The effect of money as a context on students' mental computation performance in years 3, 5, 7 and 9

Anne Paterson

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Doctoral Dissertation

The effect of money as a context on students' mental computation performance in Years 3, 5, 7 and 9.

A thesis submitted to the Graduate School in the partial fulfilment of the requirements for the Degree of Doctor of Philosophy

By

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USE OF THESIS

The Use of Thesis statement is not included in this version of the thesis.
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Abstract

The purpose of this study was to research the effect of money as a context on school students' mental computational performance and strategy choices across a range of ages. This study adds to existing research, which has compared students' mental computational methods with their written methods, by the provision of the single common context of money. The content topics of whole and other rational numbers (simple fractions, decimals, and some percentages) were covered. Forty-eight primary school students plus sixteen secondary school students were involved in this study, with equal numbers of both genders from the two primary schools and one secondary school in the Perth metropolitan area.

The method followed was both quantitative—by scoring test results—and qualitative—through tape-recorded interviews. Students' prior experiences with money were documented and performance data were collected on students' mental computation ability for the two sets of mathematically identical items presented in a money-context and context-free. Student strategy choices were also documented. The semi-structured interviews consisted of nine money experience questions such as, How often do you get pocket money or an allowance? In addition, 10, 12, 13, and 13 pairs of mental computation items for Years 3, 5, 7 and 9 respectively. Where possible, common items were used across two or more year levels to ascertain growth in mental computation skills.

Overall, results found that the context presentation did not make a difference to student performance and there was no correlation found between performance and student preference for one presentation or the other. No performance differences were found for gender. Year 3 recorded the lowest process scores, while Year 7 recorded the highest process scores although all the items used at both Year 7 and Year 9 were identical. The greatest growth in mental computation performance was found to occur from Year 3 to Year 5 and from Year 5 to Year 7. Further, for Year 3, results found that the context presentation had a negative impact on student performance. Some students were found to be using written methods mentally. Analysis of individual items revealed that context had a positive influence in some cases. However, despite the emphasis in modern curricula on the use of context, it appears that such an approach may have little value if used in contrived rather than real situations.

Recommendations for teaching practice include promoting real experiences at school by linking students' out-of-school experiences to classroom learning, such as exploring students' pocket-money purchasing power or promoting mental computation for a variety of context
tasks. It is considered likely that mental computation in classrooms tends to be non-contextual and it is recommended that teachers should make more use of context. It is further recommended that teachers use money as a context, with mental computation items presented as part of real shopping tasks. Oral presentation would remove typical school method cues—a 'sheet' and pencil—with the only visual stimuli being the goods and price labeling. Class shops could use simulations for the junior grades, while older grades could organise real money exchange experiences integrated with other curriculum areas such as raising money for charitable causes.

Research on the effect of other common contexts such as food, time, and other measurement topics should also utilise real activities, with examples of such being readily found in the media. The provision of a variety of contexts is important for students as what constitutes a meaningful context may vary from individual to individual.
Certification Statement

I certify that this thesis does not, to the best of my knowledge and belief:

(i) Incorporate without acknowledgement any material previously submitted for a degree or diploma in any institution of higher education;

(ii) Contain any material previously published or written by another person except where due reference is made in the text; or

(iii) Contain any defamatory material.

Signed

Date...
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Chapter 1: Introduction

Researchers have noted that some students are able to solve mental computation items set in a realistic context but not the same mathematically identical item presented out-of-context (Newton, 1992; Nunes, Schliemann and Carraher, 1993; van den Heuvel-Panhuizen, 1999). These researchers claimed that students' school performance does not reflect students' existing knowledge because school-taught methods are so different from out-of-school experiences. Consequently, students are experiencing difficulties transferring knowledge from one learning context to the other. According to Hughes, Desforges and Mitchell (2000), the aim of schooling is "to make the lessons of school applicable to the larger world of everyday life" (p. 9). However, school mathematics practice still commonly presents mental computation items in the abstract form--devoid of context.

1.1 Background on Contextual or Real world Settings

This section outlines recent changes within mathematics education. Current educational pedagogy recommends that the provision of context can be a means for promoting mathematics as meaningful and relevant. The Western Australian document Curriculum Framework (Curriculum Council, 1998) for example, emphasizes context throughout its new mathematics strand 'Working Mathematically'. There is therefore, a need to investigate the importance of context and its impact on the mathematics curriculum. A history of the supporting research for the provision of context within school mathematics is introduced and discussed in this chapter.

In relation to context generally, Meyer, Dekker and Querelle (2001) note the pervasive use of context in mathematics curricula in the past few years. Of the five roles they suggested that context plays, the most common two: motivating students and; a chance to apply mathematics are addressed in this chapter. The authors also suggested six characteristics of high-quality contexts. Two of these characteristics were "context should be real or at least, imaginable to the student", and "contexts should be sensitive to cultural, gender and racial norms" (pp. 526-527). Of these three aspects, gender was seriously studied for over a decade in Australia (Leder, 1990).

The epistemologist and constructivist psychologist, Piaget (1952), suggested that the younger a student is, the more a context needs to be provided to enable the connection to meaning and concrete thought, and to engage the personal interest of the student. This learning theory
follows that as students become older they are able to engage in abstract thought because they worked with many ‘concrete’ experiences earlier. Internationally, the Dutch ‘Realistic Mathematics Education’ (RME) movement has highly recommended the provision of context (Streefland, 1991; Treffers, 1998; Treffers & Beishuizen, 2000).

Lowrie and Owens (2000) outline how contexts for concepts and contexts for learning relate in their framework for space and measurement topics. While measurement topics included physical representations and space topics included outside-of-classroom activities, problem-solving activities applied to them both equally.

Irwin’s (2001) study of contextualised versus decontextualised decimal fractions with 11- and 12-year-old New Zealand students found that:

students who worked on the contextualised problems improved their competence with decimals more than did a comparable group of students who did not work on contextualised problems. (p. 415)

This was found consistent for, and independent of, the students’ mathematical ability levels. Irwin chose to contextualise the decimal fractions using volume, length, and foreign exchange rates involving more than two decimal places after interviewing the students to ascertain familiar contexts. Irwin (2001) found that “students used their everyday knowledge to make sense of numbers” (p. 417) and that lower-ranked students were more likely to use this everyday knowledge.

This may be because they were able to use ‘convenient group reasoning’, a term used by Nunes et al (1993) to explain the common sense thinking evoked by natural situations. This thinking is claimed to encourage engagement at a deeper level; one that may lead to better understanding.

Nunes et al (1993) suggested that one reason why some students were able to solve computation problems set in context but not out-of-context, was that “the problem-solving routines are different in the two situations” (p. 23). Griffiths and Clyne (1995) used the following example to illustrate how some people do not make the connection between school mathematics and real-world applications:

A small boy was about to enter Year 2. His aunt asked him if he was looking forward to this. “No” he said, “the maths will be too hard.” “Oh” she said, “what sort of maths do you think you’ll be doing?” “Sums like 30 and 30 and 30.” “So that is too hard is it?” “Yes” he said, and thought for a moment, “but if it was money it would be 90 cents”. (p. 271)
Irwin's (2001) findings substantiate Donaldson's (1978) claim that "thinking sustained by daily human sense can be at a higher level than thinking out of context in the same subject" (Nunes et al, 1993, p. 25). Irwin (2001) found that:

Students were compelled to think about the relationship...what the decimal point indicated instead of applying partially understood rules such as 'lining up the decimal points' or 'adding a zero'. (p. 417-418)

Irwin's (2001) study used peer collaboration and conflict. She recommended that, "the choice of appropriate contexts is essential if cognitive conflict is to occur" as "what amounts to everyday knowledge for one group may not be everyday knowledge for another group" (p. 418). For example, the students in Irwin's (2001) study were familiar with monetary exchange rates because "they or their parents traveled...between countries or sent money overseas" (p. 418). This suggests that cognitive conflict is less likely to occur for items presented out-of-context. The choice of which contexts are appropriate is discussed in more depth in the literature review.

1.2 Mental Computation

Mental computation involving money calculations is probably the most common calculation made by most people in their daily lives. Kirsch, Jungeblut, Jenkins and Kolstad (1993) conducted a national-wide survey of 'quantitative literacy' skills, which involved embedding numeracy tasks in texts. They found that the majority of American adults could perform lower level tasks of the scale such as adding two numbers on a bank deposit statement. These tasks involve operating on numbers, which represent money in a real-life context. That is, the answer these adults achieve has real consequences. These low-level tasks follow Plunkett's (1979) recommendation that, "it would be sensible to provide most people with means to add or subtract numbers like 54.75 and 32.80 when they have no calculator available" (p. 5). Kirsch et al (1993) claimed that success at such tasks was because little prior knowledge and few sequential steps were involved and that the task did not require an inference about the type of operation to apply to the numbers. Such a low-level task is quite suitable to be performed using mental computation.

In the Western Australian Curriculum Framework (Curriculum Council, 1998), the number strand comprises four sub-strands, all of which are directly relevant to this study. The four sub-strands are 'Understand Numbers', 'Understand Operations', 'Calculate' and 'Reason about Number Patterns' with the first three in a hierarchy. In order to 'Understand Numbers' students need to "read, write and understand the meaning, order and relative magnitudes of
numbers, moving flexibly between equivalent forms” (p. 192). For ‘Understand Operations’, students need to “understand the meaning, use and connections between addition, multiplication, subtraction and division” (p. 194). In the sub-strand ‘Calculate’, “students choose and use a repertoire of mental and calculator strategies, meeting levels of accuracy and judging the reasonableness of results” (p. 196). In the sub-strand ‘Reason about Number Patterns’, “students recognize, represent, describe and use patterns in numbers” (p. 198).

All of these sub-strands emphasise number sense or quantitative intuition (Sowder, 1992) through the need to understand the relationships between numbers and operations. Studies by McIntosh (1998) have substantiated a close link between mental computational skills and number sense.

In Western Australia, Jurat (1992) conducted a small inquiry-based comparative study. Results were “consistent with research undertaken by Carraher, Carraher and Schliemann (1985) who also found that more than half of the children who used school-taught algorithms when solving mental problems obtained a wrong answer” (pp. 58-59). One explanation for this is the ineffectiveness of using written methods, such as decomposition when subtracting mentally.

Historically, traditional mental mathematics or mental arithmetic formed a larger part of the curriculum. Once the lack of understanding associated with learning by rote became clear, less time was spent on mental arithmetic, to be replaced with more time spent on teaching standard written algorithms. Some problems associated with this change resulted in standard written algorithms not being well understood, and nor are written methods the methods most needed by adults in their daily lives (Plunkett, 1979; Wandt & Brown, 1957). Today, the predominance of calculators as computational tools requiring computational estimation is another factor that needs consideration.

It was noted from Ellerton and Clements’ (1994) criticisms of teaching fractions, that even when more time had been spent, results did not improve if the incorrect approach was followed. They suggested a qualitatively different teaching approach was needed. These criticisms for the teaching of fractions could also be applied to the teaching of mental mathematics as more time and emphasis has been found for this area (Cockcroft, 1982).

It may be useful here to distinguish between the terms, mental mathematics, mental computation, mental skills and mental strategies. The first two terms are similar, broad and encompassing, and often used by teachers to allocate curriculum times. The Cockcroft Report
(1982) used the first term when recommending that students should use self-devised methods. Mental computation is the term currently used in both the United States of America and Australia to distinguish the modern teaching approach from the more traditional mental arithmetic that used to emphasise speed and accuracy. Mental skills and mental strategies are the linkages made by students that reflect their level of understanding of the relationships between numbers and operations. Mental skills describe how flexible students may be with their existing knowledge such as to be able to derive a fact from a known fact, or to check an answer by using another method. A mental strategy is the embodiment of this as an established pathway. As students develop their mental skills, they build a repertoire of strategies that together form their cognitive construct.

The most common computational choice for adults in their daily lives is mental computation (Wandt & Brown, 1957; Reys, Reys, & McIntosh 1995; Northcote & McIntosh, 1999). Reys and Barger (1994) suggested that the contemporary view of mental computation as reported by Reed and Lave (1981) is a manipulation of quantities, rather than the old view, as a manipulation of symbols (in Reys & Barger, 1994, p. 31). This being the case, then money and measurement become appropriate contexts for mental computation activities.

In 1992, the United Kingdom National curriculum assessment (The Mathematical Association, 1992b) claimed that speed and accuracy tests were still being used to assess mental computational performance. In the United States of America, Burns (1995) claimed that timed tests were still being used as standard practice in many school districts. However, in Australia (Clarke, 1988; Clarke & Stephens 1998; Herrington, Sparrow, Herrington, & Oliver, 1997) recommended that assessment should aim to identify the degree of student constructed understanding. A variety of methods were suggested including reflective journals, teacher observation and questioning rather than just the grading of timed tests. This seems reasonable in a response to reporting for outcomes-based education. The level of detail regarding student performance was termed 'grain size' by Clarke and Stephens (1998).

Identifying the student's mental computation strategy range may indicate that we value a student's true level of understanding or number sense rather than that which Yang (1995) terms 'artificial performance', which is evident when students have learnt algorithms by rote. The question of whether Wandt and Brown's (1957) classic study that indicated adults do not use pencil and paper as much as mental methods has been reconsidered in light of the impact of calculators. A recent study by McIntosh, Northcote and Sparrow (1999) of adults' daily
use of mathematics in the home found that written methods were the least favoured of the three. Most of these mental calculations related to time or money.

It seems that a traditional teaching approach still persists in Western Australian schools as indicated by anecdotal evidence from first year student teachers returning from teaching practice in 2002. One example is the competitive game ‘Sheriff’ which has dubious educational value yet remains popular. Little learning occurs with this activity—just testing. Speed in basic fact recall is emphasised and the class divides into winners and losers. The class lines up into two rows. The teacher sets a basic fact question such as: Four sevens? The student with the fastest and most accurate response wins. The loser is ‘shot’. Losers sit down, thus ending their participation. Hence, the winning student is rewarded by further opportunities to participate and remains actively engaged, while the weaker student (and the one most in need of intervention) is only passively engaged. In contrast, an ideal learning environment seeks to engage all learners most of the time by using a variety of approaches.

To directly teach students which mental method or computational strategies to use seems unwise, because constructivist theory recognises that individuals learn best when they are able to construct their own methods. Evidence in McIntosh, Bana and Farrell (1995a) found that Australian students had not been taught mental strategies at school—rather, their strategies were self-devised. This suggests constructivist learning.

In a small survey of adults by Rosinski (1998), it was “found that none recalled ever having been taught mental computation strategies, and most used adapted forms of written computation” (McIntosh & Dole, 2000a, p. 228). This reflects the low impact of school-taught procedures, which are most inefficient for mental computation. McIntosh and Dole (2000b) further suggested that “strategy development for mental computation, therefore, can be seen to derive to some degree from an individual’s number sense” (p. 402). The study found “a link between number sense and mental computation in terms of strategy knowledge” (p. 407). This suggests that good teaching which emphasises the development of number sense may be the best way for students to develop self-discovered mental computation strategies. It also follows that out-of-school experiences might provide students with meaningful contexts in order to help them invent their own strategies.

The Nunes et al (1993) and Carraher, Carraher and Schliemann studies (1985; 1987) of street and school mathematics revealed two separate mathematical practices. The first revealed practice was that oral mathematics occurred in the streets and consisted of self-invented
mental methods. By contrast, the second practice of school mathematics consisted of written methods often presented in word problem format, which required reasonable reading skills. They noted, "the contexts in which these two occur tend not to overlap" (p. 29). This dichotomy is demonstrated in Figure 1. All out-of-school mathematical experiences by nature occur in context. The dotted line—dotted to indicate that some transfer can and does take place—in Figure 1 represents a barrier to a transfer of knowledge between the two settings.

An individual's mathematics ability

<table>
<thead>
<tr>
<th>Out of school experiences</th>
<th>School experiences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number sense/relational learning</td>
<td></td>
</tr>
<tr>
<td>Oral methods</td>
<td></td>
</tr>
<tr>
<td>Invented methods</td>
<td></td>
</tr>
<tr>
<td>Instrumental learning</td>
<td></td>
</tr>
<tr>
<td>Written algorithms</td>
<td></td>
</tr>
<tr>
<td>Role learning</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1: Mathematics Ability Defined by Situational Contexts

This barrier represents the intermittent nature by which the transfer of mathematical applications knowledge can occur, from one contextual setting to another. If personally relevant contexts were found to allow transfer, then contexts most likely to be used at school should reflect the students' out-of-school world. The problematic nature of transfer associated with mathematics applications was discussed in detail in Hughes et al (2000), and this will be outlined further in the literature review.

In contrast to Nunes et al's (1993) study, it may be useful to design a different study to check for transfer between the two experiences as successful matches in practice may enable transfer of knowledge between the two experiences. Any match for practice may indicate that out-of-school experiences (task and situational contexts) can positively influence school performance. Importantly, Nunes et al (1993) found that generalization was more likely to occur for students with out-of-school constructed knowledge than from school-taught
knowledge. This may imply that the wider the match for shared practices, the more likely the student is to develop number sense from experiencing increased estimation associated with out-of-school mental computation.

Constructivist theory suggests that all school activities should begin from the student's experiences. This suggests that one way to encourage increased transfer would be for the school's teaching approaches to mirror out-of-school experiences. Teaching should emphasise factors such as relational learning, oral methods, self-invented methods, familiar task contexts and simulated situational contexts as a bridge towards formal learning.

Presenting items in a context might encourage students to use sophisticated, higher-order methods in their solutions, thus leading to improved performance when compared to their responses for non-context items. Word items are contextualised by their nature. However, the chosen context is often of questionable relevance to students (Roth, 1996).

The issues associated with word problem formats could be addressed by removing the students' need to read; by using familiar everyday language in familiar, everyday contexts; and by the items being read to them.

Related overseas mental computation in-context studies have focused on social disability (Nunes et al, 1993), and parental influences within culturally diverse ethnic groups (Guberman, 1992). The presentation of computational items in context previously studied by Carraher et al (1985, 1987) were both conducted in South America. Similar studies by Guberman (1992) were conducted in the United States of America. The Carraher studies sampled impoverished low-achieving students who attended school irregularly. They did not sample beyond the developmental stage of grade 3 schooling, nor restrict their studies to mental computation to find out whether out-of-school experiences can aid students' school performance for money problems when compared to standard written school-taught mathematics.

A National Statement on Mathematics for Australian Schools (Australian Education Council, 1991) defined mathematics as "the science of patterns" and stated that, "mathematics provides powerful, precise and concise methods of representing patterns and relationships" (p. 4). As students become aware of these patterns and relationships, they develop flexibility with numbers—a prerequisite for 'number sense'. It has been postulated that mental computational ability improves particularly after age 12 (McIntosh, Reys, Reys, Bana, & Farrell, 1997c). Further, that both number sense and mental computation performance
improve after age 12: "Not surprisingly, the average number of items answered correctly increased with age" (McIntosh, 1998, p. 216). This suggests that an improvement in mental computation performance should have a corresponding positive impact on number sense, possibly through improved efficiencies in the students’ computational strategy choices. Sowder (1992) found a substantial positive relationship between mental computation and number sense. It remains uncertain however, how much of each student’s mental computation performance is due to more efficient strategy use or Yang’s (1995) school taught ‘artificial performance’. As Yang’s studies found, performance for written computation can be ‘artificially high’ when compared with matched tests for number sense.

In their study of student performance on mental computation for grades 3 to 6 Shigematsu, Iwasaki and Koyama (1994) also found that performance for the four operations on whole numbers generally improved with age. They further claimed that:

...the calculator will become the main computational tool...therefore, mental computation will become more than merely a basic skill...rather a meta-computation for computation by the calculator. (p. 29)

The renewed importance of mental mathematics due to the introduction of calculators (National Curriculum Council, 1989; and OFSTED—Office for Standards in Education, 1993a, 1993b) means that educators should also look at how best to assess mental mathematics performance. Advances in technology have led to the availability of calculators from the late 1970s and these are now simple, cheap and commonplace. Hence, pencil and paper routines are practically unnecessary. An increase in both mental computation and a 35 percent increase in the use of calculators in Victorian classrooms over the last decade have been identified by Mousely and Herbert (2000, p. 462).

Some confusion exists in the curriculum today regarding which mental methods students should use for which mental computation items (Swan & Bana, 2000). Swan (2002) found however, that for two computation items presented in a shopping context, students chose mental methods over both calculator and written methods. In contrast, of the three choices of computation available in the classroom today—mental, pencil and paper, and calculator—pencil and paper methods still dominate classroom work in the number strand.

Teachers mental computation programs may use either a traditional approach (emphasising speed and accuracy ‘mental arithmetic’) or a constructivist approach (emphasising invented methods). There may also be some combination of these due to their pedagogical beliefs
and/or initial teacher training. These different teaching approaches may result in different outcomes for students due to the different emphases on understanding (Jurat, 1992; Boaler, 1997).

Gender differences seem to be more pronounced in Western countries than in Eastern countries (Boaler, 2000; Easley & Easley, 1992; Leder, 1990). The mathematician and neuro-psychologist, Dehaene (1997) has suggested that attitudes towards mathematics are responsible for gender and social class differences. Dehaene claimed that in mainland China, female teenagers obtain scores that exceed those of American male teenagers—"a clear proof that the difference between men and women is small compared to the impact of educational strategies" (p. 160). The fact that Asian students often do not display gender differences in performance was also evident in Yang's (1995) study. He found that there were "no statistically significant gender differences between mean scores of number sense tests, mental computation tests and written computation tests for grades 6 and 8 at the 0.01 significance level" (p. 160). This meant that for number sense, mental computation and written computation there were no marked differences "between the Chinese girls and boys in Taiwan at grades 6 and 8" (Yang, 1995, p. 161).

An Australian study by McIntosh et al (1995a) also found some gender differences. Boys performed significantly better in Years 5 and 9, although this was only for three items (out of 30) in Year 5 and two items (out of 40) in Year 9. In Western Australia, a study by Bana and Korbosky (1995) found no significant effect on performance for automatic recall, understanding or application of basic number facts for students in Year 3–7. However, in their conclusion the authors stated that one finding indicated that the strategies used by girls and boys were somewhat different. They suggested it would be interesting to know the reasons for such differences, and that this aspect should be explored further. Reasons for any such differences could be explored by conducting a study designed to find out how much influence out-of-school experiences had on differences in strategy choices and performance results. Do males and females have different out-of-school experiences? If so, which contexts may be involved? For example, Australian males seem to spend more time involved in sport and sporting results. Perhaps the preference to use school methods over out-of-school methods is more prevalent for one gender than the other for where context experiences are similar. Alternatively, there may be no significant gender differences for mental computation performance. Any differences found have been linked to factors other than ability, such as parental expectation (Dehaene, 1997). Also, fear of success (Leder, 1980),
teacher expectation (Walkerdine, 1998), differences in learning and teaching approaches (Boaler, 1997) as well as both cultural and gender differentiated success and failure attributions (Cao & Bishop, 2001).

Some researchers have claimed that gender differences in performance are more likely to be found in low-socio economic areas, as the factors previously mentioned are more pronounced for these classes (Dehaene, 1997; Walkerdine, 1998). This suggests that gender performance difference for middle-class schools should be less significant.

1.3 A Statement of the Problem

A common question is: “How should we present mathematics in order to maximise student engagement?” The National Research Council (1989, p. 61) suggested that a natural context is best for engaging students. Both Nunes et al. (1993) and Guberman, (1992) chose to use the universally recognized quantitative and culturally relevant value of money for context in their studies.

There still appears to be a great divide between the way mathematics is taught in schools and the way mathematics is used practically in the real world (Maeir, 1980). As stated previously, schools are still spending most of their time teaching written methods, while mental methods are the preferred choice in ‘the real world’ (Northeote & McIntosh, 1999).

Applying abstract concepts to real world situations is not a problem for all students. However, Nunes et al (1993) found that there was a danger that it may be a problem for some students. Consequently, these students may be under-achieving at school and are at risk of becoming disadvantaged as career choices are denied them.

In 1978, Rathmell proposed ‘thinking strategies’ as a way of linking basic number facts via relationships rather than by independently memorising them. Schoenfeld (1987) documented that the ‘back to basics’ movement of the 1970s resulted in students who could not “use memorized procedures in the simplest applications” (p. 6). Kilpatrick (1987) suggested that it is only a matter of time (due to computers) before instruction moves “away from the memorization of standard algorithms toward a more conceptual emphasis on various operations and their use” (p. 123).

Contemporary mathematics educators emphasise the importance of number sense worldwide. For example, in the USA publication Everybody Counts, the NRC (1989) regarded number sense development as its main objective for elementary school mathematics. While many
mathematics educators recognize mental computation as a means of fostering number sense, others including English voters have been pushing for more traditional curriculum reform (Boaler, 1997). Some parents may fear that their children will become calculator dependent or the parents may be responding to the recent moves towards accountability via national testing. The traditional approach to teaching mathematics, which includes an emphasis on rote learning and drill and practice of basic facts and is devoid of context, is reflected in Skemp's (1976) notion of 'instrumental understanding'. By contrast, Sowder claimed that "it seems reasonable to expect that any computation or estimation problem set within a context will make the problem more comprehensible to most students". Therefore this should be reflected in Skemp's notion of 'relational understanding' (1992, p. 374). Part of the problem to be studied then is whether the provision of a suitably meaningful context can improve student performance at mental computation by unlocking more efficient strategy use. If so, does this correlate to students' out-of-school learning experiences for that context and does personal preference have any significant influence?

1.4 Rationale and Conceptual Framework

A prior Australian study by McIntosh et al (1995a) on mental computation was conducted for Years 3, 5, 7 and 9 for student attitudes, preference for oral or visual presentation, and performance. The McIntosh et al (1995a) tests were context-free and group-administered. As no interviews were conducted, no investigations about different mental computation strategy choices were made and therefore no conclusions were drawn in this regard. In fact, one of McIntosh et al's (1995a) study's suggestions for further research was: "If mental computation items were contextually based what difference would this make to performance?" Another suggested question was: "What is the relationship between children's mental computation skill and their overall number sense?" (p. 38).

Following the philosophy of a realistic mathematics curriculum, Plunkett (1979) stated that: "...money calculations are those most frequently used by the person in the street" (p. 5). While the Cockcroft Report (1982) identifies competence with money calculations as a mathematical need of adult life, and it also stated:

Practice in the handling of money, the giving of change by 'counting on' in the way which is commonly used in shops should all start in the primary years. (p. 92)
The United Kingdom (National Curriculum Council, 1989) Non-Statutory Guidance to Mathematics suggested four ways in which classroom activity can incorporate mental methods. The third of these was:

Consolidate knowledge through purposeful practice of recalling and using such facts in realistic contexts. (p. E3)

In Taiwan, Yang (1995) found a significant correlation between students' high scores for his number sense test and a high level of conceptual understanding as determined by their ability to estimate. Yang constructed his number sense test using items in words as a test of estimation in order to determine conceptual understanding. However, his traditional mental computation test generally required recall of non-contextualised or "ban" number facts. Examples such as $6 - $4.50 or $3 - 0.95c could be seen as contextualised abstract symbols. However, these expressions are not couched in real-world, imaginable applications. They are examples of 'money arithmetic', and devoid of 'imaginable' context. As the operation is also clearly identified, these examples are still too close to the abstract items and may not reveal any difference in performance and thinking strategies—they are merely symbolic, meaningless representations of money. If these items are not realistic to the reader, but only to the writer, are they checking the reader's number sense or the writer's, or are they checking for the connectedness of the writer to the reader's world?

Schliemann, Araujo, Cassunde, Macedo and Niceas (1998) claim that students' mathematical knowledge develops despite school, because of out-of-school experience, such as developing the commutative property of multiplication through street selling. It is also recognised that schools can be essential in developing mathematical knowledge through in-school activities. These two learning frameworks—one, in school and the other out-of-school—may develop side-by-side and therefore be 'situationally specific'. Both frameworks contribute to the learner's prior experience. Ideally, they may 'synergize' when school activities build on students' prior experiences (Dapuento & Parenti, 1999; Saxe, 1991).

Treffers and Beishuizen (2000) discussed Treffers' (1993) terms of ‘horizontal’ and ‘vertical’ mathematising, which may be useful for relating to the notion of 'number sense'. Treffers' term ‘vertical’ mathematising describes 'vertical' factors such as the process of discovering connections, finding shortcuts and applying these discoveries by using higher order mental computation strategies. Treffers' term ‘horizontal’ refers to the factors such as the provision of a context; and previous experiences with a relevant context that should help to develop the
application of short cuts or efficient mental strategies. The researcher considered that student practice with working mathematically in a context should increase with time spent practising, therefore could increase with age.

Perhaps the level of sophistication of mental strategy use could reflect the depth of understanding in mental computation. Therefore, if a student's choice of mental computational strategy and the level of understanding are linked, perhaps higher-order strategy choices reflect greater understanding. McIntosh, De Nardi and Swan (1994) developed a classification of mental computation strategies, which could be used to develop a three-tier hierarchical scale. If the provision of context correlates directly with students' use of higher order strategies, this may indicate that context promotes deeper understanding.

While it is claimed that we should “embed mathematics activities in contexts” (Burns, 1993) so that mathematics comes alive and is purposeful, there is little research evidence, which compares mental computation ability on identical items for context and non-context. This is quite apart from comparing the value of one context to another.

The Number strand of the Western Australian Curriculum Framework, (Curriculum Council, 1998) focuses on operations on numbers in order “to deal with quantitative aspects of the environment” (p. 192). Money is an obvious example of one of these aspects. Concerning money as a context and therefore student's prior experiences with money, it was considered that Year 3 students were unlikely to have any income beyond some pocket money. By contrast, it was considered that Year 9 students were the most likely to have had substantial money handling experiences and/or paid jobs.

In Boaler's (1997) award winning study of teaching styles, gender and setting, in the UK, she explains a difference in gender learning styles. When Boaler compared a traditional school of Amber Hill to Phoenix Park—a progressive one—she found that apart from textbook lessons, boys preferred the traditional methods with the emphasis on speed and accuracy and 'relative performance'. In contrast, Boaler found that girls preferred to learn at their own pace on their 'quest for understanding'. Boaler further suggested that this might be a factor of maturity (p. 120). Given these differences in learning styles, it was wondered if males might prefer non-context items while females might prefer money-contextual items. In addition, it was wondered whether a shopping context might be considered as gender neutral despite the nature of the purchase item being gender traditional, for example “If Dad bought a fishing rod...” or “If Mum bought a dress...”. School experiences might influence responses
given recent improved results for girls due to the new curriculum. However, past research in Australia from 1980 to 1990 (Leder, 1990) has suggested only minor differences for gender.

It is often considered that mathematics is more interesting to students when they are motivated by the subject matter (Burns, 1993). It may be fair to assume that when items are presented in a suitable context, this would inspire students to, rather than put students off, solving them. It was further considered that if students were motivated by a money-context then this should be reflected in them achieving superior results for mental computation items in context. It was also recognised that students may prefer non-contextual items if they are more experienced and comfortable with these items and or perform better for these items. It follows therefore that students would need to be asked whether they preferred the context items or non-contextual items or neither to find whether preference actually had any impact on their results.

There is little research regarding student preference for mental computation items presented in a context or without. Some research (McIntosh et al. 1995a) checked for preference regarding mental computation presentation modes—that is, oral versus visual. It might be assumed that weaker students (those with less strategy knowledge) might prefer working with items that are presented in a context where the meaning is maintained (Irwin, 2001). However, these same students may also prefer non-contextual items when being tested because removing ‘the trees from the woods’ provides less of a cognitive load when the pressure of speed is applied. Irwin (2001) found that “high ranking (able) partners ignored context and manipulated numbers instead” (p. 411).

Some research exists that compares children’s own mental computation strategies with standard written methods (Newton, 1992), and some research exists that identifies students’ knowledge and competence with the basic facts and associated computational strategies (Bana & Korbosky, 1995). However, Callingham and McIntosh (2001) claimed that although mental mathematics is an essential component of school mathematics, little research exists on which factors, such as context, may influence student mental computation performance and strategy choice. Cooper, Heirdsfield and Irons (1996) support this in their study of addition and subtraction word items, when they stated that there has been little research relating to mental computation and word items apart from that of Carracher et al (1987). McIntosh (1996) also claimed that “more extensive research needs to examine the role of mental computation as part of developing number sense generally” (p. 260).
In summary, little research regarding mental computation exists beyond the basic facts (Bana & Korbosky, 1995), regarding which factors influence performance and strategy choice, and student preference for context. Hence, this study on whether context could be a significant factor should add to the existing body of knowledge.

1.5 Possible Contexts

As the context chosen needed to be a common one suitable for testing students of all ages and so that teachers could apply it in the classroom, a universal or global context was needed. As prior knowledge can be attained in out-of-school experiences, students' out-of-school experiences had to be taken into consideration. Some factors that could affect the student's level of out-of-school experience for any context are the family's socio-economic status, family expectations, student age, the student's level of development and the type of school setting.

One reason given by Dapuito and Parenti (1999) for their choice of the contexts of money and calendar for their study was to integrate these contexts substantially with students' out-of-school life. This was so that students' out-of-school practices could be discussed openly in the classroom. Money and time are the two most common uses for mental computation in the home (McIntosh et al, 1999). Although both money and time are familiar contexts for students, time uses a variety of base systems, which may be confusing to master and is therefore more complex. In contrast, money only uses the decimal system and is a highly motivational context. Money is therefore a more suitable context than time. However, it is recognised that familiarity with money is limited by students' prior experiences with money.

McIntosh et al (1994) stated that when children choose their own contexts for their 'sum stories', they usually choose "themselves, money and food" (p. 37). These subjects provide a generic real-world relevant context meaningful to all students, and of which, food is often used as a context for teaching fractions (Anthony & Walshaw, 2003). Anthony and Walshaw's pizza assessment task found that "overall, the contextual nature of the problem was a significant factor in students' responses" (p. ii). Food may be suitable for some computations with whole number and common fractions, but is more limiting than money as a context for numbers generally, including decimals. For example, it may be confusing to discuss percentages in a food context, whereas students are often aware of the sales term 'percentage off' in a money context.
1.6 Why Money was Chosen

Concerning money as a context, The Mathematical Association (1992a) claimed:

To many children $1 - 0.63$ or $2 + 0.01$ look forbidding and difficult, but if they are related to the context of money, they immediately become much clearer. How much change from £1 when I spend 63p? How many pence in £2? (pp. 10-11)

Both of these examples included operating on a mixture of whole and decimal numbers to two decimal places. Given students usually find decontextualised decimals difficult, we need to ask why they should find the identical items set in a money context easier. Perhaps most students receive pocket-money—although the amount varies according to family incomes. Parents generally encourage the concept as it prevents the child from constantly asking for things by creating limits or boundaries. The child can then develop self-discipline in saving and spending. Pocket money is one step towards the student developing financial self-sufficiency. It is spent on items such as lollies, small toys, ice creams and snack foods.

As students grow, so do expectations to purchase small gifts such as Mothers’ day and Fathers’ day gifts from shops or even a school stall. Some schools encourage parents to provide small change for students to purchase these gifts or ice creams, popcorn etcetera, under adult supervision, from their first year at school. Other school-based activities may include the tuck shop, charity stalls, excursion money and school banking.

As previously noted, a few overseas studies have used money as a context: Guberman (United States 1992), Nunes et al (South America 1993), and Irwin (New Zealand 2001), but these studies have been limited in scope. There still remain areas of knowledge regarding students’ performance with mental computation in contexts that have had limited research. Three such areas include: age differences; gender; and students’ previous experiences within a money-context. A broader age ranged study that also checks for gender and previous experiences yet for one context only is needed. Such a study would add to existing but limited age-ranged studies, such as Guberman’s (1992) Years 1-3; Nunes et al’s (1993) Year 3, and McIntosh and Dole’s (2000b) study of Year 3 and Year 5.

Money was chosen as the context because it was considered to be one that teachers could use, and that all students would have had some contextual experience with this context. It was considered meaningful for all. It is the best single context suitable for all age groups because all children have had some experience with money for a variety of reasons. Of all the other contexts mentioned in curriculum outcome documents, money is the first context that requires knowledge of standard units by students. Money as a context could also be used to
create real and imagined familiar events such as 'The Show' and buying items such as lollies, drinks, ice creams, toys, clothes and birthday presents. Money should therefore be a suitable choice, although student's out-of-school experiences with money may well vary across the year levels and between individuals.

1.7 Why only One Context?

When considering suitable measurement topics for providing context, volume was an obvious topic due to students' familiarity with 3D objects at an early age. Students may have had some volume experiences at home through cooking and/or woodwork activities. This could not be assumed for all students, as grocery items have become more processed and takeaway foods more accessible. It was decided not to use measurement because formal units for length are not taught until Year 3. Formal units are usually introduced gradually from Year 3 to Year 5, as a developmental sequence is generally followed. In contrast, formal units for money are taught from Year 1. Therefore, all Year 3 children would be familiar with the context of money whereas it cannot be assumed that all students would be familiar with length as a context.

It was decided to test for only one context because students' experiences could vary from one individual to another. This is due to the various factors mentioned previously in other contexts of the family's socio-economic status, family expectations, student age, the student's level of development and the type of school setting. It seemed fair to assume that money is a context familiar to all students. Nevertheless, factors that could impinge on the student's level of experience for a money-context could be family wealth as well as those listed previously, and the types of previous school and home activities with money. In order to determine how much these factors were relevant it was also necessary to measure the type and degree of previous experience the students had with money.

In order to illustrate the process of performing mental computations that involve a context, a model was developed as illustrated in Figure 2. This model was adapted from Mason and Davis (1991a, p. 51) and altered to make it specific to suit a money-context. This model of mental computation processes for money-context items depicts the six-step cycle a learner would pass through in order to complete such an item. To be able to operate on numbers set in a context means being able to decode familiar and everyday language into mathematical language and symbols. In this model, the process of solving mental computation items in context differs from the usual four-step problem-solving process of Pólya (1990). The two
left-hand boxes represent the real or personal world, whereas the centre boxes represent the world of visual imagery and imagination. This is potentially where deeper thinking begins. Finally, the third column represents the traditional, 'algebraic-mathematical world' of mathematical formulas and techniques.

Figure 2: Cycle of Mental Computation Processes for Money Context Items

According to Newman's (1977) error analysis, students are most likely to misunderstand word items at (B) if the language is not simple, familiar or carefully read. Mason and Davis (1991a) termed this process, the 'mathematisation' of the item. They reported that students find this 'mathematisation' along with the associated 'demathematisation' at (E) extremely difficult. Students who may have difficulty with this may benefit from the use of a mental image. Without a suitable context or image, they might be expected to perform better for non-context items. Also, without the provision of context, the students may also be less likely to check their answer at (E) by asking, "Does it make sense?", because of the lack of context or suitable image to supply a sensible reference point. For students who do not have any difficulty at (B), step (E) should not be problematic. As students 'demathematisate' the item, use estimation to ask, "Does it look right?" and relate the answer back to the original item, students are developing number sense through reflection and mathematisation.

Where the set of items are presented in a non-context form, the cycle is shorter, starting and ending at (D) as the item is presented as (C). Thus, an important aspect of thinking has been removed. These abstract items are relatively straightforward to perform irrespective of
whether basic number facts have been learnt in an 'instrumental' or in a 'relational' way (Skemp, 1976). The fact that fewer steps are involved may mean fewer 'opportunities for errors' will occur and correct answers can then be attained at speed.

When items are presented in a money-context, it may be that as students learn to overcome any problems associated with step (B), they perform better with these items than for non-context items. This may be because they benefit from the reflective process of step (E). As students realise errors for themselves, they are more likely to revisit the process by 'backspacing' in an attempt to self-correct. These steps are the ones most likely to develop mental skills and number sense. However, without the presence of a context such as money at (A) and (B), there may be fewer realisations of possible errors through computational estimation, which is such an important component of number sense.

Student strategy choice may be considered in a similar manner, as efficient choices should leave less room for error. As mentioned previously, how this all links together and the differing levels of learning inefficient strategies may provide more 'opportunity for errors', as usually there is a greater number of steps involved. Pólya's (1990) classic problem solving process differs from the cycle depicted in Figure 2 in that Pólya's heuristics involve only four steps: 1) Understand the problem; 2) Devise a plan; 3) Carry out the plan and; 4) Look back.

This is sometimes simplified for primary school students as: look, plan, do, and check (Bana, Farrell, Gleeson, McIntosh & Swan, 2000, p. 66). The strategies involved in problem solving such as 'list all possibilities' and 'make a guess' are distinctly different from those involved in mental computation, such as 'bridging ten'.

1.8 Purposes of the Study

The major purpose of this study was to investigate whether a money context for mental computation items improved student performance. Improved student performance was defined as an improved score for money-context items compared to numerically identical non-context items in mental computation. The researcher hypothesized that money as a more realistic and meaningful, universal and highly motivational context would allow students to solve mental computation items with more ease, than if the same items were just presented in the traditional abstract form.
This study was also designed to examine any link between a student's background with money (prior contextualised learning) and performance for both money-context and non-context items. It was wondered what level of interest or exposure in money activities students need in order to perform better for money-context mental computation items.

It was also wondered whether as students grow older, improvements in performance for mental computation items would occur possibly as a result of practice with, or increased interest in or exposure to, money. It was also wondered whether the students' strategy choices would be broader and more sophisticated. Students may be using a variety of strategies according to whether items are presented in or out of context. As McIntosh (1998) claimed, "very little exists in the literature regarding the (mental) computation abilities or strategies of students in the middle to upper primary years" (p. 210). Irwin's (2001) interviews to ascertain familiar contexts found students as young as eight years displayed a wide knowledge of the everyday use of decimals (including currency exchange).

The impact of school-taught decimals for older children resulted in this everyday knowledge narrowing. However, this everyday knowledge still included money. This study also set out to explore whether there were any gender differences in mental computation performance for money-context and non-context items and whether there would be any gender differences in experience with money. Two important gender differences that have been identified in the area of mathematics regard both the increased performance and participation of boys in favour of girls. The 'fear of success' factor has found performance inconsistencies with a western culture focus (Leder, 1980). As current mathematics teaching methods promote gender inclusivity, this policy should be resulting in a gender-neutral affect with females and males performing equally.

It might be reasonable to assume that students might use school-taught written algorithms mentally to solve non-context mental computation items if school mathematics was the students' main exposure to mathematics. This is because mental computation items are often presented at school in a non-context format. In addition, school taught methods are predominantly written ones. It might also be reasonable to expect that students might use non-school taught methods to solve contextualised items and that this likelihood would increase with age and out-of-school mathematical experiences. As a variety of contexts would result in more variations according to individual backgrounds, a study involving only one context would be best to limit this variable, given age and gender are already variables.
being considered. Students may prefer mental computation items either presented in a context or without. This preference may affect performance, as students may be more interested in that particular set of items.

1.9 Research Questions

From the above background, the main research question arose:

What effect does the context of money have on students' mental computation performance in Years 3, 5, 7 and 9? From this, the following sub-questions arose:

1. How is mental computation performance affected by the provision of a money-context?
2. How does mental computation performance relate to students' prior experience with money?
3. How does mental computation performance relate to the levels and types of mental strategies used?
4. To what extent is year level a factor in mental computation performance for money-context and non-context items?
5. Are there any differences between genders in mental computation performance for money-context and non-context items?
6. How does a student's preference for context or non-context affect mental computation performance?

1.10 Significance of the Study

This study should add to existing research by comparing students' mental computational methods in context and without. There is increasing attention to using context in mathematics classrooms in order to make mathematics more meaningful and relevant to everyday life. As well, there is renewed emphasis on mental computation due to its recognised if not fully explained positive influence on the development of number sense. As a desired outcome of mathematics education, number sense is said to be enhanced by the provision of contexts.

This study embraces and acknowledges that curriculum reform such as the prevalence of calculators and spreadsheet technology require ability with mental computation, and number sense generally. This assumes certain teaching practices to enhance both the content and the
processes as desired outcomes of this new curriculum. Such teaching practices would foster the allocation of time spent daily on mental mathematics and that these oral methods would be openly discussed. The items presented for mental mathematics discussions may be best set in contexts such as a money context, which are relevant to the student.

1.11 Explanation of Terms

It is evident from the literature that terms can often be interpreted or used in different ways. The following terms have been explained in some detail due to their relevance to this study. Most explanations end with a definition specific to its use in this study.

