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The computation choices made by students in years 5 to 7

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The Computation Choices made by Students in Years 5 to 7

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Thesis submitted in fulfilment of the requirements for the degree of Doctor of
Philosophy, Edith Cowan University

Faculty of Community Services, Education and Social Sciences

December, 2002

Abstract

This study was designed to explore the computation choices made by 78 students in Years 5 to 7. The ability to choose and use a repertoire of computation methods is an important goal of mathematics education. While one might expect to find a great deal of research evidence outlining the computation choices students make and why they make them, this was not the case; and as such it was decided to explore what computation choices students make and why they make them.

When examining the literature dealing with computation choice few studies were found that directly discussed the issue. There were many studies of computation and discussion of factors that might affect computation choice. The literature also outlined the need for the computation focus to change from purely the development of skills, particularly with paper-and-pencil, to enhance the ability of students to make considered computation choices.

Several models of computation were reviewed along with literature dealing with metacomputation. This prompted the need for a fresh look at computation in terms of a non-linear computation model that better reflected the computation process students pass through when solving a computation problem. In particular the role of metacomputation as a means of choosing a computation method, then guiding and monitoring the computation was explored.

Students in Years 5 to 7 were chosen to participate in the study as it was felt 10–12 year-old students would have had enough exposure to various forms of computation so as to be confident and competent in using all forms of computation. Students were asked to complete a series of computation items using their preferred computation approach. Clinical interviews were conducted to determine why students made particular computation choices. Observational data and field notes were used to collect data on what computation choices were made and how successful students were in executing their chosen method of computation.

Data were analysed and it was found that students made appropriate computation choices in slightly over 50 percent of cases based on the success rate experienced when completing computation questions using their favoured method. In some cases computation choice was limited by a lack of competence in all forms of computation. In particular it was noted that many students were unable to make use of simple calculators. Interview data indicated that students make computation choices with little hesitation and based on a set of rudimentary criteria such as the magnitude of the numbers involved, or the operation required. There was little evidence to suggest that students looked beyond these simple criteria when making a decision about which form of computation to use.

The implication of the research is that teachers may better understand how students make computation choices and what hampers the making of computation choices. As a result of understanding of the process students use for making such choices, teachers should be able to raise student awareness of the process of making a computation choice.

The thesis concludes with a recommendation that in much the same way that teachers have been encouraged to focus on developing mental computation strategies, they should also encourage students to discuss their criteria for making particular computation choices. In doing so students will be encouraged to broaden their thinking about the computation process. A suggestion is also made that time spent in the classroom developing each of the computation alternatives, mental, written and calculator, needs to better reflect the usage patterns of adults. Students who have a better understanding of how to use all types of computation will be in a better position to make appropriate computation choices.

Declaration

I certify that this thesis does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any institution in higher education; and that to the best of my knowledge and belief does not contain any material previously published or written by another person except where due reference is made in the text.

Signed:



Date: 13/December/2002

Acknowledgements

I wish to thank my Principal Supervisor, Dr Jack Bana, for his help, encouragement, and patience. He managed to guide me through ten years of study in which my ideas changed and my enthusiasm for the task waxed and waned.

My thanks also go to Dr Len Sparrow, for his help and gentle encouragement to persevere. His belief in the value of my research gave me confidence to continue. Many thanks to Margaret Doust for reading the draft of this thesis and commenting on the structure.

I am grateful for the support received from my wife, Gillian. Without her personal sacrifice this thesis would never had been written.

To reach this point involved a great deal of encouragement from my parents, who having never been able to complete their secondary school education but always supported and encouraged me to be a lifelong learner. Without their initial sacrifice I would never have gone along the path that led me to writing this thesis.

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Chapter One: Introduction

Background to the Study

This thesis describes a study designed to investigate the computation choices made by students in Years 5 to 7 (ages 10–12 Years). Students were observed completing a range of computation items and after attempting each item were asked to explain why they had chosen the particular computation method, mental, written, calculator or a combination of methods used.

This chapter examines why such a study was needed. The significance of the study is outlined in terms of the broader issues impacting on mathematics education in general and more specifically computation. The focus of the research is narrowed in the section explaining the purpose of the study, to the making of computation choices. The section leads to the statement of the research questions which are used to pinpoint the research.

The chapter begins with a broad overview of mathematics education and the changes impacting on the mathematics curriculum. In particular the impact on computation of newer technologies, societal needs and emerging pedagogies are discussed.

The place of computation in the mathematics curriculum

The view of mathematics as a set of rules and procedures to be learned and retrieved when required is a common one. This view has been developed largely as a result of participation in school mathematics classes whereby the development of standard written methods of calculation have dominated the mathematics curriculum to the detriment of alternative methods. Contrast this view with the following definition of mathematics attributed to Steen (1988) and developed in *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991).

Mathematics is often defined as the science of space and number... [but] a more apt definition [is that] mathematics is the science of patterns. The mathematician seeks patterns in number, in space, in science, in computers, and in imagination. Mathematical theories explain the relations among patterns ... Applications of mathematics use these patterns to explain and predict natural phenomena ...(p. 21).

There is evidence to suggest that the focus of mathematics in many primary schools is the teaching of computation skills (Porter, Floden, Freeman, Schmidt, & Schwille, 1988). Porter (1989) described the United States curriculum in these terms: “Much of the whole number computational drill and practice instruction is focussed on skills rarely needed these days” (p. 11). The result of this is that many students leave school with a dislike of school mathematics because it is often viewed as being irrelevant to their needs. Note the following comment drawn from a current West Australian curriculum document.

There is considerable evidence that many students leave school with negative attitudes toward mathematics; some dislike the subject, others feel inadequate about it, still others feel it is irrelevant in their lives. This is an unacceptable outcome of school mathematics (EDWA, 1998, p. 9).

The narrow view of mathematics as a set of rote-learned procedures to be applied without consideration of the context has led to generations of children disliking mathematics, or at the very least disengaging from the subject. Technology has had an impact on most aspects of society, including mathematics. Computation is no longer limited to what may be completed in the head or on paper – now a third alternative, the calculator, is available.

For many years the choice was simple, either complete the calculation mentally or use paper and pencil, but the advent of simple and cheap calculators in the late seventies introduced a new option in computation choice. By this time computation approaches and teaching practices were deeply rooted in a paper-and-pencil curriculum. Teachers were familiar with paper-and-pencil algorithms, parents were comfortable with them and textbooks were focussed on them; therefore any changes to computation practices in primary school were likely to come slowly. The result is today we find that written algorithms still dominate the computation curriculum and the time devoted to teaching mathematics. Children spend years developing proficiency in paper-and-pencil routines that may have been well suited to the past but have become less relevant in the twenty-first century.

The time that is spent by children trying to perfect various standard written methods for calculation must be questioned. Current practice would suggest that the majority of primary classroom mathematics time is spent developing and practising the standard written algorithms (Porter, 1989; Sowder, 1992). The amount of time devoted to practising the standard written algorithm tends to dominate the mathematics curriculum to the point where little time is devoted to mental and calculator methods and even less time is spent helping children make choices as to the appropriate computation method to use. This has led to much debate around the following issues.

- What is the place of standard written algorithms?
- What computation choices are sensible in today's calculator age?
- What does it mean to make efficient and effective computation choices?

Recent curriculum documents indicate there is a move toward a more balanced approach to computing in the primary school. In highlighting key aspects of computation, *A National Statement on Mathematics for Australian Schools*, (AEC, 1991) made this comment

Students should develop the ability to judge the level of accuracy needed, learn to estimate and approximate, and use mental, calculator and paper-and-pencil strategies effectively and appropriately in different situations...This requires that they:

- Decide what operations to perform (formulate the calculation);
- Select a means of carrying out the operation (choose a method of calculation);
- Perform the operation (carry out the calculation);
- Make sense of the answer (interpret the results of the calculation) (p. 108).

With the introduction of calculators and a broadening of computation choice certain skills such as the calculation of a square root were no longer required. Proficiency in complicated computation routines was no longer required, but rather students needed to develop the ability to select or choose appropriate computation methods. Much of this impetus may be traced back to the introduction of calculators in the late seventies. This is highlighted in the next section.

Recommendations for computation practice in classrooms

The debate over how much instruction in computation is required and the relative emphasis that should be given to mental, written and calculator forms of computation is not a new one. As far back as 1982 the Cockcroft Report recommended that the whole question of computation needed to be re-examined in the light of the increasing availability of calculators.

There is as yet very little evidence about the extent to which a calculator should be used instead of pencil and paper for purposes of calculation in the primary years; nor is there evidence about the eventual balance to be obtained at the primary stage between calculations carried out mentally, on paper, or with a calculator. However it is clear that the arithmetical aspects of the primary curriculum cannot but be affected by the increasing availability of calculators (p. 113).

By the late eighties Willis and Kissane (1989) recognised that the emergence of calculators served to “highlight a lack of congruence between school mathematics and real mathematics” (p. 58). For example, in the ‘real world’ adults are much more reliant on mental methods of calculation than on any of the alternatives such as written or calculator-assisted methods and yet a large proportion of mathematics instructional time is taken up with the development of formal written algorithms.

While there is general agreement that the balance between the various computation alternatives needs to be adjusted, there is a lack of agreement as to the appropriate mix. The current mix in school where paper-and pencil algorithms dominate the computation curriculum is not reflected in society where the most common form of computation is mental (Northcote & McIntosh, 1999; Wandt & Brown, 1957). Redressing the balance requires that less time be spent on standard written algorithms and more time be devoted to mental computation. The addition of another computation alternative, the calculator, means that further time needs to be taken from written computation in order to better reflect the current needs of society. Further adding to the debate is speculation about the needs of a mathematically literate society well into the twenty-first century. It is often difficult to anticipate or even contemplate the needs of society in the future given the rapid changes occurring in society and the speed at which technology is impacting on day-to-day life.

Added to the changes in society, mathematics educators are debating issues such as to what extent computation alternatives should be taught as opposed to providing a classroom environment that promotes the construction of computation strategies or methods. The constructivist paradigm is at odds with the current practices associated with the teaching of rules that commonly occurs in the teaching of formal written algorithms.

The result of the foregoing has led to the recommendation that children develop the ability to “*choose and use* a repertoire of mental, paper and calculator computational strategies” [italics added] (Curriculum Council, 1998, p. 187). A great deal is implied by this rather short but all-encompassing statement. The implications are that not only are children to become proficient in the use of various computations but that they develop the ability to choose an appropriate computation method. This represents a significant departure from current practice where children are either told which computation approach should be applied or where the class text indicates the form of computation to be used.

Significance of the study

In light of the foregoing it is timely, therefore, to study how children make computation choices in the current context of a curriculum that allows the use of calculators as a computation alternative but still favours standard paper-and-pencil methods. The results of this study will prove significant in adding to the debate about the relative merits of the various computation alternatives. It will also give insight into the ways children approach making computation choices when faced with a computation problem. Further, information about children’s current facility with various computation alternatives will also aid in formulating options for future computation curricula. Groves and Stacey (1994) described the issue in these terms:

The question of the role of formal paper-and-pencil algorithms and the balance of emphasis placed on mental, paper-and-pencil and calculator computations is of critical importance in mathematics teaching at this time. Of the three available methods of computation mental and calculator computations are the ones typically used in everyday life. However, paper-and-pencil methods still receive the most emphasis in schools (p. 1).

In making an argument for the need to change the way computation is taught in school, Reys and Nohda (1994) believed that many questions needed to be answered before real progress could be made. Pragmatic questions such as, “What is number sense, mental computation, estimation, written calculation?” head their list and will be examined in the literature review. Another more complex question such as, “How are mental and written computation intertwined?” will be considered as part of a revised computation model that underpins this study. The significance of this study may also be seen in the questions Reys and Nohda (1994) proposed.

- How should computation alternatives (mental computation, estimation, written algorithms, calculators) be developed?
- When should computation alternatives be introduced?
- Should strategies and techniques be self-developed by students, as advocated by constructivists? Or should strategies and techniques be taught directly by teachers?
- How are wise choices of computation alternatives developed?
- Do students know when mental computation is appropriate?
- How can calculators be used?
- Can calculators contribute to the development of mathematical thinking? How?
- What role does the calculator play as a tool? Where? How?
- How does the development of computation alternatives contribute to number sense? (p. 5).

This abbreviated list is by no means exhaustive but gives an insight into the questions being debated by mathematics educators. This research will not directly answer all these questions but will focus on how children make computation choices and how well they execute them. As Reys and Nohda continued, “answers are needed before substantial progress can be made toward successfully implementing the array of computation alternatives” (p. 6).

Little is currently known about how children make computation choices. Once the choice has been made there is some knowledge of how it is executed. For example many mental computation strategies have been documented (McIntosh, De Nardi & Swan, 1994). Even less is known of the ways students mix computation methods when attempting to solve a problem of a numerical nature. As Shimizu and Ishida (1994) noted:

This decision making will be an important ability in situations where several computational alternatives are available. The complementary use of several computational tools will be beneficial to the validity and accuracy of computation. The decision making is crucial in a contemporary society overflowing with data. People of various abilities must learn to choose which computational tool is relevant in a situation. In addition to research on the cognitive processes associated with using alternative computation methods, the process of deciding which computational alternative should be used must be targeted for research (p. 178).

The issue of how children choose between the computation alternatives and whether the choices they make are wise is a complex one. The debate over this issue predates the introduction of calculators into the classroom, but gained momentum as a result of the inclusion of this powerful computation option. The debate, however, seems to have been raging for some time. Perhaps it is because of the various powerful lobby groups or the entrenched nature of current practice. Reys, Reys and Hope (1993) made the following comment:

If we are serious about developing mental computation and helping students make wise choices among the computational alternatives, then it is time to reexamine, rethink and redesign the entire domain of computation. Are we ready for this revolution? (p. 314).

Perhaps we were not ready for a revolution in computation practices. Teachers and educators may have been hesitant to 'change the system' due to a lack of understanding of the current situation and a fear of the unknown. Research helps to alleviate fear and to support teachers wishing to make changes in their classrooms. This research in particular has been designed to consider how children make computation choices and whether children are competent at using their chosen method. This will assist teachers wishing to help children to make better judgements about the form of computation that should be used in a particular context. The purpose of the study will be expanded in the following section.

The Purpose of the Study

The purpose of this study is to investigate the current ways in which children approach computation problems. This research is designed to help inform the debate about the role of computation in the mathematics curriculum by asking students to choose a computation method and then apply the choice. Mathematics educators can point to studies of mathematics used in the real world and to surveys of children's computation preferences but there are very few studies that have endeavoured to find

out what computation choices children make and how well those choices are executed. This study is designed to build upon the rather thin research base in this area and as such will help to inform the larger computation debate.

This thesis describes a study conducted to investigate the computation choices made by children in Years 5 to 7 (ages 10 to 12). Specifically the researcher set out to investigate:

- What computation choices were made by students in Years 5 to 7; and
- Why students in Years 5 to 7 made particular computation choices.

In addition, when examining the choices made by children the researcher also attempted to discover what computation choices the children had at their disposal and how effective they were in using them. Data were collected showing how successful individual students were in using their chosen computation method. Observations were made as to which method, mental, written or calculator, or combinations of methods were used to solve numerical problems.

This study is both timely and important given the current rethinking of computation in general and the appropriate mix of computation methods. In summary the purpose of this study was to examine:

- what computation choices were made by Year 5 to 7 students;
- how they were made;
- how well the computation was executed once the choice was made;
- whether any monitoring of the computation by the students took place; and
- whether the students really had a choice – that is, a computation alternative at their disposal.

The points raised above served to give purpose to the thesis. These primary concerns led to the development of the following specific research questions that helped to focus the research.

Research questions

The following research questions were developed to focus and drive the research. They specify the main thrust of the research and helped guide the choice of an appropriate methodology by which answers to these questions might be sought.

1. When faced with a computation question, what choices do students in Years 5 to 7 make?
2. Why do students in Years 5 to 7 make particular computation choices?
3. How successful are students in Years 5 to 7 at executing various forms of computation?

Interview data, samples of children's written methods of computation and the results of their calculations were used to answer the questions. When used in concert these data helped to provide a picture of how children make computation choices and how successful they are in applying their choices.

Summary

This chapter was designed to set the scene as to why the researcher became interested in the issue and how the issue of computation choice fits into the current debate about mathematics education. Several authorities were cited in order to provide a general overview of the main arguments related to the role of computation in the mathematics curriculum. The following chapter that reviews the literature associated with this topic will help to round out the arguments made in this chapter. Specifically, the literature review will show how curriculum developers have been trying to bring about a change in focus from simply the development of proficiency with paper-and-pencil methods of calculation to proficiency with various forms of computation, and more particularly the ability to choose an appropriate form of computation.

Organisation of the thesis

This thesis begins with a literature review in the second chapter which examines each of the computation alternatives; mental, written and calculator. Research on computation choice and factors affecting computation are also examined. The impact of this research and current learning theory is also examined in terms of the impact that has been made on the mathematics curriculum. Brief mention of computation models is made in this chapter as a prelude to Chapter 3.

Chapter 3 examines a variety of computation models. Key elements of these models are discussed and the main features of each are extracted in order to develop a comprehensive computation model. This model is used as a means for developing a conceptual framework about which the key research questions were formulated.

The fourth chapter provides details of the research methods that were used to answer the research questions posed in Chapter 2. Justification of the chosen research methods is made. The context of the study is also explained in this chapter, along with the background of the participants. This information is later referred to in the discussion of the results. The key issues of the reliability and validity of the research are also examined in this chapter along with ethical considerations.

Chapters 5 to 7 cover the data analysis and discussion associated with each of the research questions. Key points are illuminated via the use of excerpts from transcripts of student interviews and the use of tabulated data.

Chapter 8 is designed to synthesise the data from the previous three chapters and discuss the main themes that developed as a result of examining the data as a whole. A summary of the research is provided along with the key findings of the research. The chapter concludes with a discussion of the limitations of the study and some recommendations for further research.

Chapter 2. Review of the Literature

A National Statement on Mathematics for Australian Schools (AEC, 1991) indicated that a goal for primary aged children was for them to be able to “choose computation methods (mental, paper-and-pencil, calculator) and check reasonableness of results” (pp. 115, 121). In the previous chapter the issue of computation choice was raised. Several questions arose including:

- What computation choices do students make?; and
- Why do they make particular choices?

The goal of students being able to choose a computation route requires an ‘at-homeness’ (Jones & Tanner, 1998) and confidence with a variety of approaches to calculation. In this chapter various aspects of this ‘at-homeness’ are examined.

It makes sense that in order for students to be able to make a choice they must first have a variety of computation alternatives at their disposal. Prior to making a choice, however, some thought is required. While completing the calculation, further thinking is required, and once the calculation is completed even more thinking should take place. This chapter traces this thinking along with the literature on computation and computation choices in order to identify what has been clearly established and what gaps exist. The niche into which this study fits will clearly be identified as a result of the review.

The process of choosing a computation route is much more complex than simply making a simple decision to utilise a single computation method. While each form of computation exists in its own right, there are relationships between them. For example, when completing a standard written algorithm a person will draw on several mental calculations, possibly jotting down interim results on the way to the answer.

To further complicate matters the literature also refers to terms such as ‘numeracy’, ‘number sense’ and ‘meta-computation’ when describing thinking associated with choosing the type of calculation to perform and then carrying it out. The broader issues of numeracy, meta-computation and number sense will be discussed first,

followed by a review of each of the computation approaches. The links between the various methods of computation will be explored and the chapter will then finish with a discussion on the various factors that affect the computation choices that students make.

The studies of computation choice tend to fall into two broad categories:

- variables that influence computation choice; and
- examples of the choices students make.

The missing element in the literature is why students make the choices they do. This study is designed to identify the reasons behind the choices made by students. The journey begins with a brief discussion of what it means to be numerate in the calculator age.

Numeracy

The whole issue of computation and computation choice fits under the broader notion of numeracy. Willis (1990) traced the origin of the term numeracy to the Crowther report (1959) where the term was originally used as the mirror image of literacy. Numeracy came to encompass the broad idea of ‘mathematical literacy’.

Girling (1977) proposed a much more succinct definition suggesting that numeracy was “the ability to use a four-function calculator sensibly” (p. 6). He described what was meant by sensible calculator use. Girling referred to the ability to check that an answer was correct as being the key to sensible calculator use. Checking implied the use of estimation, pattern and a degree of number sense to evaluate the answer. His views were somewhat controversial at the time and the fact that his definition of numeracy has not become commonplace indicates that his proposal is still considered ‘radical’ by today’s standards. His ideas, however, had merit, especially his discussion of the understandings that would be required to underpin sensible calculator use.

Willis (1990) described how the use of the word numeracy has become corrupted in the sense that it is often used to refer to ability with computation, which in turn implies mental and paper-and-pencil calculation. The Australian Association of Mathematics Teachers (AAMT) adopted the following working definition of numeracy.

To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work and for participation in community and civic life.

In school education, numeracy is a fundamental component of learning, discourse and critique across all areas of the curriculum. It involves the disposition to use, in context, a combination of:

- Underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic);
- Mathematical thinking and strategies;
- General thinking skills; and
- Grounded appreciation of context (AAMT, 1997, p. 2).

The broad nature of numeracy is encompassed by this definition. It includes issues such as numeracy across the curriculum and broadens the notion of numeracy beyond just involving number. Of particular interest is the reference to ‘mathematical thinking and strategies’, which is related to the idea of metacomputation to be developed in the next section.

Jones and Tanner (1998) believed that numeracy “requires both mathematical knowledge and skills, and in addition, an awareness of this knowledge base so that effective choices may be made” (p. 287). The suggestion that numeracy is linked to making effective choices is of particular interest in the context of this research. They elaborated on the issue of choice.

The choice of an effective strategy for a problem is dependent not only on the knowledge, which has been learned but also on one’s awareness of that knowledge and the realisation that its use would be appropriate. To devise a strategy requires confidence, an at-homeness maybe, and a view of mathematics as a subject in which students can create their own methods (p. 287).

The implication is that to be numerate one must possess more than just a knowledge bank of facts and processes, but also be prepared to try different approaches to solving a problem, including some self-generated approaches. The approaches that a student adopts would take into account that student’s own ability and facility with various forms of computation. In defining numeracy Jones and Tanner move beyond a simple ability to compute, but rather suggest that students need to think about the problem and consider the options according to their confidence in their ability to use a particular computation approach in producing a correct answer. They described a numerate person thus:

To be numerate is to be able to mathematize situations using techniques and processes which are confidently known to generate a secure answer. Numeracy therefore involves an interaction between mathematical facts, mathematical processes, metacognitive self knowledge and affective aspects of mind including self confidence and a disposition to construct personal methods (p. 287).

The metacognitive aspect of this definition is of particular interest as the thought of metacomputation is a thread that will run through this review of the literature. The term will be examined in detail shortly but the implication is that students would be involved in thinking about the method of calculation prior to embarking on a computation and while performing the computation they would be monitoring the process.

In order to provide a framework by which computation choices may be examined a model has been provided that outlines the various computation routes and options available. The first section of the literature review follows the branches of the model and expands on what is known about each of the computation alternatives.

Computation Routes and Options

Several models of computation have been developed to explain the process of making a computation choice and performing a calculation. These models will be examined in detail in the next chapter as part of the development of a framework under which computation choice may be studied. The model shown in Figure 2.1 is presented as an example by which computation choices may be examined. The National Council of Teachers of Mathematics (NCTM, 1989) used the following model to describe the process a person goes through when deciding how to tackle a problem of a numerical nature. The model indicates where computation choices have to be made and what choices are available, but not how the choices are made. In order to simplify the complex issue of computation choice the model shows very distinct routes through the computation process, whereas in reality students may go down one path, switch to another and then return to their original path. For clarity of thought this model will be used to illustrate the process of deciding when a calculation is needed and what form of calculation to use.

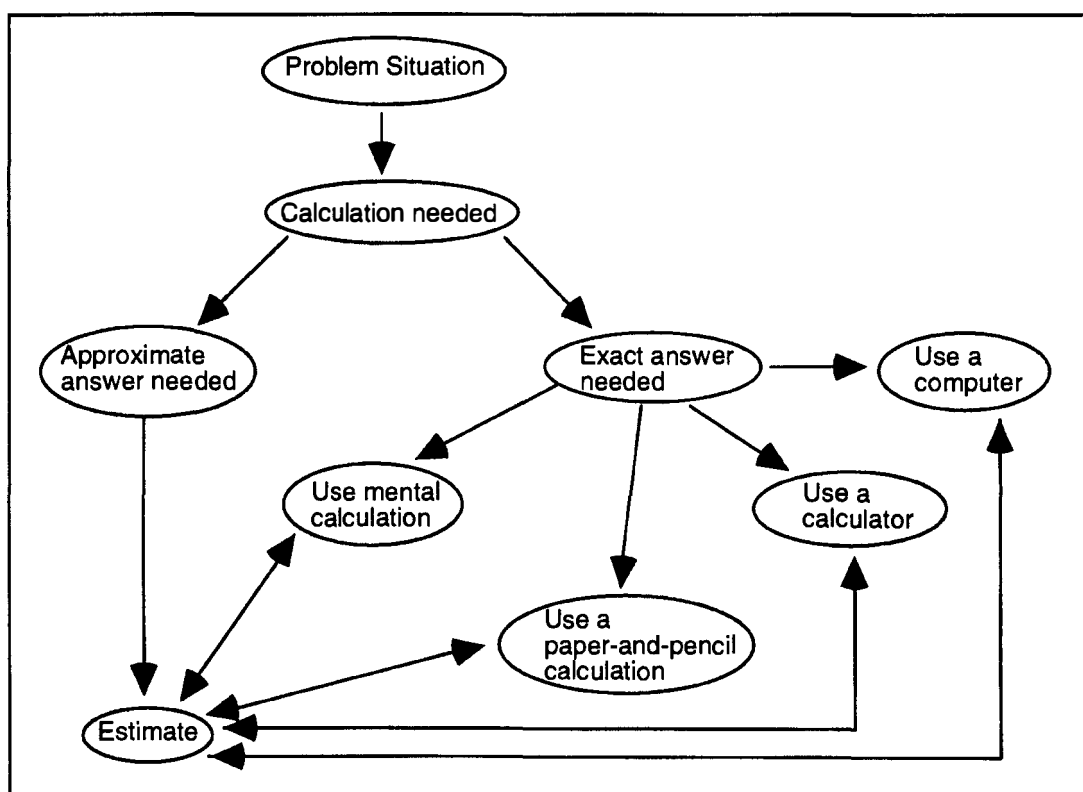


Figure 2.1: Model to describe computation choice (NCTM, 1989, p. 9).

The model indicates that after perceiving the need to perform a computation, a person has two broad choices; make an estimate or perform an exact calculation. If an exact answer is required then several alternatives are available: mental, paper-and-pencil, calculator and computer. Each of these computation alternatives with the exception of computers will be discussed. The use of computer to produce an exact answer will not be discussed as in most primary classrooms computers are rarely applied as a computation alternative.

Prior to embarking on a computation path, however, several decisions and choices have to be made, the first being whether a computation is required. How this decision is made is unclear but the presence of numbers in the problem would turn thoughts toward the need for a calculation. This research is not designed to consider how students make the decision that a calculation is required. The first decision that a calculation is required needs to be followed by a second decision – whether an approximate or exact form of calculation is required. It is likely that the context (out shopping, sitting at a school desk) and purpose behind performing the calculation would

have a bearing on the decision. It is not the purpose of this study to examine the reasons behind making the choice of exact versus approximate methods of computation.

Should the decision be to pursue an exact answer, further options are presented. The choice to use a mental, written or calculator method for computing the answer is the subject of this research. How do students make this choice? Clearly a great deal of thinking has to take place in order to reach the point where a calculation begins. This thinking about the calculation may be categorised under the heading of 'metacomputation'. As the name implies metacomputation involves higher order thinking about computation and should influence decisions about computation. In the next section the evolving understanding of the term metacomputation is discussed.

Metacomputation

Before a specific computation choice is made a considerable amount of thinking takes place. Likewise during the performance of a calculation students should monitor what is happening. Once the calculation has been completed further thought should be given to whether the answer is reasonable. This monitoring and checking function could be described as fitting under the broad construct of 'number sense'. As the term implies number sense is the equivalent to common sense as applied to number. Number sense will be discussed in detail later, suffice to say that number sense is part of the broader thinking about calculation implied in the term metacomputation.

Metacomputation appears first to have been raised in discussion about mental computation and calculator use (Shigematsu, Iwasaki, and Koyama, 1994). The term was used to describe the higher order thinking required to both plan a calculation and to check it. Mental computation was described as fulfilling this dual role.

Mental computation in the broad sense, however includes not only the computation process but also the higher order thinking and decision making that lead to the selection of the computational process... Mental computation in the narrow sense will be extremely important not only for checking the operations and the results of calculators but also for monitoring, evaluating, and controlling the whole process of computation (pp. 19-20).

In essence they were proposing that mental computation be used as a metacomputation for calculator use. The use of the term mental computation in so many different ways can be confusing. Mental computation is being used as a metacomputative tool, when applied to the

- higher order thinking;
- decision making;
- monitoring; and
- controlling

aspects of a calculation. To avoid confusion with the other ways in which the term mental computation is used it makes sense to refer to these higher order processes as metacomputation.

Shumway (1994) suggested that the use of the term metacomputation be expanded to encompass the thinking associated with making a computation choice. He stated that:

perhaps we can view metacomputation as involving processes and strategies employed to guide computational choices. It would seem that this idea of metacomputation would accommodate issues of ... number sense ... as well as ... written algorithms, mental computation, computational estimation, and calculators (p. 194).

Shumway concluded his discussion of the term metacomputation by suggesting that a clear definition for metacomputation be developed and the use of the term be explored. If metacomputation were considered to be ‘thinking about the method of computation’ then it would occur at several places in the computation process. Metacomputation would be required when first deciding whether or not a calculation is required. Once this decision is made the next decision is whether an exact or an approximate solution is required. The context in which the problem occurs will have a bearing on the decision as well as experience with the various computation alternatives.

The next example of where metacomputation takes place is when the decision to use an exact form of computation has been made. How do students decide whether to use a mental, written or calculator approach to solving a problem? This aspect of metacomputation is the focus of this research. The initial places where metacomputation has to take place are shown on the flowchart in Figure 2.2. These are not the only places where metacomputation occurs. For example, metacomputation occurs when monitoring

and checking a calculation. What the flowchart indicates are the decisions to be made prior to arriving at the point where an exact form of calculation is chosen. The shaded section indicates the focus for this research. This research, while relating to this distinct aspect of metacomputation will add to and inform the debate on metacomputation.

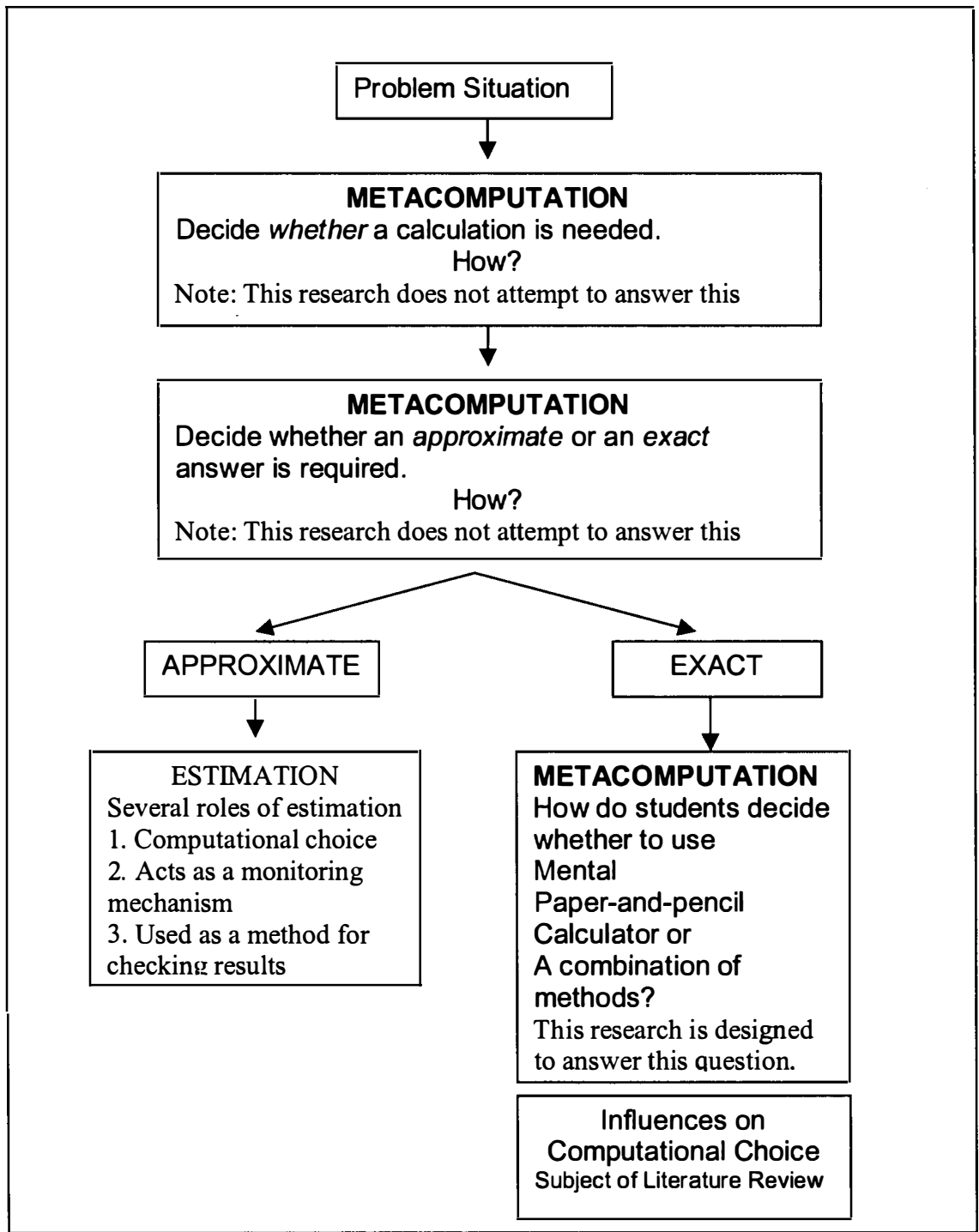


Figure 2.2: Flowchart of metacomputation.

The term metacomputation may therefore be thought of as the higher order thinking that guides the computation. Monitoring the calculation would also require higher order thinking and the application of ‘number sense’. Like the term metacomputation, the meaning of the expression number sense is still developing. Number sense is examined in the next section.

Number sense

The term ‘metacomputation’ has been used to encompass the higher order thinking associated with performing a calculation. It may be seen as fulfilling several roles, the first being to guide computation choice and the second to monitor the calculation as it progresses and third to check the results of the calculation. In the previous section the role of metacomputation in the decision making process was examined. In this section the monitoring and checking components of metacomputation will be discussed under the broad notion of number sense.

Any discussion about computation choice would not be complete without an examination of number sense. *A National Statement on Mathematics for Australian Schools* stated, “All people need to develop a good sense of number, that is, ease and familiarity with and intuition about numbers.” (AEC, 1991, p. 107). Over a decade ago, *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* noted that, “the major objective of elementary school mathematics should be to develop number sense” (National Research Council, 1989, p. 46).

The development of number sense, however, appears to have been hampered by a lack of clarity as to what constitutes number sense and how it is developed. Greeno (1991) described number sense as “several important but elusive capabilities, including flexible mental computation, numerical estimation, and quantitative judgement” (p. 170).

Sowder (1992) equated number sense with the development of ‘quantitative intuition’ or a ‘feel for number’. The thought of teaching, ‘intuition’, however, is difficult to conceive. Sowder (1992) cited the work of Resnick (1989) in this area in an attempt to ‘put some flesh on the number sense bones.’ Rather than use the term number sense, Resnick preferred to use the term ‘higher order thinking’. In the following extract Sowder substituted ‘number sense’ for Resnick’s original ‘higher order thinking’.

[Number sense] resists the precise forms we have come to associate with setting of specified objectives for schooling. Nevertheless, it is relatively easy to list some key features of [number sense] when it occurs. Consider the following:

[Number sense] is *nonalgorithmic*. That is, the path of action is not fully specified in advance.

[Number sense] tends to be *complex*. The total path is not ‘visible’ (mentally speaking) from any single vantage point.

[Number sense] often yields *multiple solutions*, each with costs and benefits, rather than unique solutions.

[Number sense] involves *nuanced judgement* and interpretation.

[Number sense] involves the application of *multiple criteria*, which sometimes conflict with one another.

[Number sense] often involves *uncertainty*. Not everything that bears on the task is known.

[Number sense] involves *self-regulation* of the thinking process. We do not recognise it in an individual when someone else ‘calls the plays’ at every stop.

[Number sense] involves *imposing meaning*, finding structure in apparent disorder.

[Number sense] thinking is *effortful*. There is considerable mental work involved in the kinds of elaborations and judgments required (p. 381).

Terms such as ‘self-regulation’ and ‘nuanced judgement’ indicate a relationship between number sense and the broader construct, metacomputation. Turkel and Newman (1988) gave a description of number sense. Of interest was their belief that a facet of number sense was associated with the ability to make choices as to the appropriate computation method. This is reflected in the later part of their definition of people with number sense, which stated,

Such people show good judgement about selecting an appropriate method of processing numbers; approximation, paper-and-pencil computation, mental estimation, or computation with a calculator (p. 53).

Number sense is difficult to define because it relates to many different behaviours rather than to a specific single behaviour. This raises the issue of identifying whether a person has number sense or is displaying number sense. A single occurrence of number sense or a single facet of number sense is not enough to suggest that a person possesses number sense.

Silver (1994) noted the difficulty in defining number sense when he stated:

Although it is difficult to define number sense precisely, behaviours like estimating before or after computing, judging the reasonableness of one's calculations, and using the relative size of numbers or numerical benchmarks (such as basic facts) to guide quantitative activity are all examples of sense-making actions associated with numbers and numerical activity (p. 158).

From his definition it is apparent that number sense is made up of many different parts and is more than the sum of its parts. Sowder (1988) defined number sense as “a well organised conceptual framework that enables a person to relate number and operation properties.” She described a person who possessed number sense as using “flexible and creative ways to solve problems involving numbers” (p. 183). The term ‘flexible’ should be noted as it arises several times in the literature and is associated with estimation, mental methods of calculation and self-generated written methods. The term flexibility implies being adaptable or being able to change according to circumstances. To be flexible in solving problems involving calculation would therefore imply students would need several methods at their disposal from which a choice may be made to fit the circumstances.

In an attempt to explain number sense McIntosh, Reys and Reys, (1992) produced a framework for number sense that included the following components:

Knowledge of and facility with number;

Knowledge of and facility with operations;

Applying knowledge of and facility with numbers and operations to computational settings (p. 4).

McIntosh et al. (1992) went on to elaborate on each of these components of number sense explaining the elements of each aspect of number sense. The focus of this explanation revolved around sense making, and the inclination to make use of relationships between numbers and strategies for calculation. Allied to this thinking was the inclination to consider whether the result from performing a calculation was reasonable and sensible.

McIntosh, Reys, Reys, Bana and Farrell (1997) refined the definition of number sense and described it as:

A person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient strategies for managing numerical situations (p. 3).

Of interest in this definition is the reference to ‘inclination’ which suggests that while students may have various computation skills at their disposal they may not be inclined to use them. For example a student may choose to use a calculator, when a mental computation would have been a better choice, given the student’s ability with mental computation, however fatigue may have meant that the student was more inclined to use a calculator in this circumstance, whereas on another occasion would calculate the result mentally. They go on to explain that while number sense is a broader term than estimation or mental computation it includes both. Silver (1994) cautioned, however, “it would be unwise for us to believe that the sum of the cognitive components would necessarily equal the complex whole of number sense” (p. 160). Therefore, a person who demonstrates excellent mental calculation and estimation skills may not possess a high degree of number sense.

The McIntosh, Reys, Reys, Bana and Farrell definition has been chosen to guide the use of the term number sense in the context of this research. This definition has a focus on number and includes the idea of number sense informing mathematical judgements.

McChesney and Biddulph (1994) noted the difficulty with the notion of number sense in that it “is not something that can be taught directly. Rather it is something that emerges from mathematical activity and exploration” (p. 10). The contrary point of view is that children need to be taught computation strategies including mental computation techniques in order to develop number sense. The risk with such an approach is that mental mathematics and in particular the development of number sense might suffer the same fate as written computation, being reduced to a set of rote-learned procedures and rules that in many cases are applied without understanding.

Number sense can therefore also be seen as helping children to develop procedures for tackling a numerical problem, and monitoring and regulating the process of solving the problem. This means that number sense is part of the broader construct of metacomputation. In the discussion about number sense the term ‘thinking strategies’ is often used in relation to starting and completing a calculation. In the following section the term thinking strategies will be explored and the relationship between metacomputation and number sense examined.

Thinking strategies

The expression ‘thinking strategies’ is not a new one, having been used by Rathmell (1978) to describe an approach to learning the basic number facts that involved more than just the memorisation of facts. In discussing the nature of mental strategies McIntosh, Reys and Reys (1997) described the purpose of a mental or thinking strategy as turning a “calculation we cannot do into a calculation we can do by employing relationships between numbers and operations” (p. 323). To do this requires some adjustments and reflections to be made prior to and while performing the calculation. For example, the problem 36×25 may seem difficult to calculate mentally but with a slight adjustment that involves the use of number properties it may be transformed to become $9 \times 4 \times 25$ or 9×100 that is easily computed mentally. In a succinct fashion McIntosh, Reys and Reys, (1997) explained mental strategies this way.

In short, mental strategies are strategies, often self-developed, for computing that are based on the user’s understanding and knowledge of mathematical properties and relationships. The thinking strategy can vary in efficiency and elegance depending on the sophistication of the student’s understanding (p. 323).

Of interest in this definition is that mental strategies are often self-developed. This does not mean they cannot be taught but in trying to teach a specific strategy it is possible that a student’s own thinking may be lost. A later definition provided by Thompson (1999) includes the thought that students would select appropriate strategies. This implies thinking about the most appropriate strategy to use, which places the use of thinking or mental strategies under the broad idea of metacomputation. His definition is reproduced below.

The application of known or quickly calculated facts in combination with specific properties of the number system to find the solution to a calculation whose answer is not known. They also incorporate the idea that, given a collection of numbers to work with, children will select the strategy that is the most appropriate for the specific numbers involved (p. 2).

This definition also includes the idea that students would have a store of known facts and use these on their own or in conjunction with other knowledge such as number properties to perform the calculation. The ability to select a strategy according to the numbers involved implies that students have a variety of strategies at their disposal and possess the ability to choose the most appropriate strategy. The earlier definition provided by McIntosh, Reys and Reys (1997) also refers to a student's knowledge of number properties and relationships and suggests that the choice and use of various strategies depends on how well the student understands the strategy and associated properties of number.

Heirdsfield (1998) reported on a study involving two children, one of whom was described as flexible in her mental methods and the other, inflexible. The flexible thinker is described as using metacognition, in that she made conscious choices and reflected on and evaluated her responses. There was little evidence to suggest the inflexible student thought much about the problem or which strategies to use.

To help understand the relationship between metacomputation, thinking strategies and number sense Figure 2.3 is provided. Each component of metacomputation draws on the next; for example, estimation draws on mental computation but involves more than just mental computation. Likewise number sense draws on both mental computation and estimation but involves more than just these two components. Thinking strategies are closely linked to conceptual understanding and make use of number sense. Metacomputation makes use of all the components and does more. It helps:

- guide decisions about whether a computation is required;
- determine which form of computation is needed;
- monitor the calculation as it progresses; and
- to determine whether the answer is reasonable.

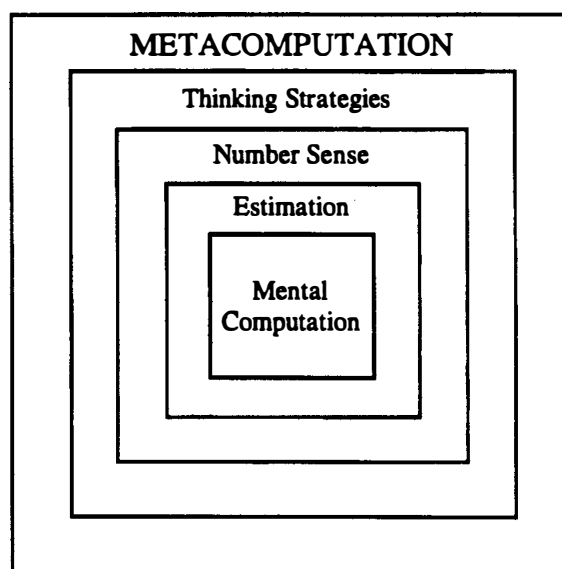


Figure 2.3: The components of metacomputation.

While metacomputation may be thought of as being made up of a number of different parts, the whole is much more than the sum of its parts. One factor that would guide the choice of computation method would be a student’s familiarity with and ability to use a variety of different methods. Estimation, mental methods, written methods and calculator methods will each be examined in turn to help explain the computation process.

The Computation Choices

Rarely is computation choice as clear-cut as the model outlined in Figure 2.1 might suggest. For example, mental computation may be a computation choice in its own right but it is also present in estimation and paper-and-pencil calculation. Some of the relationships between the various computation options such as mental calculation and estimation have already been explored in the previous sections examining metacomputation and number sense. These links as well as other relationships will be elaborated on in this section. The estimation path of the model outlined in Figure 2.1 and shown in Figure 2.4 will be examined first, followed by the approaches to exact computation.

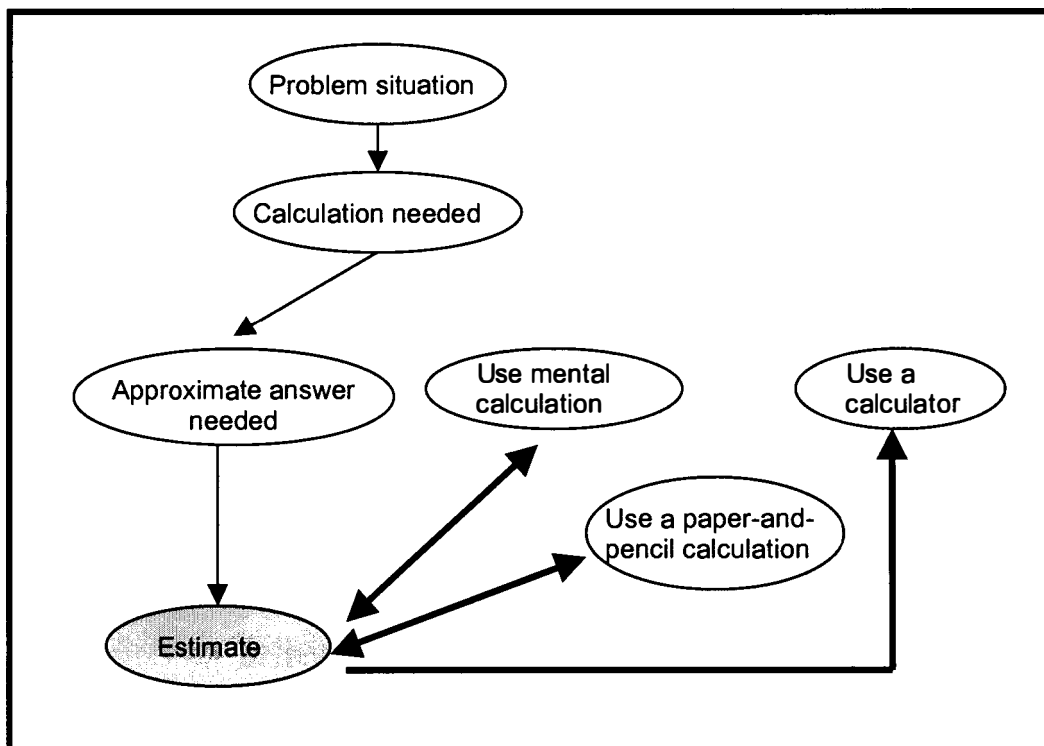


Figure 2.4: Estimation path of computation model.

Estimation

The literature refers to computational estimation to distinguish it from estimation associated with measurement. For the purpose of this discussion the use of the term estimation is restricted to computational estimation. A glance at Figure 2.4 indicates that while estimation is a computation choice in its own right it also has influence over, and is influenced by the other computation choices. The bold arrows highlight these relationships. Computational estimation may be thought of in several different ways. These include:

- Estimation as a computation choice;
- Estimation as a monitoring device for exact forms of calculation; and
- Estimation as a method of checking results of exact forms of calculation.

Primarily in this section estimation is considered in the role of a computation choice, whereby a decision is made to use an estimate in preference to an exact form of computation. The context in which the calculation occurs may only require a ‘rough answer’ be supplied and therefore an estimate is made. When estimation is used in a monitoring or checking capacity then it is no longer being used as the prime computation choice but rather as an adjunct to one of the other computation methods.

This aspect of estimation will be examined later. For now the first role of estimation as a computation choice will be examined.

The model shown in Figure 2.1 uses the terms ‘approximate’ and then ‘estimate’ when referring to a computation path where an exact answer is not required. In some cases these terms are used interchangeably. In an effort to define what it means to estimate, these terms will now be examined.

Defining estimation

There appear to be some discrepancies in the literature between the use of the terms estimation and approximation. Often these terms are used synonymously but Sowder (1992) in reviewing the literature on estimation tended to favour those definitions that separated estimation and approximation. Some of the discrepancies occurred because estimation was being used in a measurement sense.

Reys (1984) suggested there were at least four distinguishing characteristics of computational estimation:

1. it is performed mentally, generally without paper and pencil;
2. it is done quickly;
3. it produces answers that are not exact but adequate for making necessary decisions;
and
4. it often reflects individual approaches and produces various estimates and answers (p. 551).

General agreement on the use of the terms does not appear to have been reached, so for the purposes of this research the following definition of computational estimation was adopted. Estimation refers to

producing an approximate answer to a computation, one that is ‘close enough’ to allow a decision to be made. Estimation often involves the user in mental computation as a preliminary first step to forming an estimate (McIntosh, Reys & Reys, 1997, p. 322).

While this definition incorporates the term ‘approximate’ it was chosen because of the link that is made to mental computation. When making use of estimation as a computation alternative it would be expected that a student makes an ‘educated guess’, rather than simply a ‘wild guess’ so as to imply that some form of mental processing occurs.

What is known about estimation?

Sowder (1992) in her review of the research noted there was “not a rich research base on estimation” (p. 372). Trafton (1994) also noted that little research has been carried out into estimation and what studies there had been focussed on the increase in ability to estimate that resulted from children being taught various estimation strategies. In particular most attention has been focussed on:

- how students estimate;
- skilled and unskilled estimators;
- the effect of instruction; and
- affective factors associated with making estimates.

In 1982 Reys, Rybolt, Bestgen and Wyatt proposed a three-process model for computational estimation. They found that students tended to use one, or a combination of, the following methods:

- Reformulation;
- Translation; and
- Compensation.

Reformulation involved the changing the form of the numbers by using a process such as rounding to make them easier to compute. Translation, involved changing the structure of the problem to make it easier to calculate mentally. Compensation involved making adjustments after translating or reformulating the computation in order to make it simpler to handle the numbers. Sowder (1992) noted that this Reformulation, Translation and Compensation (RTC) model was often used as the basis for reporting the research in this area. Shumway (1994) suggested that the RTC model be used as a “conceptual framework for computational estimation strategies” (p. 188).

The literature comparing good and poor estimators indicates that skilled estimators:

- are flexible in their thinking;
- use a variety of estimation strategies; and
- possess a deep understanding of numbers and operations.

On the other hand poor estimators:

- are bound to applying standard written algorithm approaches;
- find it difficult to think of a problem as having more than one right answer; and
- do not value estimation and often equate estimation with guessing (Sowder, 1992, p. 386).

Estimation is a complex process that develops over a long period of time. Of interest are the findings in the literature that good estimators tend to use a variety of strategies when making estimates and tend to ignore school-taught methods such as rounding (Sowder, 1992; Trafton, 1994). Not only do skilled estimators use a variety of strategies, but also they easily switch between strategies. In general, good estimators were more flexible in their thinking. Good estimators also tended to have a sound grasp of basic number facts, properties of number, and place value. Poor estimators, however, tended to be bound to a single strategy – the application of standard written algorithms, which were used to obtain an exact result, and then adjusted to look like an estimate had been made. There seem to be several reasons for this behaviour, mostly related to beliefs that exact calculation is more highly valued than estimation. Shimizu and Ishida (1994) described high ability with mental computation as being a two-edged sword in the sense that it can inhibit estimation in much the same way as over-reliance on written methods does.

It sometimes enables students to find nice pairs of numbers quickly, and to notice when estimates are unacceptable. But it also sometimes inhibits students from estimating, and they seem to be addicted to computing mentally, even when it is impossible for them to do so (p. 176).

