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Sandra Herbert
Deakin University

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Overcoming Challenges in Assessing Mathematical Reasoning

Sandra Herbert
Deakin University

Abstract: Despite mathematical reasoning being necessary for in-depth understanding of mathematical concepts, many teacher experience difficulty in assessing it. Data were collected from 34 primary teachers at 4 Victorian government schools at two post- lesson reflective sessions following lessons with a focus on reasoning. These sessions facilitated teachers' collaborative efforts to assess their students' reasoning from students' work samples. The data included transcripts of all the reflective sessions; written work samples; and associated completed rubrics. Analysis of these data enabled identification of seven challenges teachers experienced in assessing reasoning: Limited guidance provided by curriculum documents; Teachers' knowledge of reasoning; Teacher noticing and interpretation of student reasoning; Students' difficulties in articulating their reasoning; Assessing progress in reasoning; Inadequacy of work samples; and Challenges in tracking and reporting student progress in reasoning. The discussion presents strategies to overcome these challenges.

Key words: Primary school; assessment; mathematical reasoning; professional learning

Introduction

There has been growing acknowledgement of the importance of mathematical reasoning in students' sense-making of mathematics (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2017; Brodie, 2010; Kilpatrick, Swafford, & Findell, 2001). Kilpatrick, et al. (2001) described reasoning as "the glue that holds everything together, the lodestar that guides learning" (p. 129). However, there is limited guidance for teachers in the assessment of reasoning. This article draws together research literature and data from two post-lesson reflective sessions about the assessment of student work samples with 34 primary teachers at 4 Victorian government primary schools to identify the challenges teachers face in assessing mathematical reasoning.

Mathematical reasoning has been described by many authors in a variety of ways. For example, Lannin et al. (2011) described reasoning as "an evolving process of conjecturing, generalizing, investigating why, and developing and evaluating arguments" (p. 13). In the Australian Curriculum in Mathematics (AC: M) it is described as:

Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying, and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached (ACARA, 2017, p. 5).

In this article, challenges for teachers as they strive to assess students' reasoning are explored and ways to overcome these challenges are proposed. It is guided by the research questions: What are the challenges for teachers in assessing mathematical reasoning?; and

How might these challenges be overcome? Relevant literature is reviewed to provide a theoretical framework for the study, which is followed by a description of the methodology and results. The results are then discussed with reference to existing literature and suggest various ways of overcoming the challenges identified in the results. Finally, the conclusion presents the limitations of the study and suggests some possibilities for further research.

Literature Review

Mathematical reasoning is now more visible in curriculum documents (Australia: ACARA, 2017; United States of America: Common Core State Standards Initiative (CCS), 2020; and the United Kingdom: Department for Employment and Education (DfEE), 2014) and these documents emphasise the importance of mathematical reasoning in understanding mathematics (Brodie, 2010). In the Common Core Standards (USA) aspects of mathematical reasoning appear in 5 out of 8 Standards for Mathematical Practice such as in

CCSS.Math.Practice.MP1: Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyse givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway (CCS, 2020, p.1).

This previous statement alludes to the reasoning actions of analysing and generalising, whilst CCSS.Math.Practice.MP3: Construct viable arguments and critique the reasoning of others emphasises justifying and communicating stating: “Mathematically proficient students ... justify their conclusions, communicate them to others, and respond to the arguments of others (CCS, 2020, p.2). Similarly, in the DfEE (2014) documents, the authors write that “curriculum for mathematics aims to ensure that all pupils: reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language” (p. 99).

As Brodie (2010) advises reasoning is necessary to “to understand mathematical concepts, to use mathematical ideas and procedures flexibly, and to reconstruct once understood, but forgotten mathematical knowledge” (p. 11). Reasoning assists students generate new knowledge through creating and validating mathematical ideas. This activity supports the construction of connections between logical and meaningful mathematical notions as opposed to rote learning of disconnected routine procedures through reasoning, thus (Mata-Pereira, & da Ponte, 2017). Lithner (2000) defined mathematical reasoning to be “the way of thinking, adopted to produce assertions and reach conclusions ... [and] transfer of properties from one familiar situation to another (task solving) situation” (p. 167). Brodie (2010) emphasised convincing others of claims or solutions to problems.

