"Maths outside of maths": Pre-service teachers’ awareness of mathematical and statistical thinking across teachers’ professional work

Judy Bailey  
*University of Waikato, jlbailey@waikato.ac.nz*

Bronwen Cowie  
*University of Waikato, bronwen.cowie@waikato.ac.nz*

Beverley Cooper  
*University of Waikato*

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“Maths Outside of Maths”: Pre-Service Teachers’ Awareness of Mathematical and Statistical Thinking Across Teachers’ Professional Work

Judy Bailey
Bronwen Cowie
Beverley Cooper
University of Waikato, New Zealand

Abstract: Mathematical and statistical thinking is involved across the breadth of people’s home and work life and leisure activities. This paper reports on an aspect of a project that aimed to develop pre-service teacher awareness of the mathematical and statistical thinking required across the breadth of primary teachers’ professional role. This thinking is conceptualised as the mathematics and statistics embedded in each of the curriculum learning areas, in data literacy, and administration and management tasks. Mentor meetings indicated pre-service teachers who were completing a one-year graduate diploma initially had a limited awareness of the extent of this thinking. Through focus group discussions across the year participating pre-service teachers’ commentary showed an increased awareness and appreciation of the breadth of contexts where teachers might encounter mathematics and statistics thinking beyond mathematics lessons. Given awareness is fundamental to learning and subsequent action we posit that developing this awareness during teacher education is important.

Introduction

Mathematical and statistical thinking are fundamental to effective participation in a democratic society because of the many ways they are implicated in a person’s work, social and home life, and their participation as an informed citizen (Bennison, 2015; FitzSimons, 2013; Hodgen & Marks, 2013; Karaali, Villafane Hernandez, & Taylor, 2016; PIAAC Numeracy Expert Group, 2009; Steen, 2001; Wiest, Higgins, & Frost, 2007). Most occupations, including teaching, require mathematical thinking of some kind with many of these demands specific to the particular work context (Leder, Forgasz, Kalkhoven, & Geiger, 2015; Noss, Hoyles, & Pozzi, 2000). Almost every public issue relies on the use of data to inform and/or persuade and so a person’s knowledge of how to interrogate data and data representations, and critique any conclusions and/or implications for action are important for informed participation in a democratic society (Alonzo & Starr 1987; Evans, Tsatsaroni, & Czarniecka, 2014; Jablonka, 2003; Steen, 2001). In the context of the home, mathematical thinking is central to making sensible decisions about household budgeting, deciding on which mortgage to take, scheduling exercise sessions and so on (Lusardi & Tufano, 2015). Unfortunately, the role mathematical and statistical thinking plays in these various aspects of people’s lives is largely invisible to them (Cohen, O’Donoghue, & FitzSimons, 2000; Delvin, 1998; Gal, 2000; Wedge, 2010). Teachers have an unique opportunity to address this invisibility and empower students as current and future citizens who utilise mathematical and statistical thinking to make better informed decisions.
Empowering students in this way is consistent with the New Zealand Curriculum’s (Ministry of Education, 2007) vision of young people developing “knowledge and competencies that will enable them to live full and satisfying lives”, being “effective users of communication tools”, and people who are “literate and numerate” (p. 8). To achieve this vision, teachers need to be aware of the role mathematical and statistical thinking plays in the various aspects of people’s lives, including their own work.

This paper draws on data generated as part of the Strengthening Mathematical Thinking and Reasoning Proficiency in Primary Teacher Education (MARKITE) project (Cooper, Cowie, Furness, Peter & Bailey, 2017). The MARKITE project aimed to foster pre-service primary teacher awareness of mathematical and statistical thinking across the breadth of teachers’ work, as a way to support their understanding of the role and breadth of such thinking. Within the project teachers’ mathematical and statistical thinking was envisaged as including the domains of (i) mathematics and statistics embedded across the curriculum, (ii) the collection, analysis and action on student achievement data, and (iii) non-teaching administration, planning and management tasks (Cooper et al., 2017). While the MARKITE study included a focus on curriculum mapping and lecturer and student views, in this paper we examine the research question: To what extent do pre-service primary teachers become aware and/ or more aware of the breadth of mathematical and statistical thinking within the wider professional teaching role over the course of their year-long graduate teacher education programme?

We begin by briefly reviewing research related to mathematical and statistical thinking and teachers’ work and how this thinking might develop. We then set out findings from mentor meetings from two pre-service teacher cohorts, and a series of focus group discussions with a small number of volunteer pre-service teachers from one cohort. Findings from these data indicate that opportunities to reflect on and share examples of mathematical and statistical thinking in action over time supported the expansion of pre-service teachers’ ‘personal example spaces’ (Watson & Mason, 2002, 2005) and intentions to focus on and foster children’s mathematical thinking across the curriculum. The notion of example space offers a productive means of accounting for changes in pre-service teachers’ awareness of mathematical and statistical thinking.

