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# Forecasting term structure of the Japanese bond yields in the presence of a liquidity trap

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**Abstract**

The Nelson–Siegel (NS) model is widely used in practice to fit the term structure of interest rates largely due to its high efficacy in the in-sample fit and out-of-sample forecasting of bond yields. In this paper, we compare forecasting performances of estimated yields from the Nelson–Siegel-based models and some simpler time series models, using the daily, weekly, and monthly data during a prolong period of liquidity trap in Japan. We find that the out-of-sample expanding window forecasts by NS-based models in general perform less satisfactory than the competitor models. However, the NS-based models can be useful in forecasting yields over longer horizons and can work well with GARCH-type structures in modeling the conditional volatility.

**KEYWORDS**

in-sample fitting and out-of-sample forecasting, Japanese bond yields, liquidity trap, Nelson–Siegel model

## 1 | INTRODUCTION

The term structure of interest rates, popularly known as the yield curve, is a one-to-one mapping between time to maturity and spot rate[1] of government bonds at a given point in time. In practice, the entire term structure is not observable and must be estimated from observed prices of government bonds. Because of its importance in finance and macroeconomic policy, substantial research has been conducted on modeling and forecasting the yield curve during the past four decades. A variety of diverse yield models in the literature can be roughly classified into two groups. The first group consists of models based on economics theory, for example, the no-arbitrage affine models by Vasicek (1977), Cox et al. (1985), Duffie and Kan (1996), Dai and Singleton (2000), and Piazzesi and Schneider (2006). The second group consists of statistical models without economic foundations. They include the three-factor Nelson–Siegel (NS) model by Nelson and Siegel (1987), the dynamic Nelson-Siegel (DNS) model by Diebold and Li (2006), and the affine arbitrage-free term structure model by Christensen et al. (2008), among others. However, despite tremendous advances in modeling yield curves, Duffee (2011) and Diebold and Rudebusch (2013) observe that most yield curve models tend to be either theoretically vigorous but empirically poor in forecasting, or empirically successful in forecasting but theoretically lacking.

In this paper, we adopt the NS-based model as our baseline model because our aim is to compare forecasting performances of estimated yields from the NS-based models and some simpler time series models. The NS-based models are preferred because they have been credited for their high efficacy in the in-sample fitting and out-of-sample forecasting of the term structures of interest rates. The classic NS model for a given time consists of three latent factors representing the level, slope and curvature of the yield curve. It is used to fit the cross section of term structure using a parsimonious model, reportedly with success in capturing the general shapes of the yield curves. Litterman and Scheinkman (1991) find that the three latent factors of the NS model can explain up to 97% of return variance. Svensson (1994) extends the prototypical NS model by adding a second curvature factor with a separate decay parameter to improve the smoothness of the fitted humps often present in the yield curves. Other extensions include the four factor parametric model of Björk and Christensen (1999), the five-factor generalization of the Svensson model by Christensen et al. (2008), and more recently the extended NS model with quantile autoregression by de Rezende and Ferreira (2013). According to Bank for International Settlements (2005), both the NS model and the Svensson-type extensions are heavily used in central banks to construct zero-coupon yield curves in the past two decades.

Moreover, Diebold and Li (2006) dynamize the NS model by allowing the three latent factors to be time-varying. They show that the proposed model can predict the US yield curve more accurately than other competing models, especially at longer horizons. The model is typically able to explain a large part of variance that is observed in the government bond yields. In addition, Bolder (2006) reports the superiority of the dynamized NS model for estimation and forecasting purposes when compared with other parametric models. Yu and Zivot (2011) find that the DNS factor with autoregressive lag order 1 model outperforms other competitors in the out-of-sample forecast accuracy, and the DNS factor state space model becomes appealing on the high-yield bonds in the short-term forecast horizons. Steeley (2014) engages both the DNS model and the less-structured models to forecast the UK term structure when short-term rates are near zero. It is found that the random walk (RW) and the AR(1) model have better forecasting performances. More recently, Ullah (2016) applies an extended dynamic affine NS model with macroeconomic factors to the Japanese bond market, reporting that the affine NS-type models outperform the benchmark simple time series forecasting models. Other studies based on the DNS specification are conducted in both developed and emerging markets (see Audzeyeva & Fuertes, 2018; Carriero et al., 2018; Kanjilal, 2013; Kaya, 2013; Nyholm & Rebonato, 2008). Although apparently, adding no-arbitrage restrictions and inclusion of macroeconomic variables to the DNS model have been the subject of more recent research, there is still a continuing interest in the less complicated DNS model.

The primary goal of this paper is to compare the forecasting performances of the DNS model with some simpler time series models in capturing the term structure of the Japanese government bond yields during a period of prolonged liquidity trap. We use the daily, weekly, and monthly yield data obtained from the Bank of Japan covering the period spanning from January 2000 to November 2007. The sample period was chosen as it clearly exhibits a prolonged duration of liquidity trap, which forces all short-term yields to remain close to zero for an extended period of time.

In our study, we consider two categories of models. The first group consists of NS-type models, with three specifications in the conditional mean equations with constant volatility, and two specifications in the conditional mean equations with time-varying conditional volatility. In the second group, we fit the competitors directly to the yields data. There are three specifications in the mean equations with constant volatility, and four specifications in the mean equations with time-varying conditional volatility. Forecasts of yields are generated by plugging the estimated parameters into the respective models under study.

Our empirical findings indicate that, in the presence of liquidity trap in Japan, the out-of-sample expanding window forecasts of the NS-based models in general perform inferiorly vis-à-vis competitor models, particularly those of simpler specifications such as the RW model. Moreover, within the NS-based models, the RW and AR(1) specifications of the level, slope and curvature series under the OLS estimation tend to outperform those under the NLS, and performance of the model by the state space method of estimation is ranked the least. Our results are reasonably robust to time to maturity, in-sample fitting period, and to the daily, weekly, and monthly frequency windows of the yields through a period of liquidity trap.

The rest of this paper is organized as follows. Section 2 discusses the DNS model as well as the methodology adopted in this study. Section 3 introduces the data sets, Section 4 discusses the in-sample fitting procedure and results, whereas Section 5 reports the out-of-sample forecasting procedures and performances among the NS-based and competitor models. Section 6 concludes with some remarks and implications.

## 2 | METHODOLOGY

We adopt the DNS model suggested by Diebold and Li (2006) and Diebold et al. (2006) as our baseline model.

The idea of their approach is to produce a set of estimated parameters of the monthly NS yield curves over a period of time that can be modeled and forecasted with standard time series methods. Forecasts of yields with various maturities can be generated by inserting the forecasted parameters into the structure of the yield curve. Diebold and Li (2006) show that their model is capable of replicating the main empirical facts of the term structure of interest rates in the United States over time. In what follows, we highlight the gist of the well-established DNS model proposed by Diebold and Li (2006), which is based on the classic work of Nelson and Siegel (1987) for fitting the cross section of yields. The original static Nelson–Siegel framework provides a parsimonious and flexible approximation to the spot rate with maturity $\tau$ in a yield curve, namely:

$$y(\tau) = \beta_1 + \beta_2\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_3\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right), \quad (1)$$

where $\beta_1, \beta_2, \beta_3$, and $\lambda$ are parameters, with $\lambda$ governing the exponential decay rate.

Working on the idea that the NS parameters should be time-varying if yield curves are time-varying, Diebold and Li (2006) dynamize the static NS model to produce the DNS model:

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3,t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right), \quad (2)$$

where $\beta_{1,t}, \beta_{2,t}, \beta_{3,t}$ and $\lambda_t$ are the time-varying version of parameters $\beta_1, \beta_2, \beta_3$ and $\lambda$ as specified in the static NS model.

The DNS specification in Equation (2) decomposes the spot rate $y_t(\tau)$ with maturity $\tau$ at time $t$ into three factor loadings $\left\{1, \frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}, \frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right\}$, which are flexible enough to represent a range of monotonic, humped and $S$-type shapes generally associated with the yield curve data. Diebold and Li (2006) show that the corresponding factors $\{\beta_{1,t}, \beta_{2,t}, \beta_{3,t}\}$ can be interpreted as long-term, short-term, and medium-term factors, respectively. The long-term factor ($\beta_{1,t}$) governs the yield curve level because an increase in this coefficient raises all short- and long-term yields equally, thereby changing the level of the yield curve. The short-term ($\beta_{2,t}$) and medium-term ($\beta_{3,t}$) factors are closely associated with the slope and the curvature of the yield curve.

Diebold and Li (2006) further show that the DNS parameter $\lambda_t$ controls the exponential rate of decay. For instance, smaller values of $\lambda_t$ tend to produce slower decay and can better fit the yield curve with longer maturities, while larger values of $\lambda_t$ exhibit faster decay and can better fit the yield curve at relatively shorter maturities. In addition, the decay parameter $\lambda_t$ determines the time to maturity of the yield curve such that the medium-term factor loading attains its maximum.

The DNS parameters can be estimated by three estimation procedures: Two-step OLS, one-step nonlinear least squares (NLS), and the state-space method (SSM) with the Kalman filter. These approaches are described briefly as below.

In the first step of the two-step OLS approach, the parameter $\lambda_t$ is calibrated at which the loading on the curvature factor achieves its maximum at a medium maturity. We fix this calibrated $\lambda_t$ and use it to compute values of the two regressors (short-term and medium-term factor loadings). In the second step, the OLS is used to regress the spot rates at different maturities on the long-term, short-term and medium-term regressors to produce estimates of $\beta_1, \beta_2$, and $\beta_3$, for each frequency window of the yield curves in the sample period, say $t = 1, 2, 3, \ldots, T$. This generates a three-dimensional time series of estimated factors, $\{\hat{\beta}_{1,t}, \hat{\beta}_{2,t}, \hat{\beta}_{3,t}\}_{t=1}^{T}$.

We now turn to the one-step NLS approach. The static NS model in Equation (1) is estimated using nonlinear least squares to produce estimates of the level, slope, curvature factors, and the decay parameter for each $t = 1, 2, 3, \ldots, T$. This generates a four-dimensional time series of estimated model parameters, $\{\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\lambda}_t\}_{t=1}^{T}$.

In the SSM with the Kalman filter, we follow the approach of Diebold et al. (2006) to engage a state-space framework to estimate the factors and their autoregressive parameters simultaneously. The observation equation of the state-space structure of the DNS model is

$$\begin{bmatrix} y_{\tau_1,t} \\ y_{\tau_2,t} \\ \vdots \\ y_{\tau_N,t} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} - e^{-\lambda\tau_N} \end{bmatrix} \begin{bmatrix} \beta_{1,t} \\ \beta_{2,t} \\ \beta_{3,t} \end{bmatrix} + \begin{bmatrix} w_{\tau_1,t} \\ w_{\tau_2,t} \\ \vdots \\ w_{\tau_N,t} \end{bmatrix}. \quad (3)$$

The observed spot rates are linked to the unobservable factors (state variables) and measurement errors in the transition equation via a vector autoregressive process of first order, which is specified as

$$\begin{bmatrix}\beta_{1,t}-\mu_1\\ \beta_{2,t}-\mu_2\\ \beta_{3,t}-\mu_3\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\begin{bmatrix}\beta_{1,t-1}-\mu_1\\ \beta_{2,t-1}-\mu_2\\ \beta_{3,t-1}-\mu_3\end{bmatrix}+\begin{bmatrix}\eta_{1,t}\\ \eta_{2,t}\\ \eta_{3,t}\end{bmatrix}, \tag{4}$$

where $\mu_i$ is the mean of factor $i$, for $i=1,2,3$, and $a_{ij}$ for $i,j=1,2,3$ are elements of the transition matrix. We assume that the disturbance terms in both the observation and transition equations are Gaussian white noise. In addition, $\eta_t$ and $\omega_t$ are orthogonal. The error terms in the observation equation are uncorrelated, with a diagonal covariance matrix, whereas the error terms in the transition equation are contemporaneously correlated, implying that the covariance matrix of error terms is nondiagonal. The method of maximum likelihood with the Kalman filter algorithm is used to obtain the optimal filtered and smoothed estimates of the underlying factors. As such, the state-space approach produces a three-dimensional time series of estimated factors, $\{\hat{\beta}_1,\hat{\beta}_2,\hat{\beta}_3\}_{t=1}^{T}$.

For simplicity and tractability, we model the series of the estimated NS factors according to two specifications: The conditional mean equation with time-invariant variance and the conditional mean equation with time-varying conditional variance. Models under the conditional mean specification with constant conditional variance include the RW model, univariate AR(1) model, and the trivariate AR(1) model, whereas models under the conditional mean specification with time-varying conditional variance include the GARCH(1,1) and the EGARCH(1,1,1) models. They are highlighted are as follows:

1. RW model $\hat{\beta}_{i,t}=\hat{\beta}_{i,t-1}+\epsilon_{i,t}$, where $\epsilon_{i,t}\sim\mathcal{N}(0,\sigma_i^2)$, for $i=1,2,3$.
2. AR(1) model $\Delta\hat{\beta}_{i,t}=\mu_i+\phi_i\Delta\hat{\beta}_{i,t-1}+\epsilon_{i,t}$ where $\epsilon_{i,t}\sim\mathcal{N}(0,\sigma_i^2)$, and $\Delta\hat{\beta}_{i,t}=\hat{\beta}_{i,t}-\hat{\beta}_{i,t-1}$, for $i=1,2,3$.
3. VAR(1) model $\Delta\theta_t=\mu+\Phi\Delta\theta_{t-1}+\epsilon_t$, where $\epsilon_t\sim\mathcal{N}(0,\Omega)$, where $\Delta\theta_t=[\hat{\beta}_{1,t}-\hat{\beta}_{1,t-1},\hat{\beta}_{2,t}-\hat{\beta}_{2,t-1},\hat{\beta}_{3,t}-\hat{\beta}_{3,t-1}]'$, $\mu$ is the 3 x 1 constant vector, $\Phi$ is the 3 x 3 transition matrix and $\Omega$ is the 3 x 3 nondiagonal variance–covariance matrix.
4. GARCH(1,1) model Conditional mean equation: $\Delta\hat{\beta}_{i,t}=\mu_i+\phi_i\Delta\hat{\beta}_{i,t-1}+\epsilon_{i,t}$, where $\epsilon_{i,t}\sim\mathcal{N}(0,\sigma_{i,t}^2)$, and the conditional variance equation: $\sigma_{i,t}^2=\omega_i+\alpha_{1,i}\epsilon_{i,t-1}^2+\alpha_{2,i}\sigma_{i,t-1}^2$, for $i=1,2,3$.
5. EGARCH(1,1,1) model Conditional mean equation: $\Delta\hat{\beta}_{i,t}=\mu_i+\phi_i\Delta\hat{\beta}_{i,t-1}+\epsilon_{i,t}$, where $\epsilon_{i,t}\sim\mathcal{N}(0,\sigma_{i,t}^2)$, and the conditional variance equation: $\ln(\sigma_{i,t}^2)=\omega_i+\alpha_{1,i}\left|\frac{\epsilon_{i,t-1}}{\sigma_{i,t-1}}\right|+\alpha_{2,i}\ln(\sigma_{i,t-1}^2)+\gamma_i\left(\frac{\epsilon_{i,t-1}}{\sigma_{i,t-1}}\right)$, for $i=1,2,3$.

