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Sampled Fiber Gratings for High-Resolution and high-Speed Photonic Signal Processing

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Abstract - A novel sampled grating for high-resolution, high-speed signal processing is presented. Simulation based on Sinc$^2$ sampled and rational sampled fiber grating modeling show that a large number of sub-ps time delay steps are attainable, corresponding to a sampling frequency in excess of 1THz. Design method is described for deriving sampling functions that meet specific true-time-delay profile requirements.

Index Terms – Sampled fiber gratings, true time delay, photonic signal processing.

I. INTRODUCTION

Signal processing using fiber Bragg gratings (FBGs) to realise true time delays is a powerful approach that can be employed in many applications, such as true time delay wideband beamforming and control in phased array antennas, photonic RF filters and correlators. Owing to their minimum spatial separation, the minimum time delay step achieved by discrete fibre Bragg gratings is limited to a few tens of picoseconds (ps) [1]. For signal processing requiring high-resolution and high-speed, it is desired to have sub-picosecond time delay steps, corresponding to a sampling frequency in excess of 1THz [2]. Chirped fiber Bragg gratings can provide short time delay steps [3], however, they do not support wideband signal operation because of their dispersion characteristics that result in a significant power penalty at high frequencies. Superposed FBGs have experimentally demonstrated a capability of generating ps short time delay steps [4]. However, the fabrication of a large number of superposed FBGs is not straightforward because it is difficult to accurately control the refractive index change [5].

In this paper, we demonstrate, for the first time, that Sinc$^2$ sampled gratings as well as rational sampled gratings can realise short time delay steps of 1ps ($10^{-12}$s) and much larger number of stopbands in comparison to conventional superposed gratings. Sinc$^2$ sampled gratings generate symmetrical time-delay spectrum around their stopband wavelengths, and therefore, they are instrumental for wideband signal processing. Because sampled gratings do not require accurate control of the individual refractive index change, they are much easier to fabricate than superposed gratings. The use of a sampled fiber grating in conjunction with a multi-wavelength optical source or a wideband optical source to realise a short time-delay profile or a high-speed sampler is illustrated in Fig. 1.

![Fig. 1. Illustration of a sampled fiber grating used as a ps-time-delay line. After reflection off the sampled grating, a short time delay is induced between adjacent wavelengths of the RF-modulated WDM signal.](image)

The relevant theory and analysis for Sinc$^2$ sampled gratings is presented in section II. Simulation results and discussions are presented in section III. A generalization of the design method for obtaining a sampling function that meets specific time delay profile requirements is presented in section IV. Conclusions are given in section V.

II. THEORY

A sampled grating is mathematically described as the multiplication of the refractive index function $\Delta n(z)$ of a seed grating and a sampling function $S(z)$, expressed as:

$$\Delta n(z) = S(z)\Delta n(z)$$

where $\Delta n(z)$ is the grating function representing the index variation of the sampled grating. The sampling function $S(z)$ is obtained from summation of a train of identical truncated samples $s_m(z)$ with their positions uniformly spaced along the longitudinal direction of the optical fiber, say the z-axis, and expressed as $S(z) = \sum_{m=1}^{M} s_m(z)$ where $M$ is the number of samples. It is known from Fourier transform theory that transform of a Sinc$^2$ function, defined in the form of $(\sin^2\pi z)/\pi z$, has symmetrical and linear characteristics. Thus, a sampled
grating based on a Sinc² function can generate multiple stopbands with symmetrical characteristics around their centre wavelengths and linear delay-versus-wavelength profiles. Note, however, that this is valid for ideal cases in which the Sinc² function has an infinite number of sidelobes. Therefore, as the Sinc² function has to be truncated when it is used to fabricate a sampled grating, this truncation is equivalent to imposing a rectangular function on the sampling function. According to Fourier transform, such sampling function, i.e., a multiplication of Sinc² and rectangular functions, corresponds to a convolution of the functions in the transform domain. Therefore, such sampled grating generates a reflection spectrum with its profile deviated from the ideal symmetrical linear shape. In addition, for low-loss signal transmission, the desired wavelength stopbands generated by the Sinc² sampled grating must have a reflectivity larger than 99%. This implies that a linear delay-versus-wavelength profile is not maintained in the case of high-reflectivity gratings. To simultaneously realise these conflicting conditions, we set suitable grating parameter values for designing a sampled grating and optimise these parameters to minimise the distortion caused by the truncation of the sampled grating and the use of strong grating reflectivities.

