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An Application of the Solo Taxonomy to Classify Strategies Used by Pre-Service Teachers to Solve “One Question Problems”

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Abstract: The purpose of this article is to report on the strategies of secondary mathematics pre-service teachers (PSTs) as they solved conceptually rich problems. Using the Structure of Observed Learning Outcomes by Biggs and C (1982) (SOLO) Taxonomy, 15 PSTs' solutions (in groups of 3 or 4) were analyzed by a panel of three mathematics educators. In addition, the authors studied questions posed by PSTs during their student teaching experiences through video analysis. Questions were then categorized using Crespo's criteria of problem posing. Results showed a significant majority of the problems posed were procedural while PSTs own problem solutions showed a lack of conceptual understanding and depth of knowledge. The authors found the SOLO Taxonomy, together with PSTs scores on the state licensure exam and Crespo's (2003) problem posing practices criteria to be a useful combination of tools to explore connections between PSTs' mathematical and pedagogical content knowledge.

Introduction

Jim Noble, Head of Mathematics at the International School of Toulouse (IST), France, developed the idea of ‘One Question Lessons’ while teaching mathematics to 11-18 year olds. One Question Lessons consist of tasks that begin with the expression of a single, question that subsequently takes students on multi-stepped exploratory journeys (Noble, 2013b). According to Noble (2013a), key elements of One Question Lessons include:

- The end goal of the task (not the journey/task itself) has to be easily explained and understood.
- The task has to draw people in and make them want to approach it. As such, the task has to appear possible and achievable by all students and appeal to them.
- There must be opportunities for students to make conjectures, challenge misconceptions and get feedback on their efforts straight away.
- There must be opportunities to discover some fairly profound mathematics. (para. 2)

Noble (2013a) illustrates his One Question Lessons framework through a task titled ‘Making Cones’, in which the nets of three-dimensional cones are constructed by hand given specific physical

requirements. When managed productively, the One Question Lessons framework and Making Cones task provide a space for students to explore and develop meaning for relationships between properties of cones; reason with one another about what is the same or different, what is in proportion; and test their own conjectures along with their classmates' (Noble, 2013a).

This report describes how the One Question Lessons framework and Making Cones task were presented within the context of a pre-service secondary (grades 7-12 licensure) mathematics education class. The activity was designed to demonstrate Skemp's (1976) theory of relational and instrumental understanding, and provide pre-service teachers (PSTs) with an experience at posing problems that encourage relational understanding. The purpose of the study was to examine PSTs' thinking using the *Structure of Observed Learning Outcomes by Biggs and Collis (1982) (SOLO taxonomy* following their engagement with the activity. Following this activity, during student teaching, PSTs were asked to develop (or find) and implement tasks that had the potential to engage their own students in relational understanding with a description of high and low demand tasks defined by Smith and Stein (2011).

The study addressed the following research questions:

1. When engaged in the Making Cones task, what levels of thinking did PSTs coincide with on the SOLO taxonomy?
2. How might PSTs' levels of thinking (as indicated on SOLO taxonomy) relate to their capacities to problem pose in their own classrooms?
3. What other influences may contribute to PSTs' capacities to problem pose during student teaching?

Theoretical Framework

In the United States, recent standards reforms (Board of Education Commonwealth of Virginia, 2016; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) not only provide greater focus, coherence, and rigor regarding content, but also focus on providing students with opportunities to engage in mathematical sense making, reasoning, modeling, generalizing, and communicating. According to *The Mathematical Education of Teachers II (MET II)* (Conference Board of Mathematical Sciences [CBMS], 2012), doing mathematics in ways consistent with such standards is "likely to be a new, and perhaps, alien experience for many teachers" (p. 11). Therefore, *MET II* (CBMS, 2012) recommends:

All courses and professional development experiences for mathematics teachers should develop the habits of mind of a mathematical thinker and problem-solver, such as reasoning and explaining, modeling, seeing structure, and generalizing. Courses should also use the flexible, interactive styles of teaching that will enable teachers to develop these habits of mind in their students (CBMS, 2012, p. 19).

Such flexible, interactive teaching styles that support the development of these habits of minds in students requires learning how to construct rich problems, analyze student responses, and manage activities in ways that promote student discourse, thinking, and reasoning (Crespo, 2003). Mathematics teacher preparation must play a significant role in providing mathematics PSTs with such experiences.

