Second-Year Pre-Service Teachers' Responses to Proportional Reasoning Test Items

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Abstract: A recent international study of pre-service teachers identified that proportional reasoning was problematic for pre-service teachers. Proportional reasoning is an important topic in the middle years of schooling and therefore it is critical that teachers understand this topic and can rely on their Mathematical Content Knowledge (MCK) when teaching. The focus of this paper is second-year Australian primary pre-service teachers' MCK of real number items related to ratio, rate, proportion and proportional reasoning. This paper reports on strengths and weakness of pre-service teachers' MCK when responding to test items; including a method suitable for analysing responses to five items and ranked by three levels of difficulty. The results revealed insights into their correct methods of solutions and common incorrect responses, identifying difficulty, where multiplication and division were required. The method of coding test items by difficulty ranking may assist with developing an appropriate learning trajectory, which will assist pre-service teachers develop their MCK of this and other difficult topics

Introduction

"[V]irtually the entire population is now deemed to be in need of significant mathematical power" (Kaput, 1992, p. 518). Proportional reasoning is one aspect of the significant mathematical power to which Kaput refers. Expertise in analysing the mathematics of change, such as "the relations between varying quantities [proportion] and the accumulation of those quantities" (Kaput, 1995, p. 49) has become important for all (Roschelle, Kaput, & Stroup, 2000). Hoyles, Wolf, Molyneux-Hodgson and Kent (2002), mentioned many important instances of proportional reasoning in their report to the United Kingdom's Science, Technology and Mathematics Council of mathematical skills required in the workplace. Proportional reasoning is the thinking involved when working with ratios (Reys, Lindquist, Lambdin, Smith, et al., 2012).

The aim of this study is to identify strengths and weaknesses in primary pre-service teachers' mathematical content knowledge of ratio, rate, proportion and proportional reasoning since this is an area which has been previously reported as problematic (Tatto, Schwillie, Senk, & Ingvarson, 2012). This paper interprets pre-service teachers' correct and incorrect responses to Mathematical Content Knowledge (MCK) items hence reporting their strengths and weaknesses. The results will contribute to the literature and assist with identifying aspects of real number items that pre-service teachers find least difficult, difficult or most difficult.

Tatto et al's recent study, the Teacher Education and Development Study in Mathematics (TEDS-M), which was an international study of pre-service primary and secondary teachers from 17 countries, identified proportional reasoning as problematic for

pre-service teachers. This study contributes to the discussion (e.g. Simon & Blume, 1994; Southwell & Penglase, 2005; Misailidou & Williams, 2002; Tatto, Schwillie, Senk, & Ingvarson, 2012) of strengths and weaknesses of pre-service teachers MCK of proportional reasoning.

This paper begins with a review of literature relating to ratio, rate, proportion and proportional reasoning followed by a discussion of pre-service teachers' MCK of proportional reasoning and studies of pre-service teachers' MCK. The methodology section outlines the conduct of this study within the context of a larger study investigating pre-service teachers' development of the mathematical content knowledge to teaching primary mathematics. The next section presents pre-service teachers' responses to five ratio items, including a discussion highlighting the common issues that revealed the degree to which they were able to demonstrate understanding of ratio, rate, proportion and proportional reasoning. The analysis of common incorrect responses for these items is also discussed. Finally, the conclusion summarises the strengths and weaknesses of pre-service teachers' MCK of proportional reasoning by considering item difficulty.

Background

A *ratio* is formed when one number is divided by another number, and refers "to a multiplicative relationship between two quantities" (Smith, 2002, p. 4). *Rate* is a ratio comparing two different numeric, measurable quantities. Density, for example, is a rate, which compares a measure of mass with a measure of volume. It expresses the change in the dependent variable resulting from a unit change in the independent variable. Ben-Chaim, Fey, Fitzgerald, Benedetto & Miller (1998) express *proportion* as "a statement of the equality of two ratios" (p. 249), which involves a linear, multiplicative relationship between two quantities (Karplus, Pulos & Stage, 1983; Dole, 2008). "A single proportion is a relationship between two quantities such that if you increase the size of one by a factor of *a*, then the other's measure must increase by the same factor to maintain the relationship" (Thompson & Saldanha, 2003, p.114). The importance of these concepts can be seen in their inclusion in the Australian Curriculum: Mathematics (AC: M) where students are expected to "use equivalent number sentences involving multiplication and division to find unknown quantities" (AC: M Year 3); "recognise and solve problems involving simple ratios understanding that rate and ratio problems can be solved using fractions or percentages and choose the most efficient form to solve a particular problem (AC: M Year 7); "Solve a range of problems involving rates and ratios, with and without digital technologies" (AC: M Year 8) (ACARA, 2013).

