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Abstract

This study estimates optimal hedge ratios using various econometric models. Applying daily AOIs and SPI futures on the Australian market, optimal hedge ratios are calculated from the OLS regression model, the bivariate vector autoregressive model (BVAR), the error-correction model (ECM) and the multivariate diagonal Vec GARCH Model. The hedging effectiveness is measured in terms of ex-post and ex-ante risk-return trade-off at various forecasting horizons. It is generally found that the GARCH time varying hedge ratios provide the greatest portfolio risk reduction, particularly for longer hedging horizons, but they do not generate the highest portfolio return.

Key Words: OLS Method, VAR model, Error Correction Term, M-GARCH Modelling

JEL Classification No: C5, G1, N2

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I. Introduction

Hedging has widely been viewed as a major function and also the reason for the existence of futures markets. The calculation of hedge ratios however still keeps debatable in the literature of futures trading. In order for hedgers to hedge their cash assets on hand, they need to hold a certain amount of futures contracts. Therefore, they encounter the key question of how many futures contracts should be held for each unit of cash asset, that is, how should the appropriate hedge ratio be calculated?

The Portfolio and hedging theory that has a prevailing inference today is extended by Ederington (1979) from Johnson (1960) and Stein (1961). It postulates that the objective of hedging is to minimise the variance of cash portfolio held by the investor. Therefore, the hedge ratio that generates the minimum portfolio variance should be the optimal hedge ratio, which is also known as minimum variance hedge ratio.

Despite its robustness at the early stage, Ederington's (1979) approach has undertaken increasing challenges with the recent development of time series econometrics. Many authors criticised it by drawing attention to the inefficiency of the residuals in the OLS method used to estimate the optimal hedge ratio. Herbst, Kare and Marshall (1989) argue that the estimation of the minimum variance hedge ratio suffers from problem of serial correlation in the OLS residuals. Bell and Krasker (1986) show that if the expected futures price change depends on the information set, then the traditional regression methods would yield a biased estimate of the hedge ratio[1]. Park and Bera (1987), on the other hand, point out that the simple regression model is inappropriate to estimate hedge ratios because it ignores the heteroskedasticity often encountered in cash and futures price series. Myers and Thompson (1989) argue that the covariance between the dependent and explanatory variable and variance of the explanatory variable in the optimal hedging rule should be conditional moments that depend on information set available at the time the hedging decision is made. Therefore the hedge ratio should be adjusted continuously based on conditional information and thus calculated from conditional variance and covariance. The generalised approach they developed allow for a more flexible specification of equilibrium pricing models where the conventional simple regression approaches to optimal hedge ratio estimation are special cases under particular sets of restrictions on equilibrium spot and futures price determination.

The development of generalised Autoregressive Conditional heteroskedasticity (GARCH) model and its enormous extensions and derivations in the early 1980's allow us to observe hedge ratio from a dynamic view and provide a econometric method to calculate a series of time-varying hedge ratios based on the conditional variances and covariances. However, one question remains unsure is that whether the dynamic hedge ratios calculated using more complicated GARCH method also generate better results in terms of improving hedging effectiveness. Therefore, in this paper various econometric methods are employed to calculate the hedge ratios. They include Ederington's (1979) traditional regression model, general bivariate VAR model, Lien and Luo's (1993) Error Correction Model and the more complex multivariate generalised autoregressive conditional heteroskedasticity model (M-GARCH) of Bollerslev, Bollerslev, Engle and Wooldridge (1988).

Another issue addressed by a number of authors is the important role the cointegration between spot and futures prices plays in determining optimal hedge ratios. Ghosh (1993a, 1993b), in an analysis of stock index futures and underlying stock price index incorporating in cointegrating relationship, finds that minimum variance hedge ratio estimates are biased downwards due to mis-specification if spot and futures are cointegrated and the error-correction term is not included in the regression. Lien and Luo (1994) argues that although GARCH (Generalised Autoregressive Conditional heteroskedasticity) may characterise the price behaviour, the cointegration relationship is the only truly indispensable component when comparing ex post performance of various hedge strategies. Lien (1996) provided theoretical support for the importance of the cointegrating relationship and pointed out that: “A hedger who omits the cointegration relationship will adopt a smaller than optimal futures position, which results in a relatively poor hedging performance.” In this concern, an error-correction term (ECT) is included in our VAR model, given that there is evidence of cointegrating relationship spot and futures series.

The objective of the study is to apply the multivariate-GARCH model to the calculation of hedge ratio of Australian All Ordinary stock index and corresponding SPI futures and compare its performance with constant hedge ratios calculated on the same data set using other econometric models.

The remainder of the study is as follows: the next section provides an overview of statistical tests and econometric models that are employed in the empirical estimation and analysis. The third section describes data characteristics and sources, presents the results from all models, calculates various types of hedge ratios and compares their performance. The last section concludes the paper and discusses the limitations and further research that can be explored from the results.

II. Research Methodology

In this study, four different models are described and estimated to calculate optimal hedge ratios. The performance of the hedge ratios is then compared to assess whether the more advanced time varying hedge ratios calculated from Bollerslev, Bollerslev, Engle and Wooldridge’s (1988) Multivariate-GARCH model can provide more efficiency than other constant hedge ratios from the regression model, the Bivariate VAR model and the Error-Correction Model.

A. Model 1: The Conventional Regression Method

This is simplest model of the four, which is just a linear regression of change in spot prices on change in futures prices. Let S_t and F_t be logged spot and futures prices, respectively, the one period minimum variance constant hedge ratio can be estimated from the expression:

$$DS_t = c + h^*DF_t + e_t \qquad (1)$$

where e_t is the error term from OLS estimation, DS_t and DF_t represent spot and futures price changes. The minimum hedge ratio is h^*, the slop of equation.

