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Middle School Mathematics Pre-service Teacher's Responses to a Mathematics Content and Specific Mathematics Pedagogy Intervention

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Abstract: Prospective middle school pre-service teachers' knowledge and affect in Australia has had little empirical research. In this study, 108 graduate entry pre-service teachers were surveyed for their knowledge of middle years' mathematics, confidence, and self-efficacy at the commencement of a mathematics curriculum course. It was found that their memory of middle years' mathematics was very poor and this was accompanied by low levels of confidence and self-efficacy. An intervention was undertaken to address these issues. The findings are discussed in the context of the "genericism" of pre-service teacher preparation. In particular, the findings call into question the justification for pre-service entry to such a mathematics curriculum course on the basis of proxy measures of mathematics knowledge, without which the teaching of the discipline becomes challenging.

Introduction

The main aim of this paper is to enhance practice with respect to mathematics teacher education, particularly in Australia but potentially more broadly across the West. The summary of the literature below illustrates that models of the relationship between teacher knowledge of mathematics and teacher confidence and effectiveness in classroom practice are fairly well developed. The paper presents empirical data that support the connections between knowledge of mathematics and confidence to teach it, but more importantly it gives pre-service teachers a voice as to what forms of knowledge and skills they think ought to take priority in mathematics teacher education courses.

Pre-service teachers' voice on the nature of their professional preparation is relevant because there is an assertion that Western universities – and that includes Australian teacher preparation institutions – have manifested forms of anti-intellectualism (Keeling & Hersh, 2012; Kotzee, 2012). The key factor in this accusation is that there has been a turn away from specific discipline knowledge to the embracing of "genericism" (Beck & Young, 2005, p. 183), such generic material including "thinking skills", "problem solving", and "reflective practice". The argument is that there has been too much focus on generic training rather than disciplinary education (Beck & Young, 2005; Winche, 2010) and this has the effect of emptying courses of discipline content. The focus becomes on how the professional should act, rather than what exactly the professional should do. Associated with this process is a devaluing of discipline knowledge in its own right, a corrosive popular wisdom that Young and Muller (2010) claim is becoming prevalent in the West but which is totally absent from the emerging economies of South Korea, China, and India. It is claimed that genericism is a form of anti-intellectualism and that it manifests in the dismissal of boundaries between subjects, between school and everyday knowledge, and between academic and vocational curricula (Keeling & Hersh, 2012; Young, 2011). The key point of the argument of the above-cited authors is that the pedagogic relationship between teachers and pupils in providing specialist knowledge is played down. Young and Muller argued that "the role of teachers cannot be reduced to that of a guide and facilitator rather than as a source of strategies and expertise" (p. 16). In essence, the thinking is an extension of Bernstein's theories of pedagogic discourse, where he distinguishes between esoteric and mundane knowledge forms and high levels of discipline knowledge that are needed to effectively scaffold academic discourse involving esoteric knowledge forms (Bernstein, 1999, 2000). Concerns related to the lack of sufficient focus on discipline knowledge (esoteric knowledge forms) are not new: Shulman (1986, p. 5) commented on U.S. teacher preparation programs, asking, "Where did the subject matter go?" Shulman (1986) further expressed concern that, in the main, teacher education programs focus on generic competencies such as cultural awareness, understanding youth, educational policies, recognition of individual differences, and instructional principles. Such a focus is similarly reflected in the Australian Institute for Teaching and School Leadership's Professional Standards (AITSL, 2011) where, of the seven knowledge forms, only "Know content and how to teach it" specifically focuses on the discipline.

Authors from a diversity of fields claim the dominant epistemology in the West has become hostage to interpretations of social constructivism that devalue the expertise of teachers in guiding learning and the critical nature of discipline knowledge (e.g., Chen, Kalyuga, & Sweller, 2016; Graven, 2002; Hattie, 2009; Hattie & Donoghue, 2016; Kirschner, Sweller, & Clark, 2006; Muller, 2000; Stipek, Givvin, Salmon, & MacGyvers 2001; Sweller, 2016; Tricot & Sweller, 2013). Interestingly, Depaepe and Konig (2018) in their study of German pre-service teachers found "no linear association between their domain-general pedagogical knowledge and their degree of confidence in being able to perform a diversity of teaching tasks" (p. 185) and that "GPK (general pedagogical knowledge) does not explain much variance in reported pedagogical practice" (p. 188).

Literature Review

The Importance of Content, Confidence, and Self-efficacy

While there remains controversy and debate with respect to the role of teachers in structuring effective discourse (e.g., Chen et al., 2016; Hattie, 2009; Kirschner et al., 2006; Sweller, 2016; Tricot & Sweller, 2014) the general consensus is that mathematics teachers with a deeper understanding of mathematical content are able to scaffold learning more flexibly and with purpose (Ball, Thames, & Phelps, 2008; Beswick & Goos, 2012; Burghes, 2011; Chapman, 2015; Gess-Newsome, 2013; Jacobson & Kilpatrick, 2015; Lai & Murray, 2012; Tatto et al., 2008; U.S. Department of Education, 2008; Zhang & Stephens, 2013). A recent study by Kleickmann et al. (2013) strongly linked mathematical content knowledge with mathematics pedagogical knowledge and tertiary learning experiences. Interestingly, the educational paradigm of East Asia is dominated by the view that knowledge forms such as mathematics need to be explicitly taught, and that the foundation facts and processes take substantive practice (e.g., Huang & Leung, 2004; Lai & Murray, 2012; Leung, Park, Shimizu, & Xu, 2015; Li, 2004) and East Asian teacher preparation processes reflect the paramountcy of content knowledge (Kim, Ham, & Paine, 2011; Leung et al., 2015). Recently, the Australian Government (2018, p. 71) acknowledged that the need for "A high-quality supply of specialist mathematics and science teachers is essential to turn this situation around". The Australian Government (2018, p. 77) drew on multiple sources including Hattie (2009) to state:

> *Expert teachers possess deep knowledge of pedagogical content knowledge and subject discipline, which they can employ flexibly and innovatively in their classroom teaching. Expert teachers understand reasons for individual student success, can anticipate student difficulties, can adapt with confidence in unexpected situations, and in doing so promote a student's learning growth*.