1.11.1 Context

The term 'context' needs to be explained according to its use in mathematics education. The Western Australian Curriculum Framework document (Curriculum Council, 1998) refers to 'context' in both the 'Appreciating Mathematics' and 'Working Mathematically' sub-strands. In 'Appreciating Mathematics', context is described thus: "...its terms reflect specific social and historical contexts, and (students) understand its significance in explaining and influencing aspects of our lives". While, the 'Working Mathematically', strand describes context as where: "(students) interpret and make sense of the results within the context...". Under 'Mathematical Strategies' (a sub-strand of the Working Mathematically strand), 'context' is referred to as something familiar: "The student...is prompted by a specific stimulus or familiar context" (EDWA, 1998, p. 34). The latter is the meaning assumed for this study. The Cockcroft Report (1982) also referred to the term 'context' as "familiar everyday situations". Context is just as important as flexibility (one major component of number sense) and understanding with regard to mental methods. The Mathematical Association in the UK (1992a) identified, "...three inter-related elements...of key importance in the effective use of mental methods: Understanding, Flexibility, Context" (p. 9). Contexts that will best suit students therefore use everyday language to describe familiar events in a child’s life.

1.11.2 Understanding

It has long been recognised that learning mathematics in a useful and relevant context makes learning more interesting and understanding easier to gain. In the USA, this is illustrated in the National Council of Teachers of Mathematics (2000) claims that “learning mathematics with understanding is essential” and that “learning without understanding has been a persistent problem since at least the 1930s” (p. 20). In Western Australia, understanding as a
desired goal of mathematics is indicated by the headings ‘Understand Number’ and ‘Understand Operations’ of the Number strand of the Curriculum Framework (Curriculum Council, 1998).

The term ‘understanding’ can mean different things to different people as demonstrated by Skemp’s (1976) notion of ‘instrumental understanding’ versus ‘relational understanding’. He illustrates these terms by way of metaphors of driving a car from A to B. In comparing instrumental understanding he describes how a person would find an unfamiliar place by following instructions such as “first turn on the left, then it’s second on the right”. He contrasts this with relational understanding by illustrating how a person might find their way from A to B in a familiar place by choosing from a range of possible routes available to them, depending on different criteria. For example, a different route is taken when one needs to buy supplies on the way home or to avoid traffic. While Skemp’s (1976) definition relates in a broad sense beyond mathematics, it can also be specifically related to teaching versus learning of mental strategies. These two approaches are either explicit teacher-directed instruction as to the ‘best method’, or the child-centered and class sharing of various strategies from which students choose their own preferred method.

Philosophers generally take the term understanding, to mean a private act “in the head”. Locke (1961) was an early advocate of activity as a proof of understanding (Sierpinska, 1994, p. 23), as was Piaget (1978) for whom understanding was a result of action and reflection. Current assessment methods state that students will for example, ‘demonstrate’, ‘make’, ‘say’, and ‘do’; which are all verbs. These observable outcomes are currently recommended as the best methods for assessment of student understanding.

Plunkett (1979) claimed that for mental computation tests, “a child who gets his mental calculations right almost certainly understands what he is doing” (p. 3). Others dispute this (Yang, 1995; Sowder, 1988). Interviews are generally regarded as the most reliable method to determine what a child is thinking (Ginsburg, 1977; Bell, 1999), the mental strategies they are using, and how certain they are of their answers.

According to NCTM (2000), understanding requires transfer, “the ability to use knowledge flexibly, applying what is learned in one setting appropriately in another” (p. 20). Similarly, Nunes et al (1993) used the term ‘generalization’. Further, the Mathematical Association (1992a) referred to degrees of understanding with: “Depth of understanding is perhaps indicated by the range of links and interconnections a child can make with confidence”
This definition of understanding as flexible thinking is similar to those for 'number sense' evidenced by examining students' mental computation strategy choices for the range and level of sophistication.

11.3 Estimation

It was claimed by Bana (1990) that "most everyday mathematics (about 80 percent by some estimates) requires estimation rather than precise calculation" (p. 1) and that estimation aids concept development by forcing students to think about concepts and relationships.

Researchers have suggested that poor estimators do not value estimation as they believe estimation to be inferior to exact calculation (Morgan, 1988) and even, equal to guessing (Threadgill-Sowder, 1984). Bana (1990) suggested this 'reluctance to predict' might be in part be due to students being "conditioned towards a right/wrong mentality" and "a natural fear of the unknown" (p. 1). Importantly, Sowder (1989) found that poor estimators "do not place much value on mental computation" (p. 379). In contrast, Sowder (1992) also found that good estimators will "value mental computation" (p. 379). She suggested one characteristic of skilled estimators is that they are 'flexible in their thinking' which has more recently been identified as a characteristic of number sense. She explained that the difference in the meaning of 'guessing' as wild or silly guesses compared to educated or good guesses. While the terms of estimation and number sense are different, estimation provides for the development of number sense. This may be reminiscent of the older style teaching methods where children were expected to read word perfect, rather than in today's classroom where reading for meaning by students is generally fostered.

The study in Taiwan by Yang (1995) found that students competent with written algorithms scored low for number sense. Yang found that most students scored high on the written computation test and low on the number sense test for the identical mathematical content. Answers to the number sense items needed only to be estimates, and were identical in mathematical content to their matching written computation items that required an exact answer. His research found that students who scored well on written computation tests often had little understanding of the school-taught procedures they were using. Yang claimed that this implied that their conceptual understanding of what they were doing was poor, despite good test results; something he termed 'artificial performance'. This finding suggested that the students who were currently assessed as performing well in mathematics were also those students who were good at following rules. They were not necessarily flexible thinkers, as
students who scored high for number sense generally proved to be. He suggested that this might be because too much emphasis has been placed on traditional written algorithms, without enough corresponding attention to understanding. This raised significant concerns about assessment methods. His study has far-reaching implications for curriculum change.

The use of context in estimation was a major focus of a study by Morgan (1988) that presented 'word problems' (word items) involving computation in context and found that context helped in two ways. Sowder (1992) lists both of these from Morgan's findings. According to Sowder, the first way that context helped, was that "difficulties in conceptualising operations, such as multiplying with numbers less than one, were easier to overcome in problems set in a context" (p. 374). A second way that context was found to help, as reported by Sowder was that:

> the presence of context seemed to discourage an algorithmic approach. For example, students were more likely to recognise digits after the decimal point as relatively insignificant when the decimal numbers were linked to a context. (p. 374)

Estimation can be taken to mean either a computational choice or its use in a monitoring or checking capacity. For the purpose of this study, the definition of estimation as a 'checking capacity' has been used. The importance of estimation as a checking tool for computation is discussed in Reys, Rybolt, Bestgen and Wyatt (1982).

As for mental computation and number sense, estimation skills are also reported to increase with age particularly after grade six (Sowder & Wheeler, 1987). This suggests either that there may be some relationship between these three abilities or that improvement in mathematics performance is age dependent or a combination of both. This is consistent with expectation. The definition of estimation as a checking tool includes an ability to judge the reasonableness of answers given; such as when the student is able to self-correct or uses a different method to check their work.

1.11.4 Numeracy and Number Sense

Numeracy is a term that Willis (1990) has traced back to 1959 and the UK Crowther Report. Traditionally it has been used to describe a person's ability to cope with the mathematical demands of everyday life such as the ability to read a timetable, handle money and give the correct change (McIntosh, 1996). More recently, it has been used more broadly to describe a person's ability with all aspects of mathematics not just number (DEETYA, 1997). The term
‘numeracy’ is currently used by schools in Britain and Australia as evidenced by the English and Welsh term ‘numeracy hour’ to describe the daily mathematics lesson (Hughes et al, 2000).

With regard to number, Girling (1977) defined numeracy as “the ability to use a four-function calculator sensibly” (p. 6). He further defined ‘sensibly’ as the ability to check the answer, an understanding of the relative size of numbers and the ability to perform mental calculations at speed. These first two aspects are often listed as components of number sense in definitions; and are in line with number sense principles. Girling qualified this definition later with “speed should always be considered less important than accuracy” (p. 5). Anghileri (2001) recommended the importance of context in order to develop number sense, “numbers need to be presented in a realistic setting in order to make sense to young children...they can and should also be presented orally” (p. 125).

Although controversial at the time, Girling (1977) called for encouraging and promoting mental work in order to increase mental facility. He claimed that the instruction “Don’t show your working out” illustrates an emphasis on mental methods. Today’s mathematics educators aim to encourage students to invent their own mental and written methods, which are then shown, discussed and reflected upon in order to develop number sense, which is a much broader term than traditional numeracy. Teachers aspire to greater goals for their students than merely the ability to cope with the demands of daily life. They do this through promoting the acquisition of number sense. In particular, students should be able to judge the reasonableness of their answers (McIntosh, Reys, & Reys, 1992).

A definition of innumeracy is given in Dehaene (1997) who quotes Baruk’s (1985) favourite example, the Monty Pythonesque problem: “Twelve sheep and thirteen goats are on a boat. How old is the Captain?” To which a large proportion of French first and second graders in an official survey responded, “Twenty-five years, because \(12 + 13 = 25\)” (pp. 137-138). This definition of innumeracy suggests there is a lack of understanding or making sense of numbers. McIntosh et al (1997c) claimed that the concept of number sense is recent:

(1I) refers to a person’s general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient strategies for managing numerical situations. (p. 3)

In the UK, the Cockcroft Report’s (1982) term ‘at-homeness with numbers’ may indicate where the concept of number sense began. Mathematics researchers (Resnick, 1989; Sowder
& Sowder, 1989) defined number sense as common sense—a definition supported by Dehaene (1997). In the USA, Everybody Counts: A Report to the Nation on the Future of Mathematics Education (NRC, 1989) outlined that, "the major objective of elementary school mathematics should be to develop number sense" (p. 46). The Number and Operations Standard of Principles and Standards for School Mathematics (NCTM, 2000) stated that this standard has the development of number sense as its centre.

Number sense is given similar importance in Australia. The National Statement (AEC, 1991) stated that "all people need to develop a good sense of number, that is, ease and familiarity with and intuition about numbers" (p. 107). This definition suggests that answers should make sense to students, fostering their capacity to question and revise their work through internal "checks and balances" (McIntosh et al, 1997a, p. 3). Students should be less likely to guess, write a very incorrect answer, or make no attempt.

McIntosh (1996) distinguished number sense from understanding by claiming that understanding is a means not an end, whereas number sense allows an individual the power to operate with numbers confidently and competently. Number sense is the ability to make connections or see relationships between "symbols or words or pictures or objects" (p. 61). The 'think board' (Haylock, 1984) can be used to promote this process, as mental computations are set in context (The Mathematical Association, 1992a, p. 11). Both understanding and number sense, are similar in that they develop by degrees.

McIntosh (1996) claimed that number sense provides power beyond the mathematical basics. Power can provide for change. Many would regard money as power, in the sense of possessing large sums that allow for a wider range of life's choices, but this is available to only a few. However, most people regard the ability to manage and maximise their finances as a desirable and realistic educational outcome. The term 'money sense' could be used for mathematical number sense activities set in a money context.

Definitions of number sense are as varied and as elusive as its qualities. Although a person may display a single example of number sense, this would not be sufficient to say a person possesses number sense in all instances. Table 1 indicates connecting themes in the definitions of number sense such as the word 'flexible' which is common to most definitions.
It should be noted that mental computation ability is a component of number sense and as
claimed elsewhere, number sense may be improved through better use of mental computation
strategies and estimation.

As can be gathered from the definitions in Table 1, number sense relates to observable
actions that demonstrate flexible thinking. Therefore, for each individual to be assessed for
number sense, students should be observed displaying such behaviours in different situational
contexts. Two of the number sense definitions refer specifically to mental computation
(McIntosh 1998; Greeno, 1991).

Table 1: Examples of Number Sense Definitions

<table>
<thead>
<tr>
<th>Source</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sowder (1988)</td>
<td>&quot;...flexible and creative ways to solve problems involving numbers&quot; (p. 183)</td>
</tr>
<tr>
<td>Resnick (1989)</td>
<td>Took the idea of 'higher order thinking' and substituted the term 'number sense' which involves &quot;imposing meaning, finding structure in apparent disorder&quot; (p. 381)</td>
</tr>
<tr>
<td>Greeno (1991)</td>
<td>&quot;...flexible mental computation, numerical estimation and quantitative judgement&quot; (p. 170)</td>
</tr>
<tr>
<td>Sowder (1992)</td>
<td>&quot;...flexible in their thinking and they use a variety of strategies&quot; (p. 375)</td>
</tr>
<tr>
<td>Silver (1994)</td>
<td>&quot;using the relative size of numbers or numerical benchmarks (such as basic facts) to guide quantitative activity are all examples of sense-making actions&quot; (p. 158)</td>
</tr>
<tr>
<td>McIntosh (1998)</td>
<td>&quot;...the ability to compute mentally in flexible ways is both a component and indicator of number sense&quot; (p. 211)</td>
</tr>
</tbody>
</table>

Beyond flexibility, the other elements of number sense that are suggested by McIntosh, Reys
and Reys (1992) included applying flexible thinking through knowledge, and facility with
numbers and operations. This implies the presence of context. Other elements noted by the
authors such as understanding relationships, being aware that multiple strategies exist, and an
inclination to use efficient methods could be observable when demonstrated by students
explaining their methods. For simple computations where the meaning of the operation is
obvious, mental methods should be the most efficient. An example of a mental computation
strategy often called compensation, that would demonstrate these elements, is realising that
adding 100 and subtracting one is a straightforward way to add 99.
1.11.5 Mental Computation

Mental arithmetic is a term for traditional short-answer tests of basic fact knowledge, usually ten or twenty items, often memorized by drill. The emphasis is on speed and accuracy. According to Jones, Kershaw and Sparrow (1994), school mental arithmetic is:

usually short speed and accuracy sessions...which often have little or no relationship to everyday situations (Maier, 1980)...testing rather than learning situations... (p. 13)

This can be distinguished from the broader term of mental computation used today. Interestingly, Thompson (1999) made the following observation:

there is no word for 'mental' in The Netherlands and this leads to their using terms which translate into 'working in your head' (recalling facts) and 'working with your head' (figuring out). (p. 2)

In this way, the former definition can be seen to relate more to mental arithmetic while the latter definition relates more to mental computation. A debate about which and how many basic number facts are needed for mental computation, and should be available to be recalled, is covered in Bastow (1997) and Hoffman (1997).

Mental computation, or the ability to conduct mental numerical calculations without recourse to external assistance (for example, pen and paper), is regarded as a valuable life skill for all Australians (AEC, 1991; McIntosh & Dole 2000b). The importance of mental computation has regained significance as an ultimate goal of mathematics education (McIntosh, Reys, & Reys, 1997a). Today, teachers encourage students to calculate mentally using external means such as fingers if needed, in an attempt to encourage students' use of invented procedures (Kamii, Lewis & Livingston, 1993). This is in preference to using pencil and paper (McIntosh et al 1994; Thomson, 1997). The use of aids is an indicator of the student's developmental level with number. For example, counting strategies are limited in value in the development of place value concepts. For the purpose of this study, mental computation is a calculation that can be worked out 'with your head' without any external aids such as calculators or pencil and paper, but may include fingers.

The renewed emphasis on the oral form of mathematical communication is in response to its relevance to the adult world of mental mathematics rather than traditional school mathematics. Many modern commercial texts are still designed for each student to work individually through a text by reading each question then writing the answer. While many teachers now include 'mental' items set in a realistic context, how many of us use such a
method to work out whether today's petrol is a good buy and whether it is worth filling the tank? This use of text books is not ideal for students who may have reading comprehension problems (Newman, 1977), and it may even encourage confusion with 'written method thinking' by cueing through the use of pencil and paper for recording answers. McIntosh, Reys and Reys (1997b) reported that context—in particular a money context— influences mental computation performance and the thinking strategies employed.

Traditionally, mental computation has comprised oral mathematics, mental arithmetic, mental mathematics and drill. Today, the term 'mental strategies' is used to describe the various methods individuals may choose to solve a mental computation. McIntosh, Reys and Reys (1997a) suggested that a definition of mental computation could be:

...computing an exact answer to a computation 'in the head'. Thus, no external tools, such as calculator or paper or pencil, are used in doing the computation. (p. 322)

However, for very young students some may still be making the transition from using fingers to mental computation. This may or may not be a taught method, as counting on fingers is generally no longer discouraged at school. The use of fingers however, may still lead to miscalculations if one-to-one correspondence is lacking or the counting process for larger numbers is misunderstood. Due to the age of the children involved in this study, it was decided that counting on fingers might be a strategy that some children would use despite the inefficiencies. If so, this would be important data to collect with regard to their overall performance.

1.11.6 Word Problems and Word Items

The difference between word problems, applications and problem solving needs clarification. According to Mason and Davis (1991b), a question is a problem when:

the salient characteristic of a problem (is) that the problem solvers face an unfamiliar task and that they do not know an immediate path to a solution. (p. 3)

This explanation of problem solving is distinguished from explanations of word applications or word problems, where students should already know which operation to use for these computations. Although the answer may not be immediately obvious, the pathway should be. Schoenfeld (1987) claimed that “word problems are applications of operations set in a realistic context” (p. 6). Later, Roth (1996) claimed that “a word problem (item) should be termed contextual if it gives rise to intelligible mathematical practices” (p. 520). Roth (1996) defines these practices as ones where “students can draw on previous lived experience and as
part of an observable practice" (p. 521). The USA explanation could mean that any computation item set in a word format is a word problem. An example of the term ‘word problem’ appeared in NCTM (2000) in ‘understand operations’ of ‘number and operations’ in Principles and Standards for School Mathematics:

...students should consider and discuss different types of problems...if there are 112 people travelling by bus and each bus can hold 28 people, how many buses are needed? (p. 151)

Schroeder and Lester (1989) claimed that there had been “nearly a decade of attempts to make problem solving ‘the focus of school mathematics’ (NCTM 1980)”. (p. 31). They suggested that “instead of making problem solving the focus of mathematics instruction, teachers...should make understanding their focus and goal” (p. 39) and that the two could be mutually supportive. They further claimed that understanding could aid problem solving in four ways. A first way being the monitoring and execution of procedures (such as strategies and algorithms) and a second, the types of representations the problem solver can construct. A third way involved judging the reasonableness of results; and the fourth, promoting the transfer of knowledge to related items, and its generalisability to other situations. They further claimed, “Brownell (1947), among others, has pointed out that a solution to a problem is meaningful (that is, well understood) if it transfers readily to similarly structured problems even if they are different in context” (p. 41).

By contrast, these sorts of items are traditionally known in the UK as ‘mental arithmetic problems’ (The Mathematical Association, 1992a). Australian definitions are more specific. The Australian meaning for problem solving is in line with Mason and Davis (1991b) and requires some systematic exploration of strategies such as ‘draw a diagram’ or ‘make a table’. Undertaking these problems can take time to progress through the steps or heuristics (Polya, 1990), with the emphasis on the process and the answer. Word problems are “puzzles” (Roth, 1996) or simply sentences that have a computation embedded in them. They may not be providing meaningful situations for students. The contextual items constructed for this study are ‘task-context’, intending to be meaningful, and therefore may be considered as applications set in context situations that children should be familiar with. Thus, the items in this study may generally be called ‘word problems’ in the USA or real-life applications stated in words. This should help to contextualise the mathematics because “the mathematics found in real life is always in context” (Meyer et al, 2001).
Jones et al (1994) explained that 'school maths' is mostly written computation with a few mental computation activities. Exercises begin generally with a set of 'sums' displaying only numbers and symbols (the general). Some word items that follow this require the students to use the identical operation to the one taught, for practice. Children do not have to "interpret the questions sensibly" (pp. 12-13). This suggested that apart from the need for schools to teach 'sensible' calculator use (Girling, 1977), there is an imbalance between what computation methods the school is teaching and what people need outside of school. Perhaps those students who are good at mental mathematics and have invented their own ways have done so because they have experienced out-of-school practices.

For the purpose of this study a definition for contextualised items, which distinguishes between the term 'word problems' and items needs explaining. Word problems are not really 'problems' if the student knows how to find the answer. These are generally known as applications. The mental computation items in this study (in context and non-context) have been set in order to assess current knowledge. They have not been set as problems, but rather as applications as it is assumed that students will know what to do. Therefore, both the 'word items' or applications and the 'abstract items' are referred to collectively as 'items' in order to distinguish the context and non-context mental computation component from the money experience questions.

1.12 Summary

This chapter has outlined historical and current issues along with an overview of the current position and recommendation of authorities in the USA, the UK and Australia. Six research questions have been identified along with the significance of the study. Definitions of the major terms of context, understanding, estimation, mental computation, number sense, and word items used in the study have been outlined.

The next chapter contains a comprehensive literature review covering key aspects of this study. These include: the emergence of the Realistic Mathematics Education movement; situated learning theory; and constructivist paradigms as well as contexts as real world settings—what is realistic and what is not. Number sense is overviewed along with understanding. Money as the preferred single context is also discussed by discounting alternatives along with a rationale for increased time and emphasis spent on mental computation methods as well as an outline of mental computation strategies.
Chapter 2: Review of the Literature

This chapter includes a discussion of the importance of task contexts (mental computation items) and situational contexts (environments). Appropriate contexts are discussed in response to existing research on mathematics studies using money and contexts other than money. Various learning theories such as situated learning theory, the Realistic Mathematics Education movement; and constructivist theory are overviewed with their response to context. The relevance of learning styles to mental computation strategies and the development of number sense, as well as other factors such as age and gender performances are also discussed.

2.1 Contextual or Real World Settings

In the previous chapter, the importance of context for improved learning outcomes generally by providing real-world relevance were discussed. Here, context is explored specifically concerning the advantages and disadvantages for mathematics learning outcomes through a review of previous studies.

The term 'context' often has two meanings in the literature, 'task' and 'situational'. In 1999, the international journal Educational Studies in Mathematics dedicated Volume 39 to 'teaching and learning mathematics in context'. In the editorial, Boero (1999) explained that the previous decade had seen a good deal of literature focused on 'situation contexts' (Lave, 1988; Lave & Wenger, 1991). The focus of Volume 39 however, was on 'task contexts', which is also the focus of this study. Boero (1999) made the point that while Wedega (1999) was able to make clear distinctions between the two meanings for 'context', "many studies that regard one or the other actually deal with both" (p. vii).

Nunes et al (1993) use the term 'situation', in relation to the term 'context', to differentiate between two situations where learning may occur, either in a school setting or in an out-of-school setting. The term 'task context' is conceptually different, representing the object to be performed and may be either real or imaginable. The task may be presented either in a context or not and theoretically, and either of these may occur in a school or out-of-school setting. Usually, non-context tasks such as some mental computation items (six sevens?) are presented to the learner in school settings, while task contexts are more representative of out-of-school settings. Task contexts are increasingly found in school settings as word items such as, If I had a dollar and spent 55 cents how much do I have left?, while non-context
tasks found in out-of-school settings are non-existent. An exception might be the example of Korean-American parents who were found to be supporting school-taught methods at home, although arguably, this is a simulated school setting (Guberman, 1992). Table 2 illustrates the relationship of the two contexts from Nunes et al’s (1993) studies.

### Table 2: Task and Situational Contexts

<table>
<thead>
<tr>
<th>Task Context</th>
<th>Situational Context</th>
<th>Non-Situational Context</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Simulated shop at school</td>
<td>Word items</td>
</tr>
<tr>
<td>No</td>
<td></td>
<td>Straight-forward computation</td>
</tr>
</tbody>
</table>


In Nunes et al’s (1993) studies of money as a task context, the authors also test for situational context by using a simulated shop at school, word items, and straight-forward computation. This study was a follow-up to previous studies on street vendors (Carraher et al, 1985, 1987) and found that the situational setting of the school had a stronger influence than the task contexts. The authors found that students scored better for simulated shop items than for the word problem items. The authors also found that students scored better for word items of ‘applications’ than for straightforward computational problems, suggesting that context made a difference for the tasks.

Classically, Carraher et al’s (1985) 10-year Brazilian research program is the most significant research regarding students’ use of mental computation strategies to solve real world problems. It was originally published as Na vida dez, na escola zero (for which a translation is ‘Real-life—ten, school—zero’). In 1993, Nunes et al published the study in *Street Mathematics and School Mathematics*. The study’s focus centered on nine- to 15-year-olds that worked out-of-school hours selling goods in street markets. The Carraher et al studies (1985, 1987) compared street mathematics—which being of the ‘real world’ was mostly oral and with much of the meaning preserved—with school mathematics, which was mostly written and strived for generality. They found that:

Still, it was possible to notice that the children kept the meaning of the problem in mind while solving problems in the mental mode and seemed to forget...in the written mode. This was most noticeable in problems with multiplication and division. (p. 48)
The benefits of context in mathematics teaching and learning have been recognised by movements internationally. They include the Genoa group, which is a ‘movement’ grounded in theory and practice, and has conducted research in mathematics education in a wide range of contexts from primary through to secondary school (Dapporto & Parenti, 1999). A second international movement, the Realistic Mathematics Education Movement (RME) in the Netherlands has adopted the name ‘Freudenthal Institute’ both in honour of its founder Hans Freudenthal and to celebrate the first decade of its existence (Streefland, 1991). This movement promotes the use of context, and as Van den Heuvel-Panhuizen (1999) stated, “context problems are so appropriate for providing indications for further instruction” (p. 133). She suggested three strengths of context problems including “contexts can provide strategies” as students may be inspired by the situation, and “contexts contribute to the latitude and transparency of the problems” by encouraging students to use informal methods (p. 136). The third strength the author listed was the motivational element provided by pleasant contexts. Appropriate contexts should be able to maximise these strengths. While the RME movement has not suggested that one context suits all, it has recommended the use of money, among others.

In Burns’ (1993) article, ‘The 12 most important things you can do to be a better math teacher’, her fifth suggestion is to embed math activities in contexts (p. 30). She stated that the connectedness to real life that context provides brings mathematics alive, stimulates student interest and is purposeful. She claimed to value both real-life and imaginary contexts such as may arise from children’s literature.

Along with the strengths of providing context, several authors note some difficulties. With regard to estimation, Sowder (1992) stated that context “can help in some cases”... However, [context] “can also make problems more difficult” (p. 374). Real contexts are often situationally specific to individuals and therefore are not always simple. Comments by Sparrow (2000) made to the researcher suggested that by removing real contexts and possibly making mental computation straight-forward and clinical, to fit the definition, we actually move from ‘real’ to ‘textbook real’. Roth (1996) suggested the term ‘phenomenal world’ to describe the world-of-our-experience rather than the term ‘real-world’ used by NCTM, since school is also real. Sowder (1992) warned that the use of unfamiliar words
might make problems set in context more difficult. Apart from this, Sowder (1992) stated that "it seems reasonable to expect that any computation or estimation problem set within a context will make the problem more comprehensible to most students" (p. 374).

Van den Heuvel-Panhuizen (1999) also suggested two difficulties with the provision of appropriate contexts. Firstly, students may ignore the text if their previous experience is not grounded in word applications embedded in context. This is more likely to occur if the student has had little money experience compared to having had lots of school standard written methods experience. Secondly, the problem may be rejected for not being "realistic enough" to the learner (p. 137). One instance of how contexts can encourage the use of "clever strategies" is given in an illustration comparing two boys' heights (145 cm and 138 cm) that is devoid of any formal operation signs. The author claimed that "a lot of children who cannot solve the 'bare' problem (non-context item) can solve this context problem" and suggested that this was because the context encouraged students to use a complementary addition strategy rather than subtraction by decomposition (van den Heuvel-Panhuizen, 1999, p. 133).

Hughes et al (2000) believed that while "application is at the heart of numeracy", they acknowledged that "people frequently have difficulty applying mathematical knowledge acquired in one context to problems posed in another" (p. 1). The authors discussed issues surrounding the problematic nature of application in mathematics, described as complex. These authors also compared theory with practice in England and Japan and concluded that children need to 'explain their reasoning' as required by the Numeracy Framework. They reported that in Japan, "the old Nuffield dictum of, 'I do and I understand' has been replaced with 'I explain and I understand'" (p. 113).

Current mathematics curricula recommend the provision of context. For example, in Western Australia the Curriculum Framework (Curriculum Council, 1998) has introduced a sixth strand, 'Working Mathematically' that pervades all other mathematics strands, including 'Number'. Three strands from the Curriculum Framework are relevant to this study, namely: Number, Working Mathematically, and Appreciate Mathematics. As mentioned in the section on 'understanding' with respect to number sense, two of the Number substrands are 'Understand Numbers' and 'Understand Operations'. The other two Number sub-strings are 'Calculate' which encompasses mental strategies and 'Reason about Number Patterns' which encompasses children's explanations of strategies. The Working Mathematically strand also consists of four sub-strands. Two of these sub-strands are important to this study regarding
the effect of context. One outcome from level one in ‘Contextualise Mathematics’ is that students should “explain...that the numbers enable one to exchange coins, e.g. replacing two 5 cent coins by one 10 cent coin” (p. 25). This provides a link between relational knowledge and context via everyday knowledge. ‘Mathematical Strategies’ suggests the use of a “familiar context” (p. 34) and “prices” (p. 35). The other two substands, ‘Reason Mathematically’ and ‘Apply and Verify’ seem designed to develop number sense.

The ‘Appreciating Mathematics’ strand of the Education Department of Western Australia’s (EDWA’s) (1998) Outcomes and Standards Framework: Student Outcome Statements—Mathematics Learning Area, promotes the use of context. Increasingly, students need to explain the influencing aspects of mathematics on their daily lives. Money is one context that most people use everyday (Plunkett, 1979), therefore money should be a familiar context for all students.

2.2 Contexts other than Money

Table 3 shows an overview of context topics chosen by other researchers which indicates that while there have been several different possible contexts studied, the significant common choice for all previously mentioned research was money. One reason for popularity of money may be that it is based on the decimal system. Another reason may be that it is a familiar, common context for all students. Jones et al (1994) claimed that “...context-embedded problems were more easily solved and invented mental strategies were chosen and used successfully...” (p. 22). A study of 11-year-olds in the UK by Shuard (1986) reported that children were “far more successful” finding a 17 percent improvement for a length measurement problem involving the addition of fractions when “presented in a practical situation...they did not use standard algorithms” (Jones et al, 1994, p. 22).

Sullivan, Zevenbergen and Mousley (2002) discussed the need for care when choosing contexts that are suitable, interesting and relevant, in view of some students’ socially and culturally diverse backgrounds in order not to alienate them from the invisible pedagogy. For example, contexts need to be inclusive of gender, culture and race. Some culturally positive suggestions were sports related: netball, soccer or football teams. However, it was also decided that these topics might become too sensitive if comparing heights and weights of athletes with the students’ heights and weights. Finally the authors suggested that contexts selected need to avoid being “alienating, excluding or exacerbating of disadvantage” (p. 656). Given this advice, money may be an emotive and sensitive topic, especially when asking
questions regarding personal finances. However, most adults consider being able to calculate with money—such as to give and receive the correct amount of change—to be an essential life-skill.

Table 3: Review of Researchers and Context Studies

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Money</th>
<th>Time</th>
<th>Space</th>
<th>Food</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harris (1991) study of Aboriginals' use of mathematics in contexts.</td>
<td>*</td>
<td>*</td>
<td>*</td>
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</tr>
<tr>
<td>Dapueto and Parenti (1999) calendars and money, primary schools study.</td>
<td>*</td>
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<tr>
<td>Hughes et al (1999).</td>
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<tr>
<td>McIntosh et al (1999) survey of adults' daily calculations.</td>
<td>*</td>
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<td></td>
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<td></td>
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<tr>
<td>Lowrie and Owens (2000).</td>
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<td>Irwin (2001) study of foreign exchange rates (decimals to more than two decimal places).</td>
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2.3 Money as a Context

The previous chapter outlined why money was chosen for the context of this research. Here previous research is reviewed along with the advantages and disadvantages of using money as a context.

Brown (2001) describes how, historically, before 1858 few students attending elementary school were taught arithmetic until charitable schools for the working class introduced the skills needed for personal control of money as an aspect of moral education (p. 38). It was also considered that as future shop assistants, bookkeepers, technicians and artisans, the educational requirements for these students were for accurate calculation with numbers of items, money and common measures...adding, subtracting and multiplying.

Money experiences are invaluable for students because they are often one-on-one, hands-on, with immediate responses as to whether change given is correct, as it is in both parties'
interests to safeguard their investment in the transaction. It is a common parental wish (Guberman, 1992) for their children to eventually become financially independent. To do so requires competence with money. This road to competence usually begins with the provision of pocket money and some discretionary spending. Although the amounts vary, most Year 3 children in Australia would be recipients. The quote from Charles Dickens' novel *David Copperfield* (Oxford University Press, 1979), as made by Mr. Wilkins Micawber illustrates the importance of careful money management:

Annual income twenty pounds, annual expenditure nineteen nineteen six, result happiness. Annual income twenty pounds, annual expenditure twenty ought and six, result misery. (The Oxford Dictionary of Quotations, 1979, p. 177)

The drive for pay-rises, along with better working conditions seems to be a historical-cultural one, as most Anglo-Australians experience better living conditions than our grandparents did. Harris (1991) outlines that while the drive to plan for a financially comfortable future is common to Anglo-Australian culture, it is not the case in Aboriginal-Australian culture.

Dehaene (1997) discussed the 'Right Start' program which "stresses concrete, practical and intuitive mental models of arithmetic...to enable children to relate the world of numbers to the world of quantity" (p. 142). Previously, money has been placed with measurement topics in mathematics curricula documents (Education Department of Victoria, 1981). Money differs from measurement, which is continuous and therefore approximate; whereas money can be either approximate or exact depending on the situation. Money is currently placed within the number strand of most mathematics curricula. It differs from measurement context topics because it is a measure of discrete quantity.

Haylock (2001) explained the fundamental structures of addition, subtraction, multiplication and division for primary teachers. For every instance of every structure, he suggested contexts where they might be found. For most cases, money was the primary example. For example, with 'aggregation' addition, the instance of adding two purchases or more was noted. For 'augmentation' addition, "the most important and relevant context...is again that of money, particularly the idea of increases in price or cost, wage or salary" (Haylock, 2001, p. 29). While Haylock gave other examples from measurement (mass, temperatures, time), for most structures, money was by far the most relevant.

According to Dapporto and Parenti (1999), Boero's (1999) concept of the 'field of experience' can relate to contexts in different ways (p. 9). The authors outlined how to choose and deal with contexts with reference to money and calendar 'fields of experience' used in their
primary school project. They claimed teachers worked with students in the money-context with appropriate activities to focus on and transfer conceptual knowledge explicitly to numerical problems concerning the calendar context and vice versa (p. 11). The two fields of money and calendar were chosen for three reasons. The first reason was due to their historical significance in time and commercial trading. The second reason was due to the availability of objects associated by function to these contexts, such as coins, notes, calendars and watches. The third reason was that these contexts are common to out-of-school environments, such as work, play or sport.

Ruthven (2001) warned that while gaining familiarity with monetary contexts, English students often showed:

> too literal a treatment (which) risks encouraging a view of the decimal point as a 'separator' within a system of super- and sub-ordinate units such as pounds-and-pence... (p. 181)

This could equally be applicable in Australia and other countries with similar decimal monetary systems. In New Zealand, Irwin (2001) conducted a study of lower socio-economic area students who were aged 11 and 12 to investigate the role of students' everyday knowledge of decimals on enhanced understanding. Irwin's study found that "students who worked on contextual problems made significantly more progress" and that "less able students more commonly took advantage of their everyday knowledge of decimals". Irwin's study differs from the proposed study, in that it involved only decimals, a limited age and collaborative learning. However, it may also be that students use their everyday knowledge of money as a context beyond decimals. If so, students should achieve improved performance for mental computation items that are set in a money context. Booker, Bond, Briggs and Davey (1998) emphasised the importance of acknowledging students' everyday knowledge of money as follows:

> Money transactions also provide appropriate opportunities for children to explore and develop alternative computational algorithms, for example the 'making change' algorithm for subtraction and the use of various estimation strategies...these experiences are very relevant in the development of good number sense. (p. 348)

This statement suggests that shop-keepers' addition, or complementary addition, is good training as a method of checking where an exact answer is needed, as it involves the inverse of the subtraction. Students' familiarity with decimals in everyday money contexts may help to develop number sense, as the meaning of the operations and the relationship between them
become evident. Haylock (2001) claimed that "the most common calculation we have to do
with percentage is to find a percentage of a given quantity, particularly in the context of
money" (p. 170).

Moloney and Stacey (1997) reported that "some students acquire skills in the use of
operations on decimals without understanding the comparative size of the numbers involved"
and their results showed that "many students have misconceptions that remain with them
even to year 10" (p. 25). A study by Stacey, Helme, Steinle, Baturu, Irwin and Bana (2001)
found that "a significant proportion of preservice teachers have inadequate content
knowledge of decimals" (p. 205). They also claimed that teachers "underestimation of the
extent to which students use intuitive rather than rule-based or analytical thinking" highlights
the need for "teaching that is based on accurate knowledge of students' difficulties and ways
of thinking" (p. 207). Steinle and Stacey (1998) tested students in grades 5 to 10 and found
ten incorrect ways of thinking about decimal notation. One way, 'apparent-expert, truncation
thinking' they suggest may be a result of the use of contexts such as "money or length
(m, cm) in order to make sense of decimal notation" (p. 36). They claimed one important
finding was that "the group of students who have knowledge of only the first one or two
decimal places was found to be at least 3.7 % and probably twice this size" (p. 41).

In the USA, Lave, Murnaugh and de la Rocha (1984) provided an analysis of data on
supermarket arithmetic cognition of adults—an activity in context (in a supermarket). As
mentioned previously, the term 'context' here referred to setting. The supermarket was
chosen as being a routine activity and therefore 'unproblematic'. They recommended the
value in "analyzing both the context of the activity and the activity in context" (p. 93).

According to McIntosh, Reys and Reys (1997b), "the context in which mathematical
problems are encountered influences a student's thinking" (p. viii). The authors used a
money example to illustrate this. The authors stated that a student shopping for two items
costing close to 25 cents would recognise that $5.14 is not correct. The student would use the
relative size of numbers to estimate a total of between 50 cents and one dollar. Consequently,
the authors claimed the student is more likely to check the reasonableness of the answer
because of the student's own personal stakes. They suggested that this does not happen for
'learned algorithms' as students perform these "without much thought" and when challenged,
students "often...recalculate—generally using the same method as before..." (p. viii). This
example illustrates how a computation item devoid of context is more likely to elicit the use
of a written method. However, a computation item set in a context—in particular a shopping context—may encourage the use of self-invented mental methods that have developed from out-of-school experiences (Nunes et al., 1993).

With regard to mental mathematics performance with money calculations, the degree and type of prior money experiences a student may have had needs to be considered. Each student's out-of-school experience may vary considerably depending on parental values, as was seen in Guberman's (1992) study. In addition, school experiences will vary between teachers and schools, depending on their pedagogical approaches.

2.4 Learning Theories

2.4.1 Situated Learning

So far, the nature of both 'task' and 'situational' contexts has been discussed in relation to mental computation. Another dimension of situational context is known as 'situated learning', 'situated cognition' or 'social practice'. This theory of learning as discussed in Hughes et al. (2000) refers to social aspects, such as 'communities of knowledge', which encompass both the task and the environmental settings to explain the differences between 'authentic working practices'. This theory emphasises the importance of the 'social' context in the learning process. For example, with regard to mental computation, tasks usually performed by students in the school setting make use of school-taught methods whereas tasks performed out-of-school make use of informal methods, often self-devised (Nunes et al., 1993). According to Hughes et al. (2000), situated learning theorists explain the lack of transfer between working environments as being due to the nature of knowledge between cultures being so different and requiring 'far transfer' or 'far application'. The authors suggested that situated cognition theorists claim that mathematics can be learnt through students thinking like mathematicians and working mathematically, doing "mathematics as it is practised by mathematicians" (p. 108).

The relationship between the students and their 'significant others', be they teachers or parents, needs to be taken into account as a factor that may affect performance. Lave and Wenger (1991) outlined and discussed differences in apprenticeship styles within working practices. Relationships between students and adults will be different according to whether the student is attending school or in an out-of-school environment. First, there is the relationship dynamic between the student and their teachers and/or peers to consider when at school, as well as the teachers' own pedagogical teaching styles and beliefs. Second, there is
the out-of-school relationship between the students and their family members or others to consider. Some parents consider teaching their children life skills, especially money, to be so important that education starts at home (Guberman, 1992; Hughes et al, 2000).

'Legitimate Peripheral Participation', is the defining process in a situated learning activity (Lave & Wenger 1991). This perspective of learning has moved beyond 'learning by doing' to learning as social practice by providing 'a conceptual bridge'. The authors studied five examples of apprenticeship from different cultural and historical traditions. The five apprenticeships included: Mayan Midwives, Via and Gola Tailors, Supermarket Butchers, and Alcoholics. It may be argued that the Yucatec Mayan midwives in Mexico most closely resemble the way children informally learn about money from their parents. The midwives were always daughters of experienced midwives, suggesting that gender relationships in families and child-parent relationships may be important out-of-school factors. The least successful apprenticeship schemes were clearly the supermarket butchers. The apprentices' alienation from their masters caused by the delineation of their duties and structure of the supermarket may more closely resemble the way traditional school mathematics, particularly at secondary school level is taught. As Lave and Wenger claimed, the reason the butchers' model did not work was that "union-based 'apprenticeship' programs implicitly reject an apprenticeship model and strive to approximate the didactic mode of schooling" (p. 77).

From Lave and Wenger's (1991) perspective, social learning theory involves learners' participation in 'communities of practice'. The authors further claimed that as "apprenticeship happens as a way of, and in the course of, daily life, it may not be recognized as a teaching effort at all" (p. 68). Mayan midwives for example, absorbed the essence of practice as well as the knowledge (Jordan, 1989). Midwives' apprenticeships were always with a family member and always informal. In contrast, the nature of the butchers' instructional model was formal and could be compared to traditional school models. In summary, Lave and Wenger (1991) suggested that "children are...quintessentially legitimate peripheral participants in adult social worlds" (p. 32). This historical-cultural theory of learning involves the whole person situated in a context; therefore, we need to acknowledge all out-of-school experiences as prior knowledge.

When applied to mathematics education, Lave and Wenger (1991) explained that according to social learning theory, school mathematics often results in general and abstract methods (for example, $70 + 20 = ?$). Whereas the world is concrete and particular (for example, James
had 70 cents then was given 20 cents, how much does he have now?). Lave and Wenger (1991) suggested that participation in social practice is the fundamental form of learning, although possible conflicts involved with the sustained participation of newcomers can be problematic:

Learning is never simply a process of transformation or assimilation (as assumed by traditional teacher directed-passive learner methods), learning, transformation and change are always implicated in one another. (p. 57)

Nunez, Edwards and Matos (1999) discussed the "embodied cognition" perspective (prior knowledge) to support Lave's (1991) "situated cognition perspective" that leaves open important questions such as, "What is the basis for social situatedness?" A "cultural" interpretation views Vygotsky's (1978) "zone of proximal development" as the distance between the cultural knowledge provided by the socio-historical context—usually made accessible through instruction—and the everyday experience of individuals (Davydov & Markova, 1983). Hedegaard (1988) called this the distance between understood knowledge, as provided by instruction, and active knowledge, as owned by the individual (p. 48).

Currently emerging theories, such as activity theory or the "collectivist" perspective, take into account the conflicting nature of social practice and the broader structure of the social world.

Rogoff (1984) claimed that, "increasingly, psychologists emphasize the role of context in cognitive activities" (p. 1). Social contexts are sometimes termed culture (Rogoff, 1984); an example of which is the school environment. This is similar to situational contexts mentioned previously. Rogoff further explained that "laboratory context...is not context-free as researchers frequently assume" (p. 3). This suggests that research carried out at school, with students removed from the routines and expectations of their classroom and peers may still exhibit school thinking. Students may bring this thought "training" to their school-based interviews. As interviews are by their nature partly formal—an example of a laboratory context—to conduct them out-of-school may or may not reveal different results to interviews conducted at school.