Setting the Scene

Internationally, mathematical and statistical thinking has emerged as an area of concern for international agencies and governments, for both school children and adults (Tout & Gal, 2015; National Numeracy, 2019; Mevarech & Kramarski, 2014). Within the PISA framework mathematics performance encompasses the capacity to “formulate, employ and interpret mathematics in a variety of contexts to describe, predict and explain phenomena, recognising the role that mathematics plays in the world” (OECD, 2019). The definition goes on to state that, “A mathematically literate student recognises the role that mathematics plays in the world in order to make well-founded judgments and decisions needed by constructive, engaged and reflective citizens” (OECD, 2019). Important to the context of this paper, this definition foregrounds the capacity for students to recognise the role mathematics plays in people’s lives, not just the capacity to employ mathematical thinking. The OECD Survey of Adult Skills in the Program for the International Assessment of Adult Competencies (PIAAC), referring to numeracy as a desirable capacity for adults, also focuses on the capacity “to engage in and manage the mathematical demands of a range of situations in adult life” implying the need to recognise such demands in different settings (PIAAC Numeracy Expert Group, 2009, p. 55). Seen this way, numeracy for adults is a
social practice that is central to active participation in home, community, leisure and workplace activities (Coben & Alkema, 2017). In this paper, to avoid the potential of confusion around the scope and meaning of ‘numeracy’, particularly in relation to the ‘Numeracy Project’ (Ministry of Education, 2008) in New Zealand as the site of our study, we have chosen to use the phrase ‘mathematical and statistical thinking’. This usage is consistent with Mathematics and Statistics as the label of one of the seven learning areas in the New Zealand Curriculum (Ministry of Education, 2007, p. 26) and thus comes with the potential for shared meaning with our pre-service teacher participants.

The Domains of Mathematical and Statistical Thinking in Teachers’ Work

Teachers use mathematical and statistical thinking for various aspects of their professional role (Leder et al., 2015; Watson, 2011). Most obviously, teachers use mathematical and statistical thinking as part of teaching the mathematics curriculum. Current understandings of effective practice advocate for the use of problems set in relevant contexts (Anthony & Walshaw, 2007). This demand requires teachers to recognise the mathematical and statistical thinking involved in everyday activities and to use this knowledge to develop teaching and learning activities (Boaler, 2016;Forgasz, Leder, Hall, & Clayton, 2017; Ministry of Education, 2007; National Council of Teachers of Mathematics, 2014). The entry and exit assessments being used with pre-service teachers in some jurisdictions explicitly gauge their capacity to recognise and incorporate mathematics in this way (e.g., Department for Education (UK), 2012). Teachers also need to be able to recognise the mathematical and statistical thinking in other curriculum learning areas for them to be able to identify and exploit opportunities for students to gain a fuller understanding of the role mathematics and statistics thinking plays in these learning areas, and in their lives (ACARA, n.d.; Craver, 2014; Delvin, 1998; Goos, Geiger, Dole, Forgasz, & Bennison, 1999; Lutzer, 2008). Seen this way mathematical and statistical thinking, like literacy, becomes the responsibility of all teachers (Thornton & Hogan, 2004), a proposition that is reflected in government policy from a number of countries (e.g. Human Capital Working Group, Council of Australian Governments, 2008; Learning Wales, 2014). At this time however, very little is known about how and if pre-service teachers recognise the role of mathematical and statistical thinking beyond the mathematics classroom (Geiger, Forgasz, & Goos, 2015; OECD, 2013), one exception being a pilot study by Leder et al. (2015). The Leder et al. study, conducted at a large Australian university, found that fewer than half (44%) of the 214 pre-service teacher respondents agreed that there were numeracy demands beyond the mathematics taught as a curriculum subject suggesting the wider role of such thinking is not readily recognised.

Teacher data literacy is the second domain of mathematical and statistical thinking encompassed by teachers’ professional role. The OECD, in its policy document OECD Future of Education and Skills 2030: OECD Learning Compass 2030, describes data literacy as a “foundational” literacy because it acts as a gateway for many other skills and active citizenship (OECD, 2019). The need for teacher data literacy (Bennison, 2015; Gummer & Mandinach, 2015) or professional statistical literacy (Chick & Pierce, 2013) is gaining prominence as part of a growing demand for evidence-based practice (Gummer & Mandinach, 2015). The assertion is that teachers need to be able to generate, interpret and act on qualitative and quantitative individual and class data as part of formative assessment practices focused on progressing student learning (Black & Wiliam, 2018). Increasingly, governments are expecting that teachers are able to collate and compare their student achievement data across schools and with state/national-level data and use this analysis to
inform their practice (Hardy, 2015). Based on their survey, Leder et al. (2015) found that, in general, pre-service teachers were unsure about how to interpret data indicating pre-service teacher educators face a challenge in meeting their responsibility to graduate beginning teachers who are able to make effective use of both quantitative and qualitative student data (Cowie & Cooper, 2016 for New Zealand data).

