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# **On the Evolution of Probability-Weighting Function and Its Impact on Gambling**

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## **Abstract**

It is well known that individuals treat losses and gains differently and there exists non-linearity in probability. The asymmetry between gains and losses is highlighted by the reflection effect. The non-linearity in probability is described by the curvature of the probability-weighting function. This paper studies the evolution of the probability-weighting function. It is assumed that the probability weighting for an individual follows a mean-reverting stochastic process. The Monte Carlo simulation technique is employed to study the evolution of the weighting function. The evolution of the probability-weighting function implies that an individual does not treat gains or losses consistently over time, this may be due to the change of the individual's psychological status.

*Journal of Economic Literature* Classifications: D81, L83

**Key Words:** Cumulative prospect theory, stochastic process, probability-weighting function, Monte Carlo simulation

## **I. Introduction**

The emergence of non-expected utility models since the late 70s has challenged the dominance of the von Neumann-Morgenstern expected utility theory, which is found not to be able to provide an adequate description of individuals' choices under risk and uncertainty observed in experiments, surveys, and causal observations. Among the various new paradigms, the cumulative prospect theory (Tversky & Kahneman, 1992), evolved from the earlier prospect theory (Kahneman & Tversky, 1979), has attracted the most attention. Violations of assumptions of expected utility theory are adequately accounted for in the cumulative prospect theory. They are, namely: Framing effects, non-linear preference, source dependence, risk seeking, and lose aversion.<sup>1</sup> To provide for these phenomena of choice, cumulative prospect theory deviates from the expected utility models by (1) replacing the probabilities by decision weights, which may be sub-additive, (2) replacing the utility functions by value functions, (3) treating gains and losses differently, (4) determining the value of each outcome by gains or losses instead of final assets, and finally (5) multiplying the value of each outcome by a decision weight and not by an additive probability.

A significant implication of the cumulative prospect theory is the fourfold pattern of risk attitudes, which is also envisioned by other contemporary theories (e.g., Fishburn, 1979) and observed by several experiments (see, e.g., Cohen et al., 1987; Tversky & Fox, 1995; Wu & Gonzalez, 1996). For non-mixed prospects, the shapes of the value and the weighting functions imply (a) risk aversion for gains of high probability, (b) risk seeking for losses of high probabilities, (c) risk seeking for gains of low probabilities, and (d) risk aversion for losses of low probabilities. However, the characteristic curvature of the value and weighting

functions does not imply perfect reflection in the sense that the preference between any two positive prospects is reversed when gains are replaced by losses.

This paper focuses on the part of probability weighting by individuals. Edwards (1962) suggests that there is a tendency for people to over-weight low-probability events and under-weight high-probability events. We are particularly interested in the relationship between learning and weighting. We posed the question: Would people correct their weighting functions towards the linear weighting function if they were given the opportunity to learn?

The rest of this paper is organised as follows. Next section provides a brief description of the finding of the cumulative prospect theory on probability weighting. Then we develop the stochastic model used in this paper to study learning and correction of probability-weighting function. Section IV describes the results of the Monte Carlo simulation. Section V is the conclusion.

### **Probability Weighting in the Cumulative Prospect Theory**

For each mixed prospect  $(x, p; 0, 1 - p)$ , let  $\frac{c}{x}$  be the ratio of the certainty equivalent of the prospect to the non-zero outcome  $x$ . If individuals are risk neutral,  $\frac{c}{x} = p$ . If individuals are risk averse,  $\frac{c}{x} < p$  for  $x > 0$  and  $\frac{c}{x} > p$  for  $x < 0$ . Based on their experimental data, Tversky and Kahneman (1992) come up with two curves, one showing the relation between  $\frac{c}{x}$  and  $p$  for gains and the other one for losses. They suggest fitting the two curves by the following functions:

$$w^+(p) = \frac{p^g}{[p^g + (1-p)^g]^{1/g}}, \quad w^-(p) = \frac{p^d}{[p^d + (1-p)^d]^{1/d}} \quad (1)$$

where  $w^+(p)$  is the probability-weighting function (PWF) for gains,  $w^-(p)$  is the PWF for losses,  $g$  is a parameter of gains, and  $d$  is a parameter of losses. Using their experimental data, they estimate that the median value for  $g$  to be 0.61 and  $d$  to be 0.65. Their results are summarised in the following figure.