Participation in our social world is hard to imagine without money changing hands. Adults need to manage money in a practical sense, such as purchasing goods and paying bills. Adults work for money to save for future needs, spend on current needs, donate to others in greater need, and most people have debt to repay in the form of mortgages or loans. Money is both necessary for life's needs and an item of pleasure. Both adults and children spend money on items such as entertainment, holidays, presents, food and clothes. While parents
pay for the essential needs, children often pay for the extras such as lollies, drinks and small toys. The pressure to spend on goods through advertising confronts both adults and children daily on television, radio and in the print media. Children in Australia are able to obtain part-time paid work from age 15 and some children do odd jobs for extra pocket money even before that.

Nunes et al (1993) discussed how systems of knowledge learned in everyday life, like measurement and money systems, correspond at a more advanced level to the schemas of the sensori-motor period. They include abstract logico-mathematical relations and lived-through situations in their representation. Knowledge of a monetary system used in everyday life includes knowledge of logico-mathematical principles of units, additive composition of totals, and so on (p. 139).

The Nunes et al (1993) study researched a wide range of adult occupations, with many of the adults having little education. For example, they found in a comparison between farmers and students with five years of schooling, that errors by farmers for oral representations were within a reasonable range. Further, that questions as to which operation to use were unusual in oral problem solving. In a study regarding directed numbers, Nunes et al (1993) found that:

Some students who arrived at the wrong answer in the written condition were able to provide the correct answer immediately afterward when asked to explain their procedure. (p. 146)

Both flexibility and transfer were more clearly demonstrated in everyday practices than for the school-taught proportion algorithm in Nunes et al’s (1993) fishermen study. Their results indicated that transfer might happen informally. For example, Nunes et al (1993) claimed that when solving proportions problems about agricultural variables, fishermen:

do not display knowledge that is so content bound that no transfer is possible. They clearly showed their ability to transfer the model of the weight-price relation to other variables in the fishing context and to similar variables in the new problem context of agriculture. (p. 120)

This suggests that fishermen used their everyday mathematics practices in a conceptual, rather than just a procedural way. The authors concluded by recommending that mathematics teaching should seek its inspiration in street mathematics.

When Carraher et al (1987) compared oral with written procedures by ‘situation context’ they used a simulated store, word problems, and computation. They found that “the oral procedure was significantly superior to the written procedure at the 0.002 level” (p. 89).
With regard to 'situation context', the researchers found that "overall superior performance (occurred) in the store and word problem situations" (p. 88). This suggests a comparison study of word problems or applications and identically matched computation items—both presented orally—could prove revealing. Guberman's (1992) study on mathematics and money found that Latino American students out-performed Korean American students for mathematics items in a money-context, while the reverse was found for non-money items. His research covered Years 1-3 and found that mathematical experiences at home and parental values or cultural differences were critical factors in student mathematical achievement. Latino-American students performed more successfully for money-context items because the students' parents valued day-to-day money competence and encouraged this knowledge by actively setting and supervising tasks to develop this.

Guberman’s (1992) findings are complemented by Saxe’s (1991) study of Brazilian candy sellers. Saxe compared candy sellers (usually schooled to grade 1 or 2 only) with non-sellers (identical in age and schooled children). Saxe found that:

...while sellers used more appropriate regrouping strategies across both computational and word problems than the (non-) sellers, their use of regrouping strategies was more frequent on word problems, and their use of school-linked algorithmic strategies was more frequent on computational problems. Similarly, while nonsellers used more algorithmic than regrouping strategies, non-sellers' use of regrouping strategies was more frequent on word, as contrasted with computational problems. (p. 171)

Saxe's earlier work (1982) studied a Papua New Guinean highlands group, the Oksapmin, who used a 27-body-part number system. Saxe (1982) found that their "approach to solving mathematical problems of measurement and numeration involved very different ways of thinking and very different procedures for accomplishing everyday problems" (p. 1). It is probable that the term 'regrouping strategies' indicated more sophisticated thinking, and can therefore be seen as using number sense. So where Saxe's study has shown strategy use is differentiated between word problems and symbolic computational problems for both sellers and non-sellers, it suggested that word problems rather than symbolic computational problems may promote number sense. Saxe (1991) stated: "Further, the analysis of children's strategies revealed that a source of sellers' success was their specialized knowledge of re-grouping" (p. 172) and:

With increasing school experience, sellers' strategies increasingly incorporated the use of a multiplication algorithm and shifted to the use of single unit pricing. (p. 161)
Saxe also noted that “by grade 3, nonsellers...made considerable progress in specializing adequate strategies...both adequate regrouping and algorithmic forms” (p. 173). This led to the question of how much difference in strategy use there would be for older students, and how much any such difference may be determined by problem type and prior experiences with money. In an Australian setting, where schooling is compulsory, students’ prior money experiences would need to found by way of questioning. However, it is anticipated that all children will be deemed similar to the non-sellers in the above study.

One reason that both number sense and mental computation improve with age (McIntosh et al, 1997c) might be partly due to the impact of schooling. A second reason might be students’ increasing use of mental computation outside of school, such as the use of mental methods for sport scores, and with money contexts—especially shopping. Out-of-school applications are always in-context. Thus, as a student grows older, the range and number of their mathematics-in-context experiences also increase. In addition, as the amount of students’ pocket money increases, usually with age, there may be a corresponding increase in their responsibility for managing it.

2.4.2 Constructivism

A second theory of learning, constructivist theory, views learning as a process (Cobb, 1995) “of active individual construction” (p. 364). This occurs when the learner creates new learning based on prior understandings when engaging in a new activity or ‘thinking’. By contrast, sociocultural theory (Vygotsky, 1978) views learning “as a process of enculturation into established mathematical practices” (Cobb, 1995, p. 364). These views are particularly relevant to the use of money as a context, and its importance as a cultural tool, and therefore both perspectives should be considered. Cobb (1995) claimed the relevance for both views was: socioculturally, “the influence that mastery...has on individual thought”; and constructivist, “the individual...learning to use...[it] appropriately” (p. 380). The developmental epistemologist, Piaget (1978), has inspired constructivist theories, as has Von Glasersfeld (1987) who described how children construct their own understandings. Also, Cobb (1995) who explained how “analysis...focuses on...the individual conceptual constructions in learning to use a cultural tool appropriately” (p. 380).

Cobb (1995) argued that the complementary nature of these two perspectives is such that they “encompass the actively cognizant student, the local situation of development, and the established mathematical practices of the wider community” (p. 380). Therefore, rather than
these theories being in opposition, they may co-exist to explain that 'active individual construction' is most likely to have maximum impact during sociocultural Vygotsky's (1978) 'zone of proximal development' or Piaget's (1978) 'cognitive conflict'.

Constructivist philosophy (von Glasersfeld, 1987; Cobb, Wood, & Yackel, 1992) has been recommended across the curriculum areas including mathematics. Mathematics has arguably been the learning area of most apprehension and least self-confidence for many primary school teachers. This lack of confidence may have led to teachers reverting to teaching as they were taught or to an over-reliance on textbooks, which also often promote traditional teaching approaches. Irwin (2001) stated that “textbooks are not routinely used in New Zealand elementary schools in the hope that teachers will tie mathematics to students' everyday experience” (p. 400). Sowder (1992) discussed how context could aid constructivism by providing adaptive expert experiences in preference to routine or procedural expert experiences. Sowder (1992) also explained that this provision of context allows the learner to mentally re-organize their cognitive constructs of the subject matter, a similar process happens when cognitive conflict occurs.

Current learning theory (as evidenced by modern curriculum documents) supports the provision of contexts and constructivist theory by “characterizing mathematical learning as a process of conceptual reorganization” (Cobb, 1995, p. 364).

An example of a constructivist's approach to teaching mental computation methods might be the fostering of self-generated or invented strategies. The use of open questioning by teachers or variations on the Cornwall (1993) Key Stage 2 Task Group's questions as recommended for problem solving could also be used for mental computation. For example, “What do you already know that might help?” or “If 18 is the answer, what might the question be?” Students of all abilities are able to answer this question at a variety of levels. The teacher can then group the answers according to relationships he or she wishes to emphasise. This method contrasts with the traditional transmission approach (Thorndike, 1913) or associationism (Hughes et al, 2000) where a structured learning sequence of associations is 'over-learnt' through repeated practice.

2.4.3 Transfer and the Learning Theories

Hughes, Desforges and Mitchell (1999) reported on different teachers' interpretations of applications as 'authentic activities'. Some used imagined real-life contexts, while others used applying knowledge to a 'near' real-life setting. The authors included 'Alice' as an
example of a classroom teacher who set up a practical activity (a car-boot sale) to simulate a mathematical experience close to everyday life. Not only did children find this very motivating; they also had to keep track of their spending. This example of a real-life money context was one of three discussed by the authors regarding transfer between contexts. Students could also be taken on shopping excursions to experience money activities in some defined way.

Hughes et al (2000) discussed the three learning theories of associationism, constructivism and situational theory along with the notions of far transfer and near transfer, to explain how the similarity of situational contexts may increase the likelihood of transfer. The authors claimed that according to associationist theory, more transfer occurs than should happen, while according to constructivist theory, less transfer occurs than should happen. Constructivism allows for differences of intelligence to make different connections and progress at different rates, whereas situational cognitivists believe that the similarity of contexts is important. Perhaps, therefore, associationist theorists may not have allowed for differences in individual intellect, whereas constructivist theorists believe that individuals strive to make connections beyond the immediate information that they have. Perhaps also, constructivist theorists may have discounted the effect of the dissimilarities of situational contexts.

Nunes et al's (1993) study of Brazilian children found that the children had "trouble transferring their street knowledge to the school test" (Hughes et al, 2000, p. 9). In contrast, Säljö and Wyndhamn (1990) reported on a study of Swedish children finding the cost of posting a letter. The study found that the children had "trouble applying their school knowledge to the 'street' problem" (p. 9). Hughes et al (2000) explained this as a problem of application from one contextual setting to another. Situational theorists believe that transfer of applications is problematic because of the differences between contexts. This is where good teaching needs to provide connections between the contexts.

Parents as Teachers

We cannot ignore the influence that parents and families have on their children in either the informal or the formalised way. We cannot ignore other exposures students may either have to money at school or outside of school, and therefore we need to ask questions that might give us some further information. This is in order to check for 'apprenticeship' influences of a parent or other close relative, similar to Lave and Wenger's (1991) tailor apprentices where
"the master is a close relative of the apprentice" (p. 66). There were five apprenticeships; midwives, tailors, quartermasters, butchers and alcoholics. To which the authors claimed that the first three as well as the last studied "are quite effective forms of learning; but the fourth— butchers' apprenticeship in contemporary supermarkets—often doesn't work" (p. 65). This indicated the importance of the style of teacher-student relationships as well as parent-student relationships on educational outcomes. As can be seen in this study of apprentices, parents and teachers are best placed to utilise Wood, Bruner and Ross's (1976) term 'scaffolding' to facilitate teaching moments similar to Vygotsky's (1978) 'zone of proximal development' because learning cannot be considered a non-problematic process.

As was found in Guberman's (1992) study, students were more likely to be successful at school when parental values and lifestyle mirror school values and life. Parental influence in a money context was mentioned earlier.

2.4.4 Realistic Mathematics Education

Hans Freudenthal was the founder of realistic mathematics education and after whom the Freudenthal institute is aptly named. He is remembered for his "Socratic method...to help students with re-inventing and reflection to follow, which Freudenthal called 'guided re-invention' " (Goffee, 1993, pp. 40-41). The three heuristics noted by Gravemeijer (2001) include: guided reinvention through progressive mathematisation, didactical phenomenological analysis, and emergent models [original Italics] (p. 155). With regard to the first heuristic, Gravemeijer suggested that "one needs to find contextual problems that allow for a wide variety of solution procedures" (p. 155). With regard to the second heuristic, mathematics evolved from solving practical problems; looking for "applicability and progression towards mathematisation" (Gravemeijer, 2001, p. 156). With regard to the third heuristic, Gravemeijer suggested that models used were "context-specific then became general" so that this might lead to reasoning (p. 157).

The Dutch RME reform movement is termed 'realistic'. This is not for its connection with the real world, but as van den Heuvel-Panhuizen (2001) explained, of "the emphasis that RME puts on offering the students problem situations that they can imagine" (p. 50). The "Dutch translation of 'to imagine' is zich REALISeren" (van den Heuvel-Panhuizen, 2001, p. 50). Therefore, the term 'realistic contexts' for this study aimed to create familiar situations that the children were likely to be able to 'imagine' or have experienced. Due to the six-year age
difference from 8-14 years from Year 3 through to Year 9 students, it was more important for the Year 3 students to use their imagination as they had far less of life’s experiences.

Van den Heuvel-Panhuizen (2001) stated that goals for education should address “discontinuities in the learning process” and that “understanding and skill performance are determined by the context and differ between individuals” (p. 59). Regarding money, Brown (2001), in the Dutch Key Goals of Primary School Mathematics, No. 17, under the heading ‘Measurement’, recommended that “the students can do calculations with money in daily-life context situations” (p. 58).

Treffers’ five RME learning and teaching principles were outlined by Treffers and Beishuizen (2000). The first of these principles was “learning as a constructive activity” and can be seen to support constructivist theory, while the second principle supported the “use of context problems” (p. 34). According to Treffers (1993), “realistic learning strands start with the informal context bound working methods of children, in their personal reality” (p. 102).

Beishuizen (2001) claimed that the RME view stated that “reference to real-world contextual situations should be used first to give numbers a concrete meaning for children” (p. 129). This is in an attempt to develop mental imagery. Anghileri (2001), and Beishuizen and Anghileri (1998) compared the traditional UK approach with Dutch approaches and found favourably for the Dutch with the emphasis on non-standard written methods, based on mental methods.

A similar model can be adapted to explain growth in number sense flexibility. Increased number sense flexibility may be seen as diagonal progression on an X-Y graph. Progression occurs vertically with the acquisition of higher-order strategies and horizontally through exposure to different ‘realistic’ contexts that allow for successful transfer. Increased practice in mental computation set in a variety of ‘realistic’ settings should then result in diagonal progression or increased flexibility.

2.5 Mental Computation

Of the three methods of computation (mental, written and calculator) taught in schools, the Cockcroft Report (1982) recommended that “there should be more ‘mental mathematics’ throughout the primary years” and that “all children should develop mental methods of calculation. These are likely to differ from written methods that are taught” (p. 7).
Reys et al (1995) concurred that too much emphasis has been placed on teaching written algorithms at the expense of mental computation, computational estimation, and calculator methods. They indicated that only five percent of adults' time was spent on written methods compared to 80 percent of adults' time being spent on mental methods. Schools were found to be spending 70 percent of their time on written methods, while only 10 percent of their time was spent on mental computation and ten percent on calculator methods. Different approaches are discussed by McIntosh, Nohda, Reys and Reys (1995b):

There are at least three different instructional approaches currently apparent in elementary classrooms. The first is to view mental computation as a 'topic' to be delineated into identifiable strategies that are directly presented to students. This approach is similar to the traditional teaching of pencil and paper computation algorithms. (p. 238)

This approach does not suit constructivist theory, as flexibility—possibly the most significant feature of number sense—would then be lost and may result in rule-based thinking. One example of this approach is the teaching of the 'remove the zero' rule that McIntosh (1996) claimed is "constantly misunderstood and misused" by students who do not fully understand it (p. 62). The second approach highlighted by McIntosh et al (1995b) however, matches constructivist paradigms:

A second approach for mental computation is constructivist. Students are encouraged to generate thinking strategies based on their prior experience and knowledge...some students can formulate and use a variety of strategies, both elegant and not so elegant...the likelihood of their making use of and valuing such self-generated strategies seems to be closely tied to their notion of what school is about, and in particular what mathematics is about. (p. 239)

Here the term 'thinking strategies' suggests higher order thinking skills described by Resnick (1989) or 'number sense' discussed by Sowder (1992). With this approach, students are encouraged to be creative, share, and compare their methods. For these reasons, the National Numeracy Strategy in the United Kingdom has adopted this method. The third approach to mental computation described by McIntosh et al (1995b) is as follows:

Students are taught standard written methods for computing and must extrapolate from such experiences to compute mentally. No explicit instructional attention is given to mental computation. This approach often results in students performing mental computation by applying inefficient standard, written algorithms. (p. 239)

Evidence of this approach would be if when students were asked to state their strategies after solving mental computation items, they revealed they had used a standard written algorithm mentally. This default approach could be avoided if students were encouraged to use
informal written methods consistent with constructivist paradigms expressed in the second approach.

The link between mental computation and written computation made by Reys (1984) as a reason for teaching mental computation could mean either of two interpretations. Either that the use of standard mental methods could lead to the development of standard written methods or else the use of invented mental methods could lead to the development of invented written algorithms. It is the latter which should make more sense to the learner according to constructivist principles.

According to McIntosh et al (1997b), "one benefit of mental computation is that it can lead to a better understanding of place value, mathematical operations, and basic number properties" (p. 55). These are considered to be components of number sense.

2.5.1 Understanding

Previously, in the explanation of terms, understanding was discussed as an important goal of education and mathematics generally as well as how the term can mean different things to different people (Skemp, 1976). Here a more specific classification of the various levels of understanding for mental computation is offered in Table 4.
### Table 4: Taxonomy of Mental Computation Objectives

<table>
<thead>
<tr>
<th>Category</th>
<th>Mental Computation Component</th>
<th>Mental Computation Belief</th>
<th>Classroom Practice Example</th>
<th>Key Researchers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comprehension</td>
<td>Mental Arithmetic</td>
<td>Procedural learning such as remove-the-zeros rule. One correct method and one correct answer.</td>
<td>Mental strategies taught explicitly. Student may take 99 from 264 by visualising SWA without realising that 264 - 100 + 1 (compensation) is easier.</td>
<td>Skemp’s (1976) notion of 'Instrumental Understanding'</td>
</tr>
<tr>
<td>Application</td>
<td>Estimation</td>
<td>Abstractions used in concrete situations. Relationships applied in a real-world setting.</td>
<td>Word problems or items are set in a realistic context. Validity of estimation is context dependent.</td>
<td>Sowder (1992)</td>
</tr>
<tr>
<td>Analysis</td>
<td>Number Sense</td>
<td>Relationships between numbers mean fewer facts need to be memorized as patterns in numbers are discovered.</td>
<td>Related basic fact families, such as 6 x 7 = 42, 60 x 7 = 420, and 42 + 7 = 6, are uncovered by students with teacher use of open questions.</td>
<td>McIntosh, Reys &amp; Reys (1992), Skemp’s (1976) notion of 'Relational Understanding'</td>
</tr>
<tr>
<td>Synthesis</td>
<td>Children invent their own thinking strategies</td>
<td>Combining use of the known to deduct the unknown. Constructivist.</td>
<td>“If I know that 2 x 25 = 50, could I work out 150 + 25?”</td>
<td>Rathmell (1978) Hope &amp; Sherrill (1987), Bastow (1997)</td>
</tr>
<tr>
<td>Evaluation</td>
<td>Meta-computation</td>
<td>Able to verify if own answer makes sense. Reflective.</td>
<td>“No wait, I forgot to double the 3 when I doubled 25 to make 50.”</td>
<td>Swan (2002)</td>
</tr>
</tbody>
</table>

Note: Adapted from Bloom’s Taxonomy in Good & Brophy (1977, pp. 184-185)

An outline of Bloom’s taxonomy of educational objectives in the cognitive domain found in Good and Brophy (1977, pp. 184-185) indicates a hierarchy of desired understanding levels
from knowledge of facts through to higher order desired outcomes such as judgments. In order to illustrate how this taxonomy of understanding levels relates specifically to mental computation, the following adaptation is provided in Table 4. This researcher has adapted the taxonomy with the original categories appearing on the left, by identifying the key beliefs from the learning theories, classroom examples, researchers and terms as they fit.

As a category of knowledge, the known facts component can appear misleading, as students may have simply memorised a particular fact. In which case students should be classified as working only in this category. Conversely students may have achieved full understanding of a fact to the point of ‘over learning’ it and therefore may be capable of working across a wider range of categories.

This categorisation can also be theoretically applied to Bruner’s (Good & Brophy, 1977) spiral curriculum. The spiral curriculum traditionally is described as having a wide base for knowledge from which it spirals upward towards evaluation as a higher-order objective. Bruner suggested that this was useful for teaching “the same material at several different levels by returning to it periodically” (p. 141). This has also been recommended as a model for catering for academically gifted students by allowing them to progress at their own rate. These students need to spend less time at the knowledge base before being able to progress through the spiral to the higher-order levels.

2.5.2 Rationale for Increased use of Mental Methods

In everyday settings, or real-world contexts, research indicates that adults most commonly use mental computation (McIntosh, Northcote & Sparrow, 1999; Northcote & McIntosh, 1999; Wadit & Brown, 1957). This appears to be at odds with how much class time has been spent teaching standard written algorithms compared to mental methods. McIntosh et al’s (1995a) chart in Table 5 illustrates the different classroom time allocations for four different computation methods in Australia in the past with future predictions. It is unclear when these future predictions may be achieved, as despite being obvious how to reallocate time spent on the methods of computation, the correct approach is just as important.

Porter (1989) also found 70-75 percent of teachers’ time was spent teaching textbook computation, while Duffin (1991) found that 80 percent of teachers’ time in the UK was spent teaching the methods of standard written algorithms. Kamii (1994) argued that an over-reliance on written methods of computation interferes with the development of number
sense. The amount of time students spend being taught written methods may explain why some students choose to use standard written algorithms mentally, and thus not use number sense when doing mental computation.

Table 5: Percentage of Curriculum Time Changes to Computation Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Adult Usage</th>
<th>Elementary School Experiences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Written computation</td>
<td>05</td>
<td>85</td>
</tr>
<tr>
<td>Mental computation</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>Computational estimation</td>
<td>30</td>
<td>05</td>
</tr>
<tr>
<td>Calculator</td>
<td>35</td>
<td>00</td>
</tr>
</tbody>
</table>

Note: From McIntosh et al (1995a, p. 7)

Almost half a century ago, Wandt and Brown (1957) found that formal written computation was little used by adults—rather, that three-quarters of adult calculations were performed mentally; “75 per cent of the uses reported were ‘mental’ ” (p. 152). This trend is shown in Table 6. Recent studies by McIntosh et al (1999) showed that little has changed over the years. Maier (1980) also reported that adults used mental computation for most everyday calculations. Willis and Kissane (1989) claimed that “of the three available methods of computation” (mental, calculator, written), the former two were the computations “typically used in everyday life” (p. 160).

Table 6: Comparison of Methods Used in ‘Folk’ and School Mathematics

<table>
<thead>
<tr>
<th>Environment/Situation</th>
<th>Mental</th>
<th>Calculator</th>
<th>Written</th>
</tr>
</thead>
<tbody>
<tr>
<td>Folk mathematics</td>
<td>75%</td>
<td>15%</td>
<td>10%</td>
</tr>
<tr>
<td>School mathematics</td>
<td>10%</td>
<td>5%</td>
<td>85%</td>
</tr>
</tbody>
</table>

Note: From Jones et al (1994, p. 14)

Maier (1980) used the term ‘folk mathematics’ to define mental computation and estimation that is developed by individuals through self-discovery. The nature of folk mathematics embeds it in context. In the video Real Maths—School Maths (Newton, 1992), primary school-aged students such as Aden were able to solve contextual problems mentally; yet they were unable to solve a matched item in a written format. In these cases, students often chose to use formal school-taught methods. Ten-year-old Aden calculated the correct change for a
purchase of a chocolate bar out of the five dollars tendered, a skill involving decomposition of whole numbers to two decimal places. This not only suggests that context is preferable to no context in order to encourage the use of 'intuition' (Dehaene, 1997; Stacey, 1990), but that mental methods were preferable to written methods in order to encourage self-discovery of informal methods. Lave (1988) explained the dramatic difference in Aden's performance on the mathematically identical items was due to two different communities of practice. Aden's ability to solve a chocolate bar problem mentally was probably not a school-learned skill.

As mentioned previously, mental computation has been neglected over the past twenty years due to an over-emphasis on standard written algorithms (McIntosh et al, 1997a; McIntosh 1996; Plunkett 1979; Reys, 1984; Trafton, 1986). Even by 1998, McIntosh still maintained:

It is clear that at present very few children acquire the range of mental computation strategies they possess as the result of deliberate classroom interventions or practices. Yet, eventually, most children acquire many or even most of them. Much exploratory work is needed, however in deciding how best to spread these strategies around. At present insufficient work is being directed towards this end. (p. 220)

The Cockcroft Report (1982) recommended that there needed to be a reversal in the decrease in mental mathematics trend. The Department of Education and Science (1991) recommended that pupils should use mental computation before other methods and should be encouraged to use their own methods. The move away from the rote learning of number facts towards student understanding has resulted in a shift of emphasis away from mental mathematics towards standardized written mathematics (Hope, 1986; Jones et al, 1994; Sowder & Sowder, 1989). Students have learnt procedures with little understanding that has led to 'artificial performance' on written tests with very little number sense, as indicated by Yang (1995).

Dehaene (1997) discussed the apparent genius of 'idiot savants' with calculation, to be a result of obsessed passion. He claimed that this ability has been misleadingly seen as genius while it is actually a syndrome associated with disabilities such as autism. He distinguished the mentally deficient and the idle from professional mathematicians and concluded that "today, society no longer values mental computation" as showmanship. However, he acknowledged that students in Japan are still sent to evening class to learn the 'mental abacus' (p. 164).
Decades ago, many researchers (Plunkett, 1979; Jones, 1988; Reys et al., 1995) recognised that the teaching and learning emphasis should be with developing mental methods, not standard written algorithms. Jones (1988) claimed that one benefit of mental methods is that individuals can choose their own methods.

Plunkett (1979) claimed that “a child who gets his mental calculations right, almost certainly understands what he is doing”. There are different ways of mentally calculating, and many of these are not taught at school. Therefore, it may then be possible for us to gain deeper insights into the level that students are working at, by allowing them to explain their methods.

Plunkett further stated that:

... it is fairly clear that mental methods are the ones to foster if you wish to use and develop children's understanding of number. Teaching mental techniques will not lead to children doing less calculations in school... probably... more. (p. 4)

As we increase the emphasis and time spent on mental computation (Mousley, 2000) the teaching approaches of mental mathematics should be qualitatively different to the short, sharp questions of old. In the UK, the Mathematical Association (1992a) stated that “there is a need for a range of activities and approaches which are very different from the old exclusive reliance on frequent tests of mental arithmetic” (p. 11). They concluded that there needs to be “a non-competitive, non-judgmental ethos” (p. 71).

Developing fluency requires a balance and connection between conceptual understanding and computational proficiency. On the one hand, computational methods that are over-practised without understanding are often forgotten or remembered incorrectly (Hiebert 1999; Kamii, Lewis, & Livingston, 1993). This could be the same for mental methods as well as written methods. For example, Bastow (1997) argued the need to learn only the two- and five-times basic multiplication facts because the rest may be worked out, and thus this would help to build number sense. On the other hand, understanding without fluency can inhibit the problem-solving process (Hoffman, 1997). Hoffman further argued that by Year 7 all students should know all of their basic facts instantly in order to be able to move on to extended facts and problems that are more difficult. This debate, mentioned by NCTM (2000) could be resolved by maintaining a balance between the two poles of opinion in order to develop fluency. For example, embedding mental computation items in a meaningful and familiar context such as money should strengthen conceptual understanding, and daily practice of contextualised mental computation items should develop proficiency.
As part of the Mental Arithmetic Project, McIntosh (1998) reported that “efficient mental calculators we interviewed appeared to have a range of strategies, which they used and adapted flexibly” (p. 221). This project researched mental computation strategies of students from Year 2 to Year 7. This range of strategies and flexibility indicated number sense. McIntosh (1998) claimed that there is a link between the characteristics of good number sense and the ability to choose the most efficient mental strategies and also that an improvement in number sense reflects an improvement in mental computation, and vice versa:

It is generally agreed that the ability to compute mentally in flexible ways is both a component and an indicator of number sense. It is tempting to speculate that the two may correlate closely with each other. (p. 211)

Research by McIntosh and Dole (2000b) has shown that students can score well for mental computation while scoring low for number sense. One explanation by McIntosh and Dole (2000b) suggested that high mental computation scores might reflect high general mathematical ability, but not necessarily number sense. While this was only a small-scale study (sixteen students), the interviews conducted were enlightening. It appears that such students may be relying on rote memorization or mentally performing written algorithms laboriously in their head and they may later find these methods unreliable for complex problems because they lack flexibility with numbers.

McIntosh and Dole’s (2000b) tests of number sense and mental computation and mathematical ability show that the same student may achieve differing results for two out of the three separately tested outcomes. Therefore, teachers administering tests of mental computation may need to look beyond the scores achieved. The use of questioning or interviews is recommended as techniques to probe student methods and flexibility further (Bell, 1999).

With regard to year levels, McIntosh et al (1995a) found that increases in performance for mental computation were higher from Year 3 to Year 5 than for other year levels. The authors suggested that this might be due to the peaking of mental computation ability. Students also performed better on addition than on the equivalent subtraction items or other items related to known facts.
2.6 Mental Computation Strategies

'Mathematical Strategies' is the second sub-strand of a new strand ‘Working Mathematically' in EDWA's (1998) Outcomes and Standards Framework: Student Outcome Statements: Mathematics. It was claimed that students should "show...flexibility...and call on a repertoire of...strategies..." (p. 4). McIntosh et al (1994) claimed that some strategies were more efficient than others were, and that sophisticated strategies were quicker to use.

Counting is the basic computational technique according to Ginsburg (1977) and it is students' invented methods that rely heavily on counting, as "they calculate by means of invented counting methods, often involving the use of the fingers" (p. 94). However, more efficient mental strategy choices such as bridging tens and use of relational knowledge such as doubling and place value may indicate well-developed number sense. Because these strategies use fewer steps, there is also less likelihood of error. Conversely, if students stick with inefficient strategies there is greater likelihood of them making an error as the numbers they work with become larger. For example, an efficient mental strategy for 7 x 9 is relating it to a known fact such as 10 x 7 = 70. Then by subtracting only one seven, one can obtain the answer of 63. An inefficient strategy of repeated addition of sevens allows eight opportunities for computational error rather than two. Furthermore, counting on fingers will be impossible for more complex examples such as two-digit by two-digit multiplication.

Yang (1995) found that students who performed well at written computation might not necessarily be good at estimation, although good estimators were also found to be good at mental computation. This may suggest that the type of mental computation being performed may be qualitatively different. That is, the former group of students (good at mental computation, but not estimation) may be visualising standard written algorithms. By comparison, the latter group of students (good at estimation and mental computation) may be good at applying computational estimation and a range of higher-order thinking strategies. Yang (1995) further found a significant correlation between students' high scores for his number sense test and a high level of conceptual understanding as evidenced by their ability to estimate sensibly.

From the Netherlands, Beishuizen (1999), Beishuizen and Anghileri (1998), and Anghileri (2000) suggested the use of structured aids such as an empty number line or 100-bead string to support children's intuitive mental methods before moving to written calculations. In this way, the material can provide a link between the two methods to help when difficulties arise.
Anghileri and Belshaizen (1998) also recommended teaching to "promote mental strategies" (p. 4). However, McIntosh (1996) was convinced that the answer is not to teach mental strategies directly to students—rather, that as students practised discovering relationships for themselves they were more likely to understand them. He found:

Only one strategy which has been directly taught by adults to children—the taking off and putting on of zeros—and it turns out to be the only strategy which children consistently misunderstand and misuse. (p. 62)

Sowder (1992) concurred that better mental computation performance was not a direct product of schooling. Rather, this ability was a result of out-of-school experiences, which involved models for re-grouping. Thus it remains to be seen whether better mental calculators could result from schooling practices, if the right approaches were adopted. For example, when using money as a context for calculations, the use of coins could provide models for re-grouping.

The Student Outcome Statements (EDWA, 1998) warned that as students adopt new strategies they would initially make more mistakes; therefore, speed and drill exercises will be unhelpful at this stage (p. 8). Perhaps this may even explain some students' unwillingness to take risks. Therefore, students may make mistakes with mental computation strategies for several reasons. They may be maintaining an inefficient strategy choice due to fear of failure with a new strategy or they may have tried a new strategy and made some initial errors. They may also be unable to determine which operation is appropriate (Stacey, 1990), and that "division is a notorious example" of such inappropriate choices. To which Anghilleri (1995) suggested that language may be a key factor in developing understanding of division, or it may be students' lack of everyday experience with the operation. Hope and Sherrill (1987) reported that skilled mental calculators used different strategies including distributivity and factors, avoided 'carrying', often worked from left to right, and reduced memory demands; while unskilled students used written algorithms mentally.

2.6.1 The Classroom Teacher

According to Lave and Wenger (1991), "a decentered view of the master-apprentice relations leads to an understanding that mastery resides not in the master but in the organization of the community of practice of which the master is a part" (p. 94). This principle can apply to the explosion of knowledge in our modern information age, as today's teacher cannot be expected to be the 'knower-of-all', but rather to act as a facilitator in the learning process. In the case where something may be unknown to the teacher, he or she can facilitate how to
access that knowledge, either via the library, Internet or specialist teacher et cetera. Further, Anghileri (2000) suggested working with numbers in context (in order to make sense to young children), and building on what children know when they start school. She highlighted that teachers need to listen and teach number work as a social activity in order to develop number sense.

Whitebread (1999) and Anghileri (2000) identified “Meta-cognitive awareness (thinking about your thinking) and control” (p. 127) as important features of emergent mathematics. These authors concur that emergent mathematics was an outcome of providing numbers in context.

According to Jones et al (1994):

When solving problems at school children are often not given a choice, the way children and adults are in everyday situations. Choice is not available to select...a strategy... (p. 13)

Strategy choice was the major area of research undertaken by Swan (2002). Swan and Bana’s (1998) model of computational choice is shown in Figure 3.

![Figure 3: Model of Computational Choice](image)

Note: From Swan & Bana (1998, p. 582).

Figure 3: Model of Computational Choice
It is important to note the identification of influencing factors such as previous experience and attitudes in this model, which not only relate to mathematics but may also impinge on subject content within a particular context. On the left-hand side of the model, factors influencing the individual are internal and unstable (changeable). Factors on the right-hand side of the model are external and for which change is dependent upon others; that is teachers and or parents. The metacognitive or checking strategies factor relates to higher order, reflective thinking skills associated with number sense that could be developed in the appropriate classroom environment.

The reasoning behind students' choices of written, mental or calculator methods— the focus of Swan's study—revealed students' underlying gaps of knowledge or stage of development. Students should be using mental mathematics as a first resort particularly for simple operations (Plunkett, 1976). When items increase in complexity, enough to need to use a calculator or pen and paper, students need to have a mental estimation of whether the answer is within reason or not by use of metacognitive or checking strategies. For example, students in Swan's (2002) study who chose to use a calculator for items such as $95 \times 1000$ had stated that they needed a calculator because they were working with big numbers. This choice actually revealed that they lacked an understanding of place value critical for the development of number sense. From Figure 3, the influence of written computation methods taught in schools on mental computation performance could also be detected by students choosing to use written methods to solve items which should be straightforwardly computed using mental computation.

As suggested previously, school mathematics has been synonymous with written methods, as everyday mathematics is with mental methods. Studies by Newton (1992) compared students' mental computation performance with their standard written algorithm performance. Results indicated that students who were quite capable of solving money-context problems mentally could become confused when asked to solve the identical problem by using school-taught written methods. This is, as Ginsburg illustrated in the video *Twice Five Plus the Wings of a Bird* (Campbell-Jones, 1985), due to a fundamental lack of understanding associated with the procedural way that standard written algorithms were taught. Many researchers have supported the argument against the continued teaching of standard written algorithms or written computational strategies, (Hope, Reys, & Reys, 1987; Jones 1988; Kamii & Dominick, 1998; Reys, 1984; Sowder & Sowder, 1989). It should at least be delayed until an understanding of place value is established (Ginsburg, 1977).
Plunkett (1976) claimed that one problem with standard written algorithms is that they are not flexible, or creative. This suggests that they would not be helpful in acquiring number sense. Current curriculum documents in Western Australia suggest students should use either their own invented methods or a conventional algorithm but were careful not to prescribe any particular algorithm. For example, in the Number sub-strand 'Calculate' (EDWA, 1998) students should “use their own method or a conventional algorithm to multiply” (p. 196). The debate here regarding invented mental methods parallels current written computation debates regarding invented written methods. Reys (1984) noted the link between mental computation and written computation as one of five benefits of teaching mental computation: "It (mental computation) is a prerequisite for successful development of all written algorithms" (p. 549). This could mean either of two interpretations in the classroom. Either, the teaching of standard mental methods may influence standard written methods, or else invented mental methods could encourage invented written algorithms. Non-standard written methods include examples such as the Gelosia (Venetian Grid) method or the Russian Peasant Method.

Easley and Easley (1992) have documented the loss of independence and confidence, and the development of mathematics anxiety. Mathematics anxiety has been attributed to trying to follow directions (Tobias, 1978); associated problematic memory lapses (Easley & Easley, 1992); and perceived expectations of others—both the language and image of mathematics, and rote-learning teaching styles (Haylock, 2001). The phenomenon of students becoming confused and lacking confidence in the school curriculum is not restricted to mathematics. It belongs to a teaching method founded on particular beliefs and expectations. It is illustrated in the cartoon shown in Figure 4 where the same child reads simple text in a stilted (school-taught) fashion, yet is quite capable of reading more complex vocabulary when inspired by the context. The similarities between recent mathematics and language curriculum changes reflect the same educational, psychological and philosophical bases. It is recognised that children do not come to school as 'tabula rasa' or empty slates (Shuard in Campbell-Jones, 1985).

Constructivist theory is evident in the language curriculum that espouses ‘the scientific method’ (Smith, 1985) employed by young children as the best pedagogy. Just as very young children are encouraged to approximate in speech and reading, they can be encouraged to estimate in mathematics. Just as children learn to read by reading stories (words in a context)
When Chloe was in first grade, the school expected her to grind her way through a hierarchy of school readers. She would insist on reading to us the required number of pages every night, although it was ridiculous and unnecessary—she was already reading picture books fluently.

The worst aspect of this exercise was the manner of her reading aloud. Instead of reading with the lively expression she did normally, she did it in the stilted manner of a child who’s beginning to decode the words on the first page for the first time:

“Tim • and • Pat • and Ro • ver • went • to • the • park • to • play.”

“Why are you reading like that, for heaven’s sake?” I asked.

“Because that's the way you have to read at school, silly!”

Figure 4: Differences between two Teaching Methods

rather than rote learning of words, children can learn to solve computational problems by engaging in contextual mathematical activity rather than rote learning the procedures. The rate of change within language and mathematics curricula has seen many changes that are yet more dramatic for language than for mathematics.

According to Dehaene (1997), it may be unfair to compare Western countries with Eastern countries for mathematics performance, as the language of the western numeration system is more complex than Asian numeration systems in many respects (p. 160). These claims were made in reference to Miller, Smith, Zhu, and Zhang’s (1995) study comparing American and Chinese children. Their study found amongst other conclusions that American children struggle most with numbers between 11 and 21, when reciting numbers as far as they can.
Similar teaching approaches to those used in the Netherlands (Beishuizen, 1997) have been successfully implemented in the UK. Anghileri (2000) commented that “there have been reports on the positive responses that teachers have made to the change of focus from written to mental strategies” (p. 136). Anghileri also reported that “research evidence suggests that very young children are capable of handling larger numbers in more complex ways than teachers have conventionally believed or assumed (Munn, 1994; Thompson, 1997)” (p. 126). Groves and Cheeseman (1993) reported similar findings from a significant calculator project in Victoria.

Anghileri (2001) described how teachers in Britain were being encouraged to teach effective mental strategies explicitly, with guidelines published outlining each year level expectation. Strategies such as doubling, halving and near-doubles, as well as counting backwards and forwards were encouraged. The author compared the English approach of treating mental and written methods separately with the Netherlands curriculum. The latter is founded on the development of students’ own informal mental strategies and ‘didactic context situations’ designed and sequenced as a basis for developing written methods. Therefore, the debate regarding whether students should be actively taught strategies or whether experience and maturation have more influence continues.

2.6.2 The Importance of Discourse

Anghileri (2000) discussed how talk helps students relate to the visual patterns that provide for mental imagery as does encouraging patterns and symbols (p. 8). She claimed discourse is a powerful tool. Cobb, Boufi, McClain, and Whitenack (1997) referred to discourse as ‘collective reflection’. It has also been termed ‘reflective discourse’ with the teacher mathematising discourse. The authors concluded that “children’s participation in this type of discourse constitutes conditions for the possibility of mathematical learning” (Anghileri, 2000, p. 132).

The importance of discussion about mathematics seems to be as important as discourse is to learning a foreign language. At one time students could only enter university arts courses if they had Year 12 mathematics or a foreign language. At first, the process of learning a foreign language and the process of acquiring the technical terminology that can be found in such publications as illustrated mathematics dictionaries seems more alike than not. The main differentiation though has been that foreign languages are not learnt out of context. Indeed students are first taught the most relevant everyday terms whereas, “technical
mathematical language is not used in normal everyday adult conversation" (Haylock, 2001, p. 7). If languages are best learnt by listening to them and engaging in conversation, then perhaps mathematics is best learnt by developing a community of learners, listening to others' explanations and then interacting as need be.

2.7 Age Related Differences

Two outcomes could result here. Firstly, as Schoenfeld (1987) claimed, school methods may prevent students from using their own invented methods, which might then work against any age-related improvement. Secondly, according to Piaget (1952) there is an assumption that student performance should improve with age as is consistent with developmental learning. This could be because students have assimilated more basic fact knowledge and because they have had more out-of-school experiences.

In discussing teaching approaches for developing number sense, Anghileri (2000) recommended that “numbers need to be presented in a realistic setting in order to make sense to young children” (p. 125). This suggests that the provision of context should benefit younger students such as those in Year 3. Others suggest that even where context is present, there are still age differences as Anthony and Walshaw’s (2003) study on fractions set in a food context found:

marked differences between the year levels: the development of fraction knowledge appears to be very much a function of time and associated educational experiences. (p. i)

This may suggest that students' out-of-school experiences should also increase in relation to their age, the amount of experience and complexity of tasks performed. However, it remains to be seen how much this affects positively on student performance. It also remains to be seen how much school methods influence students, positively or negatively, the longer the time they spend at school. Answers to this latter question will be found by identifying the types of strategy that students choose to use. For example, a positive impact should result not only in higher performance scores but also in strategies reflecting invented methods. Students who score well yet use written methods mentally may be reflecting a negative school impact. Students who score well and display good number sense might be indicating a positive impact from out-of-school experiences or positive in-school experiences, or both.

The chronological age range of (8-14 years) covered by this study constitutes a broader developmental age range than has been previously examined. In Australia, it is generally
regarded that there may be a range of difference of abilities of several years for any chronologically homogenous grouped class; and which varies according to each class level. In contrast, The Carraher et al studies (1985, 1987) were of students who had been developmentally assessed as performing at Year 3 standard. They ranged in age from nine to 15 years old; all placed in Year 3 as this was determined to be their ability level. In Nunes et al's (1993) study, the subjects were 16 third graders ranging in age from 8-13 years with a mean age of 11.5 years. This was because unlike most Western cultures, the students were placed according to their developmental level of achievement rather than their chronological age. Students could only graduate to the next grade once they had achieved the standard required of the previous level.

2.8 Gender Differences

Walkerdine (1998) discussed the gender perspective with regard to the greater problem of class differences within gender in the UK. She claimed that Shuard (1981) implied that when girls performed better than boys did on computation, this was discounted by the content being low-level mathematics. Walkerdine challenged this. She argued whether rule-following should be considered as low-level and whether 'real understanding' is part of school mathematics. Walkerdine maintained that gender differences are not as great as regional differences (pp. 26-27). However, she noted that gender differences in secondary school surveys are more significant, due to an improvement in boys' attainment (p. 27). Leder (1990) discussed the Australian perspective with several contributions by Fennema (1990) who compared similar outcomes for American and Australian classrooms. Leder (1990) stated that "mathematics is learned, for the most part, in classrooms" and that classroom practices, especially teacher beliefs, have influenced gender-differentiated outcomes (p. 6). Leder (1990) also noted that teacher expectation was a very important factor. Kochler (1990) claimed that:

withholding help encouraged and nourished characteristics required for independent thinking and that this, in turn, led to the higher performance of females—cannot be discounted. (pp. 193-194)

Both the 'fear of success' discussed by Leder (1980) and 'math anxiety' discussed by Brush (1978) are offered as factors affecting performance associated more with girls than with boys. Wedge (1999) linked Bourdieu's notion of 'habitus', the "often emotional relationship of adults to mathematics" (p. 211) and anthropologist Lave's theories on situational context. She showed how this relationship might result in resistance to learning, and the 'blocks'
adults can have specifically during mathematics instruction. She illustrated this by documenting interviews with her mother who viewed mathematical competence and arithmetical competence as separate entities because her school taught them as separate subjects. The former included algebra and geometry while the latter included the four operations and simple fractions (as content for mental arithmetic). Wedege suggested that "the habitus of a young woman in Denmark in the 30-40s does not automatically encompass a disposition for learning maths, or generate a conception of maths as a relevant subject" (p. 215). Indeed she stated, "in the lower secondary school there was a subject entitled 'girls' maths' " (p. 215).

