A Multilevel Analysis of the Impact of Teachers’ Beliefs and Mathematical Knowledge for Teaching on Students’ Mathematics Achievement

Adem Ekmekci  
*Rice University, ekmekci@rice.edu*

Danya M. Corkin  
*University of Houston-Downtown, corkind@uhd.edu*

Weihua Fan  
*University of Houston, wfan@Central.UH.EDU*

Follow this and additional works at: [https://ro.ecu.edu.au/ajte](https://ro.ecu.edu.au/ajte)

Part of the Science and Mathematics Education Commons, and the Teacher Education and Professional Development Commons

**Recommended Citation**

http://dx.doi.org/10.14221/ajte.2019v44n12.4

This Journal Article is posted at Research Online.  
A Multilevel Analysis of the Impact of Teachers’ Beliefs and Mathematical Knowledge for Teaching on Students’ Mathematics Achievement

Adem Ekmekci
Rice University, United States of America
Danya M. Corkin
University of Houston-Downtown, United States of America
Weihua Fan
University of Houston, United States of America

Abstract: Teachers’ content knowledge and beliefs about teaching and learning are among the key factors for effective teaching and, in turn, for student achievement-related outcomes. This study explores the extent to which K-8 math teachers—who teach in high-poverty urban schools—professional background, motivational beliefs, and mathematical knowledge for teaching (MKT) have an impact on students’ math achievement. Hierarchical linear modeling (HLM) results indicated that although students’ prior mathematics achievement was the most determining factor of their subsequent math achievement, teachers’ MKT and holding a bachelor’s degree in mathematics had significant positive effects on students’ math achievement. Results provide support for professional development (PD) to focus on improving mathematics teachers’ mathematical knowledge for teaching. Results may also have implications for education policies at both the district and state level for teacher incentives to further develop teachers’ mathematical knowledge for teaching, especially for urban school teachers.

Introduction

Being in the forefront of education, teachers are the most important key players in students’ educational outcomes. We are in a time when concerns over teacher quality have increased both nationally and internationally (Darling-Hammond, 2017; Hanushek, 2014; Leigh & Ryan, 2008; Rowe, 2003), which makes it crucial to consider the effects of teachers’ beliefs and knowledge on effective instruction (Akay & Boz, 2010; De Mesquita & Drake, 1994; Pajares, 1992). Several researchers have noted the significant role of teachers’ educational beliefs and content knowledge in teacher education (e.g., professional development) and in teacher quality (Enochs, Smith, & Huinker, 2000; Haney & Lumpe, 1995; Hill, Umland, Litke, & Kapitula, 2012; Pajares, 1992; Pintrich, 1990; Wilkins, 2008). Research has found a significant association between teachers’ beliefs and knowledge and teacher effectiveness. For example, previous findings indicate that teachers’ domain-specific knowledge for teaching have strong connections to their knowledge development, decision-making, planning, and instructional practices (Hill et al., 2012), which in turn, affects student outcomes (Rice, 2003). In
addition, previous research has found that teachers’ beliefs about their ability to successfully perform teaching-related tasks (self-efficacy) influences the type of instructional strategies they adopt as well as their instructional effectiveness (Czerniak & Chiarello, 1990; Guskey, 1988; Stipek, Givvin, Salmon, & MacGyvers, 2001).

While a multitude of studies have examined numerous teacher attributes that influence student achievement such as years of teaching experience and educational background (see Rice [2003] for review), less is known about the effects of teachers’ educational and motivational beliefs and specialized content knowledge—knowledge needed to “teach” within a specific discipline—on students’ academic achievement. Moreover, studies examining the effects of teachers’ both educational beliefs and specialized content knowledge “collectively” on students’ mathematics achievement are lacking (Hill, Charalambous, & Chin, 2018). Understanding the extent to which mathematics teaching beliefs and mathematics knowledge for teaching (MKT) play a role in students’ mathematics achievement may provide some practical implications for an urban school district related to the recruitment and retention of effective teachers. First, if certain teacher beliefs are found to have a significant role in students’ success, recruitment efforts should assess these beliefs during the teacher hiring process. Second, it may help inform professional development programs in providing strategic interventions that promote adaptive educational beliefs about mathematics teaching.

In addition to the importance of understanding, the link between teacher factors to student achievement is the importance of highlighting the disparity between qualified teachers and certain subpopulations of students (Hill et al., 2018). For example, although economically disadvantaged students are the most in need of quality teachers and quality instruction, poorer schools and school districts cannot afford to hire or keep highly-qualified teachers with their budget constraints due to the state accountability system that is based on high-stakes testing (Roza, Hill, Sclafani, & Speakman, 2004). It is well documented that economically disadvantaged students tend to not perform as well on achievement tests compared to their more affluent peers (e.g., Aikens & Barbarin 2008). However, recent Institute of Education Sciences-funded studies assessing teacher quality have consistently found that low-income students, unfortunately, receive less effective instruction on average compared to their higher income peers (National Mathematics Advisory Panel, 2008). Further support for this finding can be found in observational studies of mathematics teachers in high poverty urban schools that suggest that these teachers tend to not enact instructional approaches that are consistent with mathematics reform standards set by the National Council of Teachers of Mathematics (NCTM), which emphasize deep and conceptual learning of mathematics (Berry, Bol, & McKinney, 2009; NCTM, 2000). Instead, teachers in urban districts with a high percentage of economically disadvantaged students are more likely to ascribe to more traditional teaching practices, which are more formulaic and routine with little or no emphasis on conceptual understanding and connection of big ideas with one another, to other subjects and to the real world (Haberman, 1991, 2005). In order for teachers to adopt instructional practices that produce effective instruction and are aligned with high mathematics education standards, several researchers have contended that teachers need to possess beliefs that are aligned with the research on effective teaching of mathematics and need to have strong foundation in the subject area they teach (Borko & Mayfield, 1995; Borko & Putnam 1995; Haney, Czerniak, & Lumpe, 1996; Hill & Chin, 2018). Thus, it seems critical to examine the effects of teachers’ educational beliefs and content knowledge on students’ academic performance among mathematics teachers, especially
among those who work in large urban school districts with a high percentage of low-income students.

