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Improving Aboriginal numeracy: a book for education systems, school administrators, teachers and teacher educators

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Improving Aboriginal Numeracy

A Book for Education Systems, School Administrators, Teachers and Teacher Educators

Thelma Perso PhD

A MASTEC Publication
Improving Aboriginal Numeracy

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Preface

In this book the term 'Aboriginal' is used to refer to all Indigenous Australians who identify as Aboriginal. By contrast, the term 'non-Aboriginal' is used to refer to all other Australian people, regardless of their cultural origins. It is not the intention to put either group 'in a box' with a label or to make generalisations on that basis. Rather, this distinction is used to help in understanding differences between the two groups. The collective 'we' is most often used to mean 'teachers'; and in particular, teachers of mathematics.

As a non-Aboriginal person, I make no claim to fully understand the Aboriginal people and their culture. References and quotes are given in an attempt to understand and to assist others in their understanding, despite the fact that a large proportion of the literature on cultural differences concentrates on remote Aboriginal societies. Through these we can at best build on our understanding and therefore respect the culture of our Aboriginal students and the communities from which they come.

This book in no way attempts to synthesise all perspectives, but rather provides as many different perspectives as possible, primarily from the research, in order to assist education systems and teachers to appreciate the diversity of Aboriginal cultures and understandings. In so doing, we can attempt to make connections between these and the Western Mathematics presented in Australian schools; building a strong foundation for improving the numeracy of Aboriginal children.

In tackling this project I soon became aware of the enormity and difficulty of the task and at times almost gave up in frustration. This however, coincided with a growing sense of awe as I learned to appreciate the size and complexity of the 'gap' between what children from Aboriginal and non-Aboriginal backgrounds know, and what is often taught in schools. This resulted in me persisting with the task even though I knew I could not do it justice.

It is my hope that this work will, at the very least, provide a starting point, enabling further research and discussion, and then be built upon in order to make our schools and teachers truly inclusive in supporting real improvements in the achievement of numeracy outcomes by all children across Australia.

For many years Governments and education systems have been concerned with the widening gap between Aboriginal and non-Aboriginal numeracy achievements. Recent national testing has indicated that the situation does not appear to be improving despite a plethora of programs being implemented across Australia. In Western Australia the gap between the percentage of Indigenous students meeting the national benchmark and non-Indigenous students meeting the benchmark ranges from 31% in Year 3 to 44% in Year 7.

Research findings are documented and reports are written, but little of any practical use seems to filter through to classroom teachers. I believe that the numeracy outcomes of Aboriginal children will only improve when teachers of Aboriginal children take three things into account in their planning and teaching:
1. Aboriginal people; their culture and their transition into schools of the dominant culture;
2. The mathematical understandings brought into the classroom by Aboriginal children; and
2. Explicit mathematics teaching required by all children in our schools.
These three focus areas must overlap to form a 'layered' approach to the teaching of Western Mathematics as follows:

The above model depicts the focus of this volume. Each circle is discussed from a research perspective – what is known about each of these focus areas. It soon becomes obvious that there are many ‘gaps’ in the research. Since teachers, I believe, need to have an understanding of each of these areas it makes sense that funds should perhaps be directed toward researching these ‘gaps’ in order to increase our understandings and subsequently better support classroom teachers. Teachers should not, however wait until the ‘gaps’ in our knowledge are filled. Rather we should pay attention to what we do know from each of these focus areas, adding to this knowledge through our own action research and knowledge of the children we teach.

In my attempt at collating the available research I have applied understandings that I have gained from more than 20 years in the classroom as a teacher and researcher of Western Mathematics, and included information gained from people deeply involved with and committed to improving Aboriginal education – including Aboriginal people from a variety of Aboriginal cultures.

My attempt has been as comprehensive as possible and any omissions should not be viewed as deliberate, but rather as resulting from physical constraints such as access and resources. This volume should not be viewed as containing the answers to the question: “How do you improve Aboriginal numeracy?”, but rather as a starting point to generate discussion and further research; and to assist teachers and other stakeholders in addressing the problem with what little is known to this point.

This study arose from an attempt to understand ‘the gap’ between Aboriginal and non-Aboriginal numeracy achievements as revealed through national testing programs. I have found the task of researching this topic extremely enlightening. Despite only scratching the surface I have learned much about the Australian Aboriginal people, their cultures and their ‘ways of knowing’. I have also learned a lot about myself; my attitudes, beliefs and the values I hold about Western Mathematics and society.

I now believe that the notion of ‘closing the gap’ is arrogant and misguided. It carries with it a sense of “let’s do what we can to make these people like us” – in other words, this closing of the gap is about Western society remaining constant and Aboriginal society moving along the continuum to join it.

My personal view is that, until we as a society can view ‘closing the gap’ as being one of both groups – Aboriginal and non-Aboriginal – moving closer together and learning from each other, little will be accomplished in this area. There is a need for teachers in our schools to move beyond
‘tolerance’ to ‘respect’. This will only occur if teachers learn to understand the Aboriginal children (and children from other minority groups) that they teach.

Western Mathematics is laden with the values of Western society and I believe that, by teaching it to Aboriginal children, there is a danger that we are compromising the Aboriginal culture and particularly their ways of knowing. Research is urgently needed in this area.

Finally, when I consider the obstacles faced by Aboriginal children in Australian schools of the dominant culture I am astounded that the ‘gap’ is so narrow! I am filled with a sense of awe and incredulity that Aboriginal children perform so well on numeracy tests written by Westerners to test the facility of Aboriginal children with Western Mathematics. I admire them tremendously.

Thelma Perso
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Chapter 1
Introduction

1.1 Inclusivity

The underlying reason for this work is the principle of inclusivity. The Western Australian Curriculum Framework (1998) states that

Inclusivity means providing all groups of students, irrespective of educational setting, with access to a wide and empowering range of knowledge, skills and values. It means recognising and accommodating the different starting points, learning rates and previous experiences of individual students or groups of students. (p. 17)

Aboriginal students have the right to access the same education that is available to non-Aboriginal children in Australian schools. At present, Aboriginal children have access to the same curriculum that non-Aboriginal students have access to but it may not empower them. This might be for a variety of reasons: their different starting points, learning styles, underlying issues which are impediments to learning, and in particular, their previous experiences may not be understood by their classroom teachers.

A non-Aboriginal teacher in an early childhood class may assume that all students in their class have had similar pre-school experiences; that their parents immersed them in 'counting' experiences such as counting buttons being done up on a shirt or counting the number of weet-bix as they were placed in the child's bowl. They may assume that the child sat on the floor and played with blocks and toys and learned about shapes and textures. These experiences may not be present for many Aboriginal students (just as they may be missing for some non-Aboriginal students) and will undoubtedly impact on the ability and rate with which a child may learn to count and become familiar with shape and orientation.

It is important that we consider the impact of some of these differential experiences, bearing in mind that our schools and teaching practices aim to be inclusive. Since this is truly our aim we need to find out as much as we can about the individual experiences of children in our mathematics classes and in particular, the learning outcomes these experiences have impacted on for every child in our care.

1.2 Teaching the 'Whole Child'

Over recent decades the practice of teaching has changed dramatically. Where once the emphasis was on teachers teaching a subject or facts and skills to children, teachers are now being asked to teach children. What does this mean? It means that teachers must be inclusive in their teaching practices, taking into account what each child brings into the classroom before deciding what and how to teach. It means, as far as possible, finding out what each individual child already knows before planning teaching programs. In short, it means teaching the 'whole child' and taking into account everything that makes a child who they are: the child's culture, social background, home environment, language, and learning experiences – past and present.
The theories of Constructivism upon which many modern approaches to teaching are built, insist that children construct their own knowledge through actions on knowledge and ideas. Learners are constantly developing and 'constructing' their own knowledge through ordering, sorting, categorising information, interpreting and evaluating. The role of the classroom teacher then, is to have an understanding of 'where the child is at' in order to create a learning environment in which the child can engage in this continuous construction.

For teachers working with Aboriginal and non-Aboriginal students, to be inclusive and provide this environment, they must understand how the people of both culture groups construct their knowledge. Generally, the teachers in Australian schools are themselves non-Aboriginal and hence understand little of the 'starting points' of the Aboriginal children they teach. It might be fair to say that many non-Indigenous teachers inadvertently understand the backgrounds of their non-Indigenous students because they are similar to their own.

Teachers need to try to understand what it means for a child to be Aboriginal. There are many Aboriginal people living in Australia. They live in many types of communities in both urban and rural, in remote and isolated locations.

Many non-Aboriginal people assume that the only difference between Aboriginal people living in urban areas and those living in remote, isolated locations is the colour of their skin and their socio-economic status. Some even hold the view that Aboriginal people living in urban locations are not really Aboriginal at all.

Ngarritjan-Kessaris (1995) states that "these perceptions are based upon outward appearances such as an urban lifestyle and a seeming lack of Aboriginal language and culture". He goes on to say

Culture is more than dance, dress and artifacts. Culture has more to do with how people view the world and how they structure their lives according to that world view. It is more about how people relate to each other and to their physical and spiritual environment. Culture is about the way people do things. (p. 117)

Some people may identify themselves as Aboriginal even though they may not obviously look Aboriginal in appearance. Burney (1982), states that

being Aboriginal is not the colour of your skin or how broad your nose is. It is a spiritual feeling, an identity you know in your heart. It is a unique feeling that is difficult for a non-Aboriginal to fully understand. (pp. 10-15)

As teachers, we often see the behaviours, methods and practices of Aboriginal children, but not know or understand the attitudes and world-view experiences that underlie them. We may judge behaviours based on our standards and values, without first teaching and/or ensuring that these are known.

If we are to teach the 'whole child' we must first attempt to know the whole child and to try to 'see' as the child sees, so that we can respect and better understand their 'way of knowing' and to use this in developing the teaching and learning program. 'Ways of knowing' will be discussed later. It should be emphasised however, that merely understanding a white, middle class 'way of knowing' does not mean that white, middle class teachers understand each white, middle class child in his or her class. Similarly, understanding and accommodating Aboriginal 'ways of knowing' does not mean they can be generalised for all Aboriginal students because of the cultural diversity which exists across Australia. They will however, provide greater insight into an Aboriginal 'whole child' than currently exists.
1.3 What do Aboriginal people want for their children?

Harris (1988) states that Remote Aboriginal culture, and western culture(s) are two of the most diverse culture groups on earth. In the context of these two fundamentally different world views, Aboriginal formal schooling is facing a major dilemma, that while most Aboriginal parents have a deep desire for their children to 'grow up' Aboriginal, they also want their children to succeed in Western schooling and to perform successfully in some aspects of white culture, thereby reducing dependency on white expertise in their communities.

He sums this up in a later statement: "Aboriginal parents (want their children)...to learn the three R's and to grow up Aboriginal". (1990, p. 137)

Aboriginal education is inherently a bi-cultural undertaking. Ovington (1994) refers to this enterprise as "both-ways of education" (pp. 29-30). It is an attempt to provide educational programs for Aboriginal people that both satisfy their aspirations for dominant culture access, while at the same time maintaining their cultural integrity.

'Both-ways' or 'two-way schooling' is schooling that has both Aboriginal and non-Aboriginal input. It is about negotiation - teachers, parents, school administrators and students taking part in discussion about what each party wants to achieve from the formal and informal learning experiences of 'school'. It should however, be recognised that in spite of the positions stated in papers and made by leading Aboriginal bodies and spokespersons, there may be individual Aboriginal people who don't value the education provided by the Australian government school system and don't necessarily want it for their children. Despite this however, they are legally required to send their children to school. There needs to be flexibility for 'two-way' schooling so that individual needs and requirements can be negotiated and consequently accommodated at the school level. Aboriginal Education Officers in schools, must be involved in this process.

In considering the question "What constitutes good teaching for Aboriginal and Torres Strait Islander students?" the Schools Council responsible for writing the 1992 project paper entitled Aboriginal and Torres Strait Islander Education in the Early Years, relied on the Charter for Teaching, defined in Australia's Teachers: An agenda for the Next Decade in arguing that good teachers will value different learning styles, have a sensitivity to the context as well as an understanding of how students learn. They conclude with "Good teachers will be good teachers of any and all students" (p. x).

In schools that have predominately Aboriginal children, it is more appropriate and possible to have a 'true' two-way approach to the school programs and curriculum. At Yirrkala and the Laynhapuy Homelands there is a group of Aboriginal schools run by the Yolngu community in the Northern Territory. The Aboriginal and non-Aboriginal people have worked together to produce curriculum documents which both groups agree with and which have been successfully implemented for well over a decade.

In short, schools "should provide for the skills and knowledge from both cultures (Aboriginal and non-Aboriginal) to be learned, all involving a source of knowledge, a style of doing things, and learning context which authentically match each body of learning". (Harris, 1990, p. 4)
1.4 Aboriginal children and numeracy

Much research has been undertaken in recent years to examine and subsequently improve numeracy levels of achievement of Australian children and in particular, of Aboriginal children. One such study was conducted for the Commonwealth by the Australian Association of Mathematics Teachers (AAMT) for the Department of Education, Training and Youth Affairs (DETYA) in 1998. The study Indigenous Students and Numeracy (ISAN), attempted to address the issue of poor numeracy by Aboriginal students by undertaking action research focussing on programs used by five teachers of Aboriginal students around the country. Their task was to examine the question "What makes the difference?". One of these studies was carried out by Pam Sherrard at Kununurra District High School, in the far north of Western Australia. Pam's acclaimed work focused on the hypothesis that Aboriginal students will improve numeracy skills through good, explicit teaching of mathematics in the early years. I will be referring to this study (What Works? DETYA, 2000) and its findings in the early part this book.

It is my belief however, that in order for Aboriginal children to become proficient with Western Mathematics it is likely that their world-view may be lost to some extent. The reasons for this will become more apparent through the reading of this book.

1.5 Numeracy and mathematics

The term 'Numeracy' appears in a schooling context, to have come to mean 'Mathematics across the curriculum'. For example a student may learn a mathematics skill such as addition in a mathematics lesson and then use that skill to add the number of seeds collected or germinated in a science lesson. This connection is vague and tenuous.

The Curriculum Framework for K-12 Education in Western Australia (Curriculum Council, 1998), states that "Numerate behaviour can be thought of as the disposition and competence to use mathematics in the service of endeavours other than mathematics". It continues:

Numeracy is linked to 'what mathematics you know' but it also involves the skills, thinking processes and attitudes needed to choose and use mathematics outside mathematics. In this sense, numeracy is about practical knowledge that has its origins and importance in the physical or social world rather than in the conceptual field of mathematics itself. (p. 215)

This last sentence indicates that numeracy varies according to culture; it is a cultural construct. For example, when transferred to a remote Aboriginal community, a numerate non-Aboriginal person may find he is no longer numerate. Similarly, a numerate urban person may be innumerate in a rural environment. This is a fundamental tenet of this volume; Aboriginal children may very well be numerate in their own socio-cultural setting, but not necessarily in the white Australian setting and especially not in their performance on Numeracy 'Benchmark' tests, despite the attempts made by developers of such tests to be inclusive.

For the purpose of this volume we will assume that because Aboriginal people in general want their children to have Western education, they also want their children to be numerate in a Western context. Subsequently, the term 'numeracy' will refer to being able to cope with the mathematical demands of life in a westernised community. We should be aware however, that numeracy is about the 'maths we need' and that it is artificial to create contexts in which to use the mathematics that is learned.
Detailed studies into the mathematical needs of Western society have revealed that numerate students need to have developed the ability to:

- Read numbers and count
- Tell the time
- Pay for purchases and give change accurately
- Carry out necessary calculations, including mentally, with appropriate calculator use
- Estimate and approximate in number and measurement
- Weigh and measure
- Use geometric concepts and terminology
- Understand timetables and simple maps and charts
- Read, interpret and construct tables and graphs
- Use mathematics to make predictions
- Use a range of problem-solving strategies
- Apply mathematical skills confidently in daily life
- Read, comprehend and write in mathematical language

(Numerate Students, Numerate Adults (Education Department of Tasmania, 1995, pp. 11-28)

Some of these abilities are inappropriate for some environments. Numeracy is a cultural construct in that unless the learned mathematics is 'practised' it is not necessarily retained as a skill. For example, consider the numeracy practised in parts of rural Australia: many farmers are still comfortably numerate using imperial units of measure instead of metric units. The demands of their everyday work have not 'forced' them to implement metric units in order to cope with the work they do. In contrast, the food industry made the transition to metric units in a very short period of time because all economic and social transactions demanded it. And further, students (Aboriginal or non-Aboriginal) in remote Australia may be taught how to read a bus timetable (which may be considered part of numerate behaviour) but unless they are constantly exposed to having to use this skill their being taught it will merely be an exposure; students being aware that the skill can be performed.

The point is obvious. Teachers will have to make choices about what is taught and what is not taught in order to develop or enhance the numeracy skills of their students. This will depend on the mathematical demands of the lives and future lives of the students. There may be tensions here; what may not be useful now may be useful at a later stage in the students' lives. Exposure can be important so that at a later stage when the skill is needed, students can recollect being taught or shown and this may make is easier to 'relearn'. This is part of giving children 'confidence' to work mathematically in different situations and environments. The skills should not be practised so that they will be at the fingertips of students in case they need them one day.

It is impossible to teach numeracy. The teacher however, has a fundamental role in enhancing students' numeracy skills. For a child to be numerate they must have the disposition to draw on mathematics. There are two components to this: firstly, they must have a range or repertoire of mathematics skills and understandings upon which to draw; and secondly, they must have the 'disposition' or the attitude that will make him want to draw on that repertoire in the contexts in which it is appropriate to do so.

It is insufficient for a teacher of mathematics to teach children the mathematical skills and knowledge required in order to develop numeracy. The teachers' role is two-fold in developing numerate behaviour; they must explicitly teach the repertoire of mathematics skills from which a child may choose, and they must do all that is possible to nurture the disposition needed to draw on that repertoire by giving children opportunities to gain confidence in risk-taking and choosing and
using mathematical models with which to solve problems across a range of contexts. This disposition is fostered through successful practice in using mathematical skills outside the mathematics classroom.

It follows that opportunities which encourage children to draw on Western Mathematics skills and knowledge, must be provided outside the mathematics classroom. This should be the case in every school but may be even more essential for Aboriginal children living in environments in which Western Mathematics plays a very minor part. This is extremely important and cannot be undervalued, particularly with respect to 'making the difference' for Aboriginal numeracy.

Many teachers inadvertently have lower expectations of Aboriginal children. They make assumptions about the child's capacity to learn or about his/her academic potential. Many educated Aboriginal people attribute their 'success' to teachers who 'believed in them' or who expected the same from them as they did from their other students. Inclusivity in teaching is about providing equitable opportunity through understanding and appreciation of difference.

1.6 Summary: The purpose of this book

This book focuses on Aboriginal numeracy. The following principles underpin the proposed model for improving Aboriginal numeracy:

- That Aboriginal people want their children to attain Western numeracy and Western Mathematics;
- That this 'Western Mathematics' should be contextualised (that is, placed in contexts which assist Aboriginal children in understanding the significance of mathematics in explaining and influencing aspects of their lives) wherever possible;
- That teachers have a desire to be inclusive in their teaching practice;
- That teachers will value different learning styles, including those of Aboriginal children, and will have a sensitivity to the context as well as an understanding of different learning styles;
- That numeracy can be improved through good teaching of mathematics; and
- That numeracy is also improved through an inclusive approach to the teaching of mathematics.

These principles have assisted in the identification of the essential elements required to answer the question: "How can Aboriginal numeracy be improved?". I believe there is a need to focus on three elements as follows:

1. What is good teaching of Western Mathematics?
2. What do we know about what Aboriginal children bring to the mathematics classroom, in terms of their learning styles, cultural understandings and mathematics understandings? and
3. How can a teacher 'fuse' these two elements to create a learning environment which is conducive to fostering improved numeracy outcomes for Aboriginal children?

It is the purpose of this book to examine these three questions but not necessarily to provide exhaustive, comprehensive answers to them. In fact, what we will find is that in attempting to answer these questions enormous gaps in existing knowledge will be revealed. This has significant implications for research.
Chapter 2

Aboriginal Children and Schooling

2.1 Education and culture

Australian children are generally taught in schools steeped in the culture of the predominantly white, middle class teachers who work in them. Aboriginal children bring to these schools a cultural orientation that is not well understood and is often perceived as 'deficient'. Consequently, many non-Aboriginal teachers are not appreciative of the fact that many Aboriginal children must learn to meet two different sets of adult expectations.

Harris (1988) contends that there are marked differences between Aboriginal people and non-Aboriginal Australians in terms of their values, lifestyle, and 'world-view'. He cites the following as examples of characteristics of Aboriginal cultures:

- Knowledge is owned or looked after by particular people (i.e. not publicly available);
- The quality of relationships between individuals is more important than the quantity of things;
- Humans belong to and should 'fit into' the environment;
- A non-secular, 'religious' view of the world; and
- A more 'closed' and 'complete' society.

The National Aboriginal Education Committee (1986) claims that Aboriginal society:

- Is structured around the community;
- Tends not to be materialistic or competitive; and
- Tries to gain a group consensus in decision-making. (p. 11)

These cultural characteristics greatly impact on the education of Aboriginal children who embrace the Western world-view in schools. Non-Aboriginal children who have been reared by non-Aboriginal parents generally have similar values, life-style and beliefs as their teachers. Aboriginal children reared by Aboriginal parents however, may have different views of what is acceptable and appropriate behaviour; and each may view the behaviour of the other party as 'wrong'. Sometimes even the best attempts by the teacher to be inclusive fail, due to a lack of knowledge and/or a failure to understand the cultural background and 'ways of knowing' of Aboriginal children.

In her study on child-rearing practices in Australia, Kearens (1984) identifies some of the following differences between non-Aboriginal cultures and Australian Aboriginal cultures: Non-Aboriginal Australians tend to treat babies as helpless creatures needing all decisions to be made for them. They consider their main task as being one of training their infants in various ways (e.g. toilet, obedience) teaching their children to first understand the word 'no', and then using various forms of punishment and approval to shape behaviours. Children are taught to listen to and tend to simple instructions, help with small tasks and gradually become more and more independent. Very young children will be expected to stay close to a parent or caretaker, children being prevented from attempting tasks which may injure or over-exert them. They spend time mostly with members of their families and increasingly with age peers as they begin kindergarten and schooling. By the time they attend school they are expected to know that as 'good' children they will respect and obey adults, listening to and following instructions, and responding to constraints set by adults.

Improving Aboriginal Numeracy
Aboriginal parents however, see babies as autonomous individuals. The role of the parent is to attend to the needs of the infant on demand rather than making decisions about their needs for the infant. Aboriginal babies are treated by everyone with whom they come into contact, that is the extended family and members of the Aboriginal community, with extreme indulgence. They are not seen as helpless but are permitted to help themselves as soon as they are able to. Kearens (1984) states that the "...caretaking job of (Aboriginal) parents seems to be that of helping the young one to help himself to grow up successfully"(p. 17).

Since Aboriginal babies are not put away in a quiet place to sleep during the day, according to Kearens, they share the life around them. They stay with the group being held in an upright position which enables them to see and interact more easily than when lying flat. They mix widely and socially, and by the age of eighteen months they may be far from the side of their carer, experiencing the environment and exploring widely, gradually increasing their range of experiences. Even in city environments they are able to assess direction accurately in order to find their way home.

Aboriginal children are given more freedom to decide for themselves what they need. This parental attitude allows the children to experiment with and subsequently develop life skills without the verbal and physical restraints placed on non-Aboriginal children. The Aboriginal child's behaviours, be they dangerous or not, are seen to be in the child's hands. Mothers are less inclined to play the role of an adult providing direction; they merely watch and allow their children to develop their own sense of independence and resourcefulness. Children learn through watching and experimenting; there does not appear to be a list of age-related appropriate behaviours. Aboriginal children tend not to attempt activities that are beyond their capabilities since they do not want to be seen as foolish. Using caution and good judgement they will continue to observe and practise until they have achieved their goal.

Generally, Aboriginal children are treated as equals by Aboriginal adults. When they are at home, because of their position of independence, they can say 'no' and make decisions about what they will and won't do. Light-hearted 'teasing' is also used significantly in the learning and teaching process among individuals who have developed close relationships. Aboriginal carers may tease their children who are attempting to try and do new things when they get it wrong. They do this rather than telling them they are wrong, as previously mentioned.

Cultural differences play an important part in shaping the behaviours of Aboriginal children prior to their attending school. The children learn a set of expected behaviours that are appropriate for their home-life and the demands placed on them by the cultural community to which they belong.

2.2 The school system and Aboriginal children

The cultural differences of the Aboriginal people have consequences for both Aboriginal children attending schools, and for the non-Aboriginal teachers responsible for teaching them. Typically, Aboriginal children have less experience with the way things are done in school than their non-Aboriginal peers when they begin formal schooling. They may also be unfamiliar with the school environment.

Michael Christie and Stephen Harris, (1985) in their paper Communication breakdown in the Aboriginal classroom, classify these differences in three categories as described below.
**Phenomenological differences; differences in perspectives, expectations, understandings and interpretations**

In the Aboriginal world people and things have a spiritual quality which transcends space and time. The Aboriginal view of knowledge is related to this in that they believe that individuals have rights to certain knowledge that override the question of objective knowledge. They see knowledge as a commodity which must be handled in prescribed ways.

One consequence of this is that Aboriginal children often hold the belief that non-Aboriginal children in their school have knowledge that is inaccessible to them; they believe that they will never have that knowledge because it belongs to non-Aboriginal children.

The children in Christie's 1984 study believed that the optimum school behaviour was conforming, passive, and carefully independent. They also believed that they would obtain an education through the following means:

i. their mere presence in school ritually endows them with education;
ii. the careful performance of ritualised classroom activities (such as rote learning tables, copying from the blackboard) is efficacious; and
iii. the status of educated individuals is marked by their successful progression through the age grade stages as they moved through school. The individual creative and self-directed effort which is crucial to academic learning, is de-emphasised and, in fact, considered irrelevant.

Aboriginal children, depending on the extent to which they have taken on the non-Aboriginal culture, may believe that knowledge is conferred upon them through their participation in ritual activity. Because they see school attendance as a ritual, they then believe that they will learn merely by being in school.

The children in Christie's study believed that teachers should give them lots of ritualised work to do. If the work was not ritualised that is, if the work required them to think or talk about what they were learning, the work was seen as irrelevant and they did not wish to participate. Similarly, teachers who expected this were viewed as hostile.

Teachers of Aboriginal children frequently don't appreciate these types of behaviours as being grounded in Aboriginal cultures, instead perceiving them as laziness or parental lack of interest. Aboriginal children, and many non-Aboriginal children, may have to be taught that they need to take part in the learning experience by engaging in activities with their mind, not merely their presence. Moreover, they need to be taught how to do this, and also any rules of engagement which may exist in the school and classroom.

**Language differences**

In his work with the Milingimbi Yolngu people, Harris (1980) found a number of important ways in which Yolngu expectations about talking behaviour are distinct from those of non-Aboriginal people. Commitment to do something in the future for example, implies less of a firm commitment than it does to non-Aboriginals. It is also socially acceptable to ignore questions; in the classroom children may find it confusing to answer questions when it is clear that their teachers already know the answers. Similarly, Harris (1987b) found that many Aboriginal children irritate their teachers by asking constant procedural questions in an attempt to build security in a foreign communication setting. For some other cultural groups, questioning may not be used at all, either to clarify situations or as part of educational enquiry.
Yolngu also believe it is offensive to speak forthrightly and strongly. They are more inclined to use a 'white lie' than to meet a request that seems unreasonable. This is not considered 'dishonest' behaviour, but an acceptable way of avoiding confronting behaviour.

Harris and Christie state that "Quite apart from the problems Aboriginal children experience dealing with the surface features of classroom English, their own distinctive world view inhibits the effective understanding of the English culture and language...Aboriginal learners of English will impose their Aboriginal semantic structures upon the new word and construct an individual meaning for it. For example, to a Yolngu child the English classroom command "Try!" might mean "Take a stab at it" rather than "think carefully".

Christie and Harris also state that ".the classroom setting is unique and requires a high level of competence. This assuredly is something about the classroom context which contributes to communication breakdown quite independently from the everyday ability of its pupils". And "...even when children do understand the surface forms of classroom language, the perspectives they bring to the discernment of meaning may give rise to an interpretation very different from the teacher's intention" (p. 39). All teachers need to be explicit in the language they use to teach subject matter. They should not assume that Aboriginal children will share their understanding of the English words they use, nor that they actually know the English words that they use.

There is also the potential for significant differences in word meanings. Malcolm et al (1999, p. 31) point out that 'here' and 'there' have different ranges of meaning for Aboriginal English speakers. They also draw the distinction between 'Home Talk' by Aboriginal people using Aboriginal English to describe such things as family relationships, responsibilities, relationships with the land and shared places and knowledge, and 'School Talk' by Aboriginal students to achieve what is needed to understand their teachers, understand the books, achieve status, gain equality and gain power and self determination (Malcolm et al 1999, p. 24).

Davies et al (1997) state that because students bring their own social and cultural identity to each language interaction, they "...will also respond differently according to the language, culture and power relations present". Power relations which are not equal, for example between a student and a teacher, may cause an Aboriginal student to retreat into their own world. This is also related to differences in individual and cultural learning styles of children (p. 31).

West (1995) found that that silence is used in many ways in Aboriginal society and is very much a part of the Aboriginal communication style. He states that "Aboriginal people are much more comfortable with long periods of silence in conversations than their non-Aboriginal counterparts are" (p. 13). He also points out that it is not rude to remain silent and not answer a question when asked, according to Aboriginal cultural traditions.

West also maintains that the Western courtesy words such as 'please' and 'thank you' are not found in most Aboriginal languages due to other ways of Aboriginal people expressing courtesy, particularly through social and kinship systems.

**Differences in learning styles; which inhibit the teachers' ability to communicate effectively**

Harris (1980) found through his work with the Yolngu that Aboriginal children have different learning styles which relate to their cultural background, than those frequently expected by non-Aboriginal teachers in non-Aboriginal Australian schools. These are summarised as follows:
Yolngu learning is by observation and imitation rather than through verbal instruction, the mode most commonly used in non-Aboriginal Australian schools. Talk has more of a social function than a teaching one;

Yolngu learning is mostly through trial-and-error rather than through a combination of verbal instruction and demonstration; they learn by doing;

Yolngu learning places little focus on the sequencing of skills; the emphasis is on the whole task, not on putting the little parts or bits of the task together;

The common approach to problem-solving of the Yolngu people is persistence and repetition, and this again emphasises the relatively minor role of verbal instruction;

In the Aboriginal learning system errors are not specified by the teacher as this would be an offensive rebuke coming from someone other than a family member. Aboriginal children who are told they are wrong may produce reactions such as swearing and ripping out pages. Christie (1984) found that this was not because they were wrong but that their teachers failed to communicate the goals of the tasks being undertaken.

Harris points out that these Aboriginal learning styles are a reflection of cultural orientation and not of cognitive capacity. Aboriginal children who have been reared by non-Aboriginal parents have demonstrated that they are able to learn in other ways but must be taught these ways first.

Although most of the work by Christie and Harris cited above is based on research findings from their work with the Milimgimbi Yolngu Aborigines of North East Arnhem Land, Judith Kearans, who worked with various Aboriginal groups and peoples from Western Australia, found many similarities (Kearans, 1984). One of these similarities concerned the obedience expectations of the Aboriginal groups.

Since non-Aboriginal Australian children are taught to be obedient and non-Aboriginal Australian teachers expect all children to be so, misunderstandings are likely between teachers and Aboriginal children with regard to 'obedience' expectations. Aboriginal children are not taught to be obedient but grow up following the example set by others and respecting those whom they like and admire. They therefore should not be punished for being 'disobedient'. They should instead be taught to know about the non-Aboriginal Australian expectations particularly if they are being educated to live and work and succeed within the non-Aboriginal Australian system.

Kearens also found that, whereas non-Aboriginal Australian children are taught to listen to adults who are speaking to them as part of being obedient; Aboriginal children on the other hand, may not be taught this and consequently don't see the need to listen and obey the requests of an adult. If they are interested in an activity Aboriginal children will continue to engage with it deeply until satisfied, not paying attention to requests to 'hurry up' or to 'finish soon'.

Shellshear (1983) classified differences between traditional Aboriginals and non-Aboriginal Australians as cultural or social. One of the cultural differences he recognised was that of group orientation: "Aboriginal behaviour is governed by group decisions, and the decisions are usually made by leaders. If the leader is well informed, everything ought to be in order. Children brought up in this kind of culture develop a similar attitude to that of the adults. If a teacher asks a question, for example, "What is 10 + 5?", Aboriginal students are content if one of them knows the answer". (p. 14). This in contrast to the teacher of the dominant culture who normally expects all students to know the answer.

Shellshear also explains that meaningful personal relations are highly valued by Aboriginal people—much more than in Western society. They may not relate to anyone who is unfamiliar. Consequently, until they form a meaningful relationship with a teacher they will often pretend the subject is too difficult for them to understand.
Similarly, Partington et al. (1992) explain that Aboriginal children are 'person oriented' compared with Western children who are 'task oriented'. He goes on to say that "If (Aboriginal) children like the teacher, they will continue to attend (school) regularly and enjoy school, but if the teacher proves to be unpleasant or does not fulfil the students' expectations, they may stay away, or become difficult in class. Being with a person who is significant in the children's lives provides opportunities for them to learn even though the prime purpose for being in their company is social. The learning is almost an incidental outcome of the contact" (p. 149).

Watching rather than listening seems to be an important way that Aboriginal children learn. Gestural communication through posture and expression appear to be important to traditional Aboriginal communities and while this may be relied on less, depending on the degree of interaction with the non-Aboriginal Australian society, the visual attentiveness is important for all Aboriginal groups with respect to learning and communicating. Aboriginal children attending non-Aboriginal Australian schools might need to be explicitly taught that they need to pay attention to speech since the spoken word is important for learning in this environment.

Standard Australian English needs to be taught explicitly to all Aboriginal children attending mainstream Australian schools since even among rural and urban Aboriginal groups whose only language is English, differences of structure, vocab and semantics occur. At the same time, most Aboriginal children have a strongly developed visual sense and observational abilities which could be harnessed by their teachers to improve learning outcomes. For example, the use of looking and observing when learning how to read, and using charts, diagrams, pictures, symbols and cards rather than sounds and words in abstraction.

