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Abstract

OECD tax revenue statistics show that on average the share of consumption tax is almost equal to that of income tax. However, consumption tax hardly attracts any attention from intermediate macroeconomic textbooks, not to mention its inclusion in IS-LM model. This paper compares the IS curves under an income tax regime and a general consumption tax regime. It also examines the trade off between the income tax rate and the consumption tax rate in a dual tax regime.

Key words: General consumption tax; IS-LM model; fiscal policy; tax rate trade off; intermediate macroeconomics

JEL code: A22, E21, E62, H20

1. Introduction

An examination of the OECD government revenue statistics reveal that in the last decade, there is a spread in the use of general consumption taxes, especially the VAT type, to replace specific consumption taxes.[1] General consumption taxes are now in place in 29 of the 3O OECD countries (with the exception of the USA, which only has state and local sales taxes). As a matter of fact, a comparison of the 1985 data (Krusell et al., 1996) and the 1998 data (OECD, 2001b, p.13) on shares of tax revenue shows that the percentage of consumption taxes has slightly increased over the 13 years by about one per cent from 31 to 32 per cent while the percentage of income taxes has slightly decreased from about 36 to 35 per cent.[2]

Table 1: Percentage shares of tax revenues, OECD countries, 1985-1998

	Tax type and OECD code			
Unweighted OECD average	Income (Personal and Corporate) 1000	Social Security 2000	Goods and Services 5000	Others (incl. 3000, 4000, and 6000)
1985	36.1	24.7	31.3	7.5
1998*	34.7	24.9	32.3	7.7
Changes	- 1.4	+ 0.2	+ 1.0	+ 0.2

* Based on the 23 member countries in 1985. They were Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, UK, and USA.

Sources: OECD (2001b, p.11), OECD (2003), and Krusell et al. (1996)

There are various arguments for a general consumption tax but what I am concerned here is its incorporation into the IS-LM model in intermediate macroeconomic textbooks.[3,4] I sample a list of intermediate macroeconomic textbooks—Blanchard (2000), DeLong (2002), Farmer (1999), Hall and Taylor (1999), Mankiw (2003), Miles and Scott (2002), Pentecost (2000), and Taylor and Moosa (2000)—and as far as I am aware, general consumption tax hardly attracts any attention in the IS-LM model, which is still an important analytical tool at that level of instruction. Nevertheless, Taylor and Moosa (2000, pp. 455-7) did examine the one-off price effect of the imposition of a general consumption tax in an economic fluctuations model represented by an inflation-rate-real-GDP diagram.[5]

In my intermediate macroeconomic unit, I incorporate a general consumption tax component into the IS-LM assignment.[6,7] The students' feedback were positive and they appreciated the more realistic approach. An IS-LM model with a consumption tax component gives them a tool to analyze (a) the impact of an imposition of a general consumption tax, (b) the effect of a change in consumption tax rate, and (c) the trade off between consumption and income tax rates to maintain tax revenue neutrality. I deem that the inclusion of a general consumption tax has enriched my students' learning experience. The aims of this paper are (a) to compare the IS curves under a consumption tax regime with that under an income tax regime, and (b) to explore the trade-off between income and consumption tax rates.

In the next section, I review a typical three-sector IS-LM model with a proportional income tax. In section two, I set out the alternative IS-LM model with a consumption tax regime. I also briefly discuss the possible questions about switching from an income tax to a consumption tax regime. In section three, I examine the IS-LM model with both kinds of taxes. I focus on the trade off between the two kinds of taxes given tax-revenue neutrality. I also include a numerical example to illustrate the trade-off. Section four is the summary.

2. The IS-LM Model with an Income Tax

This section presents a typical IS-LM model for a closed economy presented in intermediate macroeconomic textbooks, for example, Mankiw (2003, pp. 307-9). The model is encapsulated in following four equations

The total demand function: $E = C(Y^D) + I(r) + G_o$

The money demand function: $(M/P)^d = m^d = L(Y, r)$

The equilibrium condition in the product market: $Y = E$

The equilibrium condition in the financial market: $m^d = m^s = m_0$

where E is total demand for goods and services, C is consumption, Y^D is disposal (real) income, I is investment, r is real interest rate, G_o is government spending, M is money, P is general price level, $(M/P)^d$ is demand for real balances, L is liquidity, Y is real income or real GDP, m^s is the supply of real balances, and m_0 is real money balances.