**Teacher Level Factors**

By focusing on *teacher qualifications* and *teacher characteristics* (Goe, 2007), this study addresses the two different traditions of research on teacher effectiveness (e.g., product function—Hanushek, 1986; between-teacher analyses—Rowan, Correnti, & Miller, 2002; for a complete review see Nye, Konstantopoulos, & Hedges, 2004). The first method of accessing teacher quality is via *teacher qualifications*, which include teachers’ degrees, coursework, and grades in higher education as well as teacher preparation routes, certification types, years of experience, and continuing education such as internships, induction, coaching support, and professional development (Barnett, 2003; Darling-Hammond, 2000; Early et al., 2006; Goe, 2007; Goe, Biggers, & Croft, 2012; Goe & Stickler 2008; Ingersoll, 2007; National Council on Teacher Quality [NCTQ], 2004; Rice, 2003, 2010; Wayne & Youngs, 2003; Zuzovsky, 2009). The second method of accessing teacher quality is via *teacher characteristics*, which encompass soft attributes such as subjective judgements, organization skills, critical thinking skills, and attitudes and beliefs (e.g., self-efficacy, epistemic beliefs, and beliefs about teaching and learning; Blanton, Sindelar, & Correa, 2006; NCTQ, 2004; Pajares, 1992).

In an extensive meta-analysis of more than 60,000 research papers about the impact of hundreds of interventions on student learning internationally, Hattie, Masters, and Birch (2016) found that teachers, and in particular teaching expertise, were the strongest predictors of student learning after controlling for student-level factors when compared to other environmental factors, including the home and school environment, principals, and peers. Additional research indicates that teachers’ educational background in a teaching discipline and other teacher attributes have significant associations to student-related outcomes (Goddard, Hoy, & Woolfolk-Hoy, 2000; Rice, 2003; Wayne & Youngs, 2003). For example, Lubienski, Lubienski, and Crane (2008) found that having been taught by certified teachers had a positive effect on student achievement-related outcomes (see also Darling-Hammond, 2000; Hanushek, Kain, O’Brien, & Rivkin, 2005).

Research also indicates that teachers’ domain-specific knowledge for teaching and their educational beliefs about teaching have strong connections to their knowledge development, decision-making, planning, and instructional practices (Hill at el., 2012; Philipp, 2007), which in turn, affects student outcomes (Hill et al., 2018; Rice, 2003). Moreover, teacher quality has been found to be more positively influential on students’ math achievement for underrepresented racial/ethnic student groups than for their non-minority counterparts (Aaronson, Barrow, & Sander, 2007; Heck, 2007). In sum, because factors deemed to be associated with highly qualified teachers strongly relate to student outcomes (e.g., Hansen, 2014; Museus, Palmer, Davis, & Maramba, 2011), one of the major goals of this study is to investigate specific, minimally explored (as collective), teacher-related factors at the K-12 level that may contribute to students’ mathematics achievement.

**Beliefs: Self-Efficacy and Epistemology**

The educational beliefs that may be relevant to mathematics teaching effectiveness are teachers’ beliefs about their ability to effectively perform mathematics teaching-related tasks
(self-efficacy; Enochs et al., 2000) and teachers’ beliefs about the nature of mathematics knowledge (epistemic beliefs; Hofer & Pintrich, 1997). The subsequent sections will first provide further descriptions of teachers’ educational beliefs. Second, rationale for why these beliefs may relate to student achievement will be discussed.

A central psychological mechanism within social-cognitive theory (SCT) is a person’s self-efficacy, which is defined as “a judgment of one’s capability to accomplish a certain level of performance” (Bandura, 1986, p. 391). According to SCT, individuals are neither solely motivated by internal influences nor regulated by environmental factors. Instead, social cognitive theorists posit that environmental events, personal factors (e.g., psychological mechanisms) and overt behavior all interact and influence each other in a reciprocal manner (Bandura, 1986). The key construct emerging from this interaction is the perceived-efficacy within a given domain, which has been found to predict performance within that domain beyond observed ability (Crombie et al., 2005; Simpkins, Davis-Kean, & Eccles, 2006).