Ngarritian-Kessaris (1995), an Aboriginal person herself, states that "we are generally careful not to be pushy or to be aggressive with our ideas, or our requests...opposing ideas are put forward in as non-threatening a manner as possible" (p. 120). Aboriginal children often perceive non-Aboriginal teachers as being 'bossy' or 'pushy'. Often this is just their way of maintaining class discipline. It can inadvertently then, cause Aboriginal children to 'switch off', refusing to do any work in the classroom.

Were teachers to fully appreciate the cultural differences between Aboriginal and non-Aboriginal children in a school, both groups of children would benefit. Teachers need to be aware of the attitudes and assumptions underlying the behaviours of Aboriginal children in their classes in order to fully appreciate these differences.

In Australia where currently systems and sectors place great emphasis on an appreciation of Aboriginal cultures, enormous gains could be made were teachers to take responsibility for attempting to understand and hence appreciate these differences.

### 2.3 What is the school's responsibility?

In the context of improving numeracy for Aboriginal students, the role of the school is two-fold:

1. to create and foster a learning environment which is inclusive and non-racist and hence facilitate a conducive attitude for Aboriginal, and indeed all students to draw on their full range of mathematical skills and understandings, and

2. to ensure that teachers of mathematics within the school have an in-depth knowledge of the mathematics they are required to teach and an understanding of how children learn the mathematics.
Creating and fostering the learning environment

The ISAN study undertaken by Pam Sherrard (DETYA, 2000, p. 95ff) indicated that a significant improvement in Aboriginal numeracy was made through the collaborative work of parents, the Aboriginal Islander Education Officer (AIEO) in the school, mathematics support teachers and classroom teachers. The AIEO liaised with the Aboriginal community and provided mentor support for both the students and their teacher. The close working partnership between the teacher and the AIEO over an extended period of time allowed the teacher to learn a great deal about the children and their community. As a consequence the teacher was supported to develop strategies which were more effective for the group and for other Aboriginal students across the school.

This liaison helped to build trust between the Aboriginal community and the classroom teacher. The other benefit was that the teacher learned more about the Aboriginal students and the community from which they came; their perspectives, learning styles, and attitudes as described above. The fact that the AIEO was also frequently in the classroom where the teacher operated with the students, meant that an element of trust pervaded the classroom and the attitude towards the learning of mathematics was enhanced. The AIEO was able to provide feedback to the classroom teacher which further enhanced her understanding of the Aboriginal perspectives and attitudes.

The school in which this study took place, implemented a mathematics homework class for its Aboriginal students. The AIEO was able to use strategies observed in class time by the classroom teacher, with the Aboriginal students during the homework period. She also learned much about the mathematics being taught, and consequently was able to further assist the students in the learning of the subject matter.

The AIEO encouraged parent participation in the program, organising and facilitating parent visits to the school to allow parents to watch their children participate in mathematics classes. Students shared their learning with their parents, thus empowering parents to participate in their children's learning. This also helped in educating the Aboriginal parents to see the importance of their children attending school regularly. Christie (1988, p. 14) saw examples of Aboriginal people seemingly not interfering with the personal freedom of other Aborigines. This, when considered with information previously documented on parents encouraging the independent behaviour of their children, could be another reason for the absenteeism of Aboriginal children not being seen as a problem by their parents.

Attention should again be drawn to the importance of relationships to the Aboriginal people; those not attending cultural ceremonies or funerals for example, may be viewed as showing a lack of respect for people and family groups from Aboriginal cultures. Should these occur regularly within any community it is not surprising that many Aboriginal children do not attend school as regularly as their teachers might wish.

Other teachers participating in the What Works? project (DETYA, 2000) attributed part of their success to the involvement of the communities; the projects were connected to the local context with a level of community understanding and support. They too were able to make use of para-professionals, these taking on an active, leadership role among their peers and in the community. In each case, "the learning environments served to build students' confidence in mathematics, and therefore their enjoyment, perseverance and willingness" to take risks (p. 7).
Ensuring that all teachers of mathematics have in-depth knowledge and understanding of the subject matter

If Aboriginal numeracy is to improve then it is imperative for teachers of mathematics to have a sound understanding of their subject matter and of the ways in which it is learned by Aboriginal (and indeed all) students.

2.4 Implications: Maximising learning for Aboriginal children in school

There is clearly a need for more research in these areas – research concerning Aboriginal children from other locations from within Australia than those given above, and of children from urban and rural locations. It would be unfair and not inclusive to generalise the behaviours identified through the above research across all Aboriginal peoples and cultural groups.

The extent to which the behaviours and practices of Aboriginal people as described above are exhibited and manifested through Aboriginal people groups depends largely on the degree of Aboriginal integration with non-Aboriginal people and the need to take them on board because of the environment. For example, some European Australians still use imperial measurements having not needed to learn metric measurements because they don't use them in their work, home or leisure time. This of course varies markedly depending also on the location of individual families and their proximity to the Aboriginal community to which families belong. It also depends on the size of that Aboriginal community and on the degree of choice by the Aboriginal people of the extent to which they embrace non-Aboriginal, and particularly Western, cultures within the home.

With this in mind it would be fair to say that many Aboriginal children are likely to have some elements of the behaviours and practices described above. The implications of these for schools and teachers of Aboriginal children have been grouped and summarised as follows, bearing in mind the ‘two-way’ schooling agenda and that Aboriginal people on the whole want the education of the non-Aboriginal cultural groups for their children:

Schools and teachers of Aboriginal children should in no way expect Aboriginal children to behave in the ways listed below nor assume that they must themselves act in these ways. To be truly inclusive, teachers need to learn and know about each individual child and not stereotype children in any way.

Teacher-Student Relationships

♦ Teachers teaching Aboriginal children will need to work hard to establish a strong relationship with the children. Successful teaching and learning for Aboriginal children depends largely on the personal relationship of trust established between child and teacher. This is imperative and clearly the most important key to any future learning.

♦ Aboriginal children may need to 'know' the adults whom they have contact with in their school; a teacher may need to explain who they are – who their parents are, who their brothers and sisters and cousins are – where they live and where they lived before that.

♦ Aboriginal children may refuse to participate in activities if they perceive the teacher as being 'too bossy'. Teachers may need to either use different classroom management styles, or at the very least, explain to their class that this is part of their management style so that the children will know what to expect and learn that what might be perceived as 'bossiness' is not personal.

♦ Aboriginal children may not depend on adults to the same extent as non-Aboriginal children. For this reason teachers should foster children offering help and requesting help from each other.
Aboriginal children may need to be taught to listen to an adult who is speaking to him or her, and to obey the requests of an adult. This may only happen when trust exists in the relationship. (It should be pointed out however, that some Aboriginal children may look like they are not listening when in fact they are).

For students who are often exposed to racial taunting, appropriate humour by the classroom teacher is a powerful means for avoiding conflict and preserving harmony. A teacher who can laugh at himself or indirectly at his own culture and ways of doing things contributes to solidarity of the group and at the same time sends indirect messages to students that no one is superior to anyone else.

Teachers teaching Aboriginal children should be aware that for some of these children, the teacher pointing out errors made, may be viewed as offensive behaviour. There may be other ways of showing an Aboriginal child that he or she is wrong other than directly being told. Teachers should also be aware that if they fail to clearly communicate the goals of a task to a child then it is unfair to rebuke a child if the teachers' expectations are not met.

Classroom Behaviours

If you wish Aboriginal children to learn and use Western forms of cultural language such as 'please', 'thank-you' then you may have to teach these explicitly as 'school way' language.

Aboriginal children may need to be made aware of the 'obedience expectations' of a classroom and/or a school, what are the rules and why are they needed?

Aboriginal children may need to be taught to respond to instructions in appropriate ways (as designated by the classroom teacher) regarding the use of time. For example, what should they do if a teacher asks them to 'hurry up' or tells them 'there is five minutes to go'? Similarly, Aboriginal children should not be hurried to complete a task since this pressure can lead to anxiety.

Aboriginal children may need to be taught as part of the school or classroom rules, what it means to make a commitment to do something in the future. For example, an Aboriginal child may not know what is meant or required when told that some homework is due the following day. This may need to be modelled by the teacher and other students before it can be made an expectation.

In a non-Aboriginal classroom Aboriginal children may not be free to decide for themselves what they need in that environment – in most cases the teacher will decide what they need.

In a classroom environment verbal and physical restraints may be used and these will need to be taught to the children.

Learning Styles

Aboriginal children may need to hear instructions repeatedly or receive them in different modes, before they 'sink in', being used to learning through the sense of sight may mean they are not used to learning through hearing.

Aboriginal children need to be made aware that they need to pay attention to speech – watching isn't sufficient – but the strong visual sense that many Aboriginal children have could be harnessed in some way by teachers to enhance their learning.

Aboriginal children may need to be made aware of how to learn and engage in different ways appropriate for Western schooling since merely watching and experimenting may be insufficient. Teachers should be aware that Aboriginal children do know how to learn, it is the role of the teacher to learn how Aboriginal children learn best and adapt to that.

Aboriginal children may need to be taught how to take risks and learn through incremental behaviours rather than waiting and observing until they themselves believe they are capable of effectively undertaking a task.

Aboriginal children may need to be made aware how to break up a task into little achievable bits where appropriate rather than approaching it holistically.

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Aboriginal children may need to be made aware of the fact that knowledge is learned and not innate; that knowledge is accessible to them and that they are capable of gaining and understanding knowledge, just like any non-Aboriginal child.

Aboriginal children may need to be taught that performance of ritualised classroom activities does not imply learning.

Aboriginal children may need to be made aware that individual creative and self-directed effort, though perhaps coordinated within the group, is crucial to academic learning and that successful movement through year grades at school is irrelevant.

Aboriginal children may need to be taught that their mere presence in school does not ritually endow them with education - they may need to be taught how to learn in different ways.

Parents of Aboriginal children may need to be informed and convinced that it is insufficient for their children to attend school on an intermittent basis - that there needs to be continuity in the learning program.

Aboriginal children may need to be explicitly taught how teachers think, and that to provide regular feedback to their teacher they need to talk about what they are learning, showing their teacher that they are engaging with their mind and not merely their presence.

Teachers teaching Aboriginal children should encourage the children to talk about what they are doing or what each other has done.

Aboriginal children are often extremely more practically competent than non-Aboriginal children due to their greater independence in the home. Teachers should try to use real life practical activities as much as possible to capitalise on this.

Teachers teaching Aboriginal children should attempt to couch all learning in immediately meaningful contexts.

Aboriginal children may be reluctant to take risks publicly, they like to try out new things privately or in groups and prefer to seek help from peers rather than the teacher; so teachers should not demand that Aboriginal children read aloud or give an answer in front of the class.

Teachers teaching Aboriginal children should be aware that these children may require many 'hands on' activities to assist learning since they may learn better by doing.

Teachers teaching Aboriginal children could perhaps harness the skills of some Aboriginal children in persistence with problem solving.

**Questioning**

Teachers teaching Aboriginal children may need to use specific questioning strategies to overcome the possibility of Aboriginal children refusing to answer questions where it is clear that the teacher already knows the answer. The teacher could say for example, "I know the answer to this question but I want to find out whether you know the answer", or perhaps more appropriately, to offer a range of questions from which the Aboriginal students can choose. (Aboriginal children tend not to like direct questions where pressure is placed on them to provide an immediate answer - open ended questions where they are given more time to respond, are better).

Aboriginal children may need to be given explanations as to when it is appropriate to ask questions and also, how these questions should be asked.

Aboriginal children may need to have the purpose of questioning explained to them, especially if the purpose of an activity or problem is not self-evident.

When asking Aboriginal children questions, teachers should try and do so in an indirect manner (not direct or forthright) and allow them time to consider whether they can answer without shame, or remain silent if they wish.
Language and semantics

- Teachers teaching Aboriginal children should not assume that the children will share their understanding of the English words they use or that they actually know the words they use.
- Aboriginal interpretation of meaning for teacher commands may vary depending on the perspectives they bring to the classroom. Teachers teaching Aboriginal children may need to speak explicitly to Aboriginal children when giving instructions since some Aboriginal people believe it is offensive to speak forthrightly and strongly. For example, say "Think carefully" instead of "Try!".
- Aboriginal children may need to be taught to understand the word 'no' in the context of the classroom and that various consequences and approvals may be used to shape their behaviours.

School responsibilities

- Schools where Aboriginal children attend should attempt to provide para-professionals such as Aboriginal Islander Education Support Officers who can provide a link between the school and the Aboriginal home.
- Para-professionals could assist in the school by being present in classrooms, liaising with parents and other community members, and organising activities such as homework groups outside class teaching time.
- If it is not possible to employ para-professionals, schools should explore other avenues for building links with the Aboriginal community to assist in the transition and help Aboriginal children to feel comfortable in schools of the dominant culture.
- Schools, through para-professionals or other avenues, may need to work with Aboriginal parents to explain why their children should attend school on a regular basis whenever possible in order to maximise learning.
- Administrators of schools where Aboriginal children attend should make it their business to ensure teachers in the school are aware of the cultural differences described above and the implications of these for teachers.
- A 'two-way' approach to teaching and learning should be negotiated whenever there are Aboriginal children in the school if possible, particularly for schools in which the Aboriginal population is the majority.

General

- Teachers need to be aware that between 25% and 80% of Aboriginal children may be affected by some form of hearing loss at any time. Clearly, a 'teacher-centred talk' style of teaching will not be helpful for these children, and possibly not for any Aboriginal children who learn best from watching and doing rather than listening. Be aware that children with hearing problems often use peer observation in class.
- Aboriginal children may need to be explicitly taught that they all need to know certain facts and skills, it is insufficient for only one person in the group to know.
- Teachers teaching Aboriginal children should be aware that the children may respond differently according to the language, culture and power relations present in the classroom.

If Aboriginal children are to achieve adequate levels of numeracy then implications suggested by the criteria above should be met by schools and teachers. Without these the learning environment, which is essential to foster the attitude needed for Aboriginal children to choose to use their mathematics, will not exist. The degree to which it exists depends on the knowledge and skill of the teacher and the support for the teacher received from the school community.
It should be pointed out however, that the above list is neither exhaustive nor comprehensive; teachers should attempt to do the best they can to address these criteria in whatever circumstances they find themselves. Indeed, some of the above criteria are essential parts of 'best practice' for teaching students of any cultural background, in particular the need to be explicit through language. That is, a teacher should not make any assumptions about what their students understand, be it words, concepts, contexts or visual forms. At the same time they should acknowledge that all students bring a wealth of understandings to the learning environment.
Chapter 3
Aboriginal Culture and Mathematics

3.1 Aboriginal ways of knowing about mathematics: Mathematics in the Aboriginal culture

Aboriginal culture and what Aboriginal children bring to the classroom are essential understandings for schools and teachers of Aboriginal children. This has further implications for teachers of mathematics in terms of their pedagogy and more importantly, in terms of the assumptions made by teachers of the mathematical understandings of their students.

The term 'Mathematics' may be vague from an Aboriginal and Torres Strait Islander perspective. Steen (1988) defines mathematics not as the science of space and number but as the science of patterns. "The mathematician seeks patterns in number, in space, in science, in computers and in imagination...Applications of mathematics use these patterns to 'explain' and predict natural phenomena..." (p. 240)

Christie (1996) states that "Mathematics is not a language, nor is it an object. It is a practice: the unseen work done by individuals and groups making sense of their lives, their territories, their histories, and economies through particular discourses which involve naming, ordering, recursion and valuing."

It is widely agreed that mathematics can enhance our understanding of the world and the quality of our participation in society. The mathematics of the Western-technological world however, is filtered by the people belonging to this cultural group. Research has shown us that "Presenting this compartmentalised decontextualised body of Western knowledge to learners with a different world view scheme invites failure for both the learner and the teacher". (Jones et al, 1995 p .2). Harris (1989, p. 91) describes this as the "...wide difference between teachers and pupils in their understanding of the nature of reality and the way they organise the world to find meaning in it...".

Ideally perhaps, in order to minimise this difference, a 'two-way' school will have Aboriginal teachers teaching Aboriginal children. In reality, we know that this is rare and even unlikely, at least in the near future, and so the next best models are teachers from the non-Aboriginal cultures working alongside Aboriginal para-professionals, or non-Aboriginal teachers who are educated in the Aboriginal 'ways of knowing'. That is, at best, making an attempt to understand the way that Aboriginal children organise the world and find meaning in it. Clearly, because of the Aboriginal 'ways of knowing', the mathematics an Aboriginal child learns will always be a different mathematics from that taught by the non-Aboriginal teacher.

We view the world through our own cultural filter. Harris (1991, p. 13) states that "A world view is the way individuals and groups see the world and how they understand that they should behave within it". She goes on to say that "...there are some parts of reality which have been organised to a much higher level of abstraction in Aboriginal culture than in Western cultures, and there are some parts of reality which in Western industrialised culture have been organised to a much higher degree than in Australian Aboriginal cultures. This is part of the difference in world-views. Each
culture develops more vocabulary and higher degrees of abstraction for those parts of reality which they consider most important" (p. 14). It is important to note that this does not make the culture that is more organised 'better' in any way, or those less organised in some way deficient—it merely makes the cultures different.

Sutherland (1999) states that "whereas Western Mathematics appears to be dominated by abstraction from reality, Aboriginal mathematical understanding centres on the functionality of concepts, irrelevant abstractions are not considered important". An Aboriginal child may have a word or language for 'five turtles' or 'five stones' but not a word for 'five', for example. The concept of 'fiveness' is abstract but may be embedded in Western culture through the activities of that culture. More will be said of this in a later chapter.

Christie, in commenting on the relationship between a traditional Aboriginal world view and Western Mathematics, states that "...the Aboriginal world-view provides for the unity and coherence of people, nature, land, and time. In a world which has not been broken down by science and mathematics, the spiritual unities which transcend Western analysis remain primary" (1985, in Harris, 1991, p. 13).

Pam Harris (1991, p. 13) suggests that Aboriginal world views "....are characterised by a very personal view of the universe in which humans are seen as united with nature rather than separate from it." By contrast, the world view dominant among non-Aboriginal Australians is marked by the separation of humans from nature which allows for the "exploitation of nature so that it serves the perceived needs and desires of humanity. This is the world-view that gives rise to the mathematics which is taught in school" (p. 11).

As a result of this, she contends that if Aboriginal people from a traditional Aboriginal community were to set up their own mathematics curricula (uninfluenced by White Australian traditions), they may give highest priorities to kinship relations and to the space strand.

Harris (1990) believes that Aboriginal world-views stem from spiritual and religious beliefs while Westernised cultures have their roots in science. Watson and Chambers (1989) state that the Aboriginal people of Australia use a genealogical pattern to make sense of their world; "By this we mean ordered ways of naming and construing the relationships of natural things according to perceived ancestral or familial linkages" (p. 31). Whereas non-Aboriginal people use number patterns based on counting and measurement, the patterns used by Aboriginal people are based on relationships between people. Number patterns are also used by Aboriginal people as a form of ordering, just as familial patterns are used by non-Aboriginal people; the difference is found in the importance placed on these.

They go on to say that, for Aboriginal people, "...the genealogical pattern explains all relationships in both the social world and the world of nature. Furthermore...the number system is involved in Aboriginal life only secondarily. Little hangs on the functioning of number, which is to say that number does not carry the deterministic weight nor the aura of objectivity and inevitability that it carries in non-Aboriginal Australia.

"In other words, in all Aboriginal-Australian communities the genealogical recursion, not the numbering system, patterns both knowledge and behaviour and carries the productive processes of the social order" (p. 32). This is difficult for a non-Aboriginal person to understand. I will draw on some of the discussion presented by Watson and Chambers (1989) in an attempt to clarify some of the understandings associated with the Aboriginal 'ways of knowing' or 'world-view'.

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The Western number system presents a sort of 'grid' that can be placed over the physical, economic and social landscape in an attempt to order and make sense of the world. The number names enable notions of equivalence and hierarchy that 'fit' with the ideas of economics and competition that dominate Western society. The patterns of Western lifestyles are strongly determined by cycles; routines pervade and organise daily, monthly and yearly cycles. A spatial grid covers the planet and enables every square centimetre to be described and labelled, this allows the landscape to be managed and controlled. The gathering, organising and summarising of data enables predictions about weather or natural disasters. Units used for measurement and the numbers attached to these when counted are powerful elements of trade in negotiation – an important part of Western society.

Contrast this with the way the world is organised by many of the Aboriginal people in Australia. The Yolngu people of Arnhemland, for example, may name a stranger among them with whom they expect to have prolonged personal contact. The name is given in order to 'locate' the person in their system of genealogical pattern. On being named the person then receives continued instruction about their relationships with other people from the group, for example "he is your brother, greet him", or "he is your son-in-law, stay away". The names not only imply emotional involvement as in Western society, they also carry with them an acknowledgement that they are 'someone'. As Watson and Chambers put it "To be a 'real' entity in Yolngu life, a person or place must be named, and thus located within the genealogical order" (p. 36).

A stranger from another people group must have the location from another group translated into their own group, and once the relation between the stranger and one person from the Yolngu has been determined, then the relationship with all other Yolngu is known. "When a non-Aboriginal person is recognised in this way he or she will have established, in one moment, formal family relations and responsibilities involving several hundreds of people and more" (Watson & Chambers, p. 36) and is expected to learn these complex, formal connections as do Yolngu children, being instructed in them from a very early age. The Yolngu people use this system to make connections to all parts of nature and across all time. Thus, the Yolngu live 'close to nature' meaning they have a "precise mathematically articulated relationship" with nature (Watson & Chambers, p. 36).

This gurrutu system of organising used by the Yolngu people is basically a series of names or labels with a recurring pattern. It enables the formal articulation of all types of relationships. Like number names, these gurrutu names from a pattern of language, the basic form of which requires alternation across three generations. The Yolngu divide the universe into two moieties based on the fact that every person has two direct ancestors, a mother and a father. In the Yolngu world, every named thing belongs to one of these two moieties. Thus the gurrutu cycle operates in two dimensions: the moieties and the generations. This system imposes order on the relationships between individuals and groups to each other as well as to the land and to all things in the Yolngu world, determining potential marriage partners for example, and designating parcels of land belonging to individuals and groups. "At a general level it is a formally articulated system of beholdenness; it orders degrees and types of indebtedness" (Watson & Chambers, p. 37). Thus from a mathematical sense every relation has a sort of negative value in contrast to the notions of value in Western society where everything has value in a positive sense based on the way the number system works.

Watson and Chambers, in this comprehensive discussion of the Yolngu gurrutu, describe how "genealogical relatedness, formally articulated in a recursive system of names, constitutes an encompassing pattern of Aboriginal life" (p. 38). Individuals play roles in hierarchies which are woven together in an orderly 'grid' which works in such a way as to emphasise co-operation instead of competition. Thus it enables both the social and the natural world to be ordered.
Jones et al (1995) contrast world-views of Aboriginal and Western cultures under headings of Life and Living, Time, Activity, and Structure of Society. These have been summarised in Table 1.

Table 1: Contrasting World Views of Aboriginal and Western Cultures

<table>
<thead>
<tr>
<th>World view</th>
<th>Aboriginal</th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life and Living</td>
<td>• Life always exists and exists in spirit form</td>
<td>• Life is transient</td>
</tr>
<tr>
<td></td>
<td>• Discussions always intertwine land, life, religion, kinship, and therefore quality is important</td>
<td>• Past is separate from present</td>
</tr>
<tr>
<td></td>
<td>• Discussions separate land, religion relationships as autonomous ideas, and therefore quantity is important</td>
<td>• Discussions separate land, religion relationships as autonomous ideas, and therefore quantity is important</td>
</tr>
<tr>
<td>Time</td>
<td>• Cyclic</td>
<td>• Linear</td>
</tr>
<tr>
<td></td>
<td>• Passage of time is related to events—nature, place and participants are important</td>
<td>• Time is divided into units to be measured as quantities</td>
</tr>
<tr>
<td></td>
<td>• Maintaining meaningful relationships is important</td>
<td>• Events are related to progress, when they occurred is important</td>
</tr>
<tr>
<td></td>
<td>• Emphasis is on shared achievement and working together</td>
<td>• Activity is concerned with setting goals and ways of achieving goals</td>
</tr>
<tr>
<td>Activity</td>
<td>• Concerned with harmonious interactions with people and the environment</td>
<td>• Completing the task is important</td>
</tr>
<tr>
<td></td>
<td>• Maintaining meaningful relationships is important</td>
<td>• Emphasis is on measuring individual achievement</td>
</tr>
<tr>
<td></td>
<td>• Emphasis is on shared achievement and working together</td>
<td></td>
</tr>
<tr>
<td>Structure of Society</td>
<td>• Living and working are synonymous; participation is important</td>
<td>• Work is related to earning money to live</td>
</tr>
<tr>
<td></td>
<td>• Learning is ritualistic e.g., participation is important in schooling</td>
<td>• School is about learning how to gain work and become employable</td>
</tr>
<tr>
<td></td>
<td>• Personal relationship with those in authority</td>
<td>• Impersonal relationship with those in authority</td>
</tr>
</tbody>
</table>

Note: Adapted from Jones et al (1995)

Tables such as these often result in stereotyping and hence it is important to understand that many Aboriginal people do not necessarily have an Aboriginal world-view and, likewise many non-Aboriginal people may have a world-view that is, or is similar to, what is categorised above as ‘Aboriginal’. More will be said of this later.

Implications for pedagogy have already been drawn from some of the differences in world-views indicated in this table in the previous chapter. There are however, other more specific implications for the teaching of mathematics; in particular the Western culture, and hence Western Mathematics, with emphasis on quantity and hence, measurement. Since the ability to quantify rests heavily on the ability to count and hence understand number, it is apparent that the capacity to be able to succeed with Western Mathematics relies heavily on understanding concepts in Number. More significantly, Aboriginal children may need to be shown or provided with a reason for needing to know these concepts since they may not exist in the cultural or economic environment in which they live.

As a result, the teaching of Western Mathematics in schools and classrooms can be problematic for many Aboriginal children. Real understanding of an abstraction comes through application and relevance; this is true for all learning. It is often difficult to provide relevance and application for abstractions for non-Aboriginal children let alone from outside this group.

The tension lies in the desire of the Aboriginal people to have Western Mathematics understandings and knowledge while at the same time not having familiar cultural contexts with which non-Aboriginal teachers can make the abstractions relevant and meaningful. For example, it is pointless to teach skills such as times-tables or reading a clock to Aboriginal children if these can not be made immediately relevant and useful for them.
In effect what this means is that in attempting to teach Aboriginal children the abstractions of Western Mathematics it may be essential to also teach the reason for wanting to know and the context in which it becomes meaningful and relevant! In teaching Aboriginal children to measure exact quantities using a measuring cup for example, a teacher may need to first give the students a piece of cake, and then say "We will now learn how to make a cake like this; it will mean that we have to measure the ingredients exactly with a measuring cup and this is how we do this". Alternately, they could all taste some cake in which too much salt or not enough sugar has been used, and the teacher may say "we will learn how to measure the ingredients so that our cake will not taste like this". Without this context an Aboriginal child may see reading measurements from a measuring cup as a pointless exercise; the abstraction of the 'degree of accuracy' required is meaningless.

This of course, may also be true for a lot of children from non-Aboriginal cultures. It is more likely however, that non-Aboriginal children will be familiar with the notion of measuring accurately, having seen the behaviour modelled within the home and possibly even taking part in similar activities while being at home. As a result the child knows it is a useful skill to have for the present and future and this provides both relevance and meaning. More will be said concerning relevance in a later chapter.

Kepert (1993) in his work with Milingimbi Aboriginal people in the Northern Territory, found that their world-view was one of relationships and not number, shapes or dimensions. As a result, contexts which related to the Aboriginal people and their culture or relationships provided an immediate context. For example, he used information provided by the students about the jobs held by people in the community to build a makeshift pie-graph. With this information in visual form it was then an easy matter for him to lead into a discussion about fraction and percentage. Similarly, Aboriginal teachers were able to understand the concept of 'one quarter' once it was related to the passage of time between sunrise and midday, a quarter of a day. This however, is more about making connections to what is known than about relevance, about which more will be said in a later chapter.

Table 1 above, clearly indicates that family and relationships, places and the land, and participation in living and life activities are of paramount importance in Aboriginal cultures. This is true for most Aboriginal communities no matter where they are located, whether remote, rural or urban. An effective teacher will draw heavily on this fact, using these as the contexts for mathematics teaching and learning wherever possible.

### 3.2 The Language of mathematics

Language reflects and facilitates the expression of the world-view of the people among whom it has developed. For this reason many readers of the above summary of world-views may find it difficult to understand the Aboriginal world-view; English does not allow for the understanding whereas many Aboriginal languages and dialects do.

Malcolm et al (1999, p. 35) states that "The way that a society interprets reality becomes fixed, over time, in its language and as a result, the way human beings represent their world view is 'very much at the mercy of the particular language which has become the medium of expression for their society" (Sapir, 1949, p. 142). But concepts will only have been identified and individually named in that language if they are important to the culture. Consequently, there may be many concepts in Western Mathematics which do not have words to describe them in Aboriginal dialects. These difficulties will be compounded for Aboriginal children learning the language of Western mathematical ideas and concepts without English as a first language.
Let me provide an illustration for this point. Supposing I were to travel to France as a speaker of English (the only language I speak fluently although I have a smattering of French). While in France I enrol in classes of German which are being taught by a French person who only speaks French. I am essentially trying to learn one foreign language in another foreign language.

For many Aboriginal children, this is what it is like when they participate in Mathematics lessons in our schools. Added to this is the fact that many words used in a mathematics sense have a different meaning than when they are used outside the mathematics classroom; words such as 'bigger' in the context of "which number is bigger?" for example. In the mathematics sense we are talking about value whereas outside the mathematics classroom we usually mean relative physical size. It should be pointed out that for many 'English as first language' speakers this is also problematic; hence the statements made in the previous chapter about the need for explicit mathematics teaching where nothing is assumed.

### 3.3 Children and cognitive differences or learning styles – a continuum

The problematic nature of Table 1 above has already been alluded to. It suggests that children can be categorised into either/or 'boxes' or categories. Many children do not belong to one cultural group only but in fact are bicultural.

We may be able to say that if children are Aboriginal then "they are likely to...", but as with any categorisation based on chance, the word 'likely' can be interpreted in many ways and can depend on many variables.

The information contained in Table 1, and other classifications like it, generally show the two extremes of a continuum. They are useful in that comprehending the understandings and behaviours of people at the extreme ends can be enlightening in attempting to understand those from within the continuum. By placing traditional Aboriginal children at one end of the continuum and Western children at the other, we can more easily examine and consider the behaviours and understandings of children between these two extremes. It is unnecessary to even attempt to define what a "Traditional Aboriginal child' might 'look like' or what a "Western child" might look like. Indeed, does either exist?

All children apparently lie somewhere in between these two extremes in terms of their mathematical understandings. The understandings of Aboriginal children will be influenced by the degree to which they, through their parents and wider family, have chosen to integrate into Western society. In a similar way, the understandings of Western children or non-Aboriginal children will be influenced by the degree of education contributed by the child's parents or carers prior to and during formal schooling.

It should perhaps be pointed out again that within the continuum described above is a secondary continuum: that of the Aboriginal people themselves. The South Australian Department for Education and Children's Services (1995) states:

> The lives and experiences of present day Aboriginal people are diverse. At one end of the continuum are those living traditional-oriented lives in remote communities in central and northern Australia who maintain traditional mathematical concepts and language but also use telephones, computers and the dollar system. At the other end of the continuum are the majority of Aboriginal people in Australia who live in cities and towns and who mostly use Western mathematical concepts but maintain degrees of values and skill more related to traditional Aboriginal culture. Many people move along the continuum, changing their focus as the circumstances permit.
In Western culture, it is more likely that children may have been taught to count, for example (as included in the Western, albeit inclusive, curriculum). It is unlikely that Aboriginal children will have been exposed to formal direct measurement using standard units of measurement. Clearly, there will also be many Western children who have not been exposed to an environment where Western Mathematics is culturally embedded.

The implications of this for teaching are obvious: the mathematical understandings of children entering the formal school system should never be assumed based on the cultural background of any child, be they Aboriginal or non-Aboriginal. Teachers should make no assumptions about what children already know.

Similarly, at an even deeper level, teachers frequently make assumptions about the mathematical understandings of their students based on what a child apparently knows. It is easy for a teacher to assume that a child understands counting for example, because a child can verbally say the number names in order. We know that many children can say the words "one, two three, ..." but not necessarily understand the process or concept of counting. This can be evident when a child is asked to start counting at 'five' instead of 'one' and cannot do it; indicating their ability to recite words but not know that to count they must continually add one on to each successive number. More will be said about this in a later chapter.

The continuum as outlined above, is a continuum of 'Mathematical understandings which children bring to the classroom', and all children are at different places on this continuum regardless of their cultural background.

Another factor that needs to be considered by the classroom teacher desirous of being inclusive, is the individual learning styles which children have. Research has told us much about the different learning styles that all children have. Learning styles may also be cultural. For example, it is likely that many Aboriginal children will have learned through their cultural environment that learning occurs primarily through observing rather than listening.

Learning styles as demonstrated in the learning of mathematics, will vary markedly for all children. Some children may learn better by interacting with 'hands on' activities rather than by listening and by being told. Many teachers were brought up in an era when mathematics teaching consisted of the classroom teacher showing students how to do some mathematical computation by way of a few examples (usually done on a blackboard) and students then practising this method until they were proficient at it. For many teachers, this model 'worked for them' and hence they continue to use this model and feel comfortable doing so. Unfortunately we now know that this method of teaching mathematics will only work for a small proportion of children. The use of this model for the majority of children merely fosters a rote, ritualistic method of learning; what children invariably learn are procedures, not mathematical understandings.

This ritualistic, observational model is likely to be favoured by many Aboriginal children due to their experiences as described previously. Teachers should not assume that Aboriginal children who favour this learning style are necessarily learning, but, as with children from other cultures who favour this learning style, teachers need to carefully assess and monitor the degree of conceptual learning (real learning) which is occurring through this process, as opposed to rote or superficial learning.

Teachers may, as a result of this monitoring process, need to teach their students -be they Aboriginal or non-Aboriginal – other learning styles which favour real learning. There may also be a need for teachers to learn these other learning styles themselves in order to teach them to their students.
3.4 Implications for teachers of mathematics

There are some implications of the above discussion for the effective teaching of mathematics in order to assist many Aboriginal children to achieve mathematics outcomes and subsequently, numeracy outcomes:

1. Where a teacher has access to para-professionals such as Aboriginal Education Officers or parents of Aboriginal children, discussions about the mathematics being taught in the classroom should occur wherever possible.

2. The teacher should consider ways of making the mathematics concepts being presented relevant and meaningful through using contexts which are relevant and meaningful.