To address the research questions, the study employed: (1) Skemp's (1976) theory on instrumental and relational understanding, (2) the SOLO taxonomy (Biggs & Collis, 1982), and (3) Crespo's (2003) problem posing practices framework.

Mathematical Understanding

Skemp (1976) posited the existence of two types of mathematical understanding that could be generated by mathematics learning and teaching in schools: instrumental and relational. For Skemp (1976), instrumental understanding was the product of rote learning through rules and theorems and specific applications. Conversely, relational understanding was the product of a learner's personal involvement with mathematical objects, situations, problems, and ideas.

At each stage in a relational learning cycle the learner is personally involved with the available data. The data are products of the learner's own investigations. In contrast, the data available in instrumental learning are given to the learner to memorize by some external source (usually the teacher, textbook, or computer). Skemp (1976) believed many students possessed only instrumental understanding of numerous mathematical concepts, having a collection of unrelated procedures for retrieval rather than an appropriate conceptual schema.

A significant portion of PSTs' methods curriculum at this university involves examining and discussing Skemp's (1976) seminal work of relational and instrumental understanding. As such, Skemp's theory was examined with PSTs as part of their participation in the study. Furthermore, PSTs were asked to develop (or find) and implement tasks they believed would engage their own students in relational understanding during student teaching.

To provide an analysis of mathematical tasks that PSTs use as they plan their lessons and select problems, the authors required students to utilize Smith and colleagues' (2008) "Thinking Through a Lesson Protocol" and Smith and Stein's (2011) "Five Practices for Orchestrating Mathematical Discussions." Using these resources, PSTs were to implement the following criteria in designing tasks:

- Lesson activities should provide opportunities for all students to be engaged in the exploration, discovery, application, practice, and/or discussion of the mathematical ideas in the lesson. Some lesson activities should provide opportunities for students to make sense of mathematical ideas, procedures, theorems, etc.
- PST's should justify that the cognitive demands of task are appropriate for achieving goals/objectives by giving attention to ensure the learning opportunities are developmentally appropriate. PST's should use support their choice of tasks with appropriate outside resources.
- The PST's should use problem solving and provide solution strategies for the lesson task(s). They should identify possible student-strategies and how they connect to the mathematical goals/objectives for the lesson. Through this process, the PST' should pay attention to students' conceptual understanding and help students develop and test conjectures.

The SOLO Taxonomy

The Structure of Observed Learning Outcomes (SOLO) was designed as an instrument for the evaluation of the quality of student responses to a problem-solving task (Biggs and Collis, (1982). There are two main features in the SOLO Taxonomy: modes of thinking and levels of response. SOLO's modes of thinking (i.e., iconic, concrete-symbolic, formal) are similar to Bruner's (1966) modes (or stages) of representation in that both develop successively in the learner but then remain simultaneously available (Pegg & Tall, 2005).

The second main feature in SOLO Taxonomy which is pertinent to the current study, is the level of response, or the individual's ability to respond with increasing sophistication to the task. Because the SOLO taxonomy describes levels of progressively complex understanding through five general stages that are intended to be relevant to all subjects within all disciplines and has been used by numerous studies (Olive, 1991), the authors deemed it an appropriate rubric for the Making Cones task. The level of response is similar to Askew, Rhodes, & William's (1997) connectionist approach to teaching. This approach bases PSTs beliefs towards learning mathematics around the methods and strategies used to establish connections within the math. Teachers who follow a connectionist orientation are more likely to have students produce greater gains in their understanding than those who believe in discovery or transmission. In this practice PSTs go beyond teaching toward memorization by having students identify relationships, find connections, develop flexible mental strategies, and hold high expectations for success (Askew, Rhodes, & William, 1997).

In SOLO, understanding is conceived as an increase in the number and complexity of connections students make as they progress from incompetence to expertise. Each level is intended to encompass and transcend the previous level (Potter & Kustra, 2012).

