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Raising Teacher Sensitivity to Key Numeracy Competencies in the Early Years

Kelly Norris
Edith Cowan University

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Raising Teacher Sensitivity to Key Numeracy Competencies in the Early Years

Kelly Norris
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ABSTRACT

Mathematical competence is a key capability for success in adult life, and yet many students do not achieve functional levels of numeracy during their school lives. Furthermore, many teachers report that they lack the confidence in teaching mathematics that they have for teaching literacy. Research indicates that it is possible to predict which students are likely to have difficulties in mathematics as early as the Pre-Primary year, and interventions can be provided which are effective in minimising such difficulties. The assumption framing this project is that raising teachers' understanding of and thus sensitivity to markers of the skills most predictive of mathematical success in the early years will result in teachers planning more targeted and responsive learning programs and positively influence classroom practice.

A professional learning intervention focussed on raising professional knowledge about the sequence of number development and the predictors of mathematical difficulties was provided to teachers of five to eight-year-old students. Tools were provided which focussed on linear tracks (board games) as an external model of number magnitude. The study utilised a pre-test post-test design and surveys, teacher interviews, reflective discussions and student estimation tests to examine effects on four outcomes: teacher self-efficacy beliefs, confidence for teaching number and aspects of Pedagogical Content Knowledge; and, student estimation skills.

The data revealed that the intervention was successful in improving teacher self-efficacy beliefs and confidence, particularly with regard to planning and providing intervention for students with mathematical difficulties. Teachers' Pedagogical Content Knowledge was improved, particularly with regard to understanding the sequence of number skills development and building mental representations of number, but the extent to which this was reflected in changes to classroom practice differed markedly between teachers. The teachers who made the greatest changes taught in the Pre-Primary year and were those who reported the most substantial changes in Pedagogical Content Knowledge. The study indicates that board game use could be beneficial in raising the accuracy of student number line estimation when combined with such changes in teacher PCK and associated practice.

Although the research supporting the use of board games to develop mental number line representations is compelling, teachers who did not experience a shift in thinking tended to use these as additional tools to complement existing programs, with little noticeable effect. The implications of these findings are discussed with regard to planning professional learning interventions for teachers which are narrow in focus, supported by a small number of specific classroom tools which can be used within existing whole-class or rotational structures, and targeted towards inducing particular changes in thinking.

DECLARATION

I certify that this thesis does not, to the best of my knowledge and belief:

i. incorporate without acknowledgement any material previously submitted for a degree or diploma in any institution of higher education;

ii. contain many material previously published or written by another person except where due reference is made in the text; or

iii. contain any defamatory material.

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CHAPTER 1: INTRODUCTION

Context

Research has confirmed that both poor mathematical and literacy skills are a significant disadvantage for citizens, reducing the likelihood of gaining and maintaining full-time employment, limiting earning potential and reducing opportunities for advancing skills (Parsons & Bynner, 1997). Early identification and intervention to address difficulties is the key to avoiding a cycle of school failure which often accompanies poor achievement and poor self-esteem (Robinson, 2002). Phonological awareness and letter knowledge have been identified as key skills which predict literacy success, and processes and programs to promote these skills are widely used and can be highly successful in preventing literacy failure (Louden et al., 2000). Conversely, students who struggle with mathematics are unlikely to be identified by their teachers at an early age (Parsons & Bynner) and far less has been known about effective methods to identify and cater for the needs of such students (Gersten, Clarke, & Mazzocco, 2007; Gersten, Jordan, & Flojo, 2005). Fortunately, research is now providing compelling evidence that mathematical difficulties can be predicted and their impact reduced through effective intervention measures (Gersten, et al., 2005).

Gersten and Chard (1999) have identified 'number sense' as a concept akin to literacy's phonological awareness; a key building block which forms the foundation of mathematics thinking and learning. Number sense is a "fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and to look at the world and make comparisons" (Gersten & Chard, 1999, p. 20). Although definitions differ, it is generally acknowledged that number sense involves skills such as understanding and applying counting principles, rote counting, subitising and comparing small quantities (Howell & Kemp, 2005, 2006). Most common diagnostic procedures which measure aspects of number sense involve the assessment of such skills on an individual basis in an interview, and hence are time-intensive and impractical for most teachers to administer to all but the most struggling students (Booker, 1994; Denvir & Brown, 1986; New South Wales Department of Education and Training, 2007; Wright, Martland, & Stafford, 2006).

Hence many researchers have attempted to determine which of these skills are most predictive of later numeracy achievement and thus develop broad screening procedures to parallel those used to prevent literacy difficulties (e.g. Chard et al., 2005; Fuchs et al., 2005; Fuchs et al., 2007; Geary, Bailey, & Hoard, 2009; Gersten, Clarke, & Jordan, 2007; Jordan, Glutting, Ramineni, & Watkins, 2010; Lembke & Foegen, 2009), enabling the provision of effective intervention targeted both to particular students and to educational needs. None of these are currently available to teachers in Australia. Furthermore, much existing research into early numeracy measures has been conducted within the framework of the Response to Intervention (RtI) approach, which aims to use progressively more intensive screening and intervention measures to ensure additional support is provided to those students who have the most need without waiting for them to fail (Marston, 2005). In this context it has been important to examine the sensitivity and specificity of screening measures to ensure that all children at risk of mathematics failure receive support, whilst minimal additional resources are allocated to those who do not require it. As a result screening methods have formal administration procedures to ensure that the predictive validity of the tests is maintained. These considerations are less relevant in the Australian context where the decision to provide additional support and in what form is left largely to individual schools and teachers, with no centralised mechanism to mandate which students will received such support and in what form.

One key construct that appears to underlie some of the most predictive skills is that of the 'mental number line'. First proposed by Case and Okamoto (1996) the mental number line (MNL) is a conceptual structure in which numerical magnitude is represented spatially on an internal number line. Studies reveal that the MNL is usually constructed from left to right (Dehaene, Bossini, & Giraux, 1993) and enables children to think about numbers separately from the objects they represent and as having their own consistent magnitude (Griffin, 2004a). The MNL makes possible calculation and counting without the presence of concrete objects and is thought to be the basis for the development of more sophisticated calculation strategies (Okamoto & Case, 1996). It also enables students to answer questions comparing numerical magnitude of the type "which is bigger, 5 or 2?", which is one of the strongest single-skill measures for predicting later mathematics achievement (Chard, et al., 2005; B. Clarke, Baker, Smolkowski, & Chard, 2008; B. Clarke & Shinn, 2004; Lembke & Foegen, 2009; Seethaler, 2008; Seethaler & Fuchs, 2010). Higher levels of mathematical achievement are correlated with increasingly linear representation of the MNL, as measured by estimation tasks on external number lines (Booth & Siegler, 2006; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Siegler & Booth, 2004).

Problem

Despite the recent advances in knowledge about key components of 'number sense', Australian teachers remain largely uninformed about how to identify mathematically at-risk children and target key weaknesses through explicit teaching. Misconceptions about the teaching of early mathematics still abound, including confusion about the role of explicit teaching, concrete materials and the Piagetian notion of mathematics readiness (J. S. Lee & Ginsburg, 2009). Furthermore, many students entering pre-service teacher training have inadequate subject matter knowledge and make errors similar to those of primary school students (Ryan & McCrae, 2006). Pedagogical Content Knowledge, including mathematical subject matter knowledge and pedagogical knowledge, has been linked to more frequent application of high quality maths instructional practices (J. Lee, 2005). Consequently less effective instructional practices may lead to an overestimation of the need for specialist intervention programs.

Rationale

Increasing knowledge about mathematical development should enable teachers to 'notice' and interpret student's performance on key measures of mathematics understanding and skill, and support them in planning appropriate experiences to promote further progress. To effectively manage this interactive formative assessment (Bell & Cowie, 2001), teachers require appropriate Pedagogical Content Knowledge (PCK) to recognise students' level of development and respond appropriately to meet their needs. Such key predictive measures should be viewed as markers within a complex web of growing mathematical knowledge. Therefore the study made accessible to teachers a trajectory of counting development to provide a framework to understand current levels of counting skill, and plan and choose appropriate learning and teaching activities. It was hypothesised that such a tool would be most useful and likely to have the greatest impact on teaching practice if accompanied by resources which are directly applicable to classroom environments; motivating, interactive and able to be conducted in small groups within the flow of the classroom program.

Purpose

The purpose of the study was to evaluate the impact of a professional learning intervention and resource kit designed around predictive early numeracy measures, on teachers' perceptions of their confidence and self-efficacy for teaching mathematics, and on students' development of key mathematical understandings and skills.

Research Questions

The study addressed the following questions:

1. To what extent do teachers perceive an increase in their confidence and self-efficacy and have increased PCK for teaching number as a result of engaging with the professional learning process?
2. What changes are made to the classroom program as a result of participating in the professional learning program?
3. Do students have more accurate mental number line representations following their teacher's participation in the professional learning intervention?

Significance

Early intervention is an efficient method to address issues when the achievement gap between proficient and struggling students is still relatively small and thus easier to manage and resource. It is far preferable to intervene before misconceptions have become entrenched in mathematical thinking, and when most young students still view themselves as capable learners.

Identifying students who are in need of such intervention in a timely and efficient fashion is therefore paramount. Numerous studies have measured the utility of written tests (e.g. Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008) and brief individual interviews (e.g. B. Clarke, et al., 2008; B. Clarke & Shinn, 2004; Lembke & Foegen, 2009; Locuniak & Jordan, 2008; Seethaler, 2008) to correctly identify students at risk of mathematics failure, the opportunity to use such measures to increase teacher knowledge and improve the specificity of teaching has as yet been largely ignored. Professional learning interventions based on increasing teachers' knowledge of mathematics development have had positive impacts on student growth (e.g. D. Clarke et al., 2002), but such an approach has not yet been applied to the growing research base in predicting mathematics failure. Observation is one of the primary methods of assessment used in early childhood classrooms, and training teachers to target their observations towards areas identified as most predictive of mathematical achievement should result in identifying problems and providing intervention at an earlier stage. The study aimed to determine whether professional learning could raise the sensitivity of teachers to key components of number sense such that they are better able to 'notice' key mathematical behaviours in their students, and to understand how to design responsive educational programs.

The present study extends previous work by examining whether raising the awareness and sensitivity of teachers to early warning signs of mathematics failure might be sufficient to have a generalised positive impact on classroom practice and teaching programs, independent of any imperative to classify children as 'at-risk' or 'not-at-risk'. The implications of such a finding would be relevant to both first-tier, high quality group instructional approaches in RtI settings, and to improving teaching in other educational systems.

By adapting the predictive tasks from previous research into motivating board-game formats, the current study will also test the feasibility of combining the benefits of assessing multiple students at once (such as in pencil-and-paper testing) with an informal environment which still allows the observation of individual solution strategies (as is the case with individual interviews). Such a format, if effective, could balance needs for manageability and detail whilst remaining non-threatening for students. The intervention which deliberately combines a focus on building proficiency in these predictive skills with using linear tracks as a model of the number line is distinctive and could hold promise in addressing the difficulties of the most mathematically vulnerable students.

CHAPTER 2: LITERATURE REVIEW

Theoretical and Conceptual Frameworks

The theory of constructivism, where knowledge is actively constructed by the learner, frames this study. Piaget's original theory of constructivism emphasised the interplay between children's existing mental structures, or schema, and external information from the environment which may necessitate development of these schemas through the process of accommodation, in concept formation (Piaget & Inhelder, 1958). The external environment, that which teachers can manipulate, is therefore of crucial importance in developing the key skills of abstraction and generalisation which underpin mathematics. Schoenfeld (1992) argues that this mathematical thinking is achieved through engagement with mathematical problems, rather than through mastery of a collection of formulae and procedures. In the context of this study, it is one of the first abstractions, that of the concept of number, which is developed first as a property of a collection of objects (quantity), as a mental model and subsequently as a concept which can be manipulated in its own right.

Sociocultural theory provides a framework for understanding the value of both language and interaction on concept formation. Vygotsky (1962) described the key role of language in concept formation as a means to direct attention and thought and facilitate the process of abstraction, and further pointed to interpersonal communication as necessary for the internalisation of higher order functions. The importance of the classroom as a social context for learning is therefore central to this view. In such a classroom, concepts can be developed through a complex network of interactions between sensory materials, internal and external language, and social interaction centred around learning experiences.

Sarama and Clements (2009) synthesised the relationship between cognitive development, language and the external environment and applied these to specific domains of mathematics through the theory of hierarchic interactionalism. This framework emphasises the developmental progression of skills within a given mathematical domain, the hierarchical nature of mathematical learning, and the interaction between intuitive mathematical skill and the experiential world in building mathematical knowledge. Sarama and Clements describe how the representation of mathematics concepts progresses from intuitive to metacognitive through an intermediary stage centred on representation through language. Of particular interest in this project is the notion of "cyclic concretisation," which describes the process by which students move from representations (in this case of number) based on sensory and concrete supports, to more abstract representations based upon generalisations and mental representations, and of how this process often cycles within domains (Sarama & Clements, 2009, p. 22).

Hierarchic interactionalism highlights the complexities involved in the process of knowledge-building, as well as the important role of the teacher in understanding the hierarchical paths of mathematical development. The teacher is responsible for planning instructional tasks which enable purposeful interaction with the child's existing knowledge, and encourage the social construction of knowledge between children and teachers.

The Role of the Teacher: Attitudes, self-efficacy and Pedagogical Content Knowledge

In discussing the development of student knowledge in mathematics it is relevant to refer to the influence of effective teaching in promoting this development. Teacher quality has been confirmed as a key variable in influencing student achievement (Hattie, 2009), with the variance in teacher effectiveness higher in mathematics than reading (Nye, Konstantopoulos, & Hedges, 2004). Hence, the use of effective instructional practices is key to student achievement. The extensive *Early Numeracy Research Project* (D. Clarke, et al., 2002) conducted over three years, involved detailed observation of the classroom practices of six teachers identified as highly effective in mathematics, and identified a number of instructional practices consistently demonstrated by these teachers. These 25 instructional practices spanned 10 areas and included using a range of materials and representations for the same concept, using teachable moments, using a variety of group structures and assessment methods, having high expectations of students and focusing of key mathematical ideas.

In addition to effective teaching practice, the construct of self-efficacy as "a teacher's individual beliefs in their capabilities to perform specific teaching tasks at a specified level of quality in a specified situation" (Dellinger, Bobbett, Olivier, & Ellett, 2008, p. 752) has been demonstrated to predict student achievement and teachers' job satisfaction (Caprara, Barbaranelli, Steca, & Malone, 2006; Klassen et al., 2009). A significant factor in predicting teacher self-efficacy is termed 'mastery experiences', with previous teaching experiences perceived as successful leading to a higher expectation of future success (Mohamadi & Asadzadeh, 2012). In novice teachers, the role of interpersonal support and teaching resources is more significant (Klassen, et al., 2009). Collaborative teaching cultures and higher student engagement are also related to higher teacher-self-efficacy (Guo, Justice, Sawyer, & Tompkins, 2011).

Specifically focussing on early childhood mathematics teaching, Lee (2005) found that Kindergarten teachers' attitudes towards the teaching of mathematics (but not towards mathematics itself) were significant in predicting developmentally appropriate classroom practice. Furthermore, teachers in general reported low confidence in teaching mathematics. In contrast, Todd Brown (2005) found no relationship between efficacy, beliefs and instructional practices, but did report a troubling lack of consensus on both the importance of mathematics instruction and on what and how mathematics should be taught to young students.

This lack of consensus can be seen as evidence of the lack of Pedagogical Content Knowledge (PCK) of teachers in the teaching of mathematics. PCK refers to how teachers apply their subject matter knowledge to teaching, including the selection of powerful examples and demonstrations that make the content intelligible to others (Shulman, 1986). PCK is a significant variable in predicting developmentally appropriate practice in mathematics, and is influenced positively by education and teaching experience (J. Lee, 2005, 2010). Furthermore, the significant positive impact of specific performance feedback on student achievement (Baker, Gersten, & Lee, 2002) underscores the importance of teachers having the necessary knowledge to understand and assess students' mathematical learning. Hence, research findings broadly indicate that teachers with higher self-efficacy beliefs for teaching, and greater knowledge of how students learn mathematics and how mathematics should be taught, have a positive impact on student learning. The remainder of this review will focus on that knowledge.

The Concept of a Learning Trajectory

It has long been acknowledged that humans are born with innate structures for understanding number (Dehaene, 1997). The evidence that so many children and adults continue to have difficulty mastering mathematics to functional levels during their school years (OECD, 2010) has led many to investigate how this innate knowledge becomes more formal and abstract through development and education.

Since the seminal work of Gelman and Gallistel (1978) articulated the understandings necessary for counting, much progress has been made in understanding how early numeracy skills develop and what mental constructs exist to support mathematics learning. Research from the fields of education, psychology and cognitive science is forming a picture of the typical developmental path, or learning trajectory, that children follow when learning to understand number concepts. Such research is informing attempts to understand which skills and understandings are necessary for acquiring number concepts (e.g. Fuchs, et al., 2005; Gersten, Clarke, & Jordan, 2007; Jordan, et al., 2010; Lembke & Foegen, 2009), and how to provide appropriate instruction to those children who lack them (e.g. Baroody, Eiland, & Thompson, 2009; Bryant et al., 2008; Clements & Sarama, 2008).

The work of Steffe and colleagues on documenting the stages of arithmetical development (e.g. Steffe, 1992; Steffe, Cobb, & von Glaserfield, 1988) has been influential in informing subsequent work on learning trajectories. One such project involved raising teachers' professional knowledge about mathematics development using a learning trajectory, which proved successful in raising student achievement and teacher knowledge. In the three-year *Early Numeracy Research Project* (D. Clarke, et al., 2002) which included 11 421 five to eight-year-old children, key advances along a learning trajectory described as 'growth points' were used as a framework to design professional learning for teachers and assessment practices for students. The study included development of a diagnostic mathematics interview, focussed professional learning and within-school support structures, and observed both teaching practices and student growth. The intervention was successful in accelerating student growth in mathematics skills and in identifying practices of effective teachers of mathematics. Similarly, positive effects on student mathematics development (Bobis & Gould, 1999) and teacher knowledge (Bobis, 1999, 2008) have been reported as a result of involvement in the *Count Me in Too* project which has been operating in New South Wales since 1996. This project uses the developmental trajectory developed by Wright and colleagues (Wright, 2000; Wright, Martland, et al., 2006) as the basis for teacher professional development in number. The *First Steps in Mathematics* program, developed in Western Australia, is also based upon matching instructional tasks to students' progress along a trajectory of maths development (Willis et al., 2004).

It is this concept of a developmental trajectory, specifically in understanding counting, which frames the current study. Whilst the aforementioned approaches have emphasised the progression of mathematical skills within a complex web of mathematical knowledge, it is the intention of the Researcher to use a narrower approach which specifically identifies and targets points along that trajectory which predict mathematics failure; those which are postulated to comprise the broad concept of 'number sense'.

The Concept of 'Number Sense'

It is well established that significant gaps in mathematical understanding are evident between social classes as early as at school entry (Ginsburg & Russell, 1981; Hughes, 1986; Huttenlocher, Jordan, & Levine, 1994; Jordan, Huttenlocher, & Levine, 1994; Jordan, Kaplan, Olah, & Locuniak, 2006; Morgan, Farkas, & Wu, 2009) and that children who start school with lower levels of mathematical proficiency are unlikely to 'catch up' to their more advantaged peers (Morgan, et al., 2009). This difference in proficiency has often been described as a lack of 'number sense' (Baroody, 1985; Berch, 2005; Gersten & Chard, 1999). Although acknowledged as a foundation for mathematical learning, definitions of number sense differ, as do interpretations of its constituent skills. According to Gersten and colleagues (Gersten, et al., 2005), number sense was a concept initially investigated by psychologists in an attempt to understand the cognitive development of children. Number sense was seen to have a biological basis and referred to so-called 'lower-order' skills including intuitions about numbers and quantities, and counting and simple arithmetic skills (Berch, 2005).

From the perspective of psychology, in five to six year-olds there are two components of number sense: counting and simple computation; and, a sense of quantity or use of a mental number line (Okamoto cited in Kalchman, Moss, & Case, 2001, p. 3). Competent counting is associated with knowledge of the 'rules' which govern counting procedures (Gelman & Gallistel, 1978); that the number words are always spoken in the conventional order (stable order), that exactly one number tag is applied to each object being counted (one-to-one correspondence), and that each counting number is inclusive of the one before it, hence the last number in the set describes the size of the entire set (cardinality). Further counting principles are the knowledge that any discrete items can be counted (abstraction) and that provided the how-to-count principles are followed, the order in which the items are counted is not important (order-irrelevance). Number sense represents a crucial achievement along the developmental path, as it signifies the amalgamation of different schemas for understanding number which are developed over the early childhood years including the worlds of actual quantities, counting language and numerical symbols (Griffin, 2004b).

Some mathematics educators have taken a broader view of number sense to include a wide variety of skills related to mathematical sense-making including the invention of computation strategies, the application of mathematical knowledge and the ability to make reasonable estimations and detect gross errors in computation (Berch, 2005). Indeed, Berch reported 30 reputed components of number sense. In a limited study of academics from Australia and overseas, Howell and Kemp (2005, 2006) identified that early counting skills are perceived as a key hallmark of number sense and predictor of mathematical success, whilst reporting a lack of consensus on what the concept represents in general.

Nevertheless, significant progress has been made in identifying skills which are highly predictive of mathematical achievement. Tasks which purport to measure 'number sense' are demonstrating moderate to strong correlational values in predicting mathematical success in subsequent school years. These tasks will be discussed presently. For the purposes of this study the term 'number sense' is hereafter used to refer to those skills which are demonstrably linked to future mathematical achievement.

The development of pre-counting skills

Humans show sensitivity to number from infancy (Dehaene, 1997). Cognitive science has revealed that the human brain has two systems for representing number: an analogue magnitude system which is shared with many animals; and, a system for processing small, exact quantities (see Noelle, Rouselle, & Mussolin, 2005, for a review). The former enables understanding of approximate magnitudes represented spatially along a mental number line, which is organised from left to right, increasingly compressed for larger numbers, and subject to size and distance effects consistent with Weber's Law (Dehaene, et al., 1993). That is to say, the comparison of two numbers becomes more difficult and associated error larger as the ratio between the two numbers decreases. The second, object-file system, is thought to enable the processing of very small numbers (up to three, possibly four) by ‘opening’ mental files which correspond to each object and store information about that object including its physical properties (Feigenson, Carey, & Spelke, 2002). This coding system is unique to humans and enables the fast processing of small numbers of objects. In mathematics education literature this ability to quickly enumerate small collections without counting is referred to as ‘subitising’ (Clements, 1999).