It is also recommended by Helme (1995), that applications need to be interesting and relevant to girls in order to make mathematics more accessible to people who have traditionally been alienated, "in particular women and girls". Barnes (1988) argued strongly to embed mathematics in people-oriented contexts and social concerns in order to appeal to girls. In light of these statements, the context items designed for this study include family members participating in several shopping activities.

Gender has recently been an issue in US mathematics equity performance as discussed in Easley and Easley (1992), when compared to Japanese students' performance (p. 19). Gender differences in performance do not seem to be an issue for eastern cultures (Yang, 1995) although gender differences in attributions of success and failure for Chinese and Australian students have been reported (Cao & Bishop, 2001). It therefore seems appropriate that the issue of gender differences needs to be investigated further for both preference and performance.

2.9 Preference

Nothing appears to have been reported in the literature regarding student preferences for mental computation items being presented in context compared to 'bare' items, not set in context.

2.10 Summary

This chapter outlined some of the most significant and related empirical and ethnographic studies on context including Carraher et al (1985; 1987), Nunes et al (1993), and Guberman (1992). Traditionally, basic numeracy has been assessed by the ability to pass paper and pencil tests. These tests emphasise de-contextualised calculations and written algorithms.
The contemporary emphasis on number sense includes recognising number relationships, the ability to move flexibly between the operations and to check answers for reasonableness. This view also accepts that number operations have more meaning when they are related to real world situations or which arise from these situations. Research has reported on the inefficiencies of teaching standard written algorithms. While some researchers have identified different mental computation or thinking strategies that students use for more practical and relevant mental mathematics, it remains to be seen what factors may influence students' choices of mental computation strategies (Swan, 2002). Far less research has examined the effect that context has on developing efficient mental computation strategies or the effect that context has on mental computation performance generally.

Regarding the notion of understanding, many mathematics educators believed that mathematics ability varies with each individual's level of understanding (Plunkett, 1979; Skemp, 1976). This is illustrated by Yang (1995) who found:

that students who could correctly carry out the exact computation using written methods were not necessarily successful in applying these skills in non-computational situations. (pp. iii-iv)

While there has also been some research on word items of computation and problem solving generally, there is little literature about money items specifically. Most word items found in commercial school texts include money as a context in mental computation applications, along with topics such as measurement, ages, food, sport and animals. However, these items are designed for individual seatwork rather than class activities as suggested by most researchers in this field, such as McIntosh et al (1994). There is also no evidence of research regarding student preference for mental computation items presented in context.
Chapter 3: Methodology

3.1 Introduction

In the previous chapters, contemporary research was reviewed to reveal both the importance of meaningful contexts such as money in mathematics and the importance of encouraging students to create their own 'toolbox' of mental methods in order to maximise understanding. The review compared the value of using different contexts to arrive at the conclusion that if only one context was to be used, the best one would be money. Different theories of knowing were outlined and a conceptual framework created, while the issue of bridging students' out-of-school experiences with their school experiences to avoid two separate 'fields of learning' was also discussed.

3.2 Background

According to Trochim (2001), most educational research now embraces a mix of qualitative and quantitative research methods, as "all quantitative data is based upon qualitative judgements; and all qualitative data can be described and manipulated numerically..." (p. 11). The methodology chosen for this study was both quantitative and qualitative.

3.2.1 Interview Rationale

Interviews were considered the best method to gather this data (Shigematsu et al, 1994). Bell (1999) recommended the interview in order to minimize students' non-response to questions. It also allows the researcher to seek immediate clarification if needed, rather than the associated problems with questionnaire use. Ginsburg (1981) recommended speaking to children rather than just observing their behaviour, in order to find out how children think. Interviews were considered particularly appropriate for the money experience questions. As data for the mental computation items needed to be attained on an individual basis, group testing with students writing down their methods was considered inappropriate for several reasons. Firstly, the students' written explanations of their method may be incomplete or unclear. Secondly, students may be tempted to write down a different method to the one that they actually used. For example, the students may use written algorithms instead of mental ones, if they have only been taught to use written methods and had little experience needing to use mental methods. Thirdly, as the extra time may allow the students longer to think about the computation items, they may be tempted to change their answers. As data was
required to determine both the method and strategies used at the time of calculation, it was considered that interviewing should be conducted immediately after each item’s answer was given. This would avoid a possible increased risk that students might forget and/or substitute a different or newly acquired strategy or method.

Due to the variety of interview types available, the type chosen was a standardised interview (Denzin, 1989) recommended by Bell (1999) for first-time interviewers because it is easier to “aggregate and quantify the results” (p. 137). It was also suitable because specific recording and verification of information linked to previous studies was required. This form of standardised open-ended interview categorised by Swan (2002) was outlined on a continuum of least to most structure as the second most structured of interviews. Characteristics belonging to this classification study included: the exact wording and sequence of questions determined in advance; all interviewees asked the same basic questions in the same order; and questions all worded in a completely open-ended format (Patton, 1990). The order of the money questions and mental computation items for this study, worded in an open-ended format, appears in the protocols.

The use of a standardised, semi-structured format allowed the researcher to leave each “interview with a set of responses that (were) easily analysed” (Bell, 1999, p. 140). These responses included students’ written answers to the mental computation items and the tape-recorded explanations, as well as the money questions and the researcher’s notes on non-verbal actions. As there were twenty hours of recordings transcribed at a rate of “ten hours for each hour recorded” (Bell, 1999, p. 140) this took around two hundred hours to complete. Interviews were conducted at a rate of up to four a day and all interviews were transcribed on the same day to make the process of transcription easier.

It was considered important that no interviewee should feel disenchanted with the process as that would be unfair for future researchers seeking volunteers. Apart from the ethics protocols, this meant that the time, place and style of interview were considered. The times chosen were school times and at secondary school always in mathematics sessions. The places chosen were either quiet interview rooms or empty classrooms with doors open in open-thoroughfare areas. The style of interview followed the protocols in Appendices II and V.
3.2.2 Oral Presentation Rationale

Nunes et al (1993) found in a comparison between farmers and students with five years of schooling, that errors by farmers for oral representations were within a reasonable range. Further, that questions as to which operation to use were unusual in oral problem solving. This suggests that context items need to be orally presented. In addition, because, according to Newman (1977), most errors are likely to occur with decoding of text, the researcher decided to present the items orally by reading the question aloud to the students, while the student would read along silently. Because some students' auditory processing development may lag behind their visual processing ability, the items were also presented to students in a visual format, as in Appendix VI.

In Nunes et al's (1993) study, computation items were presented orally. Students were allowed the choice to use pencil and paper, or to solve them orally. The answers were then only marked either correct or incorrect. By contrast, this study presented computation items that all students had to perform mentally, although they had the choice of method and strategies. Answers were scored using a process scale in order to acknowledge simple computation errors that may have been made, and to give credit for items partially correct.

3.3 The Design of the Study

Various contexts were considered for this study. However, it was necessary to use money as the one context only (other than the students themselves) that applied to all students. Money is one context familiar to students of all ages. From a constructivist view, it was considered important to determine the students' prior experience with money. As Roth (1996) claimed, the 'embodied aspects of knowing' are often neglected by the 'individualistic cognition perspective' (p. 490).

Quantitative data included three sets of results. First, each student's prior experience with money was rated (see Appendices III and IV for the procedure and outline of the three categories); and ratings given were allocated in response to students' answers to a set of nine structured questions. This was in order to quantify qualitative information. As individual students could have different backgrounds in terms of familiarity with money, it was decided to check for such differences between individuals. This was achieved with a money experience instrument as part of a qualitative data gathering one-on-one semi-structured interview. This data were to be used to answer research question 2.
The second set of quantitative data results were collected from the scored performances on the matched mental computation items both for a money-context and for non-context. For the study reported here, 'task context' is the focus with applications. The 'situational context' is the school setting with one set of items set in imagined familiar contexts. The one-on-one interview design provided for all computation to indeed be carried out mentally as Sowder (1992) claimed, "there is the difficulty of determining whether or not the computation was indeed carried out mentally" (p. 387). The McIntosh et al study (1995a), with which some of these results are compared in Chapter 4, covered similar ages and items to the study reported here, but the items were all context-free and group-administered. It seemed appropriate for this study to test for gender differences, as any information concerning subtle differences in performance or strategy choices between genders would be useful.

The third quantitative data involved noting the strategies that students chose to use. The strategies were identified and rated according to the McIntosh et al (1994) classification system given in Appendix IX. The researcher felt that this classification lent itself to grouping according to the degree of number sense observably employed. This is further illustrated in Table 11. For example, guessing or 'can't do', provides no evidence of number sense, while the use of fingers is primitive and may suggest some number sense is developing. A group-administered pencil and paper test would not have indicated which strategies a student had used or how certain they were of the answers they gave.

Lastly, students' preferences for the money-context items, the non-context items, or neither were noted, along with whether or not they had noticed any similarity in the items between the two sets.

The study described here was designed to check for transfer between the two situations of in-school and out-of-school learning environments previously discussed and illustrated in Figure 1. Thus, two sets of mental computation items, one set in-context, the other with non-context but with identical mathematical content, in order to answer research question 1, were constructed to check for differences in performance. This design is illustrated in Figure 5.
Figure 5: Design for Data Gathering during the One Interview Sitting

Figure 5 illustrates how an equal number of students of both genders were selected to check for gender differences in order to answer research question 5. Then an equal number of students per Year level were chosen to check for age differences and to answer research question 4. A money instrument was deemed necessary in order to answer research question 2. Strategies needed to be identified from the matched tests to check if context made a difference to the type of strategies chosen by students. This might explain any difference in performance results between the matched tests for the same student. It was therefore considered necessary to interview students at the time of computation and to audio-tape these interviews for checking and transcribing purposes. This design was developed in order to answer research question 3. Finally, research question 6 would be answered by asking students whether they preferred one set of items or the other. In addition, by asking if they noticed any similarities between the items on the two tests; then this might further add to research question 3.
3.3.1 Subjects

Volunteer students were selected from a secondary school in a middle class suburb of Perth and two of the feeder primary schools, from the middle spectrum of ability. The teachers were asked to select volunteers from this middle spectrum by eliminating students at either extreme of mathematics ability in order to be as representative of the average ability student in the year groups as practicable.

A total of 64 students were chosen by their teachers—eight males and eight females—from each of the four year levels of Year 3, Year 5, Year 7 and Year 9. This was in order to cover a middle spectrum of ability over a wide age range and to make connections with previous studies such as Nunes et al (1993), McIntosh et al (1995a) and McIntosh (2002).

The secondary school (Years 8-12) had streamed classes according to ability, with four first stream classes, three second stream classes, three third stream classes and one class for the 'mathematically challenged'. To select students from the middle spectrum, in consultation with the mathematics coordinator, four students were selected from first stream classes (but these were not the top performing students), with the remaining twelve students selected from second stream classes. The feeder primary school teachers selected students according to the criteria that the students represent the middle range of ability. The distribution of student samples by school is shown in Table 7.

Table 7: Distribution of Student Samples by School

<table>
<thead>
<tr>
<th>School Type</th>
<th>Sample Size</th>
<th>Year 3</th>
<th>Year 5</th>
<th>Year 7</th>
<th>Year 9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sample Size</td>
<td>Year 3</td>
<td>Year 5</td>
<td>Year 7</td>
<td>Year 9</td>
</tr>
<tr>
<td>Primary School 1</td>
<td>25</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Primary School 2</td>
<td>23</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Secondary School</td>
<td>16</td>
<td></td>
<td></td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>Totals</td>
<td>64</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Due to ethical requirements, all students who participated in this study needed to be volunteers. The Year 7 and Year 9 students were given disclosure statements and most signed their own consent forms. Class teachers noted how wonderfully unusual it was to
select students for mathematical activities based on their average range of performance. Either usually the top students are recommended for extension activities or the weaker ones are identified for remedial work.

Year 7 was the most difficult year level for obtaining volunteers. One Year 7 teacher commented that the lack of interest her students had in volunteering might have been due to a perception by the students that they were in need of 'special help'. Because they were only used to either the brightest or the weakest students being chosen for mathematics intervention, students assumed that they must belong to the latter. Year 7 may be a particularly sensitive time for students in Western Australia, as they are about to graduate from primary school and move on to secondary schooling. Either, their confidence regarding their mathematics ability may be sensitive due to this uncertainty or they may have enjoyed being able to say "no" for a change, rather than be volunteered by their parents.

The researcher noted that three Year 7 teachers at one primary school approached their classes as a group to request volunteers, and therefore group dynamics and peer pressure cannot be discounted. However, at the second primary school, the Year 7 teacher approached students on an individual basis. This individual approach proved to be more successful and it was noted that at the secondary school, the Year 9 students were approached individually by the mathematics co-ordinator. This one-on-one approach contributed to the students' willingness to volunteer. By contrast, the group request for volunteers in Year 7 seemed to work best when only parent permission slips were required, as was the case for Years 3 and 5 at both of the primary schools.

The interviews were conducted in consecutive order from Years 3-9. This order provided a theoretically logical structure for monitoring developmental changes and differences across the year levels. Commencing with Year 3—before the statewide testing—also proved practically useful, as it was anticipated that enthusiasm for any testing might dissipate immediately after the statewide testing.

3.3.2 Instruments

As mentioned previously, one single context of money was chosen as all children have experienced money—in order to be able to evaluate development throughout the year levels for a sample of this size. Two instruments were used; the money experience questions and
the two sets of matched mental computation items. Protocols for administering the interviews were adapted not only because of ethical considerations to protect the subjects, but also to ensure that each interview was consistently conducted.

Money Experience Instrument

This was considered an important instrument to ascertain the students' prior interest in, and experience with money. The instrument consisted of a series of nine questions, which can be found in Appendix II. These questions were designed to uncover each student's prior task, situational and social context history with money. Questions asked about parental influence on money use, money use in school activities, regularity of income, saving and spending habits in actual and imagined situations, and savings skills. These exposures assume certain associated skill experiences, such as counting money, giving and checking change, and multiplicative reasoning when buying more than one item or when working out a set amount to save regularly.

The rating scale shown in Table 8 represents the composite score allocated, from the various ratings awarded to students' responses to the different questions. A high exposure to money experiences was determined to be, for example, working in a shop and giving change to customers, earning a regular income from a part-time job, or being 'self-managing'.

<table>
<thead>
<tr>
<th>Money Experience</th>
<th>Rating Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Exposure</td>
<td>3</td>
</tr>
<tr>
<td>Medium Exposure</td>
<td>2</td>
</tr>
<tr>
<td>Low Exposure</td>
<td>1</td>
</tr>
</tbody>
</table>

A medium exposure to money was determined to be, for example, when a student mentioned 'planned saving' or 'real' buying or selling experiences at school. A low exposure to money experiences was determined to be, for example, the receipt of little if any pocket money, little reason for spending it, and no specific saving plan mentioned. This rated a one. Full details of the marking criteria and procedure are given in Appendices III and IV.
Mental Computation Instrument

The mental computation instrument consisted of 10, 12, 13, and 12 matched pairs of items for Years 3, 5, 7, and 9 respectively. For example, \(60 + 80 = ?\) for non-context, matched the item, \(\text{If I spent 60 cents on an icy pole then 80 cents on a chocolate bar, how much did I spend altogether?}\) The reason for the number of items being less in Year 3 and Year 5 was because the time taken needed to be of a similar duration for each year level, to meet school demands. This was consistent with Callingham and McIntosh's (2002) methodology to minimize test fatigue in Year 3.

For the purpose of this study, the context word problems are referred to as 'context items', for reasons mentioned previously. For obtaining permission notices, the term 'mental computation questions' was used rather than 'mental computation items' as it was felt that this was more meaningful to the parents and students.

Practice items were used to introduce students to the test instrument. The practice items were identical across all four of the year levels to control for any practice effect. These items were \(15 - 9 = ?\) for non-context, and \(\text{James had 20 cents then was given 70 cents. How much does he have now?}\) for context.

The items were organised by topic to cover the four operations of addition, subtraction, multiplication, and division of whole numbers. Years 5 to 9 included non-whole number operations of the addition of decimals, while Years 7 and 9 further included percentage, and subtraction and multiplication of decimal items to two decimal places. The items allowed for a progression of difficulty from addition and subtraction of whole numbers through to decimals and percentages. The items were therefore presented to the year levels according to performance expectations for each year level consistent with previous methodology in McIntosh et al (1995a). One decimal item \((6.20 \text{ plus } 4.90)\) appeared only at Years 5, 7 and 9. The percentage item \((25 \text{ percent of } 48)\) and the other two decimal items \((0.1 \times 45)\) and \((6 - 4.50)\) appeared only at Years 7 and 9. A full distribution of all items by topic appears in Appendix VII. This structure provided a limit for the content covered in this study and the full range for all four year levels is shown in Appendix VII, which also gives the wording for the items in context.

Item topics were distributed across the year levels as indicated in Table 9. Identical items across more than one year level were purposely selected to allow age-level comparisons. For example, two items, \(79 + 26\) and \(105 - 26\) span all four of the year level tests while six items
span the three year levels 5, 7 and 9. The total number of different items across all the year levels is 21. The tests for Year 7 and 9 were identical as it was considered that all the content would have been covered by the Year 7 stage. A comparison of the results could enable a check for consistency between secondary level schooling and primary level schooling.

Table 9: Non-Context Items Common across Multiple Year Levels

<table>
<thead>
<tr>
<th>All Four Year Levels (3, 5, 7, 9)</th>
<th>Three Year Levels (5, 7, 9)</th>
<th>Two Year Levels (3, 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>79 + 26</td>
<td>165 + 99</td>
<td>60 + 80</td>
</tr>
<tr>
<td>105 - 26</td>
<td>60 x 70</td>
<td>68 + 32</td>
</tr>
<tr>
<td></td>
<td>7 x 25</td>
<td>74 - 30</td>
</tr>
<tr>
<td></td>
<td>150 + 25</td>
<td>Double 26</td>
</tr>
<tr>
<td></td>
<td>6.20 + 4.90</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3500 + 35</td>
<td></td>
</tr>
</tbody>
</table>

Note: Year 7 and Year 9 items were all identical.

With regard to validity and reliability, all the items in this study were selected from mental computation test items used in the McIntosh et al. (1995a) study and adapted to provide for context. These context word items were tested in a pilot study described later in this chapter. This method of item selection ensured that all the items were known to be appropriate for mental computation for these age levels and that the results could then be compared in the analysis. For example, the item 165 + 99 was included because it can be solved mentally, using the 'add one hundred, take-away one' compensation strategy.

As discussed previously, a limit was provided for the term 'context' in this study by only using one context—money. The money-context provided for computation items used minimal words in familiar everyday language, in order to reduce errors due to language comprehension difficulties (Newman, 1977; Cockcroft, 1982; van den Heuvel-Panhuizen, 1999). As discussed in Stacey (1990), the danger of nonsense questions (a constructed context of a few words requiring a single answer response) has been avoided in this research design in line with testing reported by Stacey and Bourke (1988).
This study’s non-context tests only required students to show the second of McIntosh et al’s (1992) number sense key areas (knowledge and facility with operations). However, the items set in a money-context required students to demonstrate both the second and the third (applying this knowledge) of these key areas (p. 23). The nature of the task and situational contexts used familiar everyday events, such as shopping, that centered on family life and family members.

Each item was presented both visually (as written on the sheet) and orally (as read by the researcher). This was designed to avoid disadvantaging students who may have preferred one mode of presentation to the other, and to help overcome any reading difficulties. This was because McIntosh et al (1995a) had found a marked difference in performance for some individual items when comparing oral and visual presentations and suggested that varied presentation may encourage different strategies. In addition, because Anghileri (2000) recommended that numbers presented in a realistic setting should be presented orally in order to help develop number sense.

3.4 Procedure

All 64 students were interviewed on an individual basis by the researcher in the one sitting only over a period of several weeks. These interviews were audio-taped and transcribed later the same day by the researcher. Pseudonyms were used to identify students on the tape recordings. These administrative codes were converted to a single numeric code from one to 64 as shown in the overall results tabled in Appendix X.

First, the semi-structured set of nine questions as set out in Appendix II was asked to determine children’s prior experience with money. The protocol for conducting these questions is set out in Appendix II. Protocols were used to ensure that each interview was conducted in an identical manner, as the researcher followed the set protocol each time. The protocol for the mental computation items is set out in Appendix V. The items presented in a money-context were mathematically identical to the items presented without context.

3.4.1 Administration of Money Experience Instrument

This consisted of nine questions concerning the students’ prior experiences with money for both their in and out-of-school experiences. The questions along with the protocol are in Appendix II. These were asked before the mental computation test items. After conducting the interviews and transcribing the answers given on the tapes, student responses were rated according to the procedure designed by the researcher and outlined previously in Table 8.
Responses were rated as indicating high (3), medium (2), or low (1) exposure. Samples of interview extracts are given in Appendix IV. An overview of the three ratings is given in the analysis chapter, while the priority evaluation procedure used to determine the ratings allocated is given in Appendix III.

The flowchart in Appendix III is identical in procedure and is provided as a visual organizer to illustrate this evaluation process and it was designed for recording individual results. A sample of over 10 percent (eight out of 64) of the rated experiences was then independently validated by a mathematics educator. The mathematics educator (henceforth, the checker) checked the reliability of the scores using the priority system stated above and shown in Appendix III. The interview transcripts for the eight samples were presented to the checker in typed format as transcribed directly from the tapes by the researcher. These were accompanied with a copy of the procedure and several copies of the diagrammatic flowchart shown in Appendix III. The priority system required working through the answers to questions in a set order. The checker allocated scores by working through the interview transcripts. Once a criterion was met, a score was allocated and no further checking was needed. The elimination of possibilities provided the most experience with the highest rating through to the least experience with the lowest rating. The checker was not given the scores allocated by the researcher. The two sets of scores allocated separately by the checker and the researcher were then compared for validation purposes.

As there were 64 interviews rated, a sample size of eight, being over 10 percent, was selected to meet acceptable validation requirements. The eight selected samples comprised two transcripts from each of the four year levels and covered at least two different ratings from each of the possible three ratings. The checker perfectly matched seven out of the eight sample scores with a part match for one sample, which resulted in a final score of 94 percent. When the researcher re-checked this sample's difference, it appeared that the checker had followed an error of procedure for that sample. Three of these eight checked sample interview transcripts appear in Appendix IV. Each one chosen represents one of the three ratings (1, 2, and 3).

3.4.2 Administration of Mental Computation Instrument

Immediately following the money experiences instrument the students were given the mental computation items during a one-on-one interview of approximately thirty minutes duration.
Both sets of items set in context and non-context were presented at the same sitting. The second set of items was given directly after the first set following a short break. In order to avoid errors in decoding of text, the researcher read the question aloud to the students, while the student read along silently. Because auditory processing development may lag behind visual processing ability for some students, the items were also presented in a visual format, as in Appendix VI.

Year 3 only had addition, subtraction and doubling for whole number items with halving for non-whole number items. Because the item was designed to test for conceptual understanding, the word ‘half’ was used rather than the numerical symbol \( \frac{1}{2} \). Year 5 items included multiplication and division with one addition item of two numbers to two decimal places. Year 7 and 9 items included one percentage item, and three decimal items. Along with that mentioned previously, there was one multiplication of a whole number by a decimal fraction (a multiple of ten) and a decimal number subtraction.

The presentation order of the mental computation items was such that the contextual items were presented first for eight students at each year level, while the other eight students at each year level were presented with the non-context items first. This reversed order was designed to control for any effect of one format upon the other. In each set of eight students, there were four students of each gender.

Each student was given a copy of each set of items so that they could silently read along as the researcher read each item aloud. This was to avoid any bias for either a visual or oral presentation preference. Students were only required to write their answers, and were not permitted to record any working out. After the student had finished writing each answer, the researcher asked them to explain how they worked their answer out. Explanations were given directly after each item. This was necessary in order to determine the mental computation strategies that students had used. By removing the factor of competition with other students and the pressure of speed, fewer children engaged in writing just any response because students were informed that they would need to explain both their answers and their reasoning. This design allowed children “to show what they are able to do” (van den Heuvel-Panhuizen, 1999, p. 132).

The interviews were recorded on audio-tape. Regarding the benefits of interviews, Bell (1999) claimed that “a major advantage of the interview is adaptability” (p. 135). In particular, clarification can be sought. Bell (1999) claimed that interviews “can yield rich
material and can often put flesh on the bones” (p. 135). This, she claimed is because “the way in which a response is made (the tone of voice, facial expression, hesitation, etc) can provide information that a written response would conceal” (p. 135). For this reason, field notes were made of any observable body language or non-verbal actions, such as facial expressions and finger counting, while the students’ tone of voice and hesitation were apparent on the audio-tapes. Bell (1999) further cautioned that interviews are time-consuming. Therefore, while this data collection technique limited the number of students to be studied, the depth of information available regarding strategy choices was substantial.

The students’ answers to the two sets of items were transcribed and scored for accuracy and process—the ‘process scale’. See Table 10 for the three-point rating scale used. The usual scale used by teachers checks for accuracy only. Known a ‘basic scale’ it normally uses two scores only—a 1 (if correct) and a 0 (if incorrect). Student answers in this study were scored using both scales by adjusting the ‘basic scale’ use for correct to a 2. This was so that comparisons could be made with the ‘process scale’ to find how much of a difference the process scale actually made. The variation found is discussed in the analysis chapter.

Table 10: Process Performance Rating Scale for Mental Computation Items

<table>
<thead>
<tr>
<th>Response</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct answer</td>
<td>2</td>
</tr>
<tr>
<td>Partially correct</td>
<td>1</td>
</tr>
<tr>
<td>(Incorrect answer given but student used correct method with only one error)</td>
<td></td>
</tr>
<tr>
<td>Incorrect answer</td>
<td>0</td>
</tr>
<tr>
<td>(more than one error or no attempt)</td>
<td></td>
</tr>
</tbody>
</table>

Whitbread (1999) stated that, “often children taught to do sums vertically cannot do the same calculations when they are presented horizontally…” (p. 21). McIntosh et al (1995a) claimed that “when the item is presented visually the student is more explicitly reminded of the written algorithm, which is probably inhibiting because of its perceived complexity” (p. 29). Therefore, it was decided to present both sets of items horizontally, and orally, as it was felt that a vertical presentation format would be more likely to remind students of the school-taught written algorithm. Both the oral presentations together with the horizontal format of the written presentation should help students. Otherwise, students might rely on the visual-
presentation and try to juggle the horizontal presentation into a vertical one—to use methods learnt outside of school, as well as help students for whom reading comprehension may be a difficulty.

The individual interviews also focused on uncovering the mental computation strategies used, rather than just the answers. Participants were asked to first give their answer and then explain how they found that answer. A note was made of the strategies used then these were rated according to the degree of number sense observed as categorised in Appendix 9X. Student strategy choices were later compared to a hierarchy of mental computation strategies with some minor modifications from McIntosh et al’s (1994) classification and observation notes made by the researcher at the time and from transcripts of the tape recordings. This hierarchy is shown in Table 11. The modifications were minor because they included sub-categories that were able to be included under two existing parent code headings. These parent code headings were both classified as higher order strategies.

Students’ answers were scored correct according to Table 10. Some students were completely certain about the correctness of their answers. Other students, uncertain about their answers, may have only guessed. When students were certain of their answers, these were clearly explained by the students and the use of efficient strategies. This demonstrated number sense, while guessing was evident from the students’ uncertainty of the reasons given for their strategy choices. Partially correct answers resulted from inefficient or partly carried-out strategies, which might reflect number sense depending on the strategy or strategies chosen. Either an incorrectly applied or inefficient choice of strategy or no explanation at all accompanied incorrect answers. Some students made no attempt at all and interviews endeavored to uncover reasons for this lack of ability.

Students were asked only to write their answers, and not to do any working out on paper. Students were also removed from the class setting, their peers and class teachers to a room in the school—usually an interview room or the library or similar. This was an attempt to reduce ‘situational cues’ such as associating the use of a method taught to them by their classroom teacher by the presence of that teacher. Otherwise, the students might be more likely to use ‘school taught routines’ instead of their own thinking strategies (Carraher et al, 1987; Boaler, 2000). This was considered important, as Schoenfeld (1985) described ‘cue-based practices’ as the antithesis of mathematical thought (Boaler, 2000).
3.5 Description of the Environment

A short questionnaire was sent to each of the three schools to obtain background data on any differences in teaching styles or emphasis for mental computation between the schools involved in this study. The questionnaire is shown in Appendix XI.

Differences in environments need to be considered. LeCompte and Goetz (1982) suggested that “comparability requires that the ethnographer delineate the characteristics of the group studied...so clearly they can serve as a basis for comparison with like and unlike groups” (p. 34). A brief description of the schools, teachers, students and classroom practices with mental computation follows.

3.5.1 Primary School Classes

All primary school classes noted that they practised mental mathematics for more than ten minutes per day. Primary school (B) had a principal with an extensive mathematics background who had worked for the education department’s curriculum branch and he had published curriculum materials currently in use. This principal also took occasional mathematics classes.

Year 3

One primary school (A) had a younger teacher who used the game ‘Sheriff’ and set half of the mental computation items in a context. Program activities were sourced from the teacher’s own ideas and were ‘needs based’, using class discussions of written number sentences on the board.

The teacher at primary school (B) was more experienced, did not use ‘Sheriff’ and always set mental computation items in a context, creating her own program activities as required. This teacher used both individual seat work from a set text and class discussions for real-life, story, personal or media items, written on the board, in a number sentence.

Year 5

Both Year 5 teachers at the two primary schools were very experienced teachers who shared responsibilities for their classes with other teachers. They both specialised in mathematics. Both of these teachers presented about half of the activities in context and neither used the game ‘Sheriff’.
The teacher at school (A) used flashcards, sourced program materials from *Maths Today Series* and always used individual seat work. Patterns in tables were explicitly taught and shopping worksheets were used in conjunction with the computer program, ‘Let’s go shopping’.

The teacher at school (B) used the ‘tables’ version of the game ‘Buzz’ and her own resources to re-enforce the basic number facts. Individual seatwork was used only for revision while class discussions were used for number sentences set in contexts and written on the board. This teacher mentioned teaching strategies such as bridging, working backwards, and rounding.

**Year 7**

One class timetabled 65 minutes per week of mental mathematics made up of five 15-minute sessions. Year 7 students at both schools experienced class discussions of solutions to written number sentences on the board.

Both schools mentioned teaching mental computation strategies. These included rounding, working backwards, place value and cancelling. Both schools used individual seat work from a set text, although one school (A) always used this while the other (B) only used it sometimes. The teachers at school (A) used the game ‘Sheriff’, and presented the majority of mental mathematics items set in a context—some of these being money, some measurement.

At school (B), the teacher used his own activities, as appropriate, which sometimes included the use of money and measurement as a context for concept application with an emphasis on percentages, decimals and fractional equivalents. No ‘Sheriff’ was played at school (B) in Year 5 or Year 7.

### 3.5.2 Summary of Primary Schools’ Mental Mathematics Programs

All three primary school year levels program mental mathematics for at least ten minutes daily. Mostly items were set in a context, either money or measurement, with some individual seatwork for practice. Traditionally the game, ‘Sheriff’ is presented without context. It was only used at one school and by less senior teachers. This suggests that for mental mathematics there may still be some evidence of Thorndike’s (1913) associationist teaching principles in schools, such as repetition and practice. There were also, at every class level, discussions of solutions for context mental mathematics items written on the board.
These discussions may or may not welcome students’ invented strategies, as all teachers admitted to the teaching of mental computation strategies. However, no teacher mentioned teaching rules, such as ‘remove the zero’, as reported by McIntosh (1996).

3.5.3 Year 9 Secondary School Mathematics Program

The secondary school head of the mathematics department stated that for Year 9, the mental mathematics component of the mathematics program was used primarily as a warm-up activity. Items were usually presented on the whiteboard, to suit the topic of the lesson. There was no set program to follow, so the format and content was teacher driven. Most Year 9 mathematics teachers would spend five minutes per 45 minute lesson, with up to four lessons per week. Students did not spend this time in seat-work or working individually from written exercises in text books, but orally discussing the whiteboard item as a class allowing for students to be aware of different solutions. Some teachers would teach mental computation strategies, as needed, with around half of the mental computation items presented in context of which money was used the most often. Because the game ‘Sheriff’ was unfamiliar to the head of department, it was presumed that it was not used at this secondary school. Overall, the structure of the secondary school’s mental mathematics was similar to the primary schools’ mathematics programs, although, given the nature of the more integrated curriculums at primary school level, the opportunities for mental mathematics may occur more often.

3.6 Ethics Procedure

The first ethical requirement was for permission to be sought from the target schools. This permission was first obtained in person, then letters outlining the scope and nature of the study were sent to each of the school principals to formally request volunteers, along with sample copies of parent and student letters.

An ethics requirement of the study was that target students of a suitable age should be required to sign their own permission slips as well as seeking their parents’ permission by providing parent permission slips as shown in Appendix I. The researcher decided that Year 7 and Year 9 should be appropriate ages. This meant that parental permission slips were required only for Years 3 and 5. Although this was necessary for ethics clearance, this was problematic, as some Year 7s were reluctant to volunteer. In hindsight, this researcher would seek teacher advice before offering only student permission slips to primary school children. The problem of obtaining volunteers was the only factor to delay the research but its impact
was only minor. Interviews of volunteers were conducted in quiet areas at times negotiated to suit the schools, teachers and students. The manner of how the interviews were going to be conducted was outlined on the permission slips in Appendix I.

3.7 Pilot Study

The pilot study consisted of a small sample of students using the proposed items, conducted at one secondary school and one primary school. The trials consisted of two volunteer students of average ability, as selected by the class teachers, from each of the four year levels. Students were told to take their time, as only accuracy was important. The mental computation items took an average of half an hour. Because of the pilot study, a minor modification was made to the Year 7 mental computation instrument length by removing one item. No changes were made to the wording of the context items.

Both sets of contextual and non-contextual items were presented at the same sitting. The first student of each pair was presented with the money-context items first. The second student was presented with the context-free items first in order to avoid any bias associated with order of presentation, as was the plan for the main study. For example, if all students had scored higher for the second set of items, whether they consciously noticed any similarities or not, this could be claimed to be a factor influencing improved performance because of the repeat involved.

Students were also asked to note, which two items proved the most challenging for each set of items. This was to identify any pattern of glaring difficulty in case any items should be modified. The two items selected were not always the same for both students of the same year level. Nor were they the same two selected for the same matched items.

From the pilot testing, it was determined that the two sets of items should take approximately half an hour per year level. With regard to non-context computation items, McIntosh et al (1995a) found, during their pilot testing, that twenty seconds per question item was very generous for some. However as the McIntosh et al study items were all non-context, more time was considered necessary for the context items in this study. With regard to the context items, the pilot testing indicated that thirty seconds was sufficient for each item, so this was determined to be the maximum time allowed to answer each item.

The students' preference for either context or non-context items was noted for each student by asking, "Did you like one test (set of items) more than the other test?" and "Why is that?" at the end of the interview. Results here were mixed. A second question, "Did you notice
anything similar about the two tests (sets of items)?" was also asked. Only one Year 7 student noted, when asked, that the items were similar. Although, this was only a very small sample, it was felt that it would be appropriate to proceed with a larger-scale study to compare the two sets of items. This should determine the effect of money-context items on mental computation performance.

3.8 Preparation for Data Analysis

In the main study, results of the matched items were then compared to determine whether context, age or gender made a difference to performance on mental computation items for either non-context or context.

Due to the use of a different number of items for the three tests, which meant that the mental computation items were scored out of different totals, scores were converted to percentages for easier comparison.

Four scales were used for the study as previously illustrated in Figure 5. One rating scale, the money experience instrument, was used to rate students' prior money experiences. Two scales, basic and process rating, were used to assess performance for the mental computation items and one number sense scale was used to assess strategy use. As the money experience instrument has previously been discussed, these last three scales are discussed here in turn.

The first performance scale was a three-point process and accuracy competency rating (0, 1 and 2) as discussed previously and shown in Table 10. Students scored 2 points for a correct answer, 1 point for partially correct and 0 points for an incorrect response or no attempt. This rating score allowed for computation errors where the process was correct and was of most importance for items where a student's answer was partially correct. However, where a student gave an incorrect answer due to more than one error, this was scored as a zero. This scoring system is termed 'the process scale' for the purpose of this study.

With the second performance scale, students were rated according to the traditional basic scoring rating (0, 2) for either incorrect or correct. Thus, in order to make a comparison with the process and accuracy scale, the scores allocated were simply a 2 for each correct answer or a zero for each incorrect answer. This scoring system is termed 'the basic score' for the purpose of this study.

The number sense scale rated the level of mental computation strategies evidenced according to a three-tier sophistication scale of Low, Medium or High (L, M, H), as in Table 11.
This scale was developed by the researcher from a previous classification by McIntosh et al (1994) and was used to rate each student's answer for each item according to the type of mental computation strategies identified. A number sense rating of L, M or H was then allocated for each student's answer to each item. Where students had used more than one strategy per item, answers were coded by allocating the higher strategy rating. For example, the use of two medium level strategies and one higher order strategy resulted in converting to a rating of H for that student for that item. This number sense scale helped to identify each individual students' level of overall number sense and the level of number sense for individual items presented in context or otherwise. This allowed for cross analysis between individual students and individual items. This rating scale helped to determine two dimensions of number sense—first, 'height' represented by a sophistication ranking according to Table 11, and second, 'breadth' represented by the range of strategies used overall.

Table 11: Three-tier Classification of Mental Computation Strategies

<table>
<thead>
<tr>
<th>Parent Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Number sense is not evidenced or use of low level strategy choices</td>
</tr>
<tr>
<td>CD</td>
<td>Couldn't do</td>
</tr>
<tr>
<td>G</td>
<td>Guessed</td>
</tr>
<tr>
<td>K</td>
<td>Known fact</td>
</tr>
<tr>
<td>P1</td>
<td>Used place value instrumentally</td>
</tr>
<tr>
<td>M</td>
<td>Some number sense is evidenced by choice of medium-level strategies</td>
</tr>
<tr>
<td>A</td>
<td>Used aids (fingers)</td>
</tr>
<tr>
<td>C1</td>
<td>Counting elementary</td>
</tr>
<tr>
<td>H</td>
<td>Well-developed number sense is evidenced by choice of higher-order strategies</td>
</tr>
<tr>
<td>C2</td>
<td>Counting in larger units</td>
</tr>
<tr>
<td>I</td>
<td>Initial strategy</td>
</tr>
<tr>
<td>R</td>
<td>Used other relational knowledge (RKF) (RKS)</td>
</tr>
<tr>
<td>P2</td>
<td>Used place value relationally (USP)</td>
</tr>
</tbody>
</table>

Note: Modified from McIntosh et al (1994, p. 89)
The complete chart of McIntosh et al’s strategies within their parent codes is given in Appendix IX. The level of mental computation strategy use identified was used to determine the degree of number sense in order to check for the effect of context on strategy choices. In addition to the McIntosh et al (1994) parent code strategies identified in Table 11—because some strategies that were identified did not fit in that existing classification system—it was necessary to include some extra strategy classifications. These new categories as listed in Table 11 include RKF (relate to a known fact) and USP (used smaller parts) as well as RK$ (relate to knowledge of money). These strategies all indicated evidence of relational knowledge and were therefore placed either under the parent code of R (RKF & RK$) or P2 (USP) which rates the strategies ‘high’ for number sense.

Along with the four scales mentioned, two questions identical to those outlined in the pilot study were asked of students at the end of the interview. Firstly, in order to identify each student’s preference and any possible correlation of student preference for item type with the students’ performances, students were asked to indicate if they had a preference for either context or non-context items. This question was asked to attain evidence as to whether or not preference (attitude) might affect the students’ results. Possible answers could be either in favour of context, in favour of non-context, or in favour of neither; and could include a qualifying statement such as why such a preference was held.

Lastly, students were asked whether they noticed any similarities between the two sets of items. Answers could include noticing that all items were mathematically identical. It could also be noted that some items within the tests were related as were the pairs 165 + 99 and 264 – 99, 105 – 26 and 79 + 26.

3.9 Summary

The study reported here was designed to compare oral mental methods in a money-context both orally and visually presented, for students from Year 3 (primary school) to Year 9 (secondary school). Any marked difference in performance for context items over non-context items should suggest positive effect for context. As any improved performance for context-based items might be due to previous money experiences, the level and quality of previous money experiences—in-school or out-of-school—was determined by interview. Another reason for difference in performance might be that students chose different strategies for the different presentation formats, suggesting varying levels of number sense for individual students between tests may be context dependent.
With regard to the impact of motivation on performance, each student’s preference for a presentation format was also tested. This study’s design allowed for checking of transfer between two test items not only in cases where results for an individual student varied between context and non-context, but also for any student. Students who scored highly for both presentation formats could have their strategy choices checked to find whether they had differentiated strategy choices for the different formats. That is, whether students had used written methods mentally for the non-context items while using efficient mental methods for the context items, thus indicating lack of transfer. This may well be a transitional stage. It would be identified by a student receiving a high number sense score for context items but a low number sense score for non-context items. By comparison, low scoring students for both presentations could be expected to use low number sense strategies for both formats.

Culturally, most students revealed experience working in small businesses and shops. While the issue of gender was tested, the study design did not consider differences in cultural background, ability with English, English as a second language or social background. Anecdotal evidence from the researcher’s familiarity with the schools concerned suggested that any such differences would be minimal and, in order to consider these factors, a much larger study beyond the scope of this research would have been required.

In the next chapter, analysis of the results systematically addresses each of the specific research questions in turn. The findings are discussed within the framework of answering the main research question: What effect does the context of money have on students’ mental computation performance?
Chapter 4: Analysis of Results

The previous chapter made reference to both the qualitative and quantitative data collection methods undertaken. This was in order to answer each of the research questions. This chapter contains analysis of this data. While quantitative data was used to determine any overall difference in performance between the two sets of matched items, it was also necessary to collect qualitative data. This was in order to identify students' mental strategy methods, the degree of their previous money experiences and their preference for a particular presentation. This chapter contains an analysis of all the data from the interviews and the mental computation instrument. The data collected are discussed, first in an overview of general trends and then with more detail according to each specific research question’s emphasis. After addressing each of the specific questions in order, an explanation of how and why the data have been analysed in this way follows. To conclude, all the information is then discussed within the framework of answering the main research question: What effect does the context of money have on students' mental computation performance?

4.1 Method of Quantitative Data Analysis

The data pool of mental computation performance was analysed according to both process performance and basic scores. The first score was the three-point process score of 0, 1, 2 mentioned previously, while the second score was the more traditional basic score of either a 2 for correct or 0 for incorrect. The purpose of using both systems was in order to check for differences between the two scoring systems to see whether the process affected overall performance in the matched sets of items. Both scores were applied to each student's answer for context and non-context items. Both the performance and basic scores were then compared.

4.2 Method of Qualitative Data Analysis

Student answers to the nine money experience questions were the first set of qualitative data to be collected and given an overall money rating score of 1, 2 or 3 for money experience. Analysis occurred by grouping and classifying similar answers and then rating them according to the degree of realistic money experiences reported by each student. Then, money experience was checked against performance for mental computation.
Mental computation strategies were identified using the classification system listed in Table 11, in the previous chapter, from transcripts of the students' answers. These were then compared with each student's performance results to check for any correlation for the provision of context. The mental computation strategies identified in this study were also compared to the McIntosh (2002) students' strategy choices.