Shulman (1986) argued that the distinction between knowledge and pedagogy is a relative recent phenomenon. Shulman (1986, p. 9) defined content knowledge as

> *the amount and organisation of knowledge per se … going beyond knowledge of the facts or concepts of the domain. It requires understanding the structures of the subject matter … to include understanding the facts and structures of the subject matter to the depth of the logic behind particular propositions.*

Shulman defined pedagogical content knowledge to include "the most useful forms of representations of those ideas (mathematical concepts), the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of presenting and formulating the subject that make it comprehensible to others" (p. 9). It is hard to argue that a teacher with limited understanding of the content can enact effective pedagogy. For effective teaching the two knowledge forms are co-dependent, as illustrated by Dohrmann's (2012) use of the term "mathematical pedagogical content knowledge". Theorists from various fields argue that as the complexity of the discipline knowledge being taught increases, the importance of the teacher's content knowledge and specific pedagogy becomes more profound (Chen, Kakyuga, & Sweller, 2016; Kirschner, Verschaffel, Star, & Dooren, 2017). Unfortunately, international comparison does not paint a favourable picture of Western pre-service teachers' mathematical content knowledge (MCK), particularly in the primary teaching years but also extending into secondary teaching (e.g., Burghes 2011; Hine, 2015; Kim et al., 2011; Krainer, Hsieh, Peck, & Tatto, 2015; Ma, 1999; Tatto, Rodriguez, & Lu, 2015; Tatto et al., 2008). Those studies that have looked at the depth of MCK of middle school pre-service teachers, in the West in particular (e.g., Burghes, 2011; Kleickmann et al., 2013; Hind, 2015; Krainer et al., 2015; Tatto et al., 2015), indicate patchy MCK. There is scant empirical data on middle years' pre-service teachers' knowledge from Australia. Encouragingly, Kleickmann et al. (2013) and more recently, Depaepe and Konig (2018) found that tertiary preparation experience had the potential to make a considerable difference with respect to how well beginning teachers, including mathematics teachers, entered the classroom.

In addition to having a deep and connected knowledge of content, teachers need the communication skills and affective dispositions to convert this into productive classroom discourse. The importance of affective variables in learning mathematics has been well documented (e.g., Ingram & Linsell, 2014; Wilkins & Ma, 2003). Wilkins and Ma (2003) considered affective variables, including confidence, an important teacher attribute. Confidence has been defined as how sure a person is to perform well on a particular task (Fennema & Sherman, 1976). Confidence has been positively linked to the quality of pedagogy, acting partly through the interaction of confidence in discipline knowledge and self-efficacy, that is, a belief that their teaching will succeed (Bandura, 2006), and has been linked to more effective classroom practice (Beswick, Watson, & Brown, 2006; Graven, 2002; Lazarides, Buchholz, & Rubach, 2018). As with knowledge, there has been little empirical data on Australian middle years teachers' confidence. Beswick et al. (2006) found low levels of confidence with respect to critical aspects of pre-service teacher knowledge of middle school mathematics in their relatively small sample (N = 42) of primary and middle school teachers.

Closely related to confidence to do mathematics is self-efficacy. Hoy (2000) defined self-efficacy as a teacher's confidence to promote students' learning as distinct from their personal confidence in the mathematics. In this paper, self-efficacy is defined as a self-belief in capacity to teach particular mathematical concepts. In this regard the definition parallels self-efficacy for instructional strategies (Tschannen-Moran & Woolfolk-Hoy, 2001a). In the case of mathematics teaching, self-efficacy has been linked to persistence to achieve a mathematics teaching goal; that is, self-efficacy is task specific (Scherbaum, Cohen-Charash, & Kern, 2006). The consensus is that higher self-efficacy is related to improved classroom outcomes since self-efficacy is linked to persistence as well as a willingness to try new teaching approaches and be persistent in attempting to develop understanding (Bandura, 2006; Henson, 2001; Watt & Richardson, 2013).

Certification of Mathematics Teachers

The OECD (2014) noted that "the education requirements for entry into initial teacher training differ across OECD and partner counties" (p. 498). Duration of training can vary widely between countries; for example, for lower secondary school it can be as high as 6.5 years in Germany. Burghes (2011) reported similar variation in teacher preparation programs as well as differences in the levels of mathematics competency exhibited in his international study (China, Czech Republic, England, Finland, Hungary, Ireland, Japan, Russia, Singapore, Ukraine). In Australia, initial teacher education via the undergraduate pathway is generally 3 years of discipline-based subjects and a year of curriculum-based subjects. The graduate pathway is usually a 3-year bachelor's degree in a field considered rich in mathematics, followed by either a year in a graduate diploma teaching program or 2 years in a masters teaching program. The typical suite of subjects taught in initial teacher education programs includes practicum teaching, pedagogical studies, academic subjects, educational science subjects, child/adolescent development studies, and sometimes, research skills.

Certification of teachers may also involve the articulation of teacher standards; this practice has become relatively widespread and there is considerable similarly in wording across Western educational systems. For example, the English standards are virtually paralleled in Australia (Department of Education, 2013) and similar standards have been articulated for the United States (National Council of Teachers or Mathematics [NCTM], 2012). In Australia, knowing the discipline, in this case Mathematics, is reflected in the AITSL (2011) Australian Professional Standards for Teachers, where Standard 2 is "Know content and how to teach it" (p. 3). It is anticipated that a graduate will "demonstrate knowledge of concepts, substance and structure of the content and teaching strategies of the teaching area" (p. 10). There are seven key standards in Australia (1. Know students and how they learn; 2. Know content and how to teach it; 3. Plan for and implement effective teaching and learning; 4. Create and maintain supportive and learning environments; 5. Assess, provide feedback and report on student learning; 6. Engage in professional learning; 7. Engage professionally with colleagues, parents/carers and the community). Arguably, Standards 1, 3, and 5 are highly dependent on the graduates' knowledge (Standard 2).