1. *Pre-structural*. In this first stage, the students do not really have prior knowledge to aid in their understanding of a topic. For example, the student may not engage in the task, they may give completely unassociated data, will not know the answer, may not understand the question, may provide irrelevant information, or just repeat something they've been told.
2. *Unistructural*. During this second stage, students may have limited knowledge on the topic or know just a few isolated facts. For example, the student can use one piece of information to respond to a task but does not see connections between ideas. They may apply memorization of ideas in a procedural and predetermined manner and provide facts/concept in isolation.
3. *Multistructural*. Progressing onto stage 3, students may know a few facts about the topic but still are unable to connect them together. For example, the student may use several pieces of information but does not understand the organization and significance behind the ideas. Ideas are alienated from each other as they are concrete in nature. Student's answers may provide several relevant facts or correctly identify characteristics of a phenomenon, but these facts are not integrated.
4. *Relational*. Moving toward a higher level of thinking, students in stage 4 are able to link information together and explain several ideas pertaining to a topic. For example, the student may integrate separate pieces of information to produce a viable solution to a task. Student's answers provide explanations that relate and integrate relevant details. They may often express their answers in terms of abstract ideas with concrete facts.

Student may use prior knowledge to explain and provide context. (Note: this is not Skemp’s relational thinking)

5. *Extended Abstract.* In the final and most complex stage, students thinking is abstract. They are able to link many ideas together and connect them to larger concepts through reflection and evaluation. For example, the student can derive a general principle from the integrated data and apply it to new situations. Student’s answers go a step further, applying reasoning, anticipating possibilities, making multiple connections, and incorporating (or devising) principles to apply knowledge to new situations. (Biggs & Collis, 1982).

Crespo’s Problem Posing Practices Framework

During student teaching, PSTs were asked to develop (or find and possibly modify) and tasks to implement which contained the potential to engage their own students in relational understanding. Such tasks are often referred to as “rich” problems or tasks. Brahier (2009) describes a rich problem as “non-routine and can be solved in a variety of ways” (p. 14). For Swan (2005), a rich task (a) is accessible and extendable; (b) allows for decision making by the learner; (c) involves testing, explaining, proving, interpreting, and reflecting; (d) promotes communication and discussion; (e) encourages invention and originality; (f) encourages questions that focus on “what if” and “what if not”; and (g) is enjoyable and provides an opportunity for surprise.

In analyzing the problem posing practices of elementary pre-service teachers, Crespo (2003) utilized a six-criteria framework which emerged through meta-analysis of existing research. Table 1 is an adaptation of Crespo’s framework that was developed and utilized in the current study, including descriptions of problems, tasks, and associated support questions PSTs posed to their own students. Such samples were part of the data corpus (e.g., lesson plans, video- and audio-recordings of lessons) PSTs were asked to collect as part of a required pre-service performance-based assessment during student teaching.

Problem/Question Posing Practice	Features of the practice
<i>Simplified Problems and Questions</i>	<ul style="list-style-type: none"> - Teacher makes adaptations that narrow mathematical scope of original version of problem and uses hints to lead students to the answer - Example: Teacher asks, “What’s 5x7? (When practicing how to factor)”
<i>Familiar Problems and Questions</i>	<ul style="list-style-type: none"> - Teacher poses problems that students already know the answer to for quick interpretation - Example: “Look at the graph, what is happening?”
<i>Blind Problems and Questions</i>	<ul style="list-style-type: none"> - Teacher poses problem without fully thinking of the solution pathway or understanding the mathematics - Example: “Do you want to use FOIL or the distributive property?”
<i>Unfamiliar Problems and Questions</i>	<ul style="list-style-type: none"> - Problems are less straight-forward and more multi-step, requiring more than speed and accuracy

	- Example: "So what do you think is going to happen if we continue this pattern?"
<i>Challenging Problems and Questions</i>	- Problems introduce new ideas and challenge student understanding - Example: "How does this graph relate to the problem?"
<i>Cognition-Eliciting Problems and Questions</i>	- Problems require students to communicate their ideas and share/explain their thought process - Example: "How do you know that, what is your evidence?"

Table 1: Criteria of Problem and Question Posing Practices - Adapted from Crespo (2003)

The first three practices in Table 1 were categorized by the authors as "low-demand" problem solving practices, while the second three were categorized as "high-demand" as they required more in depth thinking on behalf of the student.

Methodology

Study Participants

Participants consisted of 15 secondary pre-service teachers enrolled in a senior capstone course for integrated mathematics majors (grades 7-12 licensure) at a large Midwestern university. Six participants were male, nine female. At the time of the study, students had completed six semester hours of mathematics methods courses and at least 30 semester hours of college-level mathematics content courses.