4.3 Overall Results for Mental Computation

Table 12: Number of Students with Correct Answer across Year Levels

<table>
<thead>
<tr>
<th>Item By Operation</th>
<th>Year 3 (N=16)</th>
<th>Year 5 (N=16)</th>
<th>Year 7 (N=16)</th>
<th>Year 9 (N=16)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>NC</td>
<td>C</td>
<td>NC</td>
</tr>
<tr>
<td>60 + 80</td>
<td>9</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>68 + 32</td>
<td>6</td>
<td>9</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>79 + 26</td>
<td>4</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>165 + 99</td>
<td>9</td>
<td>9</td>
<td>14</td>
<td>11</td>
</tr>
<tr>
<td>6.20 + 4.90</td>
<td>13</td>
<td>8</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>80 – 24</td>
<td>0</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>140 – 60</td>
<td>3</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>74 – 30</td>
<td>4</td>
<td>15</td>
<td>15</td>
<td>11</td>
</tr>
<tr>
<td>105 – 26</td>
<td>2</td>
<td>6</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>6 – 4.50</td>
<td>14</td>
<td>11</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>264 – 99</td>
<td>24</td>
<td>8</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Double 26</td>
<td>9</td>
<td>9</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>7 x 25</td>
<td>7</td>
<td>8</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>60 x 70</td>
<td>2</td>
<td>2</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>38 x 50</td>
<td>7</td>
<td>6</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>0.1 x 45</td>
<td>6</td>
<td>8</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>150 + 25</td>
<td>5</td>
<td>9</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>Half of 16</td>
<td>11</td>
<td>13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Half of 30</td>
<td>0</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3500 + 35</td>
<td>6</td>
<td>5</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>25% of 48</td>
<td>8</td>
<td>7</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Means (%)</td>
<td>30</td>
<td>50</td>
<td>60</td>
<td>60</td>
</tr>
</tbody>
</table>

Note: C = Context; NC = Non-context

Using the basic scoring system (0, 2), overall results, detailed by item and year level in Table 12, reveal no marked difference between mental computation items presented in a money-context compared with the same items not presented in context.

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Money seemed to motivate several students from Year 3 to Year 9, for whom an interest in money was evident in their love of mathematics. This love of mathematics may help them keep track of their money and enable them to watch it grow. Several Year 3 students regarded their money as treasure and were reluctant to spend any of it. Other children viewed money as a tool—something to spend when necessary.

There is however, an exception at Year 3. This is also shown as a linear correlation in Figure 6, which indicates a correlation between non-context and context. Examination of the means, given as percentages in Table 12, reveal no real differences for Years 5, 7 and 9 for context and non-context. Even when Year 3 is included, the 21 percentage point total difference is not significant. However, examination of the means for Year 3 alone accounts for 20 out of the 21 points difference. This 20 percentage points gap for Year 3 is substantial and reveals a marked difference in favour of non-context items.

Table 12 indicates the number of students out of a possible 16 per year level who scored full marks for a particular item using the traditional basic scoring system (0, 2). As can be seen in Table 12, all students received full marks on four items. That is, both items 6.20 + 4.90 for context, and 79 + 26 for non-context both at Year 7; as well as double 26, for both context and non-context at Year 5, received perfect scores of 16. Analyses of student scores in Table 12 by year level, reveal that students' scores improved with age, particularly from Year 3 to Year 5. However, Year 7 students performed better than Year 9. The lowest average scores were for context items at Year 3, while the highest average scores were for non-context items at Year 7.

While there was no marked difference found overall for context, there were substantial differences for some items. Student answers revealed that they were more likely to get some items correct when presented in a non-context format, while other items were more likely to be correct when presented in a context format. The most notable of the items favouring a context presentation for Years 5 and 7 was the addition of decimals to two places.

Results from Figure 6 indicate individual variations within the data that might be explained by individual background differences in money experiences and prior knowledge of non-school taught mental methods. For this reason, the relationship between student performance and money experience rating appear in research question two while, the relationship between student performance and strategies appears in question three.
4.3.1 Correlation of Context and Non-context Performance

Presentation of the money context items did not result in an overall improvement in mental computation performance scores for students in Years 3, 5, 7 and 9. The data revealed however, that there were some variations between items, individuals, age and gender, and these are explored further in the research questions to come.

Results of student process scores for context and non-context items were converted into a correlation chart in Figure 6. This indicates the relationship for each individual student (numbered 1-64 on the graph) between their performance for context and non-context items. Figure 6 illustrates with an $R^2$ of 0.80, firstly a strong relationship to the fitted line; and secondly, it identifies out-lying or exceptional students. Individuals who did not fit this trend are numbered 17, 20, 27, 44 and 50 for whom context made a favourable difference. Students 17, 20 and 27 were Year 5s, while student 44 was a Year 7 and student 50 was a Year 9. These students were from all four schools in the study. Non-context items showed a preferable difference in performance for students numbered 5 and 7, both Year 3s. These students were from the same school. However, it should be noted that these students had
both moved schools. Student's basic scores were generally less but sometimes equal to their process scores for both context and non-context scores. More students received identical scores for context and non-context items using the basic scoring system. The correlation between context and non-context items using the basic scoring system appears in Figure 10 as discussed in research question four along with Figure 12 which reveals that the differences between the two systems was minimal.

4.4 Research Question 1

How is mental computation performance affected by the provision of a money-context?

In Figure 7, the same data as shown previously in Figure 6 is presented by amalgamating year level results for context and non-context in a bar graph. This is in order to identify any differences across the year levels for either context or non-context with a process scoring system. The average scores for context and non-context by year level in Figure 7 illustrate that Year 5 is the only year level for which context impacted positively on process performance scores. The amount of difference is however small, being less than ten percent.

Firstly, an overall data comparison of student performance for money-context items and non-context items shall be discussed. Table 12 listed the number of students whose items were completely correct for context and non-context by operation type, across all four year levels and using the basic scoring system (0, 2). Compared with the basic scores in Table 12, the Figure 7 data that used the process scoring system (0, 1, 2), revealed only a small variation across the year levels. This indicates that Year 3 was again the only year level for which student performance was markedly higher for non-context. Year level differences will be explored more fully in the fourth research question. Two students in Year 3 scored zero for both context and non-context using the basic scoring system. While using the process scoring system they both scored 15 for context and 5 and 10 for non-context.
4.4.1 Score Differentials

The data obtained from Table 12 indicates the number of students out of a possible 16 per year level who scored full marks for a particular item. This was true using either the traditional basic scoring system (0, 2) or the process system (0, 1, 2) as a correct score for both systems was awarded a score of 2.

As can be seen in Table 12, there were two items in each of Years 5 and 7 for which all students received full marks. These items were double 26 for both context and non-context in Year 5, and in Year 7, 79 + 26 for non-context and 6.20 + 4.90 for context. Analyses of student scores in Table 12 by year level reveal that students' scores improved with age, particularly from Year 3 to Year 5. However, Year 7 students performed better than Year 9. The lowest average scores were for context items at Year 3, while the highest average scores were for non-context items at Year 7.

From Table 12, it was decided that the range of score differences as shown graphically in Figure 8 and numerically in Table 13, should be further examined due to variations in the data for individual items. Particular items were identified for which either context or non-context may have had a particular effect on student mental computation performance. These are given in Table 14, and are more fully investigated by strategy choice in the third research question.
Firstly, the maximum discrepancies in scores identified from Table 12, reveal which items were the items of most difficulty between presentation formats. Table 13 shows that the highest non-context score difference was 9 compared to the highest context score difference of 5. It was decided that any items with a score difference of 4 or above should be examined in detail, as part of the analysis. This may result in a 'favourable difference' for either context or non-context. This was explored in order to examine the positive and negative effects of money as a context on individual items. The following section—items of most difficulty—discusses the maximum scores for items overall, in order to identify both which items and which format proved most difficult for students.

Table 13: Score Differences for Context and Non-context Items

<table>
<thead>
<tr>
<th>Better Performance</th>
<th>Score Difference</th>
<th>Year 3</th>
<th>Year 5</th>
<th>Year 7</th>
<th>Year 9</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-context</td>
<td>9</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Non-context</td>
<td>6</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Non-context</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Non-context</td>
<td>3</td>
<td>4</td>
<td></td>
<td>1</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Non-context</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>Non-context</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Neither</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>Context</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>Context</td>
<td>2</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Context</td>
<td>3</td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Context</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Context</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

With regard to improved scores for context, three items were found to show a 'favourable difference' for context. Table 14 lists these items as 6.20 + 4.90; 74 - 30; and 38 x 50. The two items found for Year 5 in order of this difference were the addition of decimals 6.20 + 4.90 (score difference of 5), and the subtraction 74 - 30 (score difference of 4). As with Year 5, the addition item 6.20 + 4.90 also proved the most differential (score difference of 4) item for context in Year 7. Therefore the item that was found to have the most
'favourable difference' for context presentation for both Year 5 and Year 7 overall, was $6.20 + 4.90$. This item involved the addition of an equal number of decimal places with 'carrying'.

As discussed previously, the most marked shift from non-context performance to improved context performance occurred between Year 3 and Year 5. From Table 14, one item made the change from a score differential of difficulty of (4) for non-context at Year 3 to an equal

Table 14: Items with ‘Favourable Difference’ by Year Level and Format

<table>
<thead>
<tr>
<th>Context</th>
<th>Non-context</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 5</td>
<td></td>
</tr>
<tr>
<td>Mum spent $6.20 in the bakery then she spent $4.90 at the newsagent. How much did she spend altogether? (5)</td>
<td>Half of 30 is? (9)</td>
</tr>
<tr>
<td>Amy’s brother earned $74 in his part-time job. He gave his Mum $30. How much did he keep? (4)</td>
<td>$79 + 26 (4)</td>
</tr>
<tr>
<td>Year 7</td>
<td></td>
</tr>
<tr>
<td>Mum spent $6.20 in the bakery then she spent $4.90 at the newsagent. How much did she spend altogether? (4)</td>
<td>$74 - 30 (4)</td>
</tr>
<tr>
<td>Year 9</td>
<td></td>
</tr>
<tr>
<td>What is the total cost of 38 Harry Potter cards at 50 cents each? (4)</td>
<td>$150 + 25 (4)</td>
</tr>
</tbody>
</table>

Note: The numbers in brackets are the ‘Favourable Differences’ (from Table 13).
4.4.1 Score Differentials

The data obtained from Table 12 indicates the number of students out of a possible 16 per year level who scored full marks for a particular item. This was true using either the traditional basic scoring system (0, 2) or the process system (0, 1, 2) as a correct score for both systems was awarded a score of 2.

As can be seen in Table 12, there were two items in each of Years 5 and 7 for which all students received full marks. These items were double 26 for both context and non-context in Year 5, and in Year 7, 79 + 26 for non-context and 6.20 + 4.90 for context. Analyses of student scores in Table 12 by year level reveal that students' scores improved with age, particularly from Year 3 to Year 5. However, Year 7 students performed better than Year 9. The lowest average scores were for context items at Year 3, while the highest average scores were for non-context items at Year 7.

From Table 12, it was decided that the range of score differences as shown graphically in Figure 8 and numerically in Table 13, should be further examined due to variations in the data for individual items. Particular items were identified for which either context or non-context may have had a particular effect on student mental computation performance. These are given in Table 14, and are more fully investigated by strategy choice in the third research question.
the 'favourable difference' had found in favour of context. For two of the three most differential Year 5 items, the opposite effect to Year 3 was found as both items had higher scores for the context presentation. The third was the division item $150 + 25$ with a 'favourable difference' for non-context of (4). As can be seen in Figure 8, two items were most differentiated for Year 7. The division item $150 + 25$ was the most differentiated item for non-context at Year 7 (6) and Year 5 (4).

4.4.2 Items of Most Difficulty

Table 12 also indicated which items proved most difficult for each year level. This is interesting when compared to similar results in the McIntosh et al (1995a) studies. Both studies concur that Year 3 students found the subtraction items $105 - 26$ and $80 - 24$ very difficult, while some variation occurs for the other items. The study here found that only two students were able to solve one of these two subtraction items for context. Both items involve decomposition and it remains to be seen whether the choice of whole numbers presented as dollars rather than as cents ($1.05 - 26$ cents) may have been a difficulty. This is suggested because it is most unlikely that amounts of $105$ would be within any eight-year-olds' experience.

This issue was considered before the pilot study. However, the place value change may have resulted in several other difficulties. First, students may round the amount of 26 cents to the nearest five cents as happens in realistic shopping experiences. Second, if the items (26 cents and 24 cents) had been changed to end in a 5, this would alter the ability to make comparisons with McIntosh et al's (1995a) study. Further, students would not then have the full range of numbers to work with. Thirdly, if the Year 3 context item appeared as $1.05$, the non-context item would need to appear as 1.05 to maintain consistency and this would not be appropriate because Year 3s are not expected to work with decimals. Therefore, it was decided to use the numbers as whole dollars.

The previous explanations however, do not discount the fact that Year 3 students have had school experience working with operations to hundreds for non-context items whereas they may have only had in- and out-of-school money-context experience with lesser amounts. For example, they may bring $2$ to school to make a purchase (such as for a chocolate Freddo frog) without ever needing to see $2$ as equivalent to 200 cents, if no change is involved. In this case, each dollar appears merely as a whole number. By contrast, when students give or receive change they are making judgements of equivalence and these experiences help to
develop a relational understanding of money. Level 2.3 of the ‘Calculate’ sub-strand of number (EDWA, 1998, p. 189) lists an example of operating with hundreds of dollars by either subtraction or addition on a calculator to find the difference between $180 and $125. Money operations are usually presented to younger students for smaller amounts such as $3.05, which as hundreds of cents emphasises equivalence and place value.

Reasons why context may have made a difference for Year 5 might be explained if they were found to have more money experience. This will be examined in the second research question for which a money rating scale was devised. One Year 5 teacher commented that she noticed that by Year 5 students revealed a more personal interest in money compared to the younger students’ awareness. This interest in money was illustrated by statements made by several Year 5 students, one commented that ...counting money was better than doing sums, while another stated, ...money is more funner. As RME principles suggest, the students should be able to imagine the context—in this case, hundreds of dollars. This was illustrated by several students who indicated that they had calculated a savings plan for family entertainment toys that can cost hundreds of dollars, such as an X-box, or Play Station II.

4.4.3 Summary

Overall there was no difference found using the process scoring system, in mental computation performance for money as a context for students in Years 3, 5, 7 and 9. Rather, for Year 3, it was found that students performed somewhat better with non-context items. This has been explained as possibly due to the size or magnitude of the numbers involved in the items, such as hundreds, not matching their ‘fields of experience’ with money. Other reasons why non-context made a difference to performance for Year 3 might include the types of strategies used were of a lower level of sophistication such as counting on in ones, and perhaps the range of strategies was narrower. This will be examined in the third research question. Year 3 students may have scored better for non-context because they are unused to school mathematics being presented in a money context in word format. Either they may be unfamiliar with money (lack of experience), or they may be unfamiliar with the operation concepts in out-of school situations (lack of need).

The Year 5s performed slightly better in context than non-context, though again, not significantly. Perhaps this was, as suggested by one teacher, because students have developed ‘money consciousness’ by Year 5. This could be illustrated by the fact that although they did not personally have hundreds of dollars, many seemed to be aware of how
much value this amount of money has. For example, Year 5 was the year level where students first mentioned specific knowledge of pricing. The Year 7s performed marginally better at non-context. Again, this was not significant. By Year 9, there was no difference between the performance scores for context or no-context. The amount of improvement from Year 3 to Year 5 could also be due to improved mental computation strategies. This will be discussed in detail in the third research question.

4.5 Research Question 2

_How does mental computation performance relate to students' prior experiences with money?_

4.5.1 Money Experience Rating Scale

Values were allocated on the rating scale once the interview data had been collected. This data was analysed carefully after transcription to identify common themes. Values were allocated to the students' answers to the nine money experience questions in order to develop the money experience rating scale. All student answers were assigned values from $\frac{1}{2}$ a point to $2 \frac{1}{2}$ points value resulting in overall rating scores of 1, 2 or 3 as illustrated in the rating procedure given in Appendix III.

4.5.2 Money Rating Overviews

It was considered that 'working the till' or cash register would provide a more sophisticated experience than collecting money for fundraising. For example, it is more likely that students would receive practice in calculating change by using shop-keeper's addition (complementary addition) when working in a shop, than when fundraising which usually involves collecting people's loose change and donations of whole dollars.

The values used for the rating scales are given in Appendix III along with the procedure. The rating obtained represents the total of all values assigned to a particular student. To earn a rating, a student's total money experiences were given values ranging from a value of $\frac{1}{2}$ to a value of $2 \frac{1}{2}$ which were then added to give a total value for that student. Where a total value resulted in a $\frac{1}{2}$, the rating was rounded up so that a total value of $2 \frac{1}{2}$, would rate a 3.

Overview of a Rating 3

These students revealed a high level of exposure to real-world money experiences, such as a paid part-time job. In Appendix III, this is valued at $2 \frac{1}{2}$. In most cases, this involved working with money.
Examples of pupil responses rating a 3 included:

At the surf shop, I help at the till and use the calculator to check. I buy bigger things. I'm into investments. Unless there was something I wanted, I'd save it in the bank towards investment. My Dad's an accountant and my financial advisor. Counting money is better than sums. I don't get pocket money. I'm self-sufficient.

Generally, these students had collected money, given change, or 'worked the till', usually helping at a family-run small business and received wages. Some students were 'self-managing' worth 2 and were responsible for their own 'needs', such as clothes, school items or investments. Two students mentioned saving to invest in shares while one mentioned saving for a specific costly item—a violin. Regular pocket money or an allowance was available weekly or fortnightly, and this was worth 1/2. The students may have had some fundraising experience 'working with money' also worth 1/2 such as giving change or collecting money, either at school or for a sporting club. Therefore, a student who was classified as self-managing, received regular pocket money and mentioned having had some experience with money would get an overall rating of 3.

Overview of a Rating 2

These students may have worked with money, but had no part-time job nor were they self-managing. Their income was balanced between saving and spending mostly on wants. Students generally demonstrated 'planned saving' to save-up to spend on 'expensive' items. Their parents generally encouraged saving. School experiences may have included a class shop or worksheets.

Examples of pupil responses rating a 2 included:

I have to work for money and I get $2 every week. I have to earn it all. I save it up so I can buy something that's really expensive. Probably some clothing. I would keep on saving it up so I can go on a shopping spree with my friend. I enjoy when we get tests about money. If you need to add up something like 50 + 160 or something, you could just add it more quickly. Because if it sounds like money it sounds more funner.

Overview of a Rating 1

In this category, the students' answers appeared vague or unrealistic. Students either saved everything, with no immediate 'real' purpose or spent their birthday/tooth-fairy' money. Mostly, students saved for the sake of forming the habit. Students mentioned low-level school experiences, for example, "little", "none", "not enjoyed" or "not remembered".
Examples of pupil responses rating a 1 included:

I keep it in my purse saving for a dog. If I had a big backyard, I'd buy a horse.
Yes, at home. I'm saving for a dog. No, they won't let me spend it all the time.

Worksheets, pictures of money.

Table 15 illustrates the mean scores for students on the money experience rating scale by year level. As can be seen in Table 15, a money experience rating score of 2 was the most common score for students across all four year levels. All year levels had at least three students with a minimum score of 1, and at least two students with a maximum score of 3.

<table>
<thead>
<tr>
<th>Year</th>
<th>Score Rating</th>
<th>Frequency</th>
<th>Percent</th>
<th>Mean Rating</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>7</td>
<td>44</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7</td>
<td>44</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>16</td>
<td>100</td>
<td>1.7</td>
<td>0.70</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>4</td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4</td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>16</td>
<td>100</td>
<td>2.0</td>
<td>0.77</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>3</td>
<td>19</td>
<td></td>
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<td></td>
<td>2</td>
<td>11</td>
<td>69</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>16</td>
<td>100</td>
<td>1.9</td>
<td>0.57</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>4</td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>9</td>
<td>56</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>16</td>
<td>100</td>
<td>1.9</td>
<td>0.68</td>
</tr>
</tbody>
</table>

The highest mean rating of 2.0 was achieved by Year 5 and this year level had the greatest number of students assigned a money rating of 3. The fact that Year 5 received the highest money rating average of 2.0 may offer some explanation as to why context made a small difference for Year 5. In fact, the biggest improvement for context performance as was shown previously in Table 12, occurred from Year 3 to Year 5 with an increase of 30 percent, from 30 percent to 60 percent. This is consistent with the largest increase in the means from Year 3 to Year 5. Table 15 results may suggest that Year 5 would be expected to have performed slightly better than Year 7 and Year 9 because their average money experience rating was slightly higher. This was not the case, however. Neither does the money
experience statistical mean data explain the instance of Year 9 students achieving lower scores for identical test items compared to Year 7, when both year levels scored the same mean. What can be surmised however, is evidence to support one Year 5 class teacher’s claim that students become ‘money conscious’ at Year 5, and that this may be due to their increased experiences with money. The overall average money experience rating for students from Years 5 to Year 9 shows virtually no difference. This suggests that there were no significant factors of difference for students in these age groups. A significant factor that may affect performance was considered a part-time job that both earns money and involves money such as in pizza delivery or work in a shop. Although some students reported these activities had occurred, only two students reported that they were a regular activity for them.

Figure 9 indicates the relationship between the money experience ratings allocated and performance for context, non-context, and both. The performance scores were obtained by using the three-point process performance scale. A comparison between the process and

![Figure 9: Relationship between Money Experience Rating and Performance](image)

basic scoring systems is discussed in the fourth research question. Figure 9 reveals there was virtually no variance due to money experiences for either context (0.0366 or four percent), or for non-context (0.0172 or two percent) as indicated by the lines of fit. The line of fit for both falls in-between with 0.0276 (three percent).
4.5.3 Factors Influencing Money Experience

Some students mentioned a ‘significant other’ from whom they had requested or received financial advice; usually this mentor was a parent or a grandparent.

One of the money instrument questions (No. 7) asked, “How often do you talk to your parents about money?”, and while this was generally answered in the negative, answers to other questions revealed otherwise. For example, “I don’t spend my money unless they approve”, or “I bank my money” (this was particularly the case with the younger children). When parents or grandparents were reported as discussing money with the students it needed to be differentiated as to whether it was a formal or informal nature of instruction. When a relative is involved in the instruction process, this is reminiscent of Lave and Wenger’s (1991) midwife apprentices.

One reason it might be expected to find improved performance for a set of mental computation items when set in context compared to non-context presented at the same sitting might be due to the students’ prior experiences with that context. As Saxe (1991) claimed: “children who participate in different practices develop more sophisticated cognitive forms and functions linked to those specific practices” (p. 131). In this case, improved performance for context included those specific practices such as working with money, receiving pocket money, either saving money or spending it. It also included in-school and out-of-school experiences such as fund-raising activities where the collection of money and giving change was involved and in a few cases, the serving of customers in a shop. Why would students perform poorly for both sets of items, or less well for context, or even use identical strategies for both context and non-context items? This was more likely to occur if the student had had little money experience compared to having had lots of school standard written methods experience, in which case the context may be much less important.

According to Jones et al (1994):

Calculations in everyday employment tasks, must of course be performed accurately and efficiently...Many small places of business however do not have these sophisticated facilities and you will have experienced diverse computational methods...(p. 9).

By sophisticated facilities, the authors are referring to electronic cash registers, as older style registers are unable to calculate the change, only provide a record for the total of the transaction. Examples of this were noted in this study, when some of the students who worked in shops explained how they used a calculator to check transactions. One Year 5
A summary of this in response to money experience question 3, follows:

- Purchasing of faction prizes (1).
- Fundraising or low-level money activities (1).
- Sausage sizzle (2), Freddos (2), pig-out-pizza (1), 'smart snacks' (1), raffle tickets (1).
- Counter, or higher level money experiences, which may include a calculator to check (3).
- Gift shop (1), Mum's shop (4), Grandmother's shop (1), Tuckshop (1), Swap meet selling (1) and pizza delivery requiring counting money and giving change (1).

Question 9

How do you think that working with money helps you learn maths? This question was not used as part of the money rating procedure, as some student answers were somewhat confused, and many students answered this question the other way around. For example, one Year 7 commented, you have to add up—if you don't have a cash register, you'd have to add up the money; and another, if you work in a shop you need to get the money right.

These are more suggestive of examples to show how mathematics helps with working with money. Perhaps, for some students, money is mathematics, as mathematics often appears in a money context. Students' answers to this question will briefly be discussed in view of the information given generally.

Year 3 Reasons

Table 16 summarises the Year 3 responses to Question 9. An interesting response included:...teach you how to count in hundreds and each $2 is two hundred cents...This statement indicates that this student was already aware of the equivalent values of dollars and cents. Another response was:...help me count by my 3s and 4s and times tables. While dollar coins could be used to count by threes and two two-dollar coins to count by fours as
the student states, it is unlikely that money would be used for this purpose. It is more probable that the two-times and five-times basic facts are suited to using money by using two-dollar coins and five-cent pieces.

Table 16: Year 3 Student Responses to Money Experience Question 9.

<table>
<thead>
<tr>
<th>Categories</th>
<th>Examples of Student Reasoned Responses</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vague</td>
<td>I don't know or yes or reference to class experiences</td>
<td>4</td>
</tr>
<tr>
<td>Counting</td>
<td>Because you count money</td>
<td>3</td>
</tr>
<tr>
<td>Operations</td>
<td>Take away and add-up</td>
<td>2</td>
</tr>
<tr>
<td>Real-life</td>
<td>If you had a dollar, you might have to add three dollars</td>
<td>2</td>
</tr>
<tr>
<td>Importance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equivalence with</td>
<td>Because money is like numbers</td>
<td>5</td>
</tr>
<tr>
<td>non-context</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The only operations noted at this year level were addition, subtraction, and one student who mentioned basic facts as repeated addition. It is rewarding to see that a majority of students at this year level noted the relationship between non-context numbers and our decimal money system as being equivalent.

Year 5 Reasons

Interesting responses at Year 5 included:

Adding up 50 plus 160, you could add it up more quickly. It's more funner. If you're going to count money as an adult in your job... 'Cos [because] of the numbers and sharing out into piles.

In Year 5, reasons given for why working with money might help in the learning of mathematics included: not sure; job related; improved operations; equivalence; giving change; going shopping; sharing it out; take-away and counting. Of these, the most significant responses were equivalence and giving change, which suggest thinking that is more sophisticated.
Year 7 Reasons

Interesting responses at Year 7 included:

Because it’s numbers but in dollars. Adding, subtracting and dividing helps with the basic skills in maths. You switch on a bit more because you get it. Teaches you to add decimals. It helps you with decimals because money is decimals. You’re better at working it out in your head because you’ve got to give them the right amount.

This year level still had some students who were not sure or non-committal (two respondents). Other reasons given included: decimals (two respondents); giving or receiving change in a shop (two respondents); job related; helps with equivalence with non-context numbers; improved operations (addition, subtraction and division); incomes; rounding; relevance; improved mental mathematics performance; and worksheets.

Year 9 Reasons

Interesting comments included:

You’ve got to get it [the change] right. You can spread more out in your mind...keep track of how much you save in your bank, how much in your wallet. So you can spread things out in your head — so it helps you with sums. It’s got numbers. It’s a different way of learning, if it’s just on a [work] sheet its boring, but a tally is fun. It helps you understand easier.

By Year 9, answers given were more varied and diversified with one student mentioning percentages. The reason the majority of students gave was to check that change given was correct (five respondents) with one student terming this activity ‘interactive’ due to the immediacy of feedback. This response however, suggested that students might view mathematics as limited in relevance to money. The next most common reason for working with money was improved ease of performance and speed at mathematics generally (three respondents). Other responses included adding (two respondents); decimals; informed spending decisions; counting; subtracting; equivalence; and as a fun way to learn.

Overall, most students attempted all items. Several of these items were considered doubtful for how money helped with mathematics concepts, because they seemed to be relating working with money to understanding money. Answers indicated that most students understood money equivalence and valued money as a context for learning about rational numbers including decimals and percentages and operating on them.
4.6 Research Question 3

*How does mental computation performance relate to the levels and types of mental strategies used?*

This question sought to find out whether students would use different strategies for different items when presented differently—in context and without. For example, were students more likely to use their own strategies for the money context items yet use school-taught methods for non-context items. If so, would their own strategies be more effective, indicating higher levels of understanding and number sense?

Originally, it was assumed that students would achieve better scores for context items because it was believed that they would use more efficient, higher order strategies as previously categorised in Table 11. It was assumed that the use of better strategies would result in higher scores. It was considered that due to the young age of some of these students, they may be experiencing 'novice errors' using newly acquired efficient strategies incorrectly. Data on the strategies were compiled from the transcripts by identifying strategies per student and per item. Some students employed more than one strategy per item, in which case the more sophisticated strategy was chosen to be representative.

Analysis was made on several fronts. A first analysis was for items that stood out as receiving better performance scores for context. A second analysis examined the strategy use of the Year 3s in order to see why non-context achieved higher scores. To see whether strategy use/choice could explain this. A third analysis was made of the strategies for items found to have markedly differential scores as previously shown in Table 13. A fourth analysis was for items that received identical scores (shown as zero) for differentiation; to check if strategies used were identical. Although student scores were identical, no assumptions were made regarding student use of identical strategies. However, when the presentation formats were different, different strategies could have been in play.

4.6.1 Differential Items, Differentiated Strategies

The most difficult items were found according to their differential scores in Table 12. These were then examined by strategies, for differences in use, choice, and range. Strategy codes are given in Appendix IX categorised together with their parent code. For example, DH (used doubling/halving) and P (used pattern), both fit under R (used relational knowledge).
Improved Performance Items—Context

Both items including whole numbers and non-whole numbers resulted in some improved performance for context. Items identified previously as having either a 'favourable difference' for context will now be examined as to the strategies used, in order to explain these differences, in the sections below.

6.20 + 4.90 (Years 5 & 7)

The same students who were successful for this item presented in a money context, however, did not apply their knowledge of money to the non-context item. Instead, they chose to use the written method of addition mentally. For example, one Year 5 student: who was correct for context, stated: *Six plus four is ten dollars, then nine (ty cents) plus ten (cents) is another dollar so that's eleven (dollars), and then there's ten cents left over*. In contrast, for non-context, this student was confused with the place value: *Six plus four is ten, then two plus nine is eleven, and you can only go up to ten... I think... and so you add the ten on*. So 20.1.

Both methods were classified as working from the left.

In Year 7, one student stated *not sure, a guess* for this item in non-context and gave an incorrect answer of 1.11, while being completely correct for context, stating:

> I put 10 (cents) off the six twenty to make six ten, and added up the ten to the four dollars ninety to make five dollars, and added them both together and then I put the dollar signs on.

74 – 30 (Year 5)

For non-context, one student stated: *Because 30 take away 70 equals 40. But you take away the 4 from the 40, that equals 26*. This student's explanation showed errors both conceptually and procedurally by subtracting the four and a basic fact error to the value of ten. By contrast, for context, this student explained: *Because 30 take 70 equals 40, plus the four equals 44 dollars*.

38 x 50 (Year 9)

One Year 9 student made no attempt at this item for non-context, yet got it correct for context, stating: *76 dollars, no 76 cards, no 76 dollars... doubling that... 19. 19 dollars, halved that*. This explanation indicates the student was able to think about the reasonableness of the answer, the unit involved and self-correct confidently. Knowing that 50 cents is half of one dollar and then just halving it shows number sense was used here.
Another student who got this item incorrect for non-context explained: 1700. I found... worked out how many 50s were in ten, three times 50, then eight times 50 and added. By contrast, for context, the student explained: I halved it. Because it'd be $38 if they were one dollar each, but it's half of a dollar. This explanation indicates that the student had employed number sense for the item in context by choosing a halving strategy, while for non-context the student had attempted to use the long multiplication algorithm mentally.

Improved Performance Items—Non-context

Items identified in Table 12 as having either a 'favourable difference' for non-context were as follows.

79 + 26 (Year 3)

One Year 3 student employed different strategies for the context item giving the incorrect answer of $100, while for the non-context item the same student gave the correct answer of 105. A choice of different strategies might explain the difference in answers. For context, this student explained: I covered up the 2 and the 7 and just added 6 to 9, and I put the one up on the tens here, and added the 2 and 7 and one together, and got a hundred dollars.

This strategy resembles the written algorithm by working from the right, and 'carrying'. By contrast, for the non-context item, the same student explained: I had 79 and counted up by 1s to 20—ten twice—and then counted six on.

The student was noted to be using fingers, and strategies were classified as counting by tens and ones. Although counting strategies are generally considered inefficient, this student's results show they were more successful for this item than written algorithms used mentally.

74 - 30 (Year 3)

One Year 3 student made no attempt at this item in context, while for non-context, the same student used 'mouth-counting' and explained: I'm adding up. I had 74 and started counting until I took ten away, thirty times. I mean, three times.

Confusion regarding the use of money as a context was indicated by one Year 3 student who explained: So it's not 44 dollars, it's 45 dollars...because you can't get 44 cents, you can only get 45 cents. The context was problematic only for Year 3 students. This same student gave the correct answer of 44 for non-context by explaining you take the three from seven, which is four.
150 + 25 (Years 3 & 7)

One Year 5 student solved this item using mostly known facts, but indicated some confusion with the money context giving answers of 6 and $60. For non-context this student explained: I know that half of 50 is 25 and there's two 50s in 100...there are four 25's in 100 and there's a 50, so you add the 4 and the 2.

Strategies used in the above quotation included doubling/halving (DH), and relate to a known fact (RKF). For context however, the same student explained: 25 into 100 is 4 and 25 into 50 is 2. So 2 add 4 is six, add a zero and a dollar sign. While this also seems to employ the same strategies, it seems that the 'remove zeros' rule (RZ) had been applied incorrectly. In Year 7, one student made no attempt for context commenting: You have to work out how many 25s are in 150. I'll just leave it. Interestingly, one Year 7 student explained for context that, I made 150, made that $1.50 and that 25 cents...How many 25 cents in $1.50?...and that's six. However, for non-context, the same student explained, I counted up by 25s, until I got to 150. This is an example of using the strategy, and relating the item to known facts (RKF). This student used the same strategy differently for non-context, stating: 4 (25s) equals 100 and another 2 equals 6.

4.6.2 Strategy Use in Year 3

Year 3 strategies were examined in order to find out why the non-context items achieved higher scores. Students used mostly counting strategies for both presentations.

Analysis was made of items that had received identical scores to check if the strategies used were also identical. Where a difference of zero was recorded, perhaps transfer had occurred between the two presentations. If so, the direction could indicate which environment, school or out-of-school, was having the most impact. Zero difference items were sourced from Table 13, where nine items across all four year levels were identified as shown in Table 17.

It was decided to analyse the nine items systematically, by examining each full successful student's response, but to only report on any marked differences in the strategies that were used. Generally, most high scoring students who recorded identical scores on the two tests used the same strategy for both items. This may indicate that transfer is more likely to take place once a higher order strategy is established.

The overall range of strategies used per item was found to be a maximum of four, and a minimum of one. Bana and Korosky (1995) found that "most of the successful students had
Table 17: Items with Identical Context and Non-context Performance Scores

<table>
<thead>
<tr>
<th>Item</th>
<th>Year 3</th>
<th>Year 5</th>
<th>Year 7</th>
<th>Year 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>68 + 32</td>
<td>*</td>
<td>14</td>
<td>14</td>
<td>/</td>
</tr>
<tr>
<td>165 + 99</td>
<td>/</td>
<td>/</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>6.20 + 4.90</td>
<td>/</td>
<td>/</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>3500 + 35</td>
<td>/</td>
<td>/</td>
<td>13</td>
<td>13</td>
</tr>
</tbody>
</table>

Note: / indicates that this item was not available at this level.
* indicates that this item had differential scores.

only one strategy” (p. 26). This may be because the one strategy was the most efficient one for them. However, student knowledge of a range of strategies may not be obvious, but may be implied, if students chose only the most sophisticated ones. The use of only one strategy should result in improved performance because fewer steps are involved reducing the ‘opportunity for error’. This can be illustrated by the item, Half of 16 is? for Year 3. One student arrived at the answer of 13, because she had halved the six and then added to the ten without halving it. A second Year 3 student arrived at an answer of five, because she had halved the ten only. The strategy they were attempting required halving the tens and units separately then adding these together. Some Year 3 students were able to do this successfully. The best strategy for this item required the knowledge of basic facts such as two eight’s are 16 along with the understanding of the relatedness of the processes.

McIntosh, Reys and Reys (1997a) suggested that visual presentation might encourage students to adopt written strategies. It was considered therefore that the oral presentation of items along with the visual presentation might have been more encouraging for students to use mental computation methods rather than mental versions of standard written algorithms. Although items were presented both orally and visually, the strong tendency for some students to use school-taught written methods suggests the effects of visual presentation can not be discounted. Perhaps the visual presentation, just as the pencil, may act as a subconscious cue to think ‘school methods’, despite the student being advised to choose any method they wished. Alternatively, perhaps it was the school setting or the social
environment of being interviewed individually and questioned by an unfamiliar person. Alternatively, perhaps those who chose to use standard written algorithms did so because that is the method most familiar to them. With regard to those students who relied on school-taught methods, it is of note that Anthony and Walshaw (2003) found that:

while most students appeared quite confident with solving the problem with reference to the context, several of those who used the formal mathematics of the classroom rather than their own informal knowledge, appeared confused by fraction rules and procedures—and in some instances appeared unconcerned with their nonsensical answers. (p. ii)

Field notes made of observable body language and non-verbal actions found facial expressions; finger counting; ‘mouth counting’ reciting and pencil use to ‘air-write’ algorithms as indicators of student strategy choices.

A comparison of strategy use between two Year 9 students revealed that although the students had achieved identical performance scores for context and non-context items, one student had used higher order mental computation strategies. He had rushed his work, and consequently made careless mistakes. The second student used standard written algorithms mentally and tediously. She was meticulous and thorough in checking her work, although used the same method again, and therefore did not demonstrate flexible methods. Unfortunately, this process meant that she ran out of time and was unable to answer all items in the time allowed so could not achieve full marks, although all of her answers were correct. The fact that these two students’ achieved identical scores for context and non-context may explain why their teachers had equated them as students of the same ability level. However, the interview process that required them to explain their methods illuminated the difference in both their conceptual understanding and their work habits. This illustrated that ability level and achievement levels are different measures. The information gained at the interview indicated that the two students required two quite different interventions. The male student could be seen as underachieving due to poor work habits—that is not checking his work—whereas the female student was also underachieving due to inappropriate method use. If the basic scoring system (0, 2) had been applied to the male student’s answers, he would have scored worse because the process scoring system allowed him to get part scores for being correct in his choice of method. However, if the criteria of speed were removed for the female student she may have achieved better scores.
4.6.3 Strategy Use Summary

Overall, the strategy levels used generally reflected that when students used either lower or middle level strategies, such as counting and even known facts, their answers were often incorrect. Conversely, when students used higher order strategies such as relational knowledge and place value related knowledge their answers were more likely to be correct. Further, when students used higher order strategies, they were generally more definite about their answers, gave more detailed or better explanations, and were able to self-correct an original incorrect answer upon reflecting on the strategies that they had used. The Year 3s did not display a wide range of mental computation strategies. This was one indicator of a low level of number sense. The level of strategies used by Year 3 students was also predominantly low level, as evidenced by finger counting and counting aloud or ‘mouthing’. Because the range and sophistication of strategies increased with student age, this shows that progress towards efficient strategies and increased number sense is developmental.

4.7 Research Question 4

To what extent is year level a factor in mental computation performance for money-context and non-context items?

4.7.1 Items Common across Year Levels

Table 18 includes results for common items for both context and non-context listed and grouped across the year levels. These results show that student scores improved greatly from Year 3 to Year 5. The item showing greatest improvement was 74 – 30 with a difference of 11 for the context item. In Year 3, only four students were correct for this item for context, while by Year 5, fifteen students were correct for context. Generally, it can be seen that Year 9 students performed better than Year 5 students did. However, for the context item 6.20 + 4.90, exactly the same number of Year 5 students as Year 9 students were correct, with the best score with all students correct being achieved at Year 7.

Table 18 also shows that there was great improvement from Year 5 to Year 7. The items with the most improvement were 3500 ÷ 35 and 60 x 70. Errors for these items were found to be errors of place value and therefore conceptual errors. In some cases, students had
incorrectly applied rules relating to ‘adding zeroes’. This suggests that place value may be better understood by Year 7, along with a growing sense of the magnitude of numbers which is a component of number sense.

Table 18: Number of Correct Responses to Items Common across Year Levels

<table>
<thead>
<tr>
<th>Item By Operation</th>
<th>Year 5 (N=16)</th>
<th>Year 7 (N=16)</th>
<th>Year 9 (N=16)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>NC</td>
<td>C</td>
</tr>
<tr>
<td><strong>Four Year Levels</strong> (3, 5, 7, 9)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>79 + 26</td>
<td>4</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>105 - 26</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td><strong>Three Year Levels</strong> (5, 7, 9)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>165 + 99</td>
<td>9</td>
<td>9</td>
<td>14</td>
</tr>
<tr>
<td>60 x 70</td>
<td>2</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>7 x 25</td>
<td>7</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>150 + 25</td>
<td>5</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>6,20 + 4,90</td>
<td>13</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>3,500 + 35</td>
<td>6</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td><strong>Two Year Levels</strong> (3, 5)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 + 80</td>
<td>9</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>68 + 32</td>
<td>6</td>
<td>9</td>
<td>14</td>
</tr>
<tr>
<td>74 - 30</td>
<td>4</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td><strong>Double 26</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38 x 50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 x 45</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25% of 48</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: C = Context; NC = Non-context

Although Year 7 and Year 9 students were presented with identical items, Table 18 shows that Year 9 performed less well for some items with the greatest difference being for the item 150 + 25 for non-context. In fact for this item, the Year 9 score of eight students correct was less than the Year 5 score with nine students correct. Items with the greatest decrease in performance from Year 7 to Year 9 were 3500 + 35 for context and 0.1 x 45 for non-context. Both items had four fewer students correct. Two items with improved performance from Year 7 to Year 9 were 165 + 99 and 6 - 4.50 where both were presented as non-context.
These results suggest that it may be that Year 9 students are not getting as much classroom practice in mental computation either in context or straight computation because they are concentrating on other topics such as algebra. It may also be the case that as the primary school curriculum is often integrated, mental computation exercises may not be restricted to mathematics session times.

Figure 10 depicts trend lines for each student by year level using the process scoring system. It can be observed that as expected, Year 3s scored lower than Years 5, 7 and 9. However, it was not expected for Year 7 to have scored better than Year 9. Recent TIMSS study (2003) findings show that for Year 8 “maths is a boring subject because teachers make it so by setting repetitive, low level problems and encourage students to solve them by rote” (The World Today, 7/7/03). According to Jan Thomas of the Australian Mathematical Sciences Institute (The World Today, 7/7/03), teachers can not give good applications if they do not understand the material that they are teaching.

Figure 10: Context and Non-context Scores by Student and Year
This problem does not appear to be so marked at primary school. It may account for the performance drop-off at Year 9 found in this study. It may also be that at secondary school, less mathematics class time is spent on arithmetic as more mathematics class time is spent on geometry and algebra. Figure 10 indicates a propensity for students to score more highly for non-context although not significantly. The Year 3s scored more highly for non-context while the other three years are more balanced overall.

Importantly, some verbal responses did not seem to match written ones. This is where the interviews proved invaluable for uncovering the students’ thinking. Some Year 3 students appeared uncertain of written conventions such as where to write the dollar sign and placed decimal points unnecessarily, yet could verbalise the answer in the conventional oral form. These students’ answers were then scored as correct. This showed the importance of asking students to explain processes rather than rely solely on written results.

The researcher expected older students to perform better on the same item, but if standard written algorithm strategies were used mentally, older students’ performance may decline. However, if money experience has a positive impact on strategy choice by encouraging students to use their own developed strategies and display more number sense, then students should perform better with age, since relevant experiences should increase with age.