3. Contexts should be concerned with family, relationships, the land, nature, familiar events and life where possible. If this is not possible, other contexts which make the learning activity meaningful must not be assumed and may need to be taught.

4. Connections for mathematical concepts being taught should be made to the understandings and existing knowledge of children whenever possible.

5. The learning and practising of skills should be conceptualised wherever possible.

6. The language of Western Mathematics should not be assumed and may need to be explicitly taught. Students will need to be given opportunities through working in groups and/or with para-professionals, to discuss any words with which they are unfamiliar in order to clarify and learn essential meanings.

7. Teachers should not separate the written language of mathematics from the symbolic language of mathematics, focussing as often happens on the symbolic. The literacy of mathematics includes both forms and facility with both is essential for numeracy acquisition.

8. Teachers should be aware that all children from all cultures are on a continuum in their mathematical understandings. These differences need to be recognised and valued.

9. Aboriginal children may bring mathematical skills to the classroom which are highly developed when compared with those of their peers, and vice versa. Teachers should celebrate these in order to enhance self esteem of individuals and utilise individuals as part of the learning program wherever possible.

10. Teachers need to recognise that all children learn in different ways. Aboriginal children tend to be effective visual learners, so these strengths should be harnessed and effective pedagogies developed in order to promote such learning.

11. The emphasis on shared achievement and working together is important in Aboriginal culture. This should be acknowledged and incorporated into classroom pedagogy through group work and interactive instruction wherever possible. A method for doing this is described in Chapter 4.

This anecdote by Terry Ngarritjan-Kessaris (1995b), emphasises the contextualisation of mathematics:

At a very early age maths lessons reinforced White Australian cultural 'norms'. For example, lessons on time concepts inevitably showed a typical day for children, starting at 7 a.m. and finishing at 8 p.m. The day started with a family getting out of bed to the sound of an alarm clock. Children always wore pyjamas and they always cleaned their teeth before getting dressed for school. For breakfast children ate toast or cereal and drank milk or orange juice. Only the parents drank tea or coffee. School commenced at 8 a.m. and sandwiches and fruit were eaten for lunch at 12 noon. Classes were dismissed at 3 p.m. and the family ate their evening meal together at 6 p.m. Finally the children got back into their pyjamas, brushed their teeth and were put to bed at 8 p.m.
In our house the children generally went to bed when we wanted to. We were not forced to bed at a certain time. We didn't own toothbrushes (I bought one in high school) and we didn't always wear pyjamas to bed. The quantity, quality and timing of meals varied considerably as did the numbers of people who made up our household. Children drank tea as adults did and we did not go to school every day. The pressure for us to go to school was applied from the school, not from home. If we didn't feel like going to school we stayed home and Mum never forced us to go. Overall it seemed that time just flowed for us; it wasn't chopped up daily into distinct periods according to fixed events. I learnt early from school that we were doing many things 'wrong'.

In grade six in primary school we had to plan a 'nutritious' breakfast menu for a health and nutrition class. I couldn't rely on home experiences because there were no hard and fast rules about what to eat and when. Sometimes we ate leftover stew for breakfast and Weetbix for lunch and dinner. Sometimes we missed breakfast completely. It was important for me to appear knowledgeable about what was the 'right' food to eat so that I didn't get shamed for eating 'wrongly' at home. I took no chances, I went to the library and consulted with a White neighbour about an acceptable breakfast. There did not seem to be anything from my home life that was useable at school (p. 8).
Chapter 4

Teaching Mathematics in Schools

4.1 Perceptions of teaching mathematics – past, present and future

Most people, including a large proportion of teachers, have perceptions about what mathematics and mathematics learning and teaching, 'look like'. The present generation of adults were predominantly taught mathematics in classrooms where the teacher used a 'chalk and talk' model of curriculum delivery. That is, a mathematical concept was taught under the guise of a procedure or method, through teacher examples. Students were then asked to practise the procedure until they were efficient in its use.

Assumptions were made about this: teachers and parents and school administrators assumed that students were learning mathematics concepts because they could use the procedures required in order to find correct answers. It was somehow assumed that by knowing the procedures and routines, understanding – if it did not exist prior to the learning of the procedure – would eventually occur through the continued practice of the method. For example, we learnt about 'addition' by learning how to add up; we learnt about 'area' by learning area formulas and when to apply them; we learnt about 'algebraic equations' by learning how to solve them; we learnt about 'graphs' by learning how to draw them.

For a large proportion of teachers of mathematics, this model of teaching continues; 'it worked for us' being the justification, in many cases. Unfortunately, we now know that there are sound reasons why this method of teaching mathematics alone does not work for the majority of students. Not only that, but also this method of teaching will not prepare our children for the techno-world they will live in. We need to teach our children to understand mathematics concepts and it can not be assumed that they will be learned inadvertently through practising procedures and routines. Research has shown that there are many children who know how to 'add up' but don't understand addition; they know how to find the 'area' of a shape but don't know what 'area' is; they can draw 'graphs' but can't interpret them; they can solve 'algebraic equations' but don't understand 'variables'; they can say their 'tables' but don't understand what multiplication is, and so on.

The changing needs of society demand that we teach our children to understand mathematics concepts and to then make efficient choices about how and when to perform the necessary calculations, and that includes the appropriate use of technology such as calculators and computers. Once a child understands the concept of addition he then needs to decide for himself whether to perform an addition calculation mentally, on paper (using either an algorithm or rough working out), or with a technological tool such as a calculator. The understanding of the concept must be the focus of mathematics learning and teaching, not the methods used for findings answers.

This mind shift is very difficult for the majority of adults from the present generation. It is essential that it be made by teachers of mathematics at all phases of schooling. In order for this to occur however, it is important that parents and other members of the community also make it in order to support teachers.
Skemp (1989) believes that Western Mathematics is not like other school subjects; it relies on successive stages of classification and subsequent abstraction. For example, we start with a lot of different shapes; we sort them on the basis of what they look like visually, are they squares or triangles? We then further classify them on the basis of their properties; how many sides, how many angles? We classify them further on the basis of the lengths of the sides and the sizes of their angles; this classification relies on the understanding of the nature and measurement of length and angle. We can then prove that two shapes are similar or congruent, a further level of abstraction. Clearly, this level of abstraction cannot take place unless at some prior stage the classification of shapes into squares and triangles has occurred.

These levels of classification must be conceptualised by students, they cannot just be taught; assumptions cannot be made that the concepts of difference required in early sorting activities have been understood merely because students have been told or taught them. We need to ensure that real learning and understanding has occurred at this stage or subsequent levels of abstraction will not be learned and understood.

4.2 Process versus content; and communication skills

Bearing in mind the above discussion, it is useful to consider mathematical content and mathematical process as distinct, even though one cannot exist without the other. In the past and present much of the focus in mathematics teaching has been on content; described by Skemp as 'concepts and ideas'. Curriculum has been a list of topics and concepts, which have had to be taught in a certain time frame.

As we move into the information age people need to be able to process information; make choices about what information is needed, make decisions about how to access the information, be able to apply appropriate mathematical processes to the information and make informed judgements about results obtained. Whereas in the past we have concentrated on the teaching and learning of mathematical information, we must now concentrate also on the teaching and learning of the mathematical processes required to access, manipulate and interpret the information obtained, through the processes used.

Clearly the content or information is still an essential part of this process but only insofar that it is the vehicle for the processes; the content provides the repertoire from which to make choices about how and when and where to use it for a purpose, much of which is not important in its own right for the majority of students who may never use it outside the school context. In fact, the only group for whom it is important in its own right are the pure mathematicians, those who study mathematics for the sake of studying mathematics. For the remainder of society, mathematics is a powerful tool for the solving of real life problems. It is with the application of mathematics that we should be concerned. Children need to be taught the mathematical concepts first and then they need to be taught how to make choices about their use in various contexts. The strategy for this teaching may in fact be to confront students with the contexts and then teach the concepts required by the context, thus providing a reason for them being learned.

Another important part of this is students being able to effectively verbalise what they are doing and communicate about the decisions they have made and are making. The process of verbalising, including asking and responding to a range of questions about what they are doing, helps children to make connections between concrete and abstract thought. Harris (1984) states that "it is the verbalisation that allows children to move easily from context-specific learning to object-free principles. This is an essential goal if children are to be able to understand and apply mathematical knowledge". This is true for all children.
Harris further explains that Aboriginal children learn kinship patterns in different ways to those of other learning experiences which occur through the senses such as seeing, tasting, touching and so on. Relationships between people cannot be learned through the senses since they are abstractions. These are taught as labels, so that even when a child is very young their mother points to the person who is passing, palm up, and names the relationship so that children learn the idea through a label. This method is obviously very powerful since Aboriginal children are extremely adept at being able to deal with the complexities of the kinship systems in a purely abstract way.

Teaching very young children to use labels then could be a way of helping them to make these connections. Playing with stones, dice, counters as the concrete materials and labelling them as 'six stones', 'six dice', 'six counters' and so on will help children to understand the abstraction of 'sixness'.

As part of the verbalisation process, older children need to be able to justify choices and to interpret results using appropriate mathematical language for a particular audience. In the past the goal for many students has been to 'get the sums right' – and moreover, this has been the goal for teachers; that of teaching students to 'get the sums right'. Students have been conditioned into believing that maths is either right or wrong. They now need to be taught that it is the purpose and the context that prescribes the 'correctness' of a mathematical answer. Interpretation of purpose is about communication skills – either written or verbal.

For most children, and in particular Aboriginal and other ESL children, this process of verbalising what they are doing can be very daunting and indeed difficult, especially if they do not have the language skills that this requires. Para-professionals can help here.

Bucknell (1995) suggests:

To develop collaborative problem solving also enables a rich source of exchange as students in small groups co-operatively argue back and forth, posing possible interpretations, solutions and directions. The safety of this interaction in terms of a group identity, as opposed to an individual identity, is appreciated by Aboriginal students who are less reluctant to participate within this more familiar framework. Group authorisation of information removes the 'shame' students may experience if they have not had the opportunity to collaboratively arrive at an acceptable explanation.

Communication can also occur in ways other than verbal. Beth Graham (1984) further describes how traditional Aboriginal societies recorded their experiences through dance, stories and art, and goes on to say that children should be taught modern ways of recording experiences through 2D and 3D models and representations, and in words and symbols. These records allow for more talking and help children to remember and organise their experiences. Children should be encouraged to make their own decisions about how to record their experiences in ways that help them to remember and talk about them at a later stage. These decisions should not be made for them by their teacher all the time.

4.3 A Framework for mathematics teaching and learning: The mathematical modelling process and working mathematically

With the emphasis in the mathematics classroom ideally now being equally on processes as much as on content, the Mathematical Modelling process provides a useful framework for teachers in understanding what this means for their classroom.
For too long we have concentrated on teaching students the 'bits' and the 'tools' for applying and solving mathematical problems and have paid little if any attention to teaching children how to use them. Someone once said that if we taught English like we teach mathematics children would spend all of their time practising spelling, grammar, punctuation, and sentence structure without ever doing any creative writing. This is a very powerful analogy: we've spent most of the time teaching children how to add, subtract, multiply, calculate, and evaluate but given them little opportunity to use these in a creative way.

Problem solving, which is the creative 'goal' of mathematics, has too often been used as something 'added on' to the mathematics lesson; problems are given to the academically able students who finish their work early, or they're given to children to do for homework at the end of an exercise. Rarely are they the focus of the mathematics lesson.

The mind shift spoken of in the previous section is about teaching the repertoire of skills and the content in order that students have these to choose from when solving problems. We teach content and skills so that our students are empowered to solve problems.

The choice about what mathematical skills and knowledge to use is what empowers students. Too often in mathematics classrooms are children taught a specific skill or skills and then given problems, which require that skill. This does not empower children, it disempowers them; they do not have the choice about which mathematical models and skills to use as this choice has been made for them in the context of the lesson or the textbook exercise!

The mathematical modelling process is a problem-solving framework that will help explain what is being described here. It includes five steps:

1. The student clarifies and contextualises the problem;
2. The student chooses appropriate mathematical models (skill, method), technologies and strategies with which to solve the problem and justifies the choice;
3. The student uses the models, tools and methods chosen;
4. The student interprets the solution obtained in light of the original problem in the given context; and
5. The student communicates the process (i.e. steps 1-4) for a given audience.

These steps can be summarised as Clarify, Choose, Use, Interpret, and Communicate. Other words can be used which may be more appropriate for the age of the students or their phase of schooling, such as Read, Plan, Do, Check, and Share.

To clarify a problem children need to ask questions like:
* What do I know?
* What assumptions can I make about the context of the problem?
* What am I being asked to find out?
* What will I need to find out?

In order to answer these questions children can use various strategies like restating the problem in their own words, underlying key words, identifying any irrelevant information and so on.

On the basis of this clarification they will then need to make some choices about which mathematical skills, tools and knowledge and strategies they can use in order to solve the problem. They might ask questions like:
* How do I find out the information I need with which to solve the problem?
* What mathematics will I need to do?
* Will I need more than one mathematical model?
Which mathematical model/s shall I choose?
How should I display my results: in a table or chart or something else?
How should I organise the mathematics I use: will diagrams and calculations be helpful?
Which is the most efficient and appropriate mathematical model to use here?
What technologies could I use to enhance my work?
What strategies could I use; will trial and error be OK?

After making the choice of the mathematics, tools and strategies they will use, children then have to carry out the calculations using those choices. They will need to make decisions about how much of the actual calculation of their results they will need to show (and this is related to both the Choose and Communicate parts of the process) to the audience.

Once the calculations have been carried out students need to interpret or check their results. They should ask questions like:
- Do my answers seem reasonable? Why?
- If my answer doesn't seem reasonable could it be that I didn't clarify the problem properly? Is there something I didn't take into account?
- If my answer doesn't seem reasonable could it be that I didn't choose an appropriate model to use? Could I have chosen a more appropriate model? Could I have used more appropriate technologies or strategies?
- Does the model I chose tell me what I want to know or will I have to choose another one?
- Did I use the model/s correctly or have I made some careless errors or mistakes?
- Do the tools and technologies I chose help? Would others have been better?

At this point they might also ask some ‘What if...?’ type questions to further develop their results or the problem itself.

The communication of the processes used is an extremely important part of mathematics. For too long we have focussed on answers so that teachers and students often believe their task is complete when the answer has been produced. Unfortunately this is insufficient. Employers want people who can solve problems and communicate not only the results but also the processes used, including refinements and justification of choices made during the process, to obtain results. It is the responsibility of teachers to teach the skill of communicating both in verbal and written form, to students. Students need to ask themselves the questions:
- What did I do?
- How did I do it?
- What results did I obtain?
- Did I have to re-do anything? Why?
- What would I have done differently if I did it again?
- What assumptions did I make when clarifying the problem? Were these valid?

The responses to these questions need to be communicated in an appropriate format for the required audience. The audience may simply be the classroom teacher but it is the confidence in which to communicate mathematical thinking – so important for the development of numeracy – that is essential and clearly transferable.

**Collaboration/cooperation**

The above model, although useful for individual use, is most powerful when used in groups. Students clarify situations together, pooling ideas and perspectives that rarely are forth-coming from one person. For example, consider a situation where five people are sitting around a table drawing a bowl of fruit; they are all seeing the same bowl of fruit but each person sees it differently, from a different perspective and through different eyes. Children of different cultural backgrounds
bring a different 'world-view' to the clarification process, drawing on their individual experiences and ways of knowing. Similarly, ESL students are able to help each other understand and learn new words and their meanings in different contexts as well as developing mathematical literacies.

When choosing different models, technologies and strategies with which to solve the problem, the repertoire is increased since each person in the group has a different repertoire to draw from, thus learning may occur through the sharing of strategies and ideas. In interpreting and communicating results different vocabularies are used, different interpretations are given – again based on different perspectives and ways of knowing – and shared experiences result in greater learning.

Bucknell (1995) suggests that Aboriginal children "should be encouraged to record their findings and to use this information to explain their results to another group who in turn have an opportunity to evaluate and comment upon their findings as they interpret the findings of another group". This, she believes, enables results to be owned by the group and "removes the 'shame' students may experience if they have not had the opportunity to collaboratively arrive at an acceptable explanation". This has been previously mentioned in the context of group authorisation of information. Students are more willing to share views owned by the group rather than present them as belonging to themselves, reflecting child-rearing practices described in Chapter 2 concerning learning styles.

In summary, the mathematical modelling process of Clarifying, Choosing, Using, Interpreting and Communicating, is essentially Working Mathematically – a strand recognised as having an essential part in the desired outcomes of the teaching and learning of K-12 Mathematics in the National Statement on Mathematics for Australian Schools (1990) and more recently in the National Profile (1994).

Content then, still has an important place – indeed the processes cannot exist without it. It forms the repertoire from which the students choose what mathematics to use and how to use it. This is shown in Figure 1. It should be pointed out that although the process is shown in a linear model, the working mathematically process is in fact iterative, in that it is not a ‘lock-step’ approach.

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**Figure 1: The linking of mathematical content and process**
Students may in fact begin at the *choosing* phase and then find they need to go back to *clarify* since choosing is not possible yet. Or, they may, on *interpreting* a result, find that results are inappropriate and have to check their *using* or perhaps even have to *clarify* and *choose* again, and so on.

How would you as a teacher, use this process in your classroom? First of all you need to make the mind shift from focussing on content to focussing also on mathematical processes; you still need to teach the content but only insofar that it becomes part of the essential resource from which children can chose in order to solve mathematical problems. Using the above model then will become a continual part of the teaching environment.

You may choose to teach the skills and knowledge as they are needed in order to solve various problems or you may teach the skills and knowledge for a period of time and then use problems intermittently but constantly so that children appreciate why they are learning the skills. Degree of reinforcement of this is a personal thing and is also related to timing and classroom organisation. The mind shift made by you as the teacher will in itself affect the nature of the pedagogy or teaching style used.

The model – although drawn and described as a linear one – is in fact iterative in that students and people typically move from one step to another in any direction; they may for example *clarify* and *choose* and *clarify* and even *communicate* before attempting any *using*. Teachers need to teach their students how to do each of these components of the model and each component may be used at any time in any lesson context. It may be appropriate for example, for a teacher to teach a student or group of students how to *interpret* some classroom work that they may have just completed, or to ask a student to communicate their reasoning to the class following the performing of a calculation.

**4.4 The place of questioning**

The use of ‘questioning’ as part of *Working Mathematically* can be hugely problematic when teaching Aboriginal children. The problems can be identified through reading the above summary of what *Working Mathematically* is and the need for children to ask questions in order to clarify, choose, interpret and so on; Aboriginal children, in general, do not ask questions in this way. They may ask numerous questions *about a person*; “Who are you? Why are you here? Where are you from?” and so on as part of building relationships which are so important to Aboriginal cultures; but asking questions about processes, objects, information and so on may be totally foreign to them.

Most non-Aboriginal children *learn* through asking questions about what they see, hear and experience. Aboriginal children however, learn by watching and then ‘having a go’ themselves when they feel able to do so. If they fail, they try again and it is through this ‘holistic trial and error’ approach that Aboriginal children, in the main, learn.

It is clear that Aboriginal children – particularly those in remote areas where exposure to non-Aboriginal culture is limited – may need to be *taught* to ask questions in the context of mathematics, and indeed in every context related to ‘school’ in order to achieve desirable process outcomes. They will need to have the questioning modelled for them by their teacher and be encouraged to ask questions themselves in order to make sense of situations.

Not all Aboriginal students have the same cultural understandings and this varies, particularly in remote areas where their individual experiences with learning, language and non-Aboriginal culture has been limited.
Explicit mathematics teaching: What is it?

In previous chapters I have used the word 'explicit' in the context of the teaching of mathematics. The Collins dictionary defines 'explicit' as "clearly stated or expressed, with nothing implied". The need to be explicit when teaching mathematics is of paramount importance. Not only does mathematics have a language of its own, but English language words which are used in the context of mathematics may take on a unique set of meanings. Words such as 'bigger' and 'smaller' for example, when used in a mathematical context, can refer to value as opposed to the common English meaning with which children are most familiar, relating to relative size. For some Aboriginal cultures words such as 'bigger' and 'smaller' may not even exist and so there is an added dimension of not only teaching the use of the language in the context of mathematics, but as words.

It is important that teachers of mathematics assume nothing in particular about their students' understandings relating to mathematics or to language. This is not to say that we assume the children know nothing, that wouldn't be true. It is more likely that children who come from homes where English is the predominant language will be familiar with the words used in the mathematics context than for children where it isn't, but this should not be assumed.

I remember a few years ago asking a class of year 8 students to write down a fraction which was smaller than one half. A large proportion gave the answer: $\frac{1}{2}$. This indicated that possibly they had not understood the use of the word 'smaller' in this context; the only number they could make smaller (in terms of value) was the '2' and so they wrote a '1' in its place. It also indicated that they probably didn't understand the concept of 'fraction' despite the fact that they could all perform fraction operations efficiently.

Even mathematical terms such as 'half' have a common meaning outside the context of mathematics. The common meaning is about two pieces of relatively the same size, or even just two pieces. In mathematics the term 'half' refers to a more exact notion; the concept of 'halfness' being about two equal parts or shares. The understanding of the 'exactness' of equal shares requires a higher conceptual understanding than some common usages of the word 'half'. If students are not explicitly shown and taught the connection between the two words in contexts which require and demand 'exactness' of equal shares, they will never have a mathematical understanding of fraction but merely a common understanding.

It is for this reason that many students even into their late teens, do not understand the mathematical concept of fraction despite being able to perform operations with fraction such as addition, subtraction and so on using algorithms. In order to teach mathematical concepts 'explicitly' a good teacher will assume nothing, defining and teaching necessary terms, both mathematical and English language terms in the context of mathematics. This is essential for all children. Clearly, if children learning mathematics have Standard English as their second language or dialect, including Aboriginal Language speakers or Aboriginal English language speakers, they have an even greater need to be taught explicitly.

In mathematics the concept of equality is something that most teachers of mathematics would assume that all students would have. I recently read an anecdote (Dwyer, 1989) about a conversation with an Aboriginal boy as follows:

*I walked into a classroom one day to find the teacher at the limit of her patience with Kurt, who seemed to have no understanding of the concept of equality. To ease the tension I decided to take over, only to find that soon I too was getting frustrated and*
angry. Then I had a stroke of luck. I made one 'train' of 4 blocks and two of 2 blocks each. I placed the trains side by side so that the 2 + 2 train sat beside the 4 train. 'There,' I hissed, "look at that. What can you tell me about the two sets of trains?"

Kurt looked at me with surprise at my obvious stupidity. "Oh. They tie," he said.

Suddenly it became obvious that he did have an understanding of the concept. The difficulty had been that his teacher and I expressed that concept through the word 'equal' while he used the word 'tie', a word we would have used if we had been talking about runners who had reached the finishing line simultaneously. (p. 48)

Since 'explicit teaching' implies assuming nothing this clearly indicates a need to establish what the students you are currently teaching already know in terms of concepts, skills, and language. One of the problems in many schools is that teachers 'assume' their students know things because they have been taught them. We know that this is often not the case. Part of being 'explicit' then is to assume that each individual student brings something different to any learning situation and that this may range from knowing nothing to knowing everything. It may also range from knowing nothing that the teacher knows to knowing more than the teacher knows. Teaching to this point of need is an essential part of being inclusive as described in Chapter 1.

4.6 Relevance

Alan Bishop (1974) states that, "to the European students relevance in any subject study is important...to the Aboriginal students relevance is vital". We have since learned that relevance is an essential element in learning for all children, but for Aboriginal children who do not generally have as part of their learning styles the desire to ask 'why?' or 'how?' a practical application relevant to their needs must be established since skills and knowledge will generally only be retained where they continue to be used in meaningful contexts.

Many educationists believe that Aboriginal children from remote communities perform poorly on mathematics and numeracy tests because mathematics is not used in their community. As pointed out in the previous chapter however, mathematics is a way of ordering society and making sense of the world so it must exist in all Aboriginal communities. Unfortunately, we often assume that because the outward signs and symbols of mathematics are missing then there is no mathematics occurring.

Christie (1995), in describing some of his early teaching experiences in Arnhemland, states that "...a highly favoured solution to the apparent absence from the community (of mathematics) was to deliberately create an artificial culture of literacy and numeracy within the school (and flood the community with books, and set up artificial shops) which would somehow create a demand for our teacherly skills".

Relevance can be found by attempting to understand the ways in which the Aboriginal people order and make sense of their world (their ways of knowing) and building on this to make connections from the Aboriginal world to the Western world. Aboriginal children may indeed know an enormous amount of mathematics, but it may not be the Western Mathematics that is familiar to their teacher. This knowledge must be acknowledged and ‘tapped into’ if possible, in order to assist the teaching and learning of Western Mathematics. It should also, in my view, be shared.

This is not to say that teachers should only teach mathematics that is relevant. It means more that we should look for relevance from within the community and the culture and if we can't find it then at the very least attempt to create relevance by connecting the teaching of abstractions to familiar contexts. Bucknell (1995, pp. 25-30) states that:

Improving Aboriginal Numeracy
...opportunities to examine mathematics in practice will emerge in card games, Social Security forms and payments, taxis, banks, the processes of pooling money to buy a car, purchasing food, etc. Discovering mathematics in the students' own community (ethnomathematics) ...provides the teacher with ...

• an opportunity to be a group participant in a shared experience (allowing teachers to get to know their students and their community on their own terms)
• the mathematical knowledge base from which the students operate (thus assisting teachers to make connections between the known and the unknown)
• real life situations from which to extend a meaningful mathematics program for the students (for too long we've disempowered Aboriginal students by making decisions about what to teach and not to teach; for example, we may not teach them about concepts such as diameter since we believe they have no relevance for it. Instead, we should be teaching the concept, finding a real-life situation in which the concept exists)
• an opportunity to observe the co-operative collaborative approach adopted by the students (this can be utilised as a pedagogical approach in the classroom)
• an opportunity to observe the students' use of language, the nature of their communication with each other and the community.

Bucknell goes on to suggest that Aboriginal children (indeed all children) should be exposed to real life situations where newly acquired skills with seemingly no relevance can be utilised. She suggests visits to a bank, "a factory, a Shire meeting or other organizations as an extension of their studies into broader contexts" could be organised by the classroom teacher. She warns that these sorts of excursions should occur for Aboriginal children of all ages, not just for older children, since many younger children often can perform the mathematics correctly using rote procedures, while having "no concept that the symbols on paper could represent real-life situations".

Bucknell also suggests the inclusion of what she calls 'situational plays' where participants explore roles, as means of enabling Aboriginal children to "anticipate and respond more effectively as they interpret situations". In playing the role 'booking a visit to the cinema' for example, children are immersed in a situation with many mathematical demands, such as using telephone numbers, checking the calendar, checking the time, enquiring about costs and how payment can be made over the phone, and so on. Language can be modelled by the teacher and then used by the students in further role-play. Similar 'situational plays' can be made from numerous real-life experiences from mainstream or non-mainstream society, in order to assist children to make links into Western mathematical thinking.

It would also be of great benefit if teachers were to recognise opportunities for utilising mathematics in other learning areas such as Science and Technology and Enterprise to draw out the mathematics and hence help students appreciate the relevance of mathematics in their lives, not just in the mathematics lesson at 10 a.m. every morning.

A study by Holm and Japamangka (1976) revealed that many Aboriginal children have extremely high levels of efficacy with card games using a standard pack of cards. The children are numerate in this context but are frequently unable to make the connections between the mathematics of the card games and the abstract mathematics often taught in the schools they attend. They understand that 5 heart-cards and 4 diamond-cards give a total of 9 cards but they are unable to transfer this knowledge to understand that 5 men and 4 boys make 9 people or that 5 dogs and 4 stones make 9 objects. In a similar way that Aboriginal children may learn that there is a 'home way' and a 'school way' in the ways they behave, the things they do and the language they speak, they need to be told that the mathematics of card playing at home is not 'home maths' but rather mathematics that is true
and constant, whatever the context. In order to make these connections teachers need to help students by giving them many 'hands on' experiences before they move into formal teaching and abstract symbolisations such as \(5 + 4 = 9\).

Dawson (1991) describes a plethora of contexts that abound in the classroom that can be used to provide relevance. For example, the context of the School Bell can help in teaching number ordinality: first bell, second bell, third bell etc. What time is the bell? Was the bell early, late, on time? How long 'till the next bell? And so on. He used Roll Call as another context in which to teach many data ideas: How many children on the roll? How many are here/away? How many boys/girls? Graphing of the daily attendance on a wall chart, learning about dates through the calendar and birthdays and so on. These are not artificial contexts such as the shop mentioned by Christie above, but rather situations that are meaningful to school life. Transferability of the skills and understandings learned from these situations however, is another matter and that is why excursions – such as those mentioned by Bucknell – are so important if they are possible. Numeracy requires that children can choose to use the mathematics they learned in one context and use it in another, often unfamiliar one.

For many Aboriginal children there is often no relevance in learning 'for the future'. In other words, non-Aboriginal people may be encouraged to learn something because 'it might be important one day', or for future learning. For a people whose time is more concerned with the present and the past, the idea of learning for the future has a different relevance.

4.7 Implications for pedagogy

Pedagogy is about how we teach. The above discussion indicates a need to take many things into account in the effective teaching of mathematics in order that we prepare children for the world they live in and that they achieve the essential mathematics outcomes that we want for all children. Teachers of mathematics need to have a clear understanding of the purpose of teaching mathematics in schools. There are essential mathematics understandings and skills that children need to learn. These essential elements (in particular the mathematics content) need to be taught in an explicit fashion; teachers should make no assumptions about what their students know based on what they may or may not have previously been taught or exposed to.

Language too plays an important part in explicit mathematics teaching. Students need to be taught the symbolic mathematics language, the language (words) specific to the mathematics discipline, and the English words which may take on a specific meaning of their own in the context of mathematics.

Teachers who have used the Mathematical Modelling process described earlier with their students have found it extremely helpful, especially if students work in groups. The practice of clarifying a problem through paraphrasing it or explaining it in their own words helps children to clarify their thinking. More than that, it helps children to understand the words and the language in problems, an essential part of explicit teaching and of exposing misunderstandings in a classroom. Justifying choices of mathematical models used and the mode of calculation also helps students to clarify their thinking and learn about efficiency and appropriate usage of mathematics. Requiring students to communicate can force students to engage in the learning process.

Clearly the ineffectual use of language – both oral and written – in the mathematics classroom has for too long hidden student misunderstandings and failed to expose the need for explicit teaching in this learning area. By concentrating on 'getting the right answers' we have promoted an environment where rote learning is encouraged. This has been to the detriment of students, in failing to prepare them for life in the real world.
Results from the *Third International Mathematics and Science Study* (TIMSS) revealed that children from countries where classroom pedagogies expect children to both work individually and in groups, discuss solutions with each other, explain solutions to teachers and other class members, explain practical applications for mathematics concepts learned, and share their ideas in small groups, defending their answers to these and larger groups including the classroom teacher, achieve much higher results than students who are not expected to do these things (Geist, 2000).

We have learned through results such as these that pedagogies used by the classroom teacher are extremely important in creating an effective learning and teaching environment that fosters the achievement of mathematics and numeracy outcomes for children. These pedagogies must consider the learning styles which children – from different cultural backgrounds as well as individually, bring to the classroom, as well as the ability to teach children effective ways of learning mathematics. (p. 181)

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Chapter 5

A Model for Teaching Aboriginal Children Mathematics

5.1 What makes the difference; and the model

Previous chapters have focussed on the fundamental elements present in a learning and teaching environment where mathematics is taught to children, both Aboriginal and non-Aboriginal:

- what the Aboriginal child brings to the learning environment, both culturally and mathematically, and
- what the teacher of mathematics brings to that same environment in terms of expectations, knowledge and skills.

There are three focus areas that overlap to form a 'layered' response in the teaching of mathematics as shown in Figure 2. Clearly we are concerned with the intersection of these three circles, it is here that the learning and teaching begins, with all aspects being considered simultaneously. It is the teacher of mathematics who controls this learning environment. He/she must

1. be aware of and have an appreciation for the Aboriginal people; their culture and how it impacts on Aboriginal children in a school environment of the dominant culture;
2. be aware of the possible mathematics understandings of Aboriginal students, their strengths and weaknesses, and 'where they are at', not only in their mathematical understandings but, in their understandings of the dominant culture and the cultural environment of the school often assumed in the teaching of Western Mathematics; and
3. have a deep understanding of Western Mathematics as a way of organising and ordering the world, the desired mathematics outcomes for all children, and a repertoire of good pedagogical practice in order to assist their students in the achievement of these outcomes.

It is the combination of these aspects that makes the difference. This is a huge ask for teachers. The responsibility lies with school systems and school communities to do everything in their power to assist teachers of mathematics in creating this learning environment. Indeed, the extent to which
mathematics outcomes and hence Numeracy achievement is improved for Aboriginal children in Australia will depend on the extent to which this learning environment is created and supported by educational systems and schools.

Note that the three components outlined above represent not only a *macro* approach to the teaching of Aboriginal children but also a *micro* approach that must be considered for each lesson and each student in the mathematics classroom. This means that when choosing appropriate strategies for the teaching and learning of a concept a teacher should:

1. Consider the implications for the teaching of Aboriginal children with respect to their learning styles and cultural background as outlined in Chapter 2. For example, the use of visual and kinaesthetic approaches to teaching new concepts, and the use of para-professionals to assist with Working Mathematically techniques as described in Chapter 4.
2. Be aware of the possible mathematical understandings of Aboriginal children with respect to the particular concepts being taught, making connections to these wherever possible; and
3. Have an in-depth understanding of the mathematical concepts themselves, including the possible misunderstandings either brought to the classroom or developed through inappropriate teaching and learning.

It is the purpose of the following chapters to improve the understandings of teachers in order that points 2 and 3 above will be more effective, should this be necessary.

### 5.2 Using the model in the mathematics classroom

In order to use the model effectively in the mathematics classroom a teacher must firstly take into account the implications as suggested from the previous four chapters – that is, the strategies that should be in place, if possible, in any inclusive classroom regardless of what subject matter is to be taught. The teaching of the mathematics content – with a working mathematically focus wherever possible – should then be the final piece of the jigsaw or, to use the analogy described above, the final layer.