It should be noted that Shimizu and Ishida were discussing estimating from a Japanese perspective. Japanese students tend to show reluctance toward estimating in favour of exact methods. Schoen, Blume and Hoover (1990) in reviewing the research on how students estimate, noted that “researchers have found that students frequently compute mentally and then round their answers” (p. 61).

The ability to estimate improves with instruction in techniques for estimating and time spent learning how to estimate, but the teaching of specific techniques can hamper the development of estimation later as students become reliant on the taught methods and fail to think in flexible ways (Sowder, 1992; Trafton, 1994). A similar argument may be applied to the teaching of specific mental computation strategies. Teaching specific strategies for estimation and mental computation is at odds with the constructivist approach to teaching and learning.

Students often view estimation in a negative light and therefore tend not to estimate. This may be caused by the commonly held belief that there is only one right answer in mathematics. The traditional paper-and-pencil driven curriculum that tends to focus on a single method to produce the one correct result may retard the development of estimation. Improving estimation may assist the development of paper-and-pencil algorithms. The reverse, however, is not necessarily the case. Estimation encourages students to think about numbers and to make flexible use of the relationships between them.

In this section it has been shown that estimation may be viewed as a computation choice in its own right and that students who make good use of estimation use a variety of strategies when estimating. Some students are hampered in their ability to estimate because of a lack of flexibility in their thinking and therefore may not choose to use estimation as an alternative to exact forms of calculation. In the next section the role of estimation as a monitoring and checking mechanism for exact forms of calculation will be examined.

Estimation in a guiding and monitoring role

While estimation may be seen as a computation choice in its own right it encompasses much more. Trafton (1994) believed that estimation is the key to making sensible computation choices. Estimation encourages students to think about numbers and ways to handle them easily. Estimation therefore becomes a valuable tool for

guiding and monitoring exact computation. It also forms the basis for checking calculations and judging the reasonableness of results. As such, estimation can be seen as both a computation choice in its own right and as a form of ‘meta-computation’ that takes place before, during and after a calculation. To recap, estimation may be used purely as a computation choice in its own right, or alongside one of the other computation choices. This distinction is important because while many of the same skills are utilised, the purpose behind the estimation is completely different.

While the importance of estimation as a checking tool for exact calculation is often emphasised, Reys et al. (1982) found that students experience difficulty trying to use estimation as a mechanism for judging the reasonableness of results. What was more disturbing was that in the same study the researchers found that students placed more faith in answers generated by a calculator than in their own estimates.

Trafton (1994) provided an adaptation of the NCTM computation model (See Figure 2.5) and highlighted the role of estimation as both a monitoring and a checking device. Estimation still appears as a legitimate computation choice. The dotted line linking ‘estimate’ to ‘exact answer’ also indicates a relationship between performing an estimate and carrying out a paper-and-pencil, mental or calculator computation.

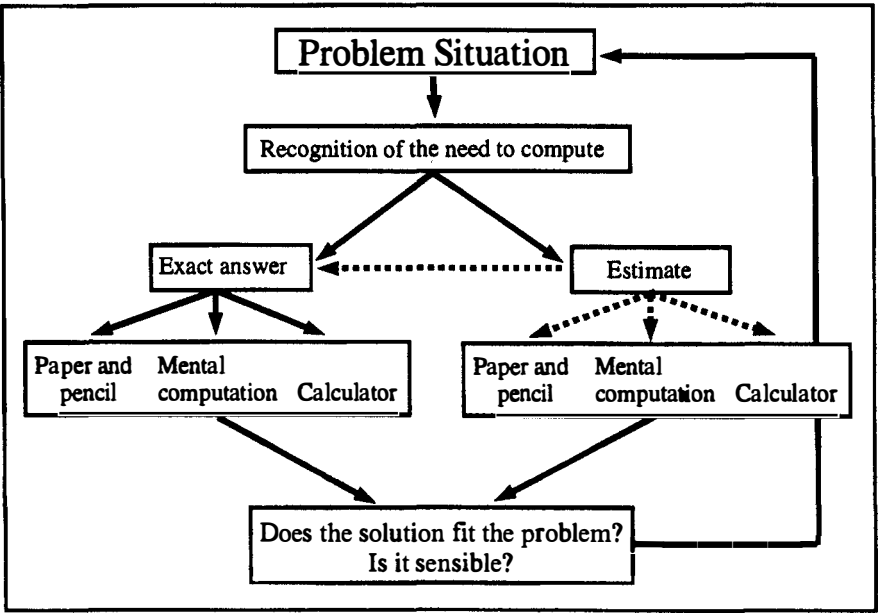


Figure 2.5: Computation model according to Trafton (1994).

By modifying the computation model Trafton managed to highlight three roles for estimation when performing a mental, written or calculator computation. This monitoring function of estimation may occur at any of three places:

- before embarking on the calculation, to establish reasonable limits within which the answer should fall;
- during the calculation to monitor progress; and
- at the end of the calculation to determine whether the answer is reasonable.

Estimation, when used to guide or monitor a computation is part of metacomputation – it is a metacognitive process. Trafton (1994) in support of the use of estimation in a monitoring role cited Hiebert (1984) who argued that estimating “encourages one to step back, to think about the structure of the problem, and to focus on the reasonableness of the solution” (p. 80). It is this ‘stepping back to think’ about the structure of the problem that is at the heart of metacomputation.

There are many similarities between estimation and mental computation. These are highlighted in the following statement by Reys (1984).

Both skills are used to check the reasonableness of results produced by hand-held calculators and computers. Each is performed mentally; each takes advantage of structural properties and relationships among numbers; and each allows individuals to use different solution processes (p. 556).

While there are many similarities, it should also be noted there are some significant differences. For example, mental computation is a vital prerequisite to computational estimation, but the opposite relationship is not necessarily true. Mental computation produces an exact answer, whereas there are several possible estimates that might be classed as correct. Reys (1984) made the following observation.

It is possible to be simultaneously competent at mental computation and very poor at estimation. However, the converse is not true; that is, people who are good at computational estimation are also good at mental computation (p. 549).

This statement does not imply a lack of thinking when calculating mentally but that an ‘extra level’ of thinking is required when performing an estimate. Students have to be comfortable with issues such as the degree of accuracy required and the fact that several answers may be considered correct.

In this section estimation has been shown to fulfil several roles, firstly as a computation choice and secondly as a vehicle by which metacomputation may take place. In performing these roles estimation draws on mental computation. Mental computation will be examined next.

Mental computation

In the previous section it was noted that estimation performed several functions. It could be used as a computation choice or in a metacomputative or monitoring and checking capacity. Likewise mental computation also fulfils several roles. Mental computation may be viewed as:

- a computation choice in its own right;
- part of estimation and paper-and-pencil methods; and
- part of the monitoring process.

The importance placed on mental computation may be seen in the following statement from *A National Statement on Mathematics for Australian Schools*

Students should regard mental arithmetic as a first resort...strategies associated with mental computation should be developed explicitly throughout the schooling years, and should not be restricted to the recall of basic facts...students should be encouraged to develop personal mental computation strategies (AEC, 1991, p. 109).

While acknowledging mental computation as an important skill McIntosh, Reys and Reys (1995) also described mental computation as a “vehicle for promoting thinking” (p. 238). There are certainly links to metacomputation that are highlighted by this comment. Metacomputation makes use of estimation, which in turn draws on mental computation.

It could be argued that there is an element of mental computation in most calculation. Figure 2.6 illustrates the links between the various computation choices and mental computation. For example, when a written algorithm is performed the student becomes engaged in a series of mental computations momentarily interrupted by jottings on paper.

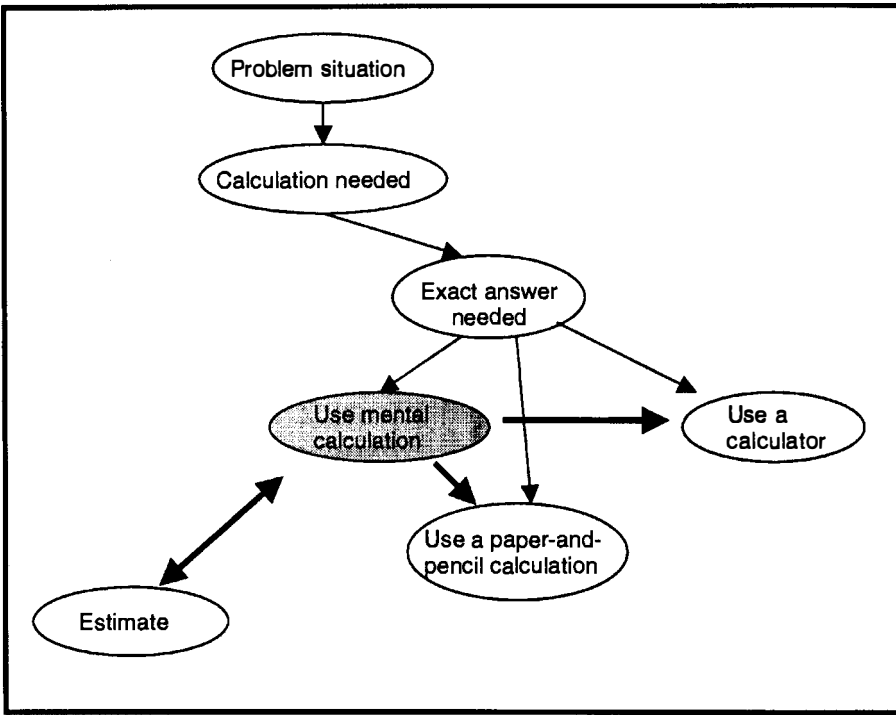


Figure 2.6: Mental computation links.

The role of mental computation as a specific computation choice will be examined first and then the separate issues of mental computation as a component of other choices will be discussed. Finally the monitoring role of mental computation will be reviewed.

Defining mental computation

It is important that a clear definition of what is meant by the term mental computation is provided because views as to what is meant by the term vary considerably. For some the term implies the drilling of the basic number facts by means of short, sharp questions, while for others the emphasis is on the development of strategies to improve mental calculation. Thompson (1999) discussed the differences between mental calculation and mental arithmetic and suggested that mental arithmetic involved recall of facts, whereas mental computation involved the use of mental strategies as well as recall. He made the following observation: “there is no word for ‘mental’ in The Netherlands and this leads to their using terms which translate into ‘working *in* your head’ (recalling facts) and ‘working *with* your head’ (figuring out)” (p. 2). In this research the term mental computation or the figuring that goes on with your head is preferred.

Several terms have been used to describe mental computation. Terms such as mental arithmetic and oral mathematics were most popular decades ago but they conjure up the idea of students being asked to respond to twenty rapid-fire questions. Drill, while the norm in many classrooms a decade ago is not what is meant by the term mental computation today. Traditionally mental computation was considered to be calculations done in the head without the use of paper-and-pencil to record, but as understanding of how people calculate mentally has grown so have the definitions of mental computation. Often reference is made to 'mental strategies' implying there is more than one way to perform a mental computation. For the purpose of this discussion 'mental computation' refers to:

computing an exact answer to a computation 'in the head'. Thus, no external tools, such as calculator or paper and pencil, are used in doing the computation. The strategy for computing may be invented by the user or borrowed from standard paper-and-pencil techniques" (McIntosh, Reys & Reys, 1997, p. 322).

This definition relates the use of mental computation to producing an exact answer and suits the purpose when mental computation is used as a computation alternative. Reference is made to the use of strategies, invented or taught as part of the process of arriving at the exact result. The definition, however, does not capture the idea of using mental computation as a step in forming an estimate. Primarily mental computation should be considered to involve thinking about the calculation and the path to solution rather than simply remembering a few basic number facts. This is borne out in the following definition.

Today there is a call for mental computation (or thinking) strategies to be born out of conceptual understanding and active problem solving rather than memorised rules or standard procedures (McIntosh, Nohda, Reys & Reys, 1995, p. 238).

Bramald (1998) asked the question,

Why is the British educational establishment so hung up on making 'mental' mean absolutely nothing but the answer? Why do we degrade any sort of thinking that uses notes or materials for intermediate stages? (p. 5).

Bramald was not suggesting some hybrid of the standard written algorithm but rather that the definition of 'mental' should include the use of external devices such as the 'empty number line'. He argued that by allowing children to jot things down some evidence is provided that may be used to work out how the children arrived at the answer and better still, these jottings might be used as prompts when the child explains

the method by which the answer was calculated. What Bramald argued for was a more liberal interpretation of what constitutes 'mental'. He concluded with the comment

We must beware of being puritanical about the meaning of the word 'mental'. We can make life difficult for ourselves and, crucially, for our children with this dogmatic interpretation of "No paper, no fingers" (p. 7).

The points Bramald makes are most valid and while the definition supplied by McIntosh, Reys and Reys (1997) is used to guide this research, the points raised by Bramald are taken up in Chapter 3 where models of computation are discussed in detail. A definition of mental computation that involves working in the head and no use of external writing devices clearly delineates between mental and informal written methods.

In recent years, particularly since the introduction of calculators into the primary classroom, the role of mental computation has received more attention. In the following section key research findings associated with mental computation are given.

What is known about mental computation?

Several different aspects of mental computation have been researched. These include chronometric research; the measurement of student reaction times to mental questions; the role of memory; and the development of mental strategies. This review focuses on mental computation strategies. The topic of mental computation strategies has a direct bearing on this research in the sense that better mental calculators tend to have a range of mental strategies at their disposal. These strategies allow for the student to make a choice as to the way a mental calculation is to be performed. The more skilled a student is at calculating mentally, the more likely he/she will be to make use of mental computation as a desired choice.

While not exhaustive Table 2.1 gives an indication of the research findings associated with mental computation and in particular the use of mental computation strategies. The table serves to highlight the many differences between mental computation and paper-and-pencil methods.

Table 2.1: Mental computation research findings

What is known about Mental Computation Strategies	References
Majority of calculations performed in real life are done using mental methods	Wandt & Brown (1957); Northcote & McIntosh (1999)
Students invent their own strategies	Kamii (1994); Kamii, Lewis & Livingston (1993)
Students use different methods in and out of school	Carraher, Carraher & Schliemann (1985, 1987); Maier (1980)
Skilled mental calculators often work left to right	Hope (1986)
Methods vary from child to child	Hope & Sherrill (1987)
The same child may use different methods to tackle similar problems	Hope & Sherrill (1987); Rathmell (1978)
Mental strategies differ from the written	Askew (1997); Hope & Sherrill (1987)
Teaching of written can stifle mental computation	Carraher & Schliemann (1985); Cooper, Heirdsfield & Irons (1996); Hope (1987); Kamii & Dominick (1998)
Some strategies more efficient than others	Hope & Sherrill (1987)
Many strategies have been identified and coded	Hope and Sherrill (1987); McIntosh, De Nardi & Swan (1994)
Presentation format, visual or oral, stimulates different approaches and performance	McIntosh, Reys & Reys (1997)
Context influences performance and thinking strategies employed	McIntosh, Reys & Reys (1997)

Mental strategies were defined earlier and in essence refer to the thinking and approaches students use to solve mental computation problems. As may be seen in Table 2:1 much of the research in mental computation has focussed on mental strategies. Hope and Sherrill (1987) who studied the characteristics of skilled and unskilled mental calculators suggested that:

Individual difference in mental calculation performance can be argued to reflect differences in choice of calculative strategy, the knowledge of useful numerical equivalents, and the capacity to process numbers (p. 99).

The expression 'choice of calculative strategy' implies that skilled mental calculators have a variety of mental strategies at their disposal. One might ask "where do these strategies come from?". For the most part they are self-generated. Some students therefore doubt the validity of their own methods and tend to abandon them once taught formal paper-and-pencil algorithms by the teacher. Cooper, Herdsfield and Irons (1996) found that before being instructed in paper-and-pencil methods children exhibited spontaneous mental strategies, but after instructions they tended to employ a mental strategy that was similar to the paper and pencil algorithm. Other students choose to use school taught methods in school and their own methods out of the school context. Young children in particular often give up their own methods, which they understand, only to adopt school taught-methods that they find hard to follow.

Hope and Sherrill (1987) examined the characteristics of skilled and unskilled mental calculators and found that those children deemed to be skilled in mental computation used a variety of strategies when tackling a question mentally. They adopted methods that reduced the cognitive load on memory such as working from left to right. This was in contrast to written methods that operate mostly from right to left. Unskilled mental calculators tended to adopt a mental version of the written algorithm that hampered the mental computation process.

Carraher, Carraher and Schliemann (1985) noted that while students may learn a particular approach to calculation at school alternative approaches are often used out of school. Computation choice differs depending on the context in which the calculation is performed. Mental computation strategies are strongly influenced by the written methods that children are taught (Swan, 1991). Written methods, when applied mentally erode children's ability rather than enhance it. Mental computation ability along with the strategies children use varies widely among children. The way computation items are presented affects performance. Some items produce better results when given orally, whereas others presented visually produce higher results. It has been suggested (McIntosh, Reys & Reys, 1997) that visual presentation may encourage children to adopt written strategies, whereas oral presentation makes it more difficult to use a written method and therefore an alternative approach is required.

The context in which a question is given also has an influence on the way a student might tackle the question. In particular, a question given devoid of context is likely to invoke a written method whereas a question given in a context, especially a shopping context, is likely to encourage the use of mental methods (McIntosh, Reys & Reys, 1997).

Plunkett (1979) listed the characteristics of mental algorithms. While the term ‘algorithms’ tends to evoke an idea of routine calculation, Plunkett was really referring to mental computation strategies. In the context of the original article Plunkett compared standard written algorithms with mental algorithms. Merttens and Brown (1997) used a table to compare and contrast the characteristics of standard written algorithms and mental algorithms as presented by Plunkett. Table 2.2 is presented as a convenient summary of Plunkett’s characteristics.

Table 2.2: Summary of Plunkett’s characteristics of written and mental algorithms

Characteristics of mental algorithms	Characteristics of standard written algorithms
fleeting, variable, not <i>designed</i> for recording	written, fixed routines
idiosyncratic, flexible, often iconic – referring to a number line or similar mental model	standardised, symbolic – operations performed directly on numerals
extended, modifiable – adjusted to deal with particular numbers	compressed, summarising
active holistic	efficient, economic (in terms of amount of detail recorded)
largest values often dealt with first	frequently start with units, then deal with tens, etc.
limited, specific – often relate to particular numbers and calculations (thus adding 21 to 70 may involve a different routine from that used to add 29 to 71)	general, exploit place value, work with any numbers
many approximations. And approximate answers appear during the algorithm	do not tend to offer approximations or give a hint of what the answer will be

Note: Taken from Merttens & Brown, 1997, p. 85.

Plunkett also spoke about mental methods being ‘active’ rather than ‘passive’. The suggestion here is that students may follow the standard written algorithm without really thinking about the steps involved, whereas mental computation involves being flexible in thinking so as to reduce the cognitive load. Flexibility was also mentioned in the previous discussions on number sense and estimation and is one of the links between them and metacomputation.

The literature indicates there are two schools of thought when discussing mental strategies. One involves teaching students particular mental strategies, much in the same way written algorithms are taught, while the other approach, based on constructivist principles involves providing students with the opportunity to invent their own strategies. These two approaches will be examined further in the next section as they may impact on computation choice.

Mental computation: Two schools of thought

Two general schools of thought prevail when mental computation is discussed in the literature. The first suggests that mental computation be viewed as a basic skill. In this guise mental computation is considered to be a prerequisite for developing estimation skills and for successfully completing written algorithms. Essentially mental computation is simply viewed as a tool for completing other forms of computation rather than a valid computation method in its own right. Mental computation under this approach is very much teacher driven, with the teaching of mental computation almost assuming algorithmic status. Teachers tend to teach and develop techniques and strategies for mental computation. Those who believe this often teach children ‘multiplication shortcuts’ and focus on drill-type activities. An example of this is the ‘shortcut’ of removing and adding zeros to make a question easier. Oftentimes students fail to comprehend why this ‘shortcut’ works but adopt the strategy because the teacher has taught it. While this strategy may work with whole numbers it can lead to misconceptions when dealing with decimal numbers (Hopkins, Glifford & Pepperell, 1996). The strategy is often applied with little real understanding and students often make mistakes when trying to apply the strategy (McIntosh, De Nardi & Swan, 1994). Under this approach an opportunity to gain insights into the structure of the number system is lost. In describing this approach McIntosh, Nohda, Reys and Reys (1995) stated:

There are at least three instructional approaches currently apparent in elementary classrooms. The first is to view mental computation as a “topic” to be delineated into identifiable strategies that are directly presented to students. This approach is similar to the traditional teaching of paper/pencil computation algorithms (p. 238).

The risk in using such an approach to teaching mental computation is that many of the positive attributes described by Plunkett (1979) would be lost. In particular the flexibility that distinguishes mental approaches from written would be sacrificed.

Early support for mental computation as a basic skill came from the thought that the mind was like a muscle that required discipline or exercise in order to keep in shape and therefore mental computation sessions at school tended to focus on periods of intense drill. While a ten-quick-questions approach to mental computation may still be found in some classrooms there appears to be a transition away from this practice to one that involves the developing of thinking strategies.

McIntosh, Nohda, Reys and Reys (1995) highlighted a second approach to mental computation that matches a constructivist paradigm.

A second instructional approach for mental computation is constructivist. Students are encouraged to generate thinking strategies based on their prior experience and knowledge... Although it is clear that some students can formulate and use a variety of strategies, both elegant and not so elegant, the likelihood of their making use of and valuing such self-generated strategies seems to be closely tied to their notion of what school is about, and in particular what mathematics is about (p. 239).

The second view of mental computation is that it is a valid computation alternative and may be used as a vehicle for developing higher order thinking about number. These higher order thinking and decision-making skills may then play a role in the process that leads to the selection of an appropriate computation method for a particular situation and in a specific context. The higher order thinking skills also come into play when children judge the reasonableness of an answer.

The development of thinking strategies is at the heart of the second approach to mental computation. The term ‘mental strategies’ is used throughout modern curriculum documents (EDWA, 1998) and has become the focus of mental computation sessions. There still appears to be some debate as to how these mental strategies are developed. In broad terms, one method of developing mental strategies involves using a constructivist approach while the second approach involves teaching a specific strategy and then practising that strategy. The constructivist approach involves encouraging students to invent and share their mental methods. This is the method adopted in the *National*

Numeracy Strategy in the United Kingdom. Children are involved in the creation and comparison of various calculation methods. This is in harmony with the *Numeracy Framework*, which in part states,

Through a process of regular explanation and discussion of their own and other people's methods they will begin to acquire a repertoire of mental calculation strategies (Department for Education & Employment as cited in Smith, 1999, p. 10).

Smith (1999) elaborated on this statement as follows:

There are two different principles underlying a commitment to comparing children's methods; either pedagogical or mathematical... A constructivist view of learning emphasises children's own methods as the necessary starting point for teachers. (p. 10).

There are various reasons for allowing children to compare and discuss their methods of calculation. Some methods are better in the sense they might be faster or apply across a range of problem types or may be easily extended to larger numbers. Some methods are simply wrong and some correction needs to take place. Smith (1999) tied in this method of developing mental strategies to the development of a repertoire of strategies from which children may choose.

It is mathematically empowering – and interesting – to use an appropriate method for a problem. Teachers and children not only need to know a range of methods but their strengths and disadvantages (p. 10).

A brief discussion of a constructivist approach to teaching and learning will help clarify the idea of students inventing their own mental strategies. It will also help explain how mental strategies developed in this way may also be linked to higher order thinking.

Constructivism

Current learning theory in mathematics has built upon the developmental theory of Piaget and has progressed to the point where the emphasis is on children constructing their own understanding of mathematics (Malone & Ireland, 1996; Steffe & Kieren, 1994; von Glasserfield, 1987). The term 'constructivism' has been used to describe this approach to teaching and learning. The constructivist views the learner as the instigator and primary director of learning. The learner assimilates and accommodates new learning with prior knowledge and experiences to form a new understanding.

The difference in teaching style and approach may be seen in the way mental computation is taught. A teacher applying constructivist principles would set up a situation whereby students are given the opportunity to generate their own mental strategies and where they would be encouraged to explain them and share them with the class. For example when adding 27 and 26 one student may choose to add 20 and 20 and then the 7 and 6, while another student may relate the question to 25 plus 25 and then compensate for the difference. Contrast this approach to one where the students are shown a particular approach to solving a problem mentally and then given many examples to practise. Self-generated mental strategies harmonise extremely well with constructivist thinking, whereas a transmission approach does not.

If mental computation is thought of simply as a topic, as strategies to be taught to students, similar to the teaching of the traditional written algorithm then many of the attributes of mental computation such as their flexibility as outlined by Plunkett (1979) will be lost. A constructivist approach to mental computation relies on the generation and sharing of thinking strategies among the class and the ability of the teacher to examine and interpret responses given by children. Teachers need to possess enough knowledge of mathematics and mental strategies to make appropriate responses to the children. Many of these responses will be in the form of questions, so teachers also need to become highly skilled in asking questions which provide a springboard for exploring the strategy and will promote higher order thinking. This process places a great deal of responsibility on teachers to be able to think on their feet.

The influence of the constructivist theory of learning may be clearly be seen in the development of modern curriculum where students are encouraged to think more about the calculation rather than simply adopt a procedure they have been taught. Methods of teaching that tend to close down thinking, such as is the case when traditional written algorithms are taught are not favoured under a constructivist approach to teaching and learning.

In the previous mental computation example students were not only encouraged to invent their own methods of calculation but also to share them with their peers. The social context of learning has long been recognised but not encouraged in classrooms where the ideal classroom was depicted as students sitting quietly at their desks working away on problems. Cobb, Yackel and Wood (1992), noted that mathematical activity in the classroom should be interactive rather than passive and involve students making sense of the mathematics they encounter. They stated that,

...we would not characterize teaching as an activity in which we attempt to focus students' attention on things we see in their environment in increasingly explicit ways. Instead, we would view it as an activity in which we guide students' constructive efforts, thereby initiating them into taken-as-shared mathematical ways of knowing. Concomitantly, learning would be viewed as an active, constructive process in which students attempt to resolve problems that arise as they participate in the mathematical practices of the classroom. Such a view emphasizes that the learning-teaching process is interactive in nature and involves the implicit and explicit negotiation of mathematical meanings (p. 10).

As Cobb, Yackel and Wood (1992) pointed out, rarely is the option as simple as choosing to use a constructivist approach or not. They noted that the more explicit teachers became the more mathematics was 'algorithmized' and the less conceptual understanding the children had. Returning to the example of teaching students to calculate mentally, teachers need to make a judgement as to whether students are instructed in various mental computation methods or immersed in them and given a choice as to which method to apply. This same idea needs to be extended when making computation choices, such as whether to use a calculator or rely on a standard written method to complete a calculation. The complex nature of the learner and the environment will have an impact on the choices made.

The teaching of a specific strategy tends to suit those teachers who consider mental computation as a basic skill to be taught rather than as a means of promoting higher order thinking. McIntosh, Nohda, Reys and Reys (1995) suggested that rather than just two approaches to mental computation there is a third default approach which they described as,

Students are taught standard written methods for computing and must extrapolate from such experiences to compute mentally. No explicit instructional attention is given to mental computation. This approach often results in students performing mental computation by applying inefficient standard, written algorithms (p. 239).

It has been shown in this section that there are distinct approaches to teaching mental computation. These approaches affect the way students perform mental computation and their choice to use mental computation.

Thus far, mental computation as a distinct computation choice has been examined along with the role mental computation strategies play in developing higher order thinking. In the next section the role mental calculation plays in estimation and paper-and-pencil and calculator computation will be developed.

Links between mental computation, estimation and written algorithms

Reys (1984) listed five benefits of teaching mental computation and linked mental computation with the development of computational estimation and written algorithms. Five widely accepted reasons for teaching mental computation are:

- it is a prerequisite for successful development of all written algorithms;

- it promotes greater understanding of the structure of numbers and their properties;

- it promotes creative and independent thinking and encourages students to create ingenious ways of handling numbers;

- it contributes to the development of better problem-solving skills; and

- it is a basis for developing computational estimation skills (p. 549).

Reys clearly links mental computation to written computation in the first point and to computational estimation in the fifth point. Points two, three and four show the relationship of mental computation to the broader notion of number sense. As stated earlier, a relationship between mental computation and estimation exists in the sense that mental computation assists in estimation and that students who are good estimators are also good at mental computation. The converse, however, is not necessarily true.

Having discussed the link between mental computation and written computation it is now appropriate to discuss the second of the exact computation choices – paper-and-pencil computation. As with the previous approaches to computation, estimation and mental computation, there are several types of paper-and-pencil computation that will be examined in the next section.

Paper-and-pencil computation

If students choose not to use estimation or mental computation, then students are left with two choices; paper-and-pencil computation or calculator. In this section the paper-and-pencil option will be examined in detail. When discussing paper-and-pencil methods standard written algorithms tend to come to the fore but paper-and-pencil methods can, at times, refer to ad hoc non-standard written methods. For the purpose of this review the terms standard written algorithm and paper-and-pencil methods are considered synonymous. Where ad hoc, self-generated, idiosyncratic or unconventional pencil-and-paper methods are mentioned they will be referred to as self-generated methods.

Defining paper-and-pencil computation

Paper-and-pencil computation may describe several approaches to computation, all of which involve jotting pieces of information on paper, not necessarily with a pencil. Often the term paper-and-pencil computation is associated with the term written-algorithm. The term algorithm is defined as:

a step-by-step process that guarantees the correct solution to a given problem, provided the steps are executed correctly (Barnett, 1998, p. 69).

Several different algorithms exist for completing calculations typically encountered in the primary school. One type of algorithm is typically chosen for each of the four operations and given the status of a 'standard written algorithm'. Who decides on the 'standard' and how the 'standard' is chosen is not always clear. Standard written algorithms may vary from state to state and country-to-country, so obviously the criteria that are used to choose the standard written algorithm will vary.

Algorithms were invented rather than discovered. Algorithms continue to be invented and modified to suit the changing needs of society. Which algorithms take on the status of the 'standard' will depend on the needs of people. Given that algorithms were invented rather than discovered the argument has been put that students should be encouraged to invent their own algorithms.

Morrow (1998) indicated there is still debate about the merits of students inventing their own algorithms. She interviewed various mathematics educators including classroom teachers and was given a mixed response to the issue of standard written algorithms and invented methods. One interviewee commented:

Learning to value student-invented algorithms is a major change. It is difficult to envision the end result – that is, what mathematics will the students have learned and how will I, the teacher, know that the student has learned something valuable? (p. 2).

The balance between standard written algorithms and self-taught methods is still to be struck. References made to written methods in the Western Australian mathematics curriculum under the number sub-strand, ‘calculate’ are reproduced below.

- Add and subtract whole numbers using their own written method or a conventional method, explaining the method ...
- use their own methods or a conventional algorithm to multiply ...
- use their own method or a conventional algorithm to divide ...
- explain why the multiplication/division method used works ...
- compare paper-and-pencil methods for ease, reliability, efficiency ...
(EDWA, 1998, p. 196).

Neither the *Curriculum Framework for Kindergarten to Year 12 Education in Western Australia* nor the *Outcomes and Standards Framework Mathematics Student Outcome Statements* prescribe a standard written algorithm. The focus is on understanding the written method being used, whether it is a self-generated method or a conventional method. The emphasis placed on written methods in general in this curriculum is very much reduced as indicated by the following statement in the *Curriculum Framework*.

They use written approaches as a backup for calculations they cannot store completely ‘in the head’. These may include diagrams, jottings, standard routines (Curriculum Council, 1998, p. 187).

The place of algorithms in a well-rounded mathematics curriculum has also been debated in the literature. The participants in the debate tend to agree that there should be a decreased emphasis on developing proficiency with paper-and-pencil algorithms and that the importance of some algorithms will change (Morrow, 1998, p. 5). Evidence of this may be seen in the removal of the square root algorithm from the curriculum. The following comment drawn from Morrow (1998) provides a reasonable balance.

We should not look at learning a particular algorithm as an all-or-nothing situation. Different levels of mastery of an algorithm may be sufficient for the needs of different students depending on their interests and talents (p. 5).

The issue of whether students should be taught a standard written method or encouraged to develop their own methods is not unlike the debate regarding the teaching of mental computation. There are two broad schools of thought; teach standard written algorithms or allow children to develop their own methods. As in the case of mental computation the issue is related to a constructivist approach. The various approaches will be elaborated on in the next section.

Paper-and-pencil computation: Approaches to teaching

Usiskin (1998) reminded us that “no matter what algorithm teachers think they are teaching, students will process it in different ways” (p. 9). He extended his argument to embrace a constructivist paradigm and argued as follows.

The construction of knowledge internally does not necessarily imply that there should not be significant external guidance. We learn language internally but would not learn any English at all if we did not hear or read it. Some of today’s algorithms (such as the quadratic formula to solve quadratic equations or the way we multiply whole numbers) have been refined over thousands of years by brilliant people in many different cultures. Thus, for the simplest tasks, it is expecting too much of students to invent efficient algorithms. However, it is not only appropriate but advisable to expect students to explore and adapt algorithms (p. 9).

Altering Usiskin’s phrase a little, one might ask, “Is it expecting too much of students to invent efficient algorithms?” Kamii and Dominick (1998) argued from a constructivist perspective and stated categorically that algorithms are harmful, especially when taught to young children. Kamii and Dominick make the distinction between algorithms or the conventional processes associated with standard written methods such as ‘borrowing’ or ‘carrying’ and procedures or child-invented methods.

McClain, Cobb and Bowers (1998) summarised the range of views from “encouraging students to invent their own algorithms with minimal guidance to teaching students to perform traditional algorithms” (p. 141). They suggested an approach that is between the two extremes.

This approach values students’ construction of non-standard algorithms. However, it also emphasizes the essential role of the teacher and of instructional activities in supporting the development of students’ numerical reasoning. In addition, this approach highlights the importance of discussions in which students justify their algorithms. It therefore treats students’ development of increasingly sophisticated algorithms as a means for conceptual learning (p. 141).

Skemp (1987) made a distinction between what he termed ‘instrumental understanding’ and ‘relational understanding’. The word ‘understanding’ was being used to mean different things so Skemp distinguished between instrumental understanding, the application of rules without reason and relational understanding – where a deeper level of understanding was meant. Students would understand why a certain procedure worked. Applying this thinking to the development of written algorithms would mean that a student given an instrumental understanding of written algorithms would simply memorise the steps required to complete the calculation, whereas a student with relational understanding of the algorithm would understand why the algorithm works.

Hope (1986) coined the phrase ‘calculative monomania’ to describe the lack of thinking often associated with the use of standard written algorithms. He described calculative monomania as “the tendency to ignore number relationships useful for calculation and, instead, resort to more cumbersome and inappropriate techniques” (pp. 50–51). He reported several examples where students used cumbersome written methods in preference to simpler mental methods. For example $5000 - 4999$ is simple to calculate mentally but involves a great deal of decomposing and renaming when completed by the standard written method for subtraction. Often students are chastised for making use of a calculator when mental methods would have been more appropriate but it is rare to hear a student being chastised for using a standard written algorithm when a mental method would have been more appropriate.

While examples of students inventing their own algorithms have been reported (Baek, 1998; Kamii & Dominick, 1998; McLain, Cobb & Bowers, 1998), further research will need to be undertaken to explore the issue of students inventing their own algorithms. Even though curriculum documents are supporting a change in the way in which written algorithms are taught anecdotal evidence would suggest that the traditional standard written algorithms are still taught in many classrooms. In the following section what is known about standard written algorithms will be reviewed.

The nature of standard written algorithms

When discussing the literature related to mental computation the work of Plunkett (1979) was reviewed. Table 2.2 outlined the key aspects of mental computation in comparison to written methods were presented. While Plunkett first raised these points over twenty years ago his work is often used in discussions relating to mental and written computation (Merttens & Brown, 1997, p. 85)

Several characteristics outlined by Plunkett have tended to make standard written algorithms popular with teachers. Paper-and-pencil calculations are permanent and therefore may be reviewed and corrected if required. This furnishes the teacher with evidence of work that has been completed and error patterns among students. The standardised nature of written algorithms allows for the algorithm to be taught in a step-by-step fashion. This is convenient in terms of planning and lesson structure. Standard written algorithms are typically ‘one size fits all’ in the sense that one algorithm may be applied over a range of situations.

Standard written algorithms can become *automatic* in the sense that someone with little understanding of how the algorithm works can apply them without thinking about the numbers in the question. It is this lack of thinking that is at the heart of Plunkett’s argument. Students tend to develop ‘cognitive passivity’ when over-exposed to standard written algorithms. Plunkett goes so far as to suggest that the teaching of standard written algorithms militates against the development of computation choice. The learning of rules without reason hinders understanding of how numbers work, which is the basis for mental computation.

Usiskin (1998) built upon the work of Plunkett and added some points in favour of standard written algorithms. He suggested several reasons for using algorithms, while at the same time noting “some of the very properties that make algorithms important – speed, reliability and instructiveness and the mental images they may generate – may create dangers” (p. 15). The points raised by Usiskin will be considered in turn.

They are powerful: The power of an algorithm derives from the breadth of its applicability (p. 10).

This strength may become a weakness if students overuse the algorithm or apply it without thinking. Usiskin referred to this as ‘overzealous application’ of the algorithm which would include using a written algorithm when a mental computation would have

been more efficient. This may also apply to using a calculator if a student made use of a calculator when a mental calculation would have been the more appropriate choice. Should the student choose to use a standard written method and achieve the correct result when a mental method would have been more efficient then it could hardly be said they were wrong, although the chosen method could be said to be unwise. Usiskin suggested that when students overuse the written algorithm they are “playing safe, worried about losing accuracy if they deviate from an algorithm” (p. 15).

He argued that the overuse of mental methods could be equally as dangerous as the overuse of standard written methods because “with mental arithmetic, we have no record of the input, so if there is an error, we cannot tell whether it is in carrying out the algorithm or in mistaken input” (p. 15). When Usiskin mentions ‘the algorithm’ in this case he is referring to the mental method used to perform the calculation.

They are reliable. When an algorithm is done correctly, it yields the correct answer time after time. When there is a possibility of error in carrying out of an algorithm, then the algorithm loses some of its utility (Usiskin, 1998, p. 10).

Kamii and Dominick (1998) referred to many research studies from the seventies and eighties that “have documented the erroneous but consistent ways in which children inadvertently change the algorithms for multidigit computation. The rules children made up showed that their focus was on trying to remember the steps” (p. 130). Their work tends to discount the suggestion that standard written algorithms are reliable. Usiskin lists another point in favour of standard written methods.

They are fast. An algorithm provides a direct route to the answer ...The ease with which the algorithm is learned or recalled is also a factor in the speed with which it can be applied. An algorithm that can be applied fast but is difficult to remember is not necessarily a good algorithm...An algorithm is less useful and less speedy to use if it is easily forgotten and if one has to find it in a book or derive it each time from scratch (p. 11).

It could be argued that mental methods and calculator methods on the whole are quite a deal faster than using paper-and-pencil algorithms. The speed at which a calculator produces results allows the user to repeat the calculation should the answer appear incorrect. Probably the most common reason given in support of the use of standard written algorithms in the primary school is outlined in Usiskin’s next comment.

They furnish a written record. The record of an algorithm is significant for teaching because we often want students to examine the process by which they obtain their answers, to share with one another what they have done, and to refine their procedures. This record also allows us to locate errors in the algorithm more easily. Consequently, an algorithm that operates without a trace, such as often happens when calculators are used, may not be as useful for learning as an algorithm that leaves a trail (pp. 11–12).

The furnishing of a paper trail can be useful in determining where a student might be experiencing trouble but similar information about a student's mental methods may be elicited by asking the student to describe how the answer was obtained. Likewise many modern calculators come with a multi-line display that allows the teacher to scroll back and forth to determine how a student arrived at a solution. The paper trail associated with a standard written algorithm may only indicate that a student does not understand the algorithm and should be using a different method.

They establish a mental image. The written record can help establish a mental image that can be used for obtaining results without pencil-and-paper (Usiskin, 1998, p. 12).

Rather than being a point in favour of using paper-and-pencil algorithms it could be argued that by developing a mental image standard written algorithms do more harm than good. One of the reasons Kamii and Dominick (1998) suggested that algorithms are harmful is because they “encourage children to give up their own thinking” (p. 135). The mental approach to many computations begins from left to right whereas most standard written algorithms work right to left. Once children learn the standard written method they often abandon their own methods in favour of taught methods. Written algorithms were designed to be completed on paper and therefore are not well suited to mental computation.

They are instructive. Some algorithms give insight into the relationship between the answer and the given information. For instance, the algorithm used for adding columns of numbers, in which one records a “carry” digit somewhere is instructive in that it applies the ideas of place value (Usiskin, 1998, p. 12).

Kamii and Dominick (1998) suggested that rather than improve understanding of place value, standard written algorithms “unteach place value, thereby preventing children from developing number sense” (p. 135). Ten years earlier Jones (1988) made similar comments.

The practising of traditional methods does not develop an awareness of the structure and properties of number. Contrary to this it will allow those with little understanding of place value to obtain right answers (p. 43).

Despite the time devoted to the teaching standard written algorithms there is evidence to suggest that children prefer not to use them. Jones as reported in Plunkett (1979, p. 3) found that when 11 year-olds were asked to perform four calculations, one of each operation, and given the opportunity to choose either written or mental methods, over half of the calculations were successfully completed using non-standard methods. While most of the calculations were relatively simple in nature, they included all four operations and the result does suggest that despite the heavy emphasis given to standard written methods many children choose not to use them.

Over twenty-five years ago Ginsburg (1977) made the following observation about the emphasis placed on standard written methods.

A good deal of elementary school education is devoted to addition, subtraction, multiplication and division with whole numbers. Children first add and subtract with small numbers and then they repeat the operations with larger ones and then larger still. These algorithms developed and codified over the course of centuries are guaranteed to achieve the correct result; applied properly they always work. So formal education tries to make available to children some powerful procedures. But what use do children make of their cultural legacy? We shall see that they often ignore the standard procedures and instead rely on methods of their own invention (pp. 90–91).

There are many documented examples of children inventing their own (Askew, 1998; Thompson, 1997). Thompson (1997) referred to examples of students' self-generated written methods and commented:

The advantages of these methods – either in idiosyncratic or formalized form – include the fact that the fundamental place value meaning of the numbers is retained, and this means that the children are manipulating quantities rather than symbols. The three methods also produce successive approximations to the answer and therefore are more likely to provide useful clues as to the accuracy of the answer. Their main strength, however, lies in the fact they model, more closely than the standard algorithm for addition, the 'natural' mental calculation heuristics of many children. It is also of interest to note that none of the methods involves 'carrying' (p. 107).

A National Statement on Mathematics for Australian Schools (AEC, 1991) acknowledged the fact that other computation methods may suffer if too much emphasis is placed on formal written methods.

"The development of flexible computational skills can be inhibited by emphasising the practice of standard paper-and-pencil methods to the exclusion of other methods. It is far more realistic to use a combination of mental and informal methods most of the time, with paper-and-pencil recording seen as providing memory support" (AEC, 1991, p.109).

It is possible that the overemphasis on written methods during the last century has interfered with the ability of children to develop higher order thinking skills. There is evidence to suggest that less skilled mental calculators rely more on using a version of the written method in their head rather than a more appropriate mental strategy. The question of what level of proficiency with paper-and-pencil methods is appropriate in the twenty-first century is still under debate. McIntosh (1990) in reviewing the three forms of exact calculation came to the following conclusion regarding written computation.

Written forms of computation should continue to have a place in the classroom provided they meet one at least of the following criteria:

- they help to illuminate the ways numbers behave;
- they provide a source of intrinsic interest (to the students!) in their own right;
- they are being developed as informal methods to extend and support the use of mental methods; or
- their development is being used as a problem solving exercise (p. 37).

Askew (1999) described the stages of calculation adopted by *The National Framework for Teaching Mathematics* in the United Kingdom. Essentially students pass through five stages before being introduced to standard written algorithms. The stages are given below.

Stage 1: Work things out mentally and, if necessary use jottings.

Stage 2: Work with a repertoire of mental strategies.

Stage 3. Have a secure knowledge of mental strategies, instant recall of number facts and good understanding of place value.

Stage 4: Move from informal jottings to standard notation.

Stage 5: Refine and make more efficient their mental and written methods.

Stage 6: Be taught standard written methods (p. 37).

Guidance is provided to explain each of these stages, but the emphasis is always on using mental methods first. Jottings are used for two purposes; as a support for short term memory and keep track of steps in a calculation. Paper and pencils methods are used to add some structure for doing calculations. Calculations are provided in horizontal, rather than vertical format, as children assume a question provided in vertical format requires the use of a standard written approach. By providing questions in horizontal format children are encouraged to choose which form of calculation,

mental or written, they will use. In line with this observation items in this research were given orally and presented in a horizontal, rather than a vertical format.

To summarise, written calculation may take two forms:

- standard written algorithms; or
- idiosyncratic, self-generated methods.

Standard written algorithms were the mainstay of most middle to upper primary mathematics programs until their role was brought into question in the late eighties and nineties. Questions arose as to whether

- students understood the standard written algorithms they were being taught;
- the standard written methods were impacting on mental methods;
- students chose to use standard written methods outside of the school setting; and
- it was logical to spend time completing tedious written sums in an age of calculators.

As a result there has been a move away from the traditional teaching of algorithms towards self-generated algorithms, an increased use of calculators and increased emphasis on mental computation. Much of the rethinking of computation was prompted by the arrival of electronic calculators in the primary classroom. The arrival of calculators added another computation choice and raised several issues. The role of the calculator as a computation choice as well as the issues surrounding calculator use is the focus of the next section.

Calculators

Calculators have been available in primary classrooms for over twenty years. Their arrival was met with scepticism in some quarters with concern being raised that students' ability with arithmetic; particularly standard written algorithms would decline. As a result much research focussed on possible negative effects associated with the use of calculators. Many of the research findings (Hembree & Dessart, 1986, 1992) associated with calculator use; therefore, tend to be couched in terms such as 'no detrimental effects' as though the researchers were looking for the negative rather than the positive aspects of calculator use. Doubts over the validity of calculators as a true computation alternative still linger in the twenty-first century and hamper efforts to allow students to have free choice as to whether to use calculators or not.

This section will first examine the nature of computation performed with the aid of a calculator and then look at the general findings of calculator studies. A closer look at studies of student achievement and two large-scale curriculum projects will follow. This section will conclude with a brief look at how calculators are used in primary classrooms and recommendations for their use.

The nature of calculator assisted computation

Much of the argument against calculator use is predicated on the assumption that you do not have to think when given a calculator to use. As Rousham and Rowland (1997) noted,

Electronic reckoning differs from the other means of calculation in a number of respects. For example, complex calculations such as 276×467 are, in principle, no more 'difficult' to execute with a calculator than trivial ones such as 2×3 (p. 61).

It is the fear that students will make use of calculators to complete trivial calculations that has sparked a number of myths surrounding the use of calculators. Swan and Sparrow (1998) as well as others (Wheatley, 1994) have documented many of the myths surrounding calculator use, most of which revolve around the issue of children losing their ability to think.

One of the myths surrounding calculator use is that students will come to depend on calculators. Use of a calculator is sometimes referred to as a crutch to support students with poor number skills or because of a perceived reduction in ability with arithmetic. The issue of dependence, however, is rarely raised when discussing the use of standard written algorithms. Coburn (1989) summed the issue up extremely well when he stated:

Dependence on a device like a calculator is inevitable to some degree. We become by nature dependent on things we use regularly; this in and of itself is not bad. The fact that many children are overly dependent on written computation is often overlooked. (A child who multiplies 300×122 using the traditional paper-and-pencil algorithm is dependent on written computation. The child who receives good instruction should decide to do this type of computation mentally, or at least take a written shortcut to the conventional algorithm.) The term crutch implies a dependency without understanding. We need to examine this issue carefully because it is a common belief that if children use calculators, they will not understand what they are doing. It is as if understanding always enters the brain on a pathway from a pencil through the fingers (p. 45).

When this argument is examined in more detail, what the term ‘thinking’ in this context means is in reality a reduction in ability to perform calculations by other means such as paper-and-pencil. As was discussed in the previous section on paper-and-pencil calculation, students using standard written algorithms do not necessarily think when applying the algorithm but simply follow a procedure without understanding how or why it works.

Ruthven (1995b) commented on the issue of thinking and calculator use.

Adoption of a calculator for computational purposes continues to call for mathematical thinking on the part of the user; albeit not exactly the same thinking as that required for alternative mental or written procedures (p. 240).

A great deal of thinking must take place to enter data into the calculator and determine the results. For example, many students experience difficulty interpreting the display on a calculator when it shows a decimal point. Ruthven (1995b) described some of the thinking that takes place when using a calculator.

...calculator use is not wholly routine. The user has to formulate the computation for input to the machine, and interpret the output. Moreover, this may involve repeated computation during which the user makes important tactical decisions in order to arrive at an acceptable answer (p. 241).

Shumway (1994) suggested that calculators provide students with the opportunity to explore various computation plans and paths to solution. The speed at which calculators perform a calculation allow for various approaches to solving a problem to be tried and evaluated.

Calculators can be quite useful in deciding how to solve a problem. In the act of devising a problem-solving plan with a calculator good problem solvers will often perform a variety of exploratory moves, trying certain computations in the process of getting to know the problem... These exploratory moves were greatly facilitated by calculators. Such exploratory methods were not observed as frequently when students did not have a how-to-proceed scheme that included calculators. When thinking of doing the computations 'by hand' many students would instead do nothing (pp. 116-117).

Rousham and Rowland (1997) go one step further by suggesting that a calculator can act as a “cognitive *reorganiser* which allows (or even obliges) the user to experience, and thus conceptualise number in a different way” (p. 68). They provided two examples of how a calculator may act as a ‘cognitive reorganiser’, but it is the second way that is of most interest. They believed the use of a calculator “may free up cognitive ‘space’ which can thus be devoted to higher-order tasks – such as monitoring how the problem is proceeding and which operation needs to be carried out next” (p. 69). This statement indicates that calculator use may assist in the development of metacomputation.

Rousham (1995) argued that many calculator activities encourage thinking because they set up what he called a ‘feedback loop’. He described the thinking associated with a feedback loop this way. It rests on the fact that the machine gives information in response to ‘questions’, provided you know how to question it (p. 97). He went on to describe the thinking behind asking questions, responding to what was displayed and then asking more questions. While this might sound like the process involved when using trial and error methods, which in itself involves a great deal of thinking, what Rousham described was clearly distinct from, and involved more than, trial and error.

Bobis (1991) linked number sense to calculator use indicating that:

A person with well developed number sense should be alert to the reasonableness of the displayed answers by monitoring the computation even before entering the numbers into the calculator (p. 42).

Figure 2.7 helps to illustrate the role played by number sense throughout a calculation. Links between estimation and number sense are also made in Figure 2.7.

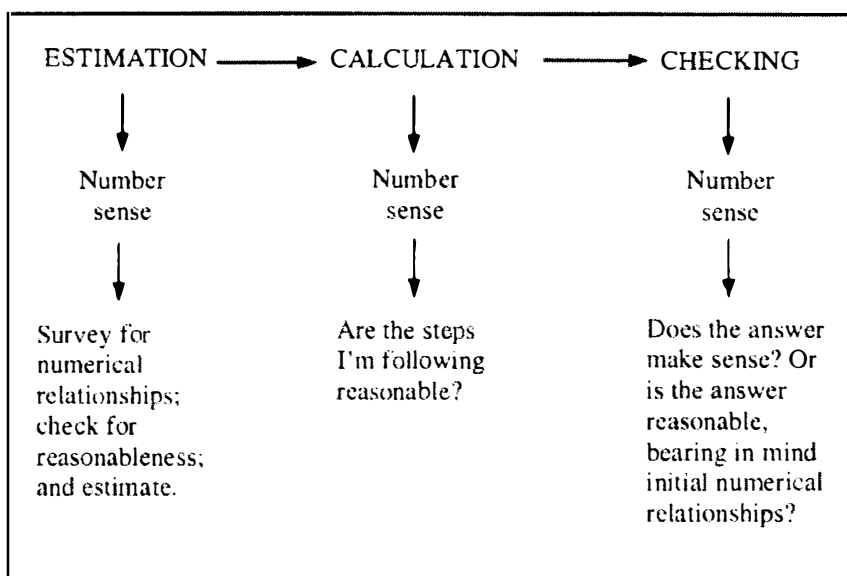


Figure 2.7: The role played by number sense during the three stages of calculation when using a calculator (Bobis, 1991, p. 42).

Suydam (1982) when reviewing the research on calculators up to that time noted, “most studies in which children were taught to estimate with the calculator have reported no significant differences”(p. 8). She noted a general reluctance on the part of students to estimate, whether it is with or without calculators. She did note, however, “students are readily willing to accept unreasonable answers from calculators” (p. 8). Given this reluctance to estimate the need to assist children to develop number sense and become metacomputative thinkers becomes extremely important.