Proportional reasoning is the application of this knowledge of the proportional relationship in problem solving. Understanding of multiplication and division is needed for proportional reasoning which is further developed along with understanding of fractions, decimals, scale drawing and ratio (Dole, 2008). Proportional reasoning enables people to make everyday choices, for example, in situations where comparisons are needed (Dole, Clarke, Wright, Hilton, et al., 2008) such as interest on a term deposit; slope of terrain taken from a map; price per orange; precision farming and many more (Swedosh, Dowsey, Caruso, Flynn, & Tynan, 2007). Proportional reasoning is the thinking involved when working with ratios and proportions. There are a variety of different kinds of ratio problems, which vary in difficulty for students (Heller, Ahlgren, Post, Behr, & Lesh, 1989). Heller et al. claimed that there are important differences in students' perceptions and thinking about different ratio types and claimed that qualitative reasoning about the direction of changes in ratios should precede numerical exercises. They also argued that an abstract understanding of ratio and proportion is not always helpful in solving problems in science contexts, such as oil consumption of furnaces. Similarly, Alatorre and Figueras (2005) concurred, "proportional

reasoning is highly context-dependent" (p. 32).

Many researchers (Bowers, Nickerson, & Kenehen, 2002; Roschelle, et al., 2000; Piaget, 1970; Schorr, 2003; Thompson, 1994) have investigated students' understanding of speed, and so their conclusions relate to speed in particular rather than proportion in general. Nevertheless their findings provide interesting background to this study, as speed is an important proportion. Speed is an intensive quantity expressing the relationship between the extensive quantities of distance and time. Allain (2001) claimed that the most common misconception was the misinterpretation of a horizontal section in a distance-time graph where many students interpreted this to mean constant speed rather than zero speed. In addition, very few students were able to make qualitative judgments about comparisons of speed in different sections of the distance-time graph. Thompson (1994) described a teaching experiment involving one ten year-old, fifth-grader. He reported that this student's initial understanding of speed as distance interfered with the way she could formulate the relationships between speed, distance and time. He asserted that introducing speed as the formula 'distance divided by time' would have little, if any, relevance to a student's initial understanding of speed and inhibits the development of a concept of speed as a ratio.

Generally there are two types of proportional problems, missing value and comparison problems. In missing value problems, two ratios are described which are equal to each other, but one ratio is missing one of the numbers, which make up the ratio (Lamon, 2007). For example, "if 2 drops of nectar are needed to feed 5 butterflies, how many butterflies could be fed with 12 drops of nectar so one ratio is 2:5". This is a missing value problem because the number of butterflies is missing from the second ratio. In comparison problems, two different proportions are considered and compared. For example, "35 feral cats were found in a 146 hectare nature reserve whilst 27 feral cats were found in a 103 hectare reserve. Which reserve had the biggest feral cat problem?"(Siemon, 2005, p.2). This is a comparison problem because the two ratios are compared.

Suggate, Davis, and Goulding (2006) claimed common mistakes in ratio and proportion are often the result of a focus on constant difference when multiplicative comparisons are more appropriate. Singh (2000) suggested posing problems, which encourage the construction of unit co-ordination schemes, such as supermarket best-buys. Within the scope and sequence of the Australian Mathematics Curriculum, students financial mathematics is regarded as an important context for the application of number and algebra, for example in Money and financial mathematics – Year 7, ACMNA174 – students are required to "investigate and calculate 'best buys'" (ACARA, 2013). Singh advised that a child's experience with ratio and proportion should not be ignored by a focus on teaching algorithms and techniques disconnected from children's everyday experiences.