B. Model 2: The Bivariate VAR Model for Hedge Ratio Estimation

As earlier noted in Herbst, Kare and Marshall (1989), one aspect of the above regression model's invalidity has been the fact that the residuals are autocorrelated. In order to eliminate the serial correlation, the spot and futures prices are modelled under a bivariate-VAR framework as presented in model 2:

$$\begin{aligned} \Delta S_t &= c_s + \sum_{i=1}^{k} b_{si} \Delta S_{t-i} + \sum_{i=1}^{k} b_{si} \Delta F_{t-i} + e_{st} \\ \Delta F_t &= c_f + \sum_{i=1}^{k} b_{fi} \Delta S_{t-i} + \sum_{i=1}^{k} b_{fi} \Delta F_{t-i} + e_{ft} \end{aligned} \qquad (2)$$

where c is the intercept, and b_s, b_f, g_s and g_f are positive parameters. e_{st}, e_{ft} are independently identically distributed (i.d.d) random vectors. The model has to decide its optimal lag length, k, which starts from one and is added up by one in each of the iteration until the autocorrelation in residuals is eliminated from the system equations. If we let var (e_{st}) = s_{ss}, var (e_{ft}) = s_{ff} , and cov (e_{st}, e_{ft}) = s_{sf} , many previous studies have shown that the minimum variance hedge ratio is

$$h^* = \frac{s_{sf}}{s_{ff}} \qquad (3)$$

C. Model 3: The Error Correction Model

It is obvious to know that Model 2 ignored the effect that the two series are cointegrated, which is further addressed in Ghosh (1993b), Lien and Luo (1994) and Lien (1996). They argue that if the two price series are found to be cointegrated, a VAR model should be estimated along with the error-correction term which accounts for the long-run equilibrium between spot and futures price movements. Thus equation (2) is modified as:

$$\begin{aligned} \Delta S_t &= c_s + \sum_{i=1}^{k} b_{si} \Delta S_{t-i} + \sum_{i=1}^{k} b_{si} \Delta F_{t-i} + g_s Z_{t-1} + e_{st} \\ \Delta F_t &= c_f + \sum_{i=1}^{k} b_{fi} \Delta S_{t-i} + \sum_{i=1}^{k} b_{fi} \Delta F_{t-i} - g_f Z_{t-1} + e_{ft} \end{aligned} \qquad (4)$$

where c is the intercept, the two terms represented by e_{st} and e_{ft} are white-noise disturbance terms and b_s, b_f, g_s and g_f are positive parameters. Z_{t-1} is the error-correct term, which measures how the dependent variable adjusts to the previous period's deviation from long-run equilibrium

$$Z_{t-1} = S_{t-1} - aF_{t-1}$$

where a is, what we call, the cointegrating vector. This two-variable error-correction model expressed in equation (3) is a bivariate VAR (*k*) model in first differences augmented by the error-correction term $g_s Z_{t-1}$ and $-g_f Z_{t-1}$. The coefficients g_s and g_f have the interpretation of speed of adjustment parameters. The larger g_s is, the greater the response of S_t to the previous period's deviation from long-run equilibrium. The constant hedge ratio can similarly calculated using equation (2).

D. Model 4: The Multivariate GARCH Model

As pointed out by Park and Bera (1987) and Pagan (1996), most economic and financial time series encounter the characteristic of heteroskedasticity (or ARCH effects) in the second movements, which partly invalidates hedge ratio estimates. Therefore, Bollerslev, Engle and Wooldridge's (1988) VEC multivariate GARCH model is employed in the fourth model to account for the ARCH effects in the residuals of error correction model.

Developed by Engle (1982) and then Bollerslev (1986), the autoregressive conditional heteroskedasticity model (ARCH) sparkled a substantial body of work which concerns with not only further examining the second moment of economic and financial time series, but also extending and generalising the initial ARCH model to better fit the situation being investigated. Bollerslev, Engle and Wooldridge (1988) generalised the univariate GARCH to a multivariate dimension to simultaneously model the conditional variance and covariance of two interacted series. This multivariate GARCH model is thus applied to the calculation of dynamic hedge ratios that vary over time based on the conditional variance and covariance of the spot and futures prices. Generalised from GARCH(1,1), A standard M-GARCH(1,1) model is expressed as:

$$\begin{bmatrix} h_{ss,t} \\ h_{sf,t} \\ h_{ff,t} \end{bmatrix} = \begin{bmatrix} c_{ss,t} \\ c_{sf,t} \\ c_{ff,t} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} e^2{}_{s,t-1} \\ e_{s,t-1}, e_{f,t-1} \\ e^2{}_{f,t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \times \begin{bmatrix} h_{ss,t-1} \\ h_{sf,t-1} \\ h_{ff,t-1} \end{bmatrix}$$

where h_{ss} , h_{ff} are the conditional variance of the errors ($e_{s,t}$ $e_{f,t}$) from the mean equations, which in this application is the bivariate VAR model (with error correction term), and h_{sf} represent the conditional covariance between spot and futures prices. In view of the excessively large number of parameters needed to be estimated in the model[2], Bollerslev (1990) proposed an assumption that matrix A_i and B_i are diagonal and the correlation between the conditional variances are to be constant. However, Bera and Roh (1991) conducted a test for the constant correlation hypothesis and found that for many financial time series, the hypothesis can be rejected. Bollerslev, Engle and Wooldridge (1988) propose a parameterisation of the conditional variance equation in the multivariate-GARCH model termed the Diagonal Vec (DVEC) model which allows for a time-varying conditional variance. Like the constant correlation model, the off-diagonal in the matrices A_i and B_i are set to zero, i.e. the conditional variance depends only on its own lagged squared residuals and lagged values. Following Bollerslev, Engle and Wooldridge (1988), the diagonal representation of the conditional variances elements h_{ss} and h_{ff} and the covariance element h_{sf} can be expressed as:

$$\begin{aligned} h_{ss,t} &= c_{ss} + a_{ss}\, e^2_{s,t-1} + b_{ss}\, h_{ss,t-1} \\ h_{sf,t} &= c_{sf} + a_{sf}\, e_{s,t-1}\, e_{f,t-1} + b_{sf}\, h_{sf,t-1} \qquad (5) \\ h_{ff,t} &= c_{ff} + a_{ff}\, e^2_{f,t-1} + b_{ff}\, h_{ff,t-1} \end{aligned}$$

This DVEC multivariate GARCH model employed in this paper explicitly incorporates a time varying conditional correlation coefficient between the spot and futures prices and hence generates more realistic time-varying hedge ratios.

III. Results and Implications

A. Data

The data used in this study is retrieved from the Datastream database. It encompasses the All Ordinaries Share Price Index (AOI) and the corresponding share price index (SPI) futures prices on a daily basis for the period of January 1th, 1988 – December 12th, 2000 summing up to totally 3139 observations. Only the first 2987 observations are used in the empirical tests, leaving the last 269 observations starting from 1st January 1999 for an ex-ante hedge ratio performance comparison.

There are four delivery months per annum for the futures on the stock price index: March, June, September and December. The three-month futures contracts are adopted and the contracts in the delivery month are rolled over to the next three-month contracts on the first day of the delivery month. For example, the March contract is renewed to the June contract on the 1st of June and hence the settlement prices of the June contracts are used in June, July and August; similarly, the September contracts are used in September, October and November.

To give a general picture of both series, the logarithm of AOIs and SPI futures are depicted in Figure 1, which strongly indicates that the two series are closely correlated. From the obvious time trend as exhibited in the diagram it is suspected that they are characterised by non-stationarity in levels. This is further tested using the ADF test and the KPSS test.

