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Learning Outcome Literacy: The Case of Five Elementary Mathematics Teachers[1]

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Abstract: Learning outcome (LO) literacy is conceptualized as a necessary skill for planning teaching and learning activities effectively. The skill has two main components: understanding and interpreting learning outcomes. The current study aimed to identify learning outcome literacy of elementary mathematics teachers with regard to this conceptualization. It is a case study based on the qualitative design. Participants were 5 mathematics teachers working at different elementary (5^{th} - 8^{th} grade) schools in Turkey. Observations and semi-structured interviews were used for data collection based on 10 learning outcomes selected from algebra and number learning domains in local mathematics curriculum. Content analysis method was used for data analysis. According to the findings, LO literacy can be regarded as a basic teacher competence and it requires the teacher to use the knowledge and skills he/she has. The results showed that participant teachers' LO literacy was insufficient. Considering the interrelation among its components, LO literacy should be developed with all the sub-components for effective teaching of LOs.

Key Words: Teacher competence, planning teaching, learning outcomes, teacher knowledge, mathematics education

Introduction

The teacher's role is complex and shaped by historical and contemporary variables (Arends, 1991). Teachers should learn the process of teaching by thinking at higher levels and improving performance in order to help students learn analytical and complex knowledge and skills. In this context, the education system should provide an effective vocational learning to teachers with the aim of developing the sophisticated teaching (Darling-Hammond & Richardson, 2009). Teacher competencies are coming into consideration as an important aspect for teachers, teacher educators and officials in Turkey. Teacher competencies are the basis for questioning and developing the qualities of teachers who play a big role in the learning and teaching process. The competencies are taken into account in teacher education, teacher training programs, in-service and pre-service training of teachers,

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evaluation of performance of teachers, and career development of teachers by the Ministry of National Education (MNE) in Turkey (MNE, 2008). For this reason, the teacher competencies and qualifications become an issue that is carefully addressed by teacher training programs.

Teachers are the professionals who make decisions based on skill, intuition, and specialty (Ball, 1991). Developments in teaching partially depend on systematic and intentional demonstration of skills outside the classroom (Morris, Hiebert, & Spitzer, 2009). One of the most important skills outside the classroom is planning teaching (Morris et al., 2009; Zazkis, Liljedahl, & Sinclair, 2009) which enables decision making that responds to the students' needs and the expectations of curriculum (Bellon, Bellon, & Blank, 1992). Curriculum includes fundamental elements such as goals, objectives, or learning outcomes which guide teachers for planning teaching. Learning outcomes (LOs) itemize the overarching "products" of the lesson. LOs are the indicators that goals or objectives are achieved. Although LOs express knowledge, concepts, skills, and actions that students should know, understand, improve, and perform after completing the learning process (Kennedy, Hyland, & Ryan, 2007), there is not much detail about the teaching and learning. Definitions of LOs can only explain a small number of variables to consider in teaching; teaching is a far more complex than curriculum-makers would believe (John, 2006). LOs are important but utilizing them is up to the teachers. There are teacher competencies that educators see compromised and indispensable. One of them is the interpretation and understanding of the LOs in the curriculum and the teacher's penetration into the philosophy, vision, and mission of the curriculum. In this context, the important competence that teachers should have is understanding and interpreting LOs to acquire them to students. This competence is termed *learning outcome literacy* in this study. The current study investigated how elementary mathematics teachers understand learning outcomes and how they interpret learning outcomes to make their students acquire them. Accordingly, the aim of the present study is to investigate elementary mathematics teachers' learning outcome literacy.

Teacher Competence

Teacher competencies are defined as "the knowledge, skills, and attitudes that teachers need to possess in order to fulfil the teaching profession effectively and efficiently" (MNE, 2008, p. VIII). Teacher competencies can also be defined as an integrated set of personal characteristics, knowledge, skills and attitudes required for effective performance in various teaching contexts (Tigelaar, Dolmans, Wolfhagen, & Vleuten, 2004). According to this definition, the teacher competencies that are useful for the purpose of organizing the vocational learning objectives for teachers are integrated and should be seen as a repertoire that the teacher has (Tigelaar et al., 2004). The competencies needed for effective mathematics teaching provide a framework for the teacher's career and clarify what kind of progress should be made. They clarify the professional characteristics that a teacher should construct and maintain in their career steps. The competencies support teachers in determining their professional development needs (Training and Development Agency (TDA), 2007).

Planning and regulation of teaching mathematics is one of the 6 basic competences identified by the MNE in Turkey (MNE, 2008). The indicators of this competence emphasize the importance of planning *in direction of* the mathematics course curriculum (MNE, 2008). Similarly, the National Council of Teachers of Mathematics (NCTM) and the Council for Accreditation of Educator Preparation (CAEP) emphasize planning teaching by applying knowledge of curriculum standards (2012). However, curriculum is only a plan in the beginning, and it is limited by itself (Kauffman, 2002; Marsh, 2009; Ornstein & Hunkin, 2004; Remillard, 2005); its effectiveness depends on how teachers understand and enact it, and these all depend on teachers' knowledge, disposition, and competences (Franke & Grouws, 1997; Remillard, 2005; Schwarz et al., 2008).

The Turkish Elementary Mathematics Curriculum

The educational system in Turkey is centrally organized, and the MNE is the main body for planning, programming, implementing, monitoring, and controlling education and training services including development of school curricula (Binbasioglu, 1995). The curriculums are prepared by commissions comprised of representatives from the relevant departments of the MNE, teachers, educational experts and academics. In this way, the development of curricula is ensured by teachers, parents, school administrators and experts from all over the country together with the officials of the MNE. The Turkish Elementary Mathematics Curriculum (MNE, 2013) is designed for students at 5^{th} - 8^{th} grade (10-13 years old). The curriculum is organized with four learning domains which are Numbers, Geometry, Measurement and Data. All learning domains are included in each grade, but some sub-learning domains are included after specific grade. Learning domains are subdivided to the sub-learning domains, it is determined which learning outcomes will be covered in these learning domains. The learning outcomes of the Turkish Elementary Mathematics Curriculum summarize the knowledge and skills for students to develop (Babadoğan & Olkun, 2006). It is stated in the curriculum that the sequential order of learning and sub-learning domains in the curriculum is not the order of teaching them. In the curriculum, the teacher is expected to make plans in such a way that he or she can use a learning outcome more than once in different learning domains, or bring together different learning outcomes out of different learning domains (MNE, 2013). The main element of the curriculum is the learning outcomes determined according to the learning domains and sub-learning domains. Since these learning outcomes can be acquired through different learning activities, Turkish teachers are relatively free to produce or choose activities (Babadoğan & Olkun, 2006). Accordingly, teachers need to understand and interpret LOs for planning and regulating teaching to enable students to acquire LOs. Therefore, as a necessary skill for planning and regulating teaching activities effectively in accordance with the curriculum, teachers should understand and interpret LOs to make their students acquire them. In this study, aforementioned competence is called *learning outcome literacy*.