General findings of calculator research

A body of literature on calculators has been built up over the last twenty years (Hembree & Dessart, 1986; 1992). Add to this the literature on calculating devices and computation in general, and one might gain the impression that there are few areas of exploration left in this area. This literature review, however, indicates that while there are some established research findings, there are other areas that require further research. Reys and Nohda (1994) acknowledged that “on no single issue has debate been more heated and emotional than on the role of calculators in school mathematics”(p. 6).

Calculator studies tend to fall into one or more of the following categories:

- the effect using calculators has on student achievement in traditional number work such as written algorithms and mental computation;
- the impact of calculators on the mathematics curriculum;
- the use of calculators as a teaching/learning aid;
- attitudinal changes that come about as a result of the introduction of calculators into the classroom.

Wheatley (1994) found evidence to suggest that “when students are engaged in problem solving with calculators, they become more persistent” (p. 116). Hembree and Dessart (1986, 1992) also found that children were more confident and persistent when it came to solving problems.

While a substantial amount of research has been carried out into calculator use, much research in this area in recent years has tended to focus on the use of graphics calculators and computers in the mathematics classroom. Williams (1987) believed that research into the use of calculators slowed when computers became more accessible in the classroom. “The initial interest in calculators waned when computers burst on the educational scene” (p. 9).

The central issue is the effect of calculator availability on paper-and-pencil computation proficiency. A great deal of value is placed on computation proficiency and calculators are often viewed as a substitute for paper-and-pencil algorithms. The view that basic skills should be mastered before calculators are introduced is based on the idea that calculators will become a substitute for mental computation. The effect of calculators on student proficiency with traditional computation will now be considered.

Student achievement

Much of the research involving primary school children using calculators is somewhat dated (Leechford & Rice, 1982; Suydam, 1982). Many of these studies were based on the premise that using a calculator may have detrimental effects on other forms of computation. The concept behind most studies carried out in the United States in particular was to evaluate the effects on achievement and attitude that resulted from the use of calculators. In the eighties much of this work was summarised by Hembree and Dessart.

Hembree and Dessart (1986) carried out a meta analysis of 79 studies and analysed the results. The results indicated that mathematical achievement was at least as high, if not higher, for those children using calculators. No detrimental effects were noted for the high and low ability groups. Hembree and Dessart updated their work in 1992, locating nine further studies to add to their original 79 studies. This gives some indication of the reduction in the amount of research carried out in this area in the late eighties and early nineties. The results from these further studies supported the previous findings. Hembree and Dessart (1992) reached the following conclusions.

The preponderance of research supports the fact that calculator use for instruction and testing enhances learning and the performance of arithmetical concepts and skills, problem solving and attitudes (p. 31).

A further advantage noted by Hembree and Dessart (1986, 1992) was that students using calculators fared better in tests involving problem solving and computation. These oft-quoted findings need to be examined carefully. Ruthven (1995b, p. 236) suggested there were numerous internal and external validity problems associated with many of the studies and they suffered from serious design flaws, so much so that the conclusions to be drawn from them would be rather dubious. Further, Goldin (1992) questioned the whole nature of meta analyses on the grounds that they are theoretically naïve.

Dessart, DeRidder and Ellington (1999) described the results of a meta-analysis of calculator results carried out by Smith (1997). This study extended the results from the previous Hembree and Dessart studies. Smith's study analysed twenty-four studies conducted from 1984 to 1995. His focus was on student achievement and attitude. In a similar way to Hembree and Dessart, Smith looked at the test results of students who used calculators and those who did not. The result is reported below.

Smith's study showed that the calculator had a positive effect on increasing conceptual knowledge. This effect was evident through all grades and statistically significant for students in third grade, seventh through tenth grades, and twelfth grade. Smith also found that calculator usage had a positive effect on students in both problem solving and computation. Smith concluded that the calculator improved mathematical computation and did not hinder the development of paper-and-pencil skills (p. 6).

Much of the research on calculator use and achievement is often couched in terms of the effect on paper-and-pencil algorithms. It is almost as if the researchers were investigating calculators in an effort to find a defence for using them in the classroom. Comments such as the one above certainly indicate that particularly in the United States paper-and-pencil skills are still highly valued.

Jones and Tanner (1997) explored the effects of calculator use on the basic arithmetic skills of Year 7 (12-year-old) students and found “the extent to which the use of calculators was restricted or discouraged had no significant impact on the students’ basic skills” (p. 33). They noted in the research that after speaking to the heads of department in the various schools studied, mental arithmetic was not a regular part of the program and declined sharply in the early years of high school. Jones and Tanner noted that in most studies that involved unrestricted calculator use that there had been an attempt to develop children’s mental computation and number sense. In their study they noted:

There was no indication that this was the case here. There was no significant difference in the frequency of practice by the mode of calculator use. This could mean that pupils were not being taught to identify when the use of a calculator was appropriate and when another strategy would be appropriate (1997, p. 34).

Jones and Tanner expressed the view that calculators are best used in concert with other strategies; that these strategies are best developed through discussion and this in turn will lead to an improvement in numeracy. The focus on discussion is a theme that runs through all forms of calculation from mental, to written methods to calculators. What appears to matter is not so much having access to calculators but rather the creative ways in which they are used and the resulting discussion that occurs.

Very little research has focussed on the positive outcomes that may come from using a calculator. Studies emanating from the United States have tended to focus on traditional curricula and approaches. It is clear, however, that both the curriculum and the way teachers approach the teaching of mathematics will need to change as a result of the availability of calculators. Two curriculum projects; one in the United Kingdom and the other in Australia were established in the late eighties and early nineties to examine the effect of a curriculum that made full use the calculator. These two projects are reviewed in the next section.

Curriculum projects involving calculator use

The Primary Initiatives in Mathematics Education (PrIME) project was launched in the United Kingdom in 1986 (Shuard, Walsh, Goodwin, & Worcester, 1991). A major part of that project was the development of a Calculator-Aware Number (CAN) curriculum. The project was directed by Hillary Shuard and involved six and seven year-old children being given access to calculators. The key elements of the project were that calculators were always available for children to use and that little or no emphasis was placed on the teaching of traditional written algorithms. Calculators were used in problem solving and investigative work and not simply in contrived circumstances. Emphasis was placed on the development of mental methods for calculating.

A brief summary of the findings is presented below.

- Children did not always make use of the calculators and would often use their own mental methods;
- The children encountered aspects of mathematics such as decimals and fractions much earlier and gained competence with them;
- The children talked more about the mathematics they were doing;
- Teachers became more open and less traditional in the way they taught mathematics; and
- Attitudes toward mathematics were more positive (Shuard, Walsh, Goodwin, & Worcester, 1991).

While not all of these changes may be directly attributed to the introduction of calculators the CAN project does provide insights into the use of calculators alongside other forms of calculation in a classroom setting where calculators were freely available. There is no evidence to suggest the children became dependant on the calculator.

The 'Calculators in Primary Mathematics' project (Groves & Cheeseman, 1995; Groves & Stacey, 1994, 1998,) sometimes referred to as 'The Victoria College Calculator Project' in the literature, was a long-term investigation into the effects of the use of calculators on the teaching and learning of primary school mathematics. The project ran from 1990–1993 and provided a wealth of data on the effect of calculator use in a primary setting.

Groves and Cheeseman (1995) summarised the findings of the Calculators in Primary Mathematics project and made the observation that children were dealing with larger numbers than would normally be expected at their age and also they were dealing with concepts such as negative numbers and decimals much earlier than would be expected. After an extensive program of testing and interviews the following conclusions were reached.

There was no evidence that children became reliant on calculators at the expense of their ability to use other forms of computation...These children also performed better on a wide range of items involving place value for large numbers, negative numbers and, more particularly, decimals. They also made more appropriate choices of calculating device and were better able to interpret their answers when using a calculator, particularly where decimal answers were involved. No detrimental effects were observed in either the interviews or written tests (p. 3).

Of particular importance to this research is the comment that the children made more appropriate choices of calculating device. This may, in part, be due to a change in the way the teachers approached the teaching of mathematics. The teachers in the project reported becoming more open in their teaching style and allowing more time for sharing and discussion. The improvement in ability to make more appropriate choices of calculating device may also be due to the children becoming better able to handle calculators – opening up a third choice when contemplating an exact calculation.

Ruthven, Rousham and Chaplin (1997) examined the long-term effects of the CAN (Calculator Aware Number) project by revisiting the cohort of children from 1989/90 who were in their final year of school in 1995/1996. The influence of the CAN project was high in the earlier years but declined as the children progressed through the school. In reaching their conclusions they noted the difficulties with naturalistic studies that take place over the long term which tend to blunt the sharpness of results. However, they were able to state with a high level of confidence:

There was no substantial long-term influence – for better or worse – on pupils' mathematical attainment, on their achievement of number concepts, or on their attitudes to number work and calculation, as a result of their following a calculator-aware number curriculum (p. 278).

In making this statement the researchers did make two minor qualifications. The first was an apparent amplification of individual differences in mathematical attainment. This occurred at both extremes. The second qualification is most interesting given the previous discussion on mental calculation.

The calculator-aware approach does seem to have resulted in more pupils seeing mental calculation as helping them to learn about numbers – perhaps because of its greater emphasis on, and broader conception of mental calculation (p. 278).

This finding seems more related to the teaching approach rather than the use of calculators. The teachers in the program were aware of the need to encourage students to think about number rather than use the calculator because it was freely available. Jones and Tanner (1998) concurred:

What does emerge from the research is that the role of the teacher is crucial. Where teachers had training and support in ways to use calculators through their involvement in research projects they placed greater emphasis on the development of students' own strategies and mental methods through the encouragement of classroom discussion (p. 290).

The role of discussion appears pivotal in all forms of work involving calculation. This is consistent with the development of metacomputation, where students are encouraged to think about the calculation. Children often complete a calculation 'without thinking' but by asking them to explain or verbalise their approach to the calculation or why they made a particular choice their awareness is raised.

Rowland (1992) played the devil's advocate in reviewing research based on curriculum projects. In discussing the CAN project carried out in the United Kingdom he made the following comment.

All of these projects, as far as I am aware, were set up and pursued more or less exclusively as curriculum development projects as opposed to research projects. They do tend to arise out of convictions, rather than questions, about how the quality of children's learning can be enhanced. From the point of view of evidence this has three shortcomings.

1. It makes dispassionate and detached reporting of outcomes improbable;
2. If there is any research associated with such projects it tends to be an underfunded afterthought; and
3. Written accounts are primarily aimed at others who might follow down the same road, and so 'research' is essentially formative evaluation or case study (p. 28).

While anecdotal evidence and reports from projects such as the CAN project may not meet all the criteria of rigorous research there is still much we can learn from observations made of children in this program. Duffin (1994), an evaluator of the project made the following observation.

One of the first things the project demonstrated was that these children did not, as the older calculator users confessed to, abandon thinking to use the calculator blindly. Indeed the children demonstrated that mental methods of calculating were being enhanced in the project, perhaps because teachers' erstwhile concentration on written down calculations had been relaxed and this had freed children to calculate in their heads (p. 26).

Duffin also found that the calculator did not stifle thinking unless it was introduced without assisting children to make appropriate computation choices. It appears that while opening up a third computation choice, the calculator may also have provided the catalyst for thinking about the catalyst for thinking about the most appropriate method for calculating.

The current situation

One might think that with such a wealth of research the issue of whether calculators should be used in primary classroom was no longer open to debate, but Lott (1999) made the following comment, drawn in part from the *Third International Mathematics and Science Study* (Mathematics Achievement in the Primary School Years: IEA's Third International Mathematics and Science Study).

For the 8th grade assessment, the majority (>50%) of the students from three of the five nations with top scores (Belgium, Korea, and Japan) never or rarely (once or twice a month) used calculators in mathematics classes. In contrast the majority of students (>65%) from 10 of the 11 nations, including the United States, with scores below the international mean, used calculators almost every day or several times a week in mathematics classes (Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith 1996). While such data don't prove that calculator use is damaging to the development of mathematical skills, it would be folly to ignore this (p. 9).

It should be noted that the substance of this text was included in the draft version of the *Mathematics Framework for Californian Public Schools K-12*. This indicates that even in the late nineties there was still some scepticism about the role of calculators in

mathematics classes in general. Surveys of primary teachers would also indicate some reluctance to use calculators in the classroom. The TIMMS report (Mullis et al., 1997) mentioned earlier surveyed teachers and students to find out how often calculators were used in class and how they were used. The data for Year 4 Australian students are presented in Table 2.3.

Table 2.3: Frequency of calculator use as reported by students in fourth grade

When Used:	Never	Some Lessons	Most Lessons
Frequency:	25%	67%	8%

Note: From Mullis et al. (1997, p. 178).

Teachers were asked to respond to a similar question about frequency of calculator use in their classrooms but were asked to differentiate usage on a four-point scale. The data are presented in Table 2.4.

Table 2.4: Frequency of calculator use as reported by teachers of fourth grade

When Used	Never or Hardly Ever	Once or Twice a month	Once or twice a week	Almost every day
Frequency	11%	33%	43%	13%

Note: From Mullis et al. (1997, p. 176).

The data indicate that calculators are not used as much as one might expect with at least a quarter of year four students and up to 44% of students rarely using a calculator in class. Sparrow and Swan (1997) sent surveys to 787 primary schools across Western Australia and collected data from Years 1, 3, 5 and 7 teachers about calculator usage. Almost three-quarters of teachers agreed with the recommendation contained in *A National Statement on the use of Calculators for Mathematics in Australian Schools* (1987) that “ALL students use calculators at ALL year levels (K-12)” (p. 1). While the response to calculator use was positive, many teachers took the opportunity to qualify the use of calculators indicating beliefs that the introduction of calculators should be delayed until a certain age or until students had mastered basic number facts. Concern was also raised that students might become reliant on calculators if allowed free access to them.

At times statistics may mask the true picture of what is occurring in classrooms. While a teacher may report using a calculator most of the time in class, the question of ‘how the calculator is being used’ needs to be asked. A calculator might be used on a daily basis in some classrooms, but only to check answers to written work. This usage differs from the use of calculators to generate patterns for discussion. As part of the TIMMS report teachers were asked not only how often calculators were used but also *how* they were used. Refer to Table 2.5.

Table 2.5: How calculators were used in fourth grade

How Used:	Never or Hardly Ever Use Calculators	Checking Answers	Test and Exams	Routine Computations	Solving Complex Problems	Exploring Number Concepts
Frequency:	11%	45%	2%	29%	35%	33%

Note: From Mullis et al. (1997, p. 177).

Teachers were free to nominate more than one category. Some fairly distinct patterns may be seen when examining the data. The data indicate that some teachers are still reluctant to use calculators in test and exam situations and for many others checking answers is the most common form of their calculator use. It is pleasing, however to see that calculators are being used to assist children to explore number concepts. These data were consistent with the findings of Sparrow and Swan (1997) who also found checking of answers to be a common calculator activity in primary school.

White (1998) surveyed teachers in New South Wales regarding their beliefs about calculator use. The teachers were teaching Year 5 or Year 6 and were asked to respond to a scenario and a set of statements about calculators relating to the scenario. Two beliefs are highlighted below.

Responses to the belief that using calculators in class would promote laziness and dependence saw 71% of teachers regarding this as unlikely, 14% undecided and only 15% regarding it as likely (p. 688).

The ‘typical teacher’ who responded to the survey was female between the ages of 36 and 45 with 11 to 15 years of teaching experience. It is a little surprising to find that 15% of these teachers felt that using calculators would promote laziness. A larger percentage felt that using a calculator would result in students just accepting answers and not thinking as indicated by the following finding.

27% of teachers felt it likely that using calculators in class would result in students just accepting answers and not thinking with 10% undecided, and 63% rated it as unlikely (p. 688).

With slightly over a quarter of teachers indicating concerns about blind acceptance of answers the argument to use calculators in an environment that fosters number sense as part of metacomputation becomes stronger. It should also be noted that a similar blind acceptance of the results of a paper-and-pencil calculation might also be found among students.

Despite all the foregoing there is still reservation about the use of calculators. In the United Kingdom where much of the pioneering work on calculator use was carried out there has been a swing away from the use of calculators, especially with young children. This in part seems to be a reaction to poor results in international numeracy comparisons such as the TIMMS report. The Association of Teachers of Mathematics calculator policy (Gammon, 1998) made the following observations

There is a danger that indiscriminate calculator use may deny students the opportunity of learning certain number facts and practising certain arithmetic skills. Therefore it is important that schools have policies, not only for use of calculators, but also for development and consolidation of number concepts and skills, paying attention to efficient and appropriate strategies, including written and mental methods (p. 12).

Reference in the policy is made to the use of appropriate mental checking strategies, which raises the thought of metacomputation and estimation once more as a means of monitoring a calculation. More specifically at primary level the policy made the following suggestion:

Calculators should be used in ways that support, rather than replace, 'in the head' or 'on paper' number work. Children need to be taught to use a calculator effectively and recognise the need to check answers through effective use of estimation and approximation (p. 13).

Houssart (2000) noted the pressure to reduce calculator use was linked to poor performance in international testing. The paradox seems to be that calculators were blamed for poor performance on these tests when in reality they were hardly being used and then only for low level activities such as checking answers. Houssart (2000) cited the *School Curriculum and Assessment Authority Report* from 1997 that indicated "pupils were only allowed to use calculators for certain lessons" (p. 15). This confirmed the findings of Warren and Ling (1995), who after surveying schools in Hertfordshire in the United Kingdom found that "the impact of calculators in the primary school setting

had not been as great as was predicted by mathematics educators in the 1980s” (p. 23). They also noted that teachers still had concerns and reservations about using calculators, particularly in the early years. Views ranged from concerns about calculators inhibiting children’s ability with paper-and-pencil and mental computation, to children becoming lazy.

Other researchers have made similar comments about the lack of calculator use (Jones & Tanner, 1997; Rousham & Rowland, 1997). Jones and Tanner (1997) reported on calculator use with older pupils in Year 7 and Year 8 and yet still found the use of calculators to be restricted. It appears as though calculators that promised so much early in the decade had been discarded because of concerns about their effect on ‘basic skills’ and a lack of clear direction on how best to use them.

Ruthven (1999) in a clever play on words suggested that schools in the United Kingdom had adopted a ‘calculator beware’ rather than a ‘calculator aware’ approach (p. 196). This in part was due to the direction coming from government, policy makers and pressure from the media. Ruthven (1999) explained that

The effect of such thinking, which assumes an antagonistic relationship between calculator use and mental calculation, has been to reinforce the ‘calculator beware’ approach to number found in many schools (p. 196).

Houssart (2000) interviewed teachers to ascertain their attitudes toward the use of calculators. She found one teacher did not allow calculator use and another expressed reservations about using calculators, one was positive about using calculators, but for the most part “others were apparently low users by default” (p. 17). She found that the lack of use of calculators was mainly due to a “lack of awareness of the teaching and learning potential of calculators” (p. 17).

Throughout history better, faster and more accurate forms of computation have been pursued. These include the development of the counting board, the abacus, various algorithms, Napier’s bones, logarithms and the electronic calculator. As a more efficient method was found the older less efficient methods gave way, albeit not without some resistance. Similarly today, the invention of the electronic calculator has the power to transform the performance of calculation in the school setting. Computers and calculators have already had an impact of the commercial world but there still seems to be resistance in the more conservative primary school setting. The invention of such a powerful calculating device cannot be ignored because it has offered students another

choice of computation method. Prior to this the choice was fairly simple; calculations that did not tax short term working memory were expected to be completed ‘in the head’, and more complex calculations on paper. Proficiency in mental, written and calculator methods of calculation is now required but another skill is also needed; the ability to choose the most appropriate form of calculation according to the context in which the calculation was to be performed.

The last word on the use of calculators is left to Shuard (1992) who stated:

History shows that newer, simpler, more powerful technology always drives out older, more cumbersome and less powerful technology. Among a great range of possible examples, ball point pens have replaced steel pen-nibs. Electric lights have replaced candles ... The telephone and wordprocessor are available in the school office. In the adult world the new technology of using a calculator has replaced the old technology of using pencil-and-paper (p. 39).

The question of ‘how do students make an initial computation choice?’ is one that is yet to be answered but there is much conjecture in the literature about what may influence computation choice. These influences are discussed in the following section.

How do Students make Computation Choices?

Throughout this chapter reference has been made to the NCTM (1989) model to describe computation choice. The various computation options outlined in the model have been discussed. In addition the broad notion of metacomputation was also raised to describe the thinking associated with making a computation choice and monitoring the calculation. Little, however, has been said about how students make computation choices.

The question of what processes students use to decide whether to use a calculator, estimate, compute mentally or use paper-and-pencil is one that is still to be answered. The NCTM model helps to trace the path a student might take when calculating but it does not really indicate how children make these choices but simply indicates what choices may be made.

Curriculum documents highlighted the need for students to be able to choose from a repertoire of computational tools (Curriculum Council, 1998; EDWA, 1998; NCTM 2000), but little direction is given as to how children make the choice as to which form of computation to use in any given situation. Suggestions as to how choices should be made or what constitutes an appropriate choice have been made.

In setting the standard for computation in Australia the *National Statement on Mathematics* (AEC, 1991) included the following comments.

All school leavers should feel confident in their capacity to deal with the computational situations which they meet daily, and number work should reflect the balance of number techniques in regular adult use...Students should develop the ability to judge the level of accuracy needed, learn to estimate and approximate, and use mental, calculator and paper-and-pencil strategies effectively and appropriately in different situations...This requires that they:

- decide what operations to perform (formulate the calculation);
 - select a means of carrying out the operation (choose a method of calculation);
 - perform the operation (carry out the calculation);
 - make sense of the answer (interpret the results of the calculation)
- (p. 108).

Jones and Tanner (1998) elaborated on the Mathematical Association (1992) suggestion that numerate people have “the ability to solve simple everyday problems involving number, by using effectively the knowledge and skills they possess by stating:

This effective use should include being able to choose and devise appropriate strategies for calculations. Numeracy here requires both mathematical knowledge and skills and, in addition, an awareness of this knowledge base so that effective choices can be made. The choice of an effective strategy for a problem is dependent not only on the knowledge that has been learned but also on one’s awareness of that knowledge and the realisation that its use would be appropriate. To devise a strategy requires confidence, an at-homeness maybe, and a view of mathematics as a subject in which students can create their own methods (p. 287).

When considering the three computation choices, paper-and-pencil, calculator and mental, available for computing the exact answer to a problem McIntosh, Reys and Reys (1997) made the following comment:

Although we are often unaware of it our most frequent choice, or instinct, is the third, to calculate mentally. Young children use this method naturally, even before written techniques become a viable option (p. 326).

This statement seems to indicate that students have a ‘natural instinct’ that inclines them toward the use of mental computation when given a choice of computation method. Reys and Reys (1998) believed, “In general, if it is possible to solve the problem mentally, then mental computation would be the natural tool of choice” (p. 238).

Rousham and Rowland (1997) cited the work of Fitzgerald (1985) when they commented:

Research such as that of Fitzgerald (1985) shows that as adults, we tend to look at a calculation and do it in our heads if we can: if we cannot, perhaps because the numbers are too big (or too small) or it would take too long, then we use a calculator. What we very seldom do nowadays is to employ the written algorithms or paper-and-pencil methods that were taught in school (p. 73).

There have been studies (Carraher, Carraher & Schlieman, 1985; Price, 1997; Reys, Reys and Hope, 1993; Swan & Bana, 1999, 1998) that have focussed on the computation choices made by students. The main findings from these studies are outlined below.

Carraher, Carraher and Schlieman (1985) studied the computation choices made by children in different settings; the school and the market place. They found children’s choice was influenced by the setting in which the calculation took place. Children tended to use school taught methods at school but abandoned these for self-taught methods in the market place.

Price (1997) found that teacher presence was an influence over computation choice. In his study he found students preferred to use written methods 56 percent of the time, calculators 26 percent, and mental methods 19 percent of the time. A 10 percent swing away from written methods was recorded when the teacher left the room.

An earlier study by Reys, Reys and Hope (1993) also found that written calculations dominated computation choice, but more mental and less calculator use than the Price study. While the researchers did not ask the students why particular choices were made, they did note a variation in computation choice according to the nature of the numbers used in each item.

Swan and Bana (1998) reported on the results of a small study and made some observations about the computation choices made by the students. The results cannot be generalised but students were given the opportunity to explain the reason behind their computation choice. Overall the students tended to favour mental methods, although this may have been due to the nature of the questions. Calculators were often used when large numbers or numbers with decimals were encountered. From the observations of, and interviews with students it was ascertained that students sometimes combine computation methods to arrive at a solution and switch methods part way through a calculation.

Several plausible reasons for switching computation approaches might be given. The original computation choice may have been poor, or the question might have been beyond the ability of the student to complete using the chosen method. Perhaps the student's knowledge base on which the path to solution relied was not extensive enough or was found lacking. Doubts may have begun to surface and so the student switched to a 'safer' method, one with which they were more familiar. Observations of 'switching' behaviour noted in this study will be reported in later chapters.

Wheatley (1994) presented a theory as to how children make computation choices. He used the expression 'thought experiment' to describe the process of making a computation choice. A brief description of what constitutes a 'thought experiment' and how it might impact on computation choice is outlined in the next section.

Thought experiments

Wheatley (1994) considered the question of how children make computation choices and introduced the notion of a 'thought experiment' (or TE), which, in simple terms may be thought of as a 'calculation plan'. He described how in an everyday sense this plan might relate to deciding on which route to take on a trip. In computation terms he suggests that in using a calculator an individual often constructs an anticipated sequence of moves and 'runs through' the activity mentally before actually entering the numbers.

The quality of the thought experiment or computation plan is likely to be related to the amount of experience the student has with the various computation alternatives. For example because of the time spent on written algorithms in many classrooms students' computation plans are likely to focus on written methods rather than mental or calculator methods. The more thought experiments or computation plans that are made the larger the repertoire a student has to call on when formulating new plans. Shumway (1994) linked the idea of thought experiments to the idea of meta-computation, discussed earlier.

Wheatley stated, "such TEs allow an individual to explore and evaluate the efficacy of using a calculator in comparison to other possible methods" (p. 121). While Wheatley used the calculator as the benchmark for making comparisons it could quite as easily be mental computation or estimation. In reality, the dominance of the written algorithm no doubt biases thought experiments toward the use of written algorithms because students have most experience of these. The value of providing the opportunity for children to carry out thought experiments or to formulate a computation plan is that these plans can add to the formulation of future plans. Wheatley continued,

The act of performing such thought experiments can provide the basis for decision making. As one performs a TE and reaches a decision, a repertoire of experiences is built that forms a general decision making scheme. When, in future, similar decisions are to be made, the results of previous thought experiments are integrated into a decision making scheme that can be used without repeating the TE at the time – perhaps some new TE will be constructed using previous experiences (p. 121).

It could be argued that in the primary school setting few students are given the opportunity to formulate computation plans. They are often told what form of computation to use by the teacher or the text. Restrictions on the use of calculators reduce the opportunity for children to produce comprehensive plans. As Reys and Reys (1998) explained, however, there are many other factors that impinge on this choice.

The computational tool applied depends on a number of factors, including the context, the particular numbers and operations involved in the computation, the tools available, and the "cognitive load" of the problem-solving process. For example, a student involved in a complex task may choose to use a low-level, non-thinking procedure, such as a calculator or standard technique, to compute so as not to distract from the more demanding cognitive problem-solving process (p. 238).

In the following section some of the factors impinging on computation choice are discussed. Some of these were alluded to earlier. Key research findings have been identified under each factor.

Factors affecting computation choice

Students exercise computation choices every day, albeit at times in an environment that might limit their choice. In this section factors that impinge on the making of computation choices are reviewed.

Time spent on computation options

In the previous discussion on the use of thought experiments as a guide to forming calculation plans, reference was made to plans being formulated based on student experience and knowledge. Student experience and knowledge is somewhat regulated in the classroom environment and therefore student choice may be shaped by this experience. McIntosh (1990) noted that while mental and calculator methods are commonplace in the ‘real world’, standard written methods are still afforded more classroom time than mental and calculator methods.

We spend the vast majority of classroom time on a form of computation – that is paper-and-pencil calculation – which is very little used by adults, and little time, and in some cases no time at all, on methods of computation – namely mental computation and calculators – which are frequently used by almost everyone (p. 24).

Research indicates (Northcote & McIntosh, 1999; Wandt & Brown, 1957) that adults when calculating in everyday settings most commonly use mental computation. When investigating classroom time allocated to computation Porter (1989) found the time allocated to teaching standard written algorithms was well above that spent on mental and calculator computation. Data from the 200 subjects surveyed as part of the Northcote and McIntosh (1999) study indicated that 84.6% of all calculations involved some form of mental mathematics.

The preference for choosing written algorithms is probably related to the amount of time devoted to the various computation alternatives in the classroom. Porter (1989) found that 70–75 percent of most teachers’ time in mathematics was spent teaching computation skills. Much of this time was spent on skill-oriented practice and completing textbook exercises. Duffin (1991) noted the results of a survey of classroom practice carried out in the United Kingdom by Shuard where “it was discovered that 80% of classroom time was devoted to the teaching and practice of the four standard arithmetical algorithms” (p. 42).

This mismatch between the time devoted to the teaching of standard written algorithms and other forms of computation has helped to focus attention on the amount of classroom time allocated to the various computation alternatives. Shumway (1994) suggested a mix of 10% of classroom time devoted to the teaching of written algorithms, 20% to mental methods, 30% to estimation and 40% to calculator use. While the exact allocation of time given to each form of computation may be debated, these figures do suggest that current allocations are inappropriate and that a reallocation of instructional time is required.

What is clear is that the amount of time allocated to each of the computation alternatives will have a bearing on the choices made by students. This becomes apparent when the effect the setting has on computation choice is considered.

Setting in which calculation takes place

It is widely recognised that often there is a lack of congruence between ‘school mathematics’ and ‘real mathematics’. Students adopt school-taught methods while in the school setting and their own methods outside of school. Carraher, Carraher and Schliemann (1985) examined the calculation habits of children in school and out of school. They suggested that children learn to operate in two different systems. When at school they use the methods taught by the teacher and when ‘out on the streets’ they adopt their own methods. Of interest is that the children in the Carraher, Carraher and Schliemann study were able to solve mental computations when posed in the naturalistic setting but failed to do so in the school setting. This led them to the conclusion that school-taught methods can interfere with the solving of a computation problem.

Context in which calculation takes place

The context in which a problem is presented will also have a bearing on the computation choice used by a student. McIntosh, Reys and Reys (1997) used the following example to show how the context may influence the thinking strategy employed by a student.

Whereas the computation $1.65 + 0.99$, devoid of a context, may trigger the application of the traditional written algorithm, embedding these values in a consumer context (e.g., asking the cost of two items priced at \$1.65 and \$0.99) is more likely to stimulate students to think “one dollar sixty-five plus another dollar is two sixty-five minus one cent is two dollars sixty-four cents” (p. 324).

Most interview items used in this research were presented devoid of context, with the exception of two items that were embedded in a consumer context. This aspect of the research will be analysed in later chapters. Research by Reys (1985) documented that the mode of presentation, oral or visual, influences student performance on mental computation. Oral and visual modes of presentation were used when interviewing students in this study.

Attitudes toward computation

The concepts of attitude and belief can be a little obtuse which leaves their use in research open to criticism. The affective domain, however, does wield some influence over computation choice so research in this area will be reviewed. Ruthven (1995a) found that,

there is a consistent pattern in which pupils rate the calculator mode favourably on each criterion and the mental mode relatively unfavourably, with the written mode rated intermediate on difficulty and reliability (p. 234).

While the majority of students in the study expressed the opinion that there were several ways of doing a number problem, they showed a preference, however, for problems where they had already been shown a method to do them. Ruthven (1995a) cited Foxman, Ruddock, McCallum and Schagen, (1991) who found that nearly 30 percent of 11-year-olds considered that use of calculators was harmful because ‘they stop you using your brain’ or ‘prevent you from learning all sorts of sums’ (p. 233). In the same study one third of 11-year-olds preferred not to use a calculator. Ruthven found similar results in his study. He also reported that 40 percent of students viewed using the calculator as a kind of cheating. Ruthven (1995a) cites an earlier study (Ruthven, 1992) where it was found that students showed a reluctance to use calculators because they felt they ‘lost control’ over their mathematics.

Many would argue that the introduction of calculators brought about a swing away from mental and paper-and-pencil calculation in favour of indiscriminate use of calculators. The following comment by Ruthven (1995b) suggested that confidence plays a role in the selection of computation approach.

Preference for not using a calculator was related to confidence in, and enjoyment of, number. In particular, for pupils with less confidence or enjoyment, the calculator seemed to provide a means of matching the demands of school work to their capabilities and interests (p. 246).

Confidence is often related to issues such as familiarity and experience. Students are more likely to choose computation methods that they feel comfortable using. Duffin (1997), an evaluator of the CAN project also believed that confidence played a role in computation choice. She stated:

It is, therefore, important to develop children's own confidence in their ability to perform mental calculations. Confident children are able to decide when a calculator is necessary (p. 138).

The research on computation choice is somewhat limited. As Reys, Reys and Hope (1993) stated in their survey of student computation choice:

The surveys were designed to provide information about preferred computational approaches of students (mental, pencil and paper, calculator), but students were not required to do any computation (p. 307).

They noted that there is a difference between stating what you would do and actually 'doing it'. They found that "a majority of students preferred to use paper and pencil on each item, with the exception of 1000×945 " (p. 310). Many of the items used by Reys, Reys and Hope (1993) were used in the present study. As they noted, it is difficult to understand why students made particular choices without interviewing them. They also acknowledge that their data do not give insight into how successful the students would have been using their chosen method of computation. The results of their survey will be examined much more closely when items common to their study and the present study are analysed.

Conclusion

The studies referred to in this review of the literature have shed some light on the computation choices made by students, but how students actually arrive at the computation choice they make is not clear. This reported research is designed to fill that gap.

Throughout this chapter computation choice has been traced along the path outlined in the NCTM (1989) model of computation choice. The points at which metacomputation takes place and each of the computation alternatives have been examined. Little discussion of the model itself was undertaken, other than the suggestion that the model was linear and simple. In reality the path to computation choice is not quite so simple, as Ruthven (1998) noted:

A refinement of the common-sense trichotomy between mental, written and calculator methods was necessary to take better account of different forms and functions of writing within computation (pp. 29–30).

As has been shown in the review of the literature, rarely are choices of computation clear-cut. Students switch between methods and adopt hybrid methods for completing a calculation. Also, it is difficult to separate the various computation options because often there are links between them, such as in the case of mental computation and standard written algorithms. This raises the question of whether a better model of computation choice exists. In the next chapter several different models of computation are examined in an attempt to answer this question and provide a framework for the research.

Chapter 3: Models of Computation

In the previous chapter a model for computation choice (NCTM, 1989) was used to guide the discussion (See Figure 2.1). The model provided a framework by which the decision-making process involving computation might be studied. Each of the computation options was examined. In doing so, links between various approaches to computation, mental, written and calculator were noted.

As the chapter progressed the paths and branches of the model were explored. Modifications were made to the NCTM model by Trafton (1994) to highlight the role of estimation as a monitoring device. The broader construct – metacomputation – was discussed and the role of estimation within metacomputation explored. The dotted line in Figure 2.5 indicates the relationship between estimation and performing an exact calculation. As Trafton stated,

It may be useful to view estimation as serving a monitoring function that can occur at three places: (1) Before exact computation, estimation can establish a ballpark sense of the answer. (2) During computation, estimation can monitor whether the work is moving in the right direction. (3) After computation, estimation helps one sense whether the answer is sensible or reasonable. Precise estimates need not be made at any of these three stages (p. 80).

Further aspects of metacomputation were noted in Trafton's model. Number sense, a component of metacomputation is clearly indicated by asking two questions when the calculation is completed: "Does the solution fit the problem?" and "Is it sensible?". The model proposed by Trafton also differs from the earlier model proposed by the National Council of Teachers of Mathematics in the way in which estimation is linked to paper-and-pencil, mental and calculator calculation. Further additions were made to the original model to indicate places where choices had to be made within the model. The decision-making process and the monitoring process were subsumed under the broader idea of metacomputation.

While the original NCTM model (1989, p. 9) in Figure 2.1, and Trafton's model in Figure 2.5 gave a good overview of the decision-making path and the computation options available to students, they did not take into account all the complexities of completing a calculation, nor all the factors impinging on computation choice. When

discussing estimation and mental computation it became apparent that while each is a computation choice in its own right, they are related to each other. The formation of an estimate, for example, calls on the use of mental computation. Mental computation is also used when performing a written calculation. For example to complete a two-digit by two-digit multiplication, several single-digit multiplication calculations need to be mentally calculated along with several additions. While computation choice may at first appear straightforward it becomes rather complex when the various relationships between forms of computation are considered. A decision was made to look for alternative models of computation to determine how these models accommodated all the complexities associated with computation choice.

Models of Computation

Several different models have been put forth to explain the computation process and the choices that exist within it. A brief review of some of these models will help develop a better understanding of the process involved in making a computation choice and then carrying out the calculation. All models contain the same essential elements but combine them in different ways to describe the computation process. Some models include a broad range of factors affecting computation choice, while others focus purely on the computation options and interplay between them.

The strengths and weaknesses of the various models will now be examined. This will be followed by the presentation of an alternative way of thinking about computation. The impact of this new model on computation choice will then be discussed.

The first model, which was developed around the same time as the NCTM model (1989, p. 9) was chosen because it presents a non-linear picture of the computation process. It presents computation within a problem solving, conceptualisation framework.

Computation – a global view

Coburn (1989) developed a model of computation that revolved around six categories of computation. In reality he referred to three categories, mental, written and calculator-assisted calculations but he made a distinction between exact calculations and

calculations used to make an estimate. His model, depicted in Figure 3.1, shows much more than just three computation approaches and two modes, exact and approximate.

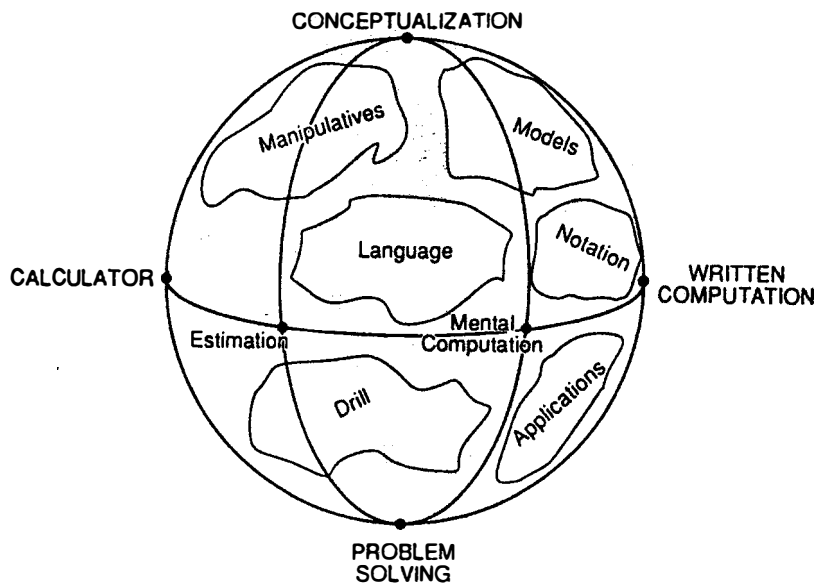


Figure 3.1: Computation – a global view.

Coburn also attempted to show the impact that those factors, such as drill and language, have on computation. Coburn (1989) described his global view in these terms.

In this global view, conceptual models and meanings of operations are foundational. All computation should relate to conceptualisation and problem solving, and these important aspects are located at the “poles.” The methods of doing computation are shown at the “equator.” Mental computation, written computation, and the use of the calculator are equally important, and children need to make appropriate choices. Manipulatives are used to deepen meaning and to connect language and symbols. Daily mixed practice and regular reviews help to further learning and maintain competence (pp. 54–55).

Of particular interest is Coburn’s reference to children making ‘appropriate choices’, although he did not elaborate on how these choices would be made. One might gain the impression that the various forms of computation are equally weighted under this model but rather Coburn argued for less written computation and more mental computation. He noted that written algorithms received the greatest amount of attention in school and that needed changing. He did not, however, wish to discard written methods altogether. He drew the analogy that even in the days of computers and word processors people still made handwritten notes and therefore he felt calculators would not eliminate the need for paper and pencil calculation, but rather, significantly reduce the need. When using the term written methods he referred to standard and non-standard methods or ‘written

shortcuts'. These 'written shortcuts' appear to be ad hoc or idiosyncratic methods developed by the children themselves. They are written in the sense of children jotting interim steps of a calculation on paper.

Coburn's vision was,

[a] curriculum in which students would also be taught to select an appropriate computational procedure depending on the problem situation. Their computational repertoire would range from mental-oral procedures for obtaining exact answers to estimation with the assistance of a calculator (p. 44).

In this statement he hinted at one of the key factors impacting on computation choice – having a repertoire of computation approaches to choose from. Throughout his discussion of the model Coburn referred to the need for understanding. He refuted the suggestion that performing a standard written algorithm develops understanding of the operation. In his view understanding was involved in deciding which operation to perform, or which sequence of keys to push on a calculator. He felt that the teaching of mental computation and estimation contributed to children's understanding of number. He broke computation choice into two components. Assuming the need for a calculation had been established students then had to decide which operation was required and then choose a mode of calculation. In reality computation choice appears to be more complex (Swan & Bana, 1998), but the relationship of operation to choice is of interest. It is feasible to imagine that because many students experience difficulty with division they might opt to use a calculator in preference to other methods.

Coburn included drill as part of his model. He did not advocate drill as an instructional activity but rather brief periods of drill after children had developed understanding. Drill was used to maintain acquired skills, but more importantly Coburn felt these sessions should include mixed questions so that the students were encouraged to consider the operation and the method of computation. This was part of the development of computation choice, where thought had to be given to the operation to be used.

Coburn's model indicated the complexities involved in teaching computation and children arriving at an appropriate computation choice. His model includes the role of manipulatives in developing understanding, the impact of drill and the use of language. These elements and many more impact on computation and the choices children make.

Coburn’s model did not fully acknowledge all the relationships between the various types of computation. Should the ‘equator’ on which calculator, estimation, mental computation and written computation lie, be viewed as a continuum, then this would go part way toward acknowledging the idea that students may use a mix of methods when solving a problem. The solution to a computation problem may involve an idiosyncratic written method that is a hybrid of mental and written methods. The relationship to conceptualisation and problem solving is certainly a strength of this model and links computation to the broader notion of mathematics as a way of thinking.

Freudenthal model

Figure 3.2 depicts the model developed by the Freudenthal Institute (van den Heuvel-Panhuizen, 2001, p. 218), as a means of showing the decision process for determining which form of computation to use. The model encompasses all forms of computation, although it down plays the role of standard written methods. Like Coburn’s model, this model has a problem solving flavour to it.

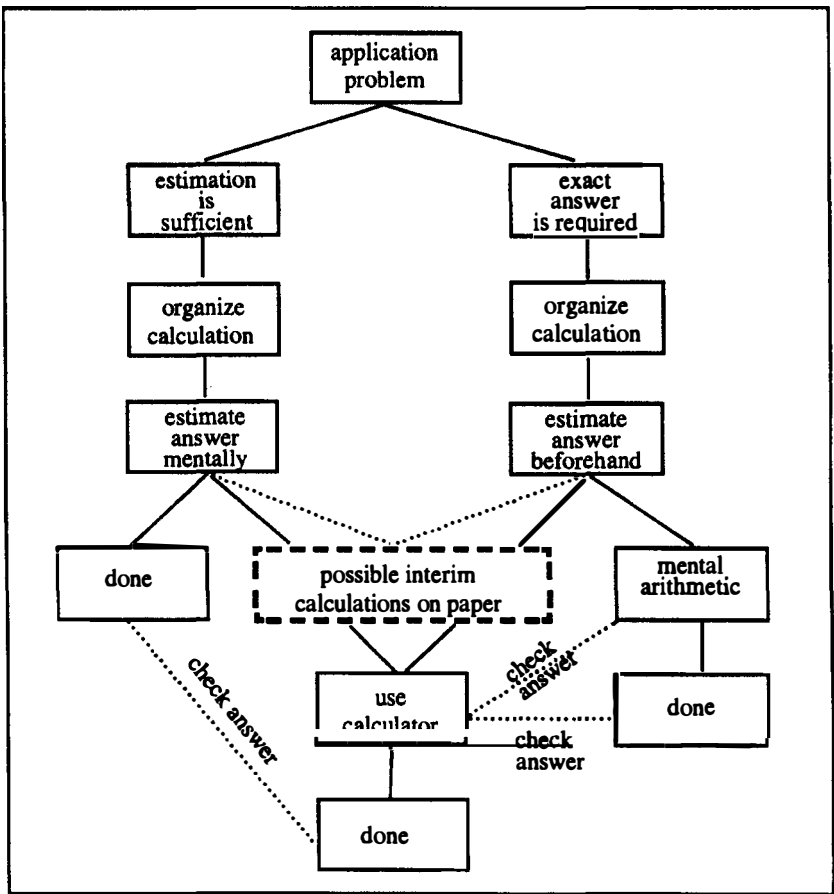


Figure 3.2: Freudenthal Institute model of computation decision process (2001, p. 218).

This model assumes that the need for a calculation has already been established and branches out to either approximate or exact forms of calculation. The various steps in the computation process are described below.

It is therefore essential that the children first analyse the situation (what is the problem?), then organise the calculation (what has to be done?), then write it down as a calculation scheme (what are the appropriate operations?), and finally carry out the actual calculations: ANALYSIS→ORGANISATION→CALCULATION SCHEME→PERFORM CALCULATION (p. 217).

This approach brings to mind the four steps outlined by Polya (1957) when solving a problem:

1. Understand the Problem (Analysis);
2. Devise a Plan (Organisation, calculation scheme);
3. Carry out the Plan (perform calculation); and
4. Look back.

While the description of the process does not include a 'looking back' phase, the diagram outlining the key features of the model indicates that an estimate would need to be made prior to performing the calculation and checking would take place after the answer was calculated. When examining the model it is worth noting features, such as, the step 'organise calculation' and the suggestion that interim calculations might need to be done on paper.

The model takes into account that every student will make different decisions about the calculation methods and the notation to use. The model shows students passing through various stages as they work through the computation process. Students must first analyse and then organise the calculation. None of the previous models considered the idea of organising a calculation prior to starting it. Previous research (Swan & Bana, 1999) found some students embarked on a computation route and then changed methods part way through a calculation. This could indicate a lack of a plan, or that things did not go according to plan. Students must step back and analyse the calculation before making a choice and organising the calculation. This is essentially a metacomputative process that involves higher order thinking. Whether students actually do step back and analyse a question before embarking on a calculation is part of this research. Once the analysis has taken place further thought is required to plan the calculation.

When using a calculator students would need to organise the calculation so that it may be entered into the calculator and then follow a plan that utilises the various keys and the logic of the machine. Ruthven (1995b) also noted the need to organise or formulate a calculation, particularly when using a calculator.

Calculator use is not wholly routine. The user has to formulate the computation for input to the machine and interpret the output. Moreover this may involve repeated computation during which the user makes important tactical decisions in order to arrive at an acceptable answer (p. 241).

The Freudenthal Institute used the term ‘insightful’ arithmetic education, which involved children developing a critical attitude toward arithmetic, when discussing the application of their model to the classroom setting. They believed this insightful approach would help to govern computation choices, especially the choice to use a calculator. Insight is also required when organising the calculation. Organising a calculation to be done with a calculator requires different thinking to completing a calculation mentally. When organising a mental calculation various mental strategies are called upon, based on a store of knowledge held by the user. When using a calculator an understanding of how the calculator works, the logic, the sequence of keys to press, all impact on the organisation of the calculation. Interpreting the result on the display is also an issue related to choosing to use a calculator. If children do not understand what is shown on the display they may shy away from using the calculator in favour of other methods. Organising a calculation also depends on experience with each of the computation alternatives.

In building a repertoire of calculation approaches students need to gain experience with all types of calculation. The common practice of providing a set of ‘addition sums’ to do may also detract from the process of making a computation choice, because the students are told what operation to use rather than decide what operation is implied by the question. One feature of the model that is quite noticeable is the lack of emphasis placed on written methods. There are several reasons for this, including the advent of the calculator, with the role of formal written algorithms in society being reduced. This should be mirrored in the classroom, where they believe formal algorithms should take on a ‘subordinate role’. Figure 3.3 describes this change in emphasis and varying line thickness is used to indicate the relative emphases associated with each form of computation.

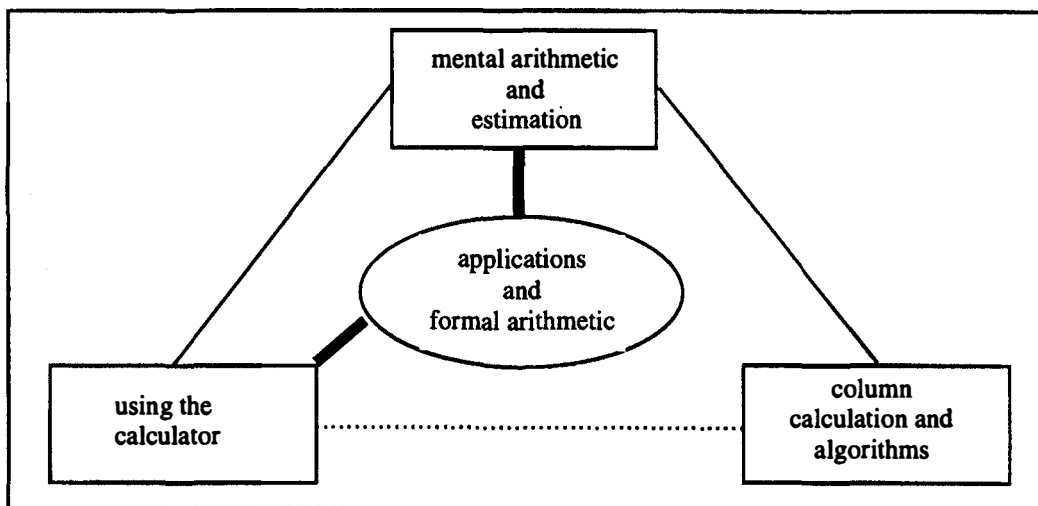


Figure 3.3: Freudenthal Institute suggested calculation emphasis (2001, p. 225).

Another feature of the model outlined in Figure 3.2 that is worth noting is the acknowledgement that a mix of calculation approaches may be used in order to find an answer. Reference is made to possible interim calculations done on paper. Children's idiosyncratic methods of calculation often make use of interim jottings to alleviate the demand on short term working memory.

The strength of the model lies in the attempt to indicate where metacomputation, thinking about the calculation and monitoring take place. The acknowledgement that forms of calculation may be mixed more closely reflects how calculations are performed. A reduced emphasis on written computation, especially standard methods are also reflected in the model with mental and calculator methods being given more prominence. The model uses a linear approach to depict the process of choosing a computation approach, using it and checking it. This helps to streamline the model, making it easy to follow, but at the same time it does not fully reflect the intricacies of the calculation process.

Teaching children to make computation choices is quite a complex task. It could be argued that it is simpler to teach children to try mental computation as a 'first resort' and then to look at either calculator or written methods. The following model is designed around the principle of 'mental first'.

Morgan model

Morgan (2000) proposed a sequence for introducing computation procedures that focussed on the role of mental computation. In reviewing computation approaches in Queensland he advocated that increased emphasis be given to mental methods. In effect, mental methods should be elevated in status to the first computation option that students should try. Typically Queensland teachers followed a teaching sequence that placed paper-and-pencil at the forefront of computation choice. This is characterised in the teaching sequence described by Morgan and shown in Figure 3.4. This sequence, shows paper-and-pencil methods coming after students have learned the basic facts, but before mental methods.

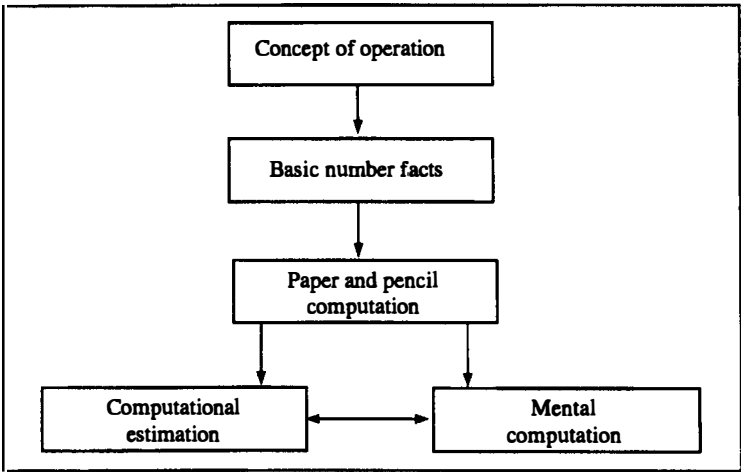


Figure 3.4: Traditional sequence for introducing computation (Morgan, 2000, p. 5, adapted from Irons, 1990).