A high number of Year 9 student responses were noted attempting to do the mental computations in “...the school way...” (Jones et al, 1994, p. 11). While for some students in this study, their visual memory was able to cope with this method, for one Year 9 student, this proved to be a barrier that she was unable to overcome. At the end of her interview, the researcher asked her to choose just one question to do in the ‘written way’. This, as expected, was the ‘school-way’ and she proved to be successful. This student might be overachieving at school, especially since most adults use mental methods most of the time (Wandt & Brown, 1957; Northcote & McIntosh, 1999). With so much emphasis on written mathematics, teachers of this Year 9 student may be unaware of her lack of mental methods ability. While this may reflect a weakness in her working memory or ability to visualise, it also indicates a lack of flexibility with numbers and therefore a low level of number sense.

An example of the school-way being used for mental methods appears in Jones et al (1994):

...Five plus five makes 10, put down the zero and carry the one, one and two are three, then three add four is seven, and nine more makes...The procedure was long, complex and required the person to try and hold many pieces of information concurrently in short term memory. (p. 11)
Further, Anghileri (2000) stated:

Ginsburg (1977) suggests that mistakes associated with written methods are often based on rules that have been misapplied, for example always subtracting the smaller digit from the larger in subtraction. (p. 66)

An example of this was evident in the Year 3 results when performing multi-digit subtraction, when several students stated *six from five; you can't do*, so they subtracted five from six instead. Carraher et al (1985) stated that, "when paper and pencil were used in simulated market place problems, school-type symbols and routines interfered with the solving process" (p. 22).

A visual presentation in a horizontal format had been selected for both sets of items as is standard practice for contextualised word items. However, this is not always standard practice for non-context items and consequently, several Year 9 students suggested they had difficulty with the setting out of the items being in a horizontal format. This is consistent with comments made by Whitbread (1999) "often children taught to do sums vertically cannot do the same calculations when they are presented horizontally..." (p. 21). This suggests that the impact of school training through the emphasis on written methods has resulted in less flexibility for students' mental methods. Even when mental computation items are presented in word form written in a horizontal format, these same items are often demonstrated by teachers in a vertical format on the board, in order to make place value connections.

In Victoria, Groves and Cheeseman (1993) identified that very young children are capable of abstract thought; suggesting that experience, rather than the age factor may accelerate the shift to abstract thought. For the purpose of this study, it was assumed that older students would choose more sophisticated computation strategies, particularly in Years 7 and 9. This was because they should have had more experience with money and other real-world contexts such as sporting scores for both in and out-of-school activities. Actual results from the money rating scores revealed little difference in money experience levels across the year levels. Netherless there was a move towards the use of more sophisticated strategies, particularly for Years 5 and 7.

During the money experience interviews, some students reported receiving pocket money regularly. Most students stated that they needed to earn either all or most of their pocket money. Several students mentioned amounts of $2 a week. The amount of students' pocket money seemed to increase with the students' age, together with an increase in their
responsibility for managing it. One student stated; *we get half our age* so he (Year 5; aged 10) received $5 a fortnight while his sister (Year 3; aged 8) received $4 a fortnight. This is one example of a differentiated experience factor for age.

Figure 11 shows the relationship between student performances using process scoring for money-as-a-context items and their money experience ratings. The lowest ranked students for performance were four Year 3 students and one Year 9 student. Two Year 3s scored over 80 percent for their context items tests. The majority of students for all years scored between 50 and 90 percent. There was no correlation for money experience and context performance at Year 3 and several students with similar performance scores rated across the different experience levels. For example, three students scored fewer than 20 percent but rated at 1, 2 and 3 respectively for money experience. In addition, two Year 3s who scored over 80 percent for performance were rated at 1 and 3. The sample sizes used in the current study are too small to be able to generalize about these statistics. However, they are able to give a descriptive representation of an actual sample of students at that time.

![Figure 11: Money Experience and Performance by Year](image_url)

Figure 12 has been included to show the affect or not of using the three-point differentiated process scoring system (with one point awarded for use of the correct process but incorrect answer). This enables comparison with the traditional or basic scoring systems. Basic scores
were only awarded if the answer was totally correct. Theoretically, this difference could give an otherwise poor-performing student half scores if he/she made one error per mental computation item.

In Figure 12, the two scoring systems were compared for non-context only. It can be seen that the one trend line that does not follow the other three year levels is Year 3 with an $R^2$ of 0.4506, which can be regarded as a moderate correlation.

![Figure 12: Basic and Process Score Comparison for Non-context by Year](image)

Table 19 lists the correlation between the two scoring systems used in the study. Comparison between the basic scores from Table 12 and process scores for Year 5, Year 7 and Year 9 are very close to a perfect fit, with correlations of 0.96, 0.97 and 0.97. These scores show that there was no marked difference found for non-context performance using either the three-point process scoring system or the basic two-point scoring system. Therefore, either scoring system could be used for Years 5, 7 and 9 because they are virtually identical in terms of differentiating between year levels.
Table 19: Basic and Process Score Averages for Non-context Items by Year

<table>
<thead>
<tr>
<th></th>
<th>Process Score</th>
<th>Basic Score</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 3</td>
<td>12.5</td>
<td>10.0</td>
<td>0.67</td>
</tr>
<tr>
<td>Year 5</td>
<td>16.9</td>
<td>14.4</td>
<td>0.96</td>
</tr>
<tr>
<td>Year 7</td>
<td>19.8</td>
<td>17.7</td>
<td>0.97</td>
</tr>
<tr>
<td>Year 9</td>
<td>18.5</td>
<td>16.6</td>
<td>0.97</td>
</tr>
</tbody>
</table>

In Figure 13, the results of comparing the two scoring systems for context reveal an almost identical pattern to the non-context comparison. Again, we find that Year 3 has the weakest fit for context items. The study reported here found that the Year 3s’ measure of goodness of fit to be the weakest fit is consistent with previous studies by McIntosh et al (1995a). The strongest fit found in this study was for Year 9 with a correlation of 0.98 shown in Table 20.
From Table 13, the greatest two score differences were both found to favour context. First, the greatest difference of 4.8—between using either the process or basic scoring systems—was found to be in Year 3. Second was Year 5, with a score difference of 3.4. All other year level comparisons for both context and non-context found a difference of around 2 to 2.5 extra points when using the process scoring system. Year 3 and Year 5 for context, therefore, are two year levels for which using the process scoring system made a difference to the student score average of more than three points. Thus as expected, it is clear that giving credit for the correct process will boost the students’ final scores.

As Table 19 indicated, process scores for non-context items were higher than basic scores for non-context items across all year levels. This was also the case for context items as shown in Table 20. Table 20 also indicates the average money experience rating scores for each level. In addition, average money experience did not influence context performance when compared with the slightly higher results for non-context in Table 19.

Table 20: Basic and Process Score Averages for Context Items by Year

<table>
<thead>
<tr>
<th>Year</th>
<th>Process Score</th>
<th>Basic Score</th>
<th>R</th>
<th>Money Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 3</td>
<td>10.8</td>
<td>6.0</td>
<td>0.62</td>
<td>1.7</td>
</tr>
<tr>
<td>(20)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 5</td>
<td>17.8</td>
<td>14.4</td>
<td>0.92</td>
<td>2.0</td>
</tr>
<tr>
<td>(24)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 7</td>
<td>19.0</td>
<td>17.4</td>
<td>0.97</td>
<td>1.9</td>
</tr>
<tr>
<td>(26)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 9</td>
<td>18.2</td>
<td>16.6</td>
<td>0.98</td>
<td>1.9</td>
</tr>
<tr>
<td>(26)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 14 shows all four year level results using the process scoring system to compare context with non-context items. Years 3, 5 and 9 results follow similar trend lines and show correlation between context and non-context performance. One noticeable feature in Year 3 was the number of simple errors of basic facts made by students, which may be an age-related phenomenon. For example, one Year 3 student, when finding half of 30, stated:...and tried out fourteen but that equalled 29, so I did sixteen which equalled 31 and I tried 15 which equalled 30.

Figure 14 shows Year 7 to have the weakest correlation of 0.54 for context and non-context items. The correlations for Year 9 and Year 3 were both on 0.79, and for which reasonable conclusions regarding correlations can be drawn. These two year levels were the ones with the most students using standard written methods.
Another feature common to Year 3 was the incorrect language used to describe a correct mathematical action. For example, when explaining $68 + 32$, one student stated: *if you double the 8 and the 2, it makes ten and if you double the 6 and the 3, it makes 9. So, double that, 100.* This answer was scored as correct, as the process was correct despite the incorrect language. The language used should have been *added*, rather than *double*. In addition, the place values were abbreviated and confusing, as the actual process was, 60 and 30 makes 90.

Overall for age, no conclusions could be drawn for improved performance for context items and non-context items for the four year levels. This may be because the context provided for items was not relevant or realistic enough to the students. It appears that context can be a disadvantage for Year 3, but this seems developmental and only temporary.
4.8 Research Question 5

Are there differences between genders in mental computation performance for money-context and non-context items?

As illustrated in Table 21, Year 3 context performance by gender indicates a trend of higher performance for males compared with females using the three-point process scoring system. This was the only year level showing any marked difference for gender. The two top females scored 13 each, while the two top performing males scored 17 each. However, and more significantly, the three lowest scoring females all scored 3 each with the next three lowest females scoring 5, 7 and 9. By comparison, the lowest three males scored 10, 11 and 12 while the next three lowest scoring males scored 14, 15 and 16. However, these results may well be due to sampling.

Table 21: Year 3 Context Performance Results by Gender

<table>
<thead>
<tr>
<th>Process Scores</th>
<th>Females</th>
<th>Males</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3-5</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>6-8</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>9-11</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>12-14</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>15-17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Means</td>
<td>6-8</td>
<td>12-14</td>
</tr>
</tbody>
</table>

As can be seen in Figure 15, average money experience ratings by gender were higher for females in Years 3, 5 and 7. However, the gender differences for Years 5-9 are very small. In fact, they appear to be somewhat constant for all students across the year levels. Only Year 9 males rated more highly for money experience than females. It is noteworthy that two Year 9 males had had exceptional experiences with money. Schools may have been encouraging money experiences at Year 5, as the highest combined gender scores for money experience were found to be in Year 5. One Year 5 teacher commented that she had noticed
Figure 15: Average Money Experience Results by Gender
that students became interested in money at this age. One Year 5 student commented: *counting money is better than sums*; while a second student claimed: *money is more funner*. Overall, there was little difference in performance for gender.

Figure 16: Percentage Process Score Averages — Females
Figure 16 shows that females scored slightly better for non-context particularly in Year 3. However, for all other year levels there was virtually no difference for performance between context and non-context items. One explanation for there being a difference for females in Year 3 maybe due to their use of different strategies for context items compared to non-context items. Nevertheless, this might also apply to the males.

As shown in Figure 17, males also scored better for non-context at Year 3. In addition, at Year 7 and Year 9, there was virtually no difference for males between process scores for the two sets of items, consistent with the results for females. However, there appears to be a gender difference at Year 5, as females exhibited no marked difference in score, while males exhibited a slightly higher score for context items. This is the only year level for either gender where context received a higher score. This was not due to the males having a higher money experience rating, since both genders received identical money rating scores in Year 5 as shown in Figure 15.

Table 22 provides a summary of both Figure 16 and Figure 17. McIntosh et al (1995a) found that girls were “less inclined to take risks than boys” (p. 14), but that gender differences were not consistent across the year levels. This may explain why females in this study, were less likely to use invented mental methods and more likely to use school methods than males were—even those designed for written algorithms.
Table 22: Context and Non-context Average Percentage Process Scores by Gender

<table>
<thead>
<tr>
<th></th>
<th>Year 3</th>
<th>Year 5</th>
<th>Year 7</th>
<th>Year 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Money Context</td>
<td>72</td>
<td>72</td>
<td>80</td>
<td>75</td>
</tr>
<tr>
<td>Non-context</td>
<td>83</td>
<td>66</td>
<td>82</td>
<td>75</td>
</tr>
<tr>
<td>Females</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Money Context</td>
<td>35</td>
<td>77</td>
<td>68</td>
<td>67</td>
</tr>
<tr>
<td>Non-context</td>
<td>43</td>
<td>77</td>
<td>70</td>
<td>67</td>
</tr>
</tbody>
</table>

Process scores were used in Figures 18 and 19. Figure 18 shows that mental computation performance for context did not vary for gender except for seven females who all scored less than 40 percent. Looking back to Figure 11 it can be seen that these females were all in Year 3.

Figure 18: Money Experience and Context Performance for Years 3–9 by Gender

The spread of process performance scores indicated in Figure 19 suggests that the males were more homogeneously grouped by ability than the females for non-context. Two males in Year 9 were actually in a class of their own regarding money experiences. They merited a very high rating of 3 but they had much more significant experience than a rating of 3 required. Both of these students scored highly for money-context but also used a variety of efficient mental strategies that demonstrated a high level of number sense.
Figure 19: Money Experience and Non-context Performance for Years 3–9 by Gender

Figure 20 shows that being a male student was more an indicator of good performance than was a money experience rating of three. This might be explained by examining the girls’ choices of strategies, which compared to Boaler’s (1997) findings, who found that more girls than boys used a mental form of standard written algorithm method for mental computation items. Figure 20 also shows that more males than females used higher order mental computation strategies, which are indicative of number sense. This suggests that males should score more highly for number sense. This is also consistent with Shuard (1982).

If, as Boaler (1997) suggested, girls are less likely to value self-invented strategies and estimation, they may be more likely to value school-taught methods, along with being diligent students. This learning style could work against females’ development of number sense. Boaler (1997) further claimed that girls are less likely to value speed as important, which should work in their favour when the class emphasis is on accuracy, rather than on both speed and accuracy. With the absence of mental methods being presented and organised in flexible, reflective, more open and less competitive ways, girls’ learning styles may not be accommodated and consequently their performance may not reflect their potential.

Some females with the top money experience rating score of 3 also scored low for context items. This, together with the fact that males with a low money experience rating score of 1 were able to score highly for context items suggests two possibilities. Perhaps the money
rating scoring system was not entirely adequate. Or, perhaps student values and beliefs about the mathematics methods, and therefore strategies that they believe they should be using, may be more influential in their performance on testing of this sort, than are their out-of-school experiences.

4.9 Research Question 6

How does a student's preference for context or non-context items affect mental computation performance?

As the students' preferences for one presentation format (either context or non-context) may have had an influence on performance, this was examined at the end of the interview. Students were asked for their preference in relation to the two sets of items. Three sets of data (context, non-context, and neither) were then recorded. The overall results in Appendix X and the statements were transcribed from the taped interviews. The number of students with no preference increased with age from none in Year 3, one in Year 5, and none in Year 7, to four in Year 9. Results in Table 23 overall reveal that students were divided equally between preferring either context or non-context. When examined by year level, it can be seen that the greatest preference for non-context (10) is at Year 3, which is consistent with their higher performance for non-context items.
Table 23: Context Preference by Year and Gender

<table>
<thead>
<tr>
<th>Year</th>
<th>Gender</th>
<th>Preference</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Context</td>
<td>Non-context</td>
</tr>
<tr>
<td>3</td>
<td>f</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>f</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>f</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>f</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>29</td>
<td>30</td>
</tr>
</tbody>
</table>

Students' personal preferences for a money context may be for either context itself or money specifically. For example, students who are intrinsically interested in money generally may respond positively to money-context items if they are pitched at the right level for them.

With respect to gender, more than 50 percent of females preferred non-context while 50 percent of males preferred context and three males had no preference. From this data, it was found that 17 females and 13 males preferred non-context; while for context, 13 females and 16 males preferred context and three males and two females preferred none.

Student preferences for either set of items did not match their highest scores for the same set of items. While this indicates there was no overall correlation between preference and performance—the exception being for Year 3—most students agreed that items set in context were more challenging. This was because they needed to think about the nature of the question and to choose an operation. This may suggest that students are thinking more deeply about items that are set in a context. However, it was found that there were greater differences between the two sets of items regarding the order of presentation, with higher scores being achieved on whichever set was given first for all four year levels. This may have been due to a fatigue factor.

Table 24 indicates that Year 3 was the only year level where students who preferred non-context items also performed better for these items. Average process scores for each year level indicate the performance. This table also shows that for the five students who indicated
no-preference, average scores were high for both context and non-context, which may suggest that these students found both sets of items just as straightforward.

Table 24: Context Preference and Average Process Scores by Year Level

<table>
<thead>
<tr>
<th>Preference</th>
<th>Context Performance</th>
<th>Non-context Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 3 (/20)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Context (n=6)</td>
<td>9.5</td>
<td>6.0</td>
</tr>
<tr>
<td>Non-context (n=10)</td>
<td>8.7</td>
<td>13.3</td>
</tr>
<tr>
<td>Year 5 (/24)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Context (n=8)</td>
<td>18.0</td>
<td>16.4</td>
</tr>
<tr>
<td>Non-context (n=7)</td>
<td>17.1</td>
<td>17.1</td>
</tr>
<tr>
<td>None (n=1)</td>
<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>Year 7 (/26)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Context (n=9)</td>
<td>21.1</td>
<td>20.7</td>
</tr>
<tr>
<td>Non-context (n=7)</td>
<td>16.4</td>
<td>18.6</td>
</tr>
<tr>
<td>Year 9 (/26)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Context (n=6)</td>
<td>18.5</td>
<td>17.2</td>
</tr>
<tr>
<td>Non-context (n=6)</td>
<td>17.0</td>
<td>17.8</td>
</tr>
<tr>
<td>None (n=4)</td>
<td>20.5</td>
<td>21.3</td>
</tr>
</tbody>
</table>

For Years 5-9, the average process scores for students who indicated a preference for context were higher for context items than the non-context scores of students who indicated a preference for non-context. The differences for context were not as marked as was found for Year 3 for non-context. It does indicate that the most marked difference in performance occurred between Year 3 and Year 5 where better performance for non-context moved towards better performance for context.

4.9.1 Similarities Noted between Test Items

An individual comparison between two students revealed a dramatic difference. While one student stated that she thought all the items on the two tests were the same items and scored 100 percent on both tests, a second student stated she did not notice any similarities. This student also struggled with both tests, scoring only eight percent for context and 15 percent for non-context.
Generally, the number of students who noted similarities between items increased with age, and that correlates with an increase in the scores—at least to Year 7. Not only did the number of students who noted similarities increase but also the number of items they noted as similar also increased. The inability to notice more than a few items as similar between the tests was most pronounced in Year 3. It may further be the case that the ability to notice the connectedness of related items within the tests as well as between them indicates ‘flexible thinking’—a component of number sense. As one student stated on several occasions, *Several ways... or you could do... several ways..., I picked the easiest way for me.* Her ability to make or see the relatedness of the answers and to remember them made getting the correct answer easier. Although initially this student’s mind was flexible enough to mentally change the horizontal format to a vertical one and compute answers using the standard written algorithmic method. This student was also able to notice the reason why so many items contained the near-compatible numbers of 9, 99 and 199 by declaring, *I’ve got the hang of 99s now.*

One high-scoring student reported that he had noted relationships between items within the tests. This result demonstrated that some learning occurred during this particular interview because of the student’s metacognition and although it was an unintentional outcome, it was not unexpected.

4.10 Research Question Review

With respect to the overall research question, *What effect does the context of money have on students’ mental computation performance in Years 3, 5, 7 and 9?* the following points are made.

It may be incorrect to assume that everyone is equally interested in any one context, as the level of interest any one individual has in that context, for example money, may be situationally specific. While one child may enjoy counting (adding) their own coins from their moneybox as part of a love of saving money, they may not care to work out change (subtracting), which is involved in spending. Another child, who enjoys shopping and is familiar with complementary addition also known as shopkeeper’s addition, may enjoy a simulated shop activity in the classroom. However, he or she may not care to solve theoretical classroom word items that do not belong in their actual world. It should be noted that the studies by Carraher et al (1985, 1987) and Nunes et al (1993) related to real life experiences whereas this study’s items were contrived imaginary situations. Imaginary
contexts recommended by Burns (1993) and RME may then be of little value for mental computation involving money as the range of imagined situations that students are able to successfully engage in is small. Indeed Burns' (1993) suggestion of the use of children's literature as a context may be effective; however, few examples involving money exist. It seems to be more important to immerse students in real activities in order for them to develop and practise their own devised mental computation strategies.

Another reason why no difference was found in favour of context items may include students' preference for non-context items because this is the most familiar presentation format. Results from teacher interviews were varied. One Year 7 class presented all mental computation items in context, half being money items. The majority of classes reported that half of the mental mathematics program was presented in a money or measurement context. This suggests that many classroom mental mathematics items may still be presented context free. Improved performance for non-context might also be due to the student's in- and out-of-school mathematics experiences consisting only of traditional computation exercises. This may occur if parents were coaching their children in the traditional methods that they remembered from school.

Although the overall results did not find any marked difference between context items and non-context items, the researcher noted several individual items for which students gave dramatically different responses with improved performance for context. One item, \(38 \times 50\) at Year 9 reported a 100 percent improved score for context as students realised that 50 cents is half of one dollar. These students were able to use the higher order doubling/halving strategy rather than long multiplication, which is classified as a lower order strategy by using place value instrumentally to solve this item. This reflected that the students' everyday knowledge of money helped them choose a more efficient strategy. Two of these items: \(6.20 + 4.90\) in Year 5 and Year 7; and \(6 - 4.50\) in Years 7 and 9, involved decimals to two places which is consistent with students' everyday knowledge of the context of money.

While overall, the context of money may not have had an effect on student mental computation performance, student interviews revealed that money had an effect on other aspects, such as student development and strategies. This was evident by some students displaying increased motivation and different strategy choices for the money context items due to their knowledge of equivalent monetary values. Overall, gender was not found to impact on paid-working experiences, although some evidence of males being paid for physical work, while females were paid for lighter duties, was noted.
Western Australian students commence their school year in February. In comparison with overseas countries, Grade 2 in the United States roughly equates with students currently in Western Australia in Year 3, for which the average age is 8 years old. The Western Australian students in the current study in Years 3, 5, 7 and 9 should be of the average ages of 8, 10, 12 and 14 respectively. This is equivalent in age to Grades 2, 4, 6 and 8 in the USA.

### 4.11.1 Introduction

Scores ranges by age group are discussed here in comparison to Reys, Reys and Hope (1993) from which some of this study's and McIntosh et al's (1995a) items were originally sourced. Reys et al's (1993) study compared students' mental computation performances for visual/oral presentation of items and a survey of fifth and seventh-grade students' preferences for computation methods (written, mental or calculator). However, Reys et al's (1993) study of application items was not restricted to a money-only context. For example, time and length measurement applications were also presented. Reys et al (1993) found that performance for second graders (n = 261) ranged from one to 98 percent for the application items, with “p-values less than 25 percent on eight of the ten items” (p. 309). By contrast, the study reported here found only four out of ten items with scores of less than 25 percent.

Student performance process scores were also more favourable for Year 3 items which ranged from 15 to 85 percent for context and ranged from 5 to 100 percent for non-context (see Appendix X).

Reys et al (1993) found that for fifth graders (n = 250) performance levels ranged from one to 64 percent. Performance on context items (applied problems) was low, "only about 1 out of 20 fifth graders correctly mentally computed the cost of four tapes, given the information that one tape cost $10.30" (p. 310). By comparison, the study reported here found that process scores for Year 5 context items ranged from 38 to 96 percent, while for non-context items process scores ranged from 33 to 96 percent. Reys et al (1993) found that for seventh graders (n = 204) performance on context items was low. They claimed that “no more than one-fourth of the seventh graders answered any of the applications correctly...the performance level was close to 10 percent” (p. 312).

While these results appear to be more favourable for the Australian students, the sizes of the samples being compared need to be taken into consideration. As do other factors such as the passage of time, improved teacher practices, as well as the likelihood that money may be a
more suitable context. The American study collected data from over 200 students at each grade level. By comparison, the study reported here had a sample size of 16 students per year level, which is too small to enable generalisations. The year difference also needs to be considered, as grade 4 in the USA is generally equivalent to Year 5 in Western Australia, with no age difference.

Reys et al (1993) also found that student preference for pencil and paper methods for items that should be straightforward to calculate mentally revealed student lack of confidence. For example, 48 percent of seventh graders stated their preference for using paper and pencil to calculate 10 percent of 750, and 49 percent preferred to use pencil and paper to calculate $1000 \times 0.123$. These both represent factors of ten and place values that are clearly not understood.

One interesting observation from the interviews in this study was the small number of students who gave correct answers while being uncertain of the correctness of their answers. This suggested that marking test items purely for correct or incorrect answers such as with the basic scoring system, might give a false representation of these students’ level of understanding. These students made a number of guesses, which were correct, but if they had been simply marked correct, without explanation, it may not be realised that the students had guessed and therefore lacked full understanding. Further, the teacher may assume that all of the correct answers are fully understood. While little difference between the basic and process scoring systems was found, suggesting that this was not a widespread problem, it does highlight the importance of students needing to give explanations for their answers.

The study reported here found that context process scores for Year 7 ranged from 46 to 100 percent, and for non-context scores ranged from 50 to 100 percent. By comparison, Reys et al (1993) ($n = 204$) found that scores for context items (although these were for mixed contexts, not only money) ranged from one to 61 percent, for grade 7. This is equivalent in age to Western Australian students in Year 8. The study reported here found for Year 9 that context process scores ranged from eight to 100 percent and for non-context items, scores ranged from 15 to 100 percent.

A study by McIntosh (2002) examined error patterns in students’ mental mathematics calculations. He found that there was a difference in error types for whole numbers, which were mostly procedural, and for fractions, decimals and percents which tended to be
conceptual. Examining the error patterns for this study reveals a difference in errors for context for whole numbers; also found by McIntosh (2002), that procedural errors occurred more often than conceptual ones.

McIntosh's (2002) conceptual errors relate to a lack of number sense, understanding about ‘the nature of the numbers’, or relational connections. Often students have trouble explaining their strategies. A conceptual error would be where student answers indicate place value confusion, such as being out by multiples of ten. By contrast, McIntosh categorised procedural errors as careless counting, mistakes when carrying, or errors of strategy execution. Examples of errors categorised as whole numbers and non-whole numbers identified in this study, revealed similar examples.

4.11.2 Whole Numbers

For the item 74 - 30, McIntosh (2002) found that a common answer given was 36. McIntosh had identified this as a procedural error, since the four was subtracted from the zero when visualising the school-taught vertical method of subtraction. In this study, this item was presented to Years 3 and 5. In Year 5 it appeared that money as a context led to an improvement as only one student made an error, giving an answer of $40. This was also classed as a procedural error. For the non-context item, five Year 5 students gave incorrect answers or made no attempt. Deeper analysis of the errors revealed the answers of 54 and 36 are examples of procedural errors as was the answer 26, which may indicate a double procedural error. In these cases, it is possible to infer that procedural errors were not influenced by the provision of a context. When the process marking guide was applied to student's answers, one procedural error would still be awarded one point if the answer had been correct. However, double procedural errors were given a score of zero.

Examples of conceptual errors that were found for whole numbers included the item 60 x 70. Many students gave an answer of 420, which is a conceptual error, as it is incorrect by a power of ten. Comparing context with non-context answers for this item revealed less than 25 percent of students got this correct at Year 5 for both modes, with students achieving higher scores for non-context in Years 7 and 9.

4.11.3 Non-whole Numbers

Overall, examples of conceptual errors for non-whole numbers in this study found that errors occurred more often with non-context items than with context items. With regard to common fractions, McIntosh (2002) claimed that:
...errors in mental computation of fractions, appear to be much less intricate and, where their reasoning can be surmised, more conceptual. Three of the common errors...can be attributed to confusion of operations. (p. 462)

In the current study, the only common fractions were in the two items, What is half of 16? and What is half of 30? for Year 3. One student gave the answer of 13 for both the context item 'My twin brother and I spent $16 on Mum's birthday present. If we paid half each, how much did I pay?' and the non-context equivalent item, Half of 15 is? For both items, the student followed the same strategy. The student halved the six to arrive at three, but omitted to halve the ten. This could be seen as a procedural error. However, it could also be argued that 13 is so close to 16, that this student does not yet understand the nature of how big half of 16 would be. Similarly, another student gave the answer of five. This student had halved the ten and forgot about the six. There were no noticeable differences between answers for context and non-context at this year level.

McIntosh's (2002) study found that for decimal fraction computation:

errors were mostly associated with the common misunderstanding noted by Hart (1981) and Stacey and Steinle (1998), namely "thinking that the figures after the decimal point represented a different number which also had tens, units etc" (Hart, 1981, pp. 51-52). (p. 463)

Three non-whole number items were found to have marked differences in favour of context due to less conceptual errors being made. These three items were: 6.20 + 4.90; 25% of 48; and 0.1 x 45.

6.20 + 4.90

This item was presented across Years 5, 7 and 9. There was a marked improvement for context in Year 5, with eight errors for non-context, while only two for context. Upon examination of the two errors for context, these were found to be procedural, out by ten cents, one error ten less, the other ten more. Non-context errors were more varied. Three students gave an answer of 10.11, two of 20.1, one of 12, and one of 11.01; which indicate conceptual errors or confusion over the meaning of decimal place values, which was not the case for context. The eighth student in the sample made no attempt at all so this was also classified as a conceptual error.

In Year 7, there were three non-context errors—no attempt, and 2.1 (conceptual), and 10.11 (procedural). However, for context, all 16 students achieved the correct answer resulting in a perfect score. In Year 9, while there were three errors for context, two errors; $11.30 and
$10.10 were procedural, while the answer of $2, was conceptual. For non-context there were three errors—two students who made no attempt and one answer of 8.1. These were found to be mostly conceptual problems.

Overall for this item, the provision of context reduced the number of conceptual errors. The total number of context errors was five—four procedural and one conceptual—compared with non-context, which had 14 errors—five were procedural and nine were conceptual. This item was the most significant item for which context reduced the conceptual errors.

0.1 x 45

This item is equivalent to the contextual item, Find ten percent of 45 was only presented to Year 7 and Year 9. There were eight Year 7 errors for non-context, and ten for context. This count included non-attempts. Excluding these, there were only four errors for non-context and five errors for context. Conceptual errors given were similar for context and non-context: 90, 0.9, 90 cents, 45.1, 0.45, 45 cents, $20.50, $3, and $5.

Discounting non-attempts in Year 9, there were seven non-context errors and five context errors. Non-context errors included five of 0.45, three of 45 cents; and one each of 45.45, $5, and $22.50. The two given money-context answers of $5 were close approximations that, while guesses, also reflect an understanding of real-world number or money-sense as it is common for shoppers to pay for $4.50 worth of goods with a five dollar note. The $22.50 amount was found by halving, so that is a conceptual error. The $20.50 answer was a combination of conceptual (halving) and procedural (forgot to add the $2) errors. Overall, these errors were mostly conceptual and were evenly spread across context and non-context.

25% of 48

McIntosh (2002) claims that percentages have not been commonly analysed in the literature, "many students would appear not to move easily between percents and their fraction equivalents (75% = ¾, 30% = 3/10) as one way of simplifying calculations" (p. 463). This was consistent in this study as a lack of fraction/percentage equivalence knowledge was noted for some students. Some students were noted finding 10 percent twice, and finding 5 percent by halving ten percent, and then adding these facts together rather than finding one quarter of 48.

This item was only presented to Year 7 and Year 9. For Year 7, there were three non-context errors: 120, 10.2 and 6; two context errors: $12.50, and $23; and six non-attempts. The three non-context errors were all conceptual. Five students who made errors for either context or non-context were the same five students who also made no attempts. This suggests that this
item was difficult for the students, conceptually. At Year 9 there were four non-context errors: 8, 9.2, 10, and 11.2. There were four errors for context: $8, $10.4, $10, and $10.80. Of these eight errors, the same students made four. Five non-attempts were made compared with only two students who did not attempt the item, 45 x 0.1. This may suggest that this item was conceptually more difficult than the other items.

4.12 Discussion of all Individual Items

A discussion of all the individual items follows, being grouped according to operation type using percentages of students with correct responses, which is equivalent to using basic scores, as in Table 12. The items discussed in this section have all been compared to results from McIntosh et al's (1995a) study involving students in the Perth metropolitan area. Generally, students in the current study scored higher for non-context than for both context and the McIntosh test items.

Item 60 + 80 (Years 3, 5)

If I spent 60 cents on an icy pole then 80 cents on a chocolate bar, how much did I spend altogether?

For this item, the in-context wording did not include obvious addition language such as 'plus'. Rather, to be consistent with the context of shopping and money, it included the wording of 'how much' and 'spend altogether'. Table 25 clearly shows progression for age

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 3 (%) correct</th>
<th>Year 5 (%) correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>36 (n=163)</td>
<td>87 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>56 (n=16)</td>
<td>81 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>75 (n=16)</td>
<td>88 (n=16)</td>
</tr>
</tbody>
</table>
across all three tests. This study proved more favourable in Year 3 for mental computation items presented in context, with a two-fold improvement for mental computation items not presented in context compared to the McIntosh study.

One counting error and also an example of a low level strategy, noted for this example was by a Year 3 student who counted 80, 90, 100, 110, 120, 130. While the student correctly counted on six tens, he clearly started from the incorrect ten. Several students revealed that they had been taught to count on from the smallest number to the biggest number as this involved a smaller number of steps and therefore was easier to do. This is an example of how the teaching of efficient strategies can still be problematic if students do not fully understand what they are doing.

Item 68 + 32 (Years 3, 5)

_When Mum brought a dress for $68, she was given $32 change. How much money did Mum give the shopkeeper?_

This item may be straightforwardly added as a two-digit number, as the two numbers add exactly to one hundred. Although students may not know this fact, they should know both number facts that two and eight make ten and that sixty and forty make one hundred. Table 26 indicates that improvements for this item from Year 3 to Year 5 are more marked than any differences between context and non-context and that this age related improvement is consistent between the two studies.

**Table 26: Comparison of Item 68 + 32 for Context and Non-context**

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 3 (% correct)</th>
<th>Year 5 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>37 (n=163)</td>
<td>79 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>38 (n=16)</td>
<td>88 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>56 (n=16)</td>
<td>88 (n=16)</td>
</tr>
</tbody>
</table>
Item 79 + 26 (Years 3, 5, 7, 9)

It cost $79 for our puppy's injections. It also cost $26 for puppy food. How much is this altogether?

Bridging tens or using compatible number two-digit addition can be used for this item. Use of this type of strategy indicates number sense. If performed as a standard written algorithm, this item requires two place-value adjustments. The results are given in Table 27.

This item was presented to all four of the year levels. In the current study, it was found that non-context scores were higher across all four year levels. The most disparate results appear in Year 3, with the Paterson non-context scores well above McIntosh et al's. This can partly be explained by this study's use of the process scoring system compared to McIntosh et al's use of basic scoring. Further examination of the answers given by Year 3 students reveal that most students gained the correct answer of 105 numerically, but assumed the answer was only one dollar and five cents—$1.05—a difference of two decimal places. This may well be due to their lack of personal experiences with amounts of money over $100. It may also be due to confusion over recently acquired knowledge that 100 cents equals one dollar. Therefore, when students add amounts of dollars to equal 100 or more, the dollars become cents. It may also be a combination of both of these suggested anomalies.

Table 27: Comparison of Item 79 + 26 for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 3 (% correct)</th>
<th>Year 5 (% correct)</th>
<th>Year 7 (% correct)</th>
<th>Year 9 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>17 (n=163)</td>
<td>66 (n=163)</td>
<td>81 (n=163)</td>
<td>89 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>25 (n=16)</td>
<td>56 (n=16)</td>
<td>88 (n=16)</td>
<td>88 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>50 (n=16)</td>
<td>63 (n=16)</td>
<td>100 (n=16)</td>
<td>94 (n=16)</td>
</tr>
</tbody>
</table>

Of the 15 Year 3 students who answered this item for context, only four achieved the correct answer of 105, while three students gave an answer of one dollar and five cents. Half of the Year 3 students gave incorrect answers ranging from 23c (achieved by adding 6 and 9 to get 15, then adding 2 and 7, which is 9, to get 23), to 100 dollars and 2 cents. This latter answer was written as $1002 (achieved by firstly applying the incorrect number fact of 9 + 6 = 12
and secondly by incorrectly writing one hundred and two dollars after adding 12 to 90 correctly. One student in Year 5 also gave the answer of $1.05, yet no Year 7 or Year 9 students were confused by the dollars and cents place values. However, one Year 9 student did not attempt this question.

This item has been classified in Callingham and McIntosh (2002) at level 5 of their mental computation competence hierarchy that they recommend as the appropriate benchmark for Year 5. They claimed National Numeracy Benchmarks currently are set slightly higher with Year 5 set at level 6. According to their studies only 20 percent of Year 3 students were at level 5, compared to 28 percent of Year 5 students at level 5 and 23 percent of Year 5 students at level 6. This suggests that Year 3 students should find this question difficult and this was substantiated by only 20 percent being correct.

Item 165 + 99 (Years 5, 7, 9)

It cost our family $165 per day for a hotel room plus $99 for a day’s meals. How much did one day cost our family on holiday?

This item can be straightforwardly performed using compensation (99 + 1 = 100 and 165 - 1 = 164. The results are shown in Table 28, which indicate only modest differences both between the tests and for context and non-context in Years 5 and 9. The greatest improvement in performance, with 32 percent, was from Year 5 to Year 7 for context. However, by Year 9 performance for non-context was slightly better than for context.

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 5 (% correct)</th>
<th>Year 7 (% correct)</th>
<th>Year 9 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>50 (n=163)</td>
<td>71 (n=163)</td>
<td>84 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>56 (n=16)</td>
<td>88 (n=16)</td>
<td>81 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>56 (n=16)</td>
<td>69 (n=16)</td>
<td>88 (n=16)</td>
</tr>
</tbody>
</table>
Item 74 – 30 (Years 3, 5)

*Amy’s brother earns $74 in his part-time job. He gave his Mum $30. How much did he keep?*

This item can be readily performed as a two-digit multiple of ten subtraction with no ‘carrying’ involved. The data in Table 29 suggests that context had a marked positive difference for Year 5 while it had a negative effect for Year 3 students. One Year 3 student stated, *because you can’t have a four, you add one more cent on.* This indicated that the student was using money-related knowledge. However, in this case the student incorrectly applied cents for dollars, which may be due to a factor of age.

Table 29: Comparison of Item 74 – 30 for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 3 (% correct)</th>
<th>Year 5 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>21 (n=163)</td>
<td>55 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>25 (n=16)</td>
<td>94 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>50 (n=16)</td>
<td>69 (n=16)</td>
</tr>
</tbody>
</table>

Item 140 – 60 (Year 3)

*Mum saved $140 then spent $60 on a present for Dad. How much did she have left?*

This item can be performed by subtracting six from 14 and compensating for the place value, or by subtracting forty then subtracting twenty. Both of these strategies would reflect number sense. Table 30 reveals low scores for both no context in the McIntosh study and for context in the current study. This shows that for both of these instances, students only achieved half of the scores that were achieved for non-context items in the current study.
Table 30: Comparison of Item 140 – 60 for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 3 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>20 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>19 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>38 (n=16)</td>
</tr>
</tbody>
</table>

Item 80 – 24 (Year 3)

*Dad had $80 and bought a shirt for $24. How much change did he have left?*

This item could be computed as $80 - 24 = 60$; and $60 - 4 = 56$. Table 31 reveals that context proved to be a disadvantage for this item at Year 3 level. Most students gave an answer of 64 indicating that they were unable to decompose the 80. This may be because they viewed the digits as separate entities rather than units of place value. This would be more likely to be due to experience with school-taught procedures than real experiences. Students made statements such as, *Zero take away four, you can’t do, so it must be four, and I put the 24 under the 80.* This may suggest that students at this age have not had enough experience calculating real world subtractions as in money exchanges and so rely on school-taught methods. For most Year 3s, money experiences would not include an amount of 24 cents, as in most shopping experiences 24 cents would be rounded to 25 cents.

Table 31: Comparison of Item 80 – 24 for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 3 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>8 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>0 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>19 (n=16)</td>
</tr>
</tbody>
</table>

150
Item 105 – 26 (Years 3, 5, 7, 9)

We took $105 to the Show but returned with $26. How much did we spend?

This item can be readily calculated as a three-digit and two-digit subtraction. One method is to subtract 25 from 105 first, then to subtract one more. Another method is to subtract 25 from 100 then compensate for the 5 by adding and compensate for the one by subtraction (100 – 25 = 75; then 75 + 5 – 1 = 79).

Table 32: Comparison of Item 105 – 26 for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 3 (% correct)</th>
<th>Year 5 (% correct)</th>
<th>Year 7 (% correct)</th>
<th>Year 9 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al</td>
<td>5 (n=163)</td>
<td>42 (n=163)</td>
<td>68 (n=163)</td>
<td>84 (n=152)</td>
</tr>
<tr>
<td>(no context)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>13 (n=16)</td>
<td>38 (n=16)</td>
<td>75 (n=16)</td>
<td>69 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>19 (n=16)</td>
<td>50 (n=16)</td>
<td>75 (n=16)</td>
<td>75 (n=16)</td>
</tr>
</tbody>
</table>

This item is the inverse of the item 79 + 26. The subtraction operation proved more difficult than the addition operation, for all four year levels. Only two students noted the relationship between the two items 105 – 26 and 79 + 26 within the test. It seems that students are not making the connection between addition and subtraction as opposites.

The results in Table 32 are consistent with trends previously mentioned. The Year 3s achieved stronger results for the Paterson non-context item than the McIntosh et al (1995a) no-context item. However, context did not make a significant difference at any year level. There was however, improvement across all Years 3, 5 and 7, with Year 9 showing some drop-off for context. One Year 9 student stated; I hate subtraction. Can I leave a dash, because I can’t work it out in my head?
Item 264 – 99 (Years 7, 9)

Alex and his Mum made $264 at their garage sale. Alex then bought a $99 play station game. How much money does Alex and his Mum have left?

This item can be straightforwardly performed as a three-digit subtraction by rounding 99 to 100 then, 264 – 100 = 164 and 164 + 1 = 165. Another observation made for this item was that some students mentally ‘decomposed’ and ‘carried’, which is consistent with performing standard written algorithms mentally. This is an example of what Hope (1986) coined ‘calculative monomania’ in order to describe the “tendency to ignore number relationships useful for calculation and, instead, resort to more cumbersome and inappropriate techniques” (pp. 50-51). The most efficient strategy used for this item was 264 – 100 + 1. This use of written methods mentally could be seen to support the arguments against teaching algorithms. They are not suited for mental computation especially if they “encourage children to give up their own thinking” and “thereby prevent...children from developing number sense” (Kamii & Dominick, 1998, p. 135).

Table 33: Comparison of Item 264 – 99 for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 7 (% correct)</th>
<th>Year 9 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>42 (n=163)</td>
<td>75 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>38 (n=16)</td>
<td>50 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>38 (n=16)</td>
<td>44 (n=16)</td>
</tr>
</tbody>
</table>

As shown in Table 33, McIntosh et al (1995a) found 42 percent correct for Year 7. This compares favourably to this study’s results of 38 for Year 7 for both presentation formats and with 50 percent for money-context and 44 percent for non-context for Year 9. The results in Table 33 indicate a substantial improvement for non-context from Year 7 to Year 9 in the McIntosh et al (1995a) results. This suggests the improvement may be age related.
Comparisons within Year 9 suggest that the Paterson Year 9s may have been weaker students with regard to this item. Context did not make a difference at either year level as scores for both the Paterson tests were similar for Year 9 and exactly the same for Year 7.

Item Double 26 is (Years 3, 5)

What is the cost of two books priced at $26 each?

This item can be computed as a two-digit by one-digit multiplication or by using doubles. Most students doubled the twenty, doubled the six, and then added 40 to 12 to reach the correct answer of 52.

This item represents a useful number fact as 26 represents the number of fortnights in a year and the number of cards in a pack (52) that are of one colour (red or black). It can be seen in Table 34 that context made no difference for this item and that students scored full marks equally for context and non-context at Year 5, while scoring the same at Year 3. As previously, the Paterson students scored higher than the McIntosh students for both context and non-context did. One Year 3 answer given for this item was $412, which indicates knowledge of doubles, but not of place values or checking for reasonableness of answers.