Regarding the mathematics teaching in particular, teachers’ self-efficacy may be defined as the degree to which teachers believe they are self-efficacious to successfully perform instructional activities in their mathematics classes (Tschannen-Moran & Hoy, 2001) and they believe in their capabilities to improve the learning of their students (Hill et al., 2018). Teachers’ self-efficacy beliefs about mathematics teaching may play a role in their students’ achievement given that previous studies have found that mathematics teachers who are less self-efficacious are more likely to ascribe to traditional mathematics classroom practices compared to their more self-efficacious peers (Guskey, 1988; Stipek et al., 2001), which in turn, may have implications for student learning. Furthermore, teachers’ self-confidence for teaching mathematics has been shown to influence students’ own self-efficacy for learning mathematics (Stipek et al., 2001)—which is associated with mathematics performance (Simpkins et al., 2006). In fact, a direct positive association between teachers’ self-efficacy beliefs and growth in student achievement has been found in previous research (Anderson, Greene, & Loewen, 1988; Ross, 1992).

Another important type of belief that influence teachers’ instructional practices relates to teachers’ mindset about what constitutes disciplinary knowledge (Hill et al., 2018). This type of belief is called epistemic beliefs—beliefs about a particular subject area—say, mathematics. This includes beliefs about the development and production of mathematical knowledge, the essence of mathematical knowledge, and how one comes to know and justify mathematical knowledge. Educational psychology research has conceptualized and measured epistemic beliefs as residing across two ends of a spectrum. Specifically, epistemic beliefs lie on a spectrum from non-availing—believing that knowledge is fixed, simple, certain, objective, and comes from a person of authority—to availing—seeing knowledge as complex, evolving, uncertain, and relies on one’s own construction of knowledge (Muis, 2004). Availing epistemic beliefs have been found to be associated with positive academic achievement and motivation (Muis, 2004). Within the teaching domain, epistemic beliefs have been shown to influence instructional approaches, and in turn, students’ own epistemic beliefs and achievement (Hofer, 2001; Muis, 2004; Muis & Duffy, 2013).

Mathematical Knowledge for Teaching

Contrary to popular beliefs regarding knowledge needed for teaching mathematics, research has revealed that measuring teachers’ knowledge using proxy variables, such as courses taken, degrees attained, or results of basic skills tests are not sufficient measures for determining
what matters most in helping students learn (Hill, Rowan, & Ball, 2005). To remedy this situation, Hill, Ball, and Schilling (2008) found a more direct measurement of teachers’ subject-matter knowledge and subject-specific teaching behaviors and later investigated its impact on student achievement. Hill et al. (2008) define mathematical knowledge for teaching (MKT) as “the mathematical knowledge that teachers use in classrooms to produce instruction and student growth” (p. 374). It should be noted that this is different than the pure mathematical knowledge (subject matter-knowledge) mathematicians or other professionals such as engineers use to perform their jobs. MKT possesses a wider scope than the traditional views that most teachers already have because it combines the knowledge of content with the ideas, knowledge, and conceptual perceptions of students as well. Moreover, in addition to subject-matter knowledge, MKT incorporates pedagogical content knowledge (PCK). In mathematics, PCK requires knowledge of content oriented towards both teachers and students and a comprehensive curriculum (Hill et al., 2008).

Specifically speaking, the MKT model details four components. The first two are subdomains of “pure” content, or subject-matter knowledge (Ball, Thames, & Phelps, 2008). The first, common content knowledge (CCK), is defined as general knowledge of mathematics that most educated people including teachers acquire. The second one is specialized content knowledge (SCK), which is mathematical knowledge that is unique to, and essential for, teaching mathematics. The last two components are subdomains of pedagogical content knowledge (PCK)—knowledge that combines content knowledge with student knowledge and knowledge that allows for the combination of content knowledge with teaching knowledge. In developing their Learning Mathematics for Teaching (LMT) instruments, Hill, Schilling, and Ball (2004) have made progress in using test items designed to identify specific knowledge and reasoning that align with the MKT model. Test items include generating representations, interpreting student work, and analyzing student mistakes (Swarz, Hart, Smith, Smith, & Tolar, 2007). These measures have been found to be valid and reliable (Hill et al., 2004).

The MKT framework is currently the most promising theory addressing the enduring question of what kind of knowledge is needed to teach mathematics effectively (Morris, Hiebert, & Spitzer, 2009), and has also laid the groundwork for studying the effects of mathematical knowledge for teaching on student learning and achievement (Hill et al., 2018). Notably, recent studies at the elementary school level have found a significant positive association between MKT and student performance (Hill et al., 2005), and mathematical quality of instruction (Hill et al., 2008).

Adding to these findings, Baumert et al. (2010) found that teachers’ domain-specific instructional knowledge seemed to be of key significance for student progress in mathematics. When studying the effects of content knowledge (CK) and PCK on student progress, researchers found that the relationship between PCK and mathematics achievement was linear. CK was less predictive of student progress than was PCK, however. These findings confirmed that PCK had greater predictive power for student progress than CK only and is pivotal for the quality of instruction.

After extensive review of the literature on teachers’ mathematical knowledge, the National Mathematics Advisory Panel’s (2008) conclusions about the relationship between mathematical knowledge for teaching and student achievement suggest that despite some mixed results, teachers’ actual content knowledge in mathematics overall is positively related to student achievements. However, evidence supporting the impact of teachers’ mathematical knowledge for teaching on students’ mathematics achievement is needed, especially at the elementary and
middle school level (National Mathematics Advisory Panel, 2008). In addition, rather than examining the impact of teachers’ mathematical knowledge on students’ achievement in isolation, more research is needed with more key teacher traits along with their mathematical knowledge (Hill et al., 2018).