Making Cones Task

PSTs were given the following task to solve (as students of mathematics) in groups that were self-selected: "You are to make a drawing on 45.72 cm. x 60.96 cm. (18 in. x 24 in.) paper that will make a cone. The radius is 10 cm. and the height is 24 cm."

Although the problem statement is clear and direct, the mathematics behind the question is more complex. A productive strategy for students (i.e., PSTs) to employ is to imagine a constructed cone of radius 10 cm and height 24 cm, as illustrated in Figure 1.

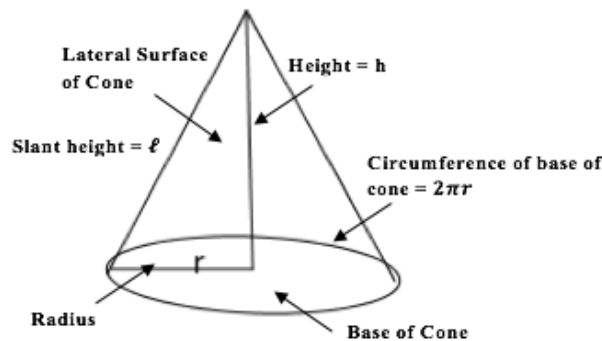


Figure 1: Illustration of constructed cone.

If PSTs imagine cutting along the slant height (ℓ), cutting out the circular base of the cone, and opening up the remaining sector, they are left with the net illustrated in Figure 2.

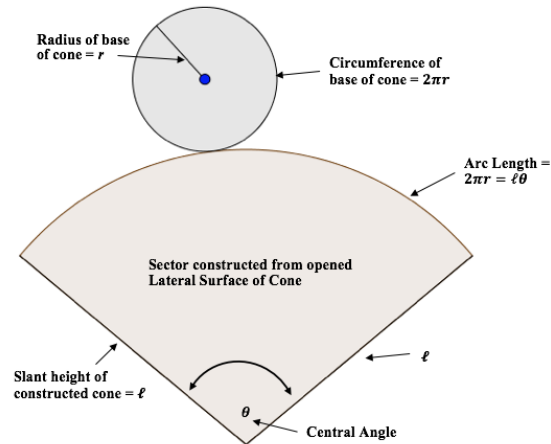


Figure 2: Net of the cone.

As illustrated in Figure 2, the net of the cone is made up a circle of radius r (the net of the base of the cone) and a sector of a circle of radius ℓ and central angle θ (constructed from the opened lateral surface of the cone). As a cone, the arc of the sector of the net wraps around the circular base of the cone. Therefore, arc length of sector = circumference of base of cone, or $2\pi r = \ell\theta$. Solving for θ yields: $\theta = \frac{2\pi r}{\ell}$ radians or $\theta = \frac{2\pi r}{\ell} \cdot \frac{360}{2\pi} = 360 \left(\frac{r}{\ell}\right)$ degrees. In Figure 1, given the radius of the base of the cone ($r = 10$ cm) and the height of the cone ($h = 24$ cm), PSTs can utilize the Pythagorean theorem to calculate $\ell = \sqrt{r^2 + h^2} = \sqrt{10^2 + 24^2} = 26$ cm. PSTs will need to construct their drawings on the 45.72 cm x 60.96 cm paper so they have a circle of radius 10 cm (Area = $\pi \cdot 10^2 \approx 314.2$ cm²) and a sector of a circle of radius $\ell = 26$ cm (Area of sector = Area of lateral surface of cone = $\pi r \ell = \pi \cdot 10 \cdot 26 \approx 816.8$ cm²). Constructing this sector requires $\theta = 360 \left(\frac{10}{26}\right) = 138.5^\circ$.

Additional relationships can be found by attempting to construct the cone from a circle of radius ℓ (Ranucci, 1990). Not only are the inherent relationships in the Making Cones task essential in creating nets, they also demonstrate a model of the problem which can displayed as a table and operationalized in order to classify students' levels on the SOLO Taxonomy.

