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Mahendra Chandra

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By

M. Chandra
Edith Cowan University

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Correspondence author and address:

Mr. M. Chandra
School of Accounting, Finance and Economics
Faculty of Business and Public Management
Edith Cowan University
100 Joondalup Drive
Joondalup WA 6027
Phone: 61+ 08 6304 5588
Fax: 61+ 08 6304 5271
Email: m.chandra@ecu.edu.au

Abstract

This paper examines the correlation estimates for some Asia-Pacific markets equity markets using the DCC-MGARCH, CC-MGARCH and a simple moving average regression based on a sliding window of 100 days. Using daily return series, the equity markets of Australia, Hong Kong, Japan and Singapore are analysed for the period 1990 to 2001. Parsimonious specifications for the multivariate GARCH framework are used to shed light on the correlation structure of these markets. The dynamic nature of the correlation between pair-wise countries is captured using the dynamic conditional correlation multivariate GARCH framework and explained. Both global as well as regional factors are seen to contribute to the correlation spikes for pair-wise markets. There is also evidence to suggest a higher comovement between markets since the Asian financial crisis. This paper provides a useful comparison of correlation estimates using a variety of specifications.

Keywords: Dynamic conditional correlation; Time-varying correlations; Asia-Pacific equity markets; GARCH models

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1. Introduction

The accurate estimation of correlations is instrumental in many financial applications. It is crucial in the evaluation of optimal portfolio weights, value-at-risk (VaR) measures and models of capital asset pricing. The covariance between national markets, for example, is a very important consideration in international portfolio diversification.

Correlation measures the association between two variables. It is essential that we understand the nature and strength of this association as it plays an important role in finance. The variance and correlations are both not observable. Hence, they need to be estimated. Estimation of these parameters can vary depending on the model specification.

For convenience, most multivariate time series techniques used in analysing returns and volatility assume the correlations to be constant, and possibly zero. For example, Bollerslev (1990) uses the constant correlation assumption to simplify the conditional covariance matrix in the multivariate generalised autoregressive conditional heteroscedasticity (GARCH) modelling process. If the non-constancy of correlation were significant, then models which assume constant correlations would be misspecified. Although some earlier studies have concluded that correlations across markets are constant, recent studies have found that the correlations are indeed time varying (see Tse, 2000; Tse and Tsui, 2002; Tsui and Yu, 1999).

There are many studies that report on the so-called 'stylised facts' about correlations. However, the specification of the models used to arrive at these facts need to be scrutinised. Solnik, et al. (1996) use a sliding window of 36 months to compute correlations between the US stock market and the markets of Germany, France, UK, Switzerland, Japan and an index comprising Europe, Asia and Far East (EAFE), and find that international correlations increase in periods of high volatility. Although they conclude that correlations between the US market and other markets are time varying, they also find that there has been a weakening in the correlations over the last decade. There are many studies that report the increase in correlations during periods of high volatility (see Karolyi and Stulz, 1996; Ramchand and Susmel, 1998; Bracker and Koch, 1999).

Longin and Solnik (1995) use a multivariate GARCH (1,1) model to test the hypothesis of constant conditional correlation, and conclude that correlations across the markets of Germany, France, United Kingdom, Switzerland, Japan, Canada and the US are dynamic and exhibit positive time trends in the conditional correlations, but not in the conditional variances. They also use a Threshold GARCH model and find that a positive or negative shock has the same impact on correlations. It is also found that when shocks to the US market are larger in absolute value than the unconditional US standard deviation, the correlation of the US with the German, French and Swiss markets increases. However, they were unable to establish asymmetry in responses to the correlations of these shocks, that is, there were no significant differences in the sensitivity of correlations between positive and negative shocks.

In understanding time-varying correlations, it also important to understand the factors that affect the cross-correlations between markets. Instead of analysing the time-varying nature of correlations, Bracker and Koch (1999) use daily data on ten markets to analyse the economic determinants of the correlation structure. They conclude that correlations are positively related to world market volatility, and negatively related to term structure differentials, real interest differentials and world market returns. Moreover, they detect a positive trend in the correlation across the markets and, as in Solnik, et al. (1996), find that this trend seems to be weakening.

Using daily returns between Japanese and US stocks, Karolyi and Stulz (1996) find strong evidence that covariances are higher when there are large contemporaneous returns shocks in national markets. They also demonstrate that there is a nonlinear relation between covariances and large market shocks, and explain this as evidence that large shocks to indices are more likely to be global shocks. Although they find that macroeconomic announcements and interest rate shocks do not significantly affect comovements, it is also found that correlations exhibit day-of-the-week effects, with Monday comovements being higher than on other days. As in Serra (2000), they find that controlling for industry effects has little or no impact on the comovements of markets.

Analysing cross-equity correlations for G7 countries, Erb, et al. (1994) conclude that correlations are also affected by business cycles. They report that correlations are highest when any two countries are in common recession, and are lower during recoveries and when business cycles in the two countries are out of phase. Such a finding is supported by Cheng (1998). Using canonical correlation analysis, Cheng finds that the comovement between the US and the UK markets is very high and that the US economic cycle is highly capable of accounting for the comovement between the US and the UK markets.

Using a multivariate switching ARCH framework, Ramchand and Susmel (1998) conclude that correlation is both time- and state-varying. They argue that the traditional ARCH and GARCH models are seriously affected by the presence of structural breaks, so that a switching ARCH model is suggested. Far East and North American markets are studied and the market in each country is characterised by high and low variance regimes. As in most studies in this area, the increase in correlations during periods of high volatility is reported.

However, using extreme value theory, Longin and Solnik (2001) argue that high volatility does not lead to an increase in conditional correlation. They find that correlation is primarily affected by market trends and that the correlation increases in bear but not in bull markets. In commenting on similar studies on correlations, they argue that it cannot be concluded that the 'true' correlation is changing over time by a simple comparison of estimated correlations conditional on different values of one return variable. The distribution of the conditional correlation must be clearly specified in order to test whether correlations increase in periods of high volatility.

Using regime-switching models, Ang and Bekaert (1999) find evidence of the presence of high volatility – high correlations regime in the US, UK and German equity returns when the market is bearish.

The time-varying nature of correlation would result in the rejection of a certain class of econometric models, such as the constant-correlation multivariate GARCH model, that assume constant correlation (see Longin and Solnik, 2001; Tse, 2000).

Recent work in this area has focused on the markets of US and Europe, and little research is available on the correlation structure in the Asia-Pacific markets. Using the multidimensional scaling technique on time-varying correlations, Groenen and Franses (2000) report that, instead of a single world market, there seem to be three distinct clusters of markets, namely, US, European markets and the Asian countries. As the first two clusters of markets have been analysed extensively, the purpose of this paper is to explore the time-varying nature of the correlation structure across the equity markets of various Asia-Pacific markets.