The task of preparing teachers to meet the AITSL Australian Professional Standards for Teachers falls to the initial teacher education providers who have had their program approved by State statutory bodies. In the study state of Queensland this is the Queensland College of Teachers (QCT). Across Australia the dominant pathways to middle school mathematics certification are an undergraduate pathway that includes six subjects in mathematics, and a graduate or masters pathway. The usual selection mechanism for the postgraduate pathway, at least for middle school mathematics accreditation, is that the prospective teachers have successfully completed four university subjects rich in mathematical concepts. The use of this measure of mathematical knowledge gives Australian mathematics teacher education providers the flexibility to structure their programs to account for the seven teacher standards. Generally, all middle school pre-service teachers will complete at least one mathematics curriculum subject and those seeking to be qualified to teach senior mathematics will undertake a second mathematics curriculum subject. Some institutions such as James Cook University have a specific middle school mathematics content subject, but this was found to be an exception. The table attached in the Appendix illustrates that tertiary providers have a great deal of flexibility in the duration of courses, the focus, and how subjects are assessed. In Australian teacher preparation institutions, most assessment is of the form of essay writing or constructing resources that will be of use to future teaching. The inference of this analysis of teacher preparation subjects is that the detail of mathematics content has been largely assumed, or that measuring such content via tests is not particularly valid or useful. If detailed knowledge of middle school mathematics is not assessed, we can reasonably assume it is not the focus of the subjects. There is considerable international support for the use of portfolio assessment of teacher artefacts as reported of American teacher education programs (Hutt, Gottlieb, & Cohen, 2018) and earlier, in the West more broadly (Burghes, 2011).

In some countries (e.g., Brazil, England, France, Finland, Korea, Israel, Mexico, Spain, Turkey, Japan, Greece, Luxembourg) competitive exams must be passed either prior to entry or at exit of teacher training (OECD, 2014). Hine (2015) reported in the United States that many states required pre-service teachers to pass a basic test of mathematics content before accreditation. Hine noted the test did not assess the mathematics they would teach in secondary classrooms. Historically, tests of teachers or pre-service teachers' content knowledge have tried to cover a spread of domain bases and difficulty levels. Australia has recently introduced a threshold numeracy test (Australian Council for Educational Research [ACER] 2018a, Literacy and Numeracy Test for Initial Teacher Education: LANTITE). The intent of the literacy and numeracy test for all pre-service teaching students is to ensure that graduates are in the top 30% of the population for literacy and numeracy. The test does not assess the content of middle school mathematics. In structure, the initial teacher education test is similarly constructed to the National Assessment Program Literacy and Numeracy (NAPLAN) tests (ACARA, 2018b) for school children, with many but not all questions set in contexts. Sample numeracy questions of the literacy and numeracy test for initial teacher education students (ACER, 2018a, p. 9) exemplify the expectations.

> Numeracy Sample Question 1
>
> Government operating expenditure on mathematics refers mainly to money spent on schools and tertiary education.
>
> The total operating expenditure on education in 2011-2012, 51% was spent on primary and secondary education and 36% on tertiary education (universities and TAFEs).
>
> What percentage of the total operating expenditure on education in 2011-2012 was spent on the remaining aspects of the education budget?

The context of percentage places the problem as upper primary, but the actual mathematical conceptualisation of this question is to identify that 51 and 36 must be added; the sum (87) then needs to be subtracted from 100 to yield 13. This level of computation is lower primary school. The last and arguably most difficult question of the sample questions is Question 10.

> The Australian Bureau of Statistics conducts a census every five years. In 2011, the population of Australia was about 22 million. About 2% of these people lived in remote or very remote areas.
>
> About how many people live in remote or very remote areas in Australia in 2011?

A) 11 000; B) 44 000; C) 110 000; D) 440,000.

Finding 1% can be done by reducing 22,000,000 by two place values to 220,000 then doubling this to 440,000. Such a computation is consistent with Year 7 minimum standards in the Australian Curriculum (ACARA, 2018) and most competent Year 7 students would do this problem mentally. Almost all the sample questions are in context and that challenges literacy as much as numeracy, but overall it can be argued that the questions are set at about the same level as a Year 7 NAPLAN test. Clearly, the test is not intended to be a reasonable threshold for teachers of middle years mathematics. The test samples do not assess the forms of mathematics middle school students struggle with, including fraction computations beyond the simplest forms, formal algebra, or any middle years formal geometry.

TIMSS (Trends in Mathematics and Science Study) (International Association for the Evaluation of Educational Achievement [IEA], 2011) assess children internationally in similar ways; some questions are embedded in contexts, others are not. It is common practice in assessing mathematics content knowledge to allocate one mark for the correct response and zero for all incorrect responses, not least because such tests are frequently dominated by multiple choice format.

Aims of the Study

With this background in mind, the key aim of the paper is to give informed middle school pre-service teachers a voice in regard to the focus of graduate entry mathematics curriculum teacher education preparation. This includes asking them what they consider important in mathematics teaching and learning, what they want from a mathematics curriculum subject, and how they evaluated attempts to meet their needs. In doing this, supporting aims are to document the starting content knowledge of graduate entry middle school mathematics pre-service teachers, their confidence and self-efficacy. These affective data help to give supporting data for the primary aim. The ethics protocol number for this study was EDN/34/14/HREC.

Method

Mixed methods were used to collect and analyse the data. The study is correlational in that the relationships between knowledge, confidence, and self-efficacy are examined. SPSS was used to calculate descriptive and correlation statistics from the data collected at the commencement of the subject through an author-constructed survey. Critical insights come from summaries of pre-service teachers' written responses at the beginning and end of a mathematics curriculum intervention that is described below.