We have experimented with other structures in modeling the series of estimated NS factors. They include the vector error correction (VECM) model in the conditional mean equation and multivariate GARCH and EGARCH models in the conditional variance equation. However, preliminary results are not promising as the out-of-sample forecasts by VECM do not outperform its counterpart VAR, and the multivariate GARCH/EGARCH do not outperform univariate GARCH/EGARCH. As such, they are not included in our study. Indeed, accumulated experience indicate that there is always a trade-off between in-sample fit and out-of-sample forecasting performance, and as remarked by Diebold (2001), parsimonious models are often more successful for out-of-sample forecasting.

## 3 | DATA

Our data set contains the daily, weekly, and monthly yield curves culled from the Bank of Japan archives, spanning almost 8 years from January 2000 to November 2007.[2] Each yield curve contains 15 spot rates with maturities of 3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240, and 360 months. There are 1888 observations for yield curves on daily frequency, 408 observations for weekly frequency, and 94 observations for monthly. As can be gleamed from Figure 1, the chosen period of study clearly exhibits the presence of liquidity trap, which forces all short-term yields to remain close to zero for an extended period. Similar to the United States, Japan applies a particular variant of the "smoothing splines" method to the estimation of zero-coupon yields. The instantaneous forward rate curves, expressed as a linear combination of cubic B-splines, are constructed from price quotes on selected risk-free fixed income assets: 3-, 6-, 120-, and 240-month bonds. The forward rate curves are interpolated by using smoothing splines, after which the spot rates can then be computed by taking an average of the forward rates. We fit the NS-based and competitor models to spot rates in each of the daily, weekly, and monthly frequencies. This enables one to check whether empirical findings of the models are insensitive to frequency window of yields.

An issue with the Japanese securities market is that, before FY2008, there was a dual system of bill issuance for the short-term securities, namely, financing bills (FB) and treasury bills (TB).[3] As such, there may be an FB and a TB issued on the same date sharing a common

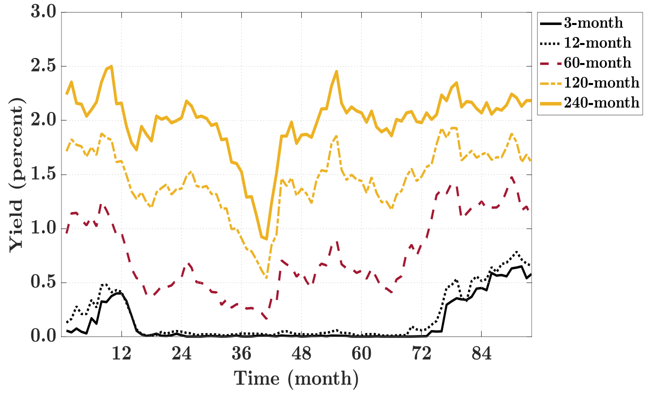


**FIGURE 1** Monthly yields with selected maturities. *Notes*: The figure depicts yields with various maturities from January 2000 to October 2007 (94 observations). Yields with maturities of 12 months and less are near zero after January 2001, followed by a mild increase after 2006

term-to-maturity, but with different yields. In this study, we only choose one set of data for the short-term bonds, such as the 3-month FB (GJFB3MO), 6-month TB (GJTB6MO), and the 1-year bond (GJGB1).[4]

Because of space limitation, we plot in Figure 1 the spot rates at maturities of 3, 12, 60, 120, and 240 months, which are based on the monthly data from January 2000 to November 2007. Two features can be observed. First, there is a low-yield period between 24 to 36 months for all five maturities after January 2000. Second, spot rates with maturities of 12 months or less are close to zero from January 2001 onwards until the end of 2006. The former observation may be associated with the privatization of the Postal Savings System as well as a complete overhaul of the existing financial structure in Japan during the period under study. The latter observation can be regarded as a classic example of liquidity trap.

Figure 2 displays a three-dimensional plot of the Japanese yield curves ranging from January 2000 to November 2007, covering 15 maturities of 3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240, and 360 months. It can be seen that the level, slope, and curvature of the term structure fluctuates over time, while taking on various humped and *S*-type shapes.

Table 1 reports the descriptive statistics for monthly, weekly and daily yields with maturities at 3, 12, 60, 120, and 240 months. The sample period is from January 2000 to November 2007. Some of the stylized facts discussed in Diebold and Li (2006) can be observed. For example, mean values of yields increase with maturities, thereby indicating a standard upward sloping shape of the average yield curve. As indicated by magnitudes of the standard deviations, yields with medium-to-long-term maturities are more volatile than those with shorter-term maturities. In addition, the sample skewness decreases with increases in maturities, and kurtosis of the shorter rates are smaller than those of longer rates. Moreover, yields for all maturities are highly persistent, as the sample autocorrelation coefficients at orders 1 and 12 are quite closer to 1, except for the monthly yields with autocorrelation coefficient at lag order 12. The observed patterns are reasonably robust to the frequency window of the yield curves.

Furthermore, the last two columns of Table 1 display, respectively, the augmented Dickey–Fuller (ADF) unit-root tests with intercept, and with intercept and a time trend under the null hypothesis that the yield series have a unit root. We use the Bayesian information criterion to choose the lag order in the ADF test. The lower tail critical values for rejection of the null hypothesis are $-2.86$ and $-3.41$ at the 5% level of significance, respectively. As expected, the ADF tests suggest that all yield series in levels are nonstationary, which provides some justification for taking the first differencing. Though not reported here, after taking the first differencing of the yield levels, all series in yield changes are stationary.

## 4 | IN-SAMPLE FITTING RESULTS

The NS-based models specified in Section 2 are fitted to spot rates using each of the daily, weekly, and monthly data for the full sample period. Both mean absolute error (MAE) and root mean square error (RMSE) as measures of forecast errors (residual statistics) are used to gauge the forecasting performances from our baseline model with different methods of estimation.

First, following Diebold and Li (2006), we employ a two-stage OLS estimation method to fit the NS model in Equation (2) with different values of the decay parameter $\lambda$. To determine the optimal decay parameter that maximizes the loading on the medium-term factor, we

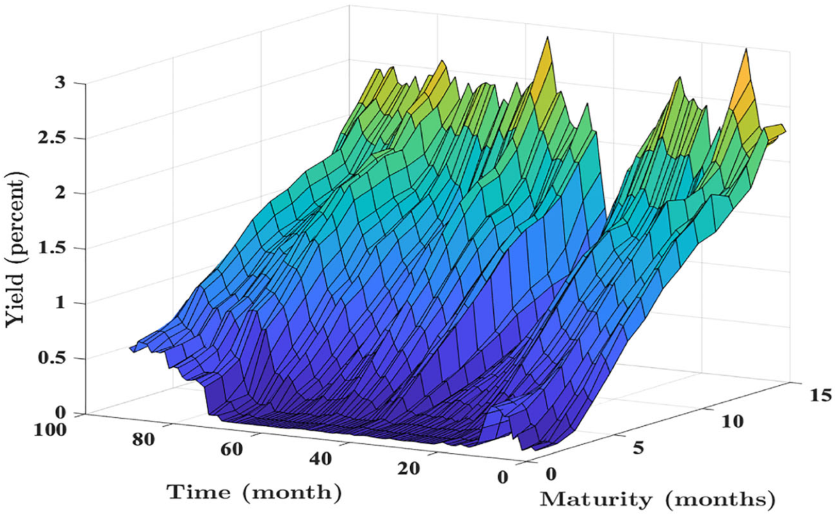


**FIGURE 2** Yield curves, January 2000 to October 2007. *Notes*: The figure plots yield curves from January 2000 to October 2007, with maturities at 3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240, and 360 months

conduct full scale in-sample estimations by systematically choosing $\lambda$ in the range of [0.005, 0.5978], with an incremental step of 0.025. The range is chosen because when $\lambda = 0.005$, it maximizes the loading on the medium-term factor with a maturity of 360 months, and when $\lambda = 0.5978$, it maximizes the loading on the medium term with a maturity of 3 months. The decay rate with the smallest RMSE and MAE and the maximum loading on the medium-term factor is selected as the optimal value. When $\lambda$ is fixed at 0.0299, the average RMSE and MAE across all maturities are minimized (results are available upon request). This optimal decay rate corresponds to maximizing the loading on the curvature (the medium-term factor) at a maturity of exactly 60 months (see Figure 3). It is interesting that our calibrated decay rate is about half of what Diebold and Li (2006) reported for maximizing the loading on the medium-term factor at the maturity of 30 months in their study of the US yield curves with maturities ranging from 3 to 120 months.

Next, we fit the baseline model by NLS estimation method to spot rates under each of the daily, weekly, and monthly data, assuming that the decay parameter $\lambda_t$ is time varying over the sample period. The NLS approach essentially fits the model via a one-step estimation procedure, though it does not necessarily provide better in-sample fitting and out-of-sample forecasting results, as demonstrated in de Pooter (2007). As can be observed from Table 2, the average of the estimated time-varying decay parameter is about 0.032 for various spot rates from the yield curves at different window frequencies, with the median ranging from 0.031 to 0.032. This is slightly higher than the optimal value of 0.0299 when using a two-stage OLS estimation method.

Finally, we fit the baseline model by the state-space method (SSM) of estimation to the same dataset as used for the OLS and NLS estimation. In this approach, we treat the decay parameter as time-invariant, which is to be estimated jointly with the three time-varying latent factors by the method maximum likelihood with the Kalman filter algorithm. Estimates of the decay parameter are 0.0307, 0.0311 and 0.0310 for monthly, weekly and daily yields, respectively. These values exceed the estimated optimal decay under the OLS method, but are smaller than the median of the estimated values under the NLS method.

The estimation results of our baseline model by method of estimation are reported in Table 2. As can be seen in this table, the summary statistics of the estimated level, slope and curvature factors are robust to sample data based on daily, weekly, and monthly frequency windows. In addition, all skewness of the long-term factor ($\hat{\beta}_1$) are negative, hovering between −1.728 and −1.381. In contrast, all skewness of the short-term and medium-term factors ($\hat{\beta}_2, \hat{\beta}_3$) are positive, with values ranging from 0.486 to 1.117. All kurtosis of the factor series are greater than 3, except for $\hat{\beta}_3$ under SSM. Based on these patterns, we may infer that the factor series follow asymmetrical distributions with heavier tails than the standard normal distribution.

Moreover, except for the monthly series, all autocorrelation coefficients at lag orders of 1 and 12 for the estimated factors are above 0.65. The persistence is most notably under SSM, followed by OLS and NLS methods. The persistence increases as data frequency increases from monthly through daily frequencies, and tends to fade away as the number of displacement periods increases. These findings are consistent with the stylized facts in Diebold and Li (2006). As for correlation between the estimated factors, the long-term factor is significantly negatively correlated with the short-term factor and is moderately and negatively correlated with the medium-term factor. In contrast, the medium-term factor is

**TABLE 1** Summary statistics of the yield curve.

| Maturity (months) | Mean | Median | Max. | Min. | Std. dev. | Skewness | Kurtosis | $\hat{\rho}(1)$ | $\hat{\rho}(12)$ | Augmented Dickey–Fuller Int. | Int. & Trend |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Panel A: Monthly data** | | | | | | | | | | | |
| 3 | 0.129 | 0.010 | 0.650 | 0.001 | 0.201 | 1.358 | 3.334 | 0.941* | 0.207* | −0.154 | −0.820 |
| 12 | 0.181 | 0.044 | 0.785 | 0.008 | 0.230 | 1.186 | 2.955 | 0.946* | 0.263* | −0.298* | −0.920 |
| 60 | 0.760 | 0.652 | 1.478 | 0.164 | 0.355 | 0.314 | 1.834 | 0.948* | 0.279 | −1.364 | −1.874 |
| 120 | 1.456 | 1.472 | 1.930 | 0.540 | 0.300 | −0.832 | 3.683 | 0.911* | 0.018 | −2.065 | −2.250 |
| 240 | 1.987 | 2.040 | 2.500 | 0.904 | 0.292 | −1.666 | 6.595 | 0.900* | −0.087 | −2.152 | −2.249 |
| **Panel B: Weekly data** | | | | | | | | | | | |
| 3 | 0.132 | 0.011 | 0.712 | 0.001 | 0.204 | 1.341 | 3.270 | 0.988* | 0.820* | −0.146 | −0.878 |
| 12 | 0.184 | 0.045 | 0.834 | 0.005 | 0.233 | 1.187 | 2.957 | 0.989* | 0.842* | −0.403 | −1.099 |
| 60 | 0.764 | 0.642 | 1.545 | 0.155 | 0.356 | 0.353 | 1.892 | 0.987* | 0.840* | −1.555 | −2.092 |
| 120 | 1.458 | 1.465 | 1.994 | 0.455 | 0.300 | −0.823 | 3.789 | 0.977* | 0.722* | −2.237 | −2.390 |
| 240 | 1.993 | 2.048 | 2.595 | 0.780 | 0.297 | −1.656 | 6.701 | 0.970* | 0.655* | −2.561 | −2.651 |
| **Panel C: Daily data** | | | | | | | | | | | |
| 3 | 0.135 | 0.010 | 0.727 | 0.001 | 0.207 | 1.315 | 3.205 | 0.997* | 0.969* | −0.236 | −0.971 |
| 12 | 0.186 | 0.045 | 0.840 | 0.005 | 0.236 | 1.167 | 2.883 | 0.998* | 0.970* | −0.166 | −0.962 |
| 60 | 0.762 | 0.645 | 1.582 | 0.155 | 0.358 | 0.383 | 1.916 | 0.997* | 0.964* | −1.626 | −2.264 |
| 120 | 1.455 | 1.460 | 2.009 | 0.448 | 0.302 | −0.822 | 3.811 | 0.994* | 0.936* | −2.361 | −2.572 |
| 240 | 1.991 | 2.043 | 2.665 | 0.764 | 0.301 | −1.663 | 6.672 | 0.993* | 0.921* | −2.643 | −2.770 |

*Note*: The sample period for monthly yield data is from end of January 2000 to end of October 2007 with 94 observations. The period for weekly yield data is from end of first week in January 2000 to end of third week in November 2007 with 408 observations, while daily yield data from January 4, 2000, to November 21, 2000, with 1888 observations. Summary statistics of yields are expressed in percentages. Sample autocorrelations are reported with lag orders 1 and 12, respectively. The augmented Dickey–Fuller (ADF) unit root test statistics with (i) an intercept, and (ii) a time trend plus an intercept are reported in the last two columns. They are based on the null hypothesis that the factors have unit roots. Asterisk * indicates rejection of null hypothesis at the 5% level of significance.