This paper uses the Constant Correlation multivariate GARCH (CC-MGARCH), the Dynamic Condition Correlations multivariate GARCH (DCC-MGARCH) and a simple correlation based on sliding window of 100 observations. Residual based diagnostics tests are employed to confirm the model specification and the correlation estimates are compared and explained.

2. Data

Daily return data series for the sample period 1990 to 2001 are used for Australia, Hong Kong, Japan and Singapore. The data are return series based on Datastream calculated Global

Equity Indices for equity markets. The Global Equity Indices are based on a representative sample of stocks covering up to 80% of the total market capitalisation. There are 3170 time series observations for each of the four indices making a total of 12,680 observations. The 5-day observations exclude Saturday and Sunday.

3. Model Development

The returns for the individual series is calculated based on the logged difference as below:

$$r_{i,t} = 100\,[\ln(P_{it}) - \ln(P_{it-1})]$$

Instead of using an autoregressive or moving average filtering process, the returns are made mean zero based on a simple demeaning using the unconditional mean of the return series. The zero mean return series, ε_{it}, is calculated for Australia (AU), Hong Kong (HK), Japan (JP) and Singapore (SP).

In extending the above to the multivariate GARCH formulation, the zero-mean return vector S_t is set to depend on the information set $\Im_{t-1}$ with a variance H_t. S_t is a 4 by 1 vector of time series where $S_t = (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, \varepsilon_{4t})$. We can express the general form of the multivariate GARCH as:

$$S_t \mid \Im_{t-1} \sim N(0, H_t)$$

where, H_t is a 4 by 4 covariance matrix. As with the univariate case, the main issue is in determining the form H_t should take.

In the univariate case, the unconditional disturbance can be expressed as:

$$\varepsilon_t = \eta_t \sqrt{h_t}$$

where, η_t ~ niid and the conditional variance, h_t, can be specified to follow the GARCH (1,1) process such that:

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$$

where, $\omega > 0,\ \alpha \geq 0, \beta \geq 0$.

In the multivariate case, H_t can be expressed as a vector form, which requires stacking the lower triangular elements of a symmetrical matrix in a column. In general, for the bivariate GARCH (1,1) model, H_t can take the form:

$$vech(H_t) = A_0 + A_i vech(\varepsilon_{t-i}\varepsilon'_{t-i}) + B_j vech(H_{t-j})$$

where, ε_t is a vector of zero mean series from some filtration process and A_0 is a $\frac{N(N+1)}{2}$ by 1 vector (for the bivariate case, it would be a 3 by 1 vector). This formulation is termed vec representation by Engle and Kroner (1995). In this representation, the covariance matrix is dependent on the *p* and *q* lagged squared residuals and past variances of all variables in the system. For a GARCH(1,1) vec model, omitting the redundant elements of the equation, the formulation is as follows:

$$vech(H_t) = \begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} a_{01} \\ a_{02} \\ a_{03} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon^2_{2,t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix}$$

The problem with the above specification is the over-parameterisation of the model. For example the parameters to be estimated for a four-market GARCH(1,1) model without any exogenous variables would be:

$$\left[\frac{N(N+1)}{2}\right] \times \left[1 + \frac{(p+q)N(N+1)}{2}\right]$$
$$= 210$$

Some restrictions, therefore, have to be imposed so that the number of parameters to be estimated is reduced. In specifying H_t, we also have to ensure that it is positive definite for all realisation.

Instead of each conditional covariance having to depend on all variables in the system, for example as in the vech specification where for the element $h_{12,t}$,

$$h_{12,t} = a_{02} + a_{21}\varepsilon^2_{1,t-1} + a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{23}\varepsilon^2_{2,t-1} + b_{21}h_{11,t-1} + b_{22}h_{12,t-1} + b_{23}h_{22,t-1},$$

the diagonal vech specifies that the conditional covariance depends on its own lagged squared residuals and lagged cross-product of the residuals. The matrices A_i and B_j are assumed to be diagonal. Thus in the diagonal representation, $h_{12,t}$ simplifies to:

$$h_{12,t} = a_{02} + a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + b_{22}h_{12,t-1}$$

In this representation, the conditional variance and covariance takes on a GARCH(1,1) type specification. Bollerslev, et al (1988) use this specification to study the returns on bills, bonds and stocks.. However, the restrictive nature of this specification is inconvenient and the positive definiteness of H_t is a difficult condition to impose during estimation.

To overcome this, Engle and Kroner (1995) introduced the BEKK (acronym for Baba, Engle, Kraft and Kroner) representation, which guarantees positive definiteness. The BEKK representation specifies H_t to be:

$$H_t = A_0 + \sum_{k=1}^{K}\sum_{i=1}^{q} A_{ik}^{*\prime}\varepsilon_{t-i}\varepsilon_{t-i}A_{ik}^{*} + \sum_{k=1}^{K}\sum_{i=1}^{q} B_{ik}^{*\prime}H_{t-i}B_{ik}^{*}$$

where, A_0, A_{ik}^* and B_{ik}^* are N by N parameter matrices. H_t is positive if A_0 is positive. For a bivariate GARCH(1,1) model, the BEKK model specification of H_t is:

$$H_t = \begin{bmatrix} a_{11}^0 & a_{12}^0 \\ a_{21}^0 & a_{22}^0 \end{bmatrix} + \begin{bmatrix} a_{11}^* & a_{12}^* \\ a_{21}^* & a_{22}^* \end{bmatrix}' \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}\varepsilon_{1,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11}^* & a_{12}^* \\ a_{21}^* & a_{22}^* \end{bmatrix} + \begin{bmatrix} b_{11}^* & b_{12}^* \\ b_{21}^* & b_{22}^* \end{bmatrix}' H_{t-1} \begin{bmatrix} b_{11}^* & b_{12}^* \\ b_{21}^* & b_{22}^* \end{bmatrix}$$

In contrast with the vec and the diagonal vech, the representation for $h_{12,t}$ in the BEKK model becomes:

$$\begin{aligned} h_{12,t} &= a_{12}^0 + a_{21}^* a_{11}^* \varepsilon_{1,t-1}^2 + (a_{11}^* a_{22}^* + a_{12}^* a_{21}^*)\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}^* a_{22}^* \varepsilon_{2,t-1}^2 + b_{21}^* b_{11}^* h_{11,t-1} \\ &+ (b_{11}^* b_{22}^* + b_{11}^* b_{21}^*) h_{12,t-1} + b_{21}^* b_{22}^* h_{22,t-1} \end{aligned}$$

The number of parameters to be estimated in the BEKK specification is still large. For a four market GARCH(1,1) the number of parameters is reduced from 210 in the vech to 42 in the BEKK.

A simpler way of specifying H_t was introduced by Bollerslev (1990). In Bollerslev's Constant Correlation multivariate GARCH (CC-MGARCH) specification, the conditional correlation matrix is assumed constant. The conditional variance matrix is specified as $H_t \equiv D_t R D_t$. In the bivariate case the representation of H_t takes the form:

$$H_t = \begin{bmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{bmatrix} \begin{bmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{bmatrix} \begin{bmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{bmatrix}$$

Hence the conditional correlation $\rho_{12} = \dfrac{h_{12,t}}{\sqrt{h_{11,t}h_{22,t}}}$ is time invariant.