Participants

The participants in this study were the pre-service teachers of a middle school mathematics curriculum subject delivered in 2017; 108 of 127 enrolled students participated, representing 85% of the cohort. The subject involved 28 hours of lectures and workshops run over 7 weeks. In this institution there are typically eight subjects spread over a 2-years master's degree and, for middle years mathematics teacher accreditation, one was mandatory. Those pre-service teachers going on to be certified to teach senior mathematics (42% in 2017) were required to take a second mathematics curriculum subject focusing on senior school mathematics curriculum, content, and pedagogy. The entry requirements for the subject included the successful completion of Year 12 mathematics, and the completion of a bachelor's degree in which four subjects were considered rich in mathematical concepts. The university in which the study was undertaken was ranked in the top 3% globally (World Universities Search Engine, 2016) and number 12 (out of 26) in Australia for graduate employability (The World Universities Ranking, 2017). The program and subject were accredited by the QCT, the formal statutory body that licences teacher training programs and registers teachers to teach.

Data-gathering Instruments

At the first workshop the pre-service teachers were given the option to complete a survey and test of MCK. The survey qualitative data reported in this paper were in response to two prompts, the first being, *"What do you think is the most important feature of quality mathematics teaching?"* and the second prompt being, *"What do you most want from this course?*" The term *course* was used rather than *subject* because at the study institution a unit of curriculum study is termed a subject; it is typically valued at 10 credit points out of 80 for a graduate diploma and out of 160 for a Masters of Education certification. The responses to the two open-ended questions above were coded according to themes. Open-ended questions such as these have the advantage over multiple-choice prompts of not channelling the responder according to the researcher's preconceptions. The survey was conducted during the same timeslot as the test of content. This is a very important methodological point: Asking the pre-service teachers what they wanted from a mathematics curriculum course at the same time as asking them to demonstrate their knowledge of middle school mathematics was bound to impact on their responses. It is probable that had the participants been unaware of the exact nature of middle school mathematics or not been confronted by their limits in mathematical knowledge, their responses may have been quite different.

Starting Mathematical Content Knowledge

Starting content knowledge was assessed via a pencil-and-paper test in the first lecture. The test contained 31 items; one mark was allocated to each mathematically correct solution and ½ mark for each nearly mathematically correct solution (i.e., the response demonstrated conceptual understanding in that the correct pathway to the solution was demonstrated, but there was a minor computational error). The allocation of part marks was a rare occurrence since almost all errors were major failures related to misunderstanding of the concepts or profound procedural errors. Six questions were very similar or identical to questions on the International Comparative Study in Mathematics Teacher Training (ICSMTT) (Burghes, 2011). The test was subdivided into five subsections along content and year level lines (whole numbers, fractions, index notation and surds, linear equations, quadratic equations). In each section there were some questions that assessed pure procedure, in that the required operation was stated; other questions were problem-solving orientated in that the required method of computation was not stated. The test and the survey were allocated 60 minutes for completion and no calculators or books were permitted, since ACARA (2017) stipulates that children must be fluent both with and without a calculator for procedures and content of the nature tested. The test was written solution format; the use of multiple-choice format has been earlier cited as a poor indicator of teacher knowledge (Hutt et al., 2018).

There were four questions related to whole number computation and problem-solving including subtraction, multiplication, division, and simple problem-solving involving multiplication and geometric thinking. This mathematics is consistent with upper primary school mathematics (ACARA, 2017). Five questions assessed fraction computation and problem-solving consistent with Year 7 and 8 mathematics (ACARA, 2018). The first of these questions was: “A car costs $50,000. You have a deposit of $2,147, how much more money is needed to buy the car?” (Success rate 85%).

Analysis of middle school students’ learning of fractions (Brown & Quinn, 2006, 2007) has been well documented and it is clear from NAPLAN and international testing analysis that the pre-service teachers would have to teach and remediate fraction misconceptions as part of their early practice. Primary pre-service teachers’ difficulties with fraction-based concepts have been relatively well reported (e.g., Chick, Baker, Phan, & Chen, 2006; Norton & Nesbit, 2011; Widjaja & Stacey, 2009), but similar challenges for middle school pre-service teachers have received relatively little empirical description, although international testing studies (e.g., Burghes, 2011; Tatto et al., 2015) suggest this is an area that warrants investigation. Sample questions assessing fraction fluency include Question 7, “What is $4\,\frac{1}{2} \div \frac{1}{3}$ “(Success rate 42%).

Nine questions probed working with index notation, surds, and logarithm conventions (Years 9 & 10; ACARA, 2018). Three questions from the ICSMTT test were duplicated in this test. One such question was “Calculate $(125)^{\frac{1}{3}}$” (success rate 35%).

Entry algebra including solving, working with simultaneous equations, and relatively simple first-order algebra problems at the Year 9 and 10 levels were assessed via six questions (ACARA, 2018). Pierce, Stacey, and Bardini (2010) are among the authors who have described the difficulties children have with understanding linear functions and the challenges involved in teaching this topic area. Question 25 was the most taxing of the linear equation questions: “There are 10 more men than women at a party. If one more woman joined the party, there will be twice as many men as women. How many men and how many women are at the party?” (success rate 19%).

Finally, there were six questions that assessed fluency and problem-solving within the context of quadratic equations, which is usually taught at the end of Year 10. The research that has been conducted on middle years school students’ struggles with quadratics suggests that it is a threshold topic area that is poorly understood by very significant portions of upper middle school students (Bosse & Nandakumar, 2005; Vaiyavutjamai & Clements, 2006; Zakaria, Ibrahim, & Maat, 2010). ICSMTT had two questions probing middle years pre-service teachers’ knowledge of quadratics; one was duplicated in this test (ICSMTT Q6; this test Q26). Question 28(a) asked the pre-service teachers to identify the roots of a quadratic from a graph (success rate was 25%).

The content of the test used in this study has a reasonable spread of the number and algebra with which students from Year 4 to 10 are expected to become fluent, thus it is argued there is content validity.