Whilst substantial research has focused on mathematical reasoning, there is no universal agreement about the meaning of the term ‘mathematical reasoning’ (Jeannotte & Kieran, 2017). Jeannotte and Kieran (2017) asserted that the various descriptions of reasoning tend “to be vague, unsystematic, and even contradictory from one document to the other” (p. 2). In seeking to bring together the diverse descriptions of reasoning, they formulated a model, consisting of two aspects: a structural aspect and a process aspect. Considerations of the formal mathematical definitions of reasoning underpin the structural aspect: these include deduction; induction; and abduction (which is more usually introduced in senior secondary school and tertiary mathematics). Jeannotte and Kieran’s (2017) process aspect is more applicable to primary schools: searching for similarities and differences; comparing and classifying; identifying a pattern; generalising and conjecturing; validating solutions including justifying and proving. This process aspect aligns well with curriculum documents and Kilpatrick et al.’s (2001) description of adaptive reasoning, that is, the “capacity for logical

thought, reflection, explanation, and justification” (p. 5).

Teacher knowledge of reasoning is a critical factor in their ability to assess their students reasoning. Previous research has identified gaps in teachers’ knowledge of reasoning (Clarke, Clarke & Sullivan, 2012; Herbert et al., 2015; Loong et al., 2017). Clarke et al. (2012) reported that teachers focused on explaining rather than other reasoning actions and that “many students appeared to have little experience in the opportunity to conjecture, justify and generalise, or certainly to articulate these processes verbally or in writing” (p. 30). Since a teacher’s knowledge of the content, they teach is an influential factor in the effectiveness of their teaching (Darling- Hammond, 2000), their knowledge of the complexity of reasoning is necessary to develop their students’ reasoning capacity (Stylianides, Stylianides, & Shilling-Traina, 2013). Stylianides, Stylianides, and Philippou (2007) asserted that “[i]f teachers’ knowledge of proof is fragile ... it is likely that teachers will teach proof poorly or will not teach proof at all” (p. 146).

Problem solving activities provide opportunities for students to reason (Wood, Williams, & McNeal, 2006), particularly where the classroom culture fosters the expectation that students will share their reasoning with others. So choosing activities that expect students to explain and justify their solutions and to look for patterns (Davidson et al., 2019) have potential to encourage the formation and verification of conjectures and verify their conjectures (Vale et al., 2017). The teachers’ role then becomes a facilitator to stimulate deeper thinking through the employment of enabling and extending prompts (Davidson et al., 2019). Mata-Pereira and da Ponte (2017) found that students’ construction of new knowledge is supported by reasoning to create and validate mathematical ideas through building connections between logical and meaningful mathematical notions rather than rote learning of poorly understood, disconnected procedures (Mata-Pereira, & da Ponte, 2017).

A deeper knowledge of the various reasoning actions enables teachers to plan to embed reasoning in lessons and foster reasoning in their classrooms (Davidson et al., 2019) by using problem solving tasks, appropriate prompts and developing supportive classroom cultures (Martino & Maher, 1999). Classroom cultures where discourse through group tasks and orchestrated discussions provide opportunities for students to build, test and refine conjectures by the necessity to convince others of the validity of solutions and conclusions (Brodie, 2010; Kilpatrick et al., 2001; Stein, Engle, Smith & Hughes, 2015; Vale et al., 2017). Careful selection of tasks that provide opportunities to conjecture, generalise and justify have the potential to develop a classroom culture that supports reasoning (Kilpatrick et al., 2001). Planning to embed reasoning-involves assessing the reasoning potential of tasks; developing effective prompts to elicit reasoning and anticipating student responses (Davidson et al., 2019).