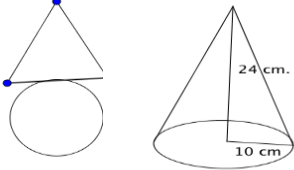
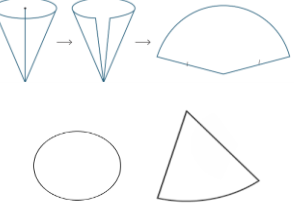
Data Analysis

To answer the research questions, the authors first classified and investigated patterns among PSTs' problem solving in groups using the SOLO taxonomy based on responses to the Making Cones task. Next, the authors compared PSTs' SOLO taxonomy levels with their levels of problem posing practices during student teaching.

Pre-service Teacher One Question Task

In order to produce contextualized SOLO descriptors and to categorize students' responses at each SOLO level for each task, it was necessary to: develop a comprehensive explanation to identify the structural complexity within which a range of possible responses might be exhibited, develop a descriptor for each SOLO category that could be operationalized for the purposes of categorizing students' responses in relation to the comprehensive explanation within each of the SOLO categories; and to identify an illustrative example from the student data for each SOLO category descriptor. The production of the comprehensive explanation involved a considerable amount of discussion around the following question: “How do secondary mathematics majors perform on one question type problems and how are they then expected to provide support to their students?” The attempt to answer this question involved an iterative dialogue: with research on secondary pre-service teachers’ knowledge of problem solving, the author’s experiences as teachers of secondary mathematics teachers and students’ responses.

Table 2 illustrates the analysis of the Making Cones task solved by the PSTs (as students of mathematics) in groups of 3 to 4. The SOLO Taxonomy was used to analyze and categorized each group’s solution to identify the structural complexity in which the task was solved.

SOLO Taxonomy	Student Responses	Evidence
Prestructural	<p><i>Group 4:</i> After cutting the net out, students (i.e., PSTs) realized this did not complete a net. They found it may be helpful to draw a 3-D view with numerical values. Students didn’t realize a net was asked for or did not think it was possible with the given information; they simply produced a drawing of an inverted cone.</p>	
Unistructural	<p><i>Group 3:</i> This group responded by drawing a circle disjoined from the triangular section of the cone. No numerical values were given, only a pictorial representation was displayed. This does not imply students completely understood the relationship between the 3-dimensional cone and its’ net. Another condition of unistructural thought was the lack of coordination of the cone's surface by wrapping items around and cutting the excess of the cone’s lateral surface.</p>	

<p>Multi-structural</p>	<p><i>Group 2:</i> Students gave a pictorial representation of the net of a cone with attached pieces. This group also considered the formulas for area of a circle, area of the net, and the diameter of the circle; and determined the diameter of the circle in the cone was equal to 10 (no unit given).</p> <p><i>Group 1:</i> This group drew the net of a cone and gave the cone dimensions with numbers such as identifying the radius of the circle or base of the cone, labeling points on the lateral section of the cone as A, B, C, and D, and determining that the length from A to B was 31.4 (no unit). This was not described as it related to the circle and no parts were identified in length on the circle.</p> <p>In both groups, students used objects such as string, protractors, and rulers, to recognize the size and length of the triangle were related to the base of the cone and instead of a triangle, the shape needed was a sector of a circle.</p>	
<p>Relating (Note: this is not Skemp’s relational thinking)</p>	<p>Students relate objects together using circumference proportions, sector area, and the Pythagorean Theorem. Students relate all content together and generalize beyond given numbers to demonstrate knowledge. No students (i.e., PSTs) used the Pythagorean theorem to identify sections of the cone or create the net of the cone. None of the PSTs identified all parts in detail relative to their measurements or connected between the lateral side and base of the cone.</p>	<p>See Figure 2 above</p>

Table 2: Analysis of Pre-service Teacher Answers to the Making Cones Task

Categorization of Pre-service Teachers Problem Posing Practices

The authors used descriptive qualitative methods to analyze the problems posed by PSTs (those who solved the Making Cones task) during student teaching. Fifteen PSTs agreed to post videos of the implementation of at least one problem with their own students. Each problem and supporting questions were coded by a panel of three university mathematics educators in terms of whether the problem or question met each of the criteria for problem-posing practices developed by Crespo (2003). To ensure trustworthiness, the videos were examined multiple times to categorized the type of problems and questions posed by PSTs. The categorization of PST’s group problem posing and question practices are presented in Table 3.

