2013

The teaching and learning of word problems in beginning algebra : a Nigerian (Lagos State) study

Omolola A. Ladele
Edith Cowan University

USE OF THESIS

The Use of Thesis statement is not included in this version of the thesis.

THE TEACHING AND LEARNING OF WORD PROBLEMS IN BEGINNING ALGEBRA: A NIGERIAN (LAGOS STATE) STUDY

Omolola Aina LADELE

B.Sc (Ed) Mathematics, Hons (Ahmadu Bello University, Zaria)

M.Phil. (University of Lagos, Lagos)

A thesis submitted in fulfilment of the requirements for the award of

the Doctor of Philosophy in the School of Education, Faculty of Education and Arts

at the Edith Cowan University, Perth, Australia.

September, 2013

“I give you thanks, O LORD, with all my heart; I will sing your praise…I will give thanks to your name for your unfailing love and faithfulness“ Psalm 138: 1-2 (Holy Bible: New Living Translation, 1996).

All the way my Savior leads me;
What have I to ask beside?
Can I doubt His tender mercies,
Who thro’ life has been my guide?
Heav’nly peace, divinest comfort,
Here by faith in Him to dwell!
For I know, whate’er befall me,
Jesus doeth all things well;
For I know, whate’er befall me,
Jesus doeth all things well.

All the way my Savior leads me;
Cheers each winding path I tread,
Gives me grace for ev’ry trial,
Feeds me with the living bread.
Tho’ my weary steps may falter,
And my soul athirst may be,
Gushing from the Rock before me,
Lo! a spring of joy I see;
Gushing from the Rock before me,
Lo! a spring of joy I see.

All the way my Savior leads me;
Oh, the fullness of His love!
Perfect rest to me is promised
In my Father’s house above;
When my spirit, clothed immortal,
Wings its flight to realms of day,
This my song thro’ endless ages:
Jesus led me all the way;
This my song thro’ endless ages:
Jesus led me all the way.

Hymn 62 (Baptist Hymnal, 1991)

ABSTRACT

At both the junior and senior secondary school levels in Nigeria, student performance in mathematics examinations has been poor. Within the context of large classes, with inadequate facilities, and teaching and learning in a second language, algebra and algebra word problems are introduced to students during their first year of junior secondary school. The transition from primary school arithmetic to the use of the algebraic letter is challenging to students and it is important that teachers should know the likely difficulties and misconceptions students may have as they begin algebra (Welder, 2012).

In this study, the impact of a teacher professional learning program on teachers' knowledge, beliefs and practice was examined. The impact on students' ability to solve word problems in beginning algebra was also investigated. To do this, a multiple case study was designed and data were collected using quantitative and qualitative methods. Thirty teachers of first year junior secondary students completed a questionnaire and this provided general information about the teachers' beliefs and algebra teaching practice. After this, 12 of the teachers actively participated and collaborated in a professional learning workshop designed as an intervention program. The program focused on enhancing the teachers' knowledge of student misconceptions about variables, expressions and equations, and language-based teaching strategies. Four teachers and their classes, two each from public and private schools, served as case studies and provided further data about the impact of the intervention program. Before and after the intervention program, lessons were observed, students completed algebra tests and some of them were interviewed using the Newman interview protocol. The data for each case study were analysed and the key findings generated from each of them were used for a cross-case analysis.

The study revealed that these Nigerian teachers had mainly traditional beliefs about mathematics teaching and that teacher-talk dominated the classroom practice. Prior to the intervention, the teachers had limited knowledge of students' algebra misconceptions and the students' main difficulty was that they did not understand the questions. The professional learning increased the teachers' knowledge of algebra, their pedagogical content knowledge and their awareness of algebra misconceptions. The teachers used more student-centred and language-based teaching strategies when working on algebra problems. There was a significant improvement in students' problem-solving success on the post-test because more students were able to understand the word problems and displayed fewer misconceptions. The incidences of ignoring the algebraic letter, believing that the algebraic letters cannot have the

same value and confusing product and sum reduced. However, the use of the letter as an object or a label and a belief that the algebraic letter had alphabetical positioning persisted.

The study demonstrated the effectiveness of the professional learning model used in this study and it should be considered for more widespread implementation with in-service teachers. There is also an implication for pre-service teacher education. Mathematics education programs should ensure that student teachers are aware of common algebra misconceptions and the language-based strategies needed to support school students’ transition from arithmetic to algebra.

DECLARATION

I certify that this thesis does not, to the best of my knowledge and belief:

i. incorporate without acknowledgement any material previously submitted for a degree or diploma in any institution of higher education;
ii. contain any material previously published or written by another person except where due reference is made in the text of this thesis; or
iii. contain any defamatory material

I also grant permission for the Library at Edith Cowan University to make duplicate copies of my thesis as required.

Signed: Date: 24 September 2013

ACKNOWLEDGEMENTS

My deep gratitude and appreciation goes to my supervisors, Dr Christine Ormond and Professor Mark Hackling, first for accepting me as their research candidate and second, for providing continuous guidance and encouragement all the way. Chris, the passion you have for beginning algebra is 'infectious' and I 'caught the bug'. You were ever ready to provide assistance and the push needed to get me going whenever I was feeling challenged. Thank you for all the smileys, Chris. Mark, you gently introduced me to the wonderful world of qualitative research and its beauty, and patiently supported me as I found my way through the maze. For your deep insights and mentoring, I thank you Mark. May the Almighty God richly bless both of you.

I am grateful to the Commonwealth of Australia for granting me the Endeavour International Postgraduate Research Scholarship (EIPRS) and to the Edith Cowan University for the International Stipend (ECUIS) which facilitated my ability to focus on my research study.

To Dr Jo McFarlane, the faculty writing consultant, you eased the transition and my adjustment to the Australian academic culture beyond writing skills. Thank you, Jo. I thank Associate Professor Jan Gray, Dr Susan Hill, Sarah Kearn, and also Heather Williams in the Graduate Research School for their help at various stages of my studies. I also acknowledge the collegial support I received from fellow postgrads Dr Femi Adedina, Dr Bolanle Olaleye, Dr Isaiah Awidi, Christina Lee, Khairiah Syahabuddin and Steve Chia. Thank you.

I thank the management of Adeniran Ogunsanya College of Education (AOCOED), Lagos for granting me permission to embark on this doctoral study. Thank you. I am grateful to the former provost of the college Mr Hakeem Ajose-Adeogun, and to Dr Benjamin, Dr (Mrs) Odigie, Dr (Mrs) Oginni, and Dr Hussain for their support. I appreciate the teachers and students who participated in the study. Thank you for allowing me into 'your space' and enabling the research to become a reality.

With deep gratitude, I acknowledge my two dads, Mr Samuel C. O. Osunkiyesi and Mr Gabriel F. F. Ladele, who believed in me, never let me forget this dream and kept on challenging me until sadly they both passed on. I also acknowledge my late mum, Mrs J. Olufunmilayo Osunkiyesi, and my second mum Mrs Gladys T. T. Ladele for their love and inspiration.

I appreciate Dr Sunmolu and Mrs Funlola Beckley for all their support and efforts towards ensuring that I had minimal distractions during my studies. Uncle and Aunty thank you, and thank you again. To my siblings ‘Modele, ‘Motayo and ‘Motola, your constant encouragement and prayer support kept me going. Thank you. Edwin Olorunju, thank you for being available.

My wonderful sons Oluwadamilola, Oluwabukunola and Oluwatomilola coped with my absence graciously, and patiently. Thank you for your understanding. My loving husband, Bola, with godly strength you bore the brunt and weight of everything. Thank you. Thank you for supporting me to fulfil this dream.

To many others in Australia and AOCOED who were instrumental and eased the completion of this research work, I am grateful. Thank you and God bless you all.

TABLE OF CONTENTS

ABSTRACT....ii
DECLARATION....iv
ACKNOWLEDGEMENTS....v
TABLE OF CONTENTS....vii
LIST OF TABLES....xiii
LIST OF FIGURES....xvii
LIST OF APPENDICES....xix
CHAPTER ONE: GENERAL INTRODUCTION....1
Background....1
Algebra....3
The Problem....5
Rationale....6
Significance....6
Purpose and Research Questions....7
CHAPTER TWO: LITERATURE REVIEW....9
Background....9
Mathematics education in Nigeria....9
Importance of mathematics education and algebra....12
Social Constructivism and Sociocultural Theory....13
Learning Algebra....15
Cognitive processes in algebra....15
Word problems....17
Misconceptions in beginning algebra....18
Diagnosis of students' problem-solving difficulties: The Newman protocol....22
Teacher and Professional Learning....25
Teacher beliefs and knowledge that affect practice....25
Teacher professional learning....28
Conceptual Framework....30
CHAPTER THREE: METHODOLOGY....33
Research Approach....33
Research Design....33
Research Participants....34
Research Procedure....35

Pilot study 35
Main study 35
Research Instruments 37
Mathematics teacher questionnaires 38
Classroom observation schedule 39
Algebra tests 39
Newman interview protocol 42
Newman error analysis guideline sheet 42
Diagnostic profile sheet 42
Teacher algebra rating sheet 42
Focus group 43
Digital recorder and camera 43
Reliability and Validity of Instruments 43
Data Analysis 44
Ethics 45
Limitations 45
CHAPTER FOUR: PROFESSIONAL LEARNING INTERVENTION 47
Introduction 47
Day One 47
Day Two 59
Day Three – Evaluation of Intervention 65
Focus Group Interviews 68
Difficulties students encountered solving word problems 68
Algebraic misconceptions 71
Key concepts in algebra 72
Suggestions for improving the teaching and learning of algebra 73
Summary 75
CHAPTER FIVE: TEACHER PERCEPTIONS 77
Introduction 77
The Teaching and Learning of Mathematics and Algebra 77
Demographic data 77
Challenges and beliefs about effective mathematics teaching 80
Teachers' confidence with mathematics teaching 82
Teachers' classroom strategies 83
Teacher beliefs about mathematics, their teaching and students' learning 86

Teachers' assessment of algebra questions 88
Professional Learning Participants Data 90
Teachers' beliefs about effective teaching and learning of mathematics 90
Teachers' confidence about mathematics knowledge and algebra teaching strategies.... 91
Teachers' classroom strategies 93
Teacher beliefs about mathematics, their teaching and students' learning 95
Teacher assessment of algebra questions 97
Importance of knowledge of students' thinking and mathematical talk........ 98
Difficulties using the Newman language-based error analysis procedure 99
Workshop Gains and Suggestions........ 100
Summary 101
CHAPTER SIX: CASE STUDY ONE – RUTH'S CLASS 103
Background 103
Pre-Intervention Beliefs and Practice 103
Beliefs........ 103
Practice........ 104
Post-intervention Beliefs and Practice........ 106
Beliefs........ 106
Practice........ 108
Changes in Ruth's Beliefs and Practice 111
Students' Algebra Pre-test Performance 112
Students' Pre-test Newman Interview Results 113
Symbolic Questions: 1- 6........ 114
Worded Questions: 7-15........ 116
Students' Algebra Post-test Performance 119
Students' Post-test Newman Interview Results 120
Symbolic Questions: 1–6........ 121
Worded Questions: 7–15 123
Changes in Ruth's Students' Performances and Error Types........ 126
Summary 127
CHAPTER SEVEN: CASE STUDY TWO - DOROTHY'S CLASS 129
Background 129
Pre-Intervention Beliefs and Practice 129
Beliefs........ 129
Practice........ 130

Post-intervention Beliefs and Practice 132
Beliefs 132
Practice 134
Changes in Dorothy's Beliefs and Practice 136
Students' Algebra Pre-test Performance 136
Students' Pre-test Newman Interview Results 137
Symbolic Questions: 1-6 139
Worded Questions: 7-15 141
Students' Algebra Post-test Performance 143
Students' Post-test Newman Interview Results 145
Symbolic Questions: 1-6 145
Worded Questions: 7-15 148
Changes in Dorothy's Students' Performance and Error Types 150
Summary 152
CHAPTER EIGHT: CASE STUDY THREE – JAMIE'S CLASS 155
Background 155
Pre-Intervention Beliefs and Practice 155
Beliefs 155
Practice 156
Post-Intervention Beliefs and Practice 157
Beliefs 157
Practice 158
Changes in Jamie's Beliefs and Practice 159
Students' Algebra Pre-test Performance 160
Students' Pre-test Newman Interview Results 161
Symbolic Questions: 1-6 162
Worded Questions: 7-15 164
Students' Post-test Performance 166
Students' Post-test Newman Interview Results 168
Symbolic Questions: 1-6 169
Worded Questions: 7-15 171
Change in Jamie's Students' Performances and Error Types 173
Summary 175
CHAPTER NINE: CASE STUDY FOUR – STEPHEN'S CLASS 177
Background 177

Pre-Intervention Beliefs and Practice 177
Beliefs 177
Practice 178
Post –Intervention Beliefs and Practice 179
Beliefs 179
Practice 180
Changes in Stephen’s Beliefs and Practice 181
Students’ Algebra Pre-test Performance 181
Students’ Pre-test Newman Interview Results 182
Symbolic Questions: 1-6 183
Worded Questions: 7-15 185
Students’ Post-test Performance 188
Students’ Post-test Newman Interview Results 189
Symbolic Questions: 1-6 190
Worded Questions: 7-15 192
Change in Stephen’s Students’ Performances and Error Types 194
Summary 195
CHAPTER 10: CROSS-CASE ANALYSIS AND DISCUSSION 197
Introduction 197
Teachers’ Beliefs and Practice before the Intervention 197
Challenges of teaching algebra 197
Teachers’ prior beliefs and knowledge 199
Observed teaching practice prior to the PL 201
Students’ General Performance and Difficulties before the Intervention 203
Pre-test performance and error frequencies prior to the PL 203
Language problems before the PL 205
Algebra misconceptions before the PL 206
Teachers’ Beliefs and Practice after the Intervention 210
Teachers’ beliefs and knowledge after the PL 210
Teachers’ observed practice after the PL 212
Students’ General Performances and Difficulties after the Intervention 213
Post-test performance and error frequencies after the PL 213
Language problems after the PL 217
Algebra misconceptions after PL 218
Fusing the Research Findings 221

CHAPTER ELEVEN: CONCLUSION AND IMPLICATIONS 223
Introduction 223
Overview 223
Conclusions 224
Implications 226
Initial teacher education 226
Professional learning for in-service teachers 226
Curricular reforms 227
Class size 227
Research 227
Original Contributions to the Literature 227
Final Note 228
REFERENCES 229
APPENDICES 240

LIST OF TABLES

Table 3.1: Summary of the research study procedure 36

Table 3.2: Relationship between research questions and data gathering instruments 38

Table 3.3: Algebra test items – concepts and required knowledge 40

Table 4.1: Teachers' mean rating scores and rankings of 15 algebra questions (n=13) 48

Table 4.2: Teachers' solutions to algebra questions (n=13) 50

Table 4.3: Teachers' reflections on the importance of Newman performance strategies (n=13) 58

Table 4.4: Teachers' self-corrected responses to algebra questions (n=13) 63

Table 4.5: Teachers' classification of students' likely misconceptions for given wrong answers (n=13) 64

Table 4.6: Teachers' use of language-based approaches (n=12) 66

Table 4.7: Strategies teachers found effective and the reasons offered (n=12) 67

Table 4.8: Strategies teachers found difficult to implement and the reasons (n=12) 68

Table 5.1: Teachers' by gender, zone and age group (n=30) 78

Table 5.2: Teachers' years of Mathematics and JS 1 teaching experience (n=30) 78

Table 5.3: Teachers' qualification (n=30) 79

Table 5.4: Challenges faced by teachers in the effective teaching of mathematics (n=30) 81

Table 5.5: Teachers' responses about effective mathematics teaching and learning (n=30) 81

Table 5.6: Teachers' responses about confidence with mathematical knowledge (n=30) 82

Table 5.7: Teachers' responses about confidence in using strategies to teach algebra (n=30) . 83

Table 5.8: Teachers' responses to ways of managing talk in the mathematics classroom (n=30) 84

Table 5.9: Frequency of teachers' use of various teaching approaches (n=30) 85

Table 5.10: Teachers' rating of the level of students' engagement in the classroom (n=30) 86

Table 5.11: Teachers' beliefs about mathematics, their teaching and students' learning (n=30) 87

Table 5.12: Ranking of algebra questions in order of difficulty by teachers (n=30) 88

Table 5.13: Teachers' reasons for questions ranked in the difficult category (n=30) 89

Table 5.14: PL teachers' beliefs about characteristics of effective mathematics teaching (n=12) 90

Table 5.15: PL teachers' beliefs about effective strategies for learning mathematics (n=12) ... 91

Table 5.16: PL teachers' self-reported confidence level with knowledge of junior school mathematics (n=12) 91

Table 5.17: PL teachers' responses about confidence in using strategies to teach algebra (n=12) 92
Table 5.18: PL teachers' responses about managing talk in the classroom (n=12) 93
Table 5.19: PL teachers' responses about the frequency of use of different approaches (n=12) 94
Table 5.20: PL teachers' belief about student engagement level in the classroom (n=12) 95
Table 5.21: PL teachers' belief about mathematics, their teaching and students' learning (n=12) 96
Table 5.22: PL teachers' ranking of algebra questions (n=12) 97
Table 5.23: PL teachers' reasons for choice of perceived difficult questions (n=12) 98
Table 5.24: PL teachers' responses about mathematical talk and knowledge of students' misconceptions (n=12) 99
Table 5.25: PL teachers' responses to difficulties faced in the use of Newman strategy (n=12) 100
Table 5.26: PL teachers' suggestions and recommendations (n=12) 100
Table 6.1: Ruth's students' pre-test total score (n=34) 112
Table 6.2: Number of Ruth's students with correct answers in each pre-test question (n=34) 112
Table 6.3: Per cent of error types made by Ruth's students on the algebra pre-test (n=4) 113
Table 6.4: Ruth's students' responses and initial errors on pre-test symbolic questions (n=4) 116
Table 6.5: Ruth's students' responses and initial errors on pre-test word problems (n=4) 119
Table 6.6: Ruth's students' post-test total score (n=34) 119
Table 6.7: Number of Ruth's students with correct answers in pre- and post-test questions (n=34) 120
Table 6.8: Ruth's students' initial errors on algebra post-test (n=4) 121
Table 6.9: Ruth's students' responses and initial errors on post-test symbolic questions (n=4) 123
Table 6.10: Ruth's students' responses and initial errors in post-test word problems (n=4) ... 125
Table 6.11: Per cent of Ruth's students' errors before and after intervention (n=4) 126
Table 7.1: Dorothy's students' pre-test total score (n=26) 137
Table 7.2: Number of Dorothy's students with correct answers in each pre-test question (n=26) 137
Table 7.3: Per cent of error types made by Dorothy's students' on the algebra pre-test (n=4) 138
Table 7.4: Dorothy's students' responses and initial errors on pre-symbolic questions (n=4). 140
Table 7.5: Dorothy's students' responses and initial errors on pre- word problems (n=4) 143
Table 7.6: Dorothy's students' post-test total score (n=26) 144

Table 7.7: Number of Dorothy's students with correct answers in pre- and post-test question (n=26) 144

Table 7.8: Per cent of error types made by Dorothy's students' on the algebra post-test (n=4) 145

Table 7.9: Dorothy's students' responses and initial errors on post-test symbolic questions (n=4) 147

Table 7.10: Dorothy's students' responses and initial errors in post-test word problems (n=4) 150

Table 7.11: Per cent of Dorothy's students' errors before and after intervention (n=4) 151

Table 8.1: Jamie's students' pre-test total score (n=54) 160

Table 8.2: Number of Jamie's students with correct answers in each pre-test question (n=54) 160

Table 8.3: Per cent of error types made by Jamie's students' on the algebra pre-test (n=4) 161

Table 8.4: Students' responses and initial errors on pre-test symbolic questions (n=4) 163

Table 8.5: Jamie's students' responses and initial errors on pre-test word problems (n=4) 166

Table 8.6: Jamie's students' post-test total score (n=54) 167

Table 8.7: Number of Jamie's students with correct answers in post-test questions (n=54) 167

Table 8.8: Per cent of error types made by Jamie's students' on the algebra post-test (n=4) 168

Table 8.9: Jamie's students' responses and initial errors on post-test symbolic questions (n=4) 170

Table 8.10: Jamie's students' responses and initial errors in post-test word problems (n=4). 173

Table 8.11: Per cent of Jamie's students' errors before and after intervention (n=4) 174

Table 9.1: Stephen's students' pre-test total score (n=67) 181

Table 9.2: Number of Stephen's students with correct answers in each pre-test question (n=67) 182

Table 9.3: Per cent of error types made by Stephen's students' on the algebra pre-test (n=4) 183

Table 9.4: Stephen's students' responses and initial errors on pre-test symbolic questions (n=4) 185

Table 9.5: Stephen's students' responses and initial errors on pre-test word problems (n=4) 187

Table 9.6: Stephen's students' post-test total score (n=67) 188

Table 9.7: Number of Stephen's students with correct answers in pre- and post-test questions (n=67) 188

Table 9.8: Per cent of error types made by Stephen's students' on the algebra post-test (n=4) 189

Table 9.9: Stephen's students' responses and initial errors on post-test symbolic questions (n=4) 191
Table 9.10: Stephen's students' responses and initial errors in post-test word problems (n=4) 194
Table 9.11: Per cent of Stephen's students' errors before and after intervention (n=4) 195
Table 10.1: Profile of the four case-study teachers 198
Table 10.2: Per cent of error types in the four case study classes prior to intervention (n=16) 204
Table 10.3: Pre-intervention language errors in the four case studies 205
Table 10.4: Pre-test algebra misconceptions in the four case studies 207
Table 10.5: Per cent of correct answers and mean scores of for pre- and post-test questions (n=181) 214
Table 10.6: Pre- and post-test mean scores of the four case studies 214
Table 10.7: Per cent of students' pre- and post-correct attempts at each stage in the four case study classes (n=16) 216
Table 10.8: Pre- and post-test language errors in the four case studies 218
Table 10.9: Pre- and post-test algebra misconceptions in the four case studies 219

LIST OF FIGURES

Figure 2.1: Mathematics classroom interactions. (Campbell et al., 2007, p. 17) 15
Figure 2.2: Newman Strategies Adapted from Newman (1983b, p. 2)................................... 23
Figure 2.3: Domains of mathematical knowledge for teaching (Ball et al., 2008, p. 403).......... 27
Figure 2.4: The conceptual framework for the study. 31
Figure 3.1: Overall plan of the research study 34
Figure 3.2: Composition of algebra test questions 39
Figure 5.1: Class sizes of teachers (n=30) 80
Figure 5.2: PL teachers' most significant workshop gain (n=12) 100
Figure 6.1: Ruth's student's working on the board 105
Figure 6.2: Ruth's students interacting at the board 109
Figure 6.3: Symbolic pre-test questions 114
Figure 6.4: Ruth's students (S1, S3 and S4) workings on Question 1 114
Figure 6.5: Worded pre-test questions 116
Figure 6.7: Ruth's students' (S1 – S4) workings on Question 15 118
Figure 6.8: Symbolic post-test questions 121
Figure 6.9: Ruth's students' (S3 and S4) workings on Question 5 122
Figure 6.10: Worded post-test questions 123
Figure 6.11: Ruth's student (S2) response on Question 7 124
Figure 6.12: Ruth's students' (S1 – S4) workings on Question 12 125
Figure 6.13: Ruth's students' performance on pre- and post-tests... 126
Figure 6.14: Distribution of errors on Ruth's students' pre- and post-test 127
Figure 7.1: Dorothy's board writings 132
Figure 7.2: Symbolic pre-test questions.. 139
Figure 7.3: Dorothy's students' (S1 – S3) workings on Question 3.. 139
Figure 7.4: Worded pre-test questions 141
Figure 7.5: Dorothy's students' (S1 – S3) on workings on Question 8.................................. 142
Figure 7.6: Symbolic post-test questions 145
Figure 7.7: Dorothy's students' workings on Question 5.. 146
Figure 7.8: Worded post-test questions 148
Figure 7.9: Dorothy's students' workings on Question 10 149

Figure 7.10: Dorothy's students' workings on Question 15 149
Figure 7.11: Dorothy's students' pre- and post-test performance 151
Figure 7.12: Distribution of errors on Dorothy's students' pre- and post-test 152
Figure 8.1: Symbolic pre-test questions 162
Figure 8.2: Jamie's students' (S1, S3 and S4) workings on Question 5 163
Figure 8.3: Worded pre-test questions 164
Figure 8.4: Jamie's students' (S1 – S4) workings on Question 11 165
Figure 8.5: Symbolic post-test questions 169
Figure 8.6: Jamie's students' (S1 – S4) workings on Question 6 170
Figure 8.7: Worded post-test questions 171
Figure 8.8: Jamie's students' performance on the pre- and post-tests 173
Figure 8.9: Distribution of errors on Jamie's students' pre- and post-tests 174
Figure 9.1: Symbolic pre-test questions 183
Figure 9.2: Stephen's students' (S1 - S4) workings on Question 1 184
Figure 9.3: Stephen's student's (S1) working on Questions 3 184
Figure 9.4: Worded pre-test questions 185
Figure 9.5: Stephen's students' working (S1, S2) on Question 10 186
Figure 9.6: Symbolic post-test questions 190
Figure 9.7: Stephen's students' (S1 – S4) working on Question 2 190
Figure 9.8: Stephen's student's (S3) working of post-test Question 5 191
Figure 9.9: Worded post-test questions 192
Figure 9.10: Stephen's students' (S1 - S4) workings on Questions 14 193
Figure 9.11: Distribution of errors on Stephen's students' pre- and post-tests 195
Figure 10.1: Pre-test performance of the four cases before intervention 203
Figure 10.2: Pre-test questions 207
Figure 10.3: Pre- and post-test performances of the four case study classes 215
Figure 10.4: Theorising the impact of professional learning 222

LIST OF APPENDICES

Appendix 1: Approval Letter from the District Office 240
Appendix 2: Principals Consent Form 241
Appendix 3: Consent Form for Teachers 242
Appendix 4: Mathematics Teacher Initial Questionnaire 243
Appendix 5: Mathematics Teacher Final Questionnaire 247
Appendix 6: Lesson Observation Sheet 251
Appendix 7: Algebra Pre-test Questions 252
Appendix 8: Algebra Post-test Questions 254
Appendix 9: Newman Interview Protocol 256
Appendix 10: Newman Error Analysis Guideline Sheet 257
Appendix 11: Newman Diagnostic Profile Sheet 258
Appendix 12: Teacher Algebra Rating Sheet 259
Appendix 13: Focus Group Interview Questions 260
Appendix 14: Per cent of Teachers' Response on Algebra Questions 261
Appendix 15: Algebra Slides about Variables 262
Appendix 16: Algebra slides about equations 270
Appendix 17: Sample of Ruth's class Lesson Observation Sheet 275
Appendix 18: Sample of Dorothy's class Lesson Observation Sheet 276
Appendix 19: Sample of Jamie's class Lesson Observation Sheet 277
Appendix 20: Sample of Stephen's class Lesson Observation Sheet 278

CHAPTER ONE: GENERAL INTRODUCTION

This study describes how beginning algebra word problems are taught in in Nigerian schools, and the impact of a teacher professional learning intervention on classroom practices and on students' learning of Beginning Algebra. The first and second sections provide background information about Nigeria and Beginning Algebra, the third section identifies the problem; the fourth and fifth sections discuss the rationale and significance of the study; and the sixth section states the research purpose and questions.

Background

Nigeria, a developing Commonwealth country with over 159 million people (United Nations Population Division, 2012) has three main ethnic groups and is reportedly "the most complex country in Africa linguistically, and one of the most complex in the world" (Blench, 1998, p. 187). Located in West Africa, Nigeria became independent in 1960, a republic in 1963, and has democracy entrenched at the Federal, State and Local Government levels. The country is subdivided into 36 states and the Federal Capital Territory (FCT). While Nigeria's administrative capital is Abuja in the FCT, the economic capital is Lagos in Lagos State. The official language of communication is English, which is not the first language for Nigerians.

A significant challenge that cannot be ignored exists in the education sector. Western education was introduced to the western and southern regions of the country by Christian missionaries while Islamic education was introduced to the northern region by the Arabs (Fafunwa, 1974). In the north, the focus was on the Arabic language needed for Islamic education; as a result, the northern region did not embrace western education as quickly as the other parts of the country (Umar, 2001). Parents were encouraged to send their children to school in the western and southern regions since education was free, resulting in these regions having an educational advantage over the northern region (Fafunwa, 1974). Successive governments have tried to bridge this gap through enacting various policies and laws to make education compulsory for all children.

The National Policy on Education (NPE) is the guide and blueprint for the implementation of the educational aspirations of the country. These aspirations are reflected in the nation's educational goals (Federal Republic of Nigeria, 2004) which include:

> the training of the mind in the understanding of the world around; and the acquisition of appropriate skills and the development of mental, physical and social abilities and competencies as equipment for the individual to live in and contribute to the development of the society. (p. 3)

One way to implement these goals according to the policy is to make the learner central in all educational activities (p. 4). A major realisation from the policy is that “no educational system may rise above the quality of its teachers” (Federal Republic of Nigeria, 2004, p. 33). Recognition of the quality and role of the teacher led to the development of a teacher education policy in 2009 that should be fully implemented by 2013. The existing challenge in the sector is to be redressed through initial training that is focussed on subject-content and pedagogy, and professional development for in-service teachers in various fields (Federal Ministry of Education, 2009).

The educational system consists of three major phases. These are the basic, senior secondary and tertiary levels of education. Although the basic level is taken as the first major phase, early childhood education has a very strong visible presence and operates within private schools. The NPE adopted six years as the entry age into the basic level, which covers a period of nine years comprising six years at a primary school and three years at a junior secondary school (JSS) (Federal Republic of Nigeria, 2004). The compulsory nine years of schooling has increased the number of students and schools, teacher shortage and an attendant increase in teacher workload. This is more obvious in Lagos State, which has 325 junior secondary schools, the highest number in a single state in the country (Akparanta & Anuforo, 2010).

On completion of this mandatory schooling period, students proceed to the Senior Secondary School (SSS) or a technical/vocational school, depending on their skills, interest and performance in the Federal and State organised examinations. The SSS spans a three-year period and terminates with external examinations conducted by the National and West African examination bodies. Successful students who are interested in further education proceed to the tertiary level of education at universities, polytechnics or colleges of education.

There are two languages of communication during the period of formal education. The first is the language spoken at the lower primary level. The NPE (Federal Republic of Nigeria, 2004, p. 11) indicated that teaching in the first three years of basic education should be in the “language of the environment of the school”. After the first three years, English is the language of communication until the end of formal education. The implementation of this policy aspect is less rigorous in the private schools, so children are taught in English not only in lower primary classes but also in the nursery schools.

Mathematics and English are the only subjects that students study throughout their basic and upper secondary education. To gain entry into a university, credit passes in English and mathematics are required. Mathematics is disliked and feared by Nigerian students for various reasons (Ifamuyiwa & Akinsola, 2008). Information, mostly negative, from senior students and

peers often causes them to have preconceived notions before attending classes. Performance in the West African School Certificate mathematics examination which is taken in the senior secondary terminal year is below average. Reports between 1997 and 2004 reflect an average of 28.5 % of students passed their mathematics examination and in 2010 only 25% of students passed (Adesina, 2010; Ifamuyiwa & Akinsola, 2008). Various means such as cooperative learning strategies, peer group tutoring, use of technology, formation of mathematics clubs, and mathematics competitions for students have been suggested to improve the situation but many students still believe that the subject is abstract and only meant for a few, especially males (Adesoji & Yara, 2007; Ifamuyiwa & Akinsola, 2008). Constraints of class space and large class sizes are also challenges to the implementation of such strategies in Nigeria (Noah, Falodun, & Ade, 2011).

Algebra

Algebra is an important domain in mathematics and it is fundamental for mathematical proficiency. Algebra is defined as "the domain consisting of operating on and with the letter, transformation of expressions with letters, formal and generalized understanding of rules and properties of operations, and using the letter for representing, proving and generalizing" (Banerjee & Subramaniam, 2012, p. 352). Previously, algebra involved the use of alphabets known as letters or literal symbols to mainly represent the unknown, but mathematical developments have led to its use to represent the known, allowing for generalisations of both the known and unknown (Kieran, 1992). For example, if we need to find the value of $5h$, and $h=2$, then $5h=10$, but if $h=0.4$, then $5h=2$; so h is a variable that can take on any value assigned to it. Similarly if $z= 4+p$, then it follows that as the value of p changes, the value of z will also change. This notion of algebra moves it away from just a representation of the unknown to generalisations of patterns and is defined by Kieran (1992, p. 391) to be "the branch of mathematics that deals with symbolizing general numerical relationships and mathematical structures and with operating on these structures".

Algebra has been described as the "gateway to higher mathematics" (Kaput, 1999, p. 134; Stacey, 2004, p. 8), so failure to understand algebra affects its application, which is needed in other areas of mathematics (Goos, Stillman, & Vale, 2007). The use of principles and generalisations in mathematics makes the knowledge of algebra fundamental for success. As a result of its various uses, various definitions exist. However, for this study, which is situated in beginning algebra which is the introductory and early aspects of algebra taught in the first few years of the secondary school, it will be viewed mainly as generalized arithmetic (Kieran, 1992). Students are introduced to the use of the algebraic letter to solve questions given in

symbolic form and word problems which are, mathematical questions written in literal form, in other words, mathematical sentences (Stacey & MacGregor, 1997). In Nigeria, Beginning Algebra is taught under the theme of algebraic processes and is offered at the JSS level.

In Beginning Algebra, concepts of the variable, expressions, equality, graphs and functions are necessary but the literature has established that students often have misconceptions about them (Kieran, 1981; Küchemann, 1981; MacGregor & Stacey, 1993b; Perso, 1993; Sfard, 1991). These misconceptions have been linked to the difficulties that students experience as they transit from arithmetic to algebra. While Herscovics and Linchevski (1994, p. 75) described the ensuing state as “a cognitive gap”, Goldin (2008, p. 186) called it a “cognitive obstacle” existing in the transition process. These difficulties translate into students committing various errors, identifiable through error analysis protocols such as Newman (1983b), that ultimately affect their ability to solve algebraic questions including word problems. Newman (1983a, p. 25) identified five “performance strategies” that have been found to be useful when solving mathematical questions. These steps are: reading recognition; comprehension; transformation; process skills and encoding. White (2005) reported the use of these steps in a professional development workshop for primary school teachers in Brunei. The Newman strategies allowed them to identify students’ processing errors which in turn would help teachers provide proper remediation.

Difficulties in Beginning Algebra may arise from the generalisations involved and the use of letters which, in algebra, differ from the everyday use that students know. For example, in Stacey and MacGregor’s (1997) study, tasks were given to over 2,000 Australian students aged 11 to 15 years. In one of the tasks, the students were told that David is 10 cm taller than Con and Con is h cm tall. Students were asked what they could write as David’s height. Answers given included: 18 (taking h as the eighth letter and computing 10 + 8), $10h$, $h10$, $h= h+10$ and 11. Their study reported misconceptions by students such as, among others, using letters as labels for objects, using sums as products and giving solutions that did not reflect an understanding of the use of equality. With these tendencies, it is no wonder that students experience difficulties in algebra and mathematics in general. As a result, students’ general interest and attitude to the subject is poor. However, teachers also have a role to play in reducing the occurrences of algebraic misconceptions.

The teachers’ methods of teaching and beliefs affect students’ learning. Mathematics and the sciences are closely linked, and science teaching in Nigeria is largely teacher-centred and traditional in approach (M. A. Benjamin, 2004). It has been noted that the quality of teaching

and learning science is compromised by students' poor background knowledge of mathematics (Ogunmade, 2005).

The Problem

Students' performances in mathematics in Nigeria over the years have remained poor. Various factors contribute to these poor performances including large class sizes, students' negative attitudes, teachers' negative beliefs, poor teaching strategies, and a lack of instructional and textual materials (Aburime, 2007; Okigbo & Osuafor, 2008; Tella, 2008). The seriousness and urgency of the situation is reflected in the results from the 2011 West African School Certificate Examinations (WASCE) in which only 39% of the more than one and a half million students who sat for the examinations scored 50% or higher in mathematics (Uwadiae, 2011). This is not surprising as the mathematics essay paper (Paper 2) presents most of the questions in words, and worded problems have been reported to be one of many students' weak areas (McClure, 2009; Rosales, Santiago, Chamoso, Munez, & Orrantia, 2012). At the JSS level also, the 2006 average national performance in mathematics was 38% (Mohammed, 2012).

Word problems often require the use of algebra to solve them. Solving word problems at levels beyond the primary school most often involves interpreting and translating the sentences into algebraic forms before mathematical operations are carried out. As a mathematics educator for more than two decades, the Researcher often saw pre-service mathematics teachers struggle to solve algebra word problems. Since the questions are either in English, algebraic letters or a combination of both, facility in both English and mathematical language is necessary for success in problem-solving.

"One way of trying to find out what makes algebra difficult is to identify the kinds of errors students commonly make in algebra and then to investigate the reasons for these errors" (Booth, 1999, p. 299). Errors can occur at any stage while solving word problems and the use of Newman's procedure has been established as a means to identify, categorise and analyse student' errors, with a view towards further action by the teacher (Chinen, 2008; Clements, 1982; A. White, 2005).

Studies have indicated that students have misconceptions in beginning algebra about variables, expressions and equality. Nigeria is no exception and Sule's (1992) study sought to identify difficult areas of the JS mathematics curriculum by giving 200 students questions from all components of the curriculum. His study reported a 30% pass in the algebra component, indicating the need for a focus on algebra teaching and learning. The Federal Government of Nigeria however, recognises that "consistent quality of teaching and learning remains a

significant challenge" (Federal Ministry of Education, 2009, p. 2) which needs to be addressed if students' performances are to improve.

Professional learning that allows teachers to gain an understanding of how students think as they engage in mathematical tasks modifies teachers' practices and improves students' understanding (Carpenter, Fennema, & Franke, 1996; Krebs, 2005; Sowder, 2007). One way of intervening is to equip mathematics teachers with an awareness of the misconceptions and difficulties experienced by some students as they start learning algebra. Equally important is the need for mathematics teachers to ensure that they are communicating with the students who are learning two languages simultaneously, that is, English language used in teaching them and the language of mathematics (Setati, 2005).

Rationale

Understanding how to solve word problems in algebra will help students to make connections between the various concepts they have learnt. Proficiency in solving algebraic word problems can also be transferred to word problems in other areas of mathematics. This kind of flexibility strengthens the ability of students to move across and between various representations in order to select the appropriate one for use. This ability also forms one of the key cognitive constructs in mathematics needed for abstractions (Goldin, 2008).

Algebra features all through secondary mathematics. One of the aims of the JSS curriculum is to provide mathematical literacy to all students (Federal Ministry of Education, 2007). As a result, students should ideally be able to solve problems represented algebraically from their first year in secondary school as they transit from arithmetic to mathematics. Competence in using algebra makes it possible to apply the knowledge to other areas of mathematics, ultimately giving them a firm foundation for problem solving in later years. If the students' poor performances are not checked, it may limit Nigeria's capacity to move towards scientific and technological development as expressed in the NPE.

Significance

The study will create new knowledge about how algebra is taught to Junior Secondary 1 (JS 1) students in Nigerian secondary schools and the difficulties experienced by the students. It is envisaged that this research will contribute to the existing knowledge of what is involved in the teaching and learning of algebraic word problems in a language of instruction which is not native to the teachers and students.

The literature is scarce concerning intervention strategies focused on enhancing secondary school teachers' awareness of students' misconceptions in Beginning Algebra as a way of reducing errors committed by students. Newman's (1983a, 1983b) procedure has been found to be useful in identifying errors committed by students and providing a strategy for solving word problems (Clements, 1982; A. White, 2005).

The use of Newman' approach in error analysis, which has its origin in Australia, enables a comparison of results between developed and developing countries. Although the approach has been used in Australia and several other countries, its use in a West African country, or its use in an adapted form as an intervention learning program for secondary school teachers, has not been reported. This study will contribute new knowledge about professional learning interventions addressing algebra learning which involves Newman error analysis.

The outcome of the study will be beneficial to stakeholders in the teacher education sector. Research findings will inform curriculum planning and training for pre-service mathematics teachers. It will benefit pre-service teachers as it will prepare them better for the world of teaching. Students at the JS 1 level will also be able to have a smoother transition from arithmetic to Beginning Algebra and subsequently improve their performance in mathematics.

Purpose and Research Questions

The study describes how word problems are taught in beginning algebra and the difficulties experienced by JS 1 students. It also examines the impact of a teacher professional learning intervention designed to address both student misconceptions and language process errors when solving algebraic word problems.

The major questions to be considered are:

1. How are word problems in JS 1 beginning algebra classes taught prior to the intervention?
2. What difficulties do students in JS 1 experience in solving algebraic word problems prior to the intervention?
3. How does the teacher professional learning intervention program impact on JS 1 mathematics teachers' beliefs, knowledge and teaching practice?
4. How does the teacher professional learning intervention program impact on students' difficulties and success in solving algebraic word problems?

The next chapter presents a more comprehensive review of the literature concerning the learning and teaching of algebra and why this Nigerian study is necessary.

CHAPTER TWO: LITERATURE REVIEW

This section is subdivided into a brief review of literature governing the study, and the conceptual framework. The literature will be reviewed under four headings, namely the background to the study, the theories that underpin the study, the learning of algebra, and the teacher and professional learning.

Background

The background to the study provides an overview of mathematics education in Nigeria, the importance of mathematics education and a reflection on the importance of algebra.

Mathematics education in Nigeria

Before gaining independence in 1960, mathematics was taught in accordance with the British curriculum using Britain's 6-5-2-3 educational system. Arithmetic was taught in primary schools and it was the only compulsory aspect of mathematics included in the training for prospective primary school teachers (Ohuche, 1978). Examinations were conducted at the end of the six years in primary school to gain admission into a secondary school. Traditional mathematics consisting of arithmetic, trigonometry, algebra and geometry was taught at the secondary school level. Students who were considered gifted or exceptional in mathematics were offered additional mathematics which comprised some topics from applied mathematics and pure mathematics as an extra subject. The secondary mathematics external examinations had two alternatives (A and B) and each school was required to choose one that her students would undergo. Topics covered in Alternative B consisted of arithmetic, trigonometry, algebra, Euclidean geometry and coordinate geometry while Alternative A excluded coordinate geometry. Learning was rote and emphasis was on developing computational skills and ensuring the correctness of answers (Ale, 1981). Textbooks used at the primary and secondary levels were published in England and had foreign examples. This system was in place until after independence.

After independence, a conference was convened in USA for invited African countries with British and American representatives to deliberate on curriculum reforms in Africa with regard to advancements in science (Ohuche, 1978). Ohuche explained that the 1961 conference gave birth to an African Mathematics Writers Program in Entebbe, Kenya in 1963. Writing workshops which continued up until 1967 led to the production of primary, secondary and teacher training mathematics textbooks for Africans (Fafunwa, 1974; Ohuche, 1978). Advancement in technology in the developed world led to the introduction of a new type of

mathematics called modern mathematics. Modern mathematics focused on understanding and on the application of mathematics to real-life situations and included fields like binary operations and set theory. This new type of mathematics was reflected in the textbooks written for both primary and secondary schools. In Lagos State, primary schools started teaching modern mathematics in 1971 but the traditional arithmetic textbooks were used alongside the modern mathematics textbooks which were known as the Entebbe series (Ohuche, 1978). Secondary schools also started teaching modern mathematics using the provided text books. However, not all schools and states participated in this endeavour as each state was free to decide for its schools. Schools had to decide whether or not their students would be offered traditional or modern mathematics in the secondary school leaving examinations. Mathematics teachers were not adequately prepared to teach modern mathematics and this genre was not familiar to the pre-service elementary teachers. The situation resulted in poor examination performances and public outcry.

A conference in Benin was convened by the Federal Government in 1977 to address the situation. The conference was held in Benin, a town in the Midwest of the country, in January 1977, and has since then been known as the Benin conference (Ohuche, 1978). The Federal Ministry of Education invited some mathematicians, mathematics educators and other stakeholders to discuss the situation and the way forward. The Federal Government "had made up its mind that modern mathematics was to be suspended for the present in elementary schools and de-emphasized in secondary schools of all the states of Nigeria" (Ohuche, 1978, p. 278). Other recommendations of the conference included: simple use of the word "mathematics" (without descriptors of Entebbe, modern or traditional) in primary and secondary schools; a new subject called further mathematics, to be taught instead of additional mathematics; provision of national mathematics curricula for all primary and secondary schools in the country and the development of objectives for mathematics teaching at all levels.

In line with the recommendations, new curricula were published for the primary and secondary levels. The six-year primary level curriculum was published in 1979; the three-year junior secondary curriculum in 1981, the three-year senior secondary curriculum in 1984 and the three-year further mathematics curriculum in 1985. The new curricula reflected an integration of both traditional and modern mathematics with stated objectives for teaching mathematics in schools. Despite the new curricula in use at all levels of primary and secondary education, students' performance in senior secondary external examinations has been poor. In the 2004 WASCE, 35% of the students passed mathematics (Ifamuyiwa & Akinsola, 2008) while 39% of the students passed in 2011 (Uwadiae, 2011). At the junior secondary level, the

triennial national assessment by the National Assessment of Learning Achievements in Basic Education (NALABE) reported that the average JS 1 national mathematics performance for 2006 was 37% (Mohammed, 2012). Mathematics remains a revered and feared subject to Nigerian students (Adesoji & Yara, 2007; Ale, 1981).

The teaching of mathematics has its challenges. An inadequate number of qualified teachers led to non-specialists teaching mathematics such as engineers and architects (Ale, 1981), although the situation seems to be gradually improving (Mohammed, 2012). Other challenges include large class sizes, inadequate resources, negative attitudes of students, inappropriate teaching styles and negative teacher beliefs (Adesoji & Yara, 2007; Ale, 1981; Igbokwe, 2000).

The language of instruction in secondary schools, English, cannot be overlooked because it is a second language which the students are concurrently learning in school. Facility with the everyday use of a language develops more easily than facility with its academic use (Cummins, 1979; Kersaint, Thompson, & Petkova, 2009). In Nigerian private primary schools teaching at all levels is in English Language, while in public schools the native language is used for teaching in the first three years. In Adetula's (1989) study concerning Nigerian Primary 4 public and private school students, there was generally a better performance on questions written in the native language than on the questions written in English language. This finding is to be expected since the students in the public schools would have experienced only one year of instruction in English and therefore would have had limited English proficiency. Students in the private schools performed better on arithmetic word problems written in English while the students in the public schools performed better when the questions were written in the native language (Adetula, 1990). While earlier assumptions were that learning in a second language limits proficiency with mathematics, recent studies interestingly suggest that students with proficiency in the first and second languages have added cognitive benefits which may allow them to record greater success than monolinguals (Clarkson, 1992; Ni Riordain & O'Donoghue, 2009). It is argued that proficiency in the first language strengthens the acquisition of mathematics in the second language (Clarkson, 1991b; Cummins, 2000).

A revision of the existing mathematics curriculum was completed and the new JSS curriculum was published in 2007 (Federal Ministry of Education, 2007). The revised version has six themes: number and numeration, basic operations, measurement, geometry and mensuration, algebraic process and everyday statistics, and these include also the application of the mathematics learnt. The objectives for mathematics teaching at the basic (primary and junior secondary) level, as stated in the Basic Education Mathematics Curriculum (Federal Ministry of Education, 2007, p. iv) are:

1. Acquire mathematical literacy necessary to function in an information age.
2. Cultivate the understanding and application of mathematics skills and concepts necessary to thrive in the ever changing technological world.
3. Develop the essential element of problem solving, communication, reasoning and connection within their study of mathematics.
4. Understand the major ideas of mathematics bearing in mind that the world has changed and is still changing since the first National Mathematics Curriculum was developed in 1977.

These objectives relate positively to current western mathematics standards which focus on understanding of both mathematical processes and literacy (Australian Association of Mathematics Teachers, 2006; National Council of Teachers of Mathematics, 2000). Achievement of the above listed objectives is conducted through the linking of mathematics content with the teaching-learning process. The importance of the field of study that facilitates this linkage, mathematics education, is next discussed.

Importance of mathematics education and algebra

Mathematics education provides not only knowledge of mathematics but the ability to use it in a practical way. Mathematical literacy implies the ability to use mathematics in everyday living, for higher studies and career development. The Programme for International Student Assessment (PISA) 2009 framework (OECD, 2010, p. 84) defined mathematical literacy as

> [A]n individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements, and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen.

The Australian process standards implies that a mathematically literate person has capabilities of investigating, conjecturing, using problem solving strategies, applying and verifying, using mathematical language and working in context (Goos et al., 2007). These mathematical literacy indices also exist in Nigeria as evidenced from the stated objectives. Described as a "tool" by Confrey (1990, p. 110), mathematics plays various roles, such as the use of arithmetic in shopping and budgeting; the use of measures in travelling and planning; and the use of statistics for planning and decision making in everyday living. Students' familiarity with mathematics in the school setting enables them to apply the knowledge in their career choices. For instance, architecture requires knowledge of numbers and spatial geometry, engineering requires algebra and calculus, and economics requires statistics and algebra.

Algebra occupies a key position in mathematics. Consisting of two aspects, namely "conventional symbol systems and … certain kinds of human activities", it is perceived through generalizations, functions and modelling (Kaput, 2008, pp. 10-11). It is described as the

“language” of higher mathematics (Fearnley-Sander, 2000, p. 77), and the “key to the characterization and understanding of mathematical structures” (Usiskin, 1999, p. 13). As a language, it allows an individual to use letters of the alphabet, known also as variables or literal symbols, in general numeric forms across other mathematical domains. The letter possesses the ability to represent whatever value a person wants to assign to it (Ely & Adams, 2012). This ability to use variables in a general form allows algebra to serve as an entry point through which other areas of mathematics can be developed (Ely & Adams, 2012). Advanced mathematics and the applied mathematics in the sciences, engineering and applied sciences all use algebraic means of representations as a tool for solving problems (Cathcart, Pothier, Vance, & Bezuk, 2006).

Algebra is defined by the Australian Concise Oxford Dictionary (1997, p. 31) as “the branch of mathematics that uses letters and other general symbols to represent numbers and quantities in formulae and equations”. This captures one of algebra’s features. Another of its features is depicted by Cathcart et al. (2006, p. 394) definition as ”the study of patterns, which forms the foundation for the logical connections in all of mathematics”. Concepts of variables, equality, functions and graphs are introduced within the first few years of secondary school in a general form as Beginning Algebra (Stacey & MacGregor, 1997; Tabach & Friedlander, 2008). Algebra has its own cultural practices (Franke, Carpenter, & Battey, 2008) and students’ lack of understanding of these concepts may affect their ability to apply the knowledge to problem solving. This is discussed in more detail further in this chapter. The following section however discusses the theories that underpin the way knowledge is constructed in the study.

Social Constructivism and Sociocultural Theory

Social constructivism has its roots in constructivism. From the root word, to “construct”, constructivism refers to individuals actively engaged in building their own knowledge (Goldin, 1990). The process of knowledge building involves making meaning from experiences in terms of existing knowledge (Cobb, Wood, & Yackel, 1990). In particular, students individually construct their patterns of reasoning which lead to the actions they take. So, in learning mathematics, persons relate known patterns to new ones and also build knowledge (Hiebert & Carpenter, 1992). Social constructivism recognises that individuals do not exist in isolation but learn within a social setting in which understanding is co-constructed with others (Franke, Kazemi, & Battey, 2007). Each individual’s present knowledge is a potential springboard to move to higher rungs of knowledge acquisition and with assistance, this is achievable (Vygotsky, 1978).

It is in the process of communicating and interacting using mathematical terms and words that mathematical concepts are learnt and learning occurs (Campbell, Adams, & Davis, 2007; Lim & Presmeg, 2011; Setati, Chitera, & Essien, 2009). Therefore, in a constructivist classroom, a mathematics teacher does not focus primarily on the correctness of the answer but on the process followed by each individual involved in arriving at it (Beswick, 2007; Hensberry & Jacobbe, 2012). Newman's (1983a, 1983b) notion of performance strategies also rests on this belief, because she sees the solution to a problem involving a series of stages, forming a pattern that each student builds on as they decide on a pathway to follow in solving problems.

Sociocultural theory emphasises that individuals do not exist in isolation but must interact within communities. As individuals grow up within a community of practice, they are introduced to established ways or patterns of working, and to the communal language of the discipline (Cobb, 1994). Social theorists draw on Leont'ev's and Vygotsky's works and believe that an individual develops his reasoning in line with the patterns of the society (Cobb, 2007). Students receive "social assistance" (van Oers, 2000, p. 141) as complex ideas and problem solutions are constructed on the social plane of the classroom and are made available for each individual to internalize and construct knowledge. This highlights the importance of collaboration in the learning of mathematics.

Mathematics is also described as a cultural activity because it uses its own language as a cultural tool to communicate (Cobb, 2000). The language has its own vocabulary, representations and symbols. Pape and Tchoshanov (2001, p. 126) asserted that, "representation is inherently a social activity. Students come to understand both the process of representation and its products through social activity". So, thinking occurs both internally by the individual, and externally in a verbal form. Students' engagement in mathematics learning in the classroom creates opportunities for knowledge building and initiation into cultural practices of the mathematical community (Cobb, 1994). Campbell et al's (2007) model in Figure 2.1 below illustrates the relationships and interactions that occur in the mathematics classroom. It seeks coordination between the world of the student learning mathematics and that of the teacher teaching mathematics. Both parties bring to the class, perceptions of each other, the classroom environment, language, culture, mathematics, their experiences and knowledge (Shirley, 2001).

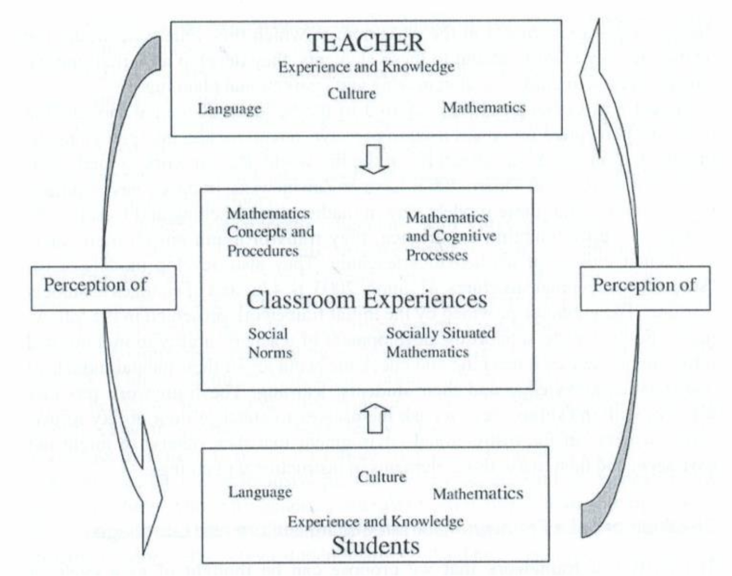


Figure 2.1: Mathematics classroom interactions. (Campbell et al., 2007, p. 17)

These two theories formed the framework within which this research took place. In the next section, the relevance of these ideas to the specific learning and teaching of algebra is discussed.

Learning Algebra

This section considers literatures pertaining to the cognitive processes in learning algebra, and some common misconceptions in beginning algebra. The focus of the latter is on concepts of variable, expressions and equations/equality. This review draws on many seminal articles written when the mathematical reforms started about three decades ago.

Cognitive processes in algebra

Learning algebra involves many cognitive activities so as to successfully learn and solve problems. These activities can be grouped as: *generalized* which involves patterns, *procedural* which involves expressions, *relational* which uses varying quantities, and *structural* which uses structural objects (Usiskin, 1999). Kieran (1981, 2007) classified them into three stages: the *generational, transformational* and the *global/meta* stages. The *generational* stage represents an understanding of algebra as arithmetic that is generalised, in that it uses mathematical language and variables with expressions and functions. The *transformation* stage consists of understanding algebra as involving representations, identities, equivalence, axioms and properties. The *global/ meta* stage applies the understanding of algebra as a tool to solve real life situations which may not be directly mathematical. These processes in algebra are all important as algebraic reasoning develops. This study employs Kieran's classifications,

focussing mostly on the generational stage but with some reference to the transformational stage since the study involves beginning algebra.

Algebra taught in schools as generalised arithmetic involves a move from the realm of specifics, in numbers, to the realm of the unknown in letters. The TIMSS report for the 2011 survey identified that algebra generally presented the most difficult content for Grade 8 students and that they only demonstrated a 37% facility this area (Trends in International Mathematics and Science Study, 2012). Studies showed that this transition is difficult because of the need to use variables, most often seen as letters, in arithmetical operations (Goldin, 2008; Herscovics & Linchevski, 1994; Linchevski & Herscovics, 1996). In using both letters and numbers in a given question, a student has to process cognitively at different levels. First the student must move from the representational format of the question to another format that is generated by the individual for use in solving the problem. A student must be able to "interpret, construct and operate" effectively use the two representational formats (Pape & Tchoshanov, 2001, p. 120). This is followed by the need for the use of computations and arithmetic skills on the generated format so as to arrive at a solution (Davis & Maher, 1990; Reed, 1999). The question form and type of solution needed, which might be in words, tables, graphs, symbols or diagrams, determines how representational shifts would be involved. Schoenfeld (2008, p. 482) defined the process of algebraic thinking as "a particular form of mathematical sense making related to symbolization". Kieran (1992, p. 394) described the situation students' encounter as follows:

> Thus, the cognitive demands placed on algebra students include, on the one hand, treating symbolic representations which have little or no semantic content, as mathematical objects and operating upon these objects with processes that do not yield numerical solutions, and, on the other hand, modifying their former interpretations of certain symbols and beginning to represent the relationships of word–problem situations with operations that are often the inverses of those that they used almost automatically for solving similar problems in arithmetic.

The likely thinking patterns of students when trying to solve algebraic problems need to be understood by their teachers. Students may select visible data in the question and perform mathematical operations on them without recourse to the problem context (Palm, 2008). It is important that as cognitive processing goes on, the student has a proper understanding of the concepts involved. The next section highlights the important role of understanding in the solving word problems.

Word problems

Word problems written in sentences, also known as verbal problems, serve as an introduction to algebra. Verschaffel, Greer and De Corte (2000, p. ix) defined them to be:

> Verbal descriptions of problem situations wherein one or more questions are raised the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement.

They are important for developing an understanding of algebra, even though students often find algebra word problems difficult (Kieran, 2007; Reed, 1999). Solving word problems requires the abilities to read, interpret and transform the stated words within their context into a symbolic form, before embarking on a search for manipulative or computational strategies (Newman, 1983a; Oviedo, 2005; Pimm, 1991). The change of representational form requires knowledge of the language text; mathematical language and mathematical knowledge (Adetula, 1989; Ormond, 2000; Oviedo, 2005). Word problems are valuable for investigating both language difficulty and conceptual understanding (Newman, 1983a; Reed, 1999).

The level of proficiency in the text language and the language of communication may also affect performance in word problems, especially in the case of bilingual and multilingual students (Adetula, 1990; Clarkson, 1991a; Ni Riordain & O'Donoghue, 2009). This is of particular importance, of course, to the Nigerian context. In the mathematics classroom, many bilinguals and multilinguals may experience difficulty with understanding the lesson when the language of instruction is not their first language and the students are not proficient in it (Setati et al., 2009). Suggested teaching strategies include explicit instruction about the differences in the everyday and mathematical meaning of the same word, a limit on the number of new words in a lesson, student collaboration, encourage students' use of new words mathematically within real-life contexts, and the use of multiple representation (Kersaint et al., 2009; Oviedo, 2005). Language-based approaches provide prompts to facilitate students' development of literacy in mathematics and subsequently should increase their word problem-solving ability. As noted by Krebs (2005, p. 409), "sometimes much is learned by and about students from an incomplete or incorrect solution".

Booth (1984) reported on mathematical tasks given to over 3,500 students aged 13 to 15 years in U.K. On a task that requested students to 'add 4 onto $3n$', only 45% of the sampled students and 22% of those aged 13 years obtained the correct answer of $3n + 4$. Others gave answers like $3n4$, $7n$ and 12, which indicate a transfer of arithmetic operations with numbers into

algebra that uses both numbers and letters. In another study, on the question "add 5 to 3*n*", only 38% of students aged 12 years correctly answered $3n + 5$ and 17% wrote $8n$ (Ryan & Williams, 2007).

A large number of the existing literature on word problems is focused on arithmetic word problems mainly at the primary school level. For students, finding solutions to multi-step problems were more difficult than single-step questions, and questions that involved comparing quantities were more difficult than those requiring changing or combining quantities (Adetula, 1989; Verschaffel et al., 2000). It is also argued that word problems framed using real-life contexts and situations enabled students to first focus on understanding the question (Chapman, 2006; Palm, 2008; Verschaffel et al., 2000). However, class observations have evidenced a prevalent practice that word problems are all about algorithms and that teaching is mainly about the mathematical structures in the questions, irrespective of the context of the question (Chapman, 2006; Depaepe, De Corte, & Verschaffel, 2010; Rosales et al., 2012).

"Algebraic reasoning depends on an understanding of a number of key ideas, of which equivalence and variable are, arguably, two of the most fundamental" (Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005, p. 68). For students to forge ahead there has to be a connection between what they already know, and the correct ways in which these patterns are used. Limited knowledge of arithmetic operations impacts negatively upon students' facility with algebra even beyond the middle school grade levels. When students are unable to correctly conceive new concepts, it might lead to misconceptions and mistakes in algebra problem-solving (Russell, O'Dwyer, & Miranda, 2009; Welder, 2012). This will now be discussed.

Misconceptions in beginning algebra

Beginning Algebra is defined as the introductory aspects of formal algebra and consists of the important and fundamental mathematical concepts of variables, expressions, equality, functions and graphs (Goos et al., 2007; Knuth et al., 2005; Nathan & Koellner, 2007; Schoenfeld & Arcavi, 1999; Sfard, 1991). The first three of these are the initial concepts introduced to students in the first year at the junior secondary school level, and are the focus of this study and the next point of discussion.

Misconceptions about the variable

Studies show that students have misconceptions about variables. A variable is a quantity that can have varying values and is represented with an alphabetical letter (Goos et al., 2007). The letter can take on different roles depending on the context of the problem at hand (Ely & Adams, 2012; Usiskin, 1999). The different uses of a letter as a variable implies that the

quantity it represents may or may not vary. It may be a single letter, a combination of letters and operations, or an abstract number of things (Linchevski & Herscovics, 1996; R. A. Philipp & Schappelle, 1999; Usiskin, 1999). The letter can represent a unique unknown value like x+8=19; have a varying quantity (ies) in an observed general expression like 2m + 3 or the final answer of an operation like 3x + 5y; represent varying quantities that vary and used to show relationships of two sides like 2f + 4 =7d. Ely and Adams (2012, p. 23) asserted that,

> two important practices required for the developed idea of variable ….are (a) the use of a letter to stand for any set of indeterminate quantities, not just a single unknown, and (b) the representation and quantification of the way one quantity changes with respect to another.

This implies that students have to go beyond 'seeing' the letter as an unknown value to its use as a placeholder and its ability to take on varying values.

Ryan and Williams (2007) noted that these various meanings and uses of the algebraic letters are a source of difficulty for many who are beginning to learn algebra and that they may bring about misconceptions based on students' misinterpretation. These misconceptions about variables are identified in the research as:

- a letter is an object/label (Booth, 1984; Küchemann, 1981; MacGregor & Stacey, 1993b; S Wagner, 1999);
- a letter is a word so it cannot be used/ ignored (Küchemann, 1981; Perso, 1993);
- a letter has a fixed value from its alphabetical position (MacGregor & Stacey, 1993b; I. Watson, 1980);
- a standalone letter has a fixed value of 1 (Perso, 1993; Stacey & MacGregor, 1997);
- a letter has a fixed/ specific known number (Knuth et al., 2005; Küchemann, 1981);
- letters have place values (Booth, 1984); and,
- different letters cannot have the same value (Booth, 1984; MacGregor & Stacey, 1993b; Perso, 1993).

Stacey and MacGregor's (1997) study was mentioned earlier, in which over 2,000 Australian students aged 11 to 15 years solved some questions including: "David is 10 cm taller than Con. Con is h cm tall. What can you write for David's height?". They reported a 50% success rate for the first year algebra students. Different answers were received. As reported, some students wrote 18 in which the alphabetical position of h was represented by its value (8) and added to 10 giving a sum of 18. Some assumed h was equal to 1 and obtained a solution of 11 while some wrote h = David + 10. Booth (1984) found that students translated '3a' as '3 apples' instead of '3 times the number of apples'. When the letter is wrongly used to represent the

object," [t]he term "fruit salad algebra" is sometimes used for this misconception" (Chick, 2009, p. 121).

Ely and Adams (2012) observed that the 8th graders in their study at times chose different but specific values to represent the general term *x* or resorted to the use of alphabetical position. The knowledge of the letter as a generalised quantity is also required to simplify expressions like "2*t* + 3*t* -9 into 5*t* - 9" (p. 22).

These appear to indicate students' common misconceptions of variables as they transit from words to symbolic representations.

Misconceptions about the concept of expressions

Similarly, students may experience difficulty accepting that the solution of a question is an expression involving two or more terms. "An algebra expression is a description of some operation involving variables, such as 3*a*, *x*+1, or *x*-*y*" (S. Wagner & Parker, 1999, p. 331). In arithmetic, the final answer is a number so it becomes difficult to accept an algebraic expression like '2*a* + 5*b*' as an answer because it is thought to add up to '7*ab*' (Booth, 1999). This misconception has several names such as "*inability to accept the lack of closure*,… *name-process dilemma*" (Chalouh & Herscovics, 1999) or 'process-product dilemma' (Sfard & Linchevski, 1994), or wrongly 'conjoining' (Tirosh, Even, & Robinson, 1998).

An answer like 3*p* + 2*a* is a process involving addition of two terms which some students wrongly 'join' together in order to close up the expression. Stacey and MacGregor's (1997) study supported the existence of this misconception as some students wrote 10*h* and *h*10 as answers to the question. Falle (2007) also asked 222 Years 8 and 9 students to "simplify 5*p* – *p* + 1". A majority (38) of the wrong answers (49) were inappropriate conjoining such as 6, 6*p* or 5*p*. On the question, "add 6 to *x* + 3", Ryan and Williams (2007) also reported that students aged 12 years in Britain had a 19% facility.

The level of awareness of teachers about this misconception must affect their practice. Tirosh et al. (1998) described how teachers' awareness of this misconception affected the algebra teaching practice of four Grade 7 teachers. Two of the observed teachers, experienced and aware of this misconception, had designed strategies to improve their students' understanding. One of the others, a beginning teacher, was unaware of the misconception. He told students to add the constants together and the ones with the same letters together. This rule was followed up with procedural examples and students continued to experience difficulties. To prevent this misconception, teachers often resort to the use of objects as labels for letters, such as 3 pears and 2 apples for the example above, and this is likely to unconsciously reinforce the variable misconception (that is, that the variable represents an

object rather than an quantity). In Banaerjee and Subramaniam's (2012) study, teachers' focus on the similarities in the structures of arithmetic and algebraic expressions resulted in a reduction of these misconceptions and increased students' understanding,

Misconceptions about the concept of equality/equations

Understanding the concept of equality is challenging for many students. Students enter the secondary school with the belief that the equal sign means they should write the final answer after completing necessary operations (Kieran, 1992), or the belief that it is a link to the next operation (Stacey & MacGregor, 1997). An equation is "any algebraic expression of equality containing a letter (or letters)" (Herscovics & Kieran, 1999, p. 185). For students, the knowledge of the equal sign (=) to represent equality of the two sides of the equation is minimal, if present (Kieran, 1992). Students write the letter in the equation as the subject (stand-alone) or engage in guess work using specific values (Egodawatte, 2011; Kieran, 1992).

Many middle school students seem to have more of an operational view of the equal sign rather than a relational view (Knuth et al., 2005). In their study, 56% of Grade 7 students' definitions of the equal sign were variations of the sign asking them to perform an operation, in contrast to the 36% who saw the sign as some form of equivalence. However, students' relational view of the equal sign improved from Grades 6 to Grade 8. This relational view "is essential to understanding that the transformations performed in the process of solving an equation preserve the equivalence relation – an idea many students find difficult, and that is not an explicit focus of typical instruction" (p. 69). The misinterpretation of the equal sign continues beyond the middle grades for some students (Egodawatte, 2011).

Since word problems are literal, translations are sometimes done directly from the left to the right, leading to the formation of wrong symbolic notations and errors including situations involving inverse operations (Kieran, 1992; Reed, 1999). Clement (1982) gave 150 first year engineering students the following question amongst others to write in algebraic notation.

> Write an equation using the variables *S* and *P* to represent the following statement: There are six times as many students as professors in this university. Use *S* for the number of students and *P* for the number of professors.

Only 63 % of the students wrote the answer correctly. Of the 47 undergraduates who were taking college algebra as a course but were not science majors, only 43% obtained the right answer, with the rest answering incorrectly with $6S = P$. The students had read and translated into numbers and letters literally without regard to the reversal built into the question (Clement, 1982). One of the reasons that the students could have made in this error was that

in their minds, they rightly 'saw' the quantities of the two objects but were unable to establish equivalence. This is known as 'static comparison' (Clement, 1982). High proportions of students using the letter as a label or having reversal errors on the same question have since then been identified. These were post-secondary students enrolled in algebra and calculus courses in colleges and universities, and pre-service teachers of various nationalities (Lochhead & Mestre, 1999; Reese, 2007; Rosnick, 1981).

The use of mathematical terms and language may bring about misconceptions of operations that need to be performed. In the course of translation, sum and product are often misinterpreted, also resulting in reversal errors (Reed, 1999). MacGregor and Stacey (1993a, p. 223) found that only 35% of the 281 Year 9 students sampled could use symbols to represent the sentence " the number y is eight times the number z" with many writing it as $z = 8y$ instead of $y = 8z$. Also, less than 30% could correctly write "s is eight more than t" as many wrote $t = s + 8$ instead of $s = t + 8$. There was also an association of y with the number 8 while z and t became the subjects of the equation. Some students may wrongly generate expressions or inequalities as answers to word problems which rightly require construction of equations. These errors were named as "lack of equation" and "inequation" (MacGregor, 1991, p. 61). Writing algebra involves using or interpreting the letter within a particular context without any extra meanings being read into it (Ormond, 2000).

In conclusion, misconceptions arising both from the use of a letter as an object, label or word, and about the meaning of the equal sign make it difficult for many students to transit from arithmetic to introductory algebra, where letters are substituted and patterns emerge. Inability to translate, perform inverse operations and develop suitable algebraic forms from word problems inhibits proper processing of questions and leads to errors before the computation and processing stages are reached (Clement, 1982; A. White, 2005).

Diagnosis of students' problem-solving difficulties: The Newman protocol

This section describes the Newman protocol and explores studies that have used the protocol in order to assess student understanding.

The Newman procedure identifies errors made by students as they solve word problems. Newman (1983b) established an interview protocol consisting of five questions, which has the intention of identifying problem solving difficulties of students in mathematics. The interview is conducted after the student has attempted to solve the problem and failed to generate a correct solution. Children may find it difficult to solve word problems because they have to process the language of the text before they embark on solving the question (Pimm, 1991). In a study of the patterns of 124 low achieving Grade 6 pupils as they solved mathematical

questions given to them, Newman (1977, 1983a) reported that 13%, 22% and 12% of the pupils had reading, comprehension and transformation errors respectively on the questions. She identified that errors could occur at each procedural step, and classified these into five main categories which are briefly described and also shown in Figure 2.2 below.

Reading recognition: Reading of the question involves recognition of both the words and symbols in the given task. A *reading error* would stall the process of answering the question.

Comprehension: The second step entails students showing an understanding of specific terms and saying the questions in their own words. Ability to extract the core issues in a question "is one of the most important skills in mathematics" (Newman, 1983b, p. 16). If they cannot paraphrase what the question is about, they are unable to move any further and this is called a *comprehension error*.

