Edith Cowan University

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REB: All right Christian. You know how to destroy the wall but that doesn't help us to find out what's going on.

CHR: Well hurry up and find it then.

[After a short time Rebecca finds the appropriate instruction page]

REB: Well it appears that we have to fire the cannon into this gap in the wall.

CHR: Easy I almost got it the first time.

REB: If it was that easy Mr Lowrie wouldn’t have chosen this activity. Let’s try to work it out anyway and remember we have to do it together Christian.

MICH: I won’t be much help I’ll watch you two do it.

CHR: You go first then.

MICH: No way.

CHR: Go on just click the mouse.

[Michelle fires]

CHR: Too low. I’ll raise the cannon up, how do I do that? Oh yeh it’s on 15 I’ll put it up to 20.

Christian fires] Now it is way too high. Try 17, Rebecca.

REB: All right but it won’t work. [Fires] See.

CHR: What’s going on here it must have something to do with the speed too?

REB: It must be a ratio type problem, the faster the speed the higher up it will hit on the wall.

CHR: Well, let’s fiddle around with it and see what happens.

REB: It’s Michelle’s turn.

MICH: No you go Christian I’ll just watch.

CHR: No, you go. But drop the speed down a little. [Lowers the speed and fires]

CHR: It hit lower but it’s still way too high. It will have to go down more.

REB: There must be more to it than that.

Christian already fires]

CHR: Now it’s too low. That’s impossible.

REB: It’s the relationship between the two. We’ll have to experiment with the two sides to balance it up.

MICH: Remember Mr Lowrie said we had to try and get it in the least amount of goes as possible.

REB: That’s right, Christian. Don’t waste the goes.

CHR: Let’s get it this time and work out a formula next time. Whose turn is it?
[33]REB: Mine I think. I’ll raise the speed and lower the cannon.

[Fires. All gasp at the close attempt]

[34]CHR: Yours Michelle we’ve got it now. [Fires]

[35]CHR: Oh, how close. [Readjusts and fires]

[36]TOGETHER: YES!

[37]CHR: Let’s do another one. It’s easy now.

[38]REB: But they’ll get harder. We’re supposed to work out what we are doing.

[39][After several minutes they agree to write down what they have already learnt.5

[40]REB: We have to figure out a formula for this. He [the teacher] likes formulas.

[41]MICH: What type of formula?

[42]REB: Well, one that shows the relationship between the angle of the cannon and the speed at which the ball comes out.

[43]MICH: You mean like the area of a triangle? That type of thing is too hard.

[44]CHR: Let’s just explain what we have done, and how we know that the speed and height need to be both changed in order to solve the problem.

[45]MICH: Anyway, don’t ask me. It was too difficult. I just did what you told me to do.


[47]CHR: Yes, so we can get on with it.

[48]MICH: What about drawing three pictures, one going too high, one too low and the one that hit.

[49]CHR: What we could do is draw the three pictures and put in the numbers that we had for the speed and angle each time.

Analysis of a Cooperative Computer-Based Session

The cooperative learning situation presented above highlighted differences in the way the three children expressed and constructed mathematical understandings. In particular the manner in which they attempted to complete the task differed.

It appears that social interaction can assist problem solving (Azmitia & Perlmutter, 1989), and can be particularly useful with tasks requiring reasoning. Ellis and Gauvain (1992) noted that the processes of interaction can vary, depending on the personal relationship between the partners, as well as age and

5. This was not unusual because they were accustomed to me requiring them to write down what they had learnt from the activity. It was a trade-off between work and fun in their eyes.
expertise, gender, and cultural background. Their "affective" dimension of problem solving has also been described by Schoenfeld (1985). The three case-study students worked together on a regular basis, and generally enjoyed solving tasks together. As already mentioned, they worked productively on most occasions, and had clearly established a friendly working relationship. During the time in which the case study was conducted they worked together at least three times a week.

The data from this computer-based session were collected at the end of the investigation (which meant that the students had been working together for over a substantial period of time). A descriptive analysis of the transcript is presented in Table 3.

Table 3
Analysis of a Cooperative Problem-Solving Task

<table>
<thead>
<tr>
<th>Rebecca</th>
<th>Michelle</th>
<th>Christian</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Attempted to identify the goal (lines 1 and 7).</td>
<td>1. Asked for guidance to identify the goal (line 13).</td>
<td>1. Acts first--goal identification not needed (line 4).</td>
</tr>
<tr>
<td>3. Remains focused (lines 7 and 27) and assists peers to focus on the task (lines 38 and 47).</td>
<td>3. Does not want to take full responsibility for the group’s actions (line 45).</td>
<td>3. Often needs peer help to remain focused (line 12).</td>
</tr>
<tr>
<td>4. Is willing to make a concluding rationale or statement (line 46).</td>
<td>4. Unable to translate actions into a logical cause and effect statement (lines 43 and 45), although is able to suggest a logical compromise (line 48).</td>
<td>4. Is willing to justify the &quot;goal state&quot; with encouragement from peers (line 49).</td>
</tr>
</tbody>
</table>

Each student played a significant role in the eventual completion of the task. Although the three students approached the problem differently, an effective solution was achieved.

Identification of the Problem

Each student identified a problem solving goal, although at different levels of intellectual development in the domain of mathematics. Rebecca was concerned about developing a solution that could justify what they had done, and what they had learnt from the process. She thought it was necessary to find a strategy, or in
her case a formula, that could be applied to all problems of this type and not just this particular problem. From past experience with this type of computer task, the students realised that examples became more difficult as a particular task was solved (this was a Level 1 problem and on most occasions a Level 2 problem would be even more difficult). Rebecca, therefore, wanted a procedure or solution that could be employed to solve more complex problems. She sought an analytic solution to the problem.

Christian, on the other hand, initially attempted to solve the problem through a trial-and-error approach. The other two students wanted to discuss what they had learnt from each subsequent attempt in order to solve the problem in the least number of moves, while Christian wanted to solve the problem in the quickest time possible in order to move to the next level. Christian assumed that through practice they would eventually work out a strategy to solve the problem.

Michelle was not willing to offer an opinion in relation to what understandings she had constructed from the exercise. She did not seem to feel that her contribution was worthwhile and was probably intimidated by the other two students despite having worked with them successfully on many occasions. Her suggestion, that they draw three pictures to represent what they had achieved during the session was, however, an important development for both the group’s understanding of the problem and as a means of reflection. Michelle was obviously not comfortable with the notion of representing the problem in an analytic manner (Rebecca’s idea), and this caused her to become an “outsider” until she realised, possibly through confidence or a growing awareness of what the task entailed, that a visual response could be used to represent the problem scene.

The Process of Problem Solving

On several occasions there were conflicts between the students’ ideas and ways of approaching the problem, and their conflicts served to stimulate cognitive growth (Piaget, 1968, 1977). Although the students considered and shared each other’s ideas, Christian initially, and Rebecca for most of the activity, pushed for their approach and ideas to be used by the group. The three students disagreed with each other in a relatively balanced pattern, allowing each student to monitor each other’s reasoning and, thereby, to accomplish mutually coordinated roles in attempting the task solution (Bearison, Magzamen & Kilardo, 1986).

The conflicts (about how to solve the task) allowed the students to express their opinions and reflect on each other’s reasoning abilities in a challenging and motivating way. In generating their final solution, the students produced a response that reflected both visual and analytic reasoning. Each stakeholder had made a significant contribution to desired goal.

6. An outsider in the sense that Michelle allowed the other two students to dominate the discussion.
Outcomes

The analysis of the cooperative computer based session has drawn attention to the fact that the students approached problem-solving tasks from a differing viewpoints, and that the three students preferred to solve problems in particular ways. Rebecca wanted to provide an analytic solution, Christian employed a means-end approach, while Michelle expressed a desire to draw diagrams to represent the task. Other computer-based activities reinforced the notion that the students valued and worked toward mathematical understandings in different ways.

Analysis of an Individual Computer-Based Session

In addition to cooperative problem-solving tasks the students completed other computer-based activities on an individual basis. It was interesting to investigate whether the students used similar mathematical language and terminology to complete tasks and represent problem scenes when not influenced by one another.

As an example, the students were asked to replicate a printed image of a clown I had designed using a computer paint program (and represented in Figure 6). The program allowed the user to manipulate geometrical shapes, which were located in folders on the screen, using a computer “mouse.” The original picture of the clown had been developed from these geometrical shapes, although some shapes had been enlarged, reduced, transposed, rotated or reflected. The students were expected to transform, translate or rotate the shapes in order to achieve the desired goal (which was to create an identical image of the clown that I had originally created and printed out).

After completing the activity each student was asked to express in writing the manner in which they solved the problem. The three students’ responses are presented below.

Rebecca’s Response

With this problem I firstly counted the sides of the base shape in the original picture. I found that it was an hexagon so looked for an hexagon on the pictures at the side. I found one and enlarged it, then put it in the middle of the page. I did similar things with the other shapes comparing, enlarging, revolving, etc. Of course I made a few mistakes and did not completely finish the problem, but that’s how I did the parts I did.

Christian’s Response

First I got the red hexagon and enlarged (it) twice then I got two yellow triangles and enlarged them once for the eyes. Then I got two orange squares for the nose. Then I got 14 yellow triangles for the mouth. When I finished that I made the eyebrows by making the triangles smaller then turning them around.
Michelle's Response

First discovered that you had to muck around with the flipping and rotating to get the shapes in the right position and angle. The growing and decreasing in size was also hard. You would click twice and it would go way too small. Then you would click three times and it would go way too big. The eyes were easy and so was the nose. The hexagon was hard to get in the right position. The mouth was the hardest part because half the time the triangles wouldn’t do what you expected.

![Image of a face with a hexagon as a head and triangles for eyes, mouth, and nose.]

Figure 1: A representation of the image used for the individual computer-based session.

Analysis of the Individual Problem-Solving Activities

Individual Computer-Based Session

The problem-solving process. It was evident that the students used different approaches to solve the task in the individual computer-based session. Although each of the three students started by manipulating a hexagon to represent the clown’s face, the other shapes were constructed in a range of ways. Rebecca and Christian were more concerned about the way specific bits fitted together to produce the whole than Michelle. Both Rebecca and Christian calculated the number of small triangles that were represented in the clown’s mouth, 14 in all, but Michelle, after reducing the icon triangle three times, moved the triangles into position one-by-one. Michelle attempted to rotate a triangle into position, then reduce another triangle, move it into position, rotated it, selected another triangle, reduce it, and so on. It appeared that she was using a one-to-one correspondence procedure to replicate the image.

By checking that 14 small triangles were required, the other two students reduced all the shapes at once then moved them all across to the approximate
position, and finally rotated each triangle and moved it into position to represent the clown’s mouth. These students, with the 14 triangles in close proximity, were able to create a “draft pattern” of the clown’s mouth before accurate manipulations took place. I would suggest that the problem-solving strategies employed by Rebecca and Christian were effective, while Michelle’s strategies may have reduced her capacity to complete the task successfully.