To facilitate the discussion, I specify the linear form of the consumption function, investment function, tax function, and money demand function as

$$C = C_o + C_Y Y^D \tag{1}$$

$$I = I_o - I_r r$$

$$T = tY \tag{2}$$

$$m^d = L_Y Y - L_r r$$

where C_o is baseline consumption, C_Y is marginal propensity to consume, I_o is baseline investment, I_r is interest sensitivity of investment, T is tax revenue, t is proportional income tax rate, L_Y is income sensitivity of demand for money, and L_r is interest sensitivity of demand for money. Without further ado, the equations of the IS and the LM curve are

$$r = \frac{C_o + I_o + G_o}{I_r} - \frac{1 - C_Y (1-t)}{I_r} Y \tag{3}$$

and

$$r = -\frac{m_0}{L_r} + \frac{L_Y}{L_r} Y$$

respectively. And the equilibrium values are $Y^* = \dfrac{C_o + I_o + G_o + \alpha m_0}{1 - C_Y (1-t) + \alpha L_Y}$ and

$r^* = \dfrac{(C_o + I_o + G_o) L_Y - [1 - C_Y (1-t)] m_0}{L_r [1 - C_Y (1-t)] + I_r L_Y}$ with $\alpha = \dfrac{I_r}{L_r}$. From equation (3), we know that a change in the income tax rate rotates the IS curve anticlockwise for $\Delta t > 0$ and clockwise for $\Delta t < 0$ with the vertical intercept as the pivot point. For the comparative statics of the IS-LM model with an income tax regime, they are well covered in any of the intermediate macroeconomic textbooks.

3. The IS-LM Model with a Consumption Tax

With a general consumption tax and no income tax, we need to rewrite the tax equation (2) to become equation (2')

$$T = v C \tag{2'}$$

where v is the proportional consumption tax rate. With a general consumption tax, the concept of disposable income Y^D and the determination of consumption level are not as clear-cut as those described in textbooks. Household consumption C depends on the level of Y^D

but the latter also depends on how much the households consume. This feedback loop makes the derivation of the IS equation more complicated for students. When I substitute the consumption, the investment, and the tax equations into the equilibrium condition of the product market, the substitution goes on forever with the following equation emerging:

$$Y = (C_o + C_Y\, Y)(1 - C_Y v + C_Y^2 v^2 + \cdots + (-1)^n C_Y^n v^n) + I_o - I_r\, r + G_o$$

With the sum of the geometric series equals $1/(1 + C_Y v)$, the above equation can be rearranged to give the equation of the consumption-tax IS curve:

$$r = \frac{C_o + (1 + C_Y v)(I_o + G_o)}{I_r (1 + C_Y v)} - \left[\frac{1 - C_Y (1 - v)}{I_r (1 + C_Y v)}\right] Y \qquad (4)$$

This equation is far more complex than the income-tax IS equation, described by equation (3); the consumption tax rate *v* appears in the numerator and the denominator of both the intercept and the slope. Consequently, a change in *v* not only rotates the IS curve but also shifts the IS curve at the same time. This contrasts to the pure rotation effect of a change in the income tax rate. Partial derivatives of the intercept and the slope with respect to *v* show that a change in the consumption tax rate shifts the IS curve (to the left for $\Delta v > 0$ and to the right for $\Delta v < 0$) and rotates it (anticlockwise for $\Delta v > 0$ and clockwise for $\Delta v < 0$) with the vertical intercept as the pivot point. With the LM curve remains the same, the equilibrium values for *Y* is $Y^* = \dfrac{C_o + (I_o + G_o)(1 + C_Y v) + \alpha m_0 (1 + C_Y v)}{1 - C_Y (1 - v) + \alpha L_Y (1 + C_Y v)}$.

3.1 Some questions about switching tax regime

There are some interesting questions about switching tax regime. Firstly, does the switch from one tax regime to another involve same tax revenue? And is tax revenue neutral equivalent to real GDP neutral? Secondly, what is the position (indicated by the intercept and the slope) of the consumption-tax IS curve as compared to that of the income-tax IS curve? Thirdly, what is the value of the tax revenue neutral consumption tax rate?