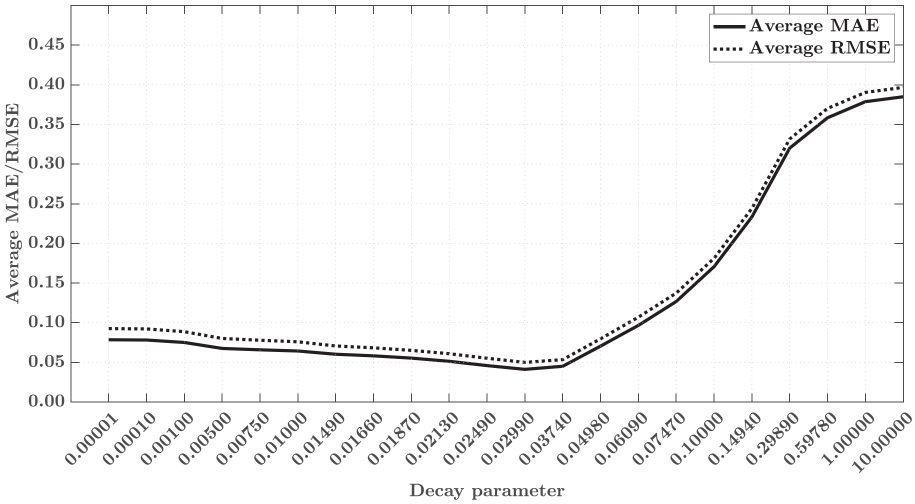


**FIGURE 3** Average MAE and RMSE versus rate of decay. *Notes*: The figure displays average MAE and RMSE of residuals fitted by the NS-based model by the two-step OLS method for monthly yields, with different values of the decay parameter. Note that when $\lambda = 0.0299$, it minimizes both the average MAE and RMSE

moderately and positively correlated with the short-term factor.

Furthermore, we perform two variations of the ADF tests, one with an intercept and the other with both an intercept and a time trend. The ADF test statistics are reported in the last two columns of Table 2. They are all insignificant at the 5% level of significance, indicating that the estimated factor series may have unit roots. As such, we find some supports for taking first differences of the series in order to make it stationary prior to modeling.

Table 3 contains summary statistics of the fitted residuals from the NS-based models for selected maturities at 3, 12, 60, 120, and 240 months. It can be observed that the NLS estimation method consistently produces the best overall fit among the other NS-based models with the lowest MAEs and RMSEs for all the maturities and at different frequencies of the yield curves, followed by the two-step OLS method and the state-space method. In addition, all three NS-based models produce reasonably close MAE and RMSE for yields with maturities at 3, 12, 60, and 120 months. In contrast, the MAE and RMSE for yields with 240 months maturity by the state space method of estimation are about threefold of those obtained by the OLS and NLS methods. Such observed patterns are robust to the window frequency of the yield curves. We note in passing that the two-step OLS method with the decay parameter $\lambda_t$ fixed at 0.0609, as proposed in Diebold and Li (2006), does not provide a good fit for the Japanese bond yields. Results are available upon request.

Panel A of Tables 4A–4C report estimation results of the first-differenced factor series using daily, weekly, and monthly data. The model specifications include AR(1) in the conditional mean equation, and GARCH(1,1) and EGARCH(1,1,1) structures in the conditional variance equations. As can be gleaned from the tables, the autoregressive coefficients are significant at the 5% level for all three methods of estimation, whereas most of the intercepts of the AR(1) structure are not significant (results available upon request). In addition, Columns 4 and 5 display magnitudes of $Q_2$ and *LM* test statistics, which are used to detect autocorrelation in the squared series of the residuals after applying the AR(1) filter to the factor series. More specifically, $Q_2$ denotes the Ljung-Box $Q$ statistic of the squares of the standardized residuals based on autocorrelation coefficients of order up to 10. In addition, LM denotes Engle's Lagrange multiplier test, which is employed to assess autoregressive conditional heteroskedastic (ARCH) effects in the residuals. We use Bayesian information criterion to choose the lag order in Engle's ARCH test. As can be seen in the tables for daily and weekly frequencies, both $Q_2$ and LM test statistics are significant at the 5% level, thereby suggesting that the nonlinear dependencies could be due to the presence of conditional heteroskedasticity.

Columns 7–9 and Columns 13–16 report estimation results of GARCH(1,1) and EGARCH(1,1,1) models specified in the conditional variance equations. The estimated values of $\alpha_1$ and $\alpha_2$ in the GARCH structure are all significant at the 5% level for all three models, which are quite robust to the weekly and daily frequency windows and with little GARCH effect from the monthly yield data. Similar patterns are observed for estimated values of $\alpha_1$ and $\alpha_2$ in the EGARCH structure, which are all significant at the 5% level for all three models. However, results are mixed for the possible presence of asymmetric volatility captured by $\gamma$ in the EGARCH structure. There are no signs of asymmetric effects in all level, slope and curvature factors for the monthly data, and some evidence for the level and slope factors estimated by the state space methods. In contrast, we detect evidence of asymmetric volatility in all three models for the daily data. There are indication of positive asymmetric volatility in the level

**TABLE 2** Descriptive statistics of factors by method of estimation.

| | Summary statistics | | | | | | | Autocorrelation | | Augmented Dickey–Fuller | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Factors** | **Mean** | **Median** | **Max.** | **Min.** | **Std. dev.** | **Skewness** | **Kurtosis** | $\hat{\rho}(1)$ | $\hat{\rho}(12)$ | **Int.** | **Int. & Trend** |
| **Panel A: OLS Estimation** | | | | | | | | | | | |
| $\hat{\beta}^{OLS}_{1,t}$ | 2.686 | 2.751 | 3.435 | 1.271 | 0.370 | −1.663 | 7.090 | 0.882* | −0.273* | −2.376 | −2.360 |
| | (2.694) | (2.753) | (3.473) | (1.157) | (0.374) | (−1.726) | (7.286) | (0.969*) | (0.622*) | (−2.566) | (−2.560) |
| | [2.696] | [2.757] | [3.553] | [1.132] | [0.381] | [−1.723] | [7.174] | [0.993*] | [0.917*] | [−2.486] | [−2.483] |
| $\hat{\beta}^{OLS}_{2,t}$ | −2.577 | −2.644 | −1.245 | −3.515 | 0.420 | 0.947 | 4.075 | 0.892* | −0.324* | −2.213 | −2.279 |
| | (−2.582) | (−2.681) | (−1.095) | (−3.515) | (0.423) | (1.006) | (4.132) | (0.974*) | (0.692*) | (−2.188) | (−2.292) |
| | [−2.579] | [−2.673] | [−1.069] | [−3.563] | [0.430] | [0.983] | [4.076] | [0.994*] | [0.928*] | [−2.213] | [−2.300] |
| $\hat{\beta}^{OLS}_{3,t}$ | −2.374 | −2.571 | 0.170 | −4.008 | 0.999 | 0.543 | 2.319 | 0.922* | 0.046 | −2.021 | −2.220 |
| | (−2.381) | (−2.538) | (0.225) | (−4.023) | (0.990) | (0.486) | (2.275) | (0.977*) | (0.769*) | (−2.214) | (−2.433) |
| | [−2.403] | [−2.546] | [0.350] | [−4.161] | [0.979] | [0.511] | [2.333] | [0.996*] | [0.929*] | [−2.081] | [−2.401] |
| **Panel B: NLS Estimation** | | | | | | | | | | | |
| $\hat{\beta}^{NLS}_{1,t}$ | 2.680 | 2.764 | 3.463 | 1.263 | 0.359 | −1.437 | 6.273 | 0.859* | −0.121 | −2.519 | −2.558 |
| | (2.685) | (2.762) | (3.556) | (1.266) | (0.362) | (−1.477) | (6.231) | (0.964*) | (0.608*) | (−2.684) | (−2.720) |
| | [2.692] | [2.771] | [3.783] | [1.238] | [0.369] | [−1.456] | [6.149] | [0.993*] | [0.905*] | [−2.556] | [−2.523] |
| $\hat{\beta}^{NLS}_{2,t}$ | −2.547 | −2.542 | −1.233 | −3.229 | 0.364 | 0.835 | 4.425 | 0.852* | −0.195 | −2.690 | −2.679 |
| | (−2.547) | (−2.541) | (−1.237) | (−3.229) | (0.367) | (0.830) | (4.287) | (0.961*) | (0.652*) | (−2.806) | (−2.814) |
| | [−2.553] | [−2.548] | [−1.212] | [−3.490] | [0.374] | [0.812] | [4.198] | [0.992*] | [0.898*] | [−2.727] | [−2.738] |
| $\hat{\beta}^{NLS}_{3,t}$ | −2.535 | −2.850 | 0.861 | −3.988 | 1.075 | 1.052 | 3.501 | 0.865* | 0.310 | −2.531 | −3.284* |
| | (−2.554) | (−2.908) | (1.803) | (−4.122) | (1.075) | (1.117) | (3.709) | (0.929*) | (0.779*) | (−3.839*) | (−4.889*) |
| | [−2.534] | [−2.895] | [1.849] | [−4.144] | [1.093] | [1.098] | [3.700] | [0.974*] | [0.886*] | [−4.969*] | [−6.297*] |
| $\hat{\lambda}^{NLS}_{t}$ | 0.032 | 0.031 | 0.059 | 0.013 | 0.009 | 0.731 | 4.106 | 0.754* | −0.151 | −3.595* | −4.146* |
| | (0.032) | (0.032) | (0.065) | (0.013) | (0.009) | (0.663) | (3.834) | (0.888*) | (0.576*) | (−4.904*) | (−5.523*) |
| | [0.032] | [0.031] | [0.066] | [0.011] | [0.009] | [0.555] | [3.896] | [0.953*] | [0.778*] | [−6.702*] | [−7.324*] |
| **Panel C: SSM Estimation** | | | | | | | | | | | |
| $\hat{\beta}^{SSM}_{1,t}$ | 2.771 | 2.834 | 3.738 | 1.204 | 0.451 | −1.381 | 5.709 | 0.912* | −0.400* | −2.044 | −2.047 |
| | (2.770) | (2.861) | (3.720) | (1.122) | (0.446) | (−1.429) | (5.963) | (0.986*) | (0.663*) | (−1.691) | (−1.708) |
| | [2.775] | [2.861] | [3.838] | [1.018] | [0.454] | [−1.410] | [5.990] | [0.998*] | [0.933*] | [−1.350] | [−1.369] |

(Continues)

TABLE 2 (Continued)

| Factors | Summary statistics: Mean | Median | Max. | Min. | Std. dev. | Skewness | Kurtosis | Autocorrelation: $\hat{\rho}(1)$ | $\hat{\rho}(12)$ | Augmented Dickey–Fuller: Int. | Int. & Trend |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{\beta}_{2,t}^{SSM}$ | −2.639 | −2.776 | −1.166 | −3.748 | 0.500 | 0.803 | 3.433 | 0.915* | −0.385* | −1.836 | −1.950 |
| | (−2.632) | (−2.780) | (−1.086) | (−3.727) | (0.497) | (0.864) | (3.515) | (0.986*) | (0.714*) | (−1.374) | (−1.541) |
| | [−2.633] | [−2.772] | [−0.992] | [−3.830] | [0.503] | [0.843] | [3.556] | [0.998*] | [0.940*] | [−1.169] | [−1.312] |
| $\hat{\beta}_{3,t}^{SSM}$ | −2.688 | −2.983 | −0.810 | −4.087 | 0.967 | 0.345 | 1.730 | 0.961* | 0.109 | −1.268 | −1.859 |
| | (−2.710) | (−2.993) | (−0.679) | (−4.124) | (0.967) | (0.406) | (1.777) | (0.992*) | (0.840*) | (−1.136) | (−1.722) |
| | [−2.727] | [−3.019] | [−0.482] | [−4.266] | [0.991] | [0.427] | [1.807] | [0.999*] | [0.961*] | [−1.012] | [−1.806] |

*Note*: Values without brackets are obtained using monthly yields, while values within round and square brackets are obtained using weekly and daily yields, respectively. The decay parameter is fixed at 0.0299 under the OLS method. Estimates of the decay parameter under the SSM are 0.0307, 0.0311 and 0.0310 for monthly, weekly and daily yields, respectively. $\hat{\rho}(1)$ and $\hat{\rho}(12)$ denote sample autocorrelation coefficients of lag orders 1 and 12. ADF unit root test statistics with (i) an intercept, and (ii) a time trend plus an intercept are reported in the last two columns, and they are based on the null hypothesis that the factors have unit roots. Asterisk * indicates rejection of null hypothesis at the 5% level of significance.

and curvature factors, whereas negative asymmetric volatility in the slope factor among the NS-based models. Moreover, it can be seen that the $Q_2$ statistics under the GARCH and EGARCH specifications in Columns 11 and 18 of Tables 4B and 4C have dropped significantly compared with those $Q_2$ under the AR specification in Column 4. Also, there are moderate improvement in the log-likelihood values as we move from AR structure to GARCH structures. Our findings provide evidence of modeling the factor series with GARCH-type structures.

Panel B of Tables 4A–4C report estimation results of the series of yield changes with maturities at 3, 12, 60, 120, and 240 months using daily, weekly, and monthly data. The model specifications include AR(1) in the conditional mean equation, and GARCH(1,1) and EGARCH(1,1,1) structures in the conditional variance equations. As can be observed from Column 2, the autoregressive coefficients are mostly significant at the 5% level for yields of weekly and daily frequencies with maturities at 12, 60, 120, and 240 months, whereas all intercepts are not significant. In addition, the $Q_2$ and $LM$ test statistics in Columns 4 and 5 are mostly significant at the 5% level. These provide support for modeling with the GARCH-type structure in the conditional variance equation.

Columns 7–9 and Columns 13–16 of Panel B of Tables 4A–4C contain estimation results of GARCH(1,1) and EGARCH(1,1,1) specifications. The estimated values of $\alpha_1$ and $\alpha_2$ in the GARCH structure are all significant at the 5% level for yields based on weekly and daily frequencies with various time to maturities. We observe similar patterns for estimated values of $\alpha_1$ and $\alpha_2$ in the EGARCH structure, which are all significant at the 5% level. Moreover, we detect evidence of asymmetric volatility, with positive $\gamma$ for yields with maturities of 60 months and above, and negative $\gamma$ for monthly and daily yields with maturity of 3 months. Apparently, this implies that positive shocks in the yield changes of longer term yields induce higher volatility than negative shocks. Similarly, there are moderate improvement in the log likelihood values as we move from AR structure to GARCH structures. As such, we find support for modeling the series of yield changes with GARCH-type structures.