The third domain of mathematical and statistical thinking within teachers’ work is to do with administration and management tasks such as lesson scheduling, budgeting, organising field trips and sporting events, and so on. We were not able to identify specific studies in this domain but did locate research on teachers’ work that documented the time teachers spend on non-instructional tasks including general administration tasks (e.g. Higton et al., 2017; OECD, 2014). Some of these administrative tasks require mathematical and or statistical thinking. One such example is the need to calculate adult-child ratios on field trips (e.g. the New Zealand, Ministry of Education, 2018) and to calculate fees, charges and donations (Ministry of Education, n.d.). The Leder et al. (2015) study indicated that pre-service teachers had some awareness of the mathematical demands involved in organisational tasks.

**What is Involved in Mathematical and Statistical Thinking?**

The literature on the development of numeracy in adults, which we are calling mathematical and statistical thinking, indicates it involves a combination of competence in the relevant knowledge and skills; confidence in evaluating, accessing and using the mathematical ideas appropriate to a particular situation; and critical awareness of the mathematical thinking embedded in an activity (Coben et al., 2000). Coben and colleagues state that critical awareness includes the person’s appreciation of their own mathematical thinking and learning needs. In terms of competency the results of the Adult Literacy and Life Skills Survey of adult numeracy indicate that this is of concern internationally (OECD, 2013). At the same time there is ample evidence that people of all ages, and from all occupations fear and or lack confidence in their mathematical abilities (e.g., Wedge, 2010), with the same true of pre-service teachers (Gresham, 2018). In this paper we do not attend to the matter of competence or confidence, rather our focus is on critical awareness.

Given evidence of the influence and general invisibility of the role mathematics plays in society and people’s daily lives that we outlined earlier, we consider critical awareness of the presence and influence of mathematics and statistics are important capacities for teachers (see also Barwell & Suurtamm, 2011). As Mason (1998) explains it, “being aware is a state in which attention is directed to whatever it is that one is aware of” (p. 254). With others (e.g., Claxton, 1984, Hewitt, 2001) Mason (1998) argues that developing awareness is the essence of learning as a social activity because “when someone else points something out our being aware alters subtly” (p. 254). In this way people become more explicitly aware of some things rather than others. Just as importantly, they become aware of the connections and inter-relationships amongst any distinctions they are able to make (see Mason,1998, 2002). In making these points Mason refers to Gattegno’s notion that only awareness is educable (Gattegno, 1987–2010, p. 220). Gattegno asserts there is value in “educating awareness” because this enhances the capacity to respond in the moment in a creative way; to take things further on our own accord and not be limited to reproducing only those things we have been told (see also Hewitt, 2001; Mason, 2002). One of our goals for our pre-service teachers was for them to develop such awareness.
Research Design

In New Zealand, the context for this paper, a range of mathematical and statistical ideas are explicit and/or implicit in each of the seven learning area statements and in the ‘key competencies’ in the national curriculum (Furness, Cowie, & Cooper, 2017; Ministry of Education, 2007). Over a period of three-years the Strengthening Mathematical Thinking and Reasoning Proficiency in Primary Teacher Education (MARKITE) study aimed to enhance pre-service teacher awareness and understanding of the role of mathematical and statistical thinking across the curriculum and the breadth of teachers’ professional work. Pre-service teachers in the one-year Graduate Diploma of Teaching (Primary) programme were the participant group for the project in each year. Ethical consent was sought and gained from the university ethics committee for the project, and informed consent was gained from all participants.

In designing the project, we were cognizant of research on the development of adult numeracy that demonstrates the value of embedded instruction (Coben et al., 2000; Taylor & Galligan, 2006). We also took account of research that indicates ‘expert’ knowledge is characterised by the capacity to see connections and to transfer ideas across contexts and that this capacity is developed through experience with ideas across a range of settings (Bransford, Brown, & Cocking, 1999). Consequently, we anticipated that pre-service teacher mathematical thinking would be enriched by a cross programme focus; that is by all lecturers in the pre-service teacher education programme making visible and paying attention to any mathematical thinking in their courses. Initially we did not know the nature of the mathematical and statistical thinking that might be in the courses within the programme and so a first step was to explain the project to the programme lecturers, ask them to identify any such thinking and to request that they made this more explicit. A small number agreed to do this; their views and experiences are reported elsewhere (Cooper et al., 2017).

During each of the project’s three years data was collected from pre-service teachers in a variety of ways, including an assessment of their mathematical and statistical thinking, and associated confidence at the beginning and end of their one-year programme. This assessment and confidence data is not the focus of this paper (see Cooper et al., 2017). Pre-service teachers were offered an opportunity to meet with a mentor to discuss their assessment results. During the mentor meetings, which were held after the pre-service teachers’ first practicum, pre-service teachers were also asked about their perceptions of the role of mathematical and statistical thinking beyond mathematics classes, i.e., across teachers’ wider professional role. Some pre-service teachers attended these mentor meetings in small groups, and some followed up on the mentor meetings by sending emails with further ideas and/or comments to the researchers. These mentor discussions constitute the first set of data reported here.