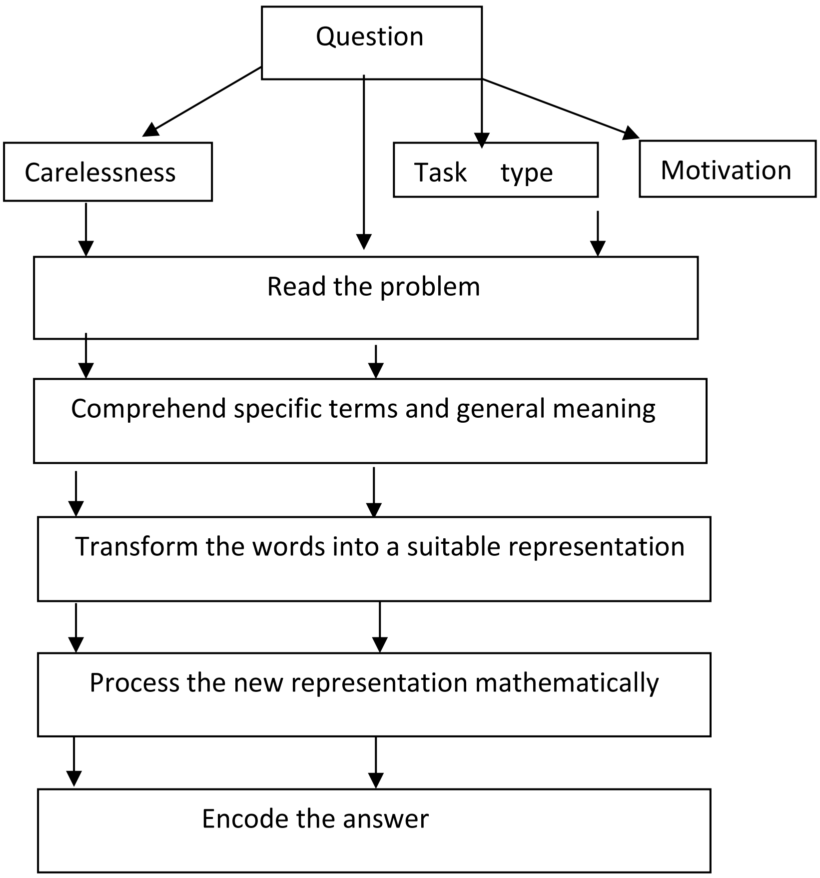


Figure 2.2: Newman Strategies Adapted from Newman (1983b, p. 2)

Transformation: The word problem once understood is written in a mathematical form. The understanding derived from the question which is in a literal form now has to be transformed into other suitable and correct representations, depending on the requirements of the task (Newman, 1983b). For this study, this would involve the symbolic form that will be needed to

solve the question in algebra. An understanding of concepts and an identification of operation(s) and method(s) to be used is examined during this stage. If an individual is unable to identify the needed operation(s) it is called a *transformation error*.

Process skills: After successful transformation of the question, it is then processed using mathematical computation and conceptual understanding. Depending on the new representation form, the arithmetical operations and computations needed to process the question are carried out. If a person does not know, guesses, uses the wrong operations, calculations or procedures, it is termed a *process skill error*.

Encoding ability: At the last step, the solution has to be written in an acceptable form. The acceptability depends on what the question asked for, that is, an answer in symbols or words or a table. If a student is unable to write the answer in a form which is acceptable, it is called an *encoding error*.

Other errors exist that may affect an individual's problem solving ability, though they are not directly related to the question being solved. Newman identified these as: careless errors, motivation errors and the task form. If an individual correctly carries out a step he missed during the first attempt of the question, it could be as a result of any one of the above three, which may occur at any of the stages.

The activities Newman outlined above for solving mathematical tasks are similar to those of Polya (1957), namely, understanding the problem, finding a plan, carrying out the plan and checking the solution. The main difference lies in Polya's first step, that of understanding of the problem. Newman subdivided this step into two, comprising reading and comprehension, and she indicated the importance of language in understanding word problems (Chinen, 2008).

Solving word problems requires an understanding of mathematics, the language of the text and the language of mathematics (Kersaint et al., 2009; Morgan, 2005) which students need to acquire and apply. However, it is often assumed that an understanding will be picked up as students' progress in their studies; hence little attention is paid to them during teaching. Everyday words such as *mean* and *product*, symbols like *+* or *:* have mathematical meanings that differ. Durkin (1991, p. 15) commented that "the language of maths is often demanding and ambiguous – pupils have ultimately to come to terms with this reality rather than to avoid it ".

Studies conducted have reported that about half of the word problem solving errors committed by students occurred before the processing stage that requires mathematical skills on selected operations. Wrong or incomplete solutions to questions provide insights into

students' thinking (An & Wu, 2012; Krebs, 2005; A. White, 2005). Children with ages less than 8 years had about 70% of errors from reading and comprehension (I. Watson, 1980) about 58% of Grade 5 pupils were reported to have comprehension and transformation errors (Clarkson, 1991b). Over 50% of Grade 7 students were identified to have transformation and process errors (Clements, 1980). In Chinen's (2008) study with Year 9 students, about 59% of the students' difficulties were comprehension and transformation errors. After a four-week language-based intervention, he reported a 27% drop in language-based errors committed by the students. Identification of errors can lead to remediation efforts by the teacher. However, the beliefs and practices of a teacher also have a strong influence on the situation.

To resolve misconceptions and ease the transition to algebra, research identified various strategies such as: introduce pre-algebraic ideas in the elementary school (Cathcart et al., 2006; Fujii & Stephens, 2008; Ormond, 2012; Warren & Cooper, 2008); use of technology (Rojano, 2008); teaching with a context-based approach (Linsell, Cavanagh, & Tahir, 2013; Tabach & Friedlander, 2008); pre-service teacher awareness of students' misconceptions (Tanisli & Kose, 2013); and, teacher professional development (Tirosh et al., 1998; Welder, 2012). Professional learning to increase teacher awareness of students' misconceptions in algebra and knowledge of the way students think might help in reducing the difficulties encountered by students in understanding algebra (Kieran, 2007).

This study examined the impact of a professional learning intervention for teachers on the way teachers teach Beginning Algebra and students' problem solving success. The last section of this review therefore considers literature about teachers' knowledge and beliefs and how they impact students' learning, and about professional learning.

Teacher and Professional Learning

Teacher beliefs and knowledge that affect practice

Teachers are members of communities of practice which share a set of cultural beliefs and practices. Teachers acquire these practices and beliefs from experiences at school and pre-service levels, in-service training, subject associations and professional learning programs. Teacher *beliefs* refer to "preferences a teacher holds as true and which affect the actions adopted in teaching", while teacher *knowledge* is defined as "the beliefs held by a teacher that are true and justified" (R. A. Philipp, 2007, p. 259). The difficulty in separating beliefs and knowledge led Thompson (1992) to refer to them as *conceptions*. Various scales have been developed to measure teacher beliefs but as noted by R. A. Philipp (2007), case studies provide insights about teacher knowledge and beliefs not always captured by scales. Teacher beliefs,

knowledge and practices are discussed in this section with particular reference to mathematics teaching and the algebra teacher.

Teachers hold individual beliefs about the curriculum, mathematics, methods of teaching mathematics and student's mathematical thinking. Stipek, Givvin, Salmon and MacGyvers (2001, p. 224) observed that "teachers had a fairly coherent set of beliefs which predicted their instructional practices". Mathematics teachers' beliefs that underpin their practice are beliefs about the nature of mathematics, mathematics learning, students and their capabilities, and teachers' beliefs about themselves and their capabilities (Beswick, 2007; Stipek et al., 2001; A. Watson & De Geest, 2005). These beliefs determine the level of focus on the students, use of dialogue and classroom tasks, and where teacher emphasis is placed (Beswick, 2007; Drageset, 2010). Teachers with mainly traditional beliefs emphasise speed, accuracy, rules and the use of teacher-directed strategies, in contrast to those with mainly constructivist beliefs who emphasise understanding, student effort, reasoning and the use of student-centred strategies (Beswick, 2006; Swan, 2006).

Teacher knowledge has several interrelated parts. Cochran-Smith and Lytle (1999) identified three components of teacher knowledge: knowledge for practice, knowledge in practice and knowledge of practice. In relation to mathematics these refer to: knowledge of mathematics, curriculum and methods of mathematics teaching; knowledge gained through experiences, reflections and situations; and, knowledge gained by participating in professional learning programs, and research. Therefore the knowledge acquired during training is to be bolstered and improved upon with teaching experience and professional support. Ball, Thames and Phelps (2008) building on Shulman's (1986) work classified and illustrated (Figure 2.3) the knowledge a mathematics teacher would need for teaching.

Mathematical knowledge for teaching (MKT) is defined as "the mathematical knowledge needed to carry out the work of teaching mathematics" (Ball et al., 2008, p. 395) and this greatly impacts students' achievement in mathematics (Ball, Lubienski, & Mewborn, 2001; Hill, Rowan, & Ball, 2005; Norton, 2012). A teacher's knowledge about the subject matter has a strong bearing on the content taught and how it is taught (Ball et al., 2008; Carpenter, Fennema, Peterson, & Carey, 1988; Drageset, 2010). It follows, therefore, that to teach algebra effectively, a teacher must understand and know the algebra that is to be taught.

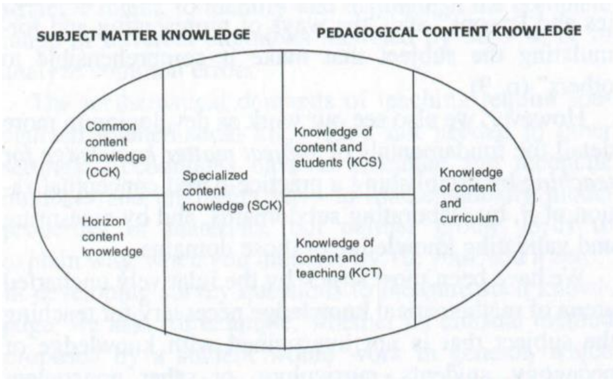


Figure 2.3: Domains of mathematical knowledge for teaching (Ball et al., 2008, p. 403)

Pedagogical content knowledge is knowledge of how to teach the subject content (Shulman, 1986) and is “critical for effective teaching” (Walshaw, 2012, p. 182) because it enables a teacher to help students understand mathematics. Ball et al. (2008) posited that the knowledge of mathematics for teaching has two subdivisions: *knowledge of content and student* and *knowledge of content and teaching*.

Knowledge of content and student is a combination of knowledge of the student and mathematics (Ball et al., 2008). It includes knowledge of students’ likely errors and misconceptions, ability to interpret students’ thinking and their likely responses (Ball et al., 2008). Teachers should also be able to determine the choice and difficulty levels of students’ tasks. The importance of this knowledge for the mathematics teacher’s practice has been exemplified in studies on algebraic expressions and representational forms of algebraic questions (Even & Tirosh, 2008; Nathan & Koedinger, 2000; Tirosh et al., 1998). Algebra teachers should be aware of common misconceptions and errors committed by students in beginning algebra (Welder, 2012) in order to help students use these experiences to enrich their learning (Ledesma, 2011).

Knowledge of content and teaching is a combination of knowledge of teaching and of mathematics (Ball et al., 2008). With this knowledge, teachers are able to sequence and scaffold instructional content to suit students’ abilities. It also involves recognition of challenges posed by various representational tasks and the teacher’s plan for handling them (Ball et al., 2008). A teacher has to know how to make use of leading questions and how to build on students’ explanations and problem solutions in order to enrich teaching. In particular, an algebra teacher needs to present different examples, and different forms of representations and approaches, and uses of concepts.

Teachers’ beliefs and knowledge affect their implementation of the subject curriculum. The mathematics curriculum serves as a guide that details the content, suggested materials and teaching approaches that should be used by a teacher. If adhered to, a curriculum should affect practice positively, but R. A. Philipp (2007) remarked that what happens in the classroom may differ from that which is expected. Teachers with years of experience develop routines which they follow, and they have identified potential trouble spots and developed their own ways of handling them, unlike beginning teachers who might turn to the curriculum or to textbooks for support, as exemplified in Tirosh’s study on algebra teaching (Tirosh et al., 1998).

Chick (2009) asked 35 teachers about their perceptions on a textbook’s explanation of questions on the distributive law in algebra. One of the questions used ‘apples and bananas’ to illustrate the operation $6a + 4b$ gives $2(3a + 2b)$. Only 26% of the teachers identified the use of the letter as an object while 74% said they would use similar explanations and provided their reasons. Chick further opined that many teachers’ pedagogy appeared to still include the ‘fruit salad’ approach and this may be traceable to some textbooks using it, and the teachers replicating how they themselves learnt it.

Teachers’ beliefs about teaching and the role of a teacher impact on classroom practice. Wilson and Cooney (2002, p. 144) noted that “the evidence is clear that teacher thinking influences what happens in the classroom, what teachers communicate to students, and what students ultimately learn”. In Nigeria, teacher beliefs and practices in mathematics and the sciences are largely traditional, and teaching is largely teacher centred as the teacher is seen as a person who has the knowledge and passes it to the student (Igbokwe, 2000; Ogunmade, 2005). Mathematics in Nigeria is hardly seen as a subject that can be creative and interesting to learn (Ladele, 2008). However, reforms in mathematics education globally have caused a significant shift from the traditional methods of teaching mathematics to one of teaching for understanding and application to real-life situations. Kieran (2007) has also called for more study into practising algebra teachers’ beliefs and knowledge about students’ thinking, misconceptions, difficulties, interpretations and ways of answering algebraic problems. This research study heeded that call.

Teacher professional learning

Professional learning for in-service teachers is needed to raise awareness of developments within their disciplines and education in general. Professional learning programs should be a blend of theory and practice in which teachers can connect new knowledge with their practice while receiving ongoing support and engaging in professional collaboration with others

(Cavanagh & Garvey, 2012; Darling-Hammond & McLaughlin, 1995). New knowledge refers to that which has emerged as a result of research, technological development or change in approaches to the subject discipline (Sowder, 2007). For example, mathematics previously taught as a system consisting of manipulations and computations requiring memorization is now more often taught for understanding and everyday use, allowing individuals to take possession of knowledge rather than it being imposed (Goos et al., 2007, pp. 36-39; National Council of Teachers of Mathematics, 2000).

There are certain activities that constitute an effective professional learning program. Ingvarson, Meiers and Beavis (2005) conducted a study with over 3,000 teachers with at least 10 years' experience. They identified that an effective teacher professional learning program should include a collaborative examination of students' work, follow-up, consideration of duration and time span, feedback, focus on content, active learning and reflection, and the building of a professional community.

An examination of students' work in professional learning provides teachers with an understanding of students' thinking as they solve mathematical problems. Teachers' understanding of students' misconceptions and problem solving difficulties can help them teach in ways that address these problems (An & Wu, 2012; Fennema et al., 1996; Krebs, 2005). Several decades ago, a Cognitively Guided Instruction (CGI) approach to professional development was used with some Grade 1 teachers in USA (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). A framework was developed concerning how students think when solving addition and subtraction problems and teachers were equipped with this knowledge and other curricular issues during a four-week workshop. Results within a year indicated both a change in teachers' beliefs and practice, and pupil performance was found to have improved. The CGI has been found by some other studies (Vacc & Bright, 1999) to modify teacher practice, although its focus is on primary mathematics teaching.

In Kreb's (2005) study, 20 middle-grade teachers participated in a professional learning program that also focused on students' thinking. The teachers answered some questions on patterns, and then examined some Grade 8 students' solutions to the same set of questions before watching video clips of the students as they solved the questions. Krebs argued that the activities enriched the teachers' understanding of how students think and understand algebra, and it also provided opportunities for the teachers to reflect upon their own pedagogy.

An and Wu (2012) used a different approach and investigated the effect of middle-grade teachers' detailed examination of their students' homework on fractions. The teachers

diagnosed and remediated the students' misconceptions individually or collectively in class. These activities increased the teachers' content knowledge and their pedagogical content knowledge.

The Newman framework, as describe earlier in this chapter, has also been used in training with in-service teachers to identify stages at which students make mistakes in problem solving with a view to remediation (Clements, 1980) although most studies were at the primary level. A professional learning program (AMIC) held in Brunei used the Newman theory as theoretical foundation for one of its workshop series (A. White, 2005). The primary school teacher participants were expected to use the Newman interview questions with their Primary 5 students. The use of Newman procedure has been found by primary school teachers to be "an easily adapted and relatively simple model" in leading students as they try to work through problems (Clarkson, 1991b, p. 245). It "has been popular with teachers'" (A. White, 2005, p. 19) and is the most preferred method for interpreting worded questions (P. White & Anderson, 2012).

Most of the existing studies about students' misconceptions analyse data collected from written work, but this research study focused also upon individualised interviews with the students. The professional learning intervention in this particular study took into consideration the activities identified in the literature for an effective learning program. It involved teachers collaboratively analysing students' incorrect solutions to algebraic questions, thus aiming to create in them an awareness of students' common misconceptions about variables, expressions and equality. It allowed for active learning with reflections on the Newman procedure and other language-based approaches to provide a connection with their practice. The program was conducted over two days and participants had the opportunity to discuss their experiences and to provide feedback afterwards at a one-day program.

Conceptual Framework

The teacher professional learning program focused on teacher awareness of research-evidenced students' misconceptions and errors in beginning algebra, and language-based teaching strategies particularly the Newman procedure. The intervention comprised activities that promote effective professional learning (Ingvarson et al., 2005). It is believed that this professional learning will impact the teachers' beliefs and their knowledge of algebra, and how it is taught. Teachers' practice would affect students' engagement and subsequently, students' understanding would reflect in their success in the algebra post-test and in responses received from the Newman interviews conducted. This is illustrated in Figure 2.4.

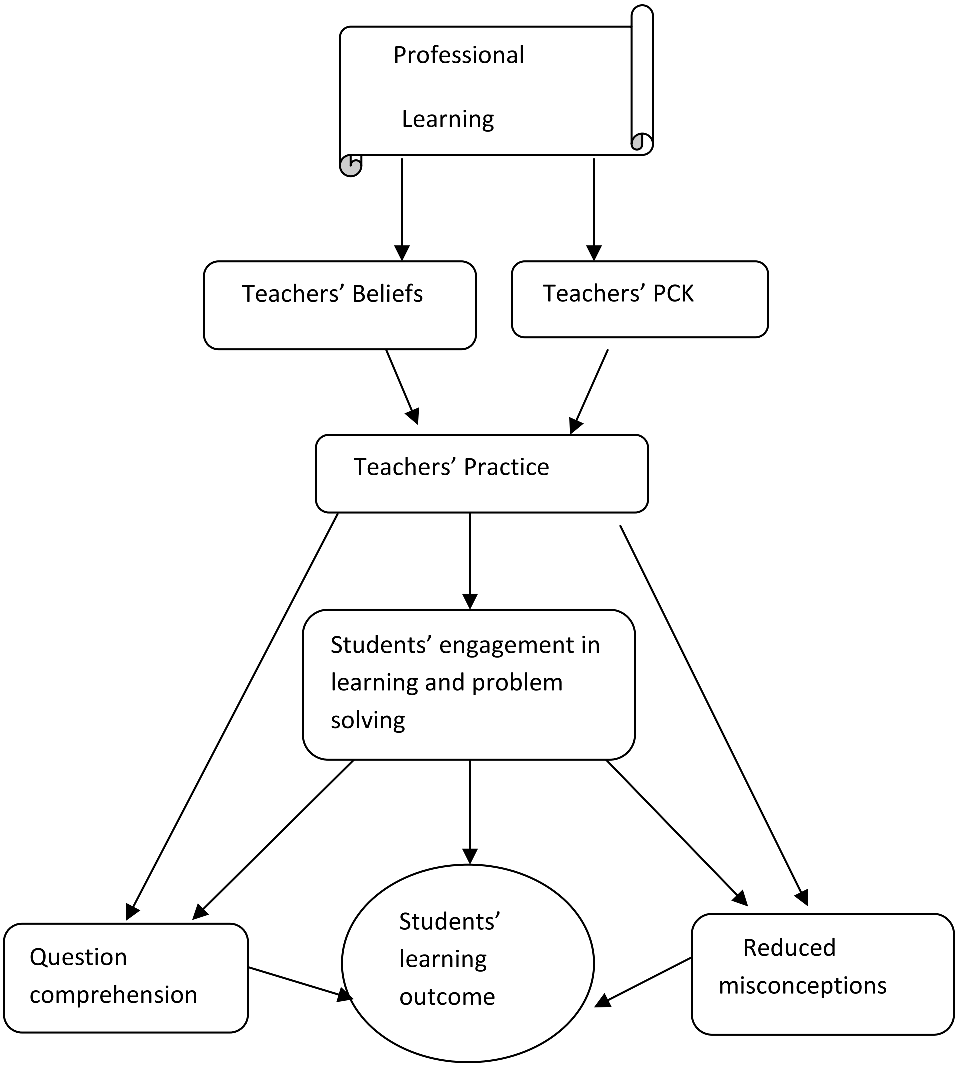


Figure 2.4: The conceptual framework for the study.

CHAPTER THREE: METHODOLOGY

This study explored the teaching and learning of word problems in beginning algebra in Nigerian schools using four case studies. To do this, teachers' beliefs, classroom practices for teaching word problems, and the difficulties experienced by students were examined. This was followed by a professional learning intervention program focused on creating an awareness of students' algebra misconceptions and language-based approaches. The impact of the intervention on teachers' practice and students' success in algebraic word problems was then investigated.

This chapter details the method used to accomplish the study and is organised into nine sections. The sections describe, in order, the research approach and design, the participants involved, the procedure followed and the instruments used in the study. Also included are descriptions about how validity and reliability of instruments were ensured, how the data were analysed, the ethical considerations, and the limitations of the study.

Research Approach

Qualitative and quantitative approaches were used to collect, analyse and interpret data. This mix of approaches complements the limitations of any one method, and findings are strengthened when different approaches result in the same findings (Cohen, Manion, & Morrison, 2011). A qualitative approach to research is focused on people within their natural settings and uses gathered information to build a full picture of the unique situation (Anderson & Arsenault, 1998; Silverman, 2011). To build this picture, classroom observations, semi-structured interviews with students, and focus group discussions with teachers were employed. Quantitative approaches, on the other hand, use measured outcomes with numeric values to evaluate the impact of an intervention (Creswell, 1994). These outcomes were obtained from the teachers' questionnaires and the students' algebra tests completed before and after the professional learning intervention.

Research Design

The research design was a multiple-case study consisting of four embedded cases (Yin, 2003). A case study is described as "an in-depth exploration from multiple perspectives of the complexity and uniqueness of a particular project, policy, institution, programme or system in a 'real life' context" (Simons, 2009, p. 21) and "can be used to document and analyse the outcomes of interventions" (Yin, 2012, p. xix). In each of the four case studies, there was a focus on the teacher's beliefs and practice and on his or her students' understanding and

success in solving word problems in Beginning Algebra. This design allowed for a rich analysis and description of the impact of the intervention on the teachers and students. Triangulation of data sources within each case and findings across cases ensured the confirmability of assertions arising from the data (Punch, 2005; Yin, 2003). A cross-case comparison revealed insights into the influence of contextual factors on the success of the professional learning intervention.

Research Participants

Participants for the study were 30 JS 1 mathematics teachers and 181 JS 1 students from secondary schools in two of the 20 Local Education Districts (LEDs) in Lagos State. The 20 LEDs are grouped into six administrative districts (1-6) and the two LEDs used in this study, Ojo and Badagry, are a part of the five LEDs that compose District 5. After six years of primary schooling, students continue their basic education for the next three years in the junior secondary school (JSS 1-3). The JS1 level was chosen for the study because that is when Nigerian students are formally introduced to algebra and learn the basic concepts of variables, expressions and equations. In each junior secondary school, two or three mathematics teachers teach the three levels of students. Classes are not often streamed by ability levels.

The teachers, from 30 different schools in the two LEDs, willingly completed the initial mathematics teacher questionnaire after the Researcher contacted them in their schools. The participants' involvement is illustrated in Figure 3.1.

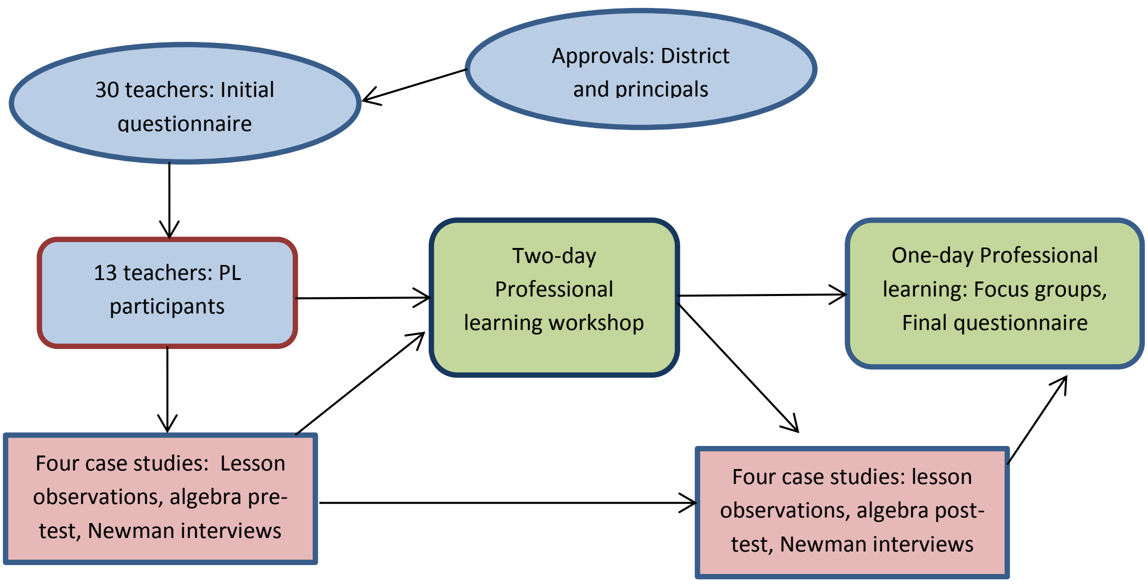


Figure 3.1: Overall plan of the research study

Thirteen of them participated in the first phase of the professional learning intervention program. Four out of the 13 teachers served as case studies and had a class each of their

students as student participants answering the algebra test items in the study. The case study schools comprised two public and two private schools with differing class sizes that totalled 181 student participants. In each class four students were interviewed before and after the professional learning intervention, giving a total of 16 interviewed students

Research Procedure

The study had two phases, the pilot study and main study.

Pilot study

A pilot study was first conducted to ascertain that the instruments were effective and captured the necessary information (Anderson & Arsenault, 1998). The Researcher's supervisors as experts in the field critically appraised draft instruments to ensure the validity of the instruments. A few JS 1 mathematics teachers and students not belonging to the educational zones used in the main study were given the initial questionnaire and algebra pre-test to complete. They were asked to bring to the attention of the Researcher items or questions that they were not clear about and any particular one that needed revision. As a result, there were slight changes of some words in the questionnaire and five questions were removed from the algebra test. This reduced the algebra test questions to 15 items. The lesson observation checklist was also trialled in two classes to ensure that it was satisfactory. The removal of inappropriate items and changes to ambiguous wording would have enhanced the reliability of the instruments.

Main study

The main study comprised six steps and each step is next described. Table 3.1 is a summary of all the steps.

The first three steps constituted the pre-intervention stage. Approval was obtained from the district on 17 February, 2011 for direct access to the schools and JS 1 mathematics teachers (Appendix 1). The principals were approached in their schools and after obtaining consents (Appendix 2), the Researcher personally met the teachers, obtained their consent (Appendix 3) and gave them the initial questionnaire to complete. This procedure enabled the Researcher to establish a cordial relationship with the teachers, and the completed questionnaires were collected on the same or next day, resulting in a 100% return rate.

The study was initially planned for only the public schools but the presence of many large classes in these schools, some above 100, led the Researcher after consulting with her supervisors to include private schools in the sample. It was felt that this inclusion would give a

more balanced and a broader perspective of the current practices in both contexts and also "clarify and deepen understanding" (Neuman, 2003, p. 211).

Table 3.1: Summary of the research study procedure

	Description of step	Period
1	Approval and consent from District 5 administrator and school principals	Feb –March, 2011
2	Distribution and collection of questionnaire from 30 JS 1 teachers. Selection of 13 teachers for professional learning program.	Feb –March, 2011
3	Selection and focus on four case studies – lesson observations, pre-test and Newman interviews with four students from each school	March, 2011
4	Two-day professional learning workshop with 13 selected teachers, including the case study teachers.	29-30 March, 2011
5	Focus on four case studies- lesson observations, post-test and Newman interviews with same set of four students from each school	April – mid June, 2011
6	One-day professional learning workshop - focus group interview and debriefing. Completion of final teacher questionnaire by 12 teachers	6 July, 2011

The criteria for selection of the 13 teachers were an expression of willingness to participate in the program, possession of a professional teaching qualification in mathematics, and having more than two years of teaching experience. The survey provided most of the data needed.

The four case study teachers were selected from the 13 teachers and comprised those who showed enthusiasm and concern about the performance of their students in algebra. Their schools were also easily accessible to the Researcher. Lesson observations in the specific classrooms chosen by the teachers started on 14 March 2011, the seventh week of the second term. In the Nigerian culture, the teachers act as parents and have the duty of care for their students within the school, so the teachers explained the research purpose to the class and the students verbally consented before observations commenced. Each teacher was observed at least twice and care was taken to ensure that all the participating classes had been taught the algebra content in the curriculum so that it was likely that the students could answer most of the algebra test questions before the pre-test was administered.

After completing the algebra pre-test, each teacher unguided by the Researcher selected four students, whom the Researcher interviewed individually using the Newman interview protocol. In three schools, the pre-tests and interviews were done on the same day while in the fourth school the interview was done on the next school day due to time constraints. The students were all very willing to participate and the interviews took place in an empty classroom within the school premises. Throughout, the interviews were non-threatening, and the students were constantly reassured of confidentiality and of the fact that the entire process had nothing to do with their school grades or performances.

The professional learning program was the intervention stage. The 13 teachers attended a professional development learning workshop that ran for two days at a suitable venue within the College of Education where the Researcher works. Permission had earlier been obtained from the management of the College of Education for its use as venue for all of the professional learning programs. The institution was easily accessible to all the participants and the program was facilitated by the Researcher.

The program focused on creating teacher awareness about common student algebraic misconceptions, and introducing them to language-based and more interactive approaches to mathematics teaching. Activities comprised discussions about students' difficulties and misconceptions about variables, expressions and equality in beginning algebra, reflections on students' solutions to algebra questions involving the use of letters as objects, the product/sum confusion and manipulation of equations, problem-solving, and active learning of the use of Newman strategy and other approaches. A detailed description on this follows in the next chapter (Chapter 4).

The post-intervention stage comprised the fifth and sixth steps outlined in Table 3.1. Each of the four case study teachers was observed thrice in their classrooms over a period of six weeks starting from 4 April. The Researcher provided support by encouraging the four case study teachers and answering their questions which were mainly about class management. After the first two weeks there was an end-of-term break before observations resumed on the 16 May (second week in the third term) for the remaining four weeks. At the end of this period, a post-test virtually identical to the pre-test was given to the participating class in each school. The same sets of students earlier interviewed on the pre-test in the schools were interviewed again on the post-test using the Newman interview protocol. The interviews were conducted on the same day the tests were taken in two of the schools and on the following day in the other two schools.

The teachers, 12 of them because one was unavoidably absent, attended a one-day program on 6 July to discuss, reflect and provide feedback on the use of the intervention in their classes. The focus group interview was replicated in two sessions instead of the planned single group because the teachers arrived at different times. At the end of the day, the teachers completed the final questionnaire.

Research Instruments

The 10 instruments that were used in the study were pre- and post-intervention teacher questionnaires; pre- and post-intervention student algebra test; an algebra lesson observation

schedule; the Newman Interview protocol, an error analysis guideline recording sheet and a student profile sheet for interview responses; a teacher algebra rating sheet; and, the focus group discussion protocol. The association between the instruments and the research questions is summarised in the Table 3.2 and this is followed by a description of each instrument.

Table 3.2: Relationship between research questions and data gathering instruments

Research question	Instruments used
1. How are word problems in JS 1 beginning algebra classes taught prior to the intervention?	Initial teacher questionnaire, classroom observations, PL discussion
2. What difficulties do students in JS 1 experience in solving beginning algebra word problems prior to the intervention?	Algebra pre-test; Newman interview, error analysis sheet, profile sheet, teacher algebra rating sheet
3. How does a teacher professional learning intervention program impact on JS 1 mathematics teachers' beliefs, knowledge and algebra teaching practice?	Final teacher questionnaire, classroom observation, focus group interview
4. How does the teacher professional learning intervention program impact on students' difficulties and success in solving algebraic word problems?	Algebra post-test; Newman interview, error analysis sheet, profile sheet

Mathematics teacher questionnaires

The questionnaires, an initial and final one (Appendix 4 and 5), were completed by the teacher participants before and after the intervention. The initial questionnaire was used to obtain data about the general context within which algebra was being taught at the junior secondary level. A questionnaire provides self-reported information about the opinion of the respondent on various issues (Johnson & Christensen, 2012). Consisting of 50 items divided into three sections, the questionnaire included closed and open-ended response items. The first section sought information on features like teachers' teaching qualifications, their mathematics and JS 1 teaching experience. The second section included Likert-type five point rating subscales about the teachers' beliefs, confidence and frequency concerning using different teaching approaches, their knowledge of JS 1 mathematics content, and their beliefs about mathematics, its teaching and learning. These subscales were mainly adapted from Hackling and Prain's (2005) and Olaleye's (2012) professional learning intervention studies with Australian and Nigerian in-service science teachers respectively, and Beswick's (2005) and Swan's (2006) mathematics teachers' beliefs survey. The Researcher created the other items. In the third section, the teachers gave reasons for the difficulty ranks they assigned to six

mathematics questions written in the questionnaire. An average of 10 minutes was taken by each of the teachers to complete the questionnaire.

The final questionnaire was completed by only the 12 teachers who participated in the two phases of the professional learning program. It included most of the items from the initial questionnaire and a few additional open-response items that evaluated the professional learning intervention learning program. The teachers' responses on both questionnaires were compared in order to detect any changes in views and beliefs.

Classroom observation schedule

The Researcher used an observation schedule to record happenings in the classroom at intervals of two minutes (Appendix 6). Cohen (2011, p. 296) noted that "at the heart of many case studies lies observation". Observations are a record in data gathering of happenings within a particular context (Anderson & Arsenault, 1998; Simons, 2009). The schedule indicated specific activities of teachers and students during the lesson that were of relevance to the study. The 14 specific activities included those related to traditional approaches such as teacher explaining, students listening and copying notes; others related to the five Newman steps, the use of mathematical language and the correction of algebraic misconceptions. Field notes were also taken and a digital recorder was used to record the observed lessons.

Algebra tests

Before the intervention stage, the participating students completed an algebra test comprising 15 short items (Appendix 7). The test was a mix of questions represented in word and symbols. The concepts treated in the questions were *variables*, *expressions,* and *equality* including *reversal operations*. Figure 3.2 and Table 3.3 describe the structure of the test items.

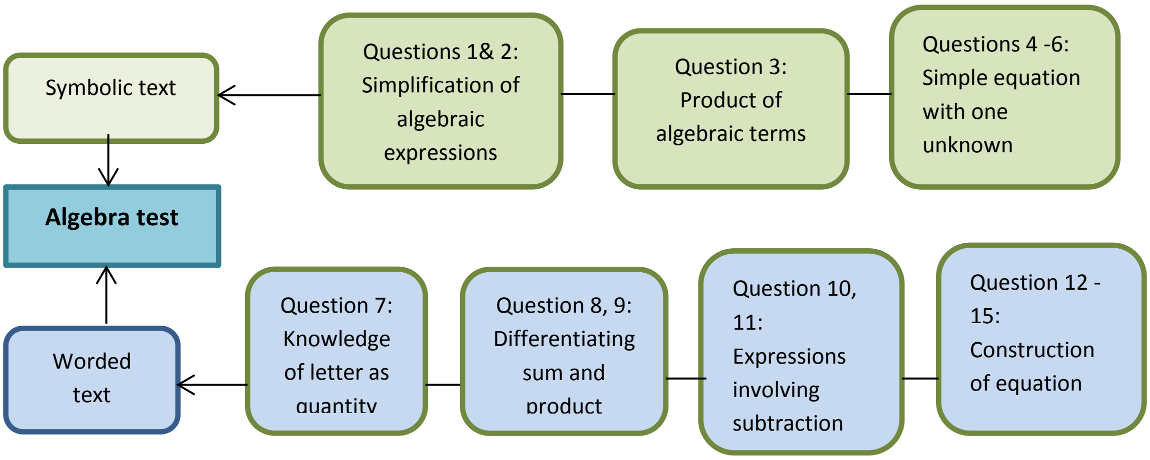


Figure 3.2: Composition of algebra test questions

Most of the questions were adapted from existing literature or sourced from the most common Nigerian mathematics textbook series, New General Mathematics (NGM).

Table 3.3: Algebra test items – concepts and required knowledge

Question text	Investigation	Questions	Concept	Needed knowledge	Reference
Symbolic	Simplification of algebraic expressions	1. Simplify as far as possible 1 + *x* + *x* 2. Simplify as far as possible 3*m* + 5*n* + 4*m* + 6*n*	Variable (GN), expressions	Letter is a quantity, collection of terms, an expression can be an answer	NGM for JS1 format
	Product of algebraic terms	3. *y*× *y* × *y* = ...	Variable (GN)	Product of algebraic term	NGM for JS1 format
	Simple equation with one unknown	4.Find the value of *x*: 7*x* = 21 5.Find the value of *x*: 2*x* – 2 = 10 6.Find the value of *x*: 21*x* = 7	Equations (SN)	Balancing an equation, inverse operations	NGM for JS1 format, question 6 - constructed
Worded	Knowledge of letter as a quantity	7. Sola has *x* bananas and Peju has *p* bananas. Peter counts the number of bananas each of them have and finds they are the same. Sola said you could write this as *x*=*p*, but Peju said that *x* and *p* are different letters and so cannot be the same. Who do you think is correct?	Variable (GN)	Letter is a quantity	Constructed
	Differentiating sum and product	8. Mary has *x* oranges and Bisi has four more oranges than Mary. How many oranges does Bisi have? 9. A basket costs eight naira and a bag costs *c* Naira more than the basket. How much does the bag cost?	Expressions (GN)	'more' in these contexts refers to addition, an answer can be an expression	Modified from MacGregor (1991), NGM for JS1 format
	Expressions involving subtraction	10. What is the number that is five less than *x*? 11.There is a *b* number of sweets in a packet. A girl has two packets of sweets and gives her friend six sweets. How many sweets does she have remaining?	Expressions (GN)	'less' in this context refers to subtraction, an answer can be an expression	Modified from Adetula (1989) and NGM for JS1
	Construction of equation	12. Write in algebra: There are twice as many pencils as biros (let *p* be the number of pencils and *b* be the number of biros). 13. If *s* is the number of students and *t* is the number of tables, write in algebra: There are three students for every table. 14. If *d* is the number of dogs and *c* is the number of cats, write in algebra: There are four more dogs than cats. 15. Write in algebra: There are three more caps than hats.	Equations (VN)	Letter is a quantity, establishing equality in relationship between quantities of the two items	Modified from Clement et al (1981), Ormond (2010)

Note. GN – Letter as a Generalised Number, SN – Letter as a Specific Number, VN – Letter as a Varying Number

Questions with given multiple answers were not used in order that that the problem-solving process could be understood through the students' workings. Word problems are most suited for evaluating students' conceptual understanding and language issues (Ni Riordain & O'Donoghue, 2009). Their difficulties were investigated through these worked solutions and performance in the questions which sought the students' understanding of the mathematical language, the algebraic letter and operating with it, and their understanding of equations. The questions examined the students' knowledge and ability to use the algebra letter as a specific unknown number (Questions 4-6), a generalised number or place holder (Questions 1-3, 7-11) or as a quantity that varies (Questions 12 – 15). The average time taken by the students to complete the test was 30 minutes.

In much of the existing literature, these sorts of questions were administered to students older than 12 years old. However, the Nigerian JS 1 curriculum content contains many of these algebra topics and this informed the Researcher's decision to use questions comparable to those found in the main JS 1 mathematics textbook. The use of standard textbook questions to examine students' understanding of word problems exists in the literature (Ni Riordain & O'Donoghue, 2009; Oviedo, 2005). It is the Researcher's belief that what the teachers teach and what the JS 1 students are expected to learn, by Nigerian standards, is beneficial to investigate.

The equation-construction questions were the only ones not specifically mentioned in the curriculum or textbook, but it was decided to explore the students' knowledge about relationships between two items with different quantities. This is because in JS 2, the students solve linear equations in two variables algebraically and graphically and they also solve word problems involving algebraic fractions (Federal Ministry of Education, 2007).

The pre- and post-test (Appendix 8) were identical tests but for the arrangement of the questions, alphabets and items. The exact questions were not used for the post-test in order to reduce students' familiarity with the questions (Chinen, 2008). Names and currency reflected those relevant to the Nigerian society. Three readability tests were employed to ascertain that the algebra questions were within the students' reading ability and these are described here.

The difficulty of the words and sentences may be measured through the syllable and word count respectively (Richardson, Morgan, & Fleener, 2006). The Fry's readability formula and accompanying graph is the most common and popular (R. G. Benjamin, 2012) and can be used to "measure the difficulty of word problems in mathematics" (Richardson et al., 2006, p. 450).

It also “measures the readability of material used in an instructional setting” (Richardson et al., 2006, p. 144). The Fry readability score for both tests fell within the Grade 6 range on the accompanying readability graph. The Flesch and Flesh-Kincaid readability tests are contained in the Windows 2010 Office software. The pre-and post-test Flesch reading ease score was 89 and 93.9 while the Flesch-Kincaid readability test was 4.0 and 3.3 respectively. The Flesch-Kincaid score indicates the grade suited for reading the text and for the algebra tests it was Grade 4, while a Flesch score of 80-90 or 90-100 means it is suited for Grade 6 or 5 respectively. The interpretation of these scores means that both tests were suitable for reading by the JS 1 students.

Newman interview protocol

The protocol (Appendix 9) was used by the Researcher with four students in each of the schools to examine the difficulties encountered in solving algebra questions. Punch (2005, p. 168) described interviews as “one of the post powerful ways of understanding people”. The Newman protocol, already used in many studies, was employed and consists of five structured questions students are asked in relation to a given problem that they have previously solved incorrectly. The interview was conducted, as quickly as possible, after the pre-test and a second attempt (on the day of interview) at solving the questions. The protocol was slightly adapted in that further questions, this time unstructured, were asked when the circumstances warranted them, in order to get a better understanding of the student’s thinking.

Newman error analysis guideline sheet

The response given by the student was noted and coded appropriately into the error analysis guideline sheet (Appendix 10). The coding corresponded to the five structured questions asked during the interview and indicated the task error on the problem by the student. There was an extension to this guideline when deemed necessary and after the task error had been recorded. This extension comprised the student’s responses to the Researcher’s unstructured questions, if asked. Such information was noted on the sheet as field notes.

Diagnostic profile sheet

Designed by Newman (1983b), the sheet is a cumulative record of a student’s incorrect responses to the questions (Appendix 11). It indicated the initial error cause for each item in the test.

Teacher algebra rating sheet

Ball et al. (2008) noted that teacher knowledge of content and student includes a teacher’s ability to predict which questions might be difficult for their students. The rating sheet (Appendix 12) contained the students’ algebra pre-test questions and a five point rating scale

from very easy (1) to very difficult (5). Each teacher who participated in the professional learning program completed a rating sheet. The teachers rated in a small box beside each question how difficult they felt it would be for their own JS 1 students to correctly answer. This concept was adapted from studies that compared teachers' (and researchers') ratings with students' actual performance in mathematics problem solving (Alexandrou-Leonidou & Philippou, 2005; Nathan & Koedinger, 2000). The purpose was to find out the teachers' opinion about the difficulty level of the questions and compare this with the students' actual performances.

Focus group

A focus group is described by Punch (2005, p. 171) as a "grouped interview" and a means of gaining insights and varied perspectives from others as the participants respond to each other's comments on issues. A 90 minute focus group discussion was used to engage the 12 teachers who were involved in the professional learning on algebra. Two group interviews were done with six teachers in each group and the same set of three questions was used in both groups (Appendix 13). The purpose of this was to receive feedback from all of the teachers on their experiences after the post -tests had been carried out in the schools.

Digital recorder and camera

A digital recorder was used to record verbal communication in the classroom, interview sessions and meetings with the teacher participants. Visuals using a digital camera captured writings of some students' mathematical work and blackboard mathematics content. Students' faces were generally not visible, but when this was occasionally unavoidable they were completely blurred to avoid the possibility of identification.

Reliability and Validity of Instruments

The Cronbach alpha reliability test was carried out for the questionnaire as a whole and for the different subscales, to establish their internal consistency (Cohen et al., 2011). An overall alpha coefficient of 0.78 was obtained for the initial questionnaire pilot test and all the subscales had alpha coefficients between 0 .72 and 0 .93, except for the frequency of use of approach scale with an alpha coefficient of 0.62. The same instruments were used to collect the data in each of the four cases and the use of digital recordings, pictures and students' work, all carefully analysed, provided a basis for validity and reliability. In the course of data collection, care was taken to confirm opinions and views expressed by participants during the professional learning program, focus group and student interviews. This measure helped to ensure that the gathered information was trustworthy and reflected the respondents' beliefs. The

Researcher's supervisors as experts in the field also supported the content validity of the instruments.

Data Analysis

All the participants were given coded identification numbers and variable names were assigned to the questionnaire and algebra test items before analysis commenced. Each correct answer in the algebra test was scored one mark while a wrong answer was scored zero. Only students who completed both the pre- and post-tests had their scores included in the analysis. The responses to the closed questions in the questionnaire were numerically coded and all documented in a coding manual. Quantitative data collected from the teachers' responses on the Likert rating scales in the questionnaire were first entered into an Excel spread sheet and checked for correctness.

All the coded data were then entered and analysed with SPSS/PASW 18 software for descriptive statistics such as percentages, means, and standard deviations. For the open-ended questions in the questionnaire, key words or descriptions were identified in the responses and used to generate categories. The multiple response descriptive analysis in the SPSS software was then used to obtain frequencies and percentages.

Wilcoxon signed-rank test, a non-parametric test was used to check for significant differences in the students' pre- and post-algebra test because the distributions were not normal but positively skewed. Since the number of students who completed the tests was sufficiently large, the t-test was also used to confirm the results of the Wilcoxon signed-rank test. Kruskal-Wallis test and Wilcoxon signed-rank tests (non-parametric tests) were also used to analyse the algebra rating sheets and the final teacher questionnaire results respectively, because of the small number (12) of teachers involved.

Qualitative data were also coded, transcribed, and read over several times, and key descriptions were identified. These descriptions resulted in emerging categories identified in each case study, the professional learning program, the focus groups and students' interviews. The students' responses to the Newman questions particularly at the transformation and processing stages were used to identify the algebra misconceptions. For example, if a student chose a value, say three, to represent x in the question "simplify $1 + x + x$" and then obtained seven as the final answer, the misconception of the letter x as having a specific value (of three in this case) was identified as the initial cause of the error. This misconception was recorded for that student as a transformation error. However, if another student responded that it added up to $1 + 2x$ and obtained $3x$ as the answer it was analysed as a processing error. In this

case, wrong conjoining was identified as the algebra misconception. The use of four case studies allowed for individual and cross-case analysis to be carried out. The data collected was triangulated to identify patterns and themes that emerged from the study. Key findings that emerged from the quantitative and qualitative analysis were interpreted drawing on the literature to generate assertions, and these were subsequently used to answer the research questions.

Ethics

The research was carried out in line with ECU ethics approval received in December 2010. Permission for access to the teachers was obtained from the administrative district involved and letters of consent were signed by teachers to indicate their willingness to participate in the study. The teachers on behalf of parents obtained a verbal affirmation from the students to participate, as is normal practice within the Nigerian school culture where teachers act in *loco parentis*. All participants were informed that they could withdraw their participation in the research at any time. The Researcher made sure the students felt at ease throughout the classroom observations and interviews, and they were at no time compelled to respond. All participants were assigned numerical codes which were used for data analysis.

Limitations

The study was case-study based and as such is limited in terms of generalizability. The number of teachers used was small and sampling was limited to one district so the research results may not be generalizable beyond the school types and district used in the study. The class sizes of the public schools used in the study were larger than those of the private schools and this may have affected the outcome. However, these large classes are typical of many of the existing public schools and it allowed the research to be carried out within a realistic setting and with consideration of the present situation. Individualised interviews with the case-study teachers would have captured much more of the teachers' knowledge and views of their respective students. The two algebra tests that the students completed were similar but necessarily slightly different and this might have affected their performance. However, using the same test twice may also affect performance because the students, especially those who were interviewed, would have likely gained familiarity with the questions.

The small number of teachers used however was advantageous in that it provided insights and ample opportunity for the teachers to share their views and beliefs about the teaching and learning of mathematics, and algebra in particular, during the professional learning intervention. This is described in the next chapter.

CHAPTER FOUR: PROFESSIONAL LEARNING INTERVENTION

Introduction

The professional learning intervention program was conducted in two phases. The first phase was for a period of two days and the second, after a six-week teaching period, was held for one day. The intervention focused on beginning algebra misconceptions and language-based approaches. As a result, the professional learning had two main purposes: first, to update teachers on algebra misconceptions about variables and equations/equality that have been reported in the literature and second, to introduce them to Newman's language-based error analysis procedure and review other language-based approaches to teaching mathematics.

While the first phase was designed to update and introduce the teachers to these issues, the second phase was directed towards receiving feedback from the teachers about the use of the acquired knowledge in their teaching practice. Thirteen JS 1teachers participated in the first phase and 12 were available to participate in the second phase. The professional learning program activities included collaborative examination of students' work; focus on content, active learning and reflection, follow-up and feedback. Ingvarson et al.(2005) identified these as necessary components for an effective professional learning program. The majority of the activities were designed to be interactive and they provided opportunities for the teachers to support/assist themselves as they engaged in meaningful learning.

This chapter describes the activities in which the teachers participated during the two phases of professional learning and reports some data about their engagement with those activities. The teachers participated in 10 activities in the first phase and three activities during the second phase.

Day One

The six activities the teachers were engaged with on the first day were rating of difficulty of some algebra questions, algebra problem solving, examining students' solutions, discussing language issues and misconceptions in beginning algebra, and learning the Newman language-based error analysis procedure. For each activity, the purpose of the activity, the teachers' responses, where relevant, and some transcript excerpts are described.

Activity One: Teachers' views about the difficulty level of algebra questions

All the teachers individually rated each of the 15 items on the students' algebra pre-test in terms of how difficult it would be for their own students to solve. To do this, each teacher was given a copy of the pre-test and asked to rate each question on a scale of one (very easy) to five (very difficult). The purpose of this activity was to assess the teachers' knowledge of the difficulties their students may face when solving algebra questions. The rating may also indicate their awareness of the abilities and conceptual understanding of their students, which Ball et al.(2008) described as important components of teachers' knowledge of content and students in mathematics teaching.

The mean rating score, standard deviation and ranks of the questions in increasing order of perceived difficulty are presented in Table 4.1.

Table 4.1: Teachers' mean rating scores and rankings of 15 algebra questions (n=13)

Question	Mean rating /5	Standard deviation	Ranking from easiest
$y \times y \times y =$	1.62	.768	1
Find the value of x: $7x = 21$	1.69	.751	2
Simplify as far as possible $1 + x + x$	2.00	.577	3
Simplify as far as possible $3m + 5n + 4m + 6n$	2.00	1.080	4
Find the value of x: $2x – 2 = 10$	2.08	.863	5
Find the value of x: $21x = 7$	2.62	1.325	6
What is the number that is five less than x?	2.85	.689	7
A basket costs eight naira and a bag costs *c* naira more than the basket. How much does the bag cost?	3.15	.689	8
If *d* is the number of dogs and *c* is the number of cats, write in algebra: There are four more dogs than cats.	3.15	.899	9
Write in algebra: There are twice as many pencils as biros (let *p* be the number of pencils and *b* be the number of biros).	3.15	.899	10
Write in algebra: There are three more caps than hats	3.23	.832	10
Mary has *x* oranges and Bisi has four more oranges than Mary. How many oranges does Bisi have?	3.31	.947	12
There is a *b* number of sweets in a packet. A girl has two packets of sweets and gives her friend six sweets. How many sweets does she have remaining?	3.38	1.121	13
If *s* is the number of students and *t* is the number of tables, write in algebra: There are three students for every table**.**	3.69	1.109	14
Sam has *x* bananas and Peju has *p* bananas. Peter counts the number of bananas each of them have and finds they are the same. Sam said you could write this as $x=p$, but Peju said that *x* and *p* are different letters and so cannot be the same. Who do you think is correct?	3.77	1.092	15

Note. Teachers rated questions on a scale of Very easy=1, Easy=2, Okay=3, Difficult=4, Very difficult=5.

All of the questions with relatively straightforward symbolic representations were rated as easier than all of the word problems involving interpretation, with more than half of the teachers categorising them as very easy or easy for their students to solve. Two-thirds of the teachers felt that most of the questions (11 /15) were easy or adequate for their students to solve (for a detailed analysis of this, see Appendix 14).

Many of the word problems that required the quantities of two objects to be equivalent to one another were rated as not particularly difficult, while the question perceived as the most difficult was the lengthiest word problem and required only the knowledge that the algebraic letter was a quantity. The questions with large standard deviations in comparison to others suggest that some of the teachers (between one and four) had differing opinions about the demands of the questions from the majority.

Key Finding 4.1 Prior to the PL, all of the word problems were rated to be more difficult to solve than mainly symbolic questions. The most difficult symbolic question was a linear equation that had a fraction as answer. Two-thirds of the teachers perceived that 11 of the 15 questions, including three word-equations with two pronumerals, were okay. The question perceived as most difficult did not need any mathematical operation, only the knowledge of a letter as a quantity.

Activity Two: Solving algebra word problems

Each teacher was given a sheet of paper with two algebra word problems; the ‘student professor’ and ‘cheesecake’ questions taken from the study by Clement, Lochhead and Monk (1981). Two words in one of the original questions, ‘cheesecake’ and ‘strudel’, which might be unfamiliar to the teachers, were changed to ‘cake’ and ‘sandwich’ respectively. The teachers individually solved the questions in the space provided and their sheets were collected. The purpose of this activity was to enable the identification of any algebra misconceptions which the teachers also might have. The purpose of this was to address them during the program because teacher subject mastery is so crucial for success in mathematics.

At the close of the first day, the Researcher examined the teachers’ written solutions. One teacher answered the first question correctly while no one had the right answer to the second question. MacGregor’s (1991) classifications were used to interpret the teachers’ incorrect answers. The questions and the teachers’ solutions are provided in Table 4.2.

Table 4.2: Teachers' solutions to algebra questions (n=13)

Question	Responses	Number (n=13)	Percentage	Interpretation
1. Write an equation using the variables *S* and *P* to represent the following statement: "There are six times as many students as professors at this university." Use *S* for the number of students and *P* for the number of professors.	$S = 6p$	1	7.7	Correct answer
	$6s = p$	5	38.5	Error-reversal
	$6ps$	3	23.1	Error-Product/ sum confusion
	$6s>p$	2	15.4	Error-Inequation
	$6s + 6p$	1	7.7	Error-Lack of equation
	$6s + p$	1	7.7	
	Representation*: S* represents students, *p* represents professor	6	46.2	Error-Letter as a label
2. Write an equation using the variables *C* and *S* to represent the following statement: "At Mindy's restaurant, for every four people who ordered cheesecake, there are five people who ordered strudel. " Let *C* represent the number of cheesecakes and *S* represent the number of strudels ordered.	$4s = 5c$	0	0.0	Correct answer
	$4c = 5s$	6	46.2	Error-Reversal
	$4c + 5s$	6	46.2	Error-Lack of equation
	cs	1	7.7	Error-Product/ sum confusion
	Representation*: C* represents cake, *s* represents sandwich	4	30.8	Error-Letter as a label

In Clement's (1982) study with 150 engineering students, 37% and 73% wrote the wrong answer for questions one and two respectively, while 68% of the wrong answers were reversal errors (that is, $6s = p$ and $4c = 5s$). In this activity with the teachers, 39% and 46% made reversal errors for the two questions. The teachers' solutions to the problems also revealed the misconception that the letter is a label for a word or an object. In some of the answers, expressions were generated instead of equations.

> Key Finding 4.2
> Many of the teachers themselves seem to have the misconception that the pronumeral letter in an equation is a label for a word or object, and many made reversal errors.

Activity Three: Examination of students' solutions to algebra question

The teachers were divided into four groups and each group was given a sheet of paper with five beginning algebra questions and some incorrect student solutions adapted from studies by Booth (1984), MacGregor (1991) and from Stacey and MacGregor (1997). The teachers were not told if the solutions they were given were right or wrong. For each question, the teachers

were required to provide the correct answer to a wrong one, or confirm a correct answer, and identify the mistakes that they believed might have led to the students' incorrect solutions. The second part required each group to choose one person who presented their findings to the larger group for discussion.

The purpose of this activity was to help the teachers see the issues from the students' perspective and to find out how much they knew about the existence of algebra misconceptions. Herbel-Eismann and E.D. Phillips (2008, p. 295) asserted:

> Through examining students' work, teachers generate evidence for claims related to what they think students know. In the process, they often find opportunities to re-examine their own knowledge.

The presentation format agreed upon was for each group to present their findings on the question number that corresponded to their group number while the other groups listened and commented afterwards. Questions one and two generated a lot of discussion and different opinions among the teachers. There was an indication that some of the teachers might indeed have some algebra misconceptions. By the third question, it seemed the teachers had a clearer understanding and reports on the remaining three questions were generally agreed upon. The five questions and the teachers' responses now reported upon.

<u>Question 1.</u> Write in algebra: The number y is eight times the number z. Students' incorrect answers: $8y = z$, $y = 8 + z$, $y + 8 = z$.

It was very surprising that some of the groups arrived at wrong answers and by the amount of discussion the question generated. It took some time before there was an agreement that that correct answer was $y = 8z$, which was not included among the provided solutions. It seemed some of the teachers (given pseudonyms T1 – T13) might have been confused about how to interpret and transform the question. The transcript below provides an insight into this.

> Researcher: What is the correct answer?
>
> T10 (Group 1 representative): The answer is $8y = z$.
>
> (Mixed responses of 'yes' and 'no' from the larger group follows)
>
> T11: No, $y = 8z$ is the answer, none of the answers is correct.
>
> Researcher: Group 1, why do you think it is $8y = z$?
>
> T3: For me, I think the number y is now greater than z by eight. To me, eight multiplied by y gives z. ....That's why I said $8y = z$. That's for me.

Researcher: The rest of us, do we think it is $8y = z$ or who supports this answer?

T13: I support. Suppose z is equal to one, what will be the value of y? If z is two, then what is y? Being specific first, and then you generalize helps.

T7: We are told that y is eight multiplied by z

(General laughter and then after some more deliberation)

T5: It is a word problem, not an assumption. It is clear. The number y is eight times the number z, so y equals eight times z.

T4: The answer is $y = 8z$

Researcher: Do we have $y = 8z$ there?

General response: No

Researcher: Do we now agree that the answer is not there?

General response: Yes

Researcher: If we can run into this type of problem =

T10: =what about the children? (Professional learning workshop, 29/3/2011)

After agreeing on the answer, the teachers concluded that misinterpretation of the problem and the use of 'more' instead of 'times' led to the students' incorrect solutions. The teachers also proffered possible reasons about why their students may also arrive at the given incorrect solutions. The excerpts are given below.

T10 (Group 1 representative): They did not interpret the question correctly.

T3: There is a language problem, their interpretation of 'is' may not be taken as 'equal'.

T7: He's thinking the question is y is eight more than z. That is, plus, not eight times

T13: Any question that involves algebra must be very, very clear. If they said 'the number y is equal to eight times the number z', then we can do it and it will be clear. I would leave it as 'number y is equal to eight times the number z' for my students.

T11: The students misunderstood the algebraic statement (Professional learning workshop, 29/3/2011)

The teachers believed that the inclusion of the word 'equal' in the question might have made it more specific and easier for the students to solve. The importance of the mathematical term 'times' and its operational representation by multiplication was noted by the teachers.

Question 2. Write in algebra: s is eight more than t. Students' incorrect answers: $t = s + 8$, $8s = t$, $s = 8t$.

There was still some deliberation and resorting to the use of specific values before the teachers agreed that the correct answer, $s = t + 8$, was not given. The transcript below captures this belief.

> Researcher: What is the correct answer?
>
> T6 (Group 2 representative): The answer is the first one.
>
> Researcher: That is, $s + 8 = t$. Do we agree?
>
> T3: The answer is not there. *S* is eight more than (emphasis) t, that is, $8 + t$.
>
> T11: Because s has eight extra, more than t
>
> T5: The answer is the first one
>
> T12: s is already more than t, so we are now adding eight
>
> T10: The statement given does not indicate =
>
> T12:= s is already eight more than t (with emphasis)
>
> T11: The answer is $t = s + 8$
>
> T4: Let us represent it with numbers, let's assume $s = 2$, then $t = 8 + 2$ with option a
>
> T12: Then t will be equal to ten.
>
> Researcher: Look at it in relation to the question, not the answer you are giving. If s is two, then what will be t?
>
> T3: If s is two, then t will be minus six.
>
> (After more specific examples and further discussion)
>
> T8: The answer is not there, it is $s = t + 8$
>
> Researcher: Is the answer there or not there?
>
> General response: Not there (General laughter) (Professional learning workshop, 29/3/2011)

The teachers claimed that they selected wrong answers because of their misinterpretation of the meaning of the letters. The teachers also proffered possible reasons about why their students might also arrive at the given incorrect solutions. The reasons were associating *eight* with variable s, resulting in t being of greater value than s, following the left-right reading, or the teacher's pedagogy. The excerpts follow.

T2: They will interpret the question from the back, from the last word. S is eight more than, seeing it as *s* plus eight is what gives *t*.

T9: They put it down after reading the question. It is what they read they put down. So they interpret it in the reverse order.

T10: Putting the statement into mathematical form is what is giving us the problem. (Professional learning workshop, 29/3/2011)

Question 3. Write in algebra: add 4 onto $3n$. Students' incorrect answers: $3n4$, $7n$, 7, 12

The answer to the question, $3n + 4$, was easily agreed upon and a lack of familiarity with the word 'onto' was said to be the reason for the mistakes. Language was identified as being important. The word 'onto' was identified by the teachers as unfamiliar to their students and they felt the use of the familiar word 'plus' would be preferable. Some teachers felt 'onto' could be thought to be a typographical error by their students or to mean multiply.

T10: They will not understand the word 'onto'. We don't normally come across it.

T3: They are not familiar with the word 'onto'. If my students see it, they'll think that I made a typographical error and that 'onto' should be 'n to' and write 7n thinking it should be 'add 4n to 3n'.

T11: They are not familiar with the word.

T5: Some may think 'onto' means multiplication. There is no reason to multiply. The word 'add' is familiar to them. (Professional learning workshop, 29/3/2011)

Question 4. David is 10 cm taller than Con. Con is *h* cm tall. What can you write for David's height? Students' incorrect answers: 18 cm, $Dh = h + 10$, $h = D + 10$, 160 cm, $h10$.

The teachers quickly agreed that the correct answer, $h + 10$, was not given. They all seemed to feel that either the students did not read the question correctly or that they just picked the values and put them together, or they misinterpreted the question and they mixed up the variables.

Question 5. Simplify if you can: 2a + 5b. Students' incorrect answers: 7ab, 8ab, 12

Agreeing readily that the answer, $2a + 5b$, was not given, the teachers believed that the students did not understand or that they had no knowledge of addition and multiplication of algebraic terms.

T11: Most don't know that 'a' cannot be added to 'b'. As teachers we should tell them that 'a' is like yam and 'b' is like a pencil. The two are not the same.

T6: In reality, can we not add them? We can add the items but here, they are not the same.

T10: The principles guiding addition of variables in algebra has not been understood by the students =

T7: = That if the variables are different they cannot be added

T3: They need to be able to identify the unknown. We tell them that...called unlike terms. We can only multiply them, we cannot add them. (Professional learning workshop, 29/3/2011)

As we concluded the activity, the teachers admitted that many teachers use few word problems because they do not understand them themselves or because of the difficulty the students experience in interpreting them. The excerpt below explains further.

T11: Most teachers don't understand word problems.

T6: Our students in general are not exposed to word problems and we use less of it in our classrooms.

T4: It is because of the interpretation

T10: Because it is difficult. Time is also another problem.

T8: We want the easy way out. When it crosses your mind that the students don't understand, we leave it out. (Professional learning workshop, 29/3/2011)

None of the teachers identified the words 'simplify' or 'algebra' as potential sources of difficulty. Their responses did not provide evidence that they were aware of many of the algebraic misconceptions about variables and expressions.

Key Finding 4.3 The teachers during the professional learning workshop do not seem to be aware that the use of the letter as a word/object is a misconception and a strong reason for the errors. Some teachers themselves appeared to initially struggle with the meaning of some questions and they also appeared to be unaware of misconceptions about variables and expressions. Teachers reported that they present few word problems in class because their students find them difficult.

Activity Four: Language issues in the teaching and learning of algebra.

The purpose of this general group discussion was to engage the teachers in thinking about the importance of mathematical and English language within the English as a Second Language (ESL) setting. The teachers admitted that ESL affects the students' learning ability because it

provides an avenue for misinterpretation of questions. The following extract typified their general belief.

> T5: In the area of interpretation, they [students] might not really get the meaning of what is being written.
>
> T2: Some JS 1 students cannot read English well, so they do not read the questions (Professional learning workshop, 29/3/2011)

Some of the teachers mentioned ways they addressed the situation in their classes. These include:

Using language suited to the students' ability level while recognising that mathematical language at the secondary and primary level differs.

> T8: In terms of maths, relating to the students in English, we have to come to the level of the students. It depends on the level of the students you are teaching, you explain so that they can understand.
>
> T5: The mathematical terms used at the primary level are what they are familiar with.
>
> T10: In JSS, level of mathematical language should be different from the primary school. The English teacher has a vital work to do, to help the maths teacher. One of the problems is the student interpreting the mathematical English. If they understand English, they will learn the meaning of words like 'twice' and build up other words with time. The first solution for word problems is the English teacher. If they [students] don't understand English, I don't know how they will understand these words in maths word problems. (Professional learning workshop, 29/3/2011)

Giving notes to the children that contains mathematical English.

> T13: English language and mathematical language are two different things. In my time, we just work maths but nowadays you have to read maths and work maths. Some people don't give notes but it is important because through it they know English and mathematical English...the way you deal with mathematical language is different from English when you transcribe it. (Professional learning workshop, 29/3/2011)

Explaining the difference between meanings in English and mathematical language before the start of the lesson.

> T6: The only thing is that it requires time. One has to be patient with the students. ... When we give statements in school, take time to tell and explain to them the difference in meaning between ordinary and mathematical English.... Not that the students are poor in English, but we need to relate the knowledge in English to mathematical terms.
>
> T11: If I want to teach a topic, ... I need to give students the meaning of related words and explain to them before the lesson starts at all. For

> algebra, I will give them the likely words, things related to algebra that they will come across. (Professional learning workshop, 29/3/2011)

Explaining the meaning of new related words and ensuring that students write them at the back of their notebooks.

> T3: ....and even tell them to take notes at the back of their exercise book so that anytime they come across it again, they can always refer to it. In the next class we should relate it; if we keep referring to it there will be continuity in what they are learning. (Professional learning workshop, 29/3/2011)

The role of an individual's first language in learning word problems was also discussed. Many of the teachers believed that the use of the first language at times to explain concepts helps students to understand. For example:

> T11: If it were in my language, I don't need to think twice before I know what you are saying.
>
> T1: If it is in their [students'] language, they will understand.
>
> T7: At times there are things you want to say, one may not have the exact words. But if it is your own language, you will know what to say. (Professional learning workshop, 29/3/2011)

Key Finding 4.4 The teachers believed that students need to be taught the language of mathematics, which is often different from English Language, and that a better understanding of the latter will facilitate understanding of word problems. That explaining concepts to students in the first language improves and accelerates understanding is also the belief of many teachers.

Activity Five: Introduction to the Newman language-based error analysis procedure

The teachers were introduced to the first part of the procedure which Newman called the "performance strategies" (1983b, p. 25). The aim of this activity was to assist the teachers to gain familiarity with, and to reflect on the importance of each strategy. The teachers were engaged through questioning to reflect on and identify the importance of each of the steps in solving word problems.

The reasons given by the teachers suggest that they think most of the strategies would prevent errors with computation rather than comprehension. The other three performance strategies, carelessness, motivation and task form, which may contribute to students' difficulties, were also mentioned. The table below provides a summary of the teachers' reflections.

Table 4.3: Teachers' reflections on the importance of Newman performance strategies (n=13)

	Performance strategy	Importance
1	Reading recognition	You cannot solve without reading Recognition of things you may not otherwise notice It demands a conscious effort on the part of the reader Self-correction is made possible when you read aloud.
2	Comprehension	Allows for interpreting key words in their own way If correct, helps to solve the problem.
3	Transformation	Many problems occur at this stage because of the variables. If done correctly, it is easy to solve the problem Choice of letter used as to represent the unknown
4	Process Skills	Shows correct use of operations and calculations
5	Encoding ability	Indicates the solution to the question Tells you if the answer is correct or not

Key Finding 4.5
Most of the teachers' reflections about the importance of the Newman strategies were directed towards obtaining a correct solution rather than ensuring understanding.

Activity Six: Misconceptions in Beginning Algebra (1)

The purpose of this activity was to engage teachers in meaningful learning of some common misconceptions about the concept of a variable. These are widely described in literature and were discussed in Chapter 2. The intention was also to help clear any possible misconceptions about the concept of a variable that might have been held by some of the teachers.

Common misconceptions about a variable that were discussed included:

- a letter is a word;
- a letter is an object/label;
- a letter has a fixed value from its alphabetical position;
- a standalone letter has a fixed value of 1;
- a letter is a fixed number;
- letters have place values; and

- different letters cannot have the same value (Booth, 1984; Küchemann, 1981; MacGregor & Stacey, 1993b; Perso, 1993)

To do this, the researcher made use of Ormond's (2011a) algebra slides which gives insights into students' perspectives about the concept of a variable, and misconceptions that may arise in presenting a particular activity (see Appendix 15).

Day Two

The four activities of the second day built upon the learning and collaborative experiences of day one. The purpose was to develop further insights into algebraic misconceptions, and the Newman procedure and other language-based approaches that might be used to teach word problems. Each of the activities is described in the next section.

Activity Seven: Misconceptions in Beginning Algebra (2)

The purpose of this activity was to engage the teachers in active whole group discussion in order to promote meaningful learning and reflection about why students may make mistakes when solving questions involving concepts of expressions and equations. Ormond's slides, which enabled teachers to put themselves in the 'shoes' of a student being introduced to algebra, and some word problems were used to introduce this activity. The common misconceptions about expressions and equations discussed were:

- the final answer must always be 'closed';
- an expression cannot be an answer;
- equations evolve from literal translation;
- equations evolve from literal visualization;
- equal sign means "I should sum up and write the total"; and
- equal sign means literal equivalence (Kieran, 1981; Ormond, 2011b; Sfard, 1991)

The activity provided an opportunity for the teachers to update their knowledge, and to address and assist one another to correct misconceptions of the letter as an object. It was remarkable that this activity generated a good deal of discussion, particularly when examining equations arising from ratio problems. The excerpt below provides an insight into the pattern of discussion, and the knowledge and beliefs of the teachers about the 'letter'.

> T13: Is there any time we can use a variable to represent an object?
>
> Researcher: Does anybody want to answer?
>
> T10: ... Since yesterday, I've been thinking of it. Using a variable to represent a quantity instead of an object...But for the students to

understand better in a simple way, like *cup* stands for *c* that's why we teachers like to use the letter.

T5: We should accept now that it is wrong.

T1: Like now, the word problem, let me say for example: I think of a number, eight is added and the result is twenty. Are you now telling me that the number I'm thinking of, that number I'm thinking of within me can be represented by any alphabet?

Many voices: Yes

Researcher: Is the number an object or quantity?

Many voices: It is a quantity

T1: That number can now be represented by any alphabet?

Many voices: Yes

T1: It is a quantity

T11: You do not say *n* is for number

T1: So it is not compulsory that it must be *n*?

T11: It is not compulsory

T12: It is now a quantity

T7: You can use any letter but there should be an indication somewhere to show let x represent... (Professional learning workshop, 30/3/2011)

The teachers also assisted each other to clarify issues about the concept of equations as we looked at ratio problems. An excerpt of a transcript given below reflects the learning that occurred.