Group	Simplified Problems and Questions	Familiar Problems and Questions	Blind Problems and Questions	Unfamiliar Problems and Questions	Challenging Problems and Questions	Cognition-Eliciting Problems and Questions
Group 1	18%	53%	13%	0%	5%	11%
Group 2	29%	23%	1%	0%	20%	27%
Group 3	19%	35%	13%	10%	10%	13%

Group 4	27%	55%	5%	7%	1%	5%
Group Avg.	23%	42%	8%	4%	9%	14%

Table 3: Categorization of PST’s Problem and Question Posing Practices

Additional Data Sources

Because connections between teachers’ mathematics and pedagogical content knowledge when solving conceptually rich problems and their own problem and question posing abilities is complex (Authors, 2014), the authors examined other available data sources to identify potential patterns. These sources included PSTs’ content knowledge of secondary mathematics (grades 7-12) as measured by the state licensure exam, type of textbook used (reform or traditional), and demographics (school and student).

The state licensure exam passing score is 220, the PST group averages in this study were as follows: Group 1: 271, Group 2: 235, Group 3: 234.5, and Group 4: 243; all participating PSTs that took the exam passed and 2 PSTs have not taken the exam. This is a 4 ½ hour exam comprised of 150 problems focusing on the areas of number, geometry, algebraic operations, data and probability, and calculus. Table 4 below indicates: (1) the percentage of low level problems and associated support questions each PST asked during their video recorded lesson, (2) whether PST was located in a middle (MS, grades 6-8) or high school (HS, grades 9-12), (3) whether PST used a textbook that was traditional or conceptual (T or C), (4) the type of school district (suburban, urban, or rural) each PST was placed during student teaching, and (5) if more than one-third of PST’s students were identified with math-specific learning disabilities (yes or no).

Students in each group	% of low level problems	Middle (M) /High School (HS)	Traditional (T) or Conceptual (C) Textbook	School Demographic	Math-specific learning disabilities (n > 1/3 class)
1A	100	M	T	Suburban	N
1B	67	HS	C	Suburban	N
1C	73	HS	C	Suburban	Y
1D	100	HS	T	Urban	N
2A	63	M	T	Suburban	Y
2B	60	M	T	Suburban	N
2C	91	M	C	Suburban	Y
2D	0	HS	C	Suburban	N
3A	100	HS	T	Urban	Y

3B	38	M	C	Suburban	N
3C	100	HS	T	Rural	Y
3D	30	HS	T	Suburban	N
4A	84	HS	T	Suburban	N
4B	92	M	T	Suburban	Y
4C	84	HS	T	Rural	N

Table 4: Problem Classification and School, Class, and Student Demographics

Findings

Only 3 PSTs (2D, 3B, and 3D) used a larger amount of high demand problems/question posing practices (i.e., problems and associated support questions) during their instruction. The remaining 12 PSTs relied solely on low demand problem-posing practice (>50% of questions asked), neglecting students' capacities for higher-order thinking. For example, in one urban Algebra I class, during a lesson on factoring polynomials, a PST (3A) asked questions such as "What is 3×5 ?" or "What are the factors of 6 and 12?" In spite of PSTs' exposure to higher-order problems and tasks throughout two semesters of mathematics methods courses, the majority of PSTs focused on asking low level problems and support questions. A number of studies align with these results, indicating teachers tend to ask more low demand than high demand questions (e.g., Long & Sato, 1983; Yang, 2006).

Comparing PSTs' SOLO Taxonomy Levels with their Problem Posing Practice Categories

Groups 1 and 2 performed at the Multistructural Level on the SOLO Taxonomy (highest level of participating PSTs as defined in Table 2). Students comprising Group 3 had the lowest group average on the state licensure exam (234.5), while students in Group 2 had the second lowest group average (235) of the four PST groups. Although one would assume that the highest average mathematics score on the state licensure exam should result in a higher-level thought process when problem solving, this was not the case. In addition, although Group 1 performed at the highest level on the SOLO Taxonomy (Multistructural Level) and had the highest group average on the state licensure exam, they asked the second fewest high demand problems and supporting questions (Table 4).

