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Probability Concepts of Secondary Pre-service Teachers in a Game Context

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Abstract: There is a rich literature on students' and teachers' intuitions and misconceptions about probability. However, less attention has been paid to the development of pre-service teachers' probabilistic thinking in teacher education. Based on this, the second author developed a lesson sequence for teaching probability. In particular, it demonstrates how a game context can be used to explore the relationship between experimental and theoretical probabilities in a collaborative learning setting. The lesson sequence integrates concepts and processes related to probability and is grounded in socio-cultural theory. We trialed the sequence with secondary pre-service teachers. This paper focuses on their understanding of the probability concepts embedded in the sequence. Video and audio data indicates that while teachers used a range of strategies and data displays to explain the ideas integrated in the lessons, some reverted to equiprobability bias. The findings also reveal that pre-service teachers can modify their thinking when engaged in rich teaching and learning tasks.

Introduction

There are different suggestions on how best to teach probability so that students leaving school may be able to interpret probabilities in a wide range of contexts. (Batanero, Chernoff, Engel, Lee, & Sánchez, 2016; Jones, Langrall, & Mooney, 2007; Kapadia, 2009). If students are to develop a meaningful understanding of probability, it is important to use effective pedagogical strategies to train teachers (Batanero, 2013; Koparan, 2019). In the area of probability, another intriguing recommendation for teaching is to use culturally diverse games to support and promote students' understanding of probability (Carlton & Mortlock, 2005; Greer & Mukhopadhyay, 2005; Naresh, Harper, Keiser, & Krumpe, 2014; Tarr, 2002). It is argued that a probability lesson embedded in a cultural context can enable students to reflect on the connections between probability and culture and as a result broaden students' perceptions of mathematics and statistics. Research in teacher education related to probability education is still limited and needs to be advanced (Groth, 2007; Leavy & Hourigan, 2014; Watson, 2006).

Different authors (Batanero et al., 2016; Batanero, 2013; Franklin, Kader, Mewborn, Moreno, Peck, Perry & Schaeffer, 2007) claim that many of the current teacher education programmes do not yet train teachers adequately to teach statistics and probability. Even though many pre-service secondary teachers have a major in mathematics, they usually study only theoretical statistics in their teacher training programmes. In other words, few mathematics teachers receive specific training in applied statistics, designing sample collections or experiments, or analysing data from real applications (Batanero, 2009). These teachers also need some training in the pedagogical knowledge related to statistics education, where general principles that are valid in mathematics cannot always be applied. Additionally, textbooks and curriculum documents developed for secondary teachers might not offer enough support (Batanero, 2013).

A number of researchers claim that pre-service teachers need to understand the probability they teach to their students (Batanero et al., 2016; Chick & Pierce, 2008). According to Batanero et al. (2016), one method is to have pre-service teachers play the role of a student and later analyse what they learnt. In this way, they will have a chance to go through a lesson as a student and at the same time look at it from the point a view of a teacher, leading to a better understanding about how the lesson will unfold later in the classroom.

Based on the literature, the second author developed a teaching sequence for teaching probability (Appendix 1). The sequence integrates the various interpretations of probability and is grounded in socio-cultural perspective (Vygotsky, 1978). The influence of socio-cultural context on a learner has been examined mostly from Vygotsky's (1978) frame of reference. The sociocultural environment incorporates use of a variety of tools such as language, sign and cultural tools (artefacts) to assist with reaching higher mental models (Vygotsky, 1978). Given the aim of the study was to explore pre-service teachers' views about the benefits of using a newly introduced probability teaching sequence (see Sharma, 2015), it was important to see how they suggest they could make use of the ideas that they could have possibly derived from the teaching sequence. The following broad research question guided the study: how do pre-service teachers understand the probability teaching sequence in small-group settings?

After presenting a literature review, a detailed description of the study's methods and participants is provided. The findings are presented and discussed next. Finally, limitations and implications for further research are examined.

Literature Review

While research into pre-service teachers' perceptions of probability and statistics generally suggest a positive attitude towards studying the subject, there are studies that confirm that pre-service mathematics teachers tend to see probability and statistics as difficult (Leavy, Hannigan, & Fitzmaurice, 2013; Hannigan, Gill, & Leavy, 2013; Estrada & Batanero, 2008; Batanero, Godino, & Roa, 2004). In particular reference to statistics education, for example, the Leavy et al., (2013) study, conducted amongst a small sample of Irish pre-service mathematics teachers noted that pre-service teachers saw statistics differently from mathematics. The pre-service teachers reported this perceived difference in terms of the uniqueness of statistical thinking and reasoning. For example, while there is usually 'one correct' answer in most mathematical situations, there was a lot of uncertainty associated with statistical scenarios. While such findings are seen as a challenge associated with probability and statistics education from pre-service teachers' perspectives, these reported 'uncertainties' could provide an important teaching and learning opportunity when viewed from a teacher educator's perspective (Batanero et al., 2004). In addition to this, the Leavy et al. (2013) findings also confirm that pre-service teachers tend to see statistics as something that is always embedded in contexts that make it interesting to study. Similar findings have been reported by Estrada and Batanero (2008) who suggest teaching probability and statistics using everyday application scenarios, both in personal and professional lives. When teachers are exposed to probability and statistics education that does that, they tend to have a more positive attitude towards probability and statistics (Estrada & Batanero, 2008).