B. Tests of Unit Roots and Cointegration

The results of unit root tests for logged stock price index and SPI futures with the first order differences are reported in Table 1. Notice that apart from the augmented Dicky-Fuller (ADF) tests, which attempt to account for temporally dependent and heterogeneously distributed errors by including lagged sequences of first differences of the variable in its set of regressors[3], the KPSS test is also used. The null hypothesis for ADF test is that the variables contain a unit root or they are non-stationary at a certain significant level. However, the power of standard unit root tests which have null hypothesis of non-stationarity has recently been questioned by Schwert (1987) and DeJong and Whiteman (1991) in that these tests often tend to accept the null too frequently against a stationary alternative. It appears that the failure to reject the null may be simply due to the standard unit root tests having low power against stable autoregressive alternatives with roots near unity. In particular, this knife-edge assumption of an exact unit root could lead to substantial biases, even in large samples. In view of the growing controversy surrounding the general tests for unit root, a different series of tests-- KPSS tests proposed by Kwiatkowski, Phillips and Shin (1992) are also employed in the context.

In the KPSS tests, the null hypothesis is that a series is stationary around a deterministic trend (TS) and the alternative hypothesis is that the series is difference stationary (DS). The series is expressed as the sum of deterministic trend, random walk, and stationary error as: $y_t = x\,t + r_t + e_t$ where $r_t = r_{t-1} + u_t$, and u_t is i.i.d.(0, s_u^2). The test is the LM test of the hypothesis that r_t has zero variance, that is, $s_u^2 = 0$. If $s_u^2 = 0$, the random walk part of the above equation, r_t, becomes a constant and thus the series $\{y_t\}$ is trend stationary. The asymptotic distribution of the statistic is derived under the null and under the alternative hypothesis. The test is based on the statistic:

$$h(u) = (1/T^2)\sum_{t=1}^{T} S_t^2 / s_k^2 \qquad \text{where} \qquad S_t = \sum_{i=1}^{t} v_i, t = 1,...T$$

with v_t being the residual term from a regression of series y_t on a intercept, and s^2 is a consistent long-run variance estimate of y_t, and T represents the sample size. Kwiatkowski et al (1992) shows that the statistic h (u) has a nun-standard distribution and critical values have been provided therein. If the calculated value of h (u) is large, then the null of stationarity for the KPSS test is rejected.

In the case of the ETA (mu) statistic, the null hypothesis is that the series $\{y(t)\}$ is stationary around a level, while in the case of the ETA(tau) statistic, the null hypothesis accepts that $\{y(t)\}$ is trend stationary (TS). These tests with the no unit root as null are used in many papers as a complement to standard unit root tests. By testing both the unit root hypothesis and the stationary hypothesis, we can distinguish series that appear to be stationary, series that appear to have a unit root, and series for which the data (or the tests) are not sufficiently informative to be sure whether they are stationary or integrated.

In Table 1, it is shown that both series are evidenced of non-stationary in their levels, as the ADF *t*-statistic is insignificant and conversely the ETA(mu) and ETA(mu) statistics significant. After being differentiated once, they all become stationary, that is, the ADF t-statistic becomes significant and both the ETA(mu) and ETA(mu) statistics turn insignificant. Therefore, it can be concluded that spot and futures prices are I (1) processes. This feature of the data forms an important precondition for the tests of a cointegrating relationship, which requires that each of the variables of concerned should be integrated to the same order great than zero (Enders (1995)). The next step is therefore to test for cointegration between these variables. Table 2 presents the results of Johansen and Juselius (1990) cointegration test and the model selection-criteria method, which is employed to test the existence of cointegrating relationship as a supplement to the Johansen's test.

The results of Johansen's cointegration test are presented in panel A, where two tests, one designed to test for the presence of *r* cointegrating vectors (the 'trace' test), and the other designed to test the hypothesis of *r* cointegrating vectors in *r+1* cointegrating vectors (the maximum eigenvalue test), are undertaken on logged spot and futures prices. When the null hypothesis is that there is no cointegrating vector existing, both eigenvalue and trace statistics strongly reject the null. When the null is that there exists a single cointegrating vector, both statistics tend not to reject it. Therefore, there is an indication of a cointegrating relationship between the variables with rank of one. Panel B uses an alternative way of selecting the number of cointegrating relationships. The values from three model selection criteria (AIC, SBC, HQC) give the same information that the rank of cointegrating vector is one in that the statistic of each criterion its reaches the largest value when the rank of cointegrating rank is one.

C. The Results from Model 1, 2, and 3

According to Schwarz Bayesian Criterion (SBC) and log-likelihood ratio statistics (LL), the appropriate lag length of the VAR model is four[4]. After checking for empirical regularities that may exist in the data[5], the estimates from the bivariate VAR (4) model is presented in Table 3.

Similarly, the error correction model can be estimated by incorporating the error correction term into the VAR(4) model. The results are presented in Table 4, which shows that for both equations of changes in spot prices and changes in futures prices, the coefficients of the error-correction term (as shown in bold characters) are significant, as indicated by the large values of the *t*-ratios. It is noticed that g_s =0.069, while g_f = 0.1. This implies that the futures price series F_t have a greater speed of adjustment to the previous period's deviation from long-run equilibrium than the spot price series. This finding is consistent with the fact that on the delivery date of each contract the futures price has to adjust itself to the prevailing spot price.

Using the variance and covariance of the residuals, the hedge ratios of both model are calculated in Table 5, together with the hedge ratio estimated from model 1, the conventional regression method. As expected and in line with most of the previous studies by Ghosh (1993b) and others, the hedge ratio estimated by the error-correction model is greater than that obtained from other models. The hedger ignorant of the cointegrating relationship between futures and spot prices is likely to take a smaller than optimal futures position.

D. Dynamic Hedge Ratio using M-GARCH Model

In this paper, Lien's (1996) study is extended to examine the efficiency of the error-correction model by further investigating the features of the residual series. The autocorrelation functions of the two streams of residuals from Equation (4) are presented in table six. For daily data in this application, the lag of 20 is chosen to correspond to a period of approximately one calendar month, and the actual residual values are plotted in Figure 2.

It is indicated clearly in Table 6 that the autocorrelation coefficients for all 20 lags are close to zero, with Q-statistic[6] probabilities well greater than 5% significance level. This leads us to believe that the estimated mean equation, that is, the bivariate VAR model incorporated in the error-correction term, has adequately accounted for the serial correlation in the logarithm of spot and futures price series. However, the plots of the actual values of the residuals in Figure 2 exhibit volatility clustering even though the mean seems constant. The variance of the series is changing through time and large (small) changes tend to be followed by large (small) changes of either sign. This characteristic has been commonly found in most economic time series by Mandelbrot (1963a, 1967), Klien (1977), Engle (1982) etc. and it is indicative of the presence of an autoregressive conditional heteroskedastic (ARCH) effect.