T11: At this point, can we now conclude that in translating questions to algebraic form, can we say what I have on my left hand side must be equal to what I am going to have on my right hand side? Is it compulsory that it must be equal? Can we say what I have on my left hand side has to be exactly equal to what I have on my right hand side?

Researcher: Let us answer.

Many voices: Yes

T11: Is it in all cases the translation has to be balanced?

T3: Yes, it has to be. I want to use simultaneous equations as an example...the answer must be the same. That is the meaning of 'equal to'. If what you have on the left hand side is not equal to what is on the right hand side then the values are wrong...

T11: So in every translation, after solving it, if I don't have, for example, if two is here (raises left hand) and I don't have two here (raises right hand), it is wrong.

T4: As long as there is an ‘equal to’ there (with emphasis)

T9: Yes

T11: Equal to (with emphasis) (Professional learning workshop, 30/3/2011)

Key Finding 4.6 On the second day of the professional learning, many of the teachers were just becoming aware for the first time that a letter represents a quantity and not a label for an object. Misconceptions that some of the teachers had about equality, equations and the reversal error surfaced on the second day of professional learning and were addressed.

Activity Eight: Newman error analysis procedure

The purpose of this activity was to introduce the teachers to the Newman interview protocol and error analysis procedure as a language-based diagnostic tool to detect students’ difficulties and to provide necessary remediation (Newman, 1983b). The teachers discussed the procedure; however, the plan for the teachers to listen to one of the recorded interviews was aborted because of insufficient time. The procedure is given below.

Newman Interview and Procedure:

- It is to be carried out with individual students immediately after an incorrect first general attempt on the questions.
- A friendly reassuring atmosphere needs to be created before the student attempts the questions a second time.
- The Newman questions are then asked which helps to classify the error type that corresponds to where the student gets off track for each problem.
- Each Newman question corresponds to an error type

1. Please read the question to me. *Reading error*
2. Tell me, what the question is asking you to do? *Comprehension error*
3. Which method do you use to get your answer? *Transformation error*
4. Show me how you get your answer and talk aloud as you do it so I can understand how you are thinking. *Process error*
5. Write down your answer. *Encoding error* (Newman, 1983b, pp. 8-14)

The procedure in its designed form as an individualised interview would be difficult to implement in many Nigerian schools due to the student population and teacher workload. Part

of the research purpose was to examine the feasibility of the use of the Newman interview procedure in a more general way so as to engage more students while at the same time identifying their errors. A general discussion followed about how the interview procedure could be adapted for general use in the classroom while teaching algebra, and how it could serve as a potential means of identifying students' difficulties and misconceptions and addressing them. The adapted form was to be trialled by the teachers during the six weeks teaching period after the professional learning program.

In the adapted form, it was decided that instead of the usual pattern where one student answers the whole question, different students would be called to answer each of the Newman questions. Some of the public schools teachers observed that that they might sometimes have to read the questions and have the students read after them because some students have limited verbal abilities. It was acknowledged that the strategy would improve student attention and engagement, as well as reduce teacher-talk.

> T3: Some of them cannot read well.
>
> T8: They will read after you.
>
> T1: All of them will be attentive
>
> T4: You will be able to correct their mistakes easily
>
> T2: It will not only be the teacher talking all the time. (Professional learning workshop, 30/3/11)

Activity Nine: Examination of solutions to algebra questions

In this activity, the teachers examined two sets of incorrect algebra solutions. The aim of the activity was to allow the teachers to analyse and reflect on the solutions, discuss these in groups, and arrive at the correct answers and the likely misconceptions which gave rise to the incorrect solutions. Each group of teachers presented its findings while the other teachers commented on their report.

The first set of solutions comprised six of the answers the teachers provided to the two questions in Activity Two completed on the first day of professional learning. This solution set was not in the initial plan as the Researcher did not anticipate then that the majority of the teachers' responses would be wrong. Their responses prompted the Researcher to include the errors they had made. The teachers were to provide the correct answer to each question and the most obvious likely misconceptions for the incorrect solutions. Most of the teachers were able to come up with the correct answers, as Table 4.4 shows.

Table 4.4: Teachers' self-corrected responses to algebra questions (n=13)

Question	Provided solution	Response	Likely misconception	Error type
Write an equation using the variables *S* and *P* to represent the following statement: "There are six times as many students as professors at this university." Use *S* for the number of students and *P* for the number of professors.	$6p = s$	Correct		
	$6s = p$	Incorrect	Letter is an object, literal translation and visualisation, use of ratio as equation	Transformation
	$6ps$	Incorrect	Left –right reading, letter is an object, use of expression as equation	
	$6s + p$	Incorrect	Letter is an object, literal translation, Use of 'more' instead of 'times as many'	
Write an equation using the variables *C* and *S* to represent the following statement: "At Mindy's restaurant, for every four people who ordered cake, there are five people who ordered sandwich. " Let *C* represent the number of cakes and *S* represent the number of sandwiches ordered.	$4c = 5s$	Incorrect	Literal translation, letter is an object	Transformation
	$4c + 5s$	Incorrect	literal visualisation, use of 'and' in literal translation, letter as an object, use of expression as equation	
		Correct response $4s = 5c$		

These activities provided an avenue for the teachers to identify, and correct their mistakes, and to apply the knowledge and skills they had been developing in the professional learning.

The second set of incorrect solutions comprised written student answers for five of the initial algebra test questions. Krebs (2005) stated that such activities provide teachers with insights to students' understanding and opportunities for reflection. The teachers' reasons suggest they now had more awareness that students' mistakes might result from misconceptions or inadequate understanding the language of mathematics. The students' solutions and the teachers' classification of their likely misconceptions are presented in Table 4.5.

Table 4.5: Teachers' classification of students' likely misconceptions for given wrong answers (n=13)

Question	Incorrect response	Likely misconception
1. Write in algebra: there are twice as many pencils as biros (let *p* be the number of pencils and *b* the number of biros).	$b + 2$	Letter is an object, 'twice' to mean 'two'.
2. A basket costs eight naira and a bag costs c naira more than the basket. How much does the bag cost?	$8 + c = 8c$	Product/sum confusion or 'closing' the answer
3. Y x y x y =	y x y x $y = 3y$	Product/sum confusion
4. Sola has *x* bananas and Peju has *p* bananas. Peter counts the number of bananas each of them have and finds they are the same. Sola said you could write this as x=p, but Peju said that *x* and *p* are different letters and so cannot be the same. Who do you think is correct?	*x* and *p* are not the same, Peju is correct	Letter is an object
5. Write in algebra: There are three more caps than hats.	$3 + c$	Letter is an object, literal translation, no equation formed

The teachers' reflections provided them the opportunity to identify some misconceptions about variable, expressions and equation amongst their students.

Key Finding 4.7
During the professional learning, the teachers identified the cause of their initial error as transformation; they self-corrected their wrong use of a letter as a label and object, and the reversal error in the word problems. The teachers were also able to identify some of the likely misconceptions that led to the students' wrong answers.

Activity Ten: Language-based approaches

This general activity was intended to update and remind the teachers about language-based activities which might help to enrich their teaching. The importance of this activity stems from the fact that English language, which is a second language to both the students and teachers, is the language of communication in the classroom and in textbooks. In addition to this,

mathematics has its own language and students have to learn and be able to use this in order to develop literacy in the subject which is core to success in mathematics.

Simple explanation of mathematical language, the use of familiar words in questions and a conscious engagement of students in learning new vocabulary as it occurs in the class (which the teachers had mentioned earlier in Activity Four) were re-emphasized. Other approaches discussed were the teacher writing and leading students to pronounce new words; limiting the number of new words introduced in a lesson; allowing students' multiple use of a new word to ascertain understanding; encouraging correct use of vocabulary and language; and teacher revoicing (Kersaint et al., 2009; Setati, 2005).

Other strategies were reviewed. Mathematical discussions and literacy may also be enriched through engaging students in class discussions; suggestions of alternate ways of verbalizing algebraic expressions; expressing information with multiple representations; and identifying the key words and the mathematical operations to be used. Emphasis was to be on transformation once comprehension of the question was established. The teachers were also encouraged to make use of the pair – share – talk strategy, and the pause and wait time (Kersaint et al., 2009; Oviedo, 2005).

Summary of day one and day two activities

The professional learning intervention program aimed at updating teachers' knowledge of misconceptions about the concept of variable, expressions and equations. It also introduced teachers to the various language-based approaches, particularly the Newman procedure which is a way of finding out why and where students make language process errors in solving problems. The teachers were actively engaged in meaningful learning and reflections through interactive discussions, group work and assistance to each other. At the end of the two days, the teachers were able to recognise their own misconceptions and correct their errors. They could also identify some likely misconceptions and errors that their students might make.

Day Three – Evaluation of Intervention

The second phase of the professional learning intervention program was held after six weeks of teaching algebra back in their classrooms. During the six weeks, the Researcher provided support in terms of encouraging the four case study teachers and answering their questions which were mainly about class management. The purpose of the day three program was for the teachers to share experiences and offer suggestions on emerging issues concerning algebra

teaching and learning. The three main activities that took place were an evaluation of trialled approaches, a focus group interview, and discussions and further suggestions by the teachers.

Activity Eleven: Review of approaches used by the teachers

The teachers in groups of two or three were given a sheet of paper with questions asking them to review the approaches used to teach algebra within the intervention period. This reflective activity was intended to allow for an assessment and initial sharing of experiences in small groups. The analysis of their written review showed that all the teachers had trialled the Newman procedure in their classes and a majority (83%) of them reported that it worked well for them. A summary of the strategies attempted by the teachers is presented in Table 4.6.

Table 4.6: Teachers' use of language-based approaches (n=12)

	Strategies used by the teachers	Number of teachers (n=12)		
		Tried	Worked well	Difficult
1	Newman procedure in class	12	10	
2	Explain mathematical language simply	9	7	
3	Engage students in class discussions	8	8	2
4	Use familiar words in questions	7	7	
5	Use of revoicing	7	7	
6	Engage students to suggest alternate ways of verbalizing algebraic expressions	7	7	5
7	Encourage correct use of vocabulary and language	5	5	
8	Write and lead students to pronounce potentially difficult new words	5	5	2
9	Express information with multiple representations	5	3	
10	Limit the number of new words introduced in a lesson	3	3	2
11	Conscious engagement of students in learning new vocabulary as it occurs in the class	3	3	2
12	Allow for students' multiple use of new words	2	2	3
13	Pair-share-talk			7
14	Frequent pausing and use of wait time			7

Note. Responses of 'difficult' were not tried by the specified number of teachers

A majority of the teachers also reported that the use of familiar words and simple language in teaching worked well for them. The teachers gave reasons for the approaches with which they had successes. The students' conceptual understanding, classroom engagement, creativity and success in problem solving were reportedly aided by the approaches. Approaches that were effective for five or more of the teachers and the reasons given are presented in Table 4.7.

Table 4.7: Strategies teachers found effective and the reasons offered (n=12)

What worked well for you?	Why
Newman procedure in class	Shows students' level of understanding of the question Allows them to learn easily The students are familiar with questions Allows the students to point out mistakes Useful for other mathematics topics
Explain mathematical language simply	Enables better success in problem solving Helps students acknowledge procedures Aids understanding of developing concept
Engage students in class discussions	It helps individual participation, Allows for new ideas
Use familiar words in questions	Aids understanding of the concept Allows easy understanding and retention Helps students to focus on the particular question
Use of revoicing	It aids retention, It enables understanding of the concept
Engage students to suggest alternate ways of verbalizing algebraic expressions	Allows for creative thinking Sometimes students provide solutions not thought of by the teacher Helps most especially those that can express themselves to ascertain their understanding
Encourage correct use of vocabulary and language	Helps in understanding concepts and questions Helps students to express themselves better
Write and lead students to pronounce potentially difficult new words	It helps recall, aids concentration, Helps mastery of pronunciation

The approaches were reported to help the students' development of the language of mathematics and their ability to use it in mathematical discussions.

Key Finding 4.8
The majority (83%) of the teachers reported they had success with the use of the Newman procedure; many also had success with the other language-based approaches. The teachers reported that the approaches facilitated students' use of the language of mathematics, their conceptual understanding, class participation and problem-solving ability.

Some teachers had difficulties in implementing some of the approaches such as group work and wait time because of the class size and their students' verbal ability. The approaches that five or more of the teachers found to be difficult and reasons given for those difficulties are presented in Table 4.8.

Table 4.8: Strategies teachers found difficult to implement and the reasons (n=12)

What was difficult?	Why?
Pair-share-talk	Overcrowded class, time consuming
Frequent pausing and use of wait time	It distracts their attention, makes the class dull, students give different meaning (something is wrong)
Engage students to suggest alternate ways of verbalizing algebraic expressions	Creates confusion, limited ability to express themselves

Key Finding 4.9
Sixty per cent of the teachers mentioned it was difficult for them to use group work and the pause and wait time approaches.

Focus Group Interviews

The focus group interview was conducted with the teachers in two separate groups. The initial plan was to employ one group but the teachers were not all available when the interview was to start, so the Researcher decided to interview them in two separate groups. The responses to the three questions asked in each group were transcribed and coded, and common categories were identified. There were also some responses peculiar to the different groups. The next section reports the themes that emerged from the data.

Difficulties students encountered solving word problems

The main difficulties mentioned by both groups were students' understanding of English and mathematical language, transforming of the question and their knowledge of mathematics. Other difficulties are grouped under 'Support System'. A summary of the teachers' beliefs about each difficulty and some of their comments in each category follows.

English language

Teachers said that some students may not understand what they read or may encounter unfamiliar words. Many teachers noted that the students often gain understanding if such words are said in the student's first language. However, teachers whose first language differed

from the one of the local community advocated for more use of familiar words, diagrams and other alternatives.

> T6: The first thing I feel the student have is the language barrier. ..Ask them to read it, they may read but they may not understand. They read just literally. They don't know the meaning...
>
> T8 ..they will be looking at you that *what is carton?*...
>
> T7: I just want to say there is nothing like the mother tongue. ...I can remember in those days, difficult words, they will use Yoruba [a first language] to explain to us...
>
> T12: If the students know...always English ..they will sit tight...if they don't understand, you use other things and ways... it's not until you bring in the vernacular...
>
> T6: When you stand in front of 150 students, you understand, and you are talking and you are sweating (emphasis). You have bitterness in your mind because the response of the students, you cannot compare (emphasis) with the level of labour you are putting. By the time you discover you need to make some sense and just say a few words in vernacular, the students get it.... (Focus group, 6/7/2011)

Mathematical language

The teachers explained that the difficulty with the mathematical language arises because at times it is unfamiliar to the students and differs from that used at the primary level. Some students also find it difficult to understand mathematical meanings of every day English words.

> T6: For instance, when you say 'a certain number is added to two and the result is divided by five'. The student is confused because number one, that *result,* they don't know what *result* means. They may understand *added* but…may relate *result* to *equal to*....but the question says the *result is*. That's where the confusion comes.
>
> T8: In a word problem 'I think of a number and the number is multiplied by two and the answer is now doubled'. Say for example, the students don't understand the word *double*. What they are familiar with is two.....they will be looking at you *what do you mean by double?*
>
> T3: So when they now come into the secondary school, that one will expose them. They start asking 'aunty, what do you mean by twice? Though it is an English word but because they don't hear it often, so they find it so, so difficult. (Focus group, 6/7/2011)

Transformation

The ability to process mathematics with variables was identified as difficult. Some said the identification of necessary keywords was the problem.

T6: The interpretation of the word problem in mathematical terms is difficult for the students.

T12: ...They still want to ask you ‘aunty, why should we bring letters into mathematics? It’s supposed to be numbers like one, two, three, four, why letters?’ ...you start telling them that since you don’t know the number, let the unknown number be the letter. That’s where the problem lies. Most of them will see the question and know the answer but they will not be able to write it down. Their problem is to interpret and write in mathematical form. (Focus group, 6/7/2011)

Knowledge of mathematics

The teachers believed that some students have a weak primary schooling background in mathematics. Students are not taught negative numbers at the primary level and the use of non-specialist teachers all cause difficulties for the child, they claimed.

T5: I think their problem is from the primary school, because their foundation is somehow weak.

T1: In primary schools in public schools, the primary teachers, there are some things they cannot teach.

T10: In some private schools also

T8: ...algebra has started from the primary school but you may not frame it as algebra form... they taught them two plus box is ten...at junior level...you now move away from the box...letter representing a value...they must have the fundamental root of word problems at the primary level.

T7: …if we say subtract seven from five or subtract seven from ten, the students will be looking at you. (Focus group, 6/7/2011)

Support system

This theme was strongly stressed in both groups and the teachers were passionate over the issues. They believed that there may be a lack of parental support in terms of encouragement, while some students were known to miss levels at the primary school. Student’s lack of interest and the teacher’s pedagogy posed impediments to the students’ word problem solving capabilities.

T3: And some of them, it is due to their parents. Some parents tell their children that ‘I did not know maths’, so telling the students that, some of the students just say ‘if don’t know maths it is not a crime. I inherit it from my parents’. Lack of interest is part of it.

T5: There is another problem at JS level. Some, they will move from primary four to JS one without doing primary five and six. In my school, we have nine years in JS one.

T10: A parent...wants the child to sit for common entrance we said no, the work the child has to know in primary five and six, if the child is not doing well in it, how does he/her want to cope in JS 1? The parent withdrew the child from the school. When they saw the child cannot cope, they had to bring him back from that school.

T1: The problem is that the students themselves are not ready to learn and they are not helping the teachers...when you go through their notes,. all those things I've been writing on the board..they did not put down anything... I was going through the class work..half of the class did not do the class work.

T10: No writing materials,..they loan from the teachers.

T6: ..majority of teachers, they use *x* as unknown number....even the textbook when it says a number, it is *x*. ....we concentrate on *x* and *y*. That's why if you use another thing they think it is another form....

T13: Some teachers, in algebra when they want to teach word problems and they cannot interpret the question themselves, they will not bother to use it as an example. They will only select the ones they know they can interpret themselves and that is what they will limit the students to. (Focus group, 6/7/2011)

Key Finding 4.10
Teachers believed that students have difficulties with understanding English and mathematical language, transformation of the question and knowledge of mathematics. They also believed that many of the students are very young and may need more support from parents and teachers.

Algebraic misconceptions

The teachers noticed many of the misconceptions about the variable, expressions and equations that were discussed in the first phase of the professional learning. They included the use of a letter as a fixed number, letters having an alphabetical position; using letters as words, objects and labels; confusing addition and multiplication (product/sum confusion), and gathering up of all terms. To remediate this, most of the teachers said they used overt explanation to the students.

T8: I noticed the use of x as a fixed number. I like using x in solving problems. When solving....*x* is equal to three, then you solve another....*x* is equal to five. But they will ask you this' but aunty, *x* is equal to three before, why is it five now'? I told them that *x* is just an alphabet, it can be any number, that it is not fixed, it is not fixed...because it is not the same question.

> T7: The first thing that most of them normally have, they have, what we call using fixed numbers for alphabets. Some of them, they believe that '*a*' is always one and '*b*' is two. So when you say *z* plus *y*, they believe that *z* will be 26 and *y* will be 25. So by the time you say $a + b = 10$ and you say '*a*' is equal to one, what is b? Some of them will say *b* is equal to two. They will not even reason that you just said *a* plus *b* is equal to ten. They just have the misconception that an alphabet has a fixed number.
>
> T5: ...i asked them to collect like terms, some added together. I tried to explain to them…There was a question whose final answer ends at $2x + 2y$. I now went further to say this will give $2(x + y)$. This student quickly raised up his hand that it is wrong. I said why. He said, how can two cats plus two elephants be two (cats and elephants)? I now told him that here we are not dealing with cats or elephants. We are dealing with algebra, common terms. What is common? Two is common. What variable is left over? *X* and *y* so accept ...He said now, if that is the case, why can't it be $4xy$?
>
> T4: So you will explain and re-explain, re-explain.
>
> T5: Yes. The student went further under which condition will the answer be $4xy$? I now said if this, plus, is not plus but times, it will give you $4xy$.
>
> T8: I saw that the students...difficult to add and multiply...they don't know ...difference between product and sum... Instead of $2x$ they will now write *x* and *x*, so when it comes to multiplication *x* times *x* they will now write $2x$. I tried to explain the difference ...
>
> T11: Letters like *V* for volume and others...discourage them that it is more than that...with questions ...it will not work, you can't solve it. You can tell them to choose any letter...go as far as Greek alphabets. It's just a choice of letter. (Focus group, 6/7/2011)

Key concepts in algebra

The teachers affirmed that the concepts of variable, expression and equality are basic, and fundamental to further work not only in algebra but almost all other aspects of mathematics. They also mentioned that the roots of these concepts exist informally at the primary level and relate to everyday living, because they are used to express situations through arithmetic word problems which are solved by the students.

> T7: I see algebra as the basics. That's why it is taught immediately in JS 1. Take for instance...perimeter, ...area,...circumference...Most of the topics later that they will be taught is based on algebra that's why they are taught at the initial stage. If not, they will have difficulties with other topics. That's how I see it. It will cause problem once they are not well grounded in algebra.
>
> T1: They learn from simple to complex. Once the needed basic thing is introduced to them in JS 1, as they are growing, they will be getting used to the system.
>
> T11: I discovered that it is only algebra that we can use to explain all these word problem questions that we have. ..the use of letters in place of

numbers, something we don't know it is algebra that will explain it better.... Using something to represent, we have started representing it in algebra...by the time we say this letter equals that, it has already turned to equation. So there is no way we cannot use it. It is the best approach that can explain this worded problem questions that we are discussing about.

T8: ....algebra has started from the primary school but you may not frame it as algebra form... they taught them two plus box is ten...at junior level they move away from the box...letter representing a value.

T6: Algebra goes along with life activities. That's why the teaching of it ... immediately they enter school, you start introducing the students to it so they can carry it through life. (Focus group, 6/7/2011)

Key Finding 4.11
The teachers noticed amongst their students the presence of many of the misconceptions we had discussed during the first phase of the professional learning program and attempted to remediate by explaining to the students. The teachers believe that the concepts of variable, expression and equality are fundamental to further work in algebra word problems and to all mathematics.

Suggestions for improving the teaching and learning of algebra

The teachers offered many suggestions about the teaching and learning of algebra, and mathematics in general. Suggestions were given concerning early algebra teaching, the content of JS 1 algebra, teachers' pedagogy and commitment, classroom size, and students' learning readiness. Some of the teachers' comments are stated below.

Early algebra teaching

It was suggested that algebra should be introduced to the students at the primary school. The teachers believe that this would reduce the pressure they face when teaching JS 1 students.

T1: …elementary algebra should be done in the primary school, so that it will make the work of the secondary teachers easy…

T4: …we have to start it from the grass root; the primary level is where we start algebra from….(Focus group, 6/7/2011)

The teachers realised that pre-algebraic ideas was a link to beginning algebra and that knowledge of it would enhance students' understanding of beginning algebra.

Algebra content

All the teachers observed that the JS 1 algebra content was heavy and not commensurate with the number of weeks allocated to teach it. As a result, some topics were always neglected or not taught satisfactorily. They suggested a shift of some of the content to JS 2 which they

noted had only algebraic fractions. Some of the teachers called for a review of textbooks seen themselves to promote some of the misconceptions discussed.

> T12: Look at the scheme of work…broken down ...have five weeks…teacher will have more time to explain in details…
>
> T1: The number of weeks should be increased.
>
> T11: …syllabus should be reduced…
>
> T3: …some topics should move to JS 2…most of what they do in JS2 is now the fraction part. ...most of the rest is now in JS 1.
>
> T4: There is a need to contact the publishers like….to change from the fruit salad approach explanation…that's the common explanation. (Focus group, 6/7/2011)

Teacher qualification, commitment and pedagogy

Suggestions were made for teaching to be more student-centred through scaffolding concepts and encouraging more word problems. They suggested that teachers should continually use familiar words, encourage the use of the dictionary and the Newman strategy to teach mathematics. There were calls for teachers to network with others and to develop themselves professionally. It was suggested that specialist teachers should teach mathematics both at the primary and secondary levels.

> T4:..should be student-centred…take time to explain…
>
> T5: ...they should be able to derive their own method of learning, discover things by themselves, not the teacher giving...adopt various methods of teaching…
>
> T11: …topics for the term come in form of worded whatever, it will assist us, we will get used to it as time goes on…
>
> T8: …we should try as much as possible to do more research ourselves, we should not restrict our knowledge to terms of what is in the textbooks….develop ourselves personally….little solution we can bring ….problem will be eased, we should not wait until the problem is over…..should register and be coming for meetings…(Focus group, 6/7/2011)

Classroom

The teachers asked that the class size should be reduced, so as to result in a classroom spacious enough to allow a teacher to move around the room and monitor students' work.

> T6: …class should be spacious to move for the teacher to help monitor individual activities ….quickly correct students…
>
> T5: ….teacher-student ratio should be reduced…

T11: ..number of students should be reduced so that it will help the teacher to monitor them closely. (Focus group, 6/7/2011)

Students' learning readiness

The teachers expressed concern that parental and societal pressure leads to 'class skipping of many children and that students' entry age impacted their performance. They suggested that the government enforces the official JS 1 entry age of 12 years.

T6: It is only the government that can monitor it. (Focus group, 6/7/2011)

Key Finding 4.12
The teachers recommended early algebra teaching at the primary level, a reduced JS 1 algebra content, improved teacher pedagogy, more spacious classrooms, smaller class sizes and enforcement of the official 12 years entry age would help improve the teaching and learning of algebra.

Summary

The professional learning intervention was a three-day program focused on updating the JS 1 teachers' knowledge about common algebra misconceptions and language-based approaches particularly the Newman procedure. The program ran in two phases and by engaging in 13 activities the teachers had opportunities to collaborate, examine, reflect, discuss and engage with one another in meaningful learning.

The program revealed that many teachers themselves appeared to have misconceptions about the letter as a label or an object and that they were not aware that this was a misconception. The misconceptions about equality and the reversal error were also present, and were addressed during the program. Given responses also suggested that the teachers were unaware of many of the algebraic misconceptions before the program began. Examination of students' solutions and their own solutions to word problems provided the opportunity for the teachers to identify their mistakes, to correct them, and to identify students' likely misconceptions. The teachers stated that many teachers do not like teaching word problems because they were unsure of themselves.

The meaningful learning of Newman's language-based error analysis procedure and discussion about other language-based approaches provided an avenue for the teachers to highlight the benefits of using the first language to teach. All of the teachers reported that they used the Newman strategy in their classes with a majority (83%) stating that it was a success. The use of familiar words in teaching was also successful, but the use of group work and pause and wait time was difficult to achieve, mainly due to class size and time constraints.

An analysis of a general survey conducted prior to the professional learning provides data about the existing teaching-learning situation described by the PL teachers. This and the impact of the PL on the teachers are discussed in the next chapter.

CHAPTER FIVE: TEACHER PERCEPTIONS

Introduction

This chapter reports the analysis of data obtained from teacher participants who were involved in the general survey or in the professional learning intervention program. The information is presented in two sections, namely: general data on the teaching and learning of mathematics and algebra; and data concerning the teachers in the professional learning intervention program. The chapter concludes with a summary of the key findings.

The Teaching and Learning of Mathematics and Algebra

The main purpose of the initial questionnaire was to obtain general information from teachers about mathematics teaching and learning at the junior secondary school level. The information relates to: demographics of the teacher sample; teachers' challenges and beliefs about effective mathematics teaching and learning; teachers' confidence about their knowledge of mathematics and algebra teaching strategies; frequency in the teachers' use of various teaching strategies; and, the teachers' assessment of the difficulty level of some algebra questions.

Demographic data

An educational district in Lagos State, Nigeria, known as District 5 was selected for the study and research participants were drawn from two zones of the five zones within this district. Thirty Junior Secondary (JS) 1 teachers from 30 schools were given questionnaires which they completed and returned giving a 100% return rate. The analysis of the demographic data which follows includes gender, zone of school, age, mathematics and JS 1 teaching experience, qualifications, classes taught and trainings attended.

Most (70%) of the teachers were between 21 and 45 years of age and the number of male respondents (57%) was slightly more than the females (43%). In relation to mathematics teaching, almost all (93.3%) the teachers had between two and 15 years of experience. However, the majority (77%) of teachers had between two and five years of JS 1 teaching experience. Table 5.1 and 5.2 present these data.

Table 5.1: Teachers' by gender, zone and age group (n=30)

		Number	Per cent
Gender	Male	17	56.7
	Female	13	43.3
School zone	Badagry	13	43.3
	Ojo	17	56.7
Age group (years)	20 or less	0	0.0
	21 to 25	5	16.7
	26 to 30	4	13.3
	31 to 35	8	26.7
	36 to 40	3	10.0
	41 to 45	6	20.0
	46 to 50	1	3.3
	Above 50	3	10.0

Table 5.2: Teachers' years of Mathematics and JS 1 teaching experience (n=30)

Years of experience	**Number (per cent) of teachers**	
	Mathematics teaching	JS 1 teaching
0 – 1	0 (0.0)	0 (0.0)
2 – 5	15 (50.0)	23 (76.7)
6 – 10	6 (20.0)	7 (23.3)
11 – 15	7 (23.3)	0 (0.0)
16 – 20	2 (6.7)	0 (0.0)
Above 20	0 (0.0)	0 (0.0)

None of the teachers had more than 10 years' experience of teaching JS 1. Half of the teachers had between two and five years' experience of teaching mathematics while no-one had less than two years or more than 20 years of experience.

Key Finding 5.1
Most (87%) of the 30 teachers were between 21 and 45 years of age and there were more males (57%) than females. Most (77%) of the teachers had between two and five years of JS 1 mathematics teaching experience.

All the teachers had undergone a three or four year training to obtain a professional mathematics teaching qualification. The National Certificate in Education (NCE), obtained after

a three year post-secondary training, is the minimum professional qualification for teaching in Nigeria. The certificate holders often have two subject specialisations and teach both primary and junior secondary school students. Half the teachers had only the NCE qualification and 11 of the remaining 15 teachers who were first degree holders also had the NCE qualification. These data are presented in Table 5.3.

Table 5.3: Teachers' qualification (n=30)

Qualification	Number	Per cent
NCE only	15	50.0
B.Sc(Ed) only	4	13.3
NCE and B.Sc(Ed)	7	23.3
NCE and B.(Ed)	2	6.7
NCE and B.Sc	2	6.7
PGDE, M.Ed, M .Sc	0	0.0

All the teachers had a first specialization in mathematics. Some of the teachers also had a second specialization and the most common of these were economics (12), integrated science (8) and physics (6). One of the teachers had a master's degree in Business Administration.

Half of the teachers reported that they had attended one or two trainings in the past two years while nine of them had not attended any. Only one teacher had attended four trainings and the remaining five teachers had attended three trainings in the past two years.

Since the teaching followed the secondary model, most teachers taught more than one class. The teachers reported on the number of JS 1 classes they taught and the number of students in each class. Almost a third (31%) of the teachers had 40 or fewer students in each class. The classes tended to be smaller in the fee paying schools, and many of the teachers taught fewer classes than in government schools. Forty per cent of the teachers taught one or two classes. However, those who reportedly taught between four and six classes were all from public schools. The official JS 1 class size is 40 (Federal Republic of Nigeria, 2004).

The graph shows a bi-modal distribution where a large group of classes, mainly from the 11 private schools, were within the regulation size and another larger group, from the 19 public schools, that were very much above the number that the policy prescribes. This information is presented in Figure 5.1.

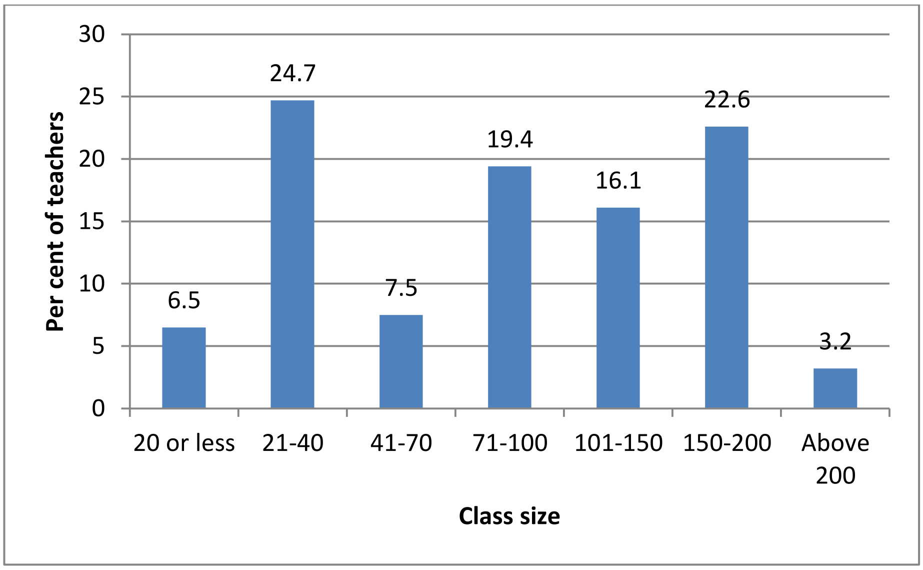


Figure 5.1: Class sizes of teachers (n=30)

Key Finding 5.2
All the teachers were professionally qualified to teach mathematics and 70% of them indicated that they had attended one or more mathematics training programs within the past two years. About two-thirds of the teachers reported they had class sizes between 41 and 200. Sixty per cent of the teachers indicated that they taught between three and six classes of JS 1.

Challenges and beliefs about effective mathematics teaching

Teachers responded to three open ended questions regarding the challenges faced and their beliefs about effective teaching of mathematics. For each question, the responses were sorted into categories and coded before further analysis was done. The questions and the teachers' responses are reported next.

The first question was "What challenges do you face in teaching upper basic mathematics effectively?" The most common challenges mentioned were lack of instructional materials, weak students' prior mathematical knowledge, inadequate facilities and large class sizes. A few stated that they had challenges with students' inability to interpret word problems, young entry age and lack of interest in mathematics. Many of the challenges mentioned have also been identified by other Nigerian science and mathematics teachers (Igbokwe, 2000; Olaleye, 2012). The responses are summarised in Table 5.4.

Table 5.4: Challenges faced by teachers in the effective teaching of mathematics (n=30)

Description of challenge	Number of responses [a]	Per cent
Lack of instructional materials	10	17.2
Students' weak prior mathematical knowledge	8	13.8
Inadequate facilities	7	12.1
Large class population	5	8.6
Students' inability to interpret word problems	4	6.9
Students' young entry age	4	6.9
Students' lack of interest in mathematics	4	6.9
Insufficient number of teachers	3	5.2
Assessment of students' learning	3	5.2
Lack of stationeries and textbooks	3	5.2
Insufficient teaching time	3	5.2
Students' fear of mathematics	2	3.4
Knowledge of appropriate teaching method	2	3.4

Note. [a] Most teachers gave more than one response to the question

Key Finding 5.3
The most common challenges to effective mathematics teaching reported by teachers were inadequate instructional materials and facilities, students' weak prior mathematics knowledge and large class sizes. A few teachers mentioned students' young entry age, their lack of interest and difficulty in interpreting word problems.

The second and third questions were "What do you believe are the characteristics of effective upper basic mathematics teaching?" and "What do you believe are the most effective teaching strategies that may help students learning mathematics?" Many teachers mentioned the use of adequate instructional materials, different teaching methods, effective class management and satisfactory organisation skills, but a third of them also believed problem solving, student activity and learner engagement were important. The responses are presented in Table 5.5.

Table 5.5: Teachers' responses about effective mathematics teaching and learning (n=30)

Number and per cent of teachers[a]					
Characteristics of teaching			**Strategies for learning**		
Instructional materials usage	15	28.3	Use of instructional materials	13	23.6
Teacher understanding of students' learning needs	10	18.9	Using practical and demonstration	8	14.5
Good class management	6	11.3	Student participation	8	14.5
Good communication skills	4	7.5	Problem solving	5	9.1
Learner engagement	1	1.9	Group work	1	1.8
Others	17	32.1	Others	20	36.5

Note. [a] Most teachers gave more than one response to the questions

The responses were mainly a reflection of teachers' solutions to the challenges of teaching resources, class size and student-related issues the teachers had identified and were stated in

Key Finding 5.3. Very few teachers reported the need to possess good communication skills or utilise problem solving strategies, and only one person mentioned group work as an effective mathematics teaching strategy. The remaining responses were grouped as 'others' because they varied in scope, with relatively few responses.

Key Finding 5.4
Very few (10%) teachers mentioned characteristics or strategies like communication skills and learner engagement or problem solving and group work. Many characteristics and strategies believed by a higher number of teachers to bring about effective mathematics teaching appeared to be solutions to previously identified challenges in Key Finding 5.3.

Teachers' confidence with mathematics teaching

Two scales, knowledge of mathematics and strategies for teaching algebra, were employed to describe the teachers' confidence. The majority (83%) of the teachers were very confident or confident about their knowledge of all aspects of mathematics taught at the junior secondary level, with confidence with algebra having the highest rating and geometry and mensuration having the lowest. This pattern of teachers' high opinions of self-competence has been established to be common in many studies before an intervention occurs, and it is believed to reflect a lack of awareness of what constitutes competence or other needed skills/pedagogy. Their responses are presented in Table 5.6.

Table 5.6: Teachers' responses about confidence with mathematical knowledge (n=30)

	Per cent of teachers					
Aspect	Very confident	Confident	Okay	Limited confidence	Not confident	Mean rating/5
Algebraic processes	76.7	23.3	0.0	0.0	0.0	4.77
Basic operations	73.3	26.7	0.0	0.0	0.0	4.73
Every day statistics	70.0	23.3	6.7	0.0	0.0	4.63
Number and numeration	66.7	30.0	3.3	0.0	0.0	4.63
Geometry and mensuration	30.0	53.3	13.3	3.3	0.0	4.10

Note. Confidence was scored on a 5 point scale Very confident=5; Confident=4; Okay=3; Limited confidence=2; Not confident=1.

Key Finding 5.5
The majority of the teachers indicated confidence in teaching all aspects of junior secondary mathematics and the highest mean rating of confidence was for algebraic processes (4.77/5).

With the ability to use some specific strategies to teach algebra, most teachers were confident in their use of traditional strategies but less confident with strategies involving student collaboration and discussion. Their responses are presented in Table 5.7.

Table 5.7: Teachers' responses about confidence in using strategies to teach algebra (n=30)

Strategies	Per cent of teachers					
	Very confident	Confident	Okay	Limited confidence	No confidence	Mean rating /5
Explaining algebra concepts	40.0	50.0	10.0	0.0	0.0	4.30
Engaging students' interest in algebra	40.0	46.7	13.3	0.0	0.0	4.27
Assessing children's learning in algebra	30.0	53.3	16.7	0.0	0.0	4.13
Managing discussions and interpretations of word problems	33.3	53.3	6.7	6.7	0.0	4.13
Developing vocabulary and terms needed for learning algebra	20.0	56.7	23.3	0. 0	0.0	3.97
Involving majority of the students in class discussions/activities	33.3	36.7	16.7	10.0	3.3	3.87
Using knowledge of students' misconceptions to plan algebra lessons	23.3	40.0	26.7	6.7	3.3	3.73
Managing group activities in algebra	20.0	0.0	26.7	10.0	3.3	3.63

Note. Confidence was scored on a 5 point scale Very confident=5; Confident=4; Okay=3; Limited confidence=2; Not confident=1

Traditional approaches like explaining and assessing learning had high ratings with more than 83% of the teachers indicating they were very confident or confident in using the strategies. Discussion and collaborative approaches like involving the students in discussion or managing group activities in algebra had lower mean ratings. Although many teachers indicated that were very confident or confident about their ability to use the knowledge of students' misconceptions to plan lessons, this item had the second lowest mean rating (3.73/5).

Key Finding 5.6
Teachers indicated least confidence in the use of class discussions, group work and knowledge of students' misconceptions in algebra lesson planning. Traditional approaches had higher mean ratings although most teachers were confident in engaging students.

Teachers' classroom strategies

Three scales were employed to describe the teachers' classroom strategies. Teachers rated how they managed classroom talk, their use of some approaches and their students' level of engagement in the classroom. Their responses on each of the scales are presented below.

Almost all of the teachers indicated that they were effective in using questions and discussions in the mathematics classroom. The responses are presented in Table 5.8.

Table 5.8: Teachers' responses to ways of managing talk in the mathematics classroom (n=30)

Strategies	Per cent of teachers: Strongly Disagree	Disagree	Not sure	Agree	Strongly Agree	Mean rating/5
I am effective in encouraging and supporting students to ask questions in my mathematics class.	3.3	0.0	0.0	30.0	66.7	4.57
I am able to respond to students answers in ways that help develop an effective discussion of mathematical ideas.	3.3	0.0	0.0	46.7	50.0	4.47
My rich knowledge of mathematics helps me respond appropriately to students' answers to my questions.	0.0	3.3	0.0	46.7	50.0	4.43
I am effective in asking questions to suit the purpose and flow of classroom discussions in mathematics.	3.3	0.0	0.0	46.7	50.0	4.40
My rich knowledge of mathematics helps me ask the right questions to develop mathematics ideas through discussion.	3.3	0.0	3.3	43.3	50.0	4.37
I am effective in engaging most students in responding to my questions during mathematics discussions.	0.0	10.0	0.0	46.7	43.3	4.23
I am effective in establishing a classroom atmosphere in which most students feel confident to give their own answers to questions.	3.3	3.3	3.3	50.0	40.0	4.20
I am effective in using questioning to identify students' prior knowledge of mathematics topics.	0.0	6.7	3.3	53.3	36.7	4.20
I am normally able to respond to students' answers in ways that maintain and promote further discussion of the mathematics ideas.	0.0	0.0	6.7	70.0	23.3	4.17
I am able to sustain discussions so that we thoroughly discuss the mathematics ideas.	0.0	0.0	6.7	73.3	20.0	4.13

Note. Confidence was scored on a 5 point scale Strongly agree=5; Agree=4; Not sure=3; Disagree=2; Strongly disagree=1

Very few(10%) of the teachers indicated that they were not effective in engaging students in responding to their questions, establishing a classroom atmosphere that makes students confident to give their own answers, or using questions to identify students' prior knowledge. These high ratings appeared to be subject to the teachers' own context as they are also at variance with their reported confidence to use classroom discussions (Key Finding 5.6).

Key Finding 5.7
Almost all of the teachers indicated they were effective in the use of questions and discussion in the mathematics classroom. However, lower mean ratings were obtained for some statements about sustaining discussions.

The teachers also indicated how frequently they used specific approaches in teaching algebra. While claiming high self-efficacy for using discussion, the teachers' ratings on approaches frequently used did not appear to privilege discussion over instruction as an approach to teaching algebra. The responses are presented in Table 5.9.

Table 5.9: Frequency of teachers' use of various teaching approaches (n=30)

Approaches	Per cent of teachers					
	Every lesson	In most lessons	In some lessons	In a few lessons	Never	Mean rating/5
Working examples on the board for students to copy	66.7	16.7	13.3	0.0	3.3	4.43
Writing notes on the board for students to copy	46.7	16.7	13.3	16.7	6.7	3.80
Explaining the meaning of equations	46.7	13.3	33.3	6.7	0.0	4.00
Asking students to identify what the question asks us to do	43.3	33.3	16.7	3.3	3.3	4.10
Asking students to identify key words and symbols in the question	43.3	26.7	16.7	13.3	0.0	4.00
Having students solving questions individually	40.0	33.3	23.3	3.3	0.0	4.10
Using different types of mathematical representations	30.0	30.0	36.7	3.3	0.0	3.87
Reminding students about the meaning of a variable	30.0	30.0	33.3	6.7	0.0	3.83
Asking students identify the plan for solving the question	26.7	36.7	30.0	6.7	0.0	3.83
Inviting students to explain the working for their answer	26.7	23.3	36.7	13.3	0.0	3.63
Having students reading aloud the question to be solved	26.7	20.0	23.3	23.3	6.7	3.37
Whole class discussion of mathematical ideas	20.0	33.3	33.3	10.0	3.3	3.57
Identifying students' misconceptions of algebra	16.7	36.7	36.7	10.0	0.0	3.60
Grouping/pairing students to solve questions in the class	0.0	23.3	50.0	20.0	6.7	2.90

The frequencies in the use of approaches seemed to be at two ends of a spectrum in which mainly teacher-directed strategies are at the higher end while the student–focused ones are at the lower end. The more frequently used approaches having higher mean ratings involve direct instruction in contrast to the less used approaches which had low mean frequency ratings. This confirms Key Finding 5.6 that traditional approaches had higher mean ratings.

The teachers also rated, on a scale of one to 10, the level to which their students were actively engaged in the classroom during lessons. More than half of the teachers rated this as seven or more. The responses are presented in Table 5.10.

Table 5.10: Teachers' rating of the level of students' engagement in the classroom (n=30)

	Very passive							Very active			Mean rating/10
Engagement level	1	2	3	4	5	6	7	8	9	10	6.67
Per cent of teachers	0.0	0.0	0.0	10.0	16.7	20.0	20.0	20.0	10.0	3.3	

It should be noted that the teachers' ratings of student engagement would be in the context of Nigerian mathematics teaching and the culture of Nigerian classrooms.

Key finding 5.8
The more teacher-directed approaches were reportedly used more regularly while approaches that elicit mathematical discussions were used less often. About half (53%) of the teachers rated their students' class engagement level to be more than six out of 10 and the overall mean rating was 6.67.

Teacher beliefs about mathematics, their teaching and students' learning

Teachers were requested to respond to statements about their beliefs regarding mathematics, their teaching, and their students' learning. Some of the statements were negatively posed (for example, 'not all students can learn mathematics') which means that agreement indicated they were generally less supportive of the study's theoretical framework of constructivism.

While most of the teachers indicated that they enjoyed teaching mathematics, many strongly believed mathematics is largely procedural, but that students have a weak mathematics background. Most of the teachers believed that mathematics, and algebra in particular, has rules that need to be learnt. More than half of the teachers reported that they used the native language at times to explain concepts, and that teaching is essentially about the teacher talking. Almost equal proportions of teachers found word problems easier to teach than symbolic questions and vice versa. Many of the teachers also believed that all students could learn mathematics. Their responses are presented in Table 5.11.

Table 5.11: Teachers' beliefs about mathematics, their teaching and students' learning (n=30)

	Per cent of teachers				
	Strongly disagree	Disagree	Not sure	Agree	Strongly agree
Nature of mathematics					
Mathematics consists of rules and procedures.[a]	3.3	0	3.3	40.0	53.3
Mathematics is mainly calculations.[a]	10.0	40.0	0	33.3	16.7
Teaching mathematics					
Teaching enjoyment					
I always enjoy my mathematics teaching.	3.3	3.3	0	40.0	53.3
If I was free to choose, I would not teach mathematics.[a]	53.3	40.0	0	3.3	3.3
Teaching practices					
If a teacher does not tell students how to solve questions, they will make mistakes.[a]	3.3	23.3	3.3	46.7	23.3
I sometimes use the native language to explain mathematical ideas in the class.	20.0	20.0	3.3	36.7	20.0
Teaching word problem					
I find it easier to teach algebraic word problems than those with symbolic notations.	20.0	33.3	3.3	36.7	6.7
Learning mathematics					
Not all students can learn mathematics.[a]	43.3	30.0	3.3	13.3	10.0
Students' mathematics background is often weak.	3.3	0	0	60.0	36.7
Students' classroom behaviour					
Students have to be attentive in a mathematics class.	0	3.3	0	26.7	70.0
Students do not like to ask questions in the class.[a]	6.7	43.3	3.3	33.3	13.3
Students' learning algebra					
Students believe algebra is difficult.	6.7	3.3	16.7	63.3	10.0
There are rules in algebra that students have to learn.	0	0	3.3	33.3	63.3

Note. Confidence was scored on a 5 point scale Strongly agree=5; Agree=4; Not sure=3; Disagree=2; Strongly disagree=1 [a]Negatively posed items

Key Finding 5.9 Almost all (over 90%) of the teachers indicated that mathematics is procedural, that they enjoy teaching it but that students often have a weak mathematics background. Many (70%) also believed that students find algebra difficult and have to be told how to solve problems, while slightly more than half of the teachers found it easier to teach symbolic questions and sometimes explained with the native language.

Teachers' assessment of algebra questions

The teachers compared six beginning algebra questions, made up of two questions presented in three different but essentially equivalent formats, and had to rank in ascending order how difficult they perceived each question would be for a JS 1 student. The questions with algebraic letters were ranked as more difficult than the box type questions but easier than the worded questions. The teachers' rankings and the median ranks are presented in Table 5.12.

Table 5.12: Ranking of algebra questions in order of difficulty by teachers (n=30)

Question type	Mathematical skill required	Question	Per cent of teachers: Difficulty rank 1	2	3	4	5	6	Median rating rank/6
Box	Understanding of place holder	$\square$ X 5 = 20. What is $\square$?	83.3	-	13.3	3.3	-	-	1
Letter	Understanding of place holder	5*x* = 20. What is *x?*	6.7	50.0	23.3	13.3	6.7	-	2
Box	Inverse	$\square$ X 20 = 5. What is $\square$?	-	33.3	20.0	30.0	13.3	3.3	3
Letter	Inverse	20*x* = 5. What is *x?*	-	6.7	40.0	43.3	-	10.0	4
Worded: "a certain number"	Understanding of place holder	5 lots of a certain number is 20. What is the number?	-	10.0	3.3	10.0	70.0	6.7	5
Worded: "a certain number"	Inverse	Twenty lots of a certain number is five. What is the number?	10.0	-	-	-	10.0	80.0	6

Note. Difficulty was ranked from one to six. Easiest =1; Easier=2; Easy=3; Difficult=4; More difficult=5; Most difficult=6

The majority of the teachers agreed on the rankings of the easiest and the two most difficult questions and recognition of the question type seemed to determine the teachers' perception

of its difficulty. For example, the question ranked as easiest (1) was in *box* form while the same question written in *words* was ranked as most difficult (5) by a majority of the teachers. All three questions requiring inverse operations were also believed to be more difficult than corresponding question types not needing inverses. Perhaps not surprisingly, the question ranked as the most difficult was a word problem involving an inverse operation.

Every teacher had to state a reason for awarding the fourth, fifth and sixth ranks to their selected questions. The majority of the teachers said it was because division, fractions, variable or words were included in the questions. Table 5.13 presents the teachers' responses.

Table 5.13: Teachers' reasons for questions ranked in the difficult category (n=30)

Question Reason for difficulty		Number	Percentage
20*x* = 5. What is *x?*	Rank 4		
Presence of a fraction or the use of division in the question		12	40.0
Presence of a variable in the question		1	3.3
Others		17	56.7
5 lots of a certain number is 20. What is the number?	Rank 5		
Interpretation of the word problem		11	36.7
Presence of a variable in the question		7	23.3
The use of division in the question		3	10.0
Others		9	30.0
Twenty lots of a certain number is five. What is the number?	Rank 6		
Interpretation of the word problem		18	60.0
The use of divisions by a coefficient in the question		4	13.3
Presence of a variable in the question		2	6.7
Others		6	20.0

Note. Difficulty category ranks are Rank 4=Difficult, Rank 5=More difficult, Rank 6=Most difficult.

The rank of four was given by the teachers because of the presence of fractions or the use of division rather than the presence of an algebraic letter in the question. The interpretations needed for solving the word problems were the main reasons for the two questions being ranked as fifth and sixth. The use of divisions was a greater contributor for choice of difficulty at the sixth rank. This ranking pattern is similar to that obtained from the rating activity done during the professional learning program (Key Finding 4.1).

Key Finding 5.10
There was a general agreement by most teachers on the ranking of the difficulty of algebra questions. Worded questions or those involving inverse operations were perceived to be more difficult for students to solve. The most difficult question was a word problem with inverse operation. Many of the teachers ranked questions to be difficult because they involved fractions, divisions, had a variable or they were word problems.

Professional Learning Participants Data

This group comprised the 12 teachers who completed both initial and final questionnaires, and participated in all the professional learning (PL) programs. The majority (75%) of the teachers had between two and 10 years mathematics teaching experience. Only one of them had taught JS 1 for more than five years and a third of them had classes within the regulation size of 40 students. Comparisons of the teachers' responses in both questionnaires are reported with other responses related to feedback from the intervention using six headings. These are: teachers' beliefs about effective mathematics teaching and learning; teachers' confidence about their knowledge of mathematics and algebra teaching strategies; frequency of the teachers' use of various teaching strategies; teachers' assessment of the difficulty level of some algebra questions; the importance of knowledge of students' thinking and mathematical talk; and, feedback from the professional learning intervention program.

Teachers' beliefs about effective teaching and learning of mathematics

There was a slight shift in beliefs between pre- and post-intervention surveys. Communication skills and teacher capability increased, while the conviction about the importance of instructional materials decreased slightly. Table 5.14 presents their responses.

Table 5.14: PL teachers' beliefs about characteristics of effective mathematics teaching (n=12)

Initial Questionnaire	Number[a]	Final Questionnaire	Number[a]
Instructional materials usage	5	Communication skills	4
Teacher understanding of students' learning needs	5	Teacher capability	4
Good class management	3	Good class management	4
Encouraging student interest	2	Instructional materials usage	4
Teacher capability	1	Effective feedback	3
Learner engagement	1	Teacher understanding of students' learning needs	3
Others	3	Learner engagement	1
		Others	3

Note. [a] Most teachers gave more than one response to the questions

Communication skills and effective feedback were new characteristics and there was a drop in some of the more traditional teacher-centred characteristics. Learner engagement remained least recognised. Language-based approaches emerged as a strategy that teachers found to be

effective for learning mathematics while the proportion of teachers who mentioned the use of instructional materials reduced by half. Table 5.15 presents this information.

Table 5.15: PL teachers' beliefs about effective strategies for learning mathematics (n=12)

Initial Questionnaire	Number [a]	Final Questionnaire	Number [a]
Use of Instructional materials	6	Student participation	4
Student participation	4	Teacher attending to student	4
Use of practicals and demonstrations	4	Language-based approaches	3
Class work	2	Use of instructional materials	3
Problem solving	2	Use of textbooks	3
Teacher attending to students	2	Problem solving	1
Others	6	Others	5

Note. [a] Most teachers gave more than one response to the questions

The importance of teachers attending to students was mentioned in some of the responses. Fewer teachers than before now mentioned the more traditional beliefs and problem solving was not reported as an effective strategy by many.

Key Finding 5.11
After the professional learning, beliefs that communication skills, feedback and language approaches were effective characteristics and strategies for teaching and learning mathematics emerged. There was a drop in the number of teachers who believe effectiveness comes from using instructional material and more teachers mentioned teacher capability and attending to students' learning needs.

Teachers' confidence about mathematics knowledge and algebra teaching strategies

The teachers' self-reported confidence level about algebra knowledge dropped slightly after the intervention. Table 5.16 presents the data.

Table 5.16: PL teachers' self-reported confidence level with knowledge of junior school mathematics (n=12)

Aspect	Mean rating/5			
	Initial Questionnaire		Final Questionnaire	
	Mean	SD	Mean	SD
Algebraic processes	4.75	.452	4.67	.492
Basic operations	4.67	.492	4.83	.389
Number and numeration	4.58	.515	4.75	.622
Everyday statistics	4.58	.669	4.67	.651
Geometry and mensuration	4.25	.622	4.25	.622

Note. Confidence was scored on a 5 point scale Very confident=5; Confident=4; Okay=3; Limited confidence=2; Not confident=1

The teachers' ratings remained generally high on all aspects as in the general survey (Key Finding 5.5). However, their initial confidence with algebra dropped from first position to tie

with statistics for the second to the last position. Geometry remained the aspect they were least confident with.

Key Finding 5.12
After the professional learning intervention period, the teachers' self-reported mean rating of confidence level for algebra knowledge dropped from first position (4.75/5) to third (4.67/5). Basic operations and geometry now had the highest and lowest mean scores respectively.

The teachers' general high ratings of self-efficacy in using different strategies for algebra teaching remained after the intervention period. The teachers' confidence in their ability to explain and develop relevant vocabulary and terms increased significantly (z = -2.121, p < 0.5) but their confidence in managing word problem discussions and group activities reduced. Although the teachers reported more confidence in assessing algebra learning, using the knowledge of students' likely misconceptions to plan lessons retained the lowest mean confidence score. Table 5.17 presents the data.

Table 5.17: PL teachers' responses about confidence in using strategies to teach algebra (n=12)

Strategies	Mean ratings/5			
	Initial		Final	
	Mean	SD	Mean	SD
Explaining algebra concepts	4.33	.651	4.83*	.389
Developing vocabulary and terms needed for learning algebra	3.83	.718	4.33*	.651
Engaging students' interest in algebra	4.42	.669	4.33	.651
Assessing children's learning in algebra	4.08	.669	4.33	.778
Managing discussions and interpretations of word problems	4.42	.515	4.08	.669
Involving the majority of the students in class discussions/activities	4.08	.996	4.08	.793
Managing group activities in algebra	4.00	.603	3.75	.754
Using knowledge of students' misconceptions to plan algebra lessons	3.75	.965	3.67	.985

Note. Confidence was scored on a 5 point scale Very confident=5; Confident=4; Okay=3; Limited confidence=2; Not confident=1

*Significant at p<.05 on Wilcoxon Signed Ranks Test

Key Finding 5.13
After the professional learning, there was significant increase in the mean rating of teachers' confidence to explain algebra concepts and develop relevant vocabulary and terms. While the mean rating on confidence for algebra assessment increased, that of managing word problem discussion, using knowledge of students' misconceptions and group work reduced.

Teachers' classroom strategies

Most teachers believed that they are able to use classroom discussions and questions effectively as a teaching tool and this belief generally increased after the intervention program. The means and standard deviations concerning these are presented in Table 5.18.

Table 5.18: PL teachers' responses about managing talk in the classroom (n=12)

Strategies	Mean ratings/5			
	Initial		Final	
	Mean	SD	Mean	SD
My rich knowledge of mathematics helps me respond appropriately to students' answers to my questions.	4.50	.522	4.75	.452
I am effective in using questioning to identify students' prior knowledge of mathematics topics.	4.08	.793	4.67	.492
I am effective in engaging most students in responding to my questions during mathematics discussions.	4.33	.888	4.58	.515
I am effective in encouraging and supporting students to ask questions in my mathematics class.	4.33	1.155	4.58	.515
I am able to respond to students answers in ways that help develop an effective discussion of mathematical ideas.	4.67	.492	4.58	.515
My rich knowledge of mathematics helps me ask the right questions to develop mathematics ideas through discussion.	4.33	1.155	4.50	.522
I am normally able to respond to students' answers in ways that maintain and promote further discussion of the mathematics ideas.	4.25	.452	4.42	.515
I am able to sustain discussions so that we thoroughly discuss the mathematics ideas.	4.25	.452	4.25	.866
I am effective in asking questions to suit the purpose and flow of classroom discussions in mathematics.	4.25	1.138	4.25	.866
I am effective in establishing a classroom atmosphere in which most students feel confident to give their own answers to questions.	3.92	1.311	4.25	1.138

Note. Agreement was scored on a 5 point scale Strongly agree=5; Agree=4; Not sure=3; Disagree=2; Strongly disagree=1

Teachers indicated that their effectiveness at managing talk in the classroom generally improved after the professional learning intervention program. Noteworthy is their use of questions to identify students' prior knowledge - this is not statistically significant but is still relatively high (z = -1.897, $p < 0.1$). The exception to the general trend was seen in the item about the teachers' ability "to respond to student answers in ways that will help develop

effective discussion of mathematical ideas". This item had a slightly reduced mean score after the intervention.

There was an increase in the use of student activity-based approaches and a slight decline in most approaches that are teacher-focused. The means and standard deviations for preferences for teaching strategies are presented in Table 5.19.

Table 5.19: PL teachers' responses about the frequency of use of different approaches (n=12)

Strategies	Mean ratings/5			
	Initial		Final	
	Mean	SD	Mean	SD
Explaining the meaning of equations	4.17	1.115	4.50	.674
Having students solving questions individually	4.50	.793	4.42	.669
Reminding students about the meaning of a variable	4.17	.835	4.42	.669
Working examples on the board for students to copy	4.67	.778	4.42	.900
Having students reading aloud the question to be solved	3.58	1.165	4.33	.651
Using different types of mathematical representations	4.17	.937	4.33	.651
Identifying students' misconceptions of algebra	3.92	.793	4.25	.754
Asking students to identify what the question asks us to do	4.08	1.240	4.17	.835
Asking students to identify the plan for solving the question	3.92	.900	4.17	.835
Inviting students to explain the working for their answer	3.92	1.165	4.00	.953
Whole class discussion of mathematical ideas	3.75	1.357	3.75	1.055
Asking students to identify key words and symbols in the question	3.67	1.155	3.75	1.055
Writing notes on the board for students to copy	4.00	1.477	3.67	1.435
Grouping/pairing students to solve questions in the class	3.17	.937	2.83	.718

Note. Frequency was scored on a 5 point scale Every lesson=5; In most lessons=4; In some lessons=3; In a few lessons=2; Never=1

There were increases in the frequency of teachers' explaining equations, reminding students about the meaning of variables and identifying students' algebra misconceptions; and a reduction in teachers' working examples for students, and writing notes. There was an increase in students' involvement in planning solutions to problems and a relatively high increase ($z = -1.897$, $p < 0.1$) for students' reading aloud of questions. However, the use of group work remained low.

Almost all (91.7%) the teachers rated their students' engagement level to be between six and eight (out of 10) after the professional learning intervention, a considerable improvement. This reflected an increase of 33.4% over the corresponding initial rating interval of six and eight, and there was no reported rating below six. The mean rating score increased by 7%. The ratings are presented in Table 5.20.

Table 5.20: PL teachers' belief about student engagement level in the classroom (n=12)

		Very passive					Very active			Mean rating/10
Engagement level of student		1 – 3	4	5	6	7	8	9	10	
Number of teachers	Initial	0	1	2	3	1	3	1	1	6.83
	Final	0	0	0	3	3	5	1	0	7.33

Key Finding 5.14
After intervention, the teachers reported increased effectiveness at managing talk in their classes through discussions and questions. The mean rating scores increased for teachers' use of explanations, identifying algebra misconceptions, students' reading aloud and identifying a solution plan for questions while teachers' working of examples and note writing reduced. Their student engagement level mean rating score increased by 7%.

Teacher beliefs about mathematics, their teaching and students' learning

The statements about teachers' beliefs regarding mathematics, teaching and learning algebra were constructed to reflect traditional and constructivist beliefs both directly and more indirectly. This necessitated writing some statements in the reverse order for teachers to reflect on the statements before indicating their level of agreement. Teaching enjoyment increased and though many teachers seemed to have a traditional belief about mathematics and student learning initially, there appeared to be a slight shift from that position in the completed final questionnaire.

There was increased use of the native language to explain mathematical ideas to students and a general agreement about the weak background of students. This confirms the teachers earlier expressed belief in Key Finding 4.4 that the use of the first language improves students understanding. Teachers enjoyed their teaching and there was some reduction in the belief that students do not like asking questions.

Analyses of the responses are presented in Table 5.21.

Table 5.21: PL teachers' belief about mathematics, their teaching and students' learning (n=12)

Strategies	Mean of ratings scores			
	Initial Questionnaire		Final Questionnaire	
	Mean	Standard deviation	Mean	Standard deviation
Nature of mathematics				
Mathematics consists of rules and procedures. [a]	4.33	.900	4.25	.622
Mathematics is mainly calculations.[a]	3.92	1.357	3.42	1.084
Teaching mathematics				
Teaching enjoyment				
I always enjoy my mathematics teaching.	4.25	1.138	4.58	.515
If I was free to choose, I would not teach mathematics.[a]	1.83	1.115	1.42	.515
Teaching practices				
If a teacher does not tell students how to solve questions, they will make mistakes.[a]	3.83	1.193	3.75	1.055
I sometimes use the native language to explain mathematical ideas in the class.	2.92	1.564	3.58	1.165
Teaching word problems				
I find it easier to teach algebraic word problems than those with symbolic notations.	2.92	1.564	2.83	1.267
Learning mathematics				
Not all students can learn mathematics.[a]	2.33	1.497	2.16	1.193
Students' mathematics background is often weak[a].	4.42	.900	4.33	.492
Students' classroom behaviour				
Students have to be attentive in a mathematics class.	4.42	.900	4.67	.492
Students do not like to ask questions in the class.[a]	3.50	1.382	2.58	1.311
Students learning algebra				
Students believe algebra is difficult.	3.50	1.382	3.92	1.084
There are rules in algebra that students have to learn.	4.50	.522	4.58	.515

Note. Confidence was scored on a 5 point scale Strongly agree=5; Agree=4; Not sure=3; Disagree=2; Strongly disagree=1 [a]Negatively posed items

Key Finding 5.15
After the intervention period, the teachers appeared to have a stronger belief that students desire to ask questions. There was an increase in the use of the native language to explain mathematical ideas.

Teacher assessment of algebra questions

The teachers had to assess the six questions in terms of how difficult they were for a JS 1 student to solve. The majority of teachers chose the same ranks for most of the questions in both questionnaires and the word problems remained the most difficult questions. Table 5.24 shows the data.

Table 5.22: PL teachers' ranking of algebra questions (n=12)

Question type	Mathematical skill required	Question	Level of difficulty /6				
			Initial		Final		
			Mean	SD	Mean	SD	Rank
Box	Understanding of place holder	$\square$ X 5 = 20. What is $\square$?	1.33	.778	1.17	.389	1
Letter	Understanding of place holder	5*x* = 20. What is *x?*	2.42	1.084	2.88	.900	2
Box	Inverse	$\square$ X 20 = 5. What is $\square$?	3.25	1.215	3.33	.888	3
Letter	Inverse	20*x* = 5. What is *x?*	3.33	.778	4.00	.953	4
Worded: "a certain number"	Understanding of place holder	5 lots of a certain number is 20. What is the number?	4.67	.651	4.92	1.084	5
Worded: "a certain number"	Inverse	Twenty lots of a certain number is five. What is the number?	6.00	.000	5.42	.996	6

Note. Difficulty was ranked from one to six. Easiest =1; Easier=2; Easy=3; Difficult=4; More difficult=5; Most difficult=6

More teachers appeared to recognise the difficulty associated with symbolic problems involving the need for inverse operations, although the increase was not significant at the $p < .05$ level, and these were still ranked third and fourth respectively. On the other hand, a few teachers seem to think that the two word problems were not as difficult as they previously thought but the ranking remained. Two-thirds of the teachers again chose a word problem that is inverse to be the most difficult question.

The reasons given for ranking difficulty in the way they did portrayed an increased awareness that the presence of variables, fractions and word problems increases the difficulty level of a question. Table 5.23 presents the responses.

Table 5.23: PL teachers' reasons for choice of perceived difficult questions (n=12)

Reason for difficulty	Number of teachers	
	initial	Final
Rank 4		
Presence of a fraction or use of division in the question	4	3
Presence of variable in the question	1	3
Multiplication has to be done	2	3
Others	5	3
Rank 5		
Interpretation of the word problem	3	9
Presence of a variable in the question	3	1
Presence of a fraction or use of division in the question	1	1
Others	5	1
Rank 6		
Interpretation of the word problem	7	8
Presence of a variable in the question	1	2
Presence of a fraction or use of division in the question	1	1
Others	3	1

The number of teachers that mentioned the presence of a variable as a reason for the difficulty of a question slightly increased, but there was an even larger increase in the choice of reason for difficulty to be word problems.

Key Finding 5.16
After the professional learning intervention, the ranking pattern remained the same as was seen in the first general survey, with the two word problems perceived as the two most difficult questions. The reason still given for the highest difficulty ranked question was because it was a word problem. The presence of variables increased as a contributing reason for the fourth rank.

Importance of knowledge of students' thinking and mathematical talk

Teachers also responded to the open ended questions "Why is mathematical talk important in the teaching and learning of algebraic word problems?" and "How has an understanding of students' misconceptions and thinking helped you to teach algebra?" Their responses indicated that they felt that it helped students to understand the language of mathematics and interpret word problems, and understanding the misconceptions in preparation of lessons

helped the teachers to identify them and be more conscious of students' learning needs in lesson delivery. Table 5.24 presents the responses.

Table 5.24: PL teachers' responses about mathematical talk and knowledge of students' misconceptions (n=12)

	Number of teachers[a]	Per cent
Importance of mathematical talk		
It helps the interpretation and understanding of word problems	4	26.7
It provide a way of identifying misconceptions	3	20.0
It increases mathematical language development	3	20.0
The nature of algebra as a topic calls for mathematical talk	3	20.0
It helps to keep the learner engaged in the classroom	1	6.7
It aids the building of knowledge	1	6.7
How understanding students' misconceptions helps algebra teaching		
A consciousness of student learning needs	6	43.0
Conscious of the need to review knowledge relevant to the topic	3	21.4
Increase in teacher subject knowledge	3	21.4
Leads to effective communication in the classroom	1	7.1
It results in a better lesson preparation	1	7.1

[a]Some teachers gave more than one response

The teachers believed that mathematical talk highlights students' misconceptions and supports their development of mathematics language and algebra knowledge. A minority also stated it is important for learner engagement. Many teachers gave reasons that seemed to focus on ensuring students' understanding of the algebra being taught. The knowledge also improved the algebra content knowledge of a few of the teachers.

Key Finding 5.17
The PL teachers reported that the knowledge of students' misconception and thinking increased their knowledge of algebra and made them more conscious of student learning needs in algebra lesson preparation and delivery. The teachers also believed mathematical talk exposes students' misconceptions and leads to an increase in mathematical language development and, to the understanding and interpretation of word problems.

Difficulties using the Newman language-based error analysis procedure

Teachers responded to the open ended question "What difficulties did you have in using the Newman strategy?" Half of the teachers reported they had no difficulties but a few mentioned that they were constrained by time to follow through all of the Newman steps and by their students' verbal ability in English Language which, as described earlier, is the official language of communication in schools. Table 5.25 presents the responses.

Table 5.25: PL teachers' responses to difficulties faced in the use of Newman strategy (n=12)

Difficulty	Number of teachers	Per cent
None	6	50.0
Some students limited verbal ability	3	25.0
Time constraints	3	25.0

Key Finding 5.18
Half of the PL teachers reported that they did not have trouble with the general use of the Newman language-based error procedure in the class. The other teachers worked with students with limited verbal abilities in English language or had insufficient time to complete all the Newman steps.

Workshop Gains and Suggestions

Teachers' responses to the open ended question "What is the most significant thing that you gained in this professional learning workshop?" indicated that the teacher gains were mostly in mathematics content knowledge and pedagogical content knowledge. Figure 5.2 below present their responses.

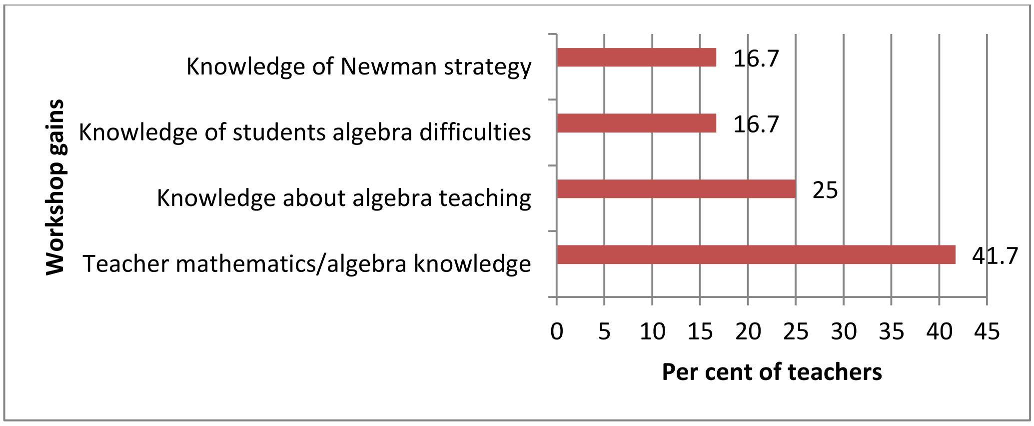


Figure 5.2: PL teachers' most significant workshop gain (n=12)

Many teachers suggested that all mathematics teachers should attend the workshop, while all teachers recommended the workshop. Table 5.26 presents the responses.

Table 5.26: PL teachers' suggestions and recommendations (n=12)

	Number of teachers *	Per cent
Suggestions		
All mathematics teachers should participate in the workshop	9	56.3
Increased time duration	2	12.5
Involvement and participation of the Ministry of Education	2	12.5
Continuous networking of participants	1	6.3
Others	2	12.5
Recommendation		
Yes	12	100.0

[a]Some teachers gave more than one suggestion

A few teachers believed that the time duration should be more than three days and that in the future; the Ministry officials should be invited to participate.