A teacher attempting to use this model may use the following checklist:

1. What mathematics do I want to teach today?
2. Are my students ready to learn this? How do I know?
3. Do my students feel comfortable about being in my classroom?
4. Do I, as a teacher, appreciate each of them as individuals?
5. Am I as aware as possible of each of their learning styles?
6. Have I done all that I can to involve their families and school community in the learning program?
7. Is the school administration supportive of me in my role?
8. Are there para-professionals available to assist me with this lesson if possible?
9. Do the para-professionals know what the desired outcomes of today’s lesson are?
10. Concerning the mathematics content I will teach today, are there any specific things I should know about how Aboriginal children might respond to this content? What understandings might they bring to the classroom?
11. Can I present this content in a way that will involve my students; what contexts may I use that will be familiar to them?
12. How can I present the material in ways that will enhance learning; for example, use of visual teaching aids?
13. Am I making use of techniques and strategies for all children and particularly Aboriginal children, to *clarify* the language which envelopes the understanding of this mathematical concept?
14. Am I able to make connections between new material and existing understandings through familiar or relevant contexts?

This checklist is not intended to be comprehensive but is merely an attempt to describe the sorts of things that a teacher must continually consider if they genuinely desire their classroom environment to capture the essence of what is described as the intersection of the three circles of the model.
Chapter 6

Teaching Key Ideas about Number

6.1 What is Number all about?

The Number strand is possibly the most important strand in Mathematics. If students don't have a sound understanding of Number concepts, what numbers are and how they work, they will arguably have difficulty with all Western Mathematics and consequently their capacity to be numerate.

The emphasis on teaching Number has changed over time in response to the changing needs of society as outlined in the previous chapter. There are three key outcomes for all children:

1. **The understanding of numbers**: What are whole numbers, decimals and fractions? How are they used?

   We know that there are many children who can perform operations with numbers correctly, but have little or no understanding of the numbers with which they are working. For example, I have taught numerous children up to 15 years of age and older, who can add, subtract, multiply and divide decimals but do not understand place value. They can similarly operate with fractions without having the conceptual understanding of what a fraction is.

2. **The understanding of number operations**: What are the meanings of each of the four key operations of addition, subtraction, multiplication and division? How are the operations used and how do they connect?

   Children need to understand the operations deeply in order to make choices about which operations to use when solving problems requiring mathematics. They need to understand that multiplication is the inverse operation of division and that subtraction is the inverse operation of addition; that multiplication is repeated addition and that division is repeated subtraction. Many of these connections are not explicitly made for children with disastrous consequences in later years.

   Many children appear to 'plateau' in their learning of mathematics and in their development of numeracy skills as a result of their inability to make choices when solving problems. They have 'success' with school mathematics while it merely involves 'doing sums' but when the time comes that they are increasingly required to make decisions about which operations to choose in order to 'do the sums' they are at a loss because they do not understand operations sufficiently in order to make informed choices.

3. **Calculation**: What sort of calculation should be used to solve a problem – mental, written or calculator? Does the solution to the calculation seem reasonable? Is the solution accurate enough for the context of the problem?

   Calculation does not merely mean the physical act of calculating. In fact, now that readily accessible technologies are available that perform calculations, the physical act of calculating becomes a very minor part of Calculation. To calculate appropriately children...
need to use the model described in the previous chapter; they need to clarify a situation, make choices about which mathematics to use in order to address the problem and take into account the required levels of accuracy. This level of accuracy determines the choices about the method of calculation used in terms of appropriateness and efficiency and the audience for the solution. With respect to efficiency, children should be taught to use mental calculation first and this should be their most efficient means of calculating. Interpreting the reasonableness of obtained results through the use of estimation skills is also an important part of calculation.

For the scope of this book it is impossible to address all aspects of the learning and teaching of the Number strand. I have chosen however, to focus on key concepts and ideas which I believe are fundamental to the successful achievement of the above Number outcomes.

### 6.2 Understanding numbers

The understanding of numbers and number forms is crucial to being numerate. The reason many children fail to have numeracy skills is that despite being fluent with mathematics computation – the ability to compute using mathematics procedures – they do not understand the numbers they are working with. It is because of this that they are unable to make choices about what mathematics to use and when which is absolutely crucial in demonstrating numeracy acquisition.

Numbers can be used in three ways:

- In a nominal sense: that is, as labels such as on busses and letterboxes;
- In a cardinal sense: that is for counting and quantification and to answer questions about 'how many'; and
- In an ordinal sense: that is, to describe order. For example the ordering of book placement in a library or the representation of 'overs' in a cricket match.

Numerate people have an ability to recognise when numbers are being used in each of these ways. Failure to recognise these differences can cause problems for children learning mathematics in schools.

Teachers should help children to make connections between the different uses of numbers in society. For example, when a cricket commentator explains that there are '4.3 overs remaining' in a cricket match, does that mean 4 and \( \frac{3}{10} \) overs? If not, why not? If these sorts of discussions do not occur, is it any wonder that children have difficulties with decimals? Many children are under the misconception that decimal numbers as used in mathematics classes are something entirely different than decimal numbers in the real world. It is these sorts of misconceptions which make children apprehensive and lacking in confidence about choosing to use mathematics outside the classroom.

For some Aboriginal children, particularly in remote communities, experiences with numbers used in an ordinal or cardinal sense may be rare; those used in a nominal sense are more likely to occur as children see these on football jumpers and car number plates. To facilitate the learning of all three ways in which numbers are used, real life experiences in and around the school community may need to be 'created' so that there is an immediate and practical purpose in learning them. We should however, be wary of creating situations that are totally foreign to students in their lives outside of school, such as the community 'shop' mentioned by Christie in the previous chapter. I believe it would be appropriate to create a 'shop' in the classroom only if students experienced a shop in their lives outside of school.
Counting and whole numbers

Counting is one of the first encounters that many children have with Number. In Western society and many other cultures, the ability to count by pre-school children is often used as an informal benchmark by parents; one often hears parents boasting about the fact that for example, "my daughter is only three and she can count up to 20". It is used as a mark of intelligence or as a promise of future achievement. Unfortunately, it is often little more than a measure of a child's ability to memorise a string of words.

When numbers are used to tell 'how many', they are being used in a cardinal context. In many Aboriginal cultures counting to find 'how many' may not have the same use as in Western society. Indeed, in some traditional Aboriginal societies counting has little value. Unfortunately, many Western societies have used the ability to count as a sign of intelligence, not recognising that counting is an activity that reflects the circumstances of the culture. As Harris (1990, p. 34) states: "people who do not need an extended number system, and therefore do not need to develop one, are not less intelligent that those who do". Harris goes on to explain that the Aboriginal people did not often have the need to count to very high numbers. They did not, for example, accrue material goods of which they had to keep record... With many variations across Australia...terms for 1, 2, 3, and 4 are used, then there are terms for 5, 10, 15 and 20. (p. 34)

Some Aboriginal languages often do not have words for numbers greater than three – this clearly indicates that the necessity for these is missing in traditional societies. After three or five it is 'big mobs' or 'lots'. Some Aboriginal people may in fact say "lots 'n' lots 'n' lots". Those I spoke to from the Noongar cultural group, said that they did have a sense that "lots 'n' lots" was double the amount in "lots", and so on. They also said that the way they said 'lots' indicated the quantity, so that if the word was drawn out and took a long time to say then this indicated there were more than if 'lots' were said as normal. They indicated that 'lots' was a manageable number such as about five or the amount they could subitise (see below). After that it was more likely to be "lots 'n' lots".

Beth Graham (1984) states that "five is another term that is often a fairly familiar concept in an Aboriginal society. People often refer to a 'hand' and in one community where turtle eggs are gathered they are distributed in groups of five".

Many Aboriginal people use what in Western Mathematics may be called a 'deficit' model when counting. When asked how many people were at a funeral for example, they may respond with "there was a big mob but Aunty Betty and Aunty Dulcie weren't there". Clearly, it is more important to know who was there rather than how many were there. It has previously been explained that for many Aboriginal peoples, relationships and people are more important than quantities. There must be a purpose for numbers and counting for them to be important. For catering purposes an extra potato may be added to the pot for each person as a sort of one-to-one correspondence as extra people turn up, but there may not be a notion of 'preparing for 10 people' for example.

Children in non-Aboriginal societies are generally immersed in a 'counting world' before entering formal schooling. Parents help their children to learn to count by teaching them to say the counting words in order. For example, a mother carrying her child up the stairs to bed may audibly count the stairs as she goes; parents driving along with their children in a car may audibly count the number of trees they pass or the number of cars on the road; a woman dishing up the dinner may count out aloud two pieces of chicken for each adult and one for each child; and a child may be helped to count how many steps they take to walk along the footpath. These experiences are often missing for many Aboriginal children since, as described before, this ability is not highly valued as either a
skill or a necessity. These children should be immersed in a 'counting world' before they can learn to count; that is, they should be surrounded by the 'saying' of the word names in sequence either deliberately or through rhymes and games. Also, they should learn the skill of counting in their own language first. If the words do not exist in the home language then they could be taught as 'words used for counting in Western society'. Once the skill is learned it can be readily transferred to the English language.

There are pitfalls here for teachers of both groups: Does the child understand counting when they say the words and moreover, is a child less intelligent or less advanced if he/she cannot say or does not know the words? Many children can learn the words up to ten with ease. They may also be fluent with words for 21, 22, 23, ...31, 32, 33, ...41, 42, 43... and so on. Indeed, the patterns within the sequence make learning numbers relatively easy for many children. Unfortunately, the numbers between 10 and 20 do not 'fit the pattern' and can cause major obstacles for children; we don't say 'tenty five' for example, we say 'fifteen'. Particular attention needs to be placed on these numbers when helping children to say, read and write them. This will be discussed in greater detail later in the chapter.

Aboriginal children are generally better than non-Aboriginal children at learning the number names in order once they are shown and can 'see' the patterns or the recursion in them. This is because of the importance of recursion in their daily lives, through kinship and environmental patterns such as the seasons and so on. This ability should be harnessed by teachers and used to help children from Aboriginal homes who may not have been immersed in a 'counting world' before coming to school.

Counting is clearly more than saying the words in the correct order. Once children are able to recite an accurate number sequence in words they then need to give the words qualitative meaning. They need to coordinate their verbal counting with actions on objects: one word for each object touched. Children need to informally learn that they are 'adding on one' each time a new number is learned. Primarily the best way to do this is to teach children through the use of objects, using a one-to-one correspondence. Activities such as counting jelly beans, matches, stones or chairs could be used. Some children may skip numbers or count objects more than once. They can be helped by actually moving each object from one place to another as they count it or, if it is a drawing or picture, cross it off as it is counted.

Clearly, in order for this to be a useful learning strategy, children need to know that it is important to be able to count – this will need to be made explicit for many Aboriginal children (and non-Aboriginal children) who don't already have a sense of this importance from their home environment. Contexts should be used where this importance is clear through the purpose. For example, count six jelly beans each so that you can eat them. Or similarly, count twenty cards each so that you can use them in your game, or count these counters so that you will know how many there are, or count these pencils so that I can find out whether you can count.

Some children can count but cannot connect this activity with cardinality – that is, they don't understand that the last number spoken tells them 'how many'. When children learn this then they clearly understand the purpose for counting. Following this they need to learn that the last number refers to the whole group – not just to the last object touched or drawing checked. This indicates that their counting knowledge has been connected to an understanding of groups.

**Subitising: Seeing numbers in groups**

Thinking in groups allows children to see relationships between numbers. Many children are able to see numbers in groups without counting (for example the way numbers are written on dice). This skill is called *subitising*. Beth Graham (1984) states that "the children in North-East Arnhem Land, if asked "how many?" will, if the group is small enough, often respond by just displaying the
appropriate number of fingers, clearly indicating their awareness of the quantity involved". She goes on to describe that this may be the reason that many Aboriginal languages only have words for one, two, and 'big mobs' since "Aboriginal societies perceive people in groups of one, two and more than two and most languages reflect this by dividing personal pronouns not only into singular and plural as we do in English, but into singular, dual and plural."

Many Aboriginal children are extremely good at subitising; and discussions with Aboriginal people and teachers of these children indicate that this also may have something to do with the amount of time they spend playing card games with a standard pack of playing cards. This may mean that these children can 'see' how many but not necessarily that they can 'count' to find how many. This is possibly also related to the fact that many Aboriginal children are strong visual learners, as described in a previous chapter.

Children should be encouraged to group numbers in different ways; six can be seen as three groups of two or two groups of three for example. Being able to think of a number as both a whole amount and as being made up of smaller parts or groups is called 'part-whole' understanding of number and provides a strong base for addition, subtraction, multiplication and division strategies. The ability to understand numbers in terms of parts and wholes is also important in assisting children to interpret numbers composed of tens, hundreds and thousands later on.

Numbers are generally attached to objects. For example, we talk about three bikes, four lollies, sixty dollars or ten fingers. A number can be vague without the noun to which it is attached. It is difficult to understand the concept of 'threeness' as merely a concept. As teachers we need to remember this as many students don't have a notion of 'three' unless it is attached to an object. Aboriginal children in particular, where traditionally concepts require an immediately useful and relevant purpose, may have greater difficulty understanding the concept of numbers greater than three unless immediate relevance can be shown. The mere fact that the words for numbers greater than three or five do not exist in many Aboriginal languages and dialects implies that the words were not needed for everyday activities or the demands of life.

Symbolic form of numbers
Being able to write numbers in their numeral form is just as important as children learning to write letters from the alphabet. In order to do this successfully children need to visualise numbers (or see them) and copy them. Teachers should be aware that whereas children from non-Aboriginal settings are surrounded by numbers in various forms – particularly as labels on phones, houses, cars and buses – Aboriginal children from remote settings may not have this exposure, making visualisation more difficult. It is helpful for children to have the numeral and a picture of the quantity on cards or charts around the schoolroom for this purpose as well as to reinforce the connection between the numeral and the quantity. Children should also be encouraged to both say and read these numerals aloud for their teacher and for other children. This should be done individually wherever possible.

Ordinal numbers
A number name which is used to describe the position of an object is being used in an ordinal sense. For example, first, second, third and so on. Understanding of ordinal numbers generally comes after understanding of cardinal numbers; once the numbers one, two, three are known children learn that the number three is third in the sequence. Children in Western homes may have this strongly reinforced through competitive games which strongly reinforce position. Very young children in Western homes may become very competitive at early ages, learning quickly whether they are first, second, third, and so on. This may not be as strongly reinforced in Aboriginal homes where cooperation may be more important than competition.
The positioning of objects can also be reinforced through activities which require children to position themselves. For example, you may ask the children to get in a line and say “Fred, you get in the fourth position”. If you don’t add “...from the left” or “...from the right” then this can create a situation where children learn that positioning may require a point of reference, so that you might say, “take the stones out of the bag and give me the fifth stone”, where the reference point is the first stone taken out of the bag.

Numbers as labels
Numbers can also be used for identification purposes, for example on football jumpers, house numbers, bus numbers, and telephone numbers. For Aboriginal children in isolated or remote schools these contexts may have little or no meaning, as some may never have seen a telephone and house numbers probably do not exist. Most however, will be familiar with numbers on football jumpers. Some of these children may even think that the player wearing number '1' on his back is the best player or the first player. This labelling function of numbers needs to be discussed and made explicit with all children.

Face value of numbers
All numbers have what is called a 'face value'. This refers to the value of the number itself, so we know that ‘8’ is more than ‘6’ and ‘2’ is more than ‘1’ for example. As teachers we often assume that children implicitly understand this if they can count, merely because the numbers of greater value are said next. So if a child says “six” after “five” we assume that they know that six has a greater value than five. Clearly, this can be a false assumption especially if children are merely saying the words as opposed to really counting, that is, saying the words whilst simultaneously understanding that they are adding a value of one each time a new word is said.

Discussions with many Aboriginal people have revealed that frequently Aboriginal children play with cards and use the face of the number (that is, the symbolic representation of the number) without understanding the face value or quantity of the number. In other words, they can say, read (and even subitise) the number '8' without understanding that 8 has a greater value than 7 or 6 for example – it is merely a symbol on a card. Thus they often don't understand the concept of 'eightness' as being connected to value and quantity.

The notion of value or quantity may need to be explicitly taught. What does it mean that 8 is greater than 6, for example? We should not assume that children know what this means. In Aboriginal societies where quality may be of greater importance than quantity in some contexts, children may not understand the notion of ‘more than’ or ‘less than' since they may not be concepts that are valued or necessary. Some Aboriginal languages reflect this in that these words are not included. These misunderstandings may be widespread among all children – both Aboriginal and non-Aboriginal. Teachers however, need to be aware that no assumptions should be made about children's abilities to count and use numbers on entering school.

The following strategies can be used in order to assist children to understand counting and whole number:

- For very young children they should be encouraged to count collections of things in different ways so that they learn that no matter which end of a row of objects they start from for example, they get the same last number which tells them how many there are, and similarly if they start in the middle of a row or a group. They can play games such as holding a small number of objects in their hand and opening it briefly asking a friend how many they have, or holding up fingers using one hand and then two, asking their friend to count how many there are and to show that many on their own hand;
Older children can decide whether they will obtain the same result if they count in twos, threes or even eights. They can discuss how many spoons or plates they need if each child and the teacher are to have one of each. They can learn to subitise larger numbers by examining different ways of displaying these as groups of dots such as arrangements on a dice. For example, is eight easier to recognise if it is shown as one group of six dots and one group of two than as two groups of four? They can use playing cards to do this by covering up the numbers and 'flashing' these to each other, so not allowing sufficient time to count the shapes on the cards.

To improve understanding of ordinal number children can say who is fifth in line or which is the third day of the week. They can order their daily routine by talking about the first thing they did when they got up. They can identify where numbers are used as labels in their community and/or in their school. These activities can increase in difficulty by using larger and larger numbers and including contexts that go beyond their known world. For example, young children need contexts that are about themselves and their families and the school room. (Note that Aboriginal children at very young ages, are often more able to use wider environments due to their well developed spatial sense, which will be discussed in detail in Chapter 8). As children get older they become more aware of the wider environment; the school, town and nearby towns that they visit. Older children can do the same activities for Western Australia and different states and countries; examining the use of numbers as post codes, zip codes, flight numbers, international phone numbers, number plates and so on.

### Place value, decimal and money

#### Two- and three-digit numbers

Understanding of place value for two- and three-digit numbers can be very difficult for many children. Counting is generally the method children use to determine 'how many'. They can use the 'touching' of objects or crossing of pictures as they count and so they need to know the sequence of numbers for the number of objects or pictures they have. The ability to count to larger numbers may precede the ability to read and write them. Learning the counting sequence however, can be assisted through the use of a hundreds chart or with the constant function on a calculator. As stated previously, children should be encouraged to say the numbers as they are read.

Writing the numbers should follow the understanding of place value using part-whole relationships. Children should be taught to count using bundles of ten. Used matchsticks, toothpicks, and straws are useful for this purpose (as opposed to MAB's) as they can be 'bundled' using rubber bands. Children counting to 26 for example, can count one group of ten, another group of ten and six more – they can see that they have 26 single straws which together make up two groups of ten and one group of six which also make a group of 26 straws. When asked, "How many tens are there in 26?" they are able to answer "two" because they can see them. When asked to show their teacher 26 using bundles of ten they understand what that means and are able to do it.

Patterns in learning to write the names of numbers can be a problem for numbers between ten and 20. For numbers such as 32 most children learn through part-whole relationships, such as that described above, that this is 30 and 2 and leads them to readily read the number as "thirty two". This pattern and part-whole relationship is often more difficult for numbers like 11, 12 13 and so on. Children need to be taught these numbers almost as a separate group since the digits used in numbers one to nine are used again but in a different way. Also, they are read from right to left as opposed to the numbers after 20; 15 for example is read as the five first and then the ten. Both children and teachers need to say these numbers clearly since for example, it is easy to mistake "forty" for "fourteen".

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**Improving Aboriginal Numeracy**
Grouping of numbers using bundles of ten, then of one hundred and of hundreds up to ten hundred can be done using straws as described before. Children should be encouraged to regroup these bundles. Six bundles of ten and five more (65) can be regrouped, for example, to show two bundles of ten and three more and four bundles of ten and two more; or one bundle of ten and zero more and five bundles of ten and five more. Proficiency and fluency at this type of task will greatly enhance understanding of place value, and later, of addition and subtraction.

To make judgements about children's understanding of place value, teachers need to ask questions of individual children. Regarding the numbers themselves, children should be asked to read and say numbers such as 13, 28, 40 and 81. Regarding quantities, children can be asked to show say thirty straws or forty-five straws, noting whether they use bundles of ten or just count single straws. A child can be shown three bundles of ten and four single straws and asked how many straws there are. A child can be asked to show three different ways of grouping thirty-four straws. The questions can also be asked using the numeral form of the number instead of the oral form; for example, showing a child the number 43 on a card or piece of paper and asking them to make that number of straws.

These experiences will give clear indications of a child's understanding of tens and ones. Diagnosis of this type can also occur for the hundreds. Note that these activities can also be conducted with children working in groups, the teacher observing individuals within the groups and listening to comments made to ascertain understandings.

Larger numbers
The relationship between numerals in the ones, tens and hundreds columns provides the basis for learning about thousands and millions. Once children have understood this critical relationship for the ones group they should then be introduced to the next 'group' which is the thousands. Often the position to the right of the hundreds is introduced as being the thousands and this can cause students to have misconceptions about numbers larger than hundreds. The relationship between one thousands, ten thousands, and hundred thousands is the same as that for the ones group (i.e. ones, tens, and hundreds) as follows:

<table>
<thead>
<tr>
<th>Millions</th>
<th>Thousands</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>T</td>
<td>O</td>
</tr>
<tr>
<td>H</td>
<td>T</td>
<td>O</td>
</tr>
<tr>
<td>H</td>
<td>T</td>
<td>O</td>
</tr>
</tbody>
</table>

The reading and saying of numbers using this format is simplified greatly for children; they understand that the words 'million' and 'thousand' occur in the gaps between the groups. A number such as 356 724 169 for example, is read as "three hundred and fifty six million, seven hundred and twenty four thousand, one hundred and sixty nine". Once again, for Aboriginal children, these patterns will assist them in learning about numbers and their attention must be drawn to them.

Decimal numbers
Many children learn how to compute with decimal numbers before they understand what decimal numbers are. Unfortunately, once computational procedures – which often are the same as for whole numbers – are learned, it is difficult for children to understand that the numbers themselves are different. Students may be able to add, subtract, multiply and divide decimal numbers using learned pen and paper strategies with fluency but do not conceptually understand what decimal numbers are. As a result, they are unable to judge the reasonableness of their answers or, for that matter, estimate an appropriate solution before they begin the task, even if they use a calculator.

It is essential therefore, that children understand the nature of decimal numbers before computing with them. As teachers we should not assume that the understanding of decimal numbers will follow or occur as children gain computational fluency with them. Indeed, children are more likely
to compute operations with decimal numbers using a calculator – and this is appropriate – and hence need to understand the decimal numbers they are working with, so that they are able to first estimate and then judge the reasonableness of the answer obtained.

It is also essential that children understand the mathematical operations they are working with before they are taught computational procedures. More will be said of this later. Some children with an immature understanding of multiplication for example, believe that when two numbers are multiplied together a larger number than either of the two will be produced. Clearly this does not hold when multiplying a whole number by a decimal number with a value less than one. They need to understand that when multiplying say 2 by 0.45 they are in fact finding two lots of 0.45 and should therefore expect an answer smaller than 2; or 0.45 of 2 is less than 2. Similarly, when finding 3.24 x 0.36 they should understand that they are finding 3 lots of 0.36 and part of one lot of 0.36; so their result will clearly be smaller than 3. The rule for placing the decimal point four places from the right of the number calculated as the answer should result from children understanding that hundredths lots of hundredths is ten thousandths, it should not be merely taught as a rule to follow! Estimation should also be used as a matter of course, so that in this case the result should be seen as about three lots of one-third or one. Multiplying 324 by 36 gives 11664 so, from the estimate, the result must be 1.1664.

There is clearly little value in children performing these algorithms if they do not firstly understand the numbers they are working with, and secondly understand the operation they are using. This may not occur until late in the primary years.

There are three similarities between decimals numbers and whole numbers (Resnick et al, 1989):

- The value of the digits decreases as you move from left to right
- Each column is 10 times greater than the column to the right
- Zero serves as a place holder

In teaching the first two of these it is important that children use materials that are continuous with the materials that were used to teach whole number. Children using MAB blocks for example, to learn whole number where a small cube was used as a unit for 'one whole', are often conceptually unable to then alter their point of reference and use a 10 cm by 10 cm by 10 cm cube to represent the unit 'one whole'. If however, they were taught about whole numbers using a straw as one unit it is much easier for them to simply use the straw cut into 10 equal lengths, each length now representing one tenth or 0.1. Cutting the straw into one hundred pieces will not be so easy although for demonstration purposes it may not be impossible!

Money is not necessarily a good concrete material to use when teaching these concepts since conservation of size and shape is not able to be used as with straws or strips of paper for example. Some children cannot 'see' that there are one hundred cents that make up a dollar, and especially since we no longer have one-cent pieces! Further discussion regarding the use of money in teaching decimal concepts will be made later.

Resnick et al (1989) similarly identified differences in comparison of whole number and decimal number knowledge as follows:

- For decimals, values decrease as you move away from the decimal point
- For decimals, names end in 'ths'
- For decimals, the naming sequence moves from left to right (e.g. tenths, hundredths)

Many children, both Aboriginal and non-Aboriginal, have numerous misconceptions concerning the similarities and differences outlined above. Unfortunately many of these find their roots in the context of money. Conversations with teachers of Aboriginal children and Aboriginal people have
revealed that some Aboriginal children have an advanced facility with money before they even begin formal schooling. This is due primarily to the greater responsibility given to these children at an early age as described in Chapter 1. Children as young as three years of age may be sent to the shop to buy goods.

As a consequence, children may come to school with a strong facility with money which may be confused by teachers as being understanding about money and hence may hinder and confuse subsequent learning about decimals and place value. The degree to which this occurs in all children is dependent on their handling and use of money in the home and, as indicated above, may be more pronounced in some cultural groups—particularly in Aboriginal children.

Pam Harris (1984b) identified many misconceptions and misunderstandings concerning money held by the Aboriginal adults and children where she worked. These include:

- Coins are easily recognised but what the relationship between different coins and notes is does not seem to be understood;
- There is a tendency to match coins to objects without thinking of calculating the total cost for the total number of items. Harris (1984b) uses the following example provided by an experienced teacher in a remote part of Western Australia.

  We cleared the area and tipped a large number of coins in the centre of a circle of children (ages approx. 10-15). I asked, "How much would it cost to buy 6 eggs? Put the money in front of you and we will see who has the correct amount". All put 6 x 5c coins out...I asked how much each person had and I was amazed to find that no one had thought it necessary to calculate the value of the coins. This could be worked out but was as an 'extra' and not part of the original question". The teacher said she had often observed this one-to-one correspondence. The method did not impede the accuracy of the answer, only the speed with which it was performed. (p. 12)

  Whereas non-Aboriginal shoppers mentally compute total numbers of goods purchased and amount needed to pay for these goods, Aboriginal shoppers have been observed purchasing several small quantities one after the other (p. 17).
- A misconception that no number of coins can ever have as much value as paper money.
- Coins are often subitised, that is, the amount of money is known by merely "looking at a group of coins rather than by counting them, e.g. a person seeing three 20c coins would know that was 60c though he may have difficulty if required to count out 20c, 40c, 60c to get the total" (p. 13). (This recognition is similar to their quickness with cards; instant recognition of patterns or relationships being used rather than any form of calculation). It is important for teachers not to assume fluency with counting in these situations but to use them instead to teach counting of the groups.
- The equivalence between the numerical value of the notes and coins and commodities is not explicitly understood. An example is given by Pyper (1978, p. 67): "...a woman, when asked how much a taxi ride from Warrabri to Tennant Creek cost, had replied 'Three reds, one blue'". And further "...residents of the Duck Creek camp will request purchases when someone is going to the store, but only give $10 for goods worth perhaps $50; there is no intention of not providing the necessary sum" (Harris, 1984b, p. 13).
- Money has both symbolic and numerical value and these may be different. While the symbolic value of money in many Aboriginal communities is the same as in non-Aboriginal communities, its use and real value might be very different.
- Coins may be regarded by many Aboriginal people as worthless or 'rubbish money', particularly in some remote shops where all prices are rounded to the nearest 10c, 20c or even the nearest $1. Teachers can teach that 5c coins have value by using them to teach the concept
of 'equivalence'; students bring in the 5c, 10c and 20c coins and add them in piles, exchanging them for $1 and $2 coins when enough have been collected or using them to buy things for the students from the school or community shop.

- Coins may be grouped and named on the basis of size, shape, colour (gold or silver); their value must be explicitly taught through exchange demonstrations and equivalence practices. At Yirrkala in the Northern Territory, the Garma maths project focuses on the use of patterns to teach equivalence. Drawing on the visual learning strengths of Aboriginal children, ten 10c coins are lined up and shown to equal one $1 coin; ten $1 coins are lined up and shown to equal one $10 note; ten $10 notes are lined up and shown to equal one $100 note. These patterns of ten help children visualise the equivalence of money values. They can also be used for units of measurement and for teaching place value of numbers (Watson-Verren, 1992).

It is important for teachers to recognise and appreciate the attitudes and values held by many Aboriginal people towards money and its traditional place in Aboriginal society as compared to that in non-Aboriginal society. These attitudes often result in different ways of handling money and have been summarised by Pam Harris as in Table 2.

Table 2: Different Ways of Handling Money

<table>
<thead>
<tr>
<th>Aboriginal Traditions, Attitudes and Values</th>
<th>Anglo-Australian Traditions, Attitudes, and Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Conservation</em> practices and the proper performance of ceremonies ensured the continuation of food supplies</td>
<td>Money is saved so that food and other necessities can also be bought in the future</td>
</tr>
<tr>
<td>'Blackfella business' is mainly concerned with ceremonies and traditions</td>
<td>'whitefella business' is mainly concerned with making money</td>
</tr>
<tr>
<td>'Work' is mostly that which must be done to perform ceremonies and maintain traditions</td>
<td>Work is mostly that which is done to earn money</td>
</tr>
<tr>
<td>Coins and notes are usually called by names that describe their appearance</td>
<td>Coins and notes are usually called by names that describe their value</td>
</tr>
<tr>
<td>Knowledge of the combined value of several coins or notes together seem often to be based on seeing a pattern</td>
<td>Knowledge of the combined value of several coins or notes together is the result of calculations using mathematical processes such as addition or multiplication</td>
</tr>
<tr>
<td>Future security is achieved through debts and obligations - 'banking on people'</td>
<td>Future security is achieved by saving money in a bank, and other kinds of investments</td>
</tr>
</tbody>
</table>

Note: After Harris (1984b, pp. 31-32).

It should be remembered that Harris's work concentrated primarily on the understandings and traditions of tribal Aboriginal groups and that as a result cannot be generalised to all Aboriginal people groups and cultures. Many of the traditions described above however, continue to be found in varying degrees throughout Aboriginal communities, both rural and urban, 'tribal' and 'non-tribal', depending again on the degree of assimilation with non-Aboriginal people groups.

There is often a tendency for teachers to believe that because children are competent with money they understand decimal and place value. Indeed, for many years our syllabuses encouraged the use of money as a context for enhancing the learning of these concepts. It can be seen from the above
discussion on money however, that 'understanding' of money is not the same as facility with its use. We should be aware that familiarity with money can provide children with a real life motivation for the learning of decimals but that connections between decimal and place value and money as a context must be explicitly made for children, and not assumed. If these are not made then lines between concept and context become blurred and provide a 'breeding ground' for misconceptions. Consider the following misconceptions.

- Some children believe that the decimal point operates as a boundary; a point used to keep the whole from the part.

  This misconception is most clearly revealed through questions such as the following which I gave to 145 thirteen-year-old students in one school where I worked:

  "What is the value of $3.156?"

Almost one quarter of the students wrote $4.56 as their answer; they perceived the decimal point as separating the dollars from the cents and hence believed $3.156 to be the same as $3 and 156 cents. They then added the 156 cents ($1.56) to the $3 to give a result of $4.56.

It is clear that these types of misconceptions are not restricted to any particular age group. In fact a misconception such as this can be widespread in children in middle schools. There are many reasons for the occurrence of this misconception. We often read $4.32 as "four dollars and thirty two cents". Clearly the dollars and the cents are separated by the decimal point. When we read 4.32 however, we say "four point three two". When children learn this they are confused; how can decimal and money have any connection at all? Children need to be taught explicitly that $4.32 is the same as "four point three two dollars" and that these are two different ways of saying the same thing. If this connection is not made for children then the following further misconceptions may result:

- The greater the number of decimal places the greater the value of the number.

  For example, more than one quarter of the 145 Year 8 students described above believed that 0.38 was worth more than 0.4. Why? The reasons were varied but most indicated that "thirty eight is bigger than four" or that "thirty eight sounds bigger".

Clearly, the saying and reading of decimal numbers is an important part of learning this concept. Owen et al (1993) suggest that children should think of a number like 0.374 as 3 tenths plus 7 hundredths plus 4 thousandths and that teachers should avoid the use of the word 'point'. I'm not sure that I agree with this. I think that children should be taught the difference in the way we say the numbers with and without a point; that digits to the right of the decimal point are read differently as a way of noting the difference in their value, that these digits are parts of whole numbers and that saying their face value as we do (for example, saying 4.37 as "four point three seven") clearly indicates that the four is a whole number and that the 3 is three tenths and the 7 is seven hundredths. The decimal point can be taught as having the function of separating the whole numbers from the part numbers, but that it is not a boundary across which number values cannot pass.

It is also clear that teachers need to be explicit about using words like 'bigger' when asking children to order numbers. For example, when we ask children “what number is bigger?” we need to be aware that this word ‘bigger’ has many meanings in different contexts. We need to be sure that children understand the concept of quantity and value as what is being talked about here and not physical size. We might ask them to list as many words as they can for meaning ‘more than’ or ‘is worth more’ so that they know what is being asked when asked to explain which number has the greater value.
I recall a situation in my classroom once when I asked students in a test question to tell me which ‘5’ was bigger in the numbers 0.05 and 0.5 to which one student explained ‘they are both the same’ – and he was right of course, in the context of the print size or face value.

♦ Zeros within a decimal number are unimportant.

Out of 145 thirteen-year-olds tested, 22 believed 5.06 to be the same as 5.6. Reasons given included
- "because zero means nothing"
- "because zero is nothing"
- "because the zero does not exit"
- "because it's just a zero before the six"

When working with money, students may indicate that $1.02 is the same as $1.2. This is clearly tied up with the discussions and arguments used above. Children need to be taught that whether they are working with money or not, each digit used has both a place value and a face value. The zero in 1.02 is used to indicate that there are no tenths; it is not used, as some children believe, to keep the 1 and the 2 apart. Many children are not taught that zero is a number; they may indicate that 'zero is nothing' and this will lead them to omit it and not write it down during calculation. This teaching should occur during the learning of whole number concepts. Children should also be taught to say it correctly; 4.06 should be read as "four point zero six" and not as "four point naught six", “four point oh six”, or “four point nothing six". Be sure children are taught that zero is a number – that 0 doesn't represent nothing.