Another way to test for the presence of ARCH effects has been suggested by McLeod and Li (1983). According to McLeod and Li (1983), a casual examination of the sample autocorrelation function of the mean equation squared residuals for a significant Q-statistic at a given lag can be used to infer the presence of ARCH effects. The (Ljung-Box) Q-statistic at lag *k* is a test statistic for the null hypothesis that there is no autocorrelation up to order *k*. Table seven presents the Q-statistic for squared residuals (e_t^2) generated from equation (3). They are all highly significant confirming the presence of ARCH effects. Therefore, a bivariate GARCH method is necessary to explicitly model the variance of the residuals of the error-correction model.

The above tests have all indicated the existence of heteroskedasticity in the VAR (with error-correction term) and thus conform the necessity of an M-GARCH modelling to estimate the conditional variance and covariance and further calculate time varying hedge ratios. For a bivariate MGARCH model in the study, the BHHH (Berndt, Hall, Hall and Hausman) optimisation method and the Simplex Algorithm optimisation method are used to estimate all the coefficients c_{ij}, a_{ij} and b_{ij} simultaneously. The Simplex method is a search procedure that requires only function evaluations, not derivatives; while the other method BHHH required twice-differentiable formulas. The use of a combination of the two methods is suggested by Doan (1996), who documents that the Simplex used in the program is to refine initial estimates before applying BHHH. The latter method is more sensitive to the choice of initial estimates. However, a disadvantage of the Simplex method is that it cannot provide standard errors for the estimated parameters. The program automatically selects parameter values that maximize the log likelihood function of the model. The results are presented in Table 8. The parameter estimates are all positive definite and highly significant. Furthermore, the sum of the coefficients for each equation is close to unit, (for example: $c_{ff} + a_{ff} + b_{ff} = 0.988$), suggesting the persistence of ARCH effects in the data sets[7]. This implies that current information remains important for forecasts of the conditional variance at all horizons. Figure 3 plots the dynamic hedge ratios obtained from the conditional variance and covariance between the spot and futures prices. Note that the hedge ratios display signs of extreme volatility and show continuous increase in the late 1980s, reflecting the irregular fluctuation in prices due to the impact of the October 1987 crash on both spot and futures markets. The hedge ratios are relatively more stable since 1991, except for 2 sharp drops in 1992 and 1996. Ranging from a minimum of –0.046 to a maximum of 0.92, the dynamic hedge ratio has a sample mean of 0.59, which is well below 1, but greater than the constant hedge ratios on average. This conclusion once again confirms the rejection of traditional 1: 1 hedging strategy. Moreover, the dynamic hedge ratio exhibits explicit random walk characteristics (non-stationarity) with its significant ADF statistic of 2.12.[8] This finding is consistent with that of Lo and MacKinlay (1988), Malliaris and Urrutia (1991), and Lindhal (1992) and others.

E. Hedging Effectiveness Comparison

So far four hedging strategies have been used to derive optimal hedge ratios, each of which is based on different econometric theories and involves different degrees of computational complexity. Then the ex post and ex ante forecasting methods are employed to compare the performance of these four types of hedge ratios.

In order to compare the performances of each type of hedging strategy, the un-hedged portfolio is constructed, consisting of shares with the same proportion as the share price index held on the spot market. Also the hedged portfolios is constructed, consisting of a combination of the share price index held on both the spot and the futures markets. The number of futures contracts held is determined by the computed hedge ratios from each hedging strategy. The hedging performance is compared in terms of the risk-return trade-off, and the percentage variance reduction in the hedged portfolio relative to the un-hedged portfolio.

The mean and variance of the returns of the hedged portfolios, and the percentage reduction in the variance of the hedged portfolio relative to the un-hedged portfolio are calculated in each forecasting horizon. According to Baillie and Myers (1991) and Park and Bera (1987: appendix), the returns on the un-hedged and the hedged portfolios are simply expressed as:

$$r_u = S_{t+1} - S_t$$
$$r_h = (S_{t+1} - S_t) - h^* (F_{t+1} - F_t)$$

where r_u and r_h are return on un-hedged portfolio and hedge portfolio, respectively. F_t and S_t are logged futures and spot prices at time period t, respectively, and h^* is optimal hedge ratio, and the return on the hedged portfolio is the difference between the return on holding the cash position and corresponding futures position.

Similarly, the variance of the un-hedged and the hedged portfolios are expressed as:

$$Var\ (U) = s_s^2$$
$$Var\ (H) = \sigma_s^2 + h^{*2}\sigma_f^2 - 2\ h^* \sigma_{s,f}$$

where Var (U) and Var (H) represent variance of un-hedged and hedged portfolios, respectively. s_s, s_f are standard deviation of the spot and futures price, respectively, and $s_{s,f}$ represents the covariability of the spot and futures price. According to Ederington (1979), the effectiveness of hedging can be measured by the percentage reduction in variance of the hedged portfolio relative to the unhedged portfolio. The variance reduction can be calculated as:

$$\frac{Var(U) - Var(H)}{Var(U)}$$

Lien and Tse (1998) propose that the performance of the models may vary according to the hedge horizon, therefore, in this context hedging effectiveness of the four models will be considered over horizons of 1, 5, 10 and 20 days.

The more reliable measure of hedging effectiveness is the hedging performance for the post-sample periods. For each out-of-sample testing period, the same parameters estimated from M-GARCH are used to forecast the conditional variance and covariance for the following day. The forecasted hedge ratio will be the one-period forecast of the conditional covariance divided by the one-period forecast of the conditional variance. Such forecasts are conducted for each day for the following 20 observations from the 16th December 1999 to 12th January 2000. For the other three models that generate constant hedge ratios, the estimated hedge ratios are used for the out-of-sample period. The results for the in-sample and post-sample performance are presented in table nine.