Batanero et al. (2004) agree to the findings from the Leavy et al. (2013) study about the challenging nature of stochastic reasoning. They argue that the nature of probabilistic and statistical reasoning is different from that encountered in mainstream mathematics lessons. In addition, they argue that probabilistic and statistical reasoning is also different from logical reasoning. The authors speculate that this makes probability and statistics a difficult subject to teach. This is mainly because teachers should not only present different models about learning, but should also go deeper in asking questions such as what knowledge is important and what knowledge can be gathered from experimental data.

One of the ways to overcome the challenge noted by Batanero et al. (2004) is through the use of challenging yet interesting teaching scenarios, such as the use of games (Batanero et al., 2004; Koparan, 2019). This idea of active learning is not a new idea and has a long and solid theoretical support in education literature in general and in mathematics education literature in particular (Cobb, 2007). The first study reviewed here (Batanero et al., 2004) has games at the fore of teaching and learning probability that have undergone trials over the past two decades. One of the activities, called *winning the games* draws on probability teaching ideas such as dependent experiments and conditional probability. Batanero et al. (2004) report that while less than half of the pre-service teachers were able to select the winning strategy at the start of the game, there was a general positive change about the concepts involved noted towards the end of the activity. Batanero et al. (2004) conclude that training of teachers must involve exposing them to similar scenarios that help them analyse real time situations using data.

In another, more recent study, Koparan (2019) explored 40 pre-service teachers' engagements with learning probability using games. The author employed the Predict-Observe-Explain (POE) strategy (Joyce, 2006; White & Gunstone, 1992) in a series of game situations, one of which is the scenario that we used in the current study. The pre-service teachers were asked to play the *difference of the dice* game. Pre-service teachers' initial predictions showed that almost 50 percent of them had made an incorrect prediction about who will win the game. The pre-service teachers were later given an opportunity to explore the chances of winning through conducting more trials and drawing up computer simulations based on more data. A majority of the pre-service teacher participants were able to come up with simulations that showed that 'lower' differences (of 0, 1, or 2) were more likely to occur. When asked to explain their models, a few teachers explained them wrongly, with the major error being failure to consider the permutation of the dices in consideration (for example, a difference of one can be observed through 5, 4 as well as 4, 5). However, a majority of the pre-service teachers were able to change their predictions upon playing the games themselves, confirming that exposing pre-service teachers to game scenarios can provide the platform to make better probabilistic and statistical reasoning.

The literature examined in this review provides a broad-brush view of the challenges in teaching probabilistic and statistical reasoning. Based on some of these findings, we speculate that pre-service teachers may form negative attitudes towards probability and statistics if they are exposed to an over-mathematised way of teaching and learning probability and statistics. Pre-service teachers in particular are able to realise that statistics presents new challenges in the form of uncertainties, which are not usually common in other mathematical topics such as algebra. For instance, in algebra, students can check their answers by substituting them in the equation. In probability and statistics, such tricks are not so useful. However, the prevalence of challenges such as the uncertainty of answers can be turned into good teaching points for exploring these ideas. The review also presents us with evidence that challenging and interesting activities can be used to challenge and build upon teachers' conceptual understanding of probability and statistics. The current study, though similar in nature to the Koparan (2019) study hopes to add to our understanding of how pre-service teachers from two different teaching contexts engage with teaching probability using games.

Research Design

In conceptualising our study, we made use of design-based research theory (Cobb & McClain, 2004) and case study approach. Design research is a cyclic process with action and critical reflection occuring in turn (Cobb & McClain, 2004; Nilsson, 2013). There are mutual benefits for both participants and researchers when undertaking a design research partnership. In addition, the research plan can be flexible and adaptable to unexpected effects or constraints (Nilsson, 2013). Further, all participants are equal partners in the research process with no hierarchy existing between researchers and participants (Kieran, Krainer, & Shaughnessy, 2013). The study itself involved three stages: a preparation and design stage, an intervention stage, and a retrospective analysis stage. Both mathematics educators were involved in the whole research process. The role of researchers involved posing questions and observing the research as it unfolded with minimal interference.