Table 34: Comparison of Item Double 26 is, for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 3 (% correct)</th>
<th>Year 5 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>34</td>
<td>80</td>
</tr>
<tr>
<td>(n=163)</td>
<td></td>
<td>(n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>56</td>
<td>100</td>
</tr>
<tr>
<td>(n=16)</td>
<td></td>
<td>(n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>56</td>
<td>100</td>
</tr>
<tr>
<td>(n=16)</td>
<td></td>
<td>(n=16)</td>
</tr>
</tbody>
</table>

Item $60 \times 70$ (Years 5, 7, 9)

If your school is fundraising by selling Grand final tickets for $60 each, and 70 tickets are sold, how much will this raise altogether?
This item can be straightforwardly performed as an extension of the basic fact, 6 x 7. While Reys et al (1993) found that only 33 percent of grade 5 students—which is equivalent to Year 6 in the current study—got this item correct, only 13 percent of Year 5 were correct for both context and non-context. It is interesting to note that the Reys study also found that 39 percent of Grade 5s and 47 percent of Grade 7s—equivalent to Year 8—preferred to do this calculation mentally. They further found that this was the only item where almost half of the seventh graders indicated this preference.

Table 35: Comparison of Item 60 x 70 for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 5 (% correct)</th>
<th>Year 7 (% correct)</th>
<th>Year 9 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>30 (n=163)</td>
<td>73 (n=163)</td>
<td>79 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>13 (n=16)</td>
<td>50 (n=16)</td>
<td>56 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>13 (n=16)</td>
<td>63 (n=16)</td>
<td>75 (n=16)</td>
</tr>
</tbody>
</table>

Table 35 shows that the biggest improvement for performance was from Year 5 to Year 7 and that this improvement was much more marked than any difference in scores for context or non-context. Some comments on strategy choices that were made by Year 9 students for context included: *Six seven's is 42 and (1) just added the zero on the end.* This resulted in an answer of 420, an example of a conceptual error because of rule-based learning and the incorrect application of the rule due to a lack of understanding. Other comments for non-context included: *I'm not sure about it because I can't visualise it, I timesed the 6 and the 7 and it has to be three digits; and two zeros because you multiply them out, it's a short cut.*

The first student’s answer was 142. This student had multiplied 6 by 7 to get 42, and then put a 1 in front to make three digits which is evidence of a conceptual error. The second student gave the correct answer of 4200, which indicates mastery of the computation.

Some teachers and parents teach children ‘shortcuts’ such as removing and adding zeros to make the question easier. McIntosh et al (1994) suggested that this might lead to the incorrect application of the rule, while Hopkins, Gifford and Pepperell (1996) noted that
'shortcuts' led to misconceptions when working with decimal numbers. This study found evidence of this rule being mis-applied to multiplication of two-digit whole numbers, so that $60 \times 70$ was often calculated as 420.

Item 7 x 25 (Years 5, 7, 9)

There are seven children that I want to buy a lollipop for. If lollipops cost 25 cents each how much will I spend in total?

This item can be straightforwardly performed as follows: $4 \times 25 = 100$, so $8 \times 25 = 200$; therefore $7 \times 25 = 200 - 25$. In Reys et al.'s (1993) study of Year 8 equivalent students' preferences for the item $36 \times 25$, 70 percent of 204 students stated that they would prefer to use pencil and paper. A similar finding was made for Year 6 equivalent students, with a preference of 71 percent for paper and pencil. Results from Table 36 reveal that only students in Year 9 scored better for context than for non-context, but the difference was small.

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 5 (% correct)</th>
<th>Year 7 (% correct)</th>
<th>Year 9 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>37 (n=163)</td>
<td>72 (n=163)</td>
<td>89 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>44 (n=16)</td>
<td>75 (n=16)</td>
<td>75 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>50 (n=16)</td>
<td>88 (n=16)</td>
<td>69 (n=16)</td>
</tr>
</tbody>
</table>

Counting by 25s proved useful here. As one Year 7 student stated after working through the item $7 \times 25$, they mentally 'carried' in the tradition of the school-taught algorithm: Oh, I should've just counted by 25s! This suggested that this student knew the multiples of 25 and this would have been quicker for him. However, as Swan and Bana (2000) found that, "students make a fairly hasty decision based on a limited set of criteria" (p. 586). Some Year 9 students used their knowledge of 25 as a quarter of 100 and 25 cents as a quarter of one
dollar. One Year 9 student commented: *I used the 25s in a hundred, there's four... 100 plus 50, is two more.* For context, one Year 9 commented: *There's four, 25s in a dollar, so I just timesed it by two to make eight, so it was two dollars and (I) took away 25 cents.*

Item 38 x 50 (Years 7, 9)

*What is the total cost of 38 Harry Potter cards at 50 cents each?*

This item can be performed by halving 38 and doubling 50 to achieve the same as 19 multiplied by 100. This strategy was more obvious to students when the item was set in context as they appeared to be more aware of the fact that 50 cents is half of one dollar. One Year 9 student’s comment for context included: *(I) Halved 38, because 50 is half of a dollar; I divided 38 by 2, because if it’s 50 cents, it’s half of a dollar.* This latter comment reflects that knowledge of money was used as money or number sense to solve this item correctly.

Table 37: Comparison of Item Double 38 x 50 for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 7 (% correct)</th>
<th>Year 9 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>31 (n=163)</td>
<td>57 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>44 (n=16)</td>
<td>50 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>38 (n=16)</td>
<td>25 (n=16)</td>
</tr>
</tbody>
</table>

Although the study reported here did not include this item for Year 3, McIntosh et al’s Year 3s scored only 7 percent for no context. This item was subsequently identified as the most difficult item for Year 3 in their study. Table 37 shows that for Year 9, this study’s non-context item achieved the lowest score, being half that for context. It also shows that for Year 7, this item achieved the better score in context.

One Year 9 student’s comment for non-context included: *Because 50 is half of a hundred, (I) divided 38 by 2 and then added a zero.* This shows that students seem to be less aware of conceptual errors regarding the relative size of numbers as numbers get into thousands,
perhaps because this is generally not in their field of experience. For context, the correct answer of $19.00, which represents 1900 cents, is much more in their field of experience as it is the equivalent of 19 dollars.

Item Half of 16 is? (Year 3)

My twin brother and I spent $16 on Mum's birthday present. If we paid half each, how much did I pay?

Halving strategies are generally used to solve this item. Students could also use their knowledge of doubling of number facts as the opposite of halving. Most students solved this item either by halving the ten, then the six, then by adding them together; or they stated that it was a known fact. Some students counted on their fingers to work out the halving. The results shown in Table 38 suggest that Year 3 students found the non-context presentation of this item easier than the contextual presentation.

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 3 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>57 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>68 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>81 (n=16)</td>
</tr>
</tbody>
</table>

Item Half of 30 is (Year 3)

Grandma had 30 cents and gave me half of it. How much did she give me?

Table 39 indicates that no Year 3 student was able to solve this item set in context. Along with the item 80 - 24 it was the second-most difficult item for Year 3. Otherwise, the two no-context test results were similar. In comparison to the previous item, where the students
were asked to find half of 16 and generally did so, this item reflected that students were not used to halving money amounts of this size. More students guessed this item than the previous item, which involved a smaller number.

Table 39: Comparison of Item, Half of 30 is, for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 3 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>58 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>0 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>56 (n=16)</td>
</tr>
</tbody>
</table>

Item 150 ÷ 25 (Years 5, 7, 9)

If 25 people share a $150 win on Lotto, how much money should each person get?

This item could be solved by counting by 25s or using relational knowledge based on the fact that four 25s make 100. One Year 9 student stated that, this one is a hard one. Table 40 indicates that while context achieved lower scores overall, Year 9 students scored much lower than Year 7 students did in the current study. Strategies used by students revealed that Year 7s used counting by 25s and the fact that four 25s make a hundred more often and more correctly than the Year 9s. This item is similar to the item, 7 x 25, as both involve the knowledge of counting by 25s. It appeared that the division symbol in the non-context item proved difficult for some students, while more students seemed unable to solve the context item by sharing. Few students were aware of the relationship between division and multiplication.

Comparing Table 40 (the division by 25s item) to Table 36 (which was the multiplication by 25s item), reveals that the McIntosh studies found students improved with age for both items with similar scores for both. However, this study found that Year 9 students had more difficulty with the division item than Year 7 students for non-context, while all three year levels found the division item to be more difficult than the multiplication item for context. This may be due to their lack of past experiences using division in real contexts such as sharing money amounts equally.
Table 40: Comparison of Item 150 + 25 for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 5 (% correct)</th>
<th>Year 7 (% correct)</th>
<th>Year 9 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>34 (n=163)</td>
<td>79 (n=163)</td>
<td>86 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>31 (n=16)</td>
<td>50 (n=16)</td>
<td>44 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>56 (n=16)</td>
<td>88 (n=16)</td>
<td>50 (n=16)</td>
</tr>
</tbody>
</table>

Item 3500 + 35 (Years 5, 7, 9)

A school fair raised $3500 for new computer programs. How many can be purchased if the price is $35 each?

Table 41: Comparison of Item 3500 + 35 for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 5 (% correct)</th>
<th>Year 7 (% correct)</th>
<th>Year 9 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>29 (n=163)</td>
<td>29 (n=163)</td>
<td>82 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>38 (n=16)</td>
<td>81 (n=16)</td>
<td>56 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>31 (n=16)</td>
<td>81 (n=16)</td>
<td>63 (n=16)</td>
</tr>
</tbody>
</table>

Results for this study as shown in Table 41 indicate a big improvement in performance from Year 5 to Year 7, and again the Year 7s scored more favourably than the Year 9s. One Year 9 student stated: Easy! Several ways, 35 goes into 35 once. I picked the easiest way for me. 35 into 35, once and put two zeros on the end.

The Year 5 finding of 35 percent success for money-context and 31 percent for non-context is consistent with the McIntosh et al studies, not just for the identical item, but also for the
inverse of this item \((100 \times 35)\). It is interesting to note that more than 40 percent of McIntosh et al's (1995) Year 5s could not compute \(100 \times 35\) mentally, suggesting a lack of conceptual understanding rather than a lack of computational skill.

**Item $6.20 + $4.90 (Years 5, 7, 9)**

*Mum spent $6.20 in the bakery and then she spent $4.90 at the newsagent. How much did she spend altogether?*

Most notably, this item was usually answered correctly when presented in a money context for Years 5, 7 and 9, as in Table 42. The greatest improvement in average performance for context was from Year 5 to Year 7. Context made a huge difference in Years 5 and 7. There was no difference between Year 5 and Year 9 for context, but Year 5 performed better for context while Year 9 scored the same for both modes of presentation. In contrast, there was a steady progression of improved scores for non-context from Years 5 to 9 that was consistent with the trend in the McIntosh et al (1995a) studies. This trend would be an expected outcome of schooling. One explanation for the Year 7 improvement could be the result of school-taught mental mathematics in a money context, as the money experience scores were not the explanation. This could be further explained by examining student strategies. For example, several Year 7 students used efficient mental strategies such as rounding, bridging to a dollar, and working from the left.

**Table 42: Comparison of Item 6.20 + 4.90 for Context and Non-context**

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 5 (% correct)</th>
<th>Year 7 (% correct)</th>
<th>Year 9 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>37 (n=163)</td>
<td>70 (n=163)</td>
<td>88 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>81 (n=16)</td>
<td>100 (n=16)</td>
<td>81 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>50 (n=16)</td>
<td>75 (n=16)</td>
<td>81 (n=16)</td>
</tr>
</tbody>
</table>
One Year 5 student worked from left to right as demonstrated in the following comment: ...because six dollars plus four dollars is ten dollars. 90 plus 10 cents would be $11 and ten cents because you do six plus four is 10 and nine plus two is 11.

From Table 42, it can be seen that the results for non-context were lower than for context in the Paterson studies except in Year 9. This was because students' answers for non-context were more often either incorrect or not attempted than when the item was presented in context. This may be because students are most familiar with decimals to two places when they appear in a money-context. It should be noted that the six Paterson items were consistent for place value with both context and non-context presented to two decimal places, while the three McIntosh items were only presented with one decimal place.

The results in Table 42 indicate that this difference may have resulted in an improved performance for non-context in Year 5. There is less of a variation in Year 7 with the reverse effect in Year 9. For context, the biggest improvement in performance was in Year 5, with optimal results in Year 7 and a drop-off in performance at Year 9. Overall, for the addition of decimals to two decimal places, the provision of context did make a difference. This is one case where the context of money was found to improve student performance.

According to Irwin (2001), the decimal system is a "multiplicative scientific concept that does not arise easily from everyday knowledge" and decimal fractions are "not intuitive or easy to learn" (p. 416). Stacey and Steinle (1998) and other studies such as Hart (1981) on students' misconceptions associated with decimal fractions have also made similar claims. Both fraction and decimal items have been highlighted in previous studies as areas of weakness for student understanding and lack of confidence (Yang, 1995; McIntosh et al, 1995a). Cockcroft (1982) also mentioned these areas for Years 7 and 9. The most difficult concepts to teach in primary school according to Irwin (2000) are:

- multiplicative processes (which) included understanding multiplicative nature of the place value system, the divisions necessary for understanding decimal fractions, common fractions and ratio. (p. 339)

McIntosh et al (1995a) suggested that perhaps students used different strategies for solving visually presented items compared to orally presented items, especially for fractions. The visual image seemed to remind the student of the written algorithm whereas the student's intuition was not impeded when the item was merely heard (p. 29).
Item 6 – 4.50 (Years 7, 9)

If you only have two-dollar coins and hand over $6 to pay for your lunch, which costs $4.50, how much change should you get?

This item found similarly to the previous decimal item $6.20 + $4.90, that performance was improved for the context item. Although, this item had only one decimal to two places, Table 43 indicates that both the Year 7 and Year 9 students’ averages had better performances for context. Examples of students’ answers showed that students were able to demonstrate how to take four whole dollars from six whole dollars and decompose the remaining two dollars. This was in order to take the remaining 50 cents away, the same was not true for non-context.

Table 43: Comparison of Item 6 – 4.50 for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 7 (% correct)</th>
<th>Year 9 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>77 (n=163)</td>
<td>92 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>88 (n=16)</td>
<td>94 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>69 (n=16)</td>
<td>88 (n=16)</td>
</tr>
</tbody>
</table>

Even some Year 9s did not know how to subtract four-point-five-zero (four and five tenths) from six because they did not see six as six and zero hundredths, subtract four and fifty hundredths. This may indicate that transfer between money-context and non-context did not happen. The presentation order did not result in any difference in performance for that particular question.

One Year 9 student explained, (I) Added it all back on to make sure, and Several ways. Six take four then half off, or you could do 600 – 450. This demonstrates flexibility in thinking. Perhaps the two Year 9 students who incorrectly answered minus 1.5 and 2.5 lacked a mental picture of decimals and experience with decimals, outside of the money context. This raises doubt about their ability with decimals outside of a money context and it might be useful to explore these particular students’ abilities working with decimals in measurement contexts such as time trials and/or distance or volume topics.

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Both the context item and non-context item in the current study were presented with two decimal places to be consistent with the place value represented by the money-context item. By contrast, the test item \((5 - 4.5)\) in McIntosh et al. (1995a) had only one decimal place. What is noticeable is that in both year levels, scores are higher for context than for either of the non-context tests. It is interesting that for non-context, both year levels performed better for the McIntosh et al. (1995a) non-context item presented with only one decimal place than for this study's two decimal places item, which more closely represents a money amount. This may suggest that where decimals are presented outside of a money context, the greater the number of decimal places, the more confused students become, even when the second place is marked with a zero. Year 7 reported the lowest of all scores, but particularly for non-context with two decimal places. This suggested that the inclusion of a zero when attached to decimal numbers outside of a money context might be confusing to students. In particular, students may not see 4.50 and 4.5 as identical values. The sample size for the study described here is however too small to claim anything emphatically.

Reys et al. (1993) found that a whole number and a mixed numeral item such as \(4 - 2 \frac{1}{2}\) was answered correctly by only 27 percent of seventh graders. This score is much lower than for either McIntosh et al. (1995a) or this study's results and may suggest that the Australian students were better mental calculators than the American students were.

Item 0.1 \(\times\) 45 (Years 7, 9)

If I want to buy 0.1 kilo of lobster that costs $45 a kilo, how much do I need to pay?

Table 44 indicates that Year 9s scored poorly for this item for non-context, compared to the Year 7s. This item was really asking students to find one-tenth or ten percent of 45, which should have been relatively straightforward. Most students appeared unaware of this equivalence. They seemed to be trying to remember some rule about moving the decimal point rather than understanding the nature of the item. Answers therefore, as discussed previously revealed that the majority of errors made were conceptual for both context and non-context.

Both fraction and decimal items have been highlighted in previous studies as areas of weakness for student understanding and lack of confidence (Ellerton & Clements' 1994; Yang, 1995; McIntosh et al., 1995a). Cockcroft (1982) makes particular mention of this for
Years 7 and 9. As stated previously, McIntosh et al (1995a) suggested that students might use different strategies for solving visually presented items compared to orally presented items, especially for fractions. The visual image seemed to remind the student of the written algorithm whereas the student's intuition was not impeded when the item was merely heard (p. 29).

Table 44: Comparison of Item 0.1 x 45 for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 7 (% correct)</th>
<th>Year 9 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>47 (n=163)</td>
<td>66 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>38 (n=16)</td>
<td>38 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>50 (n=16)</td>
<td>25 (n=16)</td>
</tr>
</tbody>
</table>

Item 25% of 48 (Years 7, 9)

My Dad had $48 and spent 25 percent of it. How much did he spend?

Table 45 indicates that context achieved a better performance in Year 7 than in Year 9. However, compared to the McIntosh study, overall this study's students found this item more difficult for both context and non-context.

Table 45: Comparison of Item 25% of 48 for Context and Non-context

<table>
<thead>
<tr>
<th>Study</th>
<th>Year 7 (% correct)</th>
<th>Year 9 (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McIntosh et al (no context)</td>
<td>81 (n=163)</td>
<td>95 (n=163)</td>
</tr>
<tr>
<td>Paterson (context)</td>
<td>50 (n=16)</td>
<td>44 (n=16)</td>
</tr>
<tr>
<td>Paterson (non-context)</td>
<td>44 (n=16)</td>
<td>50 (n=16)</td>
</tr>
</tbody>
</table>
In comparison to the McIntosh et al. (1995a) results, student answers from this study for both Year 7 and Year 9 do not compare favourably. It is impossible to compare the strategies used between the two tests. However, an examination of strategies used in the current study between successful and unsuccessful students revealed that successful students were aware of the relationship between percentages and fractions and understood the meaning of 25 percent as one quarter of the whole. They were then able to use this information to solve the item in various ways depending on their knowledge of basic facts. For example, one Year 9 student commented, "25 percent is a quarter and four times 12 is 48 (I knew that) so, 12 is a quarter of 48." Haylock (2001) described this method as ad hoc. An example of a generally efficient strategy used unsuccessfully by a Year 9 student for context was, "I calculated 20 percent of 48 and then 5 percent to arrive at $10.80. As Haylock (2001) states, "one of the easiest percentages to find is 10 percent, and most people intuitively start with this" (p. 171). However, he also warned that ten percent is a special case being the only percentage equal to its fraction of one tenth.

4.13 Summary

This study was designed to investigate the effect of money as a context on students' mental computational strategy choices across a range of ages of development. The results indicate no difference for context and non-context except at Year 3, which found improved performance for non-context. Results showed no overall difference between genders apart from Year 3 in which females scored lower than males. Further inspection of the item types revealed Year 3 student weaknesses lay with the subtraction items. Interestingly, while students' performance did improve with age, it was found that for the two year levels with identical items, Year 7 out-performed Year 9.

After students in Years 3, 5, 7 and 9 were individually interviewed and asked a set of questions to establish their background through previous experiences with money, results from rating these questions revealed little difference across the year levels. As expected, Year 3 had the lowest mean, but Year 5 had the highest. Results also show that money experience rating made little difference to student performance for context. During these interviews, it was also noted that several of the females had recently arrived from other countries, two from Africa and one from England. It is not assumed therefore that all students involved in the current study had received the same amount of schooling in Western
Australia as other students in their particular year level. It is noted however, that the students who were schooled in Africa did not possess the same range of strategies as the other students.

After students were presented with two sets of mental computation items and observed solving them, the students were asked to explain their method and or choice of mental computation strategies. Results indicated that several students, especially in Years 3 and 9 reported using school-type written methods, mentally. Overall, students who used higher order strategies generally scored higher on the items for both non-context and context. An examination of individual items revealed that student performance was improved for context for the decimal addition. Overall, comparisons with the McIntosh test results found that generally students in the current study scored higher for non-context items. It should also be noted that the number of students in the McIntosh tests was 163 compared to the 16 per age level used in the current study.

Finally, students were asked whether they had noted any similarities between the tests and asked to state whether they preferred one set of items or the other. Preference was found to bear no relationship to improved performance for one set of items or the other.
Chapter 5: Summary, Conclusions and Implications

5.1 Summary of the Effect of Context on Mental Computation Performance

This study investigated the effect of money as a context on students' mental computational performance and computational strategy choices across a range of ages from 8 to 14 years. Volunteers of equal numbers of students of each gender in Years 3, 5, 7 and 9 were interviewed. First, the students were questioned about their previous experiences with, and general interest in, money. Second, student answers and explanations for two sets of mental computation items—one set in context and one without—were observed, tape-recorded, scored and analysed. For comparison, the non-context items were identical to a selection of those used in a previous study, and the context items were developed from them.

Results from this study found that overall, money as a context did not make a significant difference to student performance for mental computation items when the items were presented in a word problem format compared to a non-context presentation. In fact for Year 3, context had a marked negative effect on performance. A few individual items were found however, for which performance was improved for context, but the difference was not consistent across all year levels or genders. The item of greatest improvement involved the addition of decimals. This result was not expected given Nunes et al's (1993) findings that students scored better for word applications than straight computation. Other influences need to be considered. The results from this study may suggest that the school environment may have had a stronger influence than first expected. Perhaps, the influence of ‘school cues’, with the tests being semi-formally conducted at school, may have led students to regard the items as set in a school task context. Results may have been different, had the context items been presented in a simulated shop situation compared with straight computation. Nunes et al (1993) reported improved student performance for context items presented in a simulated shop situation when compared to word applications. Therefore, results may have been even better for actual shopping activities in a real shop compared to simulated ones at school.

Interviews were revealing. Firstly, they revealed that performance levels were more likely to vary because of the students' individual strengths and weaknesses with computational strategy knowledge, rather than their past experiences with money. This is not to discount the effect that substantial real money experiences might have on a student's development of strategies, although it remains unclear why some students possessed better strategy
knowledge than others. Secondly, interviews revealed that there were a number of students, who gave correct answers while being uncertain about the correctness of their answers. Thirdly, interviews revealed that students at a variety of developmental levels achieved similar results. In conclusion, interviews were considered vital in assessing a student’s real level of development or number sense.

On checking each of the 64 individual students, it was found that the money experience rating allocated to each of them did not make a marked difference to either performance or strategy choices. This was except for two cases of students with exceptional experience at Year 9. If there had been more students with extensive money experiences, perhaps a more definitive conclusion could have been drawn. The only marked differences for gender appeared at Year 3. When the results were further examined by operation type, it was found that Year 3 females had performed poorly on subtraction items. Some items were found to have a marked positive effect for non-context while some were found to have a marked positive effect for context. Of these latter items, the one that was found to have the most positive effect for context was the decimal addition, *Mum spent $6.20 in the bakery then she spent $4.90 at the newsagent. How much did she spend altogether?*

### 5.2 Other Contexts

The fact that money as a context did not make a difference to performance in the current study does not mean that other contexts would not make a difference. Indeed even a money context presented differently, in a personally meaningful way such as a real shopping experience, may result in improved performance for student mental computation. The money experience rating procedure may have been too narrow in focus to gauge student performance, as prior experiences in other contexts were not considered. Significant experiences from different contexts such as sport scoring might have affected these results indirectly from the transfer of skills. For example, some high scoring students only received a rating of one for money experience.

What constitutes a meaningful context may vary from one individual to another based on personal interests and experiences. For example, one Year 3 became very animated at the item *Amy’s brother... and remarked, I have a sister called Amy! Although this student got this particular item correct for context, he did not get all the other context items correct. He did however, achieve a perfect score for the non-context item test. This shows that while he was very good at non-context mental computation, the provision of a meaningful context had
the same effect. Substituting students’ names in such items is straightforward and can make such items more relevant to the students. Another example included one student’s reference to counting by 25s when swimming laps of a 25-metre pool during the school swimming program. This could be an example of ‘scaffolding’ enabling transfer to both multiplication and division.

Simulated shopping experiences with associated motivational external devices such as play money and purchase items have found improved performance for mental computation (Nunes et al., 1993). This may be because students are more able to make links to their out-of-school learnt mental methods, thus improving transfer.

5.3 Conclusions

It was believed that a preference for the money context items might result in better performance for context. However, this was not found to be true. No difference was found for gender, nor for age overall. However, the expected difference in performance for mental computation at Years 7 and 9 was surprising, as the Year 7 students outperformed the Year 9 students for both items set in context and in non-context.

Possible reasons why context did not make a difference to performance will now be discussed. A first possible reason may be that the items presented were not personally relevant or realistic enough (as they might be in an actual shopping transaction) for the students. Perhaps, performance may be improved where the student spends part of their pocket money on lollies, or small toys of their choice compared to the contrived items used in the current study presented in ‘what if’ scenarios. For some students, even Carraher’s (1985) simulated shopping experiences did not result in improved performance. Students may have been disinterested because the activity was viewed as a school ‘task’ and not directly relevant to their world.

A second possible reason why context did not make a difference to performance may be that mental computation practice in schools is not always presented in context. This was found to be the case for some of the schools involved in the current study. While recommendations for the time spent on mental mathematics to be increased appears to be happening, perhaps there needs to be more mental mathematics that is set in context. Perhaps, there still needs to be more time given to mental mathematics, as the primary school teachers stated that mental mathematics was generally timetabled for ten minutes a day, while at high school it was only timetabled for five minutes. Most schools stated that context was provided for at least half of
their mental mathematics program, but of course, not all of this context was a money context. Therefore, students unused to mental mathematics set in a context at school, should perform less well for this presentation format. This would be expected and was found the case at Year 3.

A third possible reason may be that students in these age groupings have not had enough recent out-of-school contextualised mathematical experiences in order to practise them. For example, it was expected that Year 3 students would have received less money context experiences at home, but this expectation was not substantiated by their money experience ratings. Results improved with age to Year 7. However, Year 9 students did not perform as well as Year 7 despite having the same test items. Year 7s was the only level that mentioned recent participation in fundraising activities. Some of the Year 9s remembered participating in such activities when they were in Year 7. Perhaps the better performance results for the Year 7 students can be explained by their selling experiences being more recent or perhaps there were more of them. The Year 9s also reported less time was spent on mental mathematics in class. Recency may be important because of its nature as a revision tool for number facts and effective strategy choices.

A fourth possible reason why context did not make a difference to performance may have been that students did not use efficient mental computation strategies, for the items in context. This was evident when students used the same written method strategies mentally for both sets of items. This study found that many students did not use efficient mental strategies for either context or non-context items, especially at Year 3. Perhaps students lacked knowledge of the strategies due to either not having spent enough time working them out for themselves or a lack of class discussions involving sharing ideas with their teacher and peers. Many students were unable to use appropriate mental computation strategies consistent with highly developed number sense. Neither did these students demonstrate an understanding of the magnitude of numbers as some students confused place values, and only a few students were able to self-correct. This lack of strategy knowledge could be related back to the other three reasons, in particular, that not enough mental mathematical classroom experiences had been provided, set in familiar everyday contexts such as actual shopping and cooking activities or games.

Mental computation strategies used by the most successful students generally were of the highest order, thus indicating that these students possessed a high degree of number sense. These same students also generally demonstrated high scores for both non-context and
context, indicating that transfer had occurred. Some top-performing students also noticed similarities in items used between the tests. Further, at least one student noticed the relationship between the numbers used for items within a test. For instance, the items \(105 - 26 = 79\) and \(79 + 26 = 105\) are related facts. Knowledge of the relationship between operations can help students make more connections between number facts and operations and thus enable them to choose from a greater range of strategies. This ability to observe relationships with numbers may indicate number sense as these particular students may, although few used higher order mental computation strategies. The willingness of some students to use more than one strategy to solve items and check answers was observed by students in Years 7 and 9 in the current study. This may be because these students had been encouraged to use different mental strategies, or because they wanted to use their own methods compared to taught ones. This could be considered 'flexibility with numbers' and may be a result of classroom communities of practice using class discussions in an attempt to increase strategy knowledge, number sense, and transfer.

Some students were found to be capable of remarkable mental agility, as they mentally rearranged the numbers presented in a horizontal format into a vertical one. These students were doing this in order to apply school-taught written methods, which are not efficient mental strategies. McIntosh and Dole (2000a) also reported evidence of this mental agility in their studies. This agility cannot be considered as 'flexibility' with numbers according to definitions of number sense. An over-reliance on school-taught methods implies a lack of experience with mental methods. Although students were asked to give answers orally, the fact that students were also required to write their answers may have interfered with their ability to solve items using only mental methods. Although no working out was allowed, the very act of holding a pencil and being asked to write answers may have provided a cue to using school-taught methods especially where students had had more school mathematics experiences than out-of-school ones. Carracher et al (1985) found that for simulated market place problems, school-type symbols and routines interfered with the solving process.

3.3.1 Limitations to Generalisability

Due to the small number of students involved in the current study, there is a limited ability to generalise these results across the state, country or internationally. The nature of the context presented as formal word problems requiring imagination as opposed to actual money exchanges, such as during shopping experiences, also provided a limit to generalisability as
did the choice of wordings used. The words selected were chosen to be as consistent as possible and as common to everyday language as possible. Context may still yet provide improved performance if presented in a more realistic or interactive format.

The students' money experiences, the teachers' teaching practices and the students' socio-economic backgrounds were also limited by being homogenous due to all students residing in the same catchment area of the same secondary school.

5.3.2 Issues of Reliability and Validity

The issue of reliability is one of consistency, and according to Bell (1999) "the extent to which a procedure produces similar results under constant conditions on all occasions" (p. 103). In other words, would it be possible to produce similar results to this study's on a separate occasion and/or by a different interviewer using the same instruments and procedures.

According to Bell (1999) validity, "tells us whether an item measures or describes what it is supposed to measure or describe" (p. 104). A common problem associated with questionnaires compared to interviews is that either no response is given to a question or an inappropriate response is given, possibly due to confusion regarding the nature of the question (Bell, 1999). Hence, interviews were chosen as the method in order to maximise the amount of data collected. Bias may be a major threat to validity where interviews are used as the main data-gathering instrument (Cohen & Manion, 1980). For this reason, this study's data was compared with other data that had already been established as valid. This was straightforward because all of the test items used were similar or identical to those used in McIntosh et al (1995a). Bell (1999) claimed that interviews are:

- a highly subjective technique and therefore there is always the danger of bias.
- Analysing responses can present problems and wording the questions is almost as demanding for interviews as it is for questionnaires. (p. 135)

Further concerns raised by Bell (1999) regarding bias in interviews claimed "it is even easier to 'lead' in an interview than a questionnaire...with different emphasis and in a different tone of voice" (p. 140). This indicates that two different people may produce different emphases and therefore elicit different responses to identical questions. The data that was gathered for this study consisted of both qualitative and quantitative data. The first of the qualitative data instruments was the money experience questions. However, as the money experience instrument was developed entirely by the researcher, a mechanism to determine its validity and reliability was required. For this reason the same interviewer and set of questions was
used to interview all subjects of this study and all interviews were tape recorded. An independent checker was engaged to assess a sample of the interview answers by using the money rating guide procedure.

When adapting the McIntosh et al (1995a) mental computation items, it was necessary to reduce the number of them, as the interactiveness of the interviewing process would take more time than a pencil and paper test. Although data regarding the strategies used was collected for every item, in the case of KF (known fact) it was not possible to know how this fact had been acquired. It is possible that facts may have been learnt by rote, so it may be misleading to have KF rated as a high number sense strategy. While the interviews covered less content than other techniques could, they managed to allow more in-depth information to be collected than by using questionnaires.

5.4 Implications

5.4.1 Implications for Teaching Practice

One of the main criticisms commonly raised regarding the teaching and learning of written methods, is that the teaching has been reduced to a set of rote-learned procedures. The same should not happen with the teaching and learning of mental methods. The speed factor should not be an issue for any student until that student fully understands how to work out number facts for themselves. This could occur by way of a connectionist orientation similar to Plunkett's (1979) term 'relational', which "emphasizes the links between different aspects of mathematics" (Askew, 2001, p. 98). It should allow the class to share a wide range of informal strategies. Anghileri (2000) suggested that teachers should not worry about errors being perpetuated by children discussing their own methods. This is because "Research...shows that learning is more effective when common misconceptions are addressed, exposed and discussed in teaching (Askew & William, 1995, pp. 12-13)" (p. 66). Anghileri further acknowledged that 'faultless communication' is not possible and that the teacher must make specific efforts to uncover misconceptions. This is where the researcher saw the value of individual interviews in mathematics classes. Anghileri suggested that "addressing misconceptions during teaching does actually improve achievement and long-term retention of mathematical skills and concepts" (p. 66).

This could be achieved by providing, as McIntosh et al (1997b) suggested, "regular opportunities to develop, discuss, and apply mental computation strategies" (p. 56), which they claimed, contribute towards developing number sense. From this sharing experience,
students would then be able to choose their own preferred strategy, which may involve a mental, a written, or a calculator method. Swan (2002) found that student choices of computational method—that is written, calculator or mental—for students in Years 5 to 7, are made very quickly based on few, if sometimes superficial criteria. Swan (2002) claimed that “mental computation was favoured as the first computation choice for most items and...overall” (p. 42).

Reys et al (1993) recommended that “mental computation must be developed in a regular and systematic manner if performance...is to be improved” (p. 314). Primary school teachers in the current study revealed that mental mathematics was timetabled daily, for at least ten minutes, usually as a warm-up activity. Regularity was further recommended by McIntosh et al (1994) as IS minutes of mental computation activities to be timetabled every day. These activities might include games and be open-ended to encourage students to participate at their own level. One activity suggested by Swan (2000) that could be used as a game, involved students aiming to beat the calculator with place value calculations involving larger numbers. Therefore, one recommendation may be to increase the time spent on mental mathematics to a minimum of fifteen minutes by including some open-ended and inclusive games.

According to Anghileri (2000), games “remain a wonderfully motivating avenue to reinforce number facts and mental strategies” (p. 13). Games were recommended for individuals who stated that they did not like or understand mathematics, because games can put the fun back in. Parr (1994) wrote that games: “stimulate people to do willingly some quite demanding and not very attractive arithmetic” and furthermore that: “people...give repeated practice to...mental arithmetic...because they want to do better the second time around (p. 29)”. Hatch (1998), claimed that games “improve mental skills through repetition” (p. 32). In the current study, one Year 5 student stated when answering the practice item (9 + 15 =) for non-context, “I say, nine plus what makes fifteen...because I play cribbage with Grandma.” This quote represented evidence of a student learning and practising mental strategies in a game in an out-of-school environment. It also indicates that transfer was able to take place between the two non-context settings. Booker et al (1998) further recommend that money-based activities, games and frequent use of money transactions in real-world problems can develop number competence.

Burns’ fifth suggestion: ‘Embed math activities in contexts’ has previously been discussed. While, Burns’ tenth suggestion: “keep an eye out for instructional activities that are accessible to students with different levels of interest and experience” notes the importance of
assessing prior knowledge. It also realises that not all students will have the same interest a particular mathematical activity. With regard to this study's results, different levels of interest in money were indirectly noted. This study considered individual differences by asking students to nominate their preference for item presentation format. Students' prior knowledge, interest in the context and preference for context made little difference to the overall results.

Context is used in various areas of the mathematics curriculum today. Traditionally context was not used for the twenty rapid response mental computation exercises often presented in short, general and abstract notation where the emphasis was on speed. More recently, context has been provided in mental mathematics commercial texts usually as money or measurement. However, these exercises appear to be designed for individual seatwork and without the need to show working. As such, it may be difficult to distinguish which methods or strategy students had used without class oral discussions or interviews.

Because imaginary aspects have little or no effect when it comes to context, there is therefore a need to embed mathematics in real contexts. Suggestions for these include a school tuckshop or practical measuring, such as calculating the quantity of brick pavers required for a school courtyard. Cooking a cake provides multiple measuring, estimating and calculating experiences with money, mass and capacity when buying ingredients. Following the recipe involves accurate measuring of each ingredient's mass or capacity, and timing of the cooking, which involves temperature and finally, the mass or volume of the finished product could be measured. Generations ago, many students had apprenticeship style experiences with family members by cooking, sewing or making furniture items. For many of today's students, these at-home experiences are not possible. Generally families today have less time and this is reflected in the fact that much of today's food sold in supermarkets is pre-prepared ready to eat.

Roth (1976) suggested that in order for context to be of benefit to student mathematical practices, three school-based aspects are needed. These aspects are: a) resources; b) teacher experience; and c) classroom peer interactions; and each will be discussed in turn.

**Resources**

Reys et al (1993) reported that while there are appropriate resources such as the 1986 NCTM Yearbook, *Estimation and Mental Computation* that include practical instructional ideas, most teachers were unaware of the existence of such resources. Reys et al (1993) further
stated that, in response to the low mental computation performance reported in their study, few teachers spent time developing or teaching mental computation. Further that there was great variation among classrooms regarding the attention given to mental computation. By comparison, the study reported here found that mental computation was timetabled daily although the nature of the activities varied. It seems reasonable then to recommend that teachers be familiar with appropriate resources. It may be that in-servicing of teachers is required in order to familiarise staff with appropriate and current resources. In-servicing should include demonstrations of how to “value flexible thinking and allow for learner-constructed mental computation techniques” in order to improve results as Reys et al (1993) suggested (p. 314).

Teacher Experience

As mentioned previously with Vygotsky’s (1978) ‘zone of proximal development’, ‘Scaffolding’ is a metaphor originated by Wood et al (1976) to describe ideally what the teacher does, to extend (scaffold) the child to build on from where they already are. The teacher/adult needs to know the student well enough to comprehend the extent of their prior knowledge. This knowledge can make family members good teachers. Both the adult and the child appear to be on the same ‘wave length’ as the adult use of language familiar to the child and can act as a ‘scaffold’ by utilising shared money and other relevant real-life experiences. Responses to question 7 revealed that some children were found to talk to their ‘adult(s)/scaffold(s)” on a regular basis and this talk may be an important factor. Student responses support this, such as when one student stated that: “My Dad’s an accountant and (he is) my financial advisor” and when a second student stated that “My Grandma and I have how are your shares going? chats”.

It had been assumed that traditional teaching approaches might be more likely to be taught by older teachers who had trained when such approaches were standard practice. However, this was not found the case for the teachers involved in the current study. The older teachers were also more senior and were up-to-date with current mathematics pedagogy. Keeping up-to-date seemed to have occurred through professional development, professional reading, or in-servicing of individual teachers or whole-school staffs. Therefore the in-servicing of teachers or whole-school staffs, in the use of context, in relevant, meaningful and challenging mental mathematics activities, may be an efficient and appropriate strategy for staff to develop students’ mental strategies further.
Hughes et al (1999) claimed that an implication for teaching is that “children need to be taught from an early age how to apply their mathematical knowledge in a range of contexts and settings” (p. 76). With regard to mental computation strategies, this means encouraging the discussion of ideas to solve mental items set in a context. Numbers are rarely found in the real world devoid of context, except by a few individuals for which mathematics is their passion. As to the legitimacy of the contexts, it is up to the classroom teachers who, especially at primary school, know their students’ interests. Students in the current study were found to vary for interest shown towards the use of money as a context. This may be explained either as a developmental stage, especially at Year 3, or perhaps a variation in personalities indicating possible future careers.

Classroom Peer Interactions

The research literature generally points to child-centered learning with teachers as facilitators as opposed to ‘transmission’ teachers, constructivist learning in groups, lots of discussion (Yang, 2001) and a building up of confidence in mathematics in order to develop number sense.

Three of Burns’ (1993) suggestions from her 12 most important things you can do to be a better math teacher included: ‘Encourage children to talk with one another during mathematics class’; ‘Have your students explain their reasoning in all instances’; and ‘Take delight in students’ thinking’. All of these suggestions support whole group and small group discussions, as well as individual student interviews. Asking students to explain their thinking is a recommendation of this study as a number of students asked to change their original answers, while explaining their strategy choices. This process allowed students to spot their own errors and self-correct. Another insight that resulted from asking students to explain their thinking was that students also revealed how certain or not they were of their answers.

Summary

Good pedagogy should aim to encourage (through carefully constructed learning environments) the development of students’ mental computation skills that allow students to construct strategies for themselves. These should be appropriate to their level and at their own pace, as they work towards achieving the more efficient strategies. Teaching approaches that suit this pedagogy include estimation, open questions, looking for number patterns, the use of models and visual aids as well as including students prior out-of-school experiences in...
the provision of different contexts. A relevant and realistic money-context might be a shopping activity for a class party. This could include visual aids in the form of the objects (notes, coins and purchase item) as well as peer interaction between the subjects themselves (as buyers or sellers).

Associationist theory has been discussed in relation to explaining students' poor results while constructionist theory has been used to explain students' good performances. Recommendations for teaching practice include providing real money experiences at school, by linking students' out-of-school experiences to classroom learning, such as exploring students' pocket-money purchasing power or promoting mental computation for many context tasks. Beat the calculator activities (Swan, 2000) could be used to promote the power of mental mathematics for simple place value operations.

5.4.2 Implications for Teaching Mental Computation Strategies

McIntosh et al. (1994) reported that the one example of teaching mental strategies often taught by adults is the 'remove the zero' rule. Unfortunately, students who do not fully understand it often misapply this rule. Anghileri (2000) described the tradition in mental recall based on memorization and formulae have meant that mental strategies have not been explicitly in the classroom.

Anghileri (2000) recommended that "watching and listening to children is important for detecting errors and misconceptions" (pp. 132-133). One counting error noted by doing that, in the current study was for the non-context item 60 + 80. One Year 3 student counted "80, 90, 100, 110, 120, 130". While this student had correctly counted out six tens, they had clearly started from the incorrect ten, the 80 instead of from 90. Anghileri (2000) noted a similar instance when she stated that: "In the example '11 - 3'...the number eleven is sometimes included within the three to be counted (11, 10, 9) and the incorrect answer '9' is given" (pp. 132-133).

One remedy for this may be as Thompson (1997) proposed to "legitimate and encourage the use of fingers and counting procedures particularly for simple addition and subtraction" (p. 157). Another remedy for common counting errors recommended has been the use of the 'empty number line' as a model (Bramald, 1998; Beishuizen, 1999). Other external devices should be encouraged when engaging students in mental computation activities, such as jotting things down, and the 100-chart (Beishuizen, 1995). Concrete pedagogical materials
were discussed by Dehaene's (1997) outline of the 'Right Start' program which used concrete materials or external devices such as thermometers, board games, number lines and interactive arithmetic games such as snakes and ladders.

Easley and Easley (1992) found that "when teachers turn to discussion rather than ready-made programs, they seem mystified by the strange mathematical ideas children often have" (p. 8). Anghileri (2000) described the Realistic Mathematics Education (RME) approach to "first engage with the children's informal strategies, elaborating on them later, and move towards more formal standard procedures" as the better way to teach mental strategies (p. 135). Holloway (1997) noted positive responses in UK classrooms towards the change of focus from formal written to informal mental strategies. He described the findings of a group of primary teachers who explored the issues surrounding mental strategies in school. These teachers had found that written versions of children's mental strategies showed little similarity to standard algorithms and that they were based on a different understanding of number (In Anghileri, 2000). Saxe (1991) had earlier found the difference between the nature of informal mental strategies and formal written strategies, while Holloway (1997) reported on debates that had occurred regarding these differences.

Anghileri and Beishuizen (1998) outlined how counting and 'chunking' can be used to develop students' own written division algorithms. The authors called for "further identification of values with numbers through 'chunking'," and to "reflect pupil's naive meanings for division" which will "build on their understanding of the numbers involved" (p. 4). Anghileri (2000) recommended that standard approaches to recording when teaching mental strategies for multiplying larger numbers should be delayed and illustrated the use of 'chunks' and the distributive rule with the example 24 x 16. Anghileri (2000) explained that:

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multiplication by a two-digit number can be avoided altogether if this problem is transformed by doubling and halving...the most elegant solution to this particular problem would be using the 'fact' that 25 x 16 = 400 and then subtracting 16 from 400. (p.83)
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This study found evidence of students in Years 7 and 9 using this chunking method to solve similar items. For example, two items involving chunking of 25s for Years 5, 7 and 9 were the items, 150 + 25 and 7 x 25. Students in Year 7 scored better on both items than Year 9s except for the multiplication in context which achieved identical scores at both year levels. The Year 7s demonstrated that they had a better understanding of the fact that four 25s made 100, by referring to this fact before they calculated. One Year 9 student calculated seven 25s
first by working out two lots of four 25s as two hundred, then by subtracting 25. While the context of money did not make a difference to performance, one Year 7 student mentioned a connection with the context of swimming 25-metre laps.