Where teachers are conversant with the elements of reasoning, they are more likely to notice their students’ reasoning and hence employ suitable prompts to progress that reasoning (Llinares, 2013; Jacobs, Lamb, & Philipp, 2010; Francisco, & Maher, 2011). Llinares (2013) merged previous research on noticing into a single definition. However, teachers may not notice students’ reasoning when students begin to analyse a problem using the trial-and-error approaches (Ferrando, 2006). Llinares (2013) defined it as a teacher’s ability to “identify relevant aspects of the teaching situation; use knowledge to interpret the events and establish connections between specific aspects of teaching and learning situations and more general principles and ideas about teaching and learning” (p. 79). Similarly, Jacobs, et al. (2010) emphasised attention on students’ strategies; inferring from these strategies, students’ understanding; and formulating a suitable response. However, students sometimes have difficulty in expressing their reasoning (Bragg et al., 2016) so a teacher may not be able to provide an appropriate prompt to facilitate the student’s reasoning

Methodology

This study is part of a larger study involving 34 teachers and their students from four Victorian primary schools who collaborated with researchers in a design research project to create resources for teachers to assist in assessing mathematical reasoning through the *reSolve: Mathematics by Inquiry - Special Topic Assessing Mathematical Reasoning* (Australian Academy of Science [AAS] and Australian Association of Mathematics Teachers [AAMT], 2017). Design based research systematically employs iterative cycles to design, implement and analyse data collected in collaboration with practitioners (Wang & Hannafin, 2005). Design-based research is iterative, pragmatic, interactive, flexible and grounded in the context of practice (Wang & Hannafin, 2005) and viewed through the lens of theories teaching and learning of mathematical reasoning (Jeannotte & Kieran, 2017; Kilpatrick et al., 2001; Stylianides et al., 2013). The design process was intended to address the concern regarding the challenges of assessing mathematical reasoning.

Ethical approval for the research was granted by the relevant institutional Ethics Committee with participants giving informed consent by reading a plain language statement and signing a consent form to indicate their agreement with the plain language statement. The teachers (pseudonyms used throughout) engaged in a school-based professional learning session presented by the researchers about mathematical reasoning and its assessment. Then they taught and observed their colleagues teaching a researcher-designed task with a focus on reasoning. In post-lesson reflective assessment sessions, they and the researchers considered the assessment of students' work according to the rubric initially provided by the research team and refined through an iterative process incorporating teachers' suggestions at different schools and over time. The resources designed for this special topic divided the components of mathematical reasoning into three main reasoning actions:

Analysing includes: *Exploring the problem and connecting with known facts and properties; comparing and contrasting cases; and sorting and classifying cases.*

Generalising includes: *Identifying common properties or patterns across cases; forming conjectures, i.e. statements that are thought to be true but not yet known to be true; and communicating conjectures clearly.*

Justifying includes: *Checking the truth of conjectures; using logical argument to convince others; and refuting a claim.* (AAS & AAMT, 2017)

Two versions of the rubric were designed with key indicators of the different levels of the three aspects of reasoning—analysing, generalising and justifying: A short version for in-the-moment use in class (see Figure 1); and more detailed version to provide more assistance for teachers developing an understanding of the complexity of mathematical reasoning (for more detail of the design research please see Loong et al., 2018). These rubrics were designed to assist teachers in building their knowledge of reasoning. Eight exemplars were designed to demonstrate the assessment of reasoning of students' work using the rubric. These exemplars are based on eight different rich tasks with potential for fostering reasoning.

