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Transfer of Mathematical Knowledge: Series[1]

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Abstract: The aim of this study is to explain students' ability to transfer their knowledge about mathematical series to the problems that they encounter. The data of the study were obtained by using two different tests, namely "Problem Solving Test (PST)" and "Series Character Identification Test (SCT)" which were developed by the researchers. The study was conducted to third- grade students from department of elementary school mathematics education in the 2009-2010 academic year. In view of the analysis of the data, it was observed that the students experienced no difficulty in the SCT which required procedural knowledge. They experienced difficulty in the PST which required skill to transfer their knowledge about the series to the circumstances that they encounter. According to these results, it was determined that the students experienced difficulty in transforming real life problems to series and interpreting these problems

Introduction

There have been important changes in opinion regarding what mathematics is and how it must be taught. The teacher in the traditional mathematics instruction presents mathematical knowledge to the students. The idea that learning mathematics is making mathematics has gained importance today (Putnam, Lampert & Peterson, 1990; Toluk, 2003). Transferring the acquired knowledge to the new circumstances that we encounter stands out in the idea of making mathematics beyond the comprehension stage that can be studied as oriented towards the cognitive domain. Altun (2005) emphasizes the importance attached to problem solving by defining the aim of the mathematics instruction as; (i) making the person learn the mathematical knowledge and skills required by the real life in general, (ii) teaching him/her how to solve the problems, and (iii) making him/her learn a way of thinking that deals with the situations in the problem solving approach.

Understanding the mathematical knowledge and drawing the relationship between these pieces of knowledge stands out in the process of problem solving (Swings & Peterson, 1988). Students must combine the concepts and the procedures, and apply them on the solution of the problem during problem solving (Bernardo, 1999). Conceptual knowledge does not only consist of recognizing the concept or knowing the definition and the name of the concept, but at the same time, having the ability to recognize the mutual transitions and relationships among the concepts. Procedural knowledge is explained with the two separate

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sections that form it. The first section of procedural knowledge includes the symbols and language of mathematics. For example, the symbol y=f(x) denotes a function. The second section includes the relations used for solving the mathematics problems, procedures related to concrete objects (using base ten blocks), visual diagrams, concept images or other nonstandard objects of the mathematics system (Hiebert & Lefevre, 1986). When the procedural knowledge (or the knowledge of rules) comes between the conceptual knowledge, the individual can explain how the procedures are performed, and why they are performed. The failure to learn the conceptual basics of the procedural knowledge and draw a relationship between this knowledge and the concepts cause the failure to build the models and decide where the procedures will be used. This manifests itself as the failure in problem solving (Baykul, 2005).

Mathematical knowledge can be learned by balancing procedural knowledge with conceptual knowledge (Baki, 1998; Van De Walle, 2004). When mathematics courses are not given conceptually, an inclination towards memorization will emerge instead of learning. Generally, procedures are valued to be important instead of the concepts in teaching since the conceptual knowledge is gained by memorization only as a rule without considering the reasons and causes (Baki & Kartal, 2004). In this process, many students cannot realize that there are concepts at the basis of the procedures they use, and they do not know what mathematics means. They believe that learning mathematics is to perform operations on meaningless symbols, and they try to learn mathematics by memorization (Oaks, 1990). As for the conceptual learning, the students are problem solvers who can effectively use their own creativity, intuitions and skills in solving problems and producing mathematical knowledge. For that reason, a conceptual learning approach regards mathematics as a network of interconnected concepts and thoughts, and recommends that students structure mathematical concepts and thoughts by themselves instead of copying them from outside (Bell & Baki, 1997).

Due to various reasons, some concepts are taught at the procedural level instead of the conceptual level in mathematics courses. Because of epistemological difficulties, the series is one of these concepts. The idea that infinite sums may not give a finite value makes difficult for students to conceptualize the series. Furthermore, the series is taught by being reduced to algorithmic viewpoints (the use of the formula by memorization, the failure to take into account the relationship between this concept and other important concepts, and deficiencies in eliminating misconceptions regarding infinity) instead of being taught at the conceptual level (González-Martín, Seffah, Nardi & Biza, 2008). These approaches, which are used in teaching the series, constitute an impediment for students to learn the series at the conceptual level. It was reported that the exercises used in teaching prevent the students from constructing a correct notation for the convergence of series (Robert, 1982; reported by Gonzalez-Martin et al., 2008). It was acknowledged that the traditional teaching features very few examples of graphic representations related to the convergence of series (Boschet, 1983; cited by Gonzalez-Martin et al.). Gonzalez-Martin et al. stated that students have no visual images related to the concept of series. Alcock and Simpson (2004) stated that visualization will provide students with an advantage to comprehend the subject.