The sequence outlined in Figure 3.5 indicates that written calculation has generally been seen as the first calculation option, with estimation and mental computation acting as appendages or support for written methods. Morgan (2000) argued against this traditional sequence outlined in Figure 3.5 noting that we have moved on from the industrial age where written algorithms were so highly valued. The effect of years of teaching this way has been to stifle flexibility of thought. In his argument for a change of approach Morgan (2000) linked flexible mathematical thinking to the development of number sense. Number sense was discussed in the previous chapter but it is worth noting Morgan’s description.

Number sense is, in part, characterised by an ability to perform mental computations with non-standard strategies that take advantage of an ability to compose and decompose numbers. In so doing, students with number sense tend to analyse the whole problem first to ascertain and capitalise upon the relationships among the numbers, and the operations and contexts involved, rather than merely apply a standard algorithm (2000, p. 6).

As discussed in Chapter 2, the development of the teaching of formal written methods may hamper flexible mental approaches. This is particularly the case when written methods are taught to young children. Morgan suggested that the sequence be changed to the one outlined in Figure 3.5. Mental computation was given the pivotal role in this model. His model reflects the thinking that students should be taught to try mental computation first. He believed that strategies for calculating exact answers needed to be developed before estimation. Given that estimation draws on mental computation this suggestion appears reasonable. Standard written algorithms are given second place in this model because of the belief that they discourage thinking. Delaying the teaching of formal written techniques should, in Morgan’s opinion, help children to view standard written algorithms as one of many possible ways for calculating rather than the main method for calculating. As a result of following this sequence Morgan (2000) believed that “the ability of children to make choices between calculative methods will be enhanced” (p. 7).

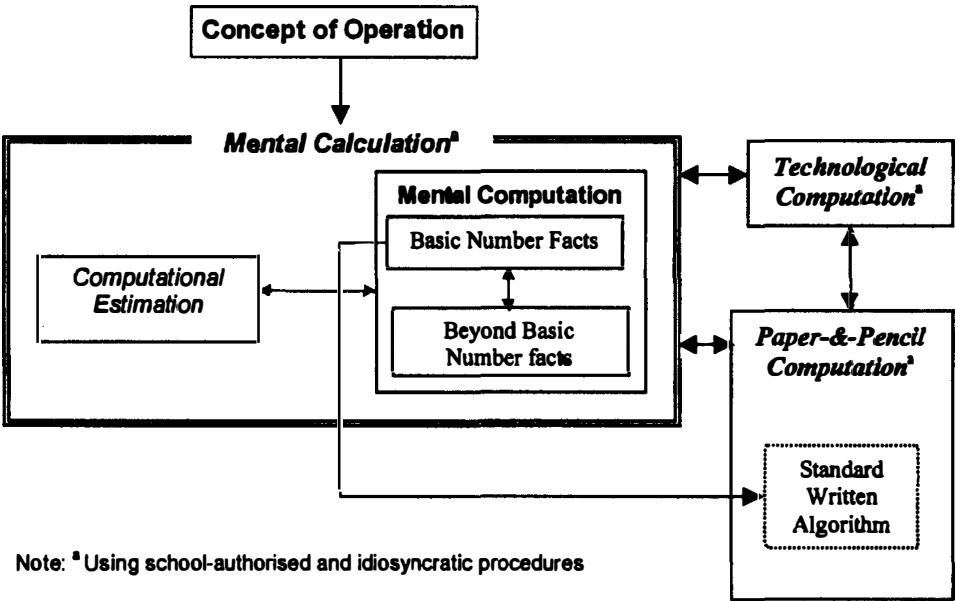


Figure 3.5: Computation model (Morgan, 2000, p. 6).

Rather than offer a range of choices, this model suggests that mental methods be tried first and then alternatives considered. Various relationships between the computation alternatives are depicted indicating a mix of methods could be used to complete a calculation.

Computational estimation is linked to mental computation which is sensible because estimation draws on mental computation. Mental computation is split into basic facts and beyond basic facts, but no mention is made of the mental strategies that might link to or draw on these components of mental calculation. The use of the term ‘technological computation’ serves to highlight that computer spreadsheets and the like as well as calculators may be used to perform calculations.

The ‘paper-and-pencil’ section takes into account both approaches to written calculation – the formal standard approach and the ad hoc or idiosyncratic methods. The suggestion that these idiosyncratic procedures be school-authorised appears to be at odds with the notion that these methods would be generated by the students rather than taught by the teacher. Certainly these methods would need to be supported by the teacher in terms of discussion and opportunities to share methods among class members, but there is no need for a school to place a stamp of approval on these methods because they are personal and not formal.

Morgan’s model, like the previous model, highlights the need for the student to have a concept of the operation before embarking on the calculation. For many years it has been assumed that because students can perform a calculation this meant they understood numbers and operations. Before leaving this model it should be noted that Morgan suggested a revised sequential framework for introducing mental, calculator and written procedures to accompany this model.

While each of the models discussed provides a picture of the computation process and the choices to be made they all fall short in one respect or another. Questions have been raised about each model. They are summarised in the next section.

Questions about the models

A problem with all the models discussed so far is that in attempting to succinctly describe computation choices and routes an oversimplification occurs. For example, the NCTM (1989) model does not identify any links between mental computation and the use of a calculator. Links between estimation and the use of a calculator are clear but

the link between mental computation and calculator use is not so clear. Hepburn (1993) noted the close links between using a calculator and mental computation. She posed the question: "Can children use calculators without doing some mathematics in their head?" (p. 13).

The Freudenthal model tried to take into account the need to organise the calculation which included elements such as choosing the appropriate operation and mode of calculation. Morgan suggested a different approach whereby students were given a 'rule of thumb' to try mental methods first. This 'mental first' approach could almost be thought of as a default model, where students are taught to try mental methods as their initial computation approach. As they build their repertoire of calculation methods and experience, this approach could be relaxed and the students could then be encouraged to 'organise the calculation' prior to embarking on it. The organization would involve considering the mental option as a first course.

The Need for a New Model of Computation

There are many factors that impinge on computation choice and these should be incorporated into any model of computation choice. Several models included metacomputative components such as estimation to guide or check a calculation. Metacomputation should certainly govern any calculation and by extension any computation model. A model that combined many of the features of previous models and attempted to include metacomputative processes and account for the complexities of calculation was required.

The foregoing models have all attempted to explain the process of using a calculator in a linear fashion but in reality the process is much more complex. Figure 3.6 depicts a computation model, devised by Swan and Bana (1998) that was developed to describe a number of factors, which impinge on computation choice.

A description of the model

Unlike many of the models discussed in this chapter, this model is not linear. It does not present computation choice as following one distinct path or another, but rather depicts computation choice as an interaction between various alternatives. Each component of the model will be discussed in turn, but a few general features are worth

noting. Mental and calculator options are listed as distinct computation options, whereas written computation is not allocated the status of a specific choice but rather the relationship between written methods and mental computation is acknowledged as a part of the recording associated with calculation. Estimation does not appear as a computation choice because this model applies to exact forms of calculation. Estimation, however, does feature in the metacognitive checking strategies associated with the performing of an exact calculation.

The complex nature of computation is such that apart from mental computation most computation activity will involve a blend of two or three of the primary processes. A Venn diagram (Swan & Bana,1998) has been used to illustrate this relationship. Each of the components of the model will now be discussed. The mental, calculator and recording components will be outlined first, followed by metacomputative strategies and finally factors that impinge on computation choice.

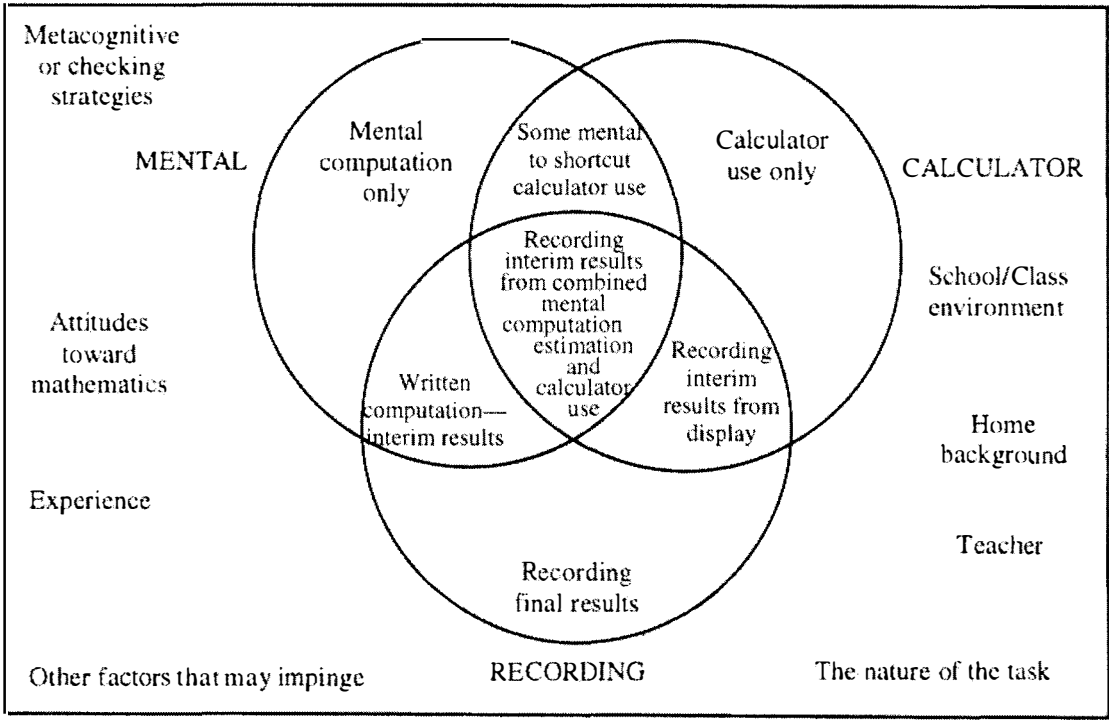


Figure 3.6: Computation model (Swan & Bana, 1998).

Mental calculation

In the proposed model the mental category refers to all work done in the head when performing a calculation. The largest component of this category is exact calculation. This refers to both the recall of stored facts and the use of mental strategies

to calculate answers. Should interim results need to be jotted down on paper as part of performing a calculation then this would be located in the intersection of the 'mental' and 'recording' components of the model. The interim results written on paper could be part of children's idiosyncratic methods of calculation or part of a standard written algorithm.

Mental calculation may be used as an adjunct to calculator use. This acknowledges that some students perform mental calculations prior to, during or after performing a calculation with the aid of a calculator. For example, a student may mentally convert a fraction such as eight tenths to 0.8 so that it may be entered into the calculator. Students without this knowledge or ability to convert, will need to key in eight divided by ten to enter the fraction into the calculator. Without this knowledge the calculation cannot proceed on a calculator and this option is denied to students.

It is feasible that children may combine mental and calculator use, entering various interim results along the way. One can imagine this being the case when completing more complex calculations, especially those involving more than one operation. Mental calculation is also associated with the various metacomputative techniques associated with estimating, checking or evaluating the answer and monitoring strategies applied throughout a calculation.

Calculator use

Calculator use refers to the process of using a calculator and this would include the ability to enter calculations into a calculator, and making use of the various functions of the calculator. Calculator use often relies on an understanding of how a calculator functions, the logic associated with entering the calculation, the way in which the calculator performs a calculation, and an ability to interpret the display. For example, various calculator models follow different forms of logic, which may impact, on the way in which a calculation is entered. This may also have a bearing on the organisation of the calculation or the calculation plan that is formulated and followed.

Children who have received little or no formal training in how to use calculators will probably not make use of the memory facility but may rely more on jotting down interim steps in the calculation. This is represented by the intersection of the calculator and recording sections of the model.

Recording

The use of the term recording is meant to denote:

- recording final results of a calculation,
- any writing down of interim steps,
- informal jottings,
- various idiosyncratic or self-generated written methods of computation, and
- the use of standard written algorithms.

The relationship between mental computation and the completion of standard written algorithms and self-generated methods is clearly located in the intersection between ‘Mental’ and ‘Recording’. Standard written algorithms and self-generated written methods rely on the combination of mental calculations, recording of interim steps and the final result. The recording of final results is located in a separate section because certain skills are associated with the recording of a final result. For example, units may need to be attached or calculator results may require rounding.

Children often write parts of calculations down to ‘keep track’ when performing a calculation. As previously mentioned, few children make use of the memory facility on a calculator (Shipley, 2002) and therefore often need to write things down when using a calculator. While the circles in Figure 3.6 contain the various forms of calculation and indicate the relationship between each, there are many factors that impinge on computation choice. Most of these have been discussed in Chapter 2, so only a brief overview follows.

Metacognitive or checking strategies

The role of estimation as a monitoring and checking device has been discussed in some detail. Estimation is a key component of metacomputation and draws on the use of mental computation. There are, however, many other intuitive forms of checking that may be included under this heading. Some checking techniques may occur subconsciously or intuitively. For example, from past experience with patterning a student may recognise that when two even numbers are multiplied then the answer is always even. To determine ‘evenness’ or ‘oddness’ one only needs to examine the units

digit in the number, regardless of the magnitude of the number. Children should recognise, therefore, that the result of a computation involving the multiplication of two even numbers produces an even number result. Number sense can be seen as playing a key role in checking the progress and final result of a calculation.

Other factors that may impinge on computation choice

Reference has already been made to the amount of time spent teaching and practising the various computation alternatives. The experience children have with each alternative will have an effect on their confidence in using a particular method. Attitudes toward using various computation options may be affected by a range of issues. The drilling and testing of mental computation may promote fear of making a mistake and therefore some children may choose the 'safer' written option. Suffice to say many different factors weigh on students' attitudes toward computation.

Little can be done in the classroom setting to offset any home background issues that impact on computation choice. Parents may view the learning of tables almost as a 'rite of passage' that all children should have to proceed on their way to adult mathematics. Views at home toward calculator use may be voiced indicating a negative attitude toward the use of calculators. The word 'cheating' may be associated with the use of calculators. Parents may feel more 'at home' with assisting their children to complete written algorithms at home, regarding 'sums' as real mathematics. Often an element of rigour is associated with the completing of a page of sums. As a brief aside, however, the completing of algorithms, particularly the subtraction algorithm can be the source of argument in many households. This generally occurs because children and parents are using different methods to perform subtraction. Both are so tied to rule-bound behaviour that neither recognises the validity of the other's method.

Previous discussion has highlighted a constructivist approach to teaching and learning. Teachers adopting a constructivist approach tend to be more open to the discussion of a variety of methods of calculation, rather than the imposition of a single method. Students are therefore encouraged to be more flexible in their approach toward calculation and as such are more likely to try different computation methods because they will have built up a repertoire of methods from which a choice may be made. The approach a teacher adopts may be seen as impacting on the development of a computation repertoire and hence on computation choice.

All the above factors and more will impinge on computation choice. This research was carried out within typical classroom settings that are described in the next chapter. Learning takes place in a social milieu and therefore it did not make sense to try to control all the variables contributing to computation choice. Rather, classes where children had access to calculators and had been given opportunity to use mental and written methods were sought.

The model in practice

The Swan and Bana (1998) model of the computation process was used to guide this research and as such will be revisited when the results of the research are discussed. Human behaviour, especially when it comes to choice, is most complicated so modifications may need to be made to the model. In particular, attention will be paid to determining:

- what triggers initial computation choice;
- whether students do use a combination of methods when solving computation problems; and
- whether they monitor the calculation as it progresses once an answer has been achieved.

Setting the scene

Prior to answering these questions and the research questions proposed in Chapter 1, a clear description of the setting for the study will be provided in the next chapter. The methodology, the instrument, the subjects and the general classroom setting will be described in order to give a better picture of how the research was conducted. This will assist in understanding the description and discussion of the results.

Chapter 4: Methodology

Introduction

In the previous chapters current thinking about computation was reviewed. This included a look at each of the computation alternatives and the broader issue of metacomputation. Various models designed to describe the computation process were discussed and a new way of thinking about computation was introduced.

In this chapter, the research methodology chosen to assist in determining what computation choices students make and why they make them is outlined. Next data gathering techniques associated with the chosen methodology, and which were most likely to provide the data required to answer the three research questions, are examined. The target population is described, along with the reasons for choosing this particular group. The data gathering instrument and procedures are explained as well as the pilot study used to refine the instrument and data gathering procedures. The process for analysing the data is then outlined. Finally the limitations of the study are discussed in light of the various threats to reliability and validity and systems for reducing these threats are outlined.

Outline of the Study

The aim of this research was to find out what computation choices students made, why they were made, and how successful they were in carrying out a calculation using their chosen method. The literature review indicated that little was known about computation choice (Hope, 1989; Price, 1995; Reys et al., 1993), therefore the study was very much an exploratory one designed to gain a better understanding about phenomena for which little was known.

Guba advised “select a paradigm whose assumptions are best met by the phenomenon being studied (1981, p. 76). Given,

- the complex and unpredictable nature of the data to be collected in this study;
- that the thinking of children was to be explored; and
- the relative lack of information on how students make computation choices;

both qualitative and quantitative data were gathered in order to gain a clearer picture of computation choice. Qualitative methods provide the best opportunity for gathering data to improve understanding of little known phenomena and to gain in-depth information and new insights (Strauss & Corbin, 1990). The researcher did not set out to prove or disprove a hypothesis, but rather was guided by research questions. The inductive nature of qualitative research allowed for meaning to be established from the data in order to explain the phenomena being studied.

The Design of the Study

This study was guided by three research questions:

1. When faced with a computation question, what choices do students in Years 5-7 make?
2. Why do students in Years 5-7 make particular computation choices?
3. How successful are students in Years 5-7 at executing various forms of computation?

The research questions dictated the use of a qualitative approach for this study, particularly when trying to answer the second research question where there was a need to find out what the students were thinking. The participants were confined to Years 5, 6 and 7 and the focus was on how these students chose to tackle a series of computation items. There were several approaches that could be used to gather this type of evidence such as stimulated recall, think aloud methods and interviews. Interviews were used as the primary data collection technique. The reasons for this choice and the supplementary data gathering techniques are examined in the next section.

A set of computation items were presented both in oral and written form, in a horizontal format. This was carried out in an individual interview situation with the student free to work mentally, with paper-and-pencil, with a calculator, or any combination of these. Students were asked to complete the calculation using whatever methods they chose. After completing the calculation, students were asked to identify the form/s of calculation used and why that particular choice had been made. Students were then invited to try the same calculation using a different method. If students indicated they could solve the problem in another way then the interviewer requested that they demonstrate this. Records were kept of successful and unsuccessful attempts, and of computation preference (See Appendix 4). Interviews were audio-taped and transcribed. In addition, field notes were made which indicated non-verbal behaviour. Comments and explanations of particular interest were noted. All written work was collected and stored for later reference.

Data gathering

There are several techniques that might be employed to gather information about cognitive processes. These include think aloud methods, stimulated recall and various types of interviews. A decision not to use think aloud methods was made because it was felt that asking students to describe their thinking prior to and while performing a calculation would interrupt the natural thought processes involved in making computation choices and executing them. When comparing think aloud and reports based on memory, Ginsburg, Kossan, Schwartz and Swanson (1983) noted that “reporting on mental states and processes might interfere with or change the very nature of the mental phenomena”. Ginsburg, et al. (1983) discussed the problem of report interference when applying research methods that involved reporting ongoing mental activity. The alternative involves relying on memory reports. These methods are not without their problems, as will be discussed later.

Stimulated recall techniques typically involve video-taping students engaged in a particular behaviour and then replaying the videotape to the subject in order to elicit responses about their thinking at the time the behaviour occurred. While this technique is powerful, the researcher lacked access to the appropriate technology and experience in using this technique.

Interviews were chosen as the best way, within the circumstances, to gather the data needed to answer the research questions. The researcher had experience in this form of data collection (Swan, 1991) and it allowed for the collection of data that would specifically answer the three research questions. Cohen and Manion (1980) described the research interview as a “two-person conversation initiated by the interviewer for the specific purpose of obtaining research-relevant information” (p. 241).

Eighteen items formed the basis of the interview. Responses were recorded, field notes were taken, that included reference to non-verbal behaviours. The interviews were audio-taped. The use of audio-tape recording is debated in the literature. Lincoln and Guba (1985) do not recommend the use of recording devices because of their intrusiveness. Patton (1990), however, stated in his opinion a tape recorder was “indispensable” (p. 348). The decision to use audio-tape was a pragmatic one as it allowed the researcher to concentrate on the interview rather than hurriedly record notes on paper. Segments of audio-tape may also be replayed to listen to the intonation, voice inflection and nuances of the student.

Data pertaining to computation choice and success rate were recorded at the time of the interview on a separate recording sheet (See Appendix 4). The sorting of this data required the use of simple statistical procedures available within a spreadsheet such as Microsoft Excel.

Sparrow (2000) noted that, “interviews may be conducted in a variety of ways: from free flowing, informal conversations to formal set question and answer styles” (p. 84). He went on to cite Herrington (1997) who reviewed the research literature on the characteristics of various types of interview. Table 4.1 outlines the general categories of interview and the broad characteristics of each type of interview.

Table 4.1: Categories of interviews.

Denzin (1989)	Patton (1990)	Characteristics
Type of interview	Type of interview	
	Least structured	
	Informal conversational interview	Questions emerge from the immediate context and are asked in the natural course of things: there is no predetermination of question topics or wording.
Non-standard interview	Interview guide approach	Topics and issues to be covered are specified in advance, in outline form: interviewer decides sequence and wording of questions in the course of the interview.
Non schedule standardised interview		Questions and probes are determined in advance but there is flexibility in the interview, for example in the sequence of questions depending on the responses of the interviewees.
Schedule standardised interview	Standardised open-ended interview	The exact wording and sequence of questions are determined in advance. All interviewees are asked the same basic questions in the same order. Questions are worded in a completely open-ended format.
	Closed fixed response interview	Questions and response categories are determined in advance. Responses are fixed: respondent chooses from among these fixed responses.
	Most structured	

Note: Denzin, 1989; Patton, 1990; Herrington, 1997 as cited in Sparrow, 1999, p. 85.

The interview technique used to gather data in this research may be located at the more structured end of the continuum of methods. The type of interview used in this research matched the criteria for what Denzin (1989) and Patton (1990) described as standardised interviews. Students were asked a sequence of questions, which were followed by a set of standard probes (See Appendix 5). Students were asked to complete 18 computation items. After completing each item the students were asked to explain what computation method they had chosen; why they had made that choice; how they had performed the calculation. This allowed the researcher to verify and clarify observations made while the student performed the calculation. While the researcher followed this set protocol, there was opportunity to probe deeper should the need arise. In this sense the interview varied from the regimen of a standardised interview. The characteristics of this data gathering method, which closely resembled the clinical interview technique used by Piaget are examined in the next section.

The clinical interview

Ginsburg (1981) noted in order to find out how children think you need to speak to them, rather than surmise from observed behaviour. He favoured the use of the clinical interview as a method for determining what was going on in a child's mind. Hunting (1983) described the clinical interview in these terms:

The clinical method usually takes the form of a dialogue or conversation held in an interview session between an adult, the interviewer, and a child, the subject of the study. Usually the discussion is centred on the task or problem which has been carefully chosen to give the child every opportunity to display behaviour from which mental mechanisms used in thinking about that task or solving that problem can be inferred. It is typical in this methodology, for the investigator to pose a verbal question to which the child makes some type of response, the investigator then asks another question, poses a variation of the problem, or in some way sets up a new stimulus situation (p. 48).

Ginsburg, et al. (1983) noted that even when using a clinical interview, "methods vary in degree of standardization" (p. 19). They go on to give two extremes of clinical interview, one where the interviewer develops the questions on the spot, to the more formal approaches where a standard set of questions is used. Ginsburg, et al. concluded:

If the questions are all given to all subjects in the same manner and order, without omissions or additions, then the result is no longer a genuine clinical interview but a standard test. Contingency defines the clinical interviewing methods (p. 19).

For the most part the interview structure in this research was rigid in terms of the questions that were asked, the order in which they were asked and the probes that were used. The probes, however, did allow for some latitude when it came to exploring the computation approaches used by students and the reasons students chose those methods.

A key part of the research involved the need to gather data relating to the thought processes involved in deciding which form of computation to use and then performing the calculation. Verbal reports are a valuable source of data when trying to determine what a student is thinking when making a choice of computation method. While the interviewer might make assumptions about the computation choice, and the thinking used when completing the calculation, this may only be verified by the student description of what occurred.

Field notes

While the primary source of data was based on the use of interviews, field notes were taken and any written work was collected. Field notes included descriptions of the school and classroom settings as well as specific notes taken during the interview. These notes included references to body language and physical actions, such as, the use of fingers that would not be recorded on audio-tape. In addition, the researcher noted interesting commentary to be reviewed later by viewing the transcript or listening to the audio-tape. Written work was collected as another source of data. The researcher was not only interested in what type of computation was used but how it was executed. For example, when using written method students may have chosen to use the standard written algorithm or an approach based on informal jottings, so it was important to gather this level of detail.

The use of multiple sources of evidence is an important facet of qualitative research as it allows for some verification of the data to occur. While the various data gathering techniques used in this research did not allow for the triangulation of data, in a strict sense, the use of some items from previous research (Hope, 1989; Price, 1995; Reys et al., 1993) did allow for comparisons to be made with a different data set, based on different samples.

Participants

The population from which the sample was drawn consisted of upper primary students in Western Australia. Students were drawn from Year 5 to Year 7 (ages 10–12) because they needed to be of sufficient maturity and have enough experience with all forms of computation, in order to provide an indication of their preferences. Younger children would have some difficulty explaining the reasons for making particular computation choices and would not have enough experience with written forms of computation to feel confident in making use of it. Similarly a lack of familiarity with the calculator was also an issue, although all children who participated in the study had their own calculator which they kept in their desk.

The participants were drawn from four classes across two primary schools in a rural district of Western Australia. Students from a Year 6/7 composite class and a Year 7 class from a state (public) primary school participated in the study. Students from a Year 5 and a Year 7 class in a local Catholic school also took part in the study, bringing the total number of participants to 78. There are no strict criteria for sample size in qualitative studies (Patton, 1990). The issue is whether there are enough participants to produce data that will help answer the research questions. Seventy-eight participants were enough to provide the data required to answer the questions.

There are several sampling strategies that might be used in qualitative research. In a quantitative study researchers typically use random or probability samples but in qualitative studies researchers typically use a purposeful sampling approach. Patton (1990) identified 16 types of purposeful sampling, each with a focus of trying to choose a sample that will provide rich data in order to help understand the phenomena being studied. A convenience sampling approach was used in this research although by spreading the sample across Years 5 to 7 an attempt was also made at maximum variation sampling. Patton (1990) described this sampling approach in these terms:

For small samples a great deal of heterogeneity can be a problem because individual cases are so different to each other. The maximum variation sampling strategy turns that apparent weakness into a strength by applying the following logic: Any common patterns that emerge from great variation are of particular interest and value in capturing the core experiences and central, shared aspects or impacts of a program (p. 172).

The sample chosen for this research allowed for enough variation to exist while still allowing trends to be examined. The focus was on establishing patterns and trends in computation choice. Where variation existed there was enough scope to allow for reporting of these variations.

Schools were initially contacted on the basis of convenience in that the researcher had worked in each of the schools on several occasions and had a working knowledge of the computation practices of each school. Two of the teachers to be referred to as S1 & S2 (i.e. State 1 and 2) had more than 20 years' teaching experience; while one C1, (Catholic 1) had 10 and the other C2, (Catholic 2) had three years experience.

Ethical considerations

Prior to commencing the research, ethics clearance was sought through the University's Ethics Committee. In order to meet the criteria set by the committee permission had to be obtained to work in the schools. Permission was also obtained from the parents and students in the target group. After initial contact with each principal, letters were subsequently sent to parents outlining the research and the extent of their child's role in the research (See Appendices 1-2 for copies of these letters). The students themselves were given the option to participate, most volunteering, but some declining the offer. Illness and absence further reduced the number of participants to 78.

Description of the environment

LeCompte and Goetz (1982) noted that for credibility and comparability purposes the characteristics of groups that were studied needed to be delineated. "Comparability requires that the ethnographer delineate the characteristics of the group studied . . . so clearly they can serve as a basis for comparison with like and unlike groups" (p. 34). In this section a brief description of the school, teachers and students in terms of classroom practices in teaching number is given.

The state school in which the research took place was situated in a middle class area. The student population was around 250. The principal saw himself as a curriculum leader having spent some time leading a mathematics project for the Education Department of Western Australia. The school had in place a 'number sense' program based on the work of McIntosh, De Nardi and Swan (1994).

The two teachers, whose students were involved in the research, were very experienced, each having more than 20 years' teaching experience. While one might imagine their computation practices to be traditional this was not the case. Teacher S1 was female and had taught in the junior primary area for much of her career. She had made the transition to upper primary, despite having experienced some trepidation toward the mathematics. She displayed a willingness to learn and try out new things and had attended mathematics conferences. As a result of attending a workshop on the use of student mathematics journals, she had incorporated them as a regular feature in her classroom. The journals gave an indication of the quality of the mathematics program being undertaken in the room. They were found to be of a high quality.

Teacher S2 was a male who had taught Year 7 students for most of his career. While he was more conservative in his approach toward mathematics than Teacher S1, he was receptive to new ideas. On a previous occasion he had participated in a trial of some fraction calculators. The interaction between Teacher S2 and the researcher at that time indicated that he was prepared to try new ideas and change his teaching approach to better reflect current thinking about computation.

Both teachers ran a number sense program, which included elements of mental computation and had a constructivist flavour in terms of the role that discussion played in the program. Observation of teaching practice and programs indicated that both teachers valued standard written forms of computation. Discussions with the teachers indicated that the parent population had conservative views toward computation, despite the principal having run explanation sessions about number sense with the parents. Teachers taught traditional written algorithms alongside their number sense program.

Calculators were freely available with students having their own calculators on their desks as well as access to some at the side of the room. Explicit instruction in calculator use was not given. The range of models of calculators varied considerably in both rooms and the teachers noted the difficulty of teaching students to use different calculators.

The second school in which the research took place was part of the Catholic school system. This school was larger than the state school having an enrolment, close to 350. It was situated in a slightly lower socio-economic area, but the student population exhibited similar characteristics to those in the state school system. The two teachers in the Catholic School were both female. One teacher (C1), who taught the Year 7 students had ten years' teaching experience and recently completed her one year B.Ed conversion in which she chose to complete some mathematics units with the researcher acting as her lecturer. She held the role of mathematics coordinator within the school and was receptive to new ideas and approaches in mathematics.

The second teacher (C2) was younger, with only three years' teaching experience. She had recently graduated from the local university where the researcher had been her mathematics education lecturer. The Catholic school appeared to be more conservative in general and this also applied to the teaching of mathematics. For example, tables charts were displayed in the classroom. Both teachers expressed a desire to adopt more open and flexible methods in teaching mathematics, but felt a little

intimidated by 'older and wiser' staff. Within this environment both teachers regularly conducted mental computation sessions which tended to be more of a blend of traditional 'tables' and 'basic facts' drills alongside more open methods as described in McIntosh, De Nardi and Swan (1994).

Students had been taught standard written algorithms, although after discussions with the teachers and reviewing their programs it was clear that time devoted to teaching formal written algorithms was more than that given to the development of written and calculators methods. Mental methods, were, however, allocated more than just simply the first few minutes of each lesson and sometimes constituted the focus of entire lessons. Both teachers attended to other parts of the curriculum such as, space and measurement, and tried to incorporate principles of the working mathematically strand, within the parameters of their respective programs. Calculators were freely available, with students having their own calculators on the desk. At times access to calculators was restricted, but for the most part students were free to use the calculator, although the teacher might challenge their use. Once again a range of calculators was in use, making it difficult for the teachers to conduct whole class lessons involving the use of the calculator.

Dress was more formal in the school and students were expected to 'show respect' to teachers and visitors, so the students tended to be slightly less open in the interview situation. The setting in which the interviews took place in the Catholic school was not as conducive to discussion as the setting in the state school. This was simply due to there being more space available, rather than a devaluing of the research. Students were interviewed in a veranda setting, which meant at time interviews were interrupted by students moving past.

Developing the Instrument

The interview was based around an instrument consisting of 18 computation items (See Appendix 3), presented in both oral and written form in horizontal format. The purpose of the research was to determine what computation choices were made, and how they were made, so it was important that the questions be set at a level that invited choice. If the items were too simple then it was likely students would choose mostly mental methods. If the questions were too difficult then it was likely that calculator methods would have been favoured.

There were several other factors that needed to be considered when designing the instrument. The appropriate mix of operations, whole number, decimal, fraction and percentage items needed to be examined. The use of items within a shopping context was explored as well as mixing two operations within the one item. The shopping items were presented in terms of purchasing two items at the shop. In addition the students were shown advertisements cut out of a supermarket catalogue that showed the items and their prices. For example, Item 14, $\$1.99 + \1.99 , was presented in terms of purchasing two bottles of Pepsi. The advertisement showed bottles of Pepsi and the price was clearly marked as $\$1.99$ per bottle. Likewise, Item 15, $\$4.93 + 39c$, involved purchasing a water noodle toy and a packet of two-minute noodles. Pictures of each item cut from a supermarket catalogue and clearly marked with the price were shown to the students. The prices were shown as $\$4.93$ and $39c$, which meant the students had to make adjustments to accommodate the mix of dollars and cents, especially if using a calculator. In addition to refining the instrument, the interview protocol and technique also needed to be fine-tuned. A pilot study was conducted to explore these issues.

Pilot study

A pilot study was undertaken to refine the instrument and interview techniques to be used in the research. Gay (1992) noted that “beginning researchers gain valuable expertise from a pilot study” (p. 112). This was certainly the case in this research. The fine-tuning of the instrument and interview protocols will be discussed in the light of the findings of the pilot study.

A sample of 12 students in Years 5-7 from two schools with similar profiles to the research schools was interviewed individually as they attempted a set of 15 computation items. The interviewer and the student sat together at a table that contained pen-and-paper and a calculator. At the beginning of the interview students were told they could solve the question using whatever method they liked. Once they had calculated an answer to each item, the interviewer asked students why they chose a particular approach to solving the problem and asked them to explain how they went about solving it. Every item was treated in this way.

Field notes were taken as the students attempted to solve the problem, and also later when they explained their strategies and processes. These interviews were not audio-taped, although the intention was to audio-tape interviews as part of the major study. The focus of the pilot study was to test the instrument and interview procedure.

As a result of the pilot study the instrument was refined, with several items being removed. Some items did not draw the range of options required and therefore little data about computation choice was collected. Some items were so close in nature that similar responses were elicited. Fatigue was less of a factor than first imagined and therefore more items could be added to the instrument. Several items based on previous research studies (Hope, 1989; Price, 1995; Reys, Reys, & Hope, 1993) were added, increasing the instrument to eighteen items. The addition of these items allowed for some comparisons to be made between data collected in the main research study and that from the previous studies.

Several items were removed from the original instrument because they did not provide useful data. The results of the pilot study have been reported in detail elsewhere (Swan & Bana, 1998). Students in the pilot study displayed a lack of number sense. For example, when attempting the following item, 'half of 5 times 2', all of the students in the pilot ($n = 12$), completed the item in the order in which it appeared, rather than considering the item as a whole.

Some interesting attitudinal data was revealed in response to the item, $300 \div 6$. One Year 6 student made the following comment.

On a test I would do it on paper because I know I am not cheating myself. Using a calculator is a bit like cheating because you don't know the answer until it comes up on the screen.

It should be noted, however, that this student eventually used a calculator to attain the correct result. There were two disturbing aspects to this vignette. The first was the association of the calculator with cheating indicating a restriction on computation choice based on attitude. The second was that it appeared as though the child had no idea as to what was going to appear on the display, indicating a lack of forethought as to the sort of answer that might occur as a result of the calculation.

The pilot study helped to refine the instrument for the current study, but it also helped to focus on some key questions. In particular, questions about what triggers particular computation choices were raised. The failure of students in the pilot group to check the results of calculations they carried out raised the question as to what monitoring strategies, if any, children use when performing a calculation. The switching of one computation method to another part way through a calculation was also noted during the pilot study.

The computation instrument

An 18-item instrument was developed based on findings from the pilot study and items used in previous research of computation preference (Hope, 1989; Price, 1995; Reys et al., 1993). The same instrument was used for all year groups as it allowed for some comparisons to be made across year groups for various items. While this was not a main focus of the current study, data for individual year groups was available from previous research from which some of the items had been used (Hope, 1989; Price, 1995; Reys et al., 1993). It was also felt that these items would offer students in Years 5-7 a reasonable range of computation options, and would be within the ability of most students to solve. The instrument is shown in Table 4.2. All but two items were presented out of context in order to focus on the computation by eliminating associated extraneous variables. The researcher wished to focus on the computation process without the added burden of the item being placed in context. Two items were given in context in an attempt to explore some of the issues associated with computation choice and contextual clues. A complete version of the instrument including the in-context items is presented in Appendix 3.

Table 4.2: The eighteen-item instrument.

Number	Item	Number	Item
1	$28 + 37$	10	$14 \times 9 \div 6$
2	$74 - 36$	11	$\frac{1}{2} + \frac{3}{4}$
3	$369 \div 3$	12	$10 - 4\frac{3}{4}$
4	36×25	13	$\frac{2}{3}$ of 45
5	70×600	14	$\$1.99 + \1.99^*
6	29×31	15	$\$4.93 + 39c^*$
7	33×88	16	$7.41 - 2.5$
8	1000×945	17	0.25×800
9	10% of 750	18	$3.5 \div 0.5$

Note: * Items were presented in a shopping context.

The instrument included a mix of operations, although half the items involved multiplication. Table 4.3 indicates which items were used in previous studies and with which year groups.

Four addition items, spread across whole number, fraction and decimal questions, were included in the instrument. The first item, $28 + 37$, was used to put participants at ease. The three subtraction items were also spread across whole numbers, fractions and decimals. The first subtraction item, $74 - 36$, was also designed to ease the students into the interview and encourage dialogue. The three division items were spread across whole numbers and decimals. Item 3, $369 \div 3$, was a relatively straightforward single-digit division where there was a clear relationship between the divisor and dividend.

Item 10, $14 \times 9 \div 6$, involved a mix of operations. There was no need to apply the order of operations in this case. Most students in Year 5 and Year 6 are not aware of the conventions surrounding the rule of order. Likewise the typical four-function calculators used by primary students are not programmed to process calculations according to the conventions of the order of operations. The division component of Item 10 involved dividing 126 by 6, which is within the ability of most students in Years 5-7.

Table 4.3: Items used in other studies.

Item No.	Item	Hope (1989) Yr 5 & 7	Reys et al. (1993) Yr 5	Reys et al. (1993) Yr 7	Price (1995) Yr 5	Price (1995) Yr 5-7
3	$369 \div 3$	√				
4	36×25		√	√	√	√
5	70×600	√	√	√	√	√
6	29×31		√		√	√
7	33×88		√		√	√
8	1000×945		√			
9	10% of 750			√		
11	$\frac{1}{2} + \frac{3}{4}$			√		
12	$10 - 4\frac{3}{4}$	√				
14	$\$1.99 + \1.99	√				
17	0.25×800			√		

Two items, 5 and 8, involved the use of zeros. Item 8 was presented as 1000×945 , rather than 945×1000 , because it was shown this way in the Reys, Reys and Hope (1993) study. Both items were included, not just because of links to previous studies but because of the difficulties students seem to experience calculating with zeros (McIntosh, De Nardi & Swan, 1994). ‘Taking off zeros’ and ‘adding zeros’ are common expressions used by students when performing this type of calculation. Rules for ‘adding’ and ‘taking’ zeros are often taught to children and the difficulties tend to stem from following ‘rules without reason’. The students lack understanding of the procedure and become confused in the process of adding and taking zeros. The value of students constructing meaning is highlighted when examples such as this arise. The data gathered from these two items, proved to be most interesting, especially for Item 8, which one might imagine to be a relatively simple mental calculation.

In Item 9, the percentage question was included for several reasons. The calculation of percentages, especially 10%, is a calculation one would expect most adults to be able to complete mentally. It is an example of a calculation often used when dealing with money. The calculation of a percentage is also an example where different model calculators utilise slightly different procedures when computing to find the answer. After pressing the percent key, some models display the answer, while others show the decimal equivalent of the percentage, in this case, 0.1. If the calculator is programmed to convert the percentage to a decimal the user must then press the 'equals' key for the answer to be displayed. If the calculator does not include a percentage key then the user has to be able to convert from a percentage to a decimal before being able to make use of the calculator. A lack of familiarity with the calculator in this instance would limit computation choice.

Items 11-13 involved fractions. The least complicated method to solve these items was mental, but the use of mental methods required knowledge of equivalents and familiarity with fractions. Many students lack understanding of fractions and therefore it was felt these items would challenge many of the students. Without knowledge of how to convert a fraction to a decimal, calculators offer little or no assistance. To use a calculator students would either need to use their knowledge of equivalents to mentally convert a fraction to a decimal or understand that a fraction may be converted to a decimal by dividing the numerator by the denominator.

Items 14-15 were both given in the context of shopping (See Appendix 3 for the exact wording). Grocery items with a value of \$1.99 were cut from a catalogue and shown to the students. Students were told they were required to purchase two items at \$1.99 each and were shown the catalogue picture. There were several interesting features to this question. The first was whether students would add \$1.99 and \$1.99 or double it. The item also lends itself to the use of a compensation strategy where \$1.99 is rounded to \$2.00 and then the amounts added and the answer adjusted to cover the extra two cents.

Item 15, \$4.93 plus 39c, also provided in a shopping context, was deliberately chosen to reflect a mix of dollars and cents. Previous work with children had alerted the researcher to the difficulties students experienced using a calculator to solve this type of question. Many students entered the numbers into the calculator as $4.93 + 39$, ignoring the relationship between dollars and cents. Similar difficulties may be found when

adding measurements such as metres and millimetres. Adding the two amounts in this way produces an answer that any reasonable estimate would reject as impossible. This item was to allow the researcher to observe how students entered this type of data into the calculator and whether they were monitoring their work and recognised an unreasonable answer.

Items 16-18 involved decimals. Item 16, $7.41 - 2.5$, was difficult, as it involved the need for decomposition. Item 17, 0.25×800 , might be viewed as relatively straightforward if the 0.25 and $\frac{1}{4}$ equivalence is recognised. This relationship is only useful if restating the question as $\frac{1}{4}$ of 800 giving the student another, perhaps simpler approach to solving the item. Earlier references to number sense (McIntosh, Reys & Reys, 1992) included elements such as the use of equivalent expressions and flexibility of approach. Item 18, $3.5 \div 0.5$, may look complicated at first but by restating the question as $35 \div 5$, '3.5, how many 0.5s?' or 'How many fives in thirty-five', or $5 \times ? = 35$, the item becomes an extension of the basic facts most students in Years 5-7 should know.

Interview procedure

The interviewer collected the student from the room and asked the student to bring a pen and paper and their own calculator. On the way to the interview students were put at ease with general comments about the nature of the interview, the approximate time it would take and the anonymity of the data. The use of the word 'test' was avoided and students were informed they would be asked to answer 18 questions as part of the interview. Appendix 6 contains the exact wording used to introduce the interview.

Once seated the interviewer explained the procedure that would be followed in the interview. The interviewer and subject sat side by side at a table on which pen, paper and calculator were available. At the commencement of the interview, subjects were told they could choose whatever method of computation they liked, mental, written or calculator, or a mixture of these. Reference was made to items on the desk such as pencils and paper and a calculator. Students were free to use their own materials, especially the calculator with which they were most familiar. Students were told that the interview would be audio-taped so the interviewer could listen to the tape at a later date to verify what was said. Students were then given the opportunity to withdraw if they felt uncomfortable about the procedure.

The interviewer observed the subject complete the calculation. The interviewer then asked for an explanation of why the student chose that particular method and then the student was asked to explain the method. Next, the student was asked whether he/she could have completed the item using a different method. If the student said yes, he/she was asked which method and invited to carry out the calculation. After completing the calculation a second time, the student was asked if he/she could calculate the answer using a different method. If the response was positive then the student was asked to complete the calculation. This process is illustrated in Appendix 5.

While completing the calculation the interviewer noted non-verbal behaviours and made notes regarding any interesting comments (See Appendix 4 for a sample recording sheet). These were later reviewed by reading through the transcript of the interview or replaying of the audio-tape. Records were kept of successful and unsuccessful attempts, and of computation preference.

All interviews took place in the third term of the year over a six-week period. The audio-taped interviews were conducted by the researcher and held in whatever private space was available in each school. Audio-tapes were transcribed for later reference and analysis.

Data Analysis

Qualitative research involves an inductive approach to data analysis, which means that categories or themes emerge from the data (Patton, 1990). The data need to be examined, worked, reworked and grouped until the groupings become clear. Decisions then have to be made as to which groupings or categories are significant in terms of the research questions being asked. The process of data analysis is one that involves tentatively formulating categories, returning to the data, modifying the categories and drawing out common themes. Bogdan and Biklen (1982) described qualitative analysis as:

Working with data, organising it, breaking it into manageable units, synthesizing it, searching for patterns, discovering what is important and what is to be learned, and deciding what you will tell others (p. 145).

At first it may appear as though qualitative data analysis is a linear process. Miles and Huberman (1984, p. 23), however, indicated that there was a strong interplay between various components of data analysis. The components and relationships between each are shown in Figure 4.1.

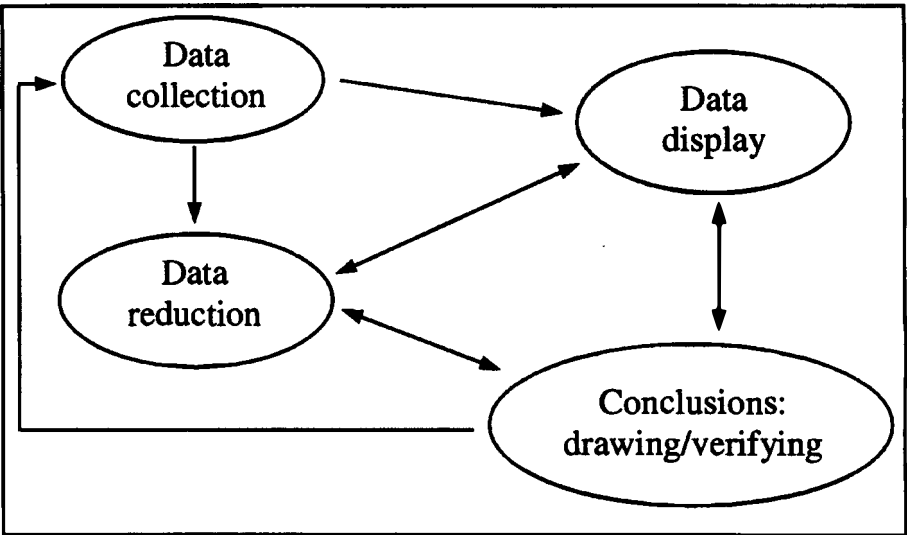


Figure 4.1: Components of data analysis (Miles & Huberman, 1984, p. 23).

As Figure 4.1 indicates, data analysis consists of three concurrent activities, data reduction, data display and conclusion drawing. As most of the data collected was in word form, rather than numbers, data needed to be organised under themes. Data reduction refers to the organising of raw data. While organisation systems may vary, the purpose was to group data so conclusions may be drawn. Assembling the data into a form that may be used to draw conclusions involves displaying or presenting the data into a manageable and understandable form. The process of data analysis allows for conclusions to be reached. To reach conclusions, patterns within the data need to be found and then tested. In this section the process of data analysis is described.

Firstly, data of computation choice were tabulated according to records kept on the interview sheets. Where this was unclear, transcripts and audio-tapes were used to clarify the method used by students to solve the item. These data were recorded in a table and simple statistics such as the mean were calculated with the assistance of a spreadsheet, used to describe the computation choices made by students. These data were used to report the answer to the first research question. The computation choices made by the students were relatively simple to compile as the choices were limited to mental, written or calculator, or a combination of these.

In order to answer the second research question, students were asked to explain why they had made the particular computation choice that they did. The data were audio-taped and transcribed. In addition, notes were made in the field. A great deal can be gained by examining raw data but the volume of data soon becomes unwieldy and decisions have to be made as to the means of reducing the data to a manageable form. Examination of raw data soon produces themes. Coding of data along with summaries help to reduce the volume of data and allow for patterns to be observed. The transcribed interviews were matched to the field notes and observations taken at the time of the initial interview were used to filter data. From this initial screening of the data broad categories were noted and data sorted according to these. So much occurs during an interview that some data may be overlooked, so it is important to revisit transcripts of the interview and at times the original audio recording. In transcribing from audio to written text, for example, voice inflections and emphases are lost and therefore where the meaning is sometimes unclear there is a need to return to the raw data.

The reasons for making computation choices were gathered under broad headings and reported as such. To ensure the clearest reporting of data, excerpts of various interviews have been reported verbatim to illustrate 'typical responses' under each category. As with any data consolidation some data are lost. A separate reporting of unusual responses or responses that did not fit specific categories has been included to provide a more complete picture of computation choice and the reasons behind the choices that were made (See Appendix 7).

The third research question was closely associated with the first, as it focussed on how successful students were in executing their chosen computation method. A spreadsheet was used to tabulate data and calculate the percentage of correct answers according to chosen strategy. Data were collected at the time students completed the eighteen items. These data were verified by referring to the interview transcripts. Data recording second and third choices, where applicable, were also kept. Some students were only able to complete certain items using a single method, while others used a different method but were unsuccessful. Still others who were unsuccessful using their first preference at times were successful when employing their second or third preferences. These data have all been tabulated to identify patterns and trends.

Limitations

The merits of quantitative and qualitative methodology have been debated over a long period (Patton, 1990). Each paradigm is based on different assumptions and as such each has its own strengths and weaknesses. One of the criticisms of qualitative research is that it is difficult to establish reliability and validity. In this section, the threats to reliability and validity related to a study employing clinical interviews as the main source of data are discussed, along with the measures taken to reduce these threats.

Issues of reliability and validity

Hammersley (1987) noted there was a large amount of literature concerned with the concepts of reliability and validity, along with techniques for reducing threats to reliability and validity. He commented: “when one looks at discussions of reliability and validity one finds not a clear set of definitions but a confusing set of ideas” (p. 73). Essentially, however, the issue of reliability is one of consistency. Would similar results be produced by the same researcher or another researcher using the same instrument and process? Validity involves examining the instrument and the process, to determine how well it measures what it is supposed to measure.

Bell (1987) defined reliability as “the extent to which a procedure produces similar results under constant conditions on all occasions” (p. 51). When this is applied to the clinical interview, the prime data collection method used in this research, then Bell (1987) suggested the researcher needed to ask, “Would two interviewers using the schedule or procedure get similar results? Would an interviewer obtain a similar picture using the procedures on different occasions?” (p. 51).

According to Bell (1987) “validity tells whether an item measures or describes what it is supposed to measure or describe” (p. 51). Cohen and Manion (1980) noted one of the major threats to validity in research that employed interviews as the main data-gathering instrument was bias. One way to help establish validity is to compare the data gathered with other data that has already been established as being valid. Eisner (1991) referred to the use of multiple sources of evidence as a means of providing credence to the data that are collected and the conclusions reached. While the use of interviews and corroborating observations and samples of student work do not allow for triangulation of data, they may be used to support each other. In addition, the 18-item

instrument developed for this research included some items from previous research studies on computation choice (See Table 4.3). The specific results for common items are compared in later chapters. The results from previous research on the computation choices made by students were used as a form of comparison to determine whether the general trends of this study were consistent with those of previous research.

Issues associated with the use of interviews

The main weakness associated with the use of the clinical interview revolves around the reliance on the verbal reflections of the subject. The quality of the data is very much dependent on the ability of the subjects to recall and then explain in precise terms what they did. This is one reason why older students were chosen for this study. Young children experience difficulty clearly enunciating their thoughts. Older students are more able to express their thoughts in a lucid manner.

Ginsburg et al. (1983) discussed some issues associated with the use of interviews. Most problems centre on the reliance of students' verbal reports. The assumption is that students are able to remember what they were thinking while solving a problem and also have the vocabulary required to describe their thoughts. Ginsburg et al. (1983) noted issues with memory decay and interference along with "the possibility that calling attention to the need for reports may make subjects self-conscious and lead them to employ different strategies or means than they might if left on their own" (p. 30). The reliance on student reflections, however, is still considered to be the best way of determining what students are thinking when solving problems. When it comes to mental activity and processes, however, Ginsburg et al. (1983) conceded "psychological research *must* rely on verbal reports, for no one else could possibly be in a *position* to observe them" (p. 23).

Hunting (1983) outlined the four key weaknesses associated with the use of the clinical interview. These weaknesses will each be considered in turn.