## 5 | OUT-OF-SAMPLE FORECASTS

In this section, we compare the forecasting performances of the $h$-period-ahead expanding window forecasts by NS-based models and the competitor models. For the NS-based models, we consider three specifications in the conditional mean equation with constant volatility, and two

**TABLE 3** Descriptive statistics for yield residuals.

| Maturities | Mean | Median | Maximum | Minimum | Std. dev. | MAE | RMSE |
|---|---|---|---|---|---|---|---|
| **Panel A: Monthly frequency** | | | | | | | |
| **OLS** ($\hat{\lambda} = 0.0299$) | | | | | | | |
| 3 | 0.781 | 0.784 | 10.141 | −4.942 | 3.389 | 0.028 | 0.035 |
| 12 | −0.341 | 0.333 | 4.314 | −14.774 | 3.190 | 0.022 | 0.032 |
| 60 | −1.959 | −2.431 | 5.102 | −8.300 | 3.175 | 0.032 | 0.037 |
| 120 | 4.583 | 4.891 | 12.682 | −3.347 | 4.389 | 0.052 | 0.063 |
| 240 | −1.146 | −1.278 | 8.244 | −9.706 | 3.764 | 0.032 | 0.039 |
| **NLS** | | | | | | | |
| 3 | −0.443 | −0.732 | 6.358 | −4.361 | 1.950 | 0.016 | 0.020 |
| 12 | 0.456 | 0.916 | 7.628 | −14.489 | 3.460 | 0.025 | 0.035 |
| 60 | −2.057 | −2.577 | 4.785 | −7.742 | 3.008 | 0.031 | 0.036 |
| 120 | 4.023 | 4.299 | 12.553 | −4.028 | 3.994 | 0.047 | 0.057 |
| 240 | −0.714 | −1.080 | 10.295 | −7.832 | 3.726 | 0.031 | 0.038 |
| **SSM** ($\hat{\lambda} = 0.0307$) | | | | | | | |
| 3 | −0.453 | −0.867 | 7.679 | −5.974 | 2.333 | 0.019 | 0.024 |
| 12 | 0.617 | 1.531 | 6.839 | −14.659 | 3.459 | 0.027 | 0.035 |
| 60 | −0.336 | −0.175 | 6.717 | −5.956 | 3.081 | 0.026 | 0.031 |
| 120 | 2.650 | 2.680 | 14.014 | −8.085 | 4.899 | 0.044 | 0.055 |
| 240 | −6.335 | −7.080 | 18.264 | −29.631 | 11.169 | 0.108 | 0.128 |
| **Panel B: Weekly frequency** | | | | | | | |
| **OLS** ($\hat{\lambda} = 0.0299$) | | | | | | | |
| 3 | 0.838 | 0.847 | 10.375 | −7.298 | 3.395 | 0.028 | 0.035 |
| 12 | −0.267 | 0.343 | 5.325 | −14.774 | 3.132 | 0.023 | 0.031 |
| 60 | −1.906 | −2.291 | 5.545 | −9.481 | 3.211 | 0.031 | 0.037 |
| 120 | 4.355 | 5.036 | 14.307 | −4.926 | 4.313 | 0.052 | 0.061 |
| 240 | −1.206 | −1.551 | 9.915 | −9.706 | 3.783 | 0.032 | 0.040 |
| **NLS** | | | | | | | |
| 3 | −0.505 | −0.708 | 6.435 | −4.464 | 1.835 | 0.015 | 0.019 |
| 12 | 0.599 | 1.108 | 8.756 | −14.489 | 3.227 | 0.025 | 0.033 |
| 60 | −2.016 | −2.425 | 5.210 | −9.001 | 3.012 | 0.030 | 0.036 |
| 120 | 3.719 | 3.914 | 12.796 | −6.142 | 4.015 | 0.045 | 0.055 |
| 240 | −0.718 | −1.250 | 12.263 | −8.552 | 3.758 | 0.031 | 0.038 |
| **SSM** ($\hat{\lambda} = 0.0311$) | | | | | | | |
| 3 | −0.635 | −1.027 | 10.732 | −5.969 | 2.119 | 0.018 | 0.022 |
| 12 | 0.652 | 1.391 | 7.534 | −14.574 | 3.513 | 0.027 | 0.036 |
| 60 | −0.506 | −0.562 | 8.318 | −7.314 | 3.084 | 0.027 | 0.031 |
| 120 | 2.071 | 2.336 | 18.175 | −10.211 | 4.534 | 0.040 | 0.050 |
| 240 | −6.371 | −6.849 | 19.117 | −34.530 | 10.805 | 0.105 | 0.125 |
| **Panel C: Daily frequency** | | | | | | | |
| **OLS** ($\hat{\lambda} = 0.0299$) | | | | | | | |
| 3 | 0.661 | 0.627 | 11.317 | −8.155 | 3.256 | 0.027 | 0.033 |
| 12 | −0.247 | 0.356 | 5.336 | −14.774 | 3.096 | 0.023 | 0.031 |
| 60 | −1.857 | −2.273 | 5.769 | −9.481 | 3.212 | 0.031 | 0.037 |

(Continues)

TABLE 3 (Continued)

| Maturities | Mean | Median | Maximum | Minimum | Std. dev. | MAE | RMSE |
|---|---|---|---|---|---|---|---|
| 120 | 4.230 | 4.929 | 14.307 | −5.208 | 4.311 | 0.051 | 0.060 |
| 240 | −1.324 | −1.632 | 10.188 | −9.706 | 3.706 | 0.032 | 0.039 |
| **NLS** | | | | | | | |
| 3 | −0.461 | −0.638 | 7.980 | −4.755 | 1.874 | 0.016 | 0.019 |
| 12 | 0.498 | 0.988 | 8.756 | −14.489 | 3.144 | 0.025 | 0.032 |
| 60 | −1.971 | −2.331 | 5.756 | −9.001 | 2.993 | 0.030 | 0.036 |
| 120 | 3.714 | 3.798 | 13.524 | −6.142 | 4.001 | 0.045 | 0.055 |
| 240 | −0.886 | −1.319 | 12.473 | −8.552 | 3.603 | 0.030 | 0.037 |
| **SSM** ($\hat{\lambda} = 0.0310$) | | | | | | | |
| 3 | −0.758 | −1.061 | 12.340 | −10.688 | 2.159 | 0.018 | 0.023 |
| 12 | 0.670 | 1.439 | 7.907 | −15.567 | 3.473 | 0.028 | 0.035 |
| 60 | −0.431 | −0.455 | 9.330 | −7.577 | 3.169 | 0.027 | 0.031 |
| 120 | 1.995 | 2.099 | 17.819 | −9.444 | 4.185 | 0.037 | 0.046 |
| 240 | −6.539 | −7.663 | 18.663 | −40.651 | 10.527 | 0.103 | 0.124 |

*Note*: This table presents various statistics that describe the in-sample fit by the NS-based models, with two-step OLS, NLS and state space method (SSM) of estimation. MAE stands for mean absolute error and RMSE stands for root mean square error. The summary statistics including mean, median, maximum, minimum and standard deviation of residuals are reported in percentages. MAE and RMSE are reported in real numbers.

specifications in the conditional mean equation with time-varying conditional volatility. As for the competitor models, we consider three specifications in the conditional mean equation with constant volatility, and four specifications in the conditional mean equation with time-varying volatility. They are described briefly as follows:

NS-based models

1. RW specification $\hat{y}_{t+h}(\tau) = \hat{\beta}_{1,t+h} + \hat{\beta}_{2,t+h}\left(\frac{1-e^{-\hat{\lambda}\tau}}{\hat{\lambda}\tau}\right) + \hat{\beta}_{3,t+h}\left(\frac{1-e^{-\hat{\lambda}\tau}}{\hat{\lambda}\tau} - e^{-\hat{\lambda}\tau}\right)$, where $\hat{\beta}_{i,t+h} = \hat{\beta}_{i,t+h-1}$, for $i = 1,2,3$. For convenience, we denote models under the RW specification as OLS-RW and NLS-RW.
2. AR(1) specification $\hat{y}_{t+h}(\tau) = \hat{\beta}_{1,t+h} + \hat{\beta}_{2,t+h}\left(\frac{1-e^{-\hat{\lambda}\tau}}{\hat{\lambda}\tau}\right) + \hat{\beta}_{3,t+h}\left(\frac{1-e^{-\hat{\lambda}\tau}}{\hat{\lambda}\tau} - e^{-\hat{\lambda}\tau}\right)$, $\Delta\hat{\beta}_{i,t+h} = \hat{\mu}_i + \hat{\phi}_i \Delta\hat{\beta}_{i,t+h-1}$ where $\Delta\hat{\beta}_{i,t+h} = \hat{\beta}_{i,t+h} - \hat{\beta}_{i,t+h-1}$, for $i = 1,2,3$. The NS-based models with AR(1) specification are denoted as OLS-AR and NLS-AR.
3. VAR(1) specification $\hat{y}_{t+h}(\tau) = \hat{\beta}_{1,t+h} + \hat{\beta}_{2,t+h}\left(\frac{1-e^{-\hat{\lambda}\tau}}{\hat{\lambda}\tau}\right) + \hat{\beta}_{3,t+h}\left(\frac{1-e^{-\hat{\lambda}\tau}}{\hat{\lambda}\tau} - e^{-\hat{\lambda}\tau}\right)$, $\Delta\hat{\beta}_{t+h} = \hat{\mu} + \hat{\Phi}\Delta\hat{\beta}_{t+h-1}$ where $\Delta\hat{\beta}_{t+h} = [\hat{\beta}_{1,t+h} - \hat{\beta}_{1,t+h-1}, \hat{\beta}_{2,t+h} - \hat{\beta}_{2,t+h-1}, \hat{\beta}_{3,t+h} - \hat{\beta}_{3,t+h-1}]'$, $\hat{\mu}$ is a 3 x 1 vector and $\hat{\Phi}$ is a 3 x 3 matrix of constants. The NS-based models with VAR(1) specification are denoted as OLS-VAR and NLS-VAR.
4. GARCH(1,1) specification $\hat{y}_{t+h}(\tau) = \hat{\beta}_{1,t+h} + \hat{\beta}_{2,t+h}\left(\frac{1-e^{-\hat{\lambda}\tau}}{\hat{\lambda}\tau}\right) + \hat{\beta}_{3,t+h}\left(\frac{1-e^{-\hat{\lambda}\tau}}{\hat{\lambda}\tau} - e^{-\hat{\lambda}\tau}\right)$, with the conditional mean equation: $\Delta\hat{\beta}_{i,t+h} = \mu_i + \phi_i \Delta\hat{\beta}_{i,t+h-1} + \epsilon_{i,t}$, where $\epsilon_{i,t} \sim \mathcal{N}(0, \sigma^2_{i,t})$, and the conditional variance equation: $\sigma^2_{i,t} = \omega_i + \alpha_{1,i}\epsilon^2_{i,t-1} + \alpha_{2,i}\sigma^2_{i,t-1}$, $i = 1,2,3$. We denote models with AR(1) and GARCH(1,1) structures as OLS-AR-GARCH, NLS-AR-GARCH, and SSM-AR-GARCH. For those with only a constant in the mean equation, we denote them as OLS-GARCH, NLS-GARCH, and SSM-GARCH, respectively.
5. EGARCH(1,1,1) specification $\hat{y}_{t+h}(\tau) = \hat{\beta}_{1,t+h} + \hat{\beta}_{2,t+h}\left(\frac{1-e^{-\hat{\lambda}\tau}}{\hat{\lambda}\tau}\right) + \hat{\beta}_{3,t+h}\left(\frac{1-e^{-\hat{\lambda}\tau}}{\hat{\lambda}\tau} - e^{-\hat{\lambda}\tau}\right)$, with the conditional mean equation: $\Delta\hat{\beta}_{i,t+h} = \mu_i + \phi_i \Delta\hat{\beta}_{i,t+h-1} + \epsilon_{i,t}$, where $\epsilon_{i,t} \sim \mathcal{N}(0, \sigma^2_{i,t})$, and the conditional variance equation: $\ln(\sigma^2_{i,t}) = \omega_i + \alpha_{1,i}\left|\frac{\epsilon_{i,t-1}}{\sigma_{i,t-1}}\right| + \alpha_{2,i}\ln(\sigma^2_{i,t-1}) + \gamma_i\left(\frac{\epsilon_{i,t-1}}{\sigma_{i,t-1}}\right)$, for $i = 1,2,3$.
Similar to the GARCH specification, we denote models with AR(1) and EGARCH(1,1,1) structures as OLS-AR-EGARCH, NLS-AR-EGARCH, and SSM-AR-EGARCH. For those with only a constant in the mean equation, we denote them as OLS-EGARCH, NLS-EGARCH, and SSM-EGARCH, respectively.

The competitor models with specifications of constant volatility and time-varying volatility are listed as follows:

Constant volatility

(a) RW on yield levels $\hat{y}_{t+h}(\tau) = \hat{y}_{t+h-1}(\tau)$.