Baillie and Bollerslev (1990) used the above specification to provide a model for exchange rate interdependencies and time dependent risk premia for four major European currencies. In order to simplify estimation, they assumed that all the variations over time in the conditional covariances are due to changes in each of the corresponding two conditional variances. They found that the conditional correlations were highly significant between the four markets.

Engle (2002) extends the multivariate GARCH process to allow for correlations to be time variant. The Dynamic Conditional Correlation multivariate GARCH (DCC-MGARCH) proposed by Engle allows for the correlation component to assume a GARCH type specification. This formulation is similar to the time-varying correlations multivariate GARCH (VC-MGARCH) model proposed by Tse and Tsui (2002).

In the DCC-MGARCH model, the conditional variance is:

$$H_t \equiv D_t R_t D_t$$

where, R_t is the time varying correlation matrix and D_t is a N by N diagonal matrix of conditional standard deviations estimated from the univariate GARCH model. The difference between the specification of H_t in this model and that of Bollerslev (1990) is that the correlation, R_t is allowed to vary with time so that the dynamic nature of the correlation can be captured.

This paper uses a four market DCC(1,1)-MVGARCH(1,1) specification. The elements of the matrix D_t will take the form:

$$D_t = \begin{bmatrix} \sqrt{h_{11,t}} & 0 & 0 & 0 \\ 0 & \sqrt{h_{22,t}} & 0 & 0 \\ 0 & 0 & \sqrt{h_{33,t}} & 0 \\ 0 & 0 & 0 & \sqrt{h_{44,t}} \end{bmatrix}$$

Unlike Tse and Tsui (2002) the DCC-MGARCH uses a two-stage estimation procedure. The first stage is the conventional univariate GARCH parameter estimation for each zero mean series. The residuals from the first stage are then standardised and used in the estimation of the correlation parameters in the second stage.

The correlation structure is given as:

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1}$$

The covariance structure is specified by a GARCH type process as below:

$$Q_t = (1 - \lambda_1 - \mu_1)\overline{Q} + \lambda_1(\eta_{t-1}\eta_{t-1}^{'}) + \mu_1 Q_{t-1}$$

where, the covariance matrix, Q_t, is calculated as a weighted average of $\overline{Q}$, the unconditional covariance of the standardised residuals; $\eta_{t-1}\eta_{t-1}^{'}$, a lagged function of the standardised residuals; and Q_{t-1} the past realisation of the conditional covariance. In the DCC(1,1) specification only the first lagged realisation of the covariance of the standardised residuals and the conditional covariance are used. This requires the estimation of two additional parameters, λ_1 and μ_1. Q_t^* is a diagonal matrix whose elements are the square root of the diagonal elements of Q_t. Hence, for a four-market specification it would take the form:

$$Q_t^* = \begin{bmatrix} \sqrt{q_{11,t}} & 0 & 0 & 0 \\ 0 & \sqrt{q_{22,t}} & 0 & 0 \\ 0 & 0 & \sqrt{q_{33,t}} & 0 \\ 0 & 0 & 0 & \sqrt{q_{44,t}} \end{bmatrix}$$

The off diagonal elements in the matrix R_t will hence take the form:

$$\rho_{12,t} = \frac{q_{12,t}}{\sqrt{q_{11,t}q_{22,t}}}$$

where, $\rho_{12,t}$ is the conditional correlation between market 1 and market 2. It follows that if $\overline{Q}$ and $\eta_{t-1}\eta_{t-1}^{'}$ are positive definite and diagonal, then Q_t will also be positive definite and diagonal. For further discussion on the conditions for positive definiteness refer to Engle and Sheppard (Engle, 2001). The log likelihood of this is given by Engle and Sheppard (2001) as:

$$L = -\frac{1}{2}\sum_{t=1}^{T}(k\log(2\pi) + 2\log|D_t| + \log|R_t| + \eta_t^{'}R_t^{-1}\eta_t)$$

where, η_t is the standardised residual derived from the first stage univariate GARCH estimation, which is assumed to be *n.i.d.* with a mean zero and a variance, R_t. That is, $\eta_t = \frac{\varepsilon_t}{\sqrt{h_t}}$ for the individual series. Hence, the variance matrix, R_t, is also the correlation matrix of the original zero mean return series.

4. Results

The summary statistics for the log differenced return series for the four markets are given in Table 1. As with most financial time series, all the series in the table exhibit excess kurtosis. The series for Australia, Hong Kong and Singapore are negatively skewed, and the series for Japan is positively skewed. The Jacque-Bera test for normality rejects normality for all the series. Q(25) is the Ljung-Box Q-statistics to test for the hypothesis of no autocorrelation up to order 25. This hypothesis is rejected for all the series. SQ(25) is the same statistics calculated for the squared values of the series.

Table 1: Summary statistics for the differenced logarithmic return series.

	Australia	**Hong Kong**	**Japan**	**Singapore**
Mean	0.0443	0.0533	-0.0264	0.0125
Maximum	5.7779	15.5518	9.3987	8.9074
Minimum	-6.9695	-13.5844	-7.2099	-8.5466
Standard Deviation	0.8397	1.6488	1.2517	1.1920
Skewness	-0.2828	-0.1366	0.1814	-0.0751
Kurtosis	7.2625	12.1923	7.2630	9.6229
Jacque-Bera	2441.2950	11167.1600	2416.9480	5794.7480
Q(25)	45.5750	64.2720	105.9000	81.9200
SQ(25)	350.4900	1082.1000	493.3900	860.8900

Note: Q(25) is the Ljung-Box Q-statistics for Ho: No autocorrelation up to order 25. SQ(25) is the Q-statistics for the squared logarithmic return series. The Jacque-Bera test statistic for normality follows a χ^2 distribution with two degrees of freedom.

Figure 1 shows the market indices and the log-differenced returns for the four markets. The diagrams for the market series indicate that Australia had the least tumultuous time out of the four markets during the last decade. The markets of Hong Kong, Japan and Singapore declined markedly during the 1997 Asian currency crisis. The diagram on the returns confirms this observation. The returns are scaled uniformly across the four markets. The returns on the Hong Kong index show the highest volatility.