Many researchers claimed that proportion is a difficult concept for many students (Adjiage & Pluvinage, 2007; Alatorre & Figueras, 2005; De Bock, Van Dooren, Janssens, & Verschaffel, 2002; Litwiller & Bright, 2002; Watson, Beswick, & Brown, 2012). Researchers Behr, Harel, Post, & Lesh (1992) called for clarity in the definitions of rational numbers and fractions, claiming that in real-world problems, fractions and rational numbers have multiple "personalities". Walter and Gerson (2007) described the emergence of connections in teachers' thinking between the notions of "additive structure, recursive linear equations, proportional relationships in discrete measurements, graphing, rise-over-run, data tables, and an embodied sense of slope as steepness of a mountainside" (p. 227). They claimed that an understanding of slope based on the calculation of the ratio 'rise over run' limits the development of connections between slope and ratio. This is consistent with Thompson's (1994) claim that calculation of speed from the formula limits the development of speed as a ratio. Rules and shortcuts for symbolic operations should emerge as generalisations from conceptual understanding rather than being taught in place of them (Thompson & Saldanha, 2003).

Various strategies have been trialled to improve students' understanding of

proportion. Adjiage and Pluvinage (2007) reported on an experiment, which tested the framework for the acquisition of fractions and solving proportionality problems they had developed by analysing the complexity of ratio problems in the middle grades. They claimed that the concepts of fractions, ratios and proportionality are important in mathematics and that the teaching approach based on their framework and computer software, supporting trial and error methods, leads to gains in students' understanding of these concepts.

Person, Berenson and Greenspon (2004) noted the disconnection between the concept of ratio and manipulations with fractions for one prospective high school teacher. This is similar to the difficulties noted by Walter and Gerson (2007) where symbolic procedures are disconnected from the conceptions students bring to mathematics classrooms. This emphasises the importance of the development of teachers' pedagogical content knowledge to the planning and designing of learning material and activities to support students' development of mathematical concepts. For example, Ben-Chaim et al. (1998) recommended an approach emphasising the association of the number of units of one variable within the given ratio with one unit of the other variable. They reported that with time to explore and discuss authentic proportion problems, many students develop their own sense-making tools to solve such problems. In their summary of early research in this area Tourniare and Pulos (1985) concluded the "research reviewed suggests that the development of proportional reasoning is much more complex than often thought" (p. 199).

Mathematical Content Knowledge

Content knowledge is one of Shulman's (1987) seven categories of teacher knowledge and is described as the "amount and organisation of knowledge in the mind of a teacher" (p. 9). Effective teachers rely on their subject matter knowledge to create good lessons and explanations for their students (Reynolds, 1995). Similarly, effective mathematics teachers rely on their mathematical content knowledge (MCK). They have a rich network of connections and select and use a range of mathematical strategies (Askew, Rhodes, Brown, William, & Johnson, 1997). Teachers of mathematics need to know procedural knowledge, procedural fluency, conceptual knowledge and mathematical connections (Australian Curriculum Assessment and Reporting Authority (ACARA), 2012; Ball & Bass, 2003). Pre-service teachers should be provided with program opportunities to extend their MCK.

Pre-service teachers' responses to MCK test items were analysed in this paper. These items were MCK of AC: M of Year 7 and 8 students. Although primary teachers may not teach this content they need to have this knowledge as part of developing their MCK understanding of the curriculum and the mathematical knowledge their students in Year 6 will be learning next. Within the literature, many researchers have referred to this knowledge as horizon knowledge. Horizon content knowledge is having a kind of peripheral vision that informs mathematical teaching practice (Ball & Bass, 2009).

However, recent Australian studies have identified weaknesses in pre-service teachers' MCK (for example ratio, Livy & Vale, 2011) with many relying on procedural methods (Goos, Smith, & Thornton, 2008). Even teachers who have studied calculus have difficulties with proportional reasoning (Thompson & Thompson, 1994). Billings and Kladerman (2000) highlighted several "cognitive obstacles" teachers may encounter as they reason about speed as represented graphically. Tall (1997) suggested that the successful application of the rules does not indicate a conceptual understanding. Tatto et al. (2012) suggested that although pre-service teachers "could solve some problems involving proportional reasoning, they often had trouble reasoning about factors, multiples, and percentages" (p. 137).

Proportional reasoning is essential knowledge for middle-years students (Siemon,

Virgona, & Corneille, 2001) and students are at risk of being left behind if their teachers have difficulty with these concepts. In their working out, pre-service teachers, like students, should demonstrate the most efficient method to solve particular problems (ACARA, 2013). Efficient methods of solution did not occur in all correct examples, and therefore it is important to understand the thinking and methods of solution these pre-service teachers demonstrate when responding to MCK items as part of extending their MCK for teaching.

