 3. RESULTS

Table 2. Results - Undiversified Transition Matrix

Model	VaR	CVaR Analytical	CVaR Monte Carlo
<b>Australia</b>			
Transition Pre-GFC	0.0242	0.0342	0.0343
Transition GFC (Static)	0.0306	0.0396	0.0363
Transition GFC (Vary)	0.0417	0.0707	0.0686
<b>US</b>			
Transition Pre-GFC	0.0197	0.0449	0.0495
Transition GFC (Static)	0.0317	0.0512	0.0519
Transition GFC (Vary)	0.0453	0.0908	0.0865

The table shows a substantial difference between VaR and CVaR which is significant at the 99% level when using an F test for differences in volatility. The Monte Carlo and Analytical CVaR methods yield similar results with no significant differences found between these two approaches. As per the methodology section, we have used two GFC measurements. The first measurement (static) is based on changes to ratings only, with no change to default probabilities in the matrix. This shows very little increase from Pre-GFC to GFC outcomes in both countries. Given the high increase in credit risk over the GFC period (fivefold increase in impaired assets), this means that models based solely on

ratings, such as the Basel standardised approach to measuring, do not reflect (nor are designed to reflect) changing economic circumstances. The second measurement (vary) includes using different default probabilities Pre-GFC and GFC even though the underlying ratings have not changed. This causes a substantial increase in the VaR and (especially) CVaR measurements for both Australia and the US.

Table 3. Results – *i*Transition

Region	VaR	CVaR Analytical	CVaR Monte Carlo
<b>Pre-GFC</b>			
Australia	1.1x	1.0x	1.0x
US	0.9x	1.0x	1.0x
<b>GFC</b>			
Australia	2.5x	2.4x	2.5x
US	1.2x	1.2x	1.1x

Rather than show absolute values, it is more meaningful (as per Table 3) to show the increase in VaR and CVaR measurements that were obtained by using *i*Transition as opposed to the undiversified Transition matrix. For example, GFC VaR (static) for Australia was 2.5x higher (0.0763 under *i*Transition as compared to 0.0306 under undiversified Transition). We note that pre-GFC, there is similarity between the two models for both Australia and the US, as indicated by all numbers being close to 1.0x. This is because neither the US or

Australian datasets had a high weighting in any industry with a particularly high or low risk. However, the picture changes markedly during the GFC, where the impact of *i*Transition was much lower for the US with a differential of only approximately 1.2x between undiversified transition and *i*Transition outcomes as compared to the 2.5x shown by Australia. The reason for this is simple: the Australian portfolio had a much higher weighting toward financial stocks than the US portfolio. Whilst financial entities had a higher weighting in the US portfolio than other industries, as reflected by the *i*Transition model transitioning slightly higher than the undiversified model, the US portfolio overall had a more balanced spread of industries than the Australian portfolio. The well known problems faced by financial institutions during the GFC caused substantial deterioration in financial entity asset values relative to other sectors. The undiversified model ignores this industry impact, with all sectors transitioning equally. The *i*Transition model factors it in and correctly measures that a portfolio weighted with financial stocks would transition differently to one that wasn't.

#### 4. CONCLUSIONS

Two innovative techniques were examined, being CVaR and *i*Transition which incorporates industry factors derived from equity price fluctuations. In both the US and Australian markets, CVaR is significantly higher than VaR, with CVaR highlighting the extreme risk that is ignored by the undiversified model. The differential between VaR and CVaR is higher in the US, reflecting the more extreme credit risk in the US as outlined in the introduction. The *i*Transition model shows how portfolio composition affects transition, with the financial industry weighted Australian portfolio transitioning differently to the more balanced US portfolio. Overall, the innovative modelling techniques are able to better measure the impact of extreme market circumstances and industry composition than traditional transition modelling.

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