Figure 1: Total market series and the return series for the four markets

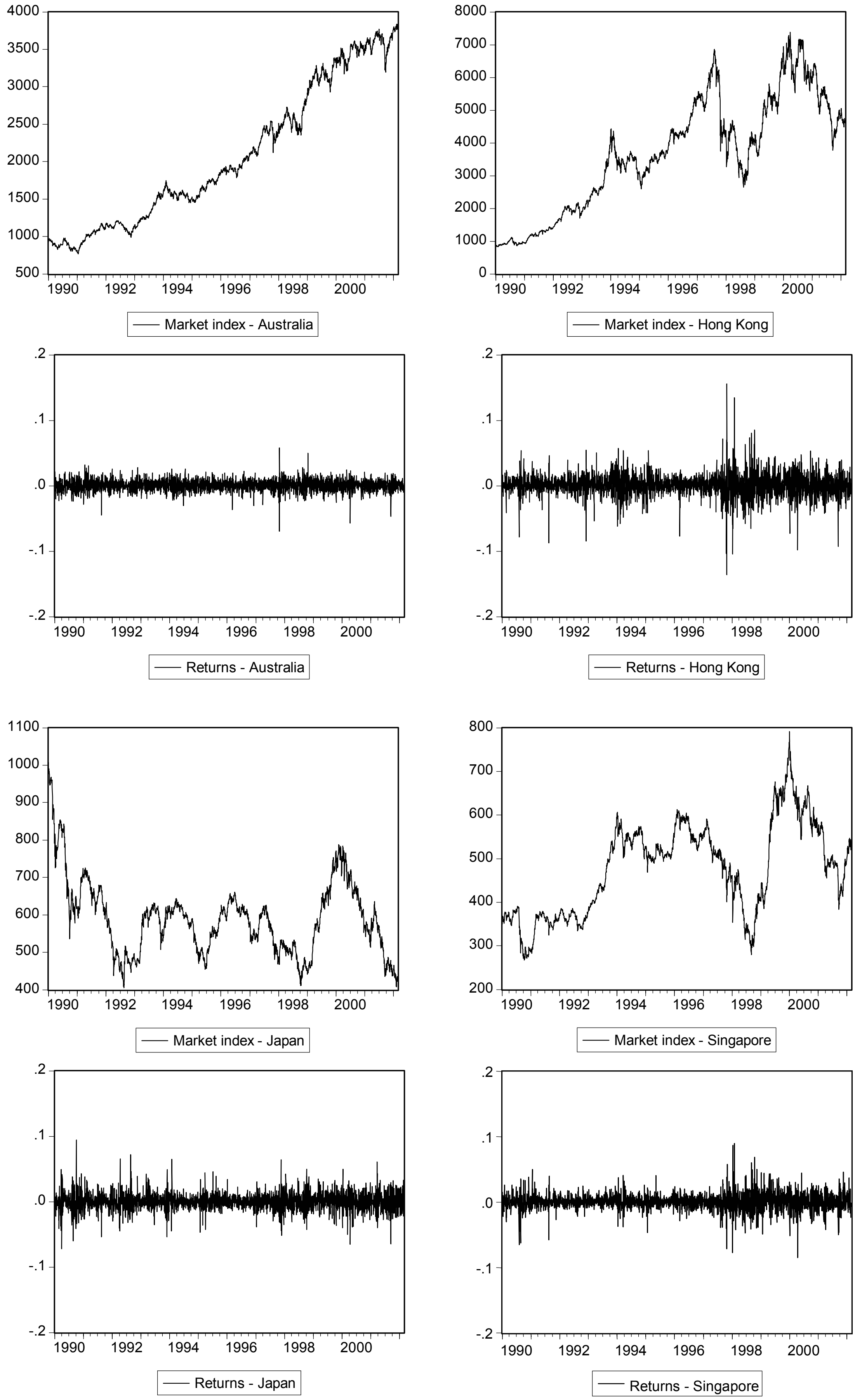


Table 2 shows the parameter estimates for the CC-MGARCH (1,1) model. The statistic reported in the parenthesis is the robust standard errors. The short run persistence α , ranges from 0.0797 for Australia to 0.1403 for Singapore. The long run persistence $\alpha + \beta$ for all the series is above 0.90, which indicates long memory processes. Also $\alpha + \beta < 1$ for all the series which indicates that the conditional variance is finite and that the series are strictly stationary and ergodic.

Table 2: Parameter estimates for the CC-MGARCH(1,1) model.

	GARCH parameters			
	ω	α	β	$\alpha + \beta$
Australia	0.0500 (0.0071)	0.0797 (0.0134)	0.8491 (0.0032)	0.9288
Hong Kong	0.0634 (0.0194)	0.1029 (0.0121)	0.8762 (0.0027)	0.9791
Japan	0.0498 (0.0105)	0.1132 (0.0120)	0.8598 (0.0031)	0.9730
Singapore	0.0353 (0.0076)	0.1403 (0.0147)	0.8443 (0.0068)	0.9846
	Constant-Correlation estimates			
	Australia	**Hong Kong**	**Japan**	**Singapore**
Australia	1.0	0.4065 (0.0160)	0.3556 (0.0159)	0.3389 (0.0182)
Hong Kong		1.0	0.3417 (0.0179)	0.4732 (0.0147)
Japan			1.0	0.3111 (0.0181)
Singapore				1.0

Note: Table 2 shows the parameter estimates for the CC-MGARCH (1,1) model. The robust standard errors are reported in parenthesis. The lower panel of the table shows the pair-wise correlation estimates for the four markets along with the robust standard errors in the parenthesis.

The correlations between the markets are all positive. The highest correlation is 0.4732, between Hong Kong and Singapore. The lowest is 0.3111, between Japan and Singapore. The CC-MGARCH(1,1) correlation estimate between Hong Kong and Singapore is lower in Tse and Tsui (2002). However, the sample period in that study is from January 1990 to March 1998.

Table 3 reports on the parameter estimates for the DCC(1,1)-MGARCH(1,1) using the same set of data for the four markets. The coefficients for all the parameters are positive. The GARCH (1,1) parameters are similar to the estimates using the constant correlation model which is also a similar finding in Tse and Tsui (2002). The long run persistence is close to but less than unity.

The DCC parameters summed to less that one which implies that the model is strictly mean reverting.