Key Finding 5.19 Mathematics and algebra content knowledge was the most significant workshop gain reported by 42% of the teachers. Other gains were more knowledge about algebra teaching, the Newman procedure and students' algebra difficulties. The program was recommended by all participants to mathematics teachers and relevant Ministry officials should be invited to participate.

Summary

All of the teachers were professionally qualified but had fewer than 10 years of JS 1 teaching experience, with 77% of them in the two to five years range. Many had class sizes between 41 and 200 and taught more than three classes of JS 1. The most common challenges to effective mathematics teaching reported by teachers were: inadequate instructional materials and facilities, students' weak prior mathematics knowledge and large class sizes. Most of the teachers indicated high self-efficacy beliefs on the use of questioning and whole-class discussions, and they were all confident of their knowledge of algebra. However, traditional approaches had the highest mean ratings and were reported to be used regularly while approaches involving mathematical discussions and identification of students' algebra misconceptions were used less often. Before the intervention started, two-thirds of the PL teachers perceived that most of the questions, including three word-equations with two pro-numerals, were adequate for JS 1 students.

After the professional learning, stronger beliefs that communication skills, feedback and language-based approaches were effective strategies for teaching and learning mathematics emerged. There was also a significant increase in the mean rating of PL teachers' confidence in explaining algebra concepts, developing relevant vocabulary and terms and using questioning, while confidence reduced about their algebra knowledge and about managing word problem discussion and group work. The mean scores for frequency of students reading aloud and identifying a solution plan for questions increased while that of teachers' working of examples and note writing reduced.

The PL teachers reported that the knowledge of students' likely misconceptions and thinking made them more sensitive to student learning needs in algebra lesson preparation and delivery, and increased their knowledge of algebra. Half of them reported that they did not experience any difficulty with the general use of the Newman language-based error procedure in the class. The other teachers worked with students with limited verbal abilities in English language or insufficient time to complete all the Newman steps. Gains from the professional

learning were seen most in mathematics and algebra content knowledge, knowledge about algebra teaching, effective use of the Newman procedure and awareness of student algebra difficulties.

CHAPTER SIX: CASE STUDY ONE – RUTH'S CLASS

The next four chapters report case studies of four of the teachers, and thus focus more closely on JS 1 algebra teaching and students' successes, and difficulties with algebra problem solving before and after the professional learning. More specific qualitative data about teachers' beliefs, knowledge and practice, and student's algebra problem-solving ability were gathered. These data seen in questionnaires, lesson observations, PL activities, interviews and algebra tests were analysed. The analysis in each of the four chapters is reported each time in two sections, namely: the teachers' pre- and post-intervention beliefs and practice; and the students' pre- and post-algebra performance and Newman interviews.

Background

Ruth (a pseudonym) was very willing and happy to participate in the study. In her early 30s and having taught mathematics for more than six years, she had fewer than five years of JS 1 teaching experience. Ruth has a first degree in mathematics education in addition to the NCE, which is a Nigerian teaching qualification obtained after three years of post-secondary training at a College of Education. She taught JS 1 and 2 in a private co-educational school located within an urban area in Ojo educational zone, and one of Ruth's three JS 1 classes participated in the study. The students did not number more than 35 in a class and were between 10 and 12 years of age.

Pre-Intervention Beliefs and Practice

Beliefs

Ruth's responses in the initial questionnaire completed before the professional learning suggest some constructivist belief about teaching. For example, she wrote that effective mathematics teaching "should be student-centred", allowing "students to solve examples by themselves" and that the "interactive method" was an effective teaching strategy. There were strong disagreements with statements like: *not all students can learn mathematics,* and *mathematics is mainly calculations.* Ruth indicated that she used the following strategies daily or often*: inviting them to explain the working for their answer* and *having them solve questions individually*.

Ruth was very self-confident about her ability to manage classroom discussions and use questions; she also expressed confidence about engaging students' interest and explaining algebra concepts to them. She rated her students' classroom engagement level to be seven out of 10. However, she never grouped or paired students in the class. She reported that the

lack of instructional materials was a challenge for teaching mathematics effectively. Ruth was confident about her knowledge of algebra but indicated limited confidence about *using the knowledge of students' misconceptions to plan algebra lessons*.

On the initial questionnaire, Ruth was asked to rank six algebra problems in terms of their difficulty for students. The two word problems were selected as the two most difficult for her students with the word problem involving an inverse operation perceived to be the most difficult of all. The three questions involving inverses were consistently ranked to be more difficult, irrespective of the question representation.

Believing that students find mathematics, and especially algebra difficult, Ruth strongly agreed with the following statements: *students don't like to ask questions in the class; students' mathematics background is often weak,* and*; if a teacher does not tell students how to solve a problem, they will make mistakes.* Ruth expressed delight and satisfaction with her teaching and indicated that she used the native language sometimes to explain mathematical ideas in the class.

Key Finding 6.1 Before the intervention, Ruth had a high self-efficacy and confidence about her knowledge of algebra and the use of questions but limited confidence with using the knowledge of algebraic misconceptions in lesson planning. She believed classroom interaction was necessary for effective teaching whilst having some traditional views of students' learning.

Practice

Activities in Ruth's classroom during two single lessons were recorded at two minute intervals. The four most frequent activities were: teacher explaining, students listening, students explaining and students' individual works (See a sample in Appendix 17). Ruth's classes often started with her giving a brief review of the previous lesson, followed by an explanation of the day's lesson and then writing a question on the blackboard as an example. To explain the concept, Ruth read the question aloud, and then proceeded to work out the answer, explaining as she moved through each solution step. The students were asked if they had any questions; if none, she explained the solution process again before another problem was written on the board.

Ruth's comments seemed to suggest that she herself used the letter as a detachable object, something that can be potentially misleading for her students. In her spoken language, she also used the phrase "add the letter" as an instruction to write the letter beside the coefficient, not as an operation. While explaining how to simplify algebraic expressions in two different lessons, she said:

When you want to add numbers like this, first of all, add the coefficients then just add the letter to it if it is the same alphabet. (Lesson observation, 17/3/2011)

You know, I told you that when you want to add, you add the coefficients and after adding them, you add the letter to it. (Lesson observation, 24/3/2011)

Students' questions were answered immediately by Ruth or other students volunteered to answer. However, it appeared that rules were followed and that faulty reasoning was not addressed as seen below.

First student: I have a question. Why can't we add 8*y* plus 5*x* to give 13*x*?

Ruth: Listen, listen everybody

Second student: I want to answer the question

Ruth: Okay, answer

Second student: The reason why we cannot add them is that they are not of the same variable

Ruth: They are not of the same variable (Lesson observation, 24/3/11)

Ruth offered no further clarification of the fact that an addition had occurred but that the resulting expression could not be gathered together.
On another occasion, a student had to simplify 8*y* + 5*x* – 5*y* - 3*s* + 1. He wrote 3*y* + 1 + 5*x* – 3*s* and then wrote final answer as 4*y* + 5*x* – 3*s* on the board (See Figure 6.1). After the student wrote the answer, she asked

Ruth: What are you collecting again? Is there any common term?

Student: No (then re-wrote as 3*y* + 5*x* – 3*s* + 1)

Another student: Since it is ordinary one, can't we add it together?

Ruth: No, we leave it like that (Lesson observation, 17/3/11)

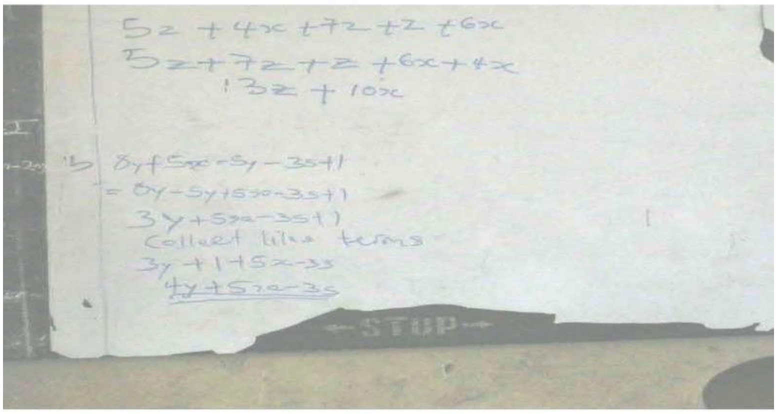


Figure 6.1: Ruth's student's working on the board

Ruth appeared to give more attention to the computational aspects than the understanding of the question. In the example, *Think of a number, subtract three from it and the result is nine. What is the number?* she called a student to work out the answer on the board. The other students were asked if the student's solution process was correct, and there were mixed responses of yes and no. The student had written nine and three inside two shapes, added them and wrote 12 as the answer.

> Ruth: The question asked you to think of a number… What is the number? The answer is correct but the steps are not in order. You missed a step before you arrive at the final answer.
>
> (Another student is called to the board to correct the working. The student does it correctly using only one unknown, subtracting three from it and balancing the equation to obtain the value of 12 as the needed number.)
>
> Ruth: She is correct. (Students copy the answer into their notebooks)
> (Lesson observation, 17/3/11)

A volunteer or an invited student answered the question on the board while Ruth watched the student's working. If correctly answered, Ruth re-explained the process before the students copied the examples into their notebooks; but if wrong, another student was called or volunteered to solve it. At times she completed the working of the question and then gave one or two problems as class work. As Ruth walked around the class, individual students showed their solutions to her and received immediate feedback. After some time, a student was called upon to show the working of the answer on the board while other students watched. The worked examples were copied by the students and then they were directed to the textbook do some exercises as classwork or homework.

Key Finding 6.2
Prior to the professional learning, the observed classes revealed Ruth's interaction with the students but teacher talk/explanation dominated. Ruth appeared to use the letter as a moveable object. When students made errors, their faulty reasoning were not addressed and it seemed there was more focus on calculating than on understanding the questions.

Post-intervention Beliefs and Practice

Beliefs

After the professional learning intervention program and the teaching period, Ruth's beliefs were analysed from her responses in the final questionnaire and the professional learning program. She seemed to have become more language-conscious in her beliefs. To her, students learnt mathematics effectively through classroom discussions in which they were the focus during the teaching process. Ruth's written reflection on mathematical talk was that it

was important for improving students' ability to understand how to interpret questions. She mentioned the importance of using familiar words to her colleagues during the final PL program.

> Teaching should not be teacher-oriented, it should be student-centred … we should not be using big words; we should use the words they are familiar with, simple ones. For example, product, if some of them don't know the meaning, it is the same thing as multiplication. Multiplication is the same thing as times. Then sum, sum is the same thing as addition, addition is the same thing as plus". (PL Workshop, 6/7/11)

Ruth still indicated a general high self-efficacy in the use of questioning in the classroom but had reduced confidence about her ability to *ask the right questions to develop mathematics ideas through discussion* and *sustain discussions so that we thoroughly discuss mathematical ideas.*

With increased confidence in her rating about her *knowledge of students' misconceptions to plan algebra lessons,* Ruth believed that a teacher has to patiently explain to help students overcome algebra misconceptions. During a period of sharing teaching experiences with other teachers, she said:

> Teachers should take time to explain to them (students) in details and he/she should not skip any step. Maybe because of time he is supposed to do this stage, he will skip it (saying) that they will do it in JS 2 (PL workshop, 6/7/11)

Ruth believed that students' use of their textbooks would improve problem-solving performances. When asked during the PL about what she required from her students to help them learn algebra better, she said

> By their practising at home; whether they give them assignments or not, they have the textbook, they have … exercises, practice at home. Any one they don't understand …. Maybe when you want to do the correction, you look at it then call them one after the other to come and do it, you will make it random. (PL workshop, 6/7/11)

Ruth wrote in her final questionnaire that her significant gains from the professional learning were her understandings that: "teaching algebra is not a fruit salad approach", and "application of Newman strategy [is useful] in teaching all topics in mathematics". By fruit salad approach, she meant that the algebraic letter should not be used as a label or to represent an object such as 'a' for 'apple', 'b' for 'banana'. When asked about her experience with the use of the Newman error analysis questions in her class, Ruth replied,

> Ruth: it worked

> Researcher: When you say it worked, I may not really understand what you mean by 'it worked'
>
> Ruth: It will be discussion because by the time you ask them to read the question……you stand up, read the question, do you understand? Who will interpret it - the question. It is discussion (and laughed).
>
> Researcher: So you were able to use it?
>
> Ruth: Ah, yes. It worked. It's not even only in teaching algebra…..it's not only for algebra (PL workshop, 6/7/11)

Ruth expressed concern that the JS 1 algebra syllabus was overloaded with content.

> It's not only the maths teachers or students alone; the scheme of work is at fault. Like now, the algebra, you have some topics about algebra. If you look at the time, you skip one, you move to another and it is not supposed to be. After giving them one example and they don't understand, you need to explain but you leave them..and say because of time when you get to JS 2, you will do it. (Focus Group, 6/7/11)

Key Finding 6.3 Following the PL, Ruth identified language and mathematical talk as important for algebra teaching and learning; she attested to the efficacy of the Newman error analysis steps in her class as opportunities to discuss mathematics. Ruth believed the duration for algebra teaching was insufficient for the JS 1 content.

Practice

During a six-week teaching period, one double and two single lessons which Ruth taught were observed and recorded by the Researcher. The most frequent lesson activities were: teacher using questions and identifying key terms; students doing board and individual work, and students asking questions.

Ruth often started with a written problem which she solved herself, explaining each step and asking students questions. Many students often volunteered to participate in solving problems on the board and sometimes two students solved the same problem on the board while she watched. The students at the board then explained the strategy they used in finding the answer and some of the other seated students were quick to point out errors, sometimes without Ruth calling on them. Sometimes the students conversed amongst themselves without reference to the teacher, as shown in the picture below.

Figure 6.2: Ruth's students interacting at the board

Ruth adapted the Newman interview procedure in her class by calling different students to answer each of the Newman questions. For example, Ruth wrote $7x - 5 = x + 7$ on the board and asked:

Ruth: Who will solve the question? (Many hands were raised) What will be the first step?

Many students: We collect like terms

Ruth: (Calls a student) Come and collect the terms

Students: He's getting it.

Ruth: Look at [name] solution, is it correct? (Student wrote $7x - x - 5 = 7$)

Students: Mixed responses of yes and no

Ruth: Look at it

Many students: It is not correct. Aunty, they are teaching him

Ruth: So, look at it now, is it correct? ($7x - x = 5 + 7$)

Many students: Yes

Ruth: Who will solve the next step? (Many eager hands are raised, calls another student) Do the next step

(Student writes $6x = 12$)

Ruth: How do you know it is $6x$?

Student: Coefficient of x is one, so $7x - 1x$ is $6x$

Ruth: Now, you want to use balance method, what do you do? (Calls another student who divides both sides by 6 to obtain $x = 2$)

Ruth: What is the sign between 6 and x?

Student: times (Lesson observation, 25/5/11)

After the solution was written, Ruth asked the students to substitute and confirm the equality before she explained again.

With the word problems, Ruth would use her students' names in the questions and then revert to use real values to explain the strategy before generalizing with the algebraic letter. For example, the question was: *In a test, Yinka gets 6 marks more than Keji. If Keji got y marks, how many marks did Yinka get?* After reading aloud the question, Ruth continued:

Ruth: If Keji got 5 marks, how many marks did Yinka get?

First student: 11

Ruth: How do you know that it is 11?

First student: 6 + 5

Ruth: Why is it plus?

Second student: Because it is more than

Ruth: That is, Yinka's score is more than Keji's mark. Now, Keji has y marks, what is Yinka's mark?

(Many students shout out y+6 and 6+y, she calls one to explain his answer)

Third student: It is y + 6 because we don't know the mark of Keji. We plus the y plus 6 (Lesson observation, 17/5/11)

Ruth explained the meaning of words she felt her students may not understand. For example, in the question *a boy is 12 years old. How old was he five years ago?* a student could not understand how "ago' transformed to "minus'.

Ruth: How old are you?

Student: 10 years

Ruth: How old were you five years ago?

Student: Five years

Ruth: How do you know?

Student: I minus, 10 – 5 (Lesson observation, 17/5/11)

After repeating the process with two other students, Ruth explained that the word 'ago' referred to the past and meant subtraction for this question. In another lesson, she mentioned 'eliminate' while explaining the balance method under simple equations but quickly asked the

students “what does it mean”, to which they replied “remove” (Lesson observation, 25/5/2011).

Her questions now drew on students’ thinking and they made her students use the mathematical language. For example,

> Ruth: What do you understand by linear equation?
>
> First student: It is an equation that has one unknown
>
> Ruth: Instead of unknown, that letter is called what?
>
> Second student: Variable
>
> Ruth: Give me an example
>
> Third student: $4x = 20$
>
> Ruth: What is the sign between 4 and x?
>
> Fourth student: Multiplication (Lesson observation, 26/5/11)

She was observed using and reminding students that the letter was a quantity and was not observed using it as a moveable object again. Most problems were provided by her and the textbook exercises were used for classwork to be submitted for marking or as homework.

Key Finding 6.4 Following the PL, Ruth was observed using more questions that engaged the students’ thinking and she adapted the Newman steps in the class. More students were engaged in classroom discussion, problem solving and asking questions. She explained the meaning of unfamiliar words and was not observed using or talking about the letter as a moveable object.

Changes in Ruth’s Beliefs and Practice

Her classes appeared to be more engaging with students freely interacting with each other. Her reported daily teaching practice included whole-class mathematics discussion which previously occurred in a few lessons. The frequency of students’ individual work reduced, from every lesson to only some lessons. In the final questionnaire, Ruth indicated that now students daily *read aloud the question* and *identify a plan for solving the question.* Ruth’s reported algebra teaching approach now included *reminding students about the meaning of a variable* and *identifying their algebra misconceptions*. Prior to the intervention, Ruth never grouped or paired students in her class, but indicated that she now used it in a few lessons.

Ruth’s beliefs about some students’ classroom behaviour appear to have slightly changed. Her opinion on the statement *students do not like to ask questions in the class*, changed from a strong agreement to a disagreement. There was a slight reduction from a strong agreement to

agreement on the statement: *students' believe that algebra is difficult*. Ruth increased her strength of disagreement, from disagree to strongly disagree; on *mathematics being mainly calculations,* and on *word problems being easier to teach than symbolic problems*. Ruth reported that she had gained confidence in lesson planning based on the knowledge of algebraic misconceptions.

Key Finding 6.5 Ruth was more confident about her knowledge of algebra misconceptions after the professional learning. She believed students liked asking questions in the classroom and daily engaged her students in whole-class mathematics discussion.

Students' Algebra Pre-test Performance

Before the professional learning intervention workshop, Ruth's students completed the algebra pre-test. The pre-test comprised 15 questions, six symbolic and nine word problems. Each correct answer was scored one point, giving a maximum total score of 15 points. The highest total score obtained was nine, by one student, while 35% of the students answered only one question correctly. Table 6.1 presents the information.

Table 6.1: Ruth's students' pre-test total score (n=34)

	Total score on pre-test /15									
	0	1	2	3	4	5	6	7	8	9
Number of students	5	12	10	5	0	1	0	0	0	1
Per cent	14.7	35.3	29.4	14.7	0.0	2.9	0.0	0.0	0.0	2.9

About 79% of the students correctly answered no more than two of the questions and only two students correctly answered one-third or more of the questions.

More correct answers were obtained for the symbolic questions than the word problems. The number of students that gave correct answers to each of the 15 questions is presented in Table 6.2.

Table 6.2: Number of Ruth's students with correct answers in each pre-test question (n=34)

	Question number and representation format														
	Symbolic						Worded								
Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of students	2	13	6	20	2	1	9	1	3	2	1	0	0	0	1
Total	**44/204**						**17/306**								

The 61 correct responses out of a total of 510 (34x15) possible responses represents a 12% overall success rate comprised of 21.6% success on the symbolic questions and 5.6% success on the worded problems. The easiest symbolic question was a linear equation for which the students had to find the value of the algebraic letter. The easiest word problem required only the knowledge of the letter as a quantity; no mathematical operation was required.

Key Finding 6.6
Ruth's class pre-test results showed that 79% of the students correctly answered no more than two questions. There was a 12% overall success rate comprised of 21.6% on the symbolic questions and 5.6% on the word problems.

Students' Pre-test Newman Interview Results

The difficulties encountered by the students were investigated using the Newman error analysis interview procedure. Four students chosen by Ruth (given codes S1, S2, S3, and S4), were individually interviewed by the Researcher concerning the questions they wrongly answered. The Researcher asked additional questions after completing the Newman interview protocol only when the student's responses needed further clarification.

One student wrongly answered all of the questions and the remaining three students gave one, three and five correct answers respectively when they completed the questions a second time as required by the Newman protocol. This led to one student being interviewed on all 15 questions, another on 14 questions and the remaining two students on 12 and 10 questions respectively. Out of the 51 incorrect responses, 39% were transformation errors, 27% were comprehension errors and 20% were processing errors. The error pattern differed with the question text format, that is, symbolic as opposed to word. (See Table 6.3)

Table 6.3: Per cent of error types made by Ruth's students on the algebra pre-test (n=4)

	Question representation		
	Symbolic	Word	All questions
Initial error	Per cent	Per cent	Per cent
Reading	0	0	0
Comprehension	23.5	29.4	27.4
Transformation	29.4	44.1	39.2
Process skills	35.3	11.8	19.6
Encoding	0	2.9	2.0
Carelessness	11.8	11.8	11.8

On the symbolic questions, the students made more process skill errors (35%) than transformation (29%) and comprehension errors (24%). This was unlike the word problems on which they made more comprehension (29%) and transformation errors (44%) than process

skill errors (12%). More students moved through the comprehension and transformation steps before errors were made in mathematical processing on the symbolic questions. However, with the word problems there were fewer successful moves through the comprehension and transformation steps. Two-thirds of all errors emanated from the word problems and there were an equal proportion of careless errors from both question formats.

Key Finding 6.7 The Newman error analysis of Ruth's students' incorrect pre-test responses showed that 39% were transformation errors and 27% were comprehension errors. Transformation errors (44%) were most common with word problems while process skills errors (41%) were most common with symbolic questions.

The analysis of the students' incorrect responses follows, and it is based on the question format. The six symbolic questions are first stated, and then followed by a description of the students' responses to the interview questions as they relate to the language-based errors and common algebra misconceptions.

Symbolic Questions: 1- 6

1. Simplify as far as possible $1 + x + x$
2. Simplify as far as possible $3m + 5n + 4m + 6n$
3. $y \times y \times y =$ ....
4. Find the value of x: $7x = 21$
5. Find the value of x: $2x - 2 = 10$
6. Find the value of x: $21x = 7$

Figure 6.3: Symbolic pre-test questions

Questions 1 and 2 examined the students' difficulties with collecting like terms in algebraic expressions. To correctly answer these questions, the knowledge of an algebraic letter as a quantity and the knowledge that the answer may be an expression without conjoining terms is needed to obtain $1 + 2x$ and $7m + 11n$ respectively. Three of the five errors that occurred were process errors involving conjoining of terms and confusing sums with products.

S4: 1 plus x plus x equals 1 plus x squared ($1 + x^2$)

S1: 1 plus the x gives $1x$, then another x, equals $1x^2$ ($1+x+x = 1x+x$)

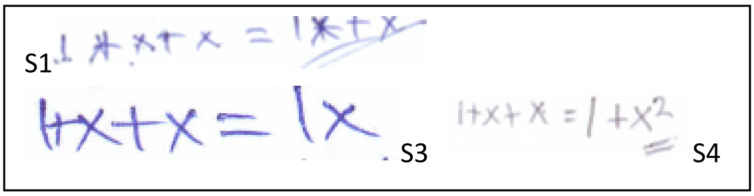


Figure 6.4: Ruth's students (S1, S3 and S4) workings on Question 1

The other two errors were transformation of the letter to a zero value and as a detachable object.

> S3: This one (points to x) is an unknown number; it is going to be zero, 1+0+0 = 1. You cannot write the two x, you write only one; 1x.
>
> S3: We are going to add them together..18*mn*: because there are two *m*'s and also two *n*s, so we are going to take one out for each. (Student interview, 28/3/11)

Question 3 required the use of the multiplication operator, not addition, to obtain y^3 and only one student gave the correct answer. The remaining three students processed the letter with a fixed value of one, giving rise to the algebraic misconception of the letter being a unit value.

> S2: It is y because when y is one, so, if you say 1x1x1 = 1. So in this case since y is one, so yxyxy = y.
>
> S3: It is equal to y because this y is just like one, then 1x1x1 =1, we are going to say yxyxy = y. (Student interview, 28/3/11)

Questions 4 to 6 had to do with finding a specific unknown value in an equation and required the knowledge of the concept of equality and the ability to perform inverse operations in order to balance an equation. Four of the nine errors related to comprehension; three occurred during transformation and the remaining two were careless errors with question 4. Most of the comprehension errors were identified with question 6 as three students thought they were meant to divide 21 by seven.

> S1: It is still the same as 21 divided by 7, so we do division
>
> S4: To get what we are going to divide by 21, we have to divide by 7 to get what is x
>
> S3: We should find the x that is there; so we are going to say, 21 divide by the answer that will give us seven. (Student interview, 28/3/11)

However, the notion of division was not carried over to Question 5 as the two transformation errors involved the students feeling compelled to use the equal sign as an action.

> S1: This x, we need to look for x. 12 minus two will give us 10. But how they write it here, I don't understand.
>
> S3: I have to minus, ten minus two to get the final answer: 10 – 2 = 8 (Student interview, 28/3/11)

The summary of all initial errors observed in the responses of the four students interviewed are in Table 6.4. The most common errors were process skills (6), transformation (5) and comprehension (4) errors.

Table 6.4: Ruth's students' responses and initial errors on pre-test symbolic questions (n=4)

Questions	Students' responses to six symbolic questions							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
Symbolic								
1	1	-	-	1	2	-	-	3
2	2	-	-	1	1	-	-	2
3	1	-	-	-	3	-	-	3
4	2	-	-	-	-	-	2	2
5	1	-	1	2	-	-	-	3
6	-	-	3	1	-	-	-	4
Overall	7	-	4	5	6	-	2	17

R- Reading, C- Comprehension, T -Transformation, P- Process skills, E- Encoding, CE - Carelessness

Key Finding 6.8
The transformation and process skill errors of Ruth's students in the pre-test symbolic questions appear to result from misconceptions about the letter as a moveable object, a specific or unit value, product –sum confusion, inappropriate conjoining of terms and an inability to use inverse operations to balance an equation.

A description of the nine word problems, the students' responses and identified errors and algebraic misconceptions follows next.

Worded Questions: 7-15

7. Sola has *x* bananas and Peju has *p* bananas. Peter counts the number of bananas each of them have and finds they are the same. Sola said you could write this as *x*=*p*, but Peju said that *x* and *p* are different letters and so cannot be the same. Who do you think is correct?
8. Mary has *x* oranges and Bisi has four more oranges than Mary. How many oranges does Bisi have?
9. A basket costs eight naira and a bag costs *c* naira more than the basket. How much does the bag cost?
10. What is the number that is five less than *x*?
11. There is a *b* number of sweets in a packet. A girl has two packets of sweets and gives her friend six sweets. How many sweets does she have remaining?
12. If *d* is the number of dogs and *c* is the number of cats, write in algebra: There are four more dogs than cats.
13. Write in algebra: There are three more caps than hats.
14. Write in algebra: There are twice as many pencils as biros (let *p* be the number of pencils and *b* be the number of biros).
15. If *s* is the number of students and *t* is the number of tables, write in algebra: There are three students for every table.

Figure 6.5: Worded pre-test questions

Question 7 required no mathematical operation and only required students to use the knowledge of the algebraic letter as a quantity to correctly answer the question. Three process errors occurred when the letter was interpreted alphabetically alongside a misconception that different letters cannot ever have the same quantities.

> S2: *X* cannot equal *p* because this thing (points to equal sign) means they are the same. Like human being and animal cannot be the same. This equal to, means they are the same; so, they cannot be equal to but they can be *x* and *p*.
>
> S3: *X* and p are different letters, so they can never be the same. They are different letters, so they cannot be written as the same variable number.
>
> S4: *X* and *p* are not the same   (Student interview, 28/3/11)

Questions 8 and 9 required students to transform the word ‘more’ into addition, then sum up the given quantities to obtain algebraic expressions ‘*x*+4’ oranges and ‘8+*c*’ naira respectively. Half of the six errors were due to carelessness; the students first substituted with real numbers but initially did not revert to the given letter to arrive at the correct answer. The other errors were: transforming the word ‘more’ as meaning multiplication (S4), “We have to say eight multiplied by *c* equals 8*c*”; processing the terms in a conjoined form (S1),”*x*+4 = 4*x*”; and, writing 4 oranges (S3) after correctly saying “4+*x*”.

Questions 10 and 11 required students to generate algebraic expressions involving the subtraction operator as correct answers. None of the students correctly answered two questions and seven of the eight errors were transformation-based. With Question 10, three students transformed ‘less’ as division to arrive at “*x* divided by five”.

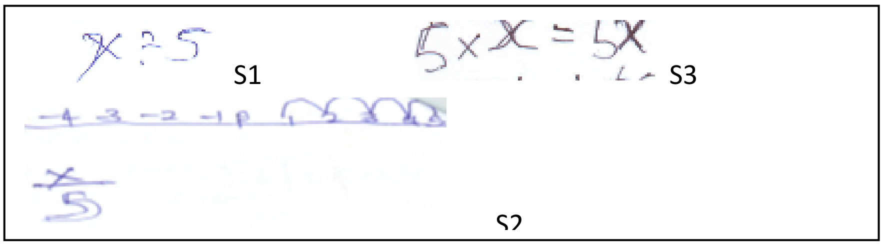


Figure 6.6: Ruth’s students’ (S1, S2 and S4) workings on Question 10

The Researcher, curious about this and wishing to understand better, asked them why they used division. The reasons were:

> S3: less means when it is lower, so we divide
>
> S2: In the number line, a number less than five, you have to move it back. Since this case is a mathematical statement, it is an unknown number, so you have to divide it since we cannot get the answer.

> S1: You need to divide to get the number that is less. We don't need to add because they say which number is less (emphasis), five less than *x*, that is, for example 10. It is five lesser than ten. You cannot say *x*+5, it will not give you the correct answer. (Student interview, 28/3/11)

In Question 11, all the students transformed the total number of sweets using specific values.

> S1: We need to do six times two; it will give us the amount, the *b*. Six times two equals twelve will be the amount of sweets inside one packet. Since she gave her friend six, it will remain one packet and six sweets.
>
> S4: *B* is the number of sweets in a packet. The girl has two packets; *b* is equal to two packets minus six sweets. Since they are not the same, we leave it like this. (Student interview, 28/3/11)

With Questions 12 to 15, the students had to construct equations showing the relationship between two objects in terms of their quantities. None of the students gave a correct answer to this set of questions. Nine of the 16 responses indicated that the students did not understand the questions or thought they had to find the quantity for one of the objects.

> S4: We have to add the *c* to four to get the number of cats
>
> S2: To know the amount of students sitting on a table

The seven transformation errors were now more concerned with students using the letter as shorthand for an object and using specific values. Examples of the typical responses were:

> S1: One chair for three students that is one over three
>
> S2: multiply *p* by two to get the twice, so it is two *p* plus this *b*, biro
>
> S1: this *d* (four), it is four more than cat, it will turn to six and this *c* will be two. So six minus 2 is four, cats is less than 4*d* (Student interview, 28/3/11)

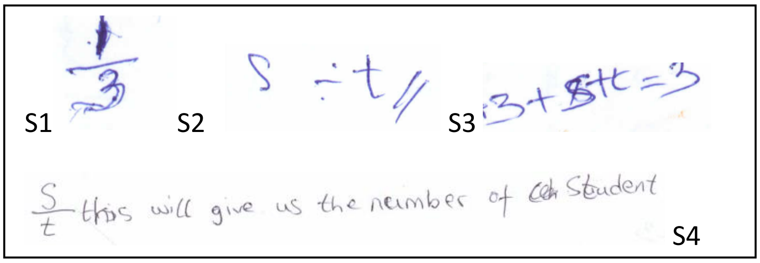


Figure 6.7: Ruth's students' (S1 – S4) workings on Question 15

The most common error types were transformation (15) and comprehension (10) errors. These data are summarised in Table 6.5.

Table 6.5: Ruth's students' responses and initial errors on pre-test word problems (n=4)

Questions	Number of responses to worded questions							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
7	-	-	-	-	3	-	1	4
8	-	-	-	-	1	1	2	4
9	2	-	-	1	-	-	1	2
10	-	-	1	3	-	-	-	4
11	-	-	-	4	-	-	-	4
12	-	-	2	2	-	-	-	4
13	-	-	4	-	-	-	-	4
14	-	-	1	3	-	-	-	4
15	-	-	2	2	-	-	-	4
Overall	2	-	10	15	4	1	4	34

R- Reading, C- Comprehension, T -Transformation, P- Process skills, E- Encoding, CE – Carelessness

Key Finding 6.9
Comprehension errors on the pre-test word problems occurred mainly with questions involving two pro-numerals; Ruth's students gave meanings limited to one of the pro-numerals. The word 'less' was also transformed to represent division. Algebraic misconceptions identified were: the use of the letter as an alphabet, as shorthand for an object; the letter has specific values; and, different letters cannot have the same value.

Students' Algebra Post-test Performance

Ruth's students completed the post-test after a post-intervention six-week teaching period. The post-test was parallel to the pre-test in that the structure and underlying concepts were unchanged; the changes only affected people's names, descriptive items and the values or quantities used. The reason for this was to reduce the bias that might influence performance, since the students had been exposed to the questions earlier.

In the post-test, 53% of the students achieved a total score of three or more and there was a significant improvement (z = -3.771) in the students' general performance even though the overall performance remained poor. Tables 6.6 and 6.7 present the details.

Table 6.6: Ruth's students' post-test total score (n=34)

	Total score on post-test /15								Pre-test		Post-test	
	1	2	3	4	5	7	8	10	Mean	SD	Mean	SD
Number of students	4	12	8	6	1	1	1	1	1.79*	1.68	3.12*	1.97
Per cent	11.8	35.3	23.5	17.6	2.9	2.9	2.9	2.9				

*Wilcoxon Signed ranks test, p<.01

The lowest total score was one, unlike the pre-test in which five students scored zero. There was a total of 101 correct answers out of a possible 510 (34x15) correct responses, representing a 21% success rate, an increase of 9% over the pre-test correct answers. The symbolic questions had more correct responses than the worded problems.

Table 6.7: Number of Ruth's students with correct answers in pre- and post-test questions (n=34)

	Question number and representation format														
	Symbolic						Worded								
Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Post-test															
Number of students	6	20	13	25	9	4	9	8	6	2	2	2	0	1	0
Total	**77/204**						**30/306**								
Pre-test															
Number of students	2	13	6	20	2	1	9	1	3	2	1	0	0	0	1
Total	**44/204**						**17/306**								

There was a 37.7% success rate with the symbolic questions and a 10% success rate on the word problems. Question 4, a symbolic-equation question, and Question 7, a word problem without mathematical operations, remained the easiest questions in their respective text formats. Further comparison of pre- and post-student performance can also be seen in Figure 6.14.

Key Finding 6.10
There was a significant improvement in Ruth's students' post-test general performance. The overall success rate Increased to 21% which comprised 37.7% success with symbolic questions and 9.8% success with word problems.

Students' Post-test Newman Interview Results

The four students earlier interviewed on the pre-test using the Newman interview protocol participated in another round of interviews on the post-test questions. The four of them jointly had a total of 13 correct responses; it consisted of five, four, three and one correct answer respectively given by the individual students. Most of the errors were transformation (44.7%) and process skill (31.9%) errors. Like the pre-test, the proportions of error type differed when the question text format was considered. There was almost an equal proportion of process skill errors found with symbolic questions, as transformation errors found with word problems. (See Table 6.8)

Table 6.8: Ruth's students' initial errors on algebra post-test (n=4)

Initial error	Question format		All questions
	Symbolic	Worded	
	Per cent	Per cent	Per cent
Reading	0	0	0
Comprehension	0	20.6	14.9
Transformation	38.5	47.1	44.7
Process skills	46.1	26.5	31.9
Encoding	0	2.9	2.1
Carelessness	15.4	2.9	6.4

The students progressed further on the symbolic questions, giving them more opportunity for mathematical processing that led to a higher percentage (46.1%) of process skill errors. On the word problems, the students' greater success with comprehension than had been seen on the pre-test enabled more activity at the transformation and processing stages.

Key Finding 6.11
Ruth's students' post-test interviews showed that the students made less comprehension errors than on the pre-test and that the majority of errors were transformation (45%) and process skills (32%) errors. The most common errors made on symbolic questions were process skill errors (46%), and transformation errors (47%) were the most common errors with word problems.

The post-test which was analysed for errors is next described according to the question format. The questions are first stated, followed by a description of the identified Newman language-based errors and algebra misconceptions evidenced from the students' responses to the Newman interview questions.

Symbolic Questions: 1–6

1. Simplify as far as possible $1 + y + y$
2. Simplify as far as possible $4z + 3p + 7z + 2p$
3. $m \times m \times m =$ ....
4. Find the value of x: $6x = 24$
5. Solve for x: $5x – 5 = 20$
6. Solve for x: $24x =6$

Figure 6.8: Symbolic post-test questions

Questions 1 and 2 responses produced five wrong answers. Most of the errors the students made on the two questions were processing errors arising from the use of the letter as a moveable object, conjoining of terms, and confusion of sums with product.

S1: We are going to remove the alphabets first and write only the numbers because we are not supposed to join the two together, we need to separate them, and then we take only one of them (and wrote 16 + *zp*)

S3: We are going to take one of these (points to *y*); we can't put the two of them together, so it is 1*y*.

Two students added and obtained 1 + *yy* and 1 + y^2 as their answers. (Student interview, 1/7/11)

Only one student (S3) wrongly answered question 3. The student processed the letter with the misconception that it had a fixed value of one saying "it is equal to *m*, because if we want to say one, one times one times one will give the same one so this *m* now will give *m*".

Questions 4 to 6 were equations requiring students to find the unknown value that will balance the equation. All the students answered Question 4 correctly, in some contrast to Question 6 that had three errors at the point of transformation. The students were quick to see some form of semblance between Questions 4 and 6 but were unable to perform the inverse operation required.

S1: I think this number (points to question 6) is the opposite of this number (points to question 4). It is just like 24 divided by six…The *x* is going to be for division so it will be 24 divided by six.

S4: It is the same as this one (points to question 4), the digits are the same. (Student interview, 1/7/11)

Two students (S3 and S4) were unable to process Question 5 and used the equal sign as meaning something to act upon. Both responses were of the form: "We are going to say five times five equals 25 minus five equals 20". The identification of the unknown as five was not however linked to the letter. The five was detached from 5*x* and multiplied with the other five (ignoring the negative sign) to obtain 25, and then five was subtracted to arrive at 20.

$5 \times 5 = 25 - 5 = 20$

$20 + 5 = 25 - 5 = 20$

Figure 6.9: Ruth's students' (S3 and S4) workings on Question 5

The most common error types were process skill (6) and transformation (5) errors. Table 6.9 presents the summary of all initial errors observed.

Table 6.9: Ruth's students' responses and initial errors on post-test symbolic questions (n=4)

Questions	Students' responses to six mainly symbolic questions							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
Symbolic								
1	1	-	-	-	3	-	-	3
2	2	-	-	2	-	-	-	2
3	3	-	-	-	1	-	-	1
4	4	-	-	-	-	-	-	-
5	1	-	-	-	2	-	1	3
6	-	-	-	3	-	-	1	4
Overall	11	-	-	5	6	-	2	13

R- Reading, C- Comprehension, T -Transformation, P- Process skills, E- Encoding, CE - Carelessness

Key Finding 6.12
The transformation and processing errors made by Ruth's students on the post-test symbolic questions were related to algebraic misconceptions of: the letter is a detachable object, the letter has a fixed value of one, conjoining of terms, inability to perform inverse operations with larger divisors and using the equal sign as a signal to act. No comprehension errors were identified.

Worded Questions: 7–15

7. Sola has *y* bananas and Peju has *x* bananas. Peter counts the number of bananas each of them have and finds they are the same. Sola said you could write this as *y*=*x*, but Peju said that *y* and *x* are different letters and so cannot be the same. Who do you think is correct?
8. Mary has *m* oranges and Bisi has three more oranges than Mary. How many oranges does Bisi have?
9. A ball costs ten naira and a shirt costs *y* naira more than the ball. How much does the shirt cost?
10. What is the number that is four less than *x*?
11. There is a *x* number of pencils in a packet. A girl has three packets of pencils and gives her friend five pencils. How many pencils does she have remaining?
12. If *p* is the number of plates and *c* is the number of cups, write in algebra: There are four more plates than cups.
13. Write in algebra: There are five more goats than dogs.
14. Write in algebra: There are twice as many books as pens (let *b* be the number of books and *p* be the number of pens).
15. If *b* is the number of boys and *g* is the number of girls, write in algebra: There are three boys for every girl.

Figure 6.10: Worded post-test questions

The three errors that occurred with Question 7 resulted from the letters being processed alphabetically instead of seen as quantities. The common reason was

S2: They are not the same letter and they are different and they cannot be equal to each other. But x can be equal to x and y can be equal to y but y cannot be equal to x

S4: They are not the same letters (Student interview, 1/7/11)

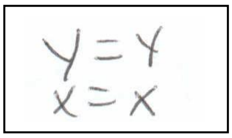


Figure 6.11: Ruth’s student (S2) response on Question 7

This work sample shows S2’s correct thinking. However, this notion is carried over incorrectly to the conclusion that it is never true also that $x = y$.

The Researcher further asked S2 “When it says that they are the same, what does that mean”? The reply was:

S2: Even though his mangoes and Sola’s mangoes are the same, but they are not of the same letter. Just like -2 and 2, they are not the same, so $y = x$, they are not the same. (Student interview, 1/7/11)

Questions 8 and 9 had three transformation and three process errors. Two students (S2 and S3) transformed the ‘more’ in Question 9 mathematically as multiplication and subtraction to obtain “y – 10” and “10y” respectively. Others chose specific values of two and three to represent the algebraic letter *‘m’* in Question 8 and processed the answers as “five” and “six oranges” respectively. A student (S1) after substituting and obtaining five oranges continued, and conjoined this by saying “but when they did not put the amount, we now put 3m”. (Student interview, 1/7/11)

Questions 10 and 11 jointly had four transformation and three process skill errors. The word ‘less’ in Question 10 was transformed to the inequality sign by one student and two others represented it as a division.

S1: Less than, this is the sign (<) and this is four. The number that is less than x. Because there is no more letter or number there, we have to write $x<4$.

S2: It is x divided by four because when they say less, they mean divide it.

To process the two questions, specific values were used.

S4: This x could be any number...I’ll choose 10; so you minus four from 10, it will be six.

S3: We are going to say three times three, it will give nine. Then we subtract from it that five, so it is going to give you four.

S2: In a packet there are 12, so now in three dozens, 12 pencils each. So if you take five out of one packet... (Student interview, 1/7/11)

There were no correct answers for Questions 12 to 15 but there were more transformation errors (9) than comprehension errors (7). The students interpreted the questions as asking them to find the quantities of the two objects in the questions. For example,

> S3: There are five more goats than dogs, so the goat is going to be 10 and the dogs will be 5.
>
> S1: We are going to assume..four is going to stand for the number of cups,…to get the answer for plates we add this four, that is four more than plates, it will give us eight. (Student interview, 1/7/11)

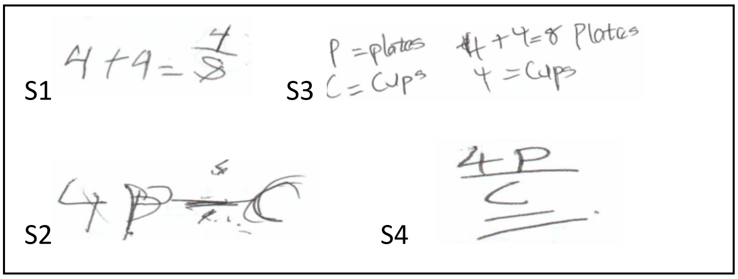


Figure 6.12: Ruth's students' (S1 – S4) workings on Question 12

Those who transformed misconceived the letter to be shorthand for the object. S2 said: "I shall put an algebraic letter to signify that this is dog and this is goat". The students continued with a literal translation of the question to obtain algebraic expressions and fractions such as "5*g* – *d*", 4*p* – *c*", "3*b* +*g*" and "4*p*/*c*", already identified as equation-construction errors (Student interview, 1/7/11).

The errors made by the four students is summarised in Table 7.10. The most common error types were Transformation (16), Process skills (9) and Comprehension (7) errors.

Table 6.10: Ruth's students' responses and initial errors in post-test word problems (n=4)

Questions	Students' responses to word questions							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
7	1	-	-	-	3	-	-	3
8	1	-	-	1	2	-	-	3
9	-	-	-	2	1	1	-	4
10	-	-	-	3	1	-	-	4
11	-	-	-	1	2	-	1	4
12	-	-	-	4	-	-	-	4
13	-	-	2	2	-	-	-	4
14	-	-	3	1	-	-	-	4
15	-	-	2	2	-	-	-	4
Overall	2	-	7	16	9	1	1	34

R- Reading, C- Comprehension, T -Transformation, P- Process skills, E- Encoding, CE - Carelessness

Key Finding 6.13
Ruth's students when working on the post-test word problems transformed 'less' as a division or an inequality sign; the letter was used as shorthand for an object, leading to literal translations and the students calculated specific quantities for the two pronumerals instead of establishing relationships between them.

Changes in Ruth's Students' Performances and Error Types

This study sought to examine the impact of the professional learning intervention on students' success in algebra problem solving. To do this, the students' pre and post-test performance and error analyses were compared.

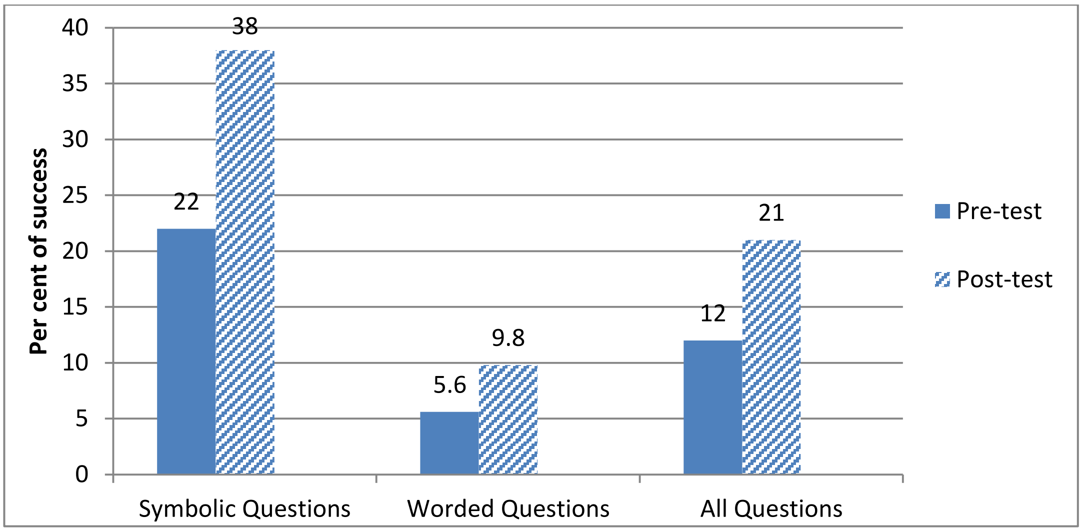


Figure 6.13: Ruth's students' performance on pre- and post-tests

There was a slight change in the types and proportions of errors made by the four interviewed students on the algebra post-test. Comprehension difficulties reduced by 44% but transformation remained the greatest challenge. Table 6.11 presents this information.

Table 6.11: Per cent of Ruth's students' errors before and after intervention (n=4)

	Per cent errors	
Error type	Pre-test	Post-test
Reading	0	0
Comprehension	27.4	17.0
Transformation	37.2	44.7
Process skills	21.6	29.8
Encoding	2.0	2.1
Carelessness	11.8	6.4

There were increased process skill errors since the four students' ability to move beyond the prior steps of comprehension and transformation enabled them to engage in mathematical

processing of the questions. Transformation and processing errors jointly accounted for 76.6% of all errors. Figure 6.14 shows the error pattern across the tests and text formats.

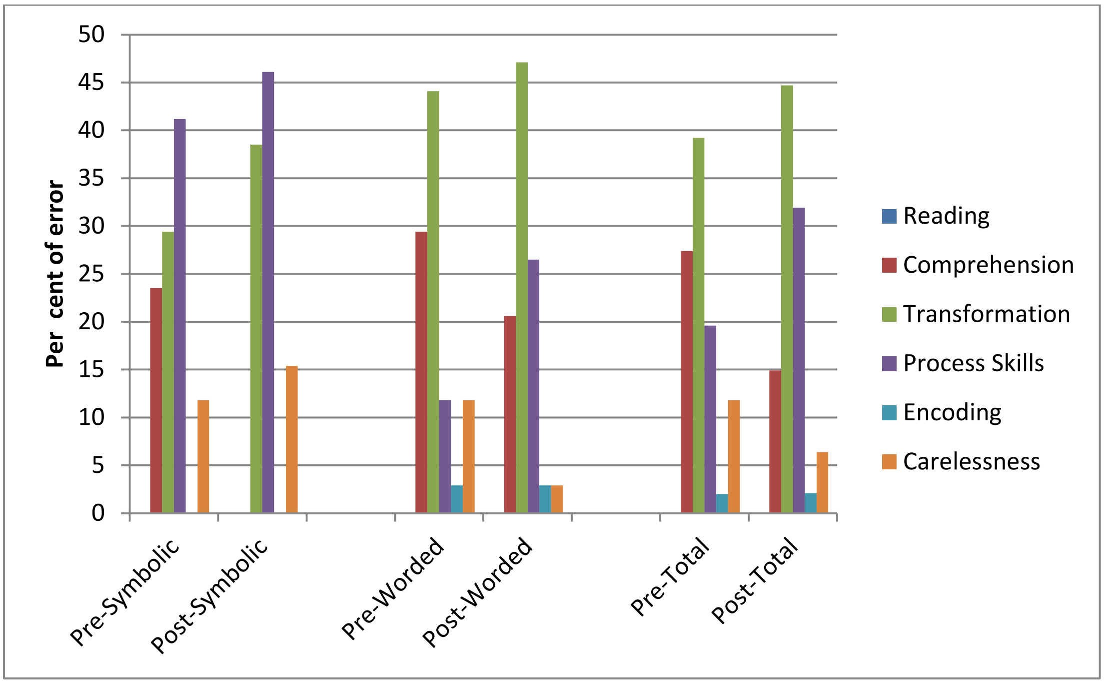


Figure 6.14: Distribution of errors on Ruth's students' pre- and post-test

Key Finding 6.14 Ruth's students overall success rate increased by 9%; success on symbolic and word problems increased by 16% and 4.2% respectively. The frequency of comprehension errors was reduced by 44% and there were consequent increases in transformation and processing errors.

Summary

Before the professional learning, Ruth believed that teaching should be interactive, focused on the student who is actively involved in problem solving during the lesson. However, she also indicated that students did not like asking questions. Whole-class mathematical discussions in her class were few, although she asked many questions in class. By the end of the intervention period, Ruth's classes were more interactive, with students asking questions and whole-class discussions taking place. Ruth had gained more confidence about her algebra knowledge, students' misconceptions, and the use of language-based approaches.

There was a significant improvement in Ruth's students' algebra performance and a decrease in the proportion of comprehension errors. Algebraic misconceptions about the letter as an

object, inappropriate conjoining of terms and the inability to do inverse operations remained.

The next case studied was also in a private school but with fewer students.

CHAPTER SEVEN: CASE STUDY TWO - DOROTHY'S CLASS

One of the teachers who was very interested in the professional learning program and indicated willingness to serve as one of the four case studies was Dorothy (pseudonym). In this chapter, Dorothy's knowledge and beliefs about mathematics, especially algebra, her teaching practice, and her students' difficulties with algebra before and after the professional learning intervention are described. This description provides information about the impact of the intervention on how algebra is taught and learnt, and the difficulties students encountered in understanding it.

Background

Dorothy, a professionally qualified mathematics teacher in her early 20s had fewer than five years of teaching experience. Her qualification of National Certificate in Education (NCE), majoring in mathematics and integrated science, is the minimum professional qualification and is obtained after three years of post-secondary training in a college of education. Dorothy taught mathematics to JS 1 and 2 students in a co-educational, private secondary school located in a suburb within Ojo Educational zone. Each of her JS 1 classes had fewer than 30 students whose ages were between 10 and 12 years.

Pre-Intervention Beliefs and Practice

Beliefs

Dorothy's responses and self-ratings on the initial questionnaire suggested that she had more of a traditional view of mathematics and how it is learnt. For example, she indicated that every lesson she taught included note writing, worked examples and problem-solving by individual students. Only in a few lessons did her students identify key words or symbols contained in the questions. At the same time, Dorothy agreed with statements like: *students do not like to ask questions in the class*; *mathematics is mainly calculations*; and, *mathematics consists of rules and procedures*. She also believed that students could learn effectively mathematics, especially algebra, if they possessed the "ability to interpret questions,. .form equations from the question.. and solve the problem" (Initial questionnaire). Dorothy reported that the lack of instructional materials hindered her ability to teach mathematics effectively.

Some constructivist beliefs were also indicated in her responses. For example, Dorothy expressed disagreement with statements such as: *not all students can learn mathematics*; and, *if a teacher does not tell students how to solve questions they will make mistakes,* even though she agreed with the statement, *students' mathematics background is often weak*. Dorothy

believed that all mathematical content was connected and that students should know this. During the professional learning workshop discussions, she mentioned that," we should let them know that there is continuity in mathematics" (PL workshop, 29/3/11). By this she meant that JS 1 mathematics was just a 'link in the chain' and that students should be told that their primary mathematics knowledge was foundational and not to be discarded.

Also reflected in the initial questionnaire were high self-efficacy ratings about Dorothy's ability to use questions and discussions to manage talk in the classroom. These ratings were matched by her confidence to teach algebra, a very high self-confidence about knowledge of algebra and an eight out of ten rating for her students' engagement level. With regard to teaching and learning algebra, Dorothy agreed *there are rules in algebra that students have to learn* though they find algebra difficult and that she found it *easier to teach word problems rather than those with mainly symbolic representation*.

Key Finding 7.1 Prior to the intervention, Dorothy was very self-confident about her knowledge of algebra and the use of questions in teaching. She indicated a mainly traditional view of mathematics and teaching, and that she found it easier to teach word problems than symbolic questions.

Practice

One double and two single algebra lessons given by Dorothy were observed by the Researcher before the professional learning program, and specific activities that occurred every two minutes were recorded (See sample in Appendix 18). The general pattern was that as Dorothy explained the concept the students listened and then she wrote notes and worked an example of a problem on the blackboard, asking students questions during the process. The students then copied the notes and the worked example into their note books. More examples were shown, with one or two students given the opportunity to work out the solution on the board, and then she explained the solution process. Following this was individual class work and Dorothy went around checking and giving individual student feedback on some of their solutions. Dorothy then wrote the correct solution on the blackboard accompanied with more explanations, the students copied it into their notebooks and sometimes she gave homework before she closed the lesson. The activities that occurred most frequently were: teacher explaining ideas/concepts, students listening and copying notes, and the teacher asking questions.

Students' knowledge of mathematical vocabulary was important to Dorothy and when she explained a new word, like coefficient, she related its importance to mathematical literacy.

> There are some words that are very common and we use them in mathematics, like coefficient. After this class now, when you will be doing other topics in mathematics, we will be referring to coefficients. We expect you to know it now and you will now be transferring this knowledge to other topics when it comes to that time. So make sure you get it now. (Lesson observation, 22/3/2011)

Dorothy appeared to connect new words with a broader future use so that the students would know its importance and usefulness in mathematical communications. She explained the meaning of words such as algebra, simplify and equation.

Ball et al. (2008) noted that mathematical knowledge for teaching includes teachers' correct use of terms and notations. Although Dorothy identified and corrected some students' misconceptions about conjoining terms, she herself did not seem to be aware of some of the misconceptions about the letter as an object or label. In a lesson, she explained that "the letters in algebra stand for something. You will understand it better when we attach the number to something". Building on this explanation, she simplified $7t - 3t$.

> Dorothy: What should we attach t to?
>
> A student: Table
>
> Dorothy: Table; you have a table before you. Seven tables minus three tables; you know the normal interpretation is seven tables take away three tables. We don't want to use tables now but t. What will be our answer?
>
> Another student: $4t$ (Lesson observation, 22/3/11)

The misconception was observed in another lesson and the names of students used as illustrations have been replaced with pseudonyms.

> Dorothy: You are all familiar with plantains. How many of you eat plantains?
>
> All the students raise their hands up.
>
> Dorothy: Curtis is having four plantains and Pauline is having three plantains. Altogether, how many plantains?
>
> Students' joint response: Seven plantains
>
> Dorothy: We can write this in algebraic form using p to denote plantain. If p represents plantain, Curtis is having four plantains and Pauline is having three plantains. If you want to use p to represent plantains, then we can say Curtis is having four p instead of writing plantain. That means we are using letter p to represent plantain. Pauline is having three p which stands for three plantains. Now, altogether how many plantains are they having?
>
> Students' joint response: Seven plantains.

Dorothy: Similarly in algebra, we can have four *p* plus three *p,* and what will it give you?

Students' joint response: Seven *p*

Dorothy: It's still the same thing. It is similar to you adding your plantains together….. to understand problems in algebra is to connect it to something around you. Once you connect it to something around you, it will be easy for you to understand. (Lesson observation, 24/3/11)

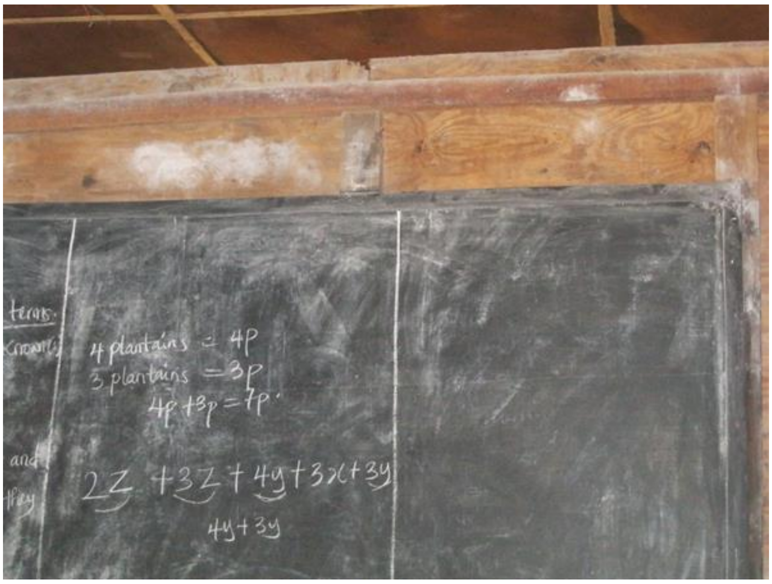


Figure 7.1: Dorothy's board writings

The observed classes lend support to a traditional teaching approach indicated in Key Finding 7.1 and portrays some incorrect usage of the algebraic letter.

Key Finding 7.2 Before the professional learning program, Dorothy's classroom teaching involved a large proportion of teacher talk and explanation. She explained the meaning of new mathematical terms to her students and was observed using the letter as shorthand and as an object during teaching.

Post-intervention Beliefs and Practice

Beliefs

Following the intervention, Dorothy seemed to believe that her algebra knowledge was limited. In the final questionnaire on completion of the PL program, she wrote that a "teacher should be trained in the discipline (mathematics) and should be vast in knowledge" in order to teach mathematics effectively. Dorothy's confidence ratings about her knowledge of algebra and that of students' algebra misconceptions dropped also. This change appeared consistent with a slight decline in her strength of agreement with questionnaire statements: *My rich knowledge of mathematics helps me ask the right questions to develop mathematics ideas through discussion*; and, *My rich knowledge of mathematics helps me to respond appropriately*

to students' answers to my questions. Dorothy's initial agreement with *I find it easier to teach algebra word problems than those with symbolic notations* changed to a disagreement in the final questionnaire.

At the end of the teaching period following the professional learning program, Dorothy's beliefs still reflected a general high self-efficacy for managing classroom talk through questions and discussions; however, her position on some traditional beliefs appeared to have changed. For example, her prior strong agreement with, *Students' don't like to ask questions in the class* became a strong disagreement. Dorothy indicated that every lesson she taught, students were asked to identify what they understood from the blackboard questions and how they planned to solve them. Her survey response also indicated an increase in her reported use of: *having students reading aloud the question*; and, *asking students to identify key words and symbols in the question.*

Key Finding 7.3 After the professional learning, Dorothy's reported beliefs suggest a reduced self-confidence about her knowledge of algebra, students' algebra misconceptions and the use of questioning. She disagreed with her initial views about word problems being easier to teach and students not wanting to ask questions.

Dorothy believed that language is very important in the teaching and learning of mathematics and should be seen to relate to real-life contexts. While sharing algebra teaching experiences with other teachers on the third day of the PL workshop, she said that "language is not isolated but transferred to other life activities". Dorothy described how she helped her students relate new words they encountered in mathematics and word problems to real-life.

> Once you know you are using the word for the first time with that set of students, you should explain the meaning, so that when next you want to use the word, you ask them…explained to you a few weeks ago. Sometimes we assume that they know the meaning of it and the word can be strange to them. I had to explain the meaning …if it is something they can relate to their day to day life. For example,… So I told them, henceforth, I want you to be using that word when you are communicating at home, so that it will become part of you and when you get to the senior….you will still come across…..so they will become familiar with it. (PL workshop, 6/7/11)

Her knowledge of her students' ability appeared to determine the extent of the content that Dorothy taught them. For example, she considered that there was too much content for the JS 1 students in the scheme of work.

> To me, it's too much. We introduce it to them in JS1, the collection of terms, like terms, use of brackets,..I skip it but it's in our scheme of work. So many things at once….I did not teach my students ……they will not understand anything. (PL workshop, 6/7/11)

Key Finding 7.4
Dorothy recognised the importance of English and mathematical language to word problems and its application to other real world situations after the PL. However, she believed that too many new terms were being introduced in the JS 1 scheme of work.

Practice

After the professional learning, Dorothy was observed by the Researcher three times, consisting of one double and two single lessons, over a six-week period. Specific activities that occurred within a two-minute interval were recorded and the most frequent activities were: teacher explaining, teacher use of questions, students asking questions, and explaining. After an initial explanation of the concept, Dorothy wrote an example to be solved on the board, read it aloud, and asked further questions to direct the students' thinking.

She often asked students to guess the unknown value or used a symbol to represent the unknown value, or both were used before generalizing with the algebraic letter when solving word problems. The guessed values are substituted into the question and checked for correctness before she reverted to the algebraic letter as the unknown value. Dorothy wrote, *What number will you multiply by three; add two and the final answer will be 11*? After ascertaining the second suggestion of three was correct, the unknown number was replaced by a box before she continued.

> Dorothy: What sign is between the three and the box?
>
> Student 1: Multiplication
>
> Dorothy (replaces the box with *x*): Can you tell me what is standing in the place of the box now?
>
> Student 2: A letter, x
>
> Dorothy: Our aim is to determine the value of the letter that we used the box to represent. That is, what number has now been replaced with that letter *x*.
>
> Another student is then called to complete the computation. (Lesson observation, 24/5/2011)

Dorothy was careful and wanted her students to interpret and translate word problems correctly. The students were to solve, *I added seven to a certain number and the result is thirteen, what is the number?* She noticed that most of the students' transformations were of the form $7 + x = 13$, and commented.

> The question says, *I added seven to a certain number*, not that *a certain number was added to seven*. Though she is correct with the answer, but I want it to be put in a correct way. You must interpret your question correctly. You are able to get the same answer because you have the addition sign here. But if you have the subtraction, it is not going to give you the same answer. (Lesson observation, 5/4/11)

Dorothy wanted the students to write the transformation as $x + 7=13$ and for them to carefully read and understand the word problem because of the operational signs that may be involved within the context of the problem, not just to select the data haphazardly.

Dorothy adapted some of the Newman questions in her lessons and expressed concern about the importance of correct interpretation of word problems. For example, after writing the question, *when a certain number is added to ten, the result is twenty five. What is the number*? She read it aloud and then continued:

> Dorothy: Two things are important. The first thing is you interpret the problem; the second thing is you solve. First step is you denote the number with a letter. Let that letter represent the number. Listen everybody, pay attention. We want to interpret what we have here. It means we have ten and a certain number was added. In other words, what do I add to ten to get 25? I don't want to write a certain number in words. What can I write instead?
>
> Students: A letter
>
> Dorothy: Denote the number with a letter. What letter do we use?
>
> Student 1: n
> Dorothy: (writes n) is added means what?
>
> Many students: plus
>
> Dorothy: The result is what you get, the answer you get when you equate it (writes 10 + n = 25). What do we do to have only the letter on one side? What number will you add to 10 in order to get 25? I know you will tell me the answer is 15 but I want you work it. (She calls a student) What can you do to both sides in order to get the certain number alone on this side?
>
> Student 2: Subtract ten from both sides
>
> Dorothy then called another student to the board to complete the process, write the answer and explain to the class. (Lesson observation, 24/5/11)

Key Finding 7.5
Dorothy was observed using the letter as a quantity after the professional learning. She tried to ensure that the students correctly interpreted and transformed word problems. More students were involved in asking and answering questions in the class.

Changes in Dorothy's Beliefs and Practice

There was some change in the type and frequency of questions asked by Dorothy in class with more concern that students should correctly understand and interpret questions. She taught with a mainly traditional approach but emphasised correct understanding and interpretation of algebra questions; students' engagement through questions was observed to increase. She wrote that mathematical talk "helps to identify areas of students' difficulties and misconceptions and promotes students understanding" (Final Questionnaire). Dorothy's use of the Newman questions did not often include students reading aloud the algebra question to be solved, as she did the reading most times.

A realization of the important fact that the algebraic letter is a quantity and not an object, and teaching this, seems to be an ongoing challenge for Dorothy, in that she still made errors herself. She described what happened during one of the lessons that the Researcher did not observe.

> We were told not to use the fruit salad approach…you now tell them *x* and *y* represent quantities and these quantities can be the same in some instances. They asked a question like…in some instances, *x* can be two, *y* can be two. So if the two of them are of the same quantity, they should be able to be added together. They will ask questions like that and that's why I was forced to still use that fruit salad when teaching them. I just had to tell them …At that junction, myself I was confused. (PL, 6/7/11)

Since the knowledge that the letter was a quantity and not an object was something that was new to Dorothy, she still erred while attempting to help her students understand the variable concept.

Key Finding 7.6 The development of Dorothy's knowledge of teaching with the letter as a quantity is ongoing. She nevertheless has developed some awareness that a letter as an object is a misconception. She also used some of the Newman questions in her lessons.

Students' Algebra Pre-test Performance

All of the students in one of Dorothy's JS 1 classes completed the algebra pre-intervention test, which comprised 15 questions containing nine word problems and six mainly symbolic questions. Each correct working and response was scored one point, giving a maximum of 15 points on the test. The highest total score was eight, obtained by one person, while half of the students answered only one question correctly. Tables 7.1 and 7.2 present the performance of the students generally and on each question respectively.

Table 7.1: Dorothy's students' pre-test total score (n=26)

	Total score on pre-test /15								
	0	1	2	3	4	5	6	7	8
Number of students	5	13	4	2	1	0	0	0	1
Per cent	19.2	50.0	15.4	7.7	3.8	0.0	0.0	0.0	3.8

About 85% of the students correctly answered no more than two of the 15 questions while only one person correctly answered one-third of the questions.

Table 7.2: Number of Dorothy's students with correct answers in each pre-test question (n=26)

	Question format														
	Symbolic						Worded								
Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of students	0	9	3	7	1	0	7	4	4	1	2	0	0	0	1
Total	**20/156**						**19/234**								

There were 39 correct responses out of a total of 390 (15x26) possible correct responses. This represents a 10% overall success in the pre-test comprised of 12.8% success rate on symbolic questions and 8.2% on word problems. The question with the highest number of correct answers required the simplification of an algebra expression, while the most correctly answered word problem did not require any mathematical operation.

Key Finding 7.7
Pre-test results showed that 85% of Dorothy's students correctly answered no more than two questions. There was a 10% overall success rate comprising 12.8% on the symbolic questions and 8.2% on the word problems.

Students' Pre-test Newman Interview Results

Since the focus of the study was not the students' scores, but identifying the difficulties they encountered and why the mistakes were made, Newman's (1983b) language-based interview protocol was used to identify the cause of the initial error made by the students. Dorothy selected four students (with codes S1, S2, S3 and S4) who were interviewed by the Researcher on all questions they wrongly answered. After completing the Newman interview protocol, further questions were asked in some cases based on the students' responses to obtain more information.

Two of these four students gave no correct answers and the other two only gave one correct answer each the second time they completed the test as required by the Newman protocol.

As a result of this, two students were interviewed on the 15 questions and the remaining two students were each interviewed on 14 questions. Although the focus of the study was on word problems, the symbolic questions provided a basis for comparison of students' language-based errors and algebra misconceptions. Of the 58 incorrect responses, 45% were comprehension errors and 24% were transformation errors. More comprehension errors were identified with word problems while more transformation errors were identified with symbolic questions. (See Table 7.3)

Table 7.3: Per cent of error types made by Dorothy's students' on the algebra pre-test (n=4)

Initial error	Question representation		
	Symbolic	Word	All questions
	Per cent	Per cent	Per cent
Reading	9.1	0	3.4
Comprehension	13.6	63.9	44.8
Transformation	40.9	13.9	24.1
Process skills	18.2	8.3	12.1
Encoding	0	2.8	1.7
Carelessness	18.2	11.1	13.8

There was a difference in the error pattern when the question representation was considered. The students had more transformation errors (41%) than process skills (18%) and comprehension (14%) errors on the symbolic questions, unlike in the case of the word problems where there were more of comprehension errors (64%) than transformation (14%) and process skills (8%) errors. More students progressed through the comprehension step successfully on symbolic problems, so that they were able to attempt the transformation step and had the opportunity to make transformation errors.

Key Finding 7.8
The Newman error analysis of Dorothy's students' pre-test wrong responses showed that 45% of the errors were comprehension-related and 24% were transformation-related. Comprehension errors (64%) were most common on the word problems while transformation errors (41%) were most common with the symbolic questions.

The questions were analysed on the basis of the text-format, that is, the six questions that are mainly symbolic and the nine questions that are word problems. The set of questions are first stated and then followed by a description of the students' responses to the interview questions as it relates to the language-based errors and algebra misconceptions.

Symbolic Questions: 1-6

1. Simplify as far as possible $1 + x + x$
2. Simplify as far as possible $3m + 5n + 4m + 6n$
3. $y \times y \times y =$ ....
4. Find the value of x: $7x = 21$
5. Find the value of x: $2x – 2 = 10$
6. Find the value of x: $21x = 7$

Figure 7.2: Symbolic pre-test questions

Questions 1 and 2 examined the students' use of the knowledge of the algebraic letter as a quantity, knowledge of addition of like terms, and the knowledge that the final answer may be an expression which cannot be 'gathered together' /conjoined. Half of the six errors that occurred during the interview were transformation errors:

> S1: 1 plus *x* plus *x* will be 1*x* plus 1*x* plus 1*x*,
>
> S4: 1 plus *x* plus *x* equals 1*x* because it has two *x*'s so we take one of them,
>
> S4: Because *m* and *n* shows two times, so we write *m* and *n*; assuming it is only *m* that shows, we will write only it. So three plus five plus four plus six equal 18*mn* (Student interview, 25/3/11)

Question 3, being syntactic in nature, investigated the student's ability to use the multiplication operator to write the answer as a product and not a sum. Two students had reading errors and read the multiplication sign as the alphabet letter x leading to the solutions of $3y2x$, and x^2y^3 , after wrong operations with addition and multiplication respectively. The other two students (S1 and S4) said "put one in front of *y"* and said 1*y* x 1*y* x 1*y* equals 3*y*. This is an algebra misconception that occurs when the terms are incorrectly added instead of multiplied (MacGregor, 1997).