Professional Background

Previous research suggests that beginner teachers lack the content and pedagogical knowledge; class-time and classroom management; an understanding of how their students learn; and their students’ degrees of success. (Harris & Sass, 2011; Hill, 2010; see Palmer, Stough, Burdenski, and Gonzales [2005] for review). Other studies found qualitative differences in teachers’ habits. For example, compared to novice teachers, experienced teachers tend to respond to student performance cues with more instructional strategies. They also establish more complex links between student performance cues and instructional responses and apply a wider range of instructional goals for classroom decisions (Fogarty, Wang, & Creek, 1983; Strahan, 1989). The evidence towards experienced teachers’ greater performance, instructional knowledge, and numerous mastery experiences (Tschannen-Moran & Hoy, 2007) justifies deeper explorations into the effects of mathematics teaching experience on student outcomes. According to the National Science Board (2018), the educational background of U.S. teachers varies in general and by grade level, in particular. Evidence suggests that teachers well-versed in the subject matter they teach are likely to be more effective (see Rice [2003]). Moreover, having a degree in the discipline area taught and type of teacher preparation program completed may contribute critically to teachers’ success and may have, in turn, an impact on student-related outcomes (e.g., Barry, 2010; Goe, 2007).

Student Level Variables

Research has documented several demographic factors that may influence students’ academic outcomes at varying levels (e.g., gender and ethnicity; Eccles, 2005; Howard et al., 2011; Riegle-Crumb, Farkas, & Muller, 2006; Seymour & Hewitt, 1997). The most common demographic factors researchers include in education research are gender, racial/ethnic background, and socioeconomic background. In addition, students’ mathematics achievement in earlier grades is another important factor that affects mathematics achievement in higher grades (Siegler et al., 2012). This section provides details on these student-level factors regarding how they relate to academic achievement.

Ethnicity

Long ago, Coleman et al. (1966) reported that apart from Asian Americans, other minority students scored significantly lower on tests than the average white pupil in first grade and this gap significantly widened when examined again at 12th grade. More than half a century later, these disparities still hold today, especially among students from racially and socio-economically different backgrounds. In fact, the latest National Assessment of Educational Progress (NAEP) mathematics report card of 2017 reports the same issues—a significant White
versus Minority (African American and Hispanic students) score difference and an increase in this difference from fourth-grade to eighth-grade. In the past 50 years, researchers have tried to bring attention, present possible explanations, and provide suggestions to eliminate these achievement gaps.

Upon analysis of PISA 2003 data, Cheema and Galluzzo (2013) confirmed the existence of the racial achievement gap, and found that White students outscore Hispanic students who outscore Black students, in terms of mean mathematics achievement. Prior to that, Bali and Alvarez (2004) and Madrid (2011) highlighted the Minority-White achievement gaps in California Public Schools. Apart from African American and Hispanic students, Pang, Han, and Pang (2011) have also identified the large differences in the achievement gaps between White American students and various Asian/Pacific Islander students. Given the significant achievement gaps among the different ethnic/racial subpopulations of students, it is important to control for ethnic/racial background in studies involving students’ achievement outcomes.

Gender

Gender disparities in mathematics achievement, are usually reported to be minor compared to the disparities due to racial and socio-economic background (Corbett, Hill, & Rose, 2008; Ellison & Swanson, 2010). The general consensus upon analysis of the data including Early Childhood Longitudinal (ECLS-K; Fryer & Levitt, 2010; Lubienski, Robinson, Crane, & Ganley, 2013, Robinson & Lubienski, 2011), Program for International Student Assessment (PISA; Cheema & Galluzzo 2013; Else-Quest, Hyde, & Linn, 2010) and NAEP (Corbett et al., 2008; Lee, /Grigg, & Deon, 2007; McGraw, Lubienski, &Strutchens, 2006) is that gender disparities in mathematics achievement are small but significant. This is consistent with Friedman (1989) who concluded upon meta-analysis of multiple studies that the gender differences in achievement are small but slightly biased in favor of boys. However, Ellison and Swanson (2010) found that there is a large gender-related gap in favor of boys among students with high achievement levels. Given the significant gender gaps in student achievement, it is important to control for gender in studies involving students’ achievement outcomes.

Socio-Economic Status

To measure achievement gaps due to socioeconomic status, researchers usually use family income or free/reduced lunch status. Reardon (2011) highlighted how the income-achievement gap is nearly twice as large as the black-white achievement gap. Additionally, Reardon (2011) has shown that the gap has widened as much as 40% over the span of twenty-five years. Duncan and Magnuson (2011) further added that this gap is large when children enter school for the first time and is quite steady until they graduate. Upon meta-analysis of relevant literature concerning the effects of socioeconomic status (SES) on mathematics achievement, Sirin (2005) found a moderate association between SES at the student-level and achievement but a large positive correlation between SES at the school level and student achievement. Perry and McConney (2013) validated this finding. Given the significant achievement gaps due to the SES level of students, it is important to control for SES in studies involving students' achievement outcomes.
Prior Mathematics Achievement