Subitising, rather than counting, is the means by which children understand the meanings of the first few number words (Benoit, Lehalle, & Jouen, 2004). Carey and colleagues (Le Corre & Carey, 2007) have demonstrated that children first learn the cardinal meanings of one, then sequentially two, three, and possibly four. In this range children are able to compare the result of the count with the cardinality of the collection attained through subitising, thus reinforcing correct counting and enabling extrapolation to counting larger quantities. Researchers from the field of cognitive psychology continue to debate whether an implicit knowledge of counting principles precedes (Gelman & Meck, 1983) or arises as a result of (Briars & Siegler, 1984; Wynn, 1990, 1992) the development of counting procedures.

Similarly, Case and colleagues (Griffin & Case, 1996; Okamoto & Case, 1996) describe the crucial stage at which the separate schemas created for understanding comparative quantity (“more/less”) through the analogue magnitude system, and number words are merged to create a single schema in which magnitudes are represented along a linear mental number line. This number line integrates understandings about numbers in the count sequence corresponding to exact quantities, and importantly that increments and decrements along the number line correspond to increases or decreases in quantity. The core ‘number sense’ ability to compare the magnitudes of numbers and move flexibly between numbers, quantities and symbols, is made possible, in large part, by this linear construct of a mental number line (Griffin, 2004b). Using such a construct it becomes reasonable to talk about numbers independently from the quantities and objects they represent, and as having magnitude if their own, with numbers further along the counting sequence representing higher quantities. Resnick (1992) described this cognitive leap in the context of a four-level progression involving this shift from the maths of quantities, where numbers are used as adjectives to describe amounts of concrete materials, to the maths of numbers, where the numbers become nouns themselves and can be conceptualised and acted upon independent of any reference to physical objects.

Demographic and Cognitive Correlates of Mathematics Achievement

Numerous studies have linked low income status with both poor mathematical skill and/or ‘number sense’ on school entry and substantially lower rates of mathematical growth than more advantaged children (Hughes, 1986; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Jordan, et al., 2006; Morgan, et al., 2009; Toll & Van Luit, 2012). Other risk factors for poor achievement include reading achievement (Jordan, et al., 2006) and difficulties with aspects of working memory (Geary et al., 2009; Geary, et al., 2007). Students with both poor mathematics and poor reading ability experience more significant problems with contextual word problem solving, greater impairment in basic calculation skills, show slower mathematical growth trajectories and are less likely to progress with instruction than their peers with maths difficulties combined with typical reading achievement (Fuchs & Fuchs, 2002; Fuchs, Fuchs, & Prentice, 2004; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000; Jordan, Hanich, & Kaplan, 2003; Jordan, et al., 2006; Jordan & Teresa, 1997). However, studies by Geary and colleagues have revealed that the most disadvantaged students in mathematics and those who show the most persistent difficulties are those with impairments in both number sense and working memory, in particular central executive function (Geary, Bailey, Littlefield, et al., 2009; Geary, et al., 2007).

Characteristic Difficulties of Children with Mathematical Difficulties/Disabilities

A large body of research has demonstrated that children with mathematical difficulties (MD) experience difficulty with a variety of mathematical processes. The source of these difficulties is a matter of some debate, with some researchers blaming the influence of poorer general cognitive capabilities, in particular aspects of working memory, which impact on students' ability to progress to more mature arithmetical procedures (Geary, Bailey, Littlefield, et al., 2009; Geary, et al., 2007; Toll & Van Luit, 2012). Others point to a weakness within specific brain circuits specialised for mathematical functions including understanding magnitude and subitising (Butterworth, 1999; Dehaene, 1997). According to Desoete and Gregoire (2006), one third of children with a clinical diagnosis of Mathematical Learning Disability in grade three continued to demonstrate a severe deficit in subitising skill, being the capacity to enumerate small collections without counting. Research by Rouselle and Noël (2007) has revealed that students with MD have difficulty accessing magnitude representations from Hindu-Arabic symbols. Desoete and Gregoire (2006) further discovered that Year Three students with MD have significantly worse knowledge of number words than digits.

Nevertheless, regardless of their source, a large body of research has established the nature of such difficulties. The use of immature counting strategies and procedures, and in particular the failure to successfully use retrieval to solve arithmetic problems, is a hallmark of students with MD (Geary, 1993). Students with MD tend to use labour-intensive counting-all and finger-counting strategies, infrequently and inaccurately use back-up calculation strategies, and commit more retrieval errors (Siegler, 1988). Such students often have poorer working memory resources than their peers (Geary, et al., 2007; Swanson & Beebe-Frankenberger, 2004), and when they use strategies which stretch these resources this leads to a high number of errors and a reduced capacity to perform higher-order mathematics (e.g. Geary, 2004; Geary, Hamson, & Hoard, 2000; Holmes & Adams, 2006).

Early Identification Measures: Multiple skill measures

Teachers need to be able to identify students who are likely to experience difficulties with mathematics in order that teaching opportunities be provided to reduce their severity and impact. Several researchers have designed batteries of mathematical tasks which are designed to measure aspects of 'number sense'. The *Number Knowledge Test* is one such instrument, developed by Okamoto and Case (1996) to measure the presence and sophistication of their postulated 'central conceptual structure' for mathematics, being the mental number line. The tasks include comparing the magnitude of collections, then single-digit and two-digit numbers, in addition to tasks which measure the development of the counting string in different ranges through 'number before' and 'number after' tasks. The *Number Knowledge Test* has since been demonstrated to show strong validity both as a concurrent and predictive measure of mathematics achievement (Chard, et al., 2005; B. Clarke & Shinn, 2004).

The advent of the Response to Intervention Approach (as described by Hempenstall, 2012) in the United States of America and similar three-wave structures in the United Kingdom have led to increased interest in designing tools to identify children for targeted instruction. The need for large scale screening procedures has prompted attempts to design assessment tools that are practical to deliver to large numbers of students, either using group-administered pencil and paper tests, through the design of more targeted individual interviews focussed on tasks most predictive of achievement, or a combination of both. Geary and colleagues designed the group-administered written *Number Sets Test* to measure fluency in recognising arithmetic combinations presented as pictures, digits and combinations of these (Geary, Bailey, & Hoard, 2009). Other researchers have designed screening measures in an attempt to differentially predict conceptual and procedural subtypes of mathematical difficulty, with mixed success (Fuchs, et al., 2007). An increasing number of studies are testing the utility of single-skill measures in predicting mathematical achievement and monitoring growth (Chard, et al., 2005; B. Clarke, et al., 2008; B. Clarke & Shinn, 2004; Fuchs, et al., 2007; Jordan, et al., 2010; Jordan, et al., 2007; Jordan, et al., 2006; Lembke & Foegen, 2009; Lembke, Foegen, Whittaker, & Hampton, 2008; Locuniak & Jordan, 2008; Vanderheyden et al., 2004).

Whilst all of these approaches have shown promise, the educational context of Australian schooling differs from that of the United States of America and the United Kingdom in that mathematics screening measures will not be used to allocate resources and funding, but to inform teaching on a classroom level and perhaps guide individual schools in tailoring preventative educational programs. Hence the need for rigorous standards of specificity is reduced, as is the need for measures delivered in formalised, standardised contexts which may not seem appropriate to the tender age of the students involved. Therefore the remainder of this review will focus on the single-skill measures which can be delivered in more informal contexts appropriate to the needs of Australian teachers.

Early Identification: Single skill measures

Ongoing research is clarifying which single mathematical skills can be used to predict mathematics difficulties in the early years. At present, the mathematical skills of magnitude comparison, missing number, number identification[1] and arithmetic combinations have been demonstrably linked to later maths performance, as has the digit span backwards measure of working memory. The reliability of the subitising skill as a predictor of mathematics difficulties is more dubious.

Magnitude comparison (quantity discrimination)

One of the most robust single predictors of later mathematics achievement is magnitude comparison, or the ability to correctly answer abstract questions comparing numerical magnitude such as "which is larger, 7 or 4?" (B. Clarke & Shinn, 2004; Lembke & Foegen, 2009; Seethaler, 2008). Such a question measures the development of a child's conceptual structure for understanding numbers and quantity. Of children who persistently struggle with mathematics in Years Two and Three, almost 70% have failed magnitude comparison tasks in the year prior to formal schooling (Mazzocco & Thompson, 2005).

Magnitude comparison tasks are subject to size effects and distance effects both in children and adults (Holloway & Ansari, 2008; Huntley-Fenner & Cannon, 2000; Zhou et al., 2007). Put simply, comparison tasks are easier and are processed more quickly and with fewer errors as the ratio between the two numbers increases. Four and two are thus compared successfully at an earlier age and with greater speed than eight and six, despite the fact that both pairs differ in value by two. This is due to internal representation of numbers on a mental number line, which is increasingly compressed as values rise (Siegler & Booth, 2004). This compression results in the perception that three and five are further apart than 73 and 75, although the difference between each pair is the same. This pattern of response times has been observed in children as young as six months old when shown dot arrays (Xu & Spelke, 2000), and five-year olds when presented with Hindu-Arabic numerals (Zhou, et al., 2007).

The ability to judge the magnitude of numbers signifies the presence of a 'mental number line' (Griffin, 2004b), and is also necessary for the development of efficient counting strategies which enable working memory resources to be devoted to higher level maths tasks involved in problem-

[1] Whilst the skill would be more correctly described as 'numeral identification', 'number identification' will be used consistently throughout this document for consistency with the research to which it refers.

solving (Geary, 2004). Counting-on, or the “Min” strategy, entails counting up from the largest number in a sum (addend), requiring the minimum cognitive effort. One of the key skills necessary for counting-on is the ability to choose which is the larger number in the sum (B. Clarke, et al., 2008).

By correctly answering an abstract question about numerical magnitude, children are demonstrating sophisticated knowledge about the nature of numbers, and as they mature this understanding extends to larger numbers and reflects an understanding of the place value system (Okamoto & Case, 1996). It is worth noting that the common language for presenting this task involves the use of the terms “bigger/larger/smaller” rather than “more/less”. Whereas “more” and “less” typically refer to collections, “larger” and “smaller” may well have been chosen as they imply that numbers have their own magnitude, independent of what is being counted or measured, a concept central to the mental number line. The exception to this trend is in the work of Bryant and colleagues, who chose the words “more” and “less” exclusively for Grade Two students to reflect vocabulary deemed appropriate to that grade level (Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008).

The magnitude comparison task was first developed by Siegler and Robinson (1982) and later adapted by Case and Okamoto (1996) for inclusion in the *Number Knowledge Test*, which was specifically designed to measure the presence of a mental number line and track its development as children became able to relate and use two and then three mental number lines as their cognitive structures matured. Magnitude comparison, along with number identification, is also a useful measure to monitor growth as both show reliably linear growth as students respond to mathematics instruction (B. Clarke, et al., 2008; Lembke & Foegen, 2009).

Missing number

Identifying the missing number in a counting sequence has been used by numerous researchers as a measure of early number sense having value in predicting mathematical achievement (Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008; Chard, et al., 2005; B. Clarke & Shinn, 2004; Fuchs, et al., 2007; Lembke & Foegen, 2009). Missing number tasks typically are presented as a written sequence of three digits, of which any one may be missing and must be supplied orally or in writing by the student (e.g. 28, __, 30). According to Clarke and colleagues (2008), missing number tasks represent a critical component of number sense due to their utility as a measure of strategic counting.

Students who are successful at mathematics use a combination of retrieval and efficient back-up strategies to solve arithmetic problems in preference to labour-intensive counting procedures (Siegler, 1988; Siegler & Booth, 2004). One of the earliest back-up strategies to develop is the Min strategy (Siegler, Adolph, & Lemaire, 1996), which entails counting up from the larger addend. After identifying the larger of the two numbers, the student must then be able to begin the count sequence at any point; hence the missing number task is thought to measure skills directly related to the development of the Min strategy (B. Clarke, et al., 2008). Students who go on to have difficulty with mathematics often struggle to acquire this and other back-up calculation procedures (Geary, et al., 2000).

Measures usually feature sequences of three numbers, with the missing number in any position. Exceptions to this trend include the measures developed by Lembke and Foegen (2009) and Fuchs and colleagues (2007) which featured four and five-item sequences respectively. Although research initially focussed on the skill in the earliest school years with the most common numerical range as 0-20, research has now demonstrated the utility of the measure for up to Year Two level with larger numbers (Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008).

Subitising (quantity array).

Subitising is “the ability to recognise the number of dots in an array without counting” (Butterworth, 1999, p. 303) and represents a sensitivity to numerosity in the human brain which is present almost from birth, far before the development of formal counting skills (e.g. Antell & Keating, 1983; Starkey & Cooper, 1980). Subitising appears to be an innate skill which has been implicated as a key component of mathematics disabilities, in part due to research with brain injured patients and those with other neurological disabilities, who show poor mathematics abilities and a complete inability to subitise (Butterworth). Indeed, Butterworth (p. 304) refers to subitising as a “key component of the Number Module” (in the brain) which gives meaning to the number words.

Subitising is thought to play a key role in the development of counting skills and in particular the cardinality principle: the concept that the last number counting represents the size of the set; and that increasing or decreasing the set will correspond to an increment or decrement along the counting sequence or ‘mental number line’ (Sarnecka & Carey, 2008). Since subitising precedes counting, this instant recognition of small sets enables children to confirm the result arrived at through counting, which leads to the conclusion that counting a collection results in knowing the size of the set (cardinality). Children thus acquire the counting principles for small numbers (one to four), which are later generalised to quantities outside the subitising range (Le Corre & Carey, 2007).

There are two forms of subitising; perceptual subitising and conceptual subitising. Perceptual subitising is the ability to “recognise a number without using other mathematical processes” (Clements, 1999, p. 400), and has a limit of around four objects. In perceptual subitising, any arrangement of between one and three or possibly four dots can be recognised and named in approximately the same time frame (around half a second) (Dehaene, 1997). Although there is evidence that this ability is sensitive to other variables besides quantity (e.g. Feigenson, et al., 2002), it has nevertheless been demonstrated to play a significant role in predicting children's counting abilities (Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & Van de Rijt, 2009).

Conceptual subitising occurs when students are presented with an arrangement of dots greater than three or four, which occur in a pattern which enables them to see the collection "as composite of parts and as a whole" (Clements, 1999, p. 401). When students recognise dots in dice patterns, or combinations of dice patterns, they are using conceptual subitising. In contrast to perceptual subitising, conceptual subitising is a learned mathematical skill which is thought to provide a platform for the development of more complex arithmetic with larger numbers (Clements).

Although subitising is thought to be a key component of number sense (Berch, 2005) and even be implicated in mathematical disabilities (Desoete & Gregoire, 2006; Reeve & Reynolds, 2004), as yet there is little empirical support for the reliability and validity of its use in predicting mathematics achievement. In one study that directly tested its utility for identifying children at risk for mathematical difficulties, Lembke and Foegen (2009) reported only moderate concurrent and predictive correlations with standardised measures and dismissed the utility of the measure on the grounds that it lacked test-retest reliability. Nevertheless, its role in early numeracy development (Clements, 1999; Le Corre & Carey, 2007) and presence in the Australian Curriculum at the precompulsory school level require that the measure not be entirely dismissed.

Digit span backwards (working memory)

According to Gathercole and Alloway, "'Working memory' is the term used by psychologists to refer to the ability we have to hold and manipulate information in the mind over short periods of time" (2008, p. 2). Generally speaking, individuals with impairments in working memory capacity (who can store in temporary memory a lesser number of 'bits' of information) tend to have poorer academic progress and struggle with inattentive and forgetful behaviour at school (Alloway, Gathercole, Kirkwood, & Elliott, 2009). Poor working memory has been implicated in many of the skills which are the hallmarks of mathematical difficulties, including inefficient counting strategies, poor knowledge of counting concepts and poor memory for arithmetic facts (Geary, Bailey, & Hoard, 2009).

According to Baddeley (1996), working memory has three components. The first is the central executive, responsible for complex higher-order tasks including planning, sequencing and directing and dividing attention. The central executive also coordinates the activities of two 'slave systems': the phonological loop and the visuospatial sketchpad. As their names suggest, these systems hold and process verbal and visual information respectively. All three components play a role in mathematics processing and in particular mental arithmetic. The phonological loop or visuospatial sketchpad encodes the information presented in the problem (depending on whether the problem is presented verbally or visually). The central executive then selects a strategy for calculation such as counting (which also involves the phonological loop) or retrieval from memory (which is governed by the central executive), whilst maintaining attention on the task, holding the numbers in memory and keeping track of which parts have been completed. The information must then be converted into an output format such as words or digits (DeStefano & LeFevre, 2004; Locuniak & Jordan, 2008). Working memory resources are also necessary to create a schema to represent and solve contextualised word problems in mathematics (Fuchs, et al., 2005).

A common method of assessing working memory is through the digit span task, in which students are asked to repeat number sequences of increasing length, for example, the. *Weschler Intelligence Scale for Children* (Wechsler, 2004) and the *Working Memory Test Battery for Children* (Pickering & Gathercole, 2001). In particular, digit span backward, in which the student repeats the digit sequence in reverse order, has been closely linked with maths achievement (Kroesbergen, et al., 2009). This task is a measure of a central executive function called "updating", which is the "monitoring and coding of information relevant to the task and replacing non-relevant information with new input" (Kroesbergen, et al., 2009, p. 227).

Evidence suggests that recalling information in forward order, that is, in the order in which it is presented, relies upon meaning cues and auditory memory, whereas backward digit span uses visual-spatial skills (Li & Lewandowsky, 1995). The link between backward digit span and mathematics is unsurprising since mathematics requires the integration of language and spatial abilities.

Arithmetic combinations

According to Locuniak and Jordan (2008, p. 451), "Dysfluent calculation is a distinguishing characteristic of children with [mathematical difficulties/disabilities]". Students who struggle with mathematics often continue to rely on immature calculation strategies and procedures such as counting on fingers, and counting all items in a collection, and have persistent problems retrieving arithmetic facts from memory (Geary, 1993). It is therefore logical that arithmetic fact retrieval features among the key predictors of early mathematical success.

A typical sequence of counting development would see students developing from effortful counting strategies, such as the Sum procedure or 'counting-all' (counting both collections to determine the total), to the use of more efficient procedures such as counting-on from the larger addend and adjusting digits or using known facts to derive answers quickly (see Baroody, 1987; Baroody & Tiilikainen, 2003, for detailed descriptions of addition strategy development). These more efficient strategies enable both the problem and answer to be active within the working memory span at the same time, forming an association which, with each correct execution, increases the likelihood that the correct answer will be stored in long-term memory with the problem (Siegler & Shrager, 1984). Hence the process of association-building is dependent on both the speed of calculation and the span of working memory. Students who continue to use slow, effortful counting strategies such as counting-on fingers and counting-all are unable to reach a solution in sufficient time to form the association, and therefore remain dependent on such strategies (Geary, 1993; Geary, et al., 2000).

Number identification

Where other measures index growth in mathematical concepts, number identification can be described as among the "gateway skills that enable a child to do mathematics" (B. Clarke, et al., 2008, p. 48). Clarke and colleagues emphasise the need to sample both informal and formal mathematical knowledge when profiling students' mathematical understanding in the early years. Skills such as counting, ordinal number, magnitude comparison and representing quantities with objects have been described as informal skills, whereas number identification is amongst the formal mathematical skills that teachers often identify as important for students to acquire (Vanderheyden, et al., 2004).

Number identification proficiency has been demonstrated to be a reliable predictor of mathematical success in a number of studies. The most common means to assess proficiency is through the use of a fluency measure, where students are presented with a page of randomly generated numbers in a given range and asked to name as many as they can within a one-minute time period. Although such a format is unsuitable for inclusion in a game format, Jordan and colleagues (Jordan, et al., 2006) demonstrated that measuring the accuracy of students' responses in naming numbers can also be predictive of success in mathematics.

As early as the pre-compulsory school year, students show differences in their rate of response to mathematics instruction (Jordan, et al., 2006). Hence it is important to be able to monitor students' responsiveness to instruction, and identify those students who are making slow progress so that additional assistance can be provided. In addition to being a strong predictor of mathematical success, number identification can be used as a brief measure of student progress as proficiency typically grows in a linear fashion over the course of instruction (Chard, et al., 2005; Methe, Hintze, & Floyd, 2008).

Intervention Approaches

When examining effective instructional interventions for students in the early years of school, it is appropriate to examine conclusions from two quite separate fields of research; early childhood education and learning difficulties and disabilities research.

Early childhood curricula

Having identified students at risk of mathematics failure it is crucial to be able to provide evidence-based approaches to remediating difficulties and building necessary cognitive structures for mathematics. Growing interest in the field of mathematics difficulties has lead to significant progress being made in recent years in identifying instructional features that are effective for students with learning difficulties. Different approaches have been taken to preventing and remediating early mathematics difficulties. Whilst some researchers have designed preventative programs for those students at risk due to their demographic features, most notably socioeconomic status (e.g. Baroody, et al., 2009; Clements & Sarama, 2008; Dyson, Jordan, & Glutting, 2013; Griffin & Case, 1996; Sarama, Clements, Starkey, Klein, & Wakeley, 2008; Sood & Jitendra, 2013; Starkey, Klein, & Wakeley, 2004), others have targeted intervention towards students identified at risk through mathematics-related tasks such as those discussed above, or performance on standardised tests (e.g. Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008; Bryant et al., 2011). Although these successful programs have differed in their scope, ranging from broad examination of such diverse mathematical fields as number, geometry, space and mathematical reasoning, instructional features common to all the programs include regular teacher-led instruction in small groups, discussion and the use of concrete materials.

Teaching students with learning disabilities and learning difficulties

A complementary approach is drawn from the field of special education, in particular research into effective practices for students with mathematical learning difficulties and disabilities. These programs typically involve assessment to identify specific areas of difficulty and the provision of targeted instruction. Several researchers have recently attempted to identify the instructional features which are most effective for this population (Baker, et al., 2002; Dowker, 2004; Fuchs & Fuchs, 2001; Gersten, Chard, et al., 2009; Gersten et al., 2008; Newman-Gonchar, Clarke, & Gersten, 2009). However, it is important to note that such studies often feature children in higher elementary grades; for example Baker, Gersten and Lee's synthesis (2002) identified studies concerning children ranging in age from Year Two to Year 11 as meeting the requirements for inclusion in the analysis. Therefore the application of these findings to early childhood contexts must be viewed with caution. Nevertheless, a number of the instructional features identified in the special education literature are also conspicuously present in the successful early childhood programs described above.