Our study used a case study design (Yin, 2014). A case study is an empirical inquiry that examines an existing phenomenon (the "case") in depth and within its real-world context. A case study relies on multiple sources of evidence and can include both single or multiple-case studies. Multiple-case studies can be used to do a comparative study. Our study is an example of a comparative case study because the intervention was carried out with two seemingly similar cohorts of pre-service teachers from rather distinct backgrounds. One of the advantages of case studies, according to Yin (2014), is that they can penetrate situations in depth. In our research we capitalise on the comparative case study design to understand pre-service teachers' pedagogical perspectives and beliefs regarding the teaching sequence.

Intervention Design

The intervention was carried out in three major phases. The phases involved in the teaching sequence (see Appendix 1) resonate with Wild & Pfannkuch's (1999) statistical PPDAC cycle mnemonic (Problem, Plan, Data, Analysis, Conclusion) with slight modifications in a probabilistic context.

The first phase, called *posing a problem,* involved pre-service teacher participants reflecting on the probability game problem. After reflecting on this problem (see table 1), the pre-service teacher participants were asked to share their answers with the whole group. Next, the pre-service teacher participants played the game in pairs with 20 trials. This phase was again followed by a short whole-group discussion on *who is the winner*. The second phase of the intervention was titled *playing the game in pairs*. The next phase of the intervention was called *planning and exploring.* In this major phase, students worked in groups to conduct an experiment with larger numbers of throws of the dice and recorded data in a convenient way. The final phase also involved deriving conclusions from their findings, followed by an additional assessment task to check if the pre-service participants could transfer their learning to new experimental contexts. Table 1 below provides a summary of the intervention design.

Research phase	**Activities**	**Reflection and discussion**
Phase 1: Posing a problem	Esha and Sarah decide to play a die rolling game. They take turns to roll two fair dice and calculate the difference (bigger number minus smaller number) of the numbers shown. If the difference score is 0, 1, or 2, Esha wins, If the score is 3, 4, or 5, Sarah wins. Is this game fair?	Why do you think the game is fair? Or unfair? Explain your thinking.
Phase 2: Playing the game in pairs	In pairs, pre-service teacher participants play the game with 20 trials and record the data.	On the basis of your results, do you think the game is fair? Why, or why not? If you wanted to win this game, which player would you choose to be? Explain your answer. If you played the game 30 more times, would the results be the same as or different from the first game? If they would be different, how?
Phase 3: Planning and exploring	In groups, students brainstorm ideas about collecting and recording more data. Main activity: data is collected, recorded and analysed. After the main activity, students are given an additional task as an assessment. Students are asked to reflect on the probability teaching sequence	***Planning stage***: Why does Esha win more often than Sarah? How can we determine if the game is fair by collecting more data? How can we record our results? ***After main activity***: What are the chances of Esha winning? What are the chances of Sarah winning? Is this game fair? Why? Discuss how knowing the expected probabilities helps us to understand why the game is unfair. ***Assessment task***: Students to decide whether the following statement is true or false and write down reasons to support their decision: *Scoring a total of three with two fair dice is twice as likely as scoring a total of two.* ***Reflections:*** Think back on the activity we did today. Did you all like the activity? Why or why not? Are there any probability teaching ideas that you can take to your classroom? Will you be using these ideas in your teaching? Suppose you were to recommend this teaching sequence to a colleague. When would you suggest he or she use it? Do you feel there are any challenges in doing this activity? What kind of support, if any, would you require?

Table 1: Summary of Intervention Design

Research Participants

The intervention phase of the study involved a total of 23 pre-service secondary mathematics teachers. 10 of our pre-service teacher participants were part of the Graduate Diploma in Teaching at the University of Waikato (UW), while 13 pre-service teacher participants were final year Bachelor of Science and Graduate Certificate in Education (BSc GCEd) students at the University of the South Pacific. A summary of our pre-service teacher participants is provided in the table below

Research Context	Research Participants	Research process
The University of Waikato (UW) is located in Hamilton and operates from two campuses, Hamilton, and Tauranga.	10 pre-service mathematics teachers completing their Graduate Diploma in Teaching programme; equal number of males and females; six New Zealanders and four international pre-service teachers; all have mathematics as their teaching major. Participants named using letters O– W.	The second author was the coordinator of a teaching methods course at the time of the study. As part of this research, students were involved in the normal tutorial activities as planned by the second author. The participants had their whole class and group discussions audio recorded during the activities. The second author also wrote field notes. Following the two lessons, the pre-service teachers participated in semi-structured interviews to reflect on the lesson. The interviews were held at a time and place convenient to the students. The participants could choose to opt out of the interview at any point. All efforts were made to be culturally and socially responsive to ensure no student was disadvantaged during this research. The research was conducted after obtaining an ethics approval from UW.
The University of the South Pacific (USP) is a regional university that is owned by 12 member countries in the Pacific and is head-quartered in Suva, Fiji Islands.	13 pre-service mathematics teachers in their final year of the four-year BSC GCED programme; seven were males and six females; all have mathematics as their teaching major; nine from Fiji; four from Kiribati. Participants named using letters A–N.	The first author was not teaching the pre-service teacher participants, but all the participants were known to the author through previous teaching contacts. The most appropriate intervention opportunity was to organise a full-day workshop for the pre-service teachers. Upon invitation and informed consent, all participants attended the workshop on a Saturday at the USP. After some short pre-intervention one-to-one interviews, the main intervention was carried out in pairs and groups. All activities were typed up and each participant was given each activity sheet as the intervention progressed. A post-intervention focus group interview was conducted with all three groups. All interviews and intervention activities were video recorded with each major group having a separate camera person. The research was conducted after getting a research ethics approval from USP.