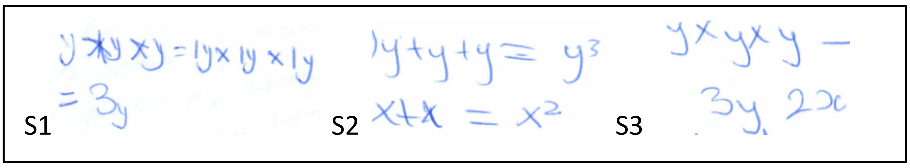


Figure 7.3: Dorothy's students' (S1 – S3) workings on Question 3

Questions 4 to 6 required knowledge of the concept of equality and how to balance an equation in order to obtain a specific unknown value. A quarter of the 12 errors were comprehension-related and half were transformational. One student said the meaning of value was "plus" and two of them did not understand Question 6. In an effort to transform,

three students detached the letter from the coefficient in Question 5, while in Question 6, the idea of a bigger divisor seemed impossible to two students. These were expressed in the students' replies to the Newman interview question: How will you find an answer to this question?

> S1: we will add the two's to get four then 10 minus four,
>
> S3: we say 10 – two equals eight, then eight divide two equals four;
>
> S2: we cannot say that 21 should divide seven. No, we can't say seven divided 21, so we say 21 divide seven to give us three…If I don't do it like this, I can never get the answer. (Student interview, 25/3/11)

The most common error types were transformation (9), process skills (4) and carelessness (4). These data are summarised in Table 7.4.

Table 7.4: Dorothy's students' responses and initial errors on pre-symbolic questions (n=4)

Questions	Students' responses to six symbolic questions							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
1	-	-	-	2	1	-	1	4
2	2	-	-	1	-	-	1	2
3	-	2	-	-	2	-	-	4
4	-	-	-	1	1	-	2	4
5	-	-	1	3	-	-	-	4
6	-	-	2	2	-	-	-	4
Overall	2	2	3	9	4		4	22

R – Reading, C – Comprehension, T – Transformation, P - Process skills, E – Encoding, CE - Careless

Key Finding 7.9
The transformation and process skill errors made by Dorothy's students in the pre-test symbolic questions appear to result from misconceptions about the letter as a moveable object, the letter having a unit value, a confusion of product and sum, and an inability to use a larger divisor or balance an equation.

The students' responses and the errors identified on the nine word problems are described below.

Worded Questions: 7-15

7. Sola has x bananas and Peju has p bananas. Peter counts the number of bananas each of them have and finds they are the same. Sola said you could write this as x=p, but Peju said that x and p are different letters and so cannot be the same. Who do you think is correct?
8. Mary has x oranges and Bisi has four more oranges than Mary. How many oranges does Bisi have?
9. A basket costs eight naira and a bag costs c naira more than the basket. How much does the bag cost?
10. What is the number that is five less than x?
11. There is a b number of sweets in a packet. A girl has two packets of sweets and gives her friend six sweets. How many sweets does she have remaining?
12. If d is the number of dogs and c is the number of cats, write in algebra: There are four more dogs than cats.
13. Write in algebra: There are three more caps than hats.
14. Write in algebra: There are twice as many pencils as biros (let p be the number of pencils and b be the number of biros).
15. If s is the number of students and t is the number of tables, write in algebra: There are three students for every table.

Figure 7.4: Worded pre-test questions

Generally, comprehension was a greater problem with the word problems.

To solve Question 7, the students only required the knowledge of the letter as a quantity because the question did not involve any mathematical operation. Three students viewed the letter literally as a member of the alphabet instead of a quantity. One student (S2) in particular had an additional view of alphabetical ordering. He initially said "since Sola has *x* and Peju has *p*, we can say *x* is equal to *p*. So we can say they are the same". After thinking for about five seconds, he continued

> I think it's true what Peju said because they are different alphabets. If *x*=*x*, then we say they are equal. There are still some letters between *x* and *p.* (Student interview, 25/3/11)

With Questions 8 and 9, the word 'more' has to be transformed to the additive operator and the given quantities added to obtain '*x* +4 oranges' and '8+*c* naira' respectively, which are expressions. There were three comprehension and three transformation errors. Comprehension errors occurred on questions 8 and 9 as shown in the students' statements:

> S2: we have known the number of Bisi's, then we are looking for Mary's own.
>
> S1: BIsi has 4 oranges, Mary has x oranges.
>
> S1: they told us the amount of the basket but they did not tell us the amount of the bag. (Student interview, 25/3/11)

One student did not know what to do after correctly explaining the two questions, while another (S1) transformed the word ‘more’ in Question 9 as subtraction and said, “If the basket cost 8 naira, then the bag will cost 8-c”. This was after correctly stating the meaning of the question. The Researcher asked “Why did you subtract?” and the response was “Because it is more”. The encoding error occurred when a student (S3) said “the answer is 4 plus *x* oranges” but went ahead to write 4+*x* =4*x* (the conjoining error). (Student interview, 25/3/11)

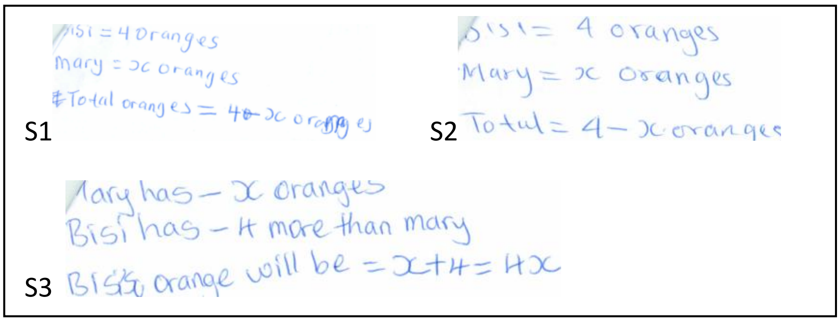


Figure 7.5: Dorothy’s students’ (S1 – S3) on workings on Question 8

Question 10 and 11 involved using subtraction and the algebraic letter in different ways. Question 10 required transforming the word ‘less’ into subtraction to generate ‘*x*-5’ as the answer. None of the students gave the correct answer but two of them said it was four. All of them had comprehension errors and expressed the meaning of ‘less’ to be: “lower than” or “we look for a number that is not more than five”. Two students explained further:

> S2: it means the number that is below. There are many numbers that are below but I don’t know the particular one we are to put in this place.
>
> S1: We have too much of numbers that can be there. (Student interview, 25/3/11)

The responses indicated their inability to use a letter as a specific unknown value and an inability to use the letter as a generalized number (Kuchemann, 1981). With Question 11, initial errors of ignoring one packet of sweets were rectified by two students while another multiplied the two ‘*b*’s’ instead of adding them.

Questions 12 to 15 required generating a relationship between two quantities expressed in the form of an equation. Some of these questions were similar to some questions in a Nigerian JS 1 textbook commonly used nationwide. None of the students appeared to have understood any of the questions and there were no correct answers. They could not explain the meaning of the word ‘algebra’, and the general meanings the students gave for questions were of the form: “we are to look for the number of dogs” and “we are to find the number of caps” which indicated that only one object was being considered by the students. The second object was ignored in most cases and the students’ solutions were mainly expressions involving one of the

variables (also known as pro-numerals) paired with the given quantity close to it in the question. A student (S3) who attempted an explanation using both objects in Question 15 said "we are to look for the number of students sitting on the whole table" (Student interview, 25/3/11).

Table 7.5 presents the summary of the identified initial errors on the nine word problems. The most common errors types were comprehension (23), transformation (5) and careless errors (4).

Table 7.5: Dorothy's students' responses and initial errors on pre- word problems (n=4)

Questions	Number of responses to worded questions							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
7	-	-	-	-	3	-	1	4
8	-	-	2	1	-	1		4
9	-	-	1	2	-	-	1	4
10	-	-	4	-	-	-	-	4
11	-	-	-	2	-	-	2	4
12	-	-	4	-	-	-	-	4
13	-	-	4	-	-	-	-	4
14	-	-	4	-	-	-	-	4
15	-	-	4	-	-	-	-	4
Overall	-	-	23	5	3	1	4	36

R-Reading, C-Comprehension, T-Transformation, P-Process skills, E- Encoding, CE-Careless

Key Finding 7.10
Comprehension errors on the pre-test word problems occurred with the meaning of the word 'algebra', and the mathematical interpretation of the words 'more' and 'less'. None of the students understood the meaning of the four questions that had two pro-numerals; all the meanings given were limited to finding the total quantity of one of the pro-numerals. Transformation and process skill errors resulted from misconceptions of the algebraic letter as a specific unknown value or as an alphabet, and that different letters cannot have the same value.

Students' Algebra Post-test Performance

After a six-week teaching period, a post-test was administered to the class. It was similar and parallel to the pre-test except that names, quantities, letters and the numbering of the questions differed. Care was taken to ensure place values were left intact, that is, one-digit quantities in the pre-test remained one-digit in the post-test. The exact pre-test questions were not used because some of the students had become familiar with them through the Newman interview process. Too much familiarity might intervene in the true assessment of the students' understanding of the questions.

In the post-test, 57.6% of the students scored a total of three or more and the Wilcoxon signed rank test showed that there was a significant improvement (z = -3.518) in the students' general performance though the mean score was still low. The total of 75 correct responses in the class represented 19.2% success, an increase of 9.2% over the pre-test overall success rate. There were more correct answers for the symbolic questions than the word problems. Tables 7.6 and 7.7 present the details.

Table 7.6: Dorothy's students' post-test total score (n=26)

	Total score on post-test/15						Pre-test		Post-test	
	1	2	3	4	5	6	Mean	SD	Mean	SD
Number of students	4	7	6	7	1	1	1.50*	1.655	2.88*	1.306
Per cent	15.4	26.9	23.1	26.9	3.8	3.8				

*Wilcoxon Signed ranks, p<.01

The lowest post-test score was one, in contrast to the pre-test where five students scored zero.

Table 7.7: Number of Dorothy's students with correct answers in pre- and post-test question (n=26)

	Question number and representation format														
	Symbolic						Worded								
Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Pre-test															
Number of students	2	9	16	16	7	0	9	4	4	3	4	0	0	0	1
Total correct answers	**50/156**						**25/234**								
Post-test															
Number of students	0	9	3	7	1	0	7	4	4	1	2	0	0	0	1
Total correct answers	**20/156**						**19/234**								

The 19.2% success comprised 32.1% of the 50 correct responses on symbolic questions and 10.7% of the 25 correct responses on word problems. Questions 3 and 4 had the highest number of correct answers but in the pre-test it was Question 2 that was answered correctly most often. The easiest word problem remained question 7 which did not require any mathematical operation. No correct response was given on most questions having two pro-numerals or requiring inverse operation with a larger divisor.

Key Finding 7.11
There was significant improvement and a 19.2% overall success rate in Dorothy's students' general post-test performance. Success on word problems and symbolic questions increased to 10.7% and 32.1% respectively.

Students' Post-test Newman Interview Results

Two of the four students gave four and three correct answers respectively and the other two students had two and one correct answer each. This totalled 50 wrong answers, consisting of 18 (38%) from symbolic questions and 32 (64%) from word problems. (See Table 7.8)

Table 7.8: Per cent of error types made by Dorothy's students' on the algebra post-test (n=4)

Initial error	Question format		All questions
	Symbolic	Worded	
	Per cent	Per cent	Per cent
Reading	5.6	0	2.0
Comprehension	5.6	28.1	20.0
Transformation	38.9	37.5	38.0
Process skills	44.4	34.4	38.0
Encoding	5.6	0	2.0
Carelessness	0	0	0

The comprehension errors reduced to a 20% proportion of all the errors on the post-test. Transformation and process skill errors were the most common error types on both question formats, since greater comprehension successes than on the pre-test created more opportunities for students to move beyond the level of understanding the sense of the questions. The process skill errors were slightly more common (44%) than the transformation errors (39%) on the symbolic questions, while with the word problems there was an almost equal proportion of transformation and process skill errors.

Key Finding 7.12
Dorothy's students post-test interview responses showed fewer comprehension errors than that of the pre-test and there was an equal proportion (38%) of transformation and processing errors. The most common errors made on symbolic questions were process skill errors (44%), and transformation errors (38%) were the most common word problems.

A description of the students' errors on each of the symbolic and worded questions follows.

Symbolic Questions: 1-6

1. Simplify as far as possible $1 + y + y$
2. Simplify as far as possible $4z + 3p + 7z + 2p$
3. $m\times m \times m =$ ....
4. Find the value of x: $6x = 24$
5. Solve for x: $5x – 5 = 20$
6. Solve for x: $24x =6$

Figure 7.6: Symbolic post-test questions

Questions 1 and 2 required the students' knowledge of the letter as a quantity, familiarity with the addition of like terms, and the knowledge that the final answer may be an expression which cannot be 'gathered together'. In answering the questions, four of the seven mistakes made by students were process skill errors that related to using the letter as a moveable object and 'closing' the expression. For example, in Question 1 a student (S2) 'closed' the expression to get 1*yy* instead of 1 +2*y* because "the last terms are the same". Another student (S4) was asked by the Researcher why he picked one '*z*' and one '*p*' to appear in the final answer for the second question; his reply was "because the *z* appears twice, the *p* appears twice so you pick one". (Student interview, 2/7/11)

Only one student (S2) had a wrong answer to question 3 which was caused by a reading error. He read the multiplication sign as the alphabetical *x* and wrote '*mmmxx*' as the final answer instead of m^3.

Questions 4 to 6 required knowledge of the concept of equality and how to balance an equation in order to obtain a specific unknown value. Six of the 10 mistakes made on this set of questions were transformation errors and the remaining four were process errors. Kieran (1992) has already noted that children see the equal sign as a directive to sum or find the total. All the students still used the letter as an object that is detachable from its coefficient in question 5 in order to balance the equation and to find the value of the letter. The Researcher asked two students (S2 and S3) why they detached the letter and subtracted in question 5. They said "if you don't subtract, you can't get it. Subtract five from each that's there and let *x* stand on its own" and "because *x* is here, then you add it". So the two fives were subtracted to isolate the letter, leaving '*x*' on one side, while on the other side the integer five was also utilized to operate on 20 indicating that the two parts were different (Student interview, 2/7/11).

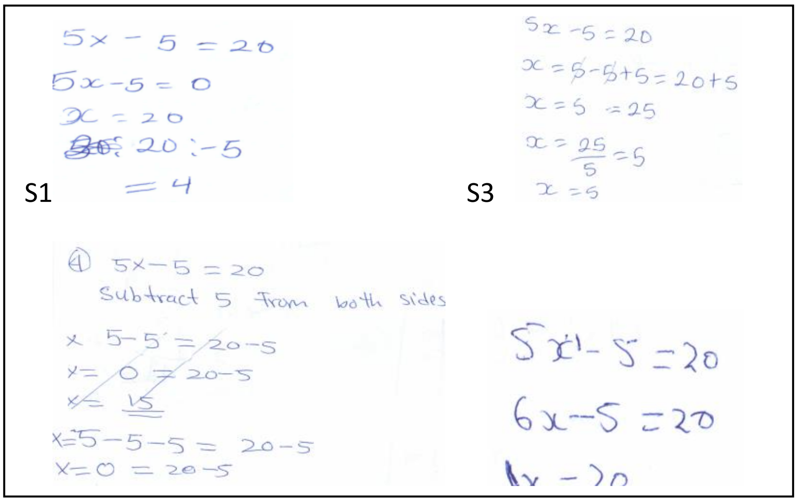


Figure 7.7: Dorothy's students' workings on Question 5

The multiplicative inverse required to correctly answer question 6 was not used. Two students (S3 and S1) seemed to believe a bigger divisor could not be used and reverted to answering it in the form of question 4. Their reasons were "because 24 comes before six" and "six is smaller, 24 is bigger than six" (Student interview, 2/7/11).

A summarised data of the students' errors is presented in Table 7.9. The most common error types were process skills (8) and transformation (7) errors.

Table 7.9: Dorothy's students' responses and initial errors on post-test symbolic questions (n=4)

Questions	Number of students' responses							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
1	-	-	1	-	3	-	-	4
2	1	-	-	1	1	1	-	3
3	3	1	-	-	-	-	-	1
4	2	-	-	1	1	-	-	2
5	-	-	-	2	2	-	-	4
6	-	-	-	3	1	-	-	4
Overall	6	1	1	7	8	1	-	18

R-Reading, C-Comprehension, T-Transformation, P-Process skills, E- Encoding, CE-Careless

Key Finding 7.13
Newman interviews with Dorothy's students on the symbolic post-test questions showed that the use of the letter as a detachable object, the inability to operate with a larger divisor and the use of the equal sign as a prompt to sum up were still evident as misconceptions. There was a reduction in reading and comprehension errors and no careless errors were identified.

A description of the nine worded post-test questions, the four interviewed students' responses and the error analysis is offered next.

Worded Questions: 7-15

7. Sola has *y* bananas and Peju has *x* bananas. Peter counts the number of bananas each of them have and finds they are the same. Sola said you could write this as $y=x$, but Peju said that *y* and *x* are different letters and so cannot be the same. Who do you think is correct?
8. Mary has *m* oranges and Bisi has three more oranges than Mary. How many oranges does Bisi have?
9. A ball costs ten naira and a shirt costs *y* naira more than the ball. How much does the shirt cost?
10. What is the number that is four less than *x*?
11. There is a *x* number of pencils in a packet. A girl has three packets of pencils and gives her friend six pencils. How many pencils does she have remaining?
12. If *p* is the number of plates and *c* is the number of cups, write in algebra: There are four more plates than cups.
13. Write in algebra: There are five more goats than dogs.
14. Write in algebra: There are twice as many books as pens (let *b* be the number of books and *p* be the number of pens).
15. If *b* is the number of boys and *g* is the number of girls, write in algebra: There are three boys for every girl.

Figure 7.8: Worded post-test questions

To solve Question 7, the students required only the knowledge of the letter as a quantity without the need for any mathematical operation. Three students still viewed the letter literally as an alphabetical entity instead of a quantity, with their pattern of response being "they have the same number but they don't have the same letter". The Researcher asked one of them (S3): "What letter should be here (pointing to *x*) for you to say they are the same"? His reply was "*y*" (Student interview, 2/7/11).

With Questions 8 and 9, the given quantities have to be added to get 'm +3 oranges' and '10+y naira' respectively, which are expressions. All the four students had wrong answers and six of the errors were evenly divided between transformation and process errors. The transformation errors were mostly on question 8 and seemed to relate to operating with the letter as two students (S1 and S2) wrote "3-*m*" as the answer while admitting that Bisi has more toys. The three process errors were seen on question 9 and generally related to the students 'closing' the expression as 10*y*. When a student (S1) was asked by the Researcher: "Why not leave it as 10 plus *y*"? He replied "because the addition of these two is 10*y*" (Student interview, 2/7/11). Once more, this is a conjoining error.

Question 10 required transforming the word 'less' into subtraction to generate '*x*-4' as the answer. None of the students gave the correct answer but gave replies like: "they've not given us the number, so *x* will be the final answer" and "4*x* - 4". A student was still seeing 'less' as a

position and when the Researcher reframed the question as “what is the number that is four less than 10”, the reply was “like one is less than 10, up to nine” (Student interview, 2/7/11).

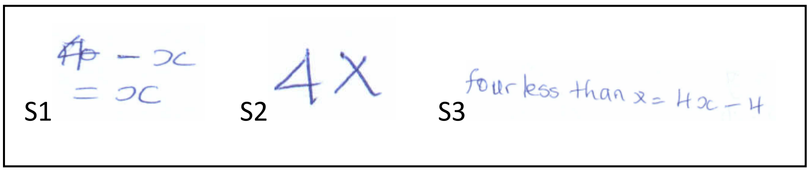


Figure 7.9: Dorothy’s students’ workings on Question 10

Half of the students did not understand Questions 12 to 15 and a few of them had transformation errors. Their lack of understanding of the questions reflected in their responses when asked “What is the question asking you to do?” The general meaning that the students gave to the questions still referred to the finding of the quantity of an item, instead of establishing a relationship between the two quantities of items (pro-numerals) given. For example: “we are to find the number of plates”; “we are asked to look for the amount of cups”; “we are not to solve anything” and “to look for the exact answer” (Student interview, 2/7/11).

In answering Question 13 where no letter was given, one of them (S2) said “in short form in algebraic process, we use *g* for goats, *d* for dog”. Interestingly, despite the fact that questions 12 to 14 stated the letter as a quantity, all of the students referred and used the letters as labels for objects in all of the questions. Three students admitted there were more boys than girls in Question 15 but two of them still did not understand it.

> S1: Boys are three, so we can guess that girls are three. That will be three boys for three girls
>
> S2: Three boys for each girl. Assuming they did not say three boys for each girl, the boys are more already. (Student interview, 2/7/11)

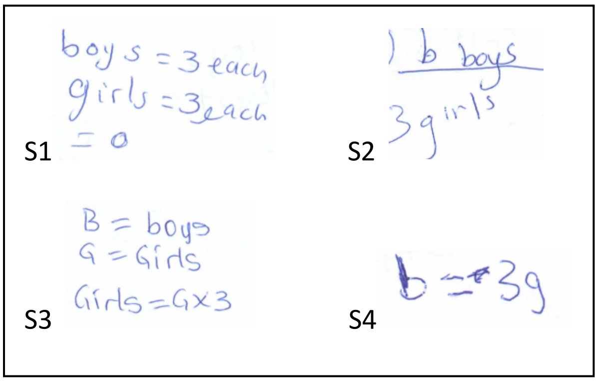


Figure 7.10: Dorothy’s students’ workings on Question 15

The only student (S4) who answered a question correctly, question 15, was asked why it was not replicated with the other questions. The reply was "I have an idea for this one because I am a girl". It is not clear if she really understood the process involved in arriving at the solution.

Data on the students' errors is summarised in Table 7.10. The most common errors were transformation (12), process skills (11) and comprehension (9) errors.

Table 7.10: Dorothy's students' responses and initial errors in post-test word problems (n=4)

Questions	Students' responses to word questions							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
7	1	-	-	-	3	-	-	3
8	-	-	-	3	1	-		4
9	-	-	-	1	3	-	-	4
10	-	-	1	1	2	-	-	4
11	2	-	-	2	-	-	-	2
12	-	-	3	1	-	-	-	4
13	-	-	2	1	1	-	-	4
14	-	-	1	2	1	-	-	4
15	1	-	2	1	-	-	-	3
Overall	4	-	9	12	11	-	-	32

R-Reading, C-Comprehension, T-Transformation, P-Process skills, E- Encoding, CE-Careless

Key Finding 7.14
Dorothy's students' post –test word problem errors stem from the misconception of the letter as: an alphabet, a specific value, a label or object, and the 'closing' of expressions. Some students were still unable to give a correct meaning to some of the questions having two pro-numerals. Three students transformed 'more' and 'less' with a sense of ordering.

Changes in Dorothy's Students' Performance and Error Types

One of the research purposes is to examine the impact of the intervention on students' success with algebra problem solving. To do this, the number of correct problem solutions, the general performances of the class, and identified causes of initial errors before and after the intervention were compared. A significant improvement in general performance was obtained (KF 7.11). Figure 7.11 presents the pattern.

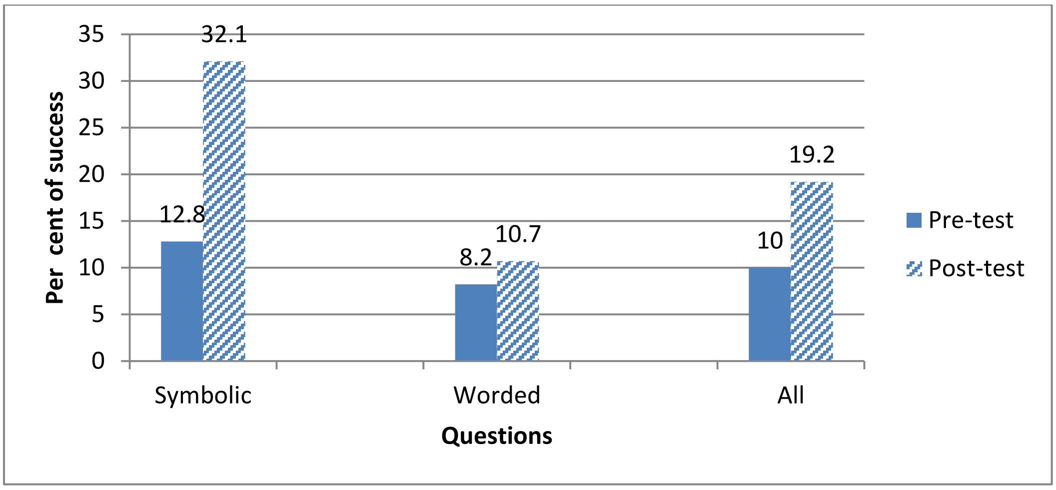


Figure 7.11: Dorothy's students' pre- and post-test performance

The number of correct answers given by the four interviewed students increased from two to 10. Initially, comprehension was the greatest difficulty, but as facility in this improved, the students progressed; opportunities previously not present paved way for transformation and process skill errors to emerge, and they jointly became the greatest difficulties. Table 7.11 illustrates this.

Table 7.11: Per cent of Dorothy's students' errors before and after intervention (n=4)

	Per cent error	
Error type	Pre-test	Post-test
Reading	2.4	2.0
Comprehension	44.8	20.0
Transformation	24.1	38.0
Process skills	12.1	38.0
Encoding	1.7	2.0
Carelessness	13.8	0.0

There was a 13.8% reduction in the total number of errors the students made, and transformation and process skill errors jointly accounted for 76% of the errors from an initial 36%. The graph below (Figure 7.12) shows the pattern of errors on the pre- and post-tests both generally and in relation to the text formats.

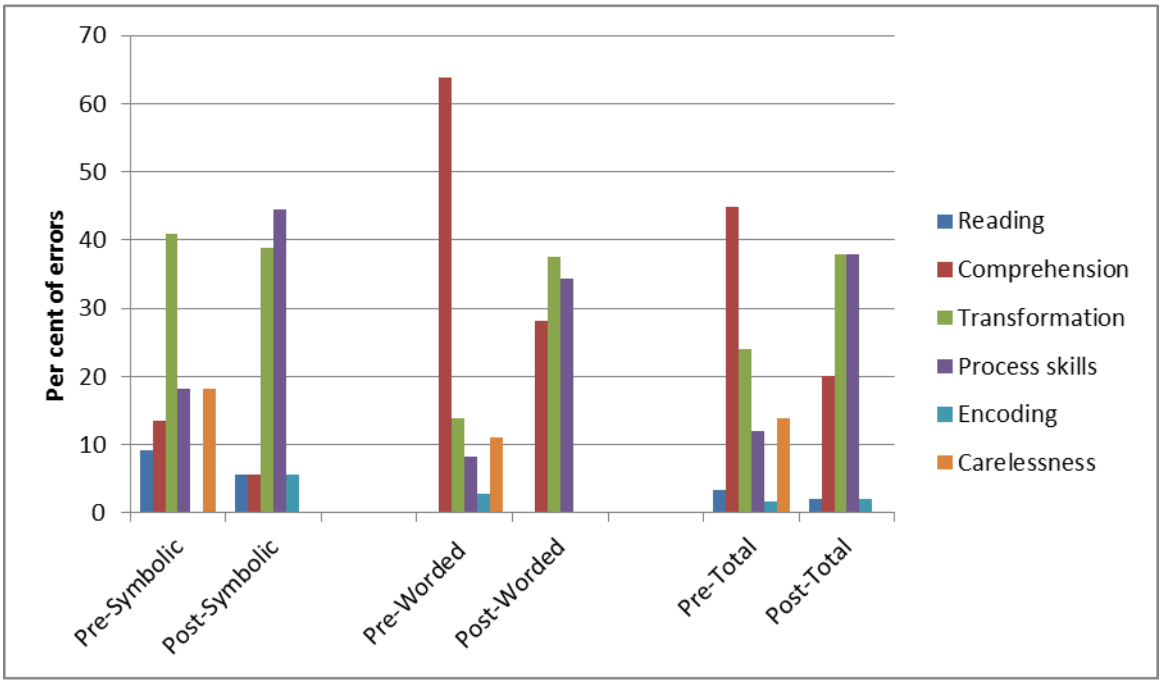


Figure 7.12: Distribution of errors on Dorothy's students' pre- and post-test

On the post-test, there was a 55% reduction in the proportion of comprehension errors, with a similar pattern found on the symbolic and word questions. With a better understanding of the questions, students were able to engage with more mathematical transformation and processing activities. No careless errors were identified from the four students interviewed on the post-test.

Key Finding 7.15
In Dorothy's class overall, symbolic and word problem success rate increased by 9.2%, 19.3% and 2.5% respectively on the post-test. The frequency of comprehension errors reduced by 55% and there were consequently increases in transformation and process skill errors.

Summary

Dorothy's initial portrait was one of a teacher with traditional beliefs, high self-efficacy in the use of questions and high self-confidence about her knowledge of algebra and its teaching. These high ratings slightly reduced after the intervention. Prior to the professional learning intervention, Dorothy was unaware that the use of a letter as shorthand or an object was a misconception. Her students had exhibited difficulties in understanding and solving algebra questions. Post-intervention class observations indicated more students asking questions and participating in problem solving as she placed emphasis on correct interpretation of word problems and the use of a letter as a quantity.

The overall performance of Dorothy's class in the post-test was still low even though there was a significant improvement and a 9.1% increase in rate of success. The four students interviewed jointly had a 55% reduction in comprehension errors on the post –test. This reduction led to increases in transformation and processing errors that in themselves were largely due to students' misconceptions about the algebraic letter, expressions and equations.

The next two chapters are case studies of teachers who taught larger classes within the public school system.

CHAPTER EIGHT: CASE STUDY THREE – JAMIE'S CLASS

In this chapter, an analysis of the teacher's beliefs, knowledge and practice before and after the professional learning intervention, and changes observed, are given. Following this, the students' general algebra performance and the Newman error analysis before and after the intervention period, and identified changes, are described. Direct statements taken from the initial or final questionnaire are written in italics.

Background

Jamie (pseudonym), a male in his early 30s, was very enthusiastic about participating in the study. A holder of the National Certificate in Education, the minimum Nigerian professional teaching qualification, Jamie's subject disciplines are mathematics and integrated Science. He had been teaching mathematics for between six and 10 years but had had fewer than five years of Year 7 teaching experience. Situated in a suburb in Ojo Educational zone, Jamie's school, a public one, has a large population because most parents want their children in schools that are close to their homes. Jamie's eight Year 7 mathematics classes each have an average class size of 70 students, between 10 and 12 years old.

Pre-Intervention Beliefs and Practice

Beliefs

Three things Jamie wrote in the initial questionnaire that constituted challenges to his effective teaching of mathematics were: an unconducive teaching environment, inadequate teaching aids and teaching methods. Jamie believed that availability of teaching aids and teacher guides was necessary for mathematics teaching.

On initial questionnaire statements that related to asking and responding to questions, and students' classroom engagement, Jamie generally had high self-efficacy ratings. However, he rated his students' engagement level to be six out of ten and disagreed with statements in the questionnaire that suggested that he was effective with asking questions that enhance the purpose of the lesson or encourage students to ask questions.

Responses given in the questionnaire suggest that Jamie had a traditional belief and approach to mathematics, its teaching and learning. For example, he agreed with statements such as: *Mathematics is mainly calculations, If a teacher does not tell students how to solve questions they will make mistakes, Not all students can learn mathematics and students do not like to ask questions in the class.* While Jamie admitted that he chose to teach mathematics, he strongly

disagreed with the statement, *I always enjoy my mathematics teaching.* Jamie's indicated daily teaching approaches included writing notes, working examples on the board and problem-solving by individual students.

Jamie was very confident about his knowledge of algebra but he indicated that he found symbolic problems easier to teach than word problems. From the six beginning algebra questions that he was asked to rank in the initial questionnaire, the two word problems had the highest difficulty ranks. The three questions requiring inverse operations were consistently ranked higher than questions not requiring them.

> Key Finding 8.1
> Before the intervention, Jamie believed that the unconducive environment, inadequate teaching aids, and methods all impeded his teaching effectiveness. His self-ratings suggested that he had traditional beliefs about mathematics and its teaching-learning process while he was confident about his ability to manage mathematical talk. Jamie did not enjoy his teaching.

Practice

Jamie's teaching approaches were mainly traditional. His classes started with a revision of the previous lesson, followed by explanations of the concept to be taught. The concept was explained as Jamie worked through a few examples while the students listened and copied the notes into their exercise books. In the two single lessons that were observed, the most frequent activities were: the teacher explaining ideas, students listening and copying notes and the teacher asking questions (See sample in Appendix 19).

Note writing was very important to Jamie. He believed it aided the students' understanding and mentioned this belief in the class.

> I am giving you these notes so you can easily remember all my explanations. You have to read them at home….These notes are very important, so that when you go through them at home you can easily remember…That's why I give you notes, so you can understand. (Lesson observation, 23/3/11)

> I will give you notes, so that when you read them at home you'll quickly understand all what we are saying…it is very important. (Lesson observation, 25/3/11)

An example of Jamie's problem solving is transcribed below. The students were being taught how to write a word problem in algebra. Jamie wrote on the board, *Decrease y by nine*.

> Jamie: This decrease *y* by nine, who can tell me what it means? I want you to change it into a mathematical form. How many of you can solve this? (One person raised his hand) It is only one person. What do you mean by decrease?

Many students: Reduce

Jamie: Now, if I put it as reduce *y* by nine, how many of you can write it mathematically? (A few more hands went up) Tell me what it means mathematically: decrease *y* by nine

Different students: Nine over *y*, nine minus *y*

Jamie: Look at it, decrease *y* by what?

Students: Nine

Jamie: All of you know the meaning of decrease, reduce, which is the same thing as minus. I'm now telling you, decrease y by nine, which means (writes) $y - 9$. That is, the value of *y*, (Is one, said a student in a low voice) reduce it by nine. It simply means *y*-9. Decrease is 'reduce'. (Lesson observation, 25/3/11)

After telling the students the answer, Jamie moved to another example and the students continued listening. He was not observed using the letter as an object.

Key Finding 8. 2 Prior to the professional learning, Jamie displayed a traditional approach to the teaching-learning process and teacher-talk occupied a large proportion of the observed lessons. Student engagement was minimal and he firmly believed that note taking aids understanding.

Post-Intervention Beliefs and Practice

Beliefs

By the end of the intervention period, Jamie's beliefs appeared to have become more language-conscious and sensitive. In the final questionnaire, completed after the intervention period, he wrote as characteristics of effective mathematics teaching, "the use of familiar words" and "simple explanation of mathematical language". Similarly, Jamie believed that students may learn mathematics effectively through a teacher's "use of revoicing" and leading students to pronounce "potentially difficult and new words" (Final Teacher Questionnaire).

Jamie's self-ratings reflected improved effectiveness with his ability to ask questions which enhance the purpose of the lesson, and his ability to encourage students to ask questions. Although ratings of Jamie's students' engagement level increased to eight out of 10, there was a drop in his self-assessment as to his ability to engage students' interest in algebra, involve most students in class discussions and handle discussions on word problems. Jamie reported reduced confidence also with planning lessons based on knowledge of students' misconceptions.

Whilst many of Jamie's beliefs remained traditional, he indicated some change in teaching satisfaction and approaches. For example: note writing now occurred in a few lessons, whole class discussion was used in most lessons, and students read the questions aloud in more lessons. Jamie also agreed strongly with the statement, *I always enjoy my mathematics teaching,* which he had previously disagreed with. With regard to algebra teaching, Jamie agreed that word problems were easier to teach than symbolic questions; a position he disagreed with before the professional learning period.

> Key Finding 8.3
> After the intervention period, language-based approaches were identified by Jamie as ways to effectively teach and learn mathematics. There was a reduction in his self-assessment on engaging students' interest, managing discussions on word problems and knowledge of students' algebra misconceptions. Teaching enjoyment now existed, students' engagement increased and there was improved effectiveness with using mathematical talk and encouraging students to ask questions.

Practice

Following the professional learning, Jamie's classes (observed three times) had a reduced proportion of teacher-talk, more questions were asked, and some of Newman's interview questions were used in an adapted form. More students were involved in the learning process but no grouped or paired work was observed. The most frequent lesson activities observed were: the teacher asking questions, the teacher using language-based approaches, students listening and the teacher explaining.

Jamie encouraged his students to discover the key words in a word problem and tried to engage more students in the problem-solving process. Some adaptation of the Newman strategies was used but he often read the question aloud after writing it on the board. The following transcript illustrates how after writing on the board, Jamie algebraically solved the question: *When a number is multiplied by five, the result is 20. What is the number?*

> Jamie (reads): When a number is multiplied by five, the result is
>
> Students (many voices): Twenty
>
> Jamie: The result is twenty. What is the number? if you remember, locate the keyword. Which means that you look for the number, and you represent the number with what?
>
> Students (many voices): A variable
>
> Jamie: It can be *x*, it can be *y*, it can be *t*, it can be any variable. Then you look for the keywords – multiplied and result. So let's get a solution. The number you think of is what?
>
> Students (many voices): *x*

Jamie: So, let the number be *x*. Now what is the next stage? (Calls two students)

First student: *x* times five

Second student: Multiply

Jamie: The number is multiplied by what?

Third student: Five

Jamie: The result is… What do you mean by this, 'result'?

Students (many voices): Answer

Jamie: And what do you mean by that?

Students (some voices): Equal to

Jamie: So we now say, five multiplied by x (writing $5x$) is

Students (many voices): 20

Jamie (writing): So $5x = 20$

Students (some voices): 20 divided by five

Jamie: So the final answer is what?

Students (many voices): Four

Jamie: The important thing is that you should be able to translate your words: multiply and result. (Lesson observation, 4/4/11)

He wrote the final answer, $x = 4$, before proceeding to write another question on the board.

Changes in Jamie's Beliefs and Practice

Jamie's teaching practice had become less teacher-dominating with more student involvement and reduced note-writing, although he still had many traditional beliefs about mathematics teaching and learning. He seemed to have more awareness about the importance of language in the class. In the final questionnaire, Jamie's response to the question W*hy is mathematical talk important in the teaching and learning of algebra word problems*? was that it will "encourage students in engagement and learning new vocabulary as it occurs in the class".

Jamie was not observed asking his students to read aloud the entire question, and wrote in the final questionnaire that the difficulties he had had with the Newman strategy were the students' "reading of the question" and "translation". He emphasized identifying keywords and had started using more questions during teaching.

Key Finding 8.4
After the professional learning, Jamie used more questions and adapted some of Newman's strategies in his teaching. There was a reduction of teacher-talk and more student involvement and language-based approaches observed in his classes.

Students' Algebra Pre-test Performance

After three classroom observations, Jamie's students completed the 15 algebra questions which comprised six symbolic and nine word problems. Each correct answer was scored one point, giving a maximum total score of 15 points. The general performance was poor with 95.4% of the students giving no more than two correct answers. The highest total score, obtained by three students was four; no student gave up to a-third correct responses. Table 8.1 presents the result.

Table 8.1: Jamie's students' pre-test total score (n=54)

	Total score on pre-test /15				
	0	1	2	3	4
Number of students	18	26	7	0	3
Per cent	33.3	48.1	13.0	0.0	5.6

More correct responses were given to the symbolic questions than the worded questions. There were 52 correct responses out of a possible (54x15) 810 correct responses, representing a 6.42% overall success rate. This comprised of 11.42% on symbolic questions and 3.8% on word problems. (See Table 8.2)

Table 8.2: Number of Jamie's students with correct answers in each pre-test question (n=54)

	Question number and representation format														
	Symbolic						Worded								
Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of students	3	26	1	7	0	0	11	0	3	1	0	0	0	0	0
Total	**37/324**						**15/486**								

For half of the questions, there were no correct responses. The easiest symbolic questions required the simplification of an algebraic expression while the easiest word problem required only the knowledge that the algebraic letter is a quantity.

Key Finding 8.5
Almost all (94%) Jamie's students gave no more than two correct answers on the pre-test. The pre-test overall success rate was 6.42%, comprised of 11.42% on symbolic questions and 3.8% on word problems.

What difficulties did the students have that gave rise to these incorrect responses? The next section examines the four students' incorrect responses and identifies the initial cause of error using the Newman error analysis.

Students' Pre-test Newman Interview Results

Following the test, Jamie chose four students (given codes S1, S2, S3, and S4) to be interviewed by the Researcher; there were no criteria used for selection to the Researcher's knowledge. It is likely that more able students would have been selected rather than a random sample. As required by the Newman error analysis interview protocol, these students completed the test again before the Researcher interviewed them on wrongly answered questions. Two students gave no correct answers while the other two gave one and two correct answers respectively. Comprehension and transformation errors were in the majority and accounted for 61.4% and 21.1% respectively of all the errors. Comprehension was the most common error type, irrespective of the question text format. Table 8.3 presents the data.

Table 8.3: Per cent of error types made by Jamie's students' on the algebra pre-test (n=4)

	Question representation		
	Symbolic	Word	All questions
Initial error	Per cent	Per cent	Per cent
Reading	4.6	-	1.8
Comprehension	50.0	68.6	61.4
Transformation	22.7	20.0	21.1
Process skills	22.7	8.6	14.0
Encoding	-	-	-
Carelessness	-	2.9	1.8

On the symbolic questions, half of the errors made were comprehension errors, a higher proportion than on the transformation (23%) and process skills (23%) errors. Comprehension errors were the most common errors made on the word problems, followed by transformation (20%) errors. The students' inability to understand the questions prevented them from progressing beyond the comprehension stage on many of their responses.

Key Finding 8.6
The Newman error analysis of Jamie's students' incorrect pre-test responses showed that comprehension (61.4%) and transformation (21.1%) errors were the most common errors. Comprehension errors were most common on both symbolic (50%) and word problems (69%).

The error analysis is reported under the two question representation forms, symbolic and worded questions. Each set of questions is stated, followed by a brief on the underlying concepts and then an analysis of the students' responses to individual questions is given.

Symbolic Questions: 1-6

1. Simplify as far as possible $1 + x + x$
2. Simplify as far as possible $3m + 5n + 4m + 6n$
3. $y \times y \times y =$ ....
4. Find the value of x: $7x = 21$
5. Find the value of x: $2x - 2 = 10$
6. Find the value of x: $21x = 7$

Figure 8.1: Symbolic pre-test questions

Questions 1 and 2 required the simplification of expressions using the knowledge that unlike terms cannot be conjoined and that the algebraic letter is a quantity not an object. Comprehension errors accounted for four of the seven errors identified on these questions. The students' typical response was that 'simplify' means "we should plus, add it together", while then going on to add the visible numbers while ignoring the algebraic letter. This generated responses such as:

> S2: 1 plus *x* plus *x*, the answer is 1
>
> S1: 3 plus 5 plus 4 plus 6, it will be 18. (Student interview, 28/3/11)

The other students processed the expression by conjoining terms or using the letter as a detachable object.

> S3: x plus x will give x^2, so we now say 1 plus x^2 will be $1x^2$ because of these two *x*'s, we cannot say 1*x*.
>
> S4: you add all of them then after you put m and n…18*mn* (Student interview, 28/3/11)

Question 3 required the students to differentiate between writing the answer as a product and as a sum. A student read the multiplication sign as an alphabet to obtain "*yxyxy*" as the final answer. The two other errors identified were mistakes due to processing the letter as a specific value or without any function.

> S4: Two times two times two equals eight
>
> S2: because we have nothing else to do, everything here is *y (*Student interview, 28/3/11)

Questions 4 to 6 examined the students' knowledge of equations, the use of the equality sign and their ability to balance equations. None of the students gave correct answers to this set of questions; seven of the errors were comprehension errors and four transformation errors. Questions 4 and 6 had more of comprehension errors, while Question 5 had more of transformation errors. Some of the students gave wrong or had no response for the meaning of 'value', while some did not see any difference between Questions 4 and 6.

S2: the value is times, when they tell us to times something

S3: This question [6] is the same thing as number [4]. They are the same but put 21 in the back and seven in the back of the other one. (Student interview, 28/3/11)

Question 5 errors arose from an inability to carry out inverse operations needed to balance the equation and using the letter as an object.

S3: we move the term to the other side, it becomes plus 2*x* – 2. The minus will not go away. If …it is meaningless in front so 10 + 2*x* = 12*x* so 12*x* – 2 = 10*x*. The answer is 10*x*.

S1: 2 in 10 will give us 5, that is, 2 divided by 5 is giving us 5. The answer is 5*x*.

S4: 2 times 5 equals 10, then *x* – 5, *x* is 10. (Student interview, 28/3/11)

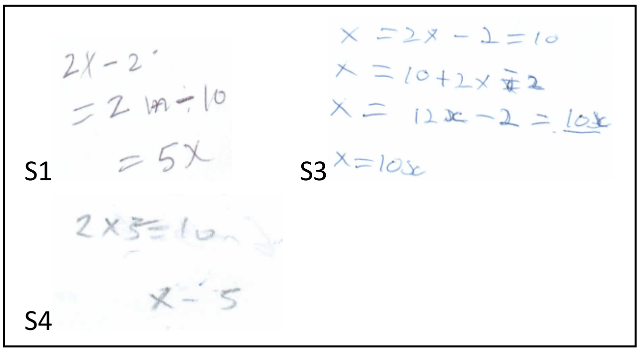


Figure 8.2: Jamie’s students’ (S1, S3 and S4) workings on Question 5

The summary of the initial cause of errors is presented in Table 8.4. The most common error types were comprehension (11), transformation (5) and process skill (5) errors.

Table 8.4: Students’ responses and initial errors on pre-test symbolic questions (n=4)

Questions	Students’ responses to six symbolic questions							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
Symbolic								
1	-	-	2	-	2	-	-	4
2	1	-	2	1	-	-	-	3
3	1	1	-	-	2	-	-	3
4	-	-	3	-	1	-	-	4
5	-	-	1	3	-	-	-	4
6	-	-	3	1	-	-	-	4
Overall	2	1	11	5	5	-	-	22

R- Reading, C- Comprehension, T -Transformation, P- Process skills, E- Encoding, CE – Carelessness

Key Finding 8.7
Comprehension errors by Jamie's students on pre-symbolic questions related mainly to the meaning of 'simplify' and 'value'. Algebraic misconceptions identified with transformation and processing errors were: the use of the letter as a detachable object, as a specific value, conjoining of terms and an inability to perform inverse operations.

The error analysis of the word problems follows and is presented in a similar pattern, that is, the questions are stated followed by a description of the students' responses to each of the nine questions.

Worded Questions: 7-15

7. Sola has *x* bananas and Peju has *p* bananas. Peter counts the number of bananas each of them have and finds they are the same. Sola said you could write this as $x=p$, but Peju said that *x* and *p* are different letters and so cannot be the same. Who do you think is correct?
8. Mary has *x* oranges and Bisi has four more oranges than Mary. How many oranges does Bisi have?
9. A basket costs eight naira and a bag costs *c* naira more than the basket. How much does the bag cost?
10. What is the number that is five less than *x*?
11. There is a *b* number of sweets in a packet. A girl has two packets of sweets and gives her friend six sweets. How many sweets does she have remaining?
12. If *d* is the number of dogs and *c* is the number of cats, write in algebra: There are four more dogs than cats.
13. Write in algebra: There are three more caps than hats.
14. Write in algebra: There are twice as many pencils as biros (let *p* be the number of pencils and *b* be the number of biros).
15. If *s* is the number of students and *t* is the number of tables, write in algebra: There are three students for every table.

Figure 8.3: Worded pre-test questions

Question 7 required the knowledge of the letter as a quantity. The letter was perceived as a member of the alphabet by two students.

> S3: *x* and p are not the same. Had it been *x* and *x*, and *p* and *p*, I will say that *x* has its own sign and p has its own, then I will say Sola is right. ….*x* and *p* are not the same thing because they are different alphabets.
> (Student interview, 28/3/11)

Questions 8 and 9 tested the students' ability to interpret 'more' as requiring the additive and not the multiplicative operator. There were four comprehension and two transformation errors.

> S3: Bisi has four, Mary does not have anything
>
> S1: to find how many oranges does Mary have

S2: more means much; the basket cost is more than the bag (Student interview, 28/3/11)

In an effort to transform 'more' in Question 9, S1 multiplied saying "we should use times".

Questions 10 and 11 required the use of the subtractive operator in establishing an expression that signifies the relationship between the algebraic letter and the given quantities. There were five transformation errors identified and the remaining two were comprehension errors. All of the students used specific values to transform the algebraic letter and obtained specific answers to Question 11. Interestingly, three of the students assumed that one packet contained 12 sweets.

S3: I don't know how many is in a packet but I'll say that 24 is for two packets of sweets

S2: The remaining one packet, he shared into two and gave is friend six, so remains half. The sweet remaining will be one packet and half. (Student interview, 28/3/11)

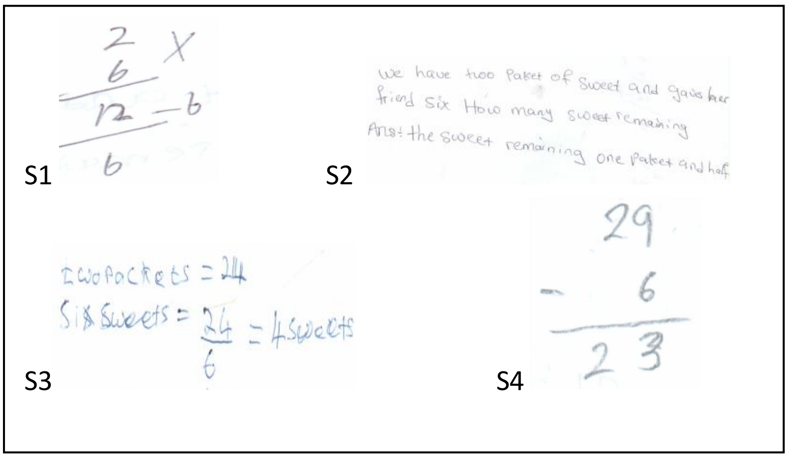


Figure 8.4: Jamie's students' (S1 – S4) workings on Question 11

Three students simply did not understand Question 10 and responded with variations of S1's response, "I have no idea of what it is asking for" while S4 said 'less' meant "the number lower than *x*" (Student interview, 28/3/11).

None of the students explained correct meanings for Questions 12 to 15. This set of questions required the construction of equations which showed a relationship between the quantities of the two items in each question. Some students gave no responses or said they did not understand what algebra meant, while the others felt they were supposed to find the number of one of the items. Samples of such responses were:

S1: I don't understand what is meant by 'write in algebra'

S2: I don't understand about algebra

S3: to find the number of cats. They've given us the number of dogs

S4: to look for the number of students (Student interview, 28/3/11)

The Researcher had to ask a student (S3) who seemed concerned about his inability to proceed with solving Question 13 "What is it that is making it difficult or disturbing you from finding an answer?" His response was, "There is no number in these hats for me to know that the cap is more than the hats" (Student interview, 28/3/11).

A summary of all the initial causes of errors made by the students is presented in Table 8.5. The most common error types were: Comprehension (24) and Transformation (7) errors.

Table 8.5: Jamie's students' responses and initial errors on pre-test word problems (n=4)

Questions	Number of responses to worded questions							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
7	1	-	1	-	2	-	-	3
8	-	-	2	1	-	-	1	4
9	-	-	2	1	1	-	-	4
10	-	-	3	1	-	-	-	4
11	-	-	-	4	-	-	-	4
12	-	-	4	-	-	-	-	4
13	-	-	4	-	-	-	-	4
14	-	-	4	-	-	-	-	4
15	-	-	4	-	-	-	-	4
Overall	1	-	24	7	3	-	1	35

R- Reading, C- Comprehension, T -Transformation, P- Process skills, E- Encoding, CE - Carelessness

Key Finding 8.8
Jamie's students did not know the meaning of algebra, they and were unable to explain relationships between two pronumerals, instead referring to just one pronumeral. Understanding the mathematical interpretation of 'more' and 'less' was a challenge as well. Algebraic misconceptions identified were, using the algebraic letter as an alphabet, as a specific value, or simply ignoring the letter.

Students' Post-test Performance

After a six-week teaching period, Jamie's class completed a post-test that was parallel to the pre-test. Both tests were similar in terms of the question context and underlying concepts and only differed in the names of people and items, and quantities used. Care was taken to ensure that the one-digit and two-digit quantities remained as one-digit and two-digit quantities respectively on both tests. The aim was to reduce the students' familiarity with the questions without losing the similarity in context and pre-requisite concepts (Chinen, 2008).

This section presents Jamie's students' post-test scores and their performances in the two question text formats. An overall significant improvement (z = -3.771) was found despite the fact that the overall performance stayed low. Table 8.6 presents the data.

Table 8.6: Jamie's students' post-test total score (n=54)

	Total score on post-test /15							Pre-test			Post-test	
	0	1	2	3	4	5	6	Mean	SD		Mean	SD
Number of students	16	23	6	3	1	4	1	0.96*	.99		1.37*	1.53
Per cent	29.6	42.6	11.1	5.6	1.9	7.4	1.9					

*Wilcoxon Signed Ranks, p<.05

Nine students correctly answered three or more questions in the post-test in contrast to three students in the pre-test. The total of 74 correct responses out of a possible 810 correct responses represents a 9.1% overall success rate on the post-test in comparison to 6.4% on the pre-test. More correct responses were obtained on the symbolic questions than on the word problems. (See Table 8.7)

Table 8.7: Number of Jamie's students with correct answers in post-test questions (n=54)

	Question number and representation format														
	Symbolic						Worded								
Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Post-test															
Number of students	3	24	7	11	8	2	11	2	5	1	0	0	0	0	0
Total	**55/324**						**19/486**								
Pre-test															
Number of students	3	26	1	7	0	0	11	0	3	1	0	0	0	0	0
Total	**37/324**						**15/486**								

Students' success rate on symbolic questions was 17% while that of word problems was 3.9%. There remained no correct responses on all of the questions that involved the construction of equations with two pro-numerals. The easiest symbolic and word problems remained at the same proportion as that obtained in the pre-test, that is, requiring the simplification of an expression and recognition of an algebraic letter as a quantity.

Key Finding 8.9
Jamie's students' post-test performance improved significantly and they had an overall success rate of 9.1% which comprised 12% success on symbolic questions and 3.9% success on word problems. There were no correct answers for the four questions having two pro-numerals.

Students' Post-test Newman Interview Results

The four students earlier interviewed by the Researcher on the pre-test were interviewed on wrongly answered questions using the Newman protocol. The information provided evidence for identifying the successes and difficulties students were having after the intervention. One student did not give any correct answer to any question, while six, three and two correct answers were given by the other three students respectively. Each of comprehension and process skill errors accounted for about one-third of the 49 wrong responses. The four students had more errors on word problems than the symbolic questions, and the pattern of errors varied with the text of the question. Table 8.8 presents the data.

Table 8.8: Per cent of error types made by Jamie's students' on the algebra post-test (n=4)

Initial error	Question format		All questions
	Symbolic	Worded	
	Per cent	Per cent	Per cent
Reading	0	0	0
Comprehension	15.8	46.7	34.7
Transformation	21.1	30.0	26.5
Process skills	52.6	20.0	32.7
Encoding	5.3	0	2.0
Carelessness	5.3	3.3	4.1

Comprehension errors (35%) were fewer than that for the pre-test and they were the most common error type. Greater success on the post-test allowed for more of processing (33%) and transformation (27%) errors. With the symbolic questions, the students' greater success with comprehension resulted in process skills error (53%) as the most common error. There was progress beyond the comprehension and transformation performance strategies, which provided an avenue for mathematical processing of the questions and more opportunities to use algebraic concepts. Comprehension errors (46.7%) were the most common errors (47%) on the word problems.

Key Finding 8.10
Jamie's students made less comprehension errors on the post-test than on the pre-test and the most common errors types were comprehension (35%) and process skill (33%) errors. Processing errors (53%) were the most common error type made on symbolic questions, while comprehension errors (47%) were the most common error type made with word problems.

The initial cause of error on each wrongly answered question was determined by the interviewed student's response. A description of the Newman error analysis done is presented in two sections namely, the symbolic questions (Questions 1-6) and the word problems

(Questions 7-15). The questions are stated, followed by a description of students' errors together with transcripts and a summary table.

Symbolic Questions: 1-6

1. Simplify as far as possible $1 + y + y$
2. Simplify as far as possible $4z + 3p + 7z + 2p$
3. $m \times m \times m =$ ....
4. Find the value of x: $6x = 24$
5. Solve for x: $5x – 5 = 20$
6. Solve for x: $24x = 6$

Figure 8.5: Symbolic post-test questions

Four out of the six errors on Questions 1 and 2 were process –based with three of them occurring on Question 1. The students processed the question with the letter used as a detachable object and found a product instead of sum.

> S1: because there are two y's here, I can't write $1 + y + y$ again. So, I now remove one y from here and add it to the one and say $1 + y$
>
> S1: everything will be 17, so the answer is $17 + z + p$
>
> S2: 1 + 1y + 1y = 3y
>
> S3: that will be $1 + y^2$ (Student interview, 1/7/11)

S4 transformed the algebraic letter as having a fixed unit value, saying, "because one is there and I have to use one to represent this y, so 1 + 1 + 1 =3" (Student interview, 1/7/11)

Two processing and one encoding error were identified as initial cause of errors on Question 3. One student processed with a random response giving the letter a specific value, and the other used a faulty algorithm arising from a confusion of product with sum.

> S4: I'll pick two, so 2 x 2 x 2 that is eight
>
> S2: We must in any letter that is in *m*, we add one making 1*m*. So 1*m* x 1*m* x 1*m* equals 3m. (Student interview, 1/7/11)

A student (S1) said the correct answer "*m* raised to power three" but wrote it wrongly as "m_3".

Four processing and three comprehension errors were identified on Questions 4 to 6. Two students, after reading these correctly, did not seem to have an understanding of the word 'value' and multiplied.

> S2: we should find the value of *x*, are looking for multiplication, 6 x 4 = 24, so 24 x 6 = 144.

S4: they said we should find the value of the times, so 24 x 6 = 144
(Student interview, 1/7/11)

Processing errors arose from an inability to carry out the inverse operations with a larger divisor as the students attempted to balance the equations.

S1: six divided by 24 is equal to 4

S3: you cannot say six in 24, you now say 24 divided by six (Student interview, 1/7/11)

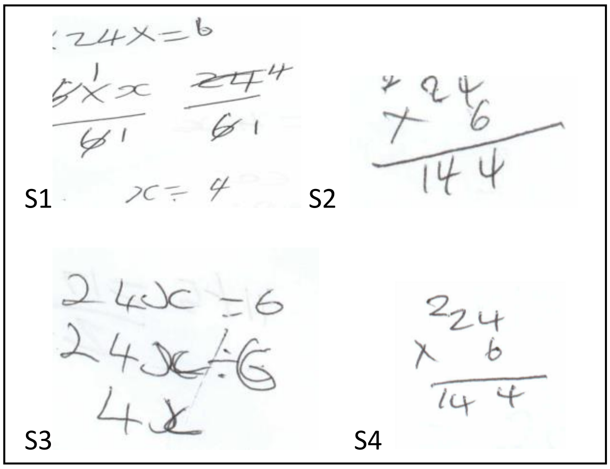


Figure 8.6: Jamie's students' (S1 – S4) workings on Question 6

The most common error types were processing (10), transformation (4) and comprehension (3) errors. The error analysis data is summarised in Table 8.9.

Table 8.9: Jamie's students' responses and initial errors on post-test symbolic questions (n=4)

Questions	Students' responses to six mainly symbolic questions							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
Symbolic								
1	-	-	-	1	3	-	-	4
2	2	-	-	1	1	-	-	2
3	1	-	-	-	2	1	-	3
4	-	-	1	-	2	-	1	4
5	2	-	-	1	1	-	-	2
6	-	-	2	1	1	-	-	4
Overall	5	-	3	4	10	1	1	19

R- Reading, C- Comprehension, T -Transformation, P- Process skills, E- Encoding, CE – Carelessness

Key Finding 8.11
Processing errors by Jamie's students on the post-symbolic questions resulted from an inability to carry out inverse operations with a bigger divisor, using the letter as a detachable object, and always giving the letter a specific value. The letter was also transformed as a fixed value of one. Some students misunderstood 'value' to mean multiplication.

The students' responses were analysed to determine the type of errors that were made on the nine post-test word questions. The questions are stated and the information is presented next.

Worded Questions: 7-15

7. Sola has *y* bananas and Peju has *x* bananas. Peter counts the number of bananas each of them have and finds they are the same. Sola said you could write this as $y=x$, but Peju said that *y* and *x* are different letters and so cannot be the same. Who do you think is correct?
8. Mary has *m* oranges and Bisi has three more oranges than Mary. How many oranges does Bisi have?
9. A ball costs ten naira and a shirt costs *y* naira more than the ball. How much does the shirt cost?
10. What is the number that is four less than *x*?
11. There is a *x* number of pencils in a packet. A girl has three packets of pencils and gives her friend six pencils. How many pencils does she have remaining?
12. If *p* is the number of plates and *c* is the number of cups, write in algebra: There are four more plates than cups.
13. Write in algebra: There are five more goats than dogs.
14. Write in algebra: There are twice as many books as pens (let *b* be the number of books and *p* be the number of pens).
15. If *b* is the number of boys and *g* is the number of girls, write in algebra: There are three boys for every girl.

Figure 8.7: Worded post-test questions

Two of the students processed the letter in Question 7 as an alphabet instead of a quantity. Their responses were explained by S4, "*y* and *x* are different letters" (Student interview, 1/7/11).

The three wrong answers on Questions 8 and 9 were processing errors that resulted from the students' conjoining of terms and the random choice of specific values for the letter.

> S2: we should use one like this with them, plus three, so $1m + 3 = 4m$
>
> S4: I'll say three plus two. This three. So I picked two representing *m*; and then adding them, so Bisi has five oranges.
>
> S4: I will put 10 plus two, 10 + 2 = 12. The two is for the *y* (Student interview, 1/7/11)

Five of the eight errors on Questions 10 and 11 were transformation errors and three of them occurred on Question 10. Three students (S1, S2, S3) understood 'less' in the question to refer to "the number smaller than x", but were unable to correctly transform it to the subtractive form. While one student transformed it as addition, another chose a specific number and the third had no understanding of what to do next. On Question 11, two students were unable to link the items within the problem context while transforming.

> S1: the five is the one she gave her to her friend, the x is – there is a x number of pencil. My answer is five times x.
>
> S3: the three packets minus the five plus x. (Student interview, 1/7/11)

S3 then went on to write $5 - 3 = x = 2 + x$ and the Researcher later asked why three minus five was written as five minus three. The reply was "because $3 - 5$ will not give us two. It is impossible". (Student interview, 1/7/11)

No correct answers were given to Questions 12 to 15 and non-comprehension accounted for 75% of the errors. Some responses indicated that the students did not understand the question, or thought that they were to find the quantity of one of the items.

> S2: I don't know
>
> S4: they say we should look for the number of goats.

Other responses identified a progression to the recognition of the two items, but with a belief that the second quantity was not given.

> S1: they are only giving us the number of plates; they did not give us the number of cups
>
> S4: they gave us the number that is for the boys, that is three; we do not know the number that is representing the girls.

For a few questions that were successfully understood, the students translated literally as they attempted the transformation step and generated expressions as answers.

> S3: The 4 will represent the plates while c represents the cups. Then went on to obtain $4 + c$ as answer
>
> S1: here they gave us 3 boys for every girl, so my answer is $3 + g$, 3 for the boys and g for the girls. (Student interview, 1/7/11)

The most common error types were comprehension (14), transformation (9) and process skills (6) errors. The summary of all the post-symbolic errors is presented in Table 8.10.

Table 8.10: Jamie's students' responses and initial errors in post-test word problems (n=4)

Questions	Students' responses to word questions							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
7	1	-	-	-	2	-	1	3
8	2	-	-	-	2	-	-	2
9	3	-	-	-	1	-	-	1
10	-	-	1	3	-	-	-	4
11	-	-	1	2	1	-	-	4
12	-	-	3	1	-	-	-	4
13	-	-	3	1	-	-	-	4
14	-	-	3	1	-	-	-	4
15	-	-	3	1	-	-	-	4
Overall	6	-	14	9	6	-	1	30

R- Reading, C- Comprehension, T -Transformation, P- Process skills, E- Encoding, CE - Carelessness

Key Finding 8.12
The majority (86%) of the comprehension errors were on word problems that required construction of equations using two pronumerals. Responses were based on the recognition of one pronumeral, or two pro-numerals with a specific quantity for one of them. Transformation attempts yielded literal translations. Algebraic misconceptions of the letter as an alphabet and as a specific value were identified. The students were unable to transform 'less' into the subtractive form.

Change in Jamie's Students' Performances and Error Types

There was a significant improvement in students' overall post-test performance although it was still low and the success rate only increased by 2.7%. The success rate on symbolic questions increased with 5.6%, while on word problems the increase was 0.8%. Figure 8.5 shows the pattern.

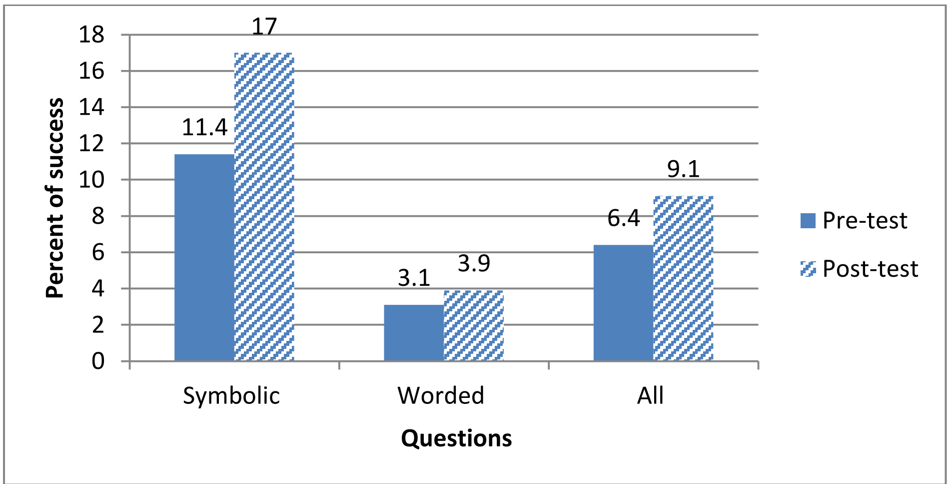


Figure 8.8: Jamie's students' performance on the pre- and post-tests

There was a change in the number and types of errors that the four students showed before and after the intervention, and a better understanding of the questions, which resulted in a 43.5% decrease in the number of identified comprehension errors. Table 8.11 presents the information.

Table 8.11: Per cent of Jamie's students' errors before and after intervention (n=4)

	Per cent error	
Error type	Pre-test	Post-test
Reading	0	0
Comprehension	61.4	34.7
Transformation	21.1	26.5
Process skills	14.0	32.7
Encoding	0	2.0
Carelessness	1.8	4.1

With increased understanding, the opportunities to transform and mathematically process the questions became available to the students. Ability to participate in these activities led to transformation and processing errors to jointly account for 59.2% of all post-test errors from an initial 35.1%. There were also some changes in error type when the text format was considered. Figure 8.9 shows the pattern.

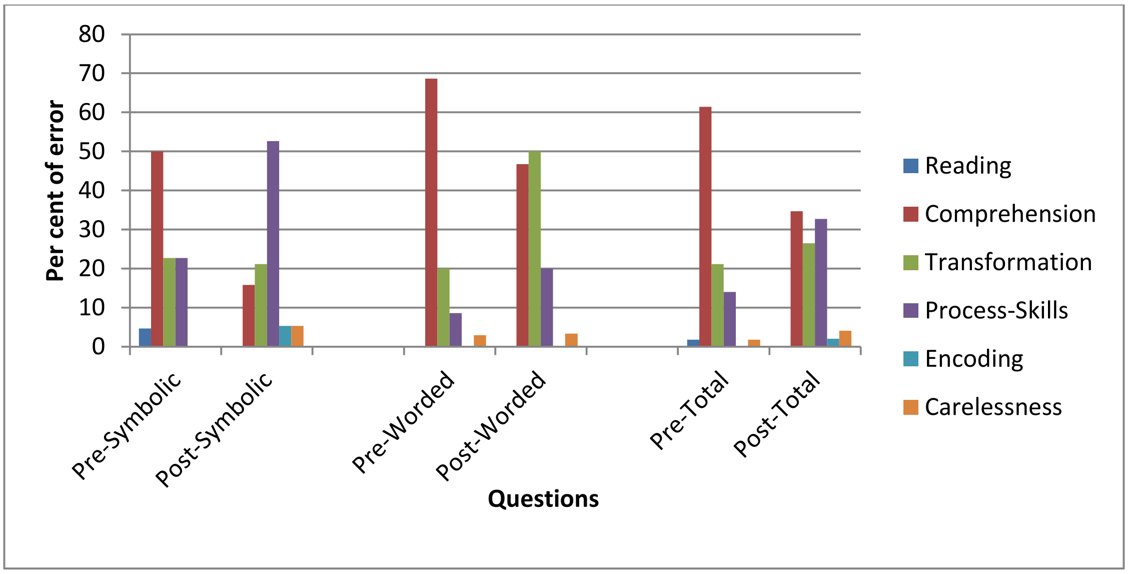


Figure 8.9: Distribution of errors on Jamie's students' pre- and post-tests

Key Finding 8.13
Jamie's students' overall success increased by 2.7%; success on symbolic and word problems increased by 5.6% and 0.8% respectively. The frequency of comprehension errors decreased by 43.5% and there were consequent increases in transformation and processing errors.

Summary

Before the professional learning, Jamie was a traditional teacher who was very confident about his ability to use questions and manage discussions but did not enjoy his teaching. A firm believer in note-writing, he believed that word problems and questions involving inverse operations with divisions were difficult for students to answer. After the intervention period, Jamie's students had become more engaged in the classroom as he used more language-based approaches and more questioning during his teaching. Jamie also had started to enjoy his mathematics teaching more.

Algebraic misconceptions however remained, although they had slightly reduced in number. The most common misconceptions still evident were: using the algebraic letter as a detachable object, as an alphabet, with a specific value or ignoring it; conjoining of terms, inability to use a larger divisor; or perform inverse operations. The students still had difficulties transforming 'less' into the subtractive operator, working with two pronumerals and constructing equations.

CHAPTER NINE: CASE STUDY FOUR – STEPHEN'S CLASS

Background

A male in his late 30s, with more than 10 years of mathematics teaching experience, Stephen (pseudonym) taught six classes of JS 1 students (10 to 12 years) with an average of 75 students in each of the classes. He taught in a public school that had a large student population; it was located in a suburban area of Ojo educational zone. Stephen had fewer than five years of experience teaching JS 1 mathematics. He holds a degree in mathematics education and also the National Certificate in Education (NCE), which is a three-year post-secondary Nigerian teaching qualification.

Pre-Intervention Beliefs and Practice

Beliefs

Stephen wrote in the initial questionnaire that he completed prior to the professional learning that his effective teaching of mathematics was constrained by inadequate facilities and the large class sizes. He believed that effective teaching was characterised by the availability of teaching aids, and a conducive learning environment where the teacher used "student-centred approaches" to help students to learn.

Prior to the professional learning, Stephen's responses suggested that he had a traditional belief about mathematics and the teaching- learning process. He agreed with statements on the initial questionnaire such as: *Mathematics is mainly calculations; Mathematics consists of rules and procedure; Students don't like asking questions;* and *A teacher has to tell students how to solve problems so that they don't make mistakes*. Stephen indicated that his daily lesson activities included writing notes, working examples for students and individual problem solving. Despite Stephen's indication of a high level of self-efficacy in the use of questioning, he rated his students' classroom engagement to be four out of ten and indicated that he enjoyed his teaching.

Stephen expressed that he found it easier to teach symbolic questions than word problems and he was very confident about his knowledge of algebra and the use of different algebra teaching strategies. Stephen believed word problems were the most difficult tasks for students, and more especially those requiring inverse operations.

Key Finding 9.1
Before the professional learning, Stephen believed his challenges to effective mathematics teaching were his large class sizes and inadequate facilities. He had traditional beliefs about the teaching and learning, a high self-efficacy in the use of questioning and confidence about algebra knowledge and strategies. Stephen rated his class engagement level as four out of ten.