[Insert Figure 1 here.]

Figure 1 shows that the PWFs for gains and losses, individuals overweight low probabilities and underweight moderate and high probabilities with probabilities in the middle of range relatively unchanged. This tendency would manifest an S-shaped probability transformation. Edwards (1962) suggests that there is a tendency for individuals to perceive an inflation of probabilities of low-probability events (e.g., winning the jackpot of a lotto game) and deflate the probabilities of high-probability events (e.g. being caught speeding by a Multinova speed camera). Psychological research attributes this distortion of probabilities to cognitive factors such as (a) illusion of control (Langer, 1975), (b) retrievability of instances (Tversky & Kahneman, 1974), and (c) near misses (Reid, 1986). Individuals who are more internally controlled are more susceptible to the illusion of control. They usually believe that they have the tools to beat the system. As a result, the perceived probability of success of winning a lotto jackpot shortens substantially. Probability distortion also results from the retrievability of instances. The constant parade of multi-million dollar lottery winners in the mass media convinces people that the odds of winning a lotto jackpot are not really as long as the objective probability would indicate. As a result, the perceived probability of winning the lotto jackpot is revised upwards. Near misses also contribute to probability distortion. They

are always taken as the encouraging signs and boost the confidence of the gamblers about their chances of winning; consequently causing them to revise the perception of probability upwards. Near misses also reaffirm perceptually that the gambler's tools are working, if not perfect, and that some fine-tuning is all that is required in order to win.

Figure 1 also shows that the PWF for gains and for losses are quite close. Experiments consistently shows that the PWF for gains is slightly more curved than the PWF for losses because risk aversion is more pronounced for gains than risk seeking for losses.

### **III. The Stochastic Model**

The cumulative prospect theory portrays static PWFs for gains and losses. In this paper, we are interested in exploring the dynamic nature of the PWFs. An individual is likely to have different PWFs over time, and the adjustment in one's PWFs would depend on a state of mind influenced by some psychological factors like those mentioned in Section II. For example, a gambler purchases lotto tickets every week and the perception of the odds of winning the jackpot is influenced by beliefs about the effectiveness of the gambler's 'system' and awareness of the numbers of winners. The gambler would revise subjective odds upward if encouraged by the near misses or media coverage of winners or both. The gambler would revise subjective odds downward if discouraged by a long losing streak or frequent rollover of jackpot or both. This random nature of events can be described by a stochastic factor. To model the evolutionary nature of the PWFs, we use a stochastic process similar to the one used in depicting asset prices in finance literature (Hull, 2000). Since the PWF for gains and the PWF for losses are similar, we shall only use the PWF for gains to illustrate our model.

More specifically, we assume that the probability weighting adjustment procedure follows a mean-reverting process<sup>2</sup>:

$$dw_t^+ = \mathbf{a} \left( \bar{w} - w_{t-1}^+ \right) dt + \mathbf{s} dz_t \quad (2)$$

where  $\mathbf{a} > 0$  is the reverting or adjustment rate,  $\bar{w}$  is the mean toward the which the probability weighting process is reverting,  $t$  is time,  $\mathbf{s}$  is the diffusion coefficient (which is the conditional standard deviation of the weighting function measuring the volatility during the  $dt$ ), and  $dz_t$  is the standard Wiener process<sup>3</sup> representing the unpredictable events that occur during  $dt$ . And the term  $\mathbf{s} dz_t$  is used to capture the impact of random events such as near misses and retrievability of instances on the mean-reverting process. Note that the parameters  $\mathbf{a}$  and  $\mathbf{s}$  can be pre-determined for each individual by experiment. Furthermore, this mean-reverting process has a long-term trend or mean, but the deviations around this trend are not entirely random. The process  $w_t^+$  can take an excursion away from the long-term trend (e.g., near misses boost the individual's confidence about a 'system' resulting in severe probability weighting or distortion). The process eventually reverts to that trend, but the excursion may take considerable time. The average length of the excursions is controlled by the reverting or adjustment rate (characterising the ability to learn or come to the senses by the individual), which is a parameter in the equation. As this parameter becomes smaller, the excursions away from the long-term trend take longer.