Instruction that is targeted to point of need and informed by an understanding of a developmental trajectory or sequence has been identified as effective in a number of studies (Dowker, 2004, 2009). In such approaches, assessment (usually in the form of an interview) identifies students' current level of development and instruction aims to scaffold children to the next level of skill/understanding. *Mathematics Recovery* is an example of such an approach, and utilises a small-group tutoring approach (Wright, 2000; Wright, Martland, et al., 2006; Wright, Stafford, & Stanger, 2006), as does *Mathematics Intervention* which was subsequently built upon the same research and shares many of its instructional features (Pearn, 1999). *The Early Numeracy Research Project* (D. Clarke, et al., 2002) similarly was the basis for the development of the *Extending Mathematical Understanding* (EMU) intervention (Gervasoni, 2003). All these approaches utilise individual diagnostic interviews as the basis to identify students' stage of arithmetical development and their learning needs, the value of which was supported by *National Numeracy Review* of 2008 (National Numeracy Review Report Panel, 2008). All Australian states and territories have recently implemented diagnostic interview processes in mathematics for children entering formal schooling in government schools, however the extent to which these are supported by teaching programs or intervention procedures greatly differs.

As a further example of the effectiveness of targeted instruction, Bryant and colleagues (Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008; Bryant, Bryant, Gersten, Scammacca, Funk, et al., 2008) have focussed on explicitly teaching to those skill areas connected to the predictive tasks themselves. Bryant and colleagues devised and implemented small group tutoring sessions for students in Years One and Two who were identified by the researcher-designed *Texas Early Mathematics Inventory – Progress Monitoring* tools as lacking fluency in areas including magnitude comparison, place value and arithmetic combinations. This work demonstrated that such tutoring, incorporating concrete to abstract instructional sequences and aids such as number lines and hundred squares, could lead to a measurable improvement in these skills and presumably a corresponding decrease in students' risk level for mathematical failure.

Instructional processes that incorporate verbalisation of students' thinking, either through formal or informal processes, have also proved successful in improving outcomes for students with learning difficulties in mathematics (Fuchs & Fuchs, 2001; Gersten, Chard, et al., 2009). Such verbalisation ranges from providing a commentary of thinking during counting and calculation activities (e.g. Griffin, 2004a) to peer tutoring (e.g. Fuchs, Fuchs, & Yazdian, 2002), use of heuristics, self-instruction protocols and strategy instruction including verbalisation (e.g. Naglieri & Gottling, 1997; Owen & Fuchs, 2002; Van Luit & Naglieri, 1999; Wood, Rosenberg, & Carran, 1993).

Similarly, is important to note that many successful intervention programs in the early years employ a small group design rather than individual instruction. In reviewing the research on early intervention, Williams (2008) recommended that due consideration be given to both approaches when developing intervention schemes. Individual settings have the advantage of facilitating highly targeted instruction which is responsive to individual needs. Examples of programs incorporating individualised instruction include *Numeracy Recovery* (Dowker, 2001), developed and renamed as

Catch-Up Numeracy, and *Supporting Children with Gaps in their Mathematical Understanding* (Department for Education and Skills, 2005), the latter developed as an individualised intervention for students in Years Two and above in response to Dowker's review of effective instructional practices for students with mathematical difficulties (Dowker, 2004).

However, such individual approaches are comparatively rare in early childhood settings and are often directed at older children where difficulties may be more pronounced and student needs are more varied. *Mathematics Intervention* (Pearn, 1999), *Extending Mathematical Understanding* (Gervasoni, 2003) and *Mathematics Recovery* (Daly, Wright, Kelly, & Martens, 1997) are examples of Australian programs targeting students struggling with mathematics in their first years of formal schooling which successfully employ small group designs. International examples include *Pre-K Mathematics* (Starkey, et al., 2004), *Building Blocks* (Clements & Sarama, 2008) and *Number Worlds* (Griffin, 2004a). Whilst some have observed that students working in small groups are more easily distracted, such contexts can also generate more relaxed atmospheres (Denvir & Brown, 1986) and enable the social construction of knowledge (Vygotsky, 1962). More specifically, small groups encourage the type of discussion and verbalisation between teacher and students, and between students themselves, which have been identified as contributing to better learning outcomes for students with difficulties (Gersten, Chard, et al., 2009).

One of the most established findings in literature on learning difficulties is the superiority of explicit instruction and direct instruction methods over embedded learning experiences (Baker, et al., 2002; Gersten, et al., 2008). Emphasis on the importance of play as a child's major work in early childhood has led to a perception that such intentional teaching activities are not appropriate for early childhood settings (J. S. Lee & Ginsburg, 2009). Conversely, early childhood interventions which feature focal mathematics teaching have proven highly successful in improving learning outcomes (e.g. Clements & Sarama, 2008; Griffin, 2004a; Starkey, et al., 2004).

The use of physical and visual representations of mathematics concepts has been recognised as a key feature of effective mathematics instructional approaches for students with learning difficulties (Fuchs & Fuchs, 2001; Gersten, Chard, et al., 2009; Gersten, et al., 2008). The use of a continuum of concrete, representational and abstract supports for learning has a long history in mathematics education, although research about which concrete and representational models are most supportive of which concepts is an ongoing field of study. Concrete materials including counters, counting toys and blocks, together with dot pictures, number lines and hundreds boards feature in many of the aforementioned successful early childhood mathematics programs.

One successful approach has been informed by examination of the mental number line as the central conceptual structure developed in early childhood which supports the learning of number concepts (Okamoto & Case, 1996). The use of board games to build an external linear representation of the number line has been successful in enhancing understandings of number magnitude and broader aspects of mathematical achievement both in isolation (e.g. Griffin, 2004a; Griffin, 2004b; Laski & Siegler, 2014; Ramani & Siegler, 2008; Siegler, 2009; Siegler & Ramani, 2009; Whyte & Bull, 2008) and as a key component of broader number curriculums or interventions (e.g. Griffin, 2004a, 2004b; Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012). The use of a number track, as in a board game, contains multiple cues to number magnitude; a higher number results in greater distance being travelled along the track (spatial cue), more individual moves along the track (kinaesthetic cue), more number words being spoken (auditory cue), and greater time taken to complete the move (temporal cue) (Ramani & Siegler, 2008). The most comprehensive attempt to using this approach was conducted by Griffin and colleagues (Griffin, 2004a; Griffin, Case, & Siegler, 1994) as the *Rightstart* program and later *Number Worlds* mentioned above, and comprised an entire Pre-K to Year Two curriculum based upon the five forms in which number lines are represented in civilised societies: as collections of objects, as dot pictures, as positions on a path, upon a vertical scale and a circular dial. The program was successful in bridging the mathematics achievement gap between students of low socioeconomic status and their more advantaged peers over the course of one year of instruction, and this level of achievement was maintained a year after the cessation of the project

The repeated use of isolated board games incorporating number tracks has also proven successful in improving the linearity of students' number line estimates in studies by Ramani and Siegler (2008; Siegler & Ramani, 2009). These studies have determined that boards involving left to right linear, numbered tracks are most effective in building these skills. However, it should be noted that these studies were conducted with boards numbered in the range 1-10. Although there is little direct evidence suggesting that external number lines promote linear representations of number in higher ranges, this would seem to be a promising line of investigation since the accuracy of mental number line representations has been linked with mathematical achievement in ranges 1-100 and 1-1000 (Booth & Siegler, 2006; Siegler & Booth, 2004). Furthermore, a number of fields of enquiry have determined that students with mathematics difficulties have weaknesses in spatial representations, some of which are specific to number (De Smedt, Verschaffel, & Ghesquière, 2009; Geary, et al., 2007; Holloway & Ansari, 2008; Holmes & Adams, 2006), and similarly others have concluded that such students lack the ability to use a mental number line for addition and subtraction (Jordan, et al., 2003). Since the use of visual and concrete representations is well-supported to promote mathematics achievement in struggling students (Fuchs & Fuchs, 2001; Gersten, Beckmann, et al., 2009; Gersten, Chard, et al., 2009), the use of a concrete instructional aid to support the number line would seem well-justified.

Implications

There are a number of ways to successfully promote mathematics achievement amongst children at risk for mathematics failure. Effective approaches and curricula should include instructional features such as the use of small groups, student verbalisation and discussion, intentional teaching of key skill areas and physical and/or visual representations of mathematical concepts. One particularly promising approach to promote accurate representations of numerical magnitude is the use of number tracks as external representations of the mental number line. The current approach aims to combine effective instructional features with explicit teaching towards key mathematical skills which predict later success, supported by an external number line which provides a motivating social context in addition to a valuable cognitive support.

Existing approaches to intervention have attempted to incorporate such scientifically validated principles into programs which teachers can be trained to deliver in their classroom contexts, or which are delivered as intensive interventions in small groups. The latter have had mixed results in influencing student achievement and some have faced challenges associated with planning for sufficient practice time and the transfer of knowledge to tasks outside the specific scope of the intervention (e.g. Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008; Bryant, Bryant, Gersten, Scammacca, Funk, et al., 2008; Bryant, et al., 2011; Fuchs, et al., 2005; Fuchs et al., 2006). Such challenges are likely to be even more difficult to overcome in an Australian context where additional teacher or paraprofessional resources are often not available to support such programs. Hence, the current project is directed at building teachers' PCK for supporting young children who are struggling with early mathematical skills, whilst providing tools which model application to teaching tasks. Such teachers who understand the developmental trajectories of different mathematical skills should be able to design their own dynamic teaching programs which take account of the science of instructional effectiveness, knowledge of mathematical development and specific knowledge of their students and their school context. Whilst resources are provided to support the explicit teaching of key predictive skills, the project's aim is to have a broader impact on educational practice by influencing teachers' planning of mathematical programs.

Conceptual Framework for this Study

The research literature reveals a complex web of factors which impact on students' development of number sense. Constructivism and sociocultural theory frame the study by drawing attention to the role of the student in constructing their own thinking in a social context that includes the verbalisation and discussion of developing concepts and strategies. Hierarchic interactionalism elaborates on this perspective by describing how such processes play out in building ever more-sophisticated models of number as a concept, supported by concrete materials.

Figure 1 demonstrates how teachers play a substantial part in influencing student learning in mathematics through their Pedagogical Content Knowledge, self-efficacy beliefs and confidence for teaching number. It was proposed that mastery experiences and rich teacher PCK would lead to higher levels of confidence and self-efficacy beliefs which would result in the selection of more effective teaching and assessment practices, for which examples and tools were provided through a professional learning intervention. These practices would include experiences based on motivating board-game formats which would raise student engagement and hence lead to successful teaching episodes (mastery experiences), further reinforcing the cycle of teacher development.

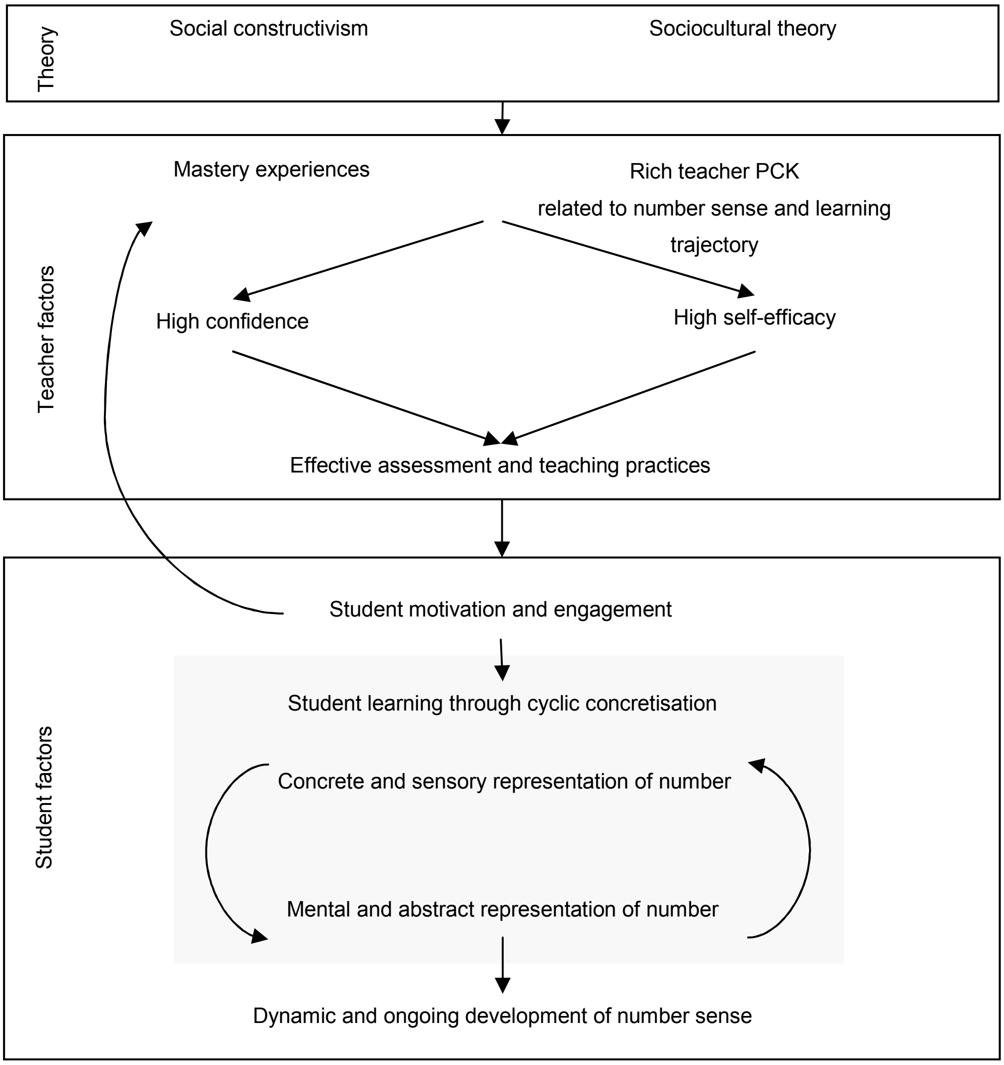

Figure 1. Conceptual framework showing the relationship between theoretical perspectives, teacher factors and student factors in developing number sense.

Student engagement would also lead to enhanced learning, in particular through the process of cyclic concretisation through which an abstract concept of number is built. External and sensory representations of number, for example manipulative materials, facilitate the construction of more abstract and internal representations of number (in particular the mental number line). The relationship between concrete and abstract representations is ongoing and reciprocal, and facilitates the ongoing development of more sophisticated number sense.

This project therefore utilised research findings on identifying and remediating mathematical difficulties to intervene at the level of teacher Pedagogical Content Knowledge, confidence and self-efficacy beliefs for teaching number, with the intention of influencing teaching and assessment practices and ultimately student learning.

CHAPTER 3: METHODOLOGY

Approach

This research project was a case study utilising mixed methods set within an interpretivist epistemology in recognition of the value of interaction between the Researcher and participants in the creation of knowledge. Teaching is a highly fluid endeavour which is responsive to many elements including interactions with students and with professional learning interventions; therefore, the context in which changes occur will be essential to describing the changes and understanding the factors influencing them. The case study method was chosen to enable rich description of the complexities and subtleties characteristic of teaching contexts, and to illuminate links and changes that occurred over the professional learning intervention (Bassey, 1999; Yin, 2003). Case study methods are powerful in describing the application of educational processes and programs within real-life contexts (Merriam, 1988).

Design

Data were collected over the course of two school terms, using a pre-test, intervention, post-test design. The initial stages of the study involved administering pre-test measures of teacher confidence and self-efficacy to the whole cohort of teachers, which were repeated following the professional learning workshops. During the pre-test period some participants (n = 5) were recruited for case studies. Semi-structured interviews with the case study teachers were used to collect data regarding their Pedagogical Content Knowledge and instructional programming prior to and following intervention. Students from within these classes also completed assessments to collect data on their mental number line representations at the beginning and end of the project. The professional learning intervention comprised three two-hour teacher workshops, which were designed to enhance the teachers' Pedagogical Content Knowledge concerning the acquisition of early number skills.

Subjects

The project was targeted towards Pre-Primary to Year Two teachers drawn from the Association of Independent Schools of WA (AISWA). 11 of the 13 teacher participants in the study taught in these grade levels, with the remaining two indicating they taught combined Year Three and Four classes, whilst one also had responsibility for maths curriculum coordination. These teachers had already opted to participate in a series of professional learning workshops conducted by the Researcher for AISWA. These teachers were given the opportunity to take part in this research associated with the professional learning program, and five elected to be case-study teachers from whom further data would be collected. Three were teachers of Pre-Primary children (whom shall be referenced as Teachers A, B and E throughout the study) and the remaining two taught Year One and Year Two classes (Teachers D and C respectively). Each of the five case-study teachers selected approximately six students within their class to complete estimation on the number line assessments (adapted from Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010) at the beginning and end of the intervention period. Teachers were encouraged to choose six students whose level of skill in number is poor to below average for stage of schooling (based on teacher judgement), to gauge the linearity of their existing mental number line representations and provide a measure of the impact of the intervention on student learning. Whilst thirty Pre-Primary students were recruited for this task, valid and complete pre- and post-test data were received for only 17 of these.

Instruments

A range of instruments was used to collect data. These are outlined in Table 3.1.

Table 3.1: Research Instruments

Question	**Instrument**	**Specific items**
To what extent do teachers perceive an increase in their confidence and self-efficacy and have increased Pedagogical Content Knowledge for teaching mathematics as a result of engagement with the professional learning process?	Survey using five point Likert-style scale (n = 13) Semi-structured interview (n = 5) Recorded plenary discussions	Teacher Survey (all) Interview questions 1-5 Plenary discussion questions 1-2
What changes will be made to the classroom program as a result of participating in the professional learning program?	Semi-structured interview Recorded plenary discussions	Interview question 1 Plenary discussion questions 2-3
Do students have more accurate mental number line representations following their teacher's engagement with the professional learning program and experience with linear board games?	Mental number line 0-20 estimation task (n = 17) Mental number line 0-100 estimation task (n = 6)	Mental number line 0-20 estimation task Mental number line 0-100 estimation task

The instruments for this study were adapted from pre-existing instruments (Berteletti, et al., 2010; Dellinger, et al., 2008; McDonough & Clarke, 2003) and are included in the appendices.

Research Procedures

The research was conducted in three phases as outlined in Table 3.2. Quantitative data were the focus of the pre-test and post-test stages, whilst qualitative data were collected through semi-structured interviews over the course of the study. Utilising mixed methods gave greater validity to the study results by ensuring that the strengths and weaknesses of each method are counter-balanced (Jick, 2008). In particular the qualitative data were used to increase sensitivity and enrich interpretation of the quantitative data.

Table 3.2: Timeline for the Research Project

Research Phase	Month	Task	Focus
Pre-test	Late February	Teachers and students completed permission forms Teachers completed confidence and self-efficacy scales Teachers selected for case-studies	
	Mid March	Students completed number line estimation tests Initial interviews with case-study teachers	
Intervention	Mid March	Workshop 1	Counting: Building the mental number line Magnitude comparison, and missing number as predictors of success
	Early April	Workshop 2	Addition, subtraction and decomposing number: Subitising and Arithmetic Combinations
	Late May	Workshop 3	Number identification and Working Memory Bringing it all together
Post-test	Late May	Teachers completed confidence and self-efficacy scales at the final teacher workshop	
	Mid June	Students completed number line estimation tests Final interviews with case-study teachers	

Each of the workshops was structured around a particular conceptual aspect of number development, which was then linked to the key predictive skill targeted through the teaching resource.

- **The first workshop** set the context for number development by introducing the concept of a learning trajectory, and then focussed on the importance of the mental number line and the utility of the magnitude comparison task as its measure in different ranges. The missing number task introduced and was linked to strategic counting development and number identification as a measure of formal mathematical skill.
- **The second session** focussed on subitising skill and development of part-part whole understandings which emerge through conceptual subitising and are later evident as knowledge of arithmetic combinations.

- **The final workshop** discussed the importance of working memory as a mental workspace, its role in mental arithmetic and in predicting mathematical achievement, alongside number identification as a measure of formal mathematical learning. The final workshop was also an opportunity to discuss key messages from the professional learning series and implementation concerns.

Teachers were encouraged to apply the knowledge gained through the professional learning sessions when planning and implementing teaching sessions over the course of the study, and when planning the subsequent term's mathematics program. For this reason, final post-test interviews were conducted after the first term break.

Data Analysis

Quantitative and qualitative data was collected to enable both triangulation and elaboration of data (Greene, Caracelli, & Graham, 2008). The data were analysed with respect to the research questions as outlined in Table 3.3.

Table 3.3: Data Analysis Methods

Question	Data collection	Data analysis
To what extent do teachers perceive an increase in their • confidence • self-efficacy; and • have increased Pedagogical Content Knowledge for teaching mathematics as a result of engagement with the professional learning process?	Teacher confidence & self-efficacy scale Semi-structured interview	Scoring of scales Comparison of average pre- and post-test responses for each item and for mean scale scores Coding of interview responses with regard to important competencies in developing number, value of intentional teaching and appropriate representations for developing number skills. Frequency of pre and post intervention responses in each coding category were compared.
What changes will be made to the classroom program as a result of participating in the professional learning program?	Semi-structured interview with reference to teaching plans Recorded plenary discussions	Themes and sub themes were teased out of the transcripts Coding of interview with regard to emerging themes such as: proportion of mathematics instructional time spent on intentional teaching Sequencing of the mathematics program Explicitness of goals for mathematics learning developed through tasks Range and appropriateness of representations used to develop number skills
Do students have more accurate mental number line representations	Estimation on the number line	Comparison in degree of error of each student's mental number line, and

following their teacher's engagement with the professional learning program and experience with linear board games?	task	prevalence of over and underestimates

Threats to Validity and Reliability

The case study technique, although perfectly suited to examining phenomena within complex contexts involving many variables, is difficult to replicate. In reference to the first research question, the collection of both qualitative and quantitative data have strengthened construct validity, since the effect of biases and limitations of each form of data is minimized when the data are triangulated. The use of qualitative techniques with a subset of participants enabled elaboration of the key trends and themes emerging from the quantitative data and rich descriptions of the context in which changes in practice occurred.