The first section of Table 9 displays the within-sample comparisons. In the one-day hedge case, a trade-off between risk and return occurs. Although the M- GARCH model generates the greatest daily return of approximately 0.07%, it incurs a considerable risk greater than any other method. It is also the poorest one in terms of percentage reduction of the variance of the un-hedged portfolio. This is not the case for the longer hedging horizons. Taking the twenty-day hedge as an example, it is shown that the greatest return is generated from the conventional regression model, and so is the greatest risk. The GARCH method tremendously reduces the overall risk in the un-hedged portfolio to a degree of 80%, but the return yielded from the hedged portfolio is the smallest. Therefore, if risk aversion is the major goal of an investor, the GARCH model hedging strategy performs the best in reducing the conditional variance of the hedged portfolio. This is consistent with most of the previous studies of Myers (1991), Baillie and Myers (1991) and Park and Switzer (1995a, 1995b) on US commodity and financial markets. Another striking feature of the in-sample results is that the longer the hedge horizon, the greater the extent to which the GARCH hedge ratios reduce the risk of the hedged portfolio relative to other alternatives.

However, if the return factor is taken into account, the M-GARCH hedging strategy does not seem to outperform the other alternatives. Although a number of previous studies of hedging effectiveness of hedging using M-GARCH optimal hedge ratios has found either marginal or substantial superiority to other alternative hedge ratios, it is based on the presumption that the hedging performance is measured in terms of the reduction in variance only. This study measures the hedging performance under a risk-return trade-off basis. It is found that the GARCH model is no longer the best choice. The results reflect a two-parameter approach in the theory of finance that was developed by Markowitz (1952): the higher the risk, the higher the return. The investor's degree of risk aversion, in this case, plays an important role in selecting the hedging method. For instance, a return oriented investor is likely to select the regression hedge ratio to form their hedged portfolio.

The post-sample comparison tells a similar story. A noticeable fact is that for one-day and five-day hedging, the dynamic hedge-ratios from the GARCH model yield both highest return and variance reduction. But as the hedging horizon increases, the return produced from this method recedes to be the poorest. It can be noted that in a twenty-day hedging strategy, the constant hedge ratios reduce the conditional variance by 64%, whereas the GARCH method reduces the variance by as much as 83%. This significant improvement seems to deserve the investor to consider a sacrifice of a part of his potential return. The GARCH out-performs the others in longer term hedging strategies.

IV. Conclusions

The futures hedge ratios have been calculated in this study using various econometric time series models and the performance of these hedge ratios have been compared in terms of a risk-return trade-offs in the ex-post and ex-ante forecasting horizons. Of the three constant hedge ratios derived from the regression model, the VAR model and the error-correction model, the error-correction model generates the hedge ratios that display the largest value in size. This finding agrees with Ghosh (1993) and Lien's (1996) demonstration that non-inclusion of a cointegration relationship leads to a hedge ratio that is biased downwards in size. The time varying hedge ratios calculated from conditional information set exhibit high degree of non-stationarity through time, though the excess volatility in the late 1980s may be due to the impact of the October 1987 crash.

In the performance of these hedge ratios, the in-sample and out-of-sample forecasts tell the similar story. The M-GARCH dynamic hedge ratios provide the greatest degree of variance reduction in most of the forecasting horizons, but also generate the smallest rate of return. On the other hand, the hedge ratio calculated from the conventional regression model performs the worst in terms reducing portfolio variance, but yields the highest rate of return. This finding implies that in selecting the most appropriate hedge ratio, the investor's degree of risk aversion plays a relatively important role. It is also found that in longer term hedging, the time varying hedge ratios out-perform the constant hedge ratios in terms of reducing portfolio variance.

Notes

1. For more example in the respect, see Castelino(1990a, 1990b), Lindahl (1992), Lo and MacKinlay (1988) and Malliaris and Urrutia (1991).

2. According to Pagan (1996), there are 21 parameters to be estimated in the simple bivariate GARCH (1, 1) model.

3. See Dickey and Fuller (1981) for the details.

4. The results for the VAR order selection can be provided on request.

5. The data is checked for January effect, holiday effect and Monday effect though no significant parameters of the dummy variables are observed using likelihood ratio tests.

6. Q-statistic is to be discussed more in the next page.

7. The particular phenomenon in GARCH model is examined by Engle and Bollerslev (1986) and termed as IGARCH.

8. The procedure of ADF test for the hedge ratios is not presented here, but can be provided on request.

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Table 1. Tests for Unit Roots

	ADF Tests:	KPSS Tests:	
	t-statistic	ETA (mu)	ETA (tau)
LAOI	-0.7746	***56.9338	***5.58186
LSPI	-1.0764	***56.73143	***5.37582
DLAOI	***-52.3981	0.04104	0.02813
DLSPI	***-59.4324	0.02495	0.02111
Critical Values:			
Level	1%	5%	10%
ADF	-3.43	-2.86	-2.57
ETA (mu)	0.739	0.463	0.347
ETA (tau)	0.21	0.146	0.119

Notes: For the ADF tests, ***means that the series is stationary at 99% confidence level. For the KPSS tests, *** represents that the series is non-stationary at 99% confidence level. The ETA (mu) statistic tests whether the series is stationary around a certain level, whereas the ETA (tau) statistic tests whether the series is trend stationary.

Table 2. Tests for Cointegration

A. Johansen's Cointagration Test

Ho:	**H_1:**	**Eigenvalue Test**			**Trace Test**		
			95%	90%		95%	90%
r = 0	**r < 1**	**165.7941	14.88	12.98	**166.3389	17.86	15.75
r = 1	**r < 2**	0.54473	8.07	6.5	0.54473	8.07	6.5

Notes: Cointegration LR Test Based on Maximal Eigenvalue of the Stochastic Matrix and Trace of the Stochastic Matrix. *r* represents the number of linearly independent cointegrating vectors. Trace statistic = $-\ T\Sigma_{i=r+1}^{n} Ln(1-\lambda);\ \lambda_{max} = -TLn(1-\lambda),$ where T is the number of observations, *n* is the dimension of x, and λ_i is the *i*th smallest squared canonical correlations in Johansen (1988, 1991) or Johansen and Juselius (1990, 1992). * and ** represent the significance level of 90% and 95% respectively.

B. Choice of the Number of Cointegrating Relations Using Model Selection Criteria

Rank	**AIC**	**SBC**	**HQC**
r = 0	25575.7	25533.6	25560.6
r = 1	*25655.6	*25604.5	*25637.2
r = 2	25654.9	25600.8	25635.4

Notes: AIC = Akaike Information Criterion, SBC = Schwarz Bayesian Criterion, HQC = Hannan-Quinn Criterion. * marks the largest statistic value for a certain criterion.