Again, we would stress that dynamic nature of the mean-reverting model differs significantly from the static PWFs in Tversky & Kahneman (1992), as described by equation (1), by capturing the dynamic nature of the PWFs. Therefore, the mean-reverting model is suitable for examining an individual's behaviour in repeated bets. Apparently, the use of the mean-reverting process to describe an individual's probability weighting adjusting process

hinges on the ability to learn from a mistake.<sup>4</sup> The assumption of  $\bar{w}(p) = p$  is plausible as long as the individual is aware of the ‘error’ after each time period and adjusts the PWF for gains accordingly. One may visualise that an individual starts with a PWF for gains given by equation (1). But as time elapse, the individual would notice a discrepancy between the PWF for gains,  $w_t^+$ , and the linear PWF,  $\bar{w}(p) = p$ . Consequently, the individual would revise the probability weights from bet to bet towards the state of linear weighting with the PWF for gains approaching the linear PWF. In the context of gambling, one can take the objective probability of winning (as described by the linear PWF) as the mean, and probability weighting by the individual to make the PWF ( $w_t^+$ ) deviate from the linear PWF because of the illusion of control, retrievability of instances and near misses. The parameter  $\mathbf{a}$  determines how fast the individual adjusts to the ‘error’ in PWF for gains. Some individuals are capable of adjusting faster (represented by larger  $\mathbf{a}$ ’s that are closer to unity) and are therefore able to achieve LPDF quicker than the others (those with smaller  $\mathbf{a}$ ’s that are closer to zero). In the next section, we shall examine the stochastic process using the Monte Carlo simulation for a set of parameters.

#### IV. The Monte Carlo Simulation

In this section, we examine the mean-reverting model by employing the Monte Carlo simulation technique. First, we need to discretize the stochastic process. It is customarily practice to discretize a stochastic process by Euler approximation. The discretized mean-reverting model is given in equation (3):

$$w_t^+ = w_{t-1}^+ + \mathbf{a}(\bar{w} - w_{t-1}^+)d_t + \mathbf{s}\sqrt{d_t}e_t \quad (3)$$

where  $\mathbf{a}$  is now interpreted as the reverting or adjustment rate per small time step,  $e_t$  is i.i.d.  $N(0,1)$ , and  $\mathbf{d}_t$  is the discretization interval or time step. Note that the discretized mean-reverting model assumes that the adjustment has two components: a mean-reverting term and a random term.

In our simulation, the initial weighting function is taken as  $w^+$  in Figure 1 and we use the following set of values for the parameters:

$$\mathbf{a} = 0.5, \mathbf{d}_t = \frac{1}{52}, \mathbf{s} = 0.4 \quad (4)$$

That is to say, we assume that the adjusting rate is 50 per cent of the ‘error’, the time step is taken as one week (because the lotto game is drawn once a week), and the conditional standard variation of the weighting function is 40 per cent per annum.

A snapshot of the simulation is depicted in Figure 2.

[Insert Figure 2 here.]

Note that the simulated PWF for gains is nothing but one of the many other possibilities. To see exactly how the PWF evolves, we need to run the simulation many times (500 times in this exercise) and take the mean of weighting functions across all simulations. This is illustrated in Figure 3. By comparing the PWFs for gains for the 30<sup>th</sup> period (week) and the 60<sup>th</sup> period (week), one can see that the PWFs for gains are indeed reverting to the linear PWF over time.

[Insert Figure 3 here.]

To see the impact of the reverting or adjustment rate, we further run the following simulations with the following set of parameters:

$$\mathbf{a} = 1, \mathbf{d}_t = \frac{1}{52}, \mathbf{s} = 0.4 \quad (5)$$

The simulated results corresponding to the set of parameters in equation (5) are depicted in Figure 4. Compared to Figure 3, Figure 4 shows a faster speed of reverting toward the linear PWF. This is exactly due to the higher reverting rate or adjustment speed assumed in equation (5) ( $\mathbf{a} = 1$  versus  $\mathbf{a} = 0.5$ ).