It is reasonable to argue that a new tax regime is at least tax revenue neutral or tax revenue enhancing in its inceptive year for the government to fulfill its financial commitments. It is more likely to be tax revenue neutral to avoid the accusation of revenue grabbing. And as long as tax revenue does not change, real GDP remains at the same level.[8]

To see this assertion, refer to the other definition of real GDP where *Y* is the sum of *C*, saving *S* and *T*. Since both *C* and *S* are functions of Y^D, which in turn depends on *T* given *Y*, a stable tax liability means stable consumption and saving, ceteris paribus. That is, tax revenue neutrality implies real GDP neutrality. The numerical example in the next section attests to the validity of this assertion. Another reason that the government may prefer to maintain neutrality is that a depletion or enhancement creates an economic shock that the government has to handle on top of the administrative problems of changing the tax system.

How does the position of a consumption-tax IS curve compare to its income tax counterpart? Since I have adopted the same set of equations except the tax equation, I can compare equations (3) and (4) to discern their differences. Apparently, the consumption-tax IS curve has a lower intercept than that of the income-tax IS curve because $C_o / I_r (1 + C_Y v) < C_o / I_r$. It is however more difficult to ascertain the relative values of the slopes of the two IS curves. With tax-revenue and real-GDP neutrality (at least in the short run and no change in monetary policy) the equilibria of the economy with the two tax regimes must be the same. A lower intercept of the consumption-tax IS curve implies it has to have a relatively flatter slope as depicted in Figure 1. Suppose the economy starts with an initial equilibrium point A, the relatively steeper and solid IS_Y is the income-tax IS curve and the relatively flatter and broken IS_C is the consumption-tax IS curve.

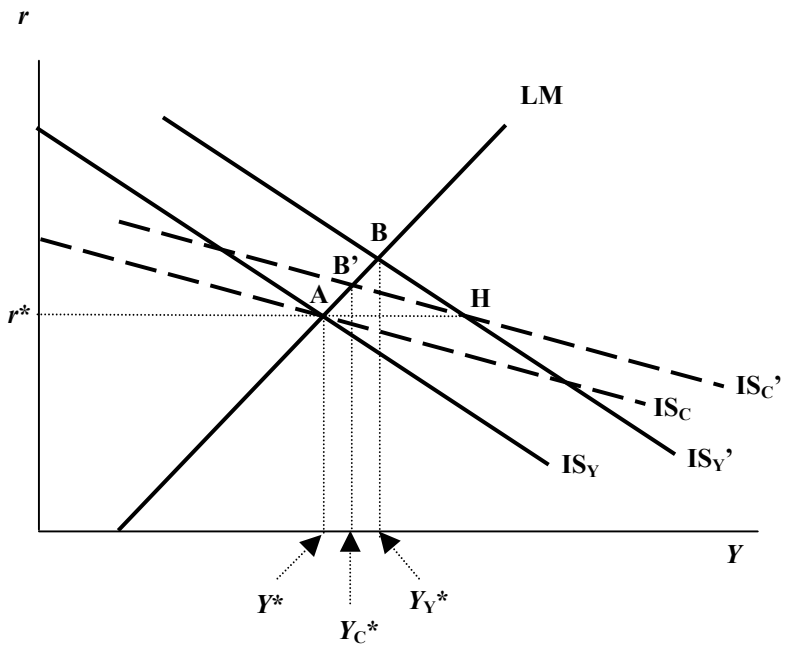

Figure 1: Effectiveness of fiscal policy under different tax regime

A relatively flatter IS_C also implies that fiscal policy is relatively less effective in an economy with a consumption tax than one with an income tax. Again refer to Figure 1. An equal increase in government expenditure G_o causes both IS curves to shift to the right by the same amount to point H. The new equilibrium for an income tax regime is point B and for a consumption tax regime is point B'. It is apparent that fiscal policy is less effective under a consumption tax regime. This observation can be safely extended to a tax regime with a consumption tax component.

As for the third question, it is best handled in a dual-tax regime, which will be discussed in the next section.

4. The IS-LM Curve with Both Taxes

Replace the tax equation (2) by one with both income and consumption taxes:

$$T = t\,Y + v\,C \tag{2"}$$

Without repeating the derivation process, the equation of the dual-tax IS curve is

$$r = \frac{C_o}{I_r\left(1 + C_Y\,v\right)} + \frac{I_o + G_o}{I_r} - \left[\frac{1 - C_Y\,(1-t) + C_Y\,v}{I_r\left(1 + C_Y\,v\right)}\right] Y \tag{5}$$

The introduction of a consumption tax into an economy with only income tax again lowers the intercept and flattens the slope of the IS curve. The observations pertain to a consumption tax regime again apply here. The higher the consumption tax rate, the flatter the IS curve and the lower its vertical intercept. Figure 2 depicts the IS curves of the three tax