The scales used to describe the constructs of confidence and self-efficacy were adapted from the *Teachers' Self-Efficacy Beliefs – Self Form* (Dellinger, et al., 2008) and evidence from the *Early Numeracy Research Project* (D. Clarke, et al., 2002) which described the practices of effective early years teachers of mathematics. Whilst the former is general in nature and describes such diverse competencies as organisation, classroom management, planning and instructional skills, the latter is specific to the population and context of the teachers being studied and therefore should provide a highly targeted picture of relevant teaching skills. As pre-existing tools, the research base informing their development has already been established, and comparisons with existing data are possible.

Ethics

Approval to conduct this research was gained from the Edith Cowan University Human Research Ethics Committee. All teacher, parent and student participants in the study received letters providing information about the study and consent forms. Data were only collected and analysed from consenting parties. All data have been de-identified, reported anonymously and stored securely to protect the confidentiality of those involved.

CHAPTER 4: RESULTS

The purpose of this project was to examine the impact of a professional learning program designed to help teachers focus on assessment and instruction targeting the development of skills predictive of mathematics difficulties, within informal game contexts. The effects on teachers' perceptions of their confidence and efficacy and changes in pedagogical knowledge were investigated, along with changes in classroom practices and impacts on students' estimation accuracy.

Data were gathered using a survey administered to 13 teacher participants, prior to and following the three professional learning sessions. Five of these teachers elected to be case-study participants. The case study teachers participated in individual semi-structured interviews pre- and post-intervention, and a recorded plenary discussion at the end of each of the three professional learning sessions, in order to gain deeper insight into their judgements and interpretation regarding the value and utility of the intervention. These teachers then chose five to six students from within their classes to complete assessments to measure the accuracy of their mental number line through completion of an estimation task prior to and after the intervention. Due to erroneous administration of the number line task and missing data, data from only three of these teachers can be reported.

In this chapter the data from these four sources will be presented and summarised; teacher surveys, case-study teacher interviews, case-study teacher plenary discussions and student estimation assessments.

Teacher Confidence and Self-Efficacy

Of the 16 teachers initially involved in the project, 13 completed both pre- and post-intervention surveys. The data from the 13 teacher surveys were collated and compared question by question to determine mean responses and standard deviations for each item at pre- and post-intervention, and the difference in pre and post mean scores. Means were tested for statistical significance using the Wilcoxon Signed Ranks test since the measures were on an ordinal scale. The response patterns observed for survey items testing confidence in teaching number are reported in Table 4.1.

Table 4.1: Teacher survey responses for items targeting confidence in teaching number (n = 13)

Aspect of teaching number	Mean of confidence ratings (/5)				
	Initial		Final		Mean Gain Initial-Final
	Mean	SD	Mean	SD	
Promote a positive classroom climate during number activities	4.23	0.576	4.38	0.738	0.15
Teach core number concepts	3.54	1.009	4.31	0.606	0.77*
Manage a range of group structures during number work as appropriate (e.g. whole class, small groups, individual work)	3.62	0.738	4.23	0.576	0.62*
Respond to 'teachable moments' in number	3.54	1.151	4.23	0.799	0.69*
Provide students with feedback during number activities to enhance learning	3.46	1.009	4.15	0.769	0.69*
Use allocated instructional time to maximise learning in number	3.31	0.910	4.08	0.615	0.77*
Accommodate individual differences in planning number work and during number activities	3.23	1.187	4.08	0.730	0.85**
Plan a sequenced and appropriate learning program in number	3.46	0.843	4.00	0.679	0.54
Diagnose students' number skills and plan specific interventions	2.77	0.890	4.00	0.679	1.23**
Manage and maximise learning during classroom and group discussions on number concepts	3.38	1.003	3.85	0.662	0.46
Mean scale score	3.454**	0.932	4.131**	0.857	

Note. Teachers rated their confidence on a five point scale VC = Very confident; C = Confident; OK; LC = Limited confidence; NC = No confidence.

* Significant p<.05 on Wilcoxon Signed Ranks test. ** Significant p<.01 Wilcoxon Signed Ranks test

Teachers were most confident on items involving general classroom teaching practices such as promoting a positive classroom climate, classroom management and what might be termed 'first wave' best practice number instruction. Items involving differentiation and sequencing of the classroom program were those at which teachers showed the least confidence at both pre- and post-intervention. The two lowest scoring items before the intervention: diagnosing skill levels and planning interventions; and, putting such accommodations into practice in the classroom, were those that showed the most growth over the course of the project (p<.01). The increase in confidence on these items was highly significant and commensurate with the focus of the professional learning project on supporting students with learning difficulties in mathematics. All items displayed an increase in confidence and for seven of the 10 items this change reached statistical significance, including the mean scale score for which the difference was highly significant ($p<.01$). Therefore, teachers had greater confidence in these aspects of teaching number as a result of the intervention.

The questionnaires also assessed teachers' perception of their efficacy in teaching number, the results of which are shown in Table 4.2.

Table 4.2: Teacher survey responses for items targeting self-efficacy for teaching number (n = 13)

Item	**Mean of agreement ratings (/5)**				
	Initial		**Final**		**Mean Gain Initial-Final**
	Mean	**SD**	**Mean**	**SD**	
I understand and am able to explicitly communicate the purpose and intended learning outcome of each activity to students	3.46	0.746	4.38	0.625	0.92**
I am able to plan purposeful learning tasks in number that are motivating and engaging	3.69	0.821	4.31	0.722	0.62*
I am able to clarify student misconceptions or difficulties in learning through appropriate teaching tasks	3.58	0.828	4.31	0.462	0.77*
My rich knowledge of mathematics in early childhood enables me to act upon students' difficulties and modify my planning	3.31	0.722	4.31	0.462	1.00**
I am able to plan and deliver effective mathematics lessons and tasks that focus on key mathematical ideas	4.08	0.640	4.23	0.576	0.15
I am able to plan opportunities for students to use a range of representations and materials to explore the same concept and build connections between representations	3.69	0.821	4.23	0.576	0.54
I am able to plan activities that accommodate the range of individual differences among my students	3.31	0.991	4.15	0.533	0.85*
I use my rich knowledge of mathematics in early childhood to ask appropriate questions to probe and promote student thinking and reasoning	3.15	1.099	4.08	0.615	0.93*
I am able to provide students with opportunities to learn at more than one cognitive or performance level	3.62	0.836	4.00	0.679	0.38
I am able to plan evaluation procedures that accommodate the range of individual differences among my students	3.31	0.821	3.92	0.730	0.62*

My rich knowledge of mathematics in early childhood enables me to notice individual students' strategies and misconceptions	3.15	1.099	3.92	0.730	0.77*
Mean scale score	3.486**	0.894	4.167**	0.655	

Note. Teachers rated the extend to which they agreed with each statement on a five point scale
SA: Strongly agree; A: Agree; UN: Undecided; D: Disagree; SD: Strongly disagree
* Significant p<.05 on Wilcoxon Signed Ranks test. ** Significant p<.01 Wilcoxon Signed Ranks test

Teachers believed they were most efficacious at items related to broader aspects of maths instruction: communicating the purpose of learning activities to students; planning purposeful, motivating and engaging lessons; and, clarifying students' misconceptions in the course of teaching. Tasks at which teachers felt least efficacious were more specific to individuals: noticing strategies and misconceptions; planning individually appropriate evaluation procedures; and, planning activities at more than one level, i.e., differentiating instruction. Of the 11 items assessing beliefs about being an effective teacher of number, the means of eight items increased significantly. The increase in ratings for two items, in addition to the overall scale, were highly significant (p<.01). In contrast to the confidence scale, these items were those at which teachers rated themselves quite highly; explicit awareness of the purpose of learning activities, and of having the rich mathematical knowledge to act upon student difficulties. The ability to notice individual students' strategies and misconceptions remained the lowest scoring item on the scale both pre- and post-intervention, although it did show a significant improvement (p<.05). The scale as a whole showed that teachers saw themselves as significantly more efficacious after the intervention (p<.01), and that there was lesser variance in responses after the intervention with the mean standard deviation reduced from 0.894 to 0.655.

Teachers were also asked to rate their confidence and knowledge for teaching number and language skills on a 10-point scale. Pre- and post-intervention means and standard deviations were calculated for each and are reported in Table 4.3.

Table 4.3: Teacher survey responses rating knowledge and confidence in teaching language vs number (n = 13)

Item	**Mean of ratings (/10)**			
	Initial		**Final**	
	Mean	**SD**	**Mean**	**SD**
Knowledge for teaching early language skills	7.23	1.671	7.85	1.292
Knowledge for teaching early number skills	6.00**	2.184	7.77**	1.423
Confidence with teaching early language skills	7.15	1.703	8.00	1.177
Confidence with teaching early number skills	6.08**	2.235	8.00**	1.414

Note. Teachers rated on a 10-point scale from Low confidence/knowledge = 1 to High confidence/knowledge = 10
** Significant p<.01 paired samples t-test

As anticipated, teachers felt generally more knowledgeable and confident about teaching language compared to number. As expected there was no significant impact on perceptions around the teaching of language. However, both knowledge and confidence for teaching number increased significantly ($p<.01$), to the point where both were comparable to feelings about the teaching of language. Furthermore, following the intervention, there was less variation in the teachers' perceptions of their knowledge and confidence for teaching number with standard deviations reduced.

Key Findings

1. Teachers were most confident in their ability to deliver high-quality number instruction at the first-wave level following the intervention.
2. Teachers had increased confidence for teaching number following the intervention, especially in catering for individual differences.
3. Teachers felt more efficacious in teaching number, especially in communicating the purpose of lessons and being responsive to student difficulties.
4. At the conclusion of the project teachers felt significantly more confident and knowledgeable about teaching early number skills, whilst there was no significant change to their knowledge and confidence for teaching early language skills.

Teachers' Beliefs and Pedagogical Content Knowledge for Number Instruction

In addition to the scaled responses on the survey, the 13 teachers also answered six short-answer questions concerning their beliefs about teaching number and responded to scenarios probing instructional approaches and knowledge of maths skill development. The questions were intended to measure the impact of the professional learning intervention on teachers' pedagogical knowledge for teaching number. Multiple responses could be provided for each question. Responses for initial and final surveys were coded into categories and their frequencies recorded. The most prevalent responses are reported in the following tables.

The first question concerned teachers' expectations about the professional learning and subsequently their perception of its personal impact (Table 4.4).

Table 4.4: Teachers' responses to the question: What do you hope to gain/feel you have gained from participating in this professional learning program? (n = 13)

Category of response	**Frequency**	
	Initial	**Final**
Increase professional knowledge	4	7
Improve assessment of students' maths understandings, especially with regard to difficulties	7	4
Increased repertoire of tools for assessing and targeting specific maths skills	2	4
Improve planning and/or teaching of students with maths difficulties	5	2
Improve maths teaching generally	4	1
Improve confidence in teaching maths	1	0
Total number of coded responses	21	18

Note. Some of the teachers provided more than one codeable response to this question

At pre-test the most common responses were that teachers intended to increase their professional knowledge related to maths teaching and difficulties, improve assessment practices and improve their planning for and teaching of students with maths difficulties. Teacher A exemplified all these themes in her response, "to be able to provide meaningful and purposeful teaching experiences and to collect real information regarding where the children are and where they need to go in mathematics learning." Two also made specific reference to gaining tools through which this could be achieved. One teacher expressed this as, "tools and strategies to identify, confirm and assess children's mathematical understandings." Following the intervention these themes continued but four teachers made specific reference to the tools they had gained for this purpose, for example, "access to tools (games) that clearly highlight gaps in young children's mathematical skills and concepts." There was a reduction in the number of teachers who focussed on improving maths teaching more generally. The greatest changes were in the number of teachers who stated they had increased their professional knowledge, for example, gaining "a much clearer understanding of the hierarchy of skills required for numeracy", increased their repertoire of tools, and in the reduction of general statements about improving maths teaching.

The second question asked teachers to make a judgement about the most important number skills students needed to learn. Each of 13 respondents listed these skills in their answers, with all giving multiple responses. Table 4.5 presents a summary of teachers' responses to this question.

Table 4.5: Teachers' responses to the question: What do you believe are the core number concepts P-2 students have to learn to succeed in maths? (*n* = 13)

Initial	**#**	**Final**	**#**
Mental calculation strategies, knowledge of number facts	9	Number magnitude, mental number line	10
Counting and counting principles	8	Knowledge of number facts and strategies	8
Knowledge of numeration, number reading, number formation	7	Verbal counting sequence	5
Reciting the number sequence, including skip counting	6	Counting collections and counting principles	5
Place value	4	Subitising	5
Subitising	4	Place value	4
Using number lines, magnitude (more/less), estimation	4	Numeration, number reading & writing	3
Matching different codes of number (verbal/numerals/words/quantities)	3	Counting strategies concrete-abstract	3
Multiplication and/or division concepts (grouping and sharing)	3	Using number lines	2
Understanding mathematical language	2	Working memory, use of long-term memory	2
Total number of coded responses	59	Total number of coded responses	47

Note. Some of the teachers provided more than one codeable response to this question

The most common responses prior to intervention concerned knowledge of number facts and arithmetic strategies, and counting collections, including counting principles. The least common responses concerned mathematical language and other responses such as problem-solving. There was a marked increase in the number of teachers who referred to comparing numbers, with many teachers making specific reference to the mental number line or number magnitude following the intervention. References to number recognition and number formation decreased markedly from seven to three. Knowledge of number facts and arithmetic strategies remained a priority.

A summary of responses to the third question, in which the 13 teachers were asked their opinion about the relative importance of informal play-based versus more explicitly planned mathematics instruction, is shown in Table 4.6. One teacher provided no answer to this question at post-test.

Table 4.6: Teachers' responses to the question: What is the place of focussed mathematics teaching in P-2, compared to learning through embedded experiences? (*n* = 13)

Initial	**#**	**Final**	**#**
Equal weighting, both important	5	Equal weighting, both important	6
Need for daily/regular explicit or focussed teaching	5	Need for daily/regular explicit or focussed teaching	2
Each as complementary to the other	4	Meaning unclear or did not respond directly to question	2
Focussed teaching to overcome misunderstandings	2	Need to explicitly teach to target certain skills	2
Focussed maths teaching to make explicit what is already known	1	Each as complementary to the other	1
Total number of coded responses	18	Total number of coded responses	14

Note. Some of the teachers provided more than one codeable response to this question

There was no strong impact on teachers' beliefs regarding the relative value of focussed teaching versus embedded experiences, with the most common response being a general statement that both were of equal importance; "there is a need for both focussed mathematics as well as play based experiences." Some teachers elaborated on this and made statements about the interaction of the two approaches: "intentional teaching, I think, might help a child understand what he was doing while playing with games (toys)." A less common response was that focussed instruction served the purpose of making implicit knowledge explicit: "focussed mathematics teaching is about making children aware of what they know and how they can use this knowledge." Responses were generally more brief on the final survey. Following the intervention, one respondent believed focussed teaching should only occur in small groups.

The fourth question presented a scenario in which a student teacher chose an unsuitable model to teach foundation counting skills (a representational model to teach concrete counting). Teachers were asked to provide feedback on the student teacher's planned lesson, and could provide multiple responses. Problems with the wording of the question ("in pairs" – intended to indicate students working with a partner rather than skip counting in twos) lead some teachers to respond without addressing this central tenet of the question. A summary of teachers' responses is presented in Table 4.7.

Table 4.7: Teachers' responses to the question: A student teacher has planned a lesson to teach counting principles to a Pre-Primary class through the use of a number line. She plans to show students how to 'jump' along a number line drawn on the whiteboard as they count aloud, using a whiteboard marker, and then ask the students to do the same in pairs. She asks for your feedback. How would you advise her? (*n* = 13)

Initial	**#**	**Final**	**#**
Have a pre-constructed physical number line	5	Use concrete materials, 1:1 counting/matching	5
Misinterpretation of counting in pairs (too difficult)	3	Number line too difficult/abstract	4
Need to explicitly teach the use of a number line	3	Assess prerequisite skills, counting objects, number sequences	3
Match number line to how many objects (concrete or visual)	2	Have a pre-constructed physical number line	3
Focus on lesson structure, e.g. pose a problem to be solved first	1	Have children construct a number line	3
Clarify learning outcomes with students	1		
Use concrete materials	1		
Pre-test for prerequisite oral counting skills	1		
Number line too difficult	1		
Total number of coded responses	17	Total number of coded responses	21

Note. Some of the teachers provided more than one codeable response to this question

The most common initial response was a reference to the need for "concrete" experiences as being a physical number line to jump along: "have a number line on the floor and jump physically along the line," and following the intervention, the counting of collections of concrete objects: "I'd suggest that prior to this lesson which is abstract that first she use concrete materials and decide if students are 'ready' for this step." This need for concrete counting, along with the recognition that using a number line is a difficult, abstract concept and that pre-requisite skills exist, were among the least common responses initially but showed a marked increase after the intervention. This awareness was exemplified by one teacher who explained, "use concrete materials first. Can they count with 1:1 correspondence? Do students know the number line? Do they have the counting sequence?"

The fifth question probed teachers' instructional strategies to advance counting development for a student who persisted with redundant counting behaviours. As before, the 13 teachers could provide multiple responses. The main categories of responses are shown in Table 4.8.

Table 4.8: Teachers' responses to the question: When he is combining two collections (e.g. 5 buttons and 2 more buttons), one of your students persists in counting out both collections separately, then counting all items starting from one, and won't move to counting-on. How would you respond? (*n* = 13)

Initial	**#**	**Final**	**#**
Explicitly teach/model counting-on	6	Count or count-on using a number line	4
Count or count-on using a number line	3	Discuss/convince student of the benefits of a more efficient method	3
Encourage or support counting as a single collection, by ones	3	Test/teach pre-requisite skills: principles of counting (conservation and cardinality)/ magnitude comparison	3
Build subitising skill, e.g. through dominoes to reduce counting by ones	2	Hidden number (collections)	1
Discuss/convince student of the benefits of a more efficient method	2	Explicitly teach/model counting-on	1
Pre-requisite skills: principles of counting (conservation)	1	Build subitising skill, e.g. through dominoes to reduce counting by ones	1
I don't know	1	I don't know	1
Total number of coded responses	19	Total number of coded responses	18

Note. Some of the teachers provided more than one codeable response to this question

Almost half of the teachers made reference to showing students how to count on through demonstration prior to the intervention, e.g. "show how to count on", whilst three made reference to using a number line to do this, e.g. "give a whole class demonstration of how we could put the larger number 'in your mind' & then count on, use a number line to start & then count on." After the intervention, teachers tended to be more specific about how to teach counting on; reference to use of a number line was the most common response (at four) and one teacher mentioned the use of hidden collections. There was an increase in the number of teachers who acknowledged the need for pre-requisite skills for counting on such as through saying, "he doesn't have conservation of number so I'd have him practice counting collections from different starting points" (from one to three responses) and the most marked change was that no teachers mentioned supporting the strategy of counting by ones, starting at one, in contrast to three at pre-test.

Similar to the fifth question, the final question was intended to elicit teachers' knowledge of stages in bridging the mathematics of objects (counting collections) with the mathematics of numbers through a scenario involving a teacher wanting to assess if her students were ready to move beyond concrete counting. Table 4.9 summarises the 13 teachers' responses to the scenario.

Table 4.9: Teachers' responses to the question: Mrs Johnson has been doing lots of counting activities involving concrete objects with her young class, and wants to find out if they are ready to move on to learning more advanced counting skills and abstract number combinations without using objects. How could she find out? (n = 13)

Initial	**#**	**Final**	**#**
Pose a verbal number problem (based on concrete objects) and observe responses	4	Solve number problems using a number line	5
Check/teach number recognition/numeration	3	Solve number problems using representations e.g. dots, drawing	4
Solve number problems using representations e.g. dots, drawing	3	Direct reference to arithmetic games in workshop	2
Use number sentences to represent number stories	2	Missing numbers, oral count sequence	2
Play subitising games	2	Play subitising games	1
Solve number problems using modelling with concrete objects	2	I don't know	1
Try it and observe what happens	1		
Total number of coded responses	19	Total number of coded responses	15

Note. Some of the teachers provided more than one codeable response to this question

Although the intent of the question was to move away from the maths of quantities into the maths of numbers, initially the most common responses referred to the use of number stories verbally and observing how students responded, such as, "give some word problems about practical/relevant situations and observe how students solve it." Three teachers saw number recognition as an important prerequisite for abstract calculation, for example writing, "I would use flash cards 1-10 first, then flash cards 11-19 10-100 and see if this will help the child." An equal number referred to the use of drawings or dots rather than concrete objects, and this increased slightly after the intervention (from three to four), for example in suggesting to move students on "by using representations of physical objects." The most noticeable change was in reference to the use of a number line, not present at the initial interviews but the most common response after the intervention, such as, "move on to representational number lines. Play games targeting counting on and counting back using number lines."

Key Findings

5. Teachers saw focussed teaching and play-based learning as equally important both prior to and after the intervention.
6. Teachers were more aware of strategies and representations they could use to bridge concrete and abstract number work following the intervention, specifically the construction of a mental number line and the use of physical number lines for calculation.

7. Teachers showed greater awareness of the need to consider pre-requisite skills when evaluating the appropriateness of lessons and planning to move students on to more advanced skills.

Case Study Teachers' Beliefs, Pedagogical Content Knowledge and Practice for Teaching Number
For five of the teacher participants, additional semi-structured interviews were conducted prior to and after the series of three professional learning sessions in order to provide elaboration for the quantitative data and gain information on changes to classroom practices. These teachers are referred to by the letters A-E, with Teachers A, B and E teaching at the Pre-Primary level, Teacher C at Year Two and Teacher D at Year One. Pre-intervention interview data were analysed to create categories of responses, for which frequencies were reported. Frequencies of the same categories were then observed and recorded from post-intervention interviews and new categories created as required. These are reported in the tables which follow.

The five teachers were first asked about their approach in teaching number, with all giving multiple responses. The varied responses are summarised in Table 4.10.