Another important factor that affects mathematics achievement in higher grades is students’ mathematics achievement in earlier grades. This has been studied across multiple grades. Various studies have come to the conclusion that the level of mathematical knowledge of students in preschool and kindergarten is associated with their later mathematics achievement in higher grades (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Bodovski & Farkas, 2007; Claessens, Duncan, & Engel, 2009; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Siegler et al., 2012; Watts, Duncan, Siegler, & Davis-Kean, 2014). Moreover, Hemmings and Kay (2010) showed that student achievement on seventh grade mathematics tests was positively correlated with student achievement on tenth grade mathematics tests. Crosnoe et al. (2010), Georges (2009), and Moller et al. (2013) explained that the differences in prior achievement among students is deeply rooted in their ethnicity and socioeconomic status. Given the significant predictive value of prior achievement in student achievement outcomes, it is important to control for prior math achievement in studies involving students' achievement outcomes (Hill et al., 2018).

Purpose and Research Questions

Based on these significant teacher and student related factors relating to student achievement found and recommended in the literature, the purpose of this study is to investigate the impact of teacher related factors "collectively" on student achievement controlling for important students background and prior achievement factors. The following research questions guided this study:

1. To what extent do high-poverty urban school district students’ demographic characteristics and prior mathematics achievement relate to their mathematics achievement?
2. To what extent do differences among high-poverty urban school district students’ mathematics achievement relate to teacher-level characteristics (e.g., teachers’ beliefs, mathematical knowledge for teaching, teaching experience, and math degree)?
3. Do the effects of students’ demographic characteristics and prior mathematics achievement on their mathematics achievement vary across teachers?

The conceptual model displayed in Figure 1 provides a graphic representation of our research questions. The arrow labeled “A” displays the direct link between the student-level variables and mathematics achievement (research question 1). The main effects of the teacher-level variables on mathematics achievement are depicted by arrow “B” (research question 2). Arrow “C” represents the effects of teacher-level variables on the relation between student-level variables and mathematics achievement (research question 3).

Method

Participants

Participants were recruited from a pool of 80 K-12 mathematics teachers who attended Rice University School Mathematics Project's summer professional development program either voluntarily or on the basis of their campus administrators’ nomination. Due to the choice in the
students’ mathematics achievement measure (Stanford 10, a norm-referenced with a nationally representative student sample, which is implemented only in a particular school district in the region in elementary and middle school grades only), we had to narrow the study participants to elementary and middle school teachers and within a particular school district. After this elimination, 45 teachers were qualified; however, the school district was not able to link 11 teachers to their student data. The final sample of teachers included in the analysis, therefore, included 34 elementary and middle school mathematics teachers from Houston Independent School District, a high poverty urban school district in Texas, U.S. The final student sample included 2,078 K-8 students. Both teachers’ and their students’ descriptive information is given in Table 1.

![Figure 1. Conceptual model for this study.](image)

**Measures**

We surveyed teachers at the end of the professional development program. The survey comprised of several sections including demographic information, mathematics background and a battery of scales measuring three constructs: teachers’ self-efficacy, internal locus of control, and non-availing epistemic beliefs. Regarding the professional background variables, mathematics teaching experience was dichotomized as experienced (dummy-coded as 1 for more than 5 years of teaching experience) and inexperienced (dummy-coded as 0 for 5 years or less teaching experience; Wolters & Daugherty, 2007). Teachers who had undergraduate or graduate degrees in mathematics were also dummy-coded (1 versus 0= no mathematics degrees). The scales consisted of 5-point Likert scale items with responses ranging from strongly disagree (1) to strongly agree (5). Higher scores for the first two constructs showed beliefs that are more positive whereas a lower score on the last construct was associated with beliefs that are more positive since its items imply a “non-availing” epistemic belief. We also measured teachers’ mathematics knowledge for teaching (MKT). More details about these scales are given below.
Self-Efficacy

The self-efficacy scale consisted of 13 items to measure the extent to which teachers believed they could successfully perform teaching-related tasks in mathematics instruction. The items were adapted from Mathematics Teaching Efficacy Beliefs Instrument (Enochs et al., 2000). The reliability analysis of this scale produced a Cronbach’s alpha of .85. An example of an item is as follows: “I understand mathematics concepts well enough to be effective in teaching mathematics.” Higher scores in items show higher presence of self-efficacy construct (more positive beliefs).

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Teachers (N=34)</th>
<th>Students (N=2,078)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>14.7</td>
<td>50.8</td>
</tr>
<tr>
<td>Female</td>
<td>85.3</td>
<td>49.2</td>
</tr>
<tr>
<td>Ethnicity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>White</td>
<td>17.6</td>
<td>8.5</td>
</tr>
<tr>
<td>Black</td>
<td>44.1</td>
<td>26.5</td>
</tr>
<tr>
<td>Hispanic</td>
<td>29.4</td>
<td>58.4</td>
</tr>
<tr>
<td>Asian</td>
<td>8.8</td>
<td>6.3</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>School Level</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elementary School</td>
<td>55.9</td>
<td>33.1</td>
</tr>
<tr>
<td>Middle School</td>
<td>44.1</td>
<td>66.9</td>
</tr>
<tr>
<td>Other Student Variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Free/Reduced Lunch</td>
<td>--</td>
<td>75.9</td>
</tr>
<tr>
<td>Minority (Black &amp; Hispanic)</td>
<td>--</td>
<td>84.9</td>
</tr>
<tr>
<td>Teachers’ Professional Background</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experienced (&gt;5 years of math teaching)</td>
<td>23.5</td>
<td>--</td>
</tr>
<tr>
<td>Mathematics Degree (B.S. or M.S)</td>
<td>5.9</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 1. Percentage of teachers and students by their demographic information.