The mathematics concepts addressed. The three students used “mathematics” terminology in their task evaluation. Each student used words like “hexagon” and “triangle” to explain the way in which they attempted to solve the task. Christian tended to be analytic making comments like “I got the red hexagon and enlarged (it) twice then got two yellow triangles and enlarged them once,” and “then I got 14 yellow triangles for the mouth.” Rebecca did not indicate the number of times she needed to compare, enlarge or revolve shapes, but often did indicate that such transformations were undertaken. Christian completed the task more efficiently and effectively than Rebecca, and was more concerned about the manner in which the shapes were changed in order to replicate the image.

Michelle used mathematics terminology including “flipping” and “rotating” to describe processes she had undertaken towards achieving the goal. Like Christian, she was also concerned with the number of times particular shapes needed to be enlarged or reduced. Michelle, who found the task to be more difficult than the other two students, used a trial-and-error approach to complete the activity. Although Michelle used visual methods to solve most of the problems reported in this investigation, she was unable to organise her response to this task in a holistic manner.

This task, then, required strategic knowledge on how to transform, rotate and manipulate shapes. A student’s capacity to complete this task effectively was directly related to such skills. A student, in this case Michelle, who often employed visual processing to solve mathematics tasks could not rely on such an approach if appropriate strategic knowledge was also required.

Summary of the Case Studies

The case studies provided data that allowed the approaches used by three Year 6 students when attempting to solve a variety of mathematics problems to be investigated. In particular the case studies detailed: (a) changes in the students’ approaches to solving mathematics problems over a period of time; (b) characteristics or methods for completing tasks that remained with each student over the 12-month period; and (c) some factors which influence problem-solving performance.

The following issues which relate to the methods or approaches used by students when attempting to solve a variety of mathematics tasks were identified and discussed:

1. Students tend to change their approaches to solving mathematics problems from visual to nonvisual methods as task complexity is reduced.
2. The approach used by students to solve mathematics problems may provide information about the students’ (a) capacity to complete tasks, and (b) their ability to apply this knowledge to similar problem-solving situations. Successful nonvisual processing of a task may suggest that the student has a “more complete” understanding of the particular problem.

3. Although students tend to change problem-solving approaches in relation to task complexity, beliefs and values about the nature of mathematics also contribute to methods selected by students when completing tasks.

4. Cooperative learning situations, which place visual and nonvisual “thinkers” together, may generate conflicts that promote higher-order reasoning within the group’s decision-making environment.

5. Visual reasoning does not always help students when confronted with spatial tasks or activities.

6. Students’ beliefs, values and personality often affect performance which highlights the impact the affective component of decision-making has on the approaches used in problem solving.

Conclusion

Early work on learning style which described learners in terms of the way in which they characteristically approached problems and general descriptions of the cause of intellectual development are clearly inadequate in capturing the complex interplay of factors which influence the case studies presented in this chapter. The case studies point to the need for a close understanding of the problem environment and the problem solvers, something which many teachers and researchers in the naturalistic tradition have long recognised.

This chapter has attempted to develop a coherent and holistic account of Year 6 problem solving in mathematics and to highlight the dynamics associated with problem solving in general.

References


THE IMPORTANCE OF VISUAL PROCESSING IN PROBLEM-SOLVING


Mental Computation: Research Aimed at Classroom Change

Alistair McIntosh

Since mathematics has been a part of universal public education at the primary school level, written paper and pencil computation has been at its centre—indeed even today many primary teachers see “the tables and the four rules” as the only really essential, fundamental goals of primary school mathematics. Any changes to this situation need to be based on a thorough foundation of research if they are to be both responsible and acceptable to all those affected by such changes—systems, schools and individual teachers.

Increasingly over the past twenty years it has been suggested (Mcintosh, 1979; Plunkett, 1980; Reys, 1984; Trafton, 1986) that written computation plays too large a part in the mathematics curriculum of the primary school, while mental computation is comparatively neglected. The continued emphasis on the teaching of written computation is criticised on the grounds that formal written computation is little used by adults (Wandt & Brown, 1957), is mistrusted and where possible avoided by children (Carraher, Carraher & Schliemann, 1987), gives false ideas about mathematical activity (Hope, 1987), and is anti-constructivist (Kamii, 1989). On the other hand it is argued that the neglect of mental computation is indefensible since it is used for most calculations by adults (Maier, 1980), is the simplest way of doing many calculations (Hope, 1986) and is an excellent way of learning how numbers work (Ewbank, 1977).

However, while a shift of emphasis from written to oral computation may be appropriate, the teaching of formal written algorithms is still seen as the central core of the primary school mathematics programme by most primary school teachers, is expected by parents and is taken for granted as the major task of the primary school by society at large. Here then, if anywhere, research appears to be a necessary and responsible forerunner to any major curriculum reforms.

Some work has been done on the mental computation strategies of primary-aged children, but often by psychologists more interested in using it as a tool for examining the role of memory than as the result of any curricular interest, and almost all research has been confined to mental strategies for the basic facts. Very little exists in the literature regarding the mental computation abilities or strategies of students in the middle to upper primary years.

Over the past twenty-five years, as I spent time in primary school classrooms observing and talking to children I became convinced that the approach to teaching computation needed to be changed because it was failing so many children, and consuming vast quantities of time and energy on the part of teachers and children without achieving commensurate results. Mental computation was being ignored
or dealt with in a sterile and threatening manner. Calculators were ignored or relegated to a trivial position. Written computation involved the transmission of a series of immutable algorithms, taught in many cases with the aid of concrete material and an intention to communicate understanding, but taught in any case with the message that these were the only correct ways to perform computations.

Trying to change such practices would clearly not be a simple matter. I therefore decided to start by placing mental computation at the forefront of attention, on the basis that changes could be made in this area without directly threatening practices in written computation. However, it seemed likely that if mental computation was accorded greater attention and approached in a constructivist manner, this was bound in the longer term to affect computation in general.

It is generally agreed that the ability to compute mentally in flexible ways is both a component and an indicator of number sense. It is tempting to speculate that the two may correlate closely with each other. It is more difficult to define number sense than to recognise its presence or absence in particular incidents. However, McIntosh, Reys and Reys (1992) have attempted to analyse the components of number sense and to arrange them in a hierarchical framework. A full account of collaborative research into the number sense of students in four countries is given in McIntosh, Reys, Reys, Bana and Farrell (1997).

In 1989 Ellita de Nardi, Paul Swan and myself started investigating the mental computation strategies of students in Years 2 to 7, in order to gain greater awareness of the range of these strategies. The purpose of these inquiries was to underpin subsequent curriculum development aimed at providing more relevant and efficient mental computation activities for primary teachers.

In 1991 I was invited by Professors Robert and Barbara Reys of the University of Columbia-Missouri to include Western Australia in a comparative study of the mental computation abilities of Japanese and American students in Years 2 to 8. The Western Australian part of this study was conducted in cooperation with Jack Bana and Brian Farrell of Edith Cowan University.

Since 1993 the scope of the study was widened to consider the number sense of the same cohorts of students in Australia and the United States as were included in the study of mental computation. This study has led to international cooperation between Edith Cowan University and the University of Missouri-Columbia in the development of an analysis of the components of number sense, the development of a classified item bank of number sense test items, and comparative data on many mental computation and number sense items from the United States, Australia, Sweden and Taiwan.

The following pages give a brief account of two research projects conducted by the author with a variety of colleagues over six years into the mental computation of students in Years 2 to 9, together with some consideration of the implications for teaching.
The Mental Arithmetic Project (MAP)

The Mental Arithmetic Project had two major goals: first, to discover, describe and analyse the mental arithmetic strategies of primary-age students. Second, to devise and trial classroom practices which might lead to more awareness of these strategies among teachers, and more proficiency and facility in flexible mental computation by students. Only that part of the project directed toward the first goal is described here. Material designed to achieve the second goal is described in McIntosh, De Nardi and Swan (1994).

Three primary schools were selected to provide a variety of situations in the Perth metropolitan area. They consisted of a large state primary school, a catholic primary school and the junior department of a boy's private school. In each school one class at each year level from Years 2 to 7 was randomly selected (except where only one class at each age existed). The class teacher of each class was asked to select four children for the study: two of "average" mathematical ability, one of "above average but not exceptional" ability and one of "somewhat below average" ability.

Clinical interviews were conducted with each of the 72 students. Each student was interviewed three times over two months, during which a total of 45 mental computation calculations involving whole numbers was given orally. The same 45 items were used for all children in all year levels. After each calculation the student was asked to describe how the calculation had been performed. Notes were taken during the interviews of visual cues such as the use of fingers and eye or head movements. All interviews were audiotaped, transcribed and analysed using a classification system devised and refined by the researchers during the study. Thus a total of 72 children (3 schools x 6 classes x 4 students) were included in the study.

The 45 items were chosen to reflect the range of calculations considered by the researchers to be within the scope of at least some of the students in the survey. Several items were also included which the researchers considered unlikely to have been given as mental calculations to students of this age but which were considered nevertheless to be possible for some students. In any event, this assumption was justified. With three exceptions (Items 22, 43 and 44) whole number items only were selected in order to not automatically exclude younger students for whom the study of fractions or decimals had not yet begun.

Four items (Items 9, 14, 37, 38) were included as an indicator of the students' short-term or working memory (STM). A 3-, 4-, 5-, and 6-digit number was presented orally to the student, whose task was to repeat the numeral backwards. The longest string correctly answered was taken as a measure of the student's STM.

Table 1 shows the full list of items.
Table 1

*Items Used in MAP Mental Computation Study*

<table>
<thead>
<tr>
<th>Item Description</th>
<th>Session 1</th>
<th>Session 2</th>
<th>Session 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 + 7</td>
<td>16</td>
<td>5 x 3</td>
<td>31</td>
</tr>
<tr>
<td>2 8 + 6</td>
<td>17</td>
<td>14 + 7</td>
<td>32</td>
</tr>
<tr>
<td>3 58 + 34</td>
<td>18</td>
<td>8 x 6</td>
<td>33</td>
</tr>
<tr>
<td>4 90 - 70</td>
<td>19</td>
<td>36 + 9</td>
<td>34</td>
</tr>
<tr>
<td>5 25 + 89</td>
<td>20</td>
<td>How many 25c in $3</td>
<td>35</td>
</tr>
<tr>
<td>6 20 + 70</td>
<td>21</td>
<td>15 + 3</td>
<td>36</td>
</tr>
<tr>
<td>7 80 - 24</td>
<td>22</td>
<td>Half of 32</td>
<td>37</td>
</tr>
<tr>
<td>8 (283 backwards)</td>
<td>23</td>
<td>2 x 7</td>
<td>38</td>
</tr>
<tr>
<td>9 14 - 6</td>
<td>24</td>
<td>360 + 9</td>
<td>39</td>
</tr>
<tr>
<td>10 9 - 7</td>
<td>25</td>
<td>4 x 9</td>
<td>40</td>
</tr>
<tr>
<td>11 74 - 30</td>
<td>26</td>
<td>150 + 30</td>
<td>41</td>
</tr>
<tr>
<td>12 80 + 60</td>
<td>27</td>
<td>48 + 6</td>
<td>42</td>
</tr>
<tr>
<td>13 114 - 30</td>
<td>28</td>
<td>How many 35c in $3</td>
<td>43</td>
</tr>
<tr>
<td>14 (49528 backwards)</td>
<td>29</td>
<td>8 x 60</td>
<td>44</td>
</tr>
<tr>
<td>15 140 - 60</td>
<td>30</td>
<td>150 lollies in bags of 30</td>
<td>45</td>
</tr>
</tbody>
</table>

Results and Discussion

**Overall Results**

Table 2 shows the number of children (maximum 12) at each year level correctly computing each item mentally. The items are grouped into clusters of linked items.