Group 3 performed at the Unistructural Level on the SOLO Taxonomy (middle level of participating PSTs) and students comprising Group 3 had the lowest group average on the state licensure exam. Although Group 3 performed at a low level on the SOLO Taxonomy (Unistructural Level) and had the lowest group average on the state licensure exam, they asked the largest percent of high demand problems and supporting questions. These findings do not align well with existing research which has found that teachers' content-specific knowledge, beliefs, and attitudes are generally assumed to influence students' learning outcomes (De Corte,

Greer, & Verschaffel, 1996; Fennema & Loef, 1992; Shulman, 1986; Verschaffel, Greer, & De Corte, 2000).

Other Influences to Pre-service Teachers' Problem Posing Practices
School Demographics

In comparing the types of support questions PSTs posed, the authors found the highest percentage of low level questions in Rural and Urban demographic settings. Only 3 of the 15 pre-service teachers asked more than 50% of their classroom questions at a higher level. In urban districts, both teachers only asked low level questions during teaching segment. The two student teachers in the rural districts asked more low level questions (>60%). All three of the teachers who asked a greater number of higher level questions were in suburban districts. Ladson-Billings (1995) analyzed characteristics and qualities of culturally relevant teaching. According to Ladson-Billings (1995), culturally relevant teachers' conceptions or beliefs about knowledge includes knowledge that "is not static; it is shared, recycled, and constructed" (p. 481).

Ladson-Billings' (1995) criteria are in stark contrast to the knowledge exhibited by PSTs in this study; the knowledge of PSTs placed in urban and rural schools was static. It did not motivate students and instead kept students at a level focused on low level problems and support questions.

Student Teaching Demographics

Six PSTs taught at middle schools, while nine taught at high schools. Three of the six PSTs in middle school placements and three of the nine PSTs in high school placements had classes where more than $\frac{1}{3}$ of the students were identified with math-specific learning disabilities. Five out of the six middle school PSTs asked more low level problems and questions (>50% of the time) while all 3 out of the 3 middle school teachers within the classes with higher populations of inclusion students asked low level problems and questions. Seven of the nine high school PSTs asked low level problems and questions (>50% of the time), while all 3 of the 3 PSTs in those classes with higher levels of inclusion asked low level problems and questions (> 50% of the time). This information indicates the PSTs with more students identified with learning disabilities asked a higher percentage of lower level questions than their counterparts (middle or high school level).

Mentors

PSTs in this study had limited influence over their instruction and the curriculum they used because of their cooperating teacher (i.e., mentor), school and district mandates. The school may have required a specific curriculum for all teachers to follow not allowing PSTs much freedom. Since this was not the PST's classroom, the rules and organization of the room, which influence the culture of the classroom, had already been in place. Therefore, students may

have been comfortable with low level problems or questions, and less-demanding expectations already put in place by the cooperating teacher that were unable to be changed by the PSTs.

Conclusion

The authors found the SOLO Taxonomy, together with PSTs scores on the state licensure exam and Crespo's (2003) problem posing practices criteria to be a useful combination of tools to explore connections between PSTs' mathematics and pedagogical content knowledge. One conclusion that can be drawn from this study is that even after receiving extensive coursework emphasizing higher-order problem posing and associated support questions, PSTs had to follow the multiple variables or constraints at work in their cooperating teacher's classroom. These constraints may have contributed to their ways of operating in the classroom. A second conclusion that can be drawn from this study is that the level of PSTs problem solving does not directly relate to the level of questions they ask in a classroom setting. This can be influenced by the environment (Rural, Suburban, Urban), the cooperating teacher's rules and expectations what were pre-established, the school atmosphere (curriculum map, expectations, textbooks, etc.), class structure (gifted, IEPs, full-inclusion, etc.), and PST's prior beliefs on teaching and learning.

SOLO Taxonomy evaluations and analyses of PSTs' student teaching data corpus (e.g., lesson plans, video- and audio-recordings of lessons) can benefit teacher educators and programs for assessment and diagnosis purposes. Further research can support this study by investigating: (1) connections between teachers' own capacities to problem solve and their problem and question posing abilities and (2) connections between teachers' mathematics and pedagogical content knowledge. Additional One Question Lesson activities, similar to the Making Cones task, and corresponding SOLO Taxonomy evaluations might lead to better alignment between PSTs' levels of thinking (as indicated on SOLO taxonomy) and their scores on the state licensure exam or indicate areas where such rich tasks are not aligned with state exams.