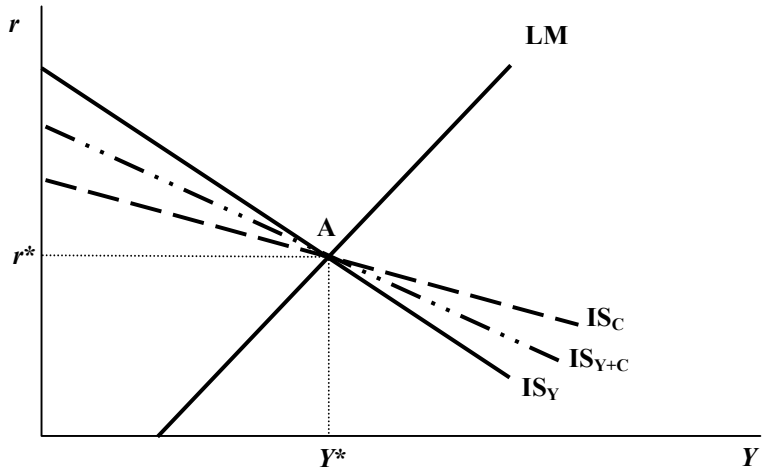

Figure 2: The IS curves under the three tax regimes

regimes under tax-revenue neutrality and IS_{Y+C} is one of the many possible IS curves (depends on the combination of *t* and *v*) of a dual-tax regime. Note that the value of *v* in equation (5) is less than that in equation (4) due to the presence of income tax and tax-revenue neutrality.

The inclusion of both taxes facilitates the study of trade off between the two tax rates under the condition of tax-revenue and real-GDP neutrality. To calculate the trade-off ratio between the two tax rates, substitute repeatedly the consumption equation (1) into tax equation (2") yields

$$T = tY + v(C_o + C_Y Y - tC_Y Y)(1 - C_Y v + C_Y^2 v^2 + \cdots + (-1)^n C_Y^n v^n)$$

Again apply the sum of the geometric series, I have

$$T = t\,Y + v\left[\frac{C_o + C_Y\,(1-t)\,Y}{1 + C_Y\,v}\right] \tag{6}$$

Total differentiate equation (6) and set $dT = dY = dC_o = dC_Y = 0$, and using the equilibrium value $Y^* = \dfrac{C_o + (I_o + G_o)(1 + C_Y v) + \alpha\, m_0 (1 + C_Y v)}{1 - C_Y (1 - v) + \alpha\, L_Y\,(1 + C_Y v) + C_Y t}$, I get

$$\frac{\partial v}{\partial t} = -\left\{\frac{C_o + B(1 + C_Y v)}{C_o\,(1 + \alpha\, L_Y) + B\,C_Y (1 - t)}\right\} < 0 \tag{7}$$

where $B = I_o + G_o + \alpha\, m_0$. There is a negative trade off between the two tax rates, indicated by the negative sign of the partial derivative $\partial v/\partial t$, to maintain tax-revenue neutrality. That is, $\Delta t < 0\,(\Delta t > 0)$ implies $\Delta v > 0\,(\Delta v < 0)$ to keep $\Delta T = 0$. I call this partial derivative the trade-off ratio and its value is determined by all the parameters in the system of equations. The trade-off ratio is less than zero because consumption tax has a narrower tax base than income tax. Let say there is a cut in the income tax by $\Delta t < 0$, the first round increase in disposable income is $\Delta Y^D = -Y\Delta t > 0$. The increase in disposable income is allocated to consumption and saving. Therefore, from the government viewpoint, $-Y\Delta t > C\Delta v$ for $-\Delta t = \Delta v$. To maintain revenue neutrality, the government has to have $-\Delta t < \Delta v$. A numerical example is given below to illustrate my point.

Equation (8) summaries the effect of a change in each parameter on the value of trade-off ratio:

$$\frac{\partial v}{\partial t} = f(\overset{-}{t}, \overset{+}{v}, \overset{+}{C_o}, \overset{+}{I_o}, \overset{+}{G_o}, \overset{-}{M_0}, \overset{+}{P_0}, \overset{+}{C_Y}, \overset{+}{I_r}, \overset{+}{L_Y}, \overset{-}{L_r}) \tag{8}$$

It is important to note that the "+" (" – ") sign on top of a parameter in the above equation indicates smaller (larger) trade off between the two tax rates. It is also important to point out that the trade-off ratio varies with the structure of the economy, represented by all the parameters. That is, a different starting point (e.g., a different combination of the initial *v* and *t*) entails a different value of trade-off ratio.