While there were only six calculation questions to which none of the youngest age group responded correctly, there were 16 items which one or more of the youngest age group calculated correctly, while one or more of the oldest age group (five years older) did not.
Table 2
Results by Year and Item Cluster

<table>
<thead>
<tr>
<th>No.</th>
<th>Problem</th>
<th>Yr 2</th>
<th>Yr 3</th>
<th>Yr 4</th>
<th>Yr 5</th>
<th>Yr 6</th>
<th>Yr 7</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 + 7</td>
<td>9</td>
<td>11</td>
<td>11</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>67</td>
</tr>
<tr>
<td>3</td>
<td>9 - 7</td>
<td>9</td>
<td>11</td>
<td>11</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>67</td>
</tr>
<tr>
<td>2</td>
<td>20 + 70</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>66</td>
</tr>
<tr>
<td>4</td>
<td>90 - 70</td>
<td>6</td>
<td>11</td>
<td>12</td>
<td>12</td>
<td>11</td>
<td>12</td>
<td>64</td>
</tr>
<tr>
<td>5</td>
<td>8 + 6</td>
<td>9</td>
<td>9</td>
<td>12</td>
<td>10</td>
<td>12</td>
<td>12</td>
<td>64</td>
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<tr>
<td>7</td>
<td>14 - 6</td>
<td>8</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>12</td>
<td>11</td>
<td>64</td>
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<tr>
<td>31</td>
<td>14 - 6</td>
<td>7</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>12</td>
<td>11</td>
<td>63</td>
</tr>
<tr>
<td>6</td>
<td>80 + 60</td>
<td>3</td>
<td>9</td>
<td>11</td>
<td>11</td>
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<td>11</td>
<td>56</td>
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<tr>
<td>15</td>
<td>140 - 60</td>
<td>2</td>
<td>8</td>
<td>9</td>
<td>11</td>
<td>12</td>
<td>12</td>
<td>54</td>
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<td>21</td>
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<td>12</td>
<td>12</td>
<td>64</td>
</tr>
<tr>
<td>20</td>
<td>5 + 3</td>
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<td>7</td>
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<td>12</td>
<td>11</td>
<td>11</td>
<td>53</td>
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<tr>
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<td>half of 30</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>48</td>
</tr>
<tr>
<td>43</td>
<td>a quarter of 120</td>
<td>0</td>
<td>3</td>
<td>7</td>
<td>9</td>
<td>5</td>
<td>10</td>
<td>34</td>
</tr>
<tr>
<td>44</td>
<td>half of 132</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>11</td>
<td>29</td>
</tr>
<tr>
<td>28</td>
<td>$3 + 25c story</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>11</td>
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</tr>
<tr>
<td>29</td>
<td>$3 + 35c story</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>7</td>
<td>17</td>
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<td>0</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>
Strategies Used

The most important finding was that the number of different strategies used overall was quite small, and the task of coming to terms with them, and being able to recognise them when used by children, would be comparatively simple—certainly not a difficult task for any primary teacher. Analysis of some 3000 responses (72 children x 45 items) revealed the following strategy clusters. The full list of strategies with examples of children's responses using these strategies is contained in McIntosh, De Nardi and Swan (1994).

I. Initial Strategy. Children often reverse the numbers in an addition or multiplication, or turn a subtraction into addition, or a division into multiplication.

C1. Elementary Counting. Children very frequently count forward or backward in ones.

C2. Counting in Larger Units. Children use a variety of forms of counting on or back in larger units, including counting on or back (most commonly in twos or tens), skip counting and reciting tables.

P1. Used Place Value Instrumentally. Using Skemp's terminology this means using aspects of place value mechanically: in particular remove and replace zeros without knowing why it works, and very many children also calculate mentally by recreating in their heads the standard written algorithm.

P2. Used Place Value Relationally. This includes any use of place value and notation which shows understanding. It includes the frequent use of “bridging” tens or hundreds (for example in calculating $87 + 25$: $87 + 13 = 100$, $+ 12 = 112$) or otherwise using tens and hundreds, working from the left or right and adding or subtracting parts of one number.

DH. Used Doubling or Halving. Children appear universally to have a special ability to double and halve and this they adapt to help in other calculations: for example $15 + 17$ will be calculated as $30 + 2$.

F. Used Fingers. While this was only a strategy which we noted incidentally, it was obvious that children have devised a wide range of (sometimes very sophisticated) ways of using their fingers to assist in calculations. For example one child, multiplying any two numbers under ten, one number being even, would picture the even number of dots arranged in rows of two on each of the required number of fingers, and then count them. However, he could not do this if both numbers were odd!

K. Known Fact. This refers to situations where children gave an answer instantaneously or otherwise demonstrated prior knowledge of the answer.

RK. Related to a Known Fact. Children commonly related the calculation to another calculation whose answer was known.

Some earlier studies had tended to describe a solution strategy by a single label. It was our experience that even with simple basic fact solution strategies, students often used a variety of strategies sequentially in the one solution. We
decided therefore to use a flexible multiple coding system. As a result solutions were coded as employing between one and four strategies.

As an example, here is Andrew calculating $36 + 9$:

I went 9, 18, 27, 36 and counted up.

The coding for this solution was I, C2, F, since Andrew changed the division into a multiplication (I), solved the multiplication by counting up in nines (C2), and was observed keeping a record of his skip-counting on his fingers (F).

**Findings Related to Age**

Table 3 shows the mean percentage score at each age level on the 41 items excluding the STM items.

Table 3

<table>
<thead>
<tr>
<th>Mean Percentage Score on MCT by Year Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yr 2</td>
</tr>
<tr>
<td>Mean Score (%)</td>
</tr>
</tbody>
</table>

Not surprisingly, the average number of items answered correctly increased with age. However, the variations between children within the same age range (though they had been selected to exclude the extremes of ability) was noticeable.

**Findings Related to Ability**

In order to look more closely at differences between the more and less capable children, the three least competent and the three most competent from the 12 children in each year group (selected on the basis of their answers to the MCT) were chosen for comparison. Comparisons were then sought between the (eighteen) least and the (eighteen) most competent mental calculators.

The average scores of the two groups of students on the short-term memory items are shown in Table 4. If the child failed to reverse the shortest (three-digit) string correctly a score of two was assigned. Thus the score could range from 2 to 6.

Table 4

<table>
<thead>
<tr>
<th>Average STM Score (Most/Least Competent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>Most Competent</td>
</tr>
<tr>
<td>Least Competent</td>
</tr>
<tr>
<td>All Children</td>
</tr>
</tbody>
</table>

There is no indication here that short-term memory is a decisive factor in the superiority of more competent mental calculators.

Table 5 shows the overall number of types of strategies used by the two groups (L = least competent, M = Most competent).
Table 5
Strategies Used by Year and Category (Least/Most Competent)

<table>
<thead>
<tr>
<th></th>
<th>Yr 2</th>
<th>Yr 3</th>
<th>Yr 4</th>
<th>Yr 5</th>
<th>Yr 6</th>
<th>Yr 7</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>5</td>
<td>9</td>
<td>8</td>
<td>21</td>
<td>15</td>
<td>8</td>
<td>129</td>
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<tr>
<td>M</td>
<td>15</td>
<td>35</td>
<td>28</td>
<td>27</td>
<td>37</td>
<td>23</td>
<td>147</td>
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<tr>
<td>Cl</td>
<td>8</td>
<td>9</td>
<td>18</td>
<td>15</td>
<td>8</td>
<td>5</td>
<td>16</td>
</tr>
<tr>
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<td>1</td>
<td>26</td>
<td>9</td>
<td>21</td>
<td>14</td>
<td>3</td>
<td>16</td>
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<tr>
<td>P1</td>
<td>2</td>
<td>2</td>
<td>19</td>
<td>16</td>
<td>22</td>
<td>16</td>
<td>52</td>
</tr>
<tr>
<td>P2</td>
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<td>25</td>
<td>19</td>
<td>34</td>
<td>18</td>
<td>44</td>
<td>104</td>
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<tr>
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<td>5</td>
<td>5</td>
<td>7</td>
<td>2</td>
<td>16</td>
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<tr>
<td>F</td>
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<td>1</td>
<td>52</td>
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<tr>
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<td>3</td>
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<td>47</td>
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<tr>
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<td>7</td>
<td>15</td>
<td>8</td>
<td>26</td>
<td>23</td>
<td>119</td>
</tr>
</tbody>
</table>

Care must be taken in interpreting the data in this table. It might appear that, in the older grades, the less competent calculators used more strategies than the more competent. However, this is largely because the more competent knew or could infer more of the results (K, RK) and therefore did not need to have recourse to other strategies. However, some observations may be made from this data:

1. Throughout Years 2 to 7 the less competent make more use of Cl strategies. That is, they rely more heavily throughout on counting on or back in ones. In contrast, the more competent make much more and much earlier use of C2 strategies (counting on and back in larger units, skip counting and reciting tables).

2. More competent children use particular strategies much earlier than less competent. For example the use of P2 strategies by the competent in Years 2 - 4 (76 occurrences) is as great as the use of these strategies by the less skilled in Years 5 - 7 (79 occurrences).

3. The skilled make rather more use of P1 strategies. Analysis shows that almost all the uses noted here refer to removing the zero (only 29 instances of the use of the written algorithm were recorded—all of them in Years 5 to 7). It may be that in some cases they were using the strategy with understanding; however, the protocols show that in most cases they were operating with the mechanical rule.

4. Except in Year 7, by which time they knew (i.e., gave an immediate correct response to) most of the answers, the more competent children made much greater use of P2 strategies. That is, they were able to manipulate numbers and exploit the place value aspects of them in a dynamic way.

5. There is not a great difference between the two groups in their use of doubling and halving; this appears to be used by both groups and its use is relatively stable throughout Year 6.

6. Except in Year 1 the less competent make more use of their fingers as a counting aid.

7. The more competent do not always appear to have a particularly wide range of strategies—rather they have a small and well understood set of strategies which they use flexibly in such a way as to put as little strain as possible on their short-term memory. In brief, they learn how to make things easy for themselves.
8. In general then the more competent students start using particular strategies earlier, make more active use of place value strategies, and know more facts. The less competent gradually make use of more strategies but rely more on elementary counting and fingers than do the more competent.