Focus group discussions with volunteer pre-service teachers were held before and after the second practicum in the second and third year of the study, before the third (final) practicum in the final year of the study, and at the end of the programme in the second and third years of the project. For all practicum placements pre-service teachers were full time in schools from four to seven weeks. They worked alongside certificated teachers gradually taking more responsibility for teaching and learning. Focus group discussions centred on pre-service teachers anticipated and experienced examples of teacher mathematical and statistical thinking outside the teaching of mathematics and statistics. With researcher prompting, the discussions ranged over the three domains of interest in the MARKITE study: cross curriculum, data literacy, and administration and management. As might be expected the initial focus and subsequent direction of the discussion varied depending on
who spoke first and about what. Typically, all participating pre-service teachers contributed to discussions, sometimes prompted by us but often encouraged by their peers to share their experiences and thoughts. In the final year of the project the team recruited and retained the involvement of nine pre-service teachers. They all attended or sent apologies to the four focus group discussions during the year. All mentor meetings and focus group discussions were audio-taped, transcribed and field notes taken. Focus group discussions in the second and third year of the project, along with data from the mentor meetings in those years are analysed in this paper because we have data on pre-service teacher views over time for these years.

### Analysis of Mentor Meetings and Focus Group Discussions

First, the transcripts and researcher notes from the mentor meetings early in the year were read and analysed deductively for reference to and examples of the three domains of interest to the study: cross curriculum, data literacy, and administration and management. Next we looked more closely at the data, viewing the conversations during the mentor meetings as reflecting a process of example-generation as described by Zaslavsky and Zodik (2014). Zaslavsky and Zodik posit that learners generating and verifying examples of a particular mathematical concept as a group activity serves as an indicator of learners’ understandings and as a catalyst for enhancing their understanding thereby expanding the example space that they associate with the particular concept. Referring to Bills et al. (2006), Zaslavsky and Zodik (2014) define an example space as “the collection of examples to which an individual has access at any moment and the richness of interconnection between those examples” (p. 527). That is, a person’s example space is the collection of examples they associate with a particular concept at a particular time or context (Watson & Mason, 2005). Their proposition is that example spaces are dynamic and evolve and so in orchestrating learning teachers need to identify if learners have limited views of certain concepts, and facilitate the expansion of their example space beyond ‘more of the same’ examples. Mason and Goldenberg (2008) note that some parts of a person’s example space may be more accessible at a given time than others with the less accessible parts requiring a trigger which, during group discussions, can be provided by another group member’s example. Significantly, they posit that each time a connection is made it is strengthened, and more likely to come to mind in the future. Seen this way, and to reiterate, we viewed the examples of mathematical and statistical thinking pre-service teachers contributed in the focus group discussions as an indicator of their understanding as well as a catalyst for educating or raising individual and collective awareness of the role of mathematical and statistical thinking in teachers’ professional work.

### Results

The results are presented in four parts. The first two parts provide evidence of the diversity of understandings/ the initial example spaces of the pre-service teachers who took part in the MARKITE. In part (i) we explain three categories (see A, B, and C in Table 1) of pre-service teacher understanding or awareness evidenced during the mentor meetings early in each year. An example of pre-service teacher thinking from each category is shared. The prevalence of pre-service teacher commentary on each of the three domains of mathematical and statistical thinking is detailed in part (ii). Parts iii and iv focus on the development of example spaces over time. Two vignettes, drawing on data from the mentor meetings and
focus group discussions, of the development of example spaces over the course of the teacher education year are shared in part (iii). We conclude in part (iv) by setting out evidence that the participating pre-service teachers considered that the research data collection discussion process had educated their awareness of the mathematical and statistical thinking in teachers’ professional work.

(i) Three Categories of Pre-Service Teacher Awareness of the Domains of Thinking

Post their first practicum 26 pre-service teachers in the second year of the project and 18 in the third and final year of the project attended mentor meetings. As the data was analysed three categories (A, B, and C) emerged. Pre-service teachers who provided examples of all three domains and thus demonstrated an awareness of the existence of mathematical and statistical thinking across the curriculum, data literacy, and administration and management were labelled category A. Those who showed an awareness of mathematical and statistical thinking across the curriculum with no reference to data literacy or administration and management purposes were labelled category B. Those pre-service teachers who primarily focused on mathematics for mathematics teaching alone were allocated to category C. There were no pre-service teachers who showed awareness of mathematical and statistical thinking for data literacy, or management and administration alone.