Learning Outcome Literacy

LO literacy is a teacher competence. LO literacy can be dealt with the competence that planning and regulation of teaching mathematics. Having this competence requires sufficient knowledge and skills. Once a teacher starts planning teaching, he/she should first consider what the learning outcome of the lesson is. Then, he/she should decide on some key questions like “Why is this LO in the curriculum? What is the most important point for students in this LO? What and how should I teach in the framework of this LO? Or what should I do to equip my students with this LO? What should my students do, say, draw, and/or represent etc. to make me confident to say that they acquired this LO as I aimed?" LO literacy can be addressed to answer these questions and to ask more specific questions about teaching a LO. It requires two main skills respectively; understanding LOs and interpreting LOs to make students acquire them.

Understanding Learning Outcomes

Teachers' mathematical knowledge is in the centre of making sensible decisions on presenting, emphasizing and using instructional materials and evaluating students' developments (Anthony & Walshaw, 2009; Ball, 2003; Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008). Understanding a LO involves understanding its mathematical structure and its relations with other LO's mathematical structures. Additionally, understanding a LO involves understanding domain specific skills in the LO. It entails mathematical content knowledge, and it is not apart from teaching. Knowing how to teach mathematics necessitates more than mathematics knowledge of any educated person (e.g. a mathematician, an engineer) (Ball et al., 2005; Hill, Schilling, & Ball, 2004). Knowing mathematics for teaching requires more than performing an algorithm truly or developing sensible explanations that explain the meanings of the algorithm and provide justifications about the steps of the algorithm (Ball & Bass, 2002; Hill, Rowan, & Ball, 2005; Hill et al., 2004). A teacher should know a wide variety of approaches to a solution and different views on an idea, and he/she should be able to distinguish their advantages and disadvantages and provide different explanations for them (Ma, 1999). A teacher should have the capacity to understand why and how a proposition is accepted as correct, what the reasons of its correctness are, in which situations beliefs about its correctness are strong or limited. Besides, a teacher should be able to define concepts and mathematical relations for students (Shulman, 1986). In this context, *a teacher should have the knowledge of mathematical structures (concept, formula, relation, operation, axiom, proof and special mathematical points) in a learning outcome*.

Teaching entails knowing about the interconnected and sensible nature of mathematical structures (Ball, 2003; Ma, 1999). A teacher should be able to explain mathematical relations inside and outside the domain in theory and practice. Having sufficient knowledge about a domain is important for pedagogical decisions about the foci points of a curriculum (Shulman, 1986). Teachers should understand and use mathematical relations of mathematical structures in a LO and their mathematical relations with the mathematical structures of other learning outcomes. Therefore, a teacher should have enough *knowledge about the mathematical relations between the mathematical structures in the outcome and those of other outcomes in the curriculum.*

A teacher should develop domain specific skills, and he/she should competently perform mathematical activities such as generalization, pattern recognition, problem solving, estimation, proving, and justification of solutions. These are other knowledge domains about mathematics (Ball & McDiarmid, 1990). Therefore, *a mathematics teacher should have the knowledge of domain specific skills in a LO.*

Interpreting Learning Outcomes to Enable Students Acquired Them

Before considering how to teach or learn mathematical knowledge and skills, teachers should make clear what they expect from their students (MacDonald, 2012). The numerous duties of teaching such as choosing meaningful learning activities, providing useful explanations, asking questions, evaluating teaching and learning and searching for evidence about students' understanding depend on teachers' perceptions about what students should learn (Ball & McDiarmid, 1990; Morris et al., 2009). These duties depend on also teachers' critical understandings about the development of particular mathematical ideas (Simon, 2006). LOs are the evidence that goals or objectives have been achieved. Therefore, teachers should reveal which goals are served by a LO. Remembering that the goals are the starting points that "set the stage for everything else" (Hiebert, Morris, Berk, & Jansen, 2007, p. 57), a teacher *should be capable of specifying goals for students' acquisition of a LO*.

After specifying the goals, the plan should be detailed to make the goals understandable and reachable for learners (Black, 2001). The teacher should explain why the goals are important for students. According to these decisions, the teacher should choose appropriate representations, useful examples, materials and models to bring key ideas of mathematics into the forefront (Ball et al., 2008; Hill et al., 2004). He/she should consider which one is appropriate for starting a lesson or going deeper into the concept in choosing examples, materials, models, etc. The teacher should use mathematically correct, complete and understandable definitions for students (Ball & Bass, 2002; Hill et al., 2004). He/she should know the reasons of relations, procedures and different variety of methods, and analyse the correctness of different methods and generalization (Ball et al., 2008; Delaney, Ball, Hill, Schilling, & Zopf, 2008; Hill et al., 2004). Therefore, a tea*cher should be capable of analysing knowledge and skills in a LO in an effort to enable students to acquire it*.

On the other hand, a teacher should know her/his students' mathematical background (Ball, 2003) and important characteristics of that grade, and how new learning can be constructed on these (NCTM, 2000). He/she should not only focus on one grade; he/she needs to have deep understanding and knowledge about all the curriculum (Grossman, 1990; Ma, 1999; Magnusson, Krajcik, & Borko, 1999). Accordingly, he/she should know about what the contributions of previous outcome are to this LO, and the students' prerequisite learning. In other words, the teacher should know about LO's instructional relations with the prior one. Similarly, he/she is required to know what the contributions of LO are to the next one, and how these two outcomes are related instructionally. Therefore, a teacher should be aware of a *LO's significance for curriculum* and should have *knowledge about the instructional relation of the outcomes with the other outcomes and learning domains in the curriculum.*

If the goals are identified clearly, and pedagogical decisions are made according to the students' educational and contextual realities, then, making instructionally right and sensible decisions will be possible (Harris & Hofer, 2009). The goals of teachers and curriculum should be coherent with the evaluation process (Black, 2001; NCTM, 2000). Teachers should be able to conjecture about the evidence showing whether the students acquire target knowledge and skills or not. Briefly, evaluation should be the mirror of learning outcomes (Kennedy et al., 2007; Marsh, 2009). Therefore, a teacher should be capable of *revealing whether students acquire LOs or not.*

As it is seen, a teacher needs to draw on content knowledge, pedagogical content knowledge, curriculum knowledge, context knowledge, etc. (Grossman, 1990; Harris & Hofer, 2009; Magnusson et al., 1999; Shulman, 1986) for being learning outcome literate. Together with the knowledge of a teacher, the way how a teacher applies his/her knowledge to teaching is important. LO literacy enables it in a systematic and explicit way.

Method

This research is a case study employing a qualitative design (Yin, 2013) investigating elementary mathematics teachers' learning outcome literacy. It portrays the current conditions of teachers' understanding and interpreting learning outcomes over which researchers have little control (Yin, 2013). Elementary mathematics teachers constitute the case of the research. Teachers' understanding and interpretation of learning outcomes are the units of analysis.

Participants

The participants were 5 elementary mathematics teachers. The elementary mathematics teachers teach students at 5th - 8th grade (10-13 years old). Teachers participated voluntarily in this study. One of the participants (Mete) was male and the other participants (Gaye, Tuba, Hale, and Sema) were female (the names are pseudonyms). While Gaye and Mete have 10 years of experience, Tuba and Sema have 8-year-experience, and Hale is in the second year of her profession. Mete works at a private teaching institution, and others work at state schools. Gaye is interested in writing an elementary mathematics textbook.