As a result of this research a curriculum development project was undertaken in order to help teachers develop ways of approaching mental computation in the classroom which would encourage, through discussion and sharing, an emphasis on the development of flexible and efficient mental computation strategies. This was seen to be particularly important for those children who, because of a lack of confidence or competence, were less able to acquire new and more sophisticated strategies for themselves without outside help and support.

The result was the Mental Arithmetic Project Schools Inservice Package (MAPSIP) which was trialed selectively by over 100 teachers and then intensively by all staff of three schools. The core of the package was a small set of formats for running mental computation sessions in ways which involve all children in the activity. The researchers observed teachers using these formats and documented the ways they adapted them to suit the age and ability of their individual classes, ranging from Year 1 classes to a remedial Year 5 group to Year 7. This package has since been revised and published (Mcintosh, De Nardi & Swan, 1994).

The Mental Computation Test/Western Australia Study

This study was designed to provide three different perspectives on the mental computation of Western Australian students in Years 3, 5, 7 and 9. First, a survey of the kinds of computations which they prefer to do mentally, second, a measure of attitude towards mental and written computation; and third, an assessment of their performance on mental computation items.

Three different survey instruments were developed for the study: a Preference Survey (PS), an Attitude Survey (AS), and a Mental Computation Test (MCT). All the instruments were administered in the order listed above during one 50-minute class period for all year levels 3, 5, 7 and 9, except that the Attitude Survey (AS) was not used in Year 3.

A “typical” metropolitan secondary school participated in the study together with three of its major “feeder” primary schools (Years K-7). Within each primary school, two classes were randomly selected at each of the year levels 3, 5 and 7. In the secondary school, where students were streamed on ability, stratified random sampling was used to select three pairs of classes, with each pair at a different level of ability.

The Preference Survey (PS) consisted of a series of numerical computations. Subjects were not asked to carry out the computations but only to decide if they would do each computation mentally if allowed a choice between calculating mentally, with pencil and paper, or with a calculator. Students indicated their answer by marking “yes” or “no.” Some items in the PS were also included in the MCT so that data reporting preferences could be compared with actual performance on the same item.
The Attitude Survey included 28 statements clustered by five dimensions: Interest and Enjoyment, Perception of Competence, Perception of Value, Perception of Use, and Perception of Source of Instruction. Typical examples to which the subjects were asked to respond were:

I think I will do written computation more than mental computation as an adult (Perception of Use).

I learned to do mental computation at school (Perception of Source of Instruction).

The Mental Computation Test was designed by the researchers for group administration. The Year 3 and Year 5 MCT versions contained 30 items, 15 administered orally and 15 administered visually. The Year 7 and Year 9 versions contained 40 items, 20 administered orally, and 20 administered visually. Table 6 shows the distribution of items by operation and number type for each year.

Table 6
Mental Computation Test Item Distribution

<table>
<thead>
<tr>
<th>Number Type</th>
<th>Operation</th>
<th>Year 3</th>
<th>Year 5</th>
<th>Year 7</th>
<th>Year 9</th>
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<td>4</td>
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<td></td>
<td>Multiplication</td>
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<td>6</td>
<td>6</td>
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<td></td>
<td>Subtraction</td>
<td>2</td>
<td>4</td>
<td>4</td>
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<tr>
<td></td>
<td>Multiplication</td>
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<td>4</td>
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<td>2</td>
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<td></td>
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<td>40</td>
<td>40</td>
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</tbody>
</table>

Results and Discussion

Preference Survey Results

The Preference Survey revealed some interesting clues to students' conceptual understanding or lack of it. For example, more than 40 per cent of Year 5 students would not do the item 100 x 35 mentally and between one third and one quarter of Year 7 and Year 9 students would not calculate mentally 945 x 1000. One third of Year 7 students would prefer not to calculate 1 - 1/3 mentally, and over half would not mentally calculate 0.1 x 45.

The Attitude Survey revealed that, in general, students felt they did more written computation than mental computation at school, but that the reverse
would be true out of school. These views became somewhat more pronounced for the older students.

In the Mental Computation Test, only in Year 3 did a t-test show a significant difference ($p < 0.05$) between the mean scores on items presented visually and orally (5.70 visual and 6.75 oral). However, parallel research with Japanese students shows that the mode of presentation can significantly affect performance (McIntosh, Reys and Reys, 1995). In addition it is suggested that the mode of presentation does affect the range of strategies used by students. When students can see the calculation they are more liable to use mental forms of written computation, whereas only hearing the calculation appears to encourage more flexible solution strategies.

In the Preference Survey students were presented with a number of computations and asked whether or not they would prefer to do them mentally. At each year level, a sub-section of these items was also included in the MCT. If we assume that those students who stated that they preferred to do the calculation mentally thought they would calculate the answer correctly, then for each of these items (nine items for each of Year 3 and Year 9 and ten for each of Year 5 and Year 7) it is possible to classify all students into four categories:

1. Those who thought they could do the calculation, and could.
2. Those who thought they could do the calculation, but couldn't.
3. Those who thought they couldn't do the calculation, but could
4. Those who thought they couldn't do the calculation, and couldn't.

Analysis of the results suggests that boys are somewhat more likely than girls to be justifiably sure of their ability to calculate mentally (category 1), while girls are somewhat more likely to be wrongly diffident of their ability (category 3). Moreover, while category 1 students were naturally more likely to be in the upper quintiles of ability, and category 2 and 4 students more likely to be in the lower quintiles, students in category 3 were spread much more evenly across the ability range. This appears an interesting area which merits further investigation.

Implications for Teaching

It is clear that at present very few children acquire the range of mental computation strategies they possess as the result of deliberate classroom interventions or practices. And yet, sooner or later, most children acquire many or even most of them. For the vast majority of primary school children the full range of strategies is potentially available—not one strategy was the province of the more competent only.

Much exploratory work is needed, however, in deciding how best to spread these strategies around. At present insufficient work is being directed toward this end. However, a great deal more support has been given recently, partly as a result of the National Statement, the Profiles, and a number of State and local initiatives, to mental computation and in particular to class discussion of the strategies used by children. Nevertheless, it is probably true to say that the majority of primary
teachers are not aware of the range of efficient mental strategies which children need to acquire, let alone how best to help children acquire them. We need a great deal of action research in this area, carefully planned and documented.

One thing does stand out from the interviews with children with regard to the teaching of mental strategies. Teaching rules (for example “removing zeros”) is as dangerous and self-defeating for mental computation as it is for written computation. This was the only strategy which children stated (and they stated it frequently) had been taught them by parents and/or teachers; and it was the only strategy which they misused through lack of understanding. There is a powerful lesson to be learned here. Helping children to acquire ownership of efficient strategies is not likely to be achieved by “teaching” these strategies at the expense of understanding. We must not make the same mistakes over the teaching of mental computation as we have made with regard to the teaching of formal written algorithms.

Even when children devise their own strategies we must expect times when these strategies are only partly formed. Children need to feel free to experiment and to verbalise their strategies. Sometimes the act of verbalising illuminates an error for the child. At other times it provides us with an insight into the process so that we can intervene effectively.

For most mental calculations there is not “one” strategy, nor is there a “best” strategy. It would be difficult to find a calculation for which all children who produced the correct answer used the same strategy. Moreover to suggest that for this or that calculation there is a “most efficient” way seems to us, even if it may be true in some isolated examples (for example 99 + 28), to be giving exactly the wrong message about how to become proficient at general mental calculation. The efficient mental calculators we interviewed appeared to have a range of strategies which they used and adapted flexibly. They did not necessarily perform the same calculation the second time around in the same way as on the first occasion: but they did show a command of relationships between numbers which they orchestrated efficiently in order to perform a calculation with the least mental strain. Thus they did not need a “better” short-term memory—they put fewer demands on it.

A crucial feature in opening up more efficient and more simple mental strategies is an operational understanding of place value and ownership of its essential features. As an obvious example, children who add ten by counting on ten in ones have set themselves a more difficult task so are therefore more prone to error than children who are aware that adding one to the tens digit achieves the same end.

It would appear that competent mental calculators become so in spite of what happens in the classroom; or, more likely, they possess or acquire an early affinity with numbers which allows them to “play” with them and they abstract from classroom practices of any kind skills and understandings which they adapt and use in mental computation and any other mathematical situations. The children who should concern us most are the less competent and less confident ones who do not receive any deliberate help from the classroom practices and do not abstract these understandings and skills for themselves. As a result they are left stranded with primitive and less efficient strategies. It is important to expose them to more
sophisticated strategies partly because these are often, in fact, simpler. How best to expose them to these strategies is not yet clear.

Future Plans

I am now developing my work aimed at helping teachers to assess the need for change and to make needed changes on four fronts. First I am continuing to run a number of school-based professional development sessions based on the case for changing mental computation practices in schools and on suggested activities arising from our research. Second, Paul Swan and I have completed work on a CD-ROM on Mental Computation, which will give pre-service and in-service teachers access to the arguments for change and show examples of children using and explaining a range of mental computation strategies. Third, I have completed work with colleagues on a series of videos which will show children performing mathematical tasks, including mental computations, at a variety of levels. The series is intended primarily to provide pre-service students with opportunities to observe closely individual children and to make professional judgments about their levels of performance and their needs. Finally, I have been working on the question of how children will move from mental computation to “informal written computation” as numbers involved become too large or too awkward.

It is hoped that all these initiatives will help to make the tasks of teachers and students alike somewhat simpler, clearer and more enjoyable.

References


Researching Learning about Teaching: Using NUD-IST to Facilitate Data Analysis

Judy Mousley and Peter Sullivan

Because of the way electronic technologies have been used in the past to support empirical research, and particularly to manipulate numerical summaries of data, it is generally assumed that computer software will not be useful for handling qualitative data. This chapter reports on the use of a computer based data analysis tool in an interpretive research project that sought to identify features of quality mathematics teaching.

Underlying Theories

Research paradigms\(^1\) represent general philosophical outlooks. They indicate that researchers have inescapable epistemological, political and social interests which shape ways of thinking about, working on, evaluating, and using the products of their inquiry (Habermas, 1973). Research paradigms are forged by researchers’ beliefs about the nature and scope of knowledge and how it can be generated, what can be known, who can be a knower, and what are the justifications against which we measure “truth” and “validity.”

If it is believed that teaching and learning phenomena exist “out there” as an objective reality, researchers will aim to capture observable entities and explain these, then to suggest ways in which such new understandings might best be used. It is likely, in this case, that researchers will use tightly controlled environments and selective collections of data as the basis for knowledge construction. If researchers aim, however, not at finding a generalisable, mind-independent reality but at the description and interpretation of contextual, subjective, multiple and socially constructed realities, then the value-laden nature of any research process can be examined more openly. In this case, researchers will take a different role in the research process (in methodological, practical, and political terms), conceive different research questions for different purposes, and use different data gathering and reporting methods. These two approaches to research display an “opposition ... built into the choice of primitives, of basic concepts for the whole argumentation” (von Wright, 1971, p. 32).