- The lack of standardised procedures;
- The inability to precisely replicate the research;
- The reliance on the skills of the interviewer; and
- The questionable reliability of one-off interviews (p. 48).

While qualitative research is sometimes seen as lacking standardised procedure, measures such as the use of an interview protocol and set interview items helped to maintain the uniformity of the interview. A pilot study assisted in streamlining the instrument and interview procedure.

It is certainly true that in qualitative research it is difficult to precisely replicate because it is impossible to control all variables. Clear descriptions of the school, the classroom setting along with the teaching style help to indicate the setting in which the research was situated and assist anyone wishing to replicate the study.

The skills of the interviewer, or lack thereof, can pose a threat to the integrity of the research. Swanson, Schwartz, Ginsburg and Kosan (1981), warned that the interviewer must avoid putting words into the subject's mouth. In this case the research involved a single interviewer who was also the researcher, so the opportunity for variation between interviewers did not exist and the possibility of variation between interviews was reduced. The interviewer was experienced, having used the clinical interview technique for gathering data in prior research (Swan, 1991). A protocol was adopted that employed predetermined probes in order to counteract this tendency. Transcripts were examined to determine whether the interviewer had 'led the subject' to a particular response. Lincoln and Guba (1985) noted that one technique for addressing credibility issues associated with drawing conclusions from qualitative data involves making samples of raw data available for others to analyse. A colleague with background in qualitative research was asked to review a sample of transcripts in order to compare conclusions that were reached. On comparison, similar conclusions were reached.

There are issues related to the use of single interviews but the purpose of this study was to determine what computation choices were being made and why. A series of interviews with the same subject, spaced over a period of time would indicate a pattern of computation choices and the level of consistency with which students made those choices. The purpose of this research, however, was to gather a knowledge base on the computation choices made by students in years 5-7 and why they made those choices.

While it may be impossible to replicate the research in precisely the same manner in which it was carried out, all procedures have been clearly documented so that another researcher could perform the same research. The school settings and classroom programs would differ, but learning takes place in a fluid social setting and this dynamic, while being impossible to control, is part of the nature of research in education settings. LeCompte and Goetz (1982) stated:

Attaining absolute validity and reliability is an impossible goal for any research model. Nevertheless investigators may approach these objectives by conscientious balancing of various factors enhancing credibility within the context of their particular research problems and goals (p. 55).

This does not mean that one should not try to control variables that may affect reliability and validity but rather that reasonable steps should be taken to do so, while acknowledging that controlling all the complex variables associated with human thinking is impossible. Hiebert (1999) acknowledged that research in education is particularly complex because of the diverse nature of classrooms, students and teachers. He stated that “most outcomes are influenced by more factors than we can identify, let alone control” (p. 6). He goes on to explain that this does not mean research is a waste of time but rather, “the clearer the results, the more confident we are that we are making good decisions. We make decisions with levels of confidence, not with certainty” (p. 6).

Herrington (1997) compiled a list of procedures, based on the literature, which may be used to reduce the threats to reliability and validity of qualitative research. The list is reproduced in Table 4.4. Sparrow (2000) applied these procedures when establishing the reliability and validity of his qualitative research. A similar approach was adopted in this research. The methods by which threats to validity were reduced in this research are outlined in the right hand column.

Table 4.4: Procedures to reduce threats to validity in qualitative research.

Procedure	Implementation in this study
Use of structural corroboration or triangulation by the use of multiple sources of data (Eisner, 1991; Miles & Huberman, 1994)	Corroboration by method, (interview and observation and collection of written work) and across data sets i.e. comparison with similar studies (Hope, 1989; Price, 1995; Reys et al., 1993)
Collection of referential materials e.g. documents, recordings, against which findings can be tested (Eisner, 1991; Guba, 1981)	Transcripts of audio-taped interviews, field notes and student work.
Consensual validation, or agreement among other researchers that the description and interpretation of data are right (Eisner, 1991; Guba, 1981).	Reviewed by colleagues with experience in this type of research.
Checking for research effects (Miles & Huberman, 1994).	Researcher did not offer opinions during the interview and tried to keep a low profile. However, the researcher would have been viewed as another teacher given the setting and circumstances under which the research took place.
Obtaining confirmatory feedback from the participants (Guba, 1981; Miles & Huberman, 1994).	Interview instrument contained similar items that could be used for comparison. When students stated they would use a particular computation method without demonstrating they could use that method they were invited to do so to confirm they could.

Note: Adapted from Herrington, 1997.

Table 4.4 provides a summary of the main techniques used to reduce the threats to reliability and validity in this research. Clearly there are general limiting factors to research of this type and this particular piece of research. There are several factors that would have influenced the data that were gathered. These included the:

- setting in which the interviews took place;
- perceived role of the interviewer as a teacher;
- request to describe the thinking behind the decision to use a particular method; and
- request to describe the way in which the problem was solved (this clearly had a bearing on the second and third choices).

Clearly the setting in which the interview took place, the request to explain their thinking, the presence of a ‘teacher interviewer’ along with the asking of probing questions are all factors that may have influenced the responses of the subjects. Asking the students to explain their way of calculating may have influenced second and third choices. For example, after completing an item with pen and paper a student might then opt to use a mental method, with the benefit of having just completed the calculation and already having the answer in mind. In the short period of time that was spent interviewing the students a relationship could not be developed, therefore it is likely students viewed the interviewer as a teacher trying to extract information. LeCompte and Goetz (1982), referred to this threat to external reliability as “researcher status position” (p. 37), which they described as the perception the subjects have of the position that the researcher holds. This may act to alter or restrict the flow of information to the researcher. As such it is possible that students may have tailored their responses to closer resemble what they thought the interviewer might wish to hear. The study was situated in a classroom environment, therefore it is only logical to conclude that this context had a bearing on the results.

These points were all taken into consideration when analysing the data. The data analysis follows in the next three chapters. Each chapter is designed to answer one of the stated research questions.

Chapter 5: Results For Question 1

In the previous chapter reference was made to the collection and sorting of quantitative and qualitative data used to answer each of the research questions. This chapter begins the analysis of those data. Research question one: *When faced with a computation question, what choices of computation method do students in Years 5-7 make?* will be addressed. Data were entered into a spreadsheet and sorted in order to answer the question. Data were then analysed according to the percentage of students choosing particular computation methods.

Firstly, general trends shown by the overall data will be discussed. As the chapter continues the data will be examined more closely. Analysis will progress from considering the initial choice for all items, to considering individual items. Data will be sorted to show patterns for year groups and question types. Where items common to previous studies were used comparisons will be made.

All data are tabulated and presented as percentages to allow for easy comparisons to be made. The sample size of 78 meant that some figures needed to be rounded as only whole numbers are reported in the tables. Thus, at times, the rounding caused some data sets to not total 100.

Large amounts of data can be confusing and hide specific trends, so to begin with, only the initial computation choice for the entire 18 items will be presented. Only first computation choice data are presented. Data were also collected indicating second and third choices. These will not be examined in detail as not all students demonstrated an ability to make and execute a second or third choice. Comments relating to second and third choice will be limited to general trends.

Initial Choice for all Items

Students were instructed to solve each item using the computation method with which they felt most comfortable. As discussed in Chapter 4 it is possible that the setting and the presence of an interviewer, regarded by most students as a ‘teacher’ may have skewed the choices made by the subjects. The first or initial choice of computation method was recorded for each item based on observations and subsequent discussion. On occasion students chose an initial approach and abandoned this approach part way through the solution process. The initial computation approach was recorded and the result noted as incorrect. Students were then free to pursue an alternate method.

If students immediately moved to a second method for calculating the answer this was noted on the recording sheet. If they stopped calculating they were prompted by the question “Can you solve it another way?”. If the prompt drew a positive response the student was asked to perform the calculation.

Table 5.1 shows overall initial computation choice data for the 18 items. The success rate for using the chosen method is not indicated and neither is the students’ ability to make use of an alternative computation method. Further analysis of that data will be provided later.

Table 5.1: Percentage distributions of initial computation for all items ($n = 78$)

Mental	Written	Calculator	Mixed	No Method
36	26	28	6	5

The data indicate a preference for mental methods. Mental methods were favoured as a first computation choice in 36% of cases. Written methods accounted for 26% of all first computation choices and 28% chose calculator methods. In some cases it was impossible to separate the mix of methods used and this represented 6% of the total. The data also indicate that 5% were unable to choose a method to start the computation. When combining data, information to explain the result is often lost. For example the ‘no method’ category was boosted by one particular set of items and also affected by one year group. Data broken down by item and year group are considered later in the chapter.

The general trend outlined in Table 5.1 suggests that students are exercising a choice. No one particular computation method dominates to the exclusion of other methods. An examination of raw data indicated students varied their methods. No evidence was found to indicate that any individual student had used a single computation method for all 18 items. The data also indicate less reliance on written methods than expected.

While data may not be directly compared, due to differences in sample size and the instruments used, various trends from two previous studies (Price, 1995; Reys et al., 1993) and raw data collected by Hope (1989) differ slightly from the above result. Hope (1989) collected data about the computation choices made by Year 5 and Year 7 students in Canada. Reys, Reys and Hope (1993) asked American students in Years 5 and Year 7 to state what method they would use to complete multiplication items. Price (1995) studied the computation choices of students in Years 5 to 7 in Queensland where students were observed completing multiplication items. Some of the items were the same across these various studies and the present study. Direct comparisons according to common items and year level will be made later in the chapter.

As a general trend students in the present study and the Price study tended to make more use of calculators than students in the American and Canadian research. Similar patterns of mental computation were noted across the present study and the Reys et al. (1993) study, although less mental methods were noted in the Queensland research. The students in the present study showed less reliance on written methods than in previous studies. It appears that rather than make more use of mental methods, however, students opted for more calculator use.

In most cases there was little or no hesitation when making the choice as to which method to use when solving an item. There was little evidence to support the notion that students carefully examine a question before choosing a computation method. On a few occasions it was observed that students, having embarked on a particular method, found that it was inappropriate and abandoned it in favour of another. When considering computation choice for individual items, certain patterns begin to emerge. These are taken up in the next section.

Computation Choice by Item

Table 5.2 outlines computation choice by item. There are some clear trends indicated in the table. These include the favouring of written methods for the multiplication items and the use of mental methods for the two items given in the shopping context. This table does not give an indication of success related to the adoption of a particular method, only the initial computation choice made by the students.

Table 5.2: Percentage distributions of initial computation choice for all items ($n = 78$)

Item number	Item	Mental	Written	Calculator	Mixed	No method
1	$28 + 37$	65	23	8	4	0
2	$74 - 36$	35	54	9	2	0
3	$369 \div 3$	32	21	46	1	0
4	36×25	15	50	31	3	1
5	70×600	37	19	37	6	0
6	29×31	15	46	35	4	0
7	33×88	9	50	35	5	0
8	1000×945	28	14	58	0	0
9	10% of 750	19	4	36	1	40
10	$14 \times 9 \div 6$	10	29	36	23	1
11	$\frac{1}{2} + \frac{3}{4}$	60	22	5	3	10
12	$10 - 4\frac{3}{4}$	77	6	4	4	10
13	$\frac{2}{3}$ of 45	27	10	9	21	33
14*	$\$1.99 + \1.99	68	26	4	2	0
15*	$\$4.93 + 39c$	49	36	11	4	1
16	$7.41 - 2.5$	35	33	28	4	0
17	0.25×800	8	8	70	12	2
18	$3.5 \div 0.5$	50	9	39	1	1
	Total	36	26	28	6	5

Note: * Item given in context.

In most cases a fairly definite pattern is formed. Initial computation choice in items 1, 2, 4, 7, 8, 11, 12, 14, 17 and 18 were very strong. In each case at least 50% of computation choices favoured one particular method. In three other items, 3, 6 and 15, computation choice was in the high 40% range. The data indicate that two items – Item 9, the percentage item and Item 13, a fraction item – caused students the most difficulty in making computation choices. If, as opponents of calculator use in primary school suggest, little thinking is required when using a calculator, then calculator use might be considered the ‘default choice’ for children unsure of how to begin or proceed with a calculation. The data for Item 13 would suggest otherwise.

Computation choice was fairly evenly spread across mental and calculator methods for Item 5, which involved zeros. Item 8, the second one involving zeros, showed a different trend. It is possible the presentation of this item, as 1000×945 , rather than 945×1000 , caused students to adopt a different computation method. A comparison of these two items indicates that students did not appear to have a standard approach for item types, but rather treated each item on its merits. Items for which students showed a preference for mental, written or calculator methods will now be examined.

Items for which mental methods were preferred

The data in Table 5.3 indicate that the students in the study preferred to use mental methods to solve the following: Item 1, $28 + 37$; Item 11, $\frac{1}{2} + \frac{3}{4}$; Item 12, $10 - 4\frac{3}{4}$; Item 14, $\$1.99 + \1.99 , given in a shopping context; Item 15, $\$4.93 + 39c$, given in a shopping context; Item 16, $7.41 - 2.5$; and Item 18, $3.5 \div 0.5$.

Table 5.3: Percentage distributions for items in which mental methods were preferred ($n = 78$)

Item number	Item	Mental	Written	Calculator	Mixed	No method
1	$28 + 37$	65	23	8	4	0
11	$\frac{1}{2} + \frac{3}{4}$	60	22	5	3	10
12	$10 - 4\frac{3}{4}$	77	6	4	4	10
14	$\$1.99 + \1.99	68	26	4	2	0
15	$\$4.93 + 39c$	49	36	11	4	1
16	$7.41 - 2.5$	35	33	28	4	0
18	$3.5 \div 0.5$	50	9	39	1	1
	Total	59	22	14	3	3

Note: Due to rounding, numbers do not always add to 100.

Table 5.3 indicates that preferences for mental computation were strongest for items 1, 11, 12 and 14, with preference dropping to around 50% for items 15 and 18. A slight preference for mental methods compared with written methods was recorded for Item 16.

Mental methods were mostly favoured in simpler questions – those involving fractions and decimals and those given in a shopping context. It may seem somewhat surprising to see mental methods as the preferred choice for most fraction related items, but it appears that most of the students were unable to solve this type of question using a paper-and-pencil algorithm and did not know how to use a calculator to assist them and hence only had one option at their disposal. The fraction item $\frac{2}{3}$ of 45, could be viewed as involving an element of division and was mostly completed mentally, whereas calculator methods were preferred for other items involving division.

Item 5, 70×600 , was unusual in the sense that computation preference was equally distributed between mental and calculator methods (See Table 5.4). This tends to indicate that students do not simply move from mental to written and finally calculator methods, but rather make a conscious decision to use a particular method. It is of concern that 37% of students used a calculator to solve an item like 70×600 but it does highlight the point that while items involving zeros may seem relatively simple to

compute, they are much more difficult for students than one might imagine. Students become confused with what to do with all the zeros, with reference being made to various half-remembered rules. Choosing not to use mental methods may be most appropriate if students are confused about what to do with the zeros, or if the process of ‘taking off and adding zeros’, as many students describe it, causes memory problems. It certainly does not make sense to complete the item with paper-and-pencil, so using a calculator may be the best option.

Table 5.4: Percentage distributions for items in which mental and calculator methods were preferred ($n = 78$)

Item number	Item	Mental	Written	Calculator	Mixed	No method
5	70×600	37	19	37	6	0

Items for which written methods were preferred

Preference for written methods was shown in the following: Item 2, $74 - 36$; Item 4, 36×25 ; Item 6, 29×31 ; and Item 7, 33×88 . Two-digit multiplication items dominate in this category. The four items that came up in this category closely resemble the types of questions typically given in mathematics textbooks as exercises, to be completed in written form. The Year 5 students recently had completed work from their school texts involving two-digit by two-digit multiplication. Students often spend a great deal of classroom time completing numerous questions of this type using standard written methods (Porter, 1989). It appears as though the cognitive demands of two-digit multiplication questions are such that students in Years 5-7 are unable to complete this type of question mentally and therefore need to use another method. Cognitive demands may also be increased because mental methods for solving this type of question are often restricted to mental versions of the standard written algorithm. Standard written algorithms are designed to be completed using external recording devices for interim steps, making them inefficient as the basis for mental methods.

The strength of preference for written methods for these items may be noted in Table 5.5. In each case almost 50% of students indicated a preference for using written methods.

Table 5.5: Percentage distributions for items in which written methods were preferred (*n* = 78)

Item number	Item	Mental	Written	Calculator	Mixed	No method
2	74 – 36	35	54	9	2	0
4	36 x 25	15	50	31	3	1
6	29 x 31	15	46	35	4	0
7	33 x 88	9	50	35	5	0
	Total	18	50	28	4	0*

Note: Data rounded.

There was little evidence of students using self-generated or invented algorithms. This is possibly a reflection that self-generated written methods are generally not encouraged in primary classrooms in Western Australia. Most students used the standard methods taught in Western Australian primary schools. It was noted that some students, particularly in Year 5, applied a faulty version of the written algorithm for multiplication. This problem is discussed in more detail in Chapter 6.

Items for which calculator methods were preferred

Calculators were the preferred choice in the following items: Item 3, $369 \div 3$; Item 8, 1000×945 ; Item 9, 10% of 750; Item 10, $14 \times 9 \div 6$; and Item 17, 0.25×800 . Table 5.6 indicates that preference for calculator use was very strong for all items, except Item 10, which involved the highest percentage of mixed methods. It is of concern that so many students opted to complete a calculation like 1000×945 using a calculator. This item appears to be relatively simple to calculate using mental methods, although one can only speculate whether a similar result would have been found if the item were presented as 945×1000 .

Table 5.6: Percentage distributions for items in which calculator methods were preferred ($n = 78$)

Item number	Item	Mental	Written	Calculator	Mixed	No method
3	$369 \div 3$	32	21	46	1	0
8	1000×945	28	14	58	0	0
10	$14 \times 9 \div 6$	10	29	36	23	1
17	0.25×800	8	8	70	12	2
	Total	20	18	52	9	1

Item 3, $369 \div 3$, involved a single-digit division where there was a relatively simple relationship between the divisor and the dividend. Interview evidence suggested, that the students focussed more on the operation than on the numbers involved in the question and therefore chose to use a calculator.

Thirty-six percent of students chose to use a calculator method to solve Item 10, $14 \times 9 \div 6$. This item was the one most likely to invoke the use of combined methods of solution, with the first part being solved using one method and the division part using another method. Twenty-three percent of students used mixed methods to solve this item. Reasons for choosing to use a calculator are discussed in the next chapter.

Items for which making a choice caused most difficulty

Table 5.7 indicates those items for which making a computation choice proved difficult. Two items caused students difficulty when trying to make a computation choice. Item 9 involved the calculation of a percentage. Forty-percent of students chose not to attempt this item. When responding to Item 9, 10% of 750, many students stated that they were unfamiliar with percentages and therefore did not know how to proceed. Some students who were unsure of what to do chose to use a calculator simply because they knew it had a percentage key. They did not necessarily know how to use the percentage key and simply assumed pressing the key in conjunction with 750 would produce the desired result.

One-third of students chose not to attempt Item 13, $\frac{2}{3}$ of 45. Students who did try to solve the item generally chose to use a mental method or a mix of methods, which always involved the use of mental computation. Of the 16 students who chose mixed methods, seven used a calculator/mental combination. The calculator/mental approach basically involved using the calculator to divide 45 by 3 and then the answer of 15 was mentally doubled.

Table 5.7: Percentage distributions for items that caused most difficulty ($n = 78$)

Item number	Item	Mental	Written	Calculator	Mixed	No method
9	10% of 750	19	4	36	1	40
13	$\frac{2}{3}$ of 45	27	10	9	21	33

Comparisons to Other Research

Items 3 to 9, 11, 12, 14 and 17 had been used in one or more previous studies (Hope, 1989; Price, 1997; Reys et al., 1993). Direct comparisons between data in the current study and previous studies cannot be made because of slight differences in the ages of the students involved in each study, the way the items were administered, and recency and sampling factors. In both the Hope (1989) and the Reys et al. (1993) studies students were asked how they would solve particular questions, but the students were not asked to perform the calculation. In the research carried out by Price (1995) and the present research, students not only chose the method of solution they preferred but also had to perform the calculation using the chosen method. The sample sizes among the three studies varied markedly. Broad comparisons, however, will be made. Table 4.3 indicated which items were used in the current research and in previous research. In some cases data are only reported for specific year levels. These are also shown in Table 4.3. In the case of research undertaken by Price, data for Year 5 and data for Years 5 to 7 combined are available.

General observations for comparative items

Price (1995) found that students in the two previous studies (Hope, 1989; Reys et al., 1993) favoured mental methods more than the students in his study. He also noted that the students in his study favoured calculator use more than students in the previous studies. Earlier studies (Hope, 1989; Reys et al., 1993) found that the students preferred written methods above mental or calculator methods. Australian students tended to make more use of calculators. This may in part be due to the fact that data in the other studies were collected in the late 1980s and early 1990s, whereas data in the Australian study was collected in the mid 1990s, possibly reflecting higher take-up rates for the technology as the decade progressed.

Similar patterns of mental computation use are evident, particularly when data from the current study and the Reys et al, (1993) study are analysed. Students in the present study showed less reliance on written methods than students in previous studies. While not too much can be drawn from this data for reasons outlined earlier, it is reasonable to suggest that Australian students tend to make less use of written methods and greater use of calculators, when completing two and three-digit multiplication items.

The study that most closely resembled the current study was the one carried out by Price (1995). Four items were common to the present study and the one made by Price (1995). Similar Year levels and sample sizes were used. Students were interviewed and asked to perform their chosen method of computation. Some aspects of the research differed, such as, in the way some items were presented, but it would be appropriate to compare data. The comparative data are presented in Table 5.8.

Table 5.8: Items common to the current study and the Price study

Item Number	Item	Mental		Written		Calculator	
		Price	Swan	Price	Swan	Price	Swan
4	36 x 25	3	15	70	50	27	31
5	70 x 600	41	37	33	19	26	37
6	29 x 31	5	15	56	46	39	35
7	33 x 88	3	9	66	50	31	35

Table 5.8 presents data for items common to the Price (1995) and current study. The data indicate that computation preference was similar in both studies, but students in the current study tended to show more preference for mental methods, except for Item 5 involving zeros. They also showed less preference for written methods and in most cases (except Item 6) more inclination to use calculator methods. Price (1995) also found that “two-digit questions were approached more often with a calculator than were other number types, and were attempted mentally much less often” (p. 58).

Year by Year Analysis

It was not the purpose of this research to make comparisons across year groups. Year 5 to 7 students were chosen because it was felt these students were experienced enough with various computation options and types to be able to make an informed choice. Data were categorised according to year group to allow comparative data from previous studies incorporating some of the same items to be reported. In doing so, some interesting trends were noted and are reported here.

The majority of the 78 students who participated in the research were drawn from Year 6, although some of these were taken from a split class of Year 6/7. Thirty-seven students in all were drawn from Year 6, 26 from Year 7 and 15 from Year 5. The overall computation choice for all items by year level is presented in Table 5.9.

Table 5.9: Computation choice percentages for all items according to year level

	Year 5 (<i>n</i> = 15)			Year 6 (<i>n</i> = 37)			Year 7 (<i>n</i> = 26)		
Method	M	W	C	M	W	C	M	W	C
% Choice	41	28	25	40	25	28	37	27	32

The data indicate a consistent pattern across year levels; with mental methods declining slightly and calculator use increasing with age. It should be noted that there are too many variables involved to interpret much into this data.

Table 5.10 shows computation choice for individual items according to year level. When computation choice, according to item is considered, some interesting comparisons may be made. Variations exist not only in strength of preference for a particular computation choice, but in some cases the preferred choices differ.

Table 5.10: Computation choice percentages according to year level and item

Item	Year 5 (<i>n</i> = 15)			Year 6 (<i>n</i> = 37)			Year 7 (<i>n</i> = 26)		
	M	W	C	M	W	C	M	W	C
28 + 37	60	27	13	84	11	3	54	34	12
74 – 36	33	60	7	49	46	5	23	62	15
369 ÷ 3	40	27	33	32	22	46	31	15	54
36 x 25	33	47	20	22	40	38	4	65	31
70 x 600	33	33	33	40	19	40	50	15	35
29 x 31	27	27	46	27	46	27	4	58	38
33 x 88	27	40	33	11	46	43	8	65	27
1000 x 945	13	40	47	32	5	62	31	12	58
10% of 750	33	0	33	13	5	35	23	4	38
14 x 9 ÷ 6	26	40	33	24	38	35	19	27	54
$\frac{1}{2} + \frac{3}{4}$	93	7	0	54	27	5	54	27	8
$10 - 4\frac{3}{4}$	87	7	0	81	8	3	73	8	8
$\frac{2}{3}$ of 45	40	7	0	35	19	22	35	8	23
\$1.99+\$1.99	53	47	0	64	32	3	88	4	8
\$4.93 + 39c	40	47	13	57	40	3	54	23	23
7.41 – 2.5	40	27	27	35	35	24	31	35	35
0.25 x 800	33	7	60	11	5	78	19	11	69
3.5 ÷ 0.5	33	13	53	54	11	32	58	4	38
Overall	41	28	25	40	25	28	37	27	32

Note: The percentages will not necessarily add to 100 as this table does not include data for mixed methods or no method.

Table 5.10 highlights the consistency of choice for the two-digit addition item, most of the two-digit multiplication items and all of the fraction items. Reference to the variation in individual item preference is made later when discussing the results for each item.

Hope's (1989) data indicated considerable variation in computation choice according to year level. A later study by Reys et al. (1993) found that a similar variation existed. Price (1995) also found variation in computation choice according to year level. He noted that Year 6 students were more likely to choose written computation and Year 7, to choose calculators. He stated:

The least variation between year levels was observed in choice of mental computation, and the greatest variation was in the use of paper-and-pencil...[Y]ear 6 students' choices varied from both year 5 and year 7 students' in all three computation methods...Year 6 students chose written methods more than either year 5 or year 7 students, and chose to use calculator or mental methods less often than students of either of the other two year levels (p. 59).

This result needs to be considered in the light of the questions that were asked as part of the Price (1995) study. Most of the items used were two-digit by two-digit multiplication questions. The results from the Price (1995) study indicated that the "type of numbers in each question did influence the choices made by the subjects" (p. 54). Items common to the current study and the Price study are shown in Table 5.9. It should also be noted that in some cases Price presented his items in symbols (i.e. 36×25) and in other cases he presented them in word form or context. Of interest is the finding by Price (1995) that "there was no significant relationship at all between question format and computation method" (p. 60). Those items where variation in computation choice existed across year levels were: Item 2, $74 - 36$, Item 3, $369 \div 3$, Item 5, 70×600 , Item 6, 29×31 , Item 9, 10% of 750, Item 10, $14 \times 9 \div 6$, Item 15, $\$4.93 + 39c$, Item 16, $7.41 - 2.5$ and Item 18, $3.5 \div 0.5$.

Item by Item Discussion

The following discussion relates to individual items and involves data from field notes that were taken at the time of the interviews and by examining the interview transcript for each item. Where required, transcripts from the interviews have been included to support the discussion.

Item 1: $28 + 37$

This item was the first the students were required to solve and therefore they tended to be a little anxious. The relatively simple nature of the item tended to make most students feel comfortable with the interview procedure. It is possible, however, that the number of students choosing to apply written methods to solve this question may have been higher than might otherwise have been the case. It appeared as though some of the students chose paper-and-pencil methods because of their anxiety and they felt safer using a 'comfortable' method. Eight percent of students chose to use a calculator. Some of these students may have chosen to use a calculator because of anxiety and others to test whether they were really allowed to use calculators. From their perspective it may have looked like a test. There were a small number of students in the study that used a calculator to solve most, but not all items. At least one of these students had a very poor understanding of computation and relied on the calculator in most, but not all instances. Price (1995) did find some students who used the calculator for every item and conversely some students who did not choose to use a calculator at all. There was no evidence in this research to support the notion that students will make indiscriminate use of calculators when given the opportunity.

The 65% of students who chose to solve the question mentally used a variety of methods including making use of the written algorithm approach in their mind, working from right to left. Many students completed the item by adding parts of one number to the other, such as $28 + 2 + 35$, to make the question simpler. An example of one mental strategy used by many of the students is given below.

S: 8 plus 7 equals 15, 15 plus 20 equals 35, 35 plus 30 equals 65.

While Item 1 reveals a consistent pattern of choosing mental computation, the Year 6 students clearly showed much more disposition toward mental methods. It is possible that Year 6 students were more experienced at completing this type of question mentally and were more confident in their ability to solve this item using mental methods. A number sense program was being explicitly taught at the school the Year 6 students attended.

Item 2: $74 - 36$

Written methods were favoured for this item. Comments from the students indicated that they found subtraction more difficult than addition. Many of the students realised that the question involved some sort of ‘borrowing’, indicating they had spent a little time looking at the item before choosing a method of computation. Students who performed the written algorithm followed the standard method taught in school, although some students made the error of taking the four away from the six, making the calculation considerably easier, though incorrect. A similar phenomenon occurred in students using mental methods, especially those who opted to use a written algorithm approach in mental form.

Note the explanation of how one student completed the problem mentally. At first it appears as though she is about to solve the item using a paper-and-pencil approach in her mind but later her explanation reveals that she is working from the left, rather than the right as one would expect in the written algorithm. Using this approach means she has to compensate later.

S: I kind of did it in my mind. I put 74 and 36 underneath, 70 take 30 is 40 and then I would take two extra because there's two left over from four take six.

This level of flexible thinking was rare. Often students used a mental version of the written algorithm and then experienced trouble with the renaming of 74 as 60 and 14 ones. Year 6 students slightly favoured mental methods over written methods for Item 2, $74 - 36$, but Year 5 and Year 7 students clearly favoured written methods. It is possible that the Year 5 students were less confident in their mental ability. Year 7 students may have become well versed in written methods by this stage of their school career and felt secure in adopting this method.

Item 3: $369 \div 3$

Forty-six percent of students chose to use calculators on this item, primarily because it involved division. Year 5 students differed in computation choice for this item, favouring mental methods, whereas Year 6-7 students mostly opted to use calculators. For some students it appears as though the division operation raises concerns about their ability to complete the problem using mental or written methods and so they opt for the safety of the calculator. Those who paused to look at the numbers tended to choose mental computation. The digits in the number are all divisible

by three and most of those who chose to use a mental method referred to the three times table and how the number 369 was made up of numbers in the three times table. This tended to give students the confidence to try solving the item mentally. Twenty-one percent of the students chose to use a written method, although in observing the students it was noted that many students simply wrote down the question and the answer, having computed the result either as they wrote the question down or prior to writing it. It appeared as though they felt more confident by writing down the question. Some children after calculating the answer with pen-and-paper or with calculator commented that they could have done it mentally. When asked to describe his computation choice one student commented “I’d do it like I was writing it in my head” indicating the adoption of a written approach, carried out mentally. The student, however, was not able to calculate the result using his preferred method and approach.

The following mental approach to Item 3 was interesting, although not common. It had been expected that more students might use this method but few did. After being asked to describe her mental method the student replied “three hundred divided into three would be 100, and then there’s twenty more threes, because two 30’s are 60 and then three more, so it’s 123”. It is of interest that the verbal description of ‘three hundred divided into three’ does not match what she actually did, which was to divide three into 300 and highlights the difficulty some children have in explaining their methods and thoughts.

Comparison with other studies

Item 3 was used by Hope (1989) when gathering preliminary data for a later research study (Reys et al., 1993). One hundred and sixty-one fifth grade students were surveyed as to computation preference for this item. Table 5.11 compares the data gathered by Hope (1989) and data gathered in the current study.

Table 5.11: $369 \div 3$, Comparison of computation preference for Year 5 students

Study	% Mental	% Written	% Calculator
*Hope (1989): $n = 161$	9	59	27
Swan: $n = 15$	40	27	33

*Note: some students unable to answer or omitted answering this item.

It should be remembered that the sample sizes and design of the two studies differed so any comparisons need to take this fact into consideration. It is, however, of interest to note the higher percentage of students in the current study making use of mental methods. It should be noted that a gap of over ten years exists between the two studies and the focus of curriculum has tended to move away from formal written methods toward mental and calculator methods. Traditional methods of calculation were taught in all classes studied but there were attempts to utilise calculators and to develop number sense.

Table 5.12 presents data comparing Year 7 preferences. These data present a clear picture indicating that both groups prefer to use calculators. Year 5 students made far less use of calculators than Year 7 students. Year 7 students in the current research still showed more reliance on mental and less on written methods than the students in the Hope (1989) study.

Table 5.12: $369 \div 3$, Comparison of computation preference for Year 7 students

Study	% Mental	% Written	% Calculator
Hope (1989): $n = 161$	9	31	59
Swan: $n = 27$	31	15	54

Item 4: 36×25

The majority of students, 51%, used paper-and-pencil to solve this item. Thirty-one percent of students made use of the calculator to find an answer. A fairly consistent pattern for Item 4 was noted with all year groups choosing written computation as their preferred option. Year 7 students were more definite in their choice, recording a 65% preference for written methods. Questions of this type abound in student texts and are commonly given as exercises to be solved using a written algorithm. Students may as a result gain the impression that written computation is the best approach for solving a two-digit multiplication problem.

A typical description of the written method used to solve this item is given below. Note in particular the use of terms such as ‘carry’ and ‘put down the’ which are typical phrases used by teachers when teaching children the written algorithm.

S: 36 times 25 and then you’d go six times five is 30. Three times 15, put the zero down. Two sixes are twelve, put down the two, carry the one. Two threes are six, that’s seven.

The same student had previously expressed the thought that he was no good at tables (basic number facts). When it was pointed out that he had made use of multiplication facts when completing this item he stated he did not feel he was using multiplication tables. This is possibly because he associates tables with speed tests.

Comparisons with other studies

This item was included in previous studies because it might be solved using aliquot parts. The use of aliquot parts involves making use of factors to make the mental calculation easier to perform. For example 36×25 may be broken up into $(9 \times 4) \times 25$, which is the same as 9×100 . Of the 16% of students who chose mental computation as a method of solution, none applied the use of aliquot parts. The computation preferences for Year 5 students completing this item are shown in the Table 5.13.

Table 5.13: 36×25 , Comparison of computation preference for Year 5 students

Study	% Mental	% Written	% Calculator
Reys, Reys & Hope (1993): $n = 250$	20	71	9
Price (1995): $n = 18$	3	70	27
Swan: $n = 15$	33	47	20

The data consistently point to a preference for written methods, although less so in the case of students in the current study. Higher preference for mental methods was noted for students in the current research. Australian students also tended to make more use of the calculator.

Table 5.14 shows data for Year 7 students completing the same item. A preference for written methods was noted for both groups of Year 7 students. Students from the earlier Reys et al., (1993) study showed a greater preference for mental methods, while the students in the current study made more use of the calculator.

Table 5.14: 36×25 , Comparison of computation preference for Year 7 students

Study	% Mental	% Written	% Calculator
Reys et al. (1993) $n = 204$	21	70	9
Swan, $n = 27$	4	65	31

Item 5: 70×600

Mental and written methods were equally favoured for this type of calculation with 37% of students choosing each method and 19% making use of a calculator. Many of the children experienced trouble with the zeros and had made up, or had been taught various rules for ‘taking off’ and ‘adding on’ zeros to try to alleviate cognitive strain. In many cases these rules were poorly understood and led to errors. Most children performed the ‘ 7×6 ’ part of the calculation without difficulty, and then had trouble with the zeros, although one child completed 7×6 on the calculator and then applied a ‘zeros rule’ to complete the calculation. It was surprising, given the student opted to use the calculator for part of the calculation, that she did not complete the entire calculation with the aid of a calculator.

The ‘take off and add zero approach’ is described below. The student gave the correct answer and when explaining the procedure, miss-stated the number of zeros.

S: You just have to do seven times six which is forty-two and then add four zeros, so it's 42 000. (Later when probed she realised her mistake.)

In the second extract the student explains the procedure of taking the zeros away. When probed about taking three zeros away but only putting two zeros back the student cannot explain why he did it. Another student had developed a rule that suggested two zeros should be added because the second number, 600, contained two zeros.

I: 70×600 .
S: 4200.
I: You did that one in your head by the looks. How did you do it?
S: I just took the 0's away and did 7×6 and then I added the 0's.
I: Right so you did 7×6 and got 42 and how many 0's did you put on.
S: 2.
I: So you put 2 back on. Okay. You took 3 off but you put 2 back on. Why was that?
S: I don't know.

Students who used a paper-and-pencil algorithm to solve this item also experienced trouble handling all the zeros. Figure 5.1 depicts two students' work and indicates the difficulty both experienced using written methods to solve 1000×945 . The student

whose work is shown on the left had demonstrated an ability to use standard written algorithms when correctly solving previous items such as 36×25 , 70×600 , 29×31 and 33×88 but became overwhelmed at all the zeros and was unsure of what to do. The student whose work is shown on the right had experienced difficulties when using a standard written algorithm to solve previous multiplication items.

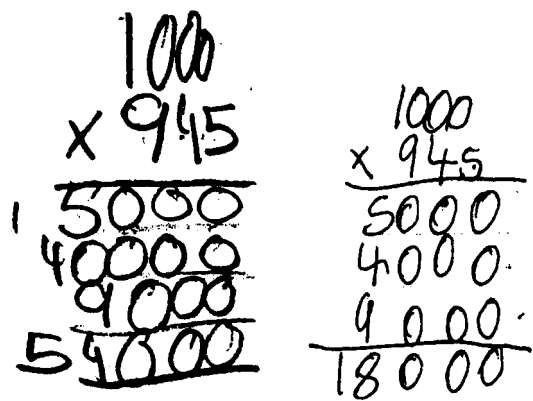


Figure 5.1: Difficulties experienced with zeros

The students who chose to use calculators often referred to the fact they were dealing with ‘big numbers’, although they could not clearly define what constituted a big number. It appears that any number in the hundreds was considered to be ‘big’.

Comparison with other studies

Table 5.15 indicates the preferences for Year 5 students across three studies for this item. Computation choice for Year 5 was equally split across the mental, written and calculator categories. Students in the first study by Reys et al. (1993) showed a preference toward written and mental methods. The later study by Price (1995) indicated more students favoured mental methods and an increase in preference toward calculator methods was noted.

Table 5.15: 70×600 , Comparison of computation preference for Year 5 students

Study	% Mental	% Written	% Calculator
Reys et al. (1993): $n = 250$	39	45	16
Price (1995): $n = 18$	41	33	26
Swan: $n = 15$	33	33	33

The results for Year 7 students are shown in Table 5.16. Students in both groups showed a preference for mental methods. The groups differed in preference for written and calculator methods. Year 7 students made more use of mental methods. It is possible that older students understand numeration better.

Table 5.16: 70 × 600, Computation preference for Year 7 students

Study	% Mental	% Written	% Calculator
Reys et al. (1993): $n = 204$	47	39	14
Swan: $n = 27$	50	15	35

Item 6: 29 × 31

Written and calculator methods were the most popular choice for this item. None of the students made use of number patterns and relationships such as $30 \times 30 = 900$ in solving this item. Those children who did try to solve the item mentally often used a mental version of the written algorithm. It appears, however, that in many cases the decision to use a mental approach was based on the use of a faulty version of the written algorithm. Several students responded with an answer of sixty-nine. When questioned as to how they obtained this result most responded by stating that ‘9 × 1 was 9’ and that ‘2 × 3 was 6’, giving a result of 69 (See Figure 5.2). Some students opting to use a paper-and-pencil method also made this same error. The adoption of this method possibly altered the student’s choice of method, because performing two simple multiplications as part of a two-digit multiplication is easier to do mentally than the four multiplications and addition required to complete the calculation correctly.

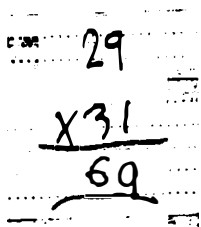


Figure 5.2: Example of student’s faulty algorithm for 29 × 31.

Comparisons with other studies

Only data for Year 5 students are available for comparison. The data were compiled and placed in Table 5.17. Year 5 students in the previous two studies showed a preference toward written methods, but in this current study, Year 5 students were more inclined to make use of the calculator. Students in the Reys et al. (1993) and current study made more use of mental methods than those in the Price (1995) study.

Table 5.17: 29 × 31, Comparison of computation preference for Year 5 students

Study	% Mental	% Written	% Calculator
Reys et al. (1993): <i>n</i> = 250	29	63	7
Price (1995): <i>n</i> = 18	5	56	39
Swan: <i>n</i> = 15	27	27	46

Item 7: 33 × 88

Computation choices for this item were similar to those made in Item 6, another two-digit by two-digit multiplication. Slightly fewer students chose mental methods and slightly more chose written methods, with the number choosing to use a calculator remaining the same. Some students felt that because the digits were repeated, this item was simpler than Item 6. It was noted that when students adopted a paper-and-pencil method they would often draw solid and broken horizontal lines across the page, prior to starting the calculation. The students in one class had been trained to set this type of calculation out in a particular way. The repetitive nature of the calculation, having to multiply 8 × 3, twice, caused some confusion and revealed a lack of understanding of place value on the part of some students.

The following scanned images of students’ work illustrate some of the difficulties experienced completing this item with pencil-and-paper methods. In the first example (See Figure 5.3) the student multiplies 3 by 8 and ‘puts down the 4 and carries the 2’. The multiplication is repeated and the 4 is written down to the left of the first 4, to make 44 and the 2 is ‘carried’ and added to the previous 2 to give an answer of 444.

$$\begin{array}{r}
 28 \\
 \times 33 \\
 \hline
 444 \\
 444 \\
 \hline
 4444
 \end{array}$$

Figure 5.3: Example one of students' written algorithm for 33×88 .

In the second example (See Figure 5.4) a similar procedure to the previous example is used but the student uses the zero as a placeholder to create an answer of 404 when multiplying the first part of 33×88 . Next she simply wrote a zero on the next line and copied her result from above onto this line and added the result to produce an answer of 4444. She did not appear to have an understanding of place value and did not know why she 'put down the zero'. Appendix 7 contains further examples that indicate students' difficulties executing the written algorithm and the problems they experienced with zeros.

$$\begin{array}{r}
 28 \\
 \times 33 \\
 \hline
 404 \\
 4040 \\
 \hline
 4444
 \end{array}$$

Figure 5.4: Example two of students' written algorithm for 33×88 .

Children who had trouble remembering the result of multiplying 3×8 tended to use a calculator to complete this part of the calculation and then reverted back to using the written algorithm. When asked about this, one student responded that he knew how to do the calculation (on paper), but had trouble with the eight times table and had forgotten what three times eight made. It was somewhat surprising, given that the student had picked up a calculator to work out 3×8 but he did not employ it to complete the entire calculation.

For this item all three year levels showed a preference for written methods. In both cases Year 7 students showed the most preference for written methods. The longer exposure to written methods may have impacted on this computation choice.

Comparisons with other studies

Comparisons may only be made with Year 5 data, as these were the only data reported in prior research. The comparative data is shown in the Table 5.18. A strong preference for written methods was noted in the earlier studies. Preference for written methods was also recorded in the current study but less so compared to the earlier studies. More students in the current study opted for mental methods. Calculator use was higher in both the Price (1995) and current research.

Table 5.18: 33 x 88, Comparison of computation preference for Year 5 students

Study	% Mental	% Written	% Calculator
Reys, et al. (1993) <i>n</i> = 250	13	73	14
Price, (1995), <i>n</i> = 18	3	66	31
Swan, <i>n</i> = 15	27	40	33

Item 8: 1000 x 945

Fifty-eight percent of students chose to use a calculator as their first method of calculation for this item. The result for this item was a little surprising. The percentage of students using a calculator was higher than expected and the results were not consistent with Item 5, the previous question involving zeros. One might argue that Item 8 was easier to calculate using mental methods than Item 5, but the data show a clear preference for calculator use. It is possible that the presentation of the question may have contributed to this. It is of concern that so many students would choose to use a calculator for an item of this nature. Also of concern were the numbers of Year 5 students opting to complete the item using written methods. This indicates a lack of number sense on the part of these students. In particular place value and understanding of the effect of multiplying by powers of ten.

The interview data indicated that students perceived the item as involving large numbers and zeros and as a result many lacked confidence in their ability to perform the

calculation mentally. The effect of large numbers and zeros on computation choice is discussed in the following chapter.

Those who chose to use a calculator often experienced difficulty reading the number on the display. Students adopting a mental approach also appeared to have difficulty explaining what answer they had reached. Some preferred to write the answer rather than say it and when asked about the result they had written, experienced difficulty stating the answer. One student when asked to explain the answer showing on the display of her calculator stated, “I can’t pronounce it.” This example highlights the need to understand numbers before being able to comfortably perform a calculation. When asked to read the number shown on the display of the calculator after completing 1000×945 , one student said “900 500, no 900 045”, neither of which was correct. When choosing to use a calculator a student had to be able to read the number in the display correctly before being judged correct.

The following student explains her reason for using a calculator. Note how she is aware of her own weaknesses and uses this knowledge to guide her decision.

S: Well because I couldn’t quite imagine it in my head and I could have written it down but the zeros get me mucked up.

Students’ lack of knowledge of place value was most evident in this item. It is also possible that poor use of language contributes to the lack of understanding of how to perform calculations involving zeros. Several students referred to o’s rather than zeros when describing their method for calculating the result.

Another student explained that she chose to use a calculator “Because I don’t really know how to do long multiplication.” When questioned a little further she explained that she could have done it mentally. She stated “I could have gone 145×1 and add three zeros.” Many students referred to ‘adding’ and ‘taking’ zeros, which appeared to hamper, rather than assist their ability to solve the problem. Some issues about second and third choices are raised in this extract. After completing the item using a calculator and then being questioned as to whether she could do the problem in another way it was almost as if this student realised that she *could have* done it mentally. Whether she would have been able to do it mentally prior to trying the question on a calculator is debatable. The incident, however, does illustrate how powerful the technique of asking students to try a different approach can be in developing thinking ability.

Comparison with other studies

This item was only used in the Reys et al. (1993) research and the current study. Data were only provided for Year 5 students. These data are recorded in Table 5.19. Considerable difference in computation preference was noted in the responses to this item. Students in the Reys et al. (1993) study stated a preference for mental methods, whereas students in the current research indicated a preference for calculator methods. It should be noted, however, that this item drew the largest percentage of students opting for calculator methods for all items given to Year 5 students in the Reys et al. (1993) study. Of concern was the small percentage of students in the present study that opted to use mental methods and of even more concern, the percentage that chose written methods.

Table 5.19: 1000 × 945, Comparison of computation preference for Year 5 students

Study	% Mental	% Written	% Calculator
Reys, et al. (1993) <i>n</i> = 250	45	29	26
Swan, <i>n</i> = 15	13	40	47

Reys et al. (1993) asked a slightly different question of Year 7 students, which incorporated the use of decimals and was given in a similar format. The item, 1000 × 0.123, produced the largest percentage of students opting for calculators than any other item. Forty-two percent of students chose to use a calculator when confronted by this item. This choice was still eclipsed by written methods, however, with 49% of students stating they would use written methods to solve this item.

Item 9: 10% of 750

Thirty-six percent of students chose to use a calculator when trying to solve this problem. Forty-percent of students did not know where to begin solving this question and chose not to try any method. Students who chose to use a calculator often did so, not because they knew how to solve the problem but rather because they knew the calculator had a percentage key on it and they thought pressing this key would somehow help them arrive at the answer. Most of the children did not know how the percentage key worked, or the required sequence of keystrokes for solving a problem of this type.

Some students who used a calculator performed an allied calculation like $750 \div 10$ or 750×0.1 , thus avoiding the use of the percentage key altogether.

An example of a student who used the calculator but not the percentage key to solve this item is shown below. The student entered $750 \div 10$ into the calculator and was asked why she had done so, and her reply follows.

- S: Because we were doing 10%, and 10% is a tenth, isn't it?
I: I'm just interested, because there is a percentage key on there, but you did divide by ten. How did you know to divide by ten?
S: I just sort of know that.
I: Could you have done it another way?
S: On paper.
I: How would you do it on paper?
S: A divide sum.

The following student demonstrates an understanding of percentages that allows him to use several methods of calculation to solve the item. Even though he is unsure of how the percentage key works, his understanding of percentages allows him to employ the calculator in a different way to solve the item. His comment about the mental method being quick and easy appears reasonable given that he displayed an efficient and effective method for calculating mentally.

- S: I went 10% of 100 would be 10 and then 10×7 is 70 and 10% of 50 would be 5, so 75.
I: Why did you do that in your head?
S: Because I knew I could do it quickly and easily in my head.
I: Is there any other way you could do it?

The student also stated he could do the calculation on a calculator and after a little hesitation explained how to complete the calculation without using the percentage key. When probed further the student described how he would complete the same calculation on paper.

- S: If I was going to do it on paper, I would just do, I'm not sure, 10% of 750, I could do it the way the calculator did, $10 \times 750 \div 100$, but that would take too long.

Of interest is the comment that it would take too long to complete the calculation on paper. Speed of calculation was often given as a reason for making a particular computation choice. This will be discussed in the next chapter.

A consistent pattern of calculator use across all year levels was noted for this item. Year 5 students showed a preference for trying mental methods as well as calculator methods. The calculator was almost used as a default choice because student's recognised calculators have percentage keys and many felt pressing the key

would produce the answer. A lack of instruction on how to use a calculator appears to have hampered the efforts of the students to solve this item.

Comparison with other studies

This item was only given to Year 7 students in the Reys et al. (1993) study. Comparative data for Year 7 students in the present study are provided in the Table 5.20. Considerable difference in computation choice may be noted when examining the data. Almost half the students in the Reys et al. (1993) study stated they would solve this item using written methods, whereas only 4% of Year 7 students in the current study chose this method.

Table 5.20: 10% of 750, Comparison of computation preference for Year 7 students

Study	% Mental	% Written	% Calculator
Reys et al. (1993): <i>n</i> = 204	16	48	36
*Swan: <i>n</i> = 27	23	4	38

Note: A large percentage of students unable to attempt this item.

This item also invoked a strong preference for calculator use from students in the Reys et al. (1993) study. Some students in the present study could not recall having learned about percentages despite the teachers involved stating that they had taught percentages. It is possible that students in the Reys et al. (1993) study had more experience with percentage calculations than the students in the current study.

Item 10: $14 \times 9 \div 6$

Thirty-six percent chose to use a calculator to help solve this problem. Most students could not recall being asked a question like this before and therefore this may have contributed to them choosing to use a calculator. The use of division in the question also seems to have contributed to the choice to use a calculator. This item also produced the largest preference for applying mixed methods to solve the question.

It is possible that the combination of two operations in one item encouraged the use of mixed methods, however, the use of mixed methods tends to indicate thought on the part of students as to how they would solve the problem. For example, some students used a calculator to complete 14×9 and then put the calculator down and

finished the item using another method. This suggested that students do not pick up a calculator and use it without thought, or use it for everything. This also tends to indicate that students using a mixed methods approach had thought about the calculation and decided what was in their ability to do mentally, on paper or with a calculator. Oftentimes students employing a mixed method would use the calculator to complete the division part of the calculation, although there were examples where 14×9 was completed with the aid of the calculator. Students who chose to use a calculator to find the result of multiplying 14 by 9 often commented that it was out of the realm of their 'tables' knowledge and hence they used a calculator. It should be noted that this item was carefully chosen to avoid problems with rule of order. The calculators used by students in primary school generally do not follow the conventions of rule of order and children will often make mistakes if they enter mixed operation questions into the calculator without first stopping to consider the order in which the information is entered into the calculator.

The following examples show how students responded to this question using a mix of methods. One student used a formal written algorithm to calculate 14×9 and then completed the remainder of the calculation via calculator. The reason given for completing the item using a calculator was that it involved division. One student used a combination of all three methods. Firstly she mentally determined that 12×9 was 108 and then worked out 2×9 and added the two results to arrive at the answer of 126. The rest of the calculation, $126 \div 6$, was completed using pen and paper. Finally the answer was checked using a calculator. Another student used a similar technique. After completing 14×9 mentally the student attempted to complete $126 \div 6$ using a formal written algorithm, but experienced difficulties and then reached for a calculator to complete the problem.

Item 11: $\frac{1}{2} + \frac{3}{4}$

This item was the first of three fraction questions given to the students. The majority of the students, 60% in all, chose to complete this item using mental methods. The 22% of students who opted to use pen-and-paper methods did not tend to use the formal methods taught in school but often made use of diagrams and symbols. Pizza circles featured in many of the diagrams that were drawn. An example of using an informal written approach to solving $\frac{1}{2} + \frac{3}{4}$ is shown in Figure 5.5.

Figure 5.5: Example of informal written algorithm.

Only 9% of the students attempted to make use of calculators. Use of the calculator appeared dependent on student knowledge of decimal equivalents. No student demonstrated an ability to convert a fraction to a decimal on the calculator. Some students confused the vinculum of the fraction with the decimal point and therefore when calculating $\frac{1}{2} + \frac{3}{4}$ they would enter $1.2 + 3.4$. This item helped to show that a calculator is of no assistance if the user does not understand the question or how to enter the numbers into the calculator.