Data Collection and Procedure

The data were collected through semi-structured interviews, nonparticipant in-class observations with audio recordings and field notes written by the present researchers during and after the observations. The important aspect of the observations was to see the consistency between the interviewees' actions, decisions and behaviours in the class and their statements during the interviews. Thus, trustworthiness of the study is established through the data triangulation- combining interviewing and observations (Patton, 2005). The interviews and observations were based on 10 learning outcomes selected from learning domains: algebra and number in 6th grade mathematics curriculum of Turkey. The reason for selecting algebra is about the observation schedule of the study. Providing variety of mathematical structures (relation, operations etc.) in the LOs was considered in the selection of other LOs. Thus, the domain of number was selected because of its close relation to the algebra.

Learning Domain	Sub-Learning Domain	Learning Outcomes
Algebra	Patterns and Relations	***LO2.*** Students can model the numerical patterns and express the relationship in these patterns by using letters.
		LO3. Students can express repeating multiplication of natural numbers as an exponential quantity and determine the value of exponential quantities.
	Algebraic Expressions	***LO1.*** Students can write appropriate algebraic expressions for specific situations.
	Equality and Equation	***LO4.*** Students can model and explain the conversation of equality.
		LO5. Students can explain the equation and set appropriate equations for the problems.
		LO6. Students can solve the first degree equation with one variable (one unknown).
Number	Fractions	***LO7.*** Students can compare, order and show fractions in a number line.
		LO8. Students can perform addition and subtraction with fractions.
		LO9. Student can perform multiplication with fractions.
		LO10. Student can perform division with fractions.

Table 1. The learning outcomes employed in the data collection

The interviews were conducted in 4-5 different sessions in a silent and appropriate environment at teachers' schools. The sessions lasted about 60-70 minutes. The example pages of explanation and activity for each LO in the curriculum were given to the teachers during the related interviews, and teachers were free to use the textbooks if they needed.

Observations were conducted in the classes of Gaye, Tuba and Mete as they were voluntary for the observation. LO literacy of these teachers differed from each other in the interviews. Thus, it was beneficial for the diversity of the observation data. Observations ranged from 6 to 8 classroom sessions for each teacher. Since number learning domain would be taught in the next semester, the lessons on algebra were observed.

Preparation of Semi-Structured Interview Forms

Before starting data collection and preparing interview forms, the following questions were examined by the researchers: How should a learning outcome be understood by a teacher in planning and evaluating a lesson for effective teaching? How should it be interpreted and analysed by a teacher to enable students to acquire it as it is expected by the curriculum? Each LO was analysed separately in the framework of these questions. According to this examination, the semi-structured interview questions were prepared. An example of the semi-structured interview questions for LOs are below:

Suppose that you are planning a lesson to make your students acquire the learning outcome: *"Students can model numerical patterns and express the relationship in these patterns by using letters[2]"*

1. What do you understand from this LO when you read it?
2. Please explain the mathematical structures (concepts, relations, operations) included in the LO. Refer to the definitions.

Prompts: What is the pattern? Is the pattern a relation? Do only numbers construct a pattern? If we consider number patterns, is each sequence a pattern?

3. What are the special key mathematical points in the LO? Explain them.

Prompts: Can you explain "modelling number patterns"? What points should we be careful about modelling? What is the meaning of expressing by letter?

4. Which learning domain involves this LO? What are the previous and the following LOs?
5. What could the reasons be for including this LO in the curriculum? Explain.
6. What do you expect from a student who acquired this LO? What should student display (representation, behaviour, discourse, action etc.) to make you satisfied that he/she acquired this LO? Exemplify.
7. What is(are) the most important point(s) in the LO for your students? What are expected from the students to acquire through the outcome? State which one is the main one among them. Explain why.

Prompts: What would be the aim of modelling number patterns? What would be the aim of finding the relation of patterns? Which one is the main goal of the LO: the modelling or expressing the relation by letters? If the goal of the LO is expressing the relation between the objects of a pattern by letters, what is the role of modelling pattern?

8. Are there any effects of student's acquisition of this LO on the previous and the following LOs? Explain.
9. What are your foci points to make your students acquire this LO? Explain why.
10. What would the effective ways of teaching this LO be? What do you pay attention to enable your students to make sense of this LO?

Prompts: What is the pattern definition that your students can understand? Could you give examples for representations, models, and examples etc. that you apply in your lessons for this LO? What are your first and last examples? Explain why. How do you facilitate and encourage your students for modelling, finding a relation and expressing by letter?

[2] This LO is the first of two LOs that is included in the sub-learning domain of patterns and relationships of the 6th grade algebra learning domain (MNE, 2013). Understanding patterns is one of the goals of 6th - 8th grade algebra in Principles and Standards for School Mathematics (NCTM, 2000). In Program Focal Points, it is stated that sequences, obtained by finding the rule of the objects or shape patterns in the form of a link to other focal points of the writing, interpreting and using mathematical expressions and equations, provide opportunities for students to develop formulas (NCTM, 2006).

11. What do you do to reveal whether your students acquired this LO or not? Exemplify. What are the properties of your activities and questions for assessment and evaluation? Explain.
12. What would other indicators be for students' acquisition of this LO? Could you give examples?

Since each LO has different mathematical structures and instructional pin points, detailed examinations were conducted. Thus, independent and different interview forms were prepared for each outcome. The pilot interviews were conducted with two research assistants who were studying for their master's degree on mathematics education and three elementary mathematics teachers. The interview forms were revised according to the pilot studies. After this revision, interview forms were submitted to two experts who have PhD degree on mathematics education. According to the pilot studies and expert opinions, the questions were revised to provide clear and explicit meanings, and some questions were added in the interview forms. For example, realizing that the teachers might answer the questions 5 and 8 by ignoring the curriculum, it was decided to ask question 4 directly to make teachers interpret the LO by considering the curriculum.

Data Analysis

The data were analysed through content analysis after teachers' words were transcribed verbatim. Data analysis was carried out for each outcome and each teacher separately to reveal the categories of teachers' understanding and interpreting outcomes. The transcripts and field notes were examined more than once to categorize the data. The patterns characterising teachers' literacy were searched for in the teachers' utterances, representations and/or drawings, to create categories. The transcriptions and field notes which were obtained from the interviews and observations were analysed for identifying repeated patterns. This process led us to identify specific categories. According to data analysis, the categories were consistent among five teachers for ten different LOs. Thus, these categories were considered as the sub-components of the LO literacy. The sub-components were compared to each other, and it was seen that they could be aggregated under two main components: understanding and interpretation of learning outcomes which were the units of analysis. Thus, LO literacy framework resulted from the analysis of research data. It is undeniable that the interview questions may influence the obtained sub-components. However, most of the sub-components were revealed from the research data. For example, the interview questions 11 and 12 might promote the showing up of the sub-component of *revealing whether students acquire LOs or not*. However, there was no direct interview question for the sub-component of *knowledge of domain specific skills in a LO* or the subcomponent of *knowledge about the mathematical relations between the LO and those of other LOs in the curriculum*. These components were obtained from the research data.

After completing the data analysis, expert opinions were gathered again from three mathematics educators. Following the experts' opinions, the category, namely the sub-component of *knowledge of domain specific skills in a LO*, was included. Figure 1 shows LO literacy with its components which are revealed through the data analysis.