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1. A research paradigm is a set of assumptions concerning reality and possibilities for coming to know about that reality.
Computer software developed over the past thirty years has been ideally suited to manipulating data collected and analysed according to the methods of the first paradigm above—the empirical-analytical paradigm. Statistical software is structured to receive findings that have been converted to numerical data and to manipulate these using standard arithmetic processes. Technology is remarkably efficient for producing statistical reports based on such data. However, such programs are based on the assumption that human behaviour can be reduced to statistical data and managed in this way.

More recently, researchers have experimented with the use of computers to manipulate the varied data from research projects in the interpretive paradigm. Typically, qualitative data are not easy to reduce to numbers—in fact, to do this would be antithetical to the nature of the paradigm. Interpretive research is likely to produce a wealth of text that expresses ideas and observations in the form of interview transcripts, field notes, responses to open-ended questionnaire items, statements in documents, and even videotaped and audiotaped incidents. These data are complex and varied enough to be difficult for researchers to compare or categorise and to decide what to report and what to omit—but it is the complexity and variety that makes the data rich and useful. There is always the fear that reduction will fail to capture personal use of terms and ideas accurately, and not make available to readers essential details or examples.

Using computer programs to help handle qualitative data has allowed raw data to be saved and called up for use in research reports or further studies. This overcomes, to some extent, the danger of loss of depth of meaning through the coalescence of discrete ideas when data is reduced. Initially, software application programs designed for other purposes were used creatively to handle qualitative data. For instance, word-processors were employed to assist with storing, editing, grouping and coding phrases from interviews; and later search functions and text-base managers were used to find coded segments as well as combinations of particular words. Spreadsheet software was also useful for managing qualitative data because people could code, order, and collate text items—then print out, compare and analyse the different categories of data so formed. The primary advantages of these processes over, say, sorting and re-sorting data on index cards are an ability to manage larger data bases without tedious handling, and that raw and categorised data can all be maintained in separate files to be recalled (and perhaps reworked) whenever necessary.

This chapter reports on a further progression—software that has been designed especially for the handling of non-numerical unstructured data. It is about using a particular computer program entitled NUD·IST—the acronym for non-numerical unstructured data indexing, searching and theorising (Richards & Richards, 1990). NUD·IST was written by Lyn and Tom Richards, of La Trobe University, Australia, and is now used worldwide as a powerful tool to assist in the organisation and analysis of on-line and off-line qualitative data. While NUD·IST facilitates all of the relevant operations supported by word-search, word-processing and spreadsheet functions, its data management capabilities also enable the development of higher-order classifications and hierarchies which assist researchers to meet their ultimate aim—theorising.
In reporting how we used NUD·IST, we will focus on how this software supported the interpretive aspects of our work and led to examination of the subjectivity of our roles during different stages of the research process. First, however, let us digress and investigate the broad nature of the research paradigm within which we were working—and some of its features that will be the focus of later discussion.

Interpretive Research

The terms interpretive (or interpretative), phenomenological, existentialist, hermeneutic and naturalistic research are all used to describe forms of research that focus on seeking understandings of natural phenomena. These different terms imply different emphases rather than different methodologies\(^2\). The features of this paradigm are described variously:

- natural setting; human instrument; utilization of tacit knowledge; qualitative methodology; purposive sampling; inductive data analysis; grounded theory; emergent design; negotiated outcome; case study reporting mode; idiographic interpretation; tentative application; focus-determined boundaries; special criteria for trustworthiness. (Lincoln & Guba, 1985, pp.34-43)

Generally, interpretive researchers aim to describe and understand human beliefs and activities holistically, in their natural (not contrived or constrained) contexts. Husén (1988) provides a useful summary:

A second strand (after positivism) was represented by the phenomenological philosophy developed by Edmund Husserl (1859-1938) in Germany. It emphasised the importance of taking a widespread perspective and of trying to "get to the roots of" human activity. The phenomenological, and later the hermeneutic, approach is holistic, it tries by means of empathy (Einfühlung) to understand the motives behind human reactions. By widening the perspectives and trying to understand human beings as individuals in their entirety and in their proper context it also tries to avoid the fragmentation caused by the positivist and the experimental approach that takes out a small slice which it subjects to closer scrutiny. (p. 18)

Burgess (1985) noted that no single attribute is present in all qualitative studies. The general characteristics he outlined, however, form a helpful framework:

1) The focus is on the observed present, but the findings are contextualised within a social, cultural, and historical framework.  
2) The research is conducted within a theoretical framework. While there may be only a small number of questions to orientate a study, further questions may arise during the investigation.  
3) The research involves close, detailed, intensive work. The researcher participates in the social situation under study.  
4) The major research instrument is the researcher who attempts to obtain a participant's account of the social setting.

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2. Tesch (1990) distinguished over forty broad types of methodologies appropriate for this paradigm, such as case study, ethnography, historical inquiry, and evaluation.
5) Unstructured or informal interviews in the form of extended conversations may complement the observational account.
6) Personal documents may give depth and background to the contemporary account.
7) … different methodologies may be integrated by the researcher.
8) The decisions regarding the collection and analyses of data take place in the field and are products of the inquiry.
9) The researcher attempts to disturb the process of social life as little as possible.

(grounds theory)

Data collected in interpretive research are generally qualitative—its value lies in the range of qualities it displays. Thus an aim is to retain the richness and holism of meanings that arise from lived experience. It is important that original data be retained and that its context be recorded, so the reductionist methods of empirical research are not used to handle it. Nevertheless, some methods must be employed to bring raw data under control.

A common method of data analysis with interpretive approaches (and the method we used with NUD-IST) is “constant comparison” within and between categories. Glaser and Strauss (1967) outline this method of data processing. First, incidents are compared to see if they exhibit any common properties, and categories begin to emerge. Further incidents are compared with these primitive categories, which are refined, articulated and integrated when necessary in light of the new data. Throughout this process, data are inspected to discover whether any theory can be developed from the patterns that they display. Theory is constructed to explore the existence of, and likely impact of, the emerging categories. Finally, the theory is refined considering the data framework and its contents. Thus any resulting theory is assumed to be “grounded” in the data, rather than in prior assumptions or generalisations. Glaser and Strauss (1967) propose that such theory should:

(First) closely fit the substantive area in which it would be used. Second, it must be readily understandable by laymen concerned with the area. Third, it must be sufficiently general to be applicable to a multitude of diverse daily situations within the substantive area, not just a specific type of situation. Fourth, it must allow the user partial control over the structure and process of daily situations as they change through time. (p. 237)

During this process, the tasks of the researcher are to conceptualise the essence of specific naturalistic interactional processes and to provide a new way of understanding the social situation under study. Although researchers attempt to take a position of “not knowing what is known” (Lincoln & Guba, 1985, p. 235), it is worth noting that they do not do so at the exclusion of previous research theories. As Wilson (1983) points out:
Seeking theory grounded in the reality of participants does not mean a disregard for previous work. The researcher must become thoroughly acquainted with related research and theory so that he (sic) can use it whenever it is helpful for explaining events. Similarly, he contributes to the development of knowledge by pointing out corroboration and contradiction of his findings with the findings of other researchers. Moreover, he uses previous research and theory to select the setting he is studying and to inform the initial focus of his information gathering. (p.16)

With interpretive research, developing theories shape further research processes so an emergent design, rather than a pre-ordinate approach, is exhibited. This allows scope for negotiated outcomes where the working hypotheses are verified and confirmed by the people involved in the research. Such an approach means that local (possibly changing) conditions as well as re-shaped values can be taken into account. Because of the emergent design, it is not possible to specify beforehand the specific focus of every research element—it is acknowledged that the emergence of successive features can be used to complement those identified earlier.

**A Focus on Social Phenomena**

Phenomena of interest to any scientific community are likely to be events whose causes can be questioned in that they need to be "squared with theory" by relating the unfamiliar to the familiar as well as the anomalous to the accepted (Toulmin, 1961). Toulmin claims that historical frameworks of understanding determine our patterns of expectation, and notes that it is those phenomena on the edge of our framework of everyday comprehension that are generally of interest—"the task of science is to extend, improve on, and refine the patterns of expectation we display every day" (p. 59).

Interpretation involves a focus on both personal activity and intention, so the social phenomena of interest to interpretive research are not external to either the individuals being studied or to the researchers themselves:

In investigating why people do what they do there is a crucial difference in describing the observable behaviour and understanding or interpreting action (which includes the intentions of the behaviour). A characteristic theme in an interpretive approach is that humans act intentionally ... if human behaviours are to be effectively understood they must be recognised as intentional and thus what people believe themselves to be doing must be a part of researching what is happening. The term interpretive is used when what a researcher is trying to do is to answer questions by producing an interpretation of how those people whose actions are being studied themselves interpret these actions. (Wiseman, 1990, p. 104)

The two levels of interpretation noted by Wiseman here are important, because it is not only the subject's renditions that are involved, but also the subjective readings of these by a researcher. While methods such as observation, interview and survey give rise to data that make the phenomena under investigation available for analysis and theory-building by the researcher, any results from analysis of such data cannot be considered "truth." They are products of the frames of reference of the researched and the researcher. It is assumed that there is no theory-free perspective on natural realities that science can discover—only "the
process of science itself, which attempts to understand natural entities from the perspective of this or that evolving theoretical framework" (Wachterhauser, 1986, p. 8).

In the interpretive paradigm, the role of subjective interpretation is overt, as it is recognised that research questions and processes cannot be totally objective and that the researcher cannot be removed from the object of research. Thus while the control techniques of the scientific paradigm aim to minimise researcher effects by establishing a climate of relative social neutrality and objectivity, interpretive inquiry recognises that the researcher is describing and discussing phenomena from a subjective position—and possibly from a participant position. It is acknowledged that the relationship between theory and evidence is not cut and dried, that "facts" can be selected and arranged in many different ways, and that researchers use their own judgement to construct the most satisfying account of the data (Connole, 1990).

A Focus on Context

Interpretation of particular phenomena relies on understanding of their contextual settings. Contexts serve to create and reify patterns of intentions, attitudes and beliefs as well as traditional ways of acting. They also serve to define what is and what is not comprehensible to researchers. In the interpretive paradigm knowledge and notions of validity, cogency and fruitfulness are considered to be relative to individual contexts. Findings are not expected to be replicable across social setting, time, place or culture. Thus while beliefs and interactions of people in specific contexts can be captured, described and interpreted, the aim is not to measure these, to generalise findings to other populations, or to use the findings to predict future behaviours.