Non-Availing Epistemic Beliefs

The epistemic beliefs scale consisted of 7 items to measure teachers’ non-availing beliefs about mathematics (i.e., where knowledge comes from, what the essence of knowledge is, and how one comes to know and justify beliefs). The items were adapted from the Problem-Solving Project Questionnaire (Schoenfeld, 1989). The reliability analysis of this scale produced a Cronbach’s alpha of .72. An example of an item is as follows: “To solve most mathematics problems you have to be taught the correct procedure.” Lower scores in items show higher presence of epistemic beliefs construct (more positive beliefs) since items imply a “non-availing” epistemic belief.
Mathematical Knowledge for Teaching

Teachers’ mathematical knowledge for teaching was measured by Learning Mathematics for Teaching (LMT) instruments, which are validated and reliable (Hill et al., 2004), at the completion of a 3-week summer professional development program. Two LMT instruments were used to measure teachers’ MKT. K-6 teachers took El NCOP 2008 (Form B) while grades 7-9 teachers took MS PFA 2007 (Form B). El NCOP instrument had 29 multiple-choice items covering numbers concepts and operations topics. MS PFA instrument had 33 multiple-choice items covering patterns, functions, and algebra topics. The total IRT scaled z-scores on the instruments were calculated. Reliability analyses produced Cronbach’s alphas of .86 and .85 for El CNOP and MS PFA, respectively.

Students’ Mathematics Achievement

Stanford Achievement Test Series (Stanford 10) is a norm-referenced measure, to evaluate the progress of student achievement and provide means of determining the relative standing of students’ academic performance when compared to the performance of students from a nationally representative sample. Mathematics portion of the Stanford 10 is used to measure student achievement in mathematics.

Analysis—Hierarchical Linear Modeling

The multilevel nature of the research questions presented in this study will be addressed by conducting hierarchical linear modeling analysis, which will evaluate both student differences and teacher effects on mathematics achievement. More specifically, addressing the main research questions will involve estimating the effects of student-level personal characteristics as well as teachers’ characteristics and educational beliefs on students’ mathematics achievement. Therefore, we will run a two-level analysis by conducting a three-step process to estimate effects on student achievement: 1) unconditional model, 2) within-teacher model, and 3) between-teacher model.

For step 1, an unconditional model will be used to estimate the amount of variance in achievement that can be explained at the individual level and at the teacher level. In step 2, a within-teacher model will be used to examine the relation between student-level factors and their respective mathematics performance on the Stanford 10. In addition, the random-effects of the student-level predictors will be estimated to determine whether there is significant variance associated with the slopes. In step 3 (between-teacher model), both student- and teacher-level variables will be included to predict students’ mathematics achievement.
Results

Unconditional Model

The unconditional statistical model is as follows:

Level 1: \( SMNCP_{ij} = \beta_{0j} + r_{ij} \)

Level 2: \( \beta_{0j} = \gamma_{00} + u_{0j} \)

Also known as empty model, the fully unconditional model is basically a one-way random effects analysis of variance (Raundenbush & Byrk, 2002), which assures that there is systematic within- and between-group variance to investigate. As Table 2 displays (see Model 1), results indicated that average student mathematics achievement by teacher was statistically different from zero (\( \gamma_{00} = 55.61, p < .001 \)).

For variance in achievement means across teachers, there were considerable variation (\( \tau_{00} = 115.94, p = .000 \)). Intra-class correlation coefficient (ICC = \( \frac{\tau_{00}}{\tau_{00}+\sigma^2} = \frac{115.94}{115.94+309.27} = .27 \)) indicated that 27% of the variability in mathematics achievement was between teachers (remaining 63% was within teacher). Level 1 (student) and level 2 (teacher) predictors were added to reduce variance within-teachers and between-teachers, respectively, as shown below.

Within-Teacher Model

The statistical model is as follows:

Level 1: \( SMNCP_{ij} = \beta_{0j} + \beta_{1j}(PFRLCH)_{ij} + \beta_{2j}(LFEMALE)_{ij} + \beta_{3j}(LMINORITY)_{ij} + \beta_{4j}(SMNCPRE)_{ij} + r_{ij} \)

Level 2: \( \beta_{0j} = \gamma_{00} + u_{0j} \)
\( \beta_{1j} = \gamma_{10} \)
\( \beta_{2j} = \gamma_{20} \)
\( \beta_{3j} = \gamma_{30} \)
\( \beta_{4j} = \gamma_{40} + u_{4j} \)