♦ The greater the number of decimal places the smaller the value.

Many children recognise that the decimal part of a number is about fractional quantity; that is, parts of a whole. Because of this they believe that the more 'pieces' there are, then because the pieces must be smaller their value must be smaller. For example, of the 145 Year 8 students tested, 24% said that 4.376 is of less value than 4.2, with some of their reasons being:
- "4.2 gives bigger pieces if you cut it like a pie"
- "because 4.2 gives a bigger chunk"

To help children with this misconception teachers can show them how to partition the number so that children look at each of the digits and learn that they have values which can be compared. For example, 4.2 = 4 ones + 2 tenths whereas 4.376 = 4 ones + 3 tenths + 7 hundredths + 6 thousandths. They can see that the one with 3 tenths must have a greater value than the one with only 2 tenths – especially if MAB blocks or straws are used to represent these numbers.

Misconceptions about the decimal value of numbers may abound in children from all cultures, backgrounds and ages. Some of these are clearly linked to the context of money and existing understandings about this context. It is not being suggested here that the use of this context be avoided in the teaching of decimal. Children need concrete materials with which to explore the 'abstractness' of the decimal concept. Connections between the concept and the context however, are of extreme importance and need to be explicitly taught.

Some possible strategies include the teacher writing a number on the board such as 367 and asking children to say the number and then write how many the six represents, for example. They can partition the number orally or in written form, saying that "three hundred and sixty seven is the
same as three hundred and sixty and seven" and then writing it as $300 + 60 + 7$. They can play games throwing dice ten times each and keeping a running score to show their total, using a place value chart such as

```
H T O
```

or by drawing bundles of ten such as \[
\begin{align*}
\text{H} & \quad \text{T} & \quad \text{O} \\
\hline \\
0 & \quad 0 & \quad 0
\end{align*}
\]

They can use a calculator, putting a number in the display such as 358 and changing the 5 to a six or the 3 to a seven without clearing the display, for example. This can also be done with decimals for older children, changing the 7 in 56.79 to an 8 without clearing the display and starting again.

Running a class shop (if children are familiar with the shop as a context) is a good way to improve understandings of money in a decimal context. Children can write their own price labels for products, learning that $4.3$ does not represent $4.03$ for example and that here, their '0' indicates there are no ten cents and that the decimal point does not separate the dollars from the cents but that the cents are part of the total amount. They will also learn that $4.03$ is the same as 403 cents and that this can be read as "four point zero three dollars" or "four dollars and three cents" or "four hundred and three cents". They can count the money they have brought to school, individually and then putting it all together, learning how to say, read and write the amount.

**Fraction**

On entering formal schooling most children have a 'common understanding' of the concept of fraction. This is derived from exposure to the hearing and the use of words such as 'half' and 'quarter'. Mothers may tell their children to "cut that in half and give the big half to your father".

Children learn that a fraction is a part of a whole. This understanding is often attached to the notion of 'fairness'. They learn that for siblings the halves are as matched as they can make them ("you cut and he can choose") but that for parents or adults the 'halves' can just be two pieces of differing size. For Aboriginal children this can be even more pronounced since personal relationships are so important, the notion of 'fairness' is not as pronounced as it often is in other cultures. It can be more about 'sharing it out until it's all gone' as opposed to 'each of us having an equal share'. It may also be related to the physical size of each person receiving a share; the bigger the person, the bigger the share they receive.

All children have to make the connection between the common understanding of fraction and the mathematical understanding of the halves being proportionately equal in value. Note that value may be a variable quantity based on area or size (as in half of a shape or object or length), or based on monetary value, or based on numerical quantity as in half of a box of toys.

Clearly, for fraction to be fully understood children need to learn that the 'whole' or 'unit' must be defined before they can find half of it. This step is often missed in the classroom so that students as old as 12 and 13 can find half of an object or shape when the unit is one object, but cannot find half of a group of objects. They may think that fraction is merely about symmetry which although a useful starting point when working with whole units such as circles, squares and so on, is only a first step in the understanding of fraction. Children must be moved on from this position in their understanding so that they make the connection that fraction is not just about parts of a whole shape, but parts of a unit and that a unit can be a collection such as a crowd, or a plate of different types of biscuits.
Children also need to learn that the defining of the 'unit' is crucial to understandings that in fact, there are instances in which a 'half' is smaller than a 'quarter' when the unit used is not the same for both fractions.

The above understandings should be consolidated before children are taught to write or manipulate fractions in the symbolic form. The word 'half' needs to be clearly understood in its mathematical sense, and similarly 'quarter' being derived from the notion of 'half of a half'. Common understanding of 'quarter' may for many children just mean 'a little bit' but not be connected to the 'half of a half' understanding as being four pieces or parts of the whole. Beware of children who can use the words 'half' and 'quarter' in a sentence and have you, as teacher, believing they understand the mathematical concept of fraction.

Many Aboriginal people talk about sharing 'alf 'n' 'alf' which can be roughly interpreted as meaning 'half each'. They might not talk about having a third or a quarter each however, and tend to use words such as 'ave a bit' each when sharing between more people and this sharing may or may not contain an element of 'equal shares'. They might say, for example, “some for Mary ‘n’ some for Colin ‘n’ some for me”.

The notion of 'fairness' in terms of 'equal shares' may have different meanings for children from different cultural backgrounds and it is essential that it is explicitly taught as being about equality in order that children gain a deep understanding of the mathematical concept of fraction. "Linguistic research in one Cape York language", explains Graham (1982), "revealed that when a back translation was done the Aboriginal word offered as an equivalent for sharing, really meant 'exchange reciprocally'. This is a vastly different concept from that which is inherent in our use of the same word". Clearly, to teach fraction (or division) in terms of sharing – and particularly 'equal shares' – would also require some explicit teaching of the words 'equal' and 'share' in context before they could be used in a mathematical, fractional sense.

Once this understanding has been established children may be exposed to the symbolic notation of unit fractions (that is, fractions with a numerator of 1), and the notion of equality can be firmly consolidated in the use of the denominator as describing 'how many equal parts or quantities'. Note however, that the unit (to be ‘fractioned’) must be defined first. Questions should be phrased in such a way that children are not left in any doubt as to the unit they are working with – such as "What is half of the number of toys in that box?" or "What is half of the length of that piece of rope?" or "What is one third of that shape?" or "What is one quarter of those circles?".

**Fraction as symmetry**

When using single shapes as units, children need to be exposed to different ways of finding fractional parts of the shapes. For example, when given a square and asked to cut it into quarters they need to learn that they are working with the area of size of the shape and that there are many ways of breaking the square into quarters. This is often a difficult concept for children when the resulting quarters do not look equal and yet they are told that they are the same. In actual fact it is the areas of the shapes that are equal not the shapes themselves.

Some children, when asked to show half of a square will usually show a half of a square using a line of symmetry as follows:

![Diagram of squares showing fractions](image)
They usually do not consider the following types of divisions, but if they do it is a clear indicator that they understand the concept of fraction more fully than children who only use a line of symmetry.

Children need to understand that fraction is about quantity not appearance; that fraction refers to equal portions. The following for example, can all represent one half:

When shading one half of the first square above, children will learn there are many ways of shading one half. They need to understand the idea that if the unit is made up of four equal quantities, as in this case, then to shade one half they need to shade one part for every two parts and that as a result, there are many ways of doing this. This idea is related again to teachers 'defining the unit' that they are working with. Here the unit is one square but it is also four squares which may or may not be joined together to form a larger square. For example, Many children could shade one half of the first square above but when asked to shade one half of four separate squares they could not do it. They would be helped in making this connection if the connection were made: "Shade one half of these squares means shade one square for every two squares". This requires them to understand however, that this means 'select two squares and shade one of them' and not 'shade one and leave two unshaded'. The following diagram represents one student's shading of two fifths:

Using shading, children learn that many different fractions can represent the same amount. Using a circle, for example, they can see that by shading $\frac{3}{4}$ of the circle and then shading $\frac{6}{8}$ of the circle they see visually that 'three parts for every four' is the same as 'six parts for every eight'.

**Using sharing as a teaching strategy**

Teachers can also use sharing strategies to teach fraction. This is particularly useful in order that children learn that fraction is not simply about 'breaking up' whole objects such as an orange, a pizza, a cake and so on. Children who learn about fraction only in continuous quantities (that is, with whole objects being partitioned into sections) will not fully understand fraction. Similarly for children who only learn fraction with discrete quantities (that is, pattern tiles, counters, pencils, toys being shared into groups); their understanding of fraction will be incomplete. Children need to be exposed to and have experiences with a range of both continuous and discrete quantities. If they don't see fractions represented in a variety of ways they may generalise that fraction is only about whole shapes or that it is only about tiles and counters. Equivalence of fraction can be taught using
this sharing strategy. By finding half of thirty using sharing (that is, you have one then I’ll have one, then you have one...) they will learn that one half is the same as fifteen out of thirty. Children should learn that the more shares that are required, the smaller each share will be.

The idea of 'equal shares' is conceptually more difficult than just 'sharing'. If this is to be used as a teaching strategy for fraction (or division), it needs to be taught explicitly, as described earlier. For some Aboriginal children, sharing can be about the ‘size’ of the recipients, as described earlier.

Fraction representations
Fraction should be taught using a variety of representations. For example, a teacher may use real world objects such as oranges or pizzas, manipulative materials such as pattern blocks and counters, pictures or drawings which can be shaded, spoken symbols – that is, saying fractions – and written symbols such as $\frac{1}{2}$. Lesh et al (1981) found that children exposed to this variety of representations and who were frequently asked to switch between these forms developed a more enhanced and flexible notion of fraction as compared with those who weren't.

Children will also learn that some groups or collections cannot be shared into equal groups. To show children that they can find half of 25 cakes for example, they can simply continue to share the pile one-for-one until they get to the end. They should then discuss what to do about the remaining cake. These discussions are invaluable in promoting student learning. We should not just use whole, 'easy' numbers but should challenge children's thinking of fraction wherever possible.

From words to symbols
Children need to learn to recognise number symbols for fractions and be able to read these symbols, just as they learn to recognise the numeral forms of numbers when they count. Teachers need to be aware however, that many children are able to recognise, read and say fractional names but do not understand the concept of fraction.

Many children have difficulties with the symbolic form of fraction. They have learnt the symbolic form of numbers and know that the numbers 1, 2, 3 and so on are in sequence so that 3 is more than 2. When they try and transfer this rule to that of fractions they are confused since $\frac{1}{3}$ is smaller in value than $\frac{1}{2}$. I recall a class of year 8 students whom I asked to write a fraction less than $\frac{1}{2}$. A large proportion of them wrote $\frac{1}{1}$ since the only thing they could 'make smaller' was the ‘2’. These sorts of misconceptions occur when children do not understand concepts and make up rules in an attempt to do so.

When teaching the language of fraction children need to use sentences prior to introduction of symbolic notation. For example, "There are three equal parts so each one is one third". They should be encouraged to speak these sentences individually.

They should learn that to find three quarters of a 'whole' unit, object or group of objects, they need to separate the object into equal parts and 'take three out of every four of them'. In this way they learn the connection between $\frac{3}{4}$ and $3 \div 4$. For example, when sharing three cakes among four people they will find this easier to understand as a possibility if they can share each of the three cakes among four people; each person will get one quarter of each cake so altogether they have $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$, a total of $\frac{3}{4}$ each.

Students need to be explicitly taught about the symbolic form of fraction; what does the top number (numerator) represent and what does the bottom number (denominator) represent? Their conceptual understanding of the denominator as the number of equal parts being considered will probably precede their conceptual understanding of the number of equal parts being focussed on (numerator), but this may not always be the case.
It is important when working with one whole unit (such as a pizza or orange) to concentrate on the idea of 'equal parts', since some children, despite having an understanding of $\frac{2}{5}$ as 'two out of five pieces' for example, may simply cut five unequal pieces and put two of them aside. Similarly, when shading $\frac{2}{5}$ of a circle they may draw five pieces of any size and shade two of them. This misunderstanding often goes by undetected, since many shapes requiring fractional partitioning are, presented by teachers with lines separating the equal portions already drawn. For example:

Shade $\frac{2}{5}$ of this shape:

Teachers can better detect their student's misunderstandings concerning the concept of denominator as being about equal or fair quantities, if they ask the same question but do not separate the fifths:

Shade $\frac{2}{5}$ of this shape:

**Different interpretations of fractions**

These different ways of interpreting fractions - as an area or measurement, a division, or ratio - are important and introducing the concept of fraction in these various forms can help children understand. It can also have an impact on helping children to understand the concepts of ratio and even division. Children who confuse $3 + 4$ with $4 + 3$ will be more able to conceptualise the difference if they can make the connection with fraction.

A fraction can also be used as a label in the sense of a number on a number line. Children need to be taught that when using it this way the *unit* they are working with is the section of the number line between 0 and 1. When children are presented with a section of a number line and asked to find, one half, they should be shown what is the unit they are working with if it is not the length from 0 to 1. If the line they are given goes from 0 to 10 for example, students can hardly be blamed for locating 0.5 on the number line as opposed to 5, if that was the teacher's intention.

**Comparing and ordering fractions**

Since fractions are numbers children learn that, as with other number forms, fractions can be compared and ordered. It is important that students conceptually understand fractions before being asked to engage in formal *comparison* of fractions. If they are able to *visualise* $\frac{1}{3}$ and $\frac{1}{2}$ they will not experience difficulties in saying which is bigger or larger or has the greatest value. If, however, they attempt to compare the numbers $\frac{1}{3}$ and $\frac{1}{2}$ without being able to visualise what these fractions look like with respect to the unit they are working with, then often the task of comparison is reduced to manipulating numbers.

This lack of understanding is often evident when students for example, say that $\frac{1}{3}$ is bigger than $\frac{1}{2}$ because 3 is bigger than 2'. Children will learn that visualisation is the most efficient way to compare fractions but that if they can't visualise then there is a method they can use which involves converting to common sized parts. Misunderstandings that arise should be exposed through discussion with both the classroom teacher and other children working together in groups. Children need to be free to voice these concerns which expose misunderstandings and consequently lead to them constructing their own knowledge with guidance and effective intervention by the classroom teacher.
Some effective strategies may include children discussing what 'half' means in different contexts such as half a pizza, half a ball, half a packet of cookies, half a page. They can talk about how they know it is a half and whether they are fair and equal halves. They should talk about how many ways there are to make half a bag of lollies or half a piece of paper. These activities can also be done for quarters and as children's understandings improve, with other unit fractions such as one third, one fifth and so on.

As children get older they can learn how to write these fractions using symbolic notation; reading fractions and drawing pictures to represent them using different contexts, such as one third of a cake, one third of a packet of Smarties, or one fifth of a square, showing different ways this can be done where possible. Children can investigate the number of different ways a square can be divided into quarters, for example.

By drawing representations of fractions using the same 'whole' or unit, they can show that there are many ways of writing the same fractional quantity; one half of a circle or square is the same amount as two quarters or three sixths of the same unit. By grouping they can show that two eights of eight lollies is the same amount of lollies as one quarter of the eight lollies. If one child folds a metre-long strip of paper into quarters then this is the same length as produced when another child folds a metre-long strip of paper into eighths and shades two of them. Thus they learn and understand about equivalent fractions.

Using similar activities they can show that fractions can be ordered; when they can see that three eights is a greater amount than two eighths they can understand and show that three eighths is greater than one quarter. Teachers should be aware however, that children need to be able to order whole numbers before they can even attempt to order fractions, so it should not be assumed that children know what ordering is.

When taking part in sharing activities such as 'share these ten pieces of paper among five people' or 'share two packets of biscuits among three people' or 'share this pizza among four people' children need to explain how they did it, justifying decisions about fairness and size. They should use words such as 'these twenty biscuits are the whole and I shared them evenly among four people by giving them five biscuits each so they would each have the same amounts. They should understand that five out of twenty biscuits make one fourth or one quarter.

One way to connect the concept of fraction to the world of Aboriginal children is to use the importance of the people in their life to place fraction in a meaningful context. For example, use the context of family as follows: children know the number of people in their family. For Aboriginal children be aware that the family 'unit' as we know it in Western society is not the same; their family 'unit' may have more than one hundred people in it.

Children can sort and classify subgroups of the family into categories such as boys, girls, grannies, uncles, and so on, which can be thought of as 'part of the family'. Using manipulatives such as coloured cubes, one colour for each group, initials for each person can be placed on the groups, helping children to make a one-to-one correspondence for each family member and in this way, not leave anyone out. They are then in the position to make statements such as "three out of 56 are grannies", or "24 out of 56 are female", and so on. This type of activity can also assist children in learning data-handling skills, which will be described in Chapter 9.

Other contexts for teaching fraction (or parts of a whole) for Aboriginal children might include:

- the seasons of the year and parts of the day; or
- people classified into people groups or cultures such as Balanda/Yolngu. (after Kepert, 1993)
6.3 Understanding operations

Traditionally a skills approach has been used to the teaching and understanding of mathematics operations in school. That is, we have focussed on the use of symbols to represent expressions, the use of algorithms, and the use of memory; teaching the method and practising the method until it became fluent.

We have worked from the belief that if children practise how to add for example, they will understand the operation of addition. Research has shown that for the majority of children, this is not true. There are many children who have difficulty with addition when performing an addition algorithm – not because they don't understand addition but because they can't remember the steps in the algorithm or they don't recognise the symbolic representation for addition.

This is likely to occur when children are taught symbolic representation or algorithmic procedures before they understand what the operations are all about and how they connect to real life experiences. We know that if children are taught the operation of addition at the same time as they are taught the addition algorithm they will confuse the two. That is, many children believe that addition is the addition algorithm; in fact, many adults believe this! It is essential that children are taught operations in isolation from formal procedures that produce answers for them.

Addition and subtraction

In Chapter 4, I retold an anecdote about an Aboriginal boy who understood the idea of 'equality' through using the words 'they tie' to mean 'they are the same length'. For many Aboriginal cultures precise ideas of equality do not exist, and hence abstract operations such as addition, subtraction, multiplication and division are non-existent.

Children should learn about addition and subtraction by using concrete materials to 'bring quantities together' and to 'take quantities away'. They can hold up three fingers on one hand and four fingers on the other and say "how many have I got altogether?" and use subitising or counting to produce an answer.

They should be exposed to problem solving situations through written stories, The Three Bears for example, and learn the concepts of addition and subtraction before being exposed to symbolic representation such as $3 + 4 = 7$. This can initially be done with small, easily visualised numbers and then with larger numbers, modelling amounts with concrete materials such as counters or Smarties. They can make up their own stories for the 'bringing together' or 'taking away' of quantities. Having listened to or read stories where 'bringing together' and 'taking away' situations arise they need to be asked "What is happening here? Are we bringing together or taking away? If she started with four and finished up with one did she take away or bring together?". These types of questions are essential for children having to think about what operation has been used. This thinking will greatly assist children later on when having to choose which operation is required when solving a problem.

Symbolic representation can be introduced through playing with a calculator. They can see that the '+' symbol can represent the words 'bring together' before they learn new words such as 'add' or 'plus' which may be abstract and meaningless in real life contexts; and similarly for subtraction.

They should learn that addition and subtraction are connected to each other; they are inverse operations. They will do this best by discussing, visualising and modelling the 'bringing together' and the 'taking away' ideas in the same context. This can further be developed through 'partitioning', a discussion of which follows later in this chapter. Children need to learn that
addition is commutative, that is for example, 3 + 4 = 4 + 3; that it doesn't matter whether 3 is added to 4 or 4 is added to 3, the result will always be the same since the 'order' that the numbers are written in doesn't affect the 'bringing together' of the numbers. They also need to learn however, that this is not the case for subtraction; that the number they start with is important and that when writing this symbolically this number must be written first. This will help if the connection between the verbal form and the symbolic form is stressed. Teachers should have children read '7 - 4' out aloud as "seven take away four".

For most children, and especially many Aboriginal children with their highly developed capacity for visual learning, connections should be made using charts, drawings to the verbal, oral and drama forms, and using stories wherever possible. This can later include the symbolic form, initially using a calculator so that children get a sense of what addition and subtraction mean through questions such as, "What is the calculator doing?", "What does the '=' mean here?", and so on.

Many children interpret the '=' symbol as an operation in its own right. For example, they can learn that the '=' sign following an operation connecting two numbers means 'do something so that there will only be one number'. Children incorrectly learn that '=' represents an operator instead of a relationship. This can cause problems for children later learning Algebra. Children should learn that the '=' sign indicates equivalence and comparison such as in 3 + 5 = 5 + 3 = 2 + 1 + 5 and so on.

Learning that '5 - 3 = 6 - 4' will help children to see the subtraction sign as useful for comparison and, in particular, difference. They should not only use it to represent 'take away' situations; indeed, if they do this will greatly affect their ability to recognise which operation to use in solving problems where difference is required. Children should also be required to read expressions such as this, interpreting the symbolic notation as "the difference between 5 and 3 is the same as the difference between 6 and 4". Practising these skills will further enhance the understanding of numbers and operations.

It should be noted that although practice of skills is an important element in the consolidation of the understanding of operations, this practice should not necessarily be in the form of formal methods; and especially not through the practising of algorithms (more on this later). Many children are switched off mathematics through boring and mundane practising of skills. Numerate behaviour, as previously stated, requires that children have positive attitudes towards mathematics and we need to be continually aware of this by exposing them to a balance between the practice of skills for computational fluency and activities which promote understanding through variety and connection.

Children who come to school with little experience with counting and using numbers informally in the home need lots of engagement with these in classrooms with their teacher before they are exposed to arithmetic instruction. For Aboriginal children this may be the case more than for non-Aboriginal children who are more likely to have many informal experiences with numbers and counting prior to attending school. Formal instruction on these operations should be introduced in terms of their existing informal knowledge about real world situations.

**Multiplication and Division**

Many children can learn 'times tables' without understanding what the process of multiplication is about. They can say "Three two's are six" without knowing that what they are indicating is that three lots of two are six, or that two add two add two equals six. This connection between addition and multiplication needs to be made clear for children, as it is the basis for understanding the operation of multiplication.
Children can 'do' $6 + 6 + 6 + 6$ on their calculator and then press $4 \times 6$ and talk or write about what they notice. This sort of discovery will assist children to understand that multiplication is repeated addition and, as they develop, that multiplication is an operation in its own right. When they use the 'x' symbol on their calculator they should interpret it as 'lots of'; when reading $5 \times 4$ from the board or on their page both teacher and students should read it as "five lots of four" in preference to "five times 4" until such time as understanding of 'x' as an operator is developed. This may be as late as 14 or 15 years of age for some children.

There are many uses for the division operation. Initial work with the operation can be done through connecting it with the operation of subtraction; just as multiplication can be seen as repeated addition, division can be seen as repeated subtraction. If we start with 8 and take away two at a time four times we have four groups of two. That is, $8 - 2 - 2 - 2 - 2$ will leave zero from the original 8 but what remains is four groups of 2. As with the multiplication symbol 'x' children should learn the division symbol '+' through exploration work with their calculator, again consolidating the '=' symbol not as an action but as a symbol of equality.

Division can be modelled through activities and much 'hands on' work, starting with a large group of objects such as pencils, lollies, stones, pop-sticks or straws, and bundling them into smaller groups. Although this may be a good starting point, children need to be guided to see the connection between subtraction and division and recognise, as a result, that division is an operation in its own right. They need much exposure to this before they are confronted with the division symbol.

Children should learn that division is useful when sharing. For example, if a box of Smarties has 24 Smarties in it, how much will each person get if there are four people? A word of caution: as described in the earlier section on fractions; the idea of sharing and equal shares may be foreign to Aboriginal children and children from other cultures. It should not be assumed that they have the same understandings concerning 'equal shares' as mainstream, non-Aboriginal children, and hence should be explicitly taught in order that the division concept can be connected with the understanding of 'fair shares'.

Children may initially work through this by starting with the 24 and taking one each at a time until the 24 have gone. They will then, with guidance, realise that each child ends up with a group of six Smarties. The connection between 'sharing' and 'grouping' needs to made here; that $24 - 6 - 6 - 6 - 6$ is the same as four people sharing 24 Smarties between themselves evenly, since '− 6' written four times symbolises each of the four people taking six Smarties. Eventually, they will understand that $24 \div 4$ in this context reads as "divide 24 Smarties by four people" and that performing this operation will tell them how many Smarties each person will get.

The connection between multiplication and division needs to be made through examples like this. It is important that children learn that if four people each have six Smarties then altogether they have four lots of six Smarties, which is 24 Smarties altogether.

They also need to learn to 'read' the symbolic form; many children have difficulty interpreting division in this form since the operator appears to work 'backwards'. For example, $8 + 2$ is read as "eight divided by two" but it can mean "divide two into eight". With addition and multiplication the order in which the numbers are written, left to right, doesn't really matter, since those operations are commutative (i.e. $3 \times 4 = 4 \times 3$ and similarly, $4 + 5 = 5 + 4$). Subtraction is not commutative but that isn't so difficult to grasp because the 'action' is read from left to right that is, '5 - 2' is read as "five subtract two". Division in its symbolic form however, can cause problems for children
because the 'action' is not apparently performed as it reads left to right, but from right to left. This can be very disconcerting for students who, up until now have been able to treat mathematics symbols in the same way as they treat words and sentences, that is from left to right.

This 'strong pull' to work from left to right almost becomes a type of conditioning in the same way that the '=' symbol can condition children into wanting an answer, as discussed previously. The conditioning power can be broken down through oral practice at hearing and saying expressions such as $8 + 2$ as "eight divided by two" or "divide eight by two". We can also say it as, "How many lots of two are there in eight?" Children should work with these expressions in order to understand the literacy associated with the concept of division.

It is essential that understanding of the operations is developed before written algorithms for computation are taught. Multiplication and division are often taught as algorithms and for many children become merely a mechanical procedure; subsequent failure to remember steps in the process can do much harm in children's attitudes towards mathematics.

Understanding of the processes of multiplication and division by children can be ascertained to a certain extent by their ability to recognise which operation to use in a story problem requiring their use. Many children who are able to use the multiplication and division algorithms are unable to decide when to use either operation in a word problem (Kouba et al, 1988). This clearly indicates the need for children to be engaged in learning how to make decisions about which operations to use, with a process such as described in the previous chapter.

Another useful strategy which children can learn from is giving them an expression such as $8 + 2$ or $5 \times 3$ and asking them to make up their own stories or situations which can be represented by these expressions. The answer does not necessarily need to be included.

It is essential that children understand the four operations addressed here in order that they can make sensible decisions about when to use them and compute with them. Clearly, this understanding involves the meaning, use and connections between the operations. These are best developed using real life contexts. Their understanding merely through computation is questionable.

### 6.4 Calculation

Mathematics instruction programs in the past have tended to concentrate on the calculation aspect of mathematics. Many teachers believed that children understand numbers and operations through calculating with them. The notion that "I will understand addition better if I practise the addition algorithm" or, "I will understand graphs better if I draw lots of graphs" formed the basis for these programs. Clearly this is ineffective for the majority of children.

This model is a carry over from a generation when computational fluency was the focus of mathematics learning and teaching in schools; in a world where the calculator did not yet exist and many people did not have access to a simple adding machine, children needed to learn how to compute with numbers quickly in order to be numerate.

In a world where this type of computation is done rapidly through technology such as calculators and computers, the emphasis and meaning of computation in school has changed. Clearly, where these computations can be done by pushing a few buttons, there is little value in teaching children fast, efficient ways of computing with paper and pencil. And, if we know that their understanding
of mathematical numbers and operations is not enhanced or improved (for most children) through the performing of paper and pencil algorithms, then it is clear that the use of algorithms as computational methods needs to be de-emphasised in the mathematics classroom.

So, what is 'calculation'? If children understand numbers and operations sufficiently in order to choose the appropriate operation, then calculation is the act of performing the operation using the appropriate numbers. In other words, given that the appropriate numbers and operations have been selected and combined to form a mathematical expression, either verbal, symbolic or written, then calculation is the actual carrying out or performing of the operation in order to produce an answer or solution.

In a real life situation where some mathematics computation is required, adults make decisions about the most efficient and appropriate method for performing the calculation, whether it be mentally, with pen and paper, or with a computational tool such as a calculator. They make the decision usually on the basis of the choices that are available to them. The role of the classroom teacher then, is to teach children a repertoire of computational techniques from which to choose.

The SAUCER (So Adults Use Computation Everyday Research) project carried out at Edith Cowan University (Northcote & McIntosh, 1999) revealed that, in a sample of 200 adults tested, 84.6% used some form of mental computation every day, 11.1% use some form of written calculation and 19.6% used some form of technology with which to compute. This has clear implications for mathematics teaching. Whereas in the past we concentrated on teaching written computation (and in particular, formal algorithms) the repertoire we should be teaching children should focus more heavily on mental strategies. There is still a place for written methods (but not necessarily formal algorithms) and we also need to teach children to make appropriate choices about when and how to use technological computational tools.

**Mental Computation**

In many classrooms mental is often just a set of mathematics computations which are asked at the beginning of the lesson; often as a means of settling students down before the real work begins. There is often little focus on how children perform mental computation but rather on the answers produced. Teachers may discuss methods used but rarely is there discussion or teaching of different strategies.

Children will rarely improve their ability to compute mentally through practising written computation. A range of mental strategies need to be taught to students. They need to at least discuss the different aspects of various strategies so that they can themselves make choices about which strategies are most efficient. These may be different for each student. When mentally computing the addition of '799 add 30', some children may find it more efficient to set up and perform an addition algorithm such as:

\[
\begin{align*}
799 \\
+30 \\
\end{align*}
\]

then working right to left by first adding the nine and the zero, then the nine and the three and finally the seven and the zero to arrive at a solution of 829. Others may recognise that 799 is one less than 800 so 799 add 30 is the same as 800 add 29. This, for most children and adults, would be more efficient than the algorithmic approach, but depends heavily on one’s ability to partition numbers (see below). In my opinion, people using the second strategy are more numerate than those using the first since, their understanding of and fluency with numbers is greater.

Clearly, for children to make choices about the most efficient mental strategy, they must have a range from which to choose. This range should be explicitly taught by the teacher and developed through discussion with children; since even teachers may not know the entire repertoire! Assisting
children to make choices based on efficiency is done through children deciding themselves; talking about different ways among their peers and 'weighing up' the benefits and disadvantages of different methods. 'Efficient' in this context usually relates to fluency or 'flowing smoothly'. This will depend on an understanding of numbers and particularly on the ability to partition numbers into manageable forms and amounts. It will also depend on an understanding of operations and how they connect with each other. For example, in computing $78 \times 3$ it is helpful to think of this as equivalent to $70 \times 3 + 8 \times 3$ or as $80 \times 3 - 2 \times 3$.

Some effective ideas for mental computation activities suitable for children of all ages, include those by Jones et al (1994, pp. 46-48):

1. **Today's number is...**
   
The teacher writes a number on the board (no paper or pencils allowed). Children call out numerical expressions which when solved would give the teacher's number as the answer. These are written on the board by the teacher. Children check these for accuracy and make corrections where necessary. These responses can be categorised or grouped, children looking for patterns and connections.

2. **Sum Story**
   
The teacher writes a numerical expression on the board. Children are then asked to make up a real life story which will make sense and describe it to the rest of the class or to their group. The best from each group can be shared with the whole class.

3. **Find my number**
   
The teacher thinks of a number. Children then ask questions about the number, to which the teacher may answer only 'yes' or 'no'. Their questions are written on the board.

For many Aboriginal children, their ability to perform mathematical operations mentally may be difficult since they are more likely to be able to learn visually than through hearing. They may need to be taught to visualise the operation through the teacher using flashcards and other visual aids and then gradually reducing their use.

**Estimation**

The community as a whole has traditionally perceived mathematics as being an exact science and as such, demanding exact answers. This expectation becomes more apparent during the middle to upper primary years and appears to be linked to the learning of mathematical procedures such as algorithms.

Early childhood experiences with mathematics are generally enjoyable, depending on the classroom teacher, and are presented in ways that allow children to feel comfortable about taking risks and discussing different ways of tackling situations that demand mathematics. Once children are taught procedures and formal methods they often begin to be conditioned to produce an exact answer for every calculation; they become fearful of producing an incorrect answer which may result in a 'cross' being placed on their work. This may be particularly true for Aboriginal children who, as described in Chapter 2, may view being told by a teacher that they are wrong, as offensive.

Children need to be taught that not only is it alright to estimate, but that it is essential. Part of being efficient with mathematics includes the ability to think critically and make judgements about the appropriateness of answers obtained. Because people are fallible then so too can be the methods they use, be they formal written methods or methods involving a technological tool such as a calculator. Effective estimation allows people to have a sense of the 'correctness' of an answer.
produced by either method. The ability to estimate effectively will depend on their understanding of numbers and operations. It is more than likely that, because of this, estimation has not featured highly in mathematics learning and teaching programs, perhaps teachers are too aware of the lack of these understandings and have not known how to deal with them effectively.

**Written Computation**

'Written computation' is referred to in *Mathematics – A Curriculum Profile for Australian schools* (Curriculum Corporation, 1994) and the *Principles and Standards for School Mathematics* from the United States (NCTM, 2000) as the developing and understanding of written methods for calculation. Nowhere in either document is there a suggestion that children learn written methods without understanding them. There are many children and adults who have learnt methods where they 'borrow and pay back' or 'invert and multiply' and so on, without understanding why. This understanding may never occur for a large proportion of the population.

These methods are generally taught as a means for producing answers quickly, for the sake of efficiency and, as described before, in a misguided attempt to indirectly improve children's understanding of numbers and operation. Formal written algorithms are rarely used in society anymore – except in the mathematics classroom and possibly by computer programmers! This is largely due to the availability of the hand held calculator.

Most people when confronted with a mathematics calculation, will attempt to perform it mentally. If this is too difficult they will usually reach for a calculator. Should a calculator not be available, most people will reach for a pen and paper (if they need a result there and then otherwise they will go in search of a calculator), but even then, what they write is very rarely an algorithm. It is more often than not 'folk maths' as opposed to school maths; that is, rough calculations based loosely on mental procedures as opposed to the formal methods learned at school.

In this context then, it is difficult to justify the heavy concentration of school mathematics on written computation, and especially written algorithms. This is not to say that there is no place for teaching algorithms anymore. If mathematics computation and calculation in schools is about children making informed choices about which method of calculation is appropriate for the task at hand, then an algorithm may be one of many written strategies and should be taught as such, not as the way which is often the approach used in classrooms.