[Insert Figure 4 about here.]

Our simulation results as shown in Figures 3 and 4 show that the stochastic model as described by equation (2) adequately captures the evolution of the PWF for gains. We anticipate similar performance would be achieved with respect to PWF for losses.

## V. Conclusion

In this paper, we are interested in the evolution of an individual's probability-weighting function. We analyse the reasons that the probability-weighting function should be dynamic rather than static as traditionally assumed in cumulative prospect theory. Furthermore, we set up a stochastic model to emulate the evolution of the probability-weighting function. Our simulation results show that an individual's probability-weighting function converges to the linear probability-weighting function over time; therefore, confirming the capability of the stochastic model in capturing the evolutionary nature of probability-weighting functions.

## Notes

1. Variations in the framing of options (e.g., in terms of gains and losses) yield systematically different preferences. Nonlinear preferences refer to the fact that the utility of a risky prospect is nonlinear in outcome probabilities. Source dependence refers to an individual's willingness to bet depending on an uncertain event but also depending on its source. Risk seeking behaviour are consistently observed in situations where a small probability of winning a large prize and in situations where people must choose between a sure loss and a large probability of a larger loss. Loss aversion refers to the phenomenon of asymmetry between gains and losses; losses loom larger than gains.
2. The mean reverting model is often used to model interest rate dynamics.
3. The Wiener process is a stochastic process that models Brownian motion.
4. The adjustment process described here is similar to adaptive expectation in economic literature.

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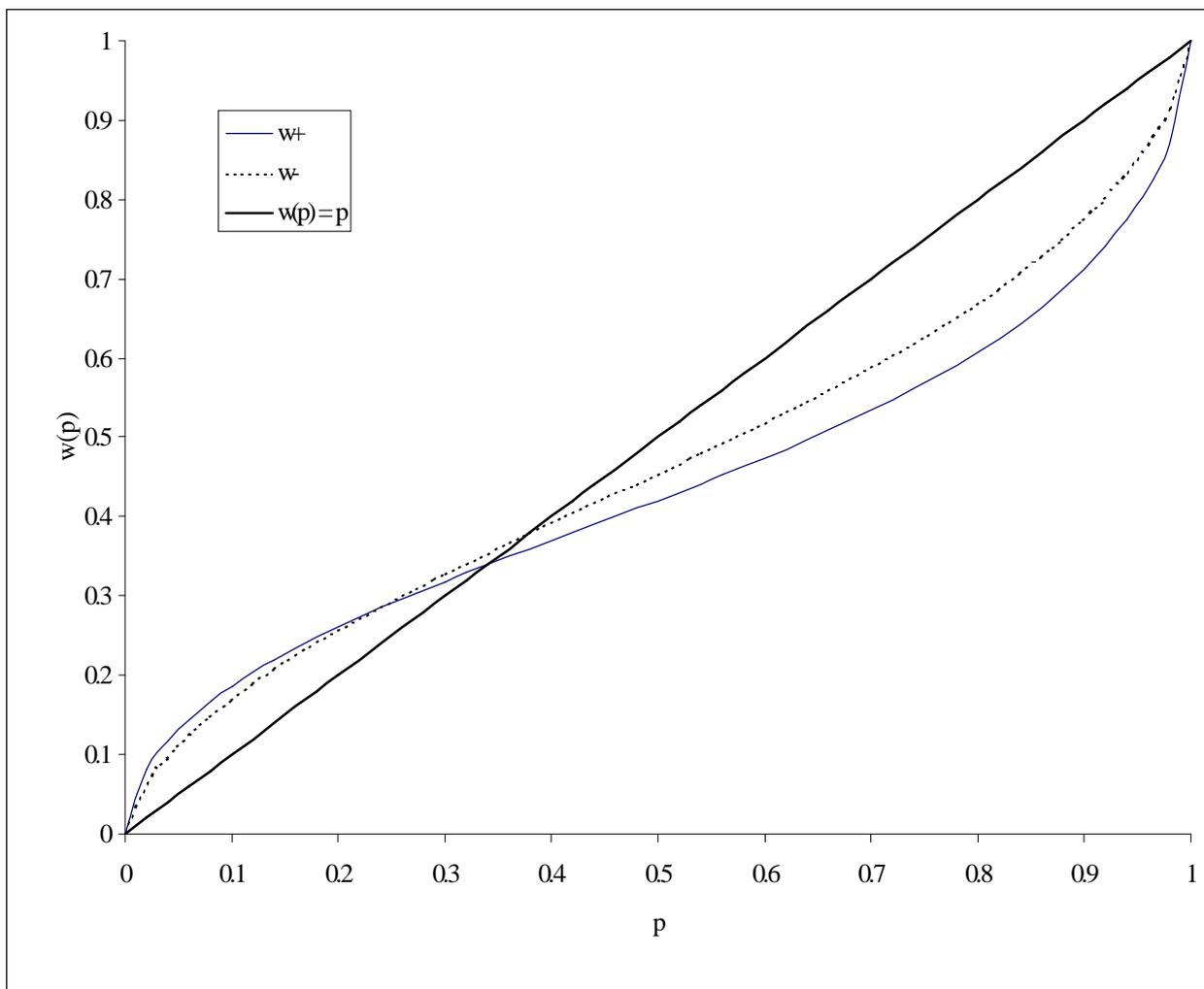
Figure 1. Probability-weighting functions for gains ( $w^+$ ) and for losses ( $w^-$ )