Table 3. Estimates of A Bivariate VAR (4) Model

	DLAOI			DLSPI		
	Coefficient	Stand. D.	t-Ratio	Coefficient	Stand. D.	t-Ratio
DLAOI(-1)	-0.3252	-0.0265	*-12.2565	0.1673	-0.0387	*-4.3218
DLAOI(-2)	-0.1959	-0.0283	*-6.9279	0.0935	-0.0413	*-2.2661
DLAOI(-3)	-0.0958	-0.0278	*-3.4412	0.0567	-0.0406	*-1.3972
DLAOI(-4)	-0.0526	-0.0235	*-2.242	-0.0157	-0.0343	-0.4578
DLSPI(-1)	0.3698	-0.0182	*-20.3156	-0.1471	-0.0266	*-5.5379
DLSPI(-2)	0.1816	-0.0214	*-8.4852	-0.1127	-0.0312	*-3.6065
DLSPI(-3)	0.1023	-0.0215	*-4.7492	-0.0904	-0.0314	*-2.8767
DLSPI(-4)	0.0674	-0.0192	*-3.5026	-0.0121	-0.0281	-0.4303
Constant	0.0001	-0.0001	-1.8597	0.0001	-0.0001	-1.4217

Notes: The results are the estimates of Equation (2), a bivariate VAR(4) model. The DLAOI(.) and DLSPI(.) represent the coefficients of each lag: 1, 2, 3, and 4 for the differenced logarithm of spot and futures prices, respectively. The standard errors and t-ratios are presented beside the corresponding coefficients to show each coefficient's relative significance at 95% level. The statistically significant coefficients are marked with *'s .

Table 4. Estimates of Error Correction Model

	D(LAOI)			D(LSPI)		
	Coefficient	Stand. D	t-Ratio	Coefficient	Stand. D	t-Ratio
Cointegrating Equation (*Z_{t-1}*)	-0.0684 (γ_s)	-0.0159	***-4.3054**	0.1014 (γ_f)	-0.0232	***-4.3768**
D(LAOL(-1))	-0.2813	-0.0283	*-9.9362	0.1041	-0.0413	*-2.5200
D(LAOL(-2))	-0.1660	-0.0290	*-5.7253	0.0510	-0.0423	*-1.2048
D(LAOL(-3))	-0.0778	-0.0281	*-2.7728	0.0319	-0.0410	*-0.7781
D(LAOL(-4))	-0.0505	-0.0234	*-2.1568	0.0173	-0.0342	-0.5060
D(LSPI(-1))	0.3179	-0.0218	*-14.6070	0.0708	-0.0318	*-2.2293
D(LSPI(-2))	0.1439	-0.0231	*-6.2346	0.0560	-0.0337	-1.6640
D(LSPI(-3))	0.0751	-0.0224	*-3.3535	0.0497	-0.0327	-1.5227
D(LSPI(-4))	0.0526	-0.0195	*-2.6965	0.0104	-0.0285	-0.3651
Cointegrating Equation						
	Coefficient	Stand. D	t-Ratio			
S_{t-1}	1.000	-	-			
F_{t-1}	-1.0047	-0.0029	351.6024			
C	0.0178	-0.0094	-1.8927			

Notes: The upper part of the results are the estimates of Equation (3), the error-correction model, or a bivariate VAR(4) model incorporated in an error-correction term. The coefficients of cointegration equation are γ_s and γ_f in Equation (3). The DLAOI(.) and DLSPI(.) represent the coefficients of each lag: 1, 2, 3, and 4 for the differenced logarithm of spot and futures prices, respectively. The standard errors and t-ratios are presented beside the corresponding coefficients to show each coefficient's relative significance at 95% level. The statistically significant coefficients are marked with *'s. The bottom part of the table presents the results estimated from the cointegration equation of spot and futures prices in levels, $Z_{t-1} = c + S_{t-1} - aF_{t-1}$.

Table 5. The Estimation Results of Regression Model

Regression Model		**VAR Model**		**Error-Correction Model**	
h***	**0.477719**	s_{ff}	2.43E-05	s_{ff}	2.42E-05
Std. Error	0.00978	s_{sf}	1.24E-05	s_{sf}	1.25E-05
t-Statistic	48.8587	s_{sf} / s_{ff}	**0.5083**	s_{sf} / s_{ff}	**0.5165**

Notes: the coefficients of the conventional regression model are estimated using ordinary least square (OLS) method. The parameter of the independent variable (the changes in the logged futures prices) is taken as the optimal hedge ratio (see equation (1)). The unconditional variances of the spot prices (s_{ss}), futures prices (s_{ff}) and the covariance (s_{sf}) of the two are calculated from the residuals of the VAR model (equation 5.1) and the error-correction model (equation 5.2), respectively. The optimal hedge ratios are thus calculated from $h^* = s_{sf} / s_{ff}$.

Table 6. The Autocorrelation Function of the Residuals

(a) Residuals of All Ordinaries Share Price Index

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.003	-0.003	0.0239	0.877
		2	-0.002	-0.003	0.0434	0.979
		3	0.003	0.003	0.0650	0.996
		4	-0.005	-0.005	0.1529	0.997
		5	-0.007	-0.007	0.3224	0.997
		6	-0.030	-0.030	3.1228	0.793
		7	-0.015	-0.016	3.8707	0.795
		8	0.022	0.022	5.4221	0.712
		9	0.001	0.001	5.4254	0.796
		10	0.015	0.015	6.1198	0.805
		11	0.025	0.024	8.0163	0.712
		12	0.039	0.038	12.685	0.392
		13	0.016	0.016	13.518	0.409
		14	0.027	0.029	15.814	0.325
		15	0.016	0.018	16.636	0.341
		16	-0.013	-0.012	17.182	0.374
		17	0.008	0.010	17.376	0.429
		18	-0.023	-0.021	19.084	0.387
		19	-0.022	-0.021	20.599	0.359
		20	-0.020	-0.020	21.838	0.349

(b) Residuals of SPI Futures

Lag	Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
1			-0.002	-0.002	0.0106	0.918
2			-0.002	-0.002	0.0180	0.991
3			0.004	0.004	0.0563	0.997
4			-0.005	-0.005	0.1194	0.998
5			-0.008	-0.008	0.3166	0.997
6			-0.035	-0.035	3.8109	0.702
7			-0.012	-0.013	4.2545	0.750
8			0.026	0.026	6.2190	0.623
9			0.005	0.005	6.2794	0.712
10			0.012	0.012	6.7241	0.751
11			0.027	0.026	8.8561	0.635
12			0.034	0.033	12.113	0.437
13			0.017	0.017	12.950	0.452
14			0.029	0.031	15.418	0.350
15			0.014	0.015	15.962	0.385
16			-0.014	-0.013	16.539	0.416
17			0.014	0.016	17.105	0.447
18			-0.022	-0.019	18.515	0.422
19			-0.022	-0.021	19.921	0.399
20			-0.019	-0.019	20.976	0.399

Notes: This table presents the plots and values of autocorrelation function (AC) and partial autocorrelation correlation function (PAC) of the residuals from Equation 5.2. The last two columns are Q-statistics for high order autocorrelation and the corresponding probability. The null hypothesis is that there is no autocorrelation at a certain order. The probabilities tell us that we have to accept the null of no autocorrelation up to 20 lags.