Table 4.10: Teachers' responses to the question: How would you describe your approach to teaching number in your classroom? (n = 5)

Category of response	**Frequency**	
	Initial	**Final**
Hands-on, using concrete materials and manipulatives	3	4
Use of maths games	2	3
Whole group mat sessions	2	3
Use of small groups	2	3
Has experienced a shift in focus or intensity as a result of the PL intervention	0	3
Explicit teaching	1	2
Worksheet recording following concrete examples	2	1
Discussion	0	1

Note. Some of the teachers provided more than one codeable response to this question

Class routines and general approaches to number learning remained much the same, with games, whole group and small group approaches described on both occasions. Teachers consistently referred to the importance of concrete materials in mathematics (Teacher D):

> I do understand that number needs to be [a] very hands on and concrete... thing for kids. So there are times when I feel that to try and get the concepts across I will try to do it in a concrete method where they can use things to build [the numbers]"

In the initial interviews all of the teachers listed concepts they were teaching as priorities in maths. For example, Teacher C said, “I think, probably, especially Term One for us is a lot of focus on number, because we’re looking so much at place value and checking that the kids can actually write numbers.” Following the intervention there was greater description of how these concepts would be developed. For example Teacher B referred to “intensive discussion” on the mat and Teacher C described “a mixture of explicit teaching and then some different, you know, hands-on activities with either manipulatives or even games … that involve manipulatives.” Teacher A described a change in her practice as an awareness of the need to delve deeper into the understanding of each concept before moving on, and a shift from a focus on number recognition and 1:1 counting to include number sequencing and number lines.

To help establish a context for how mathematics and number were taught in each classroom and what priority was given to these, teachers shared how much, if any, instructional time was routinely set aside for number activities. Table 4.11 summarises the five teachers’ responses.

Table 4.11: Teachers’ responses to the question: Do you have scheduled time for number activities, and if so, how much? (n = 5)

Category of response	**Frequency**	
	Initial	**Final**
Discrete maths sessions, at least 4 days per week	4	4
Whole days set aside for maths/number	1	1
Daily number routines e.g. counting children present/absent, calendar work	1	2

Note. Some of the teachers provided more than one codeable response to this question

All teachers had specific scheduled time for maths each week, with four of the five teachers doing maths on at least four days of each week for shorter blocks of between 45mins and 1hour 40mins, as was the case for Teacher E who stated, “It's the same every day, but usually my daily literacy groups are before recess and then I have my maths groups after recess every day except for Friday. That goes for an hour.” Teacher A differed, saying, “because of having DOTT period in the middle of the day, trying to break it up into literacy and then maths it's too small a time to really get a grasp so I have a day where we do one concept. Tomorrow we'll look at maths and we'll do all maths activities in the rotations.” Note that DOTT is a reference to time provided to teachers during the school day for Duties Other Than (face-to-face) Teaching. Two of these full days were planned with a maths focus. Following the intervention, two Pre-Primary teachers made reference to daily routines involving number and counting such as counting how many students were in the class and how many missing. Indeed, teacher A described how she had developed an entirely new morning routine which applied two of the key representations from the professional learning intervention:

assembling a number line to explore the number sequence and using a number line marked in increments for estimation. In her words, “they are really getting to know number lines so I’m thinking this, they should have done a lot better this time than they have the first time because they’ve been doing it every morning.”

Teachers were then asked about the importance of explicit teaching versus play-based learning, similar to the short answer question in the survey. Due to researcher error this question was omitted from one pre-test interview, meaning that only four responses could be coded at the initial interviews and five at final interviews (Table 4.12).

Table 4.12: Teachers’ responses to the question: Should learning experiences be embedded in play situations or planned and taught explicitly? (*n* = 4)

Category of response	**Frequency**	
	Pre	**Post**
Both	4	5
Explicit teaching of concepts, then express, reinforce or apply in play	3	3
Connecting the dots, making the maths explicit through explicit teaching	2	0

Note. Some of the teachers provided more than one codeable response to this question

On both occasions, all responses indicated that teachers valued both approaches, although teachers generally provided greater elaboration on this at the initial interviews. There was broad agreement that concepts should be introduced through explicit teaching and then consolidated or applied in less formal ‘play’ contexts: “they need to be explicitly told things in order to get the concept, but in order to get the concept they need to be able to express it in play and work out what they're doing in play” (Teacher A, Pre-Primary). Two teachers described the role of explicit teaching in making maths knowledge explicit, including Teacher C (Year Two) who asserted, “I think it needs to be explicitly taught so that they are connecting those dots, so that you're actually trying to get them to realise what they're doing.”

Teachers differed in their interpretation of ‘play’, with Teacher B (Pre-Primary) viewing maths games as part of her instructional maths session and ‘play’ as something more child-directed that happened in the course of free time in learning centres. Describing these free play times, she said, “I make it more incidental rather than trying to make maths part of their play, I don’t want to take away from their play context of ‘now let’s make it about counting’, I make it more kind of ‘oh, what are you doing, can you tell me about what you’re doing?’” Teachers A and C (Pre-Primary and Year Two respectively) interpreted ‘play’ as being more deliberately planned, such as through the use of maths games and maths manipulatives. Teacher A expressed a wish to have a greater repertoire of such experiences and said she was hoping to gain this through participation in the professional learning intervention.

An additional question was asked only at the final interviews, asking the teachers to articulate how their teaching and learning programs had been affected by the professional learning intervention. Although teachers were invited to give an example, most preferred to give more general descriptions as shown in Table 4.13.

Table 4.13: Teachers' responses to the question: [Post-test only] How have the professional development and tools provided impacted your teaching and learning program? Please give an example. (*n* = 5)

Category of response	Frequency
	Final
Increase in professional knowledge and ability to plan a sequenced learning program, especially with regard to trajectory of skills development	4
Challenges to implementation, especially collaborative teaching approaches and/or information overload	3
Incorporation of specific resources/games provided as they fit with existing teaching program	2
Planned and/or strategic incorporation of games/resources provided	2

Note. Some of the teachers provided more than one codeable response to this question

Four of the teachers described their maths programs as being more carefully planned and better sequenced as a result of the professional learning intervention and particularly the flow chart of skills that was explored within it. As Teacher B (Pre-Primary) expressed:

> Also being a lot more aware of the different components of maths… we have detailed early intervention programs for literacy but there's not a lot for maths and it kind of feels a little bit, 'should I address this now?' I just pick and choose where you're gonna go, whereas I'm finding just having the flowchart … makes it better sequenced has been the biggest benefit for me …being a bit more confident about knowing the particular skills that I'm looking for… being a bit more specific.

All teachers noted an impact on their assessment techniques, both through using the tools provided and through honing their own methods. Teacher C (Years Two) stated:

> The skills trajectory … Having something to go back to when you find an issue, not only have you got the assessments and stuff to go back to, or take it down a level if you need to, but you've also got something to look at in that and go, 'if they're not getting this then what comes before that, what do I need to maybe go back to and see have they got that?'

Teachers B and C (Pre-Primary and Year Two respectively) expressed regret that they had not been yet able to use the tools as systematically as they would have liked, due to challenges in working in a collaborative teaching environment and the need for time to enable the preparation of materials. Teacher B addressed this by designing mat sessions which addressed the core concepts discussed in the professional learning, and using the tools themselves when able. There was a move towards more deliberate teaching of key concepts that perhaps had been previously covered only incidentally. As Teacher A (Pre-Primary) expressed it:

> I guess it's bringing those elements like number before and after and planning for that and putting it into my program instead of just doing it incidentally, it's doing all of those things I was saying before but putting it into a program and actually planning for it.

Teachers were asked to list skills and concepts they felt prepared students for success in early number. All five teachers provided multiple responses, with more variation apparent prior to intervention as shown in Table 4.14.

Table 4.14: Teachers' responses to the question: What do you think students need to know in order to be successful in learning early number concepts? What are the core concepts in early number? Which core skills are important to learn? (*n* = 5)

Category of response	**Frequency**	
	Pre	**Post**
Understanding of number magnitude, e.g. a mental number line for counting-on and comparing	1	5
Number recognition and place value	4	3
Counting principles, especially 1:1 correspondence	2	3
Subitising	2	2
Keeping track strategies when counting	1	1
Number combinations and efficient counting strategies	3	0
Writing numbers including number formation	3	0
Ordering numbers	2	0
Patterns in objects or numbers	2	0

Note. Some of the teachers provided more than one codeable response to this question

At the first interviews four of the five teachers emphasised the importance of number recognition, specifically as it related understanding two-digit numbers. This was commonly the first of the listed skills, such as for Teacher D (Year One) who said, "they need to know their numbers ... if they can't recognise and put together and say what the number one sounds like and looks like they are really disadvantaged … beyond 10 they may look at a number and ask 'what number is that?'" Writing numbers was also a common response as was a focus on number combinations (Teacher C, Year Two):

> Actually starting to build some of their strategies for addition and subtraction and being exposed to some different strategies ... even just a simple ones like knowing their number bonds to ten and starting to build their number bonds to 20, and understanding there's a reason that we learn it ... and we can apply it to our addition and subtraction

A variety of other responses were provided at this time, including Teacher E (Pre-Primary) who was the only teacher who made reference to a mental number line. The importance of understanding number magnitude and/or using a mental number line was unanimously emphasised after the intervention which represented a marked change in response patterns, with Teacher A (Pre-Primary) stating, “they need to know magnitude of number… even if they don’t know that the number is a 14, they can see that because there are two-digits there that it’s bigger than 4.” Teacher E described her change in priorities thus:

> Before I pretty much had no idea, I probably would have said using materials to be able to model addition and all those sorts of things and understanding of numerals and how quantities match to that, but now I think the real background stuff behind all that number knowledge is their ability to be flexible with a mental number line.

Although the mental number line and its role in bridging concrete and abstract mathematics was emphasised in the professional learning intervention, the magnitude of this shift in focus from Teacher E was unexpected and the dismissal of skills involving concrete materials unintended. This will be discussed further in the following chapter. Table 4.14 shows that although number recognition and counting principles remained important to the teachers, only five different categories were evident post-intervention in contrast to nine at the initial interviews.

Teachers were then asked to articulate best practice in teaching number to young students, in contrast to the first interview question when teachers were asked to describe their own practice. This question was asked in tandem with the following one concerning tools and representations for number, and most teachers chose to address both at the initial interviews but were more likely to concentrate on the following at the final interviews. Hence there was lesser variation in responses following the intervention as demonstrated by Table 4.15.

Table 4.15: Teachers’ responses to the question: How would you describe the best way of teaching number concepts to young students? (n = 5)

Category of response	**Frequency**	
	Pre	**Post**
Hands-on experiences, concrete materials	3	5
Extensive concrete work must precede abstract symbols	0	3
Abstract activities to follow-on from concrete work	2	2
Discussion to initiate	1	1
Group singing, chanting and reading	1	1

Explicit teaching	1	1
Use of games to consolidate	1	0
Start with detailed assessment	1	0

Note. Some of the teachers provided more than one codeable response to this question

Table 4.15 shows that teachers made mention of similar experiences as compared to when describing their own practice (see Table 4.10). Concrete materials were consistently valued and represented the most common response on both occasions: “always playing with things first ... Give me something real, show me what it looks like ... lots and lots of different ways of doing it” (Teacher B, Pre-Primary).

The greatest change was the emergence of the idea that the move from concrete to abstract should be carefully controlled and not too rapid. All three such responses were from teachers at the Pre-Primary level. According to Teacher E:

> I think making the jump from concrete to the more abstract counting is … you can’t be too quick with that it needs to stay pretty much solidly throughout … the Pre-Primary year and then they get opportunities to use their knowledge in a more abstract way in other scenarios – you never take the materials away from them.

The next question about appropriate representations and tools was asked to elaborate upon the previous, with regard to how maths concepts were demonstrated in practice. This question, in most cases, was asked and answered in tandem with the previous. The five teachers all provided multiple responses, for which Table 4.16 provides an overview.

Table 4.16: Teachers’ responses to the question: What tools and representations are appropriate? (n = 5)

Category of response	**Frequency**	
	Initial	**Final**
Unspecified concrete materials	0	5
Countable maths manipulatives e.g. counters, teddies, fruit	4	3
Written number work or numerals	5	3
Number lines	1	2
Board games e.g. snakes and ladders	3	0
Maths materials grouped in tens, e.g. Base ten	2	1
Everyday items	1	1
Counting intangible objects e.g. sounds	1	0
General maths games e.g. Bingo, Domino formats	1	0

Note. Some of the teachers provided more than one codeable response to this question

The most commonly listed materials at the initial interviews were concrete materials, especially discrete countable materials, and numeric symbols. Two teachers described how they regularly followed a progression to symbolic (written) work after using concrete materials, including Teacher D (Year One) who explained, “I prefer to, once I’ve taught it concrete then they go and do an activity in their book so that they’re writing it as well as working with it.” Teacher B (Pre-Primary) listed concrete items and added aural stimuli: “buttons, dice, counters, pebbles, beans, playdough, sounds – so hearing sounds, shakes.” A less commonly mentioned representation was the use of number lines and this increased only slightly at the final interviews from one to two teachers. Therefore despite the increased awareness of the importance of the mental number line and magnitude judgement (evident in Table 4.14 and also the wider teacher population in Table 4.5), there was no corresponding increase in references to representations that would support the development of such a structure, namely linear tracks (boards games) and number lines (whether marked or unmarked). The greatest change over the course of the intervention was to an even greater focus on concrete materials, which teachers were less likely to specify.

Teacher A (Pre-Primary) showed a more noticeable shift in her responses. In the first interview she listed singing, chanting and reading number rhymes and stories, the latter of which she often used as an introduction to craft activities focussed on numeration. At the second interview she was focussed on exploring concepts more deeply by providing students with a variety of models for each concept in order to suit the ways different students learn, and in working from concrete to abstract:

> You do have to teach a lot of concepts explicitly, pictorially, with concrete materials and later on as you get further down the track using symbols … If you do it one way with them all the time and then they go off into the home corner and yet they can’t do that, there must be something missing with their understanding of that concept if they’re not able to take it and use it somewhere else

Subsequently, teachers gave their opinions regarding the most common number difficulties students experienced. Teachers were asked to draw from their own experience in addition to professional knowledge and all gave more than one response. A summation of these responses is given in Table 4.17.

Table 4.17: Teachers' responses to the question: Based on your knowledge and experience, what are some of the common difficulties students have in learning number skills and concepts? (n = 5)

Category of response	**Frequency**	
	Initial	**Final**
Reading numbers and/or place value	3	4
1:1 counting or counting-out	3	2
Teen numbers	2	2
Counting on or back for addition/subtraction	2	2
Cardinality principle and conservation	1	2
Understanding number magnitude, using number lines for estimation	0	2
Keeping track when counting	1	0
Assessing reasonableness of answers with respect to whether the number has increased/decreased	1	0

Note. Some of the teachers provided more than one codeable response to this question

Responses were similar prior to and following the intervention with only minor changes evident. Number recognition and place value were seen as important on both occasions, as illustrated by Teacher D (Year One) who stated:

> I think it goes back to that recognising a number, a written number, I have found, they can easily count one to fifty but if you were to show them 43 they aren't able to recognise that's 43 they need to be explicitly taught what 40 looks like and the three and that 40 and three together is 43.

Teachers also described the erroneous counting behaviours of students who struggled with object counting (Teacher A, Pre-Primary): "they can count too fast and skip numbers and need to go back and recount a lot," and contrasted verbal counting with object counting (Teacher B, Pre-Primary): "most can count to 10, orally to 10 but if you ask them to count-out you don't get much more than seven." The challenges of the teen numbers were also referred to by two teachers on both occasions, e.g., "they really struggle with the teens at this age, because we say the number first 'sixteen' they all start writing the six and then the one instead of the other way around" (Teacher A). Of some note was the emergence of the idea of understanding number magnitude. Teacher A was one of two teachers who listed this concept at the final interview thus, "the number line is a huge one … knowing where the numbers belong on the number line."

Assessing and addressing student difficulties were the foci of the following questions in each interview. Teachers responded by describing the tools they used and/or the context(s) in which they commonly used these, as reported in Table 4.18.

Table 4.18: Teachers' responses to the question: How do you help students who are experiencing difficulty with number concepts? How do you pinpoint their difficulties? (*n* = 5)

Category of response	Frequency	
	Initial	Final
Observation of performance during small group tasks which are part of the regular learning program	2	2
Use tools provided in PL intervention	0	2
Individual number interviews, teacher-designed, beginning of school year	1	1
Dissatisfied with current tools	4	0
Teacher designed baseline tasks at the beginning of the year, whole group administered (mainly written)	2	0
Performance Indicators in Primary Schools testing in addition to teacher-designed tools	2	0
Concept by concept formative assessment through written worksheets, whole group administered	1	0

Note. Some of the teachers provided more than one codeable response to this question

The most common response prior to intervention was to express dissatisfaction with the tools currently being used for assessment. For example, Teacher A (Pre-Primary) expressed the need for a greater range of assessment options in saying, "I have activities that I use but I want more, I want to find other ways of doing things," whilst Teacher B (Pre-Primary) felt dissatisfied both in her tools and her level of skill in interpreting them:

> This is where I feel I'm probably weakest at, the assessment and then knowing how to pitch things at just that level of where they're at, a bit of challenge but not too much challenge … knowing where to take them next. We're doing a lot of all-together activities but not a lot of differentiated stuff yet.

Assessment methods varied from formal and teacher-designed baseline assessments, to observational assessment in the course of small group work, to the ongoing use of written or worksheet-based tests, the latter of which was described only by the teacher of the eldest class (Teacher C, Year Two). Following the intervention, change was evident in teachers referring less to 'once-off' assessment forms and more to observation in the course of instruction, most commonly in small groups, and also specifically to their use of the assessment tools provided in the professional learning intervention.

The following question was then intended to probe what influenced teachers in choosing appropriate intervention. Only one teacher directly addressed this question, with four of the five teachers instead described how they provided intervention as summarised in Table 4.19.

Table 4.19: Teachers’ responses to the question: How do you decide what intervention is appropriate? (*n* = 5)

Category of response	**Frequency**	
	Initial	**Final**
Repetition of the concept in different ways or with greater use of concrete materials	1	3
Provide extra teacher support during classroom tasks, either in small groups or through withdrawal	3	2
Present easier concepts or with smaller numbers	1	2
Use of tools provided in PL	0	1
Not confident in designing intervention pitched at the correct level for individuals	3	0
Consult colleagues and curriculum documents	1	0

Note. Some of the teachers provided more than one codeable response to this question

Providing extra teacher support for the completion of regular classroom work was the most commonly mentioned method of addressing difficulties. Teacher B (Pre-Primary) described how she accomplished this during group work: “those ones that have been struggling a lot are supported, I try to make sure that there’s somebody not necessarily directly with them but a teacher, at least, at the table, and if it is going to be an independent task try to sit them with a stronger student.” Three of the five teachers also expressed discontent with their current level of skill at making such decisions, including Teacher E (Pre-Primary) who shared, “the intention of me engaging in this was to see what I could do with my information because I target my teaching in terms of my explicit lessons but I haven’t planned intervention so far.”

Teacher C (Year Two) addressed the question directly by explaining how she would seek assistance from the teachers of previous year levels and maths specialists, and consult the *First Steps Diagnostic Map* (Willis, et al., 2004) in order to track back and find the source of difficulties in terms of earlier mathematical content. The greatest change evident was the increase in references to repetition of concepts and use of concrete materials, e.g. (Teacher C):

> Whether it is going back and using concrete materials to reinforce the basic concept if the don’t seem to have the basic concept or how do I then take them a bit further and… start to get them to apply what they’re doing so move them from concrete and get them to work without having to use those materials.

The final question in the interview asked teachers to articulate how they made judgements on the success of their intervention. Table 4.19 presents four teachers’ responses, since Teacher E (Pre-Primary) was not asked this question as she indicated that due to her lack of experience she had not yet provided any intervention that could be evaluated.

Table 4.19: Teachers' responses to the question: How do you know when you've been successful? (*n* = 4)

Category of response	**Frequency**	
	Initial	**Final**
Repeat original task after a period of instruction, observe correct responses on repeated occasions, multiple formats	2	4
Progress as visible to teacher without specialised tools – "I can see it"	1	3
Student can verbalise their thinking	1	1
Formal written test at the end of a period of instruction	1	1
Can cope with increased task demands, greater number ranges	1	0

Note. Some of the teachers provided more than one codeable response to this question

There was great variation in responses prior to the intervention, with only one category, the repetition of the original task, evident in more than one response. In another view, Teacher C (Year Two) referred to verbalisation of reasoning as one clue, by "asking them questions about it and seeing what their responses are and whether they've actually grabbed hold of the concept just by how they explain something, or explain why they're doing something."

At the final interviews there was greater agreement that correct performance on the original task on multiple occasions or in a variety of contexts constituted progress, which represented awareness of more deliberate methods of repeatedly and specifically assessing skills causing difficulty. Teacher A (Pre-Primary) described how "they can do it and they can do it again and again, they can do it in different situations - you can actually see that understanding come together." This response also exemplified another theme which represented the greatest change in responses; the ability of the teacher to 'see' when understanding had been reached.

Key Findings

8. For the majority of teachers, classroom routines were largely unaffected following the professional learning intervention.
9. Teachers were more likely to use informal methods of assessment and to "notice" advances in students' skills following the intervention.
10. A heightened awareness of the construct of a mental representation of number magnitude, and of using a number line to bridge concrete and abstract representations of number was evident following the intervention.

Teacher Reflections Emerging from the Plenary Discussions

Plenary discussions amongst the case-study teachers following each professional learning session were of duration 5-15 minutes. Teacher D (Year One) was present for two of the three sessions and plenaries but did not take an active part in discussions, and Teacher A (Pre-Primary) was absent from one workshop. The same three questions were discussed following each:

- What did you learn/gain from the session today?
- How is this relevant to your experience and your context?
- How will you apply this knowledge/tool to your context?

Audio recordings of each of the discussions were analysed and themes identified for each question in turn across the three sessions. To some degree these reflected the different skill foci of each workshop, but through comparing responses at three different time points it was possible to see changes in the nature and particularly scope of discussions.

The first question, "what did you learn/gain from the session today?" attracted responses specific to the topic of each workshop. The idea of bridging the concrete and abstract representations of number was one which resonated with the teachers, specifically the idea of using mental representations such as imaging quantities and the mental number line. This was strongest as a theme after the first professional learning session which introduced a trajectory of counting development. As Teacher D (Year One) explained, "we teach with concrete and then we teach with numbers, but we don't do this bit here [gestures]. We miss out on the abstract thought," to which Teacher A (Pre-Primary) added, "and in the Australian Curriculum, it's just not mentioned". This lead to a further discussion about when the symbols of mathematics, such as the addition symbol, should be explicitly taught and the conflicting advice that teachers had received regarding this more formal aspect of mathematics instruction.