Table 3: Parameter estimates for DCC(1,1) – MGARCH(1,1) model

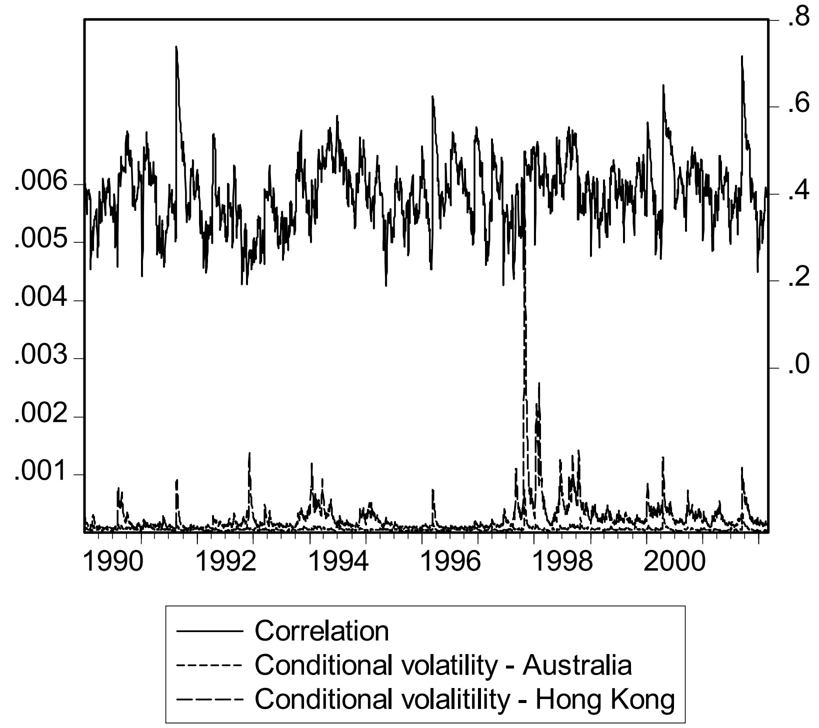
	GARCH (1,1) parameters			
	ω	α	β	$\alpha + \beta$
Australia	0.0500 (0.0071)	0.0797 (0.0135)	0.8491 (0.0033)	0.9288
Hong Kong	0.0634 (0.0194)	0.1029 (0.0120)	0.8762 (0.0027)	0.9791
Japan	0.0498 (0.0104)	0.1132 (0.0122)	0.8598 (0.0031)	0.9730
Singapore	0.0353 (0.0075)	0.1403 (0.0150)	0.8443 (0.0068)	0.9886
	DCC (1,1) parameters			
		λ	μ	$\lambda + \mu$
		0.0264 (0.0059)	0.9270 (0.0226)	0.9534

Note: The figures in parenthesis are the robust standard errors. The standard errors for the GARCH parameters are the standard Bollerslev-Woodridge robust standard errors whereas the standard errors for the DCC parameters are modified standard errors (See Engle and Sheppard (2001) for a detailed discussion on this)

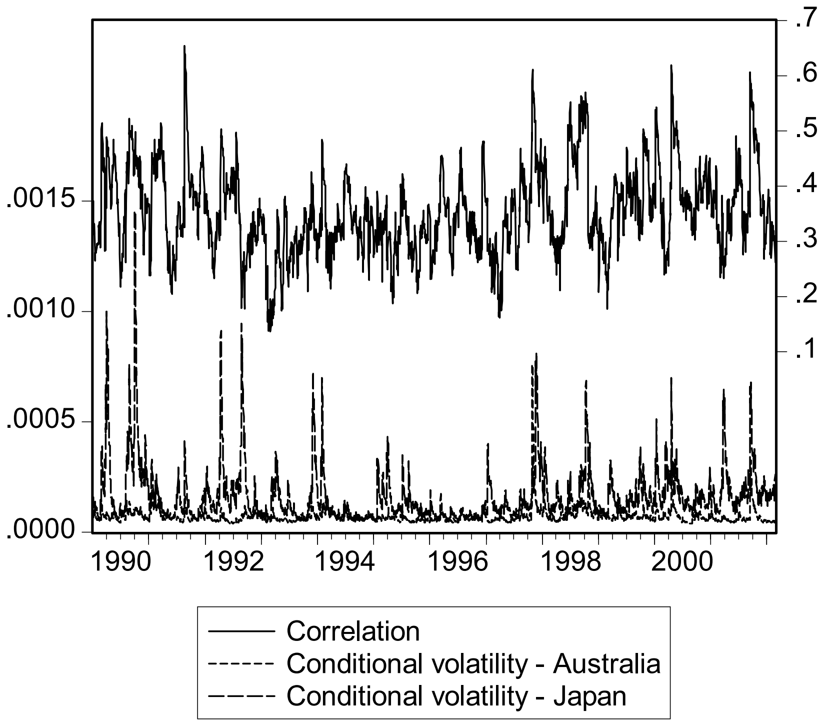
The time-varying correlation estimates using the DCC(1,1)-MGARCH(1,1) model are graphed in figure 2 along with the plots of the conditional variances of the paired markets. The scale on the left of the graph reflects the volatility of the two markets and the scale on the right reflects the correlation. Some interesting patterns are discernible from the DCC correlation estimates.

Figure 2: Time-varying correlations estimated from DCC-MGARCH

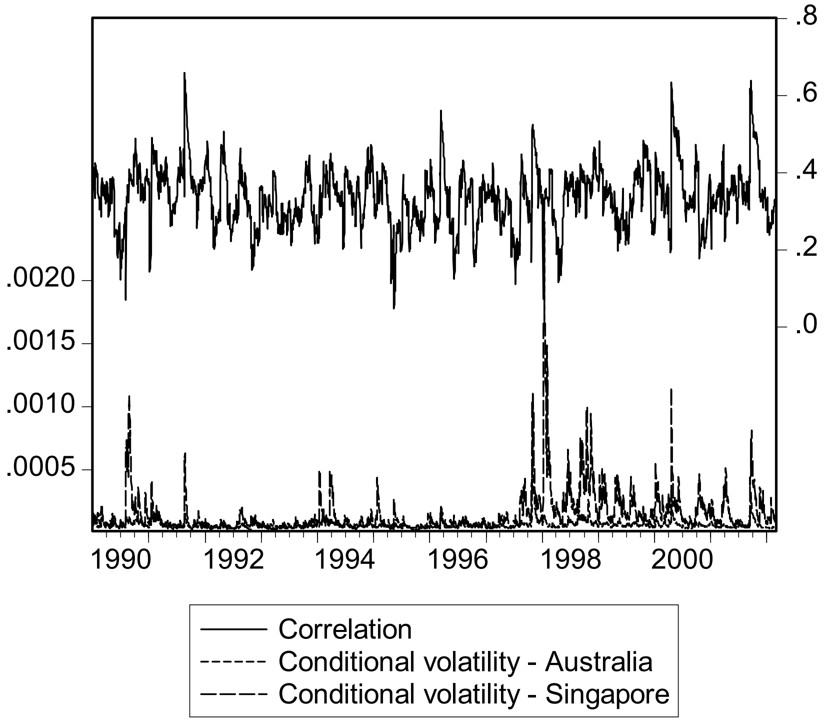
(a)



(b)



(c)