Practice

Two single lessons taught by Stephen were observed before the professional learning workshop and they were traditional in approach. HIs classes started with a review of the previous lesson followed by an explanation of the day's lesson, and then Stephen wrote notes on the board which his students copied. Using a written question as an example, Stephen then explained the solution process as he worked it out on the board. The students were asked if they had had any questions. In most cases there were none, and then Stephen explained the solution again. After copying the worked example, the students listened as Stephen explained the workings for one or two more examples. Many of the questions that he asked, he himself answered. Individual class work on similar questions followed, before Stephen explained how to find the answer to the question(s) and closed the lesson with homework sometimes given. The most frequent lesson activities were: teacher explaining, students listening, students copying notes, and teacher asking questions. (See sample in Appendix 20)

Stephen was observed using the letter as a moveable object. To compute $t + t + t$, he explained,

> When the three letters are having a coefficient of one, one, one, the coefficients are what we first of all add together, giving us 1 + 1 + 1
>
> Students: Three
>
> Stephen: All letters are common, so you just pick one of it and put it at the side, and that will become what?
>
> Students: 3*t* (Lesson observation, 23/3/2011)

Stephen's explanations also suggest that he used the letter as a label and that he did the thinking for the students during problem solving. He solved the question" Ojo is two years older than Peju. How old is Ojo?"

> Stephen: You have to solve the question by interpreting the statement. They've already given us that Ojo is older than Peju by two years but Peju's age is not given. We know that Peju's age is not given. We don't know the age of Peju. So what do we do? We find the number that will indicate the age of what?
>
> Students: Peju
>
> Stephen: So who can tell me what we use to indicate the age of Peju?

> Many voices: A letter
>
> Stephen: A letter which is known as a variable to indicate the age of Peju. So let Peju be what? *X*. Let Peju be *x*, that's for Peju…we are going to add the two years. ..the age of Peju plus that which Ojo is older with, so Ojo's age is now $x + 2$. (Lesson observation, 23/3/11)

While discussing on the second day of the professional learning program, Stephen's comment suggested that he was unaware of the misconception of the algebraic letter as a label.

> Since yesterday, I've been thinking of it, using a variable to represent a quantity instead of an object. But for the students to understand better in a simple way like 'cup' stands for 'c', that is why we teachers like to use the letter. (PL workshop, 30/3/11)

Key Finding 9.2 Stephen's teaching before the professional learning had more of teacher-talk while students listened and copied notes. He was observed using the algebraic letter as a quantity but was unaware that it was not a label or object.

Post –Intervention Beliefs and Practice

Beliefs

In the final questionnaire completed after the intervention period, Stephen's responses showed that his confidence with the use of some language-based approaches had increased. His confidence ratings on class discussions and the ability to develop algebra vocabulary and explain algebra concepts had increased. Stephen also indicated whole-class discussion and students reading aloud of questions as activities performed in every lesson, while note-writing and working blackboard examples were no longer daily lesson activities. Stephen's rating of his students' classroom engagement level improved from four to six out of ten, although he reiterated his belief that effective mathematics teaching required a conducive learning environment. (Final Questionnaire)

Stephen wrote in the final questionnaire that mathematical talk was important because it helped to debunk the notion of algebra being "tough". The most significant gains from the PL workshop for Stephen were: the knowledge that misconceptions could be a reason for students' algebra failure, and "how to teach algebra in a very simple and understanding way". This belief appeared to be buttressed by Stephen's written view that an understanding of students' misconceptions enabled him to realize students' lack of familiarity with commonly used algebra terms. (Final Questionnaire)

Key Finding 9.3
After the intervention period, Stephen's ratings on confidence in using language-based approaches and his students' classroom engagement had increased. Stephen had increased awareness of students' algebra misconceptions and believed that mathematical talk could help clarify them.

Practice

More explanations happened in Stephen's three classes that were observed after the professional learning. The lessons started with Stephen's review of the previous lesson followed by an explanation of the day's lesson and then he wrote a question on the board. Stephen often read the question aloud and identified the keywords before working out the answer using the students' responses to his questions. Stephen then explained the solution process, responded to any student's question before writing further examples on the board. Different students were called to answer Stephen's questions as he worked the examples. An explanation by him on the problem-solving process followed after the final answer was found. The most frequent lesson activities observed were: the teacher explaining, the teacher asking questions, students listening and students copying.

Stephen tried to adapt the Newman steps in his class. A question written on the board was: *Two plus a certain number is equal to twenty. What is the number?*

> Stephen: You have to be able to understand and interpret the question. (Reads the question aloud) What do we do first? First thing is to find the certain number. What do we use as our certain number?
>
> Student 1: Letter m
>
> Stephen: (Writes) let the certain number be n. What do we do next?
>
> Student 2: We add
>
> Stephen: The certain number is added to what?
>
> Student 2: Two
>
> Stephen: (Writes 2 +m) our certain number is given what letter?
>
> Student 3: *m*
>
> Stephen: Is equal to 20, it means the result is equal to 20. (Writes 2 + *m* = 20) So the statement can come in different ways but the interpretation will give you the same answer.
>
> Student 4: Why is the two in the front?
>
> Stephen: It is two plus a certain number, not a certain number plus two but the answer is the same thing. (Lesson observation, 20/5/11)

After writing the equation, Stephen explained how the students would balance the equation by subtracting two from both sides and obtained the final answer of 18. Stephen read the questions often by himself, and the use of a letter as an object was not observed in any of his classes. Many of the students seated towards the back of the classroom could not fully participate in the discussions, and the large number of students limited his physical movement around the entire classroom.

Changes in Stephen's Beliefs and Practice

After the PL, Stephen's classes remained largely traditional but he used more questions and involved more students in the problem-solving process. An understanding that students may not be familiar with mathematical vocabulary appeared to be reflected in his focus on interpretation during teaching. He had increased confidence in algebra knowledge and the use of language-based approaches.

Key Finding 9.4
After the professional learning, Stephen used more questions and explanations; more students were engaged in the problem-solving process. He was seen adapting the Newman questions in his class.

Students' Algebra Pre-test Performance

After three (a double and two single lessons) classroom observations, Stephen's students completed the 15 algebra questions containing six symbolic and nine word problems. Each correct answer was scored one point, giving a maximum total score of 15 points. The general performance was poor with 92.6% of the students giving no more than two correct answers. The highest total score was five, obtained by one student. Table 9.1 presents the result.

Table 9.1: Stephen's students' pre-test total score (n=67)

	Total score on pre-test /15					
	0	1	2	3	4	5
Number of students	31	26	5	4	-	1
Per cent	46.3	38.8	7.5	6.0	-	1.5

More correct responses were given to the symbolic questions than the worded questions. There were 53 correct responses out of a possible (67x15) 1005 correct responses representing a 5.3% overall success rate. This comprised 10.7% on symbolic questions and 1.7% on word problems. (See Table 9.2)

Table 9.2: Number of Stephen's students with correct answers in each pre-test question (n=67)

	Question number and representation format														
	Symbolic						Worded								
Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of students	3	17	2	18	3	0	5	3	1	0	1	0	0	0	0
Total	**43/402**						**10/603**								

For half of the questions, there were no correct responses. The two easiest symbolic questions, having about the same number of correct responses, required simplifying an expression and finding a specific value in a given equation. The easiest word problem did not require any mathematical operation, only the knowledge that the algebraic letter is a quantity.

Key Finding 9.5 Almost all of the students (92.6%) in Stephen's class did not answer more than two pre-test questions correctly. The overall success rate was 5.3% comprising 10.7% on symbolic questions and 1.7% on the word problems.

What difficulties did the students have that gave rise to these incorrect responses? The next section addressed this question using the Newman error analysis procedure to examine the students' incorrect responses and identify the initial cause of error.

Students' Pre-test Newman Interview Results

Following the test, Stephen chose four students (given codes S1, S2, S3, and S4) to be interviewed by the Researcher; to the Researcher's knowledge there were no specific criteria used for selection. It is likely that more able students would have been selected rather than a random sample. This set of students completed the test again and their scripts were marked before the Researcher interviewed them on wrongly answered questions using the Newman error analysis interview protocol.

After completing the test the second time as required by the Newman protocol, two of the students correctly answered only one question each while the other two gave no correct answer to any the questions. Almost half (48.3%) of the errors were comprehension errors and it remained the most common error irrespective of the question format. The students' inability to understand the questions prevented them from progressing further than the comprehension performance strategy for many of the questions. Table 9.3 presents the data.

Table 9.3: Per cent of error types made by Stephen's students' on the algebra pre-test (n=4)

	Question representation		
	Symbolic	Word	All questions
Initial error	Per cent	Per cent	Per cent
Reading	8.3	0	3.4
Comprehension	41.7	52.9	48.3
Transformation	29.2	23.5	25.9
Process skills	16.7	17.7	17.2
Encoding	0	0	0
Carelessness	4.2	5.9	5.2

The students made more comprehension errors (48%) than transformation (26%) and process skills (17%) errors. On the symbolic questions, comprehension errors (42%) were more than transformation (29%) and process skills (17%) errors. Similarly, on the word problems comprehension errors (53%) were higher in number than transformation (24%) and process skills (18%) errors.

Key Finding 9.6
The Newman error analysis of Stephen's students' incorrect pre-test responses showed that 48% were comprehension errors and 26% were transformation errors. Comprehension errors were the most common error types on both word (53%) and symbolic (42%) questions.

The error analysis interview is here reported using the two question formats, symbolic and worded questions. The set of questions is first stated, followed by a brief explanation of the underlying concepts and an analysis of the students' responses to individual questions.

Symbolic Questions: 1-6

1. Simplify as far as possible $1 + x + x$
2. Simplify as far as possible $3m + 5n + 4m + 6n$
3. $y \times y \times y =$ ....
4. Find the value of x: $7x = 21$
5. Find the value of x: $2x - 2 = 10$
6. Find the value of x: $21x = 7$

Figure 9.1: Symbolic pre-test questions

Questions 1 and 2 required the simplification of expressions using the knowledge that unlike terms cannot be conjoined and that the letter is a quantity, not an object. None of the four students correctly answered the two questions and seven of the eight errors were transformation-based. The algebraic letter was either transformed to one, or used as a detachable object.

S1: This *x* will change to one because there is no other number standing with it. So we are going to add it together, equals three.

S4: the coefficient of any letter in algebra is one. So we say one plus one plus one, then later put the *x*.

S2: I'll say 3 + 5 + 4 + 6, the answer it gives us; we choose one *m* and one *n*.

S3: First, we add the numbers, 3+5+4+6, and then the *m* and the *n*, we put it. (Student interview, 25/3/11)

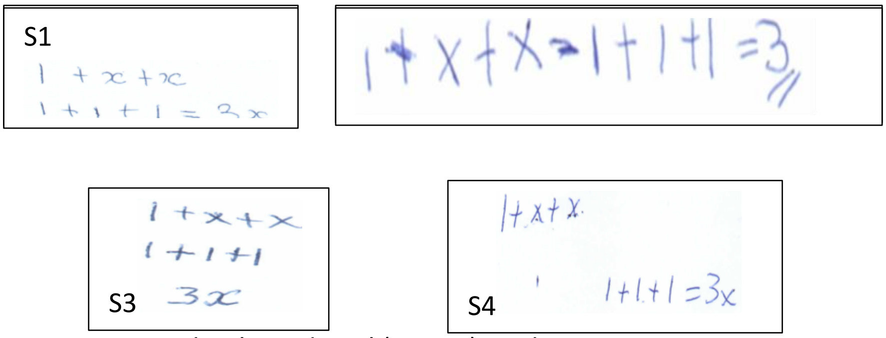


Figure 9.2: Stephen's students' (S1 - S4) workings on Question 1

Question 3 required the students to write the answer as a product not a sum. A student (S2) read the multiplication sign as an alphabet and counted the number of *y*'s to obtain "3*x*" as the final answer. The remaining three errors identified were mistakes due to processing with the sum instead of product and using the letter as a fixed value of one.

S1: This *y* will change to one because there is no number beside it; this times will change to plus also.

S3: The *y* represents 1; we say 1 x 1 x 1 = 1. We will now put the *y* at the back to get 1*y.*

S4: The coefficient of any number in algebra is one, so 1 x 1 x 1 = 1. (Student interview, 25/3/11)

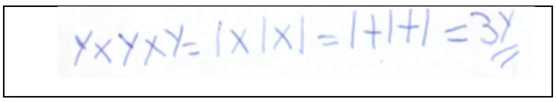


Figure 9.3: Stephen's student's (S1) working on Questions 3

Questions 4 to 6 examined the students' knowledge of equations, the use of the equality sign and their ability to balance equations. None of the students gave correct answers to this set of questions, with 10 of the 12 errors being comprehension errors. Two students (S1 and S2) did not respond when asked for the meaning of 'value' while the answers given by the other two indicated that they did not understand the questions. One student wrongly read the algebraic letter *'x'* in Question 5 as "two times minus two equals 10". (Student interview, 25/3/11)

The summary of the initial cause of errors is presented in Table 9.4. The most common error types were comprehension (10), transformation (7) and process skill (4) errors.

Table 9.4: Stephen's students' responses and initial errors on pre-test symbolic questions (n=4)

Questions	Students' responses to six symbolic questions							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
Symbolic								
1	-	-	-	3	1	-	-	4
2	-	-	-	4	-	-	-	4
3	-	1	-	-	3	-	-	4
4	-	-	3	-	-	-	1	4
5	-	1	3	-	-	-	-	4
6	-	-	4	-	-	-	-	4
Overall	-	2	10	7	4	-	1	24

R- Reading, C- Comprehension, T -Transformation, P- Process skills, E- Encoding, CE - Carelessness

Key Finding 9.7
All the comprehension errors (42%) occurred on the pre-symbolic equation questions; Stephen's students either had no understanding of the meaning of 'value' or what they were to do when addressing questions 4 to 6. Algebraic misconceptions identified with transformation and processing errors were the use of the letter as a detachable object, as a fixed value of one, and, confusing product and sum.

The error analysis of the word problems follows and it is presented in a similar pattern, that is, the questions are stated followed by a description of the students' responses to each of the nine questions.

Worded Questions: 7-15

7. Sola has *x* bananas and Peju has *p* bananas. Peter counts the number of bananas each of them have and finds they are the same. Sola said you could write this as $x=p$, but Peju said that *x* and *p* are different letters and so cannot be the same. Who do you think is correct?
8. Mary has *x* oranges and Bisi has four more oranges than Mary. How many oranges does Bisi have?
9. A basket costs eight naira and a bag costs *c* naira more than the basket. How much does the bag cost?
10. What is the number that is five less than *x*?
11. There is a *b* number of sweets in a packet. A girl has two packets of sweets and gives her friend six sweets. How many sweets does she have remaining?
12. If *d* is the number of dogs and *c* is the number of cats, write in algebra: There are four more dogs than cats.
13. Write in algebra: There are three more caps than hats.
14. Write in algebra: There are twice as many pencils as biros (let *p* be the number of pencils and *b* be the number of biros).
15. If *s* is the number of students and *t* is the number of tables, write in algebra: There are three students for every table.

Figure 9.4: Worded pre-test questions

Question 7 required the knowledge of the letter as a quantity only, without any mathematical operations. The letter was perceived as an alphabet by the four students with a typical response of "*x* and *p* are different letters". (Student interview, 25/3/11)

Questions 8 and 9 tested the students' ability to interpret 'more' as requiring the additive operator, not multiplication. There were two transformation, two processing and two careless errors. An example for each of these error type follows respectively.

> S4: Let *x* represent the number of oranges Mary has and four represent the number of oranges Bisi has more. Since they did not give us Mary's own, Bisi's is *x* - 4.
>
> S2: It is eight plus *c* equals 8*c.*
>
> S1: Mary has four oranges and Bisi has four more. We are going to use four oranges. We want to find how many does Bisi has. (Paused and reflected for a few seconds before he continued) Mary's oranges is *x* oranges and Bisi has four more oranges…..Bisi's oranges is 4 + *x. (*Student interview, 25/3/11)

Questions 10 and 11 required the use of the subtractive operator in establishing an expression that signifies the relationship between the algebraic letter and the given quantities. There were more comprehension errors with Question 10 and more transformation errors on Question 11. Two students (S1 and S3) understood Question 10 as "we look for a number that is not more than five", and said "4" was the answer. One student (S1) transformed the '*x*' as one and obtained zero as the answer. The students' transformation of Question 11 suggested that they just picked the quantities without relating them to each other.

> S1: The number of sweets is equal to six. Let b represent the number of sweets remaining, so it is 6 – *b.*
>
> S2: That is *b* + 2 + 6.
>
> S4: We will put the sweets together, so what we get is the number of sweets in a packet. 2 + 6 = 8. (Student interview, 25/3/11)

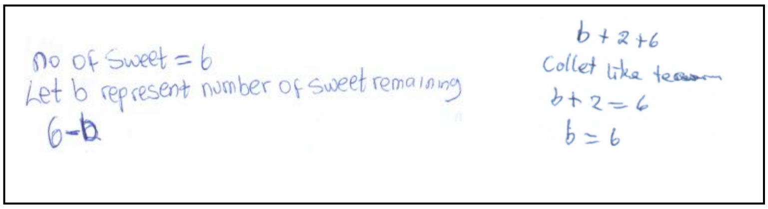


Figure 9.5: Stephen's students' working (S1, S2) on Question 10

None of the students gave correct meanings to Questions 12 to 15, which required the construction of equations to show a relationship between the quantities of the two items in

each question. Three students did not understand what it meant to write in algebra, or felt they were supposed to find the number of one of the items. Samples of such responses were:

> S1: The question says there are four more dogs than cats, so we should find the number of cats.
>
> S3: I don't understand how to write it in algebra.
>
> S2: algebra means when an answer is true or false, to work it in algebra, we say *p* +*b* +2.
>
> S3: They say we should find the number of caps. (Student interview, 25/3/11)

One student (S4) correctly attempted to transform Question 13 saying, "let the number of caps be *p*, and let the number of hats be *q*", but was unable to progress beyond the generated representations (Student interview, 25/3/11).

A summary of all the initial causes of errors made by the students is presented in Table 9.5. The most common error types were: Comprehension (18), Transformation (8) and Process skill (6) errors.

Table 9.5: Stephen's students' responses and initial errors on pre-test word problems (n=4)

Questions	Number of responses to worded questions							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
7	-	-	-	-	4	-	-	4
8	1	-	-	1	1	-	1	3
9	1	-	-	1	1	-	1	3
10	-	-	3	1	-	-	-	4
11	-	-	1	3	-	-	-	4
12	-	-	4	-	-	-	-	4
13	-	-	3	1	-	-	-	4
14	-	-	3	1	-	-	-	4
15	-	-	4	-	-	-	-	4
Overall	2	-	18	8	6	-	2	34

R- Reading, C- Comprehension, T -Transformation, P- Process skills, E- Encoding, CE – Carelessness

Key Finding 9.8
Stephen's students had no knowledge of the meaning of algebra and were unable to explain relationships between two pro-numerals. Instead, they gave the answer of one pro-numeral. Understanding the mathematical interpretation of 'less' posed a challenge as well. Algebraic misconceptions identified were using the algebraic letter as an alphabet or as a specific value.

Students' Post-test Performance

After the teaching period Stephen's class completed a post-test that was parallel to the pre-test in order to examine the impact of the intervention on the students' performances, which was the main concern of one of the research questions. The post-test was similar in terms of the question contexts and underlying concepts; the difference was in the names of items and quantities used. This was to reduce the students' familiarity with the questions. Care was taken to ensure that the one-digit and two-digit quantities remained as one-digit and two-digit quantities respectively on both tests.

This section presents the post-test general results and students' performance on the two question formats used. There was no improvement in performance. Table 9.6 presents the data.

Table 9.6: Stephen's students' post-test total score (n=67)

	Total score on post-test /15						Pre-test		Post-test	
	0	1	2	3	4	5	Mean	SD	Mean	SD
Number of students	35	21	8	2	1	0	0.79	.993	0.70	.905
Per cent	52.2	31.3	11.9	3.0	1.5	0.0				

The students' post-test mean score was lower than that of the pre-test. The total of 47 correct responses out of a possible 1,005 correct responses represents a 4.7% success rate on the post-test. More correct responses were obtained on the symbolic questions than on the word problems. (See Table 9.7)

Table 9.7: Number of Stephen's students with correct answers in pre- and post-test questions (n=67)

	Question number and representation format														
	Symbolic						Worded								
Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Post-test															
Number of students	2	13	3	13	8	0	4	1	1	0	0	1	1	0	0
Total	**39/402**						**8/603**								
Pre-test															
Number of students	3	17	2	18	3	0	5	3	1	0	1	0	0	0	0
Total	**43/402**						**10/603**								

Students' success rate on symbolic questions was 10%, while on word problems it was 1.3%. There were only two correct responses from the four questions that involved the construction

of equations using two pronumerals. The easiest symbolic and word problem questions also remained the same as seen in the pre-test, that is, simplification of an algebraic expression and finding a specific unknown in a linear equation, and the recognition of an algebraic letter as a quantity.

Key Finding 9.9 There was no improvement in Stephen's students' post-test performance. The overall success rate was 4.7% which comprised 10% success on symbolic questions and 1.3% success on word problems.

Students' Post-test Newman Interview Results

The four students earlier interviewed by the Researcher on the pre-test were interviewed on the wrongly answered questions using the Newman interview protocol. The information provided evidence for identifying successes and difficulties that the students were still having after the intervention. In their second attempt on the post-test questions, one student did not give any correct answer to any of the questions, while one, six and two correct answers were given by the other three students respectively. Transformation (59%) and processing (26%) errors were the most common error types and there were fewer comprehension (15%) errors. The four students still made more errors on word problems than the symbolic questions. Table 9.8 presents the data.

Table 9.8: Per cent of error types made by Stephen's students' on the algebra post-test (n=4)

Initial error	Question format		All questions
	Symbolic	Worded	
	Per cent	Per cent	Per cent
Reading	0	0	0
Comprehension	21.1	9.4	13.7
Transformation	52.6	62.5	58.8
Process skills	26.3	25.0	25.5
Encoding	0	0	0
Carelessness	0	3.1	2.0

Transformation errors were the most common error types in both text formats, followed by the process skills error. There were more comprehension errors on the symbolic questions (21%) than on the word problems (9%). With many questions, the students were able to progress further than the comprehension stage with greater success than on the pre-test, and this gave them an avenue to transform and process the questions and more opportunities to use algebraic concepts.

Key Finding 9.10
On the post-test interview, Stephen's students' made fewer comprehension errors than on the pre-test and the most common errors made on the post-test were transformation (59%) and process skill (25%) errors. Transformation errors were the most common error type on both symbolic questions (53%) and word problems (63%).

The initial cause of error on each wrongly answered question was determined by the interviewed student's responses. A description of the Newman error analysis is presented in two sections, the symbolic questions (Questions 1-6) and the word problems (Questions 7-15). The questions are stated, followed by a description of students' errors with transcripts and a summary table.

Symbolic Questions: 1-6

1. Simplify as far as possible $1 + y + y$
2. Simplify as far as possible $4z + 3p + 7z + 2p$
3. $m \times m \times m =$ ....
4. Find the value of x: $6x = 24$
5. Solve for x: $5x – 5 = 20$
6. Solve for x: $24x =6$

Figure 9.6: Symbolic post-test questions

No correct answers were given to Questions 1 and 2. Five out of the eight errors concerned transformation and the other three were processing errors. Three of the students still processed Question 1 using a fixed value of one for the algebraic letter, and all of them transformed the algebraic letter as a detachable object in Question 2.

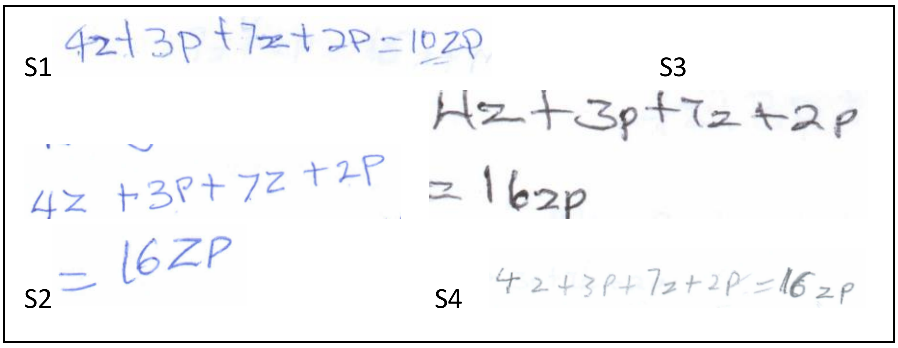


Figure 9.7: Stephen's students' (S1 – S4) working on Question 2

S1: The coefficient of these two *y*'s is two. Because there is a one there, so we are going to plus them together. So I'll pick one *y* (and wrote 1 + 1 + 1 = 3*y*).

S3: the coefficient of each *y* will be one, so we say 1 + 1 + 1, it will give us three. We pick one out of the *y*'s, so it is 3*y*.

S2: It is equal to 16, then put *zp*, will be 16*zp*. (Student interview, 2/7/11)

On Question 3, only one student processed the sum instead of the required product.

> S1: The answer is going to be 3*m*. You pick one *m* here because they are all the same. So you write it as 1 + 1 + 1 = 3, and then put *m*. (Student interview, 2/7/11)

Four comprehension, five transformation and one processing error were identified on Questions 4 to 6. One student (S1) still gave no response to the three questions and was unable to continue on any of them but the other students explained that they were to "calculate what was *x*". While attempting to transform Question 5, the algebraic letter was used as a unit value; and with Question 6, the students could not perform inverse operations with the larger divisor.

> S4: The coefficient of *x* is one and 5 x 1 is 5. So I don't understand it when 5 minus 5 again will give me 20.
>
> S3: That 5*x* – 5 will give us 1*x*, then coefficient of *x* will be 1. So 1*x* = 20.
>
> S3: 24 divided by 6, it will give us 4.
>
> S4: I think what we can calculate that can give 24. I'll say 4. 4 x 6 = 24. (Student interview, 2/7/11)

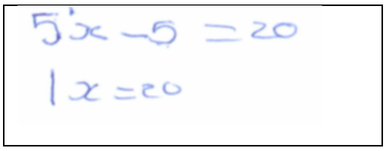


Figure 9.8: Stephen's student's (S3) working of post-test Question 5

The most common error types were transformation (10), processing (5) and comprehension (4) errors. The analysed data is summarised in Table 9.9.

Table 9.9: Stephen's students' responses and initial errors on post-test symbolic questions (n=4)

Questions	Students' responses to six mainly symbolic questions							
	Correct	Incorrect						
		Initial error cause						
		R	C	T	P	E	CE	Total
Symbolic								
1	-	-	-	1	3	-	-	4
2	-	-	-	4	-	-	-	4
3	3	-	-	-	1	-	-	1
4	2	-	2	-	-	-	-	2
5	-	-	1	2	1	-	-	4
6	-	-	1	3	-	-	-	4
Overall	5	-	4	10	5	-	-	19

R- Reading, C- Comprehension, T -Transformation, P- Process skills, E- Encoding, CE - Carelessness

Key Finding 9.11
Stephen's students' post-symbolic questions errors arose from an inability to carry out inverse operations, the use of the letter as a detachable object or with a fixed value of one. One of the interviewed students did not understand all the symbolic equation questions nor the meaning of 'value'.

The students' incorrect responses to the nine post-word questions were analysed. The questions are first stated and the information is presented next.

Worded Questions: 7-15

7. Sola has *y* bananas and Peju has *x* bananas. Peter counts the number of bananas each of them have and finds they are the same. Sola said you could write this as *y*=*x*, but Peju said that *y* and *x* are different letters and so cannot be the same. Who do you think is correct?
8. Mary has *m* oranges and Bisi has three more oranges than Mary. How many oranges does Bisi have?
9. A ball costs ten naira and a shirt costs *y* naira more than the ball. How much does the shirt cost?
10. What is the number that is four less than *x*?
11. There is a *x* number of pencils in a packet. A girl has three packets of pencils and gives her friend five pencils. How many pencils does she have remaining?
12. If *p* is the number of plates and *c* is the number of cups, write in algebra: There are four more plates than cups.
13. Write in algebra: There are five more goats than dogs.
14. Write in algebra: There are twice as many books as pens (let *b* be the number of books and *p* be the number of pens).
15. If *b* is the number of boys and *g* is the number of girls, write in algebra: There are three boys for every girl.

Figure 9.9: Worded post-test questions

> All the four students still processed the letter in Question 7 as an alphabet instead of a quantity. Their responses were of the form said by student S2 that "*y* and *x* are different letters". (Student interview, 2/7/11)

The six wrong answers on Questions 8 and 9 consisted of three each of transformation and processing errors. They resulted from the students' transforming 'more' as a product or division instead of a sum, and the faulty algorithm of ignoring the letter and conjoining terms.

> S1: The only way we can find it is by dividing the *y*. It is because of this 'more than the ball'.
>
> S2: Three times x equals 3*x.*
>
> S1: It will be three toys.
>
> S4: 10 plus *y*, and that's equal to 10*y.* (Student interview, 2/7/11)

Responses to Questions 10 and 11 were all wrong as a result of transformation errors. The students transformed the letter as a specific number rather than as a generalized number. The students understood ‘less’ to refer to a smaller quantity but were unable to transform it to represent subtraction.

> S4: The coefficient of *x* is one. There is no number less than one. The number less than one is zero.
>
> S2: You use the number in a packet that is, 12.

The Researcher asked the other two students who did not resort to specific values why they could not proceed any further. They responded:

> S1: I cannot get it because I do not know the amount of pencils in a packet.
>
> S3: This *x*, *x* is an unknown number. (Student interview, 2/7/11)

An inability to transform accounted for nine of the 14 errors identified on Questions 12 to 15. The students understood the questions and acknowledged that the two quantities were related, and then engaged in literal translations. Examples are:

> S2: 4 x *p* x *c*
>
> S4: *b* + *p* + 2 = 2*bp*
>
> S3: *b* = *g* + 3

The students wrongly processed by using wrong operations and faulty algorithm.

> S1: number of cups is 4 + *c*
>
> S1: Let *x* represent the number of dogs. So, number of dogs is 5 + *x* *(*Student interview, 2/7/11)

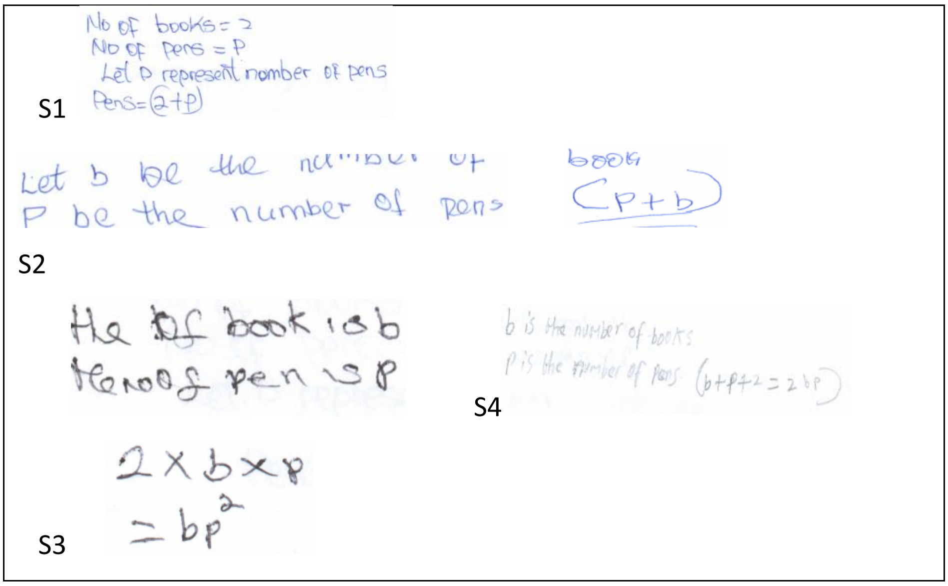


Figure 9.10: Stephen’s students’ (S1 - S4) workings on Questions 14

The most common error types were transformation (20) and process skills (8) errors. The summary of all the post-symbolic errors is presented in Table 9.10.

Table 9.10: Stephen's students' responses and initial errors in post-test word problems (n=4)

Questions	Students' responses to word questions							
	Correct	**Incorrect**						
		Initial error cause						
		R	C	T	P	E	CE	Total
7	-	-	-	-	4	-	-	4
8	1	-	-	1	2	-	-	3
9	1	-	-	2	1	-	-	3
10	-	-	-	4	-	-	-	4
11	-	-	-	4	-	-	-	4
12	1	-	1	1	-	-	1	3
13	1	-	1	2	-	-	-	3
14	-	-	1	3	-	-	-	4
15	-	-	-	3	1	-	-	4
Overall	4	-	3	20	8	-	1	32

R- Reading, C- Comprehension, T -Transformation, P- Process skills, E- Encoding, CE – Carelessness

Key Finding 9.12
For many of the post-word problem transformations, Stephen's students viewed the algebraic letter as a specific value. They acknowledged and attempted to use the two pro-numerals in equation construction but obtained expressions from literal translations. The algebraic letter was still regarded as an alphabet. The students were unable to transform 'more' and 'less' into the respective additive and subtractive form.

Change in Stephen's Students' Performances and Error Types

There was no improvement in the students' overall post-test performance and it remained very low. The success rate reduced slightly by 0.6% comprised of 1% on symbolic questions and 0.4% on word problems. However, this general pattern contradicted the performance of the four interviewed students as their combined correct responses increased from two in the pre-test to nine in the post-test.

There was a change in the proportion and types of initial errors that the four students showed before and after the intervention. In addition to the overall decrease in errors, the students were able to understand the questions, which resulted in a 71.6% decrease in the number of identified comprehension errors. Table 9.11 presents the information.

Table 9.11: Per cent of Stephen's students' errors before and after intervention (n=4)

	Per cent error	
Error type	Pre-test	Post-test
Reading	3.4	0
Comprehension	48.3	13.7
Transformation	25.9	58.8
Process skills	17.2	25.5
Encoding	0	0
Carelessness	5.2	2.0

With increased understanding, the opportunities to transform and mathematically process the questions became more available to the students. An ability to participate in these activities led to transformation and processing errors, jointly accounting for 84.3% of all the post-test errors after an initial 43.1%. The same pattern of change in error type was found when the text format was considered. Figure 9.7 shows the pattern.

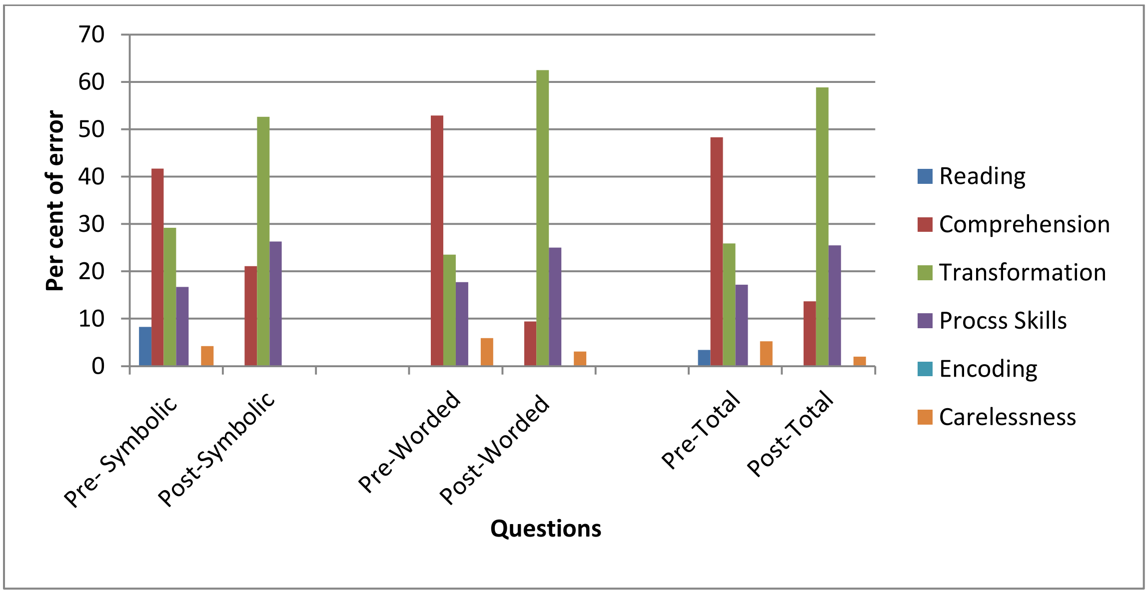


Figure 9.11: Distribution of errors on Stephen's students' pre- and post-tests

Key finding 9.13
There was a 71.6% reduction in the comprehension errors of interviewed students in Stephen's class in the post-test. The general performance of the class reduced slightly by 0.6% comprising 1% symbolic and 0.4% word problems.

Summary

Stephen professed traditional beliefs about mathematics teaching and learning and his classes equally reflected a traditional teaching approach with low student engagement. Also very

confident of his algebra knowledge and teaching before the professional learning, Stephen opined that his effectiveness was hindered by the unconducive environment in which he was teaching. After the intervention period, Stephen had increased his confidence in using language-based approaches, while also recognising that mathematical talk was important for identifying misconceptions. He also used more questioning in the class and student classroom engagement had increased.

While general students' performance did not improve, the four interviewed students had fewer comprehension errors. Students' progression into the transformation and processing stages provided an avenue for algebra misconceptions to surface. The most common misconceptions still evident were: using the algebraic letter as a detachable object, as an alphabet, with a specific value or a fixed value of one; conjoining of terms, inability to use a larger divisor or perform inverse operations. The students still had difficulties transforming 'less' into the subtractive operator, and constructing equations from two pronumerals.

CHAPTER 10: CROSS-CASE ANALYSIS AND DISCUSSION

Introduction

The purpose of this study was to investigate the teaching and learning of word problems in Beginning Algebra at the JS1 level and the effect of a professional learning program on JS1 teachers and their students. The program focused on making teachers more knowledgeable about students' Beginning Algebra misconceptions and language-based approaches in the teaching-learning process. The teachers' beliefs and practice, students' general algebra test performance, and the error analysis of the interviewed students' incorrect responses were examined.

This chapter presents a cross-case analysis and discussion, drawing on key findings from the survey data, professional learning program and the four cases. The analysis and corresponding discussion is presented using four themes generated in Chapters 4 to 9. These are the teaching context and teachers' beliefs, knowledge and practice before intervention; the students' general performance and difficulties before the intervention; the impact of the intervention on the teachers' beliefs, knowledge and practice; and, the changes in the students' general performance and difficulties after the intervention.

Teachers' Beliefs and Practice before the Intervention

Challenges of teaching algebra

About 77% of the 30 survey respondents, including the case study teachers, had between two and five years JS1 teaching experience (KF 5.1). This large proportion of 'new' teachers may be due to growth in the school population resulting from the introduction of the nine-year compulsory basic education, or to teachers leaving the profession for more financially rewarding non-teaching jobs (Ifamuyiwa, 2008). The large number of beginning teachers at the JS1 level may not be sufficiently experienced to handle students' learning difficulties, which require the knowledge of specialized mathematics content (Cady, Meier, & Lubinski, 2006; Drageset, 2010; Ormond, 2011c; Tirosh et al., 1998).

The four case study teachers were all professionally qualified and all except Dorothy were in their 30s and had more than five years' experience of teaching mathematics. Dorothy and Ruth taught in private schools while Jamie and Stephen taught in public schools. Jamie's and Stephen's large class sizes were about twice the number in Dorothy's and Ruth's classes (in private schools) respectively. Table 10.1 presents an analysis of the context within which the teachers taught, and the challenges they faced.

Table 10.1: Profile of the four case-study teachers

Descriptor	Ruth	Dorothy	Jamie	Stephen
Mathematics teaching experience	6 – 10 years	Fewer than 5 years	6 – 10 years	11 – 15 years
JS 1 teaching experience	2- 5 years	2- 5 years	2-5 years	2- 5 years
School & class size	Private, 34	Private, 26	Public, 54	Public, 67
Teaching challenges	Lack of instructional materials	Lack of instructional materials	Unconducive teaching environment, lack of instructional materials, teaching methods	Inadequate facilities, large classes
Beliefs about effective teaching & learning	Student-centred with teacher-student interaction, students able to correctly solve problems	Students being able to correctly solve problems	Availability of teaching aids and teaching guides	Availability of teaching aids, conducive learning environment, student-centred teaching

The large class population (KF 5.2) is not a new observation in Nigerian public schools (Noah et al., 2011; Ogunmade, 2005) and it has been asserted that this contributes to students' poor mathematics performances (Igbokwe, 2000). Noah et al. (2011) also note that teachers tend to adopt traditional transmissive strategies to help them to cope with the large class sizes.

Prior research indicates that large class sizes, students' lack of interest in mathematics, a lack of instructional materials and imperfect pedagogy are the main factors limiting the quality of mathematics teaching in Nigeria (Ale, 1981; Igbokwe, 2000). In the initial questionnaire, the most commonly mentioned challenges to effective teaching were a lack of instructional materials, inadequate facilities, students' weak mathematical background and large classes (KF 5.3). As expected, large classes, inadequate facilities and inappropriate teaching methods were the challenges to effective teaching mentioned by Jamie and Stephen, while Ruth and Dorothy prioritised a lack of instructional materials (KF 6.1, 7.1, 8.1, 9.1). Dorothy believed that teacher effectiveness echoes in students' ability to solve problems and Ruth agreed, also claiming that a teacher's approach should involve some engagement with students through classroom interaction (KF 6.1, 7.1). The ability to correctly solve problems as an indicator of effective learning is strongly related to a belief that mathematics is formal and all about rules (Drageset, 2010).

The weak mathematical background and young entry age of the JS1 students are note-worthy challenges that were mentioned in the survey and emphasized by participants during the

professional learning program (KF 4.10, 5.3). Arisekola's (2010) study found that JS 1 public school students and primary six private school students had not mastered up to a third of the Nigerian primary school mathematics curriculum content. Many students in the private schools do not complete the six years needed before moving to the secondary school (Chiaha, 1998). Chiaha (1998) suggested that parents' desire for their children to complete their schooling on time contributed to this trend. Secondary school mathematics builds upon primary mathematics, and beginning algebra, in particular, entails a move from operating with specifics at the primary level to a more generalized form (Kieran, 1992; Ormond, 2012). Therefore, an inadequate knowledge of primary mathematics will lead to struggling students and will impede success with beginning algebra (Hiebert & Carpenter, 1992). Interestingly though, despite the challenges, only Jamie reported that he did not enjoy his teaching (KF 7. 1).

Assertion 10.1
Large class sizes, insufficient instructional materials, a large proportion of beginning teachers teaching at the JS 1 level, JS 1 students' weak knowledge of primary mathematics and young entry age of the students are all challenges existing in the beginning algebra classroom. These would likely contribute to the use of transmissive strategies and to students' poor performance. The teachers' stated indices for effective teaching are indicators of a traditional teaching approach and view of mathematics.

Teachers' prior beliefs and knowledge

A teacher's beliefs and understanding of the mathematical content area determines the content taught, what is emphasized and what teaching methods are used (Alexandrou-Leonidou & Philippou, 2005; Beswick, 2007; Hensberry & Jacobbe, 2012). In the past 30 years, and especially recently, mathematics education reforms have resulted in a move from mathematics being viewed and taught solely as procedural manipulations of numbers resulting in knowledge of content, to an enhanced perception that mathematics is a way of thinking about and understanding relationships which results in knowledge of mathematical processes (Australian Association of Mathematics Teachers, 2006; National Council of Teachers of Mathematics, 2000). However, the survey respondents' views of mathematics were largely traditional and procedural.

The JS 1 teachers believe that word problems are the most difficult type of questions for students to answer and almost 60% of the teachers reported that questions having mainly symbolic text were easier to teach (KF 5.9). Amongst the case study teachers, only Dorothy, who had the least teaching experience, indicated that she found it easier teaching word problems rather than the symbolic aspects of algebra (KF 7.2). The next most difficult type of questions was perceived to be those problems requiring inverse operations, and the most difficult question was a word problem requiring a multiplicative inverse operation (KF 5.10).

Evidence of this belief was strengthened further by the PL teachers' rating of all the nine word problems in the algebra pre-test, regardless of length or context, as being more difficult for students to solve than the six symbolic ones (KF 4.1). Badru's (2008) study had found out that word problems are not a popular form of student assessment by Nigerian mathematics teachers.

Prior to the PL, the 30 teachers who completed the questionnaire were generally very confident of their algebra knowledge and rated highly their ability to use teaching strategies such as questioning to further mathematical discussions in the classroom; however, they also all reported using traditional teaching approaches (KF 5.5, 5.6, 5.7, 5.8). Indicators of a more traditional teaching approach are direct instruction, teachers working blackboard examples to illustrate the instruction and students practising similar questions (Hensberry & Jacobbe, 2012). The four case study teachers reported that in every lesson they directly instructed and worked examples on the board, and that their students solved questions individually in the class. Only Ruth reported that her students during every lesson had also to explain how they arrived at answers to given questions. As reported by the teachers, students' engagement levels in Stephen's and Jamie's public school classes were lower than those of Dorothy's and Ruth's students. Jamie, however, admitted that he found it difficult to use questions to enrich the purpose of the lesson or to encourage students to ask questions in the class (KF 8.1).

Assertion 10.2

The teachers' very high self-ratings of teaching effectiveness are based on their teaching styles which are largely transmissive and traditional. A belief by many of the teachers that word problems are difficult to teach and the most difficult questions for students to solve may not encourage some teachers to use them often in the class, thus limiting opportunities for students to understand beginning algebra.

The teachers rated student engagement levels lower in the public schools than the private schools and this is likely to be due to the larger class sizes and lower literacy levels experienced in the former.

The majority of those surveyed believed that the teacher's role was to give instructions and to prevent mistakes (KF 5.9). Wilson and Cooney (2002) assert that students making errors forms part of the learning process, and that it is through the process of correction that students construct their own knowledge and gain conceptual understanding. However, a teacher's ability to correct students' errors is also dependent upon the teacher knowing that an error has been made. The research indicates that teachers' knowledge of students' misconceptions is important for teaching (Chick & Baker, 2005; Welder, 2012). Ruth and many of the survey respondents indicated that they had limited knowledge of how to use an understanding of students' misconceptions in lesson planning (KF 5.6, 6.1). The teachers' ratings of the pre-test

questions and the problem-solving activity completed on the first day of the professional learning revealed that the teachers were unaware of some algebraic misconceptions, as they themselves misconceived the algebraic letter to be an object and exhibited reversal errors (KF 4.2, 4.3). This would suggest that some teachers might not understand word problems themselves, and so may avoid teaching them.

Despite their traditional beliefs, all four teachers desired that their students would gain a conceptual understanding of mathematics. They also reported that they used the native language sometimes to explain mathematics to the students. Many of the teachers in the survey and during the PL also mentioned that at times they changed to the native language, Yoruba, for explanation; although the teachers seemed to believe that the students' difficulties were caused by computational rather than comprehension-related issues (KF 4.4, 4.5, 5.9). The use of the native language is not officially allowed in classes other than the first language subject lesson, where it is studied as a subject. There is evidence that in different geographical locations in Nigeria, when the first language spoken by the majority of the students was used to teach mathematics at the upper primary classes, students' performances improved (Adetula, 1989; Ali, 2000). The first language knowledge of the students would likely serve as building blocks for the new knowledge which the teacher was attempting to develop through English.

Assertion 10.3 The professional learning revealed that the teachers had limited knowledge of students' algebra misconceptions, and that they had algebra misconceptions and reversal errors themselves. These limitations would prevent them from successfully identifying or correcting students with the misconceptions. Although the language of instruction is expected to be English, the teachers said that they sometimes use the students' first language to facilitate students' understanding of the mathematical ideas.

Observed teaching practice prior to the PL

Classroom observations revealed that teacher explanations dominated the classes of the four teachers before the intervention. Hensberry and Jacobbe (2012) note that, "in traditional mathematics classrooms, students are not typically encouraged to reflect on the problem solving strategies they have used". However, purposeful class discussions are necessary for learning mathematics (Walshaw & Anthony, 2008). Teachers who are not familiar with using students' thinking often ask questions which are intended only to elicit the correct answers (Franke et al., 2008). This was evidenced in the case studies. Most of the case study teachers' questions suggested that they were 'doing' the thinking for the students, as they asked students what they were to do next in order to arrive at the answer to a worked question. In

Jamie's and Stephen's classes, individual students solved the questions written on the board in their notebooks without any group discussion (KF 8.2, 9.2). In Dorothy's and Ruth's classes, one or two students would work the answer on the board, which was a viable strategy for their smaller classes. However, their approach differed after such student activity. Dorothy would explain afterwards, while Ruth would ask the students to explain what they had done although their faulty reasoning were not addressed (KF 6.2, 7.2). Generally, Ruth's students interacted with her more often, while Dorothy spent more time explaining terms and vocabulary to her students. In Jamie's and Stephen's classes however, student engagement and explanation of terms and vocabulary was minimal. As it is known that a teacher influences the mathematical literacy and interpretations that students have (Walshaw & Anthony, 2008), this was problematic.

Practice is very much informed by the mathematical knowledge of the teacher. Both teachers and students sometimes wrongly use the algebraic letter as a label for a word (Ely & Adams, 2012). Teachers may have misconceptions that went unnoticed during their pre-service years while others might not have been exposed to the content of what they were expected to teach the students during their training period. It appears that to make it "easier" to solve word problems, the letter was incorrectly taught and used as an object and/or label by Ruth, Dorothy and Stephen (KF 6.2, 7.2, 9.2). The concept of the variable as representing a *quantity* is very important in algebra and problem solving, which if misconceived will prevent success in higher mathematics. Focussing more on algorithms and computations does not allow for students' understanding of how to solve word problems (Rosales et al., 2012). In all classes observed, there was more concentration on the mathematical operations and manipulations of symbols involved in the problem-solving process than on thinking and understanding the processes involved.

Assertion 10.4
The observed classroom activities such as teacher talk, student listening and copying notes which dominated in lessons would not necessarily help students effectively develop mathematical literacies needed for communication, knowledge construction and conceptual understanding.
The instructional approach focused on the manipulations of symbols rather than comprehension.
The algebraic letter was used to represent a word and a label instead of a quantity by some teachers, thus passing on to the students their own misconceptions.

Students' General Performance and Difficulties before the Intervention

Pre-test performance and error frequencies prior to the PL

While the pre-test performance across all classes was less than 15%, which is disturbing, the result within the Nigerian context is not surprising (Arisekola, 2010). However, in spite of this, the general performance seems lower than the 19% to 37% success range described in studies on algebra in the middle schools in Britain, Australia and America (Booth, 1984; MacGregor & Stacey, 1993a; Ryan & Williams, 2007).

Jamie's and Stephen's students from public schools demonstrated a slightly lower performance in comparison with the students in private schools (KF 6.7, 7.6, 8.5, 9.5). The public schools most often have larger class sizes and students with lower English literacy levels, and these factors are likely to negatively impact on students' understanding and performance (Adesoji & Yara, 2007; Chiaha, 1998; Noah et al., 2011). (Figure 10.1)

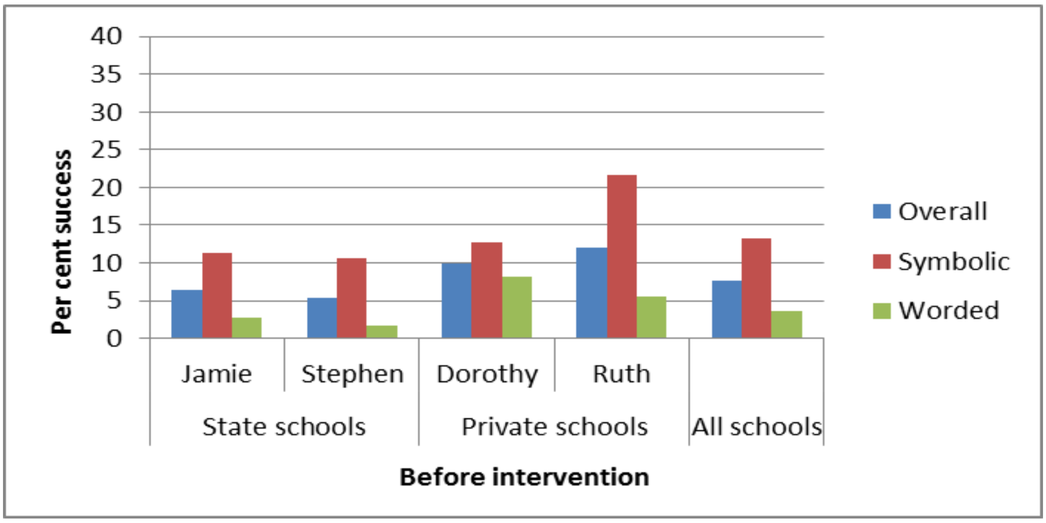


Figure 10.1: Pre-test performance of the four cases before intervention

Students from all four case study classes were more successful with symbolic questions than with worded questions. This finding is consistent with the 30 teachers' beliefs that the students would find questions with symbolic text easier to answer, and also with the difficulty ratings given by the professional learning program participants (KF 4.1, 5.10). It also appears to confirm some of the teachers' views that they found it easier to teach symbolic algebra than word problems (KF 5.9).

Word problems require "constantly translating words to gain a correct meaning for the mathematical context", and are unlike mathematical symbols which "can be considered as a form of shorthand" with often more precise meanings (Newman, 1983a, p. 6). Many Newman

studies have revealed that student difficulties occur at the comprehension level (Chinen, 2008). As in symbolic problems students do not have the English language demands of translating a written question into symbolic form, students would be expected to find them easier than questions stated in English words.

The individual teacher's approach appears to have some bearing on the students' performances, particularly in relation to the different ways that the questions were presented. Dorothy emphasized understanding of the word problems and her students fared better than the other three classes on such questions. Ruth placed more emphasis on students' engagement and teacher-student classroom interaction, and her students showed a better overall performance. Jamie who emphasized note-taking and Stephen, who taught the largest class, both taught in public schools and had lower student performances than students in the private schools. This suggests also that the more traditional that the teachers were, or the larger the classes were, the lower was the success rate of their students.

Li and Li (2008, p. 4) have stated, "Students' learning difficulties can be presented in the form of errors". The students' problem-solving errors were identified through Newman interviews conducted with four students from each class. Before the intervention, comprehension errors (46%) were the main cause of the interviewed students' wrong answers, and when combined with transformation errors (27%), it is no surprise that there were few progressions to mathematical operations and the subsequent processing errors. About 48% of the students were unable to proceed beyond the level of basic understanding of the question and this prevented them from moving into the transformational and computational steps of problem solving. Table 10.2 below summarizes the initial errors in each of the four case study classes (KF 6.8, 7.6, 8.5, 9.6).

Table 10.2: Per cent of error types in the four case study classes prior to intervention (n=16)

Error	Question representation														
	Symbolic					Worded					All questions				
	R	D	J	S	All	R	D	J	S	All	R	D	J	S	All
R	0	9	4	8	6	0	0	0	0	0	0	3	2	3	2
C	24	14	50	42	33	29	64	69	53	54	27	45	61	48	46
T	29	41	23	29	31	44	14	20	23	25	39	24	21	26	27
P	35	18	23	17	22	12	8	9	18	12	20	12	14	17	16
E	0	0	0	0	0	3	3	0	0	1	2	2	0	0	1
CE	12	18	0	4	8	12	11	3	6	8	12	14	2	5	8

R – Ruth, D – Dorothy, J –Jamie, S – Stephen; R- Reading, C – Comprehension, T – Transformation, P – Process Skills, E- Encoding, CE - Carelessness

Comprehension was more of a problem in Jamie's and Stephen's public school classes, irrespective of the question text representation. The students lacked the special mathematical

literacies required to understand the language of mathematics in English. Learning the language of mathematics is a pre-requisite for learning mathematics and having success in it (Morgan, 2005; Moschkovich, 2005; Oviedo, 2005). Ruth's class stood out by contrast with more of her students understanding the questions. This is likely to be related both to the smaller class sizes of the private school and her teaching style, which had some classroom interaction.

Assertion 10. 5
Before the intervention, few students were able to answer the algebra questions correctly and this was most commonly due to comprehension (46%) difficulties. The comprehension difficulties were more evident in the public schools, suggesting that they had a lower level of the mathematical literacies needed to understand the questions. However, there was a better performance on the symbolic questions when compared to the word problems in all of the four classes.

Language problems before the PL

Both spoken and written language, and algebraic symbolism constitute difficulties faced in algebra problem solving (Fearnley-Sander, 2000; Morgan, 2005).

In Nigeria, English language is the official language of communication and children are taught all school subjects in that language from the level of primary four. Mathematics also has its own language, implying that students need to correctly understand both English and mathematical languages in order to make meaning of the question to be solved (Kersaint et al., 2009). As would be expected, more language difficulties were identified on the questions with word text. The two issues common to all the cases were the incorrect interpretation of 'less' within the context of Question 10, and problems relating two items having varying quantities, needed for Questions 12 to 15 (See Table 10.3).

Table 10.3: Pre-intervention language errors in the four case studies

Focus	Question representation (number of students who revealed errors)					
	Usage/Descriptor	Ruth	Dorothy	Jamie	Stephen	All
Word meaning	*simplify*			S (2)		S (2)
	value		S (1)	S (2)	S (2)	S (5)
	algebra		W (4)	W (2)	W (3)	W (9)
	more		W (1)	W (2)		W (3)
	less	W (1)	W (4)	W (1)	W (3)	W (9)
Syntax	Relating two objects with varying quantities	W (4)	W (4)	W (2)	W (4)	W (14)

W- Worded text, S – Symbolic text

In all of the cases, in Question 10, some students understood 'less' to be an ordered position such as 'lower than' and one of Ruth's students transformed 'less' as division instead of subtraction (KF 6.10, 7.9, 8.8, 9.8). The few who used it as a subtraction did so outside the

context of the question. The words ‘more’ and ‘less’ have been identified as difficult for students to understand mathematically in relation to their context within a story problem (MacGregor, 1991; Verschaffel et al., 2000). Ruth’s students had the least difficulties with technical terms while Jamie’s students had difficulties with all the terms (KF 8.7, 9.7).

While acknowledging that the set of questions (Questions 12-15) that relates the number of two objects is slightly beyond the skills level of JS1 students, it was revealing to find that virtually all the students who attempted to say what the question demanded from them referred to only one of the objects, despite ‘seeing’ and acknowledging the presence of two objects. The belief in all of the cases was that they were required to find the total number of one of the objects (KF 6.10, 7.9, 8.8, 9.8). This brought a new awareness to the Researcher because her expectation was that they would have difficulty establishing equivalence in the relationship rather than actually focusing on only one object.

In Knuth et al’s (2005) study, they found that an operational interpretation of the equal sign by middle school students resulted in their simply finding the total, whereas a relational interpretation revealed knowledge of equality. This implies that the interviewed students were likely to have interpreted the questions operationally - seeking the ‘total number of one object’ - rather than relationally. The notion of equivalence is dependent on the idea of two related objects, so this particular source of misconception was not as evident as expected in these cases before the intervention.

Algebra misconceptions before the PL

The cross-case analysis of the students’ algebra knowledge is presented based on the misconceptions identified in each of the question types, that is, symbolic and word text. In all of the case studies, more misconceptions occurred with the concept of variable than with others, and Ruth’s four interviewed students revealed more algebra misconceptions than students in the other classes. As explained in this thesis, misconceptions about the variable, expressions and equations are common with students as they make the transition from arithmetic to Beginning Algebra (Knuth et al., 2005; Russell et al., 2009; Stacey & MacGregor, 1997). These misconceptions include using the algebraic letter in a variety of ways. These are, commonly, the use of a letter as a word or an object, as an alphabetical position, or as a specific value (Küchemann, 1981; Rosnick, 1981; Stacey & MacGregor, 1997). The pre-test questions are re-stated in Figure 10.2 for ease of reference during the analysis.

1. Simplify as far as possible $1 + x + x$
2. Simplify as far as possible $3m + 5n + 4m + 6n$
3. $y \times y \times y =$ ....
4. Find the value of x: $7x = 21$
5. Find the value of x: $2x – 2 = 10$
6. Find the value of x: $21x = 7$
7. Sam has x bananas and Peju has p bananas. Peter counts the number of bananas each of them have and finds they are the same. Sam said you could write this as $x=p$, but Peju said that x and p are different letters and so cannot be the same. Who do you think is correct?
8. Mary has x oranges and Bisi has four more oranges than Mary. How many oranges does Bisi have?
9. A basket costs eight naira and a bag costs c naira more than the basket. How much does the bag cost?
10. What is the number that is five less than x?
11. There is a b number of sweets in a packet. A girl has two packets of sweets and gives her friend six sweets. How many sweets does she have remaining?
12. If d is the number of dogs and c is the number of cats, write in algebra: There are four more dogs than cats.
13. Write in algebra: There are three more caps than hats.
14. Write in algebra: There are twice as many pencils as biros (let p be the number of pencils and b be the number of biros).
15. If s is the number of students and t is the number of tables, write in algebra: There are three students for every table.

Figure 10.2: Pre-test questions

Misconceptions that were common to all of the cases were using the algebraic letter as a detachable or moveable object, believing it to be a specific known value, or believing that the algebraic letters were alphabetically ordered. Table 10.4 summarises the misconceptions identified in each of the four cases.

Table 10.4: Pre-test algebra misconceptions in the four case studies

Focus	Question text (number of students presenting misconceptions)					
	Usage/Descriptor	Ruth	Dorothy	Jamie	Stephen	All
Variable	Detachable or moveable object	S (1)	S (3)	S (3)	S (3)	S (10)
	Label/shorthand	W (4)	W (3)			W (7)
	Specific known value	S (1), W (4)	W (2)	S (1), W (4)	W (3)	S (2), W (13)
	Fixed value of one	S (3)	S (1)		S (4)	S (8)
	Ignored			S (2), W (1)		S (2), W (1)
	Different letters cannot have the same value	W (3)	W (3)			W (6)
	Alphabetical ordering	W (3)	W (1)	W (2)	W (4)	W (10)
Expressions	Conjoining of terms	S (1) W (1)		S (1)	W (2)	S (2), W (3)
	Product-sum confusion	S (1) W (1)	S (2)		S (2)	S (5), W (1)
Equation	Equal sign as prompt to act	S (2)				S (2)
	Unable to do inverse operations, use a bigger divisor or balance equations	S (3)	S (2)	S (3)		S (8)

W- Worded text, S – Symbolic text

The misconceptions are now further discussed under appropriate headings.

Variable

Letter is an object. The belief that the algebra letter represented a detachable object in itself was evidenced in all of the cases, especially in Questions 1 and 2 which involved simplification of expressions (KF 6.9, 7.8, 8.7, 9.7). Instead of the students using the letter to represent the number or quantity, they viewed it literally as an object that can be moved at will or detached from the coefficient. That the letter was used as a label for an object or word by Ruth's and Dorothy's students was not surprising because, from their responses during the professional learning, all the teachers themselves seemed unaware of the fact that this was a misconception. Some of the teachers had also taught and used it in this way themselves in problem solving (KF 6.2, 7.2, 9.2).

Letter has a specific value. Many of the 16 students believed that the letter represented a specific known value in Questions 3, 10 or 11 (KF 6.10, 7.8, 7.9, 8.8, 8.9, 9.8). This misconception is also known as "letter evaluated" (Küchemann, 1981). This belief probably derives from primary school teaching where a symbol used in open sentences always has a specific value. Students found it difficult to move beyond this to a specific but *unknown* value as an answer. Some of the students appeared to believe that the unknown in Question 11 must have a value and so selected specific values for '*x*'. For example, S3 in Jamie's class said, "I don't know how many is in a packet but I'll say that 24 is for two packets of sweets" (Student interview, 28/3/11). The students were unable to move to the notion that the letter could represent a generalized number.

Letter has an alphabetical ordering. This known misconception (MacGregor & Stacey, 1993a) occurred in all of the four cases. The students gave fixed positions to the algebraic letter in Question 7 because it was 'seen' as one of the 26 alphabetical letters in English Language and not as a quantity (KF 6.10, 7.9, 8.8, 9.8).

Some other misconceptions were more commonly identified in different case studies. In Stephen's and Ruth's classes, the letter was used as a fixed value of one in Questions 1 and 3 when there was no visible coefficient (KF 7.8, 9.7). 'Letter ignored' is also known as 'letter not used' (Küchemann, 1981). This particular misconception was observed only in the responses of Jamie's students, who ignored the letter in Questions 2 and 8 (KF 8.8). Students in Dorothy's and Ruth's private schools appeared to believe that different letters must also always have different values, such as S2 in Ruth's class who said "they cannot be equal to, but they can be x and p" (Student interview, 28/3/11).

Expressions

In addition to the misconception of the letter as a detachable object, as discussed, other misconceptions about algebraic expressions occurred on the symbolic questions. The confusion between products and sums happens when a student multiplies instead of adding, or vice versa (MacGregor & Stacey, 1993a; Ormond, 2012). Students (in all cases except Jamie's) either found the sum of the terms in Question 3 instead of multiplying to arrive at the answer, or multiplied terms in Question 1 instead of adding (KF 6.9, 7.8, 9.7). This confusion was noticed only in questions that had algebraic terms with a coefficient of one. The tendency to 'close up', a common misconception with expressions, was identified among Jamie's and Ruth's students (KF 7.8, 8.7). They gave responses like "1 plus the x gives 1x..."(S1, Ruth's student interview, 28/3/11).

Equations

In all of the cases, misconceptions concerning the concept of equations occurred on the symbolic questions. It was only in Stephen's class that no misconceptions were noticed (KF 6.9, 7.8, 8.7). This said, this could have occurred because some of his students with incorrect responses had comprehension difficulty with the word 'value' as noted earlier, which prevented any further progress. In the other cases, an inability or reluctance to carry out additive or multiplicative inverse operations led to wrong answers, such as in Questions 5 and 6. For example, S2 in Dorothy's class said, "No, we can't say 7 divided by 21, so we say 21 divide 7 to give us 3" (Student interview, 25/3/11). The multiplicative inverse for Question 6 would lead to the use of a bigger divisor and ultimately balance the equation. The belief that students might find inverse questions difficult was earlier identified also in the teachers' survey (KF 5.10). Discussion about difficulties of students with fractions and inverses abounds in the literature (Robinson & LeFevre, 2012).

Assertion 10. 6 Many of the interviewed students had difficulty understanding the meaning of mathematical terms such as 'less' and in perceiving correctly a relationship between the numbers of two objects. Many algebraic misconceptions about variable, expressions and equations were also identified. These difficulties would limit students' ability to generalize mathematics, use various forms of representation and solve algebra problems.

Teachers' Beliefs and Practice after the Intervention

An effective professional learning program includes collaborative examination of students' work, a focus on content and research-based information about student learning, sufficient duration and timespan, feedback, active learning and reflection, and the support of a professional community (Ingvarson et al., 2005; Meiers & Buckley, 2010). The intervention program focused on algebraic misconceptions and language-based approaches to teaching. The teachers participated actively in small groups and joint discussions to examine and reflect on students' solutions to beginning algebra questions, and they individually solved algebra word problems. Active learning, discussions and reflections about common algebra misconceptions, algebra teaching and language-based approaches were discussed over two days. This was followed by six weeks of algebra teaching by the teachers, with the Researcher providing support. After the teaching period, a one-day workshop was held to receive feedback, reflect on experiences and provide professional support to each other.

Teachers' beliefs and knowledge after the PL

Teacher beliefs about what constitutes effective teaching differed considerably after the intervention. While availability of instructional materials was most commonly mentioned as the most crucial to effective teaching in the initial questionnaire, this was replaced by good communication skills when the final questionnaire was completed by the professional learning participants (KF 5.11). The case study teachers' responses in particular also differed from their initial traditional responses concerning availability of instructional materials and students' ability to solve problems correctly. Classroom discussions were now the focus for Ruth, and understanding what was said in the classroom was more important to Jamie than before. Dorothy's renewed focus was teacher content knowledge while Stephen showed more concern for the teaching environment. These responses suggest that the intervention made the teachers more conscious of the importance of language and discourse for effective teaching and learning of algebra.

Changes to teachers' knowledge and use of students' algebra misconceptions were observed from the teachers' responses during the professional learning program and in the final questionnaire which they completed after the intervention period. Carpenter and Lehrer (1999) state that, "teachers need to understand the mathematics they are teaching, and they need to understand their own students' thinking" (p. 30). During the professional learning program, the teachers became more aware, identified and corrected errors they made themselves in the problem-solving activity, and were able to identify the likely misconceptions in some of the students' algebra solutions (KF 4.6, 4.7). Through the intervention program, the

teachers developed a better understanding of algebra misconceptions about the variable (especially that the letter is not an object), expressions and equality, and the need to improve their students' mathematical literacy (KF 5.17, 5.19). The awareness of algebra misconceptions (especially that involving the algebraic letter standing for the number of objects and not the object in itself) is crucial to understanding and success in algebra (Ely & Adams, 2012). If teachers know this and consciously work towards helping students develop this understanding from the beginning, some of the attendant difficulties associated with word problems and algebra in general may be alleviated.

Effective professional learning can bring about awareness, a desire for change and an improvement to teaching, yet the price is often some reduction in teacher confidence. The impact of the intervention reflected in a drop in the PL teachers' initial very high self-confidence and efficacy ratings on their algebra knowledge (KF 5.12). In the final questionnaire, the PL teachers indicated less confidence about their ability to use class discussions on word problems or to plan lessons based on the knowledge of students' misconceptions (KF 5.13). The case study teachers also had reduced self-efficacy ratings about using mathematical discussions and questions appropriately in the classroom. It is likely that their greater awareness of effective questioning after the PL resulted in lower and more realistic evaluation of their self-efficacy (Hackling, Smith, & Murcia, 2011). Only Ruth, who had indicated a limitation in this aspect prior to the intervention, reported increased confidence.

Walshaw (2012) declares that "knowledge of content and knowledge of pedagogy related to content, as well as knowledge of students' thinking, all lead to more effective teaching" (p. 182). The intervention program provided an opportunity for the teachers to reflect upon their beliefs, knowledge and teaching practice. The PL teachers' written reflections indicated they realised that students need to understand the language of mathematics and that teachers need to provide opportunities for mathematical discussions before focusing on computations and algorithms (KF 5.17). The process of reflection and thinking often marks the beginning of change, which over time manifests in practice (Smith, 2012).

There also appeared to be a better appreciation and understanding of the role of language in building conceptual understanding and enhancing students' problemsolving abilities (KF 4.8). Understanding and being able to use the language of mathematics is necessary for the successful integration of new knowledge, engaging in mathematical discussions and problem-solving. A comparison of the PL teachers' responses in the two questionnaires showed an increase in the use of student activity-based approaches, a slight decline in many approaches that are purely teacher-focused, and a 7% increase in their students' class engagement levels

(KF 5.14). It also showed increased confidence in the teachers' abilities to explain, and to use mathematical vocabulary and terms. Dorothy and Ruth now seemed to be of the opinion that students were willing to ask questions. This contrasted with the prior belief of all the four teachers that students did not like asking questions.

All of the teachers' written comments reported gains in content knowledge and knowledge of content for teaching (KF 5.19). Although asked to write specifically about the most significant gain, both Dorothy and Ruth wrote about their knowledge of the Newman strategies as also being useful gains. However, a few teachers in this Nigerian study reported that they had time constraints or difficulty with some students who had limited verbal ability (KF 5.18). In Australia, Years 7 and 9 teachers reported success with using the Newman strategies as an approach, ranking it as the "most preferred strategy" to interpret context-based questions (P. White & Anderson, 2012, p. 68).

Assertion 10.7 After the intervention, teacher beliefs about effective teaching and learning centred on classroom discourse, and on ensuring students' understanding of language, and the teachers now prioritised teacher content knowledge. These would improve students' literacy and increase class engagement. There were reduced self-efficacy and confidence on some scales like algebra knowledge and questioning as teachers made informed and more realistic appraisals of their skills after the PL. Teachers' increased knowledge of algebra, algebra misconceptions, and language-based teaching approaches would likely result in better teaching, help students understand algebra better, and enhance their problem-solving skills.

Teachers' observed practice after the PL

Smith (2012, p. 319) argues that "how teachers choose to use professional development experiences is key to the impact of the professional development on a teacher's practice". In the lessons observed after the intervention, the teachers read aloud the questions on most occasions and sometimes explained the key words. Ruth's and Dorothy's students most often suggested how to solve the stated question with increased discussion after suggestions were proffered, although Ruth's classroom was the most interactive. A focus on mathematical language in the classroom is important for students' success with word problems and it has to be consciously developed by the teacher (Montague, Krawec, & Sweeney, 2008). Indeed, understanding and interpretation of the question became a new focus for Dorothy, Jamie and Stephen. Ruth's practice, on the other hand, seemed to take care of this more through student-student interaction, enabling her to turn her attention towards ensuring an

understanding of the algebra concept. Walshaw (2012, p. 185) notes that, “when teachers use their knowledge to enhance students’ learning, they are engaged in effective practice”.