Mental methods were commonly adopted. An example of one mental approach to this item is documented below.

S: Well, half is two-quarters, plus three-quarters, that's one and one-quarter.

This mental approach is based on his ability to convert one-half to an equivalent fraction. Very few students used the standard written method to solve this item. The following example indicates that even when completing a written calculation a fair amount of mental computation takes place.

S: I probably could have done it mentally in my head because two halves are one and the extra quarter makes one and a quarter.

When questioned a little further this student made reference to finding the lowest common denominator. The student completed the written algorithm without any difficulty. Some students had been studying fractions in class and applied what they had learned to solve the problem.

S: We've also been doing fractions in class recently. Well two-quarters was the same as a half so that would leave $\frac{1}{4}$ of that and put the $\frac{1}{2}$ and $\frac{2}{4}$ together making a whole and then would leave $\frac{1}{4}$, so $1\frac{1}{4}$.

An interesting approach used by some students when solving this item involved rephrasing the item as a 'whole number' question. The following student comment describes this approach: "Three quarters equals 75, and one-half is 50, so I just added them together."

Students who tried a written approach also used the pizza image. Rather than use a formal written algorithm, students who chose to write the question on paper often drew circles representing pizzas and then sliced and shaded to determine the result. Reference to pizzas was also made in student explanations.

S: Well I imagined $\frac{1}{2}$ was a pizza and then I got $\frac{3}{4}$ of a pizza and added it up together.

Students across all year levels preferred to use mental methods. What is of interest is the strong preference by Year 5 students toward mental methods. Very little calculator use was recorded. Few students were aware of how to convert a fraction into a decimal and therefore were unable to use a calculator. Similarly few indicated an awareness of the decimal equivalent for three-fourths and to a lesser extent one-half. This lack of knowledge inhibited their opportunity to use a calculator.

The choice to use mental methods for fraction items rather than formal written methods may be partly attributed to the reduced emphasis in the curriculum over the last 15 years on the teaching of formal written algorithms for fractions. Where students made use of written methods they tended to be of the less formal type, involving drawings of circular regions to represent pizza.

Comparison with other studies

This item was only given to Year 7 students participating in the Reys et al. (1993) study and all students in the current study. Data comparing the two studies are provided in Table 5.21. This was one item where students in the Reys et al. (1993) study stated that they would choose to use calculators more often than students in the current study. There is no evidence, however, to indicate whether the students could use calculators to solve a fraction item, as the students were not required to perform the calculation. Students in the current research made much more use of mental methods and less use of written methods in comparison to their counterparts in the earlier research.

Table 5.21: $\frac{1}{2} + \frac{3}{4}$, Comparison to computation preference for Year 7 students

Study	Mental	Written	Calculator
Reys et al. (1993): $n = 204$	38	39	22
*Swan: $n = 26$	54	27	8

Note: Ten percent of students overall unable to attempt this item.

Item 12: $10 - 4\frac{3}{4}$

Seventy-seven percent of students chose to solve this question mentally. The response to this item was clearly in favour of mental methods with only 6% of the students choosing written methods and 3% calculator methods. Mental methods dominated in this item more than for any other item in the instrument. The students seemed more at ease with this fraction item, possibly due to the operation, but more likely because it involved a whole number at the start of the question. The whole number at the start of the question also appears to have made the question a “simpler one” to calculate mentally. The description of how this student solved the item is very clear and concise.

- S: First I imagined the $\frac{3}{4}$ weren't there, so $10 - 4$ is 6 and also if I was going to take some more $\frac{1}{4}$'s it would be 5 and $\frac{3}{4}$ leaves $\frac{1}{4}$ to get to a whole so it was 5 $\frac{1}{4}$.

The following extract indicates the ease with which a student was able to solve this fraction item mentally. Note also the reasons given for not using an alternative method.

- S: First of all, I do ten take four, and then you have to take the three-quarters, so it's five and one-quarter.
I: Could you do it another way?
S: I think you could, you could do it written, but that would take longer, and I don't really know how to do it on a calculator.

Many students expressed similar sentiments about using a calculator for fraction questions. For most, the calculator was not an option because they did not know how to enter a fraction into the calculator. The trigger that allowed students to make use of a calculator seems to be the ability to see $\frac{3}{4}$ as the 0.75 decimal equivalent. One student entered $4\frac{3}{4}$ as 4.34 on the calculator.

Comparison with other studies

Raw data for this item were only available from the preliminary research by Hope (1989). The item was given to Year 5 and Year 7 students so comparisons across two year levels could be made. The comparative data for Year 5 and Year 7 is provided in Table 5.22. A marked difference in results was noted for this item. Clear preference for mental methods was noted for Year 5 and 7 students participating in the current research and little or no use of calculators was recorded. Calculators were the preferred option for Year 5 students in the Hope study and accounted for almost one-third of the computation preferences for Year 7 students. A swing away from calculator toward written methods by Year 7 students was noted in the Hope study.

Table 5.22: $10 - 4 \frac{3}{4}$, Comparison of computation preference for Year 5 and Year 7 students

*Study	Mental	Written	Calculator
Hope (1989), Yr 5: $n = 161$	9	25	44
Swan, Yr 5: $n = 15$	87	7	0
Hope (1989), Yr 7: $n = 161$	19	45	30
Swan, Yr 7: $n = 26$	73	8	8

Note: Many students overall unable to attempt this item.

Item 13: $\frac{2}{3}$ of 45

This item was the most difficult, with only 46% of the students prepared to attempt answering this question. Those who did attempt answering the question mostly chose written methods. Students using written methods tended not to use a standard method. Typically students would perform a written short division algorithm and then double the answer. Many students split the item into two parts as illustrated in the following extract.

S: I'd break 45 into threes, that's 15 and then I would double that, that's 30.

After observing a student use a calculator the response below was given. Note the use of the expression 'resort' as if calculators to this student are to be used as a last resort for performing calculations. This extract also reveals how a researcher may be deceived by observation. At first it appeared as though the student's first inclination was

to use a calculator when in reality the student had tried a mental method first. It is a good example of a student starting a calculation using one method, finding it too difficult and abandoning it in favour of another method.

- S: First I tried to do it in my head, but I thought it was too hard.
I: What made you think it was too hard?
S: Well first I tried to divide 3 into 45 and I got stuck for answers, so I resorted to the calculator.

When students adopted mixed methods, they often involved the combination of calculators and mental computation. Typically calculators were used for the division component of the calculation.

Item 14: \$1.99 + \$1.99

This item was the first to be administered in context. The shopping context tended to act as a prompt for the students to use mental methods. For example, many children rounded the \$1.99 to two dollars because ‘everything is rounded in the shops’. The students were asked to give an exact price, but would still double two dollars to make four and then compensate by subtracting two cents. The calculator was virtually ignored in this ‘real life’ question with only 4% of students choosing this method. The majority of students, 68%, used mental computation and 26% applied a written method. The numbers in the question made the written algorithm somewhat awkward to use as it involved ‘carrying’. It shows limited number sense if a student chose to use a method other than mental computation.

The following explanation outlines a common approach used by many students in the study. “I’d round it off, \$2 each, that’s \$4, take off two cents, that’s \$3.98.” The item lends itself to the application of a compensation approach, which in turn encourages mental computation. It is also possible that the context of money may also act as an encouragement for students to use mental methods.

A consistent pattern of preference for mental methods was recorded across all year levels. A rising trend in favour of mental methods and away from written methods was noted in the data. It is possible that the money context may have contributed to this trend. Older students tended to be more aware of shopping practices, such as, the rounding of \$1.99 to \$2. This ‘real world’ understanding seems to have contributed to the adoption of mental methods based on rounding and compensation.

Comparison with other studies

This item was given to Year 5 and 7 students in the Hope (1989) study and in the current research. There was no indication that this item was previously given in a shopping context, as was the case for this research, although it was presented in terms of money. The comparative data for Year 5 and 7 is provided in Table 5.23.

Table 5.23: \$1.99 + \$1.99, Comparison of computation preference for Year 5 and Year 7 students

Study	% Mental	% Written	% Calculator
Hope (1989), Yr 5: <i>n</i> = 161	65	31	3
Swan, Yr 5: <i>n</i> = 15	53	47	0
Hope (1989), Yr 7: <i>n</i> = 161	47	47	6
Swan, Yr 7: <i>n</i> = 26	88	4	8

A preference for mental methods is indicated across all groups. Choice of written methods also proved to be popular with all groups except the Year 7 students who participated in the current research. This group showed a strong preference for mental methods. Using a calculator to complete this question could be viewed as a poor computation choice. Likewise it could be argued that completing an item such as \$1.99 + \$1.99 using written methods is inefficient and as such represents a poor computation choice.

Item 15: \$4.93 + 39c

Item 15 was also presented in a shopping context but the mix of dollars and cents caused a few difficulties. The numbers do not present quite so obvious a case for rounding and compensation as in the previous item, although it is relatively simple to add \$5.00 and 32c mentally.

Forty-seven percent of students chose a mental method as their preferred way of solving this item. Students using a mental approach would often adopt a mental version of the standard written algorithm. A few students applied compensation-type strategies such as adding 7c to the \$4.93 and then adding 32c to the result, although this strategy was not as common as one might expect. Written methods accounted for 36% of the

first choices and calculator methods for only 12%. Students using the calculator to assist in solving this item often found a result of \$43.93. This result occurred because 39c was often entered as 39, which meant the number was interpreted by the calculator as \$39. This error tends to indicate a lack of understanding of decimal concepts. Many students were also unfamiliar with how the calculator works and how to enter such a sum into the calculator.

Mental methods were not as common as one might have expected for an item involving a money calculation. One student showed how simply the calculation could be performed using a mental method.

S: I'd do four dollars and then there would have to be seven cents to make it five, then there would be 32 left over, so its \$5.32.

Few students, however, adopted this method and often used less efficient mental methods, such as, a version of the paper-and-pencil algorithm for solving the item. The adopting of inefficient mental methods may be the reason more students did not choose to use mental methods.

Year 5 students preferred to use written methods, closely followed by mental ones. Year 6 students favoured mental above written methods but recorded a higher level of written computation. Year 7 students also preferred mental methods but written and calculator methods were evenly split. Year 7 students used a calculator more often than Year 5 or Year 6 students.

Item 16: 7.41 – 2.5

Item 16, the first of the decimal questions, produced some interesting data. Students did not appear to have a clear computation choice for this item. Even though items 14 and 15 involved decimal numbers in a money context, this item was clearly seen by students as being different because of the 'dot'. Choice of computation method was almost evenly spread across mental, 35%, written methods, 33% and calculator methods, 28%. Students would often subtract 25 from 741, indicating a lack of understanding of place value.

Another common mistake involved trying to match the number of decimal places in both numbers. Students would typically subtract 0.25 from 7.41 so that both numbers had the same number of decimal places and the decimal point would line up. This item caused concern to students because the decimal places were mismatched.

Most were unsettled by the fact that one number included two decimal places while the other included only one decimal place. In the following extract the student begins to tackle the question mentally, but opts for the calculator when she is unsure of what to do with the differing decimal places.

- S: It would be five, I think, I am not really sure, I'd probably do it on the calculator. 7.41 take 2.5, 4.91.
I: You started to do that mentally, and you changed your mind, why was that?
S: One had two decimal places and one had just one, weird. What I was going to do was take five from 41, but then I realised it was supposed to be 50 from 41 because it's in the tens.

The same student went on to express similar sentiments about Item 17 because as she says "I would probably do that on a calculator. It's pretty confusing because 0.25 is in the decimal and the other is not, kind of weird." The use of the expression 'weird' seemed to indicate a lack of experience with decimals and calculators. The student did not have the ability to make sense of the answer.

This decimal item was likely beyond the ability of most Year 5 students, and older students were also less comfortable with this item. Students were less confident with this style of question and this appears to be reflected in the computation preferences that were broadly spread across all categories. Year 5 students preferred mental methods, with preference for written and calculator methods being evenly split. Year 6 preference was split across the mental and written categories, while Year 7 preference was almost evenly spread across all three categories, with written and calculator methods equally preferred.

Item 17: 0.25×800

Over 70% of students chose the calculator as their first option when tackling this question. It appears as though the combination of 'big numbers' and decimals triggered a strong swing toward calculator use. Very few students chose to use mental methods, the most common being a quarter of 800. Similarly students who chose to use a written method did not necessarily multiply 0.25 by 800, but would often divide 800 by 4. The knowledge that the fractional equivalent of 0.25 is a quarter provided the basis for making this choice. Many students were surprised to find that a smaller number was produced as a result of the multiplication. Here is an example of how a student changed the item into division.

- I: Right, so you sort of wrote that down and then stopped and you've written 4 into 800. Can you explain why you've done that?
- S: Well 0.25 goes into 1, 4 times because it's the same as a $\frac{1}{4}$ and so I just divide 8 by 4, which made 200.

When asked whether he could do this question another way this student gave the following explanation.

- S: Well 0.25 you could do $80 \div \frac{1}{4}$. 80 divide by 4 which would be 20 and then you would just add the extra 0.

Even though he has a mental method at his disposal this student explains why he originally chose to complete the item on paper.

- S: Because I wasn't totally sure of how it would be if I did it in my head.

This item appears to have been slightly more difficult for all the students and hence computation preference was clearly centred on calculator use. Year 5 students showed a fairly strong preference for mental methods, but it was likely these students overestimated their ability to calculate the answer to this item using mental methods. Students who recognised the relationships between 0.25, $\frac{1}{4}$ and 800 were in the best position to apply mental methods. In the following example the student demonstrates a good understanding of a quarter.

- S: If there was one whole it would be 800 times one, I just did 0.5 is half so I halved it and halved it again.

Comparison with other studies

This item was only used in one previous study, Reys et al. (1993) and only with Year 7 students. Table 5.24 presents the comparative data for the current research and the previous research. Year 7 students showed far greater preference for calculator methods in the current research. Preference for mental methods was similar. For students unsure of working with decimals and unaware of the relationship between 0.25 and $\frac{1}{4}$ the choice to use a calculator was a sensible one.

Table 5.24: 0.25×800 , Comparison to computation preference for Year 7 students

Study	% Mental	% Written	% Calculator
Hope (1989), Yr 7: $n = 161$	16	53	31
Swan, Yr 7: $n = 26$	19	12	69

Item 18: $3.5 \div 0.5$

There were some concerns that student fatigue might have been a factor at this point, but most students appeared to make a considered choice about the method used to solve the item. Fifty percent of students opted for a mental method, 38% for using a calculator and only 9% tried using a written method. Many of the mental methods hinged on the understanding that 0.5 was the same as one-half and two halves made a whole. Students who did not have this understanding often chose to use the calculator and were surprised when the result of seven was displayed. This was another example of students not being able to make sense of the result shown on the calculator.

An understanding of the relationship between one-half and one-whole was the key to being able to solve this item mentally. The following student indicates a good grasp of the relationship between 0.5 and one-half and the relationship between one-half and a whole in solving this item.

S: Well that's a half, 0.5 is a half, then there would be six halves in three and one half so that's seven.

Of interest was the application of a skip-counting method to solve this item mentally.

S: I can just work that out in my head. Because 0.5 1, 1.5, 2, 2.5, 3, 3.5, 7. It could be five into 35, which is seven.

Year 5 students opted for the safety of a calculator for this item. The item was beyond the ability of most Year 5 students and therefore calculator use might be thought of as a default choice. Most Year 5 students did attempt the item using mental methods. The preference for mental methods increased in Year 6 and 7 with calculator methods becoming the second most common method for these groups. It is likely that more confident students employed mental methods and those with less confidence used a calculator. Students with a good understanding of place value would recognise this item as related to $35 \div 5$ and in turn the basic fact $5 \times ? = 35$. Very few students opted to use a written approach.

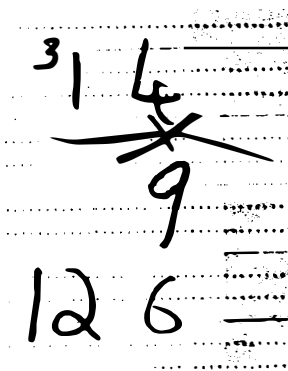
Mixed Methods

The Swan and Bana (1998) computation model outlined in Chapter 3 (See Figure 3.6) suggested that rather than use a single method of computation; students use a combination of computation approaches when solving numerical problems. The first choice data tend to indicate, however, that students favour the use of a single method of

computation. There were two items, however, where over 20% of students chose to use mixed methods. The first, Item 10, $14 \times 9 \div 6$, involved a mix of operations, which may have encouraged the adoption of mixed methods. Twenty-three percent of students chose to use mixed methods when solving this item. Different mixes of methods were used to complete this item. The following extract illustrates how a written method and the calculator were combined to produce a result.

- S: Well first I did 14×9 on the paper, which was 126 and then I divided on the calculator.
I: Why did you change?
S: I thought it would be quicker. I could have done that on the paper as well.

The associated written calculation is shown in Figure 5.6.



The image shows a handwritten calculation on lined paper. At the top, the number '31' is written above a horizontal line. Below the line, the number '14' is written. A large 'X' is drawn over the '14' and the line. Below the 'X', the number '9' is written. A horizontal line is drawn below '9'. Below this line, the number '126' is written. The calculation represents $14 \times 9 = 126$.

Figure 5.6: Written part of mixed method: $14 \times 9 \div 6$.

The next extract involves a mixed method that began with the use of a mental calculation and then concluded with the use of a calculator. Division appears to have been the trigger for using a calculator. The role of division as a trigger for calculator use is discussed in the next chapter. In explaining why he used a mix of mental and calculator methods the following student made this comment:

- S: Because with 9×14 , it's just like a table and then to divide it's a bit harder.

There were also examples of students using a calculator to multiply 14×9 and then either completing the division part of the calculation using mental or written methods. These methods were of particular interest as they indicated students do not necessarily use a calculator indiscriminately, just because they begin a calculation with the aid of a calculator.

Twenty-one percent of students used mixed methods to solve Item 13, $\frac{2}{3}$ of 45. Mixed approaches varied but in most cases students either used a calculator or written methods to divide 45 by 3 and then used mental methods to double the result. An example of using the standard written algorithm to calculate $45 \div 3$ is shown in Figure 5.7.

$$\begin{array}{r} 15 \\ 3 \overline{) 45} \\ \underline{30} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

Figure 5.7: Written part of mixed method: $\frac{2}{3}$ of 45.

The student whose work is shown in Figure 5.7 gave the following explanation of how he completed $\frac{2}{3}$ of 45.

S: Well I divided 3 into 45, so I found 15 is $\frac{1}{3}$ of it and I doubled back.

Of interest is the explanation given by the following student who used a mix of all three methods. Of particular note is his comment about writing part of the calculation down as a memory aid.

- S: 15 plus 15 equals 30
- I: I noticed you did some in your head, wrote some down, and used the calculator. Can you explain how you did that?
- S: I just figured out that, I guessed really, a third of 45 would be 15. I tested it out on the calculator, and wrote it down so I wouldn't forget it, and then I added two fifteens altogether in my head and on the calculator.

It should be pointed out that less than one-third of the students attempted this item. Mixed methods tended to be applied when the item became more complex. The more routine items tended to be attempted using either mental, written or calculator methods.

Second and Third Choices

After completing each item using their initial choice students were asked whether they could solve the item using another method. Table 5.25 indicates second preferences. As one might expect the percentage of students without a second method was high. Overall 45% of students stated they were unable to complete items using a method that differed from their first choice. The data indicate a clear move away from mental methods toward calculator methods. Calculator methods accounted for 33% of second preference methods overall. Only for two items, 1 and 3, were written methods the preferred second option.

The data tend to indicate that many students did not have a second computation choice. They were unable to apply alternative methods to solving the computation items given in the research. It appears as though the calculator is sometimes used as a ‘last resort’ or as a default calculation method. In seven items (8-13 and 17), nearly half or over half the students indicated they did not have a second method at their disposal. The percentage of students in each case except one, 1000×945 (49%), was well above 50%. This seems to indicate that the students knew their limits and were clear as to what they were able to do and what they could not do. Throughout the interviews it also became apparent that students were not used to being asked to solve a question using another method. Some students did not see the point of being able to solve an item in more than one way, expressing the feeling that all that is required is a single method that produces the correct answer. Data indicating success rate for second choices are presented in Chapter 7.

Table 5.25: Percentage distributions of second computation choice for all items ($n = 78$)

Item Number	Item	Mental	Written	Calculator	Mixed	No method
1	$28 + 37$	14	51	17	0	18
2	$74 - 36$	12	20	41	0	27
3	$369 \div 3$	12	37	19	0	32
4	36×25	4	24	44	0	28
5	70×600	9	10	38	0	43
6	29×31	5	15	49	0	31
7	33×88	6	12	51	0	31
8	1000×945	13	10	28	0	49
9	10% of 750	4	5	6	0	85
10	$14 \times 9 \div 6$	2	8	32	3	55
11	$\frac{1}{2} + \frac{3}{4}$	4	11	18	0	67
12	$10 - 4\frac{3}{4}$	0	2	22	0	76
13	$\frac{2}{3}$ of 45	0	3	10	0	87
14	$\$1.99 + \1.99	4	24	54	0	18
15	$\$4.93 + 39c$	1	17	65	0	17
16	$7.41 - 2.5$	2	17	46	0	35
17	0.25×800	1	4	18	0	76
18	$3.5 \div 0.5$	1	5	47	0	46
	Total	5	15	33	0	45

Data indicating third choices show that in all but one item most students were not able to make a third choice. The data are shown in Table 5.26. The most common response to probing about a third choice was that the student did not have one available. Some students did not see the need for more than one option or possibly two. It is probably unreasonable to suggest that students should be proficient in all mental, written and calculator forms of computation for all types of calculation. There will be

some calculations for which mental computation is clearly the best choice and others where the calculator is the preferred option. Within this range there lie many different calculations where choice will vary between mental and written, written and calculator, mental and calculator or perhaps all three or a combination of methods. The choice will depend on a number of factors including student proficiency with various computation methods.

Table 5.26: Percentage distributions of third computation choice for all items ($n = 78$)

Item Number	Item	Mental	Written	Calculator	Mixed	No Method
1	$28 + 37$	0	4	52	0	44
2	$74 - 36$	1	3	23	0	73
3	$369 \div 3$	1	4	21	0	74
4	36×25	1	1	10	0	87
5	70×600	3	6	8	0	83
6	29×31	0	5	5	0	90
7	33×88	1	3	5	0	91
8	1000×945	4	4	2	0	90
9	10% of 750	3	1	3	0	93
10	$14 \times 9 \div 6$	4	1	1	0	94
11	$\frac{1}{2} + \frac{3}{4}$	1	0	9	0	90
12	$10 - 4\frac{3}{4}$	0	1	0	0	99
13	$\frac{2}{3}$ of 45	0	0	1	0	99
14	$\$1.99 + \1.99	0	8	23	0	69
15	$\$4.93 + 39c$	1	9	10	0	79
16	$7.41 - 2.5$	0	0	8	0	92
17	0.25×800	1	0	0	0	99
18	$3.5 \div 0.5$	0	4	1	0	95
	Total	1	3	10	0	86

The data also reflect that students were keen to move on to the next item. Having tried to solve the item using two different methods many students were clear that they would not be able to perform the calculation any other way. Rather than suggest fatigue it appeared as though the students felt they had solved the item and they wanted to progress to the next one. This probably reflects typical classroom practice where the focus is on completing the maximum number of problems in a set period of time rather than discussing ways of solving problems.

Data for first, second and third choice are revisited in Chapter 7 where the success rates for using various computation methods are examined. This allows for a clearer picture of computation choice to be developed. In some cases students using their first chosen method of calculation may have produced an incorrect result, whereas when applying their second method, they produced the correct answer.

Summary

In this chapter data have been supplied to answer the first research question. The data indicate that mental computation was favoured as the first computation choice for most items and was thus the most common method of computation overall. Calculator methods were then slightly favoured over written methods. Mixed methods only accounted for a small percentage of the overall computation approaches used by the students.

The item type clearly had an impact on computation choice. Table 5.2 showed the variation in computation choice according to item. Price (1995) also found that the types of numbers in the question had an impact on computation choice. In his study the operation was restricted to multiplication but he found items that involved extended basic facts such as 70×600 were more likely to be solved mentally than items such as 36×25 . Comparisons with previous research indicated that the students in this study tended to make more use of calculator methods than did students in prior studies. Students in earlier research tended to show stronger preference for written methods.

While the purpose of the research was not to examine difference in preferences displayed by students in different year levels, some comparisons were made. Table 5.11 indicated that in many cases computation choice was consistent across Year levels, although strength of choice varied. Where variations in computation choice existed, possible reasons were suggested. These included, maturation, experience with a particular computation method and recency of practice with a particular method or type of question.

Of special interest is how students made their decision to adopt a particular computation approach. In the chapter that follows, qualitative data will be used to explain why students made the computation choices reported in this chapter.

Chapter 6: Results for Question 2

In the previous chapter an analysis of the computation choices made by students was undertaken in order to answer the first research question. Some trends in computation choice had already been established in previous research (Price, 1995; Reys et al. 1993), although the trends were limited. In this chapter the focus is turned to answering the more complex question of why students made the choices they did. In particular, the answer to Research Question 2 is provided in this chapter:

Why do students in Years 5-7 make particular computation choices?

The answer to this question should assist teachers in helping children to choose appropriate methods of calculation to suit the question, the context and the students' ability. In order to answer the second research question, use was made of the qualitative data that were gathered as part of the study. In particular audio-tapes of the interviews and associated transcripts were reviewed. As there were few previous studies of computation choice (Price, 1995; Reys et al., 1993), there were no established reasons for computation choice. Data were sorted under common themes, based on sifting through the data. These themes, along with excerpts from the interview transcripts are reported to help explain why students make particular computation choices.

For most students there was little or no hesitation when making the choice as to which method to use when solving an item. There was little evidence to support the notion that students carefully examine a question before choosing a computation method. Most students tended to use rudimentary criteria such as 'big numbers' when deciding which form of computation to use. There were a few examples of metacognitive thinking taking place and these are highlighted later in the chapter when the computation model developed in Chapter 3 is re-examined in the light of the findings.

The ability of students to choose appropriate computation approaches was at times hampered by a lack of experience with certain types of calculation and certain approaches to calculating. The 'default approach' seems to have been to use a calculator. This is an area of concern because students might have thought the calculator

would produce the correct result ‘if all else failed’, but a lack of familiarity with the way a calculator works often meant they did not obtain the correct result. A lack of understanding of the functions on a simple calculator was of concern and certainly restricted computation choice or meant that students making use of a calculator moved into unfamiliar territory and therefore made mistakes. This was most evident when students tried calculating 10% with the aid of a calculator.

Some students overestimated their ability to solve items using a specific computation approach. For example, when trying to calculate the result of multiplying two two-digit numbers (36×25 , 29×31 , 33×88), some students overestimated their ability to use mental computation. This meant that students either part way through a calculation abandoned that method in favour of ‘safer’ methods, such as using a calculator, or tried to complete the calculation but failed to achieve the correct result.

It appears that some aspects of a calculation can override other considerations when it comes to making a computation choice. The magnitude of the numbers in the question seems to be one such overriding factor. Students often chose to complete item 8, 1000×945 , using a calculator because it involved ‘big numbers’. This item could have been completed with relative ease using mental methods. When examining the way general reasons, such as, ‘big numbers’ were used, for making computation choices it was noted that:

- The same student did not necessarily use the same general reason when justifying similar computation choices even when the question was similar;
- Students might use different general reasons to justify making similar computation choices for similar questions; and
- The same general reason might be used to justify different computation choices.

The use of general filters by which students made initial computation choices was very much dependent on a range of factors, including experience. These choices were executed rapidly and were generally based on one and sometimes two features of the question. Before explaining the reasons given by students to justify their computation choices, it would be appropriate to make a few cautionary remarks about student perceptions and explanations.

Student Perceptions and Explanations

In Chapter 4 the issues of collecting data based on student recollection and explanation were discussed. Students in the study were not always able to clearly explain the reason behind the computation choices they made. Some had problems articulating their reasons while some appeared to automatically adopt a particular approach without much conscious thought. While some students gave detailed reasons for making a particular choice others made broad statements such as ‘big numbers’ to describe the reason behind making choosing a specific computation method. Even though several students may have given the same reason such as ‘big numbers’ it was most evident that students providing this explanation possessed differing viewpoints as to what constituted a big number. Similarly, other reasons given by students in the study tended to have various shades of meaning. The addition of comments made by the students is used to help clarify what was meant when a particular reason was given. It should also be noted that while students may choose a particular method of computation as their first choice, it does not necessarily mean the student is any less proficient at using another method.

Student perception and reality can be different as evidenced in the following example. The student referred to speed as the reason for using a calculator to solve Item 3, $369 \div 3$, which is a relatively simple and quick calculation to complete mentally or with paper. Possibly the thought of division invoked a vision of long and tedious calculation.

I: You used a calculator, why?

S: Because *I would have been there all day doing it in my head.*

The student was then questioned a little further and demonstrated a well executed written division algorithm.

The following extracts reveal some inconsistency in reasons given for using particular approaches to calculation. Note what the same student said in several examples described below:

I: 36×25 .

S: 900.

I: And you used the calculator. Is there a reason for that?

S: They're *really big numbers*.

I: Do you think you could have done that any other way?

S: On paper.

The student then demonstrated a proficient use of the standard written algorithm. The above example involved two-digit by two-digit multiplication and so did the following example, but the student elected to use a different computation method and cited a completely different reason. In the first case he referred to 36×25 as ‘really big numbers’ but in the following example of 29×31 he stated that the numbers were small enough to write down.

I: And you wrote that one down. How come?

S: It was a *small enough number* to write down.

The same student later chose to use a calculator to solve 1000×945 because he viewed them as ‘pretty big numbers’.

I: 1000×945 .

S: 945,000.

I: 945 000. How come you used the calculator for that?

S: They were *pretty big numbers*.

Reys, Reys and Hope (1993) also noted some interesting but perplexing information related to computation preference when the item involved two-digit by two-digit multiplication. They stated “... a greater number of fifth graders preferred to do ‘ 29×31 ’ mentally than the seemingly easier item ‘ 36×25 ’. This may reflect a misunderstanding of the task” (p. 310). It is possible that because students seem to make their mind up so rapidly as to which form of computation to use they may not fully understand the question before embarking on a particular solution path.

Not all students were able to give a reason for using a particular computation approach. For example when questioned as to why he completed Item 2, $74 - 36$, using a written method a student replied, “I wouldn’t really have a reason.” When questioned further the student demonstrated his ability to solve the item mentally using an approach that began with the tens.

The various reasons for choosing a particular computation strategy are grouped under broad themes. These themes were chosen because of the frequency with which they were used when justifying computation choices. There were no data to suggest that these reasons were restricted to particular items or item types. There was some evidence to suggest that these broad justifications were used to explain why written and calculator methods were favoured rather than why mental methods were employed. It was almost as if students felt more obliged to justify the use of calculators. It is possible that students are challenged as to their use of calculators as part of the classroom routine and therefore are more readily able to justify their use.

Computation choice appeared to be almost instantaneous, which raises some concerns. However, a deeper consideration of the explanations given indicated that while the reasons students might give seem arbitrary, students do have benchmarks or yardsticks by which they weigh up computation choices. It seems reasonable to suggest that much in the same way students seem unaware of the mental strategies used by their peers when performing a mental calculation, they may also be unaware of the strategies used by their peers to make computation choices. It was clear when interviewing students that some were concerned about the legitimacy of the reasons they gave for making a computation choice. It is possible that open discussion and sharing of reasons for making computation choices may strengthen students' thinking in this area in much the same way that discussion of mental strategies does in mental computation.

Magnitude of Numbers

A common reason for choosing a particular form of computation is related to the magnitude of numbers. While this might seem to be a rather crude method of choosing which form of calculation to use, it is a little more complex than one might imagine. Students often cited 'big numbers' as the reason for choosing not to use mental methods. In some cases the 'big numbers' prompted the student to use a written method, while in other cases they used a calculator. In the following extract the student elaborates on the 'big number' explanation. Note that in solving Item 15, $\$4.93 + 39c$, how the student links the idea of 'big numbers' to memory constraints by suggesting they are hard to remember. It is possible that the cognitive load is increased by the introduction of larger strings of numbers and that many students recognise this.

- I: And why did you choose to do that by writing it down?
S: Because *$\$4.93$ is a big number to remember*. You might just forget and put $\$4.39$, it's better just to write it down so you can remember it properly.

Similar comments were made by the following student trying to calculate 36×25 . Note the reason she gave for using a written algorithm.

- I: Why did you choose to write that one down?
S: Because I couldn't imagine it in my head, because it's *too many numbers*.

The student in the following extract makes reference to 'big numbers' as the reason for using a calculator to complete Item 3, $369 \div 3$.

- I: So you used the calculator there, why?
S: Because it's a big number, it's easier to use a calculator.

When questioned a little further this student indicated that she might use a mental method and then described the method she would use, although she was reluctant to perform the calculation. Her description of the method she planned to use to perform the calculation mentally, explains why.

S: Yes, I'd do it like I was *writing it in my head*.

There was no evidence to suggest this student could successfully complete the division item in her head, but the comment about 'writing it in her head' is interesting. Several students described mental methods based on the use of the standard written method. The mental version of the written algorithm seems to hamper ability with mental calculation and as a result influences computation choice, often away from mental methods.

Some cautionary remarks need to be made about the 'big number' explanation for using a calculator. Students using this reason for adopting a calculator-based approach to solving the item were not totally reliant on the use of a calculator but simply chose it as the most expedient option. The following extract shows an example of a student who chose to use a calculator but was also able to complete the same item mentally. Whether the use of the calculator first made the mental computation simpler is debatable, as her explanation of how to solve 70×600 mentally is most plausible.

I: And you used a calculator for that one – why?

S: Because it was a *big number*.

I: Could you have done it another way?

S: I could have gone seven times six is 42 and then put three zeros on it.

The student stated that she would not use written computation to solve this item. Later the same student also used a calculator to solve 1000×945 . This student was consistent in her choice of calculator methods for this type of question and in the reason (big number) she gave for making this choice. Another example of large numbers prompting the use of a calculator may be seen below. When solving $369 \div 3$ the student did not hesitate in picking up a calculator. Note that the student referred to the 'times table' and 'large numbers'.

I: You didn't have any hesitation going to the calculator, why was that?

S: I know my times table, *but when it gets really, really high, I always use a calculator*.

While it may appear surprising that a student would use a calculator to solve an item such as 1000×945 , clearly many students could only see the size of the numbers and failed to look at the numbers themselves. When discussing this item with the students it became apparent that they viewed the written approach as too cumbersome.

Students often cited ‘big numbers’ as the reason for choosing to use a calculator. Students who gave this reason tended not to look at the nature of the numbers but rather at how many digits were given in the item. Perceptions of ‘big numbers’ varied from student to student. Note the response to 0.25×800 . In this example the student combines the big number reason for using a calculator with the fact that the item involved decimals.

I: Why did you use a calculator for that one?

S: Because the times *was over 500, and it was in point form*, decimal form.

It should be noted that at times students gave more than one reason for making a particular computation choice. For example, students linked big numbers and decimals, as in the case above, or tables and big numbers. It was not uncommon for students to notice more than one feature of a question.

Efficiency

It appeared as though ‘speed of calculation’ was considered to be an essential part of any calculation. It is possible that certain school practices such as giving students a number of questions to complete in a set amount of time or rewarding those students who finish early may raise a student’s awareness of time as a factor in performing calculations. A further consideration for some students, who tend to dislike mathematics, is that if you calculate rapidly, then you can move on to more pleasurable tasks.

A common reason given for using a calculator involved speed. Expressions like “it’s faster”, “it’s quicker, or “it’s quick and easy” were often used in support of calculator use. Speed was also cited as a reason for not using the written algorithm. Comments such as, “it would take longer” and “it would take too long” were commonly used to explain why the written algorithm was not favoured. Comments about speed and ease of calculation tend to indicate that the students may have considered various possibilities and concluded that one particular method was better or faster than another. Note the reasons given by one student for using a calculator for 36×25 .

I: You did that on the calculator, why?

S: It’s a bit hard for my head and I didn’t want to use paper, *the calculator is easier*.

The student was then asked whether he would know how to do it on paper and continued to correctly execute the written algorithm. In the next example the student used the 'big number' reason in conjunction with a comment about the amount of time 1000×945 would take to complete on paper, to explain why he opted to use a calculator. The inflection in his voice appeared to suggest that the speed factor influenced his decision to use a calculator.

- I: I noticed that you used a calculator, why was that?
S: *Big sum.*
I: Could you do it any other way?
S: On paper, but *it would take forever.*

In the following example, also involving 1000×945 the student gave speed and the size of the numbers as reasons for using a calculator. Note the reference to the numbers being 'big'.

- I: Why did you use a calculator?
S: I would definitely use a calculator *because it takes too long on the paper, it's pretty big.*

Comments relating to speed were not restricted to calculator use. The following student cited speed as the reason for choosing mental computation for Item 1, $28 + 37$.

- I: Why did you choose to do it mentally?
S: *I could do it faster.*

In the next example the student indicated that compared to computing mentally it would be faster to write it down. Written calculation appeared to be the preferred method of calculating for this student so it was possible she was more proficient in using written methods.

- I: So you wrote that one down. Is there a reason for that?
S: I probably could have done it in my head but *it would have taken way longer* and I prefer writing than using the calculator.

In each of these examples students seemed to have made a comparison of methods based on criteria such as speed and ease of calculation. At times it is difficult to comprehend the choices made by students based on these criteria, but it must be remembered that comparisons according to speed and ease were based on students' own methods. For example it may be hard to understand why a student would choose to calculate 1000×945 using a calculator, citing that it was faster, when mental methods would seem more efficient. If, however, the student was using a mental form of the written algorithm to calculate 1000×945 one can see why a calculator would appear quicker and easier.

Recognised a Weakness

Some students appeared to be very aware of their weaknesses. Oftentimes students would comment, “I’m not good at,” or “I can’t do.” The students would name specific problems they had. For example, they would pinpoint operations such as division or problems with decimals or fractions. Other common problems included difficulties with tables or zeros. In some cases the perceived weaknesses could begin to dominate the thinking of a student to the point where computation choice became restricted. When an item involving the perceived weakness was presented to the students they tended to opt for the safest route, which in many cases involved using the paper-and-pencil algorithm or the calculator. Mental computation tended to be viewed as a slightly more risky option for many students. They would rather make use of the safest method or the method they felt would most likely produce the correct result. In the following example note how specific the explanations of why a calculator was chosen as the preferred option. These students were very clear about their areas of weakness.

S1: *I don't really understand fractions.*

S2: *Because it's a decimal times.*

S3: *Because I can't do a point times something else.*

It should be noted that students who were often critical of their own ability, at times, seemed overly harsh in their assessment of their ability to perform a particular type of calculation. There were many occasions where students stated that they could not complete an item using a particular computation approach. It was only later in the interview that they give a clear explanation of how it could be solved using an alternate method. Students cited some weaknesses more often than others. These are outlined in the next section.

Difficulties with a particular operation

Many students in the study expressed the opinion that division items were difficult. The division sign seems to dominate the thinking of the students to the point where they failed to take into account the numbers in the question. For example, one student when responding to the item $3.5 \div 0.5$ explained why she chose a written method by stating; “Well, when it comes to divide I’m not very good at it in my head.” Using a written method she calculated an incorrect result. Later, however using a calculator she managed to arrive at the correct answer.

It should be noted that difficulties were not simply restricted to division. The two examples that follow were in response to Item 16, $7.41 - 2.5$. Previously it was mentioned that this subtraction item caused difficulties among the students because of the differing number of decimal places. It appeared this added to the concern about subtraction. In the first example the student used a global statement, whereas in the second case the student was referring to a particular type of subtraction relating to decimals.

- I: How come you used the calculator?
S1: Because *I’m not very good at takes*.
- I: I noticed you used a calculator. Why was that?
S2: *I don't really understand the taking*.

In the later example the student was really expressing a lack of understanding of the subtraction algorithm for decimals. When students looked at Item 3, $369 \div 3$, most failed to consider the numbers but rather focussed on the division symbol. It is as if this feature of the question dominated their thinking so that everything else was blocked out. Previous research (Swan, 1991) has shown that when dealing with division, students tend to use versions of the written algorithm when calculating mentally. The lack of strategies for mentally calculating simple division questions may contribute to the difficulties students experience with division.

Knowledge of multiplication facts

The comments made by students in the study indicate that basic multiplication facts or ‘the tables’ as students referred to them feature prominently in their thinking about mathematics and have an influence on the computation choices they make. The following comments made by the same student indicated a preoccupation with ‘tables’.

- I: You used a calculator, why?
 S: Because 14×9 and divided by 6, *I don't know my times table up to that standard.*

In response to 36×25 the student again referred to 'tables' and 'big numbers'.

- I: You chose to use a calculator. Why?
 S: Because again it was a *big number*. *I don't really know my 36 times table.*

The last comment is of concern as it appears as though the student has a view that to perform multiplication questions requires an understanding of basic facts beyond ' 9×9 '. Negative attitudes toward basic multiplication facts also feature as a reason for avoiding mental and written methods in favour of the calculator. In response to 36×25 the student gave the following reason for using a calculator.

- S: Because *I don't like times tables very much*, and it's easier on a calculator.

The same student responded to Item 7, 33×88 , explaining why she chose to use a calculator by stating, "*because I'm not good at my eight times table.*" In the following example, when completing 70×600 the student referred to a lack of understanding of basic multiplication facts as the reason for switching from using a mental method to a calculator method.

- I: It looked as though you were trying to do that in your head to start off with and then you changed to the calculator, why?
 S: *I don't really know my seven times table.*

Some students equate mental computation with 'tables' and therefore believe that if they cannot do 'tables' then they must not be proficient mental calculators. This belief in turn influences computation choice away from mental methods in favour of written and calculator methods. The following student dismissed mental methods for calculating 36×25 in favour of using a calculator based on his perception of his ability with 'tables'.

- I: And this time you chose to use the calculator. Why was that?
 S: Because *I'm not good at my times tables* so I couldn't work it out in my head so it would be easier to work out on the calculator.

It appeared as though ability with tables was linked to confidence and this played a role in computation choice. Those students who felt more confident in their ability with tables tended to feel more at ease choosing mental methods; those with less confidence chose alternative methods.

Teacher Influence

The school influence on computation choice is quite apparent in the way students make computation choices and the reasons they give for making them. Previous research by Price (1995) indicated that the school context has an effect on choice and the relative emphasis placed on particular methods by the teacher. The recency of classroom experience with computation alternatives also appears to play a role in computation choice as the following extract indicates.

- I: How come you decided to do that one in your head?
S: *Because we've learnt about them quite a lot.*

Students made comments alluding to the fact that they did a specific type of calculation a particular way in class. For example, the influence of the teacher and the teaching of standard written algorithms may be noted in the following extract.

- I: You prefer to write it down. Is there a reason for that?
S: I just was *taught to do it* and I've always been doing it that way.

The same student also indicated an aversion to using the calculator based on the notion of 'not getting any smarter' as a result of using a calculator. It was not clear where the student developed the idea that using a calculator would not assist in learning but the impact of this type of thinking served to restrict computation choice. There are several spurious arguments suggesting that a person no longer has to think as a result of using a calculator. These arguments were refuted in Chapter 2; nevertheless comments such as the one recorded below indicate the thinking of some students. The impact of this type of thinking is that a computation alternative, the calculator, is lost.

- S: Again, it's easier because *when I do it on the calculator I think it's not using your brain and you don't work things out better and you don't get smarter*, so I write it down because it's easier and I understand better.

This same student indicated a preference for written methods in the following comment. His comments also seemed to indicate a belief that his memory would be taxed if the calculation were to be completed mentally.

- S: As again, it's pretty simple to do it that way instead of in you mind or in a calculator. Because you can write it down and *you don't have to think of two things at a time*.

Later, this student made this comment explain why he chose to make use of a calculator. In deciding to use a calculator he compared it to using a written algorithm.

- S: Because it was *faster and easier* this time because I sort of know how to do those already but if I just typed it in it would be *twice as easy than writing it down*.

While a teacher may not directly suggest that one computation method is better than another, the time allocated to a particular approach can often send a message to the students. The following student has clearly received the message that written algorithms are extremely important by the amount of class time allocated to completing written algorithms. The student now seemed to equate learning mathematics with practice. The student chose to use a standard written method to solve 33×88 .

- I: I notice that you did that with paper and pencil. Why did you choose to do that with your pencil and paper?
S: Because it's easy enough to do. *We do heaps of this in class and you get the hang of it.* I could have done it on the calculator.

Note the reference to the teacher-taught method when the student is asked to explain why she used a mental method to solve 70×600 .

- I: You did that one in your head, why was that?
S: Because *my teacher told us* to do like, 7×6 and then just write down the answer and put a 0 on it?

Many students experienced 'trouble with zeros' when tackling this item and 1000×945 . Reference was often made to being taught methods for dealing with zeros.

The influence of repeated practice in the written algorithm or possibly the recency of the students' experience with solving this type of question may account for the choice to use a written method in the following case. Notice the way the student has internalised an approach to this type of question which has been reduced to a set of procedures.

- I: Okay so you wrote that one down. Why was that?
S: *Because that's a cross out, put down, carry over.*

The influence of the teacher and class-taught methods of computation are clearly evident in the following extract. The student gave a clear explanation of how to complete a written computation. Note in particular the reference to the "O for Oscar" rule.

- S: In our class, we do it a different way, we do like the first part and then we go into, we're not really supposed to have that, but we have two columns.
I: Show me how you do it.
S: 6×5 are 30 and so took down the 0 and put the 3 up there and we go 5's, 15 and 15 plus 3 is 18 and so we put down that and then we go 2×3 are 12, and then we have to cross that out because we don't use that any more so we put that one up there and then we go 2×3 are 6 plus one is 7, and then I think we put the 0 for Oscar down and we had those two together and you get the answer.
I: And what's 0 for Oscar?
S: O for Oscar, it's a way of remembering. I'm not sure if that's right because I know that's the way that you usually do it but I sometimes don't always get it right.

The “O for Oscar” approach’ is illustrative of the statements often made by teachers to help children remember the procedure for completing the written algorithm. This student, however, had no understanding of how or why it worked and therefore lacked confidence in this approach, although he still chose to use it.

A Process of Elimination

This category describes students who made a computation choice by considering the alternatives and almost as if by a process of elimination determined the approach they would be most comfortable in using. This indicated they had thought about the item and considered various options. This implied some metacognitive thinking on the part of the students who used this approach to making a computation choice. Note how the following students used a process of elimination to arrive at the choice to use a calculator.

- I: And you did that on the calculator, why was that?
S1: I don’t know. I’m not that good at those ones; *I haven’t done it in class so I didn’t think I would do it writing it down* so I just did with the calculator.
- I: And why did you choose to use a calculator?
S2: *If I did it in my head I would get mucked up with the point* and I think I would do the same thing with writing too.

Often it was the case that students who were unsure of how to carry out a calculation opted to use a calculator. This led to the development of a ‘last resort’ sub-category, where students would almost choose a method by default, as they did not have any other methods at their disposal.

As a last resort

A slight variation of the ‘process of elimination’ theme was the ‘last resort’. For example, the calculator was often chosen by default. Comments such as “I’d have to do that on the calculator because we haven’t done that” indicated that computation choice was limited by a lack of experience. In some cases such as in the item involving percentages, students chose to use the calculator because of uncertainty with the alternatives, only to find they were unsure of how to use the calculator. Often these students chose to use a calculator in the hope that they could perform the calculation. The following explanation indicated the thinking of many students:

- S: We haven’t learnt how to do them on the page yet and I couldn’t really figure it out in my head, so I used the calculator.

The following student tried all three computation methods to calculate 33×88 before deciding to make use of a calculator.

- I: I noticed you tried to do some in your head and then you wrote something on paper and then you went to the calculator. So what was happening?
S: *In my head was a bit hard and then it was harder on paper, so I just had to use the calculator.*

The intonation in this student's voice indicated that she didn't have a choice – she just *had* to use the calculator. In some cases it appeared as though the student was a little ashamed at not having another method to complete the calculation. When faced with calculating the result of $7.41 - 2.5$ the following student chose to use a calculator. Her reasons are outlined below.

- I: I noticed you used the calculator to get the 4.91. Why did you use the calculator?
S: *Because I probably would have lost track of the numbers if I did it in my head and on paper I probably would have got a bit confused.*

It became evident, that for a range of reasons many students' computation choice was restricted. This was not simply due to a lack of experience but due to a variety of reasons outlined in the next section.

Restricted Computation Choice

Computation choice was often restricted by a lack of content knowledge, a lack of familiarity with calculators, or a lack of understanding of calculation methods. Examples indicative of these problems follow.

Lack of content knowledge

A lack of content knowledge, the ability to read numbers, is highlighted in this example. Despite using the calculator the student cannot read and state the number shown on the display of the calculator.

- I: 1000×945 .
S: That's hard.
I: What did you get for your answer?
S: *I can't pronounce that number.*
I: You show me then. Okay. 945,000. Now how come you used the calculator?
S: I was thinking that maybe it would have been something to do with the 945, but I didn't really get it.

Other examples that served to restrict computation choice included a lack of knowledge of simple fraction-decimal equivalents and a lack of place value knowledge. The previous excerpt indicated that an understanding of numbers is required when solving computation items, even with the aid of calculators.

Difficulties with zeros

Many students who tried mental methods to solve items involving zeros did not know what to do with all the zeros. Their efforts were hampered because of a lack of understanding of place value, and a reliance on half-remembered rules. Some students had developed a rule for ‘adding and taking zeros’ that involved counting the number of zeros in the multiplier. For example, when completing Item 5, 70×600 , students would ‘take three zeros off’, complete the multiplication 7×6 , and then add two zeros on once the multiplication was complete. When questioned as to why three zeros were removed but only two were added students would point out that the multiplier, 600, only contained two zeros. Item 8, 1000×945 , which included a multiplicand that contained zeros but a multiplier that did not, posed some issues for students adopting a rule based on counting the number of zeros in the multiplier. The confusion caused by zeros is most apparent in the following example, where the student described why she chose a calculator to complete 1000×945 .

- I: This time you used the calculator. Why was that?
S: Because it was pretty big and pretty tricky *because I get confused if it has lots of 0's.*

Many students recognised zeros as causing them difficulty. This difficulty is not confined to mental methods. Similar issues were noted with students who employed written methods, only to lose track of the zeros. As pointed out earlier, even students using the calculator, experienced difficulty when confronted with zeros.

Faulty reason for using mental methods

Computation choice may be affected by misinformation. Many students did not have an understanding of the distributive property of multiplication over addition and therefore when performing a two-digit by two-digit multiplication, only multiplied the units by the units and the tens by the tens. This approach reduced the multiplications required by half and often negated the need to ‘carry’, thereby reducing the cognitive load, making mental computation a more attractive option. Many students, who chose to

complete 29×31 mentally, did so based on a faulty understanding of the written algorithm. They adopted this faulty written method as their mental approach. When completing 29×31 students would multiply 1×9 and 3×2 , both simple mental calculations, to arrive at an answer of 69. This was a common occurrence, especially among Year 5 students.

Unable to use technology

Calculator use for some students was limited in part by the student's inability to use all the functions of the calculator, such as entering numbers into the calculator, or reading the display. This lack of familiarity with the calculator was particularly evident when the subjects were faced with items involving fractions, percentages, and where the number displayed was larger than some students could read. A typical result for students using a calculator to add \$4.93 and 39 cents was \$43.93. This result indicates a lack of experience with this type of calculation and confirms that a calculator does not have the capacity to think, only to follow instructions. Of concern was the number of students who accepted this result without question.

Some students became a little frustrated when trying to find 10% of 750 on the calculator. Many had assumed it would simply require pressing the percentage key. The audio recording of this exchange revealed a level of frustration in the student's voice.

- I: Can you tell me what you are doing. I noticed you used a calculator there.
What did you try?
S: I tried 750 then times the percentage I was going to put the 10 in but it went
back to zero. And then I did it the other way but it still went back to zero.
I: So you would have used a calculator if you knew?
S: Yeah *if I knew* how to do it.

The following student began to use a calculator to solve an item involving fractions, only to stop part way through and adopt a different approach. Unable to enter a fraction into the calculator she could not complete the item.

- I: Why can't you do it all on the calculator?
S: Because *I don't know how to get fractions on the calculator.*

Inefficient Approaches

Some students' computation choices were hampered due to their use of inefficient approaches. For example, the following student started using a paper-and-pencil approach to help solve $74 - 36$, but it was extremely inefficient and caused her to abandon this method part way through and opt for a calculator.

- I: What about 74 take 36?
S: 38
I: I noticed you started writing strokes on the paper and then changed your mind and went for the calculator, why?
S: It just takes too long. *I was going to write down 74 strokes and then strike out 36 and then count.*

The same student when tackling Item 3, $369 \div 3$, indicated that she had no alternative but to use a calculator.