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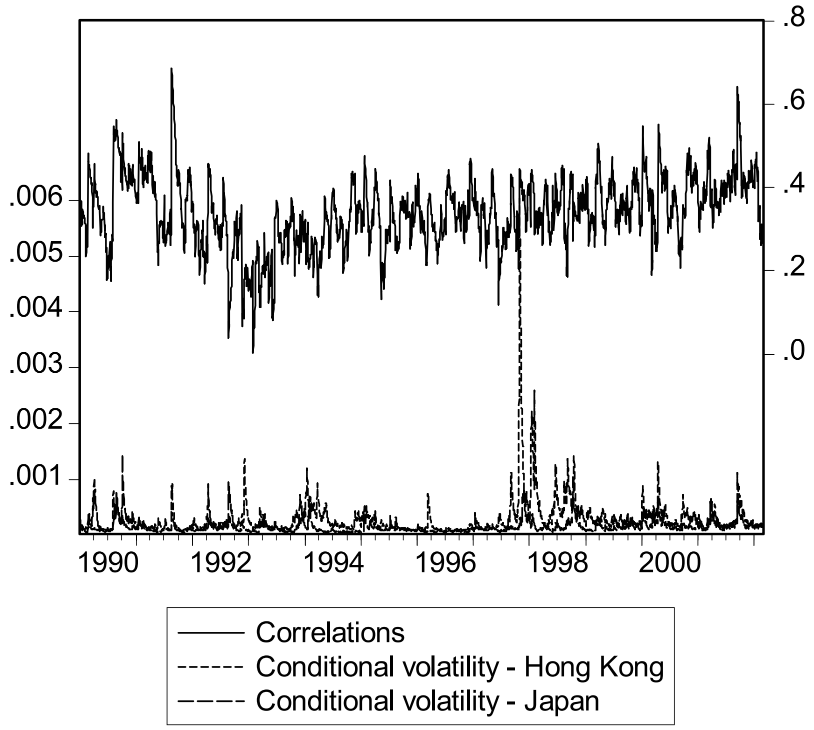
1990
1992
1994
1996
1998
2000
Correlation
Conditional volatility - Australia
Conditional volatility - Singapore

(d)



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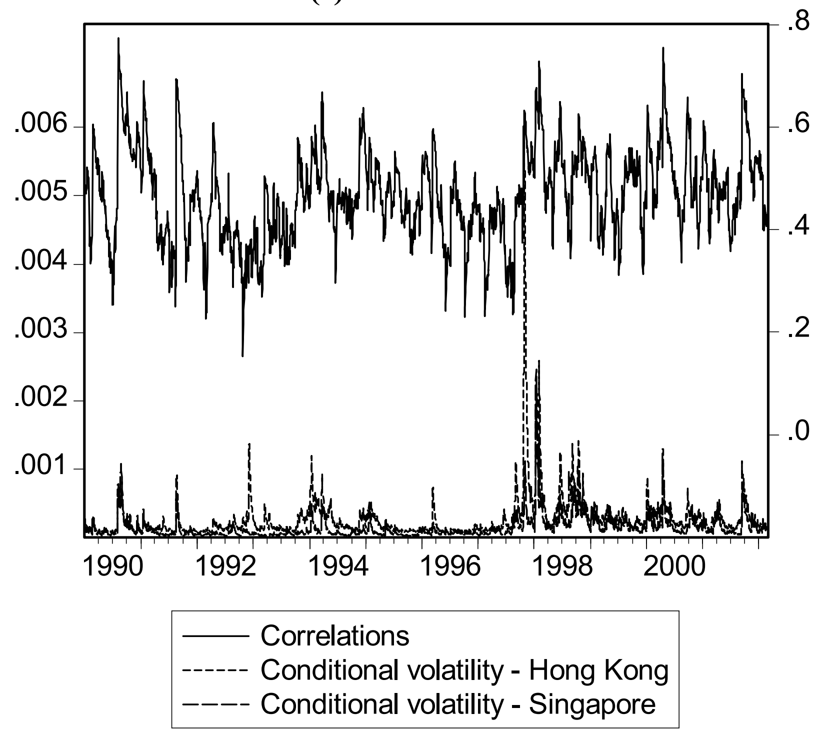
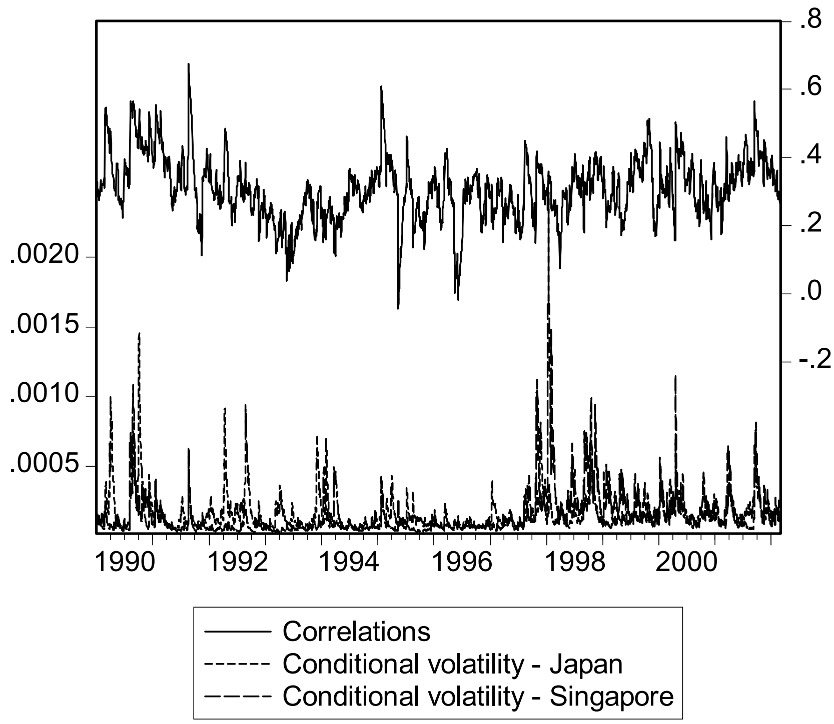
1990
1992
1994
1996
1998
2000
Correlations
Conditional volatility - Hong Kong
Conditional volatility - Japan

(e)



(f)



Note: Figure 2 – Panels (a) to (f) show the correlation and the conditional variance estimates based on the DCC(1,1)-MGARCH(1,1) model. The scale to the left of the panel refers to the volatility and the scale to the right refers to the correlation estimates.

The first panel shows the volatility of and correlation between the Australian and the Hong Kong markets. The conditional volatility of Hong Kong is much higher than that of Australia. The average correlation is 0.4022. The highest correlation was 0.7395 and the lowest was 0.1885. The correlations between the two markets show distinct spikes in August 1991, June 1994, March 1996, April 2000 and September 2001. These were significant times in the history of the two markets. The extreme correlation in August 1991 coincides with the Soviet coup in Moscow, which caused uncertainty in the markets around the world. This shock is present in all other paired correlations that are plotted in Figure 2. The period around June 1994 was a bearish period for the Hong Kong market. The market declined by 30% amid worries about rising interest rates and China's inflation problems. The Hong Kong market was nervous around March 1996 when China's handpicked Preparatory Committee voted to replace Hong Kong's Legislative Council elected in the previous September. The Tech meltdown occurred in April 2000, which saw the NASDAQ decline 25.3% in one week recording a massive loss of more than a trillion dollars. The impact of this event was felt worldwide. The correlation spike for this period is present for all paired markets. This is also true for the last spike in September 2001. The September 11 terrorist attack on the twin towers sent markets around the world tumbling.