<table>
<thead>
<tr>
<th>Category of response</th>
<th>Project Year 2 n=26</th>
<th>Project Year 3 n=18</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: Mathematical and statistical thinking in the three domains (mathematics across the curriculum, data literacy, and management and administration)</td>
<td>8 (30%)</td>
<td>10 (55%)</td>
</tr>
<tr>
<td>B: Primary focus on mathematical and statistical thinking across the curriculum (not for data literacy or management and administration)</td>
<td>9 (35%)</td>
<td>1 (6%)</td>
</tr>
<tr>
<td>C: Primary focus on mathematical and statistical thinking in mathematics education</td>
<td>9 (35%)</td>
<td>7 (39%)</td>
</tr>
</tbody>
</table>

Table 1. Initial Understandings of Mathematical and Statistical Thinking in Teachers’ Wider Professional Role

In the second year of the project around a third, and in the final year of the project around a half of the pre-service teachers interviewed during the mentor meetings identified mathematical and statistical thinking across a teacher’s wider professional role (category A), a finding generally consistent with that of Leder et al. (2015). It is not clear why a greater proportion of pre-service teachers who chose to participate in mentor meetings in the last year of the project had a more comprehensive view of the role of mathematics in teachers’ professional work, although it may be that their lecturers (having already been involved in the MARKITE project for at least one year) had been more explicit about the role of mathematical and statistical thinking in their courses.

Participating pre-service teachers from category A provided examples from each of the domains over the course of their mentor meeting, sometimes following a prompt. Of these pre-service teachers Gail (all names are pseudonyms) provided one of the more expansive example spaces. Following an explanation of the research purpose as ‘looking at
the maths that teachers need in their wider professional role and not looking at the teaching of mathematics’ the interviewer asked her, “When you were on your first practicum, did you see your teacher needing to use any maths?”. Gail provided examples of mathematical thinking occurring in other curriculum areas as in the following example.

One time the students were doing a logo, like a family crest kind of logo [for social studies]. It wasn’t really the teacher who needed to have a think about the kind of the proportions of the logo, it was more from the students.

Gail provided examples of analysing assessment data for grouping and using mathematical thinking in classroom organisation. In New Zealand primary classrooms students are commonly assigned stages or levels on the basis of teacher analysis of assessment data. Here, we can see that Gail is considering what might be the boundaries of mathematical thinking, something which she did on several occasions throughout the mentor meeting:

They were working out what stages students were at. And the levels that they were at. I’m not sure if that’s quite maths....

Being open-minded in this way may raise the possibility for the expansion of example space. Being open-minded and willing to consider options is consistent with the notion of example space expansion as a process of seeing and making new connections and relationships.

Gail provided a range of examples of where mathematical thinking would be required for administrative tasks.

Gail: Yeah, actually. There was. They were raising money and they were selling things. So, there was. And it was at the end of my three weeks, so I kind of didn’t see how it went, but they definitely were needing to be aware of the financial, the money that was there and how much they would like to raise. … … but one thing that did pop to mind was they needed often to be splitting the class up into groups. And so, they needed to have a really quick kind of idea of you know, just what size groups, or how many Year 7, or how many Year 7 would be in certain groups. … And time frames. Like I’m thinking of swimming, and they had a lot of swimming. So how much time was needed for each group and things like that.

Participating pre-service teachers from category B were able to identify mathematical and or statistical thinking across the curriculum but not for data literacy or administration. Sally was one such pre-service teacher. She identified mathematics being present in geography (graphing), art (tessellations), music (patterns) and English, referring to a series of books that combined an emphasis on mathematics within an engaging story. She also referred to science and social studies being connected to mathematical and statistical thinking. Note that while the context of this research is pre-service primary teacher education, comments provided by the research participants included subjects from the secondary education: for example, the reference to geography in the above example.

Transcripts were categorised as category C if they reflected a view of mathematical and or statistical thinking as primarily for teaching mathematics, even after several attempts to explain the focus of the MARKITE project. Krystal was typical of these students. Early on, despite being informed that the project was ‘about the mathematics that is needed for teachers in their wider role. So, we’re not talking about the mathematics you need to understand for teaching maths” Krystal responded with a focus on the teaching of mathematics. Recalling what she had on her practicum, ‘I did take a few groups of maths. Luckily it was mostly with a focus on statistics.’ There followed a discussion on her teaching of statistics, and then the interviewer reiterated the broader focus of the MARKITE study and offered an example:
**Interviewer:** Yeah, okay. So that’s an example of you being the mathematics teacher, what about using mathematics as a teacher in the wider professional role? I’ll give you an example, like sometimes teachers organise field trips, or a school camp, and they find that they need some mathematics in that process. Can you think of any examples you saw your AT [Associate Teacher] using mathematics like that? Or that you used mathematics when you were on prac?

**Krystal:** No.

**Interviewer:** No?

While it is possible that Krystal was simply reflecting not having seen examples her next statement showed she was still thinking about the teaching of mathematics as she stated, “No, it was very like old school maths as a subject”. She then began to ponder on some possibilities for mathematics being integrated across the curriculum, but quickly returned to thinking about the teaching of mathematics. Across this dialogue we can see how, despite attempts by the interviewer, Krystal continued to prioritise mathematics within the teaching of mathematics.

(ii) **Pre-Service Teacher Commentary (From Mentor Meetings) on Each of the Three Domains of Thinking**

**Pre-Service Teacher Commentary on Curriculum Learning Areas**

In both years during the mentor meetings pre-service teachers provided examples of mathematical and statistical thinking across the curriculum that they had observed during their first practicum and/or in their university course work (see Table 2). We acknowledge that it is likely that the distribution of pre-service teacher awareness was influenced by a multitude of factors including individual backgrounds, the curriculum subjects they were studying at the time of their mentor meeting (for instance, science and technology were not taught in the first semester—the time at which mentor meetings were held), and lecturer’s involvement or otherwise in the wider MARKITE project (Cooper et al., 2017) which changed from year to year.