Table 2: A Summary of Research Participants

The data reported here followed a largely descriptive analysis of what transpired during the intervention. Teacher voices from audio and video recordings are used to support the research findings.

Findings and Discussion

This section is divided according to key themes arising out of the intervention data. The discussion will be supported by the use of the participants' voice through direct quotes, examples and relevant literature.

Phase One

Before participants took part in the *posing a problem* task, the researchers had read the activity to the whole class.

The teacher participants could also view the task on the activity sheet provided or from the power point projection. The researchers thought it was important to emphasise what the term 'difference' meant in the task. The difference is calculated based on the larger number minus the smaller number when both the die are tossed at once. All participants seem to have understood this clearly as examples were provided prior to the start of the activity. In addition, the term 'fair' was also discussed by both the researchers to their respective participants. All participants seem to have understood the term properly. This was demonstrated by their utterances such as "outcomes for both players would be similar", "equally likely for both", and "equal chances for both" or "balanced outcomes for both".

Two out of the 13 USP pre-service teacher participants predicted that the game is unfair, while the remaining 11 pre-service participants stated that the game is fair. Reasons given by the two USP participants about the game being biased were to do with the chance of either smaller outcomes (0, 1, or 2) the bigger outcomes (3, 4, or 5) occurring more frequently. Only participant I was correct in her reasoning that the game is unfair. The participant explained that player one (Esha) has the three lowest numbers while player two (Sarah) has the three highest numbers. The student further argued that there should have been a mixture of numbers to make the game fair. Participant I concluded Esha has more chances of winning because she has the lower numbers which will occur more times while taking the difference. Participant D, on the other hand, felt that the game was unfair because numbers 0, 1, and 2 were less likely to occur, hence Sarah will win.

> *The game is unfair. When [the] difference is taken, there is [a] very rare chance of getting 0, 1, [or] 2 which [are] lower numbers while there is [a] higher chance of getting 3, 4, [or] (Participant D, USP)*

The remaining 11 participants initially saw the game to be fair, with all of them saying that both players had three numbers as their outcomes, hence they saw the chances of winning to be the same. These participants did not show any reason to believe otherwise. A typical response was as follows:

> *The game is fair, because both the players will have same number of outcomes, since the numbers are 0, 1, 2, 3, 4, and 5 and each player has equal numbers. Thus, the game is a fair game. (Participant G, USP)*
>
> *Esha has three numbers and similarly, Sarah has three numbers which leads [me] to say that both the players have equal chances and thus the game is fair. (Participant K, USP)*

Nine of the 10 Waikato participants predicted that the game was not fair and that Esha had more chance of winning the dice difference game than Sarah. However, their explanations varied. Four teacher participants (P, S, V, and W) showed all possible outcomes (dice differences) and used this to find out the number of ways of getting each score (Figure 1). Responses included

(0, 1, 2) = 24 outcomes; (3, 4, 5) = 12 outcomes and they concluded that Esha wins more often because her numbers (0, 1, 2) have a 2:1 chance of winning.

In summary, 9/10 of the UW cohort could explain the reasons for the unfairness of the game by pointing out the possible outcomes for each score using a two-way table as used by participant pairs PS and VW in the example above (see figure 1 below). Other ways of demonstrating were noted in all other participant pair responses that included strategies such as making a bar graph for each outcome, or simply listing the 36 pairs of possible outcomes first and then drawing a chart or graph of differences to show that the game was unfair. It is interesting to note that almost all UW participants could provide detailed explanations about their predictions using written or diagrammatic representations at the beginning of the intervention. The one participant who initially said that the game was fair provided similar reasons as the majority of the USP participants. However, the participant changed her mind during pair discussion.

It is not surprising that most of the UW participants had made the correct initial predictions about the fairness of the game when compared to the USP participants. One of the reasons is that the USP cohort has had little experience in studying probability and statistics at high school or tertiary institutions using a game-based approach, as revealed in their pre-intervention interviews. It is interesting to see that none of the teachers used a tree diagram to find the total number of combinations for dice rolls. Possibly, this was a bit cumbersome for the participants.