In this model, level 1 (student level) predictors were added to the unconditional model (see Model 2 in Table 1). Despite our interest in estimating the slopes for reduced lunch and minority status as random effects, the results showed that their random effects were not significant. Therefore, only the slope for the prior mathematics achievement was retained as the random slope. There were no level 2 predictors included in this step. After including gender, free and reduced lunch status, minority status, and prior mathematics achievement, within-teacher variability was reduced by 66.0% (\( \frac{\sigma^2_{one-way ANOVA}-\sigma^2_{within-teacher}}{\sigma^2_{one-way ANOVA}} = \frac{309.27-105.16}{309.27} = .660 \)). All student level variables were found to be significant factors in predicting students’ mathematics achievement including gender, free and reduced lunch status, minority status, and prior mathematics achievement. More specifically, female students (\( \beta = 0.94, p < .05 \)) and students with a higher prior math achievement (\( \beta = 0.78, p < .001 \)) tend to perform better compared to their respective counterparts. On the other hand, minority students (\( \beta = -3.42, p < .001 \)) and students eligible for reduced or free lunch (\( \beta = -1.90, p < .01 \)) tend to perform poorer than their respective counterparts do.
Between-Teacher Model

The statistical model is as follows:

Level 1: 
\[
SMNCPOST_{ij} = \beta_0 + \beta_1(PFRLCH)_{ij} + \beta_2(LFEMALE_{ij}) + \beta_3(LMINORITY_{ij}) + \beta_4(SMNCPRE_{ij}) + r_{ij}
\]

Level-2: 
\[
\beta_0 = \gamma_{00} + \gamma_{01}(MATHMAJOR)_{j} + \gamma_{02}(TEACHEXP)_{j} + \gamma_{03}(TEACHSE)_{j} + \gamma_{04}(TEACHEB)_{j} + \gamma_{05}(LMT)_{j} + \gamma_{06}(TEACHEXP \times TEACHSE)_{j} + (TEACHSE \times TEACHEB)_{j} + \gamma_{10}(TEACHEB \times LMT)_{j} + u_{0j}
\]
\[
\beta_1 = \gamma_{10}
\]
\[
\beta_2 = \gamma_{20}
\]
\[
\beta_3 = \gamma_{30}
\]
\[
\beta_4 = \gamma_{40} + \gamma_{41}(MATHMAJOR)_{j} + \gamma_{42}(TEACHEXP)_{j} + \gamma_{43}(TEACHSE)_{j}
\]

<table>
<thead>
<tr>
<th></th>
<th>Model 1 (unconditional)</th>
<th>Model 2 (within teacher)</th>
<th>Model 3 (between teacher)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>(SE)</td>
<td>(B)</td>
<td>(SE)</td>
</tr>
<tr>
<td>(Intercept)</td>
<td>55.61***</td>
<td>1.91</td>
<td>59.55***</td>
</tr>
<tr>
<td>Student Level</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender</td>
<td>0.94*</td>
<td>0.46</td>
<td>0.86*</td>
</tr>
<tr>
<td>Minority Status</td>
<td>-3.42***</td>
<td>0.81</td>
<td>-3.50***</td>
</tr>
<tr>
<td>Free/Reduced Lunch</td>
<td>-1.90**</td>
<td>0.65</td>
<td>-1.84**</td>
</tr>
<tr>
<td>Prior Math Achievement</td>
<td>0.78***</td>
<td>0.03</td>
<td>0.81***</td>
</tr>
<tr>
<td>Teacher Level</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Math Major</td>
<td>16.42***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experienced</td>
<td>1.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self Efficacy</td>
<td>-0.44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Epistemic Beliefs</td>
<td>5.23**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MKT</td>
<td>6.44**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interactions (Teacher Level)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experienced X Self Efficacy</td>
<td>1.48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self Efficacy X Epistemic Beliefs</td>
<td>3.6**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Epistemic Beliefs X MKT</td>
<td>3.84*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prior Math Achievement X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Math Major</td>
<td>-0.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experienced</td>
<td>-0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-Efficacy</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Epistemic Beliefs</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MKT</td>
<td>0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma^2)</td>
<td>309.269</td>
<td>105.161</td>
<td>104.548</td>
</tr>
<tr>
<td>(t_{00, Teacher ID})</td>
<td>115.944</td>
<td>107.234</td>
<td>33.699</td>
</tr>
<tr>
<td>(N_{Teacher ID})</td>
<td>34</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>ICC_{Teacher ID}</td>
<td>0.273</td>
<td>0.660</td>
<td>0.69</td>
</tr>
<tr>
<td>Observations</td>
<td>2230</td>
<td>2078</td>
<td>2078</td>
</tr>
</tbody>
</table>

In this model, level 2 (teacher level) predictors were added to the between-teacher model (see Model 3 in Table 1). Teacher level inter-interactions for variables that were found to
significantly correlate with each other were also added to the model. Prior math achievement was the only factor that was treated as the random effects while the other student-level factors were taken as fixed effects since their slopes as random effects were not significant in the within-teacher model. All student level variables remain significant in the same direction and with similar magnitude as in the within-teacher model. Among teacher level effects, math major ($\beta = 16.42, p < .01$), non-availing epistemic beliefs ($\beta = 5.23, p < .05$), and MKT ($\beta = 6.44, p < .01$) were found to be significant and to have positive predictive value for students’ mathematics achievement. This suggests that teachers possessing a math major, non-availing epistemic beliefs, and higher levels of mathematical knowledge for teaching have a positive impact on students’ mathematics achievement. Positive predictive value of non-availing epistemic beliefs was counterintuitive as the expectation was and the prior research indicates that availing epistemic beliefs are positively associated with higher teacher self-efficacy and higher student achievement (Muis, 2004). Surprisingly, their experience and self-efficacy were not found to be significant predictors of students’ performance. However, the interaction term between self-efficacy and epistemic beliefs was positive and significant ($\beta = 3.6, p < .01$) suggesting that among the teachers holding similar levels of non-availing epistemic beliefs, higher self-efficacy has greater impact on student performance. Another significant interaction term was epistemic beliefs and MKT ($\beta = 3.84, p < .05$) suggesting that among the teachers holding similar levels of non-availing epistemic beliefs, higher MKT has greater impact on student performance. None of the interactions between teacher effects and prior mathematics achievement were found to be significant.