Methodology

This paper considers the efficacy of ranking pre-service teachers' responses to five proportional reasoning test items, coded as difficult, least difficult and most difficult. Test item responses were also analysed to identify strengths and weaknesses second-year pre-service teachers demonstrated when responding to this topic. This topic was chosen because it is an important number concept related to multiplicative thinking in the middle years of teaching and is also essential mathematics for everyday life (Siemon, Virgona, & Corneille, 2001).

The test instruments were judged as valid for use to measure pre-service teachers' MCK for various reasons. These test instruments had been implemented for more than five years as part of the teacher education program with common procedures, administering and scoring. The tests used in this study were all prepared by a senior mathematics education lecturer. Throughout the program the test instruments were similar but the MCK items varied for each cohort of participants. This was to ensure validity during the program so that pre-service teachers were not able to take advantage of gaining access to any previous copies.

Coding of MCK items

Test items were coded to identify item difficulty. Grouping MCK items by difficulty rating could then be used to create codes of *least difficult*, *difficult* and *most difficult* topics that could be used to assess pre-service teachers' MCK. In addition, this may assist teacher educators and pre-service teachers consider the strengths and weaknesses of current knowledge and design learning plans, which strengthen content knowledge in order to meet teacher education standards (Australian Institute for Teaching and School Leadership, 2012). This method could be used to group and explore other topics in depth from similar test instruments.
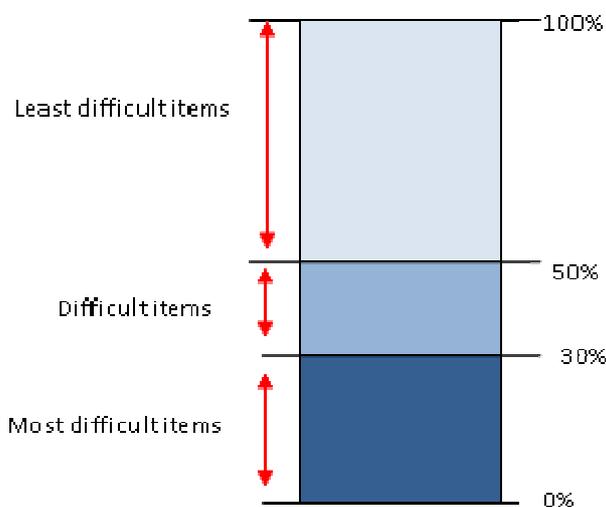
Figure 1: Ranking of items by percentage of correct item responses: Coded as least difficult, difficult and most difficult.

A quantitative method was used to rank the five MCK items by percentage of correct responses to indicate level of difficulty (Figure 1). The items were then coded as *most difficult*, *difficult* or *least difficult*. This included, least difficult MCK items when more than 50% of responses were correct; difficult if the percentage of correct responses was between 30% and 50%; and most difficult if fewer than 30% of responses were correct. Finally, for each of the five MCK item responses, data of correct responses and common misconceptions were then grouped (Table 2) for analysis and discussion.

This paper reports on a study of Australian primary pre-service teachers' MCK (N=195), firstly quantitatively by considering the percentages on correct and incorrect responses to MCK test items. Secondly, responses are analysed qualitatively to gain insights into their strategies and methods of solution. The participants were in the second year of a four-year Bachelor of Education program (Foundation to Year 12). Data was collected from three similar Mathematical, Competency Skills and Knowledge (MCSK) tests, originally designed to assess MCK, of second-year pre-service teachers at different campuses of the same university. For this paper, pre-service teachers' responses to five MCK items assessing their MCK of ratio, rate, proportion and proportional reasoning were analysed and discussed.

At the time of this study, pre-service teachers had completed three education units related to primary mathematics teaching: one during first year; and two during second year of the program, each focusing on aspects of knowledge needed for primary mathematics teaching. During the program, one of the second year primary mathematics units of study included a one-hour lecture and one two-hour tutorial focusing on rate, ratio, proportion and proportional reasoning.

A requirement of the course was for pre-service teachers to demonstrate competency in MCK by successfully completing a MCSK test. They were given opportunities to complete practice MCSK tests and/or enrol in an elective unit designed to consolidate their MCK, if required. The second-year pre-service teachers in this study had also completed at least 30 days school experience placement usually in a primary school setting, where they were expected to assist with primary mathematics lessons, teach small groups or whole class lessons with the support of their mentor teacher. They may have had the opportunity to teach or observe a primary mathematics lesson related to rate, ratio, proportion or rate.