		DICE 1					
		1	**2**	**3**	**4**	**5**	**6**
DICE 2	**1**	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

Figure 1: Two-way Table showing all possible differences.

Probabilities depend on the rules of the game. Merging simple events such as tossing two dice and noting the difference generally results in a much more complex sample space than the initial event. A single fair die has equiprobable outcomes. On the other hand, in the case of the difference of two fair dice, the outcomes are not equally likely. One reason for the lack of understanding of the dice difference game lies in the equiprobability bias (Lecoutre, 1992), which describes one’s tendency to view the probability of random events as being equal because “it reflects a process by chance”. Therefore, equiprobability bias occurs because people heuristically determine the chance of an event by simply considering the number of possible cases. In the game, this means that the probability of winning is calculated by dividing the number of possible outcomes (i.e., three) by the number of alternatives (i.e., six), which leads to the flawed judgment that the game is fair. These findings resonate with findings discussed in literature.

Phase Two

Results from phase two of the intervention suggest that nearly all the USP participants who had chosen the game to be fair during the first phase were able to realise that the game was unfair or biased. This was based on the table of outcomes that the pairs drew. Some pairs just had two columns in their table (trial and difference) while other pairs recorded both the outcomes and the differences in a three or four column table. The pre-service teachers used terms such as 'unfair' or 'biased' to describe the game. After playing 20 trials, all the participants changed their statement, agreeing that Esha has more chances of winning the game. They all stated that the probability of getting a difference of 0, 1, or 2 was more than getting a difference of 3, 4, or 5. These participants were quick to notice that the only way to get a difference of 5 is by getting 6 on one die and 1 on the other. This confirmed that upper differences (3, 4, or 5) as per the game criteria are very unusual or less likely to occur. All the USP participants confirmed that they would like to be Esha when playing this game. They stated that the outcomes would remain the same even if they played the game with a higher number of trials. A typical response included something like the following:

> *After doing 20 trials the results show a biased pattern, where more points are scored for Esha and less for Sarah. Even if more trials are done, still a similar pattern of results would be obtained, showing higher likelihood for Esha winning the game. (Participant D, USP)*

It is encouraging to note that the pre-service teacher participants from USP were able to realise their initial predictions were incorrect only with 20 trials. They could even predict that the results would remain in favour of Esha even if more trials were conducted. It was interesting to note that even Participant D – who had earlier argued that 0, 1, and 2 outcomes were less likely – was able to correct his conclusions.

Only one USP pair still seemed confused, even though they could state that after twenty trials, Esha will win. However, this pair stated that if we had conducted even more trials, any of the two players could win.

> *If more trials are done, there is a possibility that Sarah can win. The game is fair and it depends on the day it is played or simply it's about how the die shows its number (Participants M and N)*

The pair's disagreement seems to suggest that they see the probability of throwing a pair of dice and getting different outcomes as something similar to what people usually relate to in their everyday life events such as predicting weather. Their response "it depends on the day" seems to suggest a potentially ambiguous view of probability, i.e. that in real life we can never be sure about any event.

The UW participants worked in five groups of two. As they played the games, frequency tables similar to those drawn by the USP participants were used to record data. All the pairs, as expected, were able to explain why the game was unfair using explanations and representations similar to what they provided in phase one of the study. For example, one of the participants came up with the following conclusion after the pair completed their 20-throw trial:

> *Esha has a 65.56% chance of winning based on the results. And then Sarah has a 34.44% chance of winning, which is very close to the one of two to one. Sarah almost has just over a third [of a] chance, whereas Esha has just under two thirds. This is not fair and I know [Esha] has a high chance of winning. Still roughly two to one odds that she's gonna win (Participant W).*

In summary, the majority of the USP and UW participants were able to provide clear and logical explanations and representations about what will happen when 20 or more trials were to be conducted. The 20 throw trials not only helped correct the misconceptions noted in the participants' predictions but also allowed participants to generalise findings if a greater number of trials were to be conducted.

We speculate that asking the teachers to make and write predictions about the fairness of the game was a useful strategy. *Predict*, *observe and explain* is a strategy often used in science (Joyce, 2006; White, & Gunstone, 1992). It is used in *posing a problem* part of the probability lesson sequence for exploring students' original ideas and providing teachers with information about pupil's thinking. This helps in generating discussion and motivating learners towards exploring the concepts. The strategy has parallels with constructivist ideas of learning which suggest that pupils' existing understandings should be taken into account when planning and developing teaching and learning activities. For example, events that surprise are likely to create conditions where participants may be ready to start re-examining their personal theories. Explaining and assessing their initial predictions while listening to others' predictions can help participants begin to re-look at their own learning and construct new meanings.