**TABLE 4A** Estimation results of models using monthly yield data.

| | AR(1) specification | | | | | GARCH(1,1) specification | | | | | | EGARCH(1,1,1) specification | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Factors | $\hat{\phi}$ | $Q_1$ | $Q_2$ | LM | LL | $\hat{\omega}$ | $\hat{\alpha}_1$ | $\hat{\alpha}_2$ | $Q_1$ | $Q_2$ | LL | $\hat{\omega}$ | $\hat{\alpha}_1$ | $\hat{\alpha}_2$ | $\hat{\gamma}$ | $Q_1$ | $Q_2$ | LL |
| **Panel A: NS-based models** | | | | | | | | | | | | | | | | | | |
| $\hat{\beta}_{1,t}^{OLS}$ | 0.0429 | 9.80 | 10.45 | 9.71 | 28.63 | 0.0055 | 0.3094 | 0.5343 | 12.99 | 6.42 | 33.24 | −0.6986 | 0.5420 | 0.8029 | 0.0166 | 12.85 | 6.34 | 33.70 |
| | (0.1006) | | | | | (0.0049) | (0.1766) | (0.2137) | | | | (0.6109) | (0.2606) | (0.1739) | (0.1194) | | | |
| $\hat{\beta}_{2,t}^{OLS}$ | −0.0531 | 8.71 | 7.40 | 6.37 | 21.32 | 0.0148 | 0.3006 | 0.3143 | 11.08 | 8.47 | 24.32 | −3.4503 | −0.1653 | −0.0071 | −0.4785 | 7.06 | 8.19 | 27.46 |
| | (0.1053) | | | | | (0.0145) | (0.1867) | (0.4129) | | | | (0.9422) | (0.2993) | (0.2714) | (0.1775) | | | |
| $\hat{\beta}_{3,t}^{OLS}$ | 0.0312 | 6.88 | 11.49 | 10.85 | 45.13 | 0.0162 | 0.2614 | 0.6632 | 7.67 | 6.57 | 49.49 | −0.2545 | 0.4613 | 0.8632 | 0.0955 | 7.22 | 5.21 | 50.28 |
| | (0.0829) | | | | | (0.0097) | (0.1592) | (0.1501) | | | | (0.1434) | (0.2012) | (0.0632) | (0.1252) | | | |
| $\hat{\beta}_{1,t}^{NLS}$ | −0.0566 | 14.27 | 16.46 | 14.54 | 23.30 | 0.0058 | 0.2817 | 0.5734 | 13.22 | 6.51 | 29.22 | −0.6320 | 0.4607 | 0.8125 | 0.0162 | 13.63 | 8.12 | 28.63 |
| | (0.0986) | | | | | (0.0034) | (0.1458) | (0.1416) | | | | (0.4145) | (0.2092) | (0.1222) | (0.1119) | | | |
| $\hat{\beta}_{2,t}^{NLS}$ | −0.1360 | 12.93 | 11.86 | 9.97 | 20.38 | 0.0076 | 0.2541 | 0.5629 | 12.05 | 7.53 | 24.92 | −4.7401 | 0.3968 | −0.4179 | −0.1915 | 13.39 | 14.94 | 24.96 |
| | (0.1040) | | | | | (0.0041) | (0.1337) | (0.1642) | | | | (0.8090) | (0.2166) | (0.2328) | (0.1763) | | | |
| $\hat{\beta}_{3,t}^{NLS}$ | −0.3937 | 6.92 | 14.25 | 13.03 | 70.63 | 0.0184 | 0.3848 | 0.6151 | 13.22 | 8.05 | 84.12 | −1.7238 | 0.4884 | 0.0610 | 0.7411 | 7.82 | 8.61 | 91.17 |
| | (0.0618) | | | | | (0.0065) | (0.1001) | (0.0611) | | | | (0.3458) | (0.1756) | (0.1800) | (0.1323) | | | |
| $\hat{\beta}_{1,t}^{SSM}$ | 0.1383 | 6.24 | 16.55 | 17.29 | 24.60 | 0.0205 | 0.2596 | 0.1257 | 7.43 | 9.95 | 29.02 | −3.1699 | −0.6730 | 0.0915 | 0.4982 | 8.83 | 15.50 | 30.24 |
| | (0.1244) | | | | | (0.0171) | (0.3317) | (0.4857) | | | | (1.2126) | (0.3826) | (0.3372) | (0.2017) | | | |
| $\hat{\beta}_{2,t}^{SSM}$ | 0.0384 | 7.06 | 11.28 | 10.98 | 16.40 | 0.0208 | 0.1814 | 0.3305 | 8.17 | 15.66 | 17.22 | −6.2219 | −0.2027 | −0.9326 | −0.0549 | 10.13 | 9.00 | 17.73 |
| | (0.1224) | | | | | (0.0231) | (0.1637) | (0.6487) | | | | (0.6584) | (0.2124) | (0.1729) | (0.1039) | | | |
| $\hat{\beta}_{3,t}^{SSM}$ | 0.2681 | 11.08 | 7.76 | 6.57 | 6.38 | 0.0084 | 0.2085 | 0.6935 | 14.09 | 6.43 | 8.23 | −8.5735 | −0.0888 | −1.1427 | −0.0709 | 10.35 | 9.26 | 9.35 |
| | (0.1105) | | | | | (0.0064) | (0.1280) | (0.1647) | | | | (0.4648) | (0.2452) | (0.0265) | (0.0857) | | | |
| **Panel B: Competitor models** | | | | | | | | | | | | | | | | | | |
| $y_t(3)$ | 0.0384 | 10.55 | 9.53 | 10.56 | 149.35 | 0.0366 | 0.4623 | 0.5376 | 15.36 | 7.34 | 158.82 | −4.7665 | 0.4863 | 0.2173 | −0.7078 | 10.13 | 3.52 | 162.83 |
| | (0.1056) | | | | | (0.0117) | (0.1585) | (0.1100) | | | | (1.6919) | (0.2225) | (0.2609) | (0.1746) | | | |
| $y_t(12)$ | 0.0943 | 8.44 | 28.33* | 23.06* | 137.32 | 0.1041 | 0.4233 | 0.5756 | 9.59 | 16.27 | 178.66 | −0.3801 | 0.4932 | 0.9530 | 0.9004 | 12.40 | 15.62 | 196.52 |
| | (0.0815) | | | | | (0.0399) | (0.1846) | (0.1390) | | | | (0.0886) | (0.1096) | (0.0068) | (0.1923) | | | |
| $y_t(60)$ | 0.0266 | 4.36 | 6.73 | 6.41 | 76.69 | 5.6570 | 0.0013 | 0.7332 | 4.36 | 6.70 | 80.55 | −0.5165 | 0.4808 | 0.8931 | 0.1177 | 6.74 | 9.60 | 85.95 |
| | (0.1340) | | | | | (2.1451) | (0.0781) | (0.2730) | | | | (0.0080) | (0.1232) | (0.0001) | (0.0506) | | | |
| $y_t(120)$ | −0.0128 | 5.90 | 9.89 | 9.03 | 64.33 | 3.3170 | 0.0970 | 0.6899 | 5.81 | 13.85 | 64.82 | −2.7463 | 0.2287 | 0.3605 | 0.3240 | 7.55 | 14.59 | 67.43 |
| | (0.1374) | | | | | (4.4230) | (0.0674) | (0.3284) | | | | (2.4896) | (0.3947) | (0.5730) | (0.2320) | | | |
| $y_t(240)$ | 0.0923 | 8.57 | 13.94 | 12.45 | 62.13 | 2.0557 | 0.1545 | 0.7150 | 8.96 | 13.04 | 64.17 | −5.1916 | 0.2140 | 0.5593 | 0.4618 | 7.46 | 15.49 | 67.85 |
| | (0.1072) | | | | | (3.0100) | (0.1266) | (0.2750) | | | | (1.1264) | (0.2622) | (0.2792) | (0.1946) | | | |

*Note*: Estimates are obtained using monthly yields over the full sample. For the NS-based models, the decay parameter is fixed at 0.0299 for the OLS estimation method and at 0.0307 for the state-space estimation method. The reported $\hat{\omega}$ under the GARCH(1,1) specification in Panel B are multiplied by 1000. Standard errors of the parameter estimates are reported in parenthesis. $Q_1$ and $Q_2$ denote the Ljung–Box $Q$ statistics of the standardized residuals and squares of standardized residuals based on autocorrelation coefficients of order up to 10. LM denotes the Engle's ARCH-LM test statistics, and they are based on the null hypothesis that there is no residual heteroskedasticity. LL denotes the log-likelihood estimates. Asterisk * indicates rejection of null hypothesis at the 5% level of significance. The drift estimates in the conditional mean equation of the AR(1) specification in Panels A and B are not reported here as they are statistically insignificant at the 5% level.

**TABLE 4B** Estimation results of models using weekly yield data.

| Factors | AR(1) specification $\hat{\phi}$ | $Q_1$ | $Q_2$ | LM | LL | GARCH(1,1) specification $\hat{\omega}$ | $\hat{\alpha}_1$ | $\hat{\alpha}_2$ | $Q_1$ | $Q_2$ | LL | EGARCH(1,1,1) specification $\hat{\omega}$ | $\hat{\alpha}_1$ | $\hat{\alpha}_2$ | $\hat{\gamma}$ | $Q_1$ | $Q_2$ | LL |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Panel A: NS-based models** | | | | | | | | | | | | | | | | | | |
| $\hat{\beta}_{1,t}^{OLS}$ | 0.0152 | 11.98 | 29.69* | 25.45* | 388.80 | 0.0431 | 0.1237 | 0.8330 | 9.61 | 8.95 | 411.61 | −0.1462 | 0.2332 | 0.9666 | −0.0307 | 9.72 | 9.96 | 412.34 |
| | (0.0394) | | | | | (0.0150) | (0.0201) | (0.0252) | | | | (0.0777) | (0.0353) | (0.0162) | (0.0226) | | | |
| $\hat{\beta}_{2,t}^{OLS}$ | 0.0212 | 13.20 | 31.74* | 28.04* | 374.13 | 0.1620 | 0.1956 | 0.6397 | 13.34 | 13.07 | 394.60 | −0.7481 | 0.3433 | 0.8393 | −0.0562 | 13.20 | 14.12 | 394.67 |
| | (0.0382) | | | | | (0.0441) | (0.0357) | (0.0630) | | | | (0.2212) | (0.0520) | (0.0470) | (0.0352) | | | |
| $\hat{\beta}_{3,t}^{OLS}$ | −0.0206 | 10.83 | 21.42* | 23.21* | 55.73 | 0.1595 | 0.0964 | 0.8711 | 11.30 | 6.68 | 67.52 | −0.1063 | 0.1685 | 0.9660 | 0.0513 | 11.04 | 7.35 | 71.10 |
| | (0.0410) | | | | | (0.0966) | (0.0277) | (0.0368) | | | | (0.0683) | (0.0469) | (0.0212) | (0.0267) | | | |
| $\hat{\beta}_{1,t}^{NLS}$ | −0.0053 | 9.15 | 11.84 | 8.78 | 374.60 | 0.0265 | 0.0592 | 0.9134 | 8.23 | 6.14 | 384.75 | −0.0543 | 0.1269 | 0.9873 | −0.0265 | 8.50 | 6.28 | 386.99 |
| | (0.0467) | | | | | (0.0123) | (0.0184) | (0.0229) | | | | (0.0450) | (0.0326) | (0.0098) | (0.0180) | | | |
| $\hat{\beta}_{2,t}^{NLS}$ | −0.0285 | 12.48 | 26.20* | 23.13* | 353.43 | 0.0348 | 0.0575 | 0.9093 | 10.20 | 7.23 | 363.47 | −0.1040 | 0.1283 | 0.9761 | 0.0019 | 10.06 | 7.29 | 365.26 |
| | (0.0447) | | | | | (0.0165) | (0.0190) | (0.0247) | | | | (0.0595) | (0.0344) | (0.0132) | (0.0251) | | | |
| $\hat{\beta}_{3,t}^{NLS}$ | −0.4251 | 17.62 | 25.90* | 21.36* | 61.58 | 0.0958 | 0.2175 | 0.7823 | 10.40 | 2.15 | 100.41 | −0.1858 | 0.4172 | 0.9416 | 0.0504 | 5.54 | 8.41 | 102.30 |
| | (0.0171) | | | | | (0.0530) | (0.0477) | (0.0340) | | | | (0.0868) | (0.0726) | (0.0259) | (0.0539) | | | |
| $\hat{\beta}_{1,t}^{SSM}$ | 0.3047 | 16.87 | 56.98* | 45.42* | 498.30 | 0.0482 | 0.1141 | 0.7895 | 12.78 | 15.82 | 516.57 | −4.3602 | 0.2034 | 0.8689 | 0.1320 | 13.90 | 12.62 | 519.45 |
| | (0.0390) | | | | | (0.0163) | (0.0286) | (0.0516) | | | | (0.4719) | (0.0758) | (0.1107) | (0.0501) | | | |
| $\hat{\beta}_{2,t}^{SSM}$ | 0.2261 | 14.18 | 54.58* | 44.02* | 453.32 | 0.0912 | 0.1700 | 0.6890 | 16.68 | 12.88 | 466.03 | −4.3130 | 0.1690 | 0.7902 | −0.1697 | 11.49 | 10.33 | 471.38 |
| | (0.0383) | | | | | (0.0318) | (0.0561) | (0.3736) | | | | (0.5662) | (0.0812) | (0.1374) | (0.0574) | | | |
| $\hat{\beta}_{3,t}^{SSM}$ | 0.2163 | 9.59 | 24.93* | 21.58* | 284.08 | 0.0378 | 0.1093 | 0.8693 | 12.59 | 8.99 | 307.32 | −0.1597 | 0.2088 | 0.9606 | 0.0332 | 12.53 | 7.78 | 307.84 |
| | (0.0417) | | | | | (0.0158) | (0.0273) | (0.0290) | | | | (0.0619) | (0.0492) | (0.0143) | (0.0298) | | | |
| **Panel B: Competitor models** | | | | | | | | | | | | | | | | | | |
| $y_t(3)$ | 0.0267 | 17.98 | 68.56* | 40.01* | 962.61 | 0.0002 | 0.2022 | 0.5978 | 9.72 | 3.55 | 1383.48 | −0.0400 | 0.1800 | 0.9998 | 0.4257 | 13.16 | 2.60 | 1461.48 |
| | (0.0261) | | | | | (0.0003) | (0.0070) | (0.0041) | | | | (0.0182) | (0.0263) | (0.0022) | (0.0256) | | | |
| $y_t(12)$ | 0.1017 | 17.70 | 58.36* | 37.72* | 899.91 | 0.0050 | 0.1419 | 0.8580 | 14.22 | 9.32 | 1065.37 | −0.0841 | 0.0716 | 0.9881 | 0.1685 | 16.12 | 5.01 | 1091.46 |
| | (0.0351) | | | | | (0.0009) | (0.0158) | (0.0121) | | | | (0.0204) | (0.0134) | (0.0033) | (0.0125) | | | |
| $y_t(60)$ | −0.0298 | 13.59 | 28.97* | 24.32* | 602.75 | 0.0524 | 0.1496 | 0.8500 | 9.38 | 6.09 | 625.29 | 0.0125 | 0.2435 | 0.9589 | 0.1130 | 10.60 | 4.29 | 630.30 |
| | (0.0437) | | | | | (0.0240) | (0.0254) | (0.0170) | | | | (0.0768) | (0.0434) | (0.0127) | (0.0269) | | | |
| $y_t(120)$ | 0.0037 | 17.87 | 52.53* | 40.96* | 544.38 | 0.4756 | 0.3058 | 0.6136 | 13.44 | 9.19 | 572.09 | −1.1144 | 0.5653 | 0.7997 | 0.0087 | 14.23 | 11.67 | 574.19 |
| | (0.0350) | | | | | (0.1712) | (0.0577) | (0.0638) | | | | (0.3616) | (0.0873) | (0.0646) | (0.0534) | | | |
| $y_t(240)$ | −0.0071 | 13.95 | 35.52* | 32.54* | 504.98 | 0.5901 | 0.1833 | 0.7054 | 14.53 | 15.57 | 527.35 | −0.6534 | 0.3543 | 0.8765 | 0.0297 | 14.94 | 29.32 | 528.47 |
| | (0.0432) | | | | | (0.2073) | (0.0319) | (0.0540) | | | | (0.2076) | (0.0543) | (0.0390) | (0.0345) | | | |

*Note*: Estimates are obtained using weekly yields over the full sample. For the NS-based models, the decay parameter is fixed at 0.0299 for the OLS estimation method and at 0.0311 for the state-space estimation method. The reported $\omega$ estimates under the GARCH(1,1) specification are multiplied by 100 in Panel A and 1000 in Panel B, respectively. Standard errors of the parameter estimates are reported in parenthesis. $Q_1$ and $Q_2$ denote the Ljung–Box $Q$ statistics of the standardized residuals and squares of standardized residuals based on autocorrelation coefficients of order up to 10. LM denotes the Engle's ARCH-LM test statistics, and they are based on the null hypothesis that there is no residual heteroskedasticity. LL denotes the log-likelihood estimates. Asterisk * indicates rejection of null hypothesis at the 5% level of significance. The drift estimates in the conditional mean equation of the AR(1) specification in Panels A and B are not reported here as they are statistically insignificant at the 5% level.