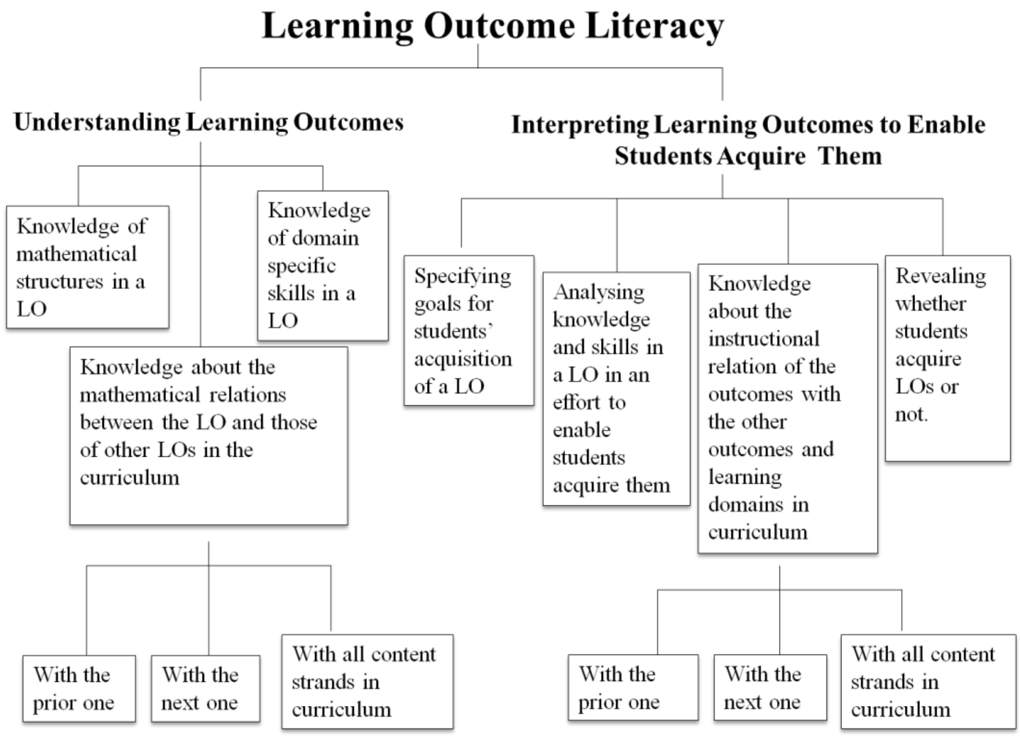

Figure 1. Learning Outcome Literacy

In the analysis, having all the components was described as the expected sufficiency. In other words, enabling to understand and interpret a LO to make students acquire it indicates strong LO literacy. However, if one of these main components was missing, then LO literacy was accepted poor or under the expected sufficiency. When we were not able to gather any information related to that subcomponent from the participant, it was concluded that this participant lacks that subcomponent, and it should be developed.

Results and Discussion

Teachers' LO literacy was found below the expected sufficiency for ten outcomes. Comparing teachers to each other, Gaye was literate of more LOs than other teachers. The literacy of one year experienced Hale is stronger than respectively eight and ten years experienced Tuba, Sema, and Mete. Therefore, it can be asserted that there could be different contributors to LO literacy other than the experience such as individual disposition or the quality of teachers' training programs (Baumert et al., 2010).

LO Literacy											
	Understanding a LO					Interpreting a LO to Enable Students Acquire It					
	Knowledge of mathematical structures	Knowledge about outcomes' mathematical relations with			Knowledge of domain specific skills	Specifying goals	Analysing knowledge and skills in LO	Knowledge about outcomes' instructional relations with			Revealing whether students acquire LO or not
		*									
Gaye	1, 2, 3, 5, 6, 9	2 ,3, 4, 5, 7, 10	1, 2, 4, 5	4 5	1 4 5	2, 3, 4, 5, 7, 8	3, 7	1,2, 4	1, 2, 4	1, 4	2, 3, 4, 5, 7, 8

Tuba	2, 3, 4, 5, 6	3, 4	4, 5		1, 5	5, 8, 9, 10	-			1, 4	-
Sema	-			2, 5	1, 5	3, 5, 7	-			2, 8, 10	-
Mete	5	1, 3	1, 5	1, 2	5	4	-	1,8, 10	1, 10	, 3, 10	4
Hale	5, 9	2, 3, 4, 5	1, 2, 3, 4, 5	2, 3, 4	1, 4, 5, 9	1, 2, 3, 4, 5	-	2, 8, 9	4	1, 2, 3, 9, 10	1, 2, 4, 5

* P. Prior LO, N. Next LO, C. All content strands

Table 2. Teachers' LO literacy with all the subcomponents for ten LOs[3]

Learning Domains and Learning Outcomes. Similar to the findings of the studies in the literature (e.g. Land & Drake, 2014), the teachers focused on mostly procedural knowledge rather than conceptual knowledge during the interpretation of the outcomes. For example, they heavily emphasized solving the equation for the outcome "Students can solve the first degree equation with one variable (one unknown)" (LO6). While Sema attached importance to the process of finding the value of the unknown, Mete seemed to have ignored it. Other teachers did not make any comments on this issue. Participants did not mention the conceptual understanding of the first degree equation with one variable and the meaning of solution, solving equations, and solution set. They did not discuss the acquisition of conceptual knowledge about rules and algorithms necessary to perform operations (Hiebert & Lefevre, 1986). For "Students can perform multiplication with fractions" (LO9), we prompted to teachers whether it was adequate for the student to be able to multiply the numerator and denominator in order to acquire LO9. Hale stated that she would like her students to notice the need for multiplication during the problem solving in addition to being able to perform the multiplication. Noticing the multiplication in the problem requires conceptual understanding. However, Mete and Sema stated that they did not aim "a detailed thing in the LO", but they expected their students to be able to perform the multiplication of fractions. Mete stated that it was important to know why the multiplication was done, however, he added that he did not "spend time for this" in his lessons. Sema, on the other hand, stated that students should know certain rules such as "multiplication of numerators is written on the numerator, multiplication of denominator is written on the denominator".

The target of the outcome should be not only performing the multiplication algorithm of fractions but also understanding the algorithm, modelling problems, solving the situations involving multiplication with fractions, and estimating the result. However, the participating teachers were not competent at reasoning the procedures, and they did not have enough knowledge about the underlying reasons of the procedures and conceptual explanations. For example, for LO10, the researchers examined the explanations of the participants on how to reach the division algorithm of fractions - invert and multiply- and why the dividend is fixed and the divisor is inverted and multiplied.

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[3] To understand Table 2, the first row includes information about Gaye. She had knowledge regarding the mathematical structures of six LOs which are 1, 2, 3, 5, 6, 9, but not the others; she was able to analyze knowledge and skills in two of ten outcomes which are 3 and 7. As another example, consider all the input in the table, there were no participants who was successful at performing all the subcomponents of the literacy regarding the LO1.