Group work was used during the activities. Students were asked to form groups to discuss the ideas and questions they might have relating to the die rolling game. Sharing student work/representation and comparing variation in experimental and theoretical probabilities are key to this sequence. Collaborative work allowed the students to collaborate in their learning and ties in with the work of Takeuchi (2016) who explains that when learners are able to work alongside a partner, they are given the opportunity for interaction and support, enhancing their learning. Collaboration afforded teachers the chance to ask questions and make mistakes in a safe setting, where they can receive direct and immediate feedback. Seen from a socio-cultural perspective, the probability teaching sequence provided our participants with opportunities to make connections to real-life gaming scenarios and to discuss and explain their findings in pairs. On most occasions, detailed explanations led to the expected learning outcomes, while there were glimpses of misconceptions.

Phase Three

For the final phase, the USP cohort was divided into three groups. Group 1 had five participants, while the other two groups had four participants each. This phase of the intervention began with researchers reminding the groups about the need to explore further and draw conclusive arguments about the nature of the probability game. The USP groups were also reminded about the need to think of data organising methods, unlike the UW group that had used various diagrammatic representations in their earlier phases.

Two of the three USP groups decided to do more trials and they came up with different methods of data recording. For example, group 1 decided to have 185 trials and record the data using a pie chart. Other group members were quick to note that a bar graph would be more useful given that they could clearly see the skewness of the outcomes using a bar graph. The group recorded their answers using a table (a two-column table is drawn and the group records the difference each time two dice are tossed). Once the trials were over, the group drew a bar graph and a lattice diagram to make sense of their findings (see figures 2 and 3 below).

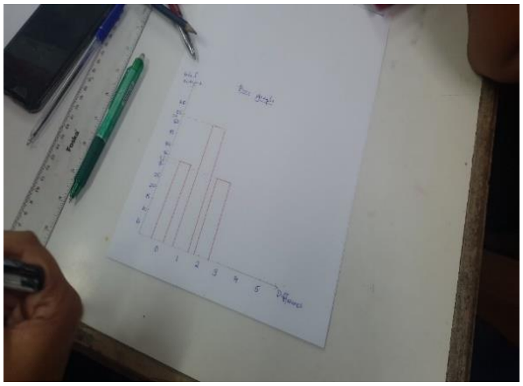


Figure 2: Bar graph

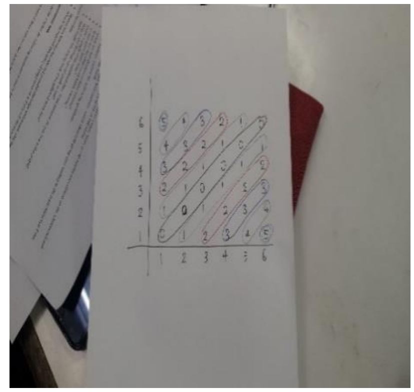


Figure 3: Lattice Diagram

One of the groups did not do the 180 throw trial because they, like group 1, were confident after the 20 throw trial that throwing a pair of dice had only 36 possible outcomes. The group argued that from these 36 outcomes, the probability of any event could be found. In summary, all three groups were able to conclude from the bar graphs and then from the lattice diagrams that the chances of Esha winning were greater than that of Sarah.

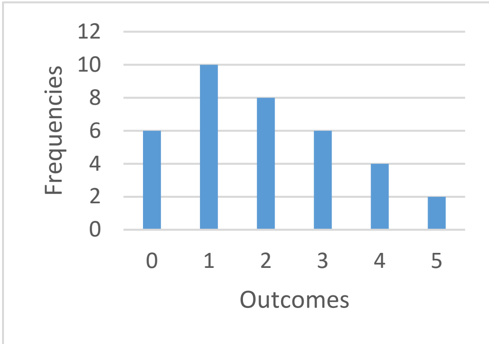
> *By looking at the lattice diagram we can say that the game is not fair. Esha has more chances of winning the game. This is for 0, 1, 2 and 3, 4, 5 (showing in the lattice diagram). We can find that there are more 0, 1, 2. Therefore we concluded, using the lattice diagram, that we do not have to throw the dice 180 times. The combined data follows a pattern which helps us to find the probabilities for larger number of trials. There are 36 possible outcomes when Sarah and Esha play the game and their difference is calculated from the rolled dice (Participant D).*

The group changed their first answer and said that the game is not fair and Esha is always going to win. The group drew a graph of the combined data.