**TABLE 4C** Estimation results of models using daily yield data.

| Factors | AR(1) specification | | | | | GARCH(1,1) specification | | | | | | EGARCH(1,1,1) specification | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\hat{\phi}$ | $Q_1$ | $Q_2$ | LM | LL | $\hat{\omega}$ | $\hat{\alpha}_1$ | $\hat{\alpha}_2$ | $Q_1$ | $Q_2$ | LL | $\hat{\omega}$ | $\hat{\alpha}_1$ | $\hat{\alpha}_2$ | $\hat{\gamma}$ | $Q_1$ | $Q_2$ | LL |
| **Panel A: NS-based models** | | | | | | | | | | | | | | | | | | |
| $\hat{\beta}_{1,t}^{OLS}$ | 0.1363 | 17.98 | 335.82* | 171.23* | 3287.63 | 0.0448 | 0.1186 | 0.8614 | 10.67 | 7.45 | 3512.83 | −0.2057 | 0.2567 | 0.9668 | 0.0210 | 11.65 | 8.15 | 3502.84 |
| | (0.0159) | | | | | (0.0448) | (0.0096) | (0.0092) | | | | (0.0326) | (0.0181) | (0.0050) | (0.0115) | | | |
| $\hat{\beta}_{2,t}^{OLS}$ | 0.1023 | 17.54 | 377.91* | 191.27* | 3150.14 | 0.0599 | 0.1191 | 0.8556 | 14.97 | 15.16 | 3359.69 | −0.2376 | 0.2544 | 0.9614 | −0.0496 | 16.06 | 15.14 | 3363.55 |
| | (0.0163) | | | | | (0.0100) | (0.0093) | (0.0092) | | | | (0.0345) | (0.0170) | (0.0054) | (0.0124) | | | |
| $\hat{\beta}_{3,t}^{OLS}$ | 0.1479 | 2.92 | 132.94* | 89.34* | 1858.57 | 0.1029 | 0.0766 | 0.9141 | 7.21 | 3.63 | 2016.54 | −0.0875 | 0.1661 | 0.9809 | 0.0245 | 8.05 | 3.92 | 2025.75 |
| | (0.0166) | | | | | (0.0227) | (0.0079) | (0.0090) | | | | (0.0199) | (0.0159) | (0.0040) | (0.0075) | | | |
| $\hat{\beta}_{1,t}^{NLS}$ | 0.2012 | 5.34 | 72.18* | 49.43* | 3287.47 | 0.0528 | 0.1137 | 0.8630 | 6.01 | 2.22 | 3476.14 | −0.1732 | 0.2093 | 0.9715 | 0.0145 | 6.52 | 2.70 | 3478.18 |
| | (0.0165) | | | | | (0.0093) | (0.0100) | (0.0120) | | | | (0.0320) | (0.0164) | (0.0050) | (0.0093) | | | |
| $\hat{\beta}_{2,t}^{NLS}$ | 0.1883 | 5.48 | 51.89* | 38.33* | 3139.87 | 0.0350 | 0.0821 | 0.9046 | 4.97 | 2.33 | 3326.77 | −0.1036 | 0.1607 | 0.9822 | −0.0269 | 5.43 | 3.29 | 3330.38 |
| | (0.0161) | | | | | (0.0067) | (0.0081) | (0.0089) | | | | (0.0212) | (0.0143) | (0.0035) | (0.0083) | | | |
| $\hat{\beta}_{3,t}^{NLS}$ | −0.2046 | 6.32 | 91.50* | 74.69* | 939.79 | 0.2585 | 0.0759 | 0.9240 | 17.75 | 0.46 | 1199.59 | −0.0056 | 0.1862 | 0.9928 | 0.0568 | 17.24 | 0.87 | 1201.27 |
| | (0.0060) | | | | | (0.0186) | (0.0052) | (0.0043) | | | | (0.0063) | (0.0090) | (0.0015) | (0.0085) | | | |
| $\hat{\beta}_{1,t}^{SSM}$ | 0.3503 | 18.30 | 530.90* | 222.23* | 4154.25 | 0.0174 | 0.1702 | 0.2780 | 12.66 | 11.14 | 4344.29 | −0.3902 | 0.3189 | 0.4991 | 0.1012 | 11.69 | 10.41 | 4351.99 |
| | (0.0170) | | | | | (0.0043) | (0.0174) | (0.1144) | | | | (0.0681) | (0.0313) | (0.1181) | (0.0216) | | | |
| $\hat{\beta}_{2,t}^{SSM}$ | 0.2599 | 16.79 | 503.51* | 226.20* | 3759.09 | 0.0219 | 0.1791 | 0.3106 | 12.24 | 12.40 | 3962.76 | −0.3628 | 0.3319 | 0.5726 | −0.0917 | 12.49 | 9.11 | 3969.76 |
| | (0.0166) | | | | | (0.0053) | (0.0177) | (0.1198) | | | | (0.0594) | (0.0333) | (0.1244) | (0.0217) | | | |
| $\hat{\beta}_{3,t}^{SSM}$ | 0.3516 | 11.90 | 386.47* | 193.59* | 2947.79 | 0.0224 | 0.1182 | 0.8813 | 11.63 | 4.67 | 3203.23 | −0.1025 | 0.2173 | 0.9798 | 0.0101 | 11.17 | 4.58 | 3210.28 |
| | (0.0160) | | | | | (0.0066) | (0.0306) | (0.2634) | | | | (0.0275) | (0.0449) | (0.0036) | (0.0102) | | | |
| **Panel B: Competitor models** | | | | | | | | | | | | | | | | | | |
| $y_t(3)$ | −0.0109 | 15.35 | 21.59* | 17.08 | 5849.78 | 0.0030 | 0.3595 | 0.6403 | 6.24 | 3.54 | 6398.37 | −0.5330 | 0.3522 | 0.9335 | −0.0532 | 14.51 | 6.91 | 6637.97 |
| | (0.0112) | | | | | (0.0014) | (0.0044) | (0.0155) | | | | (0.0640) | (0.0326) | (0.0058) | (0.0240) | | | |
| $y_t(12)$ | 0.0845 | 6.27 | 8.33 | 7.50 | 5888.49 | 0.0034 | 0.0420 | 0.9572 | 14.59 | 5.86 | 6477.74 | −0.0425 | 0.5234 | 0.9932 | −0.0724 | 14.01 | 1.14 | 6674.53 |
| | (0.0140) | | | | | (0.0008) | (0.0036) | (0.0007) | | | | (0.0036) | (0.0235) | (0.0005) | (0.0130) | | | |
| $y_t(60)$ | 0.0413 | 15.01 | 93.51* | 84.80* | 4129.23 | 0.0521 | 0.1010 | 0.8964 | 12.32 | 9.73 | 4331.27 | −0.1333 | 0.2050 | 0.9813 | 0.2002 | 14.12 | 5.25 | 4355.48 |
| | (0.0180) | | | | | (0.0141) | (0.0132) | (0.0101) | | | | (0.0219) | (0.0356) | (0.0029) | (0.0289) | | | |
| $y_t(120)$ | 0.0203 | 15.80 | 57.71* | 43.17* | 3847.76 | 0.2137 | 0.0917 | 0.8893 | 13.90 | 14.61 | 4010.87 | −0.1770 | 0.2080 | 0.9741 | 0.1626 | 16.77 | 11.36 | 4014.73 |
| | (0.0179) | | | | | (0.0434) | (0.0127) | (0.0106) | | | | (0.0364) | (0.0400) | (0.0051) | (0.0283) | | | |
| $y_t(240)$ | −0.0815 | 13.23 | 71.53* | 48.14* | 3711.42 | 0.2592 | 0.0985 | 0.8808 | 12.30 | 8.31 | 3900.45 | −0.1421 | 0.2577 | 0.9787 | 0.1194 | 12.82 | 6.91 | 3906.94 |
| | (0.0176) | | | | | (0.0536) | (0.0115) | (0.0118) | | | | (0.0326) | (0.0303) | (0.0048) | (0.0223) | | | |

*Note*: Estimates are obtained using daily yields over the full sample. For the NS-based models, the decay parameter is fixed at 0.0299 for the OLS estimation method and at 0.0310 for the state-space estimation method. The reported $\omega$ estimates under the GARCH(1,1) specification are multiplied by 1000 in Panel A and 10000 in Panel B, respectively. Standard errors of the parameter estimates are reported in parenthesis. $Q_1$ and $Q_2$ denote the Ljung–Box $Q$ statistics of the standardized residuals and squares of standardized residuals based on autocorrelation coefficients of order up to 10. LM denotes the Engle's ARCH-LM test statistics, and they are based on the null hypothesis that there is no residual heteroskedasticity. LL denotes the log-likelihood estimates. Asterisk * indicates rejection of null hypothesis at the 5% level of significance. The drift estimates in the conditional mean equation of the AR(1) specification in Panels A and B are not reported here as they are statistically insignificant at the 5% level.

(b) AR(1) on yield changes (AR) $\Delta\hat{y}_{t+h}(\tau) = \hat{\mu}(\tau) + \hat{\phi}(\tau)\Delta\hat{y}_{t+h-1}(\tau)$.
(c) VAR(1) on yield changes (VAR) $\Delta\hat{Y}_{t+h} = \hat{\mu} + \hat{\Phi}\Delta\hat{Y}_{t+h-1}$ for $\Delta Y_t = [y_t(3) - y_{t-1}(3), \quad ..., y_t(360) - y_{t-1}(360)]', \hat{\mu}$ is a 15 × 1 vector and $\hat{\Phi}$ is a 15 × 15 matrix.

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(d) GARCH(1,1) on yield changes (GARCH-C) $\Delta y_{t+h}(\tau) = \mu(\tau) + \epsilon_{t+h}(\tau), \epsilon_{t+h}(\tau) \sim \mathcal{N}(0, \sigma^2_{t+h}(\tau))$, where $\sigma^2_{t+h}(\tau) = \omega(\tau) + \alpha_1(\tau)\epsilon^2_{t+h-1}(\tau) + \alpha_2(\tau)\sigma^2_{t+h-1}(\tau)$.
(e) GARCH(1,1) with AR(1) in mean on yield changes (AR-GARCH) $\Delta y_{t+h}(\tau) = \mu(\tau) + \phi(\tau)\Delta yt + h - 1(\tau) + \epsilon_{t+h}(\tau), \epsilon_{t+h}(\tau) \sim \mathcal{N}(0, \sigma^2_{t+h}(\tau))$, where $\sigma^2_{t+h}(\tau) = \omega(\tau) + \alpha_1(\tau)\epsilon^2_{t+h-1}(\tau) + \alpha_2(\tau)\sigma^2_{t+h-1}(\tau)$.
(f) EGARCH(1,1,1) on yield changes (EGARCH) $\Delta y_{t+h}(\tau) = \mu(\tau) + \epsilon_{t+h}(\tau), \epsilon_{t+h}(\tau) \sim \mathcal{N}(0, \sigma^2_{t+h}(\tau))$, where $\ln(\sigma^2_{t+h}(\tau)) = \omega(\tau) + \alpha 1(\tau)\epsilon t + h - 1(\tau)\sigma t + h - 1(\tau) + \alpha_2(\tau)\ln(\sigma^2_{t+h-1}(\tau)) + \gamma(\tau)\left(\frac{\epsilon_{t+h-1}(\tau)}{\sigma_{t+h-1}(\tau)}\right)$.
(g) EGARCH(1,1,1) with AR(1) in mean on yield changes (AR-EGARCH-C) $\Delta y_{t+h}(\tau) = \mu(\tau) + \phi(\tau)\Delta y_{t+h-1}(\tau) + \epsilon t + h(\tau), \ \epsilon_{t+h}(\tau) \sim \mathcal{N}(0, \sigma^2_{t+h}(\tau))$, where $\ln(\sigma^2_{t+h}(\tau)) = \omega(\tau) + \alpha 1(\tau)\epsilon t + h - 1(\tau)\sigma t + h - 1(\tau) + \alpha_2(\tau)\ln(\sigma^2_{t+h-1}(\tau)) + \gamma(\tau)\left(\frac{\epsilon_{t+h-1}(\tau)}{\sigma_{t+h-1}(\tau)}\right)$.

Steps taken to compute the $h$-step-ahead forecast by the expanding window approach are highlighted as below:

1. For each of the daily, weekly, and monthly yield curves, set the initial window from the date of the first data to half of the full sample size.

2. Fit a model (either the NS-based model or the competitor model) under study to the dataset generated in Step 1. To reduce computational burden, fix the decay parameters at 0.0299 and 0.0314 for the NS-based models with OLS and NLS estimations, and (0.0310,0.0311,0.0307) for the SSM with daily, weekly, and monthly data,[5] respectively.

3. Obtain the $h$-period ahead forecasts of yields using estimation results from Step 2. For daily data, set $h = 1,2,3$ and 5 days; for weekly data, set $h = 1,2,3$, and 4 weeks; and for monthly data, set $h = 1,3,6$, and 12 months.

4. Expand the in-sample fitting window by one period, thereby increasing the sample size by 1 unit. Repeat Steps 1–3 to obtain new $h$-step ahead forecasts based on the expanded window.

5. Repeat Step 4 recursively until the appropriate terminal of the out-of-sample period.

## 5.1 | Forecasting results

The MAE and RMSE residual statistics are employed to evaluate the forecasting performances of the $h$-step-ahead forecasts of yields by both the NS-based and competitor models. For each of the daily, weekly, and monthly frequency windows, we obtain the $h$-step-ahead forecasts of yields at maturities of 3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240, and 360 months, respectively. For the daily window, we set $h = 1,2,3$, and 5 days; for the weekly window, $h = 1,2,3$, and 4 weeks; and for monthly window, we set $h = 1,3,6$, and 12 months. We mention in passing that the AR(1) and VAR(1) structures are largely adequate for the conditional mean equation and the GARCH(1,1) and EGARCH(1,1,1) structures are adequate for the conditional variance equation in the NS-based and competitor models.[6] Because of space limitation, we report selectively in Tables 5–7 the out-of-sample forecasting results at two forecasting horizons by frequency window for yields with maturities of 3, 12, 60, 120, and 240 months only. Full results are available upon request.

Constant volatility

Panel A of Tables 5–7 report the $h$-step-ahead out-of-sample forecasting results by models with constant volatility for monthly, weekly, and daily data. As indicated by MAE and RMSE measures, there is supporting evidence that the NS-based models do not perform as good as the competitor models, especially for 1- and 3-day-ahead forecasts and 1- and 4-week-ahead forecasts, during the period of prolonged liquidity trap in the Japanese bond market. However, there are improvements in the NS-based models as the forecast horizon lengthens, especially by the NLS method using the monthly data. Our findings are in contrast to some studies (such as Diebold & Li, 2006, and Nyholm & Vidova-Koleva, 2012) that the NS-based models outperform the RW when liquidity trap is not present. However, our results are largely consistent with respect to Steeley (2014), who reports that simple competitor models (in particular, the RW model) seem to provide better out-of-sample forecasts in the presence of liquidity trap.

Among the NS-based models, performances by the OLS method of estimation with the decay parameter fixed at 0.0299 is ranked the first for most of the monthly, weekly and daily frequency windows, whereas performances by SSM is ranked the lowest. In contrast, as the 'best' performer in in-sample fitting, the NLS method performs inferiorly as compared with the two-step OLS estimation method on weekly and daily forecasts, though the NLS forecasts are slightly better than the latter with the decay parameter set to 0.0299. Our findings indicate that the NS-RW model tends to provide relatively better forecasting results, while the NS-AR and the NS-VAR specifications compete similarly in most of the cases.