Assessing Confidence and Self-efficacy

Following each content question the pre-service teachers were asked to “Rate how certain you are that you can solve each of the academic problems according to the scale 0 (*cannot do at all*), 50 (indicating moderately confident the solution is correct) to 100 (*highly certain*)”. With respect to confidence, the exact explanation was presented to the participants:

"Rate how confident you are that you can solve each of the academic problems according to the scale below":

	0	0	0	0	0	0	0	0	0	1 00
	Cannot do at all				**Moderately can do**				**Highly certain**	

Further explanation was provided: "0" indicated "I cannot do at all – means you have no idea how to do this mathematics and solve the problem" and 50 means "moderately can do, means you are unsure if the solution is correct but are well on the way to a solution." "100 or highly confident can do this mathematics means you are very confident of your solution." Traditionally, Likert scales use a 4- or 5-point scale (e.g., Fennema-Sherman, 1976; Sherer et al., 1982). Bandura (2006) preferred a highly graduated scale such as the one used in this study because he believes it is more sensitive. A further difference between this study and earlier studies involving confidence and self-efficacy is that earlier studies tended to be much more generic with respect to what the subjects were confident about doing, for example, "I am sure I can learn the mathematics" or "I am a self-reliant person" (Sherer et al., 1982, p. 666). In this study participants were asked how confident they are in doing 31 very specific middle school mathematics tasks; such specificity is consistent with that modelled by Beswick et al. (2006).

Similarly, after each question the pre-service teachers were asked to rate on a 0 to 100 scale their capacity to teach the mathematics of the form inherent in each of the problems. Unlike the Tschannen-Moran and Woodfolk-Hoy (2001a; 2001b) teachers' sense of efficacy scale that tended to use generic prompts such as "To what extent can you craft good questions for your students?" this scale is specific to the teaching of particular mathematics content.

Description of the Intervention

The intervention was intended to meet the pre-service teachers' needs as perceived from the analysis of similar data collected over past years. In particular, it attempted to teach the content as well as provide specific pedagogy for a broad range of upper primary and middle years' mathematics. Frequently, the approach was to start with a typical student error and diagnose the possible underlying thinking and then plan to remediate any misconceptions with models, activities, and explicit teaching of algorithms. The organisation of the subject/course was informed by cognitive load theory with the lecturer attempting to model effective teaching of mathematics, including effective diagnosis of student errors and modelling specific mathematics pedagogy. Further detail of the specific pedagogy can be accessed at Norton (2018).

Evaluation of Intervention

Pre-service teachers' views on the nature of the intervention were assessed via standard student experience of course/subject (SEC) surveys that are instigated by the university at the end of each course. SEC is voluntary and conducted online prior to the final examination, and frequently response rates are low. In this instance, 73% (47 out of 64) on Campus A and 58% (39 out of 67) of students at Campus B responded to SEC. The generic course experience questionnaire contains prompts related to: 1) whether the course was well organised; 2) whether the assessment was clear and fair; 3) the reception of helpful feedback; 4) whether the course engaged respondents in learning; 5) effectiveness of teaching team; and

6) overall satisfaction with the quality of the course. To this list the author added questions related to 7) the learning resources supplied and of relevance to this paper; 8) whether the {focus on mathematical content knowledge} in this course assisted learning; and 9) whether the {focus on explicit mathematics teaching methods} in this course assisted learning… In this paper, responses to Questions 4, 6, 8, and 9 are reported, as the other prompts are interesting but not central to the intent of the paper. We cannot be sure that the sample is truly representative, but in the case of Campus A it is reasonably so.

Results

Scale Descriptors

The test has content validity since the content can be directly mapped to the content the pre-service teachers are expected to teach. The Cronbach's alpha statistics of .901 for the 31 items suggests a high degree of reliability. The data indicated that pre-service teachers with similar mathematical competency succeeded or failed on the same questions as their peers. Looking at these two factors together, we can be confident that the scale is a good measure of the pre-service teachers' explicit knowledge of the middle years' mathematics they will soon be expected to teach. The mean score was 10.29 out of 31 (33%) with standard deviation of 6.59, maximum score of 29, and minimum scores of 1 out of 31. The distribution of these scores is illustrated in Figure 1.

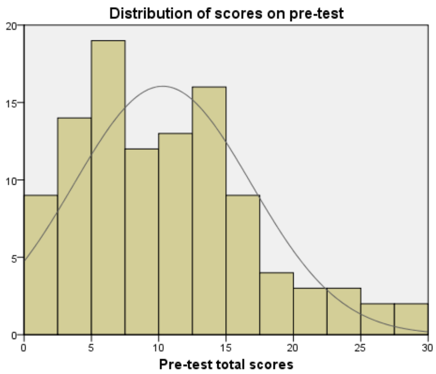

Figure 1. Distribution of scores on pre-test with total possible mark 31.

In Figure 1 a positive skew is evident, in that a few students gained high marks while a significant portion gained very low marks. Several of the students who scored less than five marks subsequently withdrew from the program. It is not known if their performance on the pre-test was a contributing factor.

There were 31 items that contributed to the expression of pre-service teachers' confidence that they could do the mathematics. The mean confidence score minimum was 16%, mean 46%, and maximum 92%. The Cronbach's alpha statistic was .968. Inter-item correlation coefficients were reasonably high between questions with similar levels of mathematical difficulty; for example, Question 26 and 27 both probe fluency in factorising a quadratic and the inter-item correlation coefficient was .888. As expected, where the mathematical demands were less matched – for example, Question 1 (subtraction of whole numbers; 85% success) and Question 16 (expressing a surd with a rational denominator; 9% success) – the inter-item correlation was very low at .138. These data suggest the confidence scale was a very good gauge of pre-service teachers' confidence to do particular middle school mathematics content over a range of difficulty levels.

As was the case with confidence, the measure of pre-service teachers' expressions that they could teach the mathematics had a very high Cronbach alpha statistic of .964. High levels of inter-item correlation existed between cognitively similar mathematics tasks. The mean confidence to teach the material was 34% with a standard deviation of 22%. That is, almost all the cohort indicated they could not teach the mathematics without a lot of background preparation.

Pre-service Teachers' Views

For the open-ended prompt, "What do you think is the most important quality of teaching mathematics?" seven themes were identified in 115 comments. Examples of the comments and how they were classified are listed below. This detail gives us confidence that the finding is grounded in the data. Comments are shown in Table 1 with relative frequency of occurrence.