Mete stated that
Go from parts to the whole. According to the result [...] If we think ½ of 1/6, for example, think 1/6 as the result [referring $\frac{1}{12} \div \frac{1}{2} = \frac{1}{6}$ *] [...] We shade one of the 6 parts [returning* $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$*]. After that, I show that as halves by dividing from top to bottom by 2. You will take one of the two big parts. So we get one of 12 parts.*

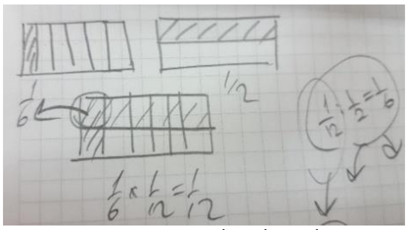

Figure 2. The model of Mete for $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$ **to explain** $\frac{1}{12} \div \frac{1}{2} = \frac{1}{6}$

Mete was aware of the inverse relation between multiplication and division and he applied this relation to explain the division algorithm of fractions (Ball, 1990). However, the purpose of Mete was to explain the algorithm by showing that the result of modelling is the same as that of the algorithm. Yet, it is an unrealistic expectation that visual tools and applications can be used to show or verify where the standard algorithm of the division with fractions comes from. These visual tools are validated in verifying the solutions obtained through the use of the symbolic algorithm (Borko et al., 1992). Additionally, he didn't explain why he models $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$ and what is the relation of it with $\frac{1}{12} \div \frac{1}{2} = \frac{1}{6}$. Other teachers also had similar statements similar with the literature (Tzur & Timmerman, 1997). Accordingly, the teachers were not competent enough to understand and interpret the outcomes to enable students to acquire procedural knowledge on the conceptual basis as expected. Thus, teachers' LO literacy was better in the algebra than in the numbers. The poorest literacy of teachers was on LO9, as seen in Table 2. Teachers' highest LO literacy was on the LO5 - "Students should be able to explain the concept of equation and they should be able to set appropriate equations to the verbal problems". There were no other outcomes for which teachers' interpretations were found competent.

Components of LO Literacy. The component in which teachers had most difficulty was analysing knowledge and skills to make students acquire it. Among five teachers, only Gaye was seen proficient for only LO3 and LO7 among ten outcomes. The other component that had poor demonstration was the knowledge about the instructional relations among the outcomes and the next one. While the teachers specified the goals of the LO, they ignored the position of that LO in the curriculum, they did not relate it with the future learning outcomes, they could not determine what students already knew and they did not obey the curriculum. Despite these, the component in which teachers were most capable was specifying the goals of the LO in contrast to the research findings of Hiebert et al. (2007). In their study, pre-service teachers had difficulty in demonstrating this skill. In this point, Hiebert et al. (2007) note that "learning goals are easier to specify if you are the teacher" (p.51). However, specifying goals does not always mean analysing LO for the acquisition of it. For some LOs, Sema specified the goals of the outcome. But for the same LOs, even if she could specify the goals, she could not demonstrate the other sub-component of interpreting the LO, especially analysing outcome. For example, the goals Sema stated for LO7 involved comparing, ordering, and showing fractions in number line as well as conceiving fraction size that require conceptual understanding. However, Sema summarized her teaching for LO7: "Immediately giving the rule, telling what they should do, and then solving the problems. All is up." However, it is obvious that this will not help students to understand the sizes of fractions.

This leads the students to acquire mathematical symbol manipulation instead of recognizing the nature of the mathematical concepts and operations associated with fractions (Steiner & Stoecklin, 1997). There were not any teachers who were incapable of specifying the goals of the LO but capable of analysing the LO.

Teachers' knowledge levels about the mathematical structures of LOs and their relations with other learning domains in the curriculum were lower than the other components of LO literacy as seen in Table 2. However, teachers' knowledge about the mathematical relations among the outcomes was generally better than their knowledge about the instructional relations among the outcomes (see Table 2). Participants had insufficient knowledge about the domain specific skills in the LOs, and they were incapable of overarching them mathematically. For example, for LO2 (expressing the relationship in the numerical patterns), Tuba stated that "using letters instead of figures by giving a letter to every unit" to express the relationship in the pattern through letters. In other words, Tuba meant giving a letter to the object used in modelling. For example, if the pattern is modelled with a ball, this relationship can be expressed by giving a letter to every ball. However, the main point in expressing the relationship by using the letters is giving a letter to number of pace or term and also expressing the relationship between numbers or figures depending on the number of pace.

While the participants focused on LOs to investigate their mathematical structures, they took into consideration LOs by focusing only the sub-content strands, and they did not consider the mathematical key points in the LOs. For example, for LO3 (exponential quantities), when teachers were asked to explain why 2^0 is equal to 1, they did make no mathematical explanations by saying they could not remember. While the teachers examined the mathematical structures in the LOs, they did not comprehensively deal with the mathematical relations of the LOs with all content strands in mathematics curriculum. For instance, Tuba did not touch on this sub-component.

As a matter of fact, there were not any participants who were incapable of understanding LOs but capable of interpreting outcomes to equip their students with them. Therefore, unless understanding of LOs, it does not seem possible to be successful at enabling students to acquire LOs as expected. At the same time, there were teachers who were good at the components of understanding the LO but incapable of interpreting LO among the participants. Thus, understanding a LO is just not enough to make students acquire it. Additionally, in line with the literature (Shechtman, Roschelle, Haertel, & Knudsen, 2010), understanding a LO does not guarantee specifying the goals. That is, as it is clear in Table 2, for LO4, Tuba was seen capable of understanding the LO4 but not specifying the goals. Additionally, some of the teachers were not considered competent for evaluating the goals even if they revealed them. Accordingly, specifying goals is not enough to be able to evaluate learning.

Classroom Observations. The findings regarding the observations of teachers' classes were mostly consistent with their answers to the questions which revealed the subcomponents of LO literacy. Namely, if there was an absent component in the LO literacy of a teacher, he/she did not apply this component to his/her teaching in the classroom. For example, Tuba and Mete did not mention the next outcome of the LO2 and their mathematical and instructional relations in the interviews. Similarly, they did not relate LO2 (numerical patterns) with the next outcome while teaching. Teachers could use the patterns to make students acquire LO3 (exponential quantities), as Gaye did; however, they did not touch upon LO2. As another example, while Tuba and Mete stated that they did not remember any representations for exponential quantity, Gaye said in the interviews that she used table representations for LO3. Similarly, Tuba and Mete did not employ any representations but

Gaye used table representation and generalization for LO3, which is one of the main skills of LO2.

Gaye whom LO literacy was stronger than other participants was well-prepared for the students' questions and aware of the key points that students would have trouble with. She organized her teaching according to these points, and readiness levels and prior knowledge of the students. For example, while teaching LO3, she emphasized such key mathematical points that what are the powers of 1, why 2^0 equals to 1, how to write the powers of 10 by utilizing the representations and generalization. Conversely, Mete who was not seen capable of demonstrating nine sub-components of the LO literacy for LO2 (see Table 2) conducted his lessons in line with his statements in the interviews. Mete wrote $a^0 = 1$ on the board, and students immediately started discussing about it. Some students (S#) perceived this equity as a question asking what the "*a*" is. They did not understand that it was different from the unknown, had a different usage/meaning, and it was a place holder. Mete did not notice this point, and he focused only on "transferring the rule". Concordantly, in the interviews, he did not talk about the different meanings of variables in understanding and interpreting LOs.