	Analysing	Generalising	Justifying
Not evident	<ul style="list-style-type: none"> • Does not notice numerical or spatial structure of examples or cases. • Attends to non-mathematical aspects of the examples or cases. 	<ul style="list-style-type: none"> • Does not communicate a common property or rule for pattern. 	<ul style="list-style-type: none"> • Does not justify. • Appeals to teacher or others.
Beginning	<ul style="list-style-type: none"> • Notices similarities across examples • Recalls random known facts related to the examples. • Recalls and repeats patterns displayed visually or through use of materials. • Attempts to sort cases based on a common property. 	<ul style="list-style-type: none"> • Draws attention to or attempts to communicate a common property or repeated components of a pattern using: <ul style="list-style-type: none"> ○ body language (gesture), ○ drawing, ○ concrete materials ○ counting or ○ oral language (metaphors) 	<ul style="list-style-type: none"> • Describes what they did and why it may or may not be correct. • Recognises what is correct or incorrect using materials, objects, or words. • Makes judgements based on simple criteria such as known facts. • The argument may not be coherent or does not include all steps
Developing	<ul style="list-style-type: none"> • Notices a common numerical or spatial property. • Recalls and repeats patterns using numerical structure or spatial structure. • Sorts and classifies cases according to a common property. • Orders cases to show what is the same or stays the same and what is different or changes. • Describes the case or pattern by labelling the category or sequence 	<ul style="list-style-type: none"> • Communicates a rule about a: <ul style="list-style-type: none"> ○ <i>property</i> using words, diagrams or number sentences. ○ <i>pattern</i> using words, diagrams to show recursion or number sentences to communicate the pattern as repeated addition. • Records other cases that fits the rule or extends the pattern using the rule. 	<ul style="list-style-type: none"> • Attempts to verify by testing cases or explaining the meaning of a conjecture using one example. • Detecting and correcting errors and inconsistencies using materials, diagrams and informal written methods. • Starting statements in a logical argument are correct and accepted by the classroom.
Consolidating	<ul style="list-style-type: none"> • Notices more than one common property by systematically generating further cases and/or listing and considering a range of known facts or properties. • Repeats and extends patterns using both the numerical and spatial structure. • Searches for and generates examples: <ul style="list-style-type: none"> ○ using tools, technology & modelling 	<ul style="list-style-type: none"> • Generalises: communicates a rule using mathematical terms, symbols or diagrams (eg. a number sentence or labelled geometric diagram) • Explains what the rule means using one example. • Extends the pattern using an example to explain how the rule works. 	<ul style="list-style-type: none"> • Verifies truth of statements by using a common property, rule or known facts that confirms each case. May also use materials and informal methods. • Refutes a claim by using a counter example. • Uses a correct logical argument that has a complete chain of reasoning to it and uses words such as ‘because’, ‘if then’, ‘therefore’, ‘and so’, ‘that leads to’ ... • Extends the generalisation
Extending	<ul style="list-style-type: none"> • Notices and explores relationships between: <ul style="list-style-type: none"> ○ common properties ○ numerical structures of patterns. • Generates examples to form a conjecture. 	<ul style="list-style-type: none"> • Generalises: communicates the rule using mathematical symbols, including algebraic symbols • Applies the rule to find further examples or cases. • Generalises properties by forming a statement about the relationship between common properties. • Compares different symbolic expressions for the same pattern or property to show equivalence 	<ul style="list-style-type: none"> • Uses a watertight logical argument that is mathematically sound and leaves nothing unexplained. • Verifies that the statement is true or the generalisation holds for <i>all</i> cases using logical argument.

Figure 1: Rubric for assessing reasoning (AAS & AAMT, 2017)

In 2016, data collected at 18 reflective sessions were the audio-recordings of the conversations between the teachers and researchers; copies of students' work samples; and copies of associated completed rubrics. This teach/observe/reflect cycle was repeated for a second lesson with a task with a focus on reasoning sourced or created by the teachers rather than the researchers and a rubric refined through consideration of previous iterations.