Theme	Example	Frequency
Content knowledge	Deep understanding of the material; deeper understanding of the content; know content enough to teach…	42%
Mathematics pedagogy	Be able to communicate maths principles so student can understand it; simple communication of maths concepts to teach effectively; teach in an appropriate way so student can learn and understand; ability to explain mathematics; can fully explain things fully	37%
Linking mathematics to the real world	Showing them how to use maths in real life; make maths relatable	8%
Understand student thinking	Be able to understand how children learn; identify weaknesses	5%
Make maths interesting	Make maths interesting	4%
Engage students in learning	Engage with students to help them learn; engage students	3%
Teaching for confidence	Getting kids mastering the basics and feeling like it is possible that they can do it.	1%

Table 1. Responses to "What do you think is the most important quality of teaching mathematics?" (n = 115)

Given the proximity of the content test and the survey it may not be surprising that depth of content knowledge emerged as the most critical variable.

For the open-ended prompt, "What do you most want from this course?" five themes were identified, as illustrated in Table 2. Many of the comments had two themes, for example, "learn the content and how to teach it" and "improve my maths in order to be able to teach it". The total number of comments was 122.

Theme	Example	Frequency
Pedagogy for teaching mathematics	Effective methods to guide students to maths knowledge; teaching methods appropriate to the content; how to teach maths	50%
Content knowledge	Skills in understanding mathematics; revision of the material I will be teaching; greater understanding/refresh of maths concepts	40%
Confidence	Confidence; confidence to teach the concepts to a class	7%
Differentiation ability	Teach maths so all different types of students can understand the processes	2%
Technology related	To learn how to use technology to make learning exciting	1%

Table 2. Responses to the prompt "What do you most want from this course?" (n = 122)

Content Knowledge, Confidence, and Self-efficacy

As illustrated in Figure 1, in terms of the participants' MCT the average mark for the total of 31 items was 10.324/31 (33%) with a standard deviation of 6.642. It is evident that a large portion of the pre-service teachers had very limited capability to successfully do the mathematics tested. Second, seeing the mathematics they would soon be accredited to teach caused most to report low self-confidence to do the mathematics and even lower self-confidence that they could teach the material without considerable preparation. However, success in the mathematics and associated confidence and self-efficacy was not uniform across the content domains, as illustrated in Table 3. Rather, pre-service teachers were more capable and more confident with primary mathematics compared to Year 10 mathematics. This is not unexpected since primary mathematics is more likely to be used in daily life and thus remembered; in any case, it is simpler.

Concept areas	Success rate in mathematics content areas	Mean confidence in mathematics solutions	Mean self-efficacy
Whole number computation and problem-solving (Years 3 to 6, ACARA, 2017) (Q1, Q2, Q3, Q4)	56% Std 30%	71% Std 26%	57% Std 26%
Fraction computation and problem-solving (Years 7 to 8, ACARA, 2017) (Q5, Q6, Q7, Q8, Q9, Q10)	50% Std 33%	69% Std 29%	50% Std 28%
Index notation and logarithm computation (Year 9 and 10, ACARA, 2017) (Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q18, Q19)	26% Std 23%	33% Std 24%	23% Std 21%
Linear equation computation and problem-solving (Year 8 and 9, ACARA, 2017) (Q20, Q21, Q22, Q23, Q24, Q25)	30% Std 28%	46% Std 33%	33% 30%
Quadratic equation computation and problem-solving (Year 10 and 10 Advanced, ACARA, 2017) (Q26, Q27, A28a; Q28b, Q28c, Q29)	17% Std 24%	24% Std 31%	17% Std 22%

Table 3. Summary of Mathematical Competency in the Different Domains of Knowledge and Associated Confidence and Self-efficacy

While the average success rates are reported according to year level and concept area, there is a great deal of variation within each concept area. For example, 85% of participants were successful with subtraction of whole numbers (Year 3; ACARA, 2018) and 41% were successful in dividing a 5-digit number by a 2-digit number (Year 6; ACARA, 2018). Clearly, the further up the grade minimum standard the questions represented, the greater the difficulty level and decreased success rates. Unfortunately, success eluded about half the intake with regard to fraction computation (typically taught in Years 7 and 8). The forms of errors the pre-service teachers made were consistent with those earlier reported by Brown and Quinn (2006, 2007). Success related to Year 8 and 9 linear algebra was at about 30% and questions related to Year 10 simultaneous equations and quadratic equations had success rates from 7% to 25%. Interpreting the word problem and solving for the roots of the quadratic is typically taught in Year 10. Question 26 was: "A triangle has an area of $20\ cm^2$. If the height is 3 cm shorter than the base, find the length of the base of the triangle?" This question had a success rate of 12%. Where there was an opportunity to compare success on particular questions (Questions 4, 10, 11, 12, 13, 22, and 26) the success rates of this sample were typically half of that cited by Burghes (2011) for pre-service teachers in the UK.

Table 3 also illustrates the pre-service teachers' declining confidence in the mathematics and confidence that they could teach the material without increasing effort in planning prior to classroom engagement. It is worth noting that success in doing the mathematics is so closely mirrored by confidence to teach the mathematics, yet the participants were consistently more confident that they could do the mathematics than was warranted by the data. As indicated by the very high reliability statistics, the same 20% or so who had a good knowledge of index laws and logs were able to succeed with questions related to quadratic equations. For the majority of students, as articulated in their survey responses, they were closer to "cannot teach this concept at all – means you have no idea where to start and would have to do a lot of background preparation before teaching this concept in the classroom" than "highly certain I can teach this concept – means you have sufficient confidence in your knowledge of mathematics and pedagogy to virtually walk in and teach this concept".

Pre-service Teachers' Evaluation of the Intervention

SEC mechanisms allow teaching academics to add prompts which are responded to on a 1 – 5 scale, where 1 indicates strong disagreement with the statement and 5 represents strong agreement. Academics have limited flexibility with the wording of these prompts, having to insert a phrase in the existing structure. The mean responses to three most relevant prompts are documented in Table 4.