In order to confirm conceptual understanding, the UW pairs were asked to explain how the findings would look if there were more trials conducted. They were sure that the findings would remain in favour of Esha. Answers provided were similar to the ones provided by the USP participants. Both the USP and UW groups used diagrammatic representations such as bar graphs, lattice diagrams and tables to explain their answers. Some responses from the UW pairs were as follows:

> *If we collect 30 more samples we will be able to see that Sarah loses and this is because each event of rolling the dice is less likely to give us a difference of 3, 4, or 5. And this will still be visible when a larger sample size is collected (Participant P).*
>
> *The heights of the bars will change relative to each other. But the bias will maintain the 2:1 ratio for 0, 1, 2, to 3, 4, 5. As we collect more data (more rolls) for the two players, the numbers will continue to show a 2:1 ratio (Participant R).*

However, when one of the UW pairs who had drawn a bar graph to represent the various outcomes was asked to draw the graph of class results if more trials were conducted, the participants said that the bars will get to the same height as all events will become equally likely (Figure 4). This misconception was clearly visible in the pair's graphs shown in figure 4 below.



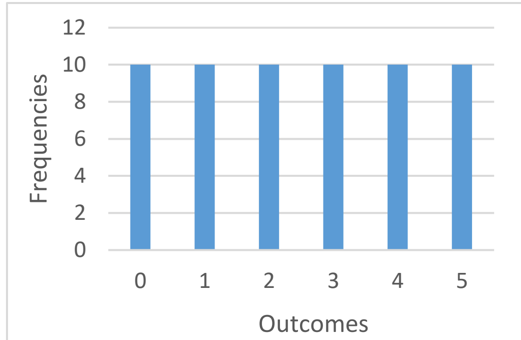


Figure 4: Correct and incorrect graphical representations

This same pair also suggested using a pie chart as one could get exact degrees of angles to represent data. The same equally likely misconception was evident in the representation as reflected in the following quote: "the more chances we take, angle of each will become 360/6 = 60".

The findings also reveal that probabilistic understanding is fallible and a few participants were still not confident about what would happen if more trials were performed. For example, the UW pair insisted on suggesting that more trials would end up in equally likely scenarios. One USP group had similar doubts as they said that things could change on a given day.

The equiprobability bias, which arises when people rely on number-of-cases intuition, may have hindered participants to develop a deep understanding of the dice difference game and its underlying probabilities in different situations. In order to make connections to appropriate displays, one should overrule erroneous heuristic reasoning and switch to correct mathematical reasoning. Our results also provide evidence that misconceptions in probability may not decrease with age. In particular, the findings confirm that equiprobability bias can strengthen with increasing age (Fischbein & Schnarch, 1997) and statistical education (Morsanyi, Primi, Chiesi, & Handley, 2009).

In addition, we believe that an extension to the current design would be to ask pre-service teachers to design a dice game that is fair. This extension activity is an important and rich problem to solve. By having multiple solutions on how to make the game fair it becomes a more cognitively demanding task. It would help deepen students' probabilistic concepts and engage them in probabilistic thinking, particularly on how to approach such a problem. However, students will need to have agreed on the theoretical probabilities (not use their experimental probabilities) before they embark on creating a fair dice difference game. We look forward to using this question in the next iteration of our study.

Limitations and Implications for Practice and Research

There are several limitations in the study. Firstly, the number of participants in the study is small, with limits on generalisability of findings. It was not possible to isolate responses related to age, qualifications or prior experience. A study with larger number of participants might be well suited to achieve these types of results, which would then have important implications for constructing support to change teacher practices.

A second limitation relates to getting student voices on the teaching sequence. While this paper only discussed data sought from the pre-service teachers, it would be valuable to know what students think about the teaching sequence. Future trialing of the sequence followed by interviews with students will help explore their thinking regarding the teaching sequence.

While several, albeit small, studies internationally have indicated the relative importance placed on statistics and probability, teachers continue to have limited awareness of issues relating to this strand. The pre-service teachers in the current study revealed a range of specific techniques consistent with research-based effective learning practice. We cannot confirm if this was a result of prior learning in teacher education or through experience in the collaborative setting provided in this study. This could be an area for future investigation.

Participants' account indicate that some were part-way to giving a complete explanation, but needed more detail or accuracy. Teacher educators need to support pre-service teachers to reveal what they already know with more precise mathematical language. In the course of such discussions, comparisons of several different answers may be made. This might result in decisions about what might constitute a reasonable explanation as well as draw attention to details that may be missing. These implications parallel those described by the New Zealand Ministry of Education (2007), where communicating mathematically is considered an essential skill in the mathematics curriculum document.

In this study, we did not intentionally look at ways in which features of cultural games as suggested in literature can help re-enforce concepts of probability. Culturally diverse games for probability exploration can be used in statistics classrooms because such activities not only provide a "legitimate case of straightforward mapping of situations onto probabilistic structures" (Greer & Mukhopadhyay, 2005, p. 316) but also allow for simulations using both cultural artefacts and technological tools. In addition, cultural games will help sustain student interest and motivation and help teachers highlight the significance of the role of culture and context in a multicultural statistics classroom (Averill et al., 2009). We certainly need to investigate how students' learning of probability can be supported by the affordances of technological tools and culturally diverse games.