In the Netherlands, van den Heuvel Panhuizen (2001) identified the main aspects of the Dutch approach. She explained how the curriculum has been developed to support mental strategies: "One must start with rich contexts demanding mathematical organisation or, in other words, 'contexts that can be mathematized'" (p. 51).

5.4.3 Implications for Curriculum

The results of this study, indicated that Year 3 students had difficulty with items in context. Therefore, Year 3 students could be given a differentiated money context curriculum involving smaller monetary values that would be more in keeping with their field of experience. Nunes (1993) explained that the usefulness of a context depends upon the student's experience with the social and empirical constraints associated with that situational context. Year 3 students would be expected to have the least amount of this experience and lesser-developed cognitive frameworks. Students could also receive practice in class shops with prices realistically marked to the cent for rounding up or down at the register.

For the later years, when teaching decimals, fractions and percentages, Anghileri has (2000) recommended against using the traditional teaching approach of starting with fractions then moving to introducing equivalent decimals and percentages. As Moss and Case (1999) stated, "the sort of confidence, flexibility, and inventiveness...called for...in number sense" (p. 143) was achieved in their teaching experiment beginning with percentages rather than fractions and decimals. This was claimed to be because "children's everyday experiences provide contexts in which percentages appear" (p. 111). With regard to a money context, this may include shopping discounts during sales promotions such as 20% off or 10% off. Anghileri (2000) further added, "Complexities also arise because two-digit decimals are commonly used for money and measures with 1.25 read as 'one twenty-five' where the five may also be considered as 'units'" (p. 112).

McIntosh and Dole (2000) asked, "where, if at all, does the assessment of mental computation occur within assessment practices at school, system or national level?" (p. 402), and recommended that "mental computation and number sense need to become integral components of curriculum and assessment procedures, at class, school, and system levels" (p. 407).
Clarke and Stephens (1998) claimed that, “what is assessed defines what is taught” (p. 77). Therefore, pencil and paper tests test only for a student's ability with pencil and paper methods. Pencil and paper mental computation tests are straightforward to administer, save time, and allow for a large amount of data to be collected in one sitting. However, they only reflect answers, rather than the methods used or how certain students were of their answers. Therefore, mental computation should not be primarily assessed by pencil and paper methods, but orally either by interview or as part of class discussions. Yang's (1995) results support Sowder (1988) who claimed that “teachers must examine more than answers and must demand from students more than answers”, as “correct answers are not a safe indicator of good thinking” (p. 227). Yang's (1995) study found that good pencil and paper performance did not correlate with high scores for number sense or understanding. In the absence of mental strategies being taught, mental computation strategies occur naturally and informally, either self-devised or borrowed. In the current study, oral explanations for these sorts of strategies were readily given. When students had obviously borrowed written methods and applied them mentally, oral explanations were harder to give.

Easley and Easley (1992) detail four basic changes for the US primary mathematics curriculum based on a Japanese alternative (Kitamaeno School, Tokyo) that avoided mathematics anxiety while catering for girls and minority groups. This was achieved by “replacing...counting by ones...with partitioning and re-grouping...directly to place value...a basis for algorithms; mental regrouping might play a stronger role in Japanese calculation than counting does (p. 25)”. Regrouping is similar to the ‘chunking’ of numbers recommended by Anghileri and Beishuizen (1998) which could use the ‘chunking’ of monetary values in order to provide a mathematised context (van den Heuvel Panhuizen, 2001).

The Kitamaeno schoolteachers' rationale used more challenging story problems in different ways, not as ‘applications’...rather groups that cut across ability levels to provide more opportunity...to discuss...with peers. The use of groups was based on the proverb that a group can achieve what an individual cannot. Students were also required to write out number sentences, give complete answers, and verbal explanations and teachers “avoid directing students in computation skills” (p. 82). Easley and Easley (1992) also found “the teachers...placing highest priority on the development of children's personal confidence and overcoming fears of being wrong” (p. 58). Anghileri (2000) has claimed that children need
opportunities to talk about their own strategies and to discuss those used by others. She further claimed that teachers were more likely to pose problems that directly address misconceptions and to encourage children to think divergently using open questions.

Plunkett's (1979) suggested that a spectrum of red, orange and yellow bands of calculations recommended for average 11-year-olds be given in a practical, motivational context. Examples of the levels of difficulty for yellow band are: $139 + 28, 83 - 26, 17 \times 3, 17 + 4$. Askew and William (1995) reported that “research results show that ‘knowing by heart’ and ‘figuring out’ support each other in children’s learning about numbers” (In Anghileri, 2000, p. 129).

If all mental computation items were presented in a realistic setting, then a large proportion of number operations could be set in contexts of time, measurement and money. Activities that encourage the use of money as a context in real and meaningful contexts include: buying school lunches, drinks and icecreams, banking, school camp costing, estimating individual spending needs and budgeting. Students could also organise stalls to raise money for a chosen cause such as a sponsored child overseas or current community need such as bushfire relief. Older students could simulate the stock market as in investments, or Lotto for chance and data studies. School-based money activities could include supervised stall shopping, such as for Mother’s Day, ice-cream days and a class shop. Excursions could include the local markets or shopping centres such as the Queen Victoria market in Victoria for ‘fruit and vegie week’ or the local bakery. Students would be supervised in small groups, counting-out money to buy specific items in single transactions. This study suggests that it is likely to be real-life contexts such as this, rather than contrived ones that make a difference.

The presentation of money context items in a game setting may have provided more incentive or fun as then students often wish to continue playing games in their free time. A money tally system is where students earn or lose money instead of points and keeps a record of this in a ‘passbook’ and maybe part of the ‘Earn n learn’ program. Some Year 7 students mentioned collecting school fundraising donations, giving change and counting the total proceeds. While it has been common practice for some teachers to begin their mathematics sessions with a game such as ‘Sheriff’, as discussed previously, the value of this playful aspect is more questionable as context and number sense gain more value. Dehaene (1997) suggested that children should be shown the playful aspects of mathematics before they are introduced to abstract symbolism. This is consistent with playing shops with five-cent and ten-cent coins that could provide the basis for informally learning the more formal
‘five times’ and ‘ten times’ number facts. In turn, this suggests that not only should teachers assess students’ prior knowledge and attitudes but also ideally, they should incorporate some pre-play experiences that are by their nature, embedded in context.

Professor Gaudry has recommended that all students be able to operate on fractions, except for division by the end of primary school. This is due to the equivalence of fractions being crucial knowledge for high school algebra (The World Today, 7/7/03). The equivalence of decimal fractions could be presented in a money context. The study reported here found improved performance for two money context items, the addition of two decimals and a multiplication of a decimal that revealed student knowledge regarding the equivalence of monetary fractions.

Whole/part/whole teaching experiences are currently being introduced in numeracy blocks in the state of Victoria. A suggested teaching sequence may include the whole class teaching of a playful aspect, commonly known as free and undirected play when introducing new materials, such as play money. 'Part teaching' caters for small ability groups, and could then focus on the more abstract and formal components such as task cards, which encourage students to explore different ways to pay for their items. An example task might be: You have $6 and need to pay $4 for your lunch. List how many different ways you could perform this transaction and note the most efficient way. Answers should include different possibilities, such as three $2-coins or six $1-coins. One group of students might systematically list every possibility, while another group may just compare two different alternatives. The students then return to a whole class discussion.

5.4.4 Implications for Research

Recommendations for further research on the provision of money as a context, include mental computation items presented as real shopping activities. For example, students could be given a task to buy and sell items (addition, multiplication and subtraction) and then split the profits equally (division). Assessment of the students could include observation, anecdotal notes and oral/written self-reporting. Class shops should not be only pretend ones for the junior grades, as older grades can organise real stalls and integrate them with other curriculum areas for charitable causes.
Further study could involve comparing a control group at Year 7 or 9 with a similar aged teaching group who are given lots of mental computation experiences set in context in a unit of work. Such a study could examine the results from this study where Year 7 students scored better than the Year 9 students, possibly because they were presented with many more recent, real money experiences at school.

Further research on the effect of context on student's mental computational performance needs to address the relevance of a particular context to the individual learner. As, what constitutes a meaningful context or presentation of a context for students' may vary from individual to individual. Contexts may be obviously mathematical in nature or embedded in a realistic situation, such as found in children's literature, in which the mathematics is not at first obvious. In addition, other common mathematical contexts such as measurement, time or food could be studied to find whether the findings of this study are applicable cross-contextually. Research on the effect of other common contexts should be embedded in real activities. Examples of such contexts can be found regularly in the media such as: litres of fuel; and quantities of materials, for measurement; and: television guides; transport timetables; and sport scores and time trials; for time. The provision of contexts other than money is important for students to understand that numbers have meaning beyond two decimal places.

Certainly, it is important to check for the effect of context when the experiences are real rather than imaginary, as provided in the current study. The imaginary play generated by junior grade students involved in class shops revolves around their imagination, not that of adults. The fact that the context items used in the current study were adult rather student generated may account for the lack of student improved performance, but equally, one student's generated idea may not be considered as relevant by other students.

5.5 Recommendations for Further Study

Possible reasons as to why context did not make a difference to student performance were previously discussed. They include; relevance for maximized engagement, problems with transfer, quality and quantity of prior context experiences, as well as opportunities for discussion and school practice. Suggestions for further study reflecting these identified areas are given below.
• Would presenting items only orally—thus removing a possible cue for the use of written methods—make a difference to student performance or the choice of strategies that students use?

• What difference would schoolwork involving real and practical money items, instead of contrived ones, make to student performance?

• To what extent are students able to demonstrate transfer of money knowledge, and thus their number sense? A comparison study could extend the context items used in the current study to related decimal and higher place values in different contexts. The situational meaning could be kept—by reducing or increasing the magnitude of the numbers by powers of ten.

• What improvement might there be for subtraction and multiplication of similar decimal items, given that the greatest improvement in student performance for context was for the addition of decimals?

• How much context experience either at school or at home, is needed to make a marked difference to student mental computation performance for items set in a context?

• Which are the most effective classroom resources for improving student mental computation performance for items set in a context?

• Which teacher strategies are the most effective for improving student mental computation performance for items set in a context?

• Which types of classroom peer interactions are the most effective for improving student mental computation performance for items set in a context?

• Would an increase of say five minutes per day in mental computation make a difference to student performance?

• To what extent does a student's ability to use efficient mental strategies and thus number sense, have on that student's choice to continue with mathematics beyond the compulsory years and on their future career choices?
5.6 Concluding Remarks

If students' performance with money context items is to be improved, then much more experience with in-context mental computation needs to occur in the classroom.

Four reasons have been suggested as to why the main finding that context did not make any difference to student mental computation performance was found by this study. It may have been a problem regarding the nature of the items being contrived and presented in a school test, rather than the choice of a money context. The context items were contrived in the same way that commercial text word applications are, and therefore may not have been relevant or real enough for the students. However, as it was also found that at Year 3, context had a negative impact for student performance, and this was probably due to an overall lack of experience with mental computation in context, both at school and elsewhere.

Money experience was found to make little difference to the results and this could have been due to the design of the instrument not allowing for how recent the experiences were. Only two students were reported to have had paid part-time jobs where they also worked with money. While both students scored better for context, only one was currently employed. This student was also found to have a significantly better score for context with a difference of 23 percent, and used higher order strategies. This may support the theory that for context to make a difference to student performance, the experience needs to be meaningful and recent.

The provision of real money experiences such as working in real shops, running real stalls either at school or for clubs, should also maximise student engagement and the transfer of strategies for use in other contexts. The use of money to develop more efficient mental computation strategies through the equivalence of money values was suggested for developing number sense.

While it was also believed that students might benefit from receiving more mental mathematics experiences generally, more examples should be set in real contexts at school, as some teachers reported using traditional word applications. This study found no difference for gender; but that for age, the Year 7s out-performed the Year 9s. Reasons given suggested that the primary school students had received more hands-on money experiences and spent more class time on mental computation. McIntosh (1994) has recommended 15 minutes of mental computation per day. Therefore, the increase need only be a matter of some ten minutes per day at the secondary school level.
Another finding was that some students were using written methods mentally. Removing school cues for the use of written methods should encourage the use of mental methods. Presenting a planned unit of work involving real and practical money experiences, where only mental computation is allowed, should result in removing school cues and lead to students using mental methods, thus improving student mental computation performance.

Extension activities that begin with real context mental computation and extend to related decimal and larger place values could help determine to what extent students are able to demonstrate transfer of money knowledge. This activity could be suited to class discussions during the conclusion of the lesson. Several suggestions have been made for better teaching for transfer, including the use of external devices such as the use of real money or play coins and notes. These materials could be used in a class shopping game as a representation of simulated shopping tasks for younger students.

While money is a significant context in all students' lives, other contexts should also be used in the classroom. They could provide a variety of situational learning experiences and cater for individual differences in the range of prior knowledge that students bring to the mathematics lessons.

Improvements to teaching and curricula require an increase to both the quality and quantity of time that students spend doing mental mathematics in context. Context experiences need to be recent, real, and relevant to the individual and be part of class discussions every day, as well as being integrated with other learning areas as appropriate.
References


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Appendix I: Ethics Protocols for Permission Notices

Letter to Secondary School

Head of the Mathematics Department

Secondary School

Dear ____________

Re: Request for volunteer students to participate in PhD studies

I am a Ph.D. student at Edith Cowan University researching mathematics education. I am currently seeking eight volunteer students over a range of ability, in Year 9 to help with a study I am conducting on students' mental computation performance. My thesis is titled: 'The effect of money as a context on students' mental computation performance for Years 3, 5, 7, and 9.'

I plan to conduct these interview sessions in second term. Each student will be individually involved in one, tape-recorded session that takes approximately 30 minutes. I will be asking students about their background knowledge about money and will ask them mental computation questions involving money. With your approval and assistance, a quiet working area is requested. Results of the study may be published, as they will form part of my doctoral thesis. In order to protect students' privacy, neither the students nor the schools will be directly identified in any publication. Pseudonyms will be used to protect each student's identity. Parents may withdraw their child from participation, at any stage in the process if they wish to do so. I would like to call you next week, to see if your school would be interested in participation in this study.

Please do not hesitate to call my supervisor, Dr. Jack Bana, Director, Mathematics, Science and Technology Centre at Edith Cowan University on 9370 6468, or myself, if you have any queries. Either of us would be happy to discuss with you any issues that you may have, before, during and/or after the session. I have enclosed copies of the parental and student permission forms for your use.

Yours faithfully

Anne Paterson

s.paterson@ecu.edu.au

Cc. Dr. Jack Bana

j.bana@ecu.edu.au
Letter to Primary School

The Principal
Primary School

Dear ___________________

Letter: Request for volunteer students to participate in PhD studies

I am a Ph.D. student at Edith Cowan University researching mathematics education. I am currently seeking 12 volunteer students over a range of ability in Years 3, 5 and 7 to help with a study I am conducting on students' mental computation performance. My thesis is titled: 'The effect of money as a context on students' mental computation performance for Years 3, 5, 7, and 9'.

I plan to conduct these interview sessions in second term. Each student will be individually involved in one, tape-recorded session that takes approximately 30 minutes. I will be asking students about their background knowledge about money and will ask them mental computation questions involving money. With your approval and assistance, a quiet working area is requested. Results of the study may be published, as they will form part of my doctoral thesis. In order to protect students' privacy, neither the students nor the schools will be directly identified in any publication. Pseudonyms will be used to protect each student's identity. Parents may withdraw their child from participation, at any stage in the process if they wish to do so. I would like to call you next week, to see if your school would be interested in participation in this study.

Please do not hesitate to call my supervisor, Dr. Jack Bana, Director, Mathematics, Science and Technology Centre at Edith Cowan University on 9370 6468, or myself, if you have any queries. Either of us would be happy to discuss with you any issues that you may have, before, during and/or after the session. I have enclosed copies of the parental permission form for your use.

Yours faithfully

Anne Paterson

Your email address
Cc. Dr. Jack Bana
j.bana@ecu.edu.au

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Letter to Parents/Guardians

Dear Parent/Guardian(s)

I am a Ph.D. student at Edith Cowan University researching mathematics education. I am seeking volunteer students in Years 3, 5, 7 and 9 to help with a study I am conducting on students’ mental computation performance. Computation will involve examples involving money, so I will also be asking each student about their experience in using money. Your child has indicated interest in becoming such a volunteer.

Each student will be involved in one, tape-recorded interview that takes approximately 40 minutes, during school time. These interviews will take place in a special room designated by the school. At the interviews are one-on-one, I need to inform you of precautions I need to implement for ethical clearance. The room is in a busy, open access area, the door will be kept ajar, and window coverings will be opened. If you would like to see the room before hand, this can also be arranged. Please also be assured that I have obtained police clearance.

I plan to conduct these interview sessions during second and third terms. Results of the study will form part of my doctoral thesis. In order to protect your child’s privacy, neither the students nor the schools will be identified in any publication. Pseudonyms will be used to protect each student’s identity. You may withdraw your child from participation, at any stage in the process if you wish to do so. If you have any concerns about the project or would prefer to speak to an independent person please call Marilyn Beresford, ECU Research Ethics Officer on 9273 8170, or my supervisor, Dr. Jack Bana on 9370 6468. Naturally, I would be happy to discuss with you and/or your child any issues that arise before, during and/or after the session. If you agree to your child’s participation in a session, please sign the consent form below and return it to your school office.

Yours faithfully

Anne Paterson

To: the School Office, ___________________________ (School),

I, ___________________________________________ (name of Parent), hereby give consent for my son/ daughter/ dependant* in Year 3/ 5/ 7/ 9* to participate in a study of mental computation by Anne Paterson as part of her studies at Edith Cowan University. I agree that the research data gathered for this study may be published if neither the students nor the schools are identified. I understand that I may withdraw my child at any stage if I wish to. My child is in class __________, Room __________.

___________________________________________

(Signature of guardian/ parent*) (Date)

*Delete as applicable
Dear Student,

I am a Ph.D. student at Edith Cowan University researching mathematics education. I am seeking volunteer students in Years 3, 5, 7 and 9 to help with a study I am conducting on students' mental computation performance. Computation will involve examples involving money, so I will also be asking each student about their experience in using money. Your school informs me that you have indicated interest in becoming such a volunteer.

Each student will be involved in one, tape-recorded interview that takes approximately 40 minutes, during school time. I plan to conduct these interview sessions during second term. Results of the study will form part of my doctoral thesis. In order to protect your privacy, neither the students nor the schools will be directly identified in any publication. Pseudonyms will be used to protect each student's identity. You may withdraw from participation, at any stage in the process if you wish to do so. If you have any concerns about the project or would prefer to speak to an independent person please call Marilyn Bercsford, ECU Research Ethics Officer on 9273 8170.

I would be happy to discuss with you any issues that arise before, during and/or after the session. If you agree to participation in a session, please sign the consent form below and return it to your school office.

Yours faithfully,

Anne Paterson

To: the School Office, __________________________ School,

I, ___________________________ (name of student), in Year 7/9 * hereby give my informed consent to participate in a study of mental computation by Anne Paterson as part of her studies at Edith Cowan University. I agree that the research data gathered for this study may be published if neither the students nor the schools are identified. I understand that I may withdraw at any stage if I wish to. I am in class __________________________, Room __________________________ 2002

(Signature of student) ___________________________ (Date)

*Delete as applicable
Appendix II: Protocol for Money Experience Instrument

Interviewer says:

Hello, my name is Anne Paterson from Edith Cowan University. Perhaps you know of it? Thank you for volunteering for this study. I am trying to find out how students at your year level do calculations mentally; therefore I need your help and appreciate your co-operation today. I shall be giving you two different sets of items to answer. The results will be kept confidential and I know you will do your best in this effort. Thank you.

As you know, this session will be tape-recorded. You will not need to write anything. All working should be done in your head. I will read each question aloud and would like you to read along silently with me. You will be given time to work out your answer, and as soon as you have worked out your answer, tell me that answer, then I will ask, 'can you tell me how you worked that answer out?' Then you need to explain how you worked it out. Is this okay? For privacy reasons, I shall identify you by a code-number rather than your real name. Your code number is ——. Before we begin the items, I need to ask you a few questions regarding your experience with money.

Some of the maths we will be doing today is about money. I want to find out what you know about money so that if you have any problems with money calculations this will help me understand why. Therefore, I need to ask you some questions. Is this okay?

How often do you get pocket money or an allowance?

Do you need to:

a) earn it all or part of it; or b) do you get money in other ways such as a part-time job?

Have you had any jobs at home or in other places where you have had to work with money? (If yes) Tell me about it.

Tell me how you use your money?

Let’s pretend you won $50 in Lotto. What would you do with it?

Do you ever save money? (If yes) Tell me about it.

How often do you talk to your parents about what to do with your money?

What sorts of school activities involving money do you enjoy?

How do you think that working with money helps you learn maths?
Appendix III: Flow Chart for Rating Money Experience

1. How often do you get pocket money or an allowance?
2. Do you need to: a) earn it all or part of it; or b) do you get money in other ways such as a part-time job?
3. Have you had any jobs at home or in other places where you have had to work with money? (If yes) Tell me about it.
4. Tell me how you use your money?
5. Let’s pretend you won $50 in Lotto. What would you do with it?
6. Do you ever save money? (If yes) Tell me about it.
7. How often do you talk to your parents about what to do with your money?
8. What sorts of school activities involving money do you enjoy?
9. How do you think that working with money helps you learn maths?

End questions Score = 3

End questions Score = 2

End questions Score = 1
Money Experience Instrument Rating Procedure

For a score of 3
[Autonomy: High exposure/High needs]

Procedure

First, check Qn 3.
If yes, the student had worked with money (worth 1/2), go to Qn 2b.
If yes, the student had a regular, paid, part-time job (worth 2 1/2) this now totals 3, allocate a 3.
If yes at Qn 3 (worth 1/2), but no at 2b, go to Qns 1 & 2a.
If yes to regular pocket money (worth 1/2), go to Qn 4. If the student was self-managing (worth 2) this now totals 3, allocate a 3.
If no to regular pocket money go to Qn 7. If yes, to parental advice (worth 1/2), go to Qn 4. If the student was self-managing (worth 2), the total is 2 1/2, round up to allocate a 3.

Note: if student answered no to Qn 7, 4b or 4c, a rating 3 was not given.

If at Qn 3 the student had not worked with money, go to Qns 1 and 2a.
If no to regular pocket money, go to Qn 2b. If yes to a part-time job (2 1/2), round this up to allocate a 3.
If yes to regular pocket money (1/2), but no to Qn 2b, go to Qn 4. If the student was self-managing, worth (2), round the 2 1/2 up to allocate a 3.
If no to Qns 3, 1, 2a and 2b, go to Qn 7. If yes to parental advice (worth 1/2), go to Qn 4. If the student was self-managing (2), round the 2 1/2 up to allocate a 3.
If at this stage, a rating of 3 has not been allocated, progress to rating 2: Novelty.
For a score of 2
[Novelty: Medium exposure/Low needs]

Procedure

[Note: Qn 2b is now eliminated]

Check Qn 3.
If yes, student had worked with money (worth \(\frac{1}{2}\)), go to Qns 1 and 2a. If yes to earns pocket money (worth \(\frac{1}{2}\)), go to Qn 4.
If balanced between saving and spending (1), this now totals 2, so allocate a 2.
If either extreme or vague (0) go to Qn 8. If either real selling (1) or class money experiences (\(\frac{1}{2}\)), allocate a 2.
If no to earns pocket money at Qns 1 and 2a, go to Qn 7. If yes (worth \(\frac{1}{2}\)), go to Qn 4.
If balanced between saving and spending (worth 1), allocate a 2.
If extreme or vague (0), go to Qn 8. If real selling experiences were evident (1) allocate a 2, or class experiences (\(\frac{1}{2}\)), round up to allocate a 2.
If no at Qn 7, go to Qn 5. If specific pricing knowledge, exact prices or budgeting (1 \(\frac{1}{2}\)) were reported, round up to allocate a 2.
If extreme or vague (0), go to Qn 8. If real selling experiences were evident (1) allocate a 2.
If the student reported class money experiences (\(\frac{1}{2}\)), go to Qn 6. If the student reported planned saving here (1), round up to allocate a 2.

If no at Qn 3, but yes to earns pocket money at Qns 1 and 2a (\(\frac{1}{2}\)), go to Qn 4.
If balanced between saving and spending (1), round this up to allocate a 2.
If extreme or vague (0), go to Qn 8. If real selling experiences, round up to allocate a 2.
If class money experiences (\(\frac{1}{2}\)) go to Qn 6. If planned saving (1), allocate a 2.

If no to earns pocket money at Qns 1 and 2a, go to Qn 7. If yes to parental advice (\(\frac{1}{2}\)), go to Qn 4.
If balanced between saving and spending (1), round up to allocate a 2.
If extreme or vague, go to Qn 8. If class money experiences (\(\frac{1}{2}\)) were reported, go to Qn 6. If planned saving was evident, allocate a 2.

If no to parental advice at Qn 7, go to Qn 5. If specific pricing knowledge was noted (worth 1 \(\frac{1}{2}\)), round up to allocate a 2.
If general or vague (0) go to Qn 8 and rate as before.
If unrealistic pricing was identified (0), go to Qn 8. If real selling experiences were reported (1), round up to allocate a 2.
If class money experiences (\(\frac{1}{2}\)) and planned saving (1) were reported, allocate a 2.

If at this stage a rating of 2 has not been allocated, progress to rating 1: Play.
For a score of 1
[Play: Low exposure/Low needs]

Procedure

[Note: This pathway is only accessed through Qn 4 or Qn 5.]

To get here from Qn 4, students may already have a minimum rating of (1/2), or a maximum of (1), for either pocket money, working with money or parent advice. While, from Qn 5, students may already have a minimum rating of (0), or a maximum of (1/2), for working with money.

First, ascertain this score from the student’s answers to Qns 3, 1, 2a and 7.

At Qn 4, if the student appeared to either be a spend-all, a save-all or was vague (0), go to Qn 8. These were considered immature experiences with limited or simplistic calculations involved.

If the student reported little or no school money experiences (0), round up if necessary, to allocate a 1.

If, class money experiences were mentioned, (1/2), allocate a 1.

[Note: The possibility of getting a (1 1/2) here was eliminated earlier].

If at Qn 5, the student demonstrated a lack of realism, for example to spend $50 to buy a horse or car (0) round up if necessary, to allocate a 1.

If the student mentioned general categories or was vague (0), go to Qn 8.

If little or no school money experiences (0), were identified, round up if necessary, to allocate a 1.

If class money experiences were mentioned (1/2), go to Qn 6. The student could have answered no or yes here—but followed this answer with a general or vague comment (0), allocate a 1.
Appendix IV:  
Sample Transcripts from Money Experience Interviews

Example of a Rating of 3  
Student 20:  
No pocket money.  
 a) N/A  
b) Cutting up boxes.  
 At the surf shop, I help at the till and use the calculator to check.  
I buy bigger things. I'm into investments.  
Unless there was something I wanted, I'd save it in the bank towards investment.  
Student had previously mentioned saving in the bank short term towards longer term investments.  
My Dad's an accountant and my financial advisor.  
Counting money is better than sums.  
The coins are worth the amounts and the notes...help with sums.

Example of a Rating of 2  
Student 32:  
I have to work for money and I get $2 every week.  
a) I have to earn it all.  
b) No.  
No.  
I save it up so I can buy something that's really expensive. Probably some clothing. I would keep on saving it up so I can go on a shopping spree with my friend.  
Student answered Yes, refer back to answers given previously for Qns 4 and 5.  
Probably, twice a week.  
I enjoy when we get tests about money.  
If you need to add up something like 50 + 160 or something, you could just add it more quickly. Because if it sounds like money it sounds more far ner.

Example of a Rating of 1  
Student 6:  
Yes.  
a) No.  
b) No.  
No.  
I keep it in my purse saving for a dog.  
If I had a big backyard, I'd buy a horse.  
Yes, at home. I'm saving for a dog.  
No, they won't let me spend it all the time.  
Worksheets, pictures of money.  
There's money and you have to add it up all right.
Appendix V: Protocol for Mental Computation Instrument

Directions to student:

Now we are ready to start the items. Student is given the first sheet of items. First, we will do a practice example. The practice examples are identical across all four year levels: First, we will be doing mental computations with (without) money. Read the practice example (in context: James had 20 cents then was given 70 cents, how much does he have now? Non-context: Fifteen take away nine equals?) You may write or say the answers but do not do any written working out. Are there any questions? Let us begin. Researcher starts the tape recording. Administer first test. Please read Number one along with me. Researcher reads question 1 aloud, waits for the student’s answer (a maximum of 30 seconds), then when the answer is given, asks: Could you tell me how you got that answer? Make notes of any non-audible observations such as body language, or use of fingers while the student explains. If no answer is given in the maximum time allowed, say time’s up and move on to the next item. At the conclusion of the first test, collect the paper and allow a few minutes’ opportunity to relax for a short time before proceeding with the second half of the items.

Now we are going to do some mental computations without (with) money. Give student the second sheet. Administer the practice example. Practice example (non-context: Fifteen take away nine equals? In context: James had 20 cents then was given 70 cents. How much does he have now?)

Administer the second test and say Please read Number one along with me. Researcher reads question 1 aloud, waits for the student’s answer (a maximum of 30 seconds), then when the answer is given, asks: Could you tell me how you got that answer? Make notes of any non-audible observations. If no answer is given in the 30 seconds allowed, say time’s up and move on.

Finally, ask the student: Did you like one test more than the other test? Why is that? Did you notice anything similar about the tests?

The test is now finished. Thank you very much for helping.

Collect the student’s second sheet. For feedback, the student is asked if they want to know of any items answered incorrectly and were then allowed to have another attempt. Alternate strategies were then discussed to assist the student, but such attempts were not recorded.
### Appendix VI: Matched Mental Computation Instrument Items by Year Level

Table A1: Year 3 Mental Computation Items in Matched Form

<table>
<thead>
<tr>
<th>Practice example:</th>
<th>Practice example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>James had 20 cents then was given 70 cents. How much does he have now?</td>
<td>15 – 9</td>
</tr>
<tr>
<td>1. I spent 60 cents on an icy pole then 80 cents on a chocolate bar. How much did I spend altogether?</td>
<td>1. 60 + 80</td>
</tr>
<tr>
<td>2. It costs $79 for our puppy's injections. It also cost $26 for puppy food. How much is this altogether?</td>
<td>2. 79 + 26</td>
</tr>
<tr>
<td>3. When Mum bought a dress for $68, she was given $32 change. How much money did Mum give the shopkeeper?</td>
<td>3. 68 + 32</td>
</tr>
<tr>
<td>4. Amy's brother earned $74 in his part-time job. He gave his Mum $30. How much did he keep?</td>
<td>4. 74 – 30</td>
</tr>
<tr>
<td>5. Mum saved $140 then spent $60 on a present for Dad. How much did she have left?</td>
<td>5. 140 – 60</td>
</tr>
<tr>
<td>6. Dad had $80 and bought a shirt for $24. How much change did he have left?</td>
<td>6. 80 – 24</td>
</tr>
<tr>
<td>7. We took $105 to the Show but returned with $26. How much did we spend?</td>
<td>7. 105 – 26</td>
</tr>
<tr>
<td>8. What is the total cost of two books priced at $26 each?</td>
<td>8. What is double 26?</td>
</tr>
<tr>
<td>9. My twin brother and I spent $16 on Mum's birthday present. If we paid half each, how much did I pay?</td>
<td>9. What is half of 16?</td>
</tr>
<tr>
<td>10. Grandma had 30 cents and gave me half of it. How much did she give me?</td>
<td>10. What is half of 30?</td>
</tr>
</tbody>
</table>
Table A2: Year 5 Mental Computation Items in Matched Form

<table>
<thead>
<tr>
<th>Presentation Format: In Context</th>
<th>Presentation Format: No Context</th>
</tr>
</thead>
<tbody>
<tr>
<td>Practice example: James had 20 cents then was given 70 cents. How much does he have now?</td>
<td>Practice example: 15 - 9</td>
</tr>
<tr>
<td>1. I spent 60 cents on an icy pole then 80 cents on a chocolate bar. How much did I spend altogether?</td>
<td>1. 60 + 80</td>
</tr>
<tr>
<td>2. It costs $79 for our puppy’s injections. It also cost $26 for puppy food. How much is this altogether?</td>
<td>2. 79 + 26</td>
</tr>
<tr>
<td>3. When Mum bought a dress for $68, she was given $32 change. How much money did Mum give the shopkeeper?</td>
<td>3. 68 + 32</td>
</tr>
<tr>
<td>4. Amy’s brother earned $74 in his part-time job. He gave his Mum $30. How much did he keep?</td>
<td>4. 74 - 30</td>
</tr>
<tr>
<td>5. We took $105 to the Show but returned with $26. How much did we spend?</td>
<td>5. 105 - 26</td>
</tr>
<tr>
<td>6. What is the total cost of two books priced at $26 each?</td>
<td>6. What is double 26?</td>
</tr>
<tr>
<td>7. It costs our family $165 per day for a hotel room plus $99 for a day’s meals. How much did one day cost for our family on holiday?</td>
<td>7. 165 + 99</td>
</tr>
<tr>
<td>8. Your school is fundraising by selling Grand Final tickets for $60 each, and 70 tickets are sold. How much money will this raise altogether?</td>
<td>8. 60 x 70</td>
</tr>
<tr>
<td>9. I want to buy a lollipop for seven children. If lollipops cost 25 cents each, how much will I spend in total?</td>
<td>9. 7 x 25</td>
</tr>
<tr>
<td>10. If 25 people share a $150 win on Lotto, how much money should each person get?</td>
<td>10. 150 + 25</td>
</tr>
<tr>
<td>11. Mum spent $6.20 in the bakery then she spent $4.90 at the newsagent. How much did she spend altogether?</td>
<td>11. 6.20 + 4.90</td>
</tr>
<tr>
<td>12. A school fair raised $3500 for new computer programs. How many can be purchased if the price is $35 each?</td>
<td>12. 3500 ÷ 35</td>
</tr>
</tbody>
</table>
## Table A3: Year 7 and Year 9 Mental Computation Items in Matched Form

<table>
<thead>
<tr>
<th>Presentation Format:</th>
<th>Presentation Format:</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Context</td>
<td>No Context</td>
</tr>
<tr>
<td>Practice example:</td>
<td>Practice example:</td>
</tr>
<tr>
<td>James had 20 cents then was given 70 cents. How much does he have now?</td>
<td>15 – 9</td>
</tr>
<tr>
<td>1. It costs $79 for our puppy’s injections. It also cost $26 for puppy food. How much is this altogether?</td>
<td>1. 79 + 26</td>
</tr>
<tr>
<td>2. It cost our family $165 per day for a hotel room plus $99 for a day’s meals. How much did one day cost for our family on holiday?</td>
<td>2. 165 + 99</td>
</tr>
<tr>
<td>3. We took $105 to the Show but returned with $26. How much did we spend?</td>
<td>3. 105 – 26</td>
</tr>
<tr>
<td>4. Alex and his Mum made $264 at their garage sale. Alex then bought a $99 play station game. How much money do Alex and his Mum have left?</td>
<td>4. 264 – 99</td>
</tr>
<tr>
<td>5. Your school is fundraising by selling Grand Final tickets for $60 each, and 70 tickets are sold. How much money will this raise altogether?</td>
<td>5. 60 × 70</td>
</tr>
<tr>
<td>6. What is the total cost of 38 Harry Potter cards at 50 cents each?</td>
<td>6. 38 × 50</td>
</tr>
<tr>
<td>7. I want to buy a lollipop for seven children. If lollipops cost 25 cents each, how much will I spend in total?</td>
<td>7. 7 × 25</td>
</tr>
<tr>
<td>8. A school fair raised $3500 for new computer programs. How many can be purchased if the price is $35 each?</td>
<td>8. 3500 ÷ 35</td>
</tr>
<tr>
<td>9. If 25 people share a $150 win on Lotto, how much money should each person get?</td>
<td>9. 150 ÷ 25</td>
</tr>
<tr>
<td>10. Mum spent $6.20 in the bakery then she spent $4.90 at the newsagent. How much did she spend altogether?</td>
<td>10. 6.20 + 4.90</td>
</tr>
<tr>
<td>11. My Dad had $48 and spent 25 percent of it. How much did he spend?</td>
<td>11. What is 25% of 48?</td>
</tr>
<tr>
<td>12. If you hand over six dollars in two-dollar coins to pay for your lunch, which costs $4.50, how much change should you get?</td>
<td>12. 6 – 4.50</td>
</tr>
<tr>
<td>13. I want to buy 0.1 kilo of lobster that costs $45 a kilo, how much do I need to pay?</td>
<td>13. 0.1 × 45</td>
</tr>
</tbody>
</table>
Appendix VII: Mental Computation Instrument Items Distributed by Topic

<table>
<thead>
<tr>
<th>Operation</th>
<th>Year 3</th>
<th>Year 5</th>
<th>Years 7 &amp; 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Numbers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+</td>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>-</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>x</td>
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<td>3</td>
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<tr>
<td>+</td>
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<td>2</td>
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<tr>
<td>Non-whole Numbers</td>
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<tr>
<td>+</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-</td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>
Appendix VIII: Samples of Matched Item Instrument

Interview Transcripts

Item: 68 + 32

Student 3, Year 3, Context
I added the 60 and the 30 and that made 90 (cents), then the 8, and the 2 to get $1. I think it's $3, I'm sure it's $3, because there's 2 more of those dollar signs and if you add 68 and 32 it makes another dollar... it makes $3.
(Note: This student's answer for this item in non-context was correct: 100)

Student 5, Year 3, Non-Context
Put 32 under the 68, and 6 add 3 is 9 and because you don't do the ten, you put the zero back on again. (Interviewer: Why don't you do the ten?) You don't do the one otherwise it would make 910.

Student 32, Year 5, Non-Context
Because two plus eight equals 10, which makes the 68, 70 and then plus, 30 makes 100.

Item: 7 × 25

Student 25, Year 5, Context
7 times 25, which is 175, because I do swimming training. I used to do it in a 25-metre pool when I was little, so I used to have to count the metres. It couldn't be $17 because they're only 25 cents each, so I just put the decimal point in.

Student 34, Year 7, Context
20 cents by seven would equal $1.40 but then it's another 5 × 7 so that's 35 cents, so it's $1.75.

Student 51, Year 9, Context
I timesed 25 times 4, would give me a dollar and then I timesed it by three to give me the 75 cents.

Item: 6.20 + 4.90

Student 32, Year 5, Context
Six and four equals 10 and then 90 and 20 equals $1.10 and then plus the dollar to the 10 and that equals 11 dollars 10.

Student 44, Year 7, Non-Context
(The answer given was incorrect for no context, correct for money-context.)
6 plus 4 is 1 whole and then 9 plus 2 is eleven so I added another whole so that's up to 10...so that would be 2 and 1 remainder. (12.10)

Student 44, Year 7, Context
4 plus 6 is 10 and then 50 add 20 is 110 so that was over 100 so add another whole dollar to the 10. 11 and then 10 cents change. ($11.10)

Student 60, Year 9, Context
(No attempt was made for non-context, while for the money-context item the student responded correctly, $11.10).
6 add 4 is 10 add on the 90 cents and the 20 cents is $1.10...

**Item: 6 – 4.50**

Student 35, Year 5, Context
Just add $1.50 to $4.50 and it equals up to $6.
(The written answer given was 4.44, hence the importance of students explaining their answers as a verbal response in cases where written responses do not make sense. The student may have made this error because 50 take away six equals 44. Note that another Year 7 student, 48, gave an identical answer).

Student 44, Year 7, Non-Context
(Zero), six take four you can’t do and the rest is not a whole number.

Student 44, Year 7, Context
**Four dollars from six dollars is two dollars and then take 50 cents.**

Student 49, Year 9, Context
I used from $6, from $4.50, it’s another 50 cents...then, it’s another dollar.

Student 49, Year 9, Non-Context
**If you take off six from 4.5, you end up in negatives.**

Student 64, Year 9, Context
**Six take four then half off. Several ways or you could do 600 take 450 (cents).**
### Appendix IX: McIntosh et al's Classification of Mental Computation Strategies

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<tr>
<th>Code</th>
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<td>CD</td>
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<td><strong>Initial Strategy:</strong>&lt;br&gt;DM: Changed division to multiplication&lt;br&gt;SA: Changed subtraction to addition&lt;br&gt;CA: Used commutative law of addition&lt;br&gt;CM: Used commutative law of multiplication</td>
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<td>C1</td>
<td><strong>Counting elementary:</strong>&lt;br&gt;CO1: Counted on in ones&lt;br&gt;CB1: Counted back in ones&lt;br&gt;CBS1: Counted back to the second number in ones</td>
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<td><strong>Counting in larger units:</strong>&lt;br&gt;CO2/10: Counted on in twos/tens&lt;br&gt;CB2/10: Counted back in twos/tens&lt;br&gt;CBS2/10: Counted back to the second number in twos/tens&lt;br&gt;RA: Repeated addition&lt;br&gt;RS: Repeated subtraction&lt;br&gt;MU: Multiples&lt;br&gt;RT: Recited tables</td>
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<td><strong>Used place value instrumentally:</strong>&lt;br&gt;RZ: Removed zero&lt;br&gt;WA: Used mental form of written algorithm</td>
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<td><strong>Used place value relationally:</strong>&lt;br&gt;ASP: Added/subtracted parts of second number&lt;br&gt;B: Bridging tens/hundreds&lt;br&gt;UTH: Used tens/hundreds&lt;br&gt;WL: Worked from the left&lt;br&gt;WR: Worked from the right</td>
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<td><strong>Used other relational knowledge:</strong>&lt;br&gt;DH: Used doubling/halving&lt;br&gt;P: Used pattern&lt;br&gt;K: Known fact:&lt;br&gt;K: Knew (that is recalled) the answer</td>
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<td><strong>Used aids:</strong>&lt;br&gt;F: Used fingers&lt;br&gt;MP: Used a mental picture&lt;br&gt;G: Guessed</td>
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Adapted from: McIntosh et al (1994, p. 89).
## Appendix X: Overall Results for all Students

Table A4: Detailed Student Results by School, Year, Gender, Presentation Order, Preference, Money Experience Rating, Basic and Process Performance Scores

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Note: Basic Scores are 0 or 2. Process Scores are 0, 1, or 2
Appendix XI: Mental Mathematics Program Questions for Schools

Mental Mathematics Questionnaire 2002

Dear Teacher,

[Please choose an answer that was the closest to your situation.]

1. How much time did you spend on mental mathematics?
   a) Daily, at least ten minutes
   b) Weekly, at least twenty minutes
   c) As appropriate to a situation that arises
   d) Other, please explain

2. Did you follow a set program?
   a) Yes, ................................................................. (please name)
   b) My own..........................................................(please give examples)

3. Did you allow individual seat work of written exercises from texts?
   a) Always
   b) Fast finishers only (or catering for special needs)
   c) Revision or testing only
   d) Never

4. Did you teach a mental computation strategy/ies?
   a) a) yes..............................................(please state)  b) no.

5. Were students encouraged to use their own invented strategies?

6. Were mental computation items set in context? For example, word problems or applications, such as If Jake had 50 cents and spent 30 cents of it...?
   a) Always
   b) Around half
   c) None
   If yes, to a) or b), How was the context of money used often?..................
   Were any other contexts used often?.........................

7. Did you use class discussions of solutions?
   a) Presented in a written context on the board (see Qn 6)
   b) Only abstract numbers
   c) Presented as a problem from a story or real-life, such as media news item or students’ personal news item.
   d) Presented pictorially
   e) No

8. Did you use the game Sheriff?
   a) Yes
   b) No
   c) Other games........................................................... (please name)

Thank-you for your time