Statement	Campus A mean	Campus B mean
This course engaged me in learning	4.1	4.6
Overall I am satisfied with the quality of this course.	4	4.5
The {focus on mathematical content knowledge} in this course assisted my learning.	4.4	4.7
The {focus on explicit mathematics teaching methods} in this course assisted my learning.	4.2	4.6

Table 4. End-of-course Student Evaluation (Campus A participation 58%; Campus B participation 73%; maximum score 5)

Across the 86 responses to SEC, two students (about 2%) did not agree with the statement valuing the focus on mathematical content and eight did not agree with the explicit teaching methods used in the course.

In total, there were 69 written responses to the university-mandated prompt, “What did you find particularly useful about this course?” The dominant themes were opportunity to learn the content in an understandable way and opportunity to see explicit pedagogy enacted. The four comments below best encapsulate the spirit of these responses:

> *I enjoyed the high level of content knowledge. This allowed me to understand mathematics concepts in a way I haven’t before. It took a lot of mathematics concepts from abstract ideas to understandable in my mind which will allow me to teach this content more effectively to students. (PST 1: Campus A)*
>
> *I found the focus on mathematical content knowledge in this course extremely helpful as I was a little rusty with some of the concepts. The focus on content has therefore made me much more confident with my own mathematics ability. The lecturer taught us the content as he would teach his students (school). I found this incredibly helpful as it not only assisted us to know the content but also how to teach it. (PST 2: Campus B)*
>
> *Having taken this course, I can now say with confidence that I am equipped with these tools, I believe it has made me a better teacher. (PST 3: Campus B)*
>
> *Quite frankly I would have felt grossly under-prepared to teach mathematics in high school if I had not attended this class. (PST 4: Campus A)*

Across the two campuses there were 61 written comments in response to the prompt, “How could this course be improved?” The sequence of commonality of the themes was: timetable-related issues (all the workshops and lectures); issues with the distribution of the text and video support; not enough time to cover all the material. There was one comment that questioned the focus on mathematical content:

> *This course teaches mathematics from primary to early secondary in a traditional didactic manner. This didactic style is the expected pedagogy. Experienced graduates have forgotten much of the maths, which is a justification for the course approach. This is a false basis, we have forgotten the detail of the maths because we do not use it and it is irrelevant. This course should be about pedagogy for how to deliver math that is engaging and relevant, and in alignment with teaching pedagogy evidence. (PST 6: Campus B)*

Discussion and Conclusions

The first finding from this study is that the level of content knowledge that the pre-service teachers brought to the course was disturbing. About a third could not multiply by a 2-digit number; less than half could divide by a 2-digit number and success tended to become increasingly elusive the more advanced the mathematics questioned. The data from this study were significantly more alarming than those reported in international studies (e.g., Burghes, 2011; Tatto et al., 2015). What is new in the data detail is the degree and spread of challenge exhibited at enrolment. The pattern of errors made tended to comprise conceptual errors (not knowing what processes and algorithm to apply) and procedural errors (not being able to apply correct processes and computations). The errors made by the pre-service teachers mimicked those made by children with respect to whole number computation (Norton, 2012), fractions (Brown & Quinn, 2006, 2007), and primary or elementary bound pre-service teachers (Chick et al., 2006; Widjaja & Stacey, 2009). The pre-service teachers experienced difficulties with early algebra not so different from that reported for children (Pierce et al., 2010) and the mathematics surrounding quadratic equations was particularly troubling, as it is for middle school children (e.g., Bosse & Nandakumar, 2005; Vaiyavutjamai & Clements, 2006; Zakaria et al., 2010). In this regard this paper adds to the emerging data on pre-service teachers' entry knowledge and concerns expressed by earlier authors (e.g., Beswick & Goos, 2012; Hine, 2015). These data expose the unsuitability of using proxy measures such as mathematics courses completed as valid estimates of depth of mathematical understanding. Concerns with regard to the use of such metrics have been articulated by other authors (e.g., Burgher, 2011; Qian & Youngs, 2016).

Almost all the middle school mathematics pre-service teachers, when shown the specific mathematics they would be expected to teach, recognised their limitations and were reasonably accurate (if somewhat optimistic) in their capacity to do the mathematics at this time. This finding has methodological implications, in that there is merit in asking about confidence and self-efficacy associated with very specific mathematics tasks. In this regard the specific questioning format supports the methods used by Beswick et al. (2006) to consider seeking teachers' views on very specific tasks, a proposition supported by Depaepe and Konig (2018). Perhaps more importantly, the dominant view expressed was that the pre-service teachers wanted to learn the mathematics and the specific didactics of how to teach it and were supportive of this approach in their SEC evaluations. This begs the question: If depth of content and specific pedagogy has the overwhelming support of cognitive load theorists and East Asian educationalists, is anticipated by Western theorists, and demanded by national accreditation standards, what can explain the absence of assessment of discipline knowledge across Australian institutions' mathematics curriculum assessment protocols? Similarly, if confidence and self-efficacy are important for structuring classroom discourse (e.g., Beswick et al., 2006; Depaepe & Konig, 2018; Hoy, 2000) why the apparent disregard of these attributes in pre-service teacher preparation? There are several potential explanations for the current focus of teacher education programs in ways that do not necessarily account for a deficit of content knowledge.

A potential explanation for the lack of relevant mathematics content knowledge at intake is that given the earlier studies completed by the pre-service teachers, middle years' content can be assumed. Clearly, at least in this instance, the assumption is flawed since only a very few pre-service teachers could demonstrate competency with even lower secondary school mathematics. Similar, results have been reported for other cohorts of middle years' mathematics teachers (Norton, 2018). It is possible, but improbable, that the study institution is unique in attracting such a large portion of pre-service teachers who have the above-reported level of mathematics knowledge. The common enrolment processes across states and institutions suggest this is improbable. In addition, the OEDC (2014) data indicate that significant portions of lower secondary pre-service teachers feel unprepared to teach the content. A range of authors' investigations of this challenge indicate the concern is relatively widespread across many Western educational systems (e.g., Burghes, 2011; Hind, 2015; Kleickmann et al., 2013; Krainer et al., 2015; Tatto et al., 2015).