Data Collection

The MCSK test instruments used in this study were designed by the primary education mathematics program co-ordinator. Each test consisted of 49 MCK items that were used to assess pre-service teachers' knowledge of number, geometry, measurement, statistics and probability. All MCSK tests were completed under exam conditions, with working out encouraged, no calculators were permitted and pre-service teachers were given 180 minutes to respond to all MCK items. Most items were closed question items and required short answers using words or symbols (numbers).
Five items (Table 1) were selected in order to identify pre-service teachers' methods of solution of proportional reasoning items including correct responses and errors or common misconceptions. They were also of particular interest because they demonstrate a range of item difficulty (see Figure 1). Most items reported in this paper occurred near the end of the test and it must be acknowledged that this may have influenced the participants' responses because they may have become tired of responding to the items. The items in Table 1 were identified as equivalent curriculum of Year 7 or 8 students (Australian Curriculum Assessment and Reporting Authority (ACARA), 2013).

Test number-Item number	Item as appeared on test	Australian Curriculum, sub-strand:	Type of situation
1-41	If the water in Container A (8 cm x 2 cm x 2 cm) is poured into container B, what height will it reach in container B (base 4 cm x 4 cm)? (see diagram Figure 2)	Develop the formulas for volumes of rectangular and triangular prisms in general. Use formulas to solve problems involving volume (Year 8).	Proportion (missing part)
1-46	How long will it take to drive 210 km if you are travelling at 100 km per hour?	Recognise and solve problems involving simple ratios (Year 7).	Rate
2-10	Which is better value: 16 songs for $24 or 12 songs for $20?	Investigate and calculate 'best buys', with and without digital technologies (Year 7).	Ratio (Comparison part-part)
2-13	A toy car is made to scale: 1:40 has a length of 8 centimetres. What is the length of the full-sized car?	Recognise and solve problems involving simple ratios (Year 7)	Scaling (Comparison whole-whole)
3-12	A cordial drink needs to be made up of syrup and water in the ratio 1:4. If you make enough cordial for 5 glasses, each containing 200 ml, how much syrup would you need for this?	Solve a range of problems involving ratios (Year 8)	Ratio (missing-part)

Table 1: Proportional reasoning items and descriptions of sub-strands of number and algebra (ACARA, 2013)

In Table 1, Item 1-41 was a proportion problem involving missing part thinking and expected knowledge at Year 8 (ACARA, 2013). For this item, the pre-service teachers had to compare the volume of water poured from one rectangular prism (Container A) to another. They calculated the missing part or height the water would reach in the second rectangular prism (Container B) given the dimensions of Container A and the base measurement of Container B. The volume of water in Container A was 32 cm³. The volume of Container B also needed to equal 36 cm³ and, given the base was 4 cm², the missing height and correct response was 2 cm.

Item 1-46 concerns a common rate, speed, where time (hr) taken was calculated given the speed (km/hr) and the distance (km). This problem involved a simple rate and would be expected knowledge at Year 7 (ACARA, 2013). Pre-service teachers had to calculate how long it would take to drive 210 km if travelling at 100 km per hour, with the correct response 2 hours and 6 minutes.

Item 2-10 was a ratio problem and involved part-part thinking and expected knowledge at Year 7 (ACARA, 2013). This ratio item compared two different parts, the

number of songs with the purchase price being the number of dollars. At this level, Year 7 students also develop understanding of money and financial mathematics by investigating and calculating 'best buys', with and without digital technologies (ACARA, 2013). For this item, pre-service teachers had to calculate the better buy, comparing 16 songs for $24 being $1.50 cents per song and 12 songs for $20 being about $1.67 per song. The first option is the cheaper and the better buy.

Item 2-13 was a scaling problem involving whole-whole thinking and expected knowledge of Year 7 (ACARA, 2013). This item compared the scale of the toy car with a full sized car. The scale or ratio was 1:40 and the length of the toy car was 8 centimetres, therefore the full sized car was 8 times larger and 320 centimetres.

The final item, Item 3-12 was a ratio problem also involving missing-part thinking and expected knowledge of Year 8 (ACARA, 2013). This was identified as a Year 8 problem because two steps were required for calculating the correct response. For this ratio item, pre-service teachers had to calculate the amount of syrup needed to make five glasses of cordial, each containing 200 ml. Given a cordial drink needs to be made up of syrup and water in the ratio of 1:4, one glass would require one fifth of 200 ml of liquid, being 40 ml of cordial per glass, and therefore five glasses would require 200 ml of cordial.