References

- Askew, M; Brown, M; Rhodes, V; Wiliam, D & Johnson, D (1997). *Effective Teachers of Numeracy: Report of a study carried out for the Teacher Training Agency*. London:King's College, University of London.
- Authors (2014). *Journal of Experiential Education*.
- Biggs, J.B., & Collis, K.F. (1982). *Evaluating the quality of learning - The SOLO taxonomy(structure of the observed learning outcome)*. Academic Press, New York.
- Board of Education Commonwealth of Virginia (2016). *Mathematics Standards of Learning for Virginia Public Schools*. Richmond, VA: Virginia Department of Education.
- Brahier, D.J. (2009). *Teaching Secondary and Middle School Mathematics, third edition*. Boston, MA: Allyn & Bacon Publishing.
- Conference Board of the Mathematical Sciences (CBMS) (2012). *The Mathematical Education of Teachers II*. Providence RI and Washington DC: American Mathematical Society and Mathematical Association of America.

- Crespo, S. (2003). Learning to pose mathematical problems: Exploring changes in pre-service teachers' practices. *Educational Studies in Mathematics* 52, 243-270.
<https://doi.org/10.1023/A:1024364304664>
- De Corte, E., Greer, B., & Verschaffel, L. (1996). Mathematics teaching and learning. In D. C. Berliner & R. C. Calfee (Eds.), *Handbook of educational psychology* (pp. 491-549). New York: Simon & Schuster Macmillan.
- Fennema, E. & Loef, M. (1992). Teachers' knowledge and its impact. In D.A. Grouws (Ed.), *Handbook of research on learning and teaching mathematics* (pp.147-164). New York: Macmillan.
- Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. *American Educational Research Journal*, 32(3), 465-491.
<https://doi.org/10.3102/00028312032003465>
- Long, M. H., & Sato, C. J. (1983). Classroom foreigner talk discourse: Forms and functions of teachers' questions. *Classroom oriented research in second language acquisition*, 268-285.
- National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, DC: Authors
- Noble, J. (2013a, May 27). *Mathagogy, one question lessons and making cones*. Retrieved from <http://www.teachmathematics.net/blog/15403/mathagogy-one-question-lessons-and-making-cones>
- Noble, J. (2013b, June 7). Mathagogy: How I Teach With One Question Lessons. Retrieved from <https://www.teachmathematics.net/blog/15403/mathagogy-one-question-lessons-and-making-cones>
- Olive, J. (1991). Logo programming and geometric understanding: An in-depth study. *Journal for Research in Math Education* 22(2), p. 90-111. <https://doi.org/10.2307/749587>
- Pegg, J., & Tall, D. (2005). The fundamental cycle of concept construction underlying various theoretical frameworks. *ZDM*, 37(6), 468-475
- Potter, M.K. & Kustra, E. (2012). A primer on learning outcomes and the SOLO taxonomy Centre for Teaching and Learning, University of Windsor. Retrieved from <http://www1.uwindsor.ca/ctl/system/files/PRIMER-on-Learning-Outcomes.pdf>
- Ranucci, E. R. (1990). On the steepness of cones. *The Mathematics Teacher*. 69(2), 140.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*. 15(2), 4-14.
<https://doi.org/10.3102/0013189X015002004><https://doi.org/10.3102/0013189X015002004>
- Smith, M. S. & Stein, M. K. (2011). 5 practices for orchestrating productive mathematics discussions. Reston, VA: National Council of Teachers of Mathematics.
- Smith, M.S. Bill, V., & Hughes. (2008). Thinking through a lesson protocol: A key for successfully implementing high-level tasks." *Mathematics Teaching in the Middle School* 14 (3): 132-38
- Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20-26.
- Swan, M. (2005). *Standards Unit-Improving learning in mathematics: challenges and strategies*. DfES, http://www.ncetm.org.uk/files/224/improving_learning_in_mathematicsi.pdf.

- Verschaffel, L., Greer, B., & De Corte, E. (2000). Making sense of word problems. *Lisse, The Netherlands*.
- Yang, Der-Ching (2006). "Investigating the strategies used by pre-service teachers in Taiwan when responding to number sense questions." *School Science and Mathematics* 107(7), 293-301. <https://doi.org/10.1111/j.1949-8594.2007.tb17790.x>