Accepting that pre-service teachers have a strong background in middle school mathematics confers flexibility upon the teacher preparation provider in that it gives the middle school mathematics educator licence to focus on generic principles. It can be argued that such an approach is consistent with the guidelines set out by AITSL (2011) in the Professional Standards, since they are relatively generic in expression. In this study, if the pre-service teachers had not been asked to demonstrate their knowledge at course commencement, no one would have been any the wiser as to their level of knowledge. As illustrated in the literature review, the threshold LANTITE test (Australian Government Department of Education and Training, 2017) is not intended to be a threshold for content for teaching middle years mathematics. Similarly, the very specific context in which the pre-service teachers were asked to reflect upon their confidence and self-efficacy was likely to be important in how they responded to the probing of their affective attributes, in particular, decreasing their reported confidence and self-efficacy at the start of the course. Arguably, had the pre-test not been administered, no remediation would be seen as necessary, a proposition supported by Depaepe and Konig (2018). Without being confronted with personal deficit in content, pre-service teachers may have been happy to have completed generalist pedagogy mathematics curriculum courses and reported relatively high levels of satisfaction. Indeed, this was the case at Campus B until 2015. It is argued that the relatively generic focus of mathematics curriculum courses across Australia has been relatively well reviewed, in part because of the way these courses are assessed and presented to pre-service teachers.

One pre-service teacher (PST 6) provided a good rationale to dismiss the data and his/her reasoning has some theoretical support and is a good justification of the current mathematics teacher program focus. PST6 did not consider this lack of knowledge important since the material had probably been forgotten, possibly because it was not relevant to their post-school lives. This participant's articulation is a justification for a generic approach to mathematics curriculum courses and a greater focus upon principles such as how to make mathematics engaging and relevant. Such a view of the primary role of pre-service teaching mathematics curriculum courses is largely reflected in the program structures which illustrate teacher preparation institutions' attempts to meet the AITSL (2011) teacher standards. As illustrated in the Appendix, there is no necessity for pre-service teachers, once enrolled – with rare exception – to demonstrate their knowledge of middle school mathematics outside of take-home assignments. In this regard the summary of assessment protocols supports the assertions of a range of authors who claim that discipline knowledge in Western tertiary institutions has been de-emphasised (e.g., Beck & Young, 2005; Bernstein, 1999, 2000; Keeling & Hersh, 2012; Young, 2011; Young & Muller, 2010). The very low level of mathematics demanded of the teacher registration test does little to alleviate this concern.

PST 6 also rejected the emphasis on focusing on the content and providing explicit models of how to teach the whole number, fractions, algebra, surds, quadratics, probability, and geometry concepts that were a focus of the intervention. PST 6 rejected what he/she described as "traditional didactic manner". In this regard PST 6 manifests the view that principles of learning, potentially emphasising facilitation and co-construction (alignment with "teaching pedagogy evidence"), are preferable to a more didactic approach favoured by the author and East Asian educators (e.g., Huang & Leung, 2004; Lai & Murray, 2012; Leung et al., 2015; Li, 2004), meta-data analysts (e.g., Hattie, 2009; Hattie & Donoghue, 2016), and cognitive load theorists (e.g., Chen et al., 2016; Kirschner, Sweller, & Clark, 2006; Sweller, 2016; Tricot & Sweller, 2014). It needs to be noted, however, that while the views of PST 6 provide a reasonably coherent justification for existing generic mathematics teacher education programs and curriculum courses/subjects, they are counter to those expressed by the majority of the pre-service teachers enrolled in the mathematics course informing this study.

What the testing data do not show, but what is implied in the student evaluation comments and ratings, is that most of the pre-service teachers improved their base level understanding of mathematics over the life of the intervention. Thus, most of them would make considerable progress in the first few years of teaching. The question is whether putting the onus to develop domain-specific knowledge and expertise onto the novice teacher – in effect, to teach themselves once in practice – is the best policy. Such an expectation is likely to have both cognitive and affective implications for themselves as beginning teachers, and for their students.

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Appendix

University*	Course code	Assessment forms	Recommended contact face to face
Edith Cowan	MSE6711	Report working mathematically 60% Case Study 40%	Unclear
Excelsia College Sydney	EDSC507	Journal task 25% Unit of work 45% Essay 30%	Unclear
Flinders University	EDUC9128F	Assignments; Tutorial presentation; oral; tutorial participation.	33 hrs
Griffith University (graduate)	EDN 7024	Closed-book exam 60% Classroom-based research assignment 40%	32 hrs
University of Queensland	EDUC 6725	Review of digital resources 33% Mathematical investigation inquiry 33% Resource, working with families 33%	24 hrs
Monash University (graduate)	EDF 5017	Tasks exploring numeracy-related issues 50% Critical reflections on numeracy 50%	24 hrs
Murdoch University	EDN554	Online assignment: interview student about some aspect of mathematics; planning a sequence of lessons; Online and in-class discussion.	Unclear
Swinbourne University of Technology	EDU600034	Presentation and report 50% Assessment folio 50%	
University of Adelaide	EDUC 4533A	Essay on the use of technology 50% Prepare teaching materials 50%	4hrs/week
Queensland University of Technology	CRB 204	Learning log 60% Teaching plan 40%	Unclear
University of Sydney	EDSE 3046	4000-word essay 60% 2000-word assignment 40%	32 hrs
University of Technology Sydney	013415	Lesson plan 30% Website comparison and report 30% Exam 40% (includes mathematics skills test)	Unclear
University of Melbourne	EDUC 90457	Two reports 50% each	36 hrs
University of New England	EDME392/393	Teaching design task 40% Written task focus on assessment 40% 20% Online quizzes X 5 20%	Unclear
University of New England	EDME393	Curriculum Investigation 45% Practical curriculum investigation 40% Online tasks 15%	Unclear
University of Newcastle	EDUC 1090	Essays/written assignments Lesson plan	8 hrs

Key: *Sourced from online university web sites.

Table A1. Sample of Course Assessment Australian Mathematics Middle Years Curriculum Courses