One of the reasons which constitute an impediment for teaching the series is that the relationship between the concept of series and the other concepts is not taken into account. Fay & Webster (1985) showed that in most of the Calculus textbooks, there is little or no relation between indefinite integrals and infinite series other than the integral test for the convergence of series. Gonzalez-Martin et al. (2008) stated students learn the concept of indefinite integral without associating it with the concept of series.

Learning mathematics does not only mean filling the mind with ready information but also using that information in solving problems in a way to reveal the individual's own thoughts (Baki & Kartal, 2004). When solving a problem, the individual must undergo a cognitive process that involves; understanding the problem sentence, making a plan for the solution, applying the plan and making an evaluation. When viewed from a cognitive perspective, mathematical problem solving is at the application level of Bloom's Taxonomy. This study examined university students' skills to transfer their knowledge about the series to the process of solving the problems related to the series.

Method

Descriptive research design was used in the study. Research using a descriptive design simply provides a summary of an existing phenomenon by using numbers to characterize individuals or groups (McMillan & Schumacher, 2010). The study was conducted with 97 third grade students from the department of elementary school mathematics education in the 2009-2010 academic year. "Problem Solving Test (PST)" and "Series Character Identification Test (SCT)" were used as data collection instruments in order to determine the students' skill in transforming their knowledge about the series to the problems that they encounter. PST is composed of four problem situations that require transferring an event about the real life to series. The first two questions in the PST include problems that require transferring the problem situation to series, identifying their characters and interpreting the results. The last two questions include only the problems that require transferring the problem situation to series and identifying their characters. The questions included in the SCT are composed of the series that give the solution of the problem situations that are featured in the PST. Expert opinions were taken for the purpose of maintaining the validity of the prepared tests. In regard with this, a pilot study was conducted. A pilot study is a "pre-study" of our fuller study. In the pilot study, SCT and PST were applied other group in checking the reliability and validity of results. Tests, whose final forms were prepared, were applied on the same day with two separate sessions. The data obtained from the PST were evaluated in the categories entitled "fully correct", "partially correct" and "incorrect". Forming the series required for the solution of the problem, identifying the character of the series and reaching a solution by correctly interpreting the series was evaluated as "fully correct". Forming the series and identifying the character of the series, but failure to reach the requested solution by making a mistake in interpretation was evaluated as "partially correct". Failure to form the series was evaluated as "incorrect". The questions included in the SCT were evaluated as "correct" or "incorrect" in accordance with being able to determine the character of the series. Students were asked to write code names instead of their real names on the answer sheets in order to compare their answers given to the questions in both tests. Here, the purpose of using code names instead of real names is to obtain more accurate information. Using code names provides the students' right to privacy through anonymity and confidentiality.

Findings

The findings were obtained regarding students' skills to transfer their knowledge about the series to problem situations about the real life. The data were presented using percentage and frequency distributions. Findings regarding students' identification of characters within the series, which are required for solving the problems in the SCT, are given in Table 1.

Series	Correct	Incorrect
	f (%)	f (%)

$\sum_{n=1}^{\infty}\frac{1}{n}$	83 (85.6)	14 (14.4)
$\sum_{n=1}^{\infty}\frac{25}{2^n}$	87 (89.7)	10 (10.3)
$\sum_{n=1}^{\infty}\frac{9}{10^n}$	89 (91.2)	8 (8.8)
$\sum_{n=0}^{\infty}(\frac{3}{4})^{2n}$	88 (90.7)	9 (9.3)

Table 1: The distribution of students' responses to SCT

In view of the findings in Table 1, it is observed that students are generally successful with a percentage around 90% in identifying the character of the series. It can be stated from the obtained findings that the students have the adequate level of procedural knowledge which is required for solving the problems.

The distribution of students' answers for the solution of problem situations using the series is given in Table 2.

Problem	Fully Correct		Partially Correct		Incorrect	
	f	(%)	f	(%)	f	(%)
1.	6	(6.2)	86	(88.6)	5	(5.2)
2.	3	(3.1)	87	(89.7)	7	(7.2)
3.	74	(76.3)	22	(22.7)	1	(1)
4.	76	(78.4)	18	(18.5)	3	(3.1)

Table 2: The distribution of students' responses to PST

In view of the data in Table 2, it is observed that the average of the "fully correct" answers given to the first two questions of the PST by the students is 4.65% whereas the average of the "fully correct" answers given to the last two questions of the PST by the students is 77.4%. It is seen from these data that there is vulnerability between the answers given to the first two questions by the students and the answers given to the last two questions in terms of students' success in problem solving. Apart from forming only the related series and correctly determining the character of the series, the errors in correctly interpreting the character of the series can be regarded as the reason for this vulnerability.