Results and Discussion

Table 2 shows the number of correct responses received for the five MCK items and gives the percentage of correct responses taken from three different second-year test instruments as well as the level of difficulty as indicated above (Figure 1). Table 2 shows that two items were least difficult, two items were difficult and one item was coded as most difficult. The least difficult items were Item 2-10 and Item 2-13; the difficult items were Item1-41 and Item1-46, both comparison problems; and the most difficult item was Item 3-12.

Test number-Item number	% correct responses according to test number (n_1=99, n_2=47, n_3=49)	Coding
1-41	37%	Difficult
1-46	44%	Difficult
2-10	78%	Least difficult
2 -13	87%	Least difficult
3 - 12	20%	Most difficult

Table 2: Correct Response Rate for Test Items

Tables 3 to 7 show the common incorrect responses received and the relative percentages for each incorrect answer. These MCK items were all worded problems and pre-service teachers provided a range of responses, which are present in in the following five tables. After each table is a discussion of the most common errors or misconceptions. Table 3 reports the results for the comparison of volume (Item 1-41) including the number and percentage of incorrect responses received. This item was coded as difficult because the

percentage of correct responses lies in the 30% to 50% range.

Incorrect response	Number and % incorrect responses (Test 1: n_1=99)
4 cm	47 (47%)
6 cm	4 (4%)
Various other	11 (11%)
Blank	1 (1%)

Table 3: Common incorrect responses and relative percentages for Item 1-41

In Table 3 and for Item 1-41 the most common error recorded was 4 cm. Figure 2 shows an example of this common misconception. The pre-service teacher has noticed the width of the container B is double container A and, therefore, halved container A's water level when calculating container B's water level. This error is a misconception of doubling and halving, which can be used to express the relationship between ratios. The pre-service teacher could have calculated the volume of both containers to make connections between knowledge of ratio and measurement to check their response.
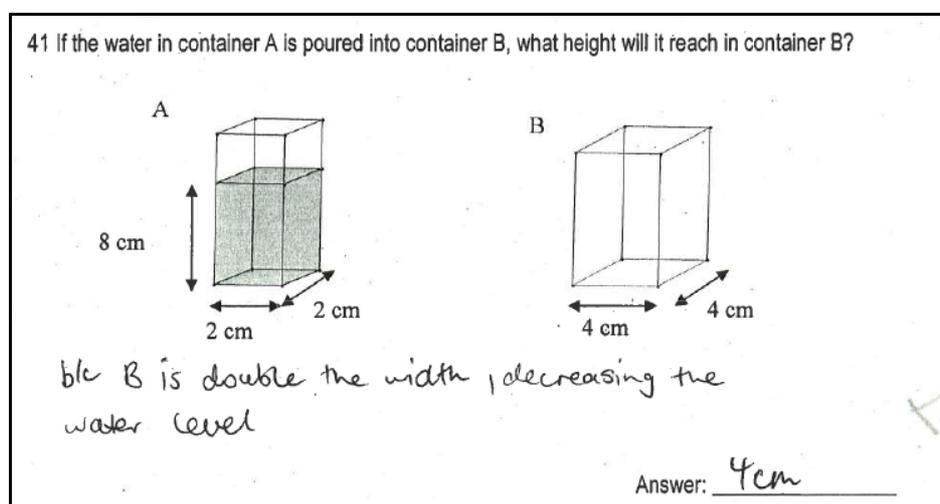

Figure 2: Example of Pre-Service Teacher Misconception for Item 1-41

Table 4 shows the number and percentage of incorrect responses received on Item 1-46. This item was coded as difficult because the percentage of correct responses lies in the range 30% to 50%. A common error for this item may have been incorrectly using a rule to calculate the answer. However, in the case seen in Figure 3 the pre-service teacher correctly applied the rule, but interpreted 2.1 hours as 2 hours and 10 minutes.