The pattern of observed practice suggested that the teachers were becoming more aware of the importance of language and were now involving the students more in the problem-solving process. This allowed students to think through the solution process, and provided both the teachers and students opportunities to identify and correct misconceptions, rather than the teachers doing all of the thinking for them. In thinking through a process, a person communicates with him/herself and when speaking that process aloud, a meaning is conveyed to which others are able to contribute, leading to a harnessing of ideas that facilitates building of knowledge individually and collectively (Sfard, Forman, & Kieran, 2001). Group work was, however, not observed in any of the classes.

Assertion 10.8 After intervention, the teachers’ observed practice shifted slightly more towards ensuring students’ conceptual understanding and reasoning, rather than a focus on manipulations only. Teacher talk, students listening and copying notes reduced in frequency. There was improvement in student engagement, teacher use of questions to draw out the mathematical knowledge of students and to identify any misconceptions, and more use of language-based approaches. The intervention seemed to serve as a platform for the students to ‘see’ mathematics as a way of thinking that involves processes built on a prior understanding of the related mathematical language and concepts.

Students’ General Performances and Difficulties after the Intervention

Post-test performance and error frequencies after the PL

The post-test was completed by the students in week eight of the third and final term for the first year of junior secondary school. While the general success rate was below 25%, there was significant improvement overall and in all of the classes (except Stephen’s class which numbered 67 students). In all of the classes, there was a better performance on the purely symbolic questions than on the word problems. Tables 10.5 and 10.6 present the percentage and mean score of pre- and post-test correct responses collapsed across the classes, and the mean scores for each class.

Table 10.5: Per cent of correct answers and mean scores of for pre- and post-test questions (n=181)

Text	Question	Pre-test		Post-test	
		Per cent	Mean	Per cent	Mean
Symbolic	1	4.4	0.04	7.2	0.07
	2	35.9	0.36	36.5	0.36
	3	6.6	0.07	21.5	0.22*
	4	28.7	0.29	35.9	0.36
	5	3.3	0.03	17.7	0.18*
	6	0.6	0.01	3.3	0.03*
All symbolic questions		**13.3**	**0.80**	**20.4**	**1.22***
Worded	7	17.7	0.18	18.2	0.18
	8	4.4	0.04	8.3	0.08
	9	6.1	0.06	8.8	0.09
	10	2.2	0.02	3.3	0.03
	11	2.2	0.02	3.3	0.03
	12	0.0	0.00	1.7	0.02
	13	0.0	0.00	0.6	0.01
	14	0.0	0.00	0.6	0.01
	15	1.1	0.01	0.6	0.01
All worded questions		**3.7**	**0.34**	**5.0**	**0.45**

*Wilcoxon Signed Rank test, $p<.05$

The z-scores showed that significant Improvements in performance occurred on the symbolic questions, and the greatest gain (14.9%) was for Question 3. The most difficult symbolic question on both tests was Question 6, *Find the value of x: 21x =7*, with a fraction as the answer while Question 2, *Simplify as far as possible* $3m + 5n + 4m + 6n$, remained the easiest in both tests. In both tests, word problems had fewer correct responses but there was a better performance on most of them, especially Question 8. The easiest word problem, Question 7, examined the misconception of the letter being used with its alphabetic position and did not require any mathematical operations; while the most difficult questions, Questions 12 to 15, examined their ability to establish equivalence between the numbers of two objects.

Table 10.6: Pre- and post-test mean scores of the four case studies

Text		Pre- test					Post-test				
		R	D	J	S	All	R	D	J	S	All
Worded	Mean	0.50	0.73	0.28	0.16	0.34	0.88	0.96	0.35	0.12	0.45
	SD	1.108	1.373	.529	.412	.819	1.297	.871	.649	.445	.853
Symbolic	Mean	1.29	0.77	0.69	0.64	0.80	2.23*	1.92*	1.02*	0.58	1.22*
	SD	1.060	.908	.797	.916	.935	1.082	.977	1.205	.781	1.199
All	Mean	1.79	1.50	0.96	0.79	1.14	3.12*	2.88*	1.37*	0.70	1.67*
	SD	1.684	1.655	.990	.993	1.303	1.966	1.306	1.533	.905	1.712

R – Ruth, D – Dorothy, J –Jamie, S – Stephen *Wilcoxon Signed Rank test, $p<.01$

After the intervention, the z-scores showed that there were significant improvements in three of the classes. The pattern of results remained the same as that obtained in the pre-test:

Dorothy's and Ruth's students in the private schools reported better performances and higher increases in success than Jamie's and Stephen's students in the public schools. It should be noted that the mean scores for word and symbolic questions in Stephen's class, with 67 students, declined. Figure 10.3 presents the comparison.

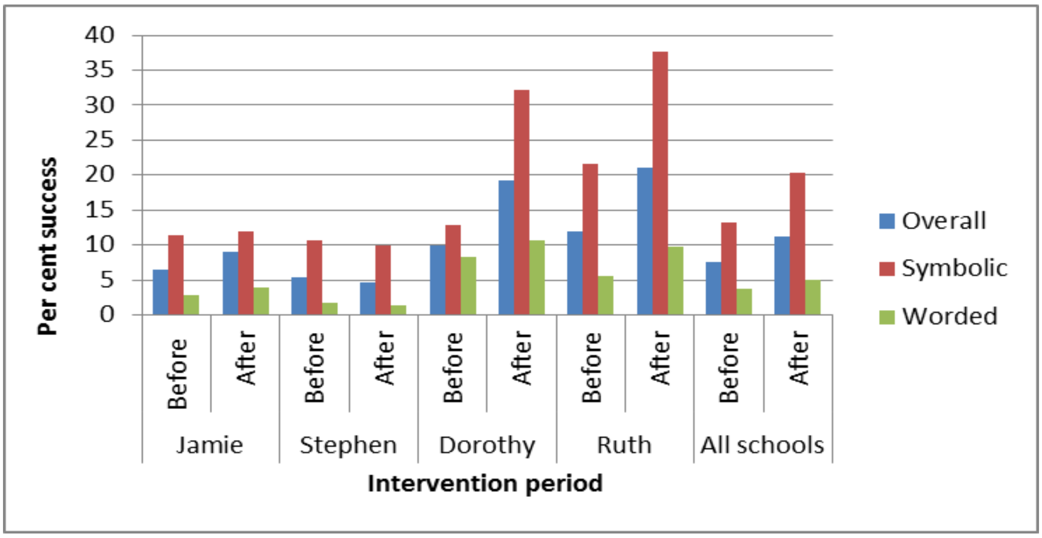


Figure 10.3: Pre- and post-test performances of the four case study classes

The trend in better performance on symbolic questions might be attributed to several reasons. The teachers had improved understanding of misconceptions about the variable, expressions and equations and could assist students more confidently. Questions with symbolic text had a lesser requirement for mathematical literacies, so students would be more successful on these questions. A study in Cyprus showed that Grade 6 students' performance in symbolic equations was also identified to be better than those concerning word problems (Alexandrou-Leonidou & Philippou, 2005). Other contributing factors to a lack of success may have been the pressure of the considerable JS 1 algebra content that needed to be taught, and students' weak pre-requisite mathematical knowledge. Perhaps the impact of the teachers' new beliefs, knowledge and practice was just unfolding and being established in the class.

The impact of the intervention was reflected in a change to the type of errors the students made. Before the intervention the main error was comprehension and only 52% of the students could go further; but after the intervention 79% of the students could continue further in the problem-solving process. This was evidenced by a reduction in comprehension errors of more than 44% in each case study class. A. White (2008, p. 58) asserts that, "student difficulties with the study of algebra include a lack of understanding of variables and formal symbolic manipulation and this acts as a barrier to success in mathematics study".

Transformation became the main error after the intervention. Table 10.7 presents a comparison of the four cases.

Table 10.7: Per cent of students' pre- and post-correct attempts at each stage in the four case study classes (n=16)

Stage	Pre-intervention						Post-intervention				
	R	D	J	S	All		R	D	J	S	All
All questions											
R	100	97	98	97	98		100	98	100	100	99
C	73	52	37	49	52		85	78	65	86	78
T	34	28	16	23	25		40	40	39	27	36
P	14	16	2	6	9		8	2	6	0	4
E	12	14	0	6	8		6	0	4	0	2
CE	12	14	2	5	8		6	0	4	2	3
Symbolic questions											
R	100	91	96	92	94		100	94	100	100	99
C	76	77	46	50	61		100	89	84	79	87
T	47	36	23	21	30		61	50	63	26	49
P	12	18	0	4	8		15	6	10	0	7
E	12	0	0	4	8		15	0	5	0	4
CE	12	18	0	4	8		15	0	5	0	4
Worded questions											
R	100	100	100	100	100		100	100	100	100	100
C	71	36	31	47	46		79	72	53	91	74
T	27	22	11	24	21		32	34	23	28	30
P	15	14	2	6	9		6	0	3	3	3
E	12	11	0	6	8		3	0	3	3	2
CE	12	11	3	6	8		3	0	3	3	2

R – Ruth, D – Dorothy, J –Jamie, S – Stephen ; R – Reading, C – Comprehension, T – Transformation, P – Process Skills, E – Encoding, CE - Carelessness

After the PL there was more progress into high levels of the procedure, seeing students make errors further down the problem solving process. Students from private schools penetrated further down the steps than those in the public schools. The most common types of errors now differed by question text format, unlike the pre-intervention errors. More transformation errors were made on the worded questions and more processing errors on the symbolic text. The language-based strategies used by the teachers had resulted in more students understanding the literacies of mathematics and having fewer comprehension errors. In

Chinen's (2008) study of Year 9 Australian students concept of time, comprehension errors reduced by 27% after a four-week language-based teaching intervention.

Ruth's, Dorothy's and Stephen's students had more process skill errors on the symbolic questions and more transformation errors on the word problems. Jamie's students were the exception; they did not understand the word problems, and were unable to mathematically operate the symbolic questions. On the other hand, Stephen's students remained fixed at the transformation stage. The main difficulty with the word problems, Jamie's students being the exception, was transforming them into a mathematical form. This is supported by the research that has found that sources of errors in word problems include an inability to translate and a failure to correctly use representations (Egodawatte, 2011; Reese, 2007). Reading of the question however did not pose a major difficulty to problem solving.

Since the teachers were just coming to terms with the awareness that the letter is not a label or an object, it seems reasonable to expect that students would still have been making mistakes in the areas of translation and processing.

Assertion 10.9 The intervention led to significant improvements in students' general performance and in particular questions with symbolic text. The use of language-based approaches by the teachers resulted in a 44% reduction in the interviewed students' comprehension difficulties, increased literacy and understanding of questions, and more students progressing to the use of mathematical transformations and operations.

Language problems after the PL

There were fewer voiced language difficulties after the intervention. Students' responses suggested that they now understood the words 'simplify' and 'algebra', despite the fact that understanding 'less' seemed to remain problematic in all the case studies. As expected, almost all the language difficulties were associated with the word problems. Saul (2008, p. 68) asserted that students need to move from "numbers to operations" as this turns their attention "away from the algorithm….toward the operation implemented by the algorithm". It has been established that 'more' and 'less' are both difficult terms for many students to mathematically interpret within the context of use. Comparisons of these language difficulties before and after the intervention are presented in Table 10.8 below.

Table 10.8: Pre- and post-test language errors in the four case studies

Focus	Question representation (number of students who revealed errors)										
		Pre-intervention					Post-intervention				
	Usage/ Descriptor	R	D	J	S	All	R	D	J	S	All
Word meaning	*simplify*			S (2)		S (2)					
	value		S (1)	S (2)	S (2)	S (5)					
	algebra		W (4)	W (2)	W (3)	W (9)					
	more		W (1)	W (2)		W (3)	W (1)	W (2)		W (2)	W (5)
	less	W (1)	W (4)	W (1)	W (3)	W (9)	W (3)	W (2)	W (2)	W (2)	W (9)
Syntax	Relating two objects with varying quantities	W (4)	W (4)	W (2)	W (4)	W (12)		W (4)	W (2)		W (6)

R – Ruth, D – Dorothy, J –Jamie, S – Stephen ; W- Worded text, S – Symbolic text

A relationship exists between 'less' and division; repeated subtraction yields a solution equivalent to division (Ormond, 2012). However, Ruth's students responded that 'less' in '*What is the number that is five less than x?'* implied division while some other students interpreted 'less' in its ordered position of 'below' the number. It could be because the word 'less' is interpreted as 'smaller' in Yoruba, the dominant first language in Lagos state, that the students perceived the question in a comparative form.

Ruth's and Stephen's students' responses suggested that they understood that a relationship existed between the two quantities of items in Questions 12 to 15, which they were required to express algebraically. This acceptance was an improvement from before, when the prior belief was often that the total quantity of one of the items should be found. A. White (2008, p. 43) noted that students "stumble when required to think relationally and algebraically". Students also have to develop knowledge that the equal sign concerns equivalence in relationships (Knuth et al., 2005). The ability to think relationally develops through exposure to problems and situations that encourage its growth, so it is not automatic but has to be learnt.

Algebra misconceptions after PL

A. White (2008, p. 57) noted that "student algebraic thinking is often confused or guided by serious misconceptions", and this has been discussed here. Misconceptions about the variable, expressions and equality persist in the middle grades and beyond (Knuth et al., 2005; Reese, 2007; Russell et al., 2009; Stacey & MacGregor, 1997). A misconception may be corrected and other errors show up but ability improves when teaching strategies correct the identified

misconceptions (Russell et al., 2009). The interviewed students' prior misconceptions that different letters cannot ever have the same value, and that the algebraic letter can be ignored, and their confusion between products and sums, appeared to have been at least partly remedied. However, some other misconceptions persisted, while some new ones occurred on the post-test interview responses. Table 10.9 presents this information across the four case studies.

Table 10.9: Pre- and post-test algebra misconceptions in the four case studies

Focus	Usage/Descriptor	Question text (number of students presenting misconceptions)									
		Pre- intervention					Post-intervention				
		R	D	J	S	All	R	D	J	S	All
Variable	Detachable or moveable object	S (1)	S (3)	S (3)	S (3)	S (10)	S (2)	S (4)	S (1)	S (4)	S (11)
	Label/shorthand	W (4)	W (3)			W (7)	W (2)	W (4)			W (6)
	Specific known value	S (1), W (4)	W (2)	S (1), W (4)	W (3)	S (2), W 13)	W (3)	W (2)	S (1), W (1)	W (2)	S (1), W (8)
	Fixed value of one	S (3)	S (1)		S (4)	S (8)	S (1)	S (1)	S (1)	S (3)	S (6)
	Ignored			S (2), W (1)		S (2), W (1)				W (1)	W (1)
	Different letters cannot have the same value	W (3)	W (3)			W (6)					
	Alphabetical ordering	W (3)	W (1)	W (2)	W (4)	W (10)	W (3)	W (3)	W (2)	W (4)	W (12)
Expression	Conjoining of terms	S (1) W (1)		S (1)	W (2)	S (2) W (3)	S (1)	S (1), W (2)	S (1)	S (1) W (1)	S (4), W (3)
	Product-sum confusion	S (1) W (1)	S (2)		S (2)	S (5) W (1)	S (1), W (1)	S (1)		S (1), W (1)	S (3), W (2)
Equation	Equal sign as prompt to act	S (2)				S (2)	S (2)				S (2)
	Unable to do inverse operations, use a bigger divisor or balance equations	S (3)	S (2)	S (3)		S (8)	S (3)	S (2)	S (3)	S (3)	S (11)
Eq-construction	Literal translation into expression						W (2)		W (2)	W (4)	W (8)
	Specific known values for items						W (2)				W (2)

R – Ruth, D – Dorothy, J –Jamie, S – Stephen; W – Worded text, S – Symbolic text

The misconceptions are further discussed under appropriate headings.

Variable

Fewer students misconceived that the algebraic letter could be ignored or that it was a specific known value. The misconception that different letters cannot have the same value was not identified in any of the students' responses. This suggests the intervention slightly improved the students' ability to use the letter in a general form or as a place holder without necessarily having a specific known value. The knowledge that the variable can be represented by more than one value has been found to increase as children move from Grade 6 to higher grades (Knuth et al., 2005).

The number of students with a misconception that the algebraic letter was an object, a label or had an alphabetical ordered position remained about the same after the intervention and suggests the misconceptions might have a persistent nature. This is in agreement with Clements et al. (1981) and Welder (2012).

Expressions

Inappropriate conjoining of terms has been identified as a common misconception in Beginning Algebra and occurred even in Year 10 students (Stacey & MacGregor, 1997). The number of students with the belief that the answer has to be 'gathered together' was minimal in each of the case study classes. The students' common responses about the confusion of products and sums were about the same as that which occurred in three classes, except Jamie's, prior to intervention. This appeared to suggest that these misconceptions about expressions were not as resistant to change as some of the others.

Equations

In all the case study classes, most of the students were unable to calculate the multiplicative inverse in the question, *Solve for x: 24x =6* (that would result in dividing by 24, a larger divisor than 6) and to balance related equations. Inverse operations with multiplication and divisions are known to be more difficult than addition and subtraction for children (Verschaffel, Bryant, & Torbeyns, 2012).

Constructions of equation

The set of word problems requiring equation-construction combines the knowledge of a letter as a quantity, the relationship of equality, and differentiating between products and sums. Misconceptions in this category were not identified in the pre-test because most of the students then did not understand the questions. After the intervention, many of the students acknowledged that there was an existing relation between the two quantities of the two objects. The knowledge of equivalence and its acceptance is reflected in the solution of

problems demanding equality (Knuth et al., 2005). Responses from Dorothy's students related only to the quantity of an item, while Jamie's students allotted a specific quantity to one of the items, seeing the second quantity as invisible or irrelevant. Ruth's students selected specific values for the two items as answers to Questions 12 to 15.

The process of equation-construction produces various types of errors, including an inability to translate, and a failure to correctly use representations and to correctly obtain expressions (MacGregor, 1991). Despite the letter being described as representing quantities in three of the four questions, in all of the cases studies, the students 'saw' and used the letters as labels for the words representing the described items/objects. As a result, the sentences were often read and interpreted literally, giving rise to literal translations that yielded expressions.

Assertion 10.10 Students' progression beyond comprehending the problem statements allowed students' misconceptions to be revealed in the transformation and processing phases of problem solving. There was a reduction in the frequency of misconceptions about the algebraic letter as representing a known value and the concept of expressions. The misconceptions of the letter as a label or object, or having alphabetical order persisted and this would hinder success with algebra problem solving.

Fusing the Research Findings

The literature concerning students' algebra misconceptions and language-based teaching procedures were employed in a professional learning program in Nigeria to enrich teachers' knowledge, beliefs and practice, and by extension, improve students' problem-solving abilities in Beginning Algebra. The professional learning led to teachers' increased algebra content knowledge, their knowledge of algebra teaching and some changes in beliefs which impacted their practice. This in turn resulted in increased student class engagement, students' development of mathematical literacies and improvements in problem-solving. These are illustrated in accompanying diagram (Figure 10.4).

Emergent Theory

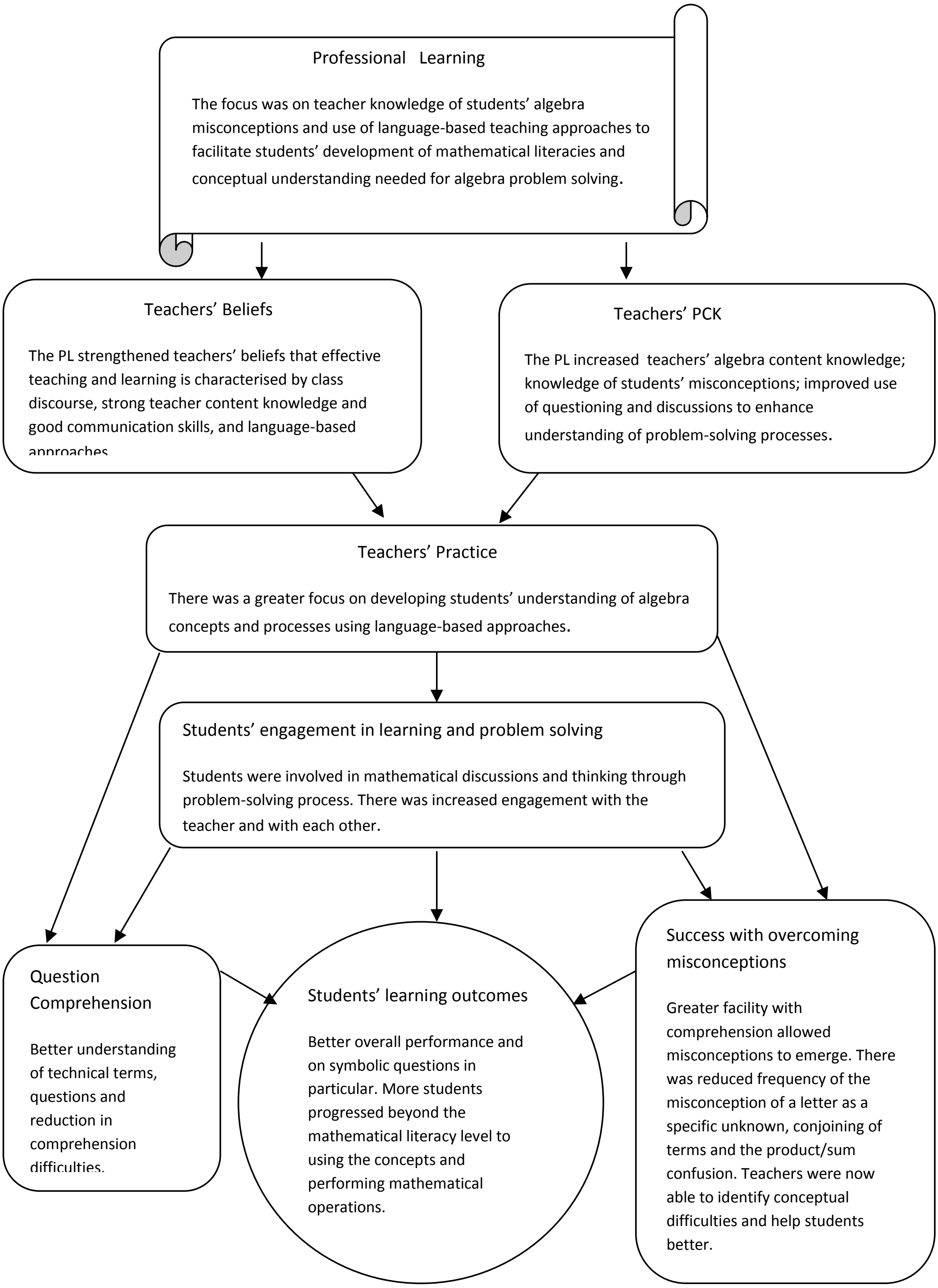


Figure 10.4: Theorising the impact of professional learning

CHAPTER ELEVEN: CONCLUSION AND IMPLICATIONS

Introduction

> Teaching and learning strategies are complex processes that interact with one another, suggesting that in-depth, context-specific analyses are necessary to fully understand each strategy's role in enhancing student performance (OECD, 2010, p. 9).

Nigerian students' poor performances in junior secondary mathematics fuelled the Researcher's desire to look at some complex teaching and learning processes within a particular Nigerian setting, and at the impact of a teacher professional learning program on both teachers and students.

Overview

The purpose of the research was to examine the impact of a professional learning program on teachers' knowledge, beliefs and teaching practice, and on students' problem-solving success in the domain of Beginning Algebra. Before embarking on this research journey, seen through the lens of four purposefully selected case study teachers and their classes, the survey responses of 30 teachers provided information about the existing teaching and learning context (Chapter 5). The professional learning focused on enriching the teachers' knowledge of algebra misconceptions, and on language-based teaching approaches (Chapter 4). The professional learning and case study participants were selected from the cohort of 30 teachers.

In each case study class, lessons were observed, students completed algebra tests and student interviews were conducted before and after the intervention (Chapters 6-9). On completion of the intervention period, the professional learning participants completed a second questionnaire. The questionnaires and algebra test scores were analysed using the SPSS software to calculate descriptive statistics and to determine if changes were significant. The students' interviews were analysed using the Newman error analysis protocol. The teachers' interview responses were coded and categorised, and from these categories themes emerged. The themes that emerged from analysing the data were interpreted after drawing on the relevant literature and this enabled assertions to be generated in Chapter 10. The findings and assertions form the basis of the conclusions to the four research questions.

Conclusions

Research Question 1: How are word problems in JS 1 beginning algebra classes taught prior to the intervention?

This question examines both the societal context within which teaching takes place in Nigeria, and the teachers' knowledge, beliefs and practice about algebra, and about the teaching and learning of mathematics.

There are large class sizes in most Nigerian public schools, many schools have insufficient instructional materials, and most of the JS 1 mathematics teachers are beginning teachers having fewer than five years of teaching experience (Assertion 10.1). Many of the JS 1 students have a weak primary mathematics background, they are very young (less than 12 years in age), and there is a considerable amount of algebra content to be learnt at the JS 1 level (Assertion 10.1). The teachers reported a traditional teaching approach and view of mathematics (formal and procedural) with low levels of student engagement in the public schools (Assertion 10.1, 10.2). The teachers also believed that word problems are difficult to teach and that they are the most difficult questions for students to solve, and indicated that students find it easier to solve symbolic questions than word problems (Assertion 10.2). Although the language of instruction is expected to be English, the teachers said that they sometimes use the students' first language to facilitate students' understanding of the mathematical ideas (Assertion 10.3).

The professional learning workshops revealed that the teachers had limited knowledge of students' algebra misconceptions and that some of the teachers exhibited algebra misconceptions themselves, especially reversal errors (Assertion 10.3). The teaching practice of the case study teachers was focused on students' knowledge of algorithmic algebra content and practice exercises, which were taught in a traditional transmissive style, while students most often listened and copied notes with limited engagement (Assertion 10.4). While Ruth's class was the most student-centred and interactive amongst the four case study classes, all four teachers demonstrated a didactic form of teaching most of the time, and some of them unknowingly used the algebraic letter to represent a word and an object instead of a quantity (Assertion 10.4).

Research Question 2: What difficulties do students in JS 1 experience in solving beginning algebra word problems prior to the intervention?

Students' general performance in algebra problems was very poor and the main difficulty was comprehension of the problem statements. They were unable to understand many of the questions because of the subsumed special mathematical language (such as 'less'), and to

understand the complex syntax of questions that asked about the relationship between numbers of two objects (ratio) (Assertion 10.5, 10.6). The individual student interviews showed that this comprehension difficulty prevented further progress on many of the questions into Newman's transformation and processing stages. There was also a better performance on symbolic questions than on word problems; students in private schools showed higher mathematical literacy skills and performed better than students in the public schools (Assertion 10.5). For some questions in which there was progress beyond comprehension into the transformation or the process skills stages, the students exhibited various misconceptions about the variable, expressions and equations/equality, and these limited their ability to generalize and solve algebra problems (Assertion 10.6).

<u>Research Question 3:</u> How does the teacher professional learning intervention impact on JS 1 mathematics teachers' beliefs, knowledge and algebra teaching practice?

After the professional learning intervention, the teachers demonstrated that they had become more conscious of the important role of language in communicating and understanding mathematics, and that they more easily utilised language-based approaches to improve their students' mathematical literacy and problem-solving abilities (Assertion 10.7). The professional learning had initiated a shift from their traditional beliefs, increased their algebra knowledge, reduced their self-efficacy ratings and improved their pedagogical content knowledge about students' thinking and algebra misconceptions (Assertion 10.7).

There was evidence that the four case study teachers' practices had increased student engagement and that they employed questioning more effectively, with the private schools having higher engagement levels. The teachers' increased knowledge of misconceptions helped them to identify and correct students' errors and to start focusing on language development and the building of conceptual understanding. The teachers had begun to move from a position of doing the 'thinking and telling' to assisting students to think for themselves (Assertion 10.8).

<u>Research Question 4:</u> How does the teacher professional learning intervention program impact on students' difficulties and success in solving algebraic word problems?

There was significant improvement in the students' general algebra problem-solving ability in questions with symbolic text, in three of the case study classes (Assertion 10.9). Students also had better success with the word problems, although this was not statistically significant. The teachers' practice appeared to have resulted in better general understanding, and there was a reduction in the occurrence of comprehension errors and in some misconceptions about the

concept of the variable and algebraic expressions (Assertion 10.9). The students in private schools fared better than their counterparts in the public schools, most probably because of smaller class sizes and students' higher levels of English literacy. Stephen's class, the largest with 67 students, was the only class that did not show any general improvements.

The students' misconceptions that the letter was an object or had alphabetical order persisted despite intervention (Assertion 10.10). Some misconceptions were partly remedied while some others were more resistant. The students had developed stronger mathematical literacies, and were then able to better understand questions and move into mathematical transformations and processing, which indicated increased conceptual understanding (Assertion 10.9, 10.10). Transforming of word problems rather than comprehending the problem statement then became the most common difficulty.

Implications

Initial teacher education

The quality of the initial mathematics teacher education can improve if pre-service teachers know more about algebra misconceptions (Ledesma, 2011) and language-based approaches. The study revealed that there were limitations to some of the teachers' knowledge and practice. Since pre-service teachers are also products of the existing Nigerian school system, many of them would likely have these misconceptions themselves. Teacher education that provides learning opportunities about algebra misconceptions has the potentials of not only exposing and addressing the pre-service teachers' limitations, but of also increasing the likelihood of the correct conceptual knowledge being passed on to their future students.

Professional learning for in-service teachers

Having a large number of beginning teachers teaching Beginning Algebra is a challenging situation because of these teachers' limited experience of what students might do and their capacity to cope with the naturally occurring challenges of Beginning Algebra students. This study showed that appropriately developed and targeted professional learning can have an impact on teachers' knowledge, beliefs and practice, and on students' algebra success. The Beginning Algebra misconceptions about variables and equality that some of the Nigerian teachers have themselves, and the language used in classrooms, were addressed during the program. The study confirmed research that suggests that providing in-service teachers with early professional learning that includes content and examination of students' work benefits the teachers' practice and their content knowledge, and helps them to develop skills (Hill, 2009; Krebs, 2005; Meiers & Buckley, 2010).

Curricular reforms

There is a need to reduce the JS 1 algebra content, to change pedagogy and to provide assessment that focuses more on understanding than on mastery of algorithms. Students need more time to understand Beginning Algebra concepts, and in this study were unable to correctly answer many of the questions because they required a deeper level of understanding. If students are to use algebra and apply it in the real-world context, then assessment at the Beginning Algebra level should expose them to such representations that will provide a balance of procedural and conceptual understanding, while also improving their mathematical literacy. Although the Nigerian curricular objectives advocate for the development of mathematical literacy and an understanding of the mathematical processes, the extensive JS 1 algebra content with its emphasis and focus on routine algorithms may make teachers more inclined towards "skimming through" the content.

Class size

The impact of the intervention was greater in the private schools than in the public schools because they had fewer students (<40) in the classes and better literacy skills. This comparative study showed, although initially not intended, that class size is a factor that can potentially impact the outcome of an intervention. Smaller classes would lead to more effective teaching and class management, and improvements in students' learning success and engagement. This study supports Cady et al. (2006) who state that it is very difficult for teachers of large classes, like Stephen, to employ pedagogies that employ students' thinking effectively.

Research

Knowledge of students' thinking and misconceptions are critical aspects of a mathematics teacher's pedagogical content knowledge (Ball et al., 2008). Further research into the mathematical literacy and algebra misconceptions of pre-service teachers in Nigeria is needed to ascertain their general understanding of algebra and to improve their pedagogy. Also it is very important to raise their awareness about the importance of mathematical literacy. Similar studies in other parts of Nigeria could indicate if the findings are the same or if they differ.

Original Contributions to the Literature

The study contributes to the literature on teacher professional development, and shows that professional learning can identify limitations in teachers' knowledge and pedagogy. Limitations were found in the Nigerian mathematics teachers' algebra knowledge and practice which the professional learning addressed. The increased teacher knowledge and the use of language-

based approaches resulted in students' better success on algebra questions and increased student classroom engagement.

The research contributes to knowledge that students' mathematical literacy and success in algebra problem solving can be improved by using language-based approaches. The modified use of Newman's (1983b) individualised interview procedure in the class, and associated research involving the use of the Newman error analysis interview protocol in a West African country is innovative and original.

This research has identified that in Nigeria, students have better success with solving symbolic algebra questions than algebra word problems. It also found that in the case of word problems, beginning algebra students' acceptance of relational knowledge (identification of two quantities in a relationship) develops after the knowledge and algebraic manipulation of a specific single quantity. The challenges and the effectiveness of language-based interventions in large classes in comparison to smaller classes were also revealed in this study.

Final Note

As this three-year research journey ends, the travails and struggles of the teacher teaching and the student learning beginning algebra weighs heavily on my mind, despite more than three decades of teaching at the secondary, sixth form and pre-service teacher levels. Because of the teachers' limited algebra teaching PCK, they found it difficult to help students with their algebra misconceptions. On the other hand, because of these misconceptions and limited mathematical literacy, the students could not achieve much success in algebra. When teachers cannot identify or unconsciously reinforce students' misconceptions, they go unnoticed and this gradually results in disillusion and failure in mathematics even as the teachers find it increasingly more difficult to help them.

In going away from this journey, it is with increased awareness that teachers should know about these misconceptions, and they themselves should not have these misconceptions if they are to assist students who have these natural tendencies to misconceive. Teachers have to develop the ability to recognize where their students are in their learning journey and what conceptual difficulties they are having. If teachers fail to recognise this, then they cannot help the students. Students' exhibitions of misconceptions are akin to red flags, and teachers should seize and utilise such golden moments to help increase students' conceptual understanding. If left uncorrected, such misconceptions may multiply into far more serious problems with worded questions in all areas of mathematics and beyond, preventing many students from fulfilling life dreams.

REFERENCES

Aburime, F. E. (2007). How manipulatives affect the mathematics achievement of students in Nigerian schools. *Education Research Quarterly, 31*(1), 3-15.

Adesina, D. (2010, August 30). The poor performance in WAEC examination, Editorial, *The Guardian*. Retrieved from http://www.guardiannewsngr.com

Adesoji, F. A., & Yara, P. O. (2007). Some school enviroment factors as correlates of mathematics achievement in southwestern Nigeria. *European Jounal of Social Sciences, 5*(2), 132-141.

Adetula, L. O. (1989). Solutiions of simple word problems by Nigerian children: Language and schooling factors. *Journal for Research in Mathematics Education, 20*(5), 489-497.

Adetula, L. O. (1990). Language factor: Does it affect children's performance on word problems? *Educational Studies in Mathematics, 21*(4), 351-365. doi: 10.1007/BF00304263

Akparanta, M., & Anuforo, E. (2010, August 19). UBEC unveils new science kits for JS students, *The Guardian* Retrieved from http://www.guardiannewsngr.com

Ale, S. O. (1981). Difficulties facing mathematics teachers in developing countries: A case study of Nigeria. *Educational Studies in Mathematics, 12*(4), 479-489.

Alexandrou-Leonidou, V., & Philippou, G. N. (2005). Teachers' beliefs about students' development of the pre-algebraic concept of equation. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Educaton* (Vol. 2, pp. 41-48). Melbourne: PME.

Ali, A. (2000). Effects of the use of Edo and Igbo for teaching and learning mathematics in Nigerain primary schools. *ABACUS: The Journal of the Mathematical Association of Nigeria, 25*(1), 51-64.

An, S., & Wu, Z. (2012). Enhancing mathematics teachers' knowledge of students' thinking from accessing and analyzing misconceptions in homework. *International Journal of Science and Mathematics Education, 10*(3), 717-753. doi: 10.1007/s10763-011-9324-x

Anderson, G., & Arsenault, N. (1998). *Fundamentals of educational research* (2nd ed.). London: The Falmer Press.

Arisekola, S. A. (2010). Cognitive entry behaviour of junior secondary school students in primary school mathematics. In G. A. Ajewole (Ed.), *Proceedings of the School of Science 2010 National Conference* (Vol. 1, pp. 100-105). Lagos, Nigeria: School of Science, Adeniran Ogunsanya College of Education.

Australian Association of Mathematics Teachers. (2006). *Standards for excellence in teaching mathematics in Australian schools (Revised edition).* Retrieved 30 October, 2012, from www.aamt.edu.au

Badru, A. S. (2008). Teacher's profile and perception of use of assessment strategies: Implications for quality assurance in junior secondary education in Ogun state. *Nigerian Journal of Curriculum Studies, 15*(4), 154-160.

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 433-456). New York: Macmillan.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it so special? *Journal of Teacher Education, 59*(5), 389-407. doi: 10.1177/0022487108324554

Banerjee, R., & Subramaniam, K. (2012). Evolution of a teaching approach for beginning algebra. *Educational Studies in Mathematics, 80*(3), 351-367. doi: 10.1007/s10649-011-9353-y

Benjamin, M. A. (2004). *Nigerian science teachers' beliefs about effective science teaching, their pedagogical content knowledge and how these influence science teaching.*

(Unpublished doctoral dissertation), Edith Cowan University, Mount Lawley, Western Australia.
Benjamin, R. G. (2012). Reconstructing readability: Recent developments and recommendations in the analysis of text difficulty. *Educational Psychology Review, 24*(1), 63-88. doi: 10.1007/s10648-011-9181-8
Beswick, K. (2005). The beliefs/practice connection in broadly defined contexts. *Mathematics Education Research Journal, 17*(2), 39-68.
Beswick, K. (2006). The importance of mathematics teachers' beliefs. . *The Australian Mathematics Teacher, 62*(4), 17-22.
Beswick, K. (2007). Teachers' beliefs that matter in secondary mathematics classrooms. *Educational Studies in Mathematics, 65*, 95-120. doi: 10.1007/s10649-006-9035-3
Blench, R. M. (1998). The status of the languages of central Nigeria. In M. Brenzinger (Ed.), *Endangered languages in Africa* (pp. 187-206). Koppe verlag: Koln.
Booth, L. R. (1984). *Algebra: Children's strategies and errors. A report of the strategies and errors in secondary mathematics project*. Windsor, London: NFER-NELSON Publishing Company.
Booth, L. R. (1999). Children's difficulties in beginning algebra. In B. Moses (Ed.), *Algebraic thinking: Grades K-12. Readings from NCTM's school-based journals and other publications* (pp. 299-307). Reston, VA: National Council of Teachers of Mathematics.
Cady, J., Meier, S. L., & Lubinski, C. A. (2006). Developing mathematics teachers: The transition from preservice to experienced teacher. *The Journal of Educational Research, 99*(5), 295-305.
Campbell, A. E., Adams, V. M., & Davis, G. E. (2007). Cognitive demands and second-language learners: A framework for analyzing mathematics instructional contexts *Mathematical Thinking and Learning, 9*(1), 3 - 30. doi: 10.1080/10986060709336603
Carpenter, T., Fennema, E., & Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 97*(1), 3-20.
Carpenter, T., Fennema, E., Peterson, P. L., & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education, 19*(5), 385-401.
Carpenter, T., Fennema, E., Peterson, P. L., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*(4), 499-531.
Carpenter, T., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 19-32). Mahwah, New Jersey: Lawrence Erlbaum Associates.
Cathcart, W. G., Pothier, Y. M., Vance, J. H., & Bezuk, N. S. (2006). *Learning mathematics in elementary and middle schools:A learner-centered approach* (4th ed.). New Jersey: Pearson Prentice Hall.
Cavanagh, M. S., & Garvey, T. (2012). A professional experience learning community for pre-service secondary mathematics teachers. *Australian Journal of Teacher Education, 37*(12), 57-75. doi: 10.14221/ajte.2012v37n12.4
Chalouh, L., & Herscovics, N. (1999). Teaching algebraic expressions in a meaningful way. In B. Moses (Ed.), *Algebraic thinking: Grades K-12. Readings from NCTM's school-based journals and other publications* (pp. 168-174). Reston, Virginia: National Council of Teachers of Mathematics.
Chapman, O. (2006). Classroom practices for context of mathematics word problems. *Educational Studies in Mathematics, 62*(2), 211-230. doi: 10.1007/s10649-006-7834-1
Chiaha, G. T. (1998). Private schools and educational development in Nigeria: A march toward the 21st century. *Journal of Education for National Development, 1*(1), 83-91.
Chick, H. L. (2009). Teaching the distributive law: Is fruit salad still on the menu? In R. Hunter, B. Bicknell & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd Annual*

Conference of the Mathematics Education Research Group of Australasia (Vol. 1, pp. 121-128). Palmerston North: Mathematics Research Group of Australasia.

Chick, H. L., & Baker, M. K. (2005). Investigating teachers' responses to students misconceptions. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 249-256). Melbourne: PME.

Chinen, G. Y. (2008). *Language process errors in year 9 mathematics problem solving: A multi-strategy language-based intervention.* (Unpublished doctoral dissertation), Edith Cowan University, Mount Lawley, Western Australia.

Clarkson, P. C. (1991a). *Bilingualism and mathematics learning*. Geelong, Australia: Deakin University.

Clarkson, P. C. (1991b). Mathematics in a multilingual society. In K. Durkin & B. Shire (Eds.), *Language in mathematical education: Research and practice* (pp. 237-246). Milton Keynes, England Open University Press.

Clarkson, P. C. (1992). Language and mathematics: A comparison of bilingual and monolingual students of mathematics. *Educational Studies in Mathematics, 23*(4), 417-429. doi: 10.1007/BF00302443

Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education, 13*, 16-30.

Clement, J., Lochhead, J., & Monk, G. S. (1981). Translation difficulties in learning mathematics. *American Mathematical Monthly, 88*, 286-290.

Clements, M. A. (1980). Analysing children's errors on written mathematical tasks. *Educational Studies in Mathematics, 11*(1), 1-21.

Clements, M. A. (1982). Careless errors made by sixth grade children on written mathematical tasks. *Journal for Research in Mathematics Education, 13*(2), 136-144.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher, 23*(7), 13-20.

Cobb, P. (2000). From representations to symbolizing: Introductory comments on semiotics and mathematical learning. In P. Cobb, E. Yackel & K. McClain (Eds.), *Symbolizing and communicating in mathematics classroms: Perspectives on discourse, tools, and instructional design* (pp. 17-36). Mahwah,New Jersey: Lawrence Erlbaum Associates.

Cobb, P. (2007). Putting philosophy to work: Coping with multiple theoretical perspectives. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 3-38). Charlotte, USA: Information Age

Cobb, P., Wood, T., & Yackel, E. (1990). Classroom as learning enviroments for teachers and researchers. In R. B. Davis, C. A. Maher & N. Noddings (Eds.), *Constructivist view on the teaching and learning of mathematics* (pp. 125-146). Virginia: National Council of Teachers of Mathematics.

Cochran-Smith, M., & Lytle, S. (1999). Relationships of knowledge and practice: Teacher learning in communities. In A. Iran-Nejad & P. D. Pearson (Eds.), *Review of research in education* (Vol. 24, pp. 249-305). Washington,DC: American Educational Research Association.

Cohen, L., Manion, L., & Morrison, K. (2011). *Research methods in education* (7th ed.). Oxon, Great Britain: Routledge.

Confrey, J. (1990). What constructivism implies for teaching. In R. B. Davis, C. A. Maher & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics. JRME Monograph No 4* (pp. 125-146). Virginia: National Council of Teachers of Mathematics.

Creswell, J. W. (1994). *Research design: Qualitative and quantitative approaches*. Thousand Oaks, CA: Sage

Cummins, J. (1979). Linguistic interdependence and the educational development of bilingual children. *Review of Educational Research, 49*, 222-251.

Cummins, J. (2000). *Language, power, and pedagogy: bilingual children in the crossfire* (Vol. 23.). Buffalo, N.Y: Multilingual Matters.
Darling-Hammond, L., & McLaughlin, N. W. (1995). Policies that support professional development in an era of reform. *Phi Delta Kappan, 76*(8), 597-604.
Davis, R. B., & Maher, C. A. (1990). The nature of mathematics: What do we do when we "do mathematics"? In R. B. Davis, C. A. Maher & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics. JRME Monograph No 4* (pp. 65-78). Virginia: National Council of Teachers of Mathematics.
Depaepe, F., De Corte, E., & Verschaffel, L. (2010). Teachers' approaches towards word problem solving: Elaborating or restricting the problem context. *Teaching and Teacher Education, 26*(2), 152-160. doi: 10.1016/j.tate.2009.03.016
Drageset, O. G. (2010). The interplay between the beliefs and the knowledge of mathematics teachers. *Mathematics Teacher Education and Development, 12*(1), 30-49.
Durkin, K. (1991). Language in mathematical education: An introduction. In K. Durkin & B. Shire (Eds.), *Language in mathematical education: Research and practice* (pp. 3-16). Milton Keynes, England: Open University Press.
Egodawatte, G. (2011). *Secondary school students' misconceptions in algebra.* (Doctoral Dissertation), University of Toronto, Canada. ProQuest Dissertations and Theses database. (NR77791)
Ely, R., & Adams, A. E. (2012). Unknown, placeholder, or variable: what is x ? *Mathematics Education Research Journal, 24*(1), 19-38. doi: 10.1007/s13394-011-0029-9
Even, R., & Tirosh, D. (2008). Teacher knowledge and understanding of students' mathematical learning and thinking. In L. D. English (Ed.), *Handbook of international research in mathematics education* (2nd ed., pp. 202-222). New York: Routledge.
Fafunwa, B. A. (1974). *History of education in NIgeria*. London: George Allen & Unwin Ltd.
Falle, J. (2007). Students' tendency to conjoin terms: An inhibition to their development of algebra. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice. Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 285-294). Hobart: MERGA Inc.
Fearnley-Sander, D. (2000). Discussion document for the twelfth ICMI study- The future of the teaching and learning of algebra. *The International Journal of Computer Algebra in Mathematics Education, 7*(1), 77-86.
Federal Ministry of Education. (2007). *9-Year basic education currriculum: Mathematics for JSS 1-3*. Abuja, Lagos: National Educational Research and Development Council.
Federal Ministry of Education. (2009). *National teacher education policy*. Abuja, Nigeria: Federal Ministry of Education.
Federal Republic of Nigeria. (2004). *National policy on education*. Abuja: NERDC.
Fennema, E., Carpenter, T., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 27*(4), 403-434.
Franke, M. L., Carpenter, T., & Battey, D. (2008). Content matters: Algebraic reasoning in teacher professional development. In J. J. Kaput, D. W. Carraher & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 333-359). New York: Lawrence Erlbaum Associates.
Franke, M. L., Kazemi, E., & Battey, D. (2007). Understanding teaching and classroom practice in mathematics. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-256). Charlotte, USA: Information Age Pub.
Fujii, T., & Stephens, M. (2008). Using number sentences to introduce the idea of variable. In C. E. Greenes & R. Rubenstein (Eds.), *Algebra and algebra thinking in school mathematics: Seventieth yearbook* (pp. 127-140). Reston, VA: National Council of Teachers of Mathematics.
Goldin, G. A. (1990). Epistemology, constructivism, and discovery learning mathematics. In R. B. Davis, C. A. Maher & N. Noddings (Eds.), *Constructivist views on the teaching and*

learning of mathematics. JRME Monograph No 4 (pp. 31-47). Virginia: National Council of Teachers of Mathematics.
Goldin, G. A. (2008). Perspectives on representation in mathematical learning and problem solving. In L. D. English (Ed.), *Handbook of international research in mathematics education* (2nd ed., pp. 176-201). New York: Routledge.
Goos, M., Stillman, G., & Vale, C. (2007). *Teaching secondary school mathematics: Research and practice for the 21st century*. Crows Nest, Australia Allen & Unwin.
Hackling, M., & Prain, V. (2005) *Research on teacher effectiveness and teacher profesional learning. Primary connections stage 2 trial research report*. Canberra: Australian Academy of Science.
Hackling, M., Smith, P., & Murcia, K. (2011). Enhancing classroom discourse in primary science: The puppets project. *Teaching Science, 57*(2), 18-25.
Hensberry, K. K. R., & Jacobbe, T. (2012). The effects of Polya's heuristic and diary writing on children's problem solving. *Mathematics Education Research Journal, 24*(1). doi: 10.1007/s13394-012-0034-7
Herbel-Eisenmann, B., & Phillips, E. D. (2008). Analyzing students' work: A context for connecting and extending algebraic knowledge for teaching. In C. E. Greenes & R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics: Seventieth yearbook* (pp. 295-311). Reston, USA: NCTM.
Herscovics, N., & Kieran, C. (1999). Constructing meaning for the concept of equation. In B. Moses (Ed.), *Algebraic thinking, grades K-12: Readings from NCTM's school-based journals and other publications* (pp. 181-188). Reston, Virginia: National Council of Teachers of Mathematics.
Herscovics, N., & Linchevski, L. (1994). A cognitive gap between arithmetic and algebra. *Educational Studies in Mathematics, 27*(1), 59-78.
Hiebert, J., & Carpenter, T. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-100). New York: Macmillan.
Hill, H. C. (2009). Fixing teacher professional development. *Phi Delta Kappan, 90*(7), 470-476.
Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.
Ifamuyiwa, S. A. (2008). Quality assurance in the preparation and production of teachers for the universal basic education (UBE) programme. *Nigerian Journal of Curriculum Studies, 15*(4), 185-195.
Ifamuyiwa, S. A., & Akinsola, M. K. (2008). Improving senior secondary school students' attitude towards mathematics through self and cooperative -instructional strategies. *International Journal of Mathematical Education in Science and Technology, 39*(5), 569-585.
Igbokwe, D. I. (2000). Dominant factors and error types inhibiting the understanding of mathematics. In M. A. G. Akale (Ed.), *Enriching science, technology and mathematics education: 41st Annual Conference Proceedings of the Science Teachers Association of Nigeria* (pp. 242-249). Ibadan, Nigeria: Heinemann Educational Books
Ingvarson, L., Meiers, M., & Beavis, A. (2005). Factors affecting the impact of professional development programmes on teachers' knowledge, practice,student outcomes and efficacy. *Education Policy Analysis Archives, 13*(10), 1-26.
Johnson, B., & Christensen, L. (2012). *Educational research: Quantitative, qualitative, and mixed approaches* (4th ed.). Thousand Oaks, California: SAGE Publications.
Kaput, J. J. (1999). Teaching and learning a new algebra. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms thst promote understanding* (pp. 133-155). Mahwah, New Jersey: Lawrence Erlbaum Associates.

Kaput, J. J. (2008). What is algebra? What is algebraic reasoning? In J. J. Kaput, D. W. Carraher & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 9-17). New York: Lawrence Erlbaum

Kersaint, G., Thompson, D. R., & Petkova, M. (2009). *Teaching mathematics to English language learners*. New York: Routledge.

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics, 12*(3), 317-326.

Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 390-419). New York: Macmillan.

Kieran, C. (2007). Learning and teaching of algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 707-762). Charlotte, USA: Information Age

Knuth, E. J., Alibali, M. W., McNeil, N. M., Weinberg, A., & Stephens, A. C. (2005). Middle school students' understanding of core algebraic concepts: Equivalence & Variable *Zentralblatt für Didaktik der Mathematik, 37*(1), 68-76. doi: 10.1007/bf02655899

Krebs, A. S. (2005). Analyzing student work as a professional development activity. *School Science and Mathematics, 105*(8), 402-411.

Küchemann, D. (1981). Algebra. In K. M. Hart (Ed.), *Children's understanding of mathematics: 11-16* (pp. 102-119). London: John Murray.

Ladele, O. A. (2008). Mathematics laboratory activities: A resource to aid delivery of the mathematics curriculum *Proceedings of the 49th Science Teachers Association of Nigeria Conference* (pp. 231-233). Ibadan: Heinmann Educational Books.

Ledesma, E. F. R. (2011). Primary and secondary teachers' knowledge, interpretation, and approaches to students errors about ratio and proportion topics. *Creative Education, 2*(3), 264-269. doi: 10.2307/749409; 10.1086/525551

Li, X., & Li, Y. (2008). Research on students' misconceptions to Improve teaching and learning in school mathematics and science. *School Science and Mathematics, 108*(1), 4-7. doi: 10.1111/j.1949-8594.2008.tb17934.x

Lim, C. S., & Presmeg, N. (2011). Teaching mathematics in two languages: A teaching dilemma of malaysian chinese primary schools. *International Journal of Science and Mathematics Education, 9*(1), 137-161. doi: 10.1007/s10763-010-9225-4

Linchevski, L., & Herscovics, N. (1996). Crossing the cognitive gap between arithmetic and algebra: Operating on the unknown in the context of equations. *Educational Studies in Mathematics, 30*(1), 39-65.

Linsell, C., Cavanagh, M., & Tahir, S. (2013). Using meaningful contexts to promote understanding of pronumerals. *The Australian Mathematics Teacher, 69*(1), 33-40.

Lochhead, J., & Mestre, J. P. (1999). From words to algebra: Mending misconceptions. In B. Moses (Ed.), *Algebraic thinking, grades K-12: Readings from NCTM's school-based journals and other publications* (pp. 321-327). Reston, Virginia: Nationsl Council of Teachers of Mathematics.

MacGregor, M. (1991). *Making sense of algebra: Cognitive processes influencing comprehension*. Geelong, Victoria, Australia: Deakin University.

MacGregor, M., & Stacey, K. (1993a). Cognitive models underlying students' formulation of simple linear equations. *Journal for Research in Mathematics Education, 24*, 217-232.

MacGregor, M., & Stacey, K. (1993b). What is x? *The Australian Mathematics Teacher, 49*(4), 28-30.

McClure, C. T. (2009). Algebraic thinking: What it is and why it matters. *District Administration, 45*(4), 44-45.

Meiers, M., & Buckley, S. (2010). Successful professional learning. *The Digest.* WACOT,2010 (1). Retrieved October 14, 2010, from http://www.wacot.wa.edu.au

Mohammed, A. M. (2012). *Laying the foundation for an enduring STM education in the universal basic education programme: Keynote address at the 53rd annual conference of the Science Teachers' Association of Nigeria.* Retrieved 1 May, 2013, from www.stanonline.org

Montague, M., Krawec, J., & Sweeney, C. (2008). Promoting self-talk to improve middle school students' mathematical problem solving. *Perspectives on Language and Literacy, 34*(2), 13-17.

Morgan, C. (2005). Communicating mathematically. In S. Johnston-Wilder, P. Johnston-Wilder, D. Pimm & J. Westwell (Eds.), *Learning to teach mathematics in the secondary school* (2nd ed., pp. 142-157). London: Routledge.

Moschkovich, J. (2005). Using two languages when learning mathematics. *Educational Studies in Mathematics, 64*(2), 121-144. doi: 10.1007/s10649-005-9005-1

Nathan, M. J., & Koedinger, K. R. (2000). Teachers' and researchers' beliefs about the development ofalgebraic reasoning. *Journal for Research in Mathematics Education, 31*, 168-190.

Nathan, M. J., & Koellner, K. (2007). A framework for understanding and cultivating the transition from arithmetic to algebraic reasoniing. *Mathematical Thinking and Learning, 9*(3), 179 - 192. doi: 10.1080/10986060701360852

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Neuman, W. L. (2003). *Social research methods: Qualitative and quantitiative approaches* (Fifth ed.). Boston, USA: Pearson Education Inc.

Newman, A. (1977). An analysis of sixth- grade pupils' errors on written mathematical tasks. *Victorian Institute for Educational Research Bulletin, 39*(31-43).

Newman, A. (1983a). *The Newman language of mathematics kit: Language and mathematics*. Sydney: Harcourt Brace Jovanovich.

Newman, A. (1983b). *The Newman language of mathematics kit: Strategies for diagnosis and remediation*. Sydney: Harcourt, Brace Jovanovich.

Ni Riordain, M., & O'Donoghue, J. (2009). The relationship between performance on mathematical word problems and language proficiency for students learning through the medium of Irish. *Educational Studies in Mathematics, 71*(1), 43-64. doi: 10.1007/s10649-008-9158-9

Noah, A. O. K., Falodun, S. A., & Ade, A. (2011). Effects of classroom size on teacher-student academic performance in junior secondary schools in Lagos state. *AOCOED Journal of Educatonal Research and Development, 6*(1), 115-119.

Norton, S. (2012). Prior study of mathematics as a predictor of pre-service teachers' success on tests of mathematics and pedagogical content knowledge. *Mathematics Teacher Education and Development, 14*(1), 2-26.

OECD. (2010). *Mathematics teaching and learning strategies in PISA: Executive summary.* Retrieved September 29, 2010, from www.pisa.oecd.org/dataoecd/15/47/46032578.pdf

Ogunmade, T. O. (2005). *The status and quality of secondary science teaching and learning in Lagos state,Nigeria.* (Unpublished doctoral dissertation), Edith Cowan University, Mount Lawley, Western Australia.

Ohuche, R. O. (1978). Change in mathematics education since the late 1950's-ideas and realisation-Nigeria. *Educational Studies in Mathematics, 9*, 271-281.

Okigbo, E. O., & Osuafor, A. M. (2008). Effect of using maths laboratory techniques in teaching mathematics on the achievement of mathematics students. *Educational Research and Review, 3*(8), 257-261.

Olaleye, B. O. (2012). *Teachers' knowledge for using multiple representations in teaching chemistry in Nigerian senior secondary schools.* (Doctoral Dissertation), Edith Cowan University, Mount Lawley, Western Australia. Retrieved from http://library.ecu.edu.au

Ormond, C. (2000). *Theorizing mathematics as a language: Implications for mature students.* (Unpublished doctoral dissertation), James Cook University, Queensland, Australia.
Ormond, C. (2011a). *Algebra: What is a pronumeral?* Algebra Lecture: Part 1. Power Point Presentation. Edith Cowan University. Perth, Western Australia.
Ormond, C. (2011b). *Algebra: What is an equation?* Algebra Lecture: Part 2. Power Point Presentation. Edith Cowan University. Perth, Western Australia.
Ormond, C. (2011c). Tailoring mentoring for new mathematics and science teachers: An exploratory study. *Australian Journal of Teacher Education, 36*(4), Article 4. Avaliable at: http://ro.ecu.edu.au/ajte/vol36/iss4/4
Ormond, C. (2012). Developing"algebraic thinking": Two key ways to establish some algebraic ideas in primary classrooms. *Australian Primary Mathematics Classroom, 17*(4), 13-19.
Oviedo, G. C. B. (2005). Comprehending algebra word problems in the first and second languages. In J. Cohen, McAlister, K. Rolstad & J. MacSwan (Eds.), *ISB4: Proceedings of the 4th International Symposium on Bilingualism* (pp. 267-295). Somerville, USA: Cascadilla Press.
Oxford University Press. (Ed.) (1997) The Australian Concise Oxford Dictionary of Current English (3rd ed.). Melbourne: Author.
Palm, T. (2008). Impact of authenticity on sense making in word problem solving. *Educational Studies in Mathematics, 67*(1), 37 - 58. doi: 10.1007/s10649-007-9083-3
Pape, S. J., & Tchoshanov, M. A. (2001). The role of representation(s) in developing marthematical understanding. *Theory into Practice, 40*(2), 118-127.
Perso, T. (1993). M is for misconception. *The Australian Mathematics Teacher, 49*(3), 36-38.
Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 257-315). Charlotte, USA: Information Age.
Philipp, R. A., & Schappelle, B. P. (1999). Algebra as generalized arithmetic: Starting with the known for a change *The Mathematics Teacher, 92*(4), 310 - 316.
Pimm, D. (1991). Communicating mathematically. In K. Durkin & B. Shire (Eds.), *Language in mathematical education: Research and practice* (pp. 17-24). Milton Keynes [England] ; Philadelphia: Open University Press.
Polya, G. (1957). *How to solve it*. New York: John Wiley & Sons.
Punch, K. F. (2005). *Introduction to social research: Quantitative and qualitative approaches* (2nd ed.). London: Sage.
Reed, S. K. (1999). *Word problems: Research and curriculum reforms*. New Jersey: Lawrence Erlbaum Associates.
Reese, M. S. (2007). *What's so hard about algebra? A grounded theory study of adult algebra learners.* (Doctoral Dissertation). ProQuest Dissertation and Theses database.
Richardson, J. S., Morgan, R. F., & Fleener, C. (2006). *Reading to learn in the content areas* (6th ed.). Belmont, LA: Thomson Wadsworth.
Robinson, K. M., & LeFevre, J. (2012). The inverse relation between multiplication and division: Concepts, procedures, and a cognitive framework. *Educational Studies in Mathematics, 79*(3), 409-428. doi: 10.1007/s10649-011-9330-5
Rojano, T. (2008). Mathematics learning in the middle school/junior secondary school: Student access to powerful mathematical ideas. In L. D. English (Ed.), *Handbook of international research in mathematics education* (2nd ed., pp. 136-153). New York: Routledge.
Rosales, J., Santiago, V., Chamoso, J. M., Munez, D., & Orrantia, J. (2012). Teacher-student interaction in joint word problem solving. The role of situational and mathematical knowledge in mainstream classrooms. *Teaching and Teacher Education, 28*(8), 1185-1195.
Rosnick, P. (1981). Some misconceptions concerning the concept of variable *Mathematics Teacher, 74*(6), 418-420.

Russell, M., O'Dwyer, L. M., & Miranda, H. (2009). Diagnosing students' misconceptions in algebra: Results from an experimental pilot study. *Behavior Research Methodds, 41*(2), 414-424.

Ryan, J., & Williams, J. (2007). *Children's mathematics 4-15: Learning from errors and misconception*. Berkshire, England: Open University Press.

Schoenfeld, A. H. (2008). Research methods in (mathematics)education. In L. D. English (Ed.), *Handbook of international ressearch in mathematics education* (2nd ed., pp. 467-519). New York: Routledge.

Schoenfeld, A. H., & Arcavi, A. (1999). On the meaning of variable. In B. Moses (Ed.), *Algebraic thinking, grades K-12: Readings from NCTM's school-based journals and other publications* (pp. 150-156). Reston, Virginia: National Council of Teachers of Mathematics.

Setati, M. (2005). Teaching mathematics in a primary multilingual classroom. *Journal for Research in Mathematics Education, 36*(5), 447-466.

Setati, M., Chitera, N., & Essien, A. (2009). Research on multilingualism in mathematics education in South Africa: 2000-2007. *Africa Journal of Research in MST Education,* (Special Issue), 65-80. www.kgethi.com/pdf/2pub.pdf

Sfard, A. (1991). 'On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides af the same coin'. *Educational Studies in Mathematics, 22*(1), 1-36.

Sfard, A., Forman, E., & Kieran, C. (2001). Learning discourse: Sociocultural approaches to research in mathematics education. *Educational Studies in Mathematics, 46*(1-3), 1-12.

Sfard, A., & Linchevski, L. (1994). The gains and pitfalls of reification-the case of algebra. *Educational Studies in Mathematics, 26*(2), 191-228.

Shirley, L. (2001). Ethnomathematics as a fundamental of instructional methodology *ZDM, 33*(3), 85-87. doi: 10.1007/BF02655699

Shulman, L. S. (1986). Those who understand : Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Silverman, D. (2011). *Interpreting qualitative data* (4th ed.). London: SAGE Publications.

Simons, H. (2009). *Case study research in practice*. Thousand Oaks, California: SAGE Publications.

Smith, W. M. (2012). Exploring relationships among teacher change and uses of contexts. *Mathematics Education Research Journal, 24*(3), 301-321. doi: 10.1007/s13394-012-0053-4

Sowder, J. T. (2007). The mathematical education and development of teachers. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157-223). Charlotte, USA: Information Age.

Stacey, K. (2004). *Trends in mathematics education: The example of algebra education*. Paper presented at the epiSTEME-1, an international conference to review research on Science, Technology and Mathematical Education, International Centre, Dona Paula, Goa,India. Abstract retrieved from http://citeeerx.ist.psu.edu/viewdoc

Stacey, K., & MacGregor, M. (1997). Ideas about symbolism that students bring to algebra. *The Mathematics Teacher, 90*(2), 110 -113.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*, 213-226. doi: 10.1016/S0742-051X(00)00052-4

Sule, A. O. (1992). An investigation into the difficulties of the current junior secondary school mathematics curriculum.   Retrieved 6 October 2010, from www.unilorin.edu.ng

Swan, M. (2006). Designing and using research instruments to describe the beliefs and practices of mathematics teachers. *Research in Education, 75*, 58-70.

Tabach, M., & Friedlander, A. (2008). The role of context in learning beginning algebra. In C. E. Greenes & R. Rubenstein (Eds.), *Algebra and algebra thinking in school mathematics:*

Seventieth yearbook (pp. 223-232). Reston, VA: National Council of Teachers of Mathematics.
Tanisli, D., & Kose, N. Y. (2013). Pre-service mathemaitcs teachers' knowledge of students about the algebraic concepts. *Australian Journal of Teacher Education, 38*(2), 1-18.
Tella, A. (2008). Teacher variables as predictors of academic achievement. *International Electronic Journal of Elementary Education, 1*(1), 16-22.
Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.
Tirosh, D., Even, R., & Robinson, N. (1998). Simplifyiing algebraic expressions: Teacher awareness and teaching approaches. *Educational Studies in Mathematics, 35*(1), 51-64.
Trends in International Mathematics and Science Study. (2012). International student achievement in the TIMSS mathematics content and cognitive domain. *TIMSS AND PIRLS International Study Centre.* Retrieved 5 April, 2013, from http://timssandpirls.bc.edu/timss201111downloads/T11_IR_M
Umar, S. (2001). Education and islamic trends in northern Nigeria: 1970s-1980s. *Africa Today, 48*(2), 126-151. doi: 10.2979/AFT.2001.40.2.126
United Nations Population Division. (2012). World population prospects: The 2012 revision. Retrieved 16 August 2013, from Population Division of the Department of Economic ansd Social Affairs of the United Nations Secretariat http://esa.un.org/unpd/wpp/index.htm
Usiskin, Z. (1999). Conceptions of school algebra and uses of variables. In B. Moses (Ed.), *Algebraic Thinking, Grades K-12: Readings from NCTM's School- Based Journals and Other Publications* (pp. 7-13). Reston, VA: National Council of Teachers of Mathematics.
Uwadiae, I. (2011). Results of the May/June 2011 West African senior school certificate examination (WASSCE). Press briefing to announce release of results. Retrieved 5 April 2013, from http://www.waecheadquatersgh.org/index.php?option=com_docman&task=dx_download&gid=57&Itemid=55
Vacc, N. N., & Bright, G. W. (1999). Elementary preservice teachers' changing beliefs and instructional use of children's mathematical thinking. *Journal for Research in Mathematics Education, 30*, 89-110.
van Oers, B. (2000). The appropriation of mathematical symbols: A psychomiotic approach to mathematics learning. In P. Cobb, E. Yackel & K. McClain (Eds.), *Symbolizing and communicating in mathematics classroms: Perspectives on discourse, tools, and instructional design* (pp. 133-176). Mahwah, New Jersey: Lawrence Erlbaum Associates.
Verschaffel, L., Bryant, P., & Torbeyns, J. (2012). Introduction. *Educational Studies in Mathematics, 79*(3), 327-334. doi: 10.1007/s10649-012-9381-2
Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems*. Lisse, The Netherlands: Swets & Zeitlinger.
Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Havard University Press.
Wagner, S. (1999). What are these things called variables? In B. Moses (Ed.), *Algebraic thinking: Grades K-12. Readings from NCTM's school-based journals and other publications* (pp. 316-320). Reston, Virginia, USA: National Council of Teachers of Mathemativs.
Wagner, S., & Parker, S. (1999). Advancing algebra. In B. Moses (Ed.), *Algebraic thinking, grades K-12: Readings from NCTM's school-basesd journals and other publications* (pp. 328-340). Reston, Virginia: Natioanl Coucil of Teachers of Mathematics.
Walshaw, M. (2012). Teacher knowledge as fundamental to effective teaching practice. *Journal of Mathematics Teacher Education, 15*, 181-185. doi: 10.1007/s10857-012-9217-0

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms *Review of Educational Research, 78*(3), 516-552.

Warren, E., & Cooper, T. (2008). Patterns that support early algebraic thinking in the elementary school. In C. E. Greenes & R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics: Seventieth yearbook* (pp. 113-126). Reston, Virginia: National Council of Teachers of Mathematics.

Watson, A., & De Geest, E. (2005). Principled teaching for deep progress: Improving mathematical learning beyond methods and materials. *Educational Studies in Mathematics, 58*(2), 209-234.

Watson, I. (1980). Investigating errors of beginning mathematicians. *Educational Studies in Mathematics, 11*(3), 319-329. doi:

Welder, R. M. (2012). Improving algebra preparation: Implications from research on student misconceptions and difficulties. *School Science and Mathematics, 112*(4), 255-264. doi: 10.1111/j.1949-8594.2012.00136.x

White, A. (2005). Active mathematics in classrooms: Finding out why children makes mistakes- and then doing something to help them. *Square One, 15*(4), 15-19.

White, A. (2008). Learning mathematics in the middle years. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W. T. Seah & P. Sullivan (Eds.), *Research in mathematics education in Australiasia 2004-2007* (pp. 41-72). Rotterdam, The Netherlands: Sense Publishers.

White, P., & Anderson, J. (2012). Pressure to perform: Reviewing the use of data through professional learning conversations. *Mathematics Teacher Education and Development, 14*(1), 60-77.

Wilson, M., & Cooney, T. J. (2002). Mathematics teacher change and development: The role of beliefs. In G. C. Leder, E. Pehkonen & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 127-148). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Yin, R. K. (2003). *Case study research: Design and methods* (3rd ed.). Thousand Oaks, California: Sage Publications.

Yin, R. K. (2012). *Applications of case study research* (3rd ed.). Thousand Oaks, California: SAGE Publications.

APPENDICES

Appendix 1: Approval Letter from the District Office

LAGOS STATE GOVERNMENT

EDUCATION DISTRICT V

(BADAGRY, OJO, AMUWO-ODOFIN & AJEROMI/IFELODUN LGAS)

OLD OJO RD.
AGBOJU,
LAGOS- NIGERIA

Fax..............................
Telephone: 01- 8987889
P.M.B..........................

EDV/11/I.R/01/VOL.I/11 17th February, 2011

The Principal

....
....
....

LETTER OF INTRODUCTION

With reference to your letter dated 27th December, 2010, I am directed to introduce you to the Principals of selected Junior Secondary School through this medium in Badagry, Ojo and Amuwo-Odofin Zones of Education District V, Agboju.

Mrs. Ladele, O. A. is a Ph.D. student in Mathematics Education at Edith Cowan University Perth Western Australia. She wants to conduct a research into the teaching and learning of beginning Algebra in Basic 7 classes.

Mrs. Ladele, O. A. is a lecturer at Adeniran Ogunsanya College of Education, Ijanikin.

Please accord her the necessary assistance.

Thank you

OZIEGBE, I. P.
FOR: TUTOR-GENERAL/PERMANENT SECRETARY

MISSION STATEMENT OF THE DISTRICT
Inspiring educational excellence in Secondary Schools through efficient service delivery, quality teaching, conducive teaching/learning environment in collaboration with all stakeholders

Appendix 2: Principals Consent Form

Title of Project: The Teaching and Learning of Word Problems in Beginning Algebra

I have read the information above regarding the research and all questions I have asked have been answered to my satisfaction. I am aware that I can contact the student, her supervisor or the Research Ethics Officer if I have further questions or concerns.

I understand that I may withdraw my school from the research study at any time.

I understand that no student, teacher, school or district will be named in any reports of this research study.

Please tick a box below to indicate your school's participation in the study.

☐ I give consent for my school to participate in this study

☐ I do not give consent for my school to participate in this study

Name of school: ____

Name of Principal: ____

Signature:__________________________________ Date: ________________

Appendix 3: Consent Form for Teachers

Title of Project: Teaching and Learning of Word Problems in Beginning Algebra

I have read the information above regarding the research and all questions I have asked have been answered to my satisfaction. I am aware that I can contact the student, her supervisor or the Research Ethics Officer if I have further questions or concerns. I understand that I may withdraw from the study at any time.

Please tick the boxes below to indicate your willingness to participate in various aspects of the study.

☐ I am willing to complete the initial teacher questionnaire.

☐ I am willing to attend the three days of training workshops, participate in the group discussion and complete the second questionnaire.

☐ I am willing to have my algebra class observed, my class to complete the algebra test and for four students to be interviewed before and after the workshops.

☐ I agree to having my voice recorded and photographs of mathematics problems solutions taken in my class.

☐ I also agree that the research data gathered for this study may be published provided that I, my school and Local Education District are not identified.