Among the competitors models, the RW model outperforms the AR and VAR models for better short-

**TABLE 5** Out-of-sample forecasting results using monthly yield data.

| Panel A: Constant volatility | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 6-month-ahead forecast | | | | | | | 12-month-ahead forecast | | | | | | |
| | AMAE | ARMSE | RMSE | | | | | AMAE | ARMSE | RMSE | | | | |
| | | | 3-mth | 12-mth | 60-mth | 120-mth | 240-mth | | | 3-mth | 12-mth | 60-mth | 120-mth | 240-mth |
| *Nelson–Siegel-type models* | | | | | | | | | | | | | | |
| OLS–RW | 0.1835 | 0.2387 | 0.1651 | **0.1385** | 0.2831 | 0.2620 | 0.2086 | 0.2432 | 0.3250 | 0.2829 | 0.2631 | 0.4073 | 0.3199 | 0.2100 |
| OLS–AR | 0.1945 | 0.2538 | 0.1611 | 0.1398 | 0.3028 | 0.2859 | 0.2289 | 0.2721 | 0.3591 | 0.2772 | 0.2703 | 0.4543 | 0.3683 | 0.2495 |
| OLS–VAR | 0.1963 | 0.2574 | 0.1538 | 0.1396 | 0.3086 | 0.2942 | 0.2337 | 0.2796 | 0.3671 | 0.2641 | 0.2704 | 0.4701 | 0.3827 | 0.2547 |
| NLS–RW | 0.1993 | 0.2576 | 0.1581 | 0.1525 | 0.2950 | 0.2973 | 0.2389 | **0.2310** | **0.2938** | 0.2797 | 0.2709 | **0.3677** | **0.2700** | **0.1632** |
| NLS–AR | 0.2011 | 0.2625 | 0.1524 | 0.1466 | 0.3016 | 0.3070 | 0.2494 | 0.2382 | 0.3041 | 0.2751 | 0.2591 | 0.3790 | 0.2896 | 0.1889 |
| NLS–VAR | 0.2070 | 0.2707 | 0.1472 | 0.1544 | 0.3107 | 0.3192 | 0.2629 | 0.2365 | 0.3073 | 0.2674 | **0.2561** | 0.3801 | 0.2979 | 0.2036 |
| SSM | 0.2028 | 0.2672 | 0.1396 | 0.1419 | 0.3094 | 0.2960 | 0.2890 | 0.2969 | 0.3928 | **0.2574** | 0.2711 | 0.4729 | 0.4110 | 0.3583 |
| *Competitor Models* | | | | | | | | | | | | | | |
| RW | **0.1812** | **0.2361** | 0.1450 | 0.1620 | **0.2767** | **0.2483** | **0.2016** | 0.2398 | 0.3249 | 0.2646 | 0.2862 | 0.4070 | 0.3076 | 0.1957 |
| AR | 0.1874 | 0.2471 | **0.1395** | 0.1590 | 0.2919 | 0.2642 | 0.2230 | 0.2636 | 0.3540 | 0.2606 | 0.2859 | 0.4479 | 0.3450 | 0.2382 |
| VAR | 0.2073 | 0.2706 | 0.1463 | 0.1698 | 0.3210 | 0.2945 | 0.2451 | 0.2958 | 0.3839 | 0.2653 | 0.3019 | 0.4855 | 0.3784 | 0.2661 |
| **Panel B: Time-varying volatility** | | | | | | | | | | | | | | |
| | 6-month-ahead forecast | | | | | | | 12-month-ahead forecast | | | | | | |
| | AMAE | ARMSE | RMSE | | | | | AMAE | ARMSE | RMSE | | | | |
| | | | 3-mth | 12-mth | 60-mth | 120-mth | 240-mth | | | 3-mth | 12-mth | 60-mth | 120-mth | 240-mth |
| *Nelson–Siegel-type models* | | | | | | | | | | | | | | |
| OLS–GARCH | 0.2020 | 0.2673 | 0.1632 | 0.1486 | 0.3172 | 0.3090 | 0.2430 | 0.3114 | 0.4003 | 0.2842 | 0.2978 | 0.5035 | 0.4270 | 0.2906 |
| OLS–AR–GARCH | 0.2053 | 0.2697 | 0.1680 | 0.1510 | 0.3169 | 0.3133 | 0.2485 | 0.3235 | 0.4083 | 0.2889 | 0.3024 | 0.5097 | 0.4385 | 0.3041 |
| OLS–EGARCH | 0.2112 | 0.2743 | 0.1859 | 0.1708 | 0.3246 | 0.3084 | 0.2360 | 0.3323 | 0.4150 | 0.3269 | 0.3376 | 0.5214 | 0.4292 | 0.2764 |
| OLS–AR–EGARCH | 0.2036 | 0.2643 | 0.1777 | 0.1659 | 0.3118 | 0.2923 | 0.2344 | 0.3164 | 0.4141 | 0.3225 | 0.3321 | 0.5109 | 0.4254 | 0.3057 |
| NLS–GARCH | 0.2238 | 0.2924 | 0.1549 | 0.1644 | 0.3319 | 0.3536 | 0.2927 | 0.3380 | 0.4079 | 0.2836 | 0.3063 | 0.4928 | 0.4402 | 0.3371 |
| NLS–AR–GARCH | 0.2434 | 0.3138 | 0.1600 | 0.1802 | 0.3570 | 0.3802 | 0.3157 | 0.3809 | 0.4471 | 0.2970 | 0.3314 | 0.5325 | 0.4906 | 0.3912 |
| NLS–EGARCH | 0.2599 | 0.3305 | 0.1734 | 0.1907 | 0.3726 | 0.3987 | 0.3407 | 0.3632 | 0.4591 | 0.3308 | 0.3529 | 0.5342 | 0.4943 | 0.4095 |
| NLS–AR–EGARCH | 0.2531 | 0.3401 | 0.2080 | 0.2183 | 0.3693 | 0.4039 | 0.3525 | 0.3998 | 0.5052 | 0.4146 | 0.4354 | 0.5644 | 0.5295 | 0.4521 |
| SSM–GARCH | 0.1991 | 0.2618 | 0.1573 | 0.1507 | 0.2988 | 0.2766 | 0.2840 | 0.2855 | 0.3802 | 0.2816 | 0.2853 | 0.4483 | 0.3758 | 0.3492 |
| SSM–AR–GARCH | 0.2018 | 0.2618 | 0.1386 | **0.1371** | 0.3014 | 0.2853 | 0.2926 | 0.2851 | 0.3764 | 0.2538 | 0.2548 | 0.4440 | 0.3863 | 0.3654 |
| SSM–EGARCH | 0.2165 | 0.2792 | 0.2040 | 0.1840 | 0.2941 | 0.2755 | 0.3338 | 0.2954 | 0.3870 | 0.3663 | 0.3396 | **0.3890** | **0.3424** | 0.4464 |

(Continues)

TABLE 5 (Continued)

| Panel B: Time-varying volatility | 6-month-ahead forecast | | | | | | | 12-month-ahead forecast | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | AMAE | ARMSE | RMSE | | | | | AMAE | ARMSE | RMSE | | | | |
| | | | 3-mth | 12-mth | 60-mth | 120-mth | 240-mth | | | 3-mth | 12-mth | 60-mth | 120-mth | 240-mth |
| SSM–AR–EGARCH | 0.2368 | 0.2997 | 0.1814 | 0.1794 | 0.3166 | 0.3177 | 0.3801 | 0.3634 | 0.4991 | 0.3809 | 0.3824 | 0.5145 | 0.5069 | 0.5836 |
| *Competitor models* | | | | | | | | | | | | | | |
| GARCH | **0.1903** | **0.2498** | 0.1426 | 0.1601 | **0.2910** | **0.2720** | 0.2268 | **0.2699** | **0.3589** | 0.2608 | 0.2830 | 0.4469 | 0.3666 | 0.2521 |
| AR–GARCH | 0.1958 | 0.2635 | 0.2965 | 0.1579 | 0.2920 | 0.2856 | 0.2303 | 0.2783 | 0.3834 | 0.5335 | 0.2766 | 0.4479 | 0.3941 | 0.2537 |
| EGARCH | 0.2011 | 0.2640 | **0.1338** | 0.1536 | 0.3382 | 0.2740 | 0.2457 | 0.3042 | 0.3982 | **0.2478** | 0.2685 | 0.5825 | 0.3817 | 0.3141 |
| AR–EGARCH | 0.1980 | 0.2624 | 0.1566 | 0.1418 | 0.3254 | 0.2862 | **0.2072** | 0.2988 | 0.4013 | 0.3683 | **0.2436** | 0.5607 | 0.4137 | **0.2198** |

*Note*: This table presents the out-of-sample forecasting performance by NS-based and competitor models, using the expanding window approach. The NS-based models are classified according to estimation methods: Two-step OLS, NLS and state space method (SSM). In addition, RW denotes the RW model on yield levels. AR denotes AR(1) model and VAR denotes VAR(1) model. They are on yield changes. As for the conditional volatility structures, GARCH and EGARCH denote GARCH(1,1) and EGARCH(1,1,1) on yield changes respectively. Moreover, MAE (mean absolute error) and RMSE (root mean square error) are reported in real numbers. AMAE and ARMSE denote average MAE and RMSE. The decay parameters used in OLS and NLS methods are fixed at 0.0299 and 0.0314. In each column, the lowest residual statistic is bolded for easy reference.

to-medium term forecasts at most of the maturities, followed by the AR and the VAR models. The possible explanation is that in the presence of liquidity trap, yields of short-to-medium term maturities tend to cluster around the lower bound which is near zero with minimal deviation. This leads to extremely highly correlated yields between the previous period and the next period. However, the RW model performs less satisfactory for yields at longer-term maturities, which in part reflects investors expectation that the liquidity traps do not sustain and affect permanently the Japanese bond market.

It is worth noting that the AR model dominates the VAR model with lower RMSE, AMAE, and ARMSE in all 1-month, 3-month, 6-month, and 12-month ahead forecasting results of yields at all maturities ranging from 3 to 360 months. The AR model still moderately outperforms the VAR model by similar measures of residual statistics in most of the forecasting horizons on weekly and daily basis. The relatively poor forecasts by the VAR model may be explained by the large number of included parameters and the not highly correlated members in the transition matrix. Our findings are robust to the in-sample fitting period.[7]

Time-varying volatility

Panel B of Tables 5–7 reports the $h$-step-ahead forecasting results of the yield models with specification of time-varying volatility for monthly, weekly, and daily data, respectively. As can be gleaned from the panels, we observe features similar to forecasting results based on models with constant volatility. For example, forecasts by the competitor models with time-varying volatility specification consistently outperform the NS-based models for most of the maturities and with forecasts over both the short-horizon and the long-horizon. Most notably, for the 6- and 12-month ahead and the 1- and 3-day-ahead forecasts, the GARCH and AR-GARCH models on yield changes generate more accurate forecasts than the EGARCH models when gauged by average MAE and RMSE on daily and monthly frequencies. However, the EGARCH models on yield changes outperform for most of the maturities on weekly frequency for the 1- and 4-week-ahead forecasts.

Within the NS-based models with GARCH-type structures, forecasting performances by the SSM-(E)GARCH are only marginally inferior to those by OLS-EGARCH when assessed by the average MAE and RMSE. However, the SSM-(E)GARCH outperform for some maturities, such as for the horizon forecasts of 6- and 12-month-ahead for maturities of 6 and/or 12 months on monthly yields, the 4-week-ahead for maturities of 12 months on weekly yields and the 1- and 3-day-ahead for maturities of 6 months on daily yields, respectively. In contrast, the

**TABLE 6** Out-of-sample forecasting results using weekly yield data.

| **Panel A: Constant volatility** | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **1-week-ahead forecast** | | | | | | | **4-week-ahead forecast** | | | | | | |
| | **AMAE** | **ARMSE** | **RMSE** | | | | | **AMAE** | **ARMSE** | **RMSE** | | | | |
| | | | **3-mth** | **12-mth** | **60-mth** | **120-mth** | **240-mth** | | | **3-mth** | **12-mth** | **60-mth** | **120-mth** | **240-mth** |
| *Nelson–Siegel-type models* | | | | | | | | | | | | | | |
| OLS–RW | 0.0553 | 0.0696 | 0.0403 | 0.0394 | 0.0685 | 0.0800 | 0.0669 | 0.0855 | 0.1116 | 0.0608 | **0.0515** | 0.1224 | 0.1312 | 0.1152 |
| OLS–AR | 0.0555 | 0.0699 | 0.0405 | 0.0396 | 0.0687 | 0.0808 | 0.0672 | 0.0859 | 0.1124 | 0.0610 | 0.0522 | 0.1227 | 0.1337 | 0.1162 |
| OLS–VAR | 0.0556 | 0.0701 | 0.0392 | 0.0406 | 0.0687 | 0.0814 | 0.0687 | 0.0862 | 0.1127 | 0.0599 | 0.0534 | 0.1228 | 0.1342 | 0.1175 |
| NLS–RW | 0.1554 | 0.1970 | 0.0309 | 0.0909 | 0.2425 | 0.2636 | 0.1875 | 0.1642 | 0.2149 | 0.0508 | 0.1010 | 0.2616 | 0.2797 | 0.2061 |
| NLS–AR | 0.1534 | 0.1947 | 0.0314 | 0.0894 | 0.2385 | 0.2614 | 0.1870 | 0.1645 | 0.2151 | 0.0512 | 0.1015 | 0.2609 | 0.2806 | 0.2074 |
| NLS–VAR | 0.1548 | 0.1966 | 0.0337 | 0.0922 | 0.2401 | 0.2633 | 0.1896 | 0.1660 | 0.2176 | 0.0529 | 0.1043 | 0.2629 | 0.2833 | 0.2111 |
| SSM | 0.0671 | 0.0865 | 0.0269 | 0.0403 | 0.0677 | 0.0811 | 0.1841 | 0.0968 | 0.1274 | 0.0478 | 0.0525 | 0.1222 | 0.1360 | 0.2090 |
| *Competitor models* | | | | | | | | | | | | | | |
| RW | **0.0391** | **0.0517** | **0.0193** | 0.0301 | **0.0595** | **0.0599** | **0.0566** | **0.0771** | **0.1025** | 0.0452 | 0.0627 | **0.1160** | **0.1166** | **0.1051** |
| AR | 0.0393 | 0.0519 | **0.0193** | **0.0300** | 0.0598 | 0.0601 | 0.0569 | 0.0774 | 0.1033 | **0.0451** | 0.0631 | 0.1170 | 0.1177 | 0.1063 |
| VAR | 0.0408 | 0.0538 | 0.0206 | 0.0310 | 0.0613 | 0.0630 | 0.0602 | 0.0786 | 0.1043 | 0.0464 | 0.0638 | 0.1176 | 0.1189 | 0.1081 |
| **Panel B: Time-varying volatility** | | | | | | | | | | | | | | |
| | **1-week-ahead forecast** | | | | | | | **4-week-ahead forecast** | | | | | | |
| | **AMAE** | **ARMSE** | **RMSE** | | | | | **AMAE** | **ARMSE** | **RMSE** | | | | |
| | | | **3-mth** | **12-mth** | **60-mth** | **120-mth** | **240-mth** | | | **3-mth** | **12-mth** | **60-mth** | **120-mth** | **240-mth** |
| *Nelson–Siegel-type models* | | | | | | | | | | | | | | |
| OLS–GARCH | 0.0554 | 0.0697 | 0.0403 | 0.0398 | 0.0684 | 0.0806 | 0.0667 | 0.0860 | 0.1126 | 0.0604 | 0.0523 | 0.1233 | 0.1339 | 0.1160 |
| OLS–AR–GARCH | 0.0560 | 0.0705 | 0.0423 | 0.0410 | 0.0692 | 0.0807 | 0.0669 | 0.0862 | 0.1129 | 0.0613 | 0.0525 | 0.1237 | 0.1339 | 0.1160 |
| OLS–EGARCH | 0.0554 | 0.0697 | 0.0403 | 0.0396 | 0.0686 | 0.0802 | 0.0671 | 0.0859 | 0.1123 | 0.0608 | 0.0520 | 0.1232 | 0.1324 | 0.1163 |
| OLS–AR–EGARCH | 0.0558 | 0.0702 | 0.0406 | 0.0397 | 0.0692 | 0.0805 | 0.0675 | 0.0864 | 0.1129 | 0.0601 | 0.0523 | 0.1242 | 0.1333 | 0.1169 |
| NLS–GARCH | 0.1561 | 0.1981 | 0.0313 | 0.0922 | 0.2439 | 0.2648 | 0.1886 | 0.1674 | 0.2190 | 0.0513 | 0.1053 | 0.2669 | 0.2842 | 0.2102 |
| NLS–AR–GARCH | 0.1528 | 0.1938 | 0.0319 | 0.0880 | 0.2368 | 0.2605 | 0.1869 | 0.1642 | 0.2147 | 0.0516 | 0.1013 | 0.2598 | 0.2801 | 0.2082 |
| NLS–EGARCH | 0.1568 | 0.1991 | 0.0312 | 0.0929 | 0.2455 | 0.2659 | 0.1893 | 0.1702 | 0.2227 | 0.0516 | 0.1077 | 0.2727 | 0.2887 | 0.2127 |
| NLS–AR–EGARCH | 0.1525 | 0.1937 | 0.0314 | 0.0876 | 0.2368 | 0.2606 | 0.1869 | 0.1641 | 0.2148 | 0.0519 | 0.1006 | 0.2602 | 0.2802 | 0.2082 |
| SSM–GARCH | 0.0667 | 0.0859 | 0.0277 | 0.0392 | 0.0680 | 0.0815 | 0.1790 | 0.0954 | 0.1260 | 0.0485 | **0.0506** | 0.1256 | 0.1388 | 0.1904 |
| SSM–AR–GARCH | 0.0684 | 0.0883 | 0.0291 | 0.0419 | 0.0697 | 0.0853 | 0.1826 | 0.0972 | 0.1279 | 0.0486 | 0.0531 | 0.1274 | 0.1410 | 0.1956 |
| SSM–EGARCH | 0.0669 | 0.0861 | 0.0277 | 0.0393 | 0.0678 | 0.0815 | 0.1799 | 0.0958 | 0.1263 | 0.0488 | 0.0512 | 0.1247 | 0.1382 | 0.1931 |
| SSM–AR–EGARCH | 0.0684 | 0.0881 | 0.0286 | 0.0424 | 0.0685 | 0.0852 | 0.1851 | 0.0968 | 0.1273 | 0.0480 | 0.0537 | 0.1239 | 0.1393 | 0.2023 |
| *Competitor models* | | | | | | | | | | | | | | |
| GARCH | **0.0392** | **0.0518** | 0.0193 | 0.0301 | 0.0597 | **0.0602** | **0.0568** | **0.0773** | 0.1035 | 0.0452 | 0.0623 | 0.1173 | **0.1191** | 0.1065 |
| AR–GARCH | 0.0394 | 0.0522 | 0.0192 | 0.0306 | 0.0599 | 0.0605 | 0.0570 | 0.0774 | 0.1036 | 0.0452 | 0.0622 | 0.1171 | 0.1197 | 0.1064 |
| EGARCH | **0.0392** | **0.0518** | **0.0190** | **0.0300** | **0.0595** | **0.0602** | **0.0568** | **0.0773** | **0.1031** | **0.0447** | 0.0622 | **0.1162** | 0.1194 | 0.1062 |
| AR–EGARCH | 0.0394 | 0.0521 | 0.0191 | 0.0302 | 0.0596 | 0.0607 | 0.0570 | 0.0773 | 0.1033 | 0.0450 | 0.0622 | 0.1163 | 0.1203 | **0.1059** |