<table>
<thead>
<tr>
<th>New Zealand curriculum learning area</th>
<th>Project year 2 n=35 mentions</th>
<th>Project year 3 n=26 mentions</th>
</tr>
</thead>
<tbody>
<tr>
<td>English</td>
<td>1 (3%)</td>
<td>3 (12%)</td>
</tr>
<tr>
<td>The Arts</td>
<td>16 (46%)</td>
<td>6 (23%)</td>
</tr>
<tr>
<td>Health and Physical Education</td>
<td>2 (6%)</td>
<td>5 (19%)</td>
</tr>
<tr>
<td>Learning Languages</td>
<td>1 (3%)</td>
<td>2 (8%)</td>
</tr>
<tr>
<td>Science</td>
<td>2 (6%)</td>
<td>2 (8%)</td>
</tr>
<tr>
<td>Social Science</td>
<td>13 (37%)</td>
<td>6 (23%)</td>
</tr>
<tr>
<td>Technology</td>
<td>0 (0%)</td>
<td>2 (8%)</td>
</tr>
</tbody>
</table>

Table 2: Numbers of examples of mathematics and/or statistics recognised across the curriculum given during Year 2 and 3 of the project’s mentor meetings (Percentages rounded to nearest whole number)
Pre-Service Teacher Commentary on Data Literacy

In the second year of the project, 12 (all eight in category A and four from category C) pre-service teachers mentioned some aspects of the analysis and/or use of assessment data in their initial mentor meetings. For example, Anna had noticed during her practicum that teachers discussed tabulated assessment data. She commented, “Like they did have tables and stuff like that, that we’d go through in their meetings, about how good their kids were doing”. Molly, who was a member of a school governing body, explained that teachers and the school as a whole needed to make sense of data to help them plan the direction for learning and strategies that a school might need to enact to achieve this. Tertia drew on her previous experience as a teacher aide, to describe how children’s oral language skills were tested, explaining this information was entered into an EXCEL spreadsheet and used to help target teacher support.

In the third year of the project, of the 18 pre-service teachers interviewed during mentor meetings at the beginning of the Year, 11 (all those in category A and one from category C) showed some awareness of student assessment data. A number commented that assessment data was gathered for the purpose of grouping. A typical comment was, “It was helping them get their Maths groups organised” (Natalie). Three pre-service teachers indicated a wider understanding of the need for teachers to analyse and act on data. Leon explained, “teachers are expected to analyse that [student test] data and make adjustments to their teaching in response”. Harry said, “I imagine testing students, and then providing feedback and analysing what they, what went wrong. Things that they did well in; areas that they could improve on”.

We were not surprised that a number of pre-service teachers recalled the use of assessment given their first practicum was at the beginning of the school year when many, if not most, teachers are using assessment data to make decisions about their teaching and learning programmes.

Pre-Service Teacher Commentary on Management and Administration Tasks

Pre-service teachers in both the second and third year of the project detailed classroom management activities such as dividing students into groups of a particular size, ensuring there were enough resources, and managing money. Most of the examples involved what some pre-service teachers described as ‘basic’ or ‘low level’ mathematical thinking such as counting. Some also discussed time management during lessons. For example, during the mentor meeting, Gail noted that in her school the Year 7 and 8 students were cross grouped for remedial maths and English so calculating the time required and remaining for different activities—games, tidying up—was important.

(iii) Vignettes Showing the Development of Example Spaces

Over the course of the year, during focus group discussions, we observed that the examples shared amongst the pre-service teachers became more specific and encompassing implying an evolving example space beyond “more of the same” examples (Zaslavsky & Zodik, 2014). We provide two vignettes to illustrate this development.
Project Year 2 Vignette: An Expanding Collective Example Space

Lucy, Shona and Norah attended the mentor meeting held early in the year as one group. Throughout the meeting their focus was on teaching mathematics, despite several attempts by the interviewer to prompt them to consider the wider role of mathematical and statistical thinking in teacher’s professional role. Debbie attended the mentor meeting on her own. Like Lucy, Shona and Norah, she did not display any real awareness of mathematics beyond the mathematics classroom, although she did recall seeing the results from maths and spelling testing while on her first practicum.

During the pre-practicum focus group all four pre-service teachers (Lucy, Shona, Norah and Debbie) discussed ideas as a group. It is surprising that only six weeks (approximately) after the mentor meeting, they evidenced a greater awareness of the need for mathematical and statistical thinking across teachers’ professional role. They listed this thinking as being a part of assessment, keeping a roll, organising groups, managing school trips, budgeting (class trips and classroom), determining ratios for parent help, and moving between digital and analog time. With respect to an awareness of mathematics across the curriculum, the list of possible applications they generated included links to science (geological time, the use of graphs and statistics), the arts (patterns, symmetry), technology (measurement to make products), social studies (critical analysis of data presented to us in our everyday lives), PE and health (teams odd and even, speed and timing, spatial awareness) and languages (counting). Our observation of the group indicated that throughout this discussion the pre-service teachers were able to construct a rich collective example space (Watson & Mason, 2005) without researcher prompting.