Incorrect response	Number and % incorrect responses (Test 1: n_1=99)
2 hours 10 minutes	23 (25%)
2.5 hours	4 (5%)
Various other	24 (26%)
Blank	1 (1%)

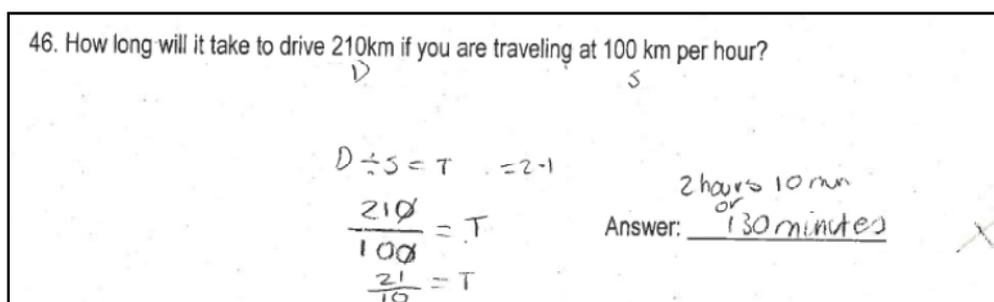**Table 4: Common Incorrect Responses and Relative Percentages for Item 1-46**

Figure 3: Example of Incorrect Response for Second-Year Teacher for Item 1-46

Table 5 shows the number and percentage of incorrect responses received for comparison of best value and Item 2-10. This item was coded as least difficult because more than 50% of the responses were correct. For Item 2-10 there were only two options, therefore a greater chance of guessing the correct response for this item when compared to the other MCK items. The most likely error was difficulty with the division algorithm when calculating a response and comparing the better value or having difficulty interpreting the results of their calculation. In addition, this MCK is particularly important as it related to the everyday application of ratio, demonstrating skill in financial literacy (ACARA, 2013).

Incorrect response	Number and % incorrect responses (Test 2: n_2=47)
12 songs for $20	22 (22%)

Table 5: Common Incorrect Responses and Relative Percentages for Item 2-10

Table 6 shows the number and percentage of incorrect responses received for the comparison of length question (Item 2-13). This item was coded as least difficult because less than 50% pre-service teachers responded incorrectly to this item.

Incorrect response	Number and % incorrect responses (Test 2: n_2=47)
Various	4 (9%)
Blank	2 (4%)

Table 6 Common Incorrect Responses and Relative Percentages for Item 2-13

For Item 2-13 in Table 6, there were no patterns of incorrect responses. A few pre-service teachers (9%) had various other answers, for example, recording the length of a full size car as too small (50 cm and 240 cm) or too large (5 m). When thinking about the relationship of these numbers and the size of a toy car compared to a full size car, these pre-service teachers did not draw on number sense or likely correct responses. They most likely made an error with a "rule" when attempting to calculate the difference in scale. Figure 4 is

an example of a *various other* response for Item 2-13. This pre-service teacher attempted to divide 40 by 8 and incorrectly recorded 50. They were unable to correctly calculate a short division or number fact, and also an incorrect method for solving this item. They possibly lacked knowledge of division or made an error with a "rule" when choosing a method of solution for this scale problem.
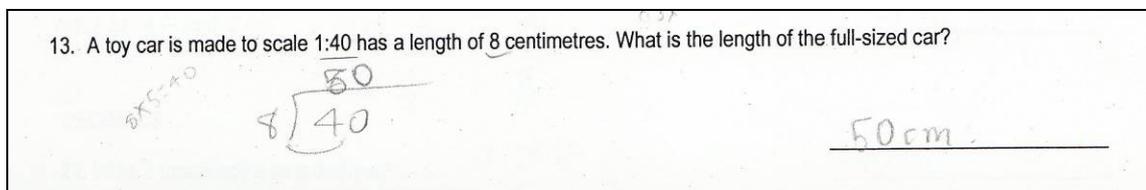

Figure 4: Example of Second-Year Pre-Service Teacher Incorrect Response for Item 2-13

Table 7 shows the number and percentage of incorrect responses received for the comparison of liquid and Item 3-12. This item was the most difficult of the five items listed in Table 1. In Table 7 there were common misconceptions, which could be identified when comparing the grouping of incorrect responses for Item 3-12. The most common error was 250 ml. For this misconception, pre-service teachers most likely incorrectly interpreted the ratio 1:4 as four parts rather than five parts. This meant they would have calculated one glass of cordial requiring one quarter of 200 ml, which is 50 ml, and then multiplied by 5, the number of glasses, to record an incorrect response of 250 ml. The second misconception also related to interpreting the problem correctly. For this error, 1000 ml, the pre-service teachers most likely calculated the total quantity of liquid, including syrup and water, required for 5 glasses. Some of the other errors related to invented strategies for calculating the answer by using the numbers within the problem and attempting the answer, for example 4 times 200 ml is 800 ml, or maybe guessing the answer as 8 ml. This item may also have been most difficult because there were two steps needed when calculating the correct response, for example, first identifying how much syrup is required for one glass then calculating the total amount of syrup required for five glasses.