The analysis was not intended to provide a comprehensive list of challenges in assessing reasoning undertaken by all teachers but rather provides insights into challenges faced by these 34 teachers in assessing reasoning as reported during the reflection sessions (Stake, 1995). "The phenomenon being researched is studied in its natural context, bounded by space and time ... [and] is richly descriptive" (Hancock & Algozzine, 2016, p. 15). The phenomenon under investigation is teachers' views of the challenges in the assessment of reasoning. The approach to analysis was "open to the use of theory or conceptual categories that guide the research and analysis of data" (Meyer, 2001, p. 331), such as Herbert et al.'s (2015) framework of mathematical reasoning. Consistent with Akerlind et al. (2005), the transcripts were read and re-read leading to "a series of iterative cycles between the transcript data, researcher interpretations of the data, and checking of interpretations back against the data" (p. 87).

Results

The analysis described above resulted in the seven themes that structure the results.

Curriculum Documents

The first challenge for teachers is making sense of Australian Curriculum: Mathematics (AC: M) statement on reasoning. Teachers seeking guidance might look to the Year Level Achievement Standards statements provided by the AC: M but these do not include reasoning. This omission limits teachers ability to assess reasoning.

When looking at the data from the reflective sessions, teachers' comments indicated that they were expecting the curriculum documents to provide guidance in assessing reasoning. For example:

School C Gloria: We're always looking at, you know [checking] against the AusVELS [Victorian Curriculum] ... how does this relate to the levels? ... If I knew whereabouts all of this stuff was plotted in the continuum that would help me.

School B Clare: Yeah but is it [the rubric] AUSVELS or is it just your own?

Each Year Level Description does describe aspects of reasoning at each level, but focuses on reasoning that is related to specific mathematical content rather than the development of a range of reasoning actions. For example, Level 2 - "reasoning includes using known facts to derive strategies for unfamiliar calculations, comparing and contrasting related models of operations and creating and interpreting simple representations of data". In this statement the content focus specific to is numeric calculations and data. Other Year Level Descriptions have a similar focus on mathematical content.

Teachers' Knowledge of Reasoning

Some teachers expressed uncertainty about the nature of mathematical reasoning. For example,

School C Lisa: I think if you're doing it as you go around the class and you really [need to] know what each of these things [reasoning actions] mean.

Other teachers referred to the rubric as a useful source of information on the different reasoning actions, intentionally using the language of the rubric to articulate their students' reasoning.

School C Robyn: *This task ... had a little bit of everything in it. It had a bit of analysing because they had to recall and repeat a pattern and had forming conjectures and generalising because they had to explain the meaning of the rule and justifying logical argument because they had to say why and because.*

School C Connie: *No, [there isn't a simpler structure that will help teachers] because I think the amount of information you've got in there helps you when you're making judgments with the kids.*

School C Robyn: *I've only got through half of them but ... most of them are at developing and consolidating. This task ... had a little bit of everything in it. It had a bit of analysing because they had to recall and repeat a pattern and had forming conjectures and generalising because they had to explain the meaning of the rule and justifying logical argument because they had to say why and because.*

Teachers in this study focused on explaining as the most visible reasoning action. For example:

School A Cathy: *I think Xxxx because she was explaining it to Yyyyy and Yyyyy is quite a lot lower and Xxxxx was using her explanations and when I pressed her on it she did have a grasp of it but sometimes she would use it. she knew she could do it but she could never explain it so it's a big step up.*

Teacher Noticing and Interpretation of Students' Reasoning

Teachers commented on issues related to students' ability to articulate their reasoning. Assessment of students' reasoning is only possible when teachers notice students' strategies and interpret students' reasoning to respond with an appropriate enabling prompt to progress their reasoning. Whilst many teachers in this study noticed that students were reasoning they struggled to understand that reasoning. For example:

School B Clare: *I'm not quite sure. [There's] an equation – I'm not quite sure what he's trying to say? He's saying small numbers make 10s and big numbers make hundreds. So if you had 57 plus 75 I don't even know where this comes from?*

School A Rosie: *The fact that she knows how many to add each time and she's realised the pattern of both of them, would you say that that's what that refers to or is that too advanced?*

In the reflective sessions the teachers often worked together to attempt to understand the reasoning shown on a worksheet.