It was found that a great majority of the students (88.6 % and 89.7%) correctly determined the characters by forming $\sum_{n=1}^{\infty}\frac{1}{n}$ and $\sum_{n=1}^{\infty}\frac{25}{2^n}$ series during the process of problem solving, but they weren't able to reach the correct solution since they misinterpreted the character of these series. It was observed that the obtained concept of infinity was interpreted as the number of discharge, not as an additive value although they determined that the sum of $\sum_{n=1}^{\infty}\frac{1}{n}$ series is infinite, and accordingly, the series is divergent. Similarly, it was observed that the students determined that $\sum_{n=1}^{\infty}\frac{25}{2^n}$ series was convergent and its sum was 25, but they were not be able to reach the correct solution since the value 25 was interpreted as the number of discharges. In regard with this condition, the answers of two students with code names "Issız

Adam" (Lonesome Man) and "Kanka-3" (Buddy-3) were scanned and given in Figure 1 and Figure 2.

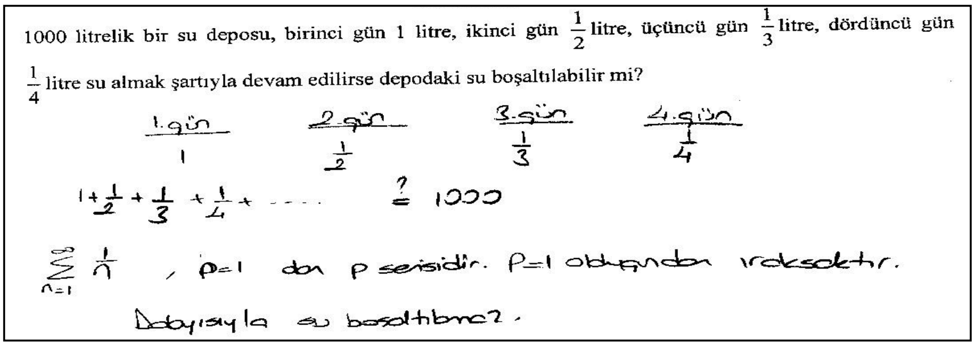


Figure 1: The answer given to the question 1 in the PST by a student

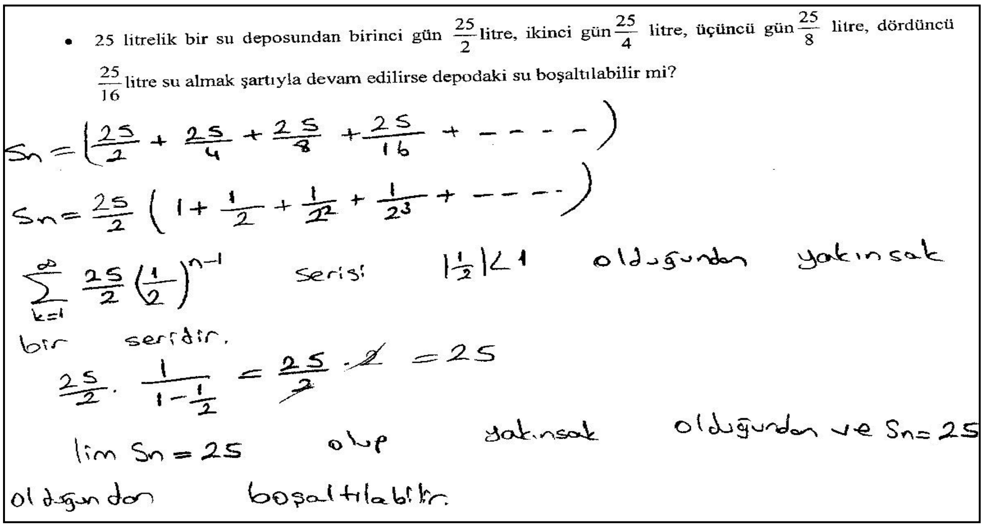
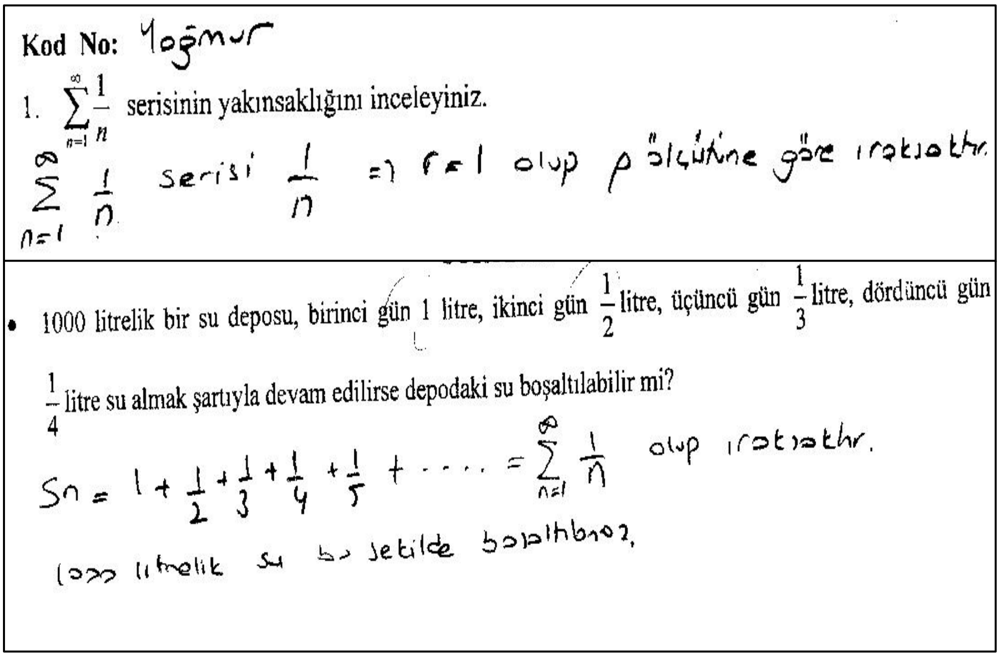