S1: I think its answer is zero... Teacher, I think its answer is zero because even if a number comes instead of "a", being multiplied with zero is zero.
Mete: Namely… you are still here; you say five exponent two is five multiply two.
Students: Aaaa...
S6: No!...
S1: Aha! ... Numbers itself also ….
S2: Teacher, "a"
S3: Either an undefined or… Undefined because…
S4: Teacher again it is zero. Because well…
S5: No! If we multiply one by one…what have you been smoking? [says to his peers]
Mete: Yes! Children, now, you will accept this right now.
S3: No! We don't!
S5: Teacher that's exactly like this! It is obvious!
Mete: When you attend the tenth grade... [Upon students' murmuring] Okay, then, don't accept now... [Students are still murmuring] You are going to learn when you are at the tenth grade in the concept of logarithm.
S3: Come on S4: Teacher! S2: Tell a little bit. S1: Don't do this please [Students reacted]
S3: Everything is at the high school!
Mete: Yes... In fact, the zeroth exponent of number "a" means that zero times "a". It says write zero times of "a" and multiply. Now zero times means there is no "a" in there. So you are going to accept…this… just its zeroth exponent... as 1.

Obviously, there were lots of opportunities to start a discussion about *the rule* with these students. The students were very curious and open to inquiry and to talk about mathematics. According to Ball and Bass (2000), learners' construction of knowledge depends on developing mathematical reasons for that knowledge, and mathematical conviction is a key for student learning, and these are mathematical demands for teaching. Mete could ask to S5 why it was obvious or, to S3, what the meaning of "undefined" was. He did not even correct S1's misunderstanding about the expression. Mete was too determined not to explain and not to talk about this in the lesson. Additionally, he did not notify that "*a*" was different than zero. During the interviews, when the researcher asked about special mathematical points in LO3 to Mete, stating that zeroth exponent of a natural number except zero is 1, Mete said that its explanation was about the logarithm function and he could not explain further. He added he “makes students learn by rote” with the statements such as “I tell students that we have definite agreements in mathematics... as it is zero times, you don’t take any… namely what will happen when you don’t take any? We will write 1”. If Mete could explain the reasons in the interviews, it could be supposed that he believed the students were limited in their capacity to understand the explanations (Bray, 2011). However, he made the same mathematically poor explanations in the process of understanding a LO. This is consistent with the literature that "teachers whose mathematical knowledge appeared to be connected and conceptual were also more conceptual in their teaching, while those without this type of knowledge were more rule-based" (Tchoshanov, 2011, p. 162). Similarly, Tuba stated that she taught this knowledge as a rule and did not prove it to avoid making students confused, and she did the same in the classroom. Because of her authority and different microculture of the classroom, students seemed accept it and did not insist on learning its reason. Teacher’s poor interpretations of mathematical points might make students feel disappointed about mathematics, as seen in the above excerpt. This may lead to students’ developing negative perceptions regarding mathematics such as viewing mathematics as a body of rules and agreements or as a field which is not entirely appropriate for them to understand. It might affect their mathematical beliefs and values negatively. As an implication for LO literacy, same mathematically poor explanations of Mete were revealed while investigating his understanding and interpreting LO, thus, this finding shows that the need for LO literacy competence can be observed explicitly inside of the classroom. Additionally, the examination of LO literacy helps to explicate the points that teachers should develop and to remind teachers the aspects that they should consider in their classes.

As another example, when some of Mete’s students asked “Can’t we write it like $2^{\backslash 3}, 2_3$?", Mete answered “We have to write it like 2^3" Another student’s question was “Why is it called exponential quantities? Can’t we call it *crosswise numbers?*”, and Mete’s answer was “No you can’t, it must be like this” Most importantly, he has a 10 years of experience, it was probably not the first time he encountered these kind of questions. Because the teacher did not endeavour analysing the knowledge in the LO in an effort to enable students to acquire them, he could not answer students’ questions in a sensible way. Contrary to Mete, Gaye whom LO literacy was stronger than other participants explained these key points without waiting for students to ask. Her explanations were “If you were the first finders of this (exponential quantities), you would demonstrate it differently. You would give different names” in the classroom. A LO literate teacher can be aware and ready of what might or could happen in the classroom. These examples of significant differences among participating teachers’ approaches in the classrooms are consistent with their LO literacy. First of all, learning outcome literacy was predicted to be a competence that a teacher should have. Thereafter, it was theoretically reinforced further with its components through analysis of the obtained data in the interviews. Additionally, the reflections of the learning outcome literacy have been observed within the classroom. Therefore, triangulation of observation and interviews enabled us to observe what the teacher, whose learning outcome literacy was measured theoretically, is doing in his/her classroom. Observations have supported the claim that LO literacy is an important teacher competence for effective teaching.

Conclusion and Implications

Learning outcome literacy was driven by the idea that teachers should be able to understand and interpret the learning outcomes, an important component of the curriculum, in the planning and regulation of teaching mathematics. When the findings of the study were examined, it was seen that the specified competence consists of certain components. This competence, which includes the components in question, is called learning outcome literacy. It was seen that the teacher who has high LO literacy reflects this competence in the classroom. The triangulation of observation and interviews has shown that LO literacy is not only theoretically viable, it can also be observed in practice and teaching, it affects teaching and it is a qualification. Observations prove the existence of such a competence by showing that the difference between the literate and illiterate teacher-in the classroom environment, in teaching, and in meeting the student needs. In this context, learning outcome literacy can be regarded as a basic teacher competence for effective teaching and it requires the teacher to use the knowledge and skills he/she has.

In this study, elementary mathematics teachers' skills of understanding and interpreting LOs in planning and regulation of teaching mathematics were identified. The results showed that teachers' LO literacy was insufficient. The reasons were both the insufficiency to understand a LO and to interpret the LO to enable students to acquire it. There were no participants who were incapable of understanding a learning outcome but capable of interpreting the outcome. Without understanding a LO, as defined in this study, teachers could not interpret this LO to enable students acquire it. In other words, it may not be possible to make students acquire an outcome without understanding it. This result is consistent with the literature (Ball, 1990; Ebert, 1993; Ma, 1999) as Baumert et al. (2010) stated "an insufficient understanding of mathematical content limits teachers’ capacity to explain and represent that content to students in a sense-making way" (p.138). Therefore, according to the findings of this study, it is necessary for teachers to develop themselves at the point of gaining knowledge about the domain-specific skills, concepts, and systems involved in a LO and mathematical relationships of a LO with other LOs in the curriculum. At the same time, some of the participating teachers were insufficient at interpreting a LO but sufficient at some sub-components of understanding the LO. Thus, understanding a LO is not solely enough for interpreting the LO to enable students acquire it. These findings emphasize the nonlinear and non-hierarchical nature of LO literacy as well as the connected nature of teacher knowledge (Krauss et al., 2008). Accordingly, considering the interrelation among its components, LO literacy should be developed with all the sub-components for effective teaching of LOs. It has already been reported that a teacher’s strong mathematical understanding does not guarantee effective teaching of mathematics (Baumert et al., 2010; Fernandez & Cannon, 2005; Kahan, Cooper, & Bethea, 2003; Mapolelo, 1999; Schoenfeld, Minstrell, & van Zee, 1999). However, it is a remarkable result that although participating teachers were able to specify the goals, they were not competent enough for interpreting the outcomes to make students acquire it. Specifying goals is not enough to explain these knowledge and skills in a mathematically complete and correct way to students. It is not enough to explain why the outcome is important for students. There were not any teachers who were incapable of specifying the goals of the LO but capable of analysing the LO in the current study. Therefore, it can be concluded that teachers may know what to teach; however, it does not mean that they know how to teach.