It is recognised that the context under investigation cannot be divorced from participant's constructions of it. In fact, Berger and Luckman (1966) go so far as to claim that the world under investigation originates in people's thoughts and actions and is maintained as real by these. In making this claim, they are focusing on the inner worlds of individuals—the "realities" that people have constructed:

When we refer to people's consciousness we are concerned with what takes place—in terms of thinking and acting—within each of us. These subjective states refer to our 'inner' world of experiences, rather than the world out there. To concentrate on subjectivity, we focus on the meanings that people give to their environment, not the environment itself ... we cannot know this independently of people's interpretations of it ... Our central interest, as researchers, is now focused upon people's understandings and interpretations of their social environments. (May, 1993, p. 8)

Interpretive research also recognises that we cannot divorce ourselves from our own reason and historical contexts in order to determine that particular knowledge-claims are true or false, or even that any particular interpretation is valid. Its development has been heavily influenced by the German philosophical tradition of hermeneutics.
A key feature of hermeneutic analysis is that authentic understanding is not detached from interpreters but constitutive of their praxis\(^3\) (Gadamer, 1976). Traditionally, there has been a bifurcation between researcher (as “subject” researching other) and researched (as “object”). But hermeneutic philosophers stress that understanding can only be an act of interpretation through the life-world of the interpreter. Text, for instance, written or spoken by one partner in the hermeneutic conversation, is expressed only through its interpreter.

The hermeneutic claim is that texts must be explored in relation to their historicity as well as that of the inquirers\(^4\). It is essential to come to understand the shared ethical principles and norms of any community to understand particular concrete situations (Gadamer, 1976). Von Wright (1971) points out that intentional behaviour is meaningful gesture that can only be understood as part of a wider “life community”:

... the understanding of action presupposes a community of institutions and practices and technological equipment into which one has been introduced by learning and training ... We cannot understand or teleologically explain behaviour which is completely alien to us. Behaviour gets its intentional character from being seen ... in a wider perspective, from being set in a context of aims and cognitions. (pp. 114-115)

This brings us to the hermeneutic notion of part–whole relationships. There are two key relationships to consider here. First, phenomena being studied by researchers are not singular experiences—they need to understood in the light of the wider community context. Heidegger (1962) developed the notion of a hermeneutic “circle” regarding the social context of phenomena:

The hermeneutic circle involves the “contextualist” claim that the “parts” of some larger reality can be understood only in terms of the “whole” of that reality, and the “whole” of that reality can be understood only in terms of its parts. That is to say that understanding any phenomenon means, first of all, situating it in a larger context in which it has its function and, in turn, it also means letting our grasp of this particular phenomenon influence our grasp of the whole context ... Thus, for example, a new argument or position against a traditional position will always be understood, at least initially, over against that traditional position, but it may also succeed in transforming our understanding of that position ... In short, the hermeneutical circle reminds us that although the past is no straitjacket or cage, there are no truly “new” beginnings. (Wachterhauser, 1986, pp. 23-24)

The research act itself is also part of that whole. A growth of understanding occurs when we find new ways of speaking and understanding that are implicit in patterns of thought and speech related to the phenomenon being studied in a particular social context. As something is examined and better understood, we think and speak about it differently and enter (and are accepted into) the

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3. People unfamiliar with this term could think of it as the impact that theory has on practice and that practice has on theory. This is a simplistic explanation of a much more complex notion of “being,” but sufficient for now.
4. History, here, is a property of community, rather than of individuals—for individuals’ horizons of understanding are formed by their experiences within social communities.
community more deeply. Researchers' knowledge and participation thus bring about small changes in the whole. This becomes a process of active, circular development into which investigators are drawn:

... making something explicit often implies a change in the understanding of it. The whole gets clearer by its being told ... Interpretation, understanding and pre-understanding are dialectically coupled with our existence as human beings. People's self-definitions and definitions of life are the result of their understanding and their understanding develops through their experiences. (Ödman, 1988, pp. 64-65)

Using NUD·IST to Manage Qualitative Data

In a recent (and continuing) research project\(^5\), we used the software program NUD·IST to support the management and organisation of qualitative data resulting from a survey question in which we sought to determine people's perceptions of the components of quality teaching of mathematics.

The survey arose from earlier studies into experiences which teacher education students had during practicum (see Mousley & Clements, 1990; Mousley, Sullivan, & Clements, 1991). We recognised that student teachers' observation and analysis skills were not well developed, but we were concerned that they were not being exposed to (or perhaps were just not appreciating) positive and varied models of mathematics education. We also knew that one-off lessons presented in the hurly-burly of classrooms do not provide opportunities for detailed observation, analysis and discussion of teaching and learning.

We decided to develop a resource that could be used to draw the attention of teacher education students to important aspects of mathematics teachers' behaviour. At this stage, we planned to make a set of videos. The first task was to seek some broader informed views on what constituted important aspects of teaching—and this was the purpose of the survey whose results we handled with NUD·IST.

The survey was first trialed by twelve teacher educators who were asked to complete the questionnaire with an observer present. They were asked to think aloud while responding to the items to determine the appropriateness of wording and layout. All comments and queries were recorded. After revision, the process was repeated with a further four teacher educators completing the survey and being interviewed.

The revised survey was mailed to three groups. There were 40 survey responses from graduate students in mathematics education (100 percent return), 56 from Victorian teacher educators (80 percent return), and 29 from teacher educators from the United States (40 percent return). These groups had been selected because they represented an informed view of current issues in teaching and learning. The graduate students were all experienced teachers who had elected

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5. The project "Using interactive multimedia to enhance awareness of key characteristics of effective teaching" is funded by the Australian Research Council, the Australian Catholic University, and Deakin University.
to complete higher degrees studies in mathematics education. The Victorian teacher educators were members of a group which meets quarterly to discuss issues related to mathematics teacher education, and their survey forms were supplemented by a few responses from experienced mathematics education consultants. The U.S. teacher educators were drawn from the membership list of the International Group for Psychology in Mathematics (PME-NA) group.

Managing Open-Ended Responses

The survey had some fixed-format items (see Sullivan & Mousley, 1994). However, the data managed with NUD·IST were gathered from one open-response item, so it is only on this item that we will focus on in this report.

The open-ended item was presented as a question on the first page of the form, as follows:

Throughout this survey we want you to imagine a mathematics lesson, at any year level, where the students are learning, for example, to estimate the mass of various objects, or to add fractions, or to record given information as a graph. Before turning the page, please write down the most important characteristics which a quality mathematics lesson on any of these concepts/skills would usually have.

Initially, all 125 responses to this item were typed, using word-processing software. Each person's response was coded the respondent's number and the information we had about that person—sex, years of teaching experience, level of teaching, and our avenue of contact with each. The initial files were then saved as "raw data" in text format—and are still available if needed.

This is one advantage of NUD·IST: every stage of the data handling process can be saved, and notes can be made periodically within the program to record thinking and processes used. A log of changes is also kept automatically, so a history of the processes is available for perusal and retracing if desired. This provides the facility for researchers to check initial interpretations or to replicate particular stages of the process. With other methods of handling textual data, looking anew at material generally involves repetitious or lengthy re-analysis and synthesis, and it is difficult to re-work data in the light of new research questions or developing understandings.

The next stage was to divide the sentences that had been typed into sensible units of text. NUD·IST can handle any unit from a single word to a chapter. It can also treat video frames or snippets (or whole films), pictures, diagrams, and audio segments as units of text. Decisions need to be made about the size of unit which will suit the purpose of the study. For instance, one sentence we faced was “Good mathematics classrooms have teachers who communicate clearly, develop relational understanding and create a positive learning environment”. This was deemed to consist of three useful phrases. We needed to decided whether the words should be reproduced to capture what we thought was the respondent's meaning:

Teachers who communicate clearly.
Teachers who develop relational understanding.
Teachers who create a positive learning environment.
Alternatively, we could have used the entire original sentence and put it into three separate categories. This would have had the advantage of preserving the context of any of the phrases, but the disadvantage of making lists of data less focused.

We decided to break sentences up into discrete phrases but keep the original wording, recognising that this would be sufficient management of the data for our purposes. This decision would not have been appropriate for all projects—each needs to be designed in response to the research questions and the purposes that researchers have for asking these. Of course, such decisions are required whether computers are being used for data handling or not.

NUDIST uses a family tree metaphor for the creation of categories and sub-categories. Initially, when we faced lists of phrases to group in some logical way we decided to start with two “children”—phrases that seemed to fall within the responsibility of the teacher and those which seemed to belong to pupils. For example:

Teacher ------ The teacher creates a warm and supportive atmosphere.

Pupils ------ Children are engaged in productive activities.

Immediately we were aware that such categories are quite arbitrary. We considered, for instance, starting with phrases that pertained to mathematics classes in particular and those which were more general. At that point, we shaped the future of the project, for we decided our eventual aim was to inform the actions of teachers so it was important to think of classroom interaction in terms of the teacher’s influence. (Already a video was shaping up in our minds—camera focused on the teacher, interview with the teacher before and after the lesson, etc.)

Notes on using NUD*IST

It may be helpful to describe one way that the NUD*IST program can operate. As with all software, there are as many ways to use the tools as there are users. Note also that this is a very brief description—for fuller details the reader should check the program manual (Richards and Richards 1990).

The basic organising structure of NUD*IST is an indexing “tree”. Data elements are attached to “nodes” and sub nodes, each of which is allocated its own discrete number.

Suppose that we asked ten people to write down their age, gender, favourite relaxation, and favourite food. The first step is to type all of the data, using a word processor, and following the format indicated in the manual, save it as a “text only” file. The units which are marked by returns are important. We prefer to have the text units being the basic ideas. In other words, if the ideas are whole sentences, then end each sentence with a return. The data are then saved and used as individual files.

The next step is to create a new project from within the NUD*IST program, and “introduce” each of the files. With a large number of files, it is not difficult to write command files which expedite repetitive tasks like this.
Next, it is necessary to create an index system. We could place the background data (age, gender) in one node with appropriate sub-nodes, and make relaxation and food separate nodes. Within the relaxation node we might have categories (e.g., passive, active) as sub-nodes of this, and so on.

Each entry at each node can be identified by file. Collectively these data sources are called documents. It is also possible to include as part of the analysis other information (e.g. drawings, photographs, videos) which is not entered into the computer (i.e. not “on line”).

It is possible to code as much as whole documents or as little as a single line to particular nodes. In most cases, it is helpful to code whole documents according to the background data (e.g., age, gender), but particular data entries (e.g., reading a book) to a specific node. In this way, when the data entries at a particular node are examined, it is possible to see all of the entries from all respondents which have been coded at that node as well as both the document source and the background codes for each entry. It is possible to recode any data elements and then re-examine the nodes. We can then explore, for example, whether for particular categories of relaxation activity, age or gender seemed to be influential. Perhaps the most important reason for examining the data in this way is to see whether the entries attached to a particular node are coherent and whether it assists us to use this particular category in subsequent description or analysis of the data. This then becomes the essential tool in the theory building components of this qualitative analysis tool.

We also discussed the fact that even this simple categorisation of the data was not “clean”. The two examples above can be used to illustrate this. Students play major roles in allowing and helping teachers to create classroom atmosphere, and teachers really have the primary responsibility for keeping children engaged in productive activities. Eventually, as the categories emerged, this point became clearer and the teacher–children division dissolved as we identified nodes based around general types of classroom actions in which all participants are involved, such as “communicating” and “problem solving.”

Key words and common phrases were useful in deciding how to classify phrases. For instance, “sharing ideas,” “time for sharing perceptions,” “sharing alternative solutions” and “listening to alternative solution strategies” seemed to be linked by common words as well as conceptually, so were all classified as sharing strategies. At times it was necessary to add further categories, or to split one as it became apparent that two distinctive groups of data were developing from one category.