At the second plenary, discussion focussed on the practicality of the tools presented through the workshop, and the utility of games as, "good ideas for some really simple things to do that you can actually see the kids would be improving on those skills" (Teacher C, Year Two). Teachers A, B, E (all Pre-Primary) and C (Year Two) discussed using the games to provide additional options for practising strategies and doing so more thoroughly: "really honing in on those earlier skills and working at higher-level skills within that skill range as opposed to going, 'okay, well we can do that so let's skip ahead to this,' - we're really drilling down on the foundation concepts at increasingly difficult levels" (Teacher E). Some mention was made of the teachers' professional growth, with Teacher B referring to "improving our skills" and Teacher C to using the new materials as a self-check to ensure that existing teaching practices were covering all the required skills.

Following the third session, teachers expressed interest in the concept of working memory and in returning to the classroom to investigate whether some students for whom they had not yet found the source of their learning difficulties may in fact have poor working memory. The working memory loads of literacy and maths were compared, and the question, “can the kids have a difficulty with memory in maths but not with literacy?” (Teacher A, Pre-Primary) was posed and discussed.

The strongest theme that emerged across the three sessions regarding the teachers’ own professional learning was the idea and utility of a trajectory of counting development particularly in assessing difficulties and identifying prerequisite skills (Teacher C, Year Two):

> How to assess some of those basic skills kids might have and how to understand the progression of where to go back to, and how you can bring them right back to where they need to go back to and know where they need to go back to… at least then I can find out what it is… and understand it better.

To this Teacher A added, “it helps you to target why – why is [the skill] hard?”, and Teacher E elaborated, “you can track it back to pre-requisite skills and see where some of the gaps are more obviously.” The role of tools presented in the workshop in this process was emphasised: “I think it’s really good to be able to have a trajectory, to be able to use the game to assess, and then go, ‘okay my child is here, what is next?” (Teacher E, Pre-Primary).

The contrasting of emphasis, teaching understanding and resourcing of literacy and numeracy was also a repetitive theme:

> There’s so much research and so much information in literacy difficulties about where you might go back or forward that maximises but there’s isn’t much [for maths]… when you go back you go back to one-to-one counting with counters (Teacher E, Pre-Primary)
>
> I bought, just last year, quite a few, games… not games - resources, because I felt I didn’t have enough. I had a hell of a lot of literacy stuff, but then – what do I do with it? But this has given me an idea of what to do with those concrete materials – I was looking at it saying ‘that’s really pretty but how do I actually use it?’ (Teacher A, Pre-Primary)

Teachers expressed that their participation in the professional learning intervention had gone some way to redressing this balance. The following comment from Teacher A (Pre-Primary), with which Teachers B, C and E can be heard agreeing was made following the final workshop:

> I’ve got so much more up in my classroom, I’ve looked around my classroom after this and I’ve had so much up for literacy and no maths, I have actually none… and also with my displays of the work we were doing, it was all literacy, I’ve actually got a wall now just called “number”… it’s that awareness.

In discussing the second focus question, “*how is this relevant to your experience and your context?”*, more consistent themes across the three workshops were evident. The games presented in the workshops were repeatedly described as “easy” and “fun” ways to explore concepts, and teachers discussed how they could fit these within classroom routines. Similarly, the play-based format of the tools appealed to teachers both for informal assessment: “you’ve got the way to assess, where to start them from and where to go … it's not just a worksheet after worksheet" (Teacher C, Year Two), and purposeful play-based learning (Teacher E, Pre-Primary):

> It's all still very hands-on, play-based, in our context there's so much … controversy, the definition of play-based learning just gets thrown around and isn't really that well understood, I don't think, in our context, especially by parents and they just think we're throwing it out to them by osmosis but this is really play-based learning – they're engaging in a game and they're learning.

Using the information and tools from the professional learning workshops as part of an assessment – teaching- assessment cycle and subsequently planning regular, cumulative review were themes that repeatedly emerged across the three sessions. "The games both assess, teach, and then reassess, so you've got your whole cycle there," Teacher E (Pre-Primary) remarked, and there was general agreement that the provision of different assessment guidelines for teachers, paraprofessionals and parents would make the games more usable in a busy classroom context. In discussing a regular review cycle for arithmetic strategies, Teacher C (Year Two) described her understanding thus: "not just teaching a strategy and then leaving it alone… once a week quickly review a strategy and as you get towards the end of the year you just add more and more strategies to that review time" (Teacher C).

The final focus question, *"how will you apply this knowledge/tool to your context?"* was interpreted in a progressively broader sense across the three workshops. Following the first and second workshops teachers discussed how they would use specific games presented in the workshop during rotations, as follow-up activities after seatwork, and as part of a review schedule as discussed in Question Two.

Following the first workshop, discussion began on the broader impacts of the professional learning intervention on their teaching in terms of the explicit teaching of skills and the deliberate targeting of skills both in assessment and teaching: "I think it's made it a lot more clear to me, of the specific number things to teach, really clear explicit teaching … and [in the past] it kind of goes from knowing your numbers to adding them together" (Teacher C, Year Two). Teacher E (Pre-Primary) remarked, "I feel much more organised in maths." According to Teacher B (Pre-Primary), "I think you can make your assessment a lot more targeted as well, picking out things before an assessment lesson, and I think you'll be able to cover a lot more… and have the knowledge to tell a parent 'this is what they can do'." By the third workshop these more strategic changes were the dominant theme of discussion, with the Pre-Primary teachers (Teachers A, B and E) describing changes to class routines that had already occurred. These teachers expressed that the idea of using estimation tasks and building a mental number line had impacted significantly on their classroom programs already with the creation of new classroom routines, which for Teacher B consisted of, "I bring out, at the end of lessons, whole class number lines, lining up … Friday mat times lining up the numbers with the dots.” Similarly, Teacher E described how “it’s been able to seep its way into all aspects of my program” such as through a new estimation routine:

> They love doing their number line, it’s just something I whip out every now and then when I’ve gotta go and do something or prepare a lesson or whatnot I’ll go ‘get into your buddy pairs (or whatever) and make a number line’, they know where the stuff is, they go and get their deck and their string and their pegs and they love it.

Teacher A had created a new display space and a new daily routine connected to recognising numbers and estimating their position on a number line.

Following the third session, teachers reflected on the value of engaging in professional learning whilst practicing in a classroom. Teacher C, who taught Year Two, contrasted what she had gained from the experience with the Pre-Primary Teachers’ descriptions by describing how important it was that she could relate the content to her current teaching context:

> When you’ve been in a year level … and then you come to something like this … you get a different understanding of your own lack and what knowledge you do have… the things that you guys have gained are different to the things that I’ve gained because of your own focus and what you are needing.

She later added:

> If you’d done this with me at university it wouldn’t have the same value as it has now … it depends on your own style of learning and for me it’s very hands-on, and now I see what you’re talking about … I focus on what I need for that year [Year Two], but I’ve got somewhere to go back to for maybe some Year One issues or whatever … then I’ve got a resource to go back to rather than keep all that knowledge in my head.

Teachers discussed how the use of board games to practice skills had increased the motivation of their students to engage with maths time, with Teacher A (Pre-Primary) remarking, “the kids love it, I say ‘we’re going to play maths games’ and the kids say ‘yes! We’re doing maths!’ and to hear that instead of [groans]” to which Teacher E (Pre-Primary) added, “they get excited about it now, they want to [do maths], they know it’s playing games, maths is now playing games.” Teachers were also motivated, with Teacher A asserting, “I want to go back and share this with the other Pre-Primary teachers”.

Key Findings

11. Teachers felt better informed about the sequence of development of number as a result of the PL intervention, specifically through use of a learning trajectory for teaching counting.
12. Pre-Primary teachers made changes to their classroom routines to emphasise comparing the magnitude of numbers and using number lines.
13. Teachers appreciated the informal design of the tools presented through the workshops and the effect these had on raising student enthusiasm for maths time.

14. Over the course of the three workshops teacher focus shifted from the utility of specific tools presented through the intervention to broader changes in emphasis for their maths programs.

Student Estimation Data

The tasks chosen to measure estimation accuracy involved students being shown a series of horizontal lines marked at both ends with a value to indicate the number range (0-20 or 0-100), each presented on its own sheet of paper. A number from within that range was written at the top of each sheet and students were instructed to make a mark on each line where the given number would sit. In the research design, all case-study teachers were to select approximately six low-achieving students who would complete pre- and post-intervention assessments measuring number estimation accuracy and the impact of the professional learning intervention on this skill. In reality, data from only three of these classes could be analysed (those of teachers A, B and C), with one teacher administering probes incorrectly and another misplacing a set of data. Each estimation attempt for each child was compared against the actual position of their mark and the disparity recorded as a positive (over-estimation) or negative (underestimation) value. Where students disregarded the instruction to make a vertical mark, the midpoint of their mark or numeral was used for measurement. The absolute value of this disparity was then used to calculate a percentage variance for each item and an average percentage variance for each student. Separate data tables were then created for pre- and post-intervention data sets for The Pre-Primary and Year Two students and average variance calculated for all students in each data set.

The number of over- and underestimates in each data set were then calculated and compared for each group, pre- and post-intervention to look for trends or changes. Furthermore, the incidence of under-and overestimates was analysed in terms of their frequency of occurrence at different ends of the tested number range.

Tables 4.20-4.21 summarise the data from the two Pre-Primary classes, Class A and Class B, in terms of average variance of estimates from the provided value, and the mean incidence of under- and over-estimates across each class of students for all trials. Pre-Primary students marked placement for nine numbers within the 0-20 number range only.

Table 4.20: Year PP Class A: Mean per cent variance of student estimates and mean numbers of under and overestimates (*n* = 5)

Range of numbers and mean scores for Class A	**Time of testing**	
	Pre-test	**Post-test**
Range 0-20		
Mean per cent variance of estimates [a]	23	10
Mean number of underestimates (/9)	4	6
Mean number of overestimates (/9)	5	3

Note [a]. Mean variances of estimates were calculated by treating all variances as positive values

Table 4.21: Year PP Class B: Mean per cent variance of student estimates and mean numbers of under and overestimates (*n* = 6)

Range of numbers and mean scores for Class B	**Time of testing**	
	Pre-test	**Post-test**
Range 0-20		
Mean per cent variance of estimates [a]	20	16
Mean number of underestimates (/9)	6	5
Mean number of overestimates (/9)	3	4

Note [a]. Mean variances of estimates were calculated by treating all variances as positive values

Estimation accuracy increased in both PP groups, but substantially more in Class A (percentage variance reduced from 23% to 10%) than Class B (reduced from 20% to 16%). In Class A students were more likely to overestimate than underestimate on the initial assessment, but this pattern reversed at the final assessment. In Class B, students were more likely to underestimate on both occasions.

Data from six students in the Year Two class, Class C, is reported in Table 4.22 for the number ranges 0-20 and 0-100 which were presented as separate tasks. Nine and 10 values were provided for placement in the 0-20 and 0-100 tasks respectively.

Table 4.22: Year Two Class C: Mean per cent variance of student estimates and mean numbers of under and overestimates (*n* = 6)

Range of numbers and mean scores for Class C	**Time of testing**	
	Pre-test	**Post-test**
Range 0-20		
Mean per cent variance of estimates [a]	12	6
Mean number of underestimates (/9)	8	7

Mean number of overestimates (/9)	1	2
Range 0-100		
Mean per cent variance of estimates [a]	12	14
Mean number of underestimates (/10)	3	2
Mean number of overestimates (/10)	7	8

Note [a]. Mean variances of estimates were calculated by treating all variances as positive values

Estimation variance was equal (12%) at the first assessment on both the 0-20 and 0-100 scales. However, at the final assessment this had halved on the 0-20 scale but increased marginally on the 0-100 scale. When using the 0-20 scale, students were far more likely to underestimate than overestimate on both occasions (ratio 8:1 initially and 7:2 following intervention). More detailed interrogation of the 0-20 data showed that all of the five overestimates across the initial testing sample were of numbers exceeding 10. Overestimates at the final assessment numbered 10 of which 5 were of items exceeding 10. If overestimates for which the error was less than 0.5 were excluded, four of these five remaining estimates of numbers were in excess of 10.

This pattern was reversed for the 0-100 scale, with students far more likely to overestimate the position of numbers. On closer examination it could be seen that there was a strong trend for higher numbers to be underestimated, which further strengthened following intervention. Fourteen of the 19 (74%) underestimates at the first assessment and 12 of the 14 (86%) underestimates at the final assessment were of numbers above 50. Only 18 of the 60 items (30%) over all six tests fell within this range.

Key Findings

15. On average, student accuracy on the estimation task improved over the course of the intervention in the 0-20 number range but not in the 0-100 number range.
16. No clear pattern was evident in over- and underestimation of numbers in the 0-20 range by Pre-Primary students.
17. For Year Two students tested in both number ranges, the pattern of under- and overestimates differed between the number ranges. Year Two students were likely to overestimate higher numbers in the range 0-20 and underestimate higher numbers in the range 0-100.

Summary

This chapter reports findings from four data sources prior to and following a professional learning intervention: teacher surveys completed by 13 teachers; individual interviews and plenary discussions in which five of these teachers also participated; and, student estimation assessments which were completed by five to six lower-performing students in each of three of the case-study teachers' classes. Teachers reported highly significant positive effects on both their confidence and self-efficacy beliefs about teaching number following the intervention. Two key ideas from the intervention had the greatest impact on teacher pedagogical knowledge, one of which was the explanation of a learning trajectory for counting which supported teachers in identifying the existence of pre-requisite skills and gave them greater confidence in planning intervention for struggling students. The construct of the mental number line also resonated strongly with teachers and impacted the practice of Pre-Primary teachers in particular. On average, student estimation skills improved in the range 0-20 over the three months between initial and final tests in both Pre-Primary and Year Two students, but not in the range 0-100.

Although teachers differed greatly in the extent to which they modified their classroom programs as a result of the intervention, overall the teachers in the study reported that their participation in the project had a positive effect on their professional knowledge and practice. The study raises important questions and issues regarding the use of teacher professional learning to assist in the prevention of mathematical learning difficulties which will be discussed in the next chapter.

CHAPTER 5: DISCUSSION

This study investigated the impact of three two-hour professional learning sessions on the ability of teachers to recognise specific mathematics difficulties and plan appropriate interventions, and whether any corresponding improvement would be evident in student estimation skills. Impacts on teachers' confidence, self-efficacy beliefs and teacher practice were also examined. Teachers were supplied with a learning trajectory for counting relevant to years Kindergarten through Year Two, assessment tools and focussed teaching activities.

The first research question comprised three parts: to establish if the professional learning intervention had an effect on teacher confidence, self-efficacy beliefs and Pedagogical Content Knowledge. The teacher survey and case-study teacher interview data confirmed a positive effect on first two aspects and revealed two important themes for the third: the value of understanding and using a learning trajectory for counting, and an increasing awareness of the need to build mental representations of number magnitude. The discussion will make reference to the Key Findings (KF) presented and numbered in Chapter 4.

Improvements in Teacher Self-Efficacy and Confidence

Previous studies have established that teachers lack confidence in teaching mathematics (J. Lee, 2005, 2010; Todd Brown, 2005), and data collected in this study prior to intervention concurred with these findings. Although the mathematics content taught in early childhood classes (typically including five to seven-year-old students) could be described as simple, teacher confidence for teaching mathematics was significantly lower at the beginning of the project than confidence for teaching language. This concurred with the findings of Lee. This pattern was also reflected in teacher perceptions of their efficacy in both subject areas.

The intervention significantly increased teachers' feelings of confidence and self-efficacy for teaching mathematics, to the point where these were commensurate with their beliefs about teaching language (KF 4). The comparison between literacy and numeracy was a recurrent theme discussed by case-study teachers, both with regard to the disparate amount of knowledge teachers felt they had and the tools and resources to which they had access in each subject area, specifically in the area of early intervention to prevent difficulties. Teachers felt that their participation in the intervention had assisted in redressing this balance, and this was reflected in their confidence and self-efficacy data post-intervention.

At the beginning of the project teachers generally felt most confident and efficacious in the management of planning and teaching at the whole-class level, and their confidence in this area continued to grow over the course of the project. Interesting patterns were observed in responses to the self-efficacy scale items, which represented the practices of highly effective teachers and hence are likely to have had a greater influence on the learning outcomes of students. The three items on this scale that referred to “my rich knowledge of mathematics” were among those at which teachers rated themselves least efficacious prior to the intervention. All three of these items showed statistically significant growth over the course of the project, as did teachers’ personal awareness of and ability to communicate the explicit purpose of lessons to students. Consequently, it is likely that teachers’ perception of their own growth in professional knowledge made a considerable contribution to their increase in feelings of efficacy.

Although teachers were most confident in whole-class strategies following the intervention (KF 1), they experienced the greatest growth in confidence in aspects of mathematics instruction associated with learning difficulties (KF 2), consistent with the focus of the professional learning workshops. The TEBS-Self scale on which many of the items were based includes five statements related to accommodating individual differences (Dellinger, et al., 2008), one of which was included in the current study. The growth in confidence on items seven and nine on the confidence scale was highly significant ($p<.01$). Item seven related to *accommodating individual differences* in planning and conducting number activities and was based upon item one from TEBS-Self, whilst item nine was specifically written to reflect the focus of the professional learning intervention on *deliberate planning* for students with mathematical difficulties based on specific assessment data. Dellinger (2001, cited in Dellinger, et al., 2008) has previously demonstrated that teachers who rate themselves more highly in accommodating individual differences also consider themselves better able to teach students of low ability. Therefore the substantial growth in confidence on both items indicates the professional learning intervention achieved its aim in this regard.

There are a number of possible explanations for the highly significant overall increase in confidence and self-efficacy beliefs demonstrated by the teachers in this study. Previous research has identified mastery experiences and teaching resources as being significant in predicting the self-efficacy beliefs of experienced and novice teachers respectively (Klassen, et al., 2009; Mohamadi & Asadzadeh, 2012). It was postulated that since the professional learning intervention also provided a number of tools with which teachers could both assess and address mathematical learning difficulties, these should help both novice and experienced teachers to have successful teaching episodes and hence lead to an increase in self-efficacy. Four of the 13 teachers referred directly to the tools provided in the workshops as representing their most significant gain over the course of the project, indicating this may indeed have been the case.

Teachers also reported that their students were more motivated to participate in mathematics lessons as a result of introducing games provided through the intervention. This reported increase in student engagement could also have contributed to higher self-efficacy, as reported by Guo and colleagues (Guo, et al., 2011). Furthermore, two of the case study teachers shared that they were in their first year of teaching, a time normally associated with a significant decline in self-efficacy beliefs (Hoy & Spero, 2005). The fact that these teachers experienced an increase in confidence and self-efficacy beliefs over the period of the intervention is encouraging.

The data indicated that the professional learning intervention had a highly significant positive effect on teachers' self-efficacy beliefs and confidence regarding the teaching of number and mathematics. This effect was most pronounced in the area of identifying, planning for and accommodating mathematical difficulties in the classroom, reflecting the specific focus of the workshops. An increase in professional knowledge for mathematics appears to be a likely source of this growth, as does increased student engagement and successful teaching episodes through the provision of tools for classroom use.

Increases in Pedagogical Content Knowledge for Number

The first research question also sought to identify if participation in the professional learning intervention had an influence on Pedagogical Content Knowledge for number. This is a key question since teachers contribute significantly to student achievement outcomes, especially in the mathematics learning area (Hattie, 2009). On the whole, teacher surveys revealed that increases in professional knowledge were the most frequently reported outcome of teachers' participation. This outcome was in contrast to expectations where teachers focussed on their improving assessment practices, planning and teaching for students with maths difficulties. Data from teacher surveys, plenary discussions and case-study interviews revealed that this growth in professional knowledge occurred in two primary fields: understanding and using a learning trajectory and building mental representations of number magnitude.

Understanding and using a learning trajectory

In describing her successful early intervention program *Number Worlds*, Sharon Griffin (Griffin, 2004a) emphasised the importance of increasing teachers' understanding of number development as a crucial focus for future research. Teachers involved in this project explained that the professional learning intervention had gone some way towards developing this knowledge, with the first obvious shift in Pedagogical Content Knowledge related to the understanding and use of a learning trajectory in counting. This trajectory made teachers aware of the prerequisite skills for common areas of difficulty and resulted in them feeling more able to track forwards and backwards in planning teaching and intervention (KF 7). This increase in confidence was reflected by case-study teachers during interview and perhaps was the source of the highly significant increase in confidence in diagnosing levels of number skill and planning interventions amongst the wider sample of teachers.

An understanding of mathematical development is particularly valuable when planning intervention considering that instruction targeted specifically at the point of need is most effective (Dowker, 2004). Several programs and documents which are based on an understanding of the developmental sequence for number learning are currently being utilised in Australia, such as the *First Steps Map of Development* (Willis, et al., 2004), *Count Me In Too* resource (State of New South Wales through the Department of Education and Training, 2009) and *Early Numeracy Research Project Growth Points* (D. Clarke, et al., 2002), however, only one of the five case-study teachers referred to any of these at interview, and she did so in the context of planning an intervention. When describing a learning trajectory for counting in this project, specific reference was made to those points along the path which corresponded to skills predictive of mathematical success as identified in the research which rendered its focus more specific than others in current use.

Teachers expressed their enthusiasm for the use of the learning trajectory and four of the five case-study teachers described their classroom programs as better sequenced as a direct result of its use. Teacher C explained that although she has not absorbed all of the content that was not directly applicable to her as a Year Two teacher, she was pleased that she could use the trajectory as a reference should she need to track back skills for a student experiencing difficulty in the future. Although the learning trajectory was used as a context for understanding the role of each of the predictive skills, it was clear that many of the case-study teachers saw its value in more general applications to planning sequenced instruction and tracking back to possible sources of difficulty. However, this was not reflected in survey data from the wider sample of teachers, where the item describing confidence in the sequencing of the mathematical learning program was one of few items on the confidence scale which increased to a non-significant degree. This could be explained by the timing of the second survey, which occurred immediately following the last professional learning session when teachers had not had sufficient time to reflect upon and apply the notion of a learning trajectory to their classroom planning. Case-study interviews occurred several weeks later, following a school break in which teachers could have more carefully considered its application to their planning and then some weeks of instruction. It is also possible that more detailed knowledge about the sequence of mathematics development caused teachers to reflect on the shortcomings of their existing programs and see them in a less favourable light.

Teachers also demonstrated their increased understanding of the sequence of mathematical development through changes in their responses to some short answer questions which dealt directly with sequencing of learning experiences. For example, when addressing the issue of whether a number line should be used to teach counting principles, teachers became increasingly aware that concrete materials were a more appropriate model to teach such a concept, and that number lines were too abstract. Similarly, references to prerequisite skills increased on all of the survey short-answer questions that dealt with hypothetical classroom teaching challenges, in preference to previous responses which had favoured re-teaching the deficient skill or teaching it in a more direct manner. This crucial awareness of the need to accurately identify the point of need should assist teachers in planning appropriate intervention in real situations.