A Sinc² sampling function can be described by the number of sidelobes, \( N_s \), controlled by the truncation using a rectangular function. The overall sampled grating is the sum of \( M \) individual sampled gratings. The seed grating is represented by a grating length \( L \) and an index change \( \Delta n_0 \). Thus, a complete sampling function for a Sinc² sampled fiber grating can be written as:

\[
S(z) = \sum_{m=0}^{M} \left[ \text{Rect}(z-(m-1/2)L_{gr}, L_{gr}/2) \right]^{2} \left[ \frac{\sin(2N_s \pi z (L_{gr}-m+1/2))}{2N_s \pi (L_{gr}-m+1/2)} \right]^{2}
\]

where \( L_{gr} \) is the length of each sampled grating, defined as \( L/M \), and the Rect function for truncation is described by its center position \( z-(m-1/2)L_{gr} \) and width \( L_{gr}/2 \). The coupled-mode equations are used to solve the sampled grating. With the complex amplitude reflectivity \( \rho(\lambda) \) of the grating obtained, its phase function \( \Theta(\lambda) \) can be derived and then the time delay spectrum \( \tau(\lambda) \) can be calculated from the following expression:

\[
\tau(\lambda) = -\left( \lambda^2 / 2\pi c \right) \left[ d\Theta(\lambda) / d\lambda \right]
\]

Where \( \lambda \) is optical wavelength, and \( c \) is light speed in vacuum. Conclusions are given in section V.

III. SIMULATION RESULTS AND DISCUSSION

A. Sinc² Sampled Fiber Grating

The objective is to obtain a Sinc² sampled grating that meets the following requirements: (i) average time delay step <1ps, (ii) number of time delay steps \( \geq 16 \), (iii) standard deviation time-delay-versus-wavelength profile from linear characteristics <5%, and (iv) stopband reflectivity >99%. Design optimisation was carried out for a Sinc² sampled fiber grating with parameter values \( L = 2cm \), \( \Delta n_0 = 0.01 \), fibre core index \( n = 1.46 \), \( N_s = 80 \) and \( s = 10 \). Simulations were implemented for calculations of the reflection spectrum and the time delay spectrum of the sampled grating. The reflectivity spectrum was used to select the wavelength channels having reflectivity of >99%. The time delay spectrum of the sampled grating from the calculation was obtained as shown in Fig. 2(a). It can be seen that the profiles on both sides of the Fig. 2(a) spectrum are approximately linear. All of the 39 channels on the left side and 35 channels on the right side exhibit reflectivities >99%. The 16 wavelength channels from 1561nm to 1567.5nm shown in Fig. 2(b) are selected arbitrarily from the right side spectrum of Fig. 2(a) in order to obtain detailed time delay information. It is important to note that the 16 channels have a linear delay profile with a standard deviation of 4.2% from linear characteristics, and an average delay step of 0.25ps as evident from Fig. 2(b). In practice, these channels can be selected by means of an optical filter.

Fig. 2. (a). Time delay spectrum of the Sinc² sampled grating. (b). Demonstration of linear delay-versus-wavelength distribution over the 1561-1567.5nm range.