Teacher education organisations will be interested in this research. Understanding the challenges and some of the opportunities pre-service teachers encounter in the classroom when teaching learners probability, will enable teacher educators to better equip teachers to work in diverse classrooms.

The lesson sequence described in this article can be explored individually or with a group of teachers who are sharing insights and reactions, working through activities together, trying things out in the classroom, and sharing experiences and next steps. Future researchers may want to teach the lesson using lesson study (Leavy & Hourigan, 2014) to examine the implementation of the sequence in secondary classrooms.

We look forward to conducting future iterations of this research to explore how consistent and useful these findings may be across diverse contexts. It is hoped that the findings reported in this paper will generate greater interest in using game contexts in probability teaching.

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Appendix

Title - A Possible Teaching Sequence to Explore Probability and Related Concepts in a Die Rolling Game

Overview of Lesson

The sequence addresses some common misconceptions relating to probability of simple and compound events. Students are asked to make predictions about the fairness of a game and then test them by gathering and examining data.
Specifically, the sequence examines:

- concepts of equally likely events, randomness, sample size, independence, probability distributions, variation (within a group and between distributions), making predictions, organising and displaying data, interpreting tables and graphs, estimating probabilities
- mathematical skills of basic facts, proportional reasoning, fractions
- mathematical practises with emphasis on reason abstractly and logically, construct viable arguments, critique the reasoning of others, making predictions and decisions, modelling, making connections, communicating statistically (verbally and in writing)

Learning Objectives

- Students are deriving and comparing experimental estimates with theoretical model probabilities for two-stage chance situations
- Students are exploring outcomes for two categorical variables in statistical investigations from a probabilistic perspective.

Lesson Background/Introduction

While there exists rich literature on students' misconceptions about probability; less attention has been paid to the development of students' probabilistic thinking in the classroom. Grounded in an analysis of research literature this article offers a lesson sequence for developing students' probabilistic understanding. In particular, it demonstrated how a game context can be used to explore the relationship between experimental and theoretical probabilities in a classroom setting. The approach integrates the content, processes and the language of probability and is grounded in socio-cultural theory. Student predictions and conclusions are examined and re-examined in interactions with small group members, whole class and the teacher as he or she monitors small group work. The sequence covers a range of criteria for a rich mathematical activity and includes suggestions for adapting the sequence.The lesson is adapted from a paper published in *Teaching Statistics* journal (Sharma, 2015).

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Lesson Outline

The phases involved in the teaching sequence resonate with Wild & Pfannkuch's (1999) statistical PPDAC cycle mnemonic (Problem, Plan, Data, Analysis, Conclusion) with slight modifications in a probabilistic context.

1. Posing a Problem

Esha and Sarah decide to play a die rolling game. They take turns to roll two fair dice and calculate the difference (larger number minus smaller number) of the showing numbers. If the difference score is 0, 1, or 2, Esha wins, If the score is 3, 4 or 5, Sarah wins. Is this game fair? Explain your thinking.

2. Playing the Game in Pairs

Pair students and have them play a round of the game described above. Explain that they are going to roll the two dice and calculate the difference of the numbers showing. With student feedback, list the possible outcomes (0, 1, 2, 3, 4, and 5) on board.

Students play the game about 20 times with a partner, and tally the results in a frequency table.

Focus Questions After the Game

- On the basis of your results, do you think the game is fair? Why, or why not?
- If you wanted to win this game, which player would you choose to be? Explain your answer.
- If you played the game 30 more times, would the results be the same as or different from the first game? If they would be different, how?

3. Planning Whole Class Explorations

Pose the following questions and brainstorm responses.

- Why does Esha win more often than Sarah?
- How can we determine if the game is fair by collecting more data?
- How can we record our results?

Students will suggest/brainstorm ideas about gathering more data and how to record data.

4/5. Data Collection and Analysis

In groups of three, data is collected and recorded. Next, group results are collated on the whiteboard and students analyse the pooled data (eg out of 180 trials).

Class results are compared with students' initial ideas and group data leading to the realization that Esha wins more often than Sarah.

In groups, students answer the following questions.

- What are the chances of Esha winning?
- What are the chances of Sarah winning?
- Is this game fair? Why?
- Draw a graph of the combined data. What patterns do you see in the graph?
- Why is this the best type of graph to use?
- How might this display look if we gathered more data?

Focus Questions

- Discuss how knowing the expected probabilities helps understand why the game is unfair.
- What is the expected frequency of (say) score of 4 if you roll the two dice 72 times and 144 times?

A Brief Assessment Task

Students to decide whether the following statement is true or false and write down reasons to support their decision.

- Scoring a total of three with two fair dice is twice as likely as scoring a total of two.