While 19 pre-service teachers attended the pre-practicum two focus group, eight came to the post practicum two focus group. Honing in on Lucy, there was evidence in her contributions that her example space had expanded since the earlier mentor meeting. She identified the mathematics within a health and physical education lesson as helping children interpret and understand nutrition labels saying, “… And so we did a focus on measurement for the time I was with them. And so, it was good, ‘cause we got to see the difference between a milligram, and—all the different measurements’”. Later in the discussion she built on the contribution of another pre-service teacher and detailed the role of mathematics in organising sports competitions saying,

*I helped her do one of the pizza days, and so I was just getting pizza for the kids—the sports leaders went around and collected numbers for the pizzas. And then we had to go through and decide how many pizzas we would order from that. Things like $2 a slice, so if they ordered four, how many should we get for this group? And how many do we get for backups in case....*

The final focus group discussion in Year 2 of the project was held as part of the wider teacher education programme debrief at the end of the year. The debrief was attended by all pre-service teachers in the programme that year, many who had not participated in mentor meetings or focus group discussions. In six groups they were asked what, if any, mathematical thinking they had observed outside of maths class while on their recent 7-week practicum; and what aspects of mathematics they felt they still needed support with for their wider professional roles. Approximately half of the written comments were about the mathematics teachers needed to teach mathematics, mirroring the lack of awareness of mathematical and statistical thinking beyond maths lessons that had been evident during many of the mentor meetings held early in the year. This focus can be contrasted with data gathered from the focus group discussions held with the research participant pre-service teachers throughout the year and provides some evidence of the potential for ongoing discussion to expand pre-service teacher example spaces.
Project Year 3 Vignette: An Expanding Individual Example Space

Harriet attended the beginning of the year mentor meeting on her own. Despite several attempts by the interviewer to explain the MARKITE focus on teachers needing mathematical and statistical thinking across their professional role, at the end of the 15-minute mentor meeting she was still not clear about what this implied, and asked, “Is that to help inside the classroom?”. Nonetheless, she showed some awareness of the use of testing in literacy and mathematics and the results being related to placing children into groups. Harriet speculated, “The knowledge I basically learnt was that she (referring to Harriet’s associate teacher) uses it to group her children and to get a level of understanding of where they’re at”. When prompted to consider curriculum areas outside mathematics and statistics, she interpreted the question as asking if she had observed the curriculum areas and did not offer any examples.

Harriet attended the pre practicum 2 focus group discussion. This time she demonstrated an awareness of mathematical thinking in other curriculum areas identifying Social Studies and Māori as two possibilities. A month later during her practicum Harriet emailed the researchers a reasonably extensive list of the mathematics she had noticed. The list included: singing songs with numbers during a singing assembly, counting when instructions were given during fitness, teachers working on classroom budgets, considering quantities for photocopying and organising resources, a parent evening where learning levels and statistics were shared with parents, running records, managing lunch orders and trip money, counting the number of objects and lines in art, calendar mathematics, Māori words for numbers and numbers being used to name groups. Harriet also attended the pre practicum 3 focus group. During that meeting, when given a curriculum map outlining the mathematics embedded within her university papers (curriculum and professional practice), she highlighted most of the opportunities as ones she had recognised during her studies.

(iv) Project Year 3 Pre-Service Teacher Reflection on their Raised Awareness

As part of the pre practicum 3 focus group in the final year of the project, we asked the pre-service teachers directly if they thought they were ‘perhaps’ more aware of mathematical and statistical thinking across teachers’ wider professional role. The consensus was that they were as the following dialogue illustrates:

Harriet: Just from coming to the first MARKITE meeting that we had, it really opened my eyes to knowing that there’s Maths outside of Maths. ‘Cause apart from just working on Maths at Maths time, or whatever you call it, that’s all my thinking was before coming to MARKITE. Like you do Maths in Maths and that’s that, kind of thing. Like without really realising there’s like reading data, and behaviour management, and there’s lots of other subjects that also encompass Maths. Yeah. So, it was just an eye-opener for me.

Joanna: I think related to that; it’s really strengthened sort of to my pedagogical approach in realising where the Maths is. So being able to incorporate it, and being able to sort of offer a Maths programme that’s a little bit more integrated, … And then my ability to use Maths as a teacher as well, in the way I collate my data and stuff.

Raewyn: As I was saying, like I first said yes at the start, because I was enjoying Maths. And then when we had that next meeting after our practicum, and how much I realised on practicum, how much we use Maths. I was like, ‘Wow.’ Like it just opened my eyes.