In summary, teachers described their instructional programs as being better sequenced as a result of the number learning trajectory discussed during the intervention. This finding was evident in teachers' responses to challenging classroom scenarios in which they showed greater awareness of the need for teaching prerequisite skills in preference to reteaching the target skills with which students are struggling.

Mental representations of number magnitude

Brown (2005) determined that teachers differ widely in their interpretations of appropriate content and methods for teaching mathematics, and this was reflected in the findings of the current study where a wide variety of skills were listed as most important in ensuring mathematical success. However, the teachers in this study shared wide consensus about how mathematics should be taught: in a hands-on fashion with concrete materials, using a balance of focussed teaching and embedded experiences.

Although the intervention made little difference to this consensus view of how mathematics should be taught, the most notable change in teacher Pedagogical Content Knowledge related to *what* should be taught, and specifically to the concept of establishing a mental representation of number magnitude (KF 6 and KF 10). The teachers themselves identified that they had previously been unaware of the need to bridge work with concrete materials and abstract symbolic work, and this was reflected in many responses which named working with concrete materials as the main pre-requisite and solution for students who struggled with abstract sums or more efficient calculation strategies. Following the intervention there was a substantial shift towards recognising of the importance of building mental representations of number magnitude, a construct demonstrated to correlate with mathematical achievement and indeed enable the acquisition of new arithmetic skills (Booth & Siegler, 2008; Laski & Siegler, 2007; Siegler & Booth, 2004; Siegler & Ramani, 2009).

Whilst some teachers referred directly to the mental number line during surveys and interviews, for others this change was reflected more in descriptions of practice than precise vocabulary. For example, one question on the teacher survey asked how teachers could ascertain if students were ready to move beyond counting concrete collections to learning abstract number combination; the mathematics of number as identified by Resnick (1992). Initial responses favoured the solving of word problems based on concrete objects in a 'try it and see' approach, and focussed on the teaching of numerals and number sentences. These responses reflect the belief that the next level of abstraction from counting collections of concrete objects is to contextualise this counting and use numerals. Following the intervention, the most common response was to move to solving problems using a number line, a tool closely aligned with a mental representation of number magnitude rather than quantity. Following the intervention, teachers were more able to discriminate between practices that would contextualise concrete mathematics and facilitate mental mathematics; perhaps in recognition that the mathematics of numbers is something that occurs in the head and not solely on paper.

Interestingly, this awareness of the mental number line was not reflected in teachers' responses when asked about the common difficulties students experience in learning mathematics, possibly since this represents an underlying difficulty which would not have been obvious without the use of specifically chosen tasks such as magnitude comparison and estimation. Similarly, board games or tracks were not mentioned prominently under key representations for teaching number concepts, even following intervention. This is disappointing since it has been demonstrated that the use of both physical and visual representations enhances mathematics achievement (Fuchs & Fuchs, 2001; Gersten, Chard, et al., 2009; Gersten, et al., 2008). Whilst teachers were acutely aware of the need for concrete materials (a physical representation) there was less attention to visual ones, although references to representations aligned with the mental number line such as number lines did increase in prominence. One explanation for this is that although teachers had gained awareness of the mental number line, they had not yet fully integrated this understanding with their existing teaching practices in order to consider what teaching tools would best support such a representation. Furthermore, it is likely that by trying to address several goals in one package, some other components of the intervention, namely the importance of board games to build number magnitude, missed their mark. This is discussed further in the following section.

The professional learning intervention seemed to deliver this one key message regarding what should be taught, that is, the importance of building mental representations of number. The interviews with case-study teachers revealed that this message resonated most strongly with Pre-Primary teachers. Teachers of older students (Teachers C and D of Years Two and One respectively) did not appear to find this message of great relevance to their contexts, which is concerning since Booth and Siegler (2004) have demonstrated that the use of an accurate and linear representation of number magnitude is related to mathematics achievement well beyond the Pre-Primary year and into Years One and Two. Future professional learning interventions must therefore also concentrate on conveying the importance of mental representations of number magnitude in higher ranges.

As a result of the intervention teachers therefore showed greater Pedagogical Content Knowledge regarding the need to build mental representations of number, and the awareness of tools which could support this construct. Unfortunately, the use of board games as a tool was largely overlooked despite being a central design feature in the vast majority of the tools provided through the intervention. This may not reflect a lack of use of board games, but perhaps as an enduring classification of 'games' and 'play' rather than a representation of number in their own right.

The second research question referred to the utility of using professional learning interventions to influence classroom practice. Semi-structured interviews with case-study teachers and plenary discussions provided data in support of the themes that larger shifts in Pedagogical Content Knowledge were associated with greater changes in practice, and that assessment practices became less formalised following the intervention.

Effects on Classroom Practice were Mediated by Changes in PCK

Interviews with case study teachers revealed that classroom routines remained largely unaffected by the professional learning intervention (KF 8). All teachers scheduled specific time for maths teaching including number concepts, and the frequency or duration of this remained constant as did the mix of whole class and small group instructional groupings. Similarly, teachers viewed focussed teaching and embedded experience as equally important and serving a complementary role both prior to and following the intervention (KF 5).

The intervention tools provided were designed to be used with small groups of students who were identified as 'at-risk' through assessment, commensurate with the instructional design of other successful second-tier interventions (e.g. Daly, et al., 1997; Gervasoni, 2003; Pearn, 1999). However, teachers within this study were more likely to use the tools within regular instruction, perhaps in the context of class-wide small group work. As anticipated, teachers implemented these tools without additional staffing which may have necessitated such a shift, although one teacher did secure the regular help of parents. Since at least some of the predictive skills being targeted were unfamiliar to the teachers or received little direct emphasis in existing programs, conscious decisions may also have been made that all children would benefit from exploration of the concepts and use of the games. The aforementioned mental number line is one such concept. This preventative approach has been used successfully in other interventions (Griffin, 2004a; Starkey, et al., 2004) where entire populations of students were identified as being at-risk of mathematics failure due to socioeconomic circumstances rather than mathematical skills.

However, there was a high degree of inconsistency with regard to the effects on classroom practice within existing routines. Whilst teachers A, B, E (Pre-Primary) and C (Year Two) expressed their enthusiasm at each plenary session and appeared inspired to make changes, such inspiration did not necessarily translate to changes in practice once this immediate effect had worn off. Teacher E communicated that she had transformed her entire maths instructional time by grouping students and planning experiences largely on the basis of assessment conducted through the professional learning intervention, and these changes extended to both rotational activities and whole-class experiences. Teacher A created a new morning mathematics routine and integrated tools for them intervention into her regular mathematics rotations, while Teacher E translated ideas from the intervention into whole class discussions. In contrast, Teachers C and D (Year Two and Year One respectively) made few changes, utilising the provided tools as a supplementary resource if they fitted with planned lessons and concepts, indicating that the tools had been fitted into existing schema for teaching number. Therefore, whilst all teachers described changes in their Pedagogical Content Knowledge and considered broader changes to mathematics programs as the intervention progressed (KF 14), it was the teachers who described the greatest changes in PCK and reflected on these changes in relation to their existing practices who made the most substantial changes to classroom practices, and indeed used their knowledge to create new routines which better suited their teaching contexts. These three teachers were all situated in Pre-Primary contexts which may have featured more flexible learning environments, making such changes more easily accomplished. This is an interesting finding since whilst the most specific interventions concerning the use of linear board games have been demonstrated to transfer to classroom contexts and small groups (Ramani, Siegler, & Hitti, 2012), such research to date has not tested whether teachers continue to use them beyond the scope of the research project without concerted attempts to change thinking. The findings of this research project are in concord with the assertion by Timperley and colleagues (Timperley, Wilson, Barrar, & Fung, 2007) that the most effective professional learning challenges teachers existing perceptions about mathematics teaching. Hence, the fact that the most commonly reported outcome of the 12 teachers involved in the project was an increase in professional knowledge is encouraging as this may lead to changes in practice.

These findings have implications for professional learning interventions in an environment where educators are constantly subject to polished sales presentations to promote resources, often of dubious value, which may or may not have any basis in educational research. Teachers feel most able to provide for the needs of students with mathematical difficulties by working within existing classroom routines and groupings. Furthermore, providing resources without a corresponding shift in thinking, results in such resources being used unsystematically and inconsistently. Based on the findings of this project, it is necessary not only to inspire teachers with tools and resources, but also to challenge the way they think about mathematics development and encourage active reflection about existing practices. The two teachers who made the greatest changes were heavily influenced by the concept of developing a mental number line, and it appears that it was this single idea which had the greatest impact on practice (KF 12).

Case study teachers described a number of barriers to changing practice, including the need for shared resources due to collaborative teaching approaches with colleagues. Some of these collaborations may have left teachers feeling beholden to established programs, to shared planning which necessitated up-skilling and providing resources for colleagues. A collaborative teaching culture has been identified as a positive predictor of teacher self-efficacy beliefs (Guo, et al., 2011), so the idea of collaborative teaching cultures being a barrier to classroom change is troubling. One solution to this problem would be for teachers to attend professional learning with all such colleagues: indeed, this was the recommendation of Clarke and colleagues (D. Clarke, et al., 2002) who recommended the establishment of Pre-Primary to Year Two professional learning teams who met regularly for professional discourse and underwent professional learning as a team.

Another challenge described was that of 'information overload' following the professional learning intervention. Indeed, Teachers C and D who made the fewest changes to their programs described the latter problem, and this idea of not knowing where to start lead to limited use of the tools. Conversely, Teachers A and E were able to extract one key idea from the intervention and used this idea to inform a course of action. Hence, it is likely that the intervention simply provided too many competing ideas in too short a time frame for all participants to frame a clear course of action. A more simplified approach which concentrated on communicating only one key idea within the context of a learning trajectory, and then provided a small number of resources to exemplify this idea, may have proved more effective in influencing not only beliefs and knowledge, but also practice. This may strike the balance between planning experiences which are simple enough to be implemented by teachers and paraprofessionals without ongoing direction (e.g. Ramani, et al., 2012), and providing more intensive and holistic learning supported by ongoing professional development and detailed implementation instructions (e.g. Clements & Sarama, 2008; Griffin, 2004a; Jordan, et al., 2012; Starkey, et al., 2004).

The results of this study indicate that efforts to meaningfully change the practice of classroom teachers are most successful if accompanied by corresponding shifts in thinking. These shifts are facilitated by a narrow focus which aims to deliver one key message for which a number of supportive tools are provided. As Griffin (2004b) noted, "even the best curricula provide nothing more than a set of tools … it is the teacher in the classroom who brings these tools to life, who decides how and when to use them, and for what purpose" (p.179). This research project demonstrated that when teachers are given such freedom, there is a high degree of inconsistency regarding to what extent and how purposefully the tools are used. Interventions which can fit within existing routines, namely whole-class mat session and class-wide small group work, appear to be most workable for teachers and most likely to be implemented. Furthermore, teachers should attend such professional development with colleagues to enable implementation in collaborative panning and/or teaching contexts.

Less Formal and More Integrated Methods of Assessment

Accurate assessment of the point of need is crucial to determining appropriate intervention for students with mathematics difficulties (Dowker, 2004). Whilst the professional learning intervention provided specific tools for assessment, it was also hoped that teachers would also become better able to ‘notice’ students’ skills and strategies within the context of classroom experiences. Anecdotal evidence indicates that many teachers find the high volume of information provided by one-to-one interviews ungainly and difficult to interpret in terms of where instructional priorities should focus, possibly due to a lack of Pedagogical Content Knowledge. Information that could be gathered in the course of instruction could complement such interviews and be more timely, more dynamic could provide feedback on the effectiveness of instruction, provided that teachers are aware of how to act on such evidence (Callingham, 2008).

Since most teachers referred to using teacher-designed methods of assessment, whether presented as assessment interviews or assessment in the course of instruction, the need for strong Pedagogical Content Knowledge to support the design of such tasks is apparent. Whilst two of the case-study teachers referred to the *Performance Indicators in Primary Schools* (PIPS) assessment (designed by the Curriculum, Evaluation and Management Centre at the University of Durham (UK) and administered in Australia by the University of Western Australia) as mandated by their schools, neither indicated they found this information particularly useful for teaching. Likewise, four of the five case-study teachers seemed to rely solely on professional judgement in deciding what intervention was appropriate for struggling students. As previously noted, very little reference was made to the use of established tools or learning trajectories in making such decisions. This is concerning considering that prior to the intervention, teachers generally considered both their confidence in teaching and professional knowledge concerning number to be a little better than adequate (average rating for both items around six out of 10). It is encouraging, then, that teachers subsequently grew in confidence in their own professional knowledge. This was likely supported by the number learning trajectory supplied through the intervention, which was identified as useful in tracking back prerequisite skills to identify sources of difficulty.

The increased emphasis on assessing competence on multiple occasions and in different contexts was confirmed by survey data indicating teachers felt more efficacious in planning for students to use and connect multiple representations; a key strategy of successful teachers of mathematics (McDonough & Clarke, 2003). However, the latter was one of the few self-efficacy items for which the increase did not reach statistical significance and case-study teachers’ responses at initial and final interviews regarding appropriate representations for number barely differed. It is also likely that since the intervention was conducted during the first term of school, a natural shift from diagnostic interviews to contextual assessment was to be expected to some degree.

Changes to assessment practices appeared subtle; although all case-study teachers indicated they had used some or all of the provided assessment tools only two of the five referred to these directly following the intervention. This is appropriate since the assessment provided was based on the indicators of mathematical difficulty rather than diagnostic tasks that could provide rich data for teaching. The skills were described as providing ‘red flags’ indicating students about whose mathematical skills teachers should seek to uncover more detail. Of concern, therefore, is that Teacher E referred to the provided tools as “all I use now”, having seemingly dismissed the need for her earlier battery of counting tasks. Individual interviews in number provide much richer information for teaching than brief screening measures and although it may be impractical to conduct interviews with every student regularly, screening measures should be seen as a means to identify students for whom more information is needed (Gersten, Clarke, Haymond, & Jordan, 2011). It should be noted here that since the participants in this study were drawn from independent schools, they did not have access to the online interview currently mandated as an on-entry assessment interview in Department of Education Schools (Department of Education Western Australia, 2010).

The dramatic shift in practice of Teacher E demonstrates the high degree to which teachers can be influenced and underscores the need for rigorous, research-based professional learning for teachers. Assessment tools which focus on a select number of mathematical skills for their predictive value also need to be seen in the context of a wider body of knowledge to ensure that ’teaching to the test’ is not a consequence of their use. Teachers need a high level of Pedagogical Content Knowledge that will equip them to understand the role of such skills and make informed judgements about how (or whether) to integrate new resources into their practice. Another factor potentially influencing the large shifts in practice made by Teacher E, is the existing professional relationship between the Teacher and Researcher which may have engendered a stronger commitment to the professional learning compared to the other teachers.

Improved Numerical Magnitude Estimation

Previous studies have established a link between interventions incorporating linear board games and improvements in number sense (Griffin, 2004a; Jordan, et al., 2012) and numerical estimation specifically, both in regard to the accuracy of estimates and the extent to which they comply with a linear representation of number magnitude. In interventions with a sole focus on board games, these improvements have been shown to be possible in a variety of contexts including one-to-one intervention administered by researchers (Ramani & Siegler, 2008; Siegler & Ramani, 2009) and in small groups led by paraprofessionals (Ramani, et al., 2012).

Although there were many aspects of the professional learning intervention that were intended to improve student achievement by influencing teacher practice, it was the integration of linear board games into the small group intervention activities which was posited to have the most direct impact on students' estimation accuracy. However, in contrast to the aforementioned studies which featured researchers or assistants trained directly by researchers to administer specific games with a high degree of fidelity, this study relied on teachers to make judgements about how to use such tools; with which children and how often. Hence there was a high degree of inconsistency regarding how board games were implemented. Furthermore, due to the small number of children in the study the results should be interpreted with caution.

The available data were consistent with previous findings in revealing that student estimation did improve in the 0-20 range for all three classes of students (KF 15); although only the two Pre-Primary classes (A and B) were provided tools which targeted this number range as part of the intervention. In analysing the little student information that was collected from these classes, it was not considered appropriate to generate linear or logarithmic fit lines for the data: indeed, no pattern was evident in under- and overestimates in any case (KF 16). However, a substantial difference was evident between the gain in estimation made by the Pre-Primary classes A and B. The average error in estimation of Class A was reduced markedly from 23% to 10%, whilst Class B, over the same period, reduced from 20% to 16%. This is interesting considering both teachers described changes to their whole-class routines as a result of the professional learning intervention, specifically targeting the use of number lines and supporting the construction of a mental number line. However, Teacher A created a daily classroom routine which closely mirrored the assessment task by judging the position of numbers on a string labelled at both ends, whereas Teacher B referred to the sequencing of counting numbers which may not have involved the judgement of relative magnitude to the same degree.

The high degree of similarity between the teaching and assessment contexts raises questions about 'teaching to the test', a common concern in education. However, the initial goal of Griffin, Case and Siegler (1994) in designing the *Rightstart* curriculum, which trialled the use of board games, was to examine if the central conceptual structure of the mental number line could indeed be taught. Research has established that the use of teaching materials that closely align with the mental representation which is the focus of instruction are most effective. This theory, variously described as 'representational congruence' (Griffin & Case, 1996) and more recently 'the cognitive alignment framework' (Laski & Siegler, 2014), has been supported by the superior numerical estimation skills of students trained using linear spatial representations of numbers (i.e. numbered board games) over non-spatial ones including Laski and Siegler's (2014) numbered cards (2014) and Siegler and Ramani's (2009) numbered circular board games. Teacher B's use of numbered cards for sequencing may have been an approximation of Laski and Siegler's number sequencing activity, which improved number recognition but not estimation skills. Teacher A's daily routine included practise of the assessment task which could be construed as 'teaching to the test.' This task involved the regular application of number magnitude encoding, similar to the intent of the board game intervention. Had this encoding not been taking place, students should not have been successful on the task regardless of their level of practice.

In addition to the use of different daily routines, perhaps more significantly, Teacher A had instigated regular use of board games in small groups, and Teacher B had not. Teacher B indicated that she had used the games, but not in any regular or systematic fashion as yet. Teacher B's description of her teaching priorities would indicate that her programs remained focussed on more 'traditional' Pre-Primary number curriculum such as verbal counting, counting collections and number recognition. The substantial improvement in the estimation skills of Teacher A's students appears to support the findings of Siegler and Ramani (2009) who established the superiority of approaches involving linear representations of number. This is significant because such research has not only demonstrated that estimation accuracy and linearity is correlated to mathematics achievement in general (Siegler & Booth, 2004), but also enhances the immediate capacity to learn other arithmetic skills (Siegler & Ramani).

Whilst previous work by Ramani and Siegler (Ramani & Siegler, 2008; Ramani, et al., 2012; Siegler & Ramani, 2009) had established the value of board games in the range 1-10, the current project also provides support for a new area of study which is establishing the utility of board games focussed on higher ranges (Laski & Siegler, 2014), since both the assessment and intervention tools administered by Teacher A were in the range 0-20. The failure of the Year Two students in the study to show any measurable improvement on 0-100 estimation task could be explained by the minimal changes Teacher C made to her classroom program, including the lack of any systematic use of board games; hence, the study was unable to ascertain if any impact on estimation would occur in this number range had such changes been implemented. The consistency of performance on the 0-100 estimation task over the period of intervention was also evident in students' propensity to overestimate the position of lower numbers in the tested range (KF 17) which was consistent with a logarithmic representation of number in the range 0-100 which has values perceived as closer together as their magnitudes increase. The persistence of this less mature representation is unsurprising since the six low-performing students concerned could have been expected to be amongst the 26% of second-graders that Booth and Siegler (2006) observed have retained logarithmic representations of number magnitude in this range.

With regard to data in the 0-20 range, one further interpretation of the substantially larger gains evident in Teacher A's students (as compared with Teacher B) is that the use of small group work, specifically focussed on board games, may add value to classroom programs over and above any gains that can be achieved through apparently equivalent whole-class work. As such this could be an indispensable feature of classrooms intended to reduce risk of mathematical failure. Indeed, even successful early childhood number sense curriculums which target whole classes of students often employ small group instruction as a primary feature of their design (e.g. Griffin, 2004a; Starkey, et al., 2004).

A limitation of the study was that, in the absence of a control group, it is not possible to conclude to what extent changes in student estimation were the result of the previously established effect of maturation (Booth & Siegler, 2006) as opposed to changes in teacher practice. However, the available student estimation data does enable the comparison of groups instructed by teachers differentially affected by the intervention. The conclusion drawn is thus that the use of instructional strategies focussed on establishing a linear representation of number, including the regular use of board games in small group contexts, has a positive effect on the accuracy of student estimation skills especially in Pre-Primary contexts.

Summary

The professional learning intervention was successful in increasing teacher confidence and self-efficacy for teaching number, and specifically for identifying and addressing the needs of students with difficulties in this area. Understanding a learning trajectory in number resulted in teachers having greater awareness of prerequisite skills and feeling more confident in the sequencing of their learning program. The most influential factor in determining whether teachers made meaningful changes to practice, however, appeared to be whether the impact on their Pedagogical Content Knowledge (PCK) resulted in a shift in thinking. The “new” knowledge acquired by Pre-Primary teachers in describing a transition phase of mental representation of number magnitude, between concrete quantities and symbolic number work, appeared to be significant in influencing the extent to which teachers changed classroom practices. The creation of this new ’schema’ for understanding number development was associated with strategic changes within classroom programs rather than as an exercise in collecting additional teaching resources.

Figure 2 represents how this process occurred. The study was influenced by the educational theories of constructivism, sociocultural theory and hierarchic interactionalism, which resulted in the design and delivery of a professional learning intervention focussed on supporting students to construct knowledge about numbers in a social context within the framework of a hypothetical learning trajectory. The pathway which follows reflects that taken by teachers who made significant changes to practice, for example teachers A and E. The professional learning intervention had the effect of raising teacher PCK which in turn had effects on confidence and self-efficacy beliefs. The result of this was the institution of more effective assessment and teaching practices (to varying degrees) which then impacted on student learning. Student learning is shown central to the learning environment, with the interaction between student learning and learning environment mediated by student motivation and engagement, which if attained not only enables greater learning but also facilitates mastery experiences for the teacher which combine with growing PCK to increase teacher confidence and self-efficacy beliefs and thereby strengthen the cycle of teacher growth and student learning.