Participant Name: ____

Signature:___________________________________ Date: _________________

Appendix 4: Mathematics Teacher Initial Questionnaire

MATHEMATICS TEACHERS INITIAL QUESTIONNAIRE (MTIQ)

This questionnaire has two sections. The first section is intended to gather background information about the teacher such as age, teaching qualification and experience. The second section addresses the teaching and learning of mathematics, teacher beliefs, confidence, self-efficacy, challenges and approaches to the teaching and learning of algebra. Your responses will help us better understand the challenges faced by teachers of algebra.

All information provided will be treated confidentially.

Please fill in the required information or put a tick in the option detail with which you identify.

SECTION 1: Background information

1.1 Gender: Male ☐ Female ☐

1.21 Local Education District: ____

1.22 Name of School: ____

1.3 Age (years): 21-25 ☐ 26-30 ☐ 31-35 ☐ 36-40 ☐ 41-45 ☐ 46-50 ☐ 50+ ☐

1.4 Completed years of mathematics teaching experience: 0-1 ☐ 2-5 ☐ 6-10 ☐ 11-15 ☐ 16-20 ☐ 20+ ☐

1.5 Completed years of Basic 7 (JSS1) teaching experience: 0-1 ☐ 2-5 ☐ 6-10 ☐ 11-15 ☐ 16-20 ☐ 20+ ☐

1.6 Please tick boxes to indicate all of your qualifications: NCE ☐ B. Ed ☐ B. Sc(Ed) ☐

B .Sc ☐ PGDE ☐ M. Ed ☐ M. Sc ☐ Others please specify): ___________

1.7 Subject Specialities (Combination): ___________ ____________

1.8 Please write the name and average class size of each Basic 7 arm you teach in the respective boxes

Name of arm						
Class size						

1.9 Number of mathematics /mathematics teaching workshops, seminars or trainings attended in the past 2 years: _______________

SECTION 2: Please write your answer to these questions in the spaces provided below

2.1 What challenges do you face in teaching upper basic mathematics effectively?

____

____

____

2.2 What do you believe are the characteristics of effective upper basic mathematics teaching?

____

_ ____
____

2.3 What do you believe are the most effective teaching strategies that may help students learn mathematics?

____

____

____

2.4 How do you manage *talk* in the mathematics classroom? Tick the option you agree with for each item.

Item		Strongly Disagree	Disagree	Not sure	Agree	Strongly Agree
1	I am effective in establishing a classroom atmosphere in which most students feel confident to give their own answers to questions.					
2	I am effective in asking questions to suit the purpose and flow of classroom discussions in mathematics.					
3	I am able to respond to students answers in ways that help develop an effective discussion of mathematical ideas.					
4	My rich knowledge of mathematics helps me ask the right questions to develop mathematics ideas through discussion.					
5	My rich knowledge of mathematics helps me respond appropriately to students' answers to my questions.					
6	I am normally able to respond to students' answers in ways that maintain and promote further discussion of the mathematics ideas.					
7	I am effective in encouraging and supporting students to ask questions in my mathematics class.					
8	I am effective in engaging most students in responding to my questions during mathematics discussions.					
9	I am able to sustain discussions so that we thoroughly discuss the mathematics ideas.					
10	I am effective in using questioning to identify students' prior knowledge of mathematics topics.					

2.5 How confident are you in using these strategies to teach algebra? (Tick a box for each item)

Item		Very	Confident	Okay	Limited	No confidence
1	Engaging students' interest in algebra					
2	Managing group activities in algebra					
3	Managing discussions and interpretations of word problems					
4	Explaining algebra concepts					
5	Developing vocabulary and terms needed for learning algebra					

6	Assessing children's learning in algebra					
7	Using knowledge of students' misconceptions to plan algebra lessons					
8	Involving majority of the students in class discussions/activities					

2.6 Please tick the option you agree with for each of the statements below

Item		Strongly disagree	Disagree	Not sure	Agree	Strongly agree
1	Not all students can learn mathematics.					
2	If a teacher does not tell students how to solve questions, they will make mistakes.					
3	Mathematics is mainly calculations.					
4	Students believe algebra is difficult.					
5	Mathematics consists of rules and procedures.					
6	If I was free to choose, I would not teach mathematics.					
7	Students do not like to ask questions in the class.					
8	Students have to be attentive in a mathematics class.					
9	Students' mathematics background is often weak.					
10	I always enjoy my mathematics teaching.					
11	There are rules in algebra that students have to learn.					
12	I sometimes use the native language to explain mathematical ideas in the class.					
13	I find it easier to teach algebraic word problems than those with symbolic notations.					

2.7 Indicate with a tick in the appropriate column how often you use the following approaches to teach upper basic algebra

Item		Every lesson	In most lessons	In some lessons	In a few lessons	Never
1	Working examples on the board for students to copy					
2	Whole class discussion of mathematical ideas					
3	Grouping/pairing students to solve questions in the class					
4	Inviting students to explain the working for their answer					
5	Having students solving questions individually					
6	Having students reading aloud the question to be solved					
7	Asking students to identify key words and symbols in the question					
8	Asking students identify what the question asks us to do					
9	Asking students identify the plan for solving the question					
10	Writing notes on the board for students to copy					
11	Reminding students about the meaning of a variable					
12	Identifying students' misconceptions of algebra					
13	Using different types of mathematical representations					
14	Explaining the meaning of equations					

2.8 Tick a box that indicates your level of confidence about the **mathematical knowledge** you need to teach these topics.

	Very confident	Confident	Okay	Limited confidence	Not confident
Number and numeration					
Basic operations					
Algebraic processes					
Geometry and mensuration					
Everyday statistics					

2.9 Please rate the extent to which your students are **actively engaged** in learning mathematics, on the scale below, with a tick in a box.

Very active								Very passive	
10	9	8	7	6	5	4	3	2	1

3.1 Rank the following questions in order of difficulty from **1 being the easiest** to **6 as the most difficult**.

Rank

A. ☐ X 5 = 20. What is ☐ ? _________

B. 5x = 20. What is x? _________

C. ☐ X 20 = 5. What is ☐ ? _________

D. 20x = 5. What is x? _________

E. 5 lots of a certain number is 20. What is the number? _________

F. Twenty lots of a certain number is five. What is the number? _________

In the space below, please explain why you think children will find questions you ranked as 4, 5 and 6 difficult.

Rank no 4

____

Rank no 5

____

Rank no 6

____

3.2 Tick a box that indicates your level of interest in attending an algebra workshop

Very interested	Interested	Not interested

Appendix 5: Mathematics Teacher Final Questionnaire

MATHEMATICS TEACHERS FINAL QUESTIONNAIRE (MTFQ)

This final questionnaire focuses on identifying changes in teachers' knowledge and beliefs about mathematics and algebra teaching and learning.

Please write your answer to these questions in the spaces provided below.

1.1 What do you believe are the characteristics of effective upper basic mathematics teaching?

____

____

____

1.2 What do you believe are the most effective ways that student may learn mathematics?

____

____

____

1.3 How do you manage *talk* in the mathematics classroom? Tick the option you agree with for each item.

Item		Strongly Disagree	Disagree	Not sure	Agree	Strongly Agree
1	I am effective in establishing a classroom atmosphere in which most students feel confident to give their own answers to questions.					
2	I am effective in asking questions to suit the purpose and flow of classroom discussions in mathematics.					
3	I am able to respond to students answers in ways that help develop an effective discussion of mathematical ideas.					
4	My rich knowledge of mathematics helps me ask the right questions to develop mathematics ideas through discussion.					
5	My rich knowledge of mathematics helps me respond appropriately to students' answers to my questions.					
6	I am normally able to respond to students' answers in ways that maintain and promote further discussion of the mathematics ideas.					
7	I am effective in encouraging and supporting students to ask questions in my mathematics class.					
8	I am effective in engaging most students in responding to my questions during mathematics discussions.					
9	I am able to sustain discussions so that we thoroughly discuss the mathematics ideas.					
10	I am effective in using questioning to identify students' prior knowledge of mathematics topics.					

1.4 How confident are you in using these strategies to teach algebra? (Tick a box for each item)

Item		Very confident	Confident	Okay	Limited confidence	No confidence
1	Engaging students' interest in algebra					
2	Managing group activities in algebra					
3	Managing discussions and interpretations of word problems					
4	Explaining algebra concepts					
5	Developing vocabulary and terms needed for learning algebra					
6	Assessing children's learning in algebra					
7	Using knowledge of students' misconceptions to plan algebra lessons					
8	Involving majority of the students in class discussions/activities					

1.5 Please tick the option you agree with for each of the statements below

Item		Strongly disagree	Disagree	Not sure	Agree	Strongly agree
1	Not all students can learn mathematics.					
2	If a teacher does not tell students how to solve questions, they will make mistakes.					
3	Mathematics is mainly calculations.					
4	Students believe algebra is difficult.					
5	Mathematics consists of rules and procedures.					
6	If I was free to choose, I would not teach mathematics.					
7	Students do not like to ask questions in the class.					
8	Students have to be attentive in a mathematics class.					
9	Students' mathematics background is often weak.					
10	I always enjoy my mathematics teaching.					
11	There are rules in algebra that students have to learn.					
12	I sometimes use the native language to explain mathematical ideas in the class.					
13	I find it easier to teach algebraic word problems than those with symbolic notations.					

1.6 Indicate with a tick in the appropriate column how often you use the following approaches to teach upper basic algebra

Item		Every lesson	In most lessons	In some lessons	In a few lessons	Never
1	Working examples on the board for students to copy.					
2	Whole class discussion of mathematical ideas.					
3	Grouping/pairing students to solve questions in the class.					
4	Inviting students to explain the working for their answer.					
5	Having students solving questions individually.					
6	Having students reading aloud the question to be solved.					
7	Asking students to identify key words and symbols in the question.					
8	Asking students identify what the question asks us to do.					
9	Asking students identify the plan for solving the question.					
10	Writing notes on the board for students to copy.					
11	Reminding students about the meaning of a variable.					
12	Identifying students' misconceptions of algebra.					
13	Using different types of mathematical representations.					
14	Explaining the meaning of equations.					

1.7 Tick a box that indicates your level of confidence about the **mathematical knowledge** you need to teach these topics

	Very confident	Confident	Okay	Limited confidence	Not confident
Number and numeration					
Basic operations					
Algebraic processes					
Geometry and mensuration					
Everyday statistics					

1.8 Please rate the extent to which your students are actively engaged in learning mathematics, on the scale below, with a tick in a box.

Very active and engaged in learning							Very passive in learning		
10	9	8	7	6	5	4	3	2	1

1.9 Rank the following questions in order of difficulty from **1 being the least difficult** to **6 as the most difficult**. Rank

A. $\boxed{}$ X 5 = 20. What is $\boxed{}$? ______

B. 5*x* = 20. What is *x*? ______

C. $\boxed{}$ X 20 = 5. What is $\boxed{}$? ______

D. 20*x* = 5. What is *x*? ______

E. 5 lots of a certain number is 20. What is the number? ______

F. Twenty lots of a certain number is five. What is the number? ______

In the space below, please explain why you think children will find questions you ranked as 4, 5 and 6 difficult.

Rank number 4____

Rank number 5 ____

Rank number 6 ____

2.0 How has an understanding of students' misconceptions and thinking helped you to teach algebra?

____

____

____

2.1 Why is mathematical talk important in the teaching and learning of algebraic word problems?

____

____

____

2.2 What difficulties did you have in using the Newman strategy?

____

____

____

2.3 What is the most significant thing that you gained in this professional learning workshop?

____

____

____

2.4 Please write your suggestions to improve the workshop

____

____

____

2.5 Will you recommend the workshop to other mathematics teachers? (Circle one) Yes No

Appendix 6: Lesson Observation Sheet

MATHEMATICS CLASSROOM OBSERVATION SCHEDULE (MCOS)

Teacher: Topic:

School: Objective:

Visit Date: Time:

Time Interval (mins)	Observed Activity Explaining idea/concept [1], Identifying variable/expression/equality misconception [2], identifying key terms and symbols [3], showing understanding that a letter is a quantity [4], resisting impulse to "inappropriately gather" terms [5], reading aloud of the question [6], reframing question in own words [7], explaining strategy for solving problem [8], finding the value of the letter (if applicable) [9], writing the answer [10], individual/paired/group work [11], use of questions [12], listening [13], note copying [14]
2	
4	
6	
8	
10	
12	
14	
16	
18	
20	
22	
24	
26	
28	
30	
32	
34	
36	
38	
40	

Note. S and T are codes used to record tallies for students' and the teacher respectively.

Appendix 7: Algebra Pre-test Questions

Initial Algebra Questions for Basic 7

Please answer the questions in the space provided after each one. Remember to write how you arrive at your answer clearly.

1. Mary has x oranges and Bisi has four more oranges than Mary. How many oranges does Bisi have?

2. Simplify as far as possible $1 + x + x$

3. Write in algebra: There are twice as many pencils as biros (let p be the number of pencils and b be the number of biros).

4. There is a b number of sweets in a packet. A girl has two packets of sweets and gives her friend six sweets. How many sweets does she have remaining?

5. A basket costs eight naira and a bag costs c naira more than the basket. How much does the bag cost?

6. If s is the number of students and t is the number of tables, write in algebra: There are three students for every table.

7. Find the value of x: $7x = 21$

8. Simplify as far as possible $3m + 5n + 4m + 6n$

9. $y \times y \times y =$ ....

10. If d is the number of dogs and c is the number of cats, write in algebra: There are four more dogs than cats.

11. Find the value of x: $2x - 2 = 10$

12. Sola has x bananas and Peju has p bananas. Peter counts the number of bananas each of them have and finds they are the same. Sola said you could write this as $x=p$, but Peju said that x and p are different letters and so cannot be the same. Who do you think is correct?

13. Find the value of x: $21x = 7$

14. What is the number that is five less than x?

15. Write in algebra: There are three more caps than hats.

Appendix 8: Algebra Post-test Questions

Final Algebra Questions for Basic 7

Please answer the questions in the space provided after each one. Remember to write how you arrive at your answer clearly.

1. A ball costs ten naira and a shirt costs y naira more than the ball. How much does the shirt cost?

2. Write in algebra: There are five more goats than dogs.

3. Mary has m toys and Bisi has three more toys than Mary. How many toys does Bisi have?

4. Solve for x: $5x - 5 = 20$

5. Sola has y mangoes and Peju has x mangoes. Peter counts the number of mangoes each of them have and finds they are the same. Sola said you could write this as $y = x$, but Peju said that y and x are different letters and so cannot be the same. Who do you think is correct?

6. Find the value of x: $6x = 24$

7. If p is the number of plates and c is the number of cups, write in algebra: There are four more plates than cups.

8. Solve for x: $24x = 6$

9. $m \times m \times m =$ ....

10. There is a x number of pencils in a packet. A girl has three packets of pencils and gives her friend five pencils. How many pencils does she have remaining?

11. If b is the number of boys and g is the number of girls, write in algebra: There are three boys for every girl.

12. Simplify as far as possible $4z + 3p + 7z + 2p$

13. Write in algebra: There are twice as many books as pens (let b be the number of books and p be the number of pens).

14. What is the number that is four less than x?

15. Simplify as far as possible $1 + y + y$

Appendix 9: Newman Interview Protocol

Newman Interview   Questions used with Students

1. Please read the question to me. If you don’t know a word or number, leave it out.
2. What does this sign/word mean? Tell me, what is the question asking you to do?
3. Show me how you start finding an answer to this question.
4. Show me how you work the answer out for this question. Tell me what you are doing as you work.
5. What is your answer?

Source: The Newman Language of Mathematics Kit (1983)

Appendix 10: Newman Error Analysis Guideline Sheet

<table>
<tr><td colspan="3">Test Items
Newman Error Analysis Guideline (NEAG)</td><td colspan="3">Numeric Code:
Second Attempt
Answer E C
Error Correct</td></tr>
<tr><td colspan="2">Strategies</td><td>Expected Response</td><td colspan="3">Pupil Response</td></tr>
<tr><td rowspan="2">1</td><td rowspan="2">Reading recognition
Please read the question to me. If you don't know a word or number, leave it out</td><td rowspan="2"></td><td colspan="2">Words E C</td><td rowspan="2">Reading Recognition (R)</td></tr>
<tr><td colspan="2">Symbols E C</td></tr>
<tr><td rowspan="2">2</td><td>Comprehension
(a) What does this sign/word mean?</td><td></td><td colspan="2">Terms E C</td><td rowspan="2">Comprehension (C)</td></tr>
<tr><td>(b)Tell me what the question is asking you to do.</td><td></td><td colspan="2">General E</td></tr>
<tr><td>3</td><td>Transformation
Tell or show me how you start finding an answer to this question.</td><td></td><td colspan="2">E C</td><td>Transformation (T)</td></tr>
<tr><td rowspan="7">4</td><td rowspan="7">Process Skills
Show me how you work the answer out for this question. Tell me what you are doing as you work.</td><td rowspan="7">Numerical Logic</td><td></td><td>C</td><td rowspan="7">Process Skills (P)</td></tr>
<tr><td>RR</td><td>E</td></tr>
<tr><td>WO</td><td>E</td></tr>
<tr><td>FA</td><td>E</td></tr>
<tr><td>FC</td><td>E</td></tr>
<tr><td>NR</td><td>E</td></tr>
<tr><td></td><td>E</td></tr>
<tr><td rowspan="2">5</td><td rowspan="2">Encoding Ability
Verbalises correct answer but writes it down incorrectly</td><td rowspan="2"></td><td colspan="2">Words E C</td><td rowspan="2">Encoding Ability (E)</td></tr>
<tr><td colspan="2">Symbols E C</td></tr>
<tr><td>6</td><td>Carelessness
Correctly done</td><td></td><td colspan="2">E</td><td>Carelessness</td></tr>
<tr><td>7</td><td>Motivation
Correctly done</td><td></td><td colspan="2">E</td><td>Motivation</td></tr>
<tr><td>8</td><td>Task Form
Correctly done</td><td></td><td colspan="2">E</td><td>Task Form</td></tr>
<tr><td colspan="3">Correct answer:</td><td colspan="3">Initial Error Cause</td></tr>
</table>

Appendix 11: Newman Diagnostic Profile Sheet

<table>
<tr><td colspan="10">Diagnostic Profile Sheet</td><td colspan="8">Name
Class
Date</td></tr>
<tr><td>Test items</td><td colspan="17">Mathematics Questions</td></tr>
<tr><td>1. Reading Recognition</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>2. Comprehension</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>3. Transformation</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>4. Process Skills</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>5. Encoding Ability</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>6. Carelessness</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>7. Motivation</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>8. Task Form</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

(Newman, 1983, Adapted from The Newman Language of Mathematics Kit, p. 123)

Appendix 12: Teacher Algebra Rating Sheet

How difficult do you think the following questions are for Basic 7 students? Using the rating scale below, indicate in the box beside each question the difficulty level you believe is appropriate.

1 -Very Easy	2 - Easy	3 - Okay	4 - Difficult	5 – Very Difficult

1. Mary has x oranges and Bisi has four more oranges than Mary. How many oranges does Bisi have?
2. Simplify as far as possible $1 + x + x$
3. Write in algebra: There are twice as many pencils as biros (let p be the number of pencils and b be the number of biros).
4. There is a b number of sweets in a packet. A girl has two packets of sweets and gives her friend six sweets. How many sweets does she have remaining?
5. A basket costs eight naira and a bag costs c Naira more than the basket. How much does the bag cost?
6. If s is the number of students and t is the number of tables, write in algebra: There are three students for every table.
7. Find the value of x: 7x = 21
8. Simplify as far as possible $3m + 5n + 4m + 6n$
9. $y \times y \times y =$ ----------------
10. If d is the number of dogs and c is the number of cats, write in algebra: There are four more dogs than cats.
11. Find the value of x: $2x – 2 = 10$
12. Sam has x bananas and Polly has p bananas. Peter counts the number of bananas each of them have and finds they are the same. Sam said you could write this as x=p, but Polly said that x and p are different letters and so cannot be the same. Who do you think is correct?
13. Find the value of x: 21x = 7
14. What is the number that is five less than x?
15. Write in algebra: There are three more caps than hats.

Appendix 13: Focus Group Interview Questions

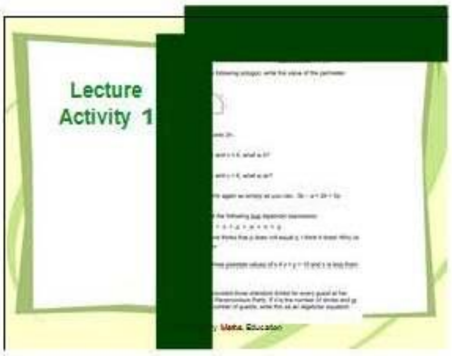
1. What difficulties can a student encounter in solving word problems?
2. Did you notice the occurrence of any of the misconceptions that your class? How did you handle it?
3. Why are variables, expression and equality important in algebra

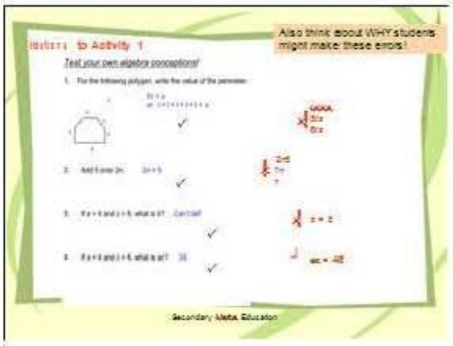
Appendix 14: Per cent of Teachers' Response on Algebra Questions

Text	Questions	Very easy	Easy	Okay	Difficult	Very difficult
Symbolic	$y \times y \times y$ = ...	53.8	30.8	15.4	0	0
	Find the value of x: 7x = 21	46.2	38.5	15.4	0	0
	Simplify as far as possible $1 + x + x$	15.4	69.2	15.4	0	0
	Simplify as far as possible $3m + 5n + 4m + 6n$	46.2	15.4	30.8	7.7	0
	Find the value of x: $2x - 2 = 10$	30.8	30.8	38.5	0	0
	Find the value of x: $21x = 7$	23.1	30.8	15.4	23.1	7.7
Word	What is the number that is five less than *x?*	0	30.8	53.8	15.4	0
	A basket costs eight naira and a bag costs *c* naira more than the basket. How much does the bag cost?	0	15.4	53.8	30.8	0
	If *d* is the number of dogs and *c* is the number of cats, write in algebra: There are four more dogs than cats.	7.7	0	69.2	15.4	7.7
	Write in algebra: There are twice as many pencils as biros (let *p* be the number of pencils and *b* be the number of biros).	0	30.8	23.1	46.2	0
	Write in algebra: There are three more caps than hats.	0	15.4	53.8	23.1	7.7
	Mary has *x* oranges and Bisi has four more oranges than Mary. How many oranges does Bisi have?	0	15.4	53.8	15.4	15.4
	There is a *b* number of sweets in a packet. A girl has two packets of sweets and gives her friend six sweets. How many sweets does she have remaining?	7.7	7.7	38.5	30.8	15.4
	If *s* is the number of students and *t* is the number of tables, write in algebra: There are three students for every table.	7.7	0	30.8	38.5	23.1
	Sam has *x* bananas and Peju has *p* bananas. Peter counts the number of bananas each of them have and finds they are the same. Sam said you could write this as $x=p$, but Peju said that *x* and *p* are different letters and so cannot be the same. Who do you think is correct?	0	15.4	23.1	30.8	30.8

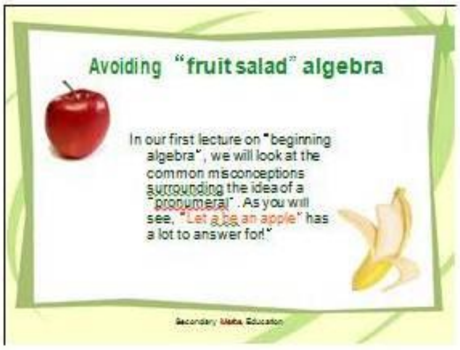
Appendix 15: Algebra Slides about Variables

Algebra Lecture (Part 1)






What is an "Unknown"?
A primary source of confusion for beginning algebra students is the question of whether or not a letter-value does, can, or should change in value. In fact, they do different things in different situations!
Such letters are also called unknowns, variables, or
Secondary Maths Education

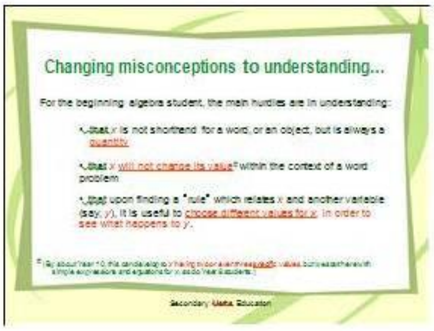

Avoiding "fruit salad" algebra
In our first lecture on "beginning algebra", we will look at the common misconceptions surrounding the idea of a "pronumeral". As you will see, "Let a be an apple" has a lot to answer for!"
Secondary Maths Education

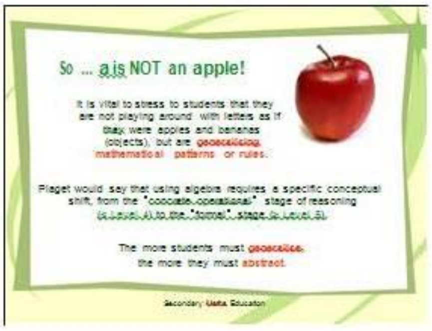

Daily language is different from mathematical language
While daily language is flexible and even sometimes ambiguous, students have nevertheless had years of practice in avoiding confusion when talking and writing.
Even students with reading or writing problems know how to make themselves understood on a daily basis.
They can do this by
– adding extra detail (elaborating)
– further clarifying meaning; or
– revealing an assumed context not explained properly
Secondary Maths Education


Daily language is different from mathematical language
In the particular mathematical context in which one is working, pronumerals do not change their individual values, whether known or not.
Try this. What is the value of t in the following?
The value of t in this equation is 3, and is so each time it appears!
Contrast this with this sentence, where the "word symbol" rook happily changes meaning within one context!
Secondary Maths Education


Daily language habits can also "interfere" with kids' understanding
Daily language can deal with ambiguity.
completely unambiguous.
we do not elaborate, clarify or shift meaning!
It means exactly what it says!
Secondary Maths Education


Daily language habits can also "interfere" with kids' understanding
Thought process...
"My pile of money is different from
Secondary Maths Education

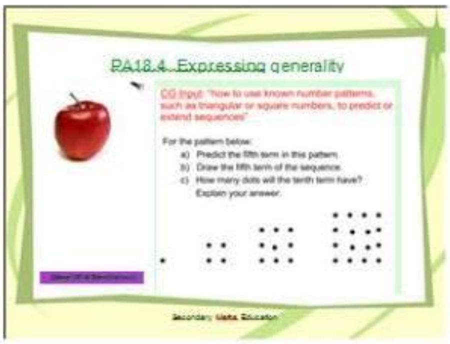

Changing misconceptions to understanding...
For the beginning algebra student, the main hurdles are in understanding:
that x is not shorthand for a word, or an object, but is always a quantity
that x will not change its value* within the context of a word problem
that upon finding a "rule" which relates x and another variable (say, y), it is useful to choose different values for x in order to see what happens to y.
Secondary Maths Education

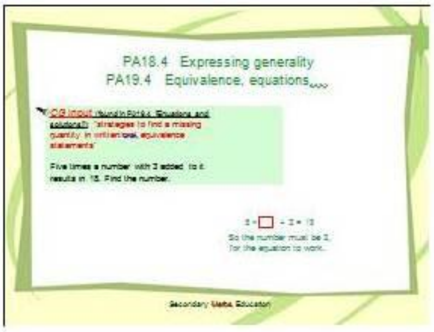

So ... a is NOT an apple!
It is vital to stress to students that they are not playing around with letters as if they were apples and bananas (objects), but are generalising mathematical patterns or rules.
Piaget would say that using algebra requires a specific conceptual shift, from the "concrete operational" stage of reasoning (≤ Level 4) to the "formal" stage (≥ Level 5).
The more students must generalise, the more they must abstract.
Secondary Maths Education


The Curriculum Guide excerpt for outcomes
PA18.4
Secondary Maths Education

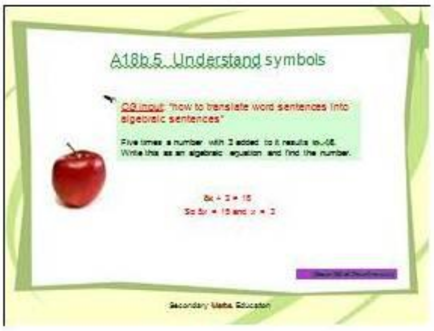

Outcome A18. Expressing generality
Note carefully that the true "algebra" outcomes only begin at Level 5! Before this the student is considered to be "pre-algebraic":
PA18.4 Expressing generality
Recognises, describes and uses spatial patterns and patterns involving whole, decimal and fractional numbers, following and describing rules for linking objects or figures by changes in size and orientation or linking successive terms in a sequence or paired quantities by a single operation.
Secondary Maths Education

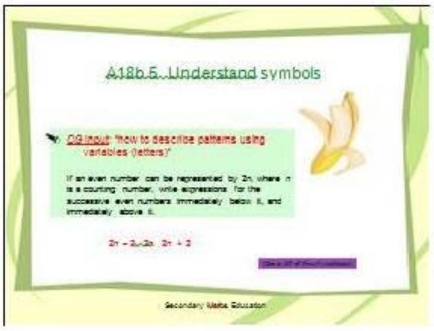

PA18.4 Expressing generality
CG input: "the language to describe number sequences"
Complete the pattern, and then write the rule in words below.
5, 13, 21, 29, ___, ___, ___
"Start with ___ , then ___ each time."
5, 13, 21, 29, 37, 45, 53, ...
"Start with 5, then add 8 each time."
Secondary Maths Education

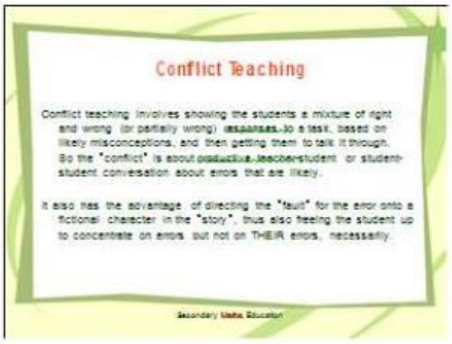

PA18.4 Expressing generality
CG input: "how to use known number patterns, such as triangular or square numbers, to predict or extend sequences"
For the pattern below:
a) Predict the fifth term in this pattern.
b) Draw the fifth term of the sequence.
c) How many dots will the tenth term have? Explain your answer.
Secondary Maths Education

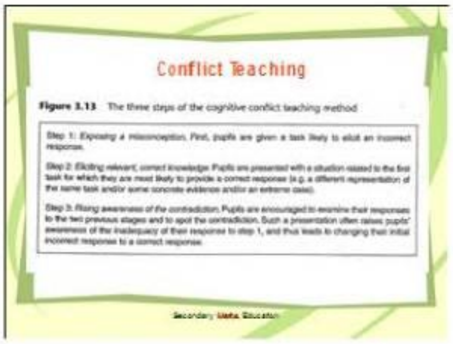

PA18.4 Expressing generality
PA19.4 Equivalence, equations
CG input (found in PA19.4 Equations and solutions): "strategies to find a missing quantity in written/oral, equivalence statements"
Five times a number with 3 added to it results in 18. Find the number.
So the number must be 3, for the equation to work.
Secondary Maths Education

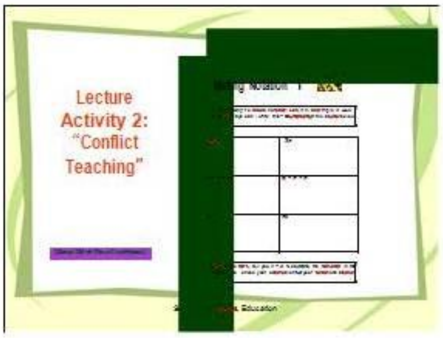

Outcome A18. Expressing generality
Today's lecture is really most concerned with A18b.5.
A18b.5. Understand symbols
Uses a letter to represent a variable quantity in an oral or written expression, uses conventional notation such as indices, links successive terms in a sequence involving one or two operations.
Secondary Maths Education


A18b.5. Understand symbols
CG input: "how to translate word sentences into algebraic sentences"
Five times a number with 3 added to it results in 18. Write this as an algebraic equation and find the number.
5x + 3 = 18
So 5x = 15 and x = 3
Secondary Maths Education


A18b.5. Understand symbols
CG input: "how to describe patterns using variables (letters)"
If an even number can be represented by 2n, where n is a counting number, write expressions for the successive even numbers immediately below it, and immediately above it.
2n – 2, 2n, 2n + 2
Secondary Maths Education


Conflict Teaching
Conflict teaching involves showing the students a mixture of right and wrong (or partially wrong) responses to a task, based on likely misconceptions, and then getting them to talk it through. So the "conflict" is about productive teacher-student or student-student conversation about errors that are likely.
It also has the advantage of directing the "fault" for the error onto a fictional character in the "story", thus also freeing the student up to concentrate on errors but not on THEIR errors, necessarily.
Secondary Maths Education


Conflict Teaching
Figure 3.13 The three steps of the cognitive conflict teaching method
Step 1: Exposing a misconception. First, pupils are given a task likely to elicit an incorrect response.
Step 2: Eliciting relevant, correct knowledge. Pupils are presented with a situation related to the first task for which they are most likely to provide a correct response (e.g. a different representation of the same task and/or some concrete evidence and/or an extreme case).
Step 3: Raising awareness of the contradiction. Pupils are encouraged to examine their responses to the two previous stages and to spot the contradiction. Such a presentation often raises pupils' awareness of the inadequacy of their response to step 1, and thus leads to changing their initial incorrect response to a correct response.
Secondary Maths Education


Lecture
Activity 2:
“Conflict
Teaching”


Answers
Detachment, and ...
“Literal translation”
“Literal visualisation”
Secondary Maths Education


The first two common
The LETTER is a WORD, or an OBJECT
The LETTER is always a FIXED NUMBER
Secondary Maths Education

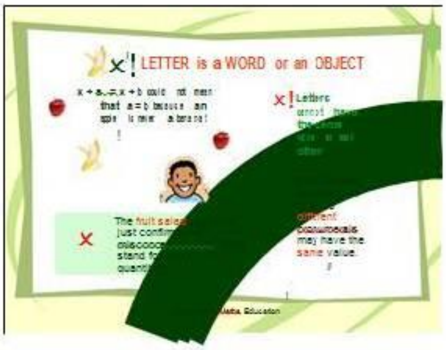

Common misconceptions about pronumerals
When students see letters such as x and a for the first time, they may make any or all of the following errors. They may believe that
the LETTER is a WORD
the LETTER is an OBJECT
the LETTER is a FIXED NUMBER (not a PRONUMERAL)
The first two are very much linked, arising because of the interference of natural language habits. This "word-object" problem becomes more troublesome still when more than one variable is involved: the subject of the next lecture.
Secondary Maths Education

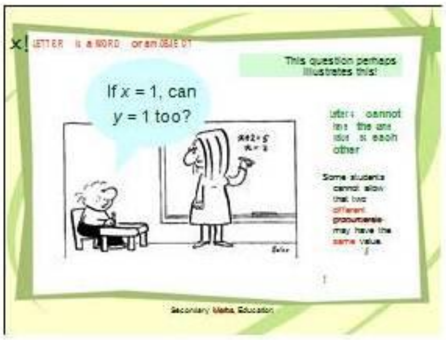

x! First common misconception
The LETTER is a WORD, or an OBJECT
they are both
from letters,
they?
should
This is what the student could well be thinking
Secondary Maths Education


x! LETTER is a WORD or an OBJECT
Fixed value in any context (a "label" for an object)!
Some students ...
... think that each letter pronumeral has a fixed value, all the time, like a label.
... believe therefore that one cannot choose values at random to "put into" an equation.
Secondary Maths Education


LETTER is a WORD or an OBJECT

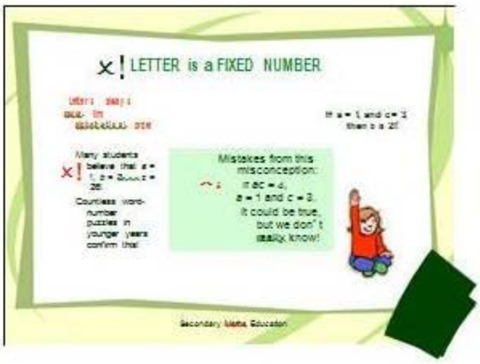

LETTER is a WORD or an OBJECT
Some students see
having several different values within the same context or question.


LETTER is a WORD or an OBJECT
may have the same value.

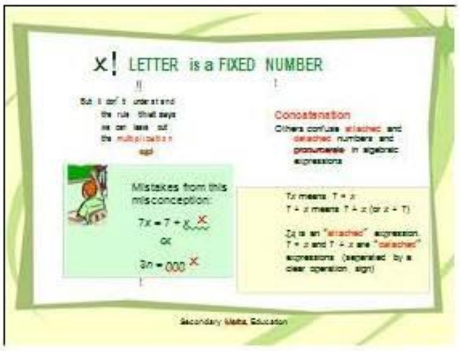

This question perhaps illustrates this!
If $x = 1$, can $y = 1$ too?
Some students cannot allow that two different pronumerals may have the same value.

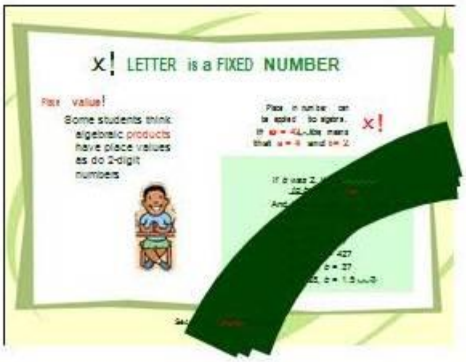

LETTER is a WORD or an OBJECT
Some students simply believe that a pronumeral can have no "value" at all, as it is a letter!


Second common misconception
This is what the student could well be thinking.

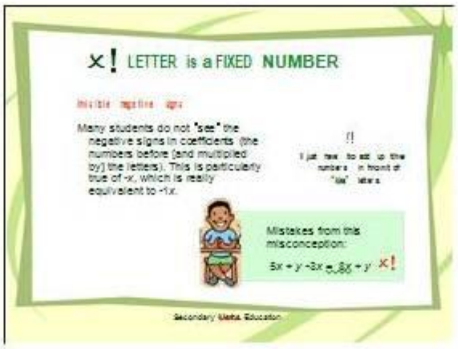

LETTER is a FIXED NUMBER
If a = 1 and c = 3
then b is 2!
Many students believe that a = 1, b = 2, … z = 26
Countless word-number puzzles in younger years confirm this!
Mistakes from this misconception:
If ac = 4,
a = 1 and c = 3.
It could be true, but we don't really know!
Secondary Maths Education

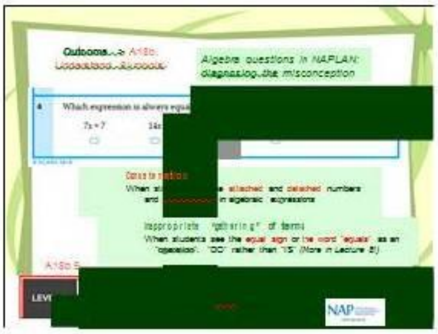

LETTER is a FIXED NUMBER
unit value
Some students believe that x is always 1.
These next five misconceptions are detachment problems.
Secondary Maths Education

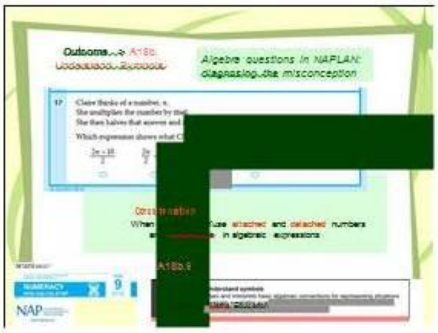

LETTER is a FIXED NUMBER
Concatenation
Others confuse attached and detached numbers and pronumerals in algebraic expressions
Mistakes from this misconception:
7x = 7 + x
or
3n = 000
7x means 7 × x
7 + x means 7 + x (or x + 7)
7x is an "attached" expression.
7 + x and 7 + x are "detached" expressions (separated by a clear operation sign)
Secondary Maths Education


LETTER is a FIXED NUMBER
value!
Some students think algebraic products have place values as do 2-digit numbers
= 427
b = 27

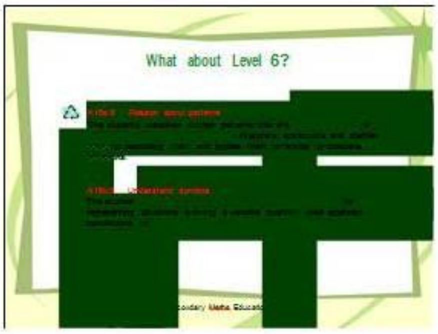

LETTER is a FIXED NUMBER
Many students do not "see" brackets, nor understand their purpose. (Brackets don't belong in speech or daily written messages, so certainly don't "belong" in mathematical language!)
2(x + 3) means
2 × (x + 3), or 2x + 6
Mistakes from this misconception:
2(x + 3) = 2x + 3
Secondary Maths Education


LETTER is a FIXED NUMBER
Many students do not "see" the negative signs in coefficients (the numbers before [and multiplied by] the letters). This is particularly true of -x, which is really equivalent to -1x.
Mistakes from this misconception:
5x + y -3x = 8x + y
Secondary Maths Education


Algebra questions in NAPLAN: diagnosing the misconception
[redacted]
Which expression is always equal
Appropriate "gathering" of terms
When students see the equal sign or the word "equals" as an "operation", "DO" rather than "IS" (More in Lecture 8)
[redacted]
NAP

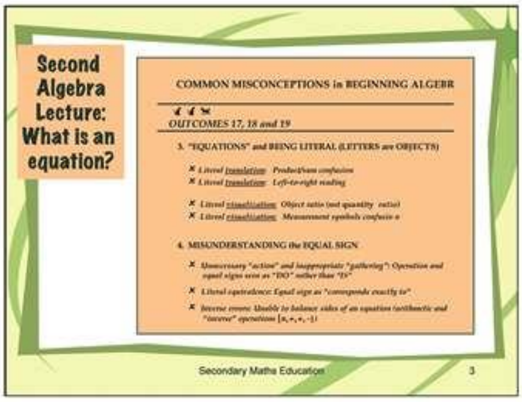

Algebra questions in NAPLAN: diagnosing the misconception
[redacted]
in algebraic expressions
NUMERACY
NAP

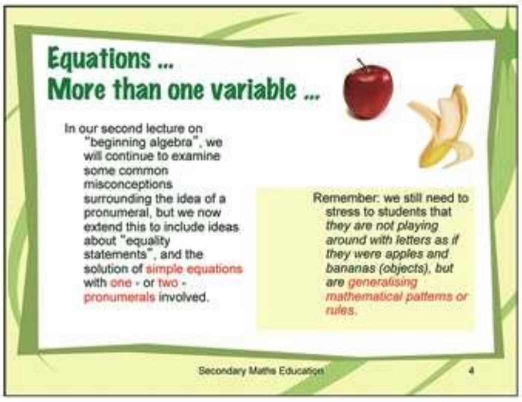

[redacted]
First Algebra Lecture
Secondary Maths Education

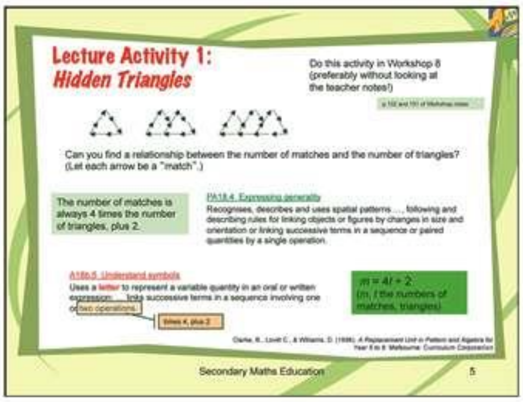

What about Level 6?
[redacted]

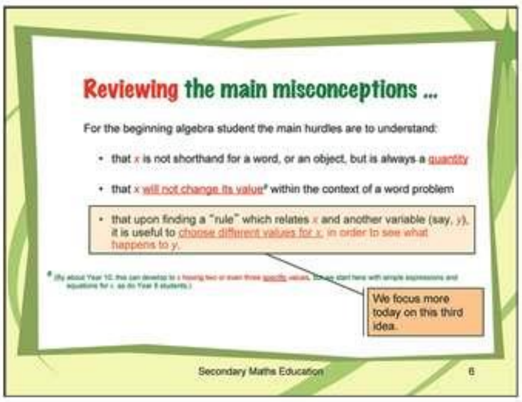

Solving equations......
More than one pronumeral ...
In the second algebra lecture we will consider situations where "equality statements" have to be written or interpreted. This involves "equations" and the "equal sign".
Thus the third and fourth set of misconceptions concerns writing algebraic relationships and are grouped as
EQUATIONS as LITERAL TRANSLATION: LETTERS are OBJECTS
MISUNDERSTANDING the EQUAL SIGN
Meanwhile, look at your summary sheet, at Nos. 1 and 2. These were the subject of today's lecture.
Secondary Maths Education

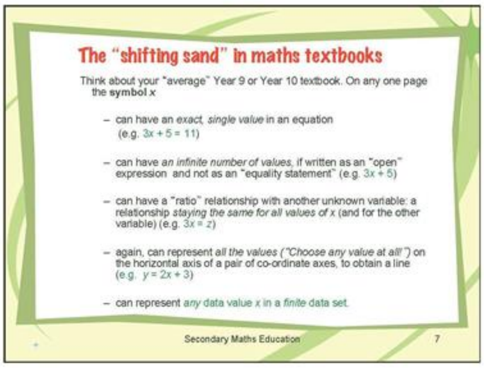
Appendix 16: Algebra slides about equations

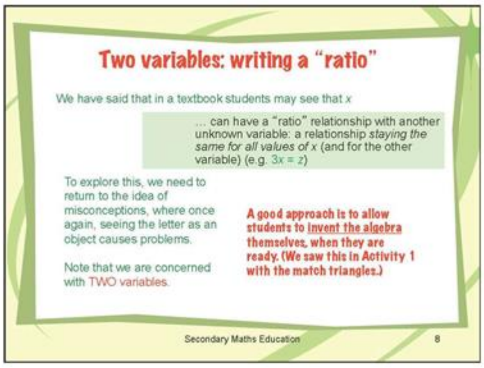


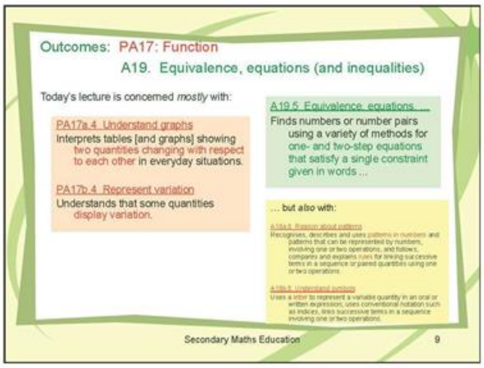


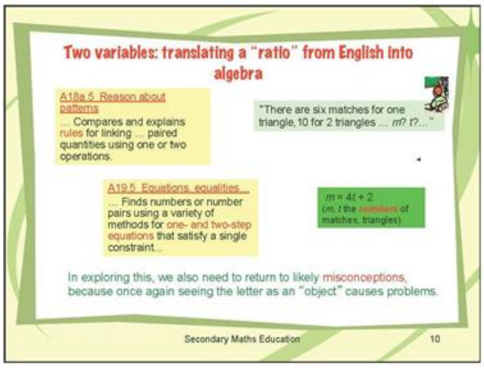


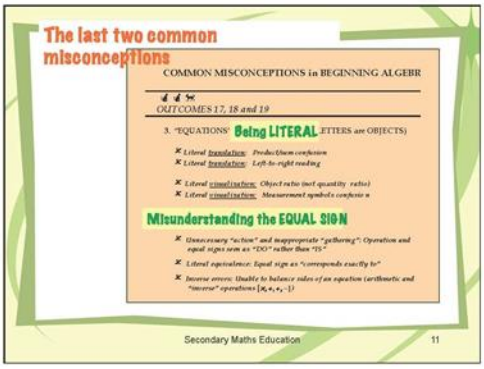


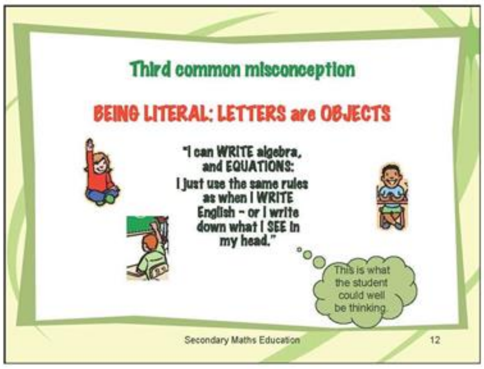

The "shifting sand" in maths textbooks
Think about your "average" Year 9 or Year 10 textbook. On any one page the symbol x
– can have an exact, single value in an equation (e.g. 3x + 5 = 11)
– can have an infinite number of values, if written as an "open" expression and not as an "equality statement" (e.g. 3x + 5)
– can have a "ratio" relationship with another unknown variable: a relationship staying the same for all values of x (and for the other variable) (e.g. 3x = z)
– again, can represent all the values ("Choose any value at all!") on the horizontal axis of a pair of co-ordinate axes, to obtain a line (e.g. y = 2x + 3)
– can represent any data value x in a finite data set.
Secondary Maths Education
7

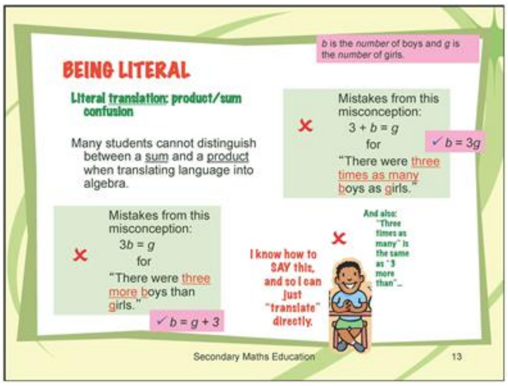

Two variables: writing a "ratio"
We have said that in a textbook students may see that x
... can have a "ratio" relationship with another unknown variable: a relationship staying the same for all values of x (and for the other variable) (e.g. 3x = z)
To explore this, we need to return to the idea of misconceptions, where once again, seeing the letter as an object causes problems.
Note that we are concerned with TWO variables.
A good approach is to allow students to invent the algebra themselves, when they are ready. (We saw this in Activity 1 with the match triangles.)
Secondary Maths Education
8

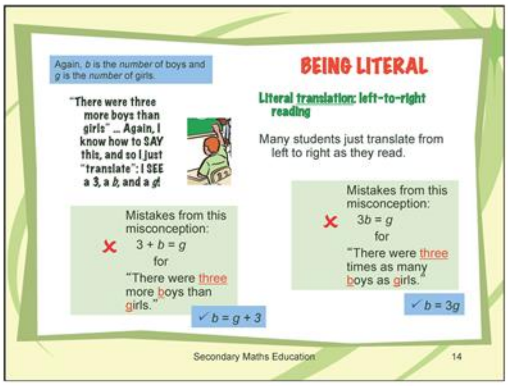

Outcomes: PA17: Function
A19. Equivalence, equations (and inequalities)
Today's lecture is concerned mostly with:
PA17a.4 Understand graphs
Interprets tables [and graphs] showing two quantities changing with respect to each other in everyday situations.
PA17b.4 Represent variation
Understands that some quantities display variation.
A19.5 Equivalence, equations, ...
Finds numbers or number pairs using a variety of methods for one- and two-step equations that satisfy a single constraint given in words ...
... but also with:
A18a.5 Reason about patterns
Recognises, describes and uses patterns in numbers and patterns that can be represented by numbers, involving one or two operations, and follows, compares and explains rules for linking successive terms in a sequence or paired quantities using one or two operations
A18b.5 Understand symbols
Uses a letter to represent a variable quantity in an oral or written expression; uses conventional notation such as indices, links successive terms in a sequence involving one or two operations.
Secondary Maths Education
9

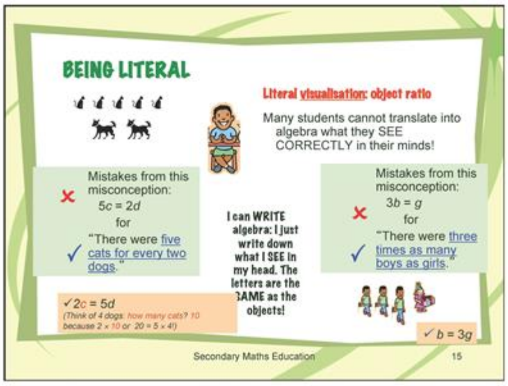

Two variables: translating a "ratio" from English into algebra
A18a.5 Reason about patterns
... Compares and explains rules for linking ... paired quantities using one or two operations.
"There are six matches for one triangle, 10 for 2 triangles ... m? t?..."
A19.5 Equations, equalities...
... Finds numbers or number pairs using a variety of methods for one- and two-step equations that satisfy a single constraint...
m = 4t + 2
(m, t the numbers of matches, triangles)
In exploring this, we also need to return to likely misconceptions, because once again seeing the letter as an "object" causes problems.
Secondary Maths Education
10

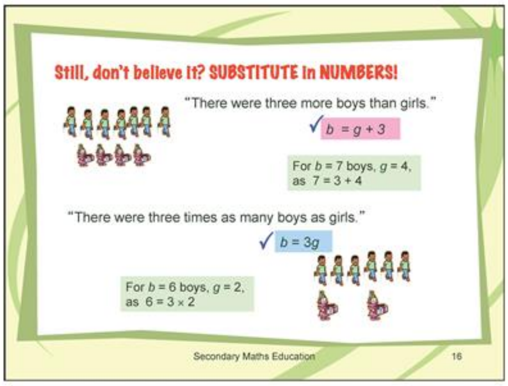

The last two common misconceptions
COMMON MISCONCEPTIONS in BEGINNING ALGEBR
OUTCOMES 17, 18 and 19
3. "EQUATIONS" Being LITERAL ETTERS are OBJECTS)
Literal translation: Product/sum confusion
Literal translation: Left-to-right reading
Literal visualization: Object ratio (not quantity ratio)
Literal visualization: Measurement symbols confusion
Misunderstanding the EQUAL SIGN
Unnecessary "action" and inappropriate "gathering": Operation and equal signs seen as "DO" rather than "IS"
Literal equivalence: Equal sign as "corresponds exactly to"
Inverse errors: Unable to balance sides of an equation (arithmetic and "inverse" operations [×, ÷, +, −])
Secondary Maths Education
11

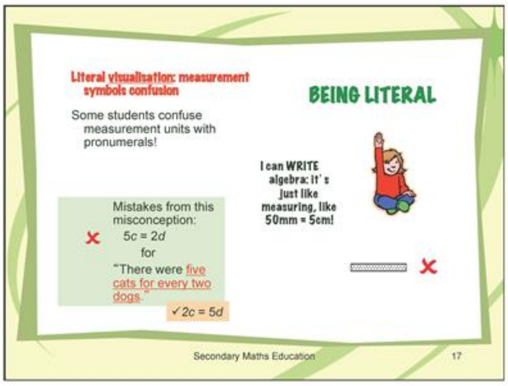

Third common misconception
BEING LITERAL: LETTERS are OBJECTS
"I can WRITE algebra, and EQUATIONS: I just use the same rules as when I WRITE English – or I write down what I SEE in my head."
This is what the student could well be thinking.
Secondary Maths Education
12

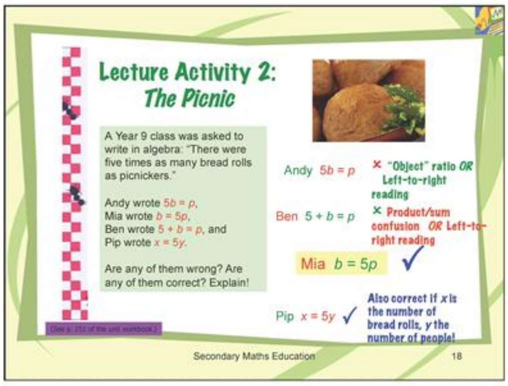

b is the number of boys and g is the number of girls.
BEING LITERAL
Literal translation: product/sum confusion
Many students cannot distinguish between a sum and a product when translating language into algebra.
Mistakes from this misconception:
3 + b = g
for
"There were three times as many boys as girls."
✓ b = 3g
Mistakes from this misconception:
3b = g
for
"There were three more boys than girls."
✓ b = g + 3
I know how to SAY this, and so I can just "translate" directly.
And also: "Three times as many" is the same as "3 more than"...
Secondary Maths Education
13


Again, b is the number of boys and g is the number of girls.
BEING LITERAL
Literal translation: left-to-right reading
"There were three more boys than girls" ... Again, I know how to SAY this, and so I just "translate": I SEE a 3, a b, and a g!
Many students just translate from left to right as they read.
Mistakes from this misconception:
3 + b = g
for
"There were three more boys than girls."
✓ b = g + 3
Mistakes from this misconception:
3b = g
for
"There were three times as many boys as girls."
✓ b = 3g
Secondary Maths Education
14

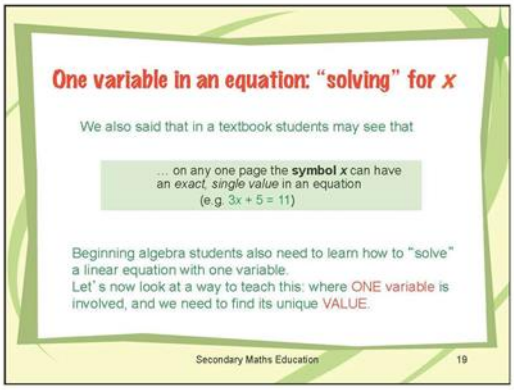

BEING LITERAL
Literal visualisation: object ratio
Many students cannot translate into algebra what they SEE CORRECTLY in their minds!
Mistakes from this misconception:
5c = 2d
for
"There were five cats for every two dogs."
✓2c = 5d
(Think of 4 dogs: how many cats? 10 because 2 × 10 or 20 = 5 × 4!)
I can WRITE algebra: I just write down what I SEE in my head. The letters are the SAME as the objects!
Mistakes from this misconception:
3b = g
for
"There were three times as many boys as girls."
✓ b = 3g
Secondary Maths Education
15

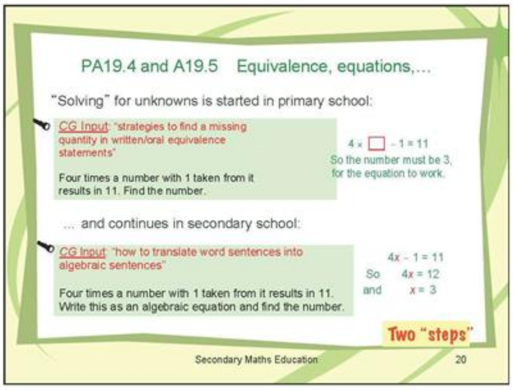

Still, don't believe it? SUBSTITUTE in NUMBERS!
"There were three more boys than girls."
✓ b = g + 3
For b = 7 boys, g = 4, as 7 = 3 + 4
"There were three times as many boys as girls."
✓ b = 3g
For b = 6 boys, g = 2, as 6 = 3 × 2
Secondary Maths Education
16

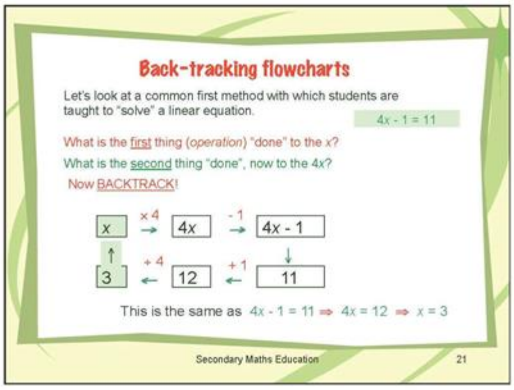

Literal visualisation: measurement symbols confusion
Some students confuse measurement units with pronumerals!
BEING LITERAL
I can WRITE algebra: it's just like measuring, like 50mm = 5cm!
Mistakes from this misconception:
5c = 2d
for
"There were five cats for every two dogs."
✓2c = 5d
Secondary Maths Education
17

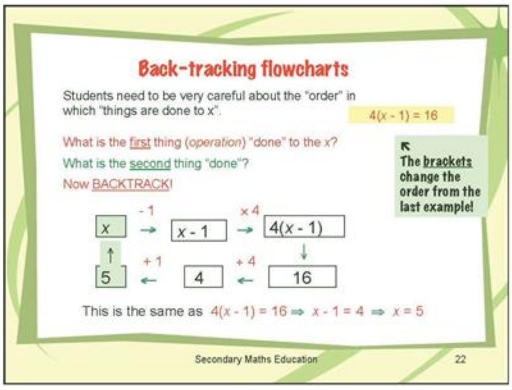

Lecture Activity 2:
The Picnic
A Year 9 class was asked to write in algebra: "There were five times as many bread rolls as picnickers."
Andy wrote 5b = p,
Mia wrote b = 5p,
Ben wrote 5 + b = p, and
Pip wrote x = 5y.
Are any of them wrong? Are any of them correct? Explain!
Andy 5b = p
× "Object" ratio OR Left-to-right reading
Ben 5 + b = p
× Product/sum confusion OR Left-to-right reading
Mia b = 5p ✓
Pip x = 5y ✓
Also correct if x is the number of bread rolls, y the number of people!
Secondary Maths Education
18

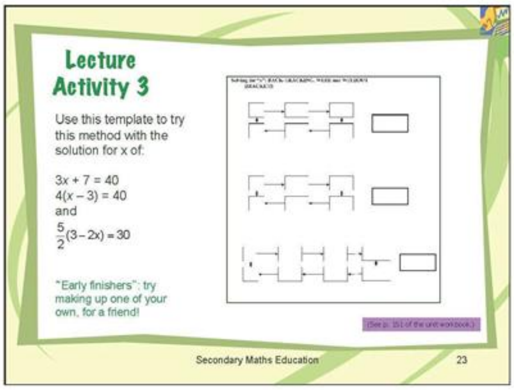
Algebra Lecture (Part 2)

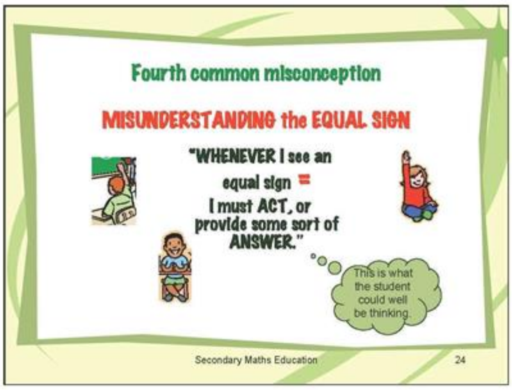




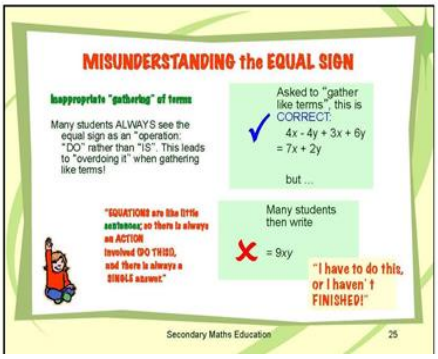


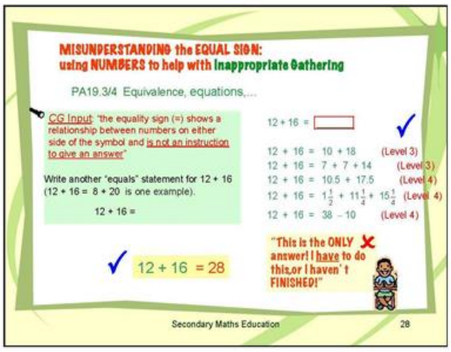


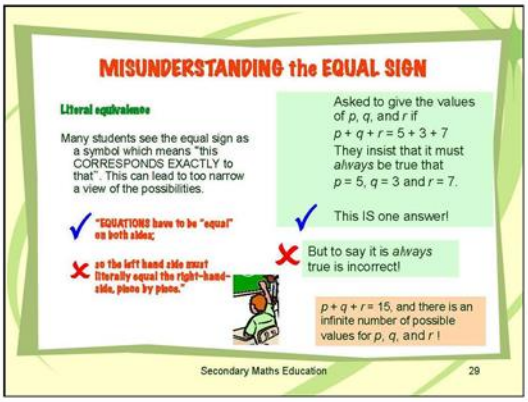


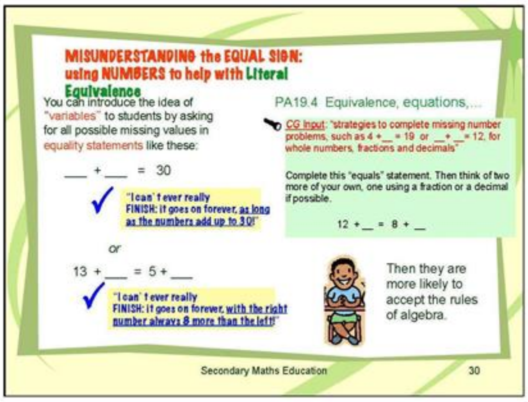


Workshop 8

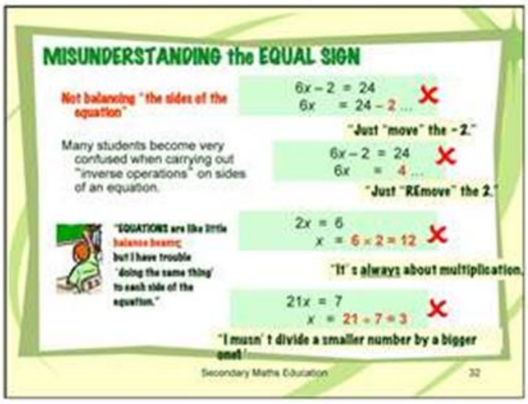








p	q	r	15
5	3	7	15 ✓
5	5	5	15 ✓
7	5	3	15 ✓
2	11	2	15 ✓
0	0	15	15 ✓
0.5	1.5	13	15 ✓
20	-3	-2	15 ✓



Appendix 17: Sample of Ruth's class Lesson Observation Sheet

MATHEMATICS CLASSROOM OBSERVATION SCHEDULE (MCOS)

Teacher: Ruth | Topic: Word problems

School: School One | Objective: To be able to change word problems into mathematical sentences and solve.

Visit Date: 17/3/11 | Time: 10.35 – 11.15am

Time Interval (mins)	Observed Activity Explaining idea/concept [1], Identifying variable/expression/equality misconception [2], identifying key terms and symbols [3], showing understanding that a letter is a quantity [4], resisting impulse to "inappropriately gather" terms [5], reading aloud of the question [6], reframing question in own words [7], explaining strategy for solving problem [8], finding the value of the letter (if applicable) [9], writing the answer [10], individual/paired/group work [11], use of questions [12], listening [13], note copying [14]
2	T(1) – review, S(13)
4	T(1) – review; S(13)
6	T(1), S(13)
8	S(14), T(1)
10	S(11) – on the board; S(8)
12	T(1), T(2), S(13), S(11) - individual
14	T(12), S(13)
16	S(11) – individual
18	S(8), T(12), T(1)
20	T(1), S(13)
22	S(8). S(14)
24	T(12), S(11) – individual
26	T(1), S(11) – individual, S(8)
28	S(11) – on the board, S(8)
30	T(1), S(13)
32	T(12), S(8)
34	S(13), S(11) – individual
36	S(11) – individual, S(8)
38	S(11) – on board, T(1), S(14)
40	S(14), S(13)

Note. S and T are codes used to record tallies for students' and the teacher respectively.

Appendix 18: Sample of Dorothy's class Lesson Observation Sheet

MATHEMATICS CLASSROOM OBSERVATION SCHEDULE (MCOS)

Teacher: Dorothy

Topic: Using algebra to solve word problems

School: School Two

Objective: Writing mathematical sentences and solving word problems.

Visit Date: 22/5/11

Time: 8-8.40am

Time Interval (mins)	Observed Activity Explaining idea/concept [1], Identifying variable/expression/equality misconception [2], identifying key terms and symbols [3], showing understanding that a letter is a quantity [4], resisting impulse to "inappropriately gather" terms [5], reading aloud of the question [6], reframing question in own words [7], explaining strategy for solving problem [8], finding the value of the letter (if applicable) [9], writing the answer [10], individual/paired/group work [11], use of questions [12], listening [13], note copying [14]
2	T(1) - review of last lesson; S(13)
4	T(1) - review; S(13)
6	T(1); S(13)
8	S(14)
10	T(1); S(13)
12	T(1); T(12)
14	T(1); T(2); T(8); S(13)
16	T(12); T(1); S(13)
18	S(14); T(3); T(8)
20	T(1); S(13);
22	T(1), T(12), S(14)
24	S(14), T(12)
26	T(12), T(1)
28	T(1), S(14)
30	S(11) - individual
32	S(11) - individual
34	T(12); S(8); T(1), S(13)
36	T(1); S(14);
38	T(12), S(13);
40	T(1), S(14)

Note. S and T are codes used to record tallies for students' and the teacher respectively.

Appendix 19: Sample of Jamie's class Lesson Observation Sheet

MATHEMATICS CLASSROOM OBSERVATION SCHEDULE (MCOS)

Teacher: Jamie

Topic: Word Problems

School: School Three

Objective: Using algebra to solve word problems.

Visit Date: 23/3/11

Time: 8:10 – 8:50 am

Time Interval (mins)	Observed Activity Explaining idea/concept [1], Identifying variable/expression/equality misconception [2], identifying key terms and symbols [3], showing understanding that a letter is a quantity [4], resisting impulse to "inappropriately gather" terms [5], reading aloud of the question [6], reframing question in own words [7], explaining strategy for solving problem [8], finding the value of the letter (if applicable) [9], writing the answer [10], individual/paired/group work [11], use of questions [12], listening [13], note copying [14]
2	T(1)-review ; S(13)
4	T(1)-review ; S(13)
6	T(1)-review ; S(13)
8	T(1) ; S(13)
10	T(12) ; S(13)
12	S(14) ; S(13) ; T(1)
14	T(8) ; S(14)
16	T(1) ; S(13) / T(12)
18	T(1) ; S(13)
20	S(12) ; T(1), T(2)
22	S(14)
24	S(14)
26	S(14)
28	T(1), T(6), T(12)
30	T(3), T(1), T(2), T(12)
32	T(8), S(13)
34	T(12), T(8) T(1)
36	T(1), S(14)
38	S(14), S(12)
40	T(1), S(13)

Note. S and T are codes used to record tallies for students' and the teacher respectively.

Appendix 20: Sample of Stephen's class Lesson Observation Sheet

MATHEMATICS CLASSROOM OBSERVATION SCHEDULE (MCOS)

Teacher: Stephen

Topic: Algebraic Processes: Writing mathematical statements from word problems

School: Four

Objective: To be able to solve word problems using symbolic form

Visit Date: 23/3/11

Time: 9.30 – 10.10 am

Time Interval (mins)	Observed Activity Explaining idea/concept [1], Identifying variable/expression/equality misconception [2], identifying key terms and symbols [3], showing understanding that a letter is a quantity [4], resisting impulse to "inappropriately gather" terms [5], reading aloud of the question [6], reframing question in own words [7], explaining strategy for solving problem [8], finding the value of the letter (if applicable) [9], writing the answer [10], individual/paired/group work [11], use of questions [12], listening [13], note copying [14]
2	T(1) – review ; S(13)
4	T(1) – review ; S(13)
6	T(1) – review ; S(13)
8	T(1) – review ; S(13)
10	T(1) ; S(14)
12	S(14) ; S(13) ; T(12)
14	T(1) ; S(13)
16	T(1) ; S(13) ; T(8)
18	T(1) ; T(8)
20	T(1) ; T(8)
22	S(14) , T(12)
24	S(14)
26	T(1) ; S(14)
28	S(13) ; T(6)
30	T(8) ; T(2) ; T(1) ; T(12) S(14)
32	T(1) ; S(13) , S(12)
34	T(1) ; S(13)
36	T(6) , S(14) T(8) , S(11)
38	T(1) , T(12) ; S(11)
40	T(12) T(1) S(14)

Note. S and T are codes used to record tallies for students' and the teacher respectively.