- I: What about 369 divided by three?
S: 123
I: You went straight to a calculator, why?
S: I just thought that's a big number, I am never going to work that out in my head.
I: Would you know how to do that on paper?
S: You could *write down 369 strokes and put a circle around the three.*

The decision to use a calculator was based on a comparison to an inefficient written method. The choice to use a calculator certainly appeared justified based on the description of the written method this student would have tried. While this student showed a propensity to use the calculator for most items in the study, she did not use it for all items. She used mental methods for two items and written methods for three items. This student, however, would be a good example of a student whose computation choice was severely limited because she had few alternatives available to her.

Relationship to Computation Model

Thus far, broad reasons for choosing specific computation approaches have been raised along with factors that impinge on computation choice. In this section data are examined in order to examine facets of the computation model proposed in Chapter 3. In particular, evidence of metacognitive thinking as well as reasons for choosing mental, written, calculator and mixed methods are examined.

Several computation options were identified in the Swan and Bana (1998) model presented in Chapter 3 (See Figure 3.6). These options included mental, written and calculator methods of calculation and various combinations of these methods. Several factors were suggested as impinging on computation choice. Explanations given by students in this study have helped to illuminate understanding of why particular choices are made.

Students in the study gave clear reasons for choosing one of the three main approaches to computation; mental, written and calculator methods. Often when giving reasons for making a particular computation choice the student would cite one or two broad reasons, such as ‘big numbers’, speed or ease of calculation, and difficulties with a particular operation as outlined earlier in this chapter.

Evidence of metacomputation

The following extract exemplifies some students who showed an awareness of the size of the expected answer.

- I: What about 33×88 ?
S: 464 042 (completed using pen-and-paper), *that's way wrong. It's too big. Way too big.*

The student was well aware that the answer should have been much smaller and later made use of a calculator to answer the question. A similar comment is made by the following student who recognised that the first result he calculated using mental methods was incorrect. It appeared that students did not have sophisticated checking methods at their disposal but relied on some rather rudimentary methods, such as the magnitude of the answer.

- I: 29×31
S: 69, oh no. Sorry 899.
I: So you started to do that one in your head and then you used the calculator, so what was going on?
S: Well I tried to work it out in my head, but I got it wrong because I got 60 something.
I: How did you know it was wrong?
S: Well because *it would have been too low* for 29×31 .
I: Is there any other way you could do that?
S: No.

After completing Item 18, $3.5 \div 0.5$, using a pencil-and-paper method one student recorded an answer of 0.1 and then repeated the calculation with the aid of a calculator. When asked why he did this, the response was, “I really didn’t think that was right, so I used the calculator.” He could not explain why he felt it was incorrect.

A common error occurred when students attempted to use a calculator to solve Item 15, $\$4.93 + 39c$. Students would often enter the 39 cents as 39 so that the calculator accepted this amount as \$39 and the result was \$43.93. Many students simply accepted this as the answer and did not question the result. Some students did, however, recognise that an answer of \$3.93 could not possibly be correct. They would not hesitate to repeat the calculation with the calculator. Rarely would a student repeat a calculation using pencil-and-paper as it would be time consuming, but calculators lend themselves to this checking approach as the calculation may be repeated quickly. On several occasions students repeated calculations made with a calculator. Note the reason given for repeating the calculation, 29×31 , in the following extract.

- I: Now you chose to use a calculator. 899 is your answer, but for some reason you did it twice, why was that?
S: Because I think I pressed something wrong in the first place.
I: What in your head told you that you had pressed something wrong?
S: Because *the answer was too small, 29×31 , it's got to be over 100.*

Not all estimations or guesses were close to the mark. It appears that some students do not have any strategies at their disposal when performing and estimate. The following example illustrates this point.

- I: 70×600
S: *I think that would be 6000*, but I'll just do it (calculator) 42 000.

The student was unable to explain how he arrived at the estimate of 6000, other than by referring to experience with other calculations. Later on in the interview the same student estimated that the result of 1000×945 to be around a million but he could not describe why he reached that conclusion.

Some students used multiple calculation methods when unsure of the solution. A popular approach involved using the calculator as a checking device. The students would complete a calculation using mental or written methods and then check it with a calculator. This is a common practice in many classrooms (Sparrow & Swan, 1997).

Reasons for Choosing to Use a Mental Strategy

When explaining why they chose mental computation most students gave reasons that indicated they had thought about alternative options and had made the decision to use mental computation. Comparative comments, such as, it was 'easier' or mental computation was 'quicker' or 'simpler'; indicated that many students had considered some alternative approaches. The comment made in the following extract was an example of the comparative comments made by students. Note the way the following student used a mental method that involved compensation when calculating the result of adding \$4.93 and 39c.

- S: Well first, \$4.93 and is 7c away from \$5 so add the 7c to 39c making \$5 and 9-7 is 2 so its 32c so its \$5.32.
I: And why did you use that method?
S: Because *it was a little easier to do than on a piece of paper.*

The mental method described by this student was certainly easier than the comparative written method. Had the student tried using a mental approach based on the standard written method, then the student's computation choice may have differed. Even when employing mental computation as the strategy of choice, it was noticeable that many students were using a form of the written algorithm by visualising it in their heads. The following extract illustrates how the student not only imagined the setting out of the item, as it would be on paper, but also the language associated with it.

- I: Can you tell me what you did?
S: Well I imagined that the 37 was on the bottom and I added 8 and 7 which was 15 and then *I put the 5 down and carried the 1* and added 2 and 3 and 1 is 6.

Reasons for Choosing to Use a Written Approach

When stating reasons for using a written approach, most students either made comparisons to alternative methods or made specific reference to an inability to remember all the parts of the calculation in their head. Some children felt more secure using the written approach, expressing some comfort in being able to see the numbers on paper.

- S: It's easier than doing it in your head *because in your head it's a lot harder to think, I forget the numbers* and if I haven't done something or not, but *on the paper I can see* if I've done something or not.

It should be noted that when many students made a comparison between using a mental approach and a written approach, the mental approach they referred to was a version of the written algorithm done in the head. This accounts for some of the memory problems mentioned by students and their inability to complete the calculation mentally. In effect, students who try to use a mental version of written algorithm have a restricted choice because the demands on short term working memory are significantly increased. The following student, who chose to perform the calculation using paper-and-pencil, indicated by her statement about trying to 'carry' in her head that she had thought about using a mental method but had dismissed it in favour of written methods because of the load on memory.

- I: You chose to do that by writing it down. Why was that?
S: Because in my head I can add up the first numbers but *I can't carry over in my head*, because *I can't remember* what the number was.

Some responses indicated that the student had at least considered using mental computation as a first resort prior to using the written algorithm. When questioned as to why written methods were adopted many students responded by referring to the difficulty of using mental methods, suggesting that written methods are often the choice by default when mental methods appear too difficult.

- I: And why did you choose to write that down?
S: Because *I couldn't imagine it in my head*.

Reasons for Choosing to Use a Calculator

Students typically explained that they chose to use a calculator because it was 'easier' or 'quicker' or as a 'last resort' when they had no other method at their disposal. It appeared that a lack of computation options, particularly a lack of mental computation ability, left students with little option but to use a calculator.

The student in the following extract provided some very good reasons for choosing to use a calculator. She recognised her limitations and was aware of the problems she had with zeros.

- I: Why did you use the calculator?
S: Well because *I couldn't quite imagine it in my head and I could have written it down but the 0's get me mucked up*.

Mixed Methods

The Swan and Bana (1998) computation model developed in Chapter 3 (See Figure 3.6) indicated that to represent computation choice as a simple trichotomy was overly simplistic. It was suggested that in reality people not only have mental, written and calculator methods at their disposal but also combinations of these methods. While combination methods were not widely used by the students in this study, accounting for close to 5% of methods used, there were several examples of combined methods. In the next section various combinations of mixed methods are examined.

Mental and informal written methods

The ability to make flexible use of number appears to improve the likelihood of a student choosing mental computation, whereas attempting to apply a mental version of the written algorithm can reduce the ability to compute mentally. Paper-and-pencil algorithms were designed to be carried out on paper, not in the head. Often a student would use some external jottings to record interim results when using a standard written approach as their mental technique.

The following student mixed mental computation with brief jottings to store interim results. The student described the reason for this by referring to memory problems as the reason for jotting two interim results down. The use of a mental version of the formal written approach is likely to have taxed the student's short-term memory.

- I: Now you seemed to do mostly mental there, but I notice you wrote two things down, what happened?
- S: *I was a bit confused, there was too much to remember.* First I started off with two, two times six equals 12 and one is left over, so two times three equals six, add one equals seven, so that's 72, so I added a zero, that's 720, and then I times five times six equals 30, then zero take three, put the three where the other three is, then five times three equals 15, and the three is 18, so that's 180 and add it up.

The student only wrote down the 720 and the 180 and completed the rest of the item mentally. The same student used a very similar approach when solving Item 6, 29×31 .

In response to Item 18, $3.5 \div 0.5$ the following student used a mix of informal jottings and mental methods. The term informal jotting covers a wide variety of external recording approaches. For example, in the following extract the student made use of tally marks in an attempt to keep track of how many 0.5's there were in 3.5.

- I: Now I see that you have put seven strokes on the paper, what was happening there?
- S: *I was counting up the parts*, like that was one, there were two, there were three, there were six halves in three, and then I added the one and that was seven.

The strokes were used as an external memory device to keep track of the number of times 0.5 had been divided into 3.5. In most cases where students made use of informal jottings, it was in an attempt to overcome problems with memory.

Combined calculator and mental method

Item 10, $14 \times 9 \div 6$, which involved two operations tended to elicit a mixed method solution. When examining student responses to this item it was evident that simply because students embarked on a particular computation path they did not necessarily use that method to complete the solution.

- S: I did 14×9 on the calculator and then divided 126 by six in my head.
- I: Why did you do the first part on the calculator?
- S: It's *a bit easier*, I could probably do it my mind, but it would *take a while*.

This approach was different to most used to solve this item. Generally where the calculator was employed it was used to complete the division part of the item. It serves to confirm the finding that students did not simply use a calculator all the time, even when they began an item using a calculator.

There were other examples of students using mixed or 'hybrid' methods of calculation. The use of all three methods combined was rare, although it was noted. The data confirm the fluid nature of calculation as proposed in the computation model, although less so than imagined. It does appear, however, that student computation choice is rather arbitrary and made quickly, suggesting that rather than applying sophisticated metacognitive skills in choosing a computation approach, students rely on rather fundamental and at times superficial aspects of the question, in order to make a computation choice. This means at times the wrong choice is made and students cannot complete the calculation, swap to an alternative method, or have to combine methods to undertake a hybrid form of calculation.

Summary

In this chapter the interview data were grouped under broad themes so sense could be made of the student responses. In doing so it became evident that students used a few rudimentary criteria when making initial computation choices. These criteria are used to allow students to make rapid decisions about which form of computation to use. These criteria were used by a large number of students although the meaning of these criteria varied from student to student. For example, while students might cite 'big numbers' as the criteria for adopting a particular computation approach, what constituted a 'big number' varied from student to student. These criteria were used in many ways when deciding what form of computation to use. In some cases they were combined in order to reach a decision, while in other cases they were used in a process of elimination.

In Chapter 4, reference was made to the loss of data whenever data reduction techniques were used. Combining data under broad categories meant that at times student explanations did not clearly fit into a particular category or categories. Likewise, some students gave interesting explanations of why they made particular computation choices. While these explanations may be of interest they were not relevant to the key themes within the chapter and could distract from these. To ensure these data were not lost, the transcripts of interesting cases, along with brief comments have been included in Appendix 7.

Data were also collected to determine the success rate based on the choices made by students. It is possible that some students may overestimate their ability to solve items using a particular computation method. In the next chapter the success rate for various computation choices is examined.

Chapter 7: Results for Question 3

In the previous chapter the reasons behind the computation choices made by students were examined in order to answer the second research question:

Why do students in Years 5-7 make particular computational choices?

It was determined that students use relatively few, sometimes superficial, criteria to make computation choices. The decision to use a particular computation approach, mental, written, calculator or a combination of these methods was made very quickly. The way these general criteria were used in deciding what form of computation to use was then outlined by examining the reasons given for choosing mental, written, calculator or mixed methods of calculation.

In this chapter the quality of the computation choices is examined in terms of the success rates experienced by students using specific computation approaches. In particular, the answer to Research Question 3 is provided in this chapter.

How successful are students in Years 5-7 at executing various forms of computation?

Examining how successful students were at applying their first computation choice provides a means by which the appropriateness of their choice may be judged. In order to arrive at this judgement, use was made of the quantitative data that were gathered as part of the study. The overall data indicated that students correctly solved the 18 computation items that made up the instrument, using their first choice, 63% of the time. This indicated that overall, the 18-item instrument was set at an appropriate level so as to challenge students in Years 5-7, but still allowed them to experience success. Table 7.1 indicates how success rate varied according to the type of computation chosen.

Success Rate for all Items by Initial Computation Choice

The 63% success rate across all items was boosted by the success rate for using a calculator. Table 7.1 indicates that overall students choosing written methods were more likely to be correct than incorrect. Students choosing mental methods were correct only 44% of the time. Students choosing written methods were only slightly more successful, producing a correct answer in 54% of cases. Students using calculator methods were most likely to be successful (79% correct). It was of concern, however, that in 21% of cases students could not calculate the correct answer when using a calculator. The 18 items making up the instrument were not complex in structure and one might expect a higher rate of correct answers.

Table 7.1: Percentage of correct answers according to chosen method (*n* = 78)

Mental	Written	Calculator
44	54	79

Of most concern was the poor performance recorded by students adopting mental methods. There are several reasons why this might be the case. Some students used a mental form of the written algorithm, which is often an inefficient mental method, and therefore they became cognitively overburdened. Interview data and observations indicated that some students used mental computation as a first resort and then realised it was beyond their mental ability and abandoned this method in favour of another method. If a method was abandoned part way through the calculation and another method adopted, then an incorrect result was recorded for the initial method and the new method was recorded as the second choice. Some students overestimated their mental computation ability and made a poor computation choice by deciding to use a mental method.

The poor success rates experienced by students employing written methods may, in part, be attributed to the use of a faulty algorithm for two-digit by two-digit multiplication items. It was a common occurrence to observe students applying faulty versions of the written algorithm. Many students, while following a standard written algorithm did not fully understand why it worked and therefore experienced difficulties. For example, when completing Item 6, 29×31 , one student who chose to use a written

algorithm and originally calculated the answer to be 59, explained his method in the following terms: “9 x 1 is 9 and 2 x 3 is 60, oh 69.” While he realised his mistake in multiplying 2 and 3, this student never realised the method he used was incorrect. The error appeared to be a systematic one because in completing Item 4, 36 x 25, he followed a similar pattern. Note his explanation of how he multiplied 36 and 25.

- I: 36 x 25
- S: 80 (student performed a written algorithm)
- I: How did you get 80 as your answer?
- S: 6 x 5 is 30, so you put down the zero and put the 3 and then 3 x 2 is 60, no 90, plus 3 is 90.

This example was not an isolated one, with many students completing two-digit by two-digit multiplication items in this way. Students may have chosen to use a written method to solve two-digit by two-digit multiplication items, but failed to calculate the correct result. Other students may have chosen to complete the two-digit by two-digit multiplication items using mental methods, based on the thought that they only had to complete two simple multiplications, rather than four. The outcome was students applying this faulty approach as their mental method failed to calculate the correct result. The following example is of interest because the student mostly favoured written methods for the two-digit by two digit multiplication items. She used the standard written algorithm to calculate the correct answer for Item 4, 36 x 25 but both for 29 x 31 and 33 x 88 her answer was incorrect. Her written calculations are shown in Figure 7.1.

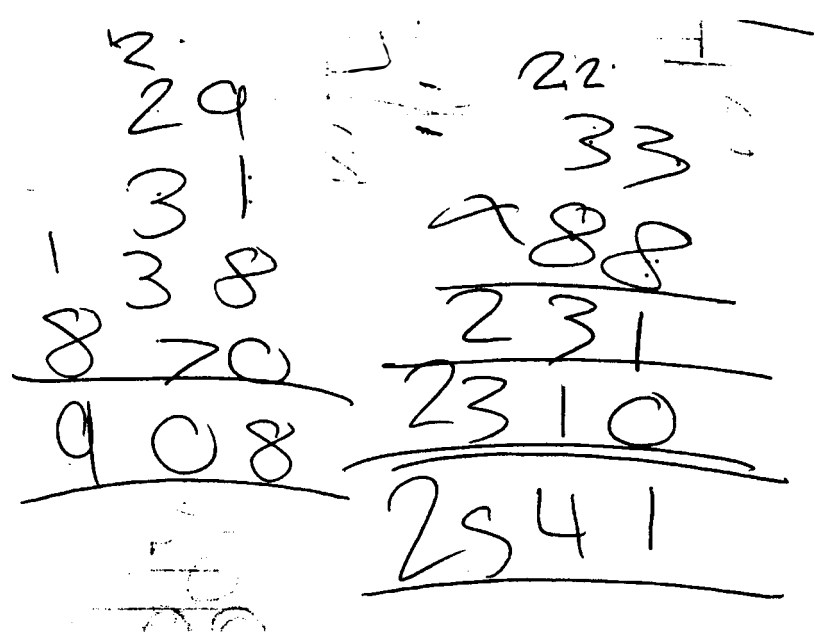


Figure 7.1: Examples of incorrect written results for 29 x 31 and 33 x 88.

The example shown in Figure 7.1 was highlighted because even though the standard written algorithm for multiplication was used, the student still made errors. Of particular interest were her comments after completing Item 7, 33×88 on paper. When it came to comparing the result she had calculated with paper and pencil and the calculator she chose to accept the result shown on the calculator. No interest was shown in finding out which answer was actually correct or where she might have made a mistake in the written calculation.

- I: You chose to write that one down. Why was that?
 S: Same reason as before, because I like doing them that way.
 I: Is there any other way you could do that?
 S: On the calculator.
 I: Do you think you could show me that please?
 S: (Student completes calculation on the calculator and says) I got the wrong answer.
 I: You got the wrong answer on the calculator or on the paper?
 S: On paper.
 I: How do you know the calculator is right and you are wrong?
 S: *Because calculators are normally right.*

While success rates for using a calculator were quite high it also appeared that students lacked an understanding of how to use a calculator and this contributed to the errors made when using it. Very few keying in errors were observed. One of the rare examples of a student keying in the wrong numbers is outlined below.

- I: $369 \div 3$
 S: I might use my calculator, I think. 12 300. (Note student entered $36\ 900 \div 3$, and never realised the mistake).
 I: Why did you use your calculator for that one?
 S: It was a bit hard because the numbers were a bit bigger.
 I: Is there any other way you could have done that?
 S: Probably on paper.
 I: How would you do that on paper?
 S: (Student executed standard written algorithm on paper) It's 123. (Student sounded a little surprised).

In this example the student failed to calculate the correct answer using his favoured method but produced a correct answer using his second choice of calculation method. The student did not realise he had made a mistake in using the calculator until his answer from the written method conflicted with the result from having used the calculator. When calculating the answer to 36×25 using a standard written method a student produced an answer of 800. His mistake may be seen in Figure 7.2. When multiplying 5 by 30 this student failed to write down the 100 part of the calculation. The example highlighted in Figure 7.2 was accompanied by an interesting explanation of why he had chosen to use a written method. Note the reason why a calculator was not employed as the first choice.

- I: Why did you choose to write that one down?
 S: Because it was a bit hard to do in my head.
 I: Is there any other way you could have done that?
 S: *I might have done it on my calculator, but that would be cheating.*
 I: You think it would be cheating if you did it on the calculator.
 S: Sure do.
 I: Would you like to show me how you would do that on the calculator?
 S: You would type in 36×25 and then would get the answer. (The display showed 900). I got it wrong.

$$\begin{array}{r}
 36 \\
 \times 25 \\
 \hline
 180 \\
 720 \\
 \hline
 900
 \end{array}$$

Figure 7.2: Incorrect written method for 36×25 .

Here was an example of the student producing an incorrect result using his first calculation method, but managing to produce the correct result using his second method. His initial choice, however, was made on his perception that making use of a calculator was “cheating.” His computation choice for this item was limited to written computation. Under normal circumstances, where a single method of calculation would have been used this student would have produced an incorrect result. Students would often use the calculator as a last resort when unable to calculate using another method, so the calculator became a ‘default’ choice and therefore the success rate was reduced.

Success Rate for Each Item

A closer look at the data revealed some interesting results. Table 7.2 shows the percentage of correct and incorrect answers for each item. This table helps to show which items caused the most difficulty for students. Table 7.2 also clearly indicates that regardless of the chosen method of computation, fraction and percentage related items were likely to cause the most difficulty. The majority of students in the study did not know how to use a calculator to solve this type of question and hence their computational choice was restricted. Typical was the response by one student who stated, “I don’t know how to get fractions on the calculator.” The thought of fractions seemed to set up a barrier in their mind that restricted thinking about the items.

Table 7.2: Percentage of students supplying correct responses to each item ($n = 78$)

Item Number	Item	Correct
1	$28 + 37$	83
2	$74 - 36$	73
3	$369 \div 3$	94
4	36×25	56
5	70×600	62
6	29×31	65
7	33×88	60
8	1000×945	79
9	10% of 750	23
10	$14 \times 9 \div 6$	74
11	$\frac{1}{2} + \frac{3}{4}$	44
12	$10 - 4\frac{3}{4}$	49
13	$\frac{2}{3}$ of 45	35
14	$\$1.99 + \1.99	68
15	$\$4.93 + 39c$	74
16	$7.41 - 2.5$	44
17	0.25×800	77
18	$3.5 \div 0.5$	71
	Total	63

Item 16, $7.41 - 2.5$, involving decimals, also caused problems for the majority of students. Similar to fractions, decimals tended to invoke a negative response on the part of the students. The items involving two-digit multiplication caused difficulties for a large number of students with close to 40% of students unable to correctly calculate an answer to this item. Ninety-four percent of students were able to correctly calculate the result to Item 3, $369 \div 3$. Students often experience difficulty with division, but the relatively simple nature of this item meant that it could be solved mentally without too much difficulty. Similarly the written algorithm was relatively easy to execute. Table 7.3, however, indicates that 46% of students used a calculator to compute the answer to this question, despite it being relatively simple to solve mentally or with paper and pencil. As outlined in the previous chapter this was probably due to the item involving division.

Success Rate According to Initial Computation Choice for all Items

Data indicating computation choice and percentage of correct answers are linked in Table 7.3. Some interesting patterns may be noted in this data.

Table 7.3: Percentage of correct answers, across all items, according to initial choice
($n = 78$)

Item	% Mental	Correct	% Written	Correct	% Calculator	Correct
$28 + 37$	65	80	23	83	8	100
$74 - 36$	35	44	54	86	9	100
$369 \div 3$	32	84	21	100	46	97
36×25	15	0	50	51	31	100
70×600	37	48	19	13	37	100
29×31	15	0	46	73	35	100
33×88	9	0	50	46	35	97
1000×945	28	73	14	27	58	100
10% of 750	19	40	4	33	36	28
$14 \times 9 \div 6$	10	37	29	65	36	93
$\frac{1}{2} + \frac{3}{4}$	60	55	22	41	5	25
$10 - 4\frac{3}{4}$	77	53	6	40	4	33
$\frac{2}{3}$ of 45	27	33	10	62	9	43
$\$1.99 + \1.99	68	60	26	85	4	67
$\$4.93 + 39c$	49	60	36	93	11	67
$7.41 - 2.5$	35	11	33	46	28	86
0.25×800	8	50	8	17	70	96
$3.5 \div 0.5$	50	64	9	14	39	97
Overall	36	44	26	54	28	80

Note: This table does not include data for mixed and no methods, hence values will not total to 100%.

Table 7.3 highlights some difficulties students experienced using:

- mental methods to calculate with zeros;
- written methods to calculate two-digit by two-digit multiplications;
- a calculator to calculate percentages;
- using mental methods to calculate multiplication of fractions and whole numbers; and
- using mental methods to subtract decimals.

These all appeared to be poor choices based on the lack of success students experienced when using them. The data also present some anomalies such as the discrepancy between success rates for employing mental methods to solve items involving zeros. The success rate for mentally calculating the result for 1000×945 was much higher than for 70×600 . A similar discrepancy may be found when examining the three items involving two-digit multiplication. In each case written methods were favoured, but the success rate for 29×31 was much higher than for 36×25 or 33×88 .

Less than half of the students were able to correctly answer extended basic fact questions such as 70×600 and 1000×945 using mental methods. The students experienced difficulties in handling the zeros. Typical methods involved removing zeros and adding zeros. In most cases students attempted to apply a half-learned rule with limited understanding of place value. The following extract illustrates the use of rule based on the number of zeros contained in the last number – in this case the 600.

- I: 70×600
S: I'll do that one in my head. 420
I: Why did you do that one in your head?
S: Because I know my 7 times tables and I can knock the zeros off and 7×6 and then add the zeros on.
I: So how do you know how many to add on?
S: I added on two because the last number is 600 and that's got two zeros.
I: Right, so that's why you added two zeros on. Did you teach yourself that did you?
S: No, my Dad told me.

The extract is of interest not only because the student explained his method but because having developed a rule the student mis-applied it. When explaining the rule the student did not realise his answer was incorrect or that it did not match the rule he was describing. This following student referred to a "times number" rule for 'adding zeros'. "I did 7 sixes, which is 42 and then I added two zeros on because the times number had two zeros on the end."

The ‘big number’ trigger appeared to have dominated in many cases, prompting the students to try calculator methods. Many students did not view items of this nature as extended basic facts but rather as a question involving big numbers. Most students who attempted to solve items such as 70×600 using written methods failed to calculate the correct result. Students appeared to have difficulty with all the zeros when performing the written algorithm, and their lack of understanding of place value also became evident.

Item 9, 10% of 750, was beyond the ability of most students. They did not appear to have a well-developed concept of percentages and were unable to make use of any of the computation methods at their disposal. The calculator was of little use due to their limited understanding of percentages and how to use the percentage function on the calculator.

Mental methods proved to be successful for the addition and subtraction of fraction items but failed when students attempted Item 13, $\frac{2}{3}$ of 45. In order to solve the fraction multiplication item mentally, the students needed to perform a division and a multiplication calculation. The need to perform two operations, one of which included division, was the cause of difficulties for many students. This difficulty is reflected in the poor success rate.

The item, $7.41 - 2.5$, caused problems for most students. This was reflected in the poor success rate. The first problem related to the mixed nature of the decimals. The first number 7.41 was given to the hundredths place, whereas 2.5 was given to the tenths place. This caused some confusion among students and probably accounted for the poor result. When completing the same item on paper, emphasis was given to aligning the decimal points. The second issue that caused difficulty was the need to decompose seven into six and ten-tenths because of the need to subtract the five-tenths from four-tenths.

A brief review of each item in terms of chosen method and success rate will now be undertaken. The most common method and success rate will be compared to alternative methods.

Good and poor computation choices

The purpose of this section is to highlight good and poor computation choices made by students in years 5-7, according to the amount of success experienced using particular methods. Reference at times is made to question type, such as those involving fractions, to indicate particular trends. The data show that in some cases the choices made by the students were not always the most appropriate. For example, in Item 4 the most common choice was to use written methods, but in only slightly over half the cases did students manage to calculate the correct result. The second most common choice, to use calculators, would have been a better option given the 100% success rate. The decision to use mental methods was a poor one considering that no student managed to calculate the correct answer. A judgement as to whether or not students in the study made appropriate computation choices was made based on the success rate for the most popular choice/s. This is shown in Table 7.4 along with brief comments supporting the judgement. The decision as to whether students had made an appropriate choice was made purely on the basis of success rate.

Table 7.4: Appropriateness of students' computation choices for each item

Item	% Choice	% Correct	Appropriate Choice	Comments
28 + 37	M 65	80	Yes	Given the success rate, mental computation was a reasonable choice, although less popular choices were also likely to produce the correct result. Given the simple nature of the item one would expect the correct answer to be produced using any of the three computation methods.
	W 23	83	Yes	
	C 8	100	Yes	
74 – 36	M 35	44	No	Written was the most common choice and based on the high rate of success this choice appears to have been an appropriate one. While calculators produced a higher success rate it was felt this was not an appropriate choice.
	W 54	86	Yes	
	C 9	100	Yes	
369 ÷ 3	M 32	84	Yes	The high success rates indicate that students seem to have chosen appropriately regardless of which form of computation was used. All choices resulted in a reasonable chance of success.
	W 21	100	Yes	
	C 46	97	Yes	
36 x 25	M 15	0	No	Most common choice, written computation, had only slightly better than a 50% success rate. Mental methods were an extremely poor choice. The best choice would have been to use a calculator.
	W 50	51	Yes	
	C 31	100	Yes	

70 x 600	M 37	48	Yes	Most common computation choice was divided between mental and calculator methods. The calculator proved to be the best choice. Mental methods were still much more successful than written methods.
	W 19	13	No	
	C 37	100	Yes	
29 x 31	M 15	0	No	Mental methods were least popular and the choice not to use mental methods was appropriate given the extremely poor success rate. Written methods were most popular and the choice to use a written approach seems reasonable. Calculator methods were also popular (35%) and the decision to use a calculator was justified by the 100% success rate.
	W 46	73	Yes	
	C 35	100	Yes	
33 x 88	M 9	0	No	The data suggest that two-digit multiplication items should not be attempted mentally. The most common choice, written computation was successful in less than half the cases in which it was used. Calculators would have been a more appropriate choice given the high success rate.
	W 50	46	No	
	C 35	97	Yes	
1000 x 945	M 28	73	Yes	The most popular computation choice was to use a calculator and this appeared justified based on the 100% success rate. Mental methods were also popular and the success rate was reasonable. One would hope that a student would opt for mental methods for this type of question but zeros tend to cause difficulties, so the calculator appears to be an appropriate choice.
	W 14	27	No	
	C 58	100	Yes	
10% of 750	M 19	40	No	The calculator was the most popular choice, but was the least successful method employed. Mental methods would have been a more appropriate choice but the success rate for mental methods was still only 40%. This item proved to be difficult regardless of chosen method.
	W 4	33	No	
	C 36	28	No	
14 x 9 ÷ 6	M 10	37	No	The most common computation choice was the calculator and this method produced the highest success rate. The second most popular choice, written computation also produced a reasonable success rate.
	W 29	65	Yes	
	C 36	93	Yes	
$\frac{1}{2} + \frac{3}{4}$	M 60	55	Yes	While the success rate for all forms of computation was not high, the choice of 60% of students to try mental methods appeared justified based on the recorded success rate of 55%.
	W 22	41	No	
	C 5	25	No	
$10 - 4\frac{3}{4}$	M 77	53	Yes	It appeared that mental methods were the most appropriate choice for simple fraction items involving addition and subtraction.
	W 6	40	No	
	C 4	33	No	

$\frac{2}{3}$ of 45	M 27	33	No	The most common choice was mental, but it produced a poor success rate. Written methods proved to be the most successful, although only 10% of students selected this method.
	W 10	62	Yes	
	C 9	43	No	
\$1.99 + \$1.99	M 68	60	Yes	The most common choice (68%) was mental. While the success rate for mental methods was less than for the alternatives it did produce a 60% success rate. Written methods, used by 26% of students produced the highest success rate. Given that the item could be solved fairly easily using a simple mental approach the low result for mental methods is of concern.
	W 26	85	Yes	
	C 4	67	Yes	
\$4.93 + 39c	M 49	60	Yes	Mental methods, the most popular, produced the lowest rate of success. Written methods the second most popular produced the highest success rate. While mental methods were favoured in items 14 and 15, written methods produced a much higher rate of success.
	W 36	93	Yes	
	C 11	67	Yes	
7.41 – 2.5	M 35	11	No	While the choice of methods was fairly close; Mental 35%, written, 33% and calculator 28%; the most popular choice had the poorest success rate and the least popular choice had the highest success rate.
	W 33	46	No	
	C 28	86	Yes	
0.25 x 800	M 8	50	Yes	The most common choice was to use a calculator. Based on the 96% success rate recorded from using a calculator this choice seems most appropriate.
	W 8	17	No	
	C 70	96	Yes	
3.5 ÷ 0.5	M 50	64	Yes	Half of all students opted to use mental methods to solve this item. This method was reasonably successful (64%), although the 39% of students who chose to use a calculator were far more successful (97%).
	W 9	14	No	
	C 39	97	Yes	

The data indicate that on the basis of success rate for chosen computation method students made slightly more appropriate than inappropriate choices. The general trends indicated that students made poor computation choices when:

- faced with two-digit by two-digit multiplication items; and
- the item was more difficult.

Students experienced trouble with the percentage item and this was reflected in the low success rate for all computation methods. This in turn impacted on the appropriateness of the methods used. This item was an example of a question that was beyond the ability of most students to solve.

The decision to classify a computation choice as appropriate, or not appropriate, was based solely on a single criterion, success rate. However, there are other factors that might be considered.

Data are not provided in the table indicating success rate for mixed methods. Item 10, $14 \times 9 \div 6$, recorded the largest percentage of students adopting mixed methods. In 79% of cases students using a mixed method for this item, managed to calculate the correct result. Students using a calculator, the most common choice, managed to compute the correct result in 93% of cases. This item, although involving multiple operations, can be entered into any type of calculator as it is shown and produce the correct result. Had the item involved a different mix of operations such that, the calculation would need to be reworked to produce the correct result on a calculator not designed to handle ‘order of operations’, then the chances of success on this item would likely have been reduced. Students attempting to use mental methods experienced difficulty multiplying 14×9 or dividing 126 by 6, both fairly simple calculations in their own right. It appears as though the extra burden of mentally tying the two steps together was too much for many students to manage. Written methods proved to be more effective because interim steps could be jotted on paper.

Delving further into the data reveal further patterns. Data for computation choice according to year level were extracted and then organised according to item and year level. The results of this analysis are presented in the next section.

Success Rate and Year Level

As explained earlier it was not the purpose of this research to examine the differences between year levels. Table 7.5 and Table 7.6 are provided, however, to give an indication of trends across each year group. Given the relatively small sample sizes little can be generalised from this data. Table 7.5 indicates, as one might expect, that students in Year 7 experienced more success than students in earlier years.

Table 7.5: Success rate (percentage correct) for each item according to year level

Item	Year 5 Correct	Year 6 Correct	Year 7 Correct
$28 + 37$	87	84	81
$74 - 36$	67	70	81
$369 \div 3$	93	92	96
36×25	27	62	65
70×600	53	62	65
29×31	53	59	81
33×88	40	68	62
1000×945	53	89	81
10% of 750	7	19	38
$14 \times 9 \div 6$	67	70	85
$\frac{1}{2} + \frac{3}{4}$	33	46	46
$10 - 4\frac{3}{4}$	33	51	54
$\frac{2}{3}$ of 45	13	43	35
$\$1.99 + \1.99	73	70	62
$\$4.93 + 39c$	67	73	81
$7.41 - 2.5$	27	46	50
0.25×800	60	81	81
$3.5 \div 0.5$	67	65	81

The two items for which Year 5 students experienced more success than Year 6 or Year 7 students were $28 + 37$ and $\$1.99 + \1.99 . It is not clear why this was the case. Each year group favoured mental methods above written or calculator methods. For Item 14, $\$1.99 + \1.99 , Year 5 students chose to use mental methods in 53% of cases and written methods in 47% of cases. The use of written methods was much higher than for either Year 6 or Year 7 students. Of more interest are the data contained in Table 7.6 which outlines the success rate for each year level according to computation method. Additional comments have been added to summarise the data.

Table 7.6: Percentage of correct answers across all items according to initial choice ($n = 78$)

Item	Year 5			Year 6			Year 7			Comments
	M	W	C	M	W	C	M	W	C	
$28 + 37$	89	75	100	80	100	100	75	78	100	High levels of accuracy regardless of method used.
$74 - 36$	20	89	100	44	88	100	67	81	100	Accuracy for mental methods improves from Year 5 to 7.
$369 \div 3$	83	100	100	83	100	94	86	100	100	All methods accurate.
36×25	0	14	100	0	60	93	0	59	88	Accuracy of written methods improves with age.
70×600	60	0	100	42	14	100	50	25	100	Mental methods decline in terms of accuracy. Calculator is safest method.
29×31	0	25	100	0	65	100	0	73	100	Written improves. Calculator safest.
33×88	0	33	80	0	65	100	-	53	100	Year 7 students have learned to do this. Written methods become more accurate. Calculator safest.
1000×945	50	0	100	75	50	100	50	67	100	Zeros cause difficulties as noted in this item and previous item involving zeros. Safest method is to use a calculator.
10% of 750	20	-	0	50	50	31	100	0	40	Mental methods produce most correct answers. They tend to improve with age.
$14 \times 9 \div 6$	0	33	100	75	69	80	0	71	100	Unusual result. Year 6 students demonstrate accuracy with mental methods unlike younger and older students. Calculators produce highest levels of accuracy.
$\frac{1}{2} + \frac{3}{4}$	36	0	-	70	20	50	54	67	0	Mental methods produce best result for most students. Written methods develop with age. Calculator of limited help.
$10 - 4\frac{3}{4}$	38	0	-	59	33	0	56	100	50	Similar result to previous fraction item.

$\frac{2}{3}$ of 45	17	–	–	45	67	50	25	50	33	Year 5 students experienced difficulty. Year 6 and Year 7 students more successful at written approaches.
\$1.99+\$1.99	50	100	–	65	75	100	59	100	50	Year 5 and Year 7 students were proficient in written methods. Result from using mental methods lower than expected.
\$4.93 + 39c	33	100	50	59	87	100	79	100	67	Similar pattern to previous item, although mental methods produced better results.
7.41 – 2.5	17	0	75	0	62	100	25	44	78	Year 6 students showed some proficiency with written methods. Calculator most likely to produce correct result.
0.25 x 800	–	0	78	33	0	100	67	33	100	Few options but calculator available to Year 5 & 6. Year 7 students had developed ability with mental methods.
3.5 ÷ 0.5	40	0	100	58	25	100	80	0	90	Calculator safest option. Mental methods develop.
Overall	31	32	66	47	57	83	49	61	78	Accuracy improves as age increases.

The data contained in Table 7.6 indicate that on nine occasions Year 5 students' most common computation choices were also their most successful. Similar matches only occurred five times for Year 6 students and four times for Year 7 students. In the case of Year 7 students this only occurred when choosing to use a calculator, whereas when examining the results for Year 5 students the matches tended to occur across all computation types. The small sample size, especially for Year 5 students meant that generalisations could not be made. It may simply mean that Year 5 students have less computation options at their disposal because of a lack of experience with mental, written and calculator methods of calculation. For example, for each of the fraction items Year 5 students did not choose to use a calculator and for the item, $\frac{2}{3}$ of 45 the only method used by Year 5 students was mental calculation. The data may indicate that once students have gained experience with all forms of computation, more time should be devoted to discussing computation choice so that students become more adept at choosing methods that are most likely to be efficient and produce correct answers.

Success Rate According to Second Computation Choice for all Items

Table 7.7 clearly indicates that calculator methods were the most common second choice of computation method made by students and that the success rate for all but a few items was high. It should be noted that some students were unable to exercise a second choice.

Table 7.7: Percentage of correct answers across all items according to second choice
($n = 78$)

Item	% Mental	Correct	% Written	Correct	% Calculator	Correct
$28 + 37$	14	82	51	98	17	100
$74 - 36$	12	44	20	88	41	100
$369 \div 3$	12	78	37	97	19	100
36×25	4	0	24	44	44	100
70×600	9	86	10	50	38	93
29×31	5	0	15	58	49	100
33×88	6	0	12	44	51	98
1000×945	13	70	10	38	20	95
10% of 750	4	33	5	25	6	60
$14 \times 9 \div 6$	2	0	8	50	32	96
$\frac{1}{2} + \frac{3}{4}$	4	33	11	55	18	64
$10 - 4\frac{3}{4}$	0	—	2	100	22	71
$\frac{2}{3}$ of 45	0	—	3	50	10	67
$\$1.99 + \1.99	4	100	24	84	54	93
$\$4.93 + 39c$	1	0	17	84	65	58
$7.41 - 2.5$	2	50	17	62	46	94
0.25×800	1	100	4	0	18	93
$3.5 \div 0.5$	1	100	5	0	47	97
Overall	5	48	15	57	33	88

Note: This table does not include data for mixed and no methods, hence values will not total to 100%.

There were only two items for which methods other than the calculator were favoured. In both cases the written methods used were highly successful. Calculators appeared to be fairly successful for calculating percentages, but only 12 students attempted this item, five using a calculator, three successfully. Similarly the item, $\frac{2}{3}$ of 45 was only attempted by 11 students, nine of whom used calculators successfully. The highest incidence of calculator use occurred in Item 15, $\$4.93 + 39c$. Students who used a calculator for this item produced the lowest success rate compared to other items for which the calculator was used. This was primarily for reasons outlined earlier, with the main reason being the mix of dollars and cents.

Success Rate According to Third Computation Choice for All Items

The data indicated that few students were able to exercise a third choice, except for simple items. When students did opt for a third computation method, the calculator was generally favoured.

Table 7.8: Percentage of correct answers across all items according to third choice
(*n* = 78)

Item	% Mental	Correct	% Written	Correct	% Calculator	Correct
28 + 37	0	—	4	100	54	100
74 – 36	1	100	3	100	23	100
369 ÷ 3	1	0	4	100	21	100
36 x 25	1	0	1	100	10	100
70 x 600	2	50	6	40	8	83
29 x 31	0	—	5	75	5	100
33 x 88	1	0	3	0	5	100
1000 x 945	3	33	4	33	3	100
10% of 750	2	50	1	0	3	50
14 x 9 ÷ 6	3	33	1	0	1	0
$\frac{1}{2} + \frac{3}{4}$	1	0	0	—	9	29
$10 - 4\frac{3}{4}$	0	—	1	100	0	—

$\frac{2}{3}$ of 45	0	–	0		1	0
\$1.99+\$1.99	0	–	8	57	23	100
\$4.93 + 39c	1	0	9	71	10	62
7.41 – 2.5	0	–	0	–	8	100
0.25 x 800	1	0	0	–	0	–
3.5 ÷ 0.5	0	–	4	33	1	0
Overall	1	15	3	45	10	62

Note: This table does not include data for mixed and no methods, hence values will not total to 100%.

Summary

In this chapter computation choice and success rate data have been linked. Success rate was then used to make a judgement as to the appropriateness of the choice that was made. While this is a rather crude and somewhat arbitrary method of determining whether a computation choice was appropriate or not, it did serve to focus the discussion on how successful students in Years 5-7 were at executing various forms of computation, which was the basis of the research question guiding this study.

The indications were that for at least half the items used in the research, students made appropriate computation choices based on the rate of success they experienced using those forms of computation. Students tended to make better choices when the items were more familiar to them, such as in the first three items. When the items became more challenging and computation choice was restricted by a lack of proficiency with all forms of computation, students became less adept at choosing computation approaches that would lead to success. A better match between computation choice and success was found for Year 5 students and several reasons were suggested for this finding. Calculator methods were found to dominate the second computation choices made by students. Based on success rate, the use of the calculator could be considered to be a good choice on all occasions. In some cases students were found to be more successful in employing their second choices than their first, indicating that students need to be given more opportunity to discuss and examine computation choice in class.

In the final chapter all three research questions will be re-examined as a whole. Limitations of the research will be outlined along with recommendations and suggestions for further research discussed.

Chapter 8: Conclusions and Implications

Clearly, the question of what computation choices students make and how they make them is of significance to teachers and mathematics educators. While teachers are advised to assist students in choosing between computation alternatives there is little evidence of how and why students make these choices.

This research was designed to answer three questions.

1. When faced with a computation question, what choices do students in Years 5 to 7 make?
2. Why do students in Years 5 to 7 make particular computation choices?
3. How successful are students in Years 5 to 7 at executing various forms of computation?

The investigation of computation choice began with a review of the literature as outlined in Chapter 2. Various computation options; estimation, mental methods, written methods and calculator methods were reviewed. The term metacomputation was introduced as a means of describing the higher order thinking associated with computation, particularly the choosing of appropriate computation methods and the monitoring of a calculation. The role of estimation as a computation alternative and also as a form of metacomputation was discussed.

The literature included calls for a re-examination of computation practices in the light of student access to calculators, although the actual desired balance of computation methods that should be employed was unclear. There was general agreement that time spent on teaching formal written methods should be reduced, but few authors (Plunkett, 1979) were prepared to clearly state a preference. By collecting data on the computation choices made by students the current use of computation methods could be documented and used as a starting point for further debate on the issue.

While the suggestion that students develop the ability to choose appropriate computation methods may seem like a reasonable expectation of primary school mathematics programs a review of the literature indicated that there was little evidence to describe how students make decisions relating to which form of computation to use. The literature review indicated that little direct research had been carried out indicating how students made computation choices (Price, 1995; Reys et al., 1993). The study by Reys, Reys and Hope (1993) only involved asking students to state how they would solve a particular computation item, but did not require the students to perform the calculation. A later study by Price (1995) required students to perform calculations using their chosen method. This study, however, only involved observing students as they solved multiplication items. As a result of considering this early research, a decision was made to observe and interview students in Years 5-7 as they solved a range of computation items. After completing each item, students were questioned about their ability to solve the item using alternative computation methods.

Computation choice was considered in the light of various models of computation that had been produced. The NCTM (1989, p. 9) model of computation was used to guide much of the discussion in Chapter 2 (See Figure 2.1). This model along with others was examined in Chapter 3 and an alternative computation model, Swan and Bana (1998), was introduced (See Figure 3.6). Unlike previous computation models that depicted computation choice as a linear process the Swan and Bana (1998) model was based on a Venn diagram and allowed for a much more fluid description of the computation process and the way in which students made computation choices. The Swan and Bana (1998) model allowed for a mixture of methods to be used in solving computation items and included factors, such as, home background and teacher influence, that might impinge on computation choice.

In Chapter 4 the three research questions used to guide the research were re-examined in the light of the methodology and data gathering techniques that were most likely to answer the three research questions were also examined. A qualitative methodology, employing interview methods was chosen as it was felt this methodology would provide the most appropriate data for answering the research questions. Qualitative methodology was reviewed along with the use of the clinical interview as the prime data gathering technique. The development of the instrument (See Appendix 3) was outlined along with the pilot study used to refine the instrument and the

protocols and questions used in the study. These are reproduced in Appendices 3, 4 and 5. Chapter 4 also included a discussion of potential limitations of the research methodology as applied to this research and the measures that were taken to reduce threats to the reliability and validity of the research.

The first research question, *When faced with a computation question, what choices do students in Years 5 to 7 make?*, was answered in Chapter 5. Data for computation choice were tabulated to indicate trends. Data were grouped:

- to indicate for which items students favoured mental, written, calculator or mixed methods of calculation; and
- to compare computation choices made in previous studies to computation choices made in this study.

Data pertaining to individual items were also discussed. Second and third computation choices were also examined. The data indicated that computation choice was spread across mental, written and calculator methods and to a far lesser extent mixed computation methods. Mental methods were most popular, accounting for 36% of students' computation choices. Calculator and written methods were chosen on 28% and 26% of occasions, respectively. Mixed methods were employed, but mostly for Item 10, $14 \times 9 \div 6$, and Item 13, $\frac{2}{3}$ of 45. When choosing written methods students mostly used a standard written algorithm. No student used a single method to solve all items. Each item was solved using a variety of methods, mental, written or calculator – although the use of specific methods was favoured for particular items. These are summarised later.

The second research question, *Why do students in Years 5 to 7 make particular computation choices?*, was answered in Chapter 6 by following the data analysis procedures outlined in Chapter 4. Transcripts of student interviews were examined to determine why students made particular computation choices. Common themes running through the transcript were grouped together in order to arrive at the answer to the second research question. Excerpts from students' interview transcripts and copies of students' written work were used to explain the computation choices that were made and that illustrated each theme. Student responses that were of interest but not directly related to the discussion in Chapter 6 were presented in Appendix 7.

The data indicated that students made fairly hasty decisions about which form of computation to use, based on some rather rudimentary criteria. The criteria are summarised later but tend to indicate that students rarely considered relationships between numbers when making computation choices.

The third research question, *How successful are students in Years 5 to 7 at executing various forms of computation?*, was answered in Chapter 7. Data pertaining to how successful students were in applying their chosen strategy were tabulated to indicate trends. A judgement as to whether appropriate computation choices had been made, was made, based on how successful students were when employing those strategies. Data for second and third choices were also presented. The data indicated that students often did not have a third computation method at their disposal.

The success rates for utilising various computation alternatives when solving each item were used to decide the appropriateness of the choices that were made. The indications were, that at times, students over-estimated their ability to use certain computation methods in solving particular items. At other times students were more successful using their second computation method than in performing the calculation using their first method. The data indicated that improvement could be made in the way students chose and executed their computation methods.

In this chapter the data from Chapters 5-7 are reviewed as a whole to explain what computation choices students in Years 5-7 made, why they made them and how successful they were in employing them. Recommendations for changes in classroom practice based on the findings of this research will also be made. The limiting factors of this research, as outlined in Chapter 4, will be discussed in the light of the findings and recommendations made for further research.

Student Computation Choice

The data indicated that given a choice students preferred to use mental methods. Mental methods were preferred for eight of the 18 items given to students. Mental methods were used in 36% of cases. Calculator methods were favoured in five items and accounted for 28% of choices, while written methods were favoured in four items and accounted for 26% of choices. Over all 18-items, mixed methods were used in 6% of cases.

Mental methods tended to be favoured for items involving addition, fractions and decimals. Calculators were preferred for items involving division, the percentage item and items where the numbers were perceived to be 'big'.

Previous studies (Price, 1995; Reys et al., 1993) found that students choosing to use mental methods to solve computation items would do so with questions that they felt confident solving this way, but would resort to another method of computation once they started to feel unsure about the calculation. In this research confidence was also a factor in students making computation choices. Confidence, or rather a lack of confidence in using one method often meant students would choose an alternative method, almost by default. When considering the items for which students chose particular computation methods, it became apparent that in addition to the broad criteria, such as, the size of the numbers involved, students chose safe options or at least the option in which they felt most confident.

Preference for mental methods

Students favoured the use of mental methods when solving addition items. Items $28 + 37$, $\frac{1}{2} + \frac{3}{4}$, $\$1.99 + \1.99 and $\$4.93 + 39$ were mostly completed using mental methods. The two in-context items (14 & 15) both involved money and this may have influenced the choice to use mental methods. Earlier, reference was made to student confidence as playing a role in computation choice. It appeared, however, in deciding to solve the two fraction items, $\frac{1}{2} + \frac{3}{4}$ and $10 - 4\frac{3}{4}$, it was possibly a lack of confidence or experience that played a part in the choice to use mental methods. Students were unable to employ a written method for solving fraction items and were unaware of how to enter fractions into a calculator and therefore chose mental methods almost as a last resort. The same, however, could not be said for the two decimal items, $7.41 - 2.5$ and $3.5 \div 0.5$, in which students also favoured the use of mental methods. Items involving decimals may easily be entered into a calculator, indicating that other factors influenced the computation choices made by the students.

There was no evidence in this research to indicate an over-reliance on the use of a calculator. The data also indicated that while mental methods were often chosen, students were not always able to perform the calculation mentally. It was noted that students might initially try mental methods but revert to written or calculator methods, or perhaps a mix of methods, when they found they were unable to complete the calculation.

Preference for written methods

Written methods were favoured for each of the three two-digit by two-digit multiplication items 36×25 , 29×31 and 33×88 , and for the two-digit subtraction item, $74 - 36$. While the students were chosen because they came from classes where they had easy access to calculators they were still taught standard written algorithms for two-digit subtraction and two-digit multiplication. The choice to use written methods to solve items of this nature appeared, in part, to have been influenced by this practice. Several students made comment that they had been taught this approach. This finding concurs with that of Price (1995) who found the teacher to have an influence on computation choice.

It should be noted, however, given the nature of the items for which students chose to use written methods, and the success rate experienced, the choices were appropriate. In at least three cases, $74 - 36$, 36×25 and 29×31 , alternative mental methods exist that are within the ability level of Year 5-7 students. For example, by making use of flexible mental strategies that utilise the properties of number, Item 4, may be thought of as $9 \times 4 \times 25$, which is fairly simple to calculate mentally. Granted this would require a fair amount of number sense, but if students were used to 'playing around with numbers' then it is possible they might adopt such mental methods. Likewise Item 6, 29×31 , may be restated as $30 \times 30 - 1$, and as such becomes a much simpler question to solve using mental methods. This relationship is one that might be investigated as part of a mathematics program that had as its aim the understanding of number and the development of flexible approaches to calculation. Rather than make use of relationships between numbers and number properties that would make mental computation easier, many students used a mental version of the standard written algorithm, which made mental computation more difficult. As mentioned in Chapters 5-7 the decision to use mental methods was, at times, not only based on not simply using a

mental version of the standard written algorithm, but rather on using a faulty version of the standard written algorithm.