Katy: It’s almost kind of like PD, an undergrad reflection thing.
Multiple Voices: Yeah, yeah.
Julia: I think if this hadn’t been going on while we were here, I don’t think I would have necessarily thought about it.
Raewyn: … it’s definitely changed our thinking. And I think more people would have been on board if they had have known what we got out of it.

We can see here that for these pre-service teachers the process of focusing on the possibility of mathematical and statistical thinking outside of maths lessons had helped make visible the hitherto invisible role of mathematical thinking to the extent that one of them described the discussions as ‘kind of like PD (professional development)’. The group also followed up on Joanna’s comment on the shift in her pedagogical thinking as a consequence of realising ‘where the Maths is’. They discussed some of the ways they might integrate mathematical and statistical thinking more deliberately, and overtly, into learning activities across the curriculum.

There were also indications that this group of pre-service teachers was developing the capacity and inclination to critique the use of mathematics and statistics. An example arose in relation to a campus-based lesson on assessment where these pre-service teachers had made sure that they understood the statistical thinking that underpinned the established procedure for using data. One explained, “Like with [deleted] as an example, it is important that we know what those numbers mean, ‘cause otherwise why are we writing them down if we don’t understand them?”

We acknowledge that the mentor meetings and focus group discussions are not the only activities the pre-service teachers were engaged in during their year of teacher education (and this research). The mathematics education course and practica are also activities that could have influenced their awareness of mathematical and statistical thinking across the wider professional role. However, the dialogue above does suggest the research process of sharing examples during mentor meetings and focus group discussions had at least some role in raising awareness and adding to their sample space.

Discussion

The mathematical and statistical thinking involved across the breadth of people’s everyday lives tends to be invisible to them. The same is true of teachers. This is of concern given there is ample research on the value of teachers using everyday contexts for the mathematics activities that they use with children. It also poses a challenge when teachers are considered to have a responsibility to help students understand and use mathematical and statistical thinking across the curriculum. Recent research and policy initiatives also highlight that teachers need to be able to collect, analyse and act on individual and collective student assessment data, which also comes with implications for teacher mathematical and statistical understanding (Cowie & Cooper, 2016). The mathematical and statistical thinking involved in management and administration tasks is not often made explicit but can be inferred as underpinning teaching related tasks such as allocating students to groups and organising field trips.

All of the pre-service teachers who participated in the project Year-3 focus group discussions reported that their involvement in the research alerted them to the existence of mathematical and statistical thinking across the curriculum, in the analysis of student achievement data, and in administrative and management tasks. That is, for these pre-service teachers the mathematical and statistical thinking used across the breadth of their professional work was no longer invisible. Time and again during the focus group discussions we witnessed pre-service teachers’ awareness change as a result of another pre-service teacher’s
contribution to discussion. Hence, our data indicates that sharing and discussing examples was a useful strategy for raising the awareness of our pre-service teachers of the presence and role of mathematical and statistical thinking outside of mathematics classes. These findings resonate with Gattegno’s (1987–2010) proposal that awareness is educable: that is, the propensity to attend to and note mathematical and statistical thinking can be developed and refined. Put another way, the discussions (in mentor meetings and focus groups) served as a productive means for extending the personal example space the pre-service teachers had to draw on when considering the presence and role of mathematical and statistical thinking. This finding echoes the assertion by Watson and Mason (2002) that deliberately prompting students to generate examples can be a powerful tool for alerting them to the role/s mathematical thinking has, or might play, in a context. At the same time, it extends their work, and that of Zaslavsky and Zodik’s (2014), to discussions amongst adults and about the wider role of mathematical and statistical thinking in teachers’ professional work.

Given Mason’s (1998) and Hewitt’s (2001) contention that awareness is a precursor to action, particularly to enabling fresh and creative responses, the data showing an expanding example space and awareness of the wider role of mathematical and statistical thinking opens the possibility that pre-service teachers will be better prepared to support student learning. In particular, during the final focus group discussion in the project Year-3 pre-service teachers spoke of collecting and using data, offering a more integrated programme (i.e., being aware of mathematics across the curriculum) and being able to engage critically with the use of statistical data: all actions likely to enhance their future teaching practice. Whether this happens in practice is yet to be investigated.

If we value the role of mathematical and statistical thinking as an important aspect of well-being and active participation in democratic society, then the question for us as pre-service teacher educators is how best to embed discussions that make visible and extend pre-service teachers’ personal example spaces. Literature on the teaching and learning of transversal concepts, or concepts that run across the curriculum such as assessment has identified a number of models for teaching these ideas. These models range from dedicated elective courses to dedicated compulsory courses to programmes that embed ideas across all curriculum areas, courses, and an institution (DeLuca & Lam, 2014; Evans, Stevenson, Lasen, Ferreira, & Davis, 2017). Indications from the wider MARKITE study are that teacher educators making visible and paying attention to any mathematical thinking embedded in their courses can be productive (Cooper et al., 2017). Further research is needed to explore in detail the benefits of this experience, and how and if such discussions support beginning teachers to make deliberate use of this thinking across their professional work.

References