School B Terry: *[it's difficult] to describe exactly what he did there because they've just said you know 'I've counted with my fingers and the number chart so I know the answer' and then he's provided place value but he's actually done it for ones like that and just represented the number but not his actual calculation of how he got that number. So described what he did. He recognised it was incorrect using materials, objects, or words but it wasn't really coherent.*

School B Clare: *It's not that he's not coherent, he is coherent he's just not – he's only got one argument really and then he's done the place value. Well that's two arguments I suppose.*

Terry and Clare struggled to understand the students' reasoning as presented on the worksheet and did not identify the nature of the reasoning or its level.

Students' Difficulties in Articulating their Reasoning

Some teachers found it difficult to interpret their students reasoning because of the challenges students faced in communicating their reasoning, especially if they had limited mathematical language on which to draw.

School B Clare: *He used the words like – the numbers were too heavy. He said 'that's too heavy to have these lighter numbers'. He didn't use mathematical language.*

In the following quote the teacher acknowledged this struggle and provided the enabling prompt, suggesting students first try to verbalise their reasoning.

School C Ann: *I gave them [a chance] ... to explain this as well orally because I said sometimes you don't say exactly what you want to say when you write, so have a go at explaining.*

Assessing Progress in Reasoning

Many teachers expressed concern that the complexity of mathematical reasoning made it difficult to observe and record each student's progress.

School A June: *[Assessing reasoning] is time consuming. You really need to sit down and have a think about what it's asking you, but then if I was to use it [the rubric] I would just at a glance go, well [for these students] next time I'm going to do that. But I probably wouldn't sit there and go to each one and say, well, he can't do this one, so I'm going to do that.*

Consequently, teachers found that noticing and assessing students' reasoning was only possible if they focused on the reasoning of just a few students' reasoning each lesson and recorded their observations on the short version of the rubric available from the reSolve website (see Figure 1). Other teachers supplemented the written work samples with student videos of their reasoning.

Inadequacy of Work Samples

Many teachers commented on the lack of information on the worksheets to use to assess a student's reasoning. They often talked about interactions they had had with the students and relied on these conversations to assist in assessment of reasoning. Teachers considered listening to students' attempts to articulate reasoning was important to understand their students' reasoning especially when it was not available in the students' written work.

School C Kerry: *I must've picked up quite a bit from just the conversations on the floor they were watching and listening because they haven't actually shown it here [on the work sample].*

School D Elizabeth: *it's hard just looking at their random working out.*

School A Rosie: *I think her verbal explanation is very good but probably didn't have time to write it. I think with the next session she will. She'll just be given that extra time to work on the formula.*

Challenges in Tracking and Reporting Student Progress in Reasoning

Teachers acknowledged that the complexity of mathematical reasoning made it difficult to observe and record a student's progress particularly when this was absent from the work samples. They advised concentrating on just a few students using the short version of the rubric over time rather than all students in a single lesson.

School A June: [Assessing reasoning] is time consuming. You really need to sit down and have a think about what it's asking you, but then if I was to use it [the rubric] I would just at a glance go, well [for these students] next time I'm going to do that. But I probably wouldn't sit there and go each one and say, well he can't do this one, so I'm going to do that. It's almost for me a little bit overwhelming because there's so much on the page.

In addition, many teachers viewed the task as summative assessment to rate students' reasoning performance rather than formative assessment used to guide subsequent lessons. For example:

School C Con: How would you mark someone that has terrible reasoning for one task and then really good for another? Does that mean they just know one task better than another reasoning task?

School B Terry: That's how I score.

These results exemplify seven different themes evident in the data: unhelpful curriculum documents; limited teachers' knowledge of reasoning; inadequacy of teacher noticing and interpretation of student reasoning; difficulties in assessing progress in reasoning; inadequacy of work samples; students' difficulties in articulating reasoning; and challenges in tracking and reporting student progress in reasoning. This study affirms some of the challenges generally encountered by students and teachers with regards to reasoning, and particularly of assessing mathematical reasoning as evidenced in the research literature. This research extends this work by considering the data collected in the post lesson reflections for this project to identify other challenges not previously noted in the literature.