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The capability of generating sub-ps time delay steps by the sampled grating can be understood from Fourier transform and calculation using (1) and (2). The specified refractive index distribution results in discrete reflectivity spectral channels with small differences in channel reflectivities, which means small differences in the time delay minima. The available number of the stopband channels is mainly affected by the number of the individual seed sampled gratings, M. A smaller number of seed sampled gratings M allows generating a larger number of delay channels. Linearity of the time delay profile is largely determined by the number of the sidelobes Ns of the seed sampled grating. Therefore, there is a trade-off between Ns and the maximum refractive index change since a large Ns results in low use of index change during the grating fabrication. The other grating parameters L, \( \delta n_0 \), and sampling parameters M influence the generated time delay steps. Reduction in delay step corresponds to decreasing L, M and increasing \( \delta n_0 \), and vice versa.

B. Rational Sampled Fiber Grating

Based on the Fourier analysis for the case of Sinc\(^2\) sampled gratings, some other functions which can give symmetrical, monotonic increasing and decreasing variations can also be used to generate sampled gratings, thus achieving linear short time delays. An example is to use a 2\(^{nd}\) order rational function of Fourier transform described by an exponential function as follows:

\[
\frac{2a}{a^2 + 4\pi^2 z^2} \Leftrightarrow \frac{1}{\sqrt{2\pi}} e^{-\frac{|p|}{2}}
\]

(4)

Where the parameter \( a \) is for controlling of the spectral width of the 2\(^{nd}\) order function. The exponential spectral profile can be modified by truncation and strong grating index change, and consequently it is possible to generate a linear time delay profile for such grating. Taking similar consideration for a 2\(^{nd}\) order rational sampled grating as in Sinc\(^2\) sampled grating, the sampling function can be written as:

\[
S(z) = \sum \text{Re} \{ \text{e}^{-iL_m \frac{a^2}{a^2 + 4\pi^2 (m - \frac{1}{2} L_m)^2 + \left[ 2\pi \left( m - \frac{1}{2} L_m \right) \frac{\delta n_0}{L_m} \right]^2} \}
\]

(5)

where \( L_m \) is defined as in (2).

Assuming that the requirements for a 2\(^{nd}\) order rational sampled fiber grating are basically same as those of a Sinc\(^2\) fiber grating, therefore, in order to obtain a high linearity short time delay profile, a viable design was based on the parameter values \( L = 2cm, \ \delta n_0 = 0.01, a = 10^{-5} \) and \( M = 30 \). Simulation results show that a high linearity time delay profile in a wavelength range of 40nm is achieved with 26 channels on left hand side and 24 channels on right hand side having reflectivity \( > 99\% \). These are depicted in Fig. 3(a) that shows a near ideal linear spectral profile. Detailed information is shown in Fig. 3(b) for arbitrarily selected 16 channels on the left hand side of the spectrum. It is seen that a high linearity of 2\% standard deviation for the 16 channels with 0.5ps time delay steps is achieved. These results meet our design requirements.

A modification design for increasing the bandwidths of the generated stopband channels has also been implemented for a 2\(^{nd}\) order rational function sampled fiber grating. In this case, the number of seed sampled gratings was reduced to 5 and the shape factor \( a \) was increased to \( 5 \times 10^{-5} \). This modification was based on the facts that a large shape factor is beneficial for larger channel width generated and fewer seed sampled gratings give more stopband channels. Calculation results show that with the new value of \( a \), the full width at half maximum (FWHM) bandwidths of the stopbands increase from 0.065 nm to 0.13 nm. The penalties are lower linearity of 3.3\% standard deviation with 0.44ps delay steps obtained for the 16 stopband channels on the time delay profile. Besides, the minimum channel reflectivity is decreased to 90\%.