4.1 A numerical example

Suppose $C_o = \$10\text{b}$, $C_Y = 0.75$, $I_o = \$400\text{b}$, $I_r = \$800\text{b}$, $L_Y = 0.9$, $L_r = \$900\text{b}$, $M_0 = \$400\text{b}$, $P_0 = 4$ (represents an index of 400), $G_0 = \$200\text{b}$. With $t = 20\%$, the equilibrium values of the economy are: $Y^* = \$582.41\text{b}$, $T^* = \$116.48\text{b}$, $Y^{D}* = \$465.93\text{b}$, $C^* = \$359.44\text{b}$, public saving $T^* - G^* = -\$83.52\text{b}$, share of government = 34.34%, $r^* = 0.47\%$, $I^* = \$22.96\text{b}$, private saving = \$106.48%, and national saving rate = 3.94%.

The trade off between income and consumption tax rates can be obtained by applying the values of the parameters to equation (7). The result is $\partial v/\partial t = -1.6203$, which means a one per cent reduction in income tax rate requires a 1.6203 per cent increase in consumption tax rate to maintain the tax revenue. To ascertain the validity of the concept, I calculate the equilibrium set of values for the following three cases with $t = 20\%$ and $v = 0\%$ as the baseline: (a) dual tax regime with $t = 15\%$ and $v = 8.10\%$, (b) dual tax regime with $t = 13.83\%$ and $v = 10\%$, and (c) single tax regime with $v = 32.41$. Lo and behold, their equilibrium sets of values are the same (allowing for rounding error).[9] The equations of the four IS curves are tabulated in Table 2 and the results agree with the pattern shown in Figure 2.

Table 2: IS equations under different tax regimes

Tax regime	Equation of IS curve
Income tax regime with $t = 20\%$	$r = 0.7625 - 0.0005000Y$
Dual tax regime with $t = 15\%$ and $v = 8.1015\%$	$r = 0.7618 - 0.0004988Y$
Dual tax regime with $t = 13.8283\%$ and $v = 10\%$	$r = 0.7616 - 0.0004985Y$
Consumption tax regime with $v = 32.4060\%$	$r = 0.7601 - 0.0004958Y$

5. Summary

I consider that the incorporation of a general consumption tax in the IS-LM model brings it more in line with the reality that students experience. Even though the mathematics involved is more complicated, the inclusion of a general consumption tax allows students to appreciate (a) the effect of a change in the consumption tax rate, and (b) the trade off between income and consumption tax rates. It is also found that fiscal policy is less effective with a consumption tax component than without.

NOTES

1. The most recent example is Australia. In July 2000, the Australian Commonwealth Government introduced the Goods and Services tax (GST)—a VAT-type general consumption tax—to replace its wholesale sale taxes (six different rates), financial institution duties, debit tax, conveyancing duties on business property and five other stamp duties.

2. Quite a few economists, e.g. Krusell et al. (1996), regard social security tax as income tax because it is income based. If we take this position, then the share of income taxes has decreased from 61 per cent to 60 per cent over the 13-year period.

3. Normally, the base of a general consumption tax should logically include all goods and services, which may not be politically acceptable. In fact, the Australian GST has exemption (i.e., zero rating) on several kinds of goods and services (food, health, education, childcare services, hospitals and nursing homes, local government rates, water and sewerage charges, and charitable activities) to increase the progressivity of the tax.

4. Consumption is regarded as a greener and equitable tax than income tax in the sense that economic agents are taxed according to the resources they use up (i.e., actual consumption) rather than their potential consumption. For a survey, see Atkinson and Stiglitz (1987, pp. 563-6).

5. When Australia introduced its GST in July 2000, the impact was a one-off increase in quarterly inflation rate by 2.3%. Similar patterns were observed in both New Zealand (1987) and Canada (1991).

6. DeLong (2002) is the adopted textbook for my intermediate macroeconomic unit.

7. It is an analytical exercise based on a case study using a four-sector IS-LM spreadsheet model.

8. This is only valid in a very short-run model such as the IS-LM where prices are assumed to be rigid. Inflation statistics show that the introduction of a general consumption tax creates a one-off direct effect on prices paid by consumers, which in turn lowers real GDP, ceteris paribus. Also see end note 5.

9. I did the calculation by an IS-LM spreadsheet model.

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