Some children are extremely efficient at using mental calculations and their own informal written methods but would argue that they are not good at maths because they don't do it 'the right way'. This 'right way' is usually just an algorithm taught to them by their classroom teacher. Many adults will say they couldn't do maths at school, and often that simply means they couldn't master or remember the algorithms they were taught.

With respect to Aboriginal children, it has previously been highlighted that many Aboriginal children find success in ritualistic learning. In general, they like learning algorithms, as do many non-Aboriginal children, but unfortunately come unstuck when the time comes for them to critically analyse their methods or apply their learning to solving problems. In some schools they may appear to be progressing at an acceptable rate due to their ability to accurately perform these methods based on memory. They may then appear to 'plateau' or 'fall off' in their mathematics ability; their lack of understanding often being the cause.

'Written methods' also refers to informal algorithms which, if developed by children themselves, are often better understood and consequently more effective for children, Aboriginal or non-Aboriginal. Pam Sherrard, as part of her work with the ISAN project described earlier (DETYA, 2000), had a great deal of success working with Aboriginal children developing and using their own subtraction
algorithms. When subtracting double and treble digit numbers, the children drew bundles of ones and crossed them off as groups of ten. For example, in subtracting 36 from 54, they drew five bundles of ten and 4 more to represent 54. They then crossed off three bundles of ten and six single ones and were able to demonstrate that 18 ones were left as one bundle of ten and eight more. This approach may be more time consuming for some, but clearly this strategy indicated a far greater understanding of place value and subtraction as an operation than the subtraction algorithm, with terms like 'borrowing and paying back' used by children; but with no understanding of what they are doing or why.

Some children can become very proficient at using their own informal written algorithms. Sometimes these can be mental procedures written in steps on paper as opposed to holding all working 'in your head'. For example, the same calculation, 54 subtract 36, could look like this:

\[
\begin{align*}
54 & - 36 \\
54 & - 30 = 24 \\
24 & - 6 = 18
\end{align*}
\]

Children should learn that this is not an inappropriate method for performing these calculations if they can't perform them mentally and/or there is no calculator available. In fact, for some children this method may be more efficient than performing it mentally or using a calculator.

Some teachers insist that one particular method be used by all their children but often this is so that they can find out which parts their students can do or, if they are producing incorrect answers, where they are going wrong. Clearly, the issue is more about teaching children how to communicate their methods so that we can determine where they are going wrong through their explanation and not solely through their written methods.

If children find a formal written algorithm the most efficient for them then they should not be discouraged from using it, as long as they have some understanding of why it works, and more importantly, are able to estimate first in order to judge the reasonableness of the answer produced.

**Calculator Computation**

Some teachers of mathematics believe that by giving their students calculators they will no longer have to teach them mathematics! This may in fact be true if their definition of teaching mathematics is one of teaching children how to perform the simple computations that a calculator performs. Clearly, the role of the mathematics teacher is much more specialised and complex than this.

Girling (1977) suggested the following definition for numeracy "Basic numeracy is the ability to use a four-function electronic calculator sensibly" (p. 4). There is a lot of truth in this definition. Many children aged thirteen or more, when given a calculator to use in mathematics classes, believe that they will now 'be good at maths'. Unfortunately they soon learn that this is not the case because many do not know which buttons to push. On reading problems that require mathematical computation they ask questions of their teacher such as "Is it a plus?" and "Do I multiply or divide?". This clearly supports earlier discussion about the inability of children to effectively perform mathematical computations without a sound understanding of both numbers and operations.

The perception that "I will be better at maths if I have a calculator" is true to the extent that it has been shown that students from all year grades and ability levels who are permitted to use a calculator in mathematics lessons generally have a more positive attitude to the learning of mathematics. This is partly because they are less afraid of making computational errors. (Hembree and Dessart 1986, p. 96) By removing the fear children are more likely to take risks and this risk-taking promotes learning.
It is also due to the fact that when children invest a lot of time performing calculations using written methods such as formal algorithms, they can be very demoralised to find that their answer is wrong. A calculator can free them up from these feelings by providing immediate feedback and an opportunity to try again with the time factor being minimised. Students can learn about appropriateness of answers, the order in which keystrokes are made, and different means for achieving the same answers through discussing and showing what they have done.

Using a calculator can also 'free children up' from any possible embarrassment caused by being 'exposed' to possible reactions from peers when they make an error. In this way a calculator can foster mathematical thinking and investigation. It can also foster trial and error approaches to problem solving so that when selecting or 'choosing' an appropriate mathematical model with which to solve a problem (as described in the previous chapter) they are more inclined to take risks which include selecting models which may be inappropriate and then trying again.

If the hand-held calculator is used as an instructional tool, teachers must make the connections for children between the calculator, manipulative and concrete materials, and symbolic representation. Children – particularly young children – should not use a calculator in isolation, particularly since using a calculator is mostly an individual activity. Children need to discuss answers obtained with peers or teachers and should be able to demonstrate what they have found using other forms including writing.

Bell et al (1978, p. 31) state that "...three basic computational skills seem essential when the calculator is used freely:
(a) facility in single-digit arithmetic
(b) a good understanding of place value, including decimals
(c) ability to estimate and check."

Using a calculator with young children can enhance their mathematical understandings. It can allow children to concentrate on real mathematical thinking instead of getting 'bogged down' with the technicalities of algorithmic calculation. As described earlier, this can often get in the way of finding answers.

The dates of the last three quotations indicate that these are not new messages. Unfortunately however, these messages have yet to penetrate for a large proportion of the teaching profession, particularly for the K-8 grades. Teachers need to view the hand-held calculator not merely as a 'number-cruncher' removing the tedium from calculation, but as a useful teaching-aid in assisting conceptual understanding of mathematical concepts. For example, in giving a child a calculator, asking them to put 357 in the display and then to change the 5 to a 6, the child clearly must have an understanding that the '5' represents fifty or five tens. This simple activity has the potential to greatly enhance children's understanding of place value. To help children understand that 0.39 is less than 0.4 children can begin with 0.35 in their display, adding 0.01 constantly so that they can visually 'see' that 0.39 + 0.01 = 0.4. This can be done by entering 0.35, then + 0.01 = = = = = = and thus getting the calculator to 'count' by 0.01 after starting at 0.35. For children who have the misconception that 'the more decimal places the greater the value of the number', as described previously, this visual confrontation is often sufficient to eradicate the misconception and hence improve understanding of decimal and place value.

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A calculator can be used to show children who believe that 'multiplication makes bigger' that this is not always true, or that division and subtraction are not commutative – for example, 3 - 2 ≠ 2 - 3, and 5 ÷ 2 ≠ 2 + 5.

Included in the effective use of a calculator is being able to judge the reasonableness of answers shown in the display. Some children, even as old as 17, are unable to do this. They believe that the answer is right because "the calculator did it". Children need to be taught that the using of the mathematical process or model selected is not the end of the calculation; as outlined in the previous chapter interpretation is a major part of the calculation process.

Clearly, without the ability to know which buttons to push or be able to judge the reasonableness of answers obtained through pushing the buttons, a calculator is of little use to anyone. This highlights the need for children to understand numbers and operations in order to facilitate estimation and give them a sense of the size or magnitude of the result expected when performing a calculation.

Teachers of Aboriginal children indicate that calculators provide enormous intrinsic value for motivation in the classroom. This may be due to them often being very visual learners. Their effective use by classroom teachers in using them to enhance mathematical understanding has the potential to greatly assist the achievement of numeracy outcomes for all children.

6.5 Partitioning

Partitioning refers to the ability to 'make up' and 'break down' numbers. An ability to partition numbers provides the basis for effective mental strategies and the sound understanding of the meaning, use and connections between the four mathematical operations of addition, subtraction, multiplication and division. Its importance in the achievement of mathematics outcomes by children cannot be overemphasised.

Partitioning should initially be encouraged and learned through concrete objects. Children can have experiences with discovering partitions themselves using working mathematically strategies such as those described in the previous chapter. Children will learn for example, that 9 is the same as 5 + 4 or 6 + 3 or 7 + 2 or 8 + 1 or the reverse of each of these. They will learn that 16 = 9 + 7 or 8 + 8 or 6 + 2 + 8 and so on. As their understanding of larger numbers increases they will learn that 1 298 is the same as 1 000 + 200 + 90 + 8. It is this skill which enables them to mentally calculate 65 - 42 by breaking down the numbers through the use of various strategies such as 65 - 40; or recognising that 42 + 23 results in 65, so 23 must be the solution. Students need to be encouraged to use their own strategies and to compare them with those used by other students in order to make decisions about the most efficient partitions to use.

These 'building up' and 'breaking down' strategies should be practised with all kinds of numbers. Understanding of decimal numbers is enhanced through understanding that 0.45 is 4 tenths and 5 hundredths, for example. Understanding of fractions likewise is enhanced through learning that for example 5/6 can be broken down to be 3/6 and 2/6 or 1/6 and 4/6. This can also consolidate understanding of like parts (i.e. sixths in this case).

It is the skill of estimation through partitioning which enables children to make reasonable estimates in order to judge the appropriateness of calculations obtained with a calculator. Indeed, children should learn that wherever possible calculation should be done using mental strategies; a written method or use of a calculator should only be used when that is perceived to be a more efficient way of operating. By partitioning children can use known facts to work out facts which are not yet known. For example, by using the fact that 5 + 5 = 10 children can work out that 5 + 6 = 5 + 5 + 1 = 11. These strategies are essential for children's fluency with mental calculation. When adding 63
and 45 for example, most people recognise 63 as $60 + 3$ and 45 as $40 + 5$ and then add the 60 and the 40 followed by the 3 and the 5. Similarly, when subtracting 32 from 65 we might subtract 30 from 65 to obtain 35 and then subtract 2 more, based on breaking down 32 into 30 and 2. By partitioning, we can use $100 + 25$ when calculating $97 + 28$.

For multiplication, we use the fact that $3 \times 30 = 90$ to calculate $3 \times 27$ and conclude that $3 \times 27$ is $90 - 9 = 81$. Both multiplication and division are understood better if we build on partitioning skills. Using the words 'lots of', we can describe 20 as '5 lots of 4', or '4 lots of 5', or '2 lots of 10', or '10 lots of 2'. We can further partition these numbers by recognising that '4 lots of 5' is the same as '2 lots of 2 lots of 5', and so on, which helps children understand factoring and assists in learning multiplication and division.

The development of any standard algorithms for the four operations should follow the development of mental and informal written approaches and be introduced to extend children's existing strategies to help them deal with bigger numbers. Facility with partitioning can avoid unnecessary use of written and calculator methods. Some children for example, may as their first choice attempt an algorithmic procedure when calculating 508 - 497. This is bizarre and can be traced back to lack of confidence and facility with number partitioning as well as poor number sense.

Some teaching strategies for younger children may include holding up fingers in different ways to show a total of eight fingers; one child holds up five fingers on one hand and three on the other while another holds up four on one hand and four on another. Discuss 'how many different ways are there? How do we show no fingers on one hand?" Teachers can also use stories with numbers in them to provide meaning and relevance. Make sure the stories are culturally appropriate so that children understand the context. There are quite a few story books around with both Aboriginal and non-Aboriginal contexts, otherwise teachers could make them up. A story about seven boys and girls: "how many ways can they fit into two cars?" for example.

Stories using larger numbers can also be used for older children. Partitioning numbers like 48 into 2 tens and 3 ones, added to 2 tens and 5 ones using straws, wooden cubes, single strokes drawn on a page or other concrete representations are useful activities. Children could investigate in groups the number of partitions they can find using tens, fives etc. Older children should also be encouraged to write the partitions they find using symbols in their exploration, such as $53 = 49 + 4 = 48 + 5 = 20 + 20 + 20 - 7$ and using multiple operations such as $3 \times 25 = 20 + 20 + 20 + 10 + 5$ and so on. They should also partition decimal and fractional numbers and combinations of these.

### 6.6 Being numerate in Number

The *Numerate Students, Numerate Adults* study (Education Department of Tasmania, 1995, pp. 11-28) stated that to be numerate in Number people need to be able to:

- Interpret numbers when they are used for different purposes
- Understand how numbers can be expressed
- Use estimation techniques appropriately to make and check calculations
- Use a variety of calculation methods
- Choose and use appropriate technology

Clearly, the above discussion indicates some of the reasons why children may not become numerate in these areas and suggests some strategies to assist teachers in dealing with these. In doing so we must be aware of specific needs that many students have, particularly if they are from different cultural backgrounds. We cannot assume for example, that all children have been exposed to the same experiences with numbers despite the environments that we may be familiar with.

Improving Aboriginal Numeracy
Chapter 7

Teaching Key Ideas about Measurement

7.1 What is Measurement all about?

When Europeans arrived in Australia they failed to realise or appreciate the long evolving history behind the ways Aboriginal people had established and maintained land boundaries in response to the dynamic and trying environment. They also did not appreciate the "abstract knowledge, the practical and conceptual skills, the structures of authority, the belief systems or the social mechanisms through which control was exerted" (Watson & Chambers, 1989, p. 44).

The boundaries used by Aboriginal peoples are validated by social practices. By contrast, the Europeans set about creating a set of boundaries which did not take into account the topography, climate, flora or fauna of the land itself. Rather they were derived from a strong knowledge-power network that enabled administrators (some thousands of miles away) to implement European concepts of order on a land which they claimed as their own. Land was parcelled up and distributed and titles and maps kept in banks and titles offices.

These practices of assigning boundaries are still in use today: "...they are tightly bound to European notions of individual and community, ownership and law, hierarchies of social class, systems of agricultural practice, and attitudes towards nature. They are not easily separated from the use of Indo-European languages and the number system" (Watson & Chambers, 1989, p. 45).

The number system and units of measurement allow the land to be divided into sections, each section being larger or smaller than the other based on the counting of these standard units. The qualities or attributes of 'length', 'width' and 'area' and other concepts of measurement, are entirely abstract. One cannot see 'length' or 'area'; they are visual metaphors, 'social facts', "made in talk and practice, then used in good faith as real things to construe as divided into units", according to Watson and Chambers (1989, p. 45). The perimeters or boundaries of properties describe who has the right to use the land enclosed.

So we see that units of measurement have been created to enable descriptions and order founded in social constructs. These do not, as a very broad generalisation, have meaning for most Aboriginal cultures, since order and structure are described through kinship patterns; an Aboriginal person might look at the land and see the trees, grasses, animal life and so on, in contrast to a non-Aboriginal person who, with a completely different world view, might see the length, width, and resulting area as described by the perimeter, or boundary.

Measurement in Western cultures is an essential skill for all people in order to function effectively in society. The teaching of Measurement often focuses on the use of standard units. The emphasis should be more about teaching students to use measurement and estimation skills in order to describe, compare, evaluate and construct. There are three key outcomes desired for all children:

1. **Carry out measurements;** by first deciding what needs to be measured and to what level of accuracy. Children need to know that measurements are only as accurate as the unit chosen to measure with. Based on their understanding of attributes of what needs to be measured
they should be able to make decisions about what measuring tool is needed in order to measure the attribute. They may use a 'span' or a 'cubit' if measuring for their own purposes but when comparison is required they may need to use a standard unit. They need to make decisions about what attributes need to be measured (for example, length, area, volume and capacity, mass, time and angle) and for what purpose. In making comparisons they can use language such as 'bigger', 'wider', and 'the same' based on the unit used with which to make the measurement.

2. Measure indirectly; by deciding when it is inappropriate to measure directly (i.e. themselves) they can then make decisions about which measurements can be combined and which measurement relationships and/or formulas can be used to determine further measurements. The concept of 'perimeter' for example relies on the understanding that in order to find the perimeter of a shape a number of lengths or distances need to be measured. The accuracy of the perimeter required will determine the level of accuracy used when measuring the lengths. Many children can apparently use formulas for finding volume, area and so on without understanding that these formulas are merely 'short cuts' used to make calculation more efficient. They confuse the 'short cut' with the attribute being measured.

3. Make sensible estimates of quantities; estimation in measurement is impossible unless students 'have a feel' for the units they are using. How can a student make an estimate of the area of a floor surface for example, if he/she does not have a visual sense of what a square metre looks like? The ability to estimate measurements is essential in order for students to judge the reasonableness of their measurements and results obtained through measurement calculation.

The understanding of units in the measurement strand has often been neglected in the mathematics classroom. Too often teachers focus on standard unit conversions and formulas while neglecting to teach students about units – standard or otherwise. Clearly the emphasis should be on the processes as opposed to the content. What we want is for our students to be able to understand the units used to measure the attributes of length, area, volume, capacity, mass, time, angle, so that they can estimate and subsequently measure these attributes (either directly or indirectly). They can then judge the reasonableness of the results they obtain based on their understandings of the units used.

In Western Mathematics, Measurement focuses on concepts such as length, area, mass and so on. All of these attributes are quantifiable. We compare and order trees mathematically by comparing their heights, the width of the trunk, the density of the foliage and so on. For an Aboriginal person these may not be the attributes used for comparison, if indeed there is a purpose for comparison. The Aboriginal person may use different qualities to order trees, talking about trees; almost in a personal way.

For some Aboriginal cultures measurement concepts as listed above are totally foreign. Research by Pam Harris (1980) revealed that for many traditional groups in northern Australia there are almost no abstract terms for measurement concepts such as length, height, distance, speed, weight, area or volume. The language for one cultural group she worked with indicated that "how long" was used for height and length but that it was applied only to people and features of the landscape. Another group referred to a word meaning 'to weight down' but no abstract word for 'weight'. Yet another explained that each separate area of land had a specific name but there was no abstract word for 'area'.

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In contrast to the above, Harris found that seven out of twelve groups had an abstract word for 'time'; some used it in connection with a place or location and others used it in the sense of a time period of era. One group that did not have an abstract term for time did however have words meaning 'long time' and 'short time' (p. 50).

As Pam Harris explains: "...there will be no concise way of explaining it in the child's own language. Thus the child will be required to learn new vocabulary and a new concept, both at the same time..." (1980, p. 75); and for some Aboriginal children they will also be required to do so in a second language. Clearly, this is a huge ask.

For many of Aboriginal children, particularly from remote communities, many of these concepts will not be part of the environment outside of school and so clearly cannot be assumed as prior knowledge. In terms of constructivist teaching practices and 'building on what children already know', teachers of Aboriginal children (and non-Aboriginal children) need to be aware that they might have to start from scratch! This means providing contexts in order to provide a purpose and reason for wanting or needing to measure.

It is important then to first understand that children need to learn about the ways in which Western society orders and compares – based on quantifiable attributes. Aboriginal comparative language is usually relative. For example, when comparing the heights of two trees they might say, "that tree is not very high" (in comparison with other trees of the same type not in comparison with all types of trees), or "that fish is big" (in comparison with fish of the same type, not in comparison with all fish). "How big" might in fact mean 'how tall' or 'how high'. I was first made aware of this when speaking with a young Aboriginal boy at Nullagine in Western Australia who explained that he had been fishing on the weekend. When I asked him how big the fish was that he caught he said "real big!" and so I asked him to show me with his hands and he indicated about 20 cm long. One Aboriginal person I have since spoken to indicated that the 20 cm may have in fact been referring to the width of the fish as opposed to its length. Still another explained that the day and the situation are important so that the boy may have been telling me that the fish he caught was big on the day or big for that stretch of water in which it was caught.

Even in terms of 'sameness' as a measure of comparison, some Aboriginal languages have a word 'same' which is relative. They may say 'those two fish are the same' but that may mean they are both big (which we've just seen can mean they are similar but not necessarily the same as in the way a European may use the word).

In some Aboriginal languages comparisons such as 'small, smaller, smallest' may not occur. When considering abstract concepts such as volume which are non existent in some Aboriginal cultures, it is important to be aware that not only does this concept need to be explicitly taught; so too do comparisons for its use. In the same way, adjectives such as 'big', 'tall', 'high', 'small' need to be explicitly taught so that children learn how to use them in the context of Western Mathematics and Measurement attributes.

### 7.2 Components of measurement

There are three components of Measurement that are fundamental – they are understanding units, conservation, and transitivity.

**Understanding units**

If there are words for abstract measurement concepts then there are usually words for the units used to measure them. For example, if there is a concept of time in any culture one would expect that terms such as 'day', 'year' or 'second' might exist for measuring the attribute.
Since some Aboriginal cultures do not have abstractions for some measurement concepts it is clear that these as well as the abstraction, need to be explicitly taught.

The measurement concepts described above have, in the past, tended to be taught with a focus on using standard units. We have taught students about length, for example, by teaching them how to measure length directly using standard metric units, and based on the skill of converting one standard unit of length to another.

There are some fundamental understandings about units which are essential for children to know if they are to be able to achieve measurement outcomes. These do not necessarily apply only to standard metric units. Clearly, many students will come to the mathematics classroom with a common understanding of standard metric units based on the degree to which they are immersed in them in the home environment. This will often be greater for non-Aboriginal children since there is more likely to be an emphasis on quantification of measures in Western households, as described in Chapter 3.

Many children however, can use the language of standard metric units without having an understanding of the units themselves and teachers need to be aware of this. They use words such as 'kilometres' and 'minutes' in sentences without really having a 'feel' for these as units of measurement. Many children as old as fifteen can use the word 'kilometre' in a sentence in the correct context without really having any sense of how far a kilometre is. Similarly, many adults can use words such as 'hectare' in an appropriate way without having a feel for the size of the unit, which would enable them to visualise the size of a playing field or block of land given the size in hectares.

The understandings of units in Measurement that we want children to have are as follows:

1. Some objects work better as units to measure with than others (due to gaps and overlaps created when using some objects);
2. The bigger the unit chosen for making measurement the smaller the number of units measured;
3. For comparison you need to use the same unit;
4. The purpose for measuring tells us which unit and how much care (level of accuracy) is needed;
5. Standard units are no more correct when measuring than non-standard units; and
6. Using standard units is more helpful for communicating purposes (especially written).

Comparing and ordering things using their attributes is an important part of Measurement. Teachers should not assume that children know and understand the language of comparison. For Aboriginal children, words such as 'bigger', 'smaller', 'lighter', 'further' may be familiar in the context of magnitude but not measurement, especially since the emphasis in their culture is more likely to be on quality than quantity. These words may need to be explicitly taught and probably should be for all children.

Estimation of units

Estimation techniques help children to both understand units and understand measurement. Effective estimation is impossible unless students understand and 'have a feel for' the units they are working with. For example, a student who looks at a shoe-box and, given its height, length and width calculates the capacity of the box using a formula, will have no sense of the reasonableness of the calculated solution unless they can visualise the size of a cubic centimetre. Many students I have taught would have no problem accepting that a shoe-box has a capacity of 30 cubic centimetres (cm$^3$) if that was what they calculated as their answer.
Students must be taught to *visualise* measurements so that even before commencing calculation, such as finding the capacity of a shoe-box, they have a rough idea of the size of the answer they are expecting. Estimation should always precede calculation and may in fact be the first step in the calculation process.

Most estimation questions presented to children are *closed* in that they can be answered using little understanding of units and can be marked only right or wrong. For example, "estimate the length of this eraser (or this book, or the height of the door)". It is better to use *open* estimation questions that stem from the measurement rather than the object. For example, questions such as "find something in the room that is about 12 centimetres (or 130 centimetres, or 2.6 metres) long", or "suggest some 3D shapes that have a capacity or volume of 3 000 cubic centimetres". These questions allow the teacher a clearer insight as to whether students really understand and 'have a feel for' the units.

**Conservation**

Students who understand that an object is still the same shape and size, even if it is moved or broken up into parts are said to be able to *conserve*. For example, if a student can look at two pieces of string like the following:

![String Diagram](image)

and say "they are the same length", they are said to be able to conserve length. Similarly, a child may have two straws, side by side, and says they are the same length.

![Straw Diagram](image)

One straw is moved so that it is further along as follows:

![Straw Diagram](image)

If the child now says that one is longer than the other then he/she probably cannot yet conserve length.

A student who can look at the following diagrams and see that the shaded areas are the same may be said to be able to *conserve area*.

![Area Diagram](image)

Conservation of volume is generally more difficult for children. Most children can conserve length and area well before they are able to conserve volume. To look at the amount of water in two different-shaped bottles and know that the amounts of water are the same is connected to an understanding of space and occupying space.
Teachers can help their students to conserve length, area and volume through engaging them in many hand-on activities requiring them to pour liquid or sand from one container to another and other similar activities which require measuring, talking with teachers and peers and reflecting on what is happening during the activities.

Transitivity

Most children develop transitive reasoning along with their understandings of conservation. Transitivity is, broadly speaking, about comparing lengths, areas and volumes of objects using indirect comparison. For example, if you wanted to compare the width of a wardrobe with the width of a door it is unlikely you would compare the widths directly because that would mean moving the wardrobe. You would be more likely to use a piece of string or your hand span to measure the width of each and indirectly compare the widths obtained with the string or hand span. The ability to do this requires an understanding that the comparison of the attribute can be made using a third object.

Again, teachers can best help their students to develop this ability through many measuring activities where students are actively engaged in the measuring, talking and reflecting process with each other and their teacher.

7.3 Length

The concept of length is one which teachers often assume as being understood by children, particularly when they use words such as 'kilometres' or 'k's' in appropriate ways in sentences. As described above, many children from Western cultures are immersed in this language even prior to formal schooling. Aboriginal children however - their cultures being less concerned with quantity and accuracy - are unlikely to bring this understanding to a classroom situation. Measures of distance are more likely to be referred to in general terms such as 'not far' or 'a little way' than in specifics. For some culture groups, it might be the direction that is more important then the distance, so an Aboriginal person might for example, point in a direction and say “close up” (meaning not very far) or “long way” and the length of time it takes to say the word 'long' may be an indication of the distance. Similarly, lengths in Aboriginal culture are frequently more about individual and personal length; when making a spear for example the length of the arm belonging to the person using the spear may be used as a reference. This is appropriate and children should learn this as described above.

To teach children what the concept of length is - as opposed to distance, which may not be in a straight line and which may be measured in term of units such as 'days', 'stubbies', 'lots of Play School' depending on the context and reason for knowing of the person asking the question - teachers need to engage their students in activities which give children a 'feel' for length using units which allow for comparison. To ensure that students have a 'feel' for the standard unit of kilometre it is useful to take students for a walk - a kilometre there and a kilometre back if time permits. It should be pointed out that it is often essential to come back using a different route as some children may otherwise think that a kilometre is one particular and specific route as opposed to an abstract length or distance. This is often easier to teach with units such as metres and centimetres since children can more readily engage in measurement activities requiring them to directly measure things.

Children need to become familiar with a range of measuring tools which measure length to differing degrees of accuracy, such as rulers, trundle wheels, measuring tapes, strips of paper, paper clips, Minties, pens and other implements. They should be observed making choices about which object,
tool or implement to use when given specific purposes for why the length is required. They should also be given opportunities to work collaboratively on tasks which require measurement to be made, teachers paying attention to the ensuing discussion between children and reasons given for choices made.

Making these activities relevant is essential for all children. For Aboriginal children who often have an acute awareness of the location in which they live, it makes sense to teach units such as kilometre and metres by talking about distances between places in their community. The units of distance in some traditional Aboriginal groups are not specific but very general; words for 'far, very far, quite a long way, long long way', for example.

**Perimeter**

Perimeter is a form of length. In fact, the perimeter of an object or shape is found by measuring the length around the object or shape. Perimeter is best taught through active participation; children can walk around shapes such as the school football field or netball court as well as buildings such as the school library. They can be given a piece of string and asked to make and draw as many shapes as they can that have a perimeter the length of the string. In this way they learn that all shapes have perimeter (not just rectangles!), that many different shapes can have the same perimeter, and that these perimeters can be made up of lengths, widths, or curved shapes.

Formulas for finding perimeters of shapes such as squares and rectangles should not be formally taught too early, but children should be allowed to develop their own methods or rules when they reach that level of conceptual understanding. Children should be asked to measure perimeter using both formal and informal units and should be encouraged to talk about their findings and solutions. This can be done effectively using the strategies discussed in Chapter 4.

They should be given real problems to solve, including those where it may be seemingly impossible to measure lengths and perimeters themselves, so that they can explore ways of measuring indirectly. Formulas for finding the perimeter should evolve from the teaching and learning experiences – they are 'short cuts' for counting units of length. Teachers should facilitate children in finding their own shortcuts rather than explicitly teaching these.

Open-ended tasks are great ways of facilitating explorations that allow discovery of short cuts and consolidation of concepts. For example, asking children to draw as many shapes as they can which have a perimeter of 25 centimetres allows children to discover certain short cuts for rectangles (and maybe even triangles) and also consolidates their understandings of the concept of perimeter. They could even work with a 25 cm piece of string in order to assist their understanding of transitivity – don't limit them by giving them grid paper unless you only want them to only explore rectangles and squares.

For Aboriginal children, particularly those from remote communities, the notion of fences may be unnecessary and foreign. Boundaries are for them, more likely to be natural boundaries such as rivers, roads, mountain ranges and so on. Fences and other man-made boundaries are usually used in connection with ownership, to mark out the boundary of what belongs to whom, and may only occur in rural or urban settings. Consequently, the need for measurement of a perimeter may need to be explicitly taught for many children.

### 7.4 Area

Some of the traditional Aboriginal groups surveyed for Pam Harris's (1980) research indicated that area was related more closely with space; each family needing space or area for their camp. This may still in fact be the case and could be a starting point for the teaching of the concept of area.
Aboriginal children however, may not necessarily be at a disadvantage here since many children, even those who may have been taught *area* at school, have little if any understanding of area as a mathematical concept. Some children as old as 15 will argue that area is 'length times width'; a clear indication that they believe the concept to be the short cut used to find the area of a rectangle.

It is important that children are taught from a very early age, that *area* is an attribute of a surface and can be measured through coverage of the surface. Teachers need to initially develop the concept of area using informal units so that students can appreciate the need for standard units which tessellate. Children should be engaged in discussions about measurement of coverage in the context of using units such as a book, a round plate, and irregular shapes such as a leaf, the teachers saying for example "How many of these books side by side will cover this bench?" Discussions directed and facilitated by teachers should consider gaps and overlaps so that children can see reasons for using regular shapes which tessellate when making decisions about what units to use in order to measure area.

It should be noted that continuous attributes such as area and length require an ability to count since units of area and length must be counted in order to provide a measurement. It is possible that if students are unable to measure length and area it may be because they can’t count!

Children should engage in activities such as finding the area of their desk or table using regular and non-regular units. As they understand that standard units are needed in order for comparisons to be made they can begin to use standard metric units such as square metres and square centimetres, making and using models of these which they can use as templates for direct measurement purposes. They need to have a 'feel' for metric units of area being able to visualise them in order to estimate and judge reasonableness of results in calculation.

Once these skills and understandings are developed students can be given problems such as, "What is the area of the floor in my home?". They should as a matter of course (as a result of the way these concepts are taught) consider:

- what units they should use;
- an estimation of their answer based on their understanding of the units chosen to work with; and
- whether or not *direct* measurement is appropriate and, if not, whether a method of *indirect* measurement might be used.

Formulas should not be introduced for finding the areas of shapes too early - treatment should continue to be informal for as long as possible. Students should be encouraged to develop their own 'short cuts' and formulas for counting units before these are formally taught. It is important that children understand what *area* is before they consider any short cuts used to determine it for any particular shape.

Open-ended tasks are a wonderful way of allowing children to explore the concept of *area*. They can be given a piece of grid paper (or blank paper) and asked to draw as many shapes as they can with an area of 40 cm$^2$. Alternatively, the teacher could give them one transparency each with a 1 cm$^2$ grid copied onto it and a page of shapes of any size and made up of any manner of closed curves, and asked to find the area of each of them. This will assist children to understand that all closed shapes have *area* – not just squares and rectangles! This sort of discovery activity will also help them develop short cuts for finding the areas of squares and rectangles.

### 7.5 Mass

Many children have difficulty understanding the difference between *Mass* and *Weight*. Mass is concerned with the quantity of matter in a solid or object, while its weight concerns the force of gravity acting on it. Students need to understand that the mass of an object is constant, but its
weight will vary depending on the force of gravity on it. Thus an object on the Moon and on Earth have the same mass despite the fact that on the Moon the object will have less weight due to the different gravitational forces.

Students should be encouraged to have a 'feel' for weights and standard units of weight. 'Hefting' is clearly an important strategy which can be used to develop these understandings, students being engaged in activities requiring them to lift objects and make comparisons with respect to benchmark weights which they are familiar with such as one kilogram of sugar or 500 grams of margarine.

Students considering units of mass estimate the mass of objects and use a variety of instruments to determine the mass. They need to be given opportunities to decide which measuring instruments to use in order to weigh objects. Clearly they need to be familiar with different types of scales and beam balances in order to make these choices. Collaboration again is important with teachers observing and listening as students make decisions about what to use and justify these.

The attribute of mass may be completely new for many children, including Aboriginal children. Aboriginal people from remote communities tend to make comparative statements comparing the weight of a fish with the weight of other fish of the same type. For example "that is light for a fish like that". They would rarely, if ever, make a statement such as "that is light" since the concept of 'lightness' is abstract. Contexts used should be relevant and meaningful and related to the children themselves wherever possible.

7.6 Volume and capacity

Many students (and indeed teachers!) are confused about the difference between these two attributes. Whereas Capacity is the ability of a container to contain or hold a bulk, mass or liquid, Volume is an attribute of the bulk, mass or liquid. For example, a one-litre jug has a capacity of one litre. If it is half full of milk then the volume of the milk is one half of a litre or 500. A block of wood has no capacity, it has volume, which we measure in cubic centimetres, and mass, which we measure in grams. Two bricks the same size, one made of mortar and the other of wood, have the same volume (measured in cubic centimetres) but different masses (which depends on the density of the material of which they are made) measured in grams.

Of the 20 Aboriginal language groups surveyed in Pam Harris's research (1980) none had any abstract word for volume. Consequently there were no local units used for measuring it. The only terms that were used were words for 'full' and 'empty' when used with respect to cars and petrol.

For any container needing to be filled the container was usually just brought to the shop-keeper who filled it up. A person may have brought an empty bag to a shop and asked that it be filled with potatoes. When children were asked to buy milk or juice and so on, they were usually instructed just to buy a 'big one' or a 'small one'; standard units were not generally used. Similarly and also for Aboriginal children living in urban settings, they may talk about 'a big drink' or 'a little drink' and this may be about the length of time taken to drink it. The length of time taken to say the word 'big' may also indicate the volume. As with length and distance, 'a big, big bag' may have a sense of being twice as big as 'a big bag'.