It is interesting to note that the correlations did not spike during Hong Kong's most volatile period during the Asian financial crisis. This is not surprising as Australia was one of the least affected markets during the crisis in Asia.

Panel (b) shows the volatility and correlations between Australia and Japan. The average correlation is 0.3526 with the highest being 0.6590 and the lowest, 0.1354. The correlation spikes between Australia and Japan occur in August 1991, September 1997, April 2000 and September 2001. These coincide with the explanation given above. It is interesting to note that the correlation between Australia and Japan was high during the Asian financial crisis.

The conditional correlation between Australia and Singapore is shown in panel (c). The highest correlation between the two countries is 0.6590 and the lowest is 0.0471 with the average being 0.3352. The correlation spikes in August 1991, April 2000 and September 2001 which coincide with major world events.

The average correlation between Hong Kong and Japan is 0.3394 with the highest being 0.6861 and the lowest being 0.0029. Again the extreme volatility occurs in August 1991, April 2000 and September 2001.

Among the four markets, the Hong Kong and Singapore markets show the highest correlation. The average correlation between the two markets is 0.4667. The highest is 0.7740 and the lowest, 0.1533. Panel (e) shows that correlation spikes occur more frequently between the two markets. There are seven notable spikes in August 1990, January 1991, August 1991, March 1994, January 1995, February 1998, April 2000 and September 2001. The extreme correlation in August 1990 coincides with Iraq's invasion of Kuwait and the subsequent oil shock. Most of the correlations between the other markets were generally high around this period. There is a big increase in the correlations between the markets around 1994. There were spikes in the conditional volatility of these two markets as the Singapore market reacted to the rather steep decline in the Hong Kong market. Again the spikes in volatility in 1995 is common to both markets as much of it was due to Hong Kong poor economic performance. The Kobe earthquake and the subsequent decline in the Japanese market which resulted in the Barings disaster also played a part.

Japan and Singapore show the lowest correlation among the four markets. The average correlation between the two markets is 0.3075. There are fewer correlation spikes and these occur in August 1991, January 1995 and September 2001.

In examining the correlation and the volatility between the markets, it can be observed that extreme volatility not necessarily results in extreme correlations. Japan seems to be the least correlated market among the four. Correlation between Australia and Singapore is also low.

The timing of the correlation spike suggests that some of the extreme correlations were a result of global factors that impacted on all the markets. For example, all the markets show a correlation spike during the September 11, terrorist attack and the tech disaster in 1994.

For markets that are closely linked like the Hong Kong and Singapore markets, there are regional factors that might impact on its correlation. The term 'extreme correlation' is used loosely here. The extreme correlations identified in this paper were based on visual inspection. The idea of extreme value for a bounded variable like the correlations is a concept that needs further research.

Table 4 shows the summary statistics for the correlations calculated using the DCC(1,1)-MGARCH(1,1) model.

Table 4: The average correlations calculated using the conditional correlations estimate over the sample period.

	Average correlations – DCC(1,1)-MGARCH(1,1) model			**Post 1997 correlations**
	Mean	**Maximum**	**Minimum**	**Mean**
$\rho_{Aus,HK}$	0.4022	0.7395	0.1885	0.4183
$\rho_{Aus,JP}$	0.3526	0.6528	0.1354	0.3840
$\rho_{Aus,SG}$	0.3352	0.6590	0.0471	0.3546
$\rho_{HK,JP}$	0.3394	0.6861	0.0029	0.3804
$\rho_{HK,SG}$	0.4667	0.7740	0.1533	0.4957
$\rho_{JP,SG}$	0.3075	0.6743	-0.0444	0.3347

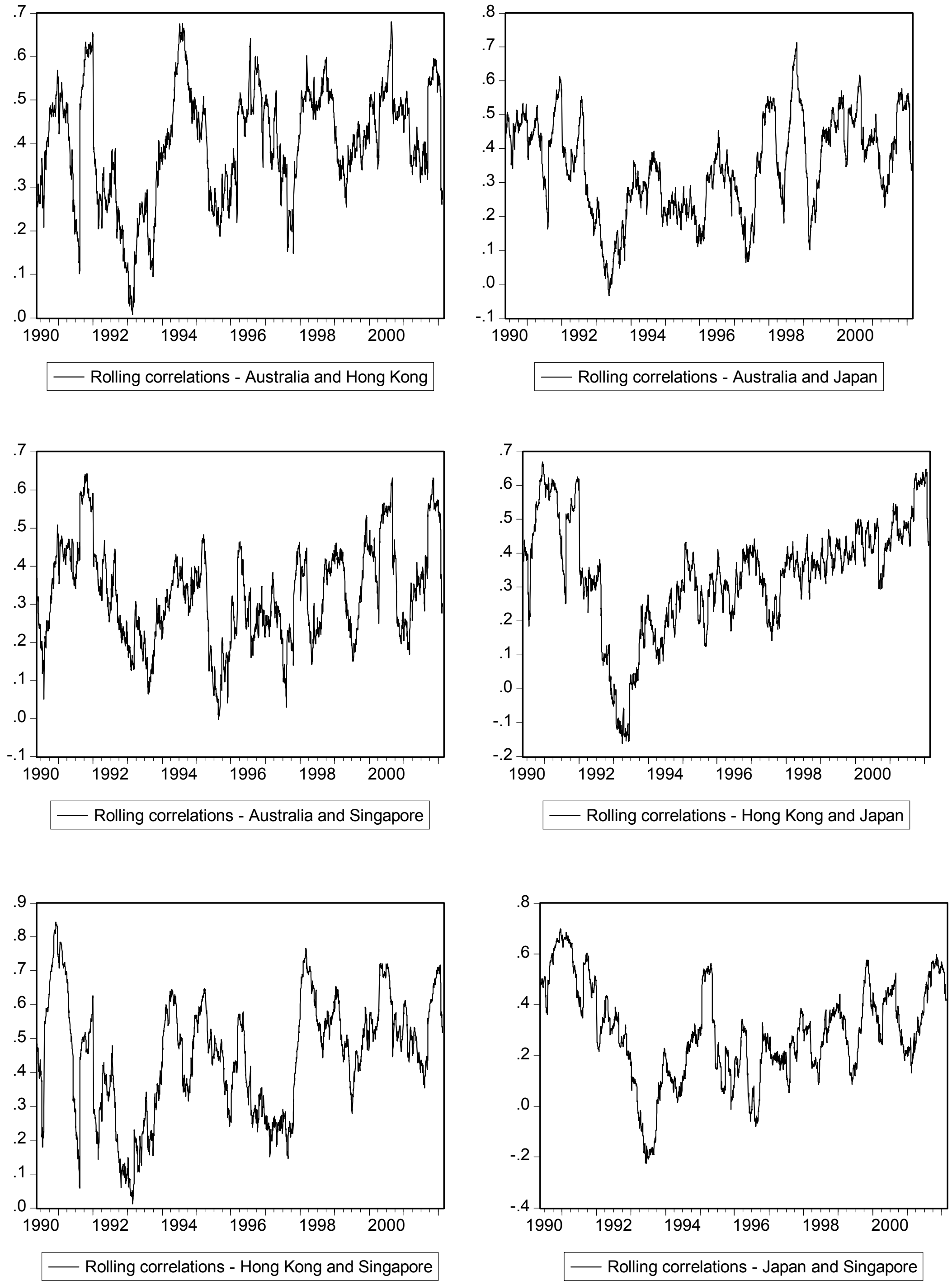
Note: The post-1997 correlations are the arithematic mean of the correlations from 01 July 1998. There were 953 correlations for this sample period.