Teachers’ knowledge about the instructional relations among learning outcomes and curriculum influence specifying the goals and analysing knowledge and skills of LOs. Accordingly, knowledge about the instructional relations of the LOs with all the curriculum is important for specifying the goals and other pedagogical decisions. While other components are thought to be inadequate due to various misconceptions or mistakes of teachers, some of the teachers never addressed the sub-components of *knowledge of the mathematical and instructional relationship of the LO with the other LOs of the curriculum*. Therefore, at this point, it can be said that the participants’ knowledge about the curriculum is inadequate. It turns out that teachers need to improve themselves at the point of mathematical and instructional relationships with the other LOs and knowledge of the concepts and systems involved in the LO.

The LO literacy of the teachers was more sufficient in the algebra which mostly has conceptual knowledge than in the domain of number which is heavily based on procedural knowledge. The reason for this could be teachers' insufficient conceptual understandings of procedures and algorithms. Although teachers focused mostly on procedural knowledge when interpreting LOs, their mathematical knowledge about the procedures and the underlying justification of procedures and their skills of interpreting procedural knowledge to enable students to acquire them were not at an expected level. As a result of this, teachers' LO literacy was more efficient in the algebra than number learning domain. Similarly, it is stated in the study of Land and Drake (2014), which elaborates pre-service teachers' learning trajectory for the curriculum use, that pre-service teachers mostly expressed procedural goals in this process. This indicates that effects of the experience on conceptual understanding are not major. Thus, the conceptual understanding should be improved in the teacher training programs without leaving its development to the time and profession.

Focusing on and researching teacher's enactment and interpretation of curriculum would provide useful advices for teacher training programs (Charalambous & Hill, 2012). The LO literacy is thought to be a useful instrument to make systematic and elaborate examination of planning and regulating teaching possible and to reveal the points that teachers and prospective teachers need to develop. It provides insights for the studies that investigate ways to examine teacher knowledge (e.g. Kersting, Givvin, Sotelo, & Stigler, 2010). By considering its components, teachers can have a more systematic and comprehensive way of self-evaluation and/or reflection. Being aware of LO literacy and focusing its components while planning teaching and learning activities would make teachers think deeply about what they really do or should do in the classroom. The components of LO literacy might present a framework for the activities employed in planning and regulating teaching for teachers, prospective teachers and their instructors. It might be useful to consider and remember every detail for teaching and to introduce the curriculum to the prospective teachers. The current study might be one of the studies that helps for how teacher educators can "support and foster preparation for the practice of teaching a lesson, without turning that preparation into an activity of filling tables of rubrics" (Zazkis et al., 2009, p. 40). An important feature of this study is to determine, identify learning outcome literacy and how this competence can be measured. In the current study, LO literacy has been investigated both in practice and in theory, and observations and interviews have proved the existence and necessity of this competence. It was seen that the teachers' classroom practices on LO literacy were consistent with the interviews. Therefore, it can be concluded that the teacher who has acquired LO literacy and performed it cognitively and theoretically reflects this in his/her teaching, and this also makes noticeable differences in his/her teaching. This conclusion promises hope for the studies and attempts to be done to achieve the LO litarecy competence.

References

Anthony, G., & Walshaw, M. (2009). Characteristics of effective teaching of mathematics: A view from the West. *Journal of Mathematics Education, 2*(2), 147-164.

Arends, R. I. (1991). *Learning to teach (second edition).* New York, NY: McGraw-Hill.

Babadoğan, C., & Olkun, S. (2006). Program development models and reform in Turkish primary school mathematics curriculum. *International Journal for Mathematics Teaching and Learning*, *1*(1), 1-6.
Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449-466. https://doi.org/10.1086/461626
Ball, D. L. (1991). *Implementing the NCTM standards: Hopes and hurdles* (Issue Paper 92-2). East Lansing: Michigan State University, The National Center for Research on Teacher Learning.
Ball, D. L. (2003). *What mathematical knowledge is needed for teaching mathematics?* Paper presented at the US Department of Education, Secretary's Mathematics Summit, Washington, DC.
Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. *Multiple Perspectives on the Teaching and Learning of Mathematics*, 83-104.
Ball, D. L., & Bass, H. (2002, May). Toward a practice-based theory of mathematical knowledge for teaching. In *Proceedings of the 2002 Annual Meeting of the Canadian* Mathematics *Education Study Group* (pp. 3-14). Edmonton Alberta.
Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, *29*, 14-22
Ball, D. L., & McDiarmid, G. W. (1990). The subject matter preparation of teachers. In W. R. Houston (Ed.), *Handbook of Research on Teacher Education* (pp. 437–449). New York: Macmillan.
Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407. https://doi.org/10.1177/0022487108324554
Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., … Tsai, Y.-M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, *47*(1), 133-180. https://doi.org/10.3102/0002831209345157
Bellon, J. J., Bellon, E. C., & Blank, M. A. (1992). *Teaching from a research knowledge base: A development and renewal process*. New York, NY: Macmillan.
Binbasioglu, C. (1995). *Türkiye'de eğitim bilimleri tarihi* (*History of Turkish educational sciences*). *The series of research and examination*. Ankara, Turkey: MNE.
Black, P. (2001). Formative assessment and curriculum consequences. *Curriculum and Assessment*, *1*, 7.
Borko, H., Eisenhart, M., Brown, C. A., Underhill, R. G., Jones, D., & Agard, P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education*, 194-222. https://doi.org/10.2307/749118
Bray, W. S. (2011). A collective case study of the influence of teachers' beliefs and knowledge on error-handling practices during class discussion of mathematics. *Journal for Research in Mathematics Education, 42*(1), 2-38. https://doi.org/10.5951/jresematheduc.42.1.0002
Charalambous, C. Y. & Hill, H. C. (2012). Teacher knowledge, curriculum materials, and quality of instruction: Unpacking a complex relationship. *Journal of Curriculum Studies, 44*(4), 443-466. https://doi.org/10.1080/00220272.2011.650215
Darling-Hammond, L., & Richardson, N. (2009). Research review/ Teacher learning: What matters? *How Teachers Learn, 66*(5), 46-53.

Delaney, S., Ball, D. L., Hill, H. C., Schilling, S. G., & Zopf, D. (2008). "Mathematical knowledge for teaching": Adapting US measures for use in Ireland. *Journal of Mathematics Teacher Education, 11*(3), 171-197. https://doi.org/10.1007/s10857-008-9072-1

Ebert, C. L. (1993). *An assessment of prospective secondary teachers' pedagogical content knowledge about functions and graphs*. Paper presented at the annual meeting of the American Educational Research Association, Atlanta, GA, April 12–16, 1993.

Fernandez, C., & Cannon, J. (2005). What Japanese and US teachers think about when constructing mathematics lessons: A preliminary investigation. *The Elementary School Journal*, *105*(5), 481-498. https://doi.org/10.1086/431886

Franke, M. L., & Grouws, D. A. (1997). Developing Student Understanding in Elementary School Mathematics: A. *Handbook of Academic Learning: Construction of Knowledge*, 307.