Figure 2: A snapshot illustrating the stochastic process for a probability-weighting function for gains

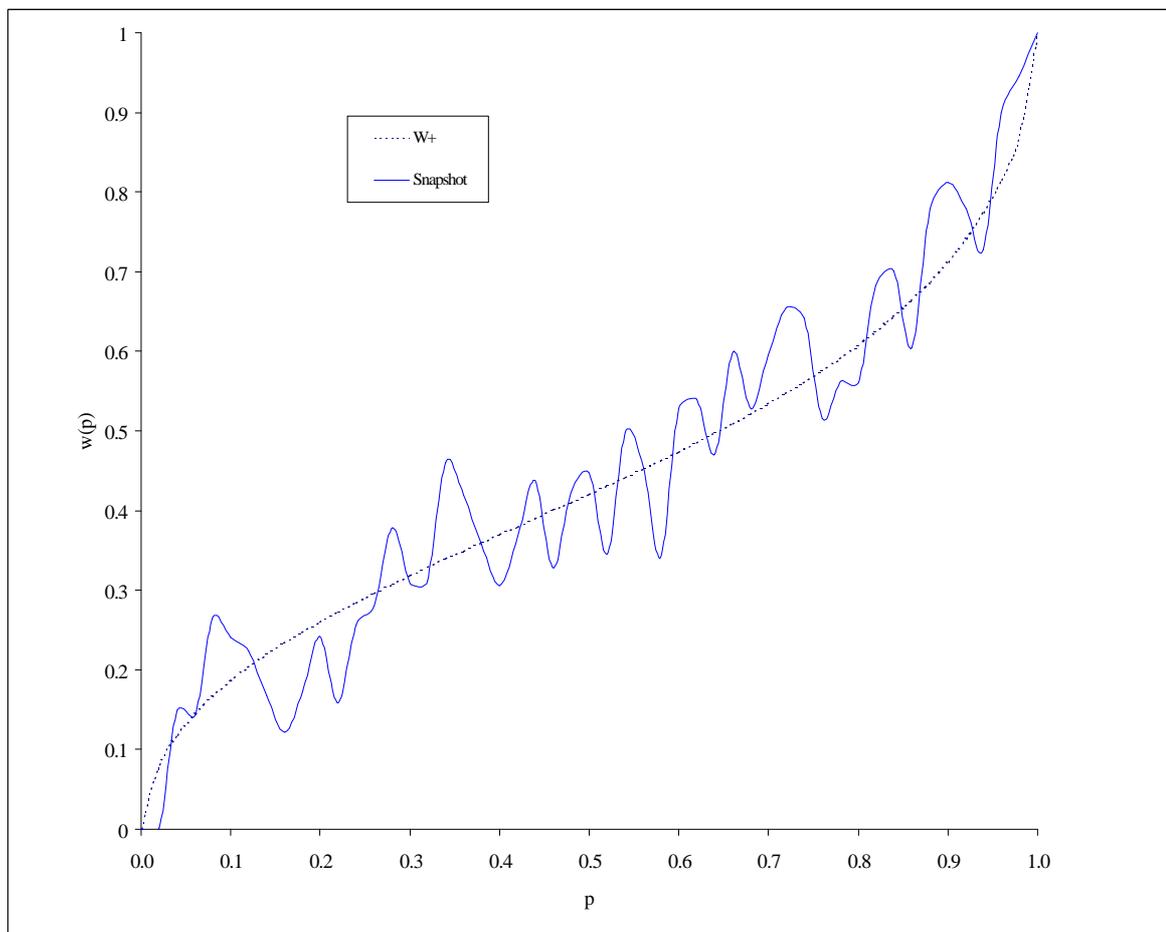
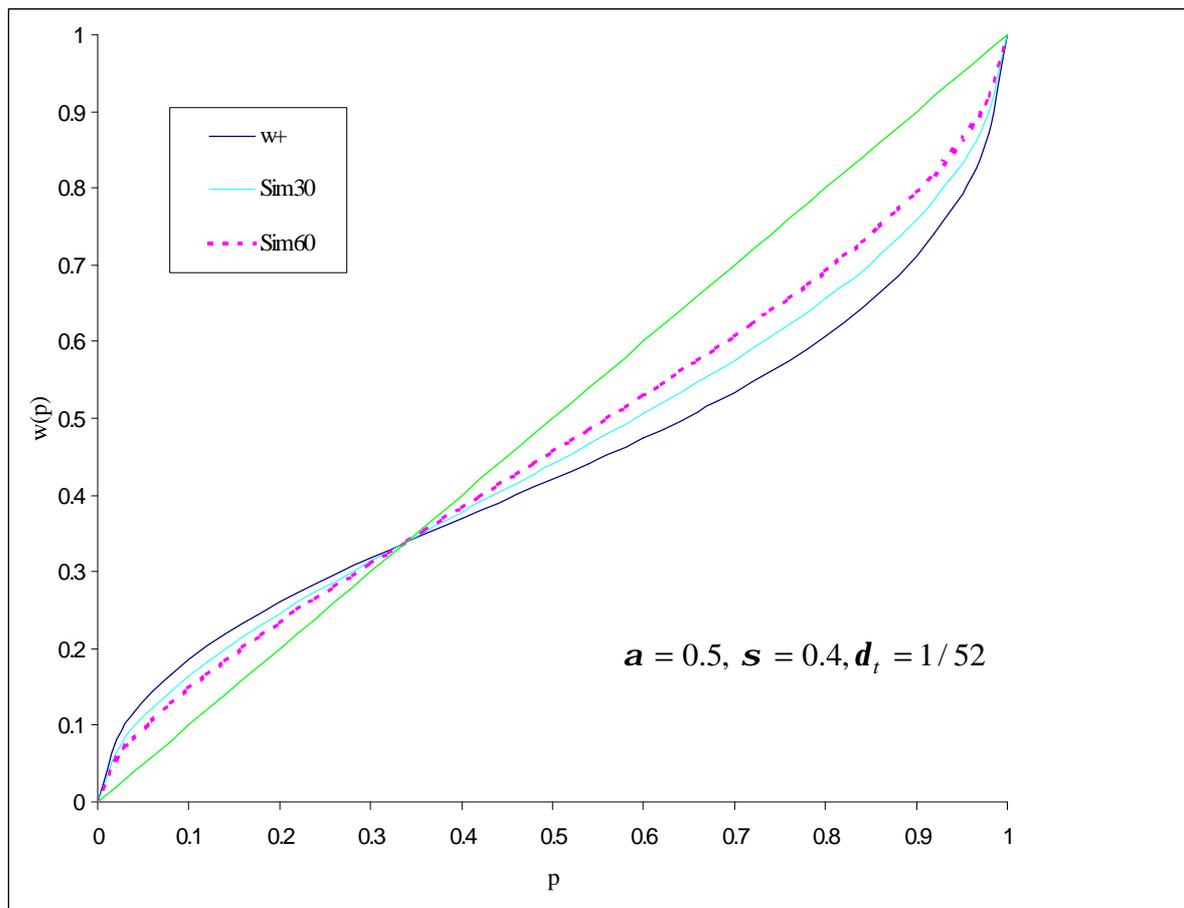
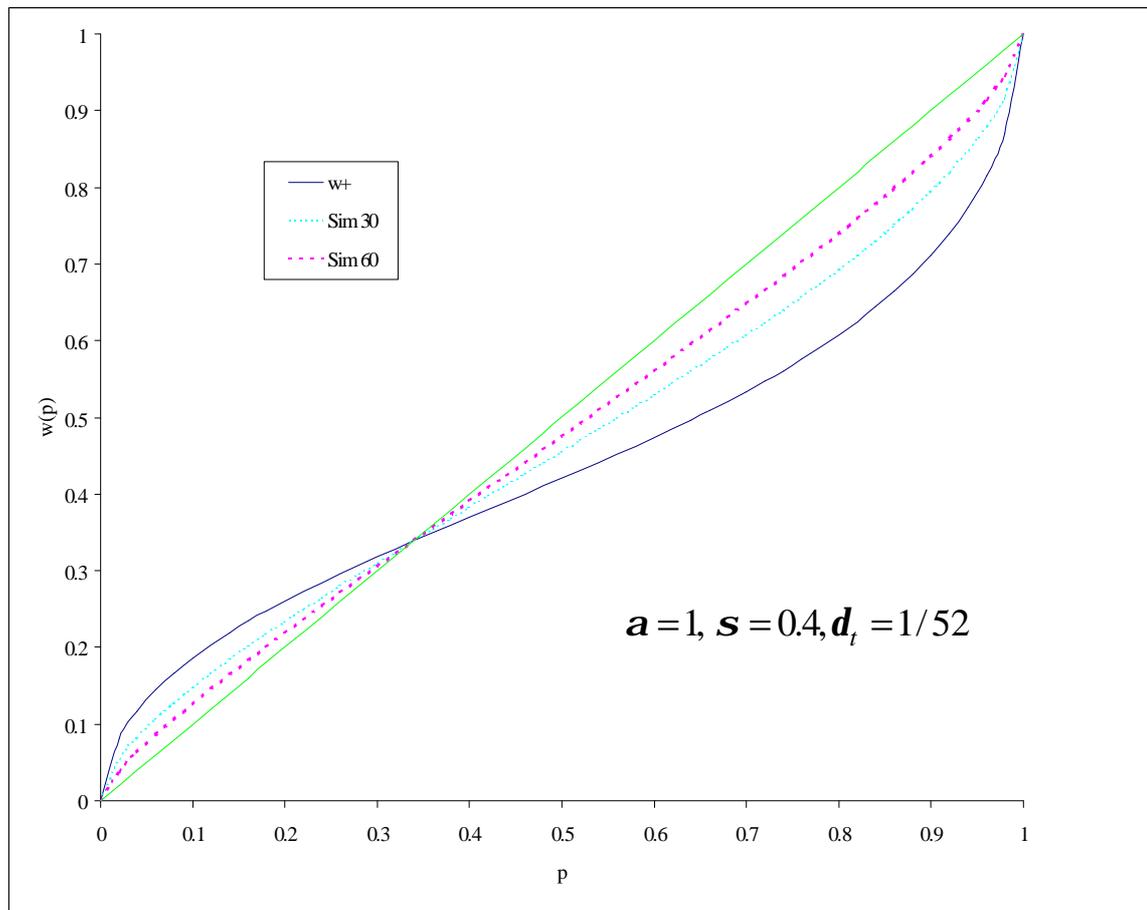


Figure 3: The mean of the simulated probability-weighting functions for gains with  $a = 0.5$ 

Notes: Sim 30 refers to the mean weighting function for 30th period and Sim60 refers to the mean weighting function for 60th period.

Figure 4: The mean of the simulated probability weighting functions for gains with  $\mathbf{a} = 0.5$ 

Notes: Sim 30 refers to the mean weighting function for 30th period and Sim60 refers to the mean weighting function for 60th period.