Incorrect response	Number and % incorrect responses (Test 3: n_3=49)
250 ml	18 (36%)
1000 ml	5 (10%)
160 ml	2 (4%)
5:20	2 (4%)
Various	12 (24%)
Blank	1 (2%)

Table 7: Common incorrect responses and relative percentages for Item 3-12

This study was restricted because of the small number of items analysed, so limited conclusions can be drawn when comparing the difficulty rating of these items. However, some findings can be suggested. Item 2-10 was most likely the least difficult because there was a chance of guessing the answer given only two choices. The other items in this paper involved a calculation of proportion, rate, scale and ratio including the choice of an appropriate method of solution. Some misconceptions related to other mathematical skills demonstrating misconceptions when relying on a rule (Item 1-41), multiplication (Item 2-13), calculating time (Item 1-46) or calculating division (Item 2-10). Item 3-12 may have been the most difficult item because this problem was more difficult to interpret, since it had more description in the question and could have involved more than one step when calculating the correct response.

In summary, many errors of all items related to making connections when interpreting

the problem or errors related to MCK of a mathematical process and qualitative reasoning (Heller et al., 1989). Askew et al.'s (1997) study reported that highly effective teachers make connections between mathematical concepts. Misalilidou and Williams' (2002) study of pre-service teachers' awareness of middle-years students' knowledge suggested trainee teachers may develop their knowledge of ratio and proportion by completing the same diagnostic test middle-year students complete so they may compare and learn from the different responses and to make connections with their thinking about the correct response and the method used by the students to solve the problems. In the study reported in this paper, the researchers were also concerned with helping pre-service teachers to become effective teachers of mathematics. The method chosen to classify MCK items could be used to assist teacher educators and pre-service teachers to scaffold their learning, by developing understanding of least difficult, difficult, then difficult items during teacher education program by using a larger sample of items related to this troublesome topic. This may also assist with determining if pre-service teachers had difficulties with ratio and proportion or other mathematical skills required when solving these items.

Conclusion

The findings from the study show that second-year, pre-service teachers demonstrated a lack of knowledge of multiplicative thinking, in particular, where multiplication and division were required within the items. These findings were not surprising and consistent with those noted by Livy and Vale (2011) of first-year, pre-service teachers' responses to a difficult ratio item and other studies (e.g. Dole, 2008). The results of this study and other similar studies will assist with development of pre-service teachers' MCK who may be expected to have difficulties with these topics.

The second-year, pre-service teachers in this study had completed half their program that is, three mathematics education units of study and a range of mathematics experiences when participating in primary school visits. So it is surprising that many had difficulties related to interpreting these items as well as demonstrating a lack of knowledge of multiplicative thinking, division and multiplication skills. These items relied on qualitative reasoning as the pre-service teachers had to interpret written problems as noted by Heller et al., (1989) as the first step in the development of proportional reasoning. In addition, there were examples in their working out which demonstrated errors in execution of the algorithms.

The identification of item difficulty was significant in facilitating the analysis of the responses to these items, having the potential to pinpoint the level of difficulty of items in MCK tests. Consideration of difficulty ranking of items may provide lecturers with benchmarks regarding the level of pre-service teachers' MCK so that an appropriate learning trajectory may assist their preparation for primary mathematics teaching. A much larger sample of items would be useful and could also be used with students so that pre-service teachers extend their MCK by interpreting the correct and incorrect methods of solutions that students use. This is similar to the method used by Misailidou and Williams (2002) in their study of raising teachers' awareness of middle-years students' thinking. This may assist their development as effective teachers (Askew et al., 1997) of numeracy as they make connections with what they know and how others may respond to similar items. This may build their MCK for solving these problems using more than one method and assist pre-service teachers to consider the different methods from a conceptual focus rather than rely on a rule (Tall, 1997). In addition they should be encouraged to model answers with pictures, look for patterns and record ratio patterns in tables as this would develop their MCK and also assist their pedagogical content knowledge for teaching this topic to their students in the future.

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