The capacity of different containers can be compared and ordered using activities requiring children to fill them with water or sand (or any material which adjusts to 'fill' the container). In this way they can compare capacities of containers and volumes of the amounts of materials used. They can talk about the capacity of a fridge box for example using children as the units; "everyone pile in!"
Teachers can engage in discussions about children being uniform units (the same size) or standard or non-standard units and talk about gaps and spaces created when the units don't adjust to completely 'fill' the container.

Students need to initially understand the attributes, have a 'feel' for and be able to estimate with, the units in which they are measured – particularly standard units – and be able to make decisions about whether to use direct or indirect measurement depending on the degree of accuracy required as demanded by the context and the accessibility of what it is that needs to be measured.

It is likely that these attributes will be completely new to most children. For comparison purposes teachers should not take it for granted that any of their children understand the language of comparison; language such as 'holds more than' or 'holds less than', for example, should be explicitly taught for Aboriginal and non-Aboriginal children in contexts which are meaningful and relevant. Comparisons should be visual and activity-based as opposed to pencil and paper examples.

7.7 Time

Outcomes relating to the attribute of time are concerned with students understanding what time and the passing of time is. They are also concerned with students developing time as an attribute, considering units of time, using a variety of clocks to read time and a range of instruments to measure elapsed time, and obtaining relevant information from timetables and calendars.

Many students have a great deal of difficulty understanding time because it is not something they can feel or see. As with other units they need to have a 'feel' for units of time such as one minute, one hour and so on. Teachers can help their students develop this sense of the passing of time through activities which students can relate to such as "How long does it take me to walk down the school verandah?" or "How long does it take me to walk to school?", or "suggest some activities which take about an hour to do".

For many Aboriginal children it is likely that there has been little emphasis placed on the importance of specific time in their home environments. Time is generally not measured in mathematical terms. Underlying the concept of time is the 'Dreaming'; a time in which man and nature came to be as they are, so that the present and past are fused since man and nature continue to evolve together.

In traditional Aboriginal cultures any times within a 24-hour period which may be considered significant, might be labelled by the names given to the movement of the sun and the moon. Daily times might be 'lunch time' or 'when the bell goes'; weekly times might be 'when the mail plane comes in' or 'when we get our cheque'; and yearly times might be 'when the rains come' or 'when the mud cracks' or 'when the turtles breed', or 'when the leaves drop from the trees'.

In classrooms where non-Aboriginal teachers work and where Western Mathematics syllabuses dominate, lessons about time tend to concentrate on the measurement of time using fixed standard units. In contrast, Pam Harris states that, "when people live close to nature as the Aborigines have traditionally done, the dominant experience of time is in the rhythms and cycles of nature as observed in the changing seasons bringing different supplies of food, the waxing and waning of the moon, the changing patterns of the stars, the rise and fall of the rides, and the never-ending alternation of day and night" (1984a, p. 10).

The passing of time may be considered by the passing of events and not necessarily by mathematical quantities. The passage of time is marked by most people groups by 'day' and 'night'. Pam Harris points out that in English the use of the term 'day' is ambiguous since in reality the start
of the day is at sunrise while in a mathematical sense the 'day' starts at midnight. For many Aboriginal people living in urban and rural settings, sun-up is about breakfast time and sun-down is about tea time. This is often the reason that Aboriginal children tend to be late for school in the winter time but turn up early during the summer months.

Sharon Cooke, a Noongar lady and friend of the author recalls being taught along with her siblings, that lunch time was when there was no shadow on the ground. She recalls older Aboriginal people organising meetings so that they were between meals that is, between sun-up and lunch time, and between lunch time and sundown.

Sharon, now a school teacher in Western Australia, describes how Noongar Aboriginal children she works with quickly develop an understanding of hours as units but have difficulty understanding or seeing the need for minutes or seconds. She explains that this is very much tied to the cultural backgrounds of the children where the hours are emphasised, giving an example where, if a meeting or appointment is made for 10 a.m., an Aboriginal person more often than not in her experience, understands that to mean any time between 10 and 11 am and usually half way between these two times. She describes how Aboriginal students at the high school she recently worked at, would often arrive for form-room period just before 9 a.m. which was when it finished! These students, she explained, understood that there was no urgency or requirement to ‘be on time’ and that this concept was foreign to them.

Tribal Aborigines referred to in Pam Harris’s work (1985a, p. 13) "usually count days in terms of 'camps' (overnight stops) 'suns' or 'sleeps'. J. Harris (1979, p. 108) points out that for the Anindilyakwa Aborigines, the new day begins at sunrise, not at midnight, which is a concept directly related to the use of clocks. Thus, a person who, at 9.00 a.m. is describing an incident which occurred at 4.00 a.m. will say that it happened yesterday, while at 4.00 a.m. a person will refer to the previous day's events as happening today. In a personal communication to Pam Harris (1985a) Street, a linguist working in the Murrinpatha language in Port Keats, Northern Territory, explained that "the word that can be translated 'day' also means time, camp or location" (p. 13).

The passage of time is experienced in every person’s life cycle but these have a different significance for different cultural groups. In Western cultures, special birthdays such as 18th, 21st, 40th, and 50th mark milestones in a persons’ life and with which come particular rights and responsibilities (for example the right to vote and drink at reaching 18 years of age). Harris (1985a) points out that traditionally the Australian Aboriginal people did not keep track of birthdays beyond a few years; that formally keeping track of age in years and birthdays was a practice introduced to them through formal schooling.

Aboriginal people may however, approximate their age with reference to particular events or aspects in their lives such as physical development or marital status. As with other traditional peoples the need to remember specific dates as part of their history is not necessarily important. As Moiti (1969, p. 19) points out that “time is meaningful at the point of the event and not at the mathematical moment”.

Malcolm et al (1999) state that:

...in Aboriginal English time is not measured with the same degree of precision (as in Standard Australian English). Time is not measured using standardised calendar measurements or other standard units of time but by relating an event to other familiar events: for example, when Nan 'ad 'er baby, When uncle got 'is new motacar"...quantity in time and space tend to be expressed with non-specific words (drekli, mobs) rather than specific words (hours, minutes, years) and it is the responsibility of the listener to interpret the particular meaning from a context of other past events. (p. 30)
Time may also be considered as distance. The measurement of space, time and distance are interrelated since it takes time to cover distance. An Aboriginal person, when asked how far it is to some place, may respond “three days”. This is not uncommon in Western cultures depending on the context. If asked the distance between two towns that are relatively close, we might respond with “sixteen kilometres” whereas when asked the distance between two major cities such as Perth and Melbourne, we would rarely respond with the exact number of kilometres but are more likely to respond with “about three hours”. Clearly, it is not necessarily the cultural group to which one belongs that prescribes the likelihood of the unit used to measure distance but the environment and circumstances in which one lives.

Time may, in Western cultures, be considered as a limit or boundary during which activities are defined or delineated. Activities such as cooking, working, and sitting for examinations for example, are often defined by the amount of time needed or given in which to complete the task. In some societies, particularly Western, ‘time is money’ and is counted and reckoned as such.

As mentioned previously, in Aboriginal societies – past and present – the idea of ‘being on time’ may be a foreign one since the event of journeying to a place for an event can be just as important as the event itself. The ideas of children completing a task in a given time frame were discussed in an earlier chapter; Aboriginal children may have difficulty with this concept since they are often more familiar with continuing with a task until they wish to stop – not because someone else tells them to or because a school bell has rung.

Pam Harris (1984a, p. 38) stresses the need for Aboriginal children to be taught first about time as a social and cultural construct before teachers commence lessons on how to tell the time using a specific measuring instrument such as a clock. In other words, they need to understand and know the relevance of “Why would you want to measure the time in such an exact manner?” Teachers can do this through modern technologies such as television programs; (“We need to know exactly when this program starts or we will miss it”) or through activities such as cooking; (“If we let this cake cook for longer than 30 minutes it will burn and we won’t be able to eat it”).

Teaching children how to read the time on an analog clock should not be done by physically manipulating the hands on the clock and asking students to ‘tell the time’. This can lead to an artificial understanding of the passing of time. Instead it is better done by a classroom teacher who refers to the clock and reads the time continually during the course of the day. Children from non-Aboriginal homes have this modelled to them as they are growing up in their home environments although teachers should never assume this to be the case. Beth Graham (1984) suggests that comments such as "We’ll go out to play in two minutes", or "It will be lunch time when both hands point to the twelve" need to be made for perhaps many years before such things are formally taught.

Aboriginal children, and indeed all children, will only retain specific skills concerning the reading of clocks and so on, when the children continue to use these skills in meaningful contexts. For some Aboriginal children in remote settings, the skills learned in the classroom are not retained for this reason, they have no use for them outside of the classroom situation.

To the question "When did you travel to Perth?" an Aboriginal person may respond "a while ago"; on closer questioning this may in fact turn out to fifteen years before! The words "a while ago" may also be the answer to a question about something that was done last week. Often, as with distance, measures of time are more likely to be referred to in general terms such as 'a little while' or 'long time' as compared with more accurate time used in Western cultures.
Sutherland (2000) suggests that:

Spending inordinate amounts of time recording time on paper clock faces is a dubious way of enabling children to recognise time sequences. Being aware of clues Aboriginal students use to tell the time would be a valid way initially to assess their ability and then expose them to situations such as watching television or attending an organised event...awareness of a lack of exact time in traditional Aboriginal culture is the key to (teachers) understanding the lack of urgency in this skill acquisition area. (p. 11)

We cannot assume that all children are familiar with 24-hour clocks. Homes may not have clocks in them – either analog or 24-hour. These contexts should never be assumed for children of any culture.

7.8 Angle

Children need to be able to measure angles to required levels of accuracy. The concept of an angle is probably best taught in the context of a 'turn'. Children can stand facing the front and turn ninety degrees so that they are now facing either left or right of their initial position. They can in this way learn that a full turn will result in them facing the same way that they started and that this is $360^\circ$ in the same way that a half turn is $180^\circ$. They need to learn that the size of the angle is the same no matter whether they turn to the left or right as direction (in this context) is not important unless they are given specific instructions to turn to the right or left. Young children will learn that angles can in this way be compared.

As children grow older they will learn a more formal approach to angles; angles can be represented on paper and measured. Students frequently have problems measuring angles; they want to measure the distance between the rays of an angle with a ruler for example and obviously find this frustrating. Early experiences with ‘turning’ help students to focus on the size of the angle getting larger depending on the degree of turning that is made. An understanding of fraction may also help as they visualise that 'half a quarter turn' is smaller than 'quarter of a full turn'. Clearly their ability to both hear and visualise angle sizes is important.

Teachers need to be aware however, that for many Aboriginal children the concepts of ‘left and right’ may be totally foreign. Aboriginal children are often taught direction using the compass points of north, south, east and west, that is, the sun is the reference point for direction, not the body – as is the case when using left and right. Non-Aboriginal children are more likely to be taught ‘leftness’ and ‘rightness’ at home before coming to school in the context of telling their hands apart and in putting on their shoes. Be sure that children know these concepts before including the additional instruction of the degree of turning, for example, “Turn $90^\circ$ to the left". (More will be said of this in the following chapter).

Formalisation of angles through representing them on paper can often lead to misunderstandings due to the way they are presented by the teacher on the blackboard. Some students for example, could be excused for thinking that a right angle always looked like this if every time their teacher drew one it had this orientation.:
Many children in fact believe that a right angle looks like the one above and a 'left angle' looks like this:

An angle which looks like neither of these understandably can throw children into a spin! What would an angle that looks like this be called, I wonder?

In helping children understand what they are measuring teachers should focus on the degree of turn. This can be assisted, even for much older children, by getting them to use their arms to indicate the turn or the opening; children hold both arms out in front of them and 'make an angle of about 30° or angle of about 150°. The teacher could make the angle with his/her arms and get children to estimate its size. They could similarly be asked to make acute, obtuse, straight or right angles in the same manner before formally having to draw them on paper. Teachers should be sure to vary the orientation of the angles, and the lengths of the rays of the angles when drawing them on the board or as models for children.

7.9 Being numerate in Measurement

The Numerate Students, Numerate Adults study (Education Department of Tasmania, 1995, pp. 11-28) stated that to be numerate in Measurement people need to be able to:

- Use the language of measurement appropriate to the task
- Choose and use measuring tools and instruments appropriate to the task
- Use estimation techniques
- Use measurement techniques to solve problems
- Recognise that some measures are obtained by combining two or more other measures.

The above discussion draws attention to some of the possible reasons for failure of children to become numerate in Measurement, and suggest some strategies to assist teachers in dealing with them.

Teaching concepts in Measurement should provide a holistic approach to the teaching of measurement attributes such as length, area, volume, capacity, mass, angle, and time. This holistic perspective can be summarised as follows:

Students will make decisions about whether to measure directly or indirectly in a particular context, estimating first based on their understanding of the units they choose to measure with; the purpose for measuring will provide the context for the decision to use standard units or not, and the level of accuracy required.
Chapter 8
Teaching Key Ideas about Space

8.1 What is Space all about?

'Space' is everywhere. Figures and objects, besides taking up space, break it up into natural sections which enable us to talk about and describe particular spaces. The ways in which each culture organises space depend on that culture and its rules. For example, some Aboriginal cultures may not allow women in certain spaces or places. Western cultures may place artificial grids on top of the natural features to define space using grid reference points. So divisions of space can be physical or natural, artificial, social (political and administrative), personal, and sacred.

Unlike *Number* and *Measurement* which are grounded in value (more/less, bigger/smaller, heavier/lighter and so on) and taught through quantity, *Space* is not grounded in value. Watson-Verran (1992) states that "Yolngu children have grown up in a community where 'knowing the value' and comparisons of 'objective value' are de-emphasised. Instead the manifold types of relatedness between humans, and between the human and non-human are emphasised." (p. 31).

Considerable research, carried out over recent decades, supports this statement, indicating that many Aboriginal people have an acute facility with spatial concepts and in particular, an acute sense of direction and location in space, (Dasen, 1973; Shellshear, 1983; Graham, 1985; Harris, 1991). This is because Aboriginal cultures place greater importance on direction than on measurement. Since the study of space includes the understanding the spatial features of environments this is not surprising when considering previous discussion on child rearing practices.

There are two key spatial outcomes for all children:

1. **Being able to represent location, shape and arrangement;** using visualisation, drawing, modelling techniques, and being able to predict and show the effect of transformations on them. Students are, on the basis of these skills, able to sort and classify shapes and arrangements in order to generate useful properties such as "all triangles which have two sides the same length are called isosceles triangles".

2. **Use properties and generalisations** to reason about shapes, transformations and arrangements to solve problems and justify solutions.

The use of properties and generalisations can not occur unless children have developed 'spatial sense' which is the essence of the first outcome above. In short, spatial sense can be defined as "an intuitive feel for one's surroundings and the objects in them", (National Council of Teachers of Mathematics, 1989). "To develop spatial sense", it continues, "children must have many experiences that focus on geometric relationships: the direction, orientation, and perspectives of objects in space, the relative shape and sizes of figures and objects, and how a change in shape relates to a change in size" (p. 49).
8.2 Location

Orientation
From a very early age many Aboriginal children develop an acute spatial awareness. This is developed largely through the responsibility given them through the freedom allowed in understanding and learning about the physical environment as described in Chapter 2.

For an Aboriginal child living in an urban setting 'home' may be six houses spread over four suburbs. Consequently the child learns at a very early age how to use landmarks and recognise arrangements between landmarks and places in order to find their way home. Laughren (1978) in her work with Warlpiri in Central Australia, found that children as young as three years old could handle directional terminology such as 'here', 'there', 'up' and 'down', including the points of the compass and positions and directions with a great deal of competence.

Whereas in many Western cultures parents are proud of their pre-school children if they can count, an Aboriginal parent may have the same degree of pride for their toddler if he/she has a good sense of direction and place. In particular, Aboriginal people tend to use words which are about cardinal direction (north, south, east and west) as compared with a strong orientation to the left and right sides of the body used by non-Aboriginals, as explained in the previous chapter. Harris (1991, p. 28) relates a story about an eighteen month-old Aboriginal child who was told to look to the east to see someone and did so. This ability can of course vary from group to group and between different locations.

These special skills, often peculiar to the Aboriginal child, if they are noticed, should be celebrated by the classroom teacher and the children in the class. Children need to appreciate these cultural differences and learn from them.

Other sources tell me that language is not used to specifically describe direction; children may have an idea of 'east' and 'west' by knowing which side of the tree the shade is on, but not use those words. Anecdotes related by Harris (1989) indicate that Aboriginal people don't get their sense of direction by the stars, sun and other external features, or depend on being in familiar locations; they 'just know' (pp. 10-12).

Harris also uses examples that indicate that many Aboriginal people use compass points over short and even personal distances. She gives one example of a woman talking about her 'east leg', (p. 13) and describes a personal example from her teaching days in Yuendumu, of Warlpiri students using "the compass points (according to their own orientation at the moment) in situations where I would have used up, down, left and right" (p. 12).

For some Aboriginal language groups statements about movement may be incomplete unless locative and/or directional language is included. By contrast, Anglo-Australians have a strong orientation towards the two sides of their bodies and their own personal orientation is about being 'on the left hand side or on the right hand side'. An Aboriginal person giving directions may be likely to use cardinal points of the compass, whereas a non-Aboriginal person might say something like "Go down this street and then turn left".

Teaching children about their left hand side or their right hand side can be difficult since it is a concept which is about personal orientation. Harris (1989) suggests we should not be too quick to teach this concept to Aboriginal children for whom it may be totally unnatural. She also strongly points out that orientational language for different Aboriginal people may only be local and teachers need to be aware that they should determine this prior to teaching specific Western Mathematics ideas and concepts (pp. 15-16).
In Standard Australian English we use the word 'here' to describe things which are close and 'there' to describe things which are more distant. Malcolm et al (1999) describe situations where, speakers using Aboriginal English used the word 'here' to describe things which were up to 10 km away, or "'here' might (also) refer to a place that the Aboriginal speaker 'has not yet reached but will soon". By way of explanation for this, they explain that for the non-Aboriginal child "space or one's own area is contained within walls or boundary fences so that the non-Aboriginal child divides space into what is available and here and what is not available and there" (p. 31). They cite work by Coombs et al (1983) which showed that space is not necessarily viewed in the same way by Aboriginal people whose world consists more of open and shared spaces. Aboriginal parents may only use words like 'here' and 'there' to describe position and location and, even then, not in the same sense that non-Aboriginal parents would use them.

For non-Aboriginal children, the language of location, from a Western Mathematics perspective, is familiar, having been used in the home. These children are generally immersed in a world where words and phrases such as 'under', 'behind', 'in front of', 'left', 'right', 'on', 'near' and so on are used all the time. For Aboriginal children these words may be totally new since they may not be used in the home at all.

Words of position and placement (such as on, under, in, at) are an important part of understanding spatial concepts in Western Mathematics. In some Aboriginal languages these words do not exist and prepositions are often indicated through suffixes – short words added to the end of word stems that indicate location. For example, (in the following sentences, the Warlpiri locative suffix – ngka could be translated by the English prepositions ‘on’ or ‘in’, or ‘at’ or ‘by’.

- Wati ka pirlngka nyinami The man is sitting on the rock
- Juripu ka manjangka nyinami The bird is sitting in the mulga tree
- Kerdjka ngapangka nyinami The child is sitting in the water

(Harris, 1989, p. 43)

Other Aboriginal languages have "postpositions – words that come after the name of the person or thing" (Harris, 1989, p. 44). Vaszolyi (1976) provides examples of this from the Wunampal language spoken in the Kimberley region of Western Australia: 'ko:ya' is the word for 'crocodile', and so one can say:

- Ko:ya pale ‘behind the crocodile’
- Ko:ya arangu ‘atop the crocodile’
- Ko:ya mintatj ‘across the crocodile’ (p. 39)

Work collated by Anne Shinkfield (1996) with Aboriginal children from the Central Desert region in Western Australia, revealed that words such as 'side', 'centre' and 'forward' had either no concept or no direct translation in the local language while other positional words such as 'around', 'over', 'between', 'behind', and 'above' had Aboriginal phrases that had similar but not direct meanings. For example, the word 'above' was translated as 'moving above' or 'is above' but not the concept 'above'.

An Aboriginal mother indicating to a child that the object is ‘on the chair’ might simply point at the chair with her chin so that the child will then look in the direction indicated in order to locate the object. In English, one preposition, such as ‘over’ can be used in many different ways and contexts, such as ‘over the ground’ or ‘put something over the food’; we make sense of the word through the context. Harris (1989, p. 46) relates an anecdote about a teacher who gave instructions in the
Aboriginal children's own language to ‘write over (on top of) the dotted lines’ on a worksheet. The children had translated ‘over’ in the context as ‘above’ and had proceeded to write higher up the page.

These words need to be modelled and explicitly taught for all children as we should not assume that they are known. The language can be taught by the teacher giving directions and by the students interpreting and carrying out instructions, such as "go and stand behind the bin", or "draw a picture of the bird flying around the tree". Remember that the visual and modelling approach should be utilised as much as possible as opposed to an oral instructional model.

Maps

Most maps are either topological (that is, showing the arrangement of places or features with respect to other places or features but not necessarily having any sense of exact measurement or scale), or exact representations, with a scale which enables the reader or user to know exactly where places and features are. These two types of maps, one stylised and abstract (such as a map showing bus routes or maps of underground railway stations) and one having precise reduction of distances to scale and accurate compass point alignment) have advantages and disadvantages depending on the use to which they are put.

Whereas in Western Mathematics children develop in their understanding of location in the level of sophistication with grids and map references through to latitude and longitude, for many Aboriginal children the development of their understandings of location is more about the detail in their maps using an analog approach (that is, showing where things are in relation to other things or points). The maps used therefore, concentrate on arrangement as opposed to exact location and placement. This clearly is related to the understanding of measurement since generally, it is the sense of scale that appears to be missing in most Aboriginal maps. Aboriginal people might generally be more concerned with the arrangements between spatial features such as trees, roads, houses, rivers and so on, than they are about the distances between them.

Aboriginal children may be taught at a very young age, to find their way home using landmarks and features; they find the next landmark from the one before and, having found that one, find the next one and so on. In this way, they build up a ‘mental map’. Aboriginal maps too may need to be translated through ‘the same eyes’ as the person who drew the map. Lewis (1976, p. 271) suggests that the sacred and emotional significance attached to certain places is an important aid to Aboriginal memory and this is why maps often have a sort of ‘freedom’ in the way aspects of the terrain are shown.

Some Aboriginal people have difficulty reading Western-style maps (as do non-Aboriginal people!). People working with Aboriginal people have indicated however, that often this is due to the orientation of the maps and that when the map is turned so that the north point on the map is facing true north, the maps could be used well enough (Harris, 1989, p. 24). Harris relates an incident where some 8-14 year old Aboriginal children in school were looking at an atlas in order to try and find some named places.

So long as they oriented the map and thought about the directions, they were able to find the places and enjoyed the activity. But when they discovered they could read the names of the places, and began to use the method of reading the names to find the places, they were no longer so successful and lost interest in the activity. (p. 25)

Schooling then, may in fact interfere with Aboriginal children’s natural ability to read maps. Kearens, in her 1977 unpublished report on work with 10-12 year old (and later with Year 2) Aboriginal children in Meekatharra, found that the ability of the children to locate where they lived
on a map showing their school, the creek and the railway station, were far superior in their ability to do this than the non-Aboriginal children. The Aboriginal children “looked carefully and either moved a finger from the school to their home in a direct line, or simply put a finger on the place. White children in almost all cases followed streets carefully with a finger turning all corners ... as they might walk”. This was not found to be the case with other Aboriginal children from other locations, and once again points to the need to be wary of over-generalisation to Aboriginal cultures.

Aboriginal paintings from northern parts of Australia, are frequently 'birds'-eye-views' of environments and locations. Lines might depict waterways or physical boundaries, and circles or dots depict people or trees. This is not necessarily true for all Aboriginal culture groups; Noongar paintings are not ‘birds-eye-view’ but tend to be drawings made from a front view, slightly elevated.

For numeracy acquisition, all children need to be taught how to describe space using coordinates and compass bearings. It is therefore imperative that this be taught with measurement concepts so that children develop a sense of scale. Coordinate points can be taught using a grid marked on the ground, children being asked to go and stand on certain points. They obviously need to have an understanding of words like 'left' and 'right' which should not be assumed but also explicitly taught, as previously mentioned.

The analog ways of describing Space should not be seen as inferior, only different. They can be taught alongside the coordinate approach. For example, "Go and stand two points to the left of the chair" might be an instruction given that uses both approaches. All children should appreciate and learn an analog referencing system as part of appreciating the mathematics of different cultures.

8.3 Shape

Shapes surround people in any location. In Western Mathematics, part of imposing order on the world we live in is to sort and classify shapes. The idea of a triangle or a quadrilateral for example, is abstract. There is no use for a plastic model of a triangle other than to learn what a triangle looks like and its label 'triangle' so that we can recognise and appreciate where the shape occurs in our environment. In some environments these shapes do not naturally occur in nature; they are used in building and construction because of the properties they possess which result from an analysis and description of shapes.

Because of the way Western society has ordered the world, children from ‘westernised’ homes are often immersed in the language of spatial classification. Parents may talk about triangles, circles, squares and rectangles frequently and show children where they occur in the home: windows, the TV screen, wheels, books etc. It is unlikely that these objects are labelled and spoken of in Aboriginal homes using this Western mathematical classification. As indicated before, this is primarily because the focus is placed on people and relationships rather than on things and objects.

Early learning experiences should be through hands-on visual approaches. Children learn about shapes through handling and seeing which then leads them to remember and imagine. Lessons can begin with pictures of shapes such as circles and then through visual contexts such as showing a picture of a car and asking children to say where there are circles on the car; thus making the connection from the abstract concept of a circle to real world applications with which children are familiar.

Children need lots of free play in spatial 'environments'. These environments can include 'the block corner', a sand pit with many objects of different shapes and sizes in it, or even on a desk in a classroom with children having access to two and three-dimensional shapes. They need to be
immersed in the language of shape – not just in terms of labels for different shapes, but with words such as 'smooth, flat, pointy, curved'. They should have opportunities to make shapes out of plasticine, cardboard, play-dough and so on, and opportunities to draw shapes through copying and then by imagining and from memory.

They also need many opportunities to sort shapes based on different attributes and criteria: for very young children these sorting activities may simply be on the basis of size, colour, or surface texture; and later on the basis of shape names such as circle, square, triangle. As they develop their understandings they will sort on the basis of attributes such as length: "put all the triangles having three sides of different lengths together", and so on in order to develop understandings of properties of shapes.

To make connections with their environments, children should be given opportunities to recognise shapes, not just from pictures, but also from the real environment. A nature walk can provide the opportunity to ask children questions such as "What trees have a trunk like a rectangle?" or "What shape does a berry have?", and "Who can see a cloud that has a round shape?" Shapes can also be seen in animals; a kangaroo could be seen as having 'a triangle shape'. The school grounds can similarly be used: ask children to draw as many things as they can see that look like a square or have straight sides, for example. They need to be asked questions like "Why do you think it is a rectangle?" for example, in order help them learn the language and correct terminology and also to assist the teacher in diagnosing misconceptions.

The language of shapes is a unique language that is not necessarily used outside of the world of Western Mathematics. Words such as pentagon, hexagon, dodecahedron, isosceles, rhombus, and so on are rarely encountered elsewhere in real-life contexts.

Once children have a good grasp of this spatial language they are able to talk about and understand some basic properties of shapes, such as 'will roll', 'will stack' and talk about why this is possible or not possible. They will learn that if they are making a see-saw this task would not be possible if they were to use a square or rectangular box as the base; a triangular or circular shape would be needed. They will learn that if they are to make a 'tower' to stand on in order to climb up next to a wall to see over it they could not use blocks that did not stack or have flat faces. They are able to select plastic shapes from a pile to make a 'picture' of their house, and make a bird or a fish using Tangram pieces.

Aboriginal cultures have not placed such importance on classifying and naming abstract shapes as have Western cultures. Whereas the ancient Greeks and modern mathematics might see a circle as Harris (1989) points out that

an abstract shape having a circumference everywhere equidistant from its centre, and that may be important to them, to an Aborigine from Central Australia a circle is more likely to be seen as a symbol for a water hole or camping place, or as an integral part of a design which recalls the story of some ancestor hero who formed the country in the Dreamtime, and that symbolic spiritual meaning is what is important to the Aboriginal person. (p. 33)

There are many shapes used in Aboriginal art and these can be used to help Aboriginal children make connections with shapes in Western Mathematics as well as be a context for the teaching of appreciating mathematics in different cultures to non-Aboriginal children. Morphy (1980) analysed the art of the Warlpiri and found that the circle and the line are the dominant signs. A straight vertical line might have a range of meanings such as a spear, fighting stick, a dog or kangaroo lying
stretched out or a human actor lying straight. A circle might depict a nest, waterhole, tree, hill, fire, billycan, egg, dog curled up in camp or the act of circling an object (pp. 34-35). Similarly, an arc may represent a person sitting, a hill, windbreak, mia-mia, or dog sleeping.

Morphy (in Harris, 1989) explains that the circle and line when used together as follows, signify:

the idea of fixed points or discrete objects and the connections between them, rather than a representation of specific objects or places. The difference between them is that the Warlpiri designs are linked to ancestral events, many of which took place at waterholes, which are seen as one of the main indications of ancestral creativity. (p. 33)

The Yirritja moiety in North East Arnhem Land has clan designs which consist of sequences of diamonds as follows. Each clan has a variation on the design (Morphy, 1980, Figure 1). For example,

Harris (1989, pp. 34-37) describes some of the results of her research that indicate that in some Aboriginal languages shapes are described using verbs such as ‘sitting’, ‘standing’, ‘lying’ and ‘leaning’; if something is on the ground and long it may be described as ‘lying’.

In a personal correspondence to Harris (1989) Fay Bell, writing about Pitjantjatjara, says,

The four verbs ‘sitting’, ‘standing’, ‘lying’, and ‘leaning’ describe the state of ‘being’ for all things. When talking of variously shaped objects which in English tend to ‘lie’ on the table, on the floor, etc, it would appear that relative proportions within a three-dimensional frame of reference determine which verb describes the state of being. (p. 36)
Laughren gave a diagrammatic representation of the shapes implied in the use of the four Warlpiri verbs.

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\end{tabular}}\]

'\text{to sit, stay, to be}'

\[\text{\begin{tabular}{c}
| \ \ \\
\hline
\end{tabular}}\]

'\text{to lie, to extend horizontally}'

\[\text{\begin{tabular}{c}
| \ \ \\
\hline
\end{tabular}}\]

'\text{to stand}'

\[\text{\begin{tabular}{c}
\hline
\hline
\end{tabular}}\]

'\text{to crouch, to be humped}'

(English glosses abbreviated from Hale's Warlpiri-English Vocabulary 1974, from Harris, 1989, p. 36)

According to Harris (1989),

It would seem that shape is not something separate from the object which can be talked about in the abstract. Reference to the object automatically carries reference to the shape within the categories described. The fact that highly educated English-speaking people find it difficult to use the categories correctly is an indication of the difficulties which Aboriginal school children must have when required to use unfamiliar English classifications such as those for shape … (p. 37)

Clearly, the concept of 'shape' in Western Mathematics, is an abstract one and this is highly problematic for a large proportion of Aboriginal children, particularly since many Aboriginal languages do not have this abstract term.

The Gurindji people of Wave Hill have terms for 'circle,' 'concentric circles' and 'make a circle', but no terms for 'ellipse' or any shapes made with straight lines (McNair, personal communication to Harris, 1989, p. 38).

An Iwaidja woman described the following shapes in her own language to Pym (personal communication to Harris, 1989, p. 38) as follows:

\[\text{\begin{tabular}{c}
| \ \ \\
\hline
\end{tabular}}\]

'\text{flat, smooth}'

\[\text{\begin{tabular}{c}
\hline
\end{tabular}}\]

'\text{round}'

\[\text{\begin{tabular}{c}
\hline
\hline
\end{tabular}}\]

'\text{long}'

\[\text{\begin{tabular}{c}
\ \\
\hline
\end{tabular}}\]

'\text{having sides}'
Similarly, McNair, writing about the Gurindji language, in a personal communication to Harris (1989, p. 38) says: “There appears to be three main shapes – flat, round and long. Books, mangoes and boxes are considered flat. Berries, eggs and balls are considered round. Yams, swags, and tin cans are considered long.

In the Western Mathematics taught in most schools, there is also a focus on the classification of lines; curved, straight, horizontal, vertical, oblique, intersecting, diagonal, parallel, and so on. Results of Harris’s research (1989, Appendix A) revealed that the use of lines in the art of some Aboriginal groups, varies markedly. Some Pitjantjatjara people described this series of straight lines as ‘drawing straight’.

Their word, tjukaruru, (straight) can also be translated as ‘good’ as in ‘a good man’. Plain t- shape crosses drawn with different orientation, were described by these people as ‘having two heads’ and may explain why Pitjantjatjara children (and possibly children from other Aboriginal groups) have difficulty remembering the difference between the symbols for addition and multiplication.

A zig-zag line was described as a boomerang and a wavy line likened to curly hair.

Teachers who are aware of these differences are able to build on them and use them in their teaching programs in order to assist children to make connections. Awareness of these differences is paramount. At no point should a teacher make assumptions about the words and abstractions of Western Mathematics with respect of abstract concepts of shape for any child let alone for Aboriginal children. The language of shape needs to be specifically taught and enhanced through ‘hands-on’ experiences. Children should use objects and manipulatives and then connections between these and the real, familiar environment should be made wherever possible. Familiarity with the real environment of the children is essential as is a deep appreciation of the difficulties which many children may have in learning the language and abstractions of shape.

8.4 Movement

Movement in a spatial sense can be thought of as shapes ‘flipping’, sliding’ and 'turning' and includes enlargement and dilation. The ideas of orientation are important here: that is, in common terms "which way is it facing?". If I walk around a tree I will finish facing the same direction that I was facing before I started – my orientation is the same. If I walk half way around a tree I will finish facing in the opposite direction to that when I started: my orientation has changed.
The language of spatial movement is as follows:

- The *translation* transformation — causes a shape to move vertically or horizontally while maintaining its original orientation (that is, vertices or key points in the image are in the same position as vertices in the original shape);
- The *reflection* transformation — causes a shape to be reflected in a line. The line acts as a mirror in that the vertices or key points of the image have the reverse orientation to the vertices in the original shape. (that is, clockwise-labelled vertices become anticlockwise);
- The *rotation* transformation — causes a shape to rotate around a fixed point, or centre of rotation. The orientation of the vertices or key points remains unchanged under rotation;
- The *enlargement* transformation — causes a shape to be enlarged by a factor of some whole number;
- The *dilation* transformation — causes a shape to be dilated. This transformation is really an enlargement transformation which causes shapes to be enlarged by a factor between zero and one (i.e. a fractional value which causes a reduction in size).