Grossman, P. L. (1990). *The making of a teacher: Teacher knowledge and teacher education*. Teachers College Press.

Harris, J., & Hofer, M. (2009). *Instructional planning activity types as vehicles for curriculum-based TPACK development.* Paper presented at the Society for Information Technology & Teacher Education International Conference.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. *Conceptual and Procedural Knowledge; The Case of Mathematics*, 1-23.

Hiebert, J., Morris, A. K., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of Teacher Education, 58*(1), 47-61. https://doi.org/10.1177/0022487106295726

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406. https://doi.org/10.3102/00028312042002371

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal*, *105*(1), 11-30. https://doi.org/10.1086/428763

John, P. D. (2006). Lesson planning and the student teacher: re-thinking the dominant model. *Journal of Curriculum Studies, 38*(4), 483-498. https://doi.org/10.1080/00220270500363620

Kahan, J. A., Cooper, D. A., & Bethea, K. A. (2003). The role of mathematics teachers' content knowledge in their teaching: A framework for research applied to a study of student teachers. *Journal of Mathematics Teacher Education*, *6*(3), 223-252. https://doi.org/10.1023/A:1025175812582

Kauffman, D. (2002). *A search for support: Beginning elementary teachers' use of mathematics curriculum materials.* Harvard Graduate School of Education.

Kennedy, D., Hyland, A., & Ryan, N. (2007). *Writing and using learning outcomes: A practical guide*. University College Cork.

Kersting, N. B., Givvin, K. B., Sotelo, F. L., & Stigler, J. W. (2010). Teachers' analyses of classroom video predict student learning of mathematics: Further explorations of a novel measure of teacher knowledge. *Journal of Teacher Education, 61*(1-2), 172-181. https://doi.org/10.1177/0022487109347875

Krauss, S., Brunner, M., Kunter, M., Baumert, J., Blum, W., Neubrand, M., & Jordan, A. (2008). Pedagogical content knowledge and content knowledge of secondary mathematics teachers. *Journal of Educational Psychology*, *100*(3), 716. https://doi.org/10.1037/0022-0663.100.3.716

Land, T. J., & Drake, C. (2014). Understanding preservice teachers' curricular knowledge. In *Research Trends in Mathematics Teacher Education* (pp. 3-22). Cham: Springer https://doi.org/10.1007/978-3-319-02562-9_1

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates

MacDonald, J. (2012). Identify learning outcomes for learning agreements. *Education for Primary Care, 23*(2), 128-130. https://doi.org/10.1080/14739879.2012.11494087

Magnusson, S., Krajcik, J., & Borko, H. (1999). Nature, sources, and development of pedagogical content knowledge for science teaching. In *Examining Pedagogical Content Knowledge* (pp. 95-132). Dordrecht: Springer.

Mapolelo, D. C. (1999). Do pre-service primary teachers who excel in mathematics become good mathematics teachers? *Teaching and Teacher Education, 15*(6), 715-725. https://doi.org/10.1016/S0742-051X(99)00012-8

Marsh, C. J. (2009). *Key concepts for understanding curriculum*. London, Routledge. https://doi.org/10.4324/9780203870457

Ministry of National Education. (2008). *Öğretmen yeterlikleri: Öğretmenlik mesleği genel ve özel alan yeterlikleri*. Ankara: Öğretmen Yetiştirme ve Eğitimi Genel Müdürlüğü

Ministry of National Education. (2013). *Ortaokul matematik dersi (5, 6, 7 ve 8. sınıflar) öğretim programı.* Retrieved from http://ttkb.meb.gov.tr/program2.aspx?islem=1&kno=219

Morris, A. K., Hiebert, J., & Spitzer, S. M. (2009). Mathematical knowledge for teaching in planning and evaluating instruction: What can preservice teachers learn. *Journal for Research in Mathematics Education, 40*(5), 491-529.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: NCTM.

National Council of Teachers of Mathematics and the Council for the Accreditation of Educator Preparedness. (2012). *NCTM CAEP Standards- Elementary Mathematics Specialists (Advanced Preparation)*. Retrieved from http://www.nctm.org/Standards-and-Positions/CAEP-Standards/

Ornstein, A. C., & Hunkin, F. (2004). *Foundations, principles and issues*: New York, NY: Allyn.

Patton, M. Q. (2005). *Qualitative research*. Wiley Online Library. https://doi.org/10.1002/0470013192.bsa514

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211-246. https://doi.org/10.3102/00346543075002211

Schoenfeld, A. H., Minstrell, J., & van Zee, E. (1999). The detailed analysis of an established teacher's non-traditional lesson. *The Journal of Mathematical Behavior, 18*(3), 281-325. https://doi.org/10.1016/S0732-3123(99)00035-8

Schwarz, C. V., Gunckel, K. L., Smith, E. L., Covitt, B. A., Bae, M., Enfield, M., & Tsurusaki, B. K. (2008). Helping elementary preservice teachers learn to use curriculum materials for effective science teaching. *Science Education, 92*(2), 345-377. https://doi.org/10.1002/sce.20243

Shechtman, N., Roschelle, J., Haertel, G., & Knudsen, J. (2010). Investigating links from teacher knowledge, to classroom practice, to student learning in the instructional system of the middle-school mathematics classroom. *Cognition and Instruction, 28*(3), 317-359. https://doi.org/10.1080/07370008.2010.487961

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14. https://doi.org/10.3102/0013189X015002004

Simon, M. A. (2006). Key developmental understandings in mathematics: A direction for investigating and establishing learning goals. *Mathematical Thinking and Learning, 8*(4), 359-371. https://doi.org/10.1207/s15327833mtl0804_1

Steiner, G. F., & Stoecklin, M. (1997). Fraction calculation—A didactic approach to constructing mathematical networks. *Learning and Instruction, 7*(3), 211-233. https://doi.org/10.1016/S0959-4752(96)00026-6

Training and Development Agency (TDA). (2007). *Professional Standards for Teachers in England from September*. Retrieved from http://www.connaught-school.co.uk/cs/staff/professional_standards adresinden 03.06.2013

Tigelaar D., Dolmans, D., Wolfhagen I., & Van Der Vleuten, C. (2004). The development and validation of a framework for teaching competencies in higher education. *Higher Education. 48*, 253-268. Netherlands: Kluwer Academic. https://doi.org/10.1023/B:HIGH.0000034318.74275.e4

Tzur, R., & Timmerman, M. (1997). Why do we invert and multiply? Elementary teacher's struggle to conceptualize division of fractions. In J.A. Dosey, J.O. Swafford, M. Parmantie & A. E. Dossey (Eds.), *Proceedings of the 19th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, 2*, 553-559.

Tchoshanov, M. A. (2011). Relationship between teacher knowledge of concepts and connections, teaching practice, and student achievement in middle grades mathematics. *Educational Studies in Mathematics, 76*(2), 141-164. https://doi.org/10.1007/s10649-010-9269-y

Yin, R. K. (2013). *Case study research: Design and methods*. Sage publications.

Zazkis, R., Liljedahl, P., & Sinclair, N. (2009). Lesson plays: Planning teaching versus teaching planning. *For the Learning of Mathematics, 29*(1), 40-47.