Figure 2: The answer given to the question 2 in the PST by a student

When the data in Table 1 and Table 2 are compared, it is seen that the ratio of correct answers is approximately 90% in the SCT whereas the ratio of fully correct answers is approximately 41% in the PST. It is observed that the difference between these ratios is higher among the first two questions in the SCT and the corresponding first two questions in the PST. It is seen that the ratio of correct answers for the first two questions is approximately 88% in the SCT whereas the ratio of fully correct answers for the corresponding first two questions is 4.65% in the PST. In regard with this condition, the answer given to the first question in the SCT and PST by the student with code name "Ya mur" (Rain) is given in Figure 3.


Kod No: Yağmur

1. $\sum_{n=1}^{\infty}\frac{1}{n}$ serisinin yakınsaklığını inceleyiniz.

$\sum_{n=1}^{\infty}\frac{1}{n}$ serisi $\frac{1}{n}$ => r≤1 olup p ölçütüne göre ıraksaktır.

• 1000 litrelik bir su deposu, birinci gün 1 litre, ikinci gün $\frac{1}{2}$ litre, üçüncü gün $\frac{1}{3}$ litre, dördüncü gün $\frac{1}{4}$ litre su almak şartıyla devam edilirse depodaki su boşaltılabilir mi?

$S_n = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\dots=\sum_{n=1}^{\infty}\frac{1}{n}$ olup ıraksaktır.

1000 litrelik su bu şekilde boşaltılamaz.

Figure 3: The answer of a student who correctly found the series character, but wasn't able to perform the knowledge transfer.

Conclusion and Discussion

In view of the obtained findings, it was observed that the ratio of students' correct answers in the SCT, which required procedural knowledge, was high. Thus, it can be stated that the students experienced no difficulty in using procedural knowledge related to the series. On the other hand, it was determined that high success, which was observed in the procedural knowledge dimension, was not observed in the conceptual knowledge dimension that requires solving problems and interpreting. According to Sabella and Redish (1995), the real difficulty for students is to learn the concepts related to the given subjects, not to learn algorithmic calculations. Nonetheless, mathematical knowledge of many of the students in the world, primarily including the students in the USA, is at the procedural stage. The result obtained from our research supports this result of Sabella and Redish. This case is seen in the first and second problems of the PST which require transferring the problem situation to series, identifying their characters and interpreting the result. However, it was observed that the success levels in the third and fourth problems of the PST, which require only transferring the problem situation to series and identifying their characters, is similar to the success level in the procedural dimension. At this point, it can be said that the students experienced difficulty in transforming real life problems about the series into series and interpreting these problems. According to Gonzalez and Martin (2008), the idea that infinite sums may not give a finite value makes difficult for students to conceptualize the series. This result corresponds to the findings obtained from the first and second problems of the PST.

The concept of mathematical series has many areas of application such as medicine, physical sciences and economics. Particularly, series constitute the focal point of Riemann sum and certain integral calculations. The fact that the procedural aspect of this concept is frequently studied causes its conceptual dimension to be neglected. The chief reasons for students failing to form conceptual knowledge are frequent application of the formulae in a careless way, the failure to draw its relationship with other concepts, and misconceptions about the concept of infinity. In this regard, students must be taught the skills such as interpretation, reasoning, association, and critical thinking. This can be achieved by including real life problems in the process of teaching the subject of series rather than solely identifying the character.

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