Young children learn movement using the processes of *flip*, *slide*, *turn*. In other words, students are able to look at a given drawing of a shape in whatever position and, holding the shape in their hand, flip, slide, or turn it so that it looks like the one in the picture. Note that *flip* is a word which describes the process of reflection, *slide* describes the process of translation, and *turn* describes the process of rotation. Clearly, children should be learning about shapes at the same time so that if they are asked to "slide the round shape along the page" they will identify the 'round shape' first in order to 'slide it'.

At the next level of development children are able to construct multiple copies of the results of these processes to produce patterns; they can draw a 'frieze' pattern for example, using the same shape and sliding it along, flipping it over, turning it around and drawing around it each time to form a repetitive pattern. They can do this on their own as well as following instructions, and can use the words, *flip*, *slide*, *turn*.

The next level of conceptual development indicates children being able to look at a pattern or frieze and describe how it was produced: "This pattern was made using a triangle which was flipped the first time and turned the next time, flipped the next, turned the next over and over". They can recognise shapes as having line symmetry — a fold line — without necessarily having to fold along the line to check. If they are given multiple copies of a shape they can decide whether it will tessellate with no gaps or overlaps, and explain why using informal language. They need to be able to recognise shapes in various orientations, not just in standard positions, and that a rectangle for example, has *four* sides, not just two and a top and a bottom. Similarly, that a triangle has three sides and does not stop being a triangle even if it looks 'skinny'.

Some children for example, cannot recognise a square if it is drawn with the following or similar orientation:
Can some children recognise a triangle if it is drawn like this:

Connections with the real world should be made wherever possible. Children can see a car for example, sliding along the road, or a child sliding down a see-saw: their orientation or the way the car or child look doesn’t change; they are merely moving along a straight line. They can play games on a merry-go-round or simply hold hands around a tree trunk and move around it so that they finish facing in the opposite direction or in the same place. Some Aboriginal children may be able to connect rotation with people sitting around a camp-fire; always facing the centre so that each person can see each other person and the entire surroundings beyond the circle of people can be seen (for safety purposes).

The ideas of line symmetry (as reflection or ‘flip’) can be seen in designs, boomerangs and other Aboriginal artefacts and these should be used to make connections with these ideas as much as possible both in remote and urban communities. There are shapes, both two dimensional and three-dimensional, in all natural and man-made environments that exhibit this property.

Technology has taken on a major role in all Australian schools—even remote ones. Since Aboriginal children are very visual learners it makes sense to use the computer to teach spatial concepts whenever possible. The environment created by individual use of a computer can remove the 'shame' aspect that often inhibits individual participation and individual response by Aboriginal children. Children can engage without the fear sometimes imposed by the possibility of getting incorrect answers or making incorrect responses.

There are many good software packages currently on the market—for example, the Space CD ROM developed by Olssen and Walsh (1996) and the interactive programs developed by Creative Educational Software (Lamb, 1999)—which should be used by schools for the purpose of helping children understand spatial movement. It is often a difficult concept to teach since it cannot easily be modelled in a classroom. Computer graphics are an extremely valuable asset to be harnessed in the teaching of spatial concepts since they allow movement to be shown.

8.5 Spatial Reasoning

Earlier I made the statement that children are unable to reason spatially unless they also understand location, shape and movement. This is because spatial reasoning involves making connections between location, shape and movement. Children use the correct nomenclature for shapes such as square, circle and triangle and adjectives such as 'flat, curved, pointy, straight, round' to sort shapes as being alike or different.

The concepts of 'same' and 'different' need to be explicitly taught using collections of objects with which children are familiar. In the same way that children learn that teddy bears are alike because they all are fluffy, have two eyes and a flat nose, and that flowers are different because they are different colours and have different numbers of petals, so they learn that two rectangles have the same number of sides and square angles and that a square is not the same as a triangle because one has four sides and the other has three.
Aboriginal children may even have a stronger, more developed sense of same and different; some Aboriginal languages have over 50 words for 'kangaroo', demonstrating a recognition of the differences of different types (or characteristics) of kangaroos. Aboriginal children are often taught the similarities of trees, clouds, leaves and so on, and can compare with a great deal of accuracy not in evidence in non-Aboriginal children. This is taught as part of their desire to recognise objects and places with accuracy in order to define their location. It may not be enough for example, to know that the shop is near the trees; better to be able to say the shop is near 'the trees with the silver leaves'. This connection can be made to help children learn about sorting and characteristics or 'properties' that make shapes the same or different.

Once again, many hands-on experiences are recommended including touching and feeling shapes, drawing shapes, making shapes so that properties can be explored through the senses. They should learn to connect various representations of shapes (drawing, verbal descriptions, models and identified shapes in the environment) and be encouraged to connect location, shape and movement in each of the representations. For example, they can identify a rectangle in some brickwork, explaining why it is a rectangle, and be able to show how it has been moved to form the 'tessellation' using the brick as a tile to cover the wall surface. They might similarly recognise a triangle-shape in a wrought iron fence and describe (either verbally or through drawing) how the shape has been turned and flipped to produce the pattern along the fence.

8.5 Being numerate in Space

The *Numerate Students, Numerate Adults* study (Education Department of Tasmania, 1995, pp. 11-28) stated that to be numerate in Space people need to be able to:

- Recognise and describe common shapes
- Use shapes appropriate to the task
- Choose and use appropriate equipment for a particular purpose
- Recognise and interpret the conventions of visual representation
- Use spatial techniques to solve problems

The above discussion presents some ideas for developing these numeracy skills in children – both Aboriginal and non-Aboriginal – and draws particular attention to the skills and abilities that both groups bring to the classroom. Teachers should not assume that any of their students have any of these specific skills but should be aware that they may have and that these should be identified and built on. If not, they will need to be explicitly taught.

It also draws attention to the language of space and how crucial this is in developing spatial sense. Teachers should attempt to understand ‘where their students are at’ in their knowledge and understandings of the spatial environment and in particular, their ability to make sense of it and describe it using appropriate language. The language of space as used in Western Mathematics, can be complex and abstract and teachers need to be aware that for many Aboriginal cultural groups (and perhaps for non-Aboriginal children) this language may not be used in the home.

The benefits of spatial language immersion should not be underestimated in preparing children for higher levels of mathematics understandings, and that should these experiences be missing for children, they need to be created in the school particularly during the kindergarten and pre-primary years.
Chapter 9

Teaching Key Ideas about Chance and Data

9.1 What is Chance and Data all about?

Chance and Data is a relatively new inclusion in most school mathematics curricula, which have tended in the past to concentrate on Number, Measurement and Space, particularly in the early years of school. Recently the amount of qualitative information has increased at an exponential rate and the need to be able to manage that information and access it in readily Usable forms has meant that systems and curriculum developers have been forced to recognise data handling processes as essential skills for all children; hence its inclusion and increased status over the last decade or so.

Chance and Data could be argued as one of the most important strands for numeracy in the mathematics outcomes. To a certain extent, this is true. For the majority of the population, the average Joe and Mrs Blow, dealing with data and chance are the most common of all mathematical activities which occur in daily life; in particular the interpretation of statements relating to chance and data. The media is riddled with statements related to the use and misuse of chance and data.

Chance is essentially about situations in which uncertainty or unpredictability exists. Data is about information: gathering it, organising it, summarising and representing it, and interpreting it. Every cultural group attempts to do this in various ways. For traditional Aboriginal cultures information is handled principally through tradition and ceremony and passed on through dance, songs, stories, paintings and drawings, and other forms of expression, except for the written word. This practice continues throughout Australia and is a very valuable part of Aboriginal culture. In recent years the Aboriginal people themselves have become aware of the need to preserve some of this information in written forms.

The Western way of handling information is generally through the written word. This should be taught as just one way of representing information, but not as the 'best' way or the 'only' way. Children who are not Aboriginal should be taught to appreciate the data handling methods used by other cultures, just as Aboriginal children should be taught to appreciate those of Western societies. Both groups can learn from each other and it is appropriate that connections between the two be made.

There are three key outcomes for children from the Chance and Data strand:

1. To be able to understand the language of Chance; and as a result, be able to make statements about the likelihood of an event occurring. Children should be able to use their knowledge of Chance processes when dealing with situations in which uncertainty is involved.
2. To be able to plan and undertake data collection and organise, summarise, represent for the purpose of interpreting and communicating it.
3. To locate, interpret, analyse and draw conclusions from data; students should be able to locate data (collected, organised and summarised by themselves or someone else), interpret
it and communicate their interpretations and any conclusions made from it. This discussion should include critical analysis of methods of data handling and chance processes used and comments about the subsequent validity of the data.

The interpretation, analysis and communication cannot effectively occur if children do not understand the methods used and decisions made when choosing these methods. Hence, children need to be involved in this decision-making process, and not merely taught or shown the ways of doing it.

9.2 Chance

The notion that Chance is an important part of our daily lives has the notion of 'the everyday language of chance' at the root of it. Many cultures use words of chance all the time; words such as 'maybe, might, could, probably, will, won't, can't' and so on. If the average person in the street were to make a list of the words of chance used in their everyday vocabulary the list would probably be a kilometre long! And that isn't including the colloquialisms that most Australians use in their speech, such as:

- Pigs might fly
- You've got Buckley's
- In your dreams
- Once in a blue moon
- Fat chance
- Fifty-fifty chance

These statements and words that we use are related to our own or someone else's experience. They also relate to experiments and analysis of data. For example, when we watch the weather at night on the TV we hear a weather forecast which is made on the basis of past experience, long term trends and prediction. The experience and long term trends are, in this case, determined formally using statistical analysis. Similarly, financial planning is based on experience of trends. Those who do this for a living provide advice to investors on the basis of analysing figures gained from the past and on making predictions aimed at minimising risks. In a court of law, the jury makes a decision of guilty or not guilty 'beyond reasonable doubt', a statement implying that there is a small amount of doubt but insufficient to pronounce the person on trial as innocent. Sometimes these decisions have been shown to be wrong, resulting in people spending years of their life in jail until new evidence comes forward.

We need to teach our children what these things mean and why they are important from a mathematical viewpoint. Clearly, many decisions made on the basis of mathematics, have an element of risk and children need to know about risk so that when they make plans, either financial or as simple as planning a wedding day in spring, they need to minimise the risk of losing all their money or of it raining in spring in order to maximise their financial security or their happiness.

The language of Chance as a concept might appear to be foreign to many Aboriginal cultures. In a personal correspondence to the writer, Dr Michael Christie, well-known authority on Aboriginal cultures and an Aboriginal linguist of international reputation, explains that

> It is true that what you call a subjunctive in English (could, would, should, might etc.) looks very strange in Aboriginal languages. That is basically I think because the focus of the relation between language and experience in European languages is...to do with cause and effect, whereas in Aboriginal languages it is more to do with correlation, that is the relation between events.
He goes on to say that

If we accept the gross over simplification that hunter-gatherer types are predisposed to the analysis of patterns of co-occurrence in the environment, we can assume that in their natural state, Aborigines would be much better predictors of the outcomes of, or consequence of the configurations they observe. In other words, making predictions about the likelihood of future events is precisely what kept the Aboriginal cultures alive here for 50 000 years; without that ability they would have died out. So we can assume they have the skills, it's just the language and the context which might trip them.

So the ideas of prediction are part of Aboriginal tradition. The words of Chance may not be. The unpublished work by Anne Shinkfield (1996) quoted earlier, revealed that the language group from the Central Desert region in Western Australia where she worked do not have concepts for 'always' or 'never'.

The implications concerning the language of chance and lack of some chance ideas for teachers are challenging. To teach the language and words when the concepts are not part of the 'ways of knowing' is difficult. Children from Western societies are immersed in the language of chance on a daily basis prior to coming to school. It would not be rare to hear a four-year-old explain to her teacher, for example, that "Mum might pick me up after school", or "we might go to the pool for a swim this afternoon".

Malcolm Sercombe, who worked with the Wongi Aboriginal people in the eastern goldfields of Western Australia for some time, indicated in a personal correspondence with the author, that many of the Aboriginal men would say in Aboriginal English "Maybe we go out hunting, boss?" but that it was said more as meaning "Can we go out hunting?" than as a possibility. It is likely that this language developed in response to the reply being 'maybe', to the question "Can we...?"

Teachers will need to model the use of the language for the children they teach and should not assume that the children from non-Aboriginal backgrounds have this knowledge and understanding either. The learning of terms and the language of chance must be the starting point; real learning of the terms will only come through understanding the concepts. The concepts will no doubt need to be taught using situations in the classroom and school and future learning in chance will occur through making connections to these situations and contexts.

In the early years children should learn to be able to recognise the element of chance in familiar daily activities. This means that they know the difference between statements like "It might happen" and "It won't happen"; they can use words like will, won't, might, could, couldn't in sentences. Later their understanding increases slightly in that they have a stronger sense of certainty. Distinguishing possible from impossible is a big step; most small children from Western cultures believe that anything is possible since a big part of their world is fantasy. They don't yet understand that a dinosaur couldn't walk into their bedroom or that the frog won't turn into a prince. As they develop in their understanding they know that these things are impossible because they have a stronger sense of what is real and what isn't.

Similarly, they understand that some things are more likely to happen than others. For example, if they live in Perth they know that it is less likely that it will rain in January than in July. As they develop further the understanding of chance is stronger in that they can decide between two events that are unlikely and say 'this is more likely than that' for example, they might know that it is unlikely that they will go to the beach on Saturday and it is unlikely that they will go to the circus...
on Saturday, but it is more likely they will go to the beach than the circus and can give legitimate reasons for that. When using instruments of chance they know that it is equally likely they will get a six as a one when throwing a die.

We must however, be aware that many children develop misconceptions about chance through playing with dice. Some children believe, because of their own experiences, that:

- They are more likely to get a 2, 3, 4, or 5 with a die than a 1 or a 6;
- It's harder to get a 6 with a die than any other number; or
- You're more likely to get a 3 or a 4 with a die than the other numbers.

Of course these misconceptions are caused by the fact that when children are playing with dice they rarely seem to throw what they want! There are other misconceptions concerning chance that even adults have, such as:

- An event is more likely if it happens to someone you know (this is why newsagents publicise the fact that the winning lotto ticket was bought from them); or
- After a run of bad luck your luck will eventually change.

One way to remove these misconceptions (in adults or children) is through cognitive conflict; that is creating conflict to the beliefs of those who hold them so that they can reorganise their thinking. Teachers can generate class discussions in order to reveal the misconceptions in their students and create cognitive conflict. They can play ‘devil's advocate’ and ask questions such as, "Is it more likely I'll get a six with the die than a four?", or "If we threw a die fifty times would we get proportionately more sixes than if we threw a die ten times?".

Still later in their development, upper primary, lower secondary, children begin to associate a hierarchy of 'more likely' and 'less likely'; they can associate 'impossible' with a numerical value of zero and 'certain' with a numerical value of one, and place other events on a continuum between these two values. They are able to use available data to order events. For example, they can use a table showing how many students there are in each year at school to explain which year group a student drawn at random is more likely to come from.

As a result of these informal experiences with chance they are able to then use formal probability (that is, the probability of an event occurring as being the quotient of the number of favourable outcomes divided by the number of possible outcomes) and can hence use fractions to assign probabilities.

Michael Christie (1984), in his thesis titled The Classroom world of the Aboriginal Child studied the propensity of Milingimbi children to use feedback of success or failure in their assessment of risk of subsequent success or failure. He found that, compared with the non-Aboriginal children in the classroom, they tended not to use their feedback to ameliorate their risk taking in subsequent tasks. Christie interpreted this as neither a mathematical nor a language failure but as a failure to compete in favour of cooperation, to see tasks as competitive rather than individualised, or to think that winning was probably more fun that doing things together. In the personal correspondence to the writer mentioned earlier, Christie makes the point that it is difficult to encourage Aboriginal children to articulate their appreciation of risk and uncertainty in the classroom context and to make these sorts of assessments relevant to their own lives which strongly focus on cooperation as part of their world view, without compromising their Aboriginality. He concludes with the statement "...and is it possible and why exactly do we want to do that?".

My response would be that we do it in the context of assisting Aboriginal children to be numerate in the world in which they live; to recognise and appreciate the part chance plays in everyday life, and to recognise and interpret estimates of chance events (see section 9.4 below). Again we are faced
with the tensions concerning the definition of 'the world in which they live', described earlier. We should remember that Western Mathematics is simply a way of organising the world and that Aboriginal children are taught it as such; it isn't in any way superior to Aboriginal ways of organising the world – just different (see Chapter 3.1).

9.3 Data handling

**Collecting and organising data**

The ideas of 'sorting' were presented in the previous chapter on Space. Classifying collected objects is an important part of managing information. Young children need to be engaged in classifying objects on the basis of familiar criteria. To connect this with a familiar context for Aboriginal children a teacher might draw on the Aboriginal kinship groups with which even very young children are familiar.

Every Aboriginal person belongs to a class group and the patterns between the classes determine and govern every aspect of their life, and particularly their choice of marriage partner. It is extremely important for the children to learn these kinship patterns so that they know which group they belong to and which groups others belong to. It would be rare that Aboriginal children not know these classifications. At a different level, this process of classifying can be transferred to the classification of leaves for example, based on length or colour, or trees based on height, shape or colour. They can also classify information from their own lives such as what type of car their family owns, what type of dog they have, what colour hair they have, the types of food they eat, and so on.

It is important that children are involved in making decisions about what data to collect and how it should be organised. This should arise naturally from questions for which the answer is required or for which children are interested in. Children should be encouraged to make their own decisions about how to record their experiences in ways that help them to remember and talk about them at a later stage. These decisions should not be made for them by their teacher all the time.

Aboriginal people are themselves realising that the traditional ways of recording Aboriginal experiences through dance, stories and art are at risk of not being passed on to younger generations and there seems to be a national renewal to ensure that Aboriginal parents teach their children these ways.

Kepert (1993) describes an activity he observed with Palumpa Primary School students in the Northern Territory who use little English language outside the classroom. The teacher had been questioning children about the jobs done by people in the community:

> When questioned about who was doing what job the students easily listed the work force of the small Aboriginal run cattle station. Taking a bag of multi-link cubes we wrote the initials of each worker on a cube as they went through the names a second time according to occupation. Starting with red for stockmen we were able to assemble a small pile representing these workers. They very quickly picked up the system and helped classify all workers. The initials were more helpful as a memory aid for me as the students seemed to have no difficulty in remembering which cube represented whom and where everyone worked. All the cubes were sorted according to their colours (occupation).

In the past we taught students how to collect and organise data; we taught different ways of collecting and different ways of organising and that was where our part finished. Now, the emphasis is on teaching the students different methods of collecting and organising so that they can
make their own decisions about how to collect and organise the data based on the purposes for doing so. Collecting and organising data then is about students:

- Contributing to discussions about how to collect and organise the data;
- Participating in decision-making about the methods they will use; and
- Collaborating with their peers to plan data collection methods, and so on.

In fact, in the early years it is about students working collaboratively in one form or other until they are much older and can make the decisions both collaboratively and independently. Class work should emphasise the collaboration and this should provide enjoyable learning experiences for all children.

This being the case, as teachers we need to be aware that there is a need for teacher guidance and that what is developing in the children is the degree of autonomy, the ability to plan, coordinate, and make decisions about the hows and whys of data collection on their own. Initially, in the early years, much of the decision-making is done by the teacher who guides and prompts. The student merely participates; whereas later the student starts to think about the process and is able to contribute to the discussion, which the teacher is still guiding. Later on the teacher may still be contributing but taking a 'back-seat' role as the students work together.

**Summarising and representing data**

As described above, the data handling processes of collecting, organising, representing and summarising are very closely linked. Data is collected and organised, summarised and represented, for the purpose of interpreting. Data can be collected and organised for the purpose of interpretation only, no summarising or representing are necessary. Data can also be summarised and represented for the purposes of interpreting. In this case, it is secondary data (that is, data that has been collected and organised by someone else).

What does 'summarise and represent' mean when referring to data? Representing data is about showing it in some way. So this is where all the graphing occurs; we represent data in graphical form. Summarising is about using one or more process or 'statistic' to summarise a set of data. For example, we can summarise a set of data by describing it using its mean and its standard deviation. We can describe the relationship between two sets of data using the correlation coefficient.

We should be questioning the need to teach students how to draw graphs by hand in the technological world we live in. When students, even in remote schools, have access to computers with spreadsheets and graphing packages where, at the push of a button or keypad they have more than twenty graphs to choose from, why on earth should they be practising how to draw graphs? This is not to say there is no place for drawing graphs by hand in the mathematics classroom. When teaching what the graphs are and how they work it doesn't hurt for students to be engaged in these activities. It's the practice of drawing graphs when they have access to a computer that is not productive.

Some teachers may argue this point on the basis that many of their students don't understand how to draw scales. There are many students who don't know how to draw scales but will they learn how to draw scales better by drawing graphs? Teachers need to get to the source of the problem; the fact that students can't draw scales is rarely because they can't draw graphs. It's more likely that they don't understand fractions or can't work with a number line or don't understand the spread or range of the data they are working with. Once these specific skills and understandings are isolated they need to be addressed. Rarely will these skills and understandings be improved through the practice of drawing graphs, and what's more, why waste twenty minutes of a student's time in completing a graph when two or three minutes might be all that is required to draw the scale?
We need to keep the desired outcomes at the forefront of our teaching and learning programs in this regard: students are taught to summarise and represent data for effective interpretation and communication. In other words, when we teach students different graphs we are teaching them some ways of representing data so that they can make decisions about which graph represents the data best so that they can interpret it and communicate what it shows.

Clearly, being able to 'click' on twenty different icons for graphs and make a decision about which graph shows the data the best for the purposes of interpretation, is the most efficient and expedient way to achieve this outcome. When we teach students how to draw this graph and that graph as an end in itself, they get so caught up in the whole process of labelling, titling, calibrating scales and so on, that by the time that is done there is no time left for interpretation! I think this is why I often call graph-drawing 'busy-work'; it's something that students can get engrossed in but there is often very little learning occurring.

A summary may simply be saying a total arrived at by counting, such as "I have eight books", and representing this might be drawing a picture of the books. Children in the early years will 'make' a graph using objects or themselves; they can get into rows to show which way they came to school that morning or they might use their lunches to show what types of lunches the class brings to school. It could be one unifix cube representing each person, cubes sitting end to end in a row. It could be children 'being' their own graph by standing in rows or in segments of a circle drawn in the school yard.

The cubes collected and labelled by Kepert's students were then arranged by the students to form a graph – originally in lines to form a bar graph, and then, by arranging them in a circle, as a form of pie graph. The children divided the circle in segments according to the different occupations and used textas to put a label on each of the segments, which included such information as names, fractions and categories (Kepert, 1993).

As children develop in their understandings of representing data a degree of abstraction is shown in that students can use other objects to represent the real thing. For example, they might use coloured counters – one for each student – to represent the coloured shirts that the children in the class are wearing. Note that a one-to-one correspondence is part of the understanding whereas at a later level of understanding they are able to use a many-to-one correspondence. For example, they might use a red counter to represent two or five students wearing red shirts. As a result of using the tallying method to organise the data they have collected, they can summarise each tally by counting each tally mark to find a total. They are also able to display data in a pictograph where each symbol represents more than one unit.

Still later, students are able to use simple scales on axes, and some grouping – grouping data involving whole numbers only – into class intervals with assistance. They can use fractions and decimals to summarise data, for example, say "about one quarter of the students in our class like wearing red shirts", and they can find the mean and use this summary statistic to describe a set of data.

**Interpreting Data**

Interpreting data is becoming an increasingly important skill for daily living. With technology able to be programmed to collect, organise, summarise and represent data for the purpose of interpreting data, it can be seen that the interpretation is the culmination of a process, that collecting, organising, summarising and representing are rarely end processes in themselves but are in the main, means to the end.
We live in a world where interpretation of information in the form of data is not only important, but essential. Newspapers, television broadcasts, Internet displays and other multimedia are full of information which needs to be interpreted for it to mean anything. More than all the other aspects of data handling, interpretation – which includes the ability to communicate what is interpreted – is not a skill that technology can necessarily be programmed to do well. This is because statistical analysis does not allow for the human element that says "but what if...?".

It is easy to use and abuse statistics to manipulate them to say what we want them to say. For example, using statistics we could show that there is a relationship between the number of good looking guys who use a particular brand of tooth paste and who get good looking girlfriends. An advertising company may quote this statistic in order to make the broader public believe that good-looking guys who use X brand of toothpaste will get good-looking girlfriends. The relationship between these two variables however, may be completely spurious. In other words, statistically there is a relationship but this doesn't mean that one variable causes the other; by using X brand of toothpaste the good-looking guy won't always get the good-looking girl; and in fact, the brand of toothpaste used may have little, if anything, to do with it.

So we need to teach our students to be discerning about how statistics can be used and abused, particularly by politicians, advertising companies and businesses. This discernment is part of being able to interpret data.

Kepert's students (see previous section) talked about the graph they had made and found this very easy since they were basically relaying what they knew about their relatives and the jobs they held. "By looking at the relative size of each segment in the pie chart, the students were able to give the same information and use numbers, fractions and areas. Questions such as, 'Who is represented by the largest segment?' and 'How many men do stock work?' were all answered correctly and sometimes by interpreting from the graph. Students were able to accurately reproduce individual graphs from the class one" (Kepert, 1993).

Students need to interpret data in terms of 'what is means for them', which is why it is important to connect the activity of interpreting to primary data (data that they have collected). If secondary data (data collected by somebody else) is used then this data, at least initially, should be relevant to the world in which the students live. Data should be drawn from students’ own personal environment or the environment of the community in which they live.

Being able to interpret data, that is, make statements about what it means, is a higher level of development than simply collecting, organising, representing and summarising it. Students can say for example, "My picture shows that all the people in my family eat toast for breakfast", but they couldn't necessarily make interpretational statements about that, such as, "All my family eat toast because it is the easiest breakfast to prepare" or "Toast is the most popular breakfast food in our family". These statements involve a level of abstraction which younger students have generally yet to achieve.

As they develop, students are able to make these kinds of statements. They can interpret total numbers in a tally table and say "This shows that five people in our class like bananas". They can explain what their simple one-to-one graph shows. At this level their graphing is about using objects to represent real life things, so they might use a red counter to represent a red shirt. Correspondingly, students can interpret by explaining that one red counter represents one person wearing a red shirt and that 15 red counters represent 15 people wearing red shirts.
The holistic nature of Chance and Data

Some teachers do not see the connection between *Chance* and *Data handling*. It is important to note that there is not a distinction here; that the power of chance processes rests in the relative long-term frequency of the results of events. What this means is that the reason we can say for example, that there is a fifty percent chance of getting a head when you toss a coin is because of the mathematics of tossing a coin many times. You know yourself that if you toss a coin 100 times you will very rarely get 50 heads and 50 tails. 'Long term relative frequency' means that if you toss a coin 1,000 times you are more likely to get around 500 heads and around 500 tails than you are of getting 5 heads and 5 tails on 10 tosses; and if you toss it 1,000,000 times you will get very much closer to a half of each. So the more times the experiment is carried out the more likely the prediction to be accurate.

The power of prediction lies in collecting, organising, summarising, representing and interpreting frequency data over a long period of time. We are able to predict the weather based on our records of the weather and the longer we have been collecting, organising, summarising and representing weather statistics the more accurate our ability to interpret and make a chance prediction about what the weather will be like tomorrow.

This means that students should be involved in collecting, organising, summarising and representing data from the perspective of chance—it is not something separate. In order to determine whether or not a coin is biased for example, they should make decisions about how to collect, organise, summarise and represent data from an experiment which might involve tossing a coin 500 times.

The Chance and Data strand provides an excellent opportunity for lots of *Working Mathematically* work with students. The whole data handling process can start from a problem such as, "What sorts of lunches do students in this school have?" or "What TV programs do the children in this class like to watch?" Children should be asked to put forward their own questions to be answered and in this way learn something about what is important to each cultural group represented in the class.

From this context students in any grade can be involved in making decisions about what data to collect, whether they will just include their class, ask every student or survey all classes in the school. They can then make decisions about what data to collect, how to organise it, how to represent and summarise it if appropriate, and how to interpret it in order to answer the original question.

9.4 Being numerate in Chance and Data

The *Numerate Students, Numerate Adults* study (Education Department of Tasmania, 1995, pp. pp. 11-28) stated that to be numerate in Chance and Data people need to be able to:

- Recognise and understand the part chance plays in everyday life
- Recognise and interpret estimates of chance events
- Judge the quality and appropriateness of data collection
- Understand and use common methods of summarising and displaying data
- Make and question judgements based upon data presented
- Make predictions based upon data presented

In order to develop skills and understandings concerning data handling processes in Aboriginal children it is important to teach them in the context of Aboriginal knowledge. In other words, use information about the Aboriginal people themselves, sorting and classifying, organising,
representing and summarising information about the Aboriginal people. When these skills have been developed they can be transferred to other more abstract contexts from the wider environment. For developing the understandings of chance teachers need to be aware that many Aboriginal children may have nothing to connect these ideas to, especially if the language of chance is not part of the child's world. This language needs to be taught in order to develop the concepts and should be done through immersion in the classroom, the school and the wider community if possible, using technology and the media to assist.
The writing of these materials has been an interesting and evolutionary project for me. I have taken a journey down the path towards understanding some of the true meaning of 'inclusivity' and what it means for teachers. I say 'some of', because the more I learn about inclusivity in this context, the more I realise what I don't know and the more I realise how difficult it is for teachers to be inclusive in their classrooms.

10.1 The Model

I have focused on each of the following areas:
1. The Aboriginal people; their culture and their transition into schools of the dominant culture;
2. The Aboriginal children; the mathematical understandings they bring into the classroom; and
3. Explicit mathematics teaching required by all children in our schools.

I have attempted to describe the intersection of these three focus areas as it occurs in the classroom — seeing this as a 'layered' response to the teaching and learning of mathematics to include Aboriginal children:

![Model of layered focus areas for teaching mathematics to Aboriginal children](image)

The description of each of the three focus areas has provided the main part of the discussion. The description of the intersection of them has been more difficult. In fact, I believe that it is impossible to do it justice. This is because describing the intersection is about 'what does combining these three focus areas look like in the classroom?' The combining of the three areas is not, I believe, something tangible or visible; it has to occur in teachers' heads.
As teachers focus on explicit mathematics teaching whilst attempting to take into account the
different learning styles and understandings of their students, their knowledge and skills become
intertwined and 'fused' to generate an inclusive learning environment which 'flows' from these
understandings.

This 'fusing' occurs at the subconscious level of the teacher and as a result, I believe a description
cannot be written or produced. For the teacher desiring to produce this type of inclusive learning
and teaching environment, he/she must attend to a deep understanding of each of the three areas.
This may be bad news for teachers wanting to 'do it' in a hurry. In my opinion, and as a result of my
teaching experience and my knowledge of this area of research, there is no 'recipe' for effective
inclusive teaching that by-passes a deep understanding of both the desirable learning outcomes and
the children being taught. Both of these are essential, and it is only through understanding them
separately that the 'fusing' will occur.

10.2 Implications for further research

There are many 'gaps' in what is known about Aboriginal cultures and their mathematics. The
research cited in previous chapters has indicated that a great deal of study is necessary with
Aboriginal culture groups from the southern parts of Australia and with Aboriginal peoples from
urban and rural locations, particularly with respect to their mathematics and numeracy. There is
also the need for more recent research from other parts of Australia where there may have been
research undertaken in the past.

Also, while much research has been done in some mathematical strands such as Measurement and
Space – albeit in remote parts of Australia and now somewhat dated – there is a need to investigate
more thoroughly the Chance and Data strand, particularly the topic of Chance in Aboriginal cultures
from all parts of Australia.

I would urge Federal and State governments to wisely reconsider the allocation of funding to
address problems in improving Aboriginal numeracy. Research undertaken in partnerships with
education systems, professional educational groups and universities is essential in my view, if we
are to make progress and truly include Aboriginal people in our education programs.

10.3 Concluding Remarks

I will again reiterate the fact that the examples used from different Aboriginal cultural groups
within Australia are just that – examples from different cultural groups. I have used every possible
means to stress that what is true for one cultural group may not be true for another, and within that,
what is true for one individual is not true for another. The onus is on the teacher to firstly be aware
that all children bring different understandings into the classroom and secondly, that they should try
to elicit these understandings whenever possible in order to build on them.

The goal of these materials has not been to suggest that 'this is how you do it' – in fact it would be
foolish to suggest that there is one way of teaching mathematics to improve Aboriginal numeracy.
The goal is that teachers become inclusive in their teaching for all children – not just for those from
Aboriginal cultures.

In order to be inclusive we need to acknowledge, appreciate and respect differences while at the
same time provide equality of opportunity for all children regardless of their cultural or ethnic
backgrounds. We need to appreciate that all children should have access to the same learning
outcomes and that wherever possible, this should not be compromised by us, their teachers.
As teachers we have a responsibility to ensure that we understand to the best of our ability:

- the subject matter we are responsible for teaching;
- the desired outcomes for our students of our teaching; and
- the cultural backgrounds, world views and understandings that our students bring with them to the learning environment.

I will also stress again that Numeracy is not the same as Mathematics (see Chapter 1.5). Improving the mathematics of children does not automatically improve their ability to be numerate. Numeracy involves fostering a disposition to use mathematics outside the mathematics classroom; and this is fostered by the pedagogies used by teachers to enhance that disposition.

In the case of teaching Aboriginal students it is imperative that teachers build a rapport based on personal understanding, empathy and respect. This is true for all children but it would seem that Aboriginal children are at a greater risk of not becoming numerate than non-Aboriginal children if this is not done, due to the importance of personal relationships in their cultures.

The tools for numerate behaviour are learned through mathematics, so it is the teacher of mathematics who has the greatest responsibility for developing these skills. *Teacher numeracy* allows teachers to recognise opportunities for encouraging numerate behaviour in other learning areas by their students.

Finally, teachers can only do so much in the classroom. They must be supported by their education system and by their school administration through the provision of appropriate professional development, and through the provision of para-professionals to provide in-class support. Through access to these resources they can improve the understandings of their students, their subject matter, and the way to connect these in order to encourage and enhance the learning and teaching that occurs in the school environment. The school community can also play a major part, and school leaders and principals need to work strategically to ensure that these stakeholders are included in the decision-making process.

I have raised my own awareness of the real meaning of 'inclusivity' through the writing of this book. My hope is that, in reading it, you will also become more aware; enabling you to say not "I treat all of my students the same", but "I treat all of my students differently".
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