This stage took a long time, not in terms of physical shifting of data (it takes only a few seconds to “send” a phrase to a specific node or to create new nodes) but because of the discussions we had along the way. There were some lively debates about possible interpretations of responses, and possible meanings of terms such as “non-threatening”. We became very aware of our roles in this secondary interpretive stage. The exercise taught us as much about our own (as well as each
others') concepts of quality lessons, and about the act of writing a questionnaire item and interpreting the results as it did about the beliefs of people who had responded to the survey.

Throughout this process, we grew more conscious of the illusiveness of meaning in pedagogical terms that we use frequently, as well as the fact that we did not know how our survey respondents would have grouped the terms that they had used. Take the term "problem solving" for instance. What does it mean? What is a problem—and whose responsibility is it to create problems. What are the reasons for it to be included in a mathematics lesson? Is problem posing different from problem solving, and does the former fit more readily in the teacher's area and the latter in the children's? (Perhaps this is so in traditional practice, but we would prefer to challenge this notion than to reify it. This, and similar, discussions raised the political nature of educational research.) Does problem posing belong in a subsidiary set of problem solving or form a set in its own right? If we were merely recording how many people had mentioned these terms, we would not have been faced by such questions. But we were not interested in the quantitative aspects of the responses, but how we would interpret and use (in the videos) the qualities of the different responses.

When considering questions such as whether a phrase such as "problem posing and problem solving" should be placed in one or two categories, we decided that we were going to make a break with the traditions of quantitative data handling. We determined that we would include a phrase in any category where it seemed to fit—and in two or three categories if necessary. Hence the total of phrases eventually printed out in groups would be larger than the number of original phrases. This, of course, gives extra weighting to particular phrases, but we were not planning to carry out any statistical comparisons so this was not an important consideration.

Once categories were half formed, our choices seemed to be constrained somewhat by existing categories. Faced with the phrase "Asking thought-provoking questions", for instance, we were more inclined to ask whether it would fit with Problem Posing or with Questioning—two existing categories—than we were to think of possible new classifications. At that stage, we did not even talk about whether we should have a set of phrases to do with provoking thinking, although some later phrases led us to create one and review the data to see what phrases we had missed. Such a review is facilitated by the "Search" capacity of NUD·IST. Searching for individual words (think*, thought, reflect*, imagin*) will bring up phrases that contain words such as imagination, reflection, and thought-provoking. It is also possible to search for strings of words and words that occur within a certain distance of each other, such as "children" and "working" within five words.

Eventually, we had created very rough categories that seemed to be inclusive of most of the data, although we were aware that we would need to refine the groups further, as some of the phrases had been "squeezed" into categories they only half suited. When we were only half way through the data but satisfied that the classifications were inclusive of most of what had been written, we split the rest of the raw data in half to complete at home. When we got together again to review what had been put into each category, further discussions (some arguments) and
Further refinement of the sub-categories followed. NUD-IST facilitates movement of data with ease. It is possible to move whole branches of the tree (using standard cut and paste functions), split categories or join them, call up the contents of any section to examine them more closely, and make notes about any concerns as well as about why particular decisions have been made.

Asking NUD-IST for a "report" on the final product resulted in lists of sorted data that we printed out and examined. As an example of what was produced, two sub-nodes had the following amongst their data:

<table>
<thead>
<tr>
<th>THINKING (2 12 19)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aims to develop understanding opportunity for student thinking</td>
</tr>
<tr>
<td>Lesson requires thinking rather than repetition and mimicking</td>
</tr>
<tr>
<td>Promotes thinking</td>
</tr>
<tr>
<td>Lots of thinking by students about task</td>
</tr>
<tr>
<td>Speculating on how best to solve them and solving</td>
</tr>
<tr>
<td>Thought about the problem</td>
</tr>
<tr>
<td>Asking thought-provoking questions</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHALLENGING (2 12 9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Challenging</td>
</tr>
<tr>
<td>Challenging task within reach</td>
</tr>
<tr>
<td>Challenging activities</td>
</tr>
<tr>
<td>Challenge</td>
</tr>
<tr>
<td>Should have ability for extension</td>
</tr>
<tr>
<td>Challenging but caters for individual differences</td>
</tr>
</tbody>
</table>

We took half of the categories each and worked through the phrases, checking their placement and recommending changes where necessary. After swapping categories and repeating this process independently, we shifted phrases where there was agreement between our independent suggestions. We discussed incidents where one person only had suggested a change, or where the recommended destinations differed. Six colleagues were then asked to examine the phrases in each category to determine whether they formed a coherent set, and this led to a few further changes. They were also to create a name for that set, and from that point on we used the category title that was suggested by most people.

The sharing of perspectives between two researchers, and later within a wider group, was a significant control over researcher bias. However, it would be foolish to think that bias had not been present from the initial stages of the project and influential at every stage—a point we will take up again below.
We did not use the full potential of NUD·IST for data handling. For instance, it supports Boolean comparisons of data sets (A and B, A or B, A but not B, etc.). Instructions for finding data of particular interest can be strung together to create precise, powerful search requests. We did not ask for comparisons between groups of respondents (e.g. what male respondents said compared with female respondents) and did not record particular numbers of occurrences of particular strings of text or particular sets of ideas. However, it is important to note here how we followed two key principles of research. First, we used the software as a tool for our own purposes at the time. Second, the raw data (plus categories of data formed at each stage of the project) remain available for further interrogation as new interests or needs arise.

It would also be possible to add more data in the future. For instance, we did consider asking the same question in a range of countries to see if the videos we were planning would be useful for teacher education in other areas, and to find out if some interesting comparisons could be made between perceptions of quality mathematics teaching in different countries.

A Hierarchy of Categories

Six key components of quality mathematics teaching emerged. They, and general descriptions we constructed as summaries of the phrases included in each category, were as follows:

Building Understanding. Building Understanding seemed to refer to a role which the teacher assumes in order to convey some pre-determined meaning to students. It is a recognition of particular understandings to be developed, and of strategies to achieve this end by building on existing knowledge, using materials to explain and clarify concepts, choosing appropriate sequences, helping students to make connections, forming relationships, and knowing the meaning of terms. There is a strong inference of teacher decision, teacher direction, teacher explanations and teacher control. The data attributed to Building Understanding was allocated to sub-nodes of Materials, Prior Knowledge, Mathematical Thinking, Reflection, Sequencing, Review, and Conceptual Understanding.

Communicating. Communication is widely acknowledged as an essential component of learning mathematics. Yet the term is often used without an adequate definition or explanation that would allow an appreciation of what is meant. In grouping the responses, this node was used to include statements related to opportunities for talking, explaining, describing, listening, asking, clarifying, sharing, writing, reporting, and recording. The data attributed to Communicating was allocated to Discussion Between Pupils, Sharing Strategies, Co-Operative Situations, and Recording.

Engaging. This node is about the students' involvement in their own learning. The data attributed to Engaging was allocated to sub-nodes of Active Involvement, Enjoyment, Real World Examples, Motivation, and Variety.

Problem Solving. The term problem solving has been used to mean many things in many situations. It is not surprising that some variation of the term problem solving was used frequently by respondents. The data attributed to Problem Solving...
was allocated to sub-nodes of Investigation, Problem Solving, Open-Ended Questions, Challenging Situations, and Problem Posing.

**Nurturing.** A recurring thread in the listed features of a quality lesson were characteristics which suggested that the teacher is sensitive to the needs of the students as individuals. Nurturing refers to that concern and caring for students as individuals that we associate with intra-family relationships. The data attributed to Nurturing was allocated to sub-nodes of Catering For Abilities, Non-Threatening, Rapport, Relationships, Goal Setting, and Enthusiasm.

**Organising for Learning.** This component refers to actions designed to keep students working towards achieving the lesson’s goals. We took it to mean decisions made by the teacher about a specific focus for what would happen in the lesson and a commitment to pursuing that focus and to communicating the focus to the students. The data attributed to Organising for Learning was allocated to Clear Purpose, Clear Instructions, Organisation, Clear Instruction, Organisation, Questions, and Assessment.

While these six components presented a summary of the responses, they were different in both the focus and the locus of responsibility. For example, the component “building understanding” was outstanding in terms of the number of times respondents referred to its features. Upon reflection, it seemed that each of the other nodes could be considered a vehicle for building mathematical understanding. For instance, the phrases listed under Organising for learning seemed directed at teacher actions which in turn would lead to building understanding. As we mulled over the connections between the categories, we mapped out a conceptual structure which fitted the data and the way we had organised it (see Figure 1).

![Figure 1](image)

**Figure 1:** Initial data tree: Components of quality teaching.

In this model, building understanding became the focus of each of the other nodes. Our theorising which informed this structure was that if we seek to describe features of quality teaching, we must be clear on the real goals. For example, the use of materials can become a goal in itself. The use of materials in a lesson, per se, is not necessarily grounds for a positive learning environment. Materials should be linked with symbols or as aids to the development of generalisations or abstract
mathematical ideas embodied in the materials. Likewise, engagement, by itself, may not be enough, nor is communication. Communication in mathematics classrooms is for building understanding, (and indeed for enhancing engagement and problem-solving which in turn lead to building understanding). By itself, it is not evidence of effective learning.

The model above, however, suggested that building understanding stands apart from the other factors such as teacher concerns and communication, so eventually we developed the following model which seemed to represent a more interactive processes (see Figure 2).

![Figure 2: Final model: Six components of quality teaching.](image-url)

This model, the descriptions reproduced above and the lists of phrases put into each classification were used in planning lessons to be videotaped. Over the previous few weeks we had moved from a stage of having a large amount of textual data which was of little use to us to a situation where we had a set of powerful descriptors and a key lesson focus which could be used to inform future stages of the research and development project.

**Using the Research Findings**

Four lessons, aimed at building mathematical understanding and based on the six categories of good teaching practice which had been identified, were videotaped. During the data analysis process we had realised that it would be difficult to convey much of what we had discovered (and especially the contested
and conflicting nature of some of the data we had discussed) in videotapes. Videos could not portray the problematic nature of some of the pedagogical issues we had discussed. They could not include the informative readings we had come across. They could not express the varied interpretations of quality mathematics teaching that we had collected.

Thus we decided to make the tapes the core element of a much broader resource—an interactive multimedia program. After writing and collecting appropriate readings to support this basic element, we worked with a variety of media to draw out specific elements of the teaching and learning process portrayed so that students could undertake a close, informed analysis of the lessons captured. We also wrote questions which appear at appropriate times to prompt thought and discussion about specific features of the lessons.

The product we have now developed consists of a CD-ROM disc and a book entitled Learning About Teaching. The first disc completed includes video and audio components of a full mathematics lesson, interviews with the teacher before and after the lesson, documents and readings related to particular aspects of the lesson, graphic representations of the classroom interaction, transcripts and other lesson documentation, some associated readings and bibliographic data bases, sets of thought-provoking questions, and other resource materials.