IV. GENERALIZATION DESIGN METHOD

The sampled gratings described above were aimed at generating linear short time delays from the stopband wavelength channels with nonuniform coupling strengths.
This means that sensitivity to variations in grating index distribution is high. Generalization of the design method for the short time delay sampled gratings provides an understanding in the characteristics of the grating index change distribution and the time delay spectrum. It is a practical requirement that a sampled grating has to be designed to meet a specific time delay profile on which the delay steps may be constant or may not be constant. Therefore, it is necessary to have a sampled grating that provides a specific time delay distribution. Starting from the key requirements, namely the desired time delay steps and the number of the delay steps, a set of coupling coefficients can be obtained from the relationship of time delay and coupling coefficient of the fiber gratings. The set of discrete data is performed for a discrete Fourier transform. The generated data are then used to construct a new function. This function can be used to sample a seed fiber grating and obtain a sampled fiber grating that can give desired time delay spectrum.

This method is further described by means of an example using the results of 2cm fiber gratings. It is assumed for simplicity that 32 linear time delays of 0.5ps step are required. Starting from the relationship of time delay and coupling coefficient of the fiber gratings shown in Fig. 4 (a), a section on time delay \( \tau \) axis is chosen for 31 equal intervals of 0.5ps. Assuming that the weakest channel has 99% reflectivity, the section can be chosen from 16.5ps to 32ps. These points on the \( \tau \) axis correspond to coupling coefficient values on the \( \kappa \) axis, which are unequally spaced. A symmetrical coupling coefficient distribution is obtained and shown on Fig. 4 (b). Note that the discrete data in Fig. 4(b) can be expressed in a continuous function that can be obtained by data fitting. Then, a discrete Fourier transfrom can be applied to the coupling coefficients in order to obtain a set of transform data. These complex data represent a function for a sampled grating. This function can be used for periodically sampling the index distribution of a uniform fiber grating. The generated sampled fiber grating must have a reflectivity >99% for all stopband channels and a constant delay steps of 0.5ps between adjacent stopbands. The generated sampling function for one period is depicted in Fig. 5.

**Fig. 4.** (a). Time delay versus coupling coefficient of the single fiber gratings; (b). Obtained coupling coefficients for 63 channels.

**Fig. 5.** The derived sampling function shown in one period is symmetrical and has Sinc²-like features.

Generally, it is difficult to obtain an analytical expression of the generated index data as shown in Fig. 5. However, a refractive index change profile can still be determined effectively by the data. Consequently, a sampled fiber grating using this function can be determined. If different time delay steps are required, a similar operation can be implemented to obtain a new sampling function, where a specific refractive index change expression can be found for the new sampled grating. Therefore, high-resolution arbitrary time delay compensation, such as the non-uniform delay profile required in conformal phased array antennas, can be achieved.

**V. CONCLUSIONS**

Sampled fiber gratings have been shown to be capable of generating a high linearity time delay distribution with <1ps time steps, corresponding to sampling frequencies in excess of 1THz. Two types of sampled grating have been investigated, namely Sinc² sampled fiber gratings and 2\(^{nd}\) rational sampled fiber gratings, which demonstrated 0.25ps of delay steps and 4.2% standard deviation, and 0.5ps and 2% standard deviation, respectively, for 16 stopband channels. We have also discussed other functions which may be suitable for specific sampled gratings such as Sinc²-Sinc combined...
function. We have shown that in practical applications, where specific time delay distributions are required, a generalized design can be used to find the optimum seed sampled grating that meets a specific time delay profile and can be implemented using a practical fabrication process.

This sampled gratings reported in this paper can also be implemented in planar optical waveguides having larger refractive index change than that of fiber gratings, thus realising a higher number of time delay channels. Fabrication of Sinc sampled fibre gratings with arbitrary index profile and high accuracy has been reported [6], demonstrating the feasibility for making Sinc sampled fiber gratings. Recently, a technique for fabricating a sampled grating in a chalcogenide-based As$_2$S$_3$ rib-waveguide has been developed with a typical available refractive index change up to 0.04 [7]. The sampled grating can provide significant capabilities for signal processing applications to meet high-resolution in time delay and high-speed in sampling and timing.

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