Taking an arbitrary date of July 01, 1998, post-Asian financial crisis correlations are calculated for all the paired markets. There is an increase in the correlations for all pairs. The increases were notably higher Hong Kong/Japan, Australia/Japan and Hong Kong/Singapore and Japan/Singapore, which might provide some anecdotal evidence to claim that the region is becoming more integrated although caution must be exercised.

To contrast the time-varying estimates from the DCC model, Figure 3 shows the correlations estimates using a sliding window of 100 observations based on the standardised residuals of a GARCH(1,1) process.

One of the criticisms of using a sliding window is that equal weighting is given to all the observations hence distorting the true conditional correlation structure. There is higher dispersion in the correlations calculated using this method. Table 5 reports on the summary statistics for the correlations for the six pairs of markets.

Figure 3: Correlation estimates based on a sliding window of 100 observations



Note: Figure 3 plots the correlation estimates based on a sliding window of 100 observations. The standardised residual from a GARCH(1,1) process is used.

Table 5: The average correlations calculated based on standardised residuals using a 100-day sliding window.

	Average correlations – 100-day sliding window		
	Mean	**Maximum**	**Minimum**
$\rho_{Aus,HK}$	0.4000	0.6800	0.0082
$\rho_{Aus,JP}$	0.3458	0.7134	-0.0339
$\rho_{Aus,SG}$	0.3351	0.6424	-0.0034
$\rho_{HK,JP}$	0.3354	0.6703	-0.1658
$\rho_{HK,SG}$	0.4463	0.8433	0.0115
$\rho_{JP,SG}$	0.2873	0.6980	-0.2255

Again, the highest correlation is between Hong Kong and Singapore. A comparison of the estimates of correlations from the three models is provided in Table 6. The correlation estimates for the three models are similar. The 100-day sliding window method of estimation seems to consistently under estimate the correlation for all pairs of markets compared to the DCC model. The CC-MGARCH(1,1) model seems to consistently over estimate the correlations for all pairs compared to the DCC model.

Table 6: Comparison of average correlations

	Average correlations		
	DCC(1,1)-MGARCH(1,1)	**CC-MGARCH(1,1)**	**100-day sliding window**
$\rho_{Aus,HK}$	0.4022	0.4065	0.4000
$\rho_{Aus,JP}$	0.3526	0.3556	0.3458
$\rho_{Aus,SG}$	0.3352	0.3389	0.3351
$\rho_{HK,JP}$	0.3394	0.3417	0.3354
$\rho_{HK,SG}$	0.4667	0.4732	0.4463
$\rho_{JP,SG}$	0.3075	0.3111	0.2873

The ranking of the strength of the correlations is somewhat similar. Hong Kong and Singapore is consistently ranked the most correlated markets followed by Australia and Hong Kong. All three models show that the lowest correlation is between Japan and Singapore.

Table 7 reports on the diagnostics conducted on the standardised residuals of the DCC(1,1)-MGARCH(1,1) and the CC-MGARCH(1,1) models. Both the models report reduced kurtosis in the standardised residuals.

Table 7: Diagnostics on standardised residuals

	Australia	**Hong Kong**	**Japan**	**Singapore**
DCC(1,1)-MGARCH(1,1)				
Mean	-0.0006	0.0027	-0.0154	-0.0047
Standard Deviation	1.006	0.9944	0.9995	0.9953
Skewness	-0.0015	-0.4024	0.2052	0.2137
Kurtosis	4.487	6.2834	4.9977	5.6171
Q(25)	39.3840	45.4420	71.3020	56.5710
SQ(25)	26.0510	10.1380	24.5340	22.5770
LM	5.8912	2.2532	1.1218	8.6641
p-value	0.3170	0.8131	0.9522	0.1232
CC-MGARCH(1,1)				
Mean	0.0043	0.0029	-0.0140	-0.0057
Standard deviation	0.9995	0.9988	1.0009	1.0005
Skewness	0.0106	-0.4051	0.2072	0.2337
Kurtosis	4.2995	6.2018	5.0487	5.6997
Q(25)	40.9130	50.7800	71.6950	59.8820
SQ(25)	26.0510	9.5352	24.7700	24.6860
LM	5.4665	2.2151	1.8507	9.0126
p-value	0.3616	0.8186	0.8694	0.1086

The summary statistics is computed from the standardised residuals of the models. Q(25) is the Ljung-Box Q-statistics for Ho: No autocorrelation up to order 25, calculated using the standardised residuals. SQ(25) is the Ljung-Box Q-statistics based on the squared standardised residuals. LM is the Engle's LM test for no ARCH up to 5 lags. This statistic is the product of the number of observations multiplied by the R^2. The p-values are the corresponding chi-square probability values for the LM test statistics.

The Q(25) statistics for the standardised residuals for both models are lower than that reported in Table 1. The SQ(25) statistics is based on the squared standardised residuals and is markedly lower that those reported in Table 1. The Lagrange Multiplier (LM) tests on the standardised residuals of both models indicate no ARCH effects in the residuals hence there are no misspecifications in the models.

5. Conclusion

This paper provides estimates of correlations for the equity markets of Australia, Japan, Hong Kong and Singapore using a variety of models. Conditional correlation estimates based on the DCC(1,1)-MGARCH(1,1) model are compared to the conventional CC-MGARCH and a sliding window estimation procedure. It can be observed from the DCC model that extreme volatility shocks does not necessarily result in extreme correlation shocks. There are correlation shocks that are common to all markets, which suggest that there are global factors impacting on all the markets - a conclusion similar to Karolyi and Stulz (1990). For markets such as Hong Kong and Singapore, which showed the highest correlations, regional factors also explain some of the correlation shocks.

The global shocks which coincide with the correlations shocks observed in the data are the Gulf war of 1990, the Soviet coup of 1991, the tech meltdown of 2000, The Asian financial crisis of 1997 and the September 11 terrorist attack.

The highest correlation is reported between Hong Kong and Singapore. The correlation for these markets is also affected by regional factors that impact on the conditional volatility of those markets. There is also an increase in the correlations for all the markets after the Asian financial crisis.

The paper finds that the 100-day sliding window under-estimates and the CC-MGARCH(1,1) over-estimates the mean correlation for all pairs of the markets compared to the DCC(1,1)-MGARCH(1,1) model.

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