Table 7. Autocorrelation Function of the Squared Residuals

(c) Squared Residuals of All Ordinaries Share Price Index

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.381	0.381	456.28	0.000
		2	0.124	-0.025	504.45	0.000
		3	0.112	0.086	543.85	0.000
		4	0.092	0.027	570.42	0.000
		5	0.046	-0.004	577.00	0.000
		6	0.015	-0.010	577.68	0.000
		7	0.044	0.041	583.71	0.000
		8	0.050	0.018	591.49	0.000
		9	0.013	-0.017	592.04	0.000
		10	0.021	0.020	593.43	0.000
		11	0.026	0.006	595.57	0.000
		12	0.012	-0.005	596.01	0.000
		13	0.042	0.045	601.68	0.000
		14	0.022	-0.015	603.22	0.000
		15	0.016	0.008	604.02	0.000
		16	0.012	-0.002	604.46	0.000
		17	0.022	0.016	605.94	0.000
		18	0.027	0.012	608.32	0.000
		19	0.000	-0.018	608.32	0.000
		20	0.020	0.025	609.56	0.000

(d) Squared Residuals of SPI Futures

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.158	0.158	78.430	0.000
		2	0.066	0.043	92.283	0.000
		3	0.039	0.023	97.029	0.000
		4	0.039	0.028	101.90	0.000
		5	0.030	0.018	104.77	0.000
		6	0.049	0.039	112.28	0.000
		7	0.049	0.033	119.83	0.000
		8	0.013	-0.006	120.35	0.000
		9	0.047	0.039	127.16	0.000
		10	0.022	0.005	128.73	0.000
		11	0.013	0.001	129.28	0.000
		12	0.022	0.014	130.75	0.000
		13	0.046	0.035	137.36	0.000
		14	0.048	0.032	144.67	0.000
		15	0.011	-0.009	145.05	0.000
		16	0.034	0.024	148.67	0.000
		17	0.019	0.006	149.85	0.000
		18	0.040	0.029	155.01	0.000
		19	0.014	-0.005	155.60	0.000
		20	0.025	0.013	157.57	0.000

Notes: This table presents the plots and values of autocorrelation function (AC) and partial autocorrelation correlation function (PAC) of the residuals from Equation 5.2. The last two columns are Q-statistics for high order autocorrelation and the corresponding probability. The null hypothesis is that there is no autocorrelation at a certain order. The probabilities tell us that we have to accept the null of no autocorrelation up to 20 lags.

Table 8. The Estimates of MGARCH Model

Variable	Coefficient	Std. Error	t-statistic	Probability
c_{ss}	4.13E-07	4.56E-08	9.0512	0.0000
c_{sf}	3.84E-07	3.58E-08	10.7204	0.0000
c_{ff}	3.88E-07	4.57E-08	8.4900	0.0000
b_{ss}	0.07491	0.00338	22.1889	0.0000
b_{sf}	0.0716	0.00305	23.4462	0.0000
b_{ff}	0.07262	0.00278	26.1227	0.0000
a_{ss}	0.89672	0.00571	157.1213	0.0000
a_{sf}	0.90729	0.00428	212.2021	0.0000
a_{ff}	0.91578	0.00382	239.9111	NA

Notes: This table report the results estimated from the MGARCH Model in Equation 5.5. c_{ss} , c_{sf} and c_{ff} are constants. b_{ss},, b_{sf} and b_{ff} are coefficients of the conditional variances and covariances, respectively. a_{ss}, a_{sf} and a_{ff} are coefficients of the squared error terms, respectively.

Table 9. Hedging Performances Comparison

In Sample Comparison:

Forecast Horizons	Mean of the Hedge Ratio	Mean of the Return of the Hedged Portfolio	Variance of the Return of the Hedged Portfolio	Percentage in variance Reduction
One – Day				
OLS	0.4778	0.069%	0.00009%	88.58%
VAR	0.5083	0.067%	0.00009%	89.01%
Error-Corre.	0.5147	0.067%	0.00009%	89.12%
M-GARCH	0.5922	0.071%	0.00010%	87.95%
Five – Day				
OLS	0.4778	0.127%	0.00026%	67.99%
VAR	0.5083	0.126%	0.00026%	68.97%
Error-Corre.	0.5147	0.125%	0.00025%	69.22%
M-GARCH	0.5922	0.123%	**0.00024%**	71.08%
Ten – Day				
OLS	0.4778	0.145%	0.00029%	64.63%
VAR	0.5083	0.138%	0.00027%	67.92%
Error-Corre.	0.5147	0.136%	0.00026%	68.76%
M-GARCH	0.5922	0.090%	**0.00014%**	83.56%
Twenty – Day				
OLS	0.4778	0.136%	0.00028%	66.20%
VAR	0.5083	0.130%	0.00026%	68.61%
Error-Corre.	0.5147	0.129%	0.00025%	69.22%
M-GARCH	0.5922	0.108%	**0.00017%**	79.73%

Table 9. Continued

Post-Sample Comparisons:

Forecast Horizon	Mean of the Hedge Ratio	Mean of the Return of the Hedged Portfolio	Variance of the Return of the Hedged Portfolio	Percentage in variance reduction
One –Day				
OLS	0.4778	0.031%	0.00002%	99.04%
VAR	0.5083	0.032%	0.00002%	99.03%
Error-Corre.	0.5147	0.032%	0.00002%	99.03%
M-GARCH	0.5922	0.050%	0.00005%	97.60%
Five – Day				
OLS	0.4778	0.133%	0.00037%	82.11%
VAR	0.5083	0.132%	0.00035%	82.99%
Error-Corre.	0.5147	0.132%	0.00035%	83.21%
M-GARCH	0.5922	0.136%	**0.00028%**	86.57%
Ten – Day				
OLS	0.4778	0.130%	0.00031%	84.88%
VAR	0.5083	0.129%	0.00030%	85.47%
Error-Corre.	0.5147	0.129%	0.00030%	85.62%
M-GRCH	0.5922	0.123%	**0.00024%**	88.23%
Twenty – Day				
OLS	0.4778	0.189%	0.00079%	61.48%
VAR	0.5083	0.182%	0.00074%	64.16%
Error-Corre.	0.5147	0.181%	0.00072%	64.85%
M-GARCH	0.5922	0.132%	**0.00035%**	83.19%

Notes: The return on the hedge portfolio is calculated using Equation 5.6 for each hedge horizon. The percentage of variance reduction is calculated by substituting the Var(H) and Var(U) in Equation 5.7, 5.8 to Equation 5.9.