We have been researching the way that undergraduate and postgraduate Education students use the resource—and indeed the ways it has been used by a number of lecturers in education studies and mathematics education as well as in preparing students for practicum experience in schools. It is being used to handle a very large data base which includes transcripts of pre-and post-use interviews, videos of students working with the resource, a screen-by-screen log of its use, observation notes, audiotaped sessions, students' journals, pre- and post-use naturalistic descriptions of lessons, students' assessment items and other relevant information. With this more varied and extensive data, the capabilities of NUD·IST (and ourselves) are really being put to the test.

Conclusion

Our experience in this project has enabled us to understand further the claim that the whole gets clearer by being told. The categories have given us a framework for studying and evaluating teaching (including our own tertiary teaching) as well as guidance about aspects that should be included in resources for learning about teaching. As they formed, we saw new data through this formation—but we also saw potential audiences for the principle findings and the problematic issues raised along the way.

It is clear that categorisation of the data collected with the survey instrument manipulated it into a structure where it could form the basis of a) a theory about features of quality mathematics teaching, and b) a useful product which not only resulted from research but could be used in further research.

It is also evident, though, that at every stage of the process our own theories about learning and teaching shaped both the way we perceived these data and the
way we used them. In this respect, the results and major research product are a
mirror of our own theories—products of our social group (the professional
community) and its past and present histories. Any interpretations made were
conceptions informed by our own experiences and beliefs in that community, for it
is not possible for researchers to step outside of their own personal worlds and see
phenomena from any other perspective.

As we analysed the data in our project, we could not hold conversations with
the respondents, or even to observe their body language and hear inflections. Thus
while we were attempting to capture participant’s interpretations of quality
teaching, there was always a secondary level of interpretation involved. We put
our own meanings into their text.

Sometimes it became clear that the two of us had interpreted a phrase quite
differently—and yet our backgrounds (age, teacher training and experience,
interests, fields of reading, etc.) have been very similar. Resulting dialogues with
ourselves, each other and our colleagues made us see the data differently. They also
made us see each other differently. If we had involved the original respondents in
this process, we are sure that more distinctive and challenging ideas would have
been raised and our perceptions of their ideas as well as our knowledge of the
participants as experts in the field would have been broadened further.

The results of our project were also very much a product of a limited
community. The survey population was basically Western, middle class
professionals vitally interested in a relatively narrow professional field—a
population which shares a relatively common history in mathematics as well as in
education. This raises two questions. The first is the issue of whether people from
other cultural traditions, social groups or historical times would have produced
similar raw data. The second question we must ask is whether people from
distinctly different backgrounds would have read and managed the data in the
same way.

The research results were also quite clearly a product of the times. For example,
co-operative group work, constructivist principles for mathematics pedagogy and
basing mathematics classes around open-ended questions which are relevant to the
students’ lives are modern trends. These features are emphasised in national
curriculum documents and teacher education courses of the three countries of the
participants. They are also featured of many current journal articles and books,
conference papers and staff-room discussions. When answering the survey
questions, participants would have been drawing on their recent experiences in
this professional community. It is likely, too, that they would have considered
(even unconsciously) their perceptions of the answers that would be expected or
sought by ourselves.

Problems related to the subjective nature of decision-making and interpretation
of human beliefs and actions are not in any way specific to the use of NUD·IST.
Computer software can only be a tool for handling data: it cannot interpret that
data. Meaning is framed as individuals view the real world in terms of their
personal understandings and that these interpretations are made in the light of
their understandings of the theories, ideas and concepts developed in their
geographical and temporal fields. Results are framed by epistemological, political
and social interests of researchers—they are products of the historicity of all
participants in the research activity.
References


About the Authors

Jennie Bickmore-Brand

Jennie Bickmore-Brand is currently President of the Australian Council for Adult Literacy and the Director of the Western Australian Adult Literacy Research Network. Although trained as a primary teacher and subsequently teaching in teacher-education programmes in South Australian and Western Australian universities for seventeen years, she has in recent years been concerned for the language, literacy and numeracy needs of adults. Jennie’s key interest is in breaking down disciplinary barriers and her doctoral study took an interdisciplinary approach to language and mathematics. Her thesis, “Teachers of mathematics teach mathematics differently: A case study of two teachers,” investigated two different methodologies for teaching mathematical content at the upper primary level of education. Jennie continues to work in mathematics education and provide her services to communities and schools across the state.

Tony Herrington

Tony Herrington is coordinator of mathematics education at Edith Cowan University and lectures to students in undergraduate and postgraduate mathematics education courses. His research interests have focused on two areas: metacognition in students’ mathematics learning and the professional development of teachers. In 1992 he completed his PhD titled “Student beliefs and strategies for learning mathematics” and has been involved in projects and graduate supervision in the areas of action research and multimedia approaches to professional development. During 1994 and 1995 he led a research project that focused on implementing action research studies to investigate professional development of primary mathematics teachers. In 1997 he led a team of researchers in the development and study of CD-ROM interactive multimedia that focused on professional development of mathematics teachers in the areas of teaching and assessment strategies.

Tom Lowrie

Tom Lowrie started his teaching career in primary schools in Sydney and Wollongong over a ten year period. For the past four years he has been lecturing at Charles Sturt University in Wagga Wagga. His PhD focused on the way students use visual imagery to solve mathematics problems. His current research interests include the way children reflect upon and monitor their own problem-solving processes. In addition, he is co-authoring books in mathematics education and problem solving. Tom is a nationally accredited professional tennis coach who has a strong desire to reduce his golf handicap.
Agnes Macmillan

Agnes Macmillan has taught in a variety of positions in independent and primary schools in New South Wales, particularly with children in the first years of school. The challenges of having the major responsibility for introducing formal knowledge domains to children led her to question and reflect on her own teaching practices and renew her qualifications. These studies permitted opportunities to pursue many questions relating to the social dynamics of classroom life. At the same time, an abiding interest in language and early literacy directed and informed recent research projects in mathematics education. She is currently a lecturer in early childhood mathematics and research at the Albury Campus of Charles Sturt University.

Alistair McIntosh

Alistair McIntosh was for seventeen years Principal Mathematics Adviser to the Leicestershire Education Authority in England, and was a member of the Government Committee of Inquiry into Mathematics in Schools in England and Wales (the Cockcroft Committee). For the past nineteen years he has been at Edith Cowan University in Western Australia, where he is currently Senior Lecturer in Education and Director of AIRPME (Australian Institute for Research in Primary Mathematics Education). He has been involved in research and professional development at local, national and international levels particularly in the fields of mental computation and number sense. He has recently led teams of researchers comparing performance on mental computation and number sense in Australia, the United States, Japan, Sweden and Taiwan, and is completing a set of six videos on performance assessment of primary children.

Ramakrishnan Menon

Ramakrishnan Menon, born in Malaysia, currently teaches mathematics education and computers in education at the Ohio State University at Lima, Ohio, USA. Prior to that, he was at the National Institute of Education, Nanyang Technological University, Singapore, from 1993 to 1997. He began teaching in Malaysia in 1968, and has taught mathematics, science and English language. He has been involved with pre-service and in-service mathematics education since 1974, mainly in Malaysia but also in Australia, Canada and Singapore. Believing that research should inform practice, he collaborates with teachers in school-based research, involving teachers, mathematics and language education. He is married with two children and his hobbies including reading, badminton, and lawn and table tennis.
Judy Mousley

Judy Mousley taught in primary and secondary classrooms for fifteen years before undertaking Honours research on Reading Mathematics. Her Masters research and current PhD work are on teachers’ perceptions of mathematics and hence of their roles in mathematics education. Judy has taught a range of education studies, mathematics education, philosophy of education, and research methodology units since 1983. She has written and edited a number of books, and participates in a range of professional associations and research projects.

Joanne Mulligan

Joanne Mulligan is a senior lecturer in Education and co-ordinator of the Primary Teacher Education Programme at Macquarie University in Sydney. She supervises postgraduate research students and contributes to the Master of Education (Mathematics Education). She is an experienced primary teacher, professional development leader, author and consultant in mathematics education. Her research interests focus on children’s development of multiplication and division strategies, the representation of mathematical ideas and the role of imagery in number learning. She investigates children’s mathematical development through classroom studies and clinical interviews, and has contributed to the development of early number assessment in the New South Wales “Count Me In Too Project” since 1997. Joanne is an active member of the Mathematics Education Research Group of Australasia and is well known for her work in professional development and community projects.

Mal Shield

Mal was a secondary school mathematics teacher and department head in Queensland schools for 16 years before becoming a lecturer in mathematics education at the Queensland University of Technology in 1986. In the past ten years his research interests have involved various aspects of language use in mathematics learning with a particular emphasis on written mathematics. He has been co-author of a series of three secondary school mathematics textbooks. Mal’s PhD thesis involved an investigation of the use of student expository writing in mathematics learning. In this and subsequent research a scheme for examining written mathematical presentations has been developed. His current research is aimed at the development of more effective textbook presentations of mathematical material.

Elena Stoyanova

After a secondary teaching career in Bulgaria, where she worked as a mathematics teacher and as a superintendent and expert of mathematics in the Bulgarian Ministry of Education, Elena took up the challenge to undertake a doctoral study in Western Australia. Her PhD focused on extending and exploring students’ mathematical performance via problem solving. Elena’s professional interests also include working with gifted and talented students. She has attended several International Mathematical Olympiads and was a member of the Australian Mathematical Olympiad Committee (1995-1996).
Peter Sull

Peter Sull attended university and is currently Head of the School of Education (Victoria) at the Australian Catholic University. Peter has been a consultant to a range of Australian and international institutions and government departments, including projects in Malaysia, Papua New Guinea and the United States, and is an author of a primary mathematics text series and professional development programme for all schools in Papua New Guinea. Peter’s major research interests include a project on the use of interactive media to enhancing teachers’ awareness of key characteristics of effective teaching, which he is conducting with Judy Mousley of Deakin University. He also researches and publishes on the use of open-ended questions in mathematics, and on mathematics and technology.

Margaret Taplin

Margaret Taplin spent eight years as a primary school teacher before becoming a lecturer in mathematics education at the University of Tasmania in 1987. She has also worked in the Centre for Mathematics Education at the Open University (UK) and the Department of Curriculum Studies at the University of Hong Kong. She is currently a research fellow in the Centre for Research in Distance and Adult Learning at the Open University of Hong Kong. Her research interests have included gender issues in mathematics education, problem-solving with school-aged children and pre-service teachers, computer-assisted instruction, and problem-based learning.

Noel Thomas

Noel Thomas is a lecturer in mathematics education at Charles Sturt University at Bathurst. A former mathematics teacher, he completed a Master of Science degree with research in Oceanology at the then Wollongong University College (University of NSW) and has just completed a doctoral programme in mathematics education at Macquarie University. He has taught university programmes in early childhood and primary education, developed professional development programmes and conducted and published research into children’s learning of numeration, use of imagery, and problem solving strategies.
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Mathematics, Science and Technology Education Centre
Edith Cowan University, Perth, Western Australia