Preference for calculator methods

Calculators were preferred when solving items involving division, items classed as containing 'big numbers' and items for which the students were unfamiliar or unsure. For example, Item 9, 10% of 750, was an item for which students were unsure of how to proceed and because the calculator contained a percentage key many students thought that pressing this key would produce an answer. Item 10, $14 \times 9 \div 6$, was unusual in the sense that it involved a mix of operations and students were a little unsure of how to proceed and therefore often chose to use a calculator. Here again was an example where confidence was a factor in choosing to use a calculator in preference to alternate methods. Mixed computation methods were also used to solve this item, many of which included the use of a calculator. Students preferred to solve Item 17, 0.25×800 using a calculator because it involved two features, decimals and 'big numbers', that caused students to avoid mental and written methods.

An over-reliance on the use of calculators was not noted in this research. It appeared that in some cases the opposite was true – students would have been wiser to choose to use a calculator. A lack of familiarity with how to use a calculator appeared to hamper students' ability to use calculators and it was felt that this had a bearing on the choice of whether to use a calculator. This was another example where confidence played a role in making a computation choice.

Evidence of Metacomputation

In Chapter 2, the term metacomputation was used to describe the higher order thinking associated with making a computation choice and monitoring the calculation. Metacomputation was also a feature of the Swan and Bana (1998) computation model (See Figure 3.6). Firstly the role of metacomputation when making a computation choice will be discussed.

The data indicated that the students made a quick decision about which form of computation to use. There was little evidence to suggest that students considered any relationships between the numbers or reformulating the question to make it simpler to calculate an answer. The decision to use a particular computation method was made, in most cases, based on a limited set of criteria. These criteria included the following:

- magnitude of numbers;
- efficiency or speed of calculation;
- recognised a weakness (operation, table facts);
- process of elimination (as a last resort); and
- restricted computation choice (lack of content knowledge, difficulty with zeros, faulty algorithm, unable to use technology).

When discussing computation choice, Price (1995) noted that mental methods were not applied as often as they might have been, because, “the majority of children did not recognise the aspects that make these questions easy to solve mentally” (p. 66). A similar phenomenon was noted in this research. Students would focus on the size of the numbers in the calculation rather than notice relationships between the numbers. Students seemed so pre-occupied with one or two features of an item, such as whether it included decimals or involved big numbers, that they failed to recognise other aspects of the item that may have made it simpler to calculate using an alternate method. For example, when considering how to calculate 1000×945 the large numbers appeared to dominate students’ thinking and they chose to use a calculator when mental methods may have been applied. It is also possible that a lack of confidence played a part in the choice to use a calculator to solve this item. This lack of confidence, was noted in the confusion that many students who chose mental methods experienced, when trying to apply rules for ‘taking off zeros’ and ‘adding zeros’. This underlying lack of number sense and by extension metacomputation may be seen as having had an influence on computation choice.

The second aspect of metacomputation, as outlined in Chapter 2, involved the monitoring of a calculation. There was little evidence to suggest that students gave much consideration to the result of a calculation. There were occasions where students noted mistakes, such as the answer was too large or too small, but it did not occur very

often. When completing the same item using an alternative computation approach some students noted that they had calculated a different answer and queried the result, whereas others seemed oblivious to the difference or were not concerned about it. For many students the prime concern appeared to be completing the items as quickly as possible. This meant making a rapid decision about which form of computation to use; executing the calculation as quickly as possible and noting the answer without considering whether the result was reasonable or made sense. The students displayed a desire to move onto the next item. Some students seemed reluctant to solve the same item using an alternative computation method and it appeared as though they viewed computation as something to be completed as rapidly as possible.

The perception that computation items need to be solved as rapidly as possible seemed to militate against the application and possibly development of metacomputation. Students wished to make a quick decision about which computation method to use, without considering the numbers and the relationships between the numbers. They did not appear to make an estimation prior to embarking on the calculation, which reduced the methods available for checking the result. Once the calculation had been completed, students did not seem to consider the result, even when it might conflict with a previous result. Students appeared mainly concerned with moving on to the next calculation. In order to encourage the development of metacomputation it is recommended that students be encouraged to spend more time completing less calculations.

The criteria by which students made computation choices are reviewed in the next section. While the magnitude of numbers category is a rather superficial method of making a computation choice, categories such as 'recognised a weakness' and 'process of elimination' do suggest that students were either comparing alternate computation methods or taking into account their own calculation ability and experience when making a computation choice. This at least suggests that a low level of metacomputation was being used. Each of the criteria students used when making computation choices is briefly reviewed.

Magnitude of numbers

The magnitude of numbers criterion was fairly broad and student explanations of this criterion varied. Many students considered numbers in the hundreds as 'big'. The 'big number' criterion tended to block out students' perception of the item. This was particularly the case for Item 8, 1000×945 , which students preferred to solve with the use of a calculator. Students tended to focus on the 1000 and did not consider how simple it might be to solve using mental methods. Students would often cite the presence of 'big numbers' as the reason for choosing to use a calculator.

While the use of a calculator to solve items such as 1000×945 and 70×600 might seem inappropriate, the data indicated that students had difficulties when calculating with zeros. When calculating with zeros, many students who chose to use mental methods failed to calculate the correct result. Likewise, students who attempted items involving zeros using written methods also experienced difficulty. A mathematics program that emphasised the patterns associated with multiplying by powers of ten and focussed on place value may help to alleviate some of the problems students experienced calculating with zeros. Students who chose mental methods to calculate the answer to items involving zeros, tended to rely on rules they had been taught or had developed. The rules that students attempted to apply, such as 'take off all the zeros and then add the zeros contained in the second number', indicated a lack of understanding on the part of the students. More work in place value and in examining patterns when multiplying by powers of ten would assist students to become more confident in their ability to use mental methods to calculate with items such as, 70×600 and 1000×945 .

Efficiency

Two considerations, ease of calculation and speed of calculation were foremost in students' minds when they used these criteria for choosing a particular computation method. Often the terms speed and ease were used to justify choosing to use a calculator, but students also referred to them when using mental methods. It appeared that the classroom practice of completing a specific number of calculations in a set period of time might raise awareness of time as a factor when completing calculations. It was unclear whether this was a common practice in the classrooms from which the students for this study were drawn.

The expression “it’s easier” was mostly used as a term where one computation method was compared to another. The idea that a student compared or weighed up computation alternatives does tend to indicate that some metacognitive thinking was taking place. When comparisons were made they generally only included comparing two computation methods, such as, mental or written. At times these comparisons were made based on a faulty understanding of the standard written algorithm or inefficient mental methods.

Recognised a weakness

Students appeared to have a good understanding of their shortcomings when it came to calculating. Students openly indicated that they experienced difficulties with decimals or lacked basic fact knowledge as the reason for avoiding one form of computation in favour of another – generally mental or written computation, in favour of using a calculator. The ability to recognise a weakness suggested that some higher-level thinking was occurring that assisted in the making of a computation choice.

Process of elimination

Evidence of some metacognitive thinking may also be seen in the way students used a process of elimination to decide which form of computation to use. Students who used this approach would consider several computation alternatives and choose the one they felt safe or comfortable using. At other times student comments indicated that they chose a particular computation method by default, as they were unable to use one or more computation methods or they lacked confidence in the alternative methods.

Restricted computation choice

It became evident by the comments made by students that some students did not have a range of computation choices available. A lack of understanding of how to use a calculator, for example, restricted the use of calculators on certain items, such as those involving fractions. At times, it also appeared as though students lacked confidence in certain computation methods and this also served to restrict their computation choices.

Recommendations

The data indicated that students need to be taught to make better computation choices. The choices they do make tend to be based on a fairly limited set of criteria. Computation choices are made quickly without considering the relationships between the numbers. The success rates for first and second computation choices also indicated that students also need to consider which computation methods are more likely to produce a correct result. At times students lacked confidence in their ability to solve particular items using specific computation approaches.

It is important that students be given the opportunity to make computation choices. Small decisions made by teachers can impact on the opportunity students have to make computation choices. For example, restricting access to calculators would reduce student computation choice. Firstly, students would not gain experience in making decisions to use a calculator within the controlled environment of the classroom. The result may be that when students finally gain free access to calculators in secondary school, they make indiscriminate use of them. The second possibility is that students may not become familiar enough with calculators to be able to make use of them when the opportunity arises.

The decision by the teacher to use a textbook as a teaching vehicle may also have an impact on students' computation choice. In many cases prompts to solve questions using a particular computation method are contained in student texts. For example, the prompt "check with a calculator" implies that the calculation should have been performed using a different method.

The amount of time allocated to various forms of computation is yet another example of how decisions made by a teacher can impact on computation choice. To assist students to become better at making computation choices, recommendations for changing teaching practice have been made.

Encourage discussion about computation choice

In much the same way that students are encouraged to explain how they completed a calculation using mental or informal written methods, they should be encouraged to explain why they chose to use a particular computation method or mix of methods. Teachers adopting constructivist principles in their teaching have focussed more on how students calculate rather than the speed of calculation. In addition to asking students how they performed a particular calculation it is recommended that students be challenged as to their choice of method. Currently in most classrooms it is more likely that a student is challenged when using a calculator than when performing a mental or written calculation. The justification of computation methods should become a regular part of number lessons.

Redress the balance of time devoted to all forms of computation

Evidence was presented in Chapter 2 to suggest that most classroom time devoted to computation was spent developing standard written methods (Porter, 1989). This was despite research indicating that mental methods, which included a large component of estimation, were mostly used by adults in real life (Northcote & McIntosh 1999; Wandt & Brown, 1957). This does not mean that students should be given more rapid-fire mental computation sessions that emphasise speed over thinking, but rather more time needs to be devoted to the development of number sense. The notion of number sense was discussed at length in Chapter 2, where various components of number sense, including the ability to estimate were outlined. The discussion of number sense indicated that it takes time for students to develop number sense. A reduction in the time spent on teaching standard written methods should allow more time to be spent on:

- improving mental computation;
- improving estimation;
- developing mixed methods of calculation;
- developing informal written calculation methods;
- learning to make efficient use of a calculator;
- discussing how calculations are performed; and
- discussing computation choices and the reasons behind making them.

A greater focus on using calculators

While students in this study and previous studies made use of calculators, it was clear by the explanations given and the way calculators were used that students' knowledge of how to use a calculator was limited. Greater attention needs to be given to help students choose when to use a calculator and how to use a calculator. Many students in this research did not know how to use the percentage key on the calculator. Previous research (Shiple, 2002) indicated that students make inefficient use of calculators and have little understanding of how to use functions such as the memory. Observations made during this research confirm this finding.

Rather than suggest students be given formal lessons on how to use a calculator it is recommended that calculators be used as a tool to explore number. Exploring number and number relationships will undoubtedly lead to questions about how to use a calculator that may be answered in the context of the calculation.

One item, $14 \times 9 \div 6$, given as part of this research, involved a mixture of operations. When entered into a simple calculator of the type often used in primary school, the correct result is produced, but had the calculation involved a different mix of operations, then students would have needed to cope with the rule of order of operations. It is suggested that rather than teach students a rule to be applied in this situation, a variety of calculators be used to stimulate discussion of how calculators are made to deal with issues, such as, which operation to perform first. What is recommended is that rather than view instruction in calculator use as another topic to be added to an already full curriculum, that calculators be used as a catalyst to stimulate discussion of number related issues. Discussion centring around when to use a calculator and how to use a calculator, may be used to improve students overall number sense.

More emphasis on understanding numbers and number properties

Students are often expected to perform calculations before having gained an understanding of number and the properties of number. For example a student may learn to multiply 8×3 by rote methods but when questioned as to the result of multiplying 3×8 , may explain that he/she has not learned this specific calculation. A sound knowledge of number properties such as the commutative property would mean that a student would be able to connect 8×3 with 3×8 . A lack of understanding such as this often occurs when it is assumed that because students can calculate, they must understand numbers and number properties.

The *Outcomes and Standards Framework: Mathematics Student Outcome Statements* (EDWA, 1998) specifically lists the need to 'Understand Numbers' and 'Understand Operations' prior to the teaching of calculation. A lack of understanding of place value by students was noted in this research. This served to inhibit their ability to perform mental calculations involving zeros. The development of a better understanding of place value would assist students when performing calculations, particularly mental calculations. Greater emphasis needs to be placed on the study of number prior to starting formal calculation work. Time is often cited as the reason for not providing more opportunities for students to explore numbers and the number system. The next recommendation deals with the issue of time devoted to formal calculation methods.

Less emphasis on formal written algorithms

There were several instances in this research where students chose to use standard written algorithms as their first choice when solving computation items, especially those involving two-digit by two-digit multiplication. There were very few instances where informal written methods were used. Where these methods were used they tended to be highly inefficient, involving the use of tally marks or strokes. Pictures of circular regions depicting pizza were used when solving items involving fractions.

A main area of concern was the number of students who chose to use standard written methods but used a faulty version of the algorithm. This was particularly the case for multiplication where many students appeared to lack an understanding of the distributive property. When multiplying to solve 29×31 , students would multiply 1×9

and 3×2 and calculate the result to be 69. This misunderstanding meant that students were basing their computation choices on a faulty premise, as the calculation, while incorrect appeared much easier than it really was. Price (1995), noted similar problems when students tried to use mental methods to solve two-digit by two-digit multiplication items. He stated that, “few students attempted to solve two-digit by two-digit questions mentally ... students who attempted these questions mentally all used faulty methods and produced incorrect results” (p. 67). Price described one of the methods used to multiply 29 by 31. The method was similar to the one described earlier. He suggested that students who adopted this approach were treating multiplication in a similar way to addition.

Based on the evidence that students experienced difficulty completing two-digit by two-digit multiplication items using mental methods it may be tempting to suggest that students not be taught to attempt mental methods to complete this type of calculation. It would be over-simplistic, however, to suggest that students be given a set of guidelines to use when making computation choices, because there are too many factors involved. Plunkett (1979) suggested that calculations might fit within certain categories that could be considered more conducive to certain methods of calculation, but such an approach is not recommended here. Returning to the two-digit by two-digit multiplication example, it can be seen that producing a guideline suggesting this type of calculation be completed using paper-and-pencil methods would be counter-productive. Firstly, a set of guidelines would produce a layer of rules to be remembered when performing a calculation and not assist in the development of metacomputation. As with any set of rules or guidelines they would be open to misinterpretation and there would always be exceptions. For example, it has already been explained that the item 36×25 may be solved using mental methods by considering the calculation as $9 \times 4 \times 25$, so it would not make sense to set a rule that suggested two-digit by two-digit multiplication questions be performed using paper-and-pencil methods. Finally even if students had followed a guideline to complete two-digit by two-digit multiplication items using paper-and-pencil methods, students who used a faulty algorithm would not have calculated the correct answer.

Standard written methods appear to have impacted on students mental methods to the point where student descriptions of mental methods were often accounts of how the standard written algorithm would be performed. The result is that students used

inefficient mental methods and the strain on short term working memory was increased, making the calculation more difficult to perform. This meant that students using standard written methods as a mental approach experienced difficulty calculating the correct result. Computation choice was also skewed away from using mental methods because the mental methods being used (mental versions of the standard written algorithm) were difficult to execute because of the strain on short term working memory.

Students should be encouraged to explore a range of written approaches to calculate, of which standard written methods are just one of many. In Chapter 2 an explanation of how written methods could be developed starting from students' mental methods was described. This approach should add legitimacy to the methods developed by students and encourage them to use their own mental strategies rather than abandon them in favour of standard written methods.

This change would require support, as teachers and parents would be concerned about a perceived reduction in the ability to calculate. A decrease in time spent on teaching standard written algorithms would need to be compensated for by an increase in the time allocated to estimation, mental and informal written methods.

Increased focus on estimation

In Chapter 2 estimation was shown to fulfil two roles:

- first as a computation alternative, and
- second as a monitoring tool – part of metacomputation.

Even though a decision was made not to study estimation as a computation choice, the researcher was interested to see how estimation contributed to metacomputation.

Students in this research, while not being offered estimation as a computation choice, appear to have made little use of estimation as a method of monitoring a calculation. This research indicated that few students thought about the result of a calculation or were interested whether they had calculated the correct result. It appeared that the students mainly focused on completing a calculation as rapidly as possible, with accuracy being a secondary concern.

While teachers often encourage students to check their work it is recommended that more emphasis be placed on developing students' ability to estimate. Students should be encouraged to use estimation as computation alternative and as a means of monitoring the results of a calculation. Checking methods need to be discussed and shared. Checking methods may range from formal approaches such as repeating a calculation, to less formal approaches such as making a rough estimate of the magnitude of the answer. Students' awareness of these methods need to be raised. Given students interest in completing a calculation as quickly as possible, it is likely that they will prefer to adopt the less formal, quicker methods of checking a calculation.

In Chapter 2, the research related to estimation was reviewed. While the techniques used to make an estimate have been established, how estimation is used to monitor the results of a calculation is less clear. In addition, less formal methods for determining the reasonableness of an answer, such as considering whether the result will be odd or even, need to be investigated.

Less emphasis on speed

It is recommended that less emphasis be placed on completing a large number of calculations in a short period of time. This practice seems to militate against the development of metacomputation and checking an answer. It is recommended that fewer calculations be attempted and more time be devoted to discussion of the calculation process. This discussion should include the making of computation choices and measures that could be taken to monitor a calculation.

Students' comments indicated that speed of calculation was an important factor when deciding which form of calculation to use. It is likely that students who focus on speed of calculation fail to consider other aspects of a calculation. There are several reasons that might account for this thinking. Many mental computation sessions focus on speed of response and therefore students associate mental approaches with short simple calculations that are completed almost instantaneously. Likewise, students are often given sets of written calculations to complete within a limited time period and therefore speed is also associated with written computation. Less emphasis should be placed on speed and more attention be given to discussing how a calculation may be performed. By emphasising how a calculation is performed, rather than on how fast it is performed, students might develop better metacomputation ability.

Change emphasis on basic facts

Basic number facts, which are often emphasised in mental sessions associated with speed of response, featured prominently in student explanations of computation choice. Students would use basic number facts, particularly ‘tables’ as a benchmark for deciding whether a calculation was within their ability to calculate mentally, with paper and pencil or whether they would need to use a calculator. Two factors appeared to be involved, the first being speed of response, which has already been discussed, but the second related to confidence. Students tended to judge their general ability to calculate according to their perception of their ability with basic number facts, particularly the multiplication facts. Considering these facts account for such a small amount of number work, they appear to have a large impact on computation in general and on computation choice in particular.

It is recommended that students be given the opportunity to develop proficiency with basic number facts, but in developing this proficiency the emphasis should not be placed on speed of response. The focus, rather, should be on developing confidence and flexibility when calculating with small numbers. The data from this research also indicated that students experienced difficulty with extended basic fact items, such as, 70×600 . In particular, students experienced trouble with zeros. Many of the problems associated with calculating with zeros related to the students adopting rules that they did not understand.

Avoid teaching rules

There was considerable evidence indicating that students lacked confidence when performing calculations involving large numbers, which for the most part involved zeros. The difficulties students experienced were not confined to the use of mental methods but extended to written methods and even to an inability to read numbers on the display of the calculator. It may be tempting to teach students certain rules involving ‘taking off’ and ‘adding zeros’ but the evidence from this research and previous research (McIntosh, et al., 1994) indicates this is counter-productive. Students rarely understand these rules and therefore mis-apply them.

Student descriptions of “taking zeros off” and “adding zeros” also revealed many misconceptions that had been developed. These misconceptions appear to have been generated as a result of observing patterns that have worked on some occasions, or as a result of being alerted to a pattern, but not fully understanding it.

It is recommended that students be given exposure to patterns involving zeros as part of an overall number sense program. It is also important that students are exposed to patterns involving zeros in a carefully structured way and those students make sense of the data that are generated. It is important that students be given the opportunity to discuss and explain their understandings so that any misconceptions may be dealt with before becoming deeply rooted.

Limitations

The foregoing results, implications and recommendations of the research need to be viewed in the light of the limitations of the research. These limitations were discussed in Chapter 4.

Various issues of reliability and validity associated with qualitative research in general, and the use of clinical interviews to gather data in particular, were outlined. Although measures were put in place to reduce the threats to reliability and validity some aberrations may have occurred. The setting in which the research took place, along with the perception of the researcher as a teacher, may have influenced the computation choices made by students. At times, when describing what decisions had been made, or how calculations had been performed, students may have said what they felt the researcher wanted to hear. The difficulties associated with recalling the method used, and a lack of ability to clearly describe the method that had been used, may also have affected the data that was collected and hence the conclusions reached. Comparative data from previous studies, although only relating to a sample of items, however, did tend to confirm many of the findings from this research.

The relatively small sample of students, drawn from four classes across two schools in a regional setting makes it difficult to generalise the results to any large extent. As this research was exploratory in nature it does, however, help to establish a base upon which further research may be carried out.

The items that were used to make up the instrument may have encouraged the use of particular computation methods. The instrument included a range of items involving all four operations, a mix of operations, fractions, percentages and decimals. While a broad range of items were used the instrument lacked depth in some areas. There were several two-digit by two-digit multiplication items but only one item that involved a mix of operations. Small numbers of items in any one category such as mixed operations meant that patterns of computation choice could not be examined to any depth. Even though the instrument was the subject of a pilot study, it appeared that some items were a little more difficult and some students were unable to respond, or their responses were limited.

Several variables were acknowledged as impacting on computation choice. These were noted on the Swan and Bana (1998) computation model (Figure 3.6). No attempts were made to control these variables. The students were chosen from classes and schools where teachers allowed easy access to calculators and included mental computation sessions as part of their daily routine, but variations existed among classes within the schools and across schools. Descriptions of the teachers, schools and class settings did, however, provide a picture of the conditions under which students were taught number.

The reliance on observation and students' descriptions of how they solved each item also raised some issues as outlined in Chapter 4. Essentially, it was impossible to observe what was going on in a student's mind, so the interviewer had to infer what was happening. While observations were backed up by interview data, students may not have been fully aware of what they had done or else may not have been able to explain why they chose a particular computation method, or how they performed the calculation.

Despite these limitations on the research, there is still much to be gained from the results. This research was very much exploratory in its nature. Little prior research had been carried out into computation choice despite recommendations that students make computation choices (Price, 1995; Reys et al., 1993). Of the research that had been undertaken on students' computation choices, most had focussed on calculations involving multiplication. In the study by Reys, Reys and Hope (1993) students were only required to state their preferred computation method and were not required to actually perform calculations using their stated method. In this research students were

asked to perform the calculation using their chosen method which revealed that some students were unable to do so and that others changed their method part way through the calculation. Students in this research were also asked whether they had a second and third method at their disposal, and were required to perform the calculation using this method. This allowed for comparisons to be made between the success rate for the first method that was chosen and subsequent methods. At times the data indicated that students' second choices were more likely to be successful than their initial choices. Some students were unable to exercise a second computation choice and many were unable to go beyond this to exercise a third method.

Much of the discussion of computation research centred on work carried out in Australia, Canada, The United States and The United Kingdom. It is acknowledged that computation practices vary from country to country and in particular Asian and European computation practices may not be the same as those discussed in the literature review and found in Australian classrooms. Therefore the results may not be generalised across differing cultures.

Subjects were chosen from typical school populations from The South West of Western Australia and represent the same general mix of students found in Western Australia and the more general Australian school population. Year levels for entry into high school vary across Australia, therefore Year 7 data may not be generalised as much as data for Year 5 and Year 6. Overall it would be reasonable to generalise the results for this study across the student population from Years 5-7 in Australia. Generalising beyond Australia becomes more problematic as instructional practices vary from country to country but the general trends noted in this study could be applied to The United States, Canada and The United Kingdom. The extent to which trends may be applied to other countries and settings would depend on how closely instructional practices and conditions mirror those described in this thesis.

Implications for Further Research

This research, while adding to the limited knowledge of how students make computation choices, also raised several issues, which require further investigation. In Chapter 3 a new model for computation (Swan & Bana, 1998) was proposed (See Figure 3.6). Further research needs to be carried out to consider the ways in which

students mix computation methods. Replicating this research with a larger sample should help to clarify the results. Extending the instrument to include more items of a similar nature would help develop the breadth of understanding of how students make computation choices when faced with particular types of computation items.

As with any piece of research, while this study has answered questions about what computation choices students make, why they make them, and how successful they were in applying them – several questions remain unanswered. There was little evidence of students checking the results of a calculation. This raises questions such as

- what checking strategies do students know? and
- what checking strategies do students employ?

It is possible that the checking strategies students use are fairly rudimentary rather than the sophisticated estimation techniques that one might expect. In much the same way that the computation choice strategies were found to be rather simple and broad it may be that students use broad, rather than precise means, to determine whether a calculation is correct. An opportunity also exists for a teaching experiment to monitor the results of teaching students various checking strategies and observe whether students adopt these strategies as part of their regular classroom practice.

The difficulties students experienced calculating with zeros appears worthy of further research. Many estimation techniques rely on rounding the numbers so they include trailing zeros. The idea behind this is to simplify the calculation, but because students experience trouble calculating with zeros this process may not be helpful.

In Chapter 1 several questions raised by Reys and Nohda (1994) were outlined. It is appropriate at the end of this study to return to these questions and examine how this research has contributed to a better understanding of computation in general and what questions remain to be answered. The questions posed by Reys and Nohda (1994) are summarised below.

- How should computation alternatives (mental computation, estimation, written algorithms, calculators) be developed?
- When should computation alternatives be introduced?
- Should strategies and techniques be self-developed by students, as advocated by constructivists? Or should strategies and techniques be taught directly by teachers?

- How are wise choices of computation alternatives developed?
- Do students know when mental computation is appropriate?
- How can calculators be used?
- Can calculators contribute to the development of mathematical thinking?
How?
- What role does the calculator play as a tool? Where? How?
- How does the development of computation alternatives contribute to number sense? (p. 5).

While the author has suggested that computational strategies be developed via a constructivist approach, research is needed to clarify whether such an approach produces a better outcome than directly teaching students computation strategies. Finding the right mix of computation alternatives needs to be considered. Perhaps students should be taught to try mental computation as their first resort. The role of calculators still requires clarification despite the research that was considered in Chapter 2 indicating that calculators are not detrimental to the development of computation skills. Teachers and parents still seem to have doubts as to the place of the calculator among the computation alternatives. This research indicated that students were unable to make the best use of the technology and therefore were hampered in making use of calculators and hence their computation choice was reduced.

The terms number sense and metacomputation require further clarification. While these terms are unclear in teachers' minds there will be a consequent lack of direction in how number sense and metacomputation is to be developed in students. In Chapter 2 an attempt at showing the relationship between these terms and others was made. Figure 2.3 showed the various components of metacomputation and how number sense was a key aspect of metacomputation. The whole issue of metacomputation and number sense requires further debate, discussion and clarification.

A different model of computation (Figure 3.6) was presented in Chapter 3. This model, while building upon previous computation models, differed in many ways. Further research is required to consider whether this model provides an adequate description of the computation process. In particular, research on mixed computation methods needs further study. Why students chose to mix computation methods and how they choose to blend these methods is of interest in the light of recommendations that students develop their own informal calculation methods.

One might ponder as to why in the twenty-first century, nineteenth century computation practices are still the norm in many classrooms. Why have calculators not had more impact on computation in primary school? As Reys and Nohda (1994) noted, for any real change to occur in classrooms, “answers are needed before substantial progress can be made toward successfully implementing the array of computation alternatives” (p. 6). This research has answered three questions:

- When faced with a computation question, what choices do students in Years 5 to 7 make?
- Why do students in Years 5 to 7 make particular computation choices?
- How successful are students in Years 5 to 7 at executing various forms of computation?

The results from this research will allow for some progress to be made in the area of computation choice. However, before substantial progress can be made toward the aim of students being able to choose and use a repertoire of computation methods much more research is needed.

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Appendix 1: Letter to Principals

Dear Principal

I am writing this letter to provide you with some information about a research project in which I am engaged and to ask if you would be willing for your school to be involved in the project. The research would involve interviewing students in Years 5 to 7 so I realise teachers and parents would need to be consulted. I have already received ethics clearance from Edith Cowan University and have spoken to the EDWA District Director and Catholic Education Office about the research.

The research is part of a PhD Thesis that I am working on as part of my studies with Edith Cowan University. The purpose of the research is to gain more detailed information about the computation choices made by students in Years 5 to 7. The information gained from the research will aid in the development of materials to assist students in making computation choice.

A well known academic, Dr Jack Bana, a senior lecturer in mathematics education at Edith Cowan University, is supervising the project. Dr Bana is well qualified to supervise this research, having spent many years studying children's numeracy.

Having taught in both primary and secondary schools I realise that the demands placed on teachers are great. The data collection phase has therefore been designed to cause as little disruption as possible to the school and will not involve the relevant staff in any extra work.

I would be happy to discuss any matter with yourself and/or your staff prior to you making a decision if you wish. The anonymity of the school, teachers and students would be guaranteed. I have attached a letter designed to gain parent permission. Students would also be asked for permission, prior to being interviewed.

Yours sincerely

Paul Swan

Appendix 2: Letter to Parents

Dear Parent or Caregiver

I am writing this letter to provide you with some information about a research project in which I am engaged and ask if you would be willing to allow your child to take part. I have already received approval from Edith Cowan University and the Education Department through the Principal and I am now seeking your permission to work with your child.

The research is part of a PhD Thesis that I am working on as part of my studies with Edith Cowan University. The purpose of the research is to gain more detailed information about the computation choices made by students in Years 5 to 7. The information gained from the research will aid in the development of materials to assist students in making computation choice.

The project is being supervised by a well known academic, Dr Jack Bana, a senior lecturer in mathematics education at Edith Cowan University. Dr Bana is well qualified to supervise this research, having spent many years studying children's numeracy.

Should you agree to allow your son/daughter to participate in the research their involvement would normally be restricted to a single interview of approximately 30 minutes' duration. It is possible that a second, follow up interview, may be required. The interviews would be conducted at the school within class time. All interviews will be audio-taped for further analysis. The anonymity of the students and schools involved in the research will be guaranteed and data stored in a secure location. Thus complete confidentiality is assured.

Should you have any concerns please feel free to contact me through the school. Prior to the commencement of interviews students will also be given the opportunity to decide whether they wish to be part of the research.

Yours sincerely

Paul Swan

Appendix 3: The Eighteen-Item Instrument

Number	Item	Number	Item
1	$28 + 37$	10	$14 \times 9 \div 6$
2	$74 - 36$	11	$\frac{1}{2} + \frac{3}{4}$
3	$369 \div 3$	12	$10 - 4\frac{3}{4}$
4	36×25	13	$\frac{2}{3}$ of 45
5	70×600	14	$\$1.99 + \1.99^*
6	29×31	15	$\$4.93 + 39c^*$
7	33×88	16	$7.41 - 2.5$
8	1000×945	17	0.25×800
9	10% of 750	18	$3.5 \div 0.5$

Item 14

Imagine you went to a shop to buy two bottles of Pepsi (show picture – Note picture cut from supermarket catalogue depicts two bottles of Pepsi (2L) with a caption of \$1.99 ea). Each bottle costs \$1.99. Exactly how much would two bottles cost?

Item 15

Imagine you went to another shop and bought a water noodle (a toy used to splash water) for \$4.93 and a packet of two-minute noodles for 39 cents (show pictures from catalogue illustrating both items and showing the prices). How much would it cost?

Appendix 4: Field Notes

Name: _____ Date: _____ Age _____

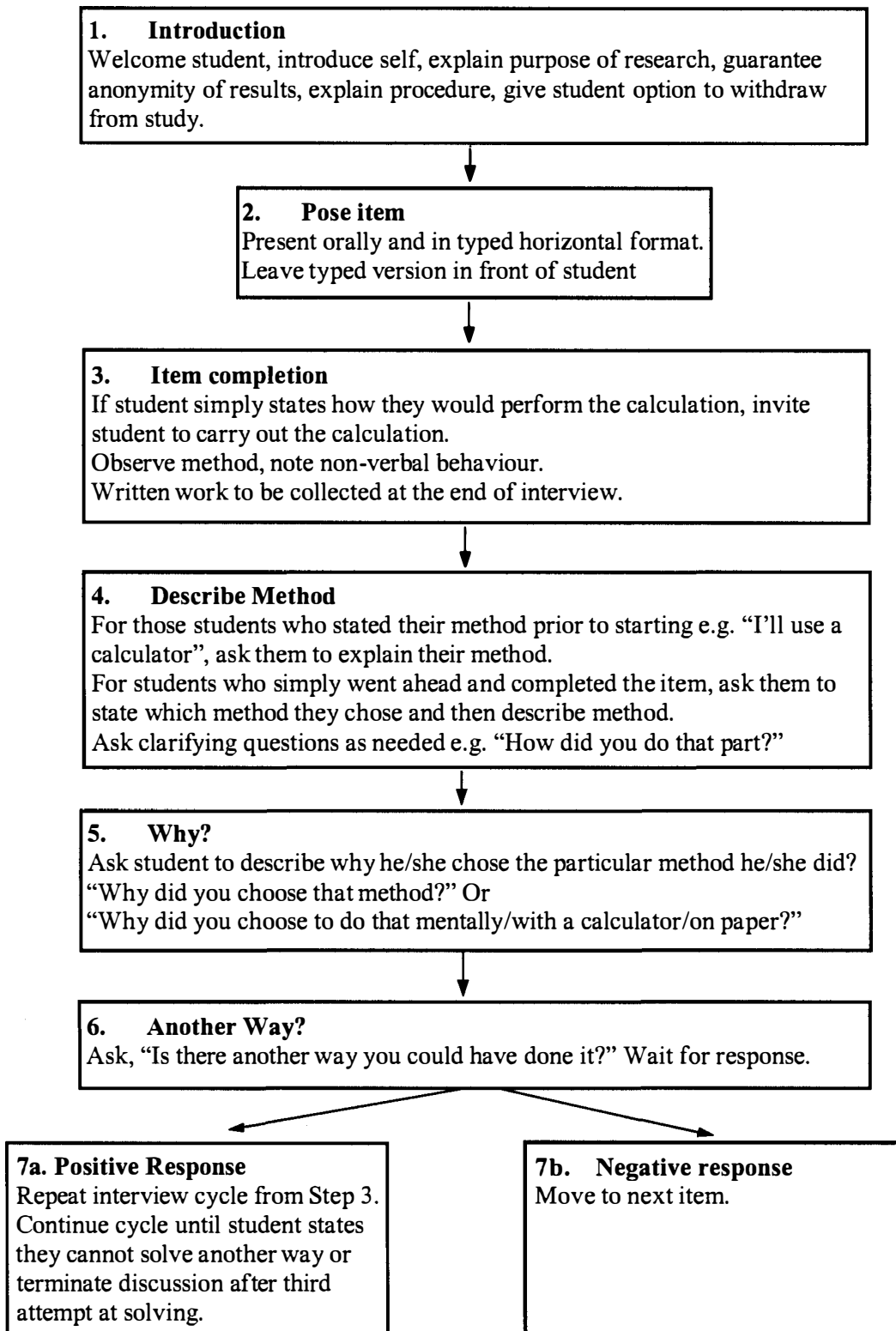
Year _____ School _____

No	Question	Preferred First Choice	Right/Wrong	Could solve another way?	Could solve another way?
1	$28 + 37$	M W C			
2	$74 - 36$	M W C			
3	$369 \div 3$	M W C			
4	36×25	M W C			
5	70×600	M W C			
6	29×31	M W C			
7	33×88	M W C			
8	1000×945	M W C			
9	10% of 750	M W C			
10	$14 \times 9 \div 6$	M W C			
11	$\frac{1}{2} + \frac{3}{4}$	M W C			
12	$10 - 4\frac{3}{4}$	M W C			
13	$\frac{2}{3}$ of 45	M W C			
14*	$\$1.99 + \1.99	M W C			
15*	$\$4.93 + 39c$	M W C			
16	$7.41 - 2.5$	M W C			
17	0.25×800	M W C			
18	$3.5 \div 0.5$	M W C			

circle list answer method r/w method r/w

Notes:

Appendix 5: Interview Flowchart and Probes



Appendix 6: Introductory Comments to Participants

Thank you for offering to help me out with this research. What I want to do is find out how students in Years 5, 6 and 7 calculate. On the desk you will notice a calculator and paper and pencil which you are free to use anytime you wish. You may also use your own calculator if you wish. What I will do is ask you a question and show you a piece of paper with the question printed on it and you will be given time to answer the question. You may choose to calculate the answer any way you like and use any of the material on the table. If you can not work out the answer just let me know and we will move on.

After you have worked out the answer I will ask you a few questions about the method you used to solve the question and why you chose that method. You have probably noticed that I have a tape-recorder and I will be taping this session. That way I can concentrate on what you are saying and replay the tape later to see if I have missed anything. Now and again I will also be writing down notes to remind me of what you said.

Your teacher will not be given any information about what we discuss in this room. Do you have any questions? If you no longer wish to take part you can leave now and if you wish to leave part way through the interview you may.

Appendix 7: Interview Extracts

The following excerpts from the student interviews are used to illustrate examples of unusual computation approaches or computation approaches that illustrate aspects of the Swan and Bana computation model outlined in Chapter 3. Examples have been grouped and where appropriate explanatory comments made. The examples were chosen to supplement those used to illustrate key points in Chapters 5, 6 and 7.

Changing approach part way through a calculation

Students are prepared to depart from their initial choice when they realise it is inappropriate. The following example shows how students who change their approach part way through a question often make use of a calculator. Note the student tried mental then could not proceed and changed his approach.

- I: 70×600 .
S: 42.
I: I noticed that you started to look as though you were doing it in your head and then you used the calculator, so what was happening there?
S: I was sort of like 7×6 and then got a bit stuck so swapped to the calculator.

Reasons for changing approach

The following example helps explain why students would change their computation approach part way through a calculation. Many students cited memory difficulties when explaining why they changed from mental to alternative methods.

- I: $28 + 37$
S: 65
I: I noticed that you started to do it in your head and then changed to pencil and paper. Was there a reason for that?
S: Because I got a bit confused.

Use of fingers

There were some examples of students using their fingers when completing mental and written calculations. Often the use of fingers was carried out in a covert fashion under the desk. The following student freely admitted to using her fingers to support her calculation.

- I: $28 + 37$

S: I would do it with my fingers. 65.

She went on to explain in response to Item 3, $369 \div 3$, that “Sometimes I might use paper and sometimes I might use my fingers. This one I think I’d use my fingers and it would be 123”. This was not an isolated case, particularly when it came to solving Item 1. Several students volunteered the information that to solve the question in their head they would use their fingers.

Using the calculator as a checking device

The use of a calculator as a checking device is most common in many primary classrooms. It appears as though this teaching practice has influenced the computation choices of some students. The following student indicated that he made use of the calculator as a checking device.

S: I did it in my head, but then just checked with the calculator.

Evidence of metacognitive thought

Some students indicated that they had thought about the item prior to starting the computation, although it does appear to be a rare event. In this example the student had considered an alternative.

I: I noticed that you decided to write that one down. Was there a reason for that?

S: Well if it had been 30×31 it would have been easier, but there were no zeros.

Realises could have done another way

When probed as to the reason behind the choice to complete 1000×945 using a standard written approach the following student made a discovery. The trigger for the realisation appeared to be related to the student being asked to try the item using another method.

I: 1000×945 .

S: They’re big numbers.

I: All right so you choose to do that by writing it down. Why was that?

S: Because it’s easier.

I: And you got 945 000 as your answer. Is there another way you could have done that question do you think?

I: Yes *I just realised it just then. I could have just times that*, I could have just gone 945 000.

Explanation triggers self correction

Note how the following student self-corrects after calculating an answer of 5.36 for the item $7.41 - 2.5$. The self-correction, however, did not take place until the student

began to explain his method. In giving his explanation the student placed the decimal question in the context of money.

- I: Would you explain how you did it?
S: You'd just do 7.41 take 2.5, no 2.5, no it would be 4.91.
I: So you changed your mind there?
S: Yes because I thought it was like cents, that would be five cents and that would be 41 cents, that's half and that's a bit under half. (Later on the student indicated the five meant 50 cents)

An example of a student realising he was wrong

While it appears that many students do not estimate the possible result of a calculation there were some who indicated they had thought about the size of the possible answer. Rather than applying typical estimation techniques it appears by the comment made in the following extract that the result was 'too low' that the student simply had a feeling about the size of the result.

- I: What about 29×31 ?
S: 69, oh no. Sorry. 899.
I: So you started to do that one in your head and then you used the calculator, so what was going on?
S: Well I tried to work it out in my head, but I got it wrong because I got 60 something.
I: How did you know that was wrong?
S: Well because it would have been *too low* for 29×31 .

Influence of previous item

There was little evidence throughout the interviews of students referring to previous items. The following student was one of the few who made reference to a previous item when giving a reason for the computation choice he made. In this particular case he had previously used a written method to solve the previous two-digit by two-digit multiplication only to find later when using a calculator that he had made a mistake. The conflict between the two results immediately caused him to question the written result rather than the one computed with the aid of a calculator. There were several instances where the answers for the same item varied according to the computation approach that was taken. Rarely did the students question the existence of differing results.

- I: 33×88 .
S: 2904.
I: Right, how come you used a calculator?
S: Because I got that one wrong before and I thought I should have used a calculator.

Rules used by students when calculating

The difficulty experienced by children tackling an item containing more than one operation is highlighted in the following extract. Clearly this student had developed some rules to help him remember how to calculate. The reference to 'start backwards like the Chinese' was made to indicate that in all operations except division the calculation begins on the right with the units and progresses toward the left, whereas in division the opposite is true. It appeared this student has learnt this without explicitly being taught it. His lack of understanding of this approach, however, prompts him to make use of a calculator.

- I: $14 \times 9 \div 6$.
S: I know that 10×9 is 90 and 4×9 are 36 so 36 plus 90 is 126 and then divided by 6 would probably be, so I have to turn 126 into divided by now, start backwards like the Chinese. Do you go backwards or forwards with this?
I: Right I didn't quite understand. You said go backwards like the Chinese. What does that mean?
S: Because like usually in maths you start from this end, no that end usually but I wasn't sure.
I: Where did you learn this thing to go backwards?
S: Because in normal maths when you do times and everything you always start with the littlest numbers because when they get bigger you can put the numbers up there and add onto them.
I: So you're not sure where to start on this one?
S: So I think I might do this one on the calculator. Yeah $126 \div 6$ is 21.

Use of prior knowledge to assist in calculation

The following student demonstrated a clear understanding of the question. When she chose to make use of a calculator the choice was based on expediency and not laziness. This extract also shows how computation methods may be mixed in a natural way to arrive at a solution in an effective and efficient manner.

- I: 0.25×800 .
S: 200.
I: Right, so now you were doing some of that in your head. Can you explain to me what was happening?
S: First I was doing it in my head and I was thinking how am I going to tackle this one and then I thought it's just like a $\frac{1}{4}$ of 800, so first I divided 800 by 8 so I would just be 100 and then $\frac{1}{4}$ of 100 is 25 and then times back by 8 which worked out at 200.
I: But you used the calculator for part of it. What did you use the calculator for?
S: First I tried to divide it back by 8 in my head and I thought it would take too long, so I did it by the calculator.

Difference between stating a method and using a method

The following conversation has been included for several reasons. Firstly it illustrates that students may state they can solve a particular type of question using a particular approach, but when asked to use it, find they are unable to do so. This highlighted a weakness of some previous research where students were only asked to state which method they would prefer to use but did not have to demonstrate their use of the method. The student also refers to removing the zero when explaining how to solve the item.

- I: 10% of 750.
S: 75.
I: Why did you do that in your head?
S: Because you just take the zero off.
I: And is there another way you could do that?
S: I could have done it on the calculator.
I: How would you do that on the calculator?
S: No you can't do it on the calculator.

When completing Item 3, $369 \div 3$, several students made comments similar to those recorded below.

- I: $369 \div 3$.
S: 123.
I: And you wrote that one down as well. Why was that?
S: Because there all in the 3 times tables and so I just did it that way because it's easy to get the answers because there's no remainders.
I: And is there any other way you could have done it?
S: On the calculator, but I wouldn't be able to do it in my head.

When observing students and this student in particular it was clear that after writing the item down on paper in the standard form for the division algorithm they immediately wrote the answer above. It certainly appeared to the interviewer that rather than complete the question using the standard algorithmic approach, she had mentally calculated the result. At the end of the discussion, however, she clearly stated she would be unable to perform the calculation mentally. It appears that when students use written algorithm approaches in simple questions they are really using mental computation techniques.

Non-standard written methods

When categorising computation choice as written it was not always the case that a formal written algorithm was used. In particular when solving items involving fractions students rarely used formal written algorithms. Often students made use of

pictures as a means of working out a solution. An example of this method may be seen below.

- I: $\frac{2}{3}$ of 45.
S: 30.
I: Okay so you did that one by pictures by the looks of things. What did you do?
S: I drew 45 circles and then I did 3 divided 45 is 15 so then I put 15 to three groups and I circled 2 of them and two 15's are 30.

Mental computation as a first resort

The following comments made by the same student indicate that rather than look at the particular question this student used paper-and-pencil as her first resort rather than mental computation, and then used a calculator for more difficult questions. An examination of the entire interview transcript indicated that she followed this rule throughout. The extracts that follow outline further comments made by the same student. She uses the same approach to solve another two-digit by two-digit multiplication and also for the in-context question $\$1.99 + \1.99 .

- I: 36×25 .
S: 900.
I: And you chose to write that one down. Why was that?
S: Well, I wouldn't be able to do it in my head and it would have been quickest to do it on the calculator, but I like to try it on paper first, then if I get confused I do it on the calculator.
- I: What about 29×31 .
S: 899.
I: All right and once again, you wrote that down. Why was that?
S: Because it's not very complicated and I'm used to writing it down because we don't use calculators all that often.
- I: I want you to imagine now that you go to the shop and you want to buy some Pepsi and at the shop they've got 2 litre bottles of Pepsi on special for \$1.99 and you want to buy 2 of those bottles. How much exactly would that cost?
S: \$3.98.
I: And I noticed you wrote that one down. Why was that?
S: I probably could have done it easy in my head, but sometimes when I first look at them they look a bit tricky so I just write them down so I can work them out.

Difficulties with zeros

Several students experienced difficulties with zeros. Note the comments made by students in the following extracts. The trigger for using a mental method in the following example appears to be the known fact ' 6×7 '. His comment about doing the tables backwards are of interest. In several examples that follow the students turn the ' 7×6 ' around to ' 6×7 '. The student then indicated that two zeros are added because the second number in the multiplication, 600, contains two zeros.

- I: 70×600 .
 S: 4,200.
 I: That was fairly quick. So you did that one in your head too. Can you tell me how you did it. Why did you choose to do it in your head first?
 S: Because I'm used to the 6 times tables and I'm not as good in the 7's so I done it backwards with the 6×7 and that equaled 42 and then with the two 0's here, I added them on.

Previous research by McIntosh De Nardi and Swan (1994) uncovered similar difficulties with zeros. Students appear to acquire the 'trick' of 'taking zeros off and adding them on' from parents or teachers but it can cause difficulties. Rather than develop a relational understanding of place value many students appear only to have developed instrumental understanding. They are aware of the 'trick' but unsure of why it works.

- I: 70×600 .
 S1: 70×600 . I'd just use the calculator.
 I: What answer did you get?
 S1: 42 000.
 I: Why did you choose to use a calculator?
 S1: It's faster and it's a lot easier.
 I: And could you do it another way?
 S1: Yeah I could have just gone 70×600 I would have gone 6×7 equals 42, plus three zeros.

 I: 600×700
 S2: 42 000
 I: Before you started writing it down, it looked like you were trying to do it in your head.
 S2: Yes, I was just thinking times that by 100 and then taking a couple off that, but I kept getting mixed up.

Bias against calculator use

This extract is illustrative of comments made indicating a bias against calculator use. A reluctance to use calculators, regardless of the reason, only serves to reduce the computation options available, thus restricting computation choice.

- S: Again, its easier because when I do it on the calculator I think its not using your brain and you don't work things out better and you don't get smarter, so I write it down because its easier and I understand better.

Lack of experience

This extract has been included to give some insight into the way students think. This student does not believe she has the experience to complete the item using paper-and-pencil methods. She is confident, however, that she will go on to learn to complete more difficult written calculations when in high school.

- I: 1000×945 .
 S: I'll definitely use a calculator for this one. 945 000.

- I: Right and you said you would definitely use the calculator for this one. Why is that?
- S: Maybe when I get a bit older like in high school I should be able to do it maybe on paper but at my year level I'm not really used to doing those types of sums either just in my head or writing it down.

Fraction items

Items involving fractions proved to be difficult for many students. Computation choice was often restricted because the students did not understand how to use the formal written algorithm and they did not know how to enter a fraction into the calculator. Some students like the one featured in the second extract below knew the decimal equivalent for fractions such as one-half and were able to use this knowledge in combination with the calculator to solve the item. Most students who adopted a written approach made use of diagrams rather than the formal written algorithm.

- I: Right let's have a look at the next one. One-half plus three-quarters.
- S: That would equal one-and-a-quarter which equals one whole and one-quarter.
- I: So you did that one in your head. Why was that?
- S: Because I can't really explain it on paper so I just did it in my head. $\frac{1}{2}$, $\frac{3}{4}$, 1 plus 3, like $\frac{1}{2}$ and then put another $\frac{1}{4}$ in that, that's 1 whole and then you've got $\frac{1}{4}$ left, so 1 $\frac{1}{4}$.

The ability to remember decimal equivalents for common fractions such as one-half and three-quarters was a key factor in determining whether a student made use of a calculator to solve a fraction item. No student in the study demonstrated an ability to convert a fraction to a decimal using a calculator.

- I: One-half plus three-quarters
- S: 1.25.
- I: You used the calculator for that one. Can you explain how you used the calculator?
- S: I did 0.5 which is a half, plus 0.75 which is three quarters and added them together.

The following student demonstrates a good understanding of fractions and decimals. This understanding allows the student to complete the calculation without difficulty.

- I: Ten take four and three quarters.
- S: 5.25.
- I: So you did that one in your head. But you gave it to me not as a fraction but as a decimal. How come?
- S: Because they're the same. Fractions and decimals are the same.

Informal written approaches

The following example shows how one student made use of informal jottings and mental methods to solve the item $\$1.99 + \1.99 . This item clearly lends itself to the

use of a mental computation strategy. The student combines this approach with some jottings on paper to relieve the cognitive load

- I: Now I want you to think about this. If you're going to buy a 2 litre bottle of Pepsi and it's a \$1.99 and you buy 2 of those, how much is it going to cost you exactly?
- S: That would be \$3.98.
- I: And would you tell me how you did that?
- S: Well first I wrote down the two 99's and made it \$2 and then since there's 1 of a \$1, two 99's, I made them add another dollar making \$4, but since they were one off and there's two of them, I took 2c off \$4 making \$3.98.
- I: And why did you choose to use that method?
- S: It was the easiest.

The last resort

The calculator was often chosen by default. Comments such as "I'd have to do that on the calculator because we haven't done that" indicate that computation choice is limited by a lack of experience. In some cases such as in the item involving percentages students chose to use the calculator because of uncertainty with the alternatives, only to find they were unsure of how to use the calculator.

The computation choice of some students was limited because they had not yet learned to calculate using a particular approach. This does not mean they have not been taught how to calculate but simply they cannot remember how to calculate. Often these students choose to use a calculator in the hope they can perform the calculation. The following explanation indicated the thinking of many students. "We haven't learnt how to do them on the page yet and I couldn't really figure it out in my head, so I used the calculator". This student tried all three computation methods before deciding to make use of a calculator.

- I: 33×88 . What did you end up getting?
- S: 2 904.
- I: I noticed you tried to do some in your head and then you wrote something on paper and then you went to the calculator. So what was happening?
- S: In my head was a bit hard and then it was harder on paper, so *I just had to use the calculator.*

The difference in computation approach and ability

The following explanations of how to solve Item 18, given by two students indicate the diverse background and range of abilities displayed by students in the study. The first student used the relationship between 0.5 and one-whole to calculate the answer mentally, whereas the second student knew he had difficulty handling questions involving decimal points and therefore made the choice to use a calculator. This

example also highlights the problems associated with trying to set guidelines for which types of calculations should be performed mentally, on paper, or with the aid of a calculator. When completing $3.5 \div 0.5$ the first student used a mental method while the second reached for a calculator.

I: That was very quick. How did you do that?

S: Well 5 goes into a whole twice, so 3×2 was 6 and then there was the extra point 0.5 so that made 7.

I: Why did you do it in your head?

S: *I found it easiest.*

I: How come you used a calculator for that one?

S: *I didn't know how to put the points in.*

The first student used the relationship between 0.5 and one whole to calculate the answer mentally, whereas the second student clearly understood his limitations and opted to use the calculator.