The process of cyclic concretisation is shown at the core of student learning where concrete representations of number facilitate the building of number concepts, most notably a mental representation of number. The development of number as an object of thought is supported by the use of representations in the instructional environment which mirror the mental number line, for example linear tracks and number lines. This mental representation then enables the development of efficient calculation skills which are first understood through the use of concrete materials which enable growing sophistication of number as a concept in a continual cycle. Discussion with teachers and peers facilitates the continual development of such strategies and is influenced by teachers intentionally planning opportunities for students to articulate and discuss thinking, including through targeted small group work which is informed by understanding of a hypothetical learning trajectory in number as postulated by hierarchic interactionalism (Sarama & Clements, 2009). This understanding also results in more careful sequencing of learning experiences to ensure strategies and representations used in instruction are closely aligned with students' current thinking.

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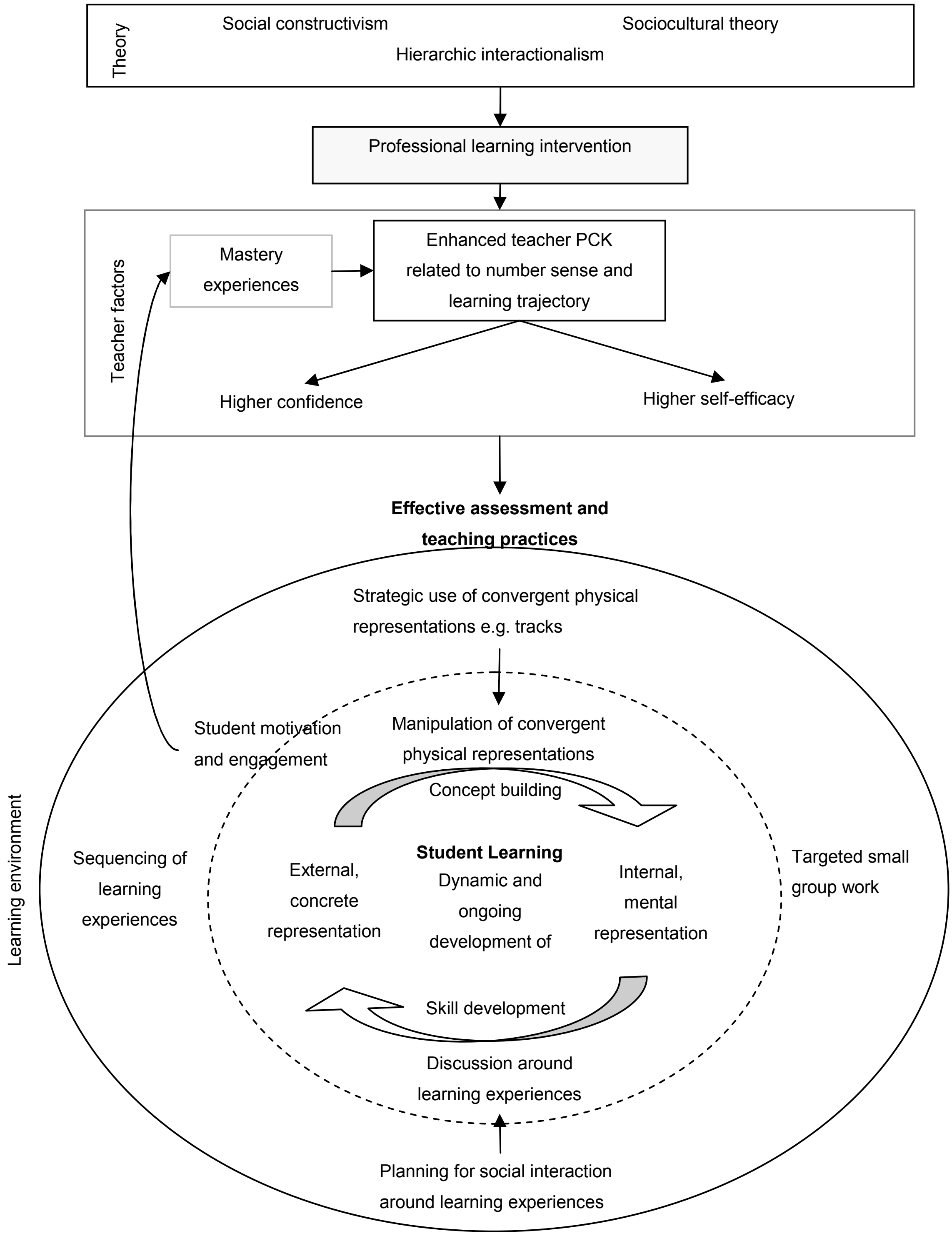

Figure 2: Modified conceptual framework showing the interaction between the intervention, teacher factors, the learning environment and student learning.

Another pathway not shown in Figure 2 allows for the professional learning to impact on teacher confidence and self-efficacy, but with a less extensive impact on teacher PCK. In this case the impact on teaching practice would be appreciably reduced, as was the case with Teachers C and D. In the absence of a new 'schema' for understanding number development, teaching and assessment tools provided through the professional learning intervention were used as additional resources within existing teaching approaches.

CHAPTER 6: CONCLUSION

The purpose of this study was to examine the impact of a professional learning intervention focussed on targeting skills predictive of mathematical difficulties, on teacher confidence, self-efficacy beliefs and practice. The study investigated whether research identifying such skills could be successfully used to design teacher workshops and instructional tools which increased teacher knowledge to the extent where practice and student achievement (estimation) could be positively impacted, within existing school budgets and resources. This was in contrast to previous research which had measured the effectiveness of intensive intervention provided to such students in the context of a Response to Intervention system which funds such initiatives. A pre-test/post-test, mixed-method design was employed with a group of early childhood teachers who completed surveys to measure the constructs of confidence, self-efficacy beliefs and Pedagogical Content Knowledge prior to and following intervention which took place in three, two-hour sessions over three months. Qualitative data were also collected from five case-study teachers who volunteered to participate in interviews and plenary discussions and who tested lower-achieving students on an estimation task before and after the series of workshops. This chapter summarises the conclusions from the study and their implications.

Conclusions

The research project investigated three questions which will be examined in turn.

> *To what extent do teachers perceive an increase in their confidence and self-efficacy and have increased PCK for teaching number as a result of engaging with the professional learning process?*

Teachers were significantly more confident in teaching language than number at the beginning of the project. The increase in teachers' confidence and self-efficacy beliefs for teaching number over the course of the project was highly significant and by the end of the project were similar to their feelings about language. Although teachers were most confident about aspects of quality instruction delivered at the whole-class level, the most significant gains in confidence occurred in areas specifically concerned with identifying and addressing learning difficulties in mathematics.

Teachers identified an increase in professional knowledge as the most notable impact of the professional learning intervention, and this was evident in growth in their understanding of appropriate instructional responses to classroom scenarios. Using a learning trajectory for counting which enabled them to pinpoint students' current levels of skill, target instruction and identify prerequisite skills was regarded as particularly valuable. The concept of bridging the maths of quantities with the maths of numbers with mental representations, and more specifically of the mental number line, resonated particularly strongly with Pre-Primary teachers.

What changes are made to the classroom program as a result of participating in the professional learning program?

The extent to which teachers changed classroom practices varied greatly. Existing routines were maintained and teachers were most likely to use strategies from the professional learning sessions within whole-class contexts, or small-group activities through which all students rotated. There was some evidence that teachers were beginning to use less formal methods of assessment and to 'notice' more in the course of regular classroom activities.

Increases in confidence and self-efficacy alone were not sufficient to ensure changes to practice, and in isolation were associated with use of ideas and tools from the intervention to supplement existing programs. Substantial changes in Pedagogical Content Knowledge that challenged existing thinking about number development were required for meaningful and strategic changes in practice. The aforementioned concept of creating a mental number line was that which lead to such changes in case-study teachers.

Do students have more accurate mental number line representations following their teacher's participation in the professional learning intervention?

The data collected indicated that only Pre-Primary students developed more accurate mental number line representations, as measured by estimation tasks, following the intervention. Students taught by Teacher A, who reported a substantial influence on her practice following the intervention including deliberate planning to target development of the mental number line, showed a noticeably larger improvement than their counterparts in Teacher B's class, on whom the impact appeared lesser.

Year Two students, the only other age group for which data were collected, improved their accuracy of estimation in the 0-20 but not 0-100 number range. Since their teacher reported only limited use of specific strategies intended to influence the development of the mental number line (e.g. board games and number line tasks), it is unsurprising that no impact on estimation skills was observed.

Implications

The current study, although small, verified the application of research on predicting children's number learning difficulties to the design of professional learning for teachers. Positive effects on teacher confidence, self-efficacy and practice were evident which appeared to result in improved number magnitude representations in young children. The study observed rather than controlled for the variable of teacher judgement and choice when implementing instructional strategies following a professional learning intervention. Two key themes emerged which can inform the design of professional learning interventions intended to change teacher practice: the need to impact teacher Pedagogical Content Knowledge; and, the importance of communicating a clear key message with support for application within existing class routines. Such an approach could help to overcome limitations in the design of workshops delivered through this project which attempted to investigate and support too many skills and concepts within too short a time-frame. As a result some teachers felt overwhelmed, resorted to a 'grab-bag' approach towards using the given tools and strategies and missed key messages which underpinned the design of these tools.

Future professional learning should consider delivering one key message at a time, for example the construction of the mental number line, and developing the significance of this idea across different stages of mathematical development and in different applications across the mathematics conceptual area. Furthermore, subsequent efforts should specifically plan to demonstrate how these ideas can be explored within existing class structures such as mat sessions, whole class lessons and small group tasks which can be differentiated for students of diverse abilities. Tools provided should be few and teachers given support to demonstrate and apply their understanding of the key message through the design of unique routines and activities to suit their students. Professional learning which intended to target each of the predictors of mathematical difficulty might therefore take place over a much longer period to ensure changes in practice could be implemented strategically and maintained.

This research project has demonstrated that it is possible to positively impact upon teaching practice through professional learning designed to highlight the skills predictive of mathematical difficulties. Because the intent was to increase teachers' awareness of such skills rather than reliably classify students according to risk, these predictive skills were presented in informal board game contexts. Future research should establish the extent to which such practice decreases students' risk of mathematics difficulties, as measured by fluency with these predictive skills as previously established in the literature.

One question which the research project was unable to answer was if educating teachers in the use of congruent representations to build the mental number line (e.g. linear numbered tracks and partially completed and empty number lines) was associated with improved numerical estimation skills of students beyond the Pre-Primary year. Future efforts could seek to answer this question.

The findings of this research have demonstrated the importance of equipping classroom teachers with timely information about educational research that can inform classroom practice. Teachers who are appropriately informed and see opportunities for distinct changes in practice as relevant to their students' needs are capable of interpreting such research in ways that are workable and sustainable in their classroom contexts. Whilst there will always be a need for intervention programs for some students, effectively designed professional learning programs may better equip teachers with the knowledge they need to devise a preventative layer of instruction which reduces the number of students requiring such support.

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APPENDIX A: RESEARCH INSTRUMENTS

Raising Teacher Sensitivity to Key Numeracy Competencies in Years PP-2

Teacher Survey (all teacher participants)

Your responses to the following will help us to tailor the professional learning program to your needs, evaluate its success and improve upon the program for the benefit of other teachers.

Please answer each question honestly and frankly, stating things as they are rather than how you would like them to be seen. Your name and school will only be seen by the researcher and will not be published in any study or report.

Name: _______________

School: _________________Year level taught: _____________

The project

What do you expect to gain/feel you have gained from participating in this professional learning program?

Confidence with teaching number

Please rate your confidence with the following aspects of teaching number skills by ticking the appropriate box after each statement.

VC: Very confident
C: Confident
LC: Limited confidence
NC: No confidence

Please tick INSIDE boxes rather than on the line between boxes.

Aspect of teaching number	VC	C	OK	LC	NC
Promote a positive classroom climate during number activities					
Plan a sequenced and appropriate learning program in number					
Accommodate individual differences in planning number work and during number activities					
Manage and maximise learning during classroom and group discussions on number concepts					
Respond to ‘teachable moments’ in number					
Provide students with feedback during number activities to enhance learning					

Manage a range of group structures during number work as appropriate (e.g. whole class, small groups, individual work)					
Use allocated instructional time to maximise learning in number					
Teach core number concepts					
Diagnose students' number skills and plan specific interventions					

Beliefs about being an effective teacher of number in Early Childhood

Please indicate the extent to which you agree or disagree with each of the statements below which relate to planning and teaching early number experiences.
Key:
SA: Strongly agree
A: Agree
UN: Undecided
D: Disagree
SD: Strongly disagree

Please tick INSIDE the boxed and not on the line between boxes.

Item	SA	A	UN	D	SD
I am able to plan and deliver effective mathematics lessons and tasks that focus on key mathematical ideas					
I understand and am able to explicitly communicate the purpose and intended learning outcome of each activity to students					
I am able to plan purposeful learning tasks in number that are motivating and engaging					
I am able to plan activities that accommodate the range of individual differences among my students					
I am able to plan opportunities for students to use a range of representations and materials to explore the same concept and build connections between representations					
I am able to provide students with opportunities to learn at more than one cognitive or performance level					
I use my rich knowledge of mathematics in early childhood to ask appropriate questions to probe and promote student thinking and reasoning					
My rich knowledge of mathematics in early childhood enables me to notice individual students' strategies and misconceptions					
I am able to clarify student misconceptions or difficulties in learning through appropriate teaching tasks					
I am able to plan evaluation procedures that accommodate the range of individual differences among my students					
My rich knowledge of mathematics in early childhood enables me to act upon students' difficulties and modify my planning					

Effective number instruction

What do you believe are the core number concepts P-2 students have to learn to succeed in maths?

What is the place of focussed mathematics teaching in P-2, compared to learning through embedded experiences?

Teaching number skills and concepts

A student teacher has planned a lesson to teach counting principles to a Pre-Primary class through the use of a number line. She plans to show students how to 'jump' along a number line drawn on the whiteboard as they count aloud, using a whiteboard marker, and then ask the students to do the same in pairs. She asks for your feedback. How would you advise her?

When he is combining two collections (e.g. 5 buttons and 2 more buttons), one of your students persists in counting out both collections separately, then counting all items starting from one, and won't move to counting-on. How would you respond?

Mrs Johnson has been doing lots of counting activities involving concrete objects with her young class, and wants to find out if they are ready to move on to learning more advanced counting skills and abstract number combinations without using objects. How could she find out?

Please rate your knowledge for and confidence with teaching early language and number skills by placing ticks on the scales below

Knowledge for teaching ...	Low knowledge						High knowledge			
	1	2	3	4	5	6	7	8	9	10
early language skills										
early number skills										

Confidence with teaching ...	Low confidence						High confidence			
	1	2	3	4	5	6	7	8	9	10
early language skills										
early number skills										

Thank you for your time.

Structured individual interview questions

(case study teacher participants)

1. How would you describe your approach to teaching number in your classroom?

- Do you have scheduled time for number activities, and if so, how much?
- Should learning experiences be embedded in play situations or planned and taught explicitly?
- [Post-test only] How have the professional development and tools provided impacted your teaching and learning program? Please give an example.

2. What do you think students need to know in order to be successful in learning early number concepts?

- What are the core concepts in early number?
- Which core skills are important to learn?

3. How would you describe the best way of teaching number concepts to young students?

- What tools and representations are appropriate?

4. Based on your knowledge and experience, what are some of the common difficulties students have in learning number skills and concepts?

5. How do you help students who are experiencing difficulty with number concepts?

- How do you pinpoint their difficulties?
- How do you decide what intervention is appropriate?
- How do you know when you've been successful?

Plenary Discussion Questions

(case study teacher participants)

1. What did you learn/gain from the session today? (2 mins)

2. How is this relevant to your experience and your context? (3 mins)

3. How will you apply this knowledge/tool to your context? (5 mins)

Student Task: Estimation on the Number Line

Teacher's Instructions

Cut sheets apart as indicated.
Shuffle in random order.
Place cover page at the front and "10" page next in booklet and staple on left (10 is practise item).

Please write the child's initials and date on the front page of the booklet and tick a box to indicate their classroom performance in learning aspects of number <u>before completing the estimation task</u>. Please note this is asking for your professional judgement based on your knowledge of the child's classroom performance, not on their skill on this particular estimation task.

Say to students:
We will now play a game with number lines. Look at this page, you see there is a line drawn here. I want you to tell me where some numbers are on this line. When you have decided where the number I will tell you has to be, I want you to make a mark with your pencil on this line.

For the practise item, say:
This line goes from 0 to 20. If here is 0, and here is 20, where would you position 10?
Do not give feedback.
Turn over the page so that for each trial the previous items can not be seen.

For each subsequent page, say:
This line goes from 0 to 20. If here is 0, and here is 20, where would you position (x)?

Student Task: Estimation on the Number Line

Child's initials: ____________________ Date: ________________________

Please tick one box below to indicate this child's level of skill in number (teacher judgement):

Causing concern	Below average	Average

Practice item:

This line goes from 0 to 20. If here is 0, and here is 20, where would you position 10?

10

0 20

2

0 20

4

0 20

6

0 20

7

0 20

13

0 20

15

0 20

16

0 20

18

0 20

Student Task: Estimation on the Number Line Years 1 and 2

Teacher's Instructions

Cut sheets apart as indicated.
Shuffle in random order.

Please write the child's initials and date on the front page of the booklet and tick a box to indicate their classroom performance in learning aspects of number <u>before completing the estimation task</u>. Please note this is asking for your professional judgement based on your knowledge of the child's classroom performance, not on their skill on this particular estimation task.

Say to students:

We will now play a game with number lines. What I'm going to ask you to do is to show me where on the number line some numbers are. When you decide where the number goes, I want you to make a line through the number line like this [making a vertical hatch mark].

For each page, say:

This number line goes from 0 at this end to 100 at this end. If this is 0, and this is 100, where would you put (x)?

Student Task: Estimation on the Number Line

Child's initials: ____________________ Date: ________________________

Please tick one box below to indicate this child's level of skill in number (teacher judgement):

Causing concern	Below average	Average

Demonstration item:

We will now play a game with number lines. What I'm going to ask you to do is to show me where on the number line some numbers are. When you decide where the number goes, I want you to make a line through the number line like this [making a vertical hatch mark].

0 100

2

0 100

3

0 100

4

0 100

6

0 100

18

0 100

25

0 100

42

0 100

67

0 100

71

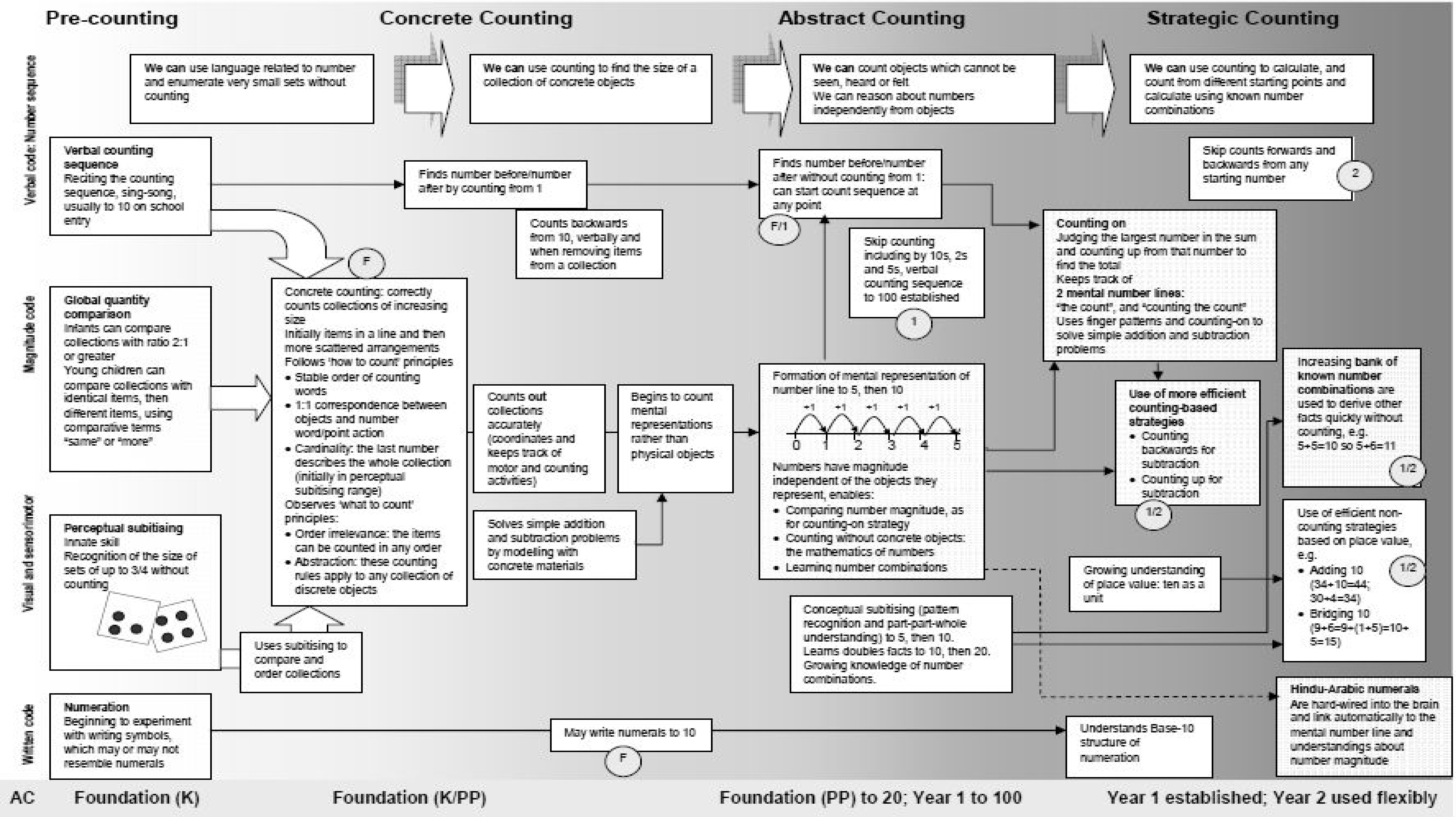
0 100

86

0 100

APPENDIX B: LEARNING TRAJECTORY FOR COUNTING

Development of Counting Skills



Note: AC Refers to Australian Curriculum Foundation (F), Years 1 and 2 levels