Figure 1. The Logarithm of AOI and SPI Series

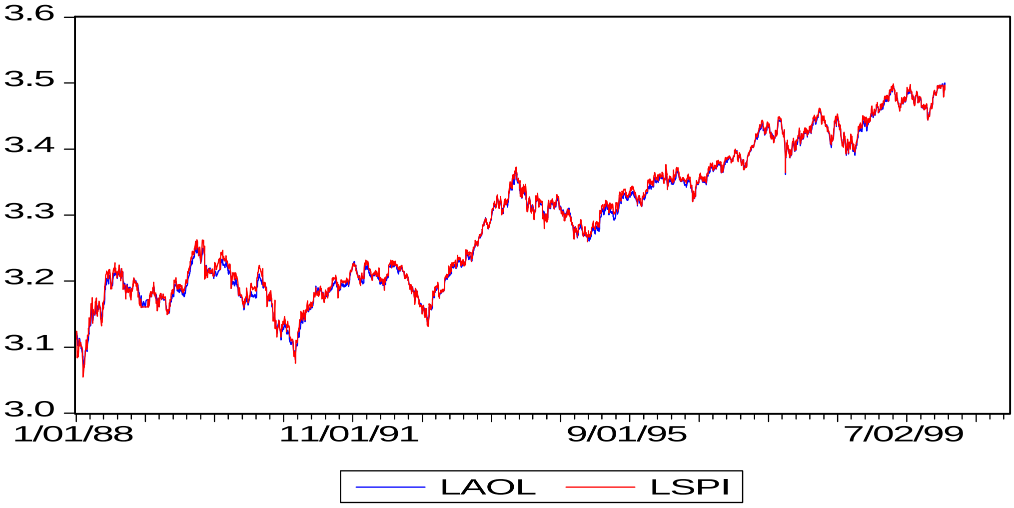


Figure 2. The Plot of Residuals

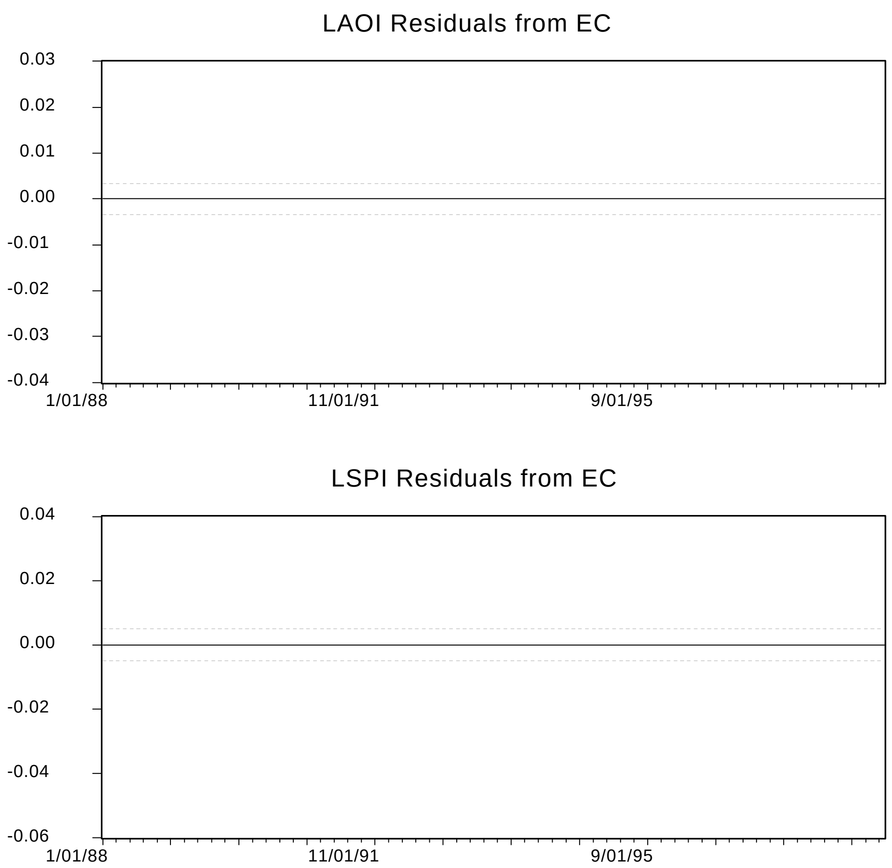
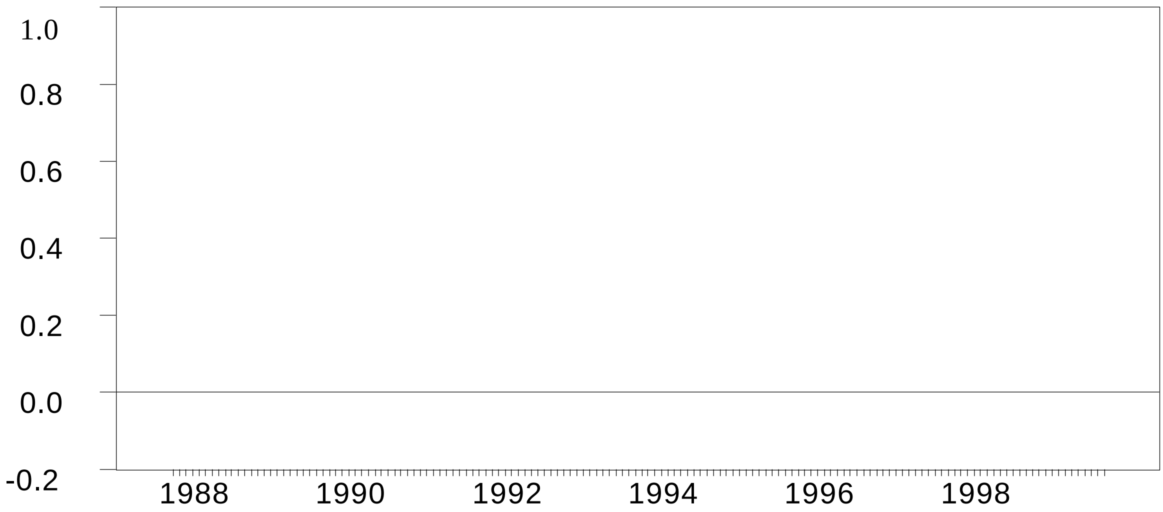


Figure 3. The time-Varying Hedge Ratios



Notes: The diagram plots the estimates of the time varying hedge ratios from the MGARCH model. It is shown that they are highly non-stationary and are mostly positive.