*Note*: This table presents the out-of-sample forecasting performance by various models using the expanding window approach. Refer to description of notations in Table 5.

TABLE 7 Out-of-sample forecasting results using daily yield data.

**Panel A: Constant volatility**

| | 1-day-ahead forecast | | | | | | | 3-day-ahead forecast | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | AMAE | ARMSE | RMSE | | | | | AMAE | ARMSE | RMSE | | | | |
| | | | 3-mth | 12-mth | 60-mth | 120-mth | 240-mth | | | 3-mth | 12-mth | 60-mth | 120-mth | 240-mth |
| *Nelson–Siegel-type models* | | | | | | | | | | | | | | |
| OLS–RW | 0.0415 | 0.0518 | 0.0346 | 0.0367 | 0.0422 | 0.0569 | 0.0447 | 0.0494 | 0.0622 | 0.0380 | 0.0377 | 0.0576 | 0.0700 | 0.0599 |
| OLS–AR | 0.0416 | 0.0520 | 0.0345 | 0.0370 | 0.0423 | 0.0572 | 0.0449 | 0.0494 | 0.0624 | 0.0379 | 0.0380 | 0.0576 | 0.0705 | 0.0603 |
| OLS–VAR | 0.0415 | 0.0518 | 0.0344 | 0.0370 | 0.0421 | 0.0571 | 0.0449 | 0.0494 | 0.0622 | 0.0377 | 0.0380 | 0.0575 | 0.0704 | 0.0602 |
| NLS–RW | 0.1555 | 0.1958 | 0.0251 | 0.0931 | 0.2429 | 0.2627 | 0.1856 | 0.1563 | 0.1985 | 0.0280 | 0.0928 | 0.2462 | 0.2648 | 0.1888 |
| NLS–AR | 0.1547 | 0.1948 | 0.0253 | 0.0923 | 0.2411 | 0.2617 | 0.1853 | 0.1560 | 0.1980 | 0.0284 | 0.0926 | 0.2451 | 0.2644 | 0.1891 |
| NLS–VAR | 0.1545 | 0.1945 | 0.0251 | 0.0922 | 0.2406 | 0.2613 | 0.1850 | 0.1558 | 0.1977 | 0.0282 | 0.0925 | 0.2446 | 0.2640 | 0.1888 |
| SSM | 0.0488 | 0.0653 | 0.0217 | 0.0368 | 0.0410 | 0.0817 | 0.1212 | 0.0565 | 0.0749 | 0.0258 | 0.0381 | 0.0572 | 0.0915 | 0.1267 |
| *Competitor models* | | | | | | | | | | | | | | |
| RW | **0.0174** | **0.0239** | **0.0074** | 0.0121 | **0.0281** | **0.0287** | **0.0284** | **0.0313** | **0.0417** | 0.0147 | 0.0227 | **0.0482** | **0.0493** | **0.0482** |
| AR | 0.0175 | **0.0239** | **0.0074** | 0.0120 | 0.0282 | **0.0287** | 0.0286 | 0.0314 | 0.0418 | 0.0147 | 0.0226 | 0.0483 | 0.0494 | 0.0486 |
| VAR | 0.0176 | 0.0240 | 0.0076 | **0.0117** | 0.0284 | 0.0291 | 0.0288 | 0.0315 | 0.0418 | **0.0145** | **0.0224** | 0.0485 | 0.0495 | 0.0488 |

**Panel B: Time-varying volatility**

| | 1-day-ahead forecast | | | | | | | 3-day-ahead forecast | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | AMAE | ARMSE | RMSE | | | | | AMAE | ARMSE | RMSE | | | | |
| | | | 3-mth | 12-mth | 60-mth | 120-mth | 240-mth | | | 3-mth | 12-mth | 60-mth | 120-mth | 240-mth |
| *Nelson–Siegel-type models* | | | | | | | | | | | | | | |
| OLS–GARCH | 0.0415 | 0.0518 | 0.0344 | 0.0369 | 0.0422 | 0.0572 | 0.0444 | 0.0494 | 0.0623 | 0.0374 | 0.0383 | 0.0576 | 0.0708 | 0.0591 |
| OLS–AR–GARCH | 0.0416 | 0.0520 | 0.0343 | 0.0372 | 0.0425 | 0.0573 | 0.0448 | 0.0495 | 0.0625 | 0.0373 | 0.0386 | 0.0580 | 0.0708 | 0.0599 |
| OLS–EGARCH | 0.0415 | 0.0518 | 0.0344 | 0.0369 | 0.0422 | 0.0571 | 0.0445 | 0.0494 | 0.0623 | 0.0376 | 0.0382 | 0.0576 | 0.0706 | 0.0594 |
| OLS–AR–EGARCH | 0.0416 | 0.0520 | 0.0344 | 0.0373 | 0.0424 | 0.0571 | 0.0451 | 0.0495 | 0.0625 | 0.0375 | 0.0388 | 0.0579 | 0.0702 | 0.0606 |
| NLS–GARCH | 0.1556 | 0.1960 | 0.0253 | 0.0934 | 0.2430 | 0.2628 | 0.1856 | 0.1566 | 0.1989 | 0.0287 | 0.0937 | 0.2467 | 0.2652 | 0.1888 |
| NLS–AR–GARCH | 0.1573 | 0.1993 | 0.0255 | 0.0963 | 0.2484 | 0.2661 | 0.1872 | 0.1589 | 0.2034 | 0.0289 | 0.0974 | 0.2539 | 0.2696 | 0.1911 |
| NLS–EGARCH | 0.1554 | 0.1958 | 0.0252 | 0.0933 | 0.2428 | 0.2625 | 0.1856 | 0.1562 | 0.1984 | 0.0284 | 0.0934 | 0.2459 | 0.2644 | 0.1888 |
| NLS–AR–EGARCH | 0.1563 | 0.1983 | 0.0258 | 0.0961 | 0.2470 | 0.2647 | 0.1864 | 0.1572 | 0.2044 | 0.0296 | 0.0992 | 0.2561 | 0.2693 | 0.1908 |
| SSM–GARCH | 0.0487 | 0.0652 | 0.0219 | 0.0368 | 0.0413 | 0.0814 | 0.1199 | 0.0563 | 0.0746 | 0.0258 | 0.0383 | 0.0577 | 0.0908 | 0.1235 |
| SSM–AR–GARCH | 0.0497 | 0.0662 | 0.0219 | 0.0374 | 0.0420 | 0.0831 | 0.1220 | 0.0581 | 0.0768 | 0.0258 | 0.0395 | 0.0600 | 0.0941 | 0.1275 |
| SSM–EGARCH | 0.0487 | 0.0652 | 0.0219 | 0.0366 | 0.0414 | 0.0814 | 0.1199 | 0.0563 | 0.0747 | 0.0261 | 0.0378 | 0.0580 | 0.0907 | 0.1237 |
| SSM–AR–EGARCH | 0.0497 | 0.0663 | 0.0219 | 0.0375 | 0.0420 | 0.0831 | 0.1224 | 0.0581 | 0.0769 | 0.0259 | 0.0397 | 0.0598 | 0.0942 | 0.1287 |
| *Competitor models* | | | | | | | | | | | | | | |
| GARCH | **0.0174** | **0.0239** | **0.0074** | 0.0121 | **0.0281** | 0.0288 | **0.0285** | **0.0313** | **0.0418** | **0.0147** | **0.0227** | 0.0484 | 0.0496 | 0.0484 |
| AR–GARCH | 0.0175 | 0.0240 | 0.0075 | 0.0126 | 0.0282 | 0.0288 | 0.0286 | 0.0314 | 0.0419 | 0.0148 | 0.0232 | 0.0484 | 0.0496 | 0.0486 |
| EGARCH | 0.0182 | 0.0415 | **0.0074** | 0.0932 | **0.0281** | **0.0287** | **0.0285** | 0.0338 | 0.0977 | **0.0147** | 0.2787 | **0.0483** | **0.0494** | **0.0483** |
| AR–EGARCH | 0.0176 | 0.0264 | **0.0074** | **0.0120** | 0.0282 | **0.0287** | 0.0286 | 0.0321 | 0.0582 | **0.0147** | **0.0227** | **0.0483** | 0.0495 | 0.0485 |

*Note*: This table presents the out-of-sample forecasting performance by various models using the expanding window approach. Refer to description of notations in Table 5.

NLS-(E)GARCH consistently perform the worst with various horizon forecasts at all maturities in terms of MAE and RMSE as well as their average values. In summary, we find evidence that in the presence of liquidity trap, the competitor models with simpler structures outperform the NS based models. Our findings are reasonably robust to yields with different window frequencies and to yields with a wide range of maturities.

## 6 | CONCLUDING REMARKS

In this study, we have examined the term structure of the Japanese government bond yields by employing both the NS-based models and competitor models in a period of prolonged liquidity trap. Our dataset consists of yields with various maturities at daily, weekly, and monthly frequencies. We have conducted both the in-sample fitting and out-of-sample forecasting exercises with different decay parameters. Our in-sample fit results show that the NS-based models are capable of not only producing accurate fits for the yield data, but also replicating the common stylized facts of the yield curve as identified in the literature. It is also found that the long-term and the medium-term factors are more persistent than the short-term factor and the spread dynamics are less persistent. Our findings are consistent with the stylized facts discussed in Diebold and Li (2006) and others.

Our *h*-step-ahead out-of-sample forecasting results indicate that, in the presence of liquidity trap, the competitor models always outperform the NS-based models when assessed by the residual statistics. Our findings are robust to the specification of volatility structures. Most notably, the short-to-medium term forecasts of yields by the RW model outperform those forecasts by other models under study at most of the maturities. Moreover, AR-GARCH models on yield changes produce more adequate forecasts than their counterpart NS-based models on monthly and daily yields, while the AR-EGARCH models outperform for most of the maturities on weekly yields. Nevertheless, the NS-based models can be useful in forecasting yields over longer horizons and can work well with the GARCH-type structure of time-varying volatility, especially when using the OLS and SSM estimation procedures. Comparatively, the NS-based OLS-GARCH models with fixed decay parameter at 0.0299 produce relatively more adequate out-of-sample forecasting results, while the NS-based SSM-GARCH models show better performance only for monthly yields, and the performance by the NS-based NLS model is ranked the least. Our findings have important implications for forecasting the Japanese bond yields in the presence of a liquidity trap.

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## DATA AVAILABILITY STATEMENT

Data will be made available upon request.

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## ENDNOTES

[1] Note that the yield (yield-to-maturity) and the spot rate of a zero-coupon bond are the same, and hence the terms will be used interchangeably.

[2] End-of-month yield rates are culled from January 2000 to October 2007 instead.

[3] According to the Ministry of Finance of Japan, FBs and TBs were unified from FY2008 to ensure common term-to-maturity and issuance date.

[4] Currently, 2-, 5-, and 10-year bonds are issued every month; 15-year bonds are issued quarterly; 20-year bonds are issued every other month; and 30-year bonds are issued twice a year. The Japanese government issued 4-year bonds in February 2001, 6-year bonds in March 2001 but has stopped the issuance of these bonds ever since. For discounted government bonds, September 2000 marked the final issuance of 5-year discounted bonds, though 3-year discounted bonds continue to be issued every other month.

[5] Nelson and Siegel (1987) found that in-sample fit is not degraded much when the sample median of the decay parameter is used in place of the actual estimates. Similarly, de Pooter (2007) experimented with the most recent decay parameter and the mean estimates, and found that the median estimate is comparatively more stable than others.

[6] Based on the Bayesian information criteria, we find that AR(1) models are largely suitable for the mean equation, and the Ljung-Box Q-statistics provides some support for the GARCH(1,1) and EGARCH(1,1,1) structures in the conditional variance equation. To avoid computational complexity, we do not consider the VAR(2) model and higher order GARCH and EGARCH structures.

[7] In order to further compare the performances of the AR and VAR models, we divide yield data of different frequencies into the pre-evaluation and forecasting sub-samples. We then perform in-sample fitting exercises using both two models in both subsamples. The pre-evaluation period is taken from the first observation

to half of the sample period, while the forecasting period contains the remaining half. Based on the residual analysis, we find that the summary statistics generated by both models are largely consistent in the two periods, with similar MAE and RMSE statistics particularly for the shorter term maturities across all data frequencies. We have also conducted an out-of-sample forecasting exercise in which 2/3 of the yield sample are treated as in-sample window, while the remaining 1/3 as forecasting window. Our results are qualitatively consistent to those reported in Tables 5–7. These results are available upon request. We thank one anonymous referee for pointing out this issue.

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