Discussion

The findings of this study are consistent with many of the difficulties teachers face in assessing mathematical reasoning evident in the research literature. In this study there were challenges associated with teachers' knowledge that had also been noted in previous research (Clarke et al., 2012; Herbert et al., 2015; Loong, et al., 2017; Hilton et al., 2016), such as noticing, interpreting, and assessing students' reasoning. Creating a classroom cultures that support and expect student to articulate and justify their reasoning and evaluate the ideas of other students, have the potential to foster the development of students' reasoning. Students' reasoning capacity is enhanced through the communication of ideas through discussion with others (ACARA, 2017; Brodie, 2010; Jeannotte. & Kieran,2017). Orchestrated discussions (Stein, et al., 2015) may assist in refining conjectures and convincing others of the validity of conclusions (Brodie, 2010; Kilpatrick et al., 2001). Indeed, Long, De Temple and Millman (2012) suggested that students' reasoning grows when "students are encouraged to put forth their own ideas for examination [where] ... students need to explain and justify their thinking and learn how to detect fallacies and critique others' thinking" (p. 49). These opportunities to explain and justify thinking arise during problem solving activity (Wood, Williams, & McNeal, 2006) with convincing of the validity of solutions (Kilpatrick et al., 2001; Vale et al., 2017).

The choice of problem-solving tasks is important in providing opportunities for students to think more deeply. Many of the open tasks on the NRICH (University of

Cambridge), AAMT (2020) and reSolve websites (AAS & AAMT, 2017) are a good starting point, providing tasks that provide opportunities to conjecture, generalise and justify, to develop a culture supporting reasoning in their classes (Kilpatrick et al., 2001). However, for these tasks to be effective a teacher's ability to use appropriate prompts (such as those available in the resolve Teachers' Guide) to elicit further reasoning, is also important (Martino & Maher, 1999). The teaching approaches offered in these documents have potential to overcome many of the challenges of assessing reasoning. "If students are consistently expected to explore, question, conjecture and justify their ideas, they learn that mathematics should make sense rather than believing that mathematics is a set of arbitrary rules and formulas" (Reys, Lindquist, Lambdin, Smith, Rogers, Falle, Frid, & Bennett, 2012, p. 97).

The Australian curriculum (ACARA, 2017) some Year Level Descriptions mention *communicating* reasoning through various modes of communication are appropriate – verbal, drawings, written, symbolic which is consistent with the research literature to clarify their own thinking about their reasoning (Brodie, 2010), for example Foundation, Level 1, Level 2, Level 3 and Level 4. *Analysing* involving comparisons, processes and strategies (Pedemonte, 2007) is evident in the Level 4 Description. Reasoning required to interpret and evaluate others' representations, conjectures, explanations (Pedemonte, 2007) is evident in Foundation, Level 1, Level 6. *Justifying*, where strategies and results are presented as evidence (Pedemonte, 2007) can be seen in Level 1 and Level 6. However, the Year Level Descriptors focus on reasoning in particular content areas, for example "reasoning includes investigating strategies to perform calculations efficiently, continuing patterns involving fractions and decimals, interpreting results of chance experiments, posing appropriate questions for data investigations and interpreting data sets" (ACARA, 2017).

The important reasoning action of *generalising* is not emphasised in the Year Level Descriptions. Mata-Pereira and da Ponte (2017) considered generalising to be fundamental to mathematics. Likewise, Carpenter Franke and Levi (2003) stressed the necessity of creating opportunities for students to explore, generalise, and form and test conjectures.