**Discussion and Conclusions**

This study provides evidence regarding the importance of teacher-related factors in students’ mathematics achievement. Rather than focusing on a single teacher factor and exploring its connection to student achievement, this study included several key teacher factors identified in previous theories and research. More specifically, this study investigated the extent to which K–8 math teachers’ professional background, adaptive educational beliefs, and MKT would have an effect on their students’ mathematics achievement. A collective investigation of several teacher factors contributes to the body of knowledge about the relation of teacher characteristics to student achievement because the extant research in this area typically focuses on only a single isolated factor or fewer than needed characteristics (Hill et al., 2018).

In line with prior theoretical expectations based on the previous research, several teacher factors predicted the students’ achievement outcomes. Among the teacher-level factors, whether teachers held a bachelor’s degree in mathematics had a very strong and direct effect on students’ math achievement. This finding becomes even more significant considering only a small portion of the teachers had a degree in mathematics (less than 10%). The second strongest predictive value was produced by teachers’ MKT. Non-availing epistemic beliefs about mathematics—i.e., the belief that knowledge is fixed, simple, certain, objective, and comes from a person of authority in mathematics rather than seeing the discipline of mathematics as evolving, complex, and uncertain at times—were found to be positively associated with higher student mathematics achievement. This may be due to the way the standardized mathematics assessments were constructed (more factual than cognitively rich test questions; Popham, 2001) and teachers’ reflexive strategies (i.e., test preparation) against high-stakes testing that may result in less
availing views of mathematics and less ambitious teaching of mathematics (Blazar & Pollard, 2017).

That said, proper attention must be paid towards the development of teachers’ beliefs about mathematics and teaching of mathematics and knowledge in mathematics because the self-efficacy and epistemic beliefs are associated with MKT (Corkin, Ekmekci, & Papakonstantinou, 2015; Hill et al., 2005). However, this relation is not well-established (Hill et al., 2018); and further research is needed to understand whether the development of adaptive forms of these beliefs is necessary for teachers to increase their MKT through PD (Stevens & Wenner, 1996).

The results of this study support policy initiatives designed to improve students’ success in mathematics by improving teachers’ mathematical knowledge as well as attracting teachers who have strong mathematics backgrounds (by virtue of extensive coursework or a relevant degree). Because of the degree to which these factors can influence students’ success in math, school districts need to include them in their teacher hiring processes and use them to determine the type of support systems to needed for current teachers (Goe, 2007). Some schools already do this by screening the applications of potential teachers (Hill, Blazar, & Lynch, 2015; Hill et al., 2018). However, more should be done to improve students’ success in math by focusing on currently employed teachers. School districts should offer math teachers opportunities in PD programs centered on mathematical content and pedagogical knowledge. Furthermore, if these programs are aligned with a constructivist philosophy (Stevens, Aguirre-Munoz, Harris, Higgins, & Liu, 2013), then teachers already well-versed in math could attain proficiency for effective teaching of math. School districts could encourage teachers without an adequate math background to complete additional coursework in mathematics instead (Hill et al., 2015; Hill et al., 2018).

The implications of PD programs and continuing education are incredibly significant for teachers in high-poverty urban schools and districts. Admittedly, the direction of these programs in these environments is not as clear-cut, not to mention comparative difficult to implement due to budget restrictions. Still, urban districts can rethink their existing PD programs to incentivize growth in their teachers. Urban school districts need quality teachers most; it is important to minimize their teachers’ limitations as much as possible to increase student achievement.

Limitations of This Study and Direction for Future Research

The biggest challenge in this study was to determine the most reliable measure for student achievement in mathematics. This led to a reduced sample size for teachers included in the study. Replication research of this sort is needed, especially with inclusion of more teachers’ key characteristics and with a larger and a more representative datasets. With a larger sample of teachers, analyses by the school level of teachers (i.e., elementary, middle) would be possible and provide more fine-grained evidence for the connection between teacher factors and student achievement outcomes at different school levels. Furthermore, additional measurable factors that may contribute to the quality of mathematics teachers’ instructions and that may be related to other dimensions of teacher quality (e.g., effort invested in non-instructional activities) should be considered for a more comprehensive investigation of the relation between teacher factors and student achievement outcomes.
References


Acknowledgement

This study is made possible by Houston Education Research Consortium’s (HERC) support that provided access to the HERC Database for Houston Independent School District and is based, in part, on a project partially funded by Teacher Quality Grants Program at the Texas Higher Education Coordinating Board under Grant #531. The Teacher Quality Grants Program is supported through federal funds under NCLB Title II, Part A.

We would also like to thank Rice University student research assistant Elizabeth White and Rice University graduate research assistant Miten Shah for their assistance in editing this paper.