One aspect of effective teaching is teacher knowledge of the content they teach (Darling-Hammond, 2000). There has been much written about teacher knowledge of reasoning (Clarke, et al., 2012; Herbert et al., 2015; Loong et al., 2017; Hilton et al., 2016). Clarke et al.'s (2012) teacher survey revealed that "many students appeared to have little experience in the opportunity to conjecture, justify and generalise, or certainly to articulate these processes verbally or in writing" (p. 30). In order for teachers to better understand the AC: M reasoning statements a more complete understanding of the complexity of mathematical reasoning would be required (Stylianides, Stylianides, & Shilling-Traina, 2013). Perhaps as Jacobs et al. (2010) suggested professional development, emphasising the development of noticing students thinking is likely to be the key to strengthening teachers' knowledge of reasoning so that they may be able to provide their students with opportunities to reason and through these experiences cater for the assessment of reasoning, but what form should this take? The reSolve site has potential to support a teacher's individual professional learning about reasoning. Additionally, Herbert and Bragg (2020) suggested that planning together in peer learning teams with peer-observation of lessons might be effective.

Conclusion

Whilst Mathematical reasoning now has higher prominence in the Australian Curriculum with the expectation that it will be embedded in all topics, little guidance is given regarding assessment of reasoning. This paper has highlighted some of the challenges faced by primary teachers when assessing mathematical reasoning, such as knowledge of the

complicated nature of reasoning; noticing and interpreting students' reasoning; students' ability to explain their reasoning; limited guidance in curriculum documents; inadequacy of work samples; and challenges related to the tracking and reporting student progress in reasoning.

Given the renewed interest in mathematical reasoning in curricula, such as the Australian Curriculum, where it is listed as one of the four key ideas to be embedded in all content areas, teachers need sufficient knowledge and experience in the difficult task of assessing reasoning. This study has revealed the many challenges primary teachers experience when attempting to assess mathematical reasoning and builds on the previously identified challenges by confirming those already in the literature (Clarke et al., 2012; Herbert et al., 2015; Loong et al., 2017; Llinares, 2013; Jacobs, Lamb, & Philipp, 2010; Francisco, & Maher, 2011) and adding to the list new challenges not previously identified. Another opportunity for future research is further clarification of the role played in planning and task selection to enable students to develop reasoning (Davidson et al., 2019).

This study confirmed challenges previously identified in the research literature: Teachers' knowledge of reasoning (Clarke et al., 2012; Herbert et al., 2015; Loong et al., 2017; Hilton et al., 2016); Teacher noticing of reasoning (Jacobs et al., 2010); Students' difficulties in articulating their reasoning (Bragg et al., 2016). Building on previous literature we also identified four additional challenges to assessing students' mathematical reasoning not previously reported. Assessing progress in reasoning; Inadequacy of work samples; Challenges in tracking and reporting student progress in reasoning and Lack of direction/support in curriculum documents. Identifying these challenges begins the conversation about strategies to overcome them.

The teachers in this study have also suggested possible ways of overcoming these challenges. Their role in the design of the *Special Topic: Assessing Mathematical Reasoning* on the *reSolve: Mathematics by Inquiry* website (AAS & AAMT, 2017) ensured that the resources provided are suitable for busy teachers to assess reasoning. The Teachers' Guide (AAS & AAMT, 2017) explains three main reasoning actions, analysing, generalising and justifying and suggests appropriate enabling and extending prompts teachers could use to foster students' reasoning. The detailed rubric provides further support for teachers in broadening their knowledge of the nature of reasoning, whilst the short version (See Figure 1) could be used on-the-run during class. Teachers wishing to know more about assessing could access the exemplars of annotated work samples (AAS & AAMT, 2017). In general, many of the rich tasks on reSolve website (AAS & AAMT, 2017) are a useful starting point as there is potential in them for teachers to draw out students' reasoning using the suggested enabling and extending.

Whilst the reSolve Special Topic: Assessing Mathematical Reasoning goes some way to assisting teachers with this tricky task of assessing students' development in mathematical reasoning, further work needs to be done in exploring effective strategies to overcome these challenges. Assessment of reasoning needs to be easier to do and more successful in identifying progress in students' reasoning.

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