Stability analysis of anchored rock slopes against plane failure subjected to surcharge and seismic loads

Md M. Hossain
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Stability Analysis of Anchored Rock Slopes against Plane Failure Subjected to Surcharge and Seismic Loads

Md Monir Hossain

This thesis is presented in fulfilment of the requirements for the degree of Master of Engineering Science

Faculty of Computing, Health and Science
Edith Cowan University

May 2011
USE OF THESIS

The Use of Thesis statement is not included in this version of the thesis.
ABSTRACT

The stability analysis of rock slopes has been a challenging task for engineers because the rock mass constituting the slope often has discontinuities in various forms, resulting in different types of slope failures. The plane failure is one of the rock slope failures observed in field situations when the discontinuity is in the form of joint planes. There are several parameters including surcharge and seismic loads that govern the stability of the rock slope against plane failure in field projects. The limit equilibrium approach for the estimation of the factor of safety of the rock slope against plane failure has been well accepted by the engineers in the past. Very recently, attempts have been made to present analytical expressions for the factor of safety of the rock slopes against plane failure, which are not in a generalised form because they do incorporate most field parameters. Therefore, in the present work, the analytical expression for the factor of safety of a single-directional anchored rock slope (SDARS) is derived, along with a discussion of its special cases in view of different practical situations. Parametric studies and design charts for the stability of the SDARS are presented, and an illustrative example is included to explain the calculation steps for the factor of safety. In order to investigate the effect of multi-directional rock anchors on the factor of safety, an analytical expression for a multi-directional anchored rock slope (MDARS) is also presented.

The graphical presentations for typical values of governing parameters indicate that the factor of safety of a rock slope increases with an increase in both angle of shearing resistance and cohesion of the joint material. The rate of increase in the factor of safety increases with an increase in angle of shearing resistance, whereas it remains constant for any increase in cohesion. The vertically upward direction of the inertial seismic force results in an increase in factor of safety, but the vertically downward direction of the inertial seismic force causes a decrease. For a higher factor of safety of the rock slope, greater values of shear strength parameters and/or the stabilizing force should be available. It is also observed that surcharge and water pressure in the tension crack decrease the factor of safety significantly. It is noted that for the stability analysis of rock slopes, it is essentially required to consider realistic values of all these parameters based on the actual field conditions.
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I also wish to acknowledge the dedication of the administrative staff, especially Muriel Vaughan, Kim Gardiner and Pat McGinley. All these people have tremendous impact on my work. Indeed they understood the needs of a research student and were always available to meet them.

Finally, I wish to thank many people. I have had the pleasure of meeting during my work. Without the help and the friendship of these people, this thesis would never be completed. I, as well as, anyone who gains from reading the words herein owe them a debt of gratitude.

Md Monir Hossain
DECLARATION

I certify that this thesis does not, to the best my knowledge and belief:

I. incorporate without acknowledgement any material previously submitted for a degree or diploma in any institution of higher education;

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NOTATION

Basic SI units are given in parentheses.

$A$ base area of the sliding block ($\text{m}^2/\text{m}$)

$B$ distance between the crest of slope and the tension crack ($\text{m}$)

$c$ cohesion of the joint material along the sliding surface ($\text{N}/\text{m}^2$)

$c^*$ nondimensional cohesion along sliding surface ($= c/\gamma H$) (dimensionless)

$C$ total cohesive force on the failure plane ($\text{N}/\text{m}$)

$N$ normal force acting on the failure plane ($\text{N}/\text{m}$)

$F$ total frictional resisting force acting along the joint plane

$F_i$ force available to induce sliding ($\text{N}/\text{m}$)

$F_r$ force tending to resist sliding ($\text{N}/\text{m}$)

$FS$ factor of safety against sliding (dimensionless)

$H$ height of the rock slope ($\text{m}$)

$k_h$ horizontal seismic coefficient (dimensionless)

$k_v$ vertical seismic coefficient (dimensionless)

$q$ surcharge pressure ($\text{N}/\text{m}^2$)

$q^*$ nondimensional surcharge ($= q/\gamma H$) (dimensionless)

$T$ stabilizing force ($\text{N}/\text{m}$)

$T^*$ nondimensional stabilizing force ($= T/\gamma H^2$) (dimensionless)

$T_i$ stabilizing force ($\text{N}/\text{m}$) in the $i$th set ($i = 1, 2, 3, \ldots n$)

$T_i^*$ nondimensional stabilizing force ($= T/\gamma H^2$) (dimensionless)

in the $i$th set ($i = 1, 2, 3, \ldots n$)

$U_1$ horizontal force due to the water pressure in the tension crack ($\text{N}/\text{m}$)

$U_2$ uplift force due to the water pressure on failure plane ($\text{N}/\text{m}$)

$V$ volume of the sliding rock mass block

$W$ weight of the sliding block ($\text{N}/\text{m}$)

$z$ depth of the tension crack ($\text{m}$)

$z^*$ nondimensional depth of tension crack ($= z/H$) (dimensionless)

$z_w$ depth of water in the tension crack ($\text{m}$)
$z_w^*$  nondimensional depth of water in tension crack (m)
$
\alpha
$
angle of inclination of stabilizing force to the normal at the failure plane (degrees)
$
\alpha_i
$
angle of inclination of stabilizing force to the normal at the failure plane (degrees) in the $i$th set ($i = 1, 2, 3, \ldots n$)
$
\phi
$
angle of shearing resistance of the joint material (degrees)
$
\gamma
$
unit weight of rock (N/m$^3$)
$
\gamma^*
$
nondimensional unit weight of rock ($=\gamma/\gamma_w$) (dimensionless)
$
\gamma_w
$
unit weight of water (N/m$^3$)
$
\theta
$
an angle equal to $\tan^{-1}\left(\frac{k_n}{(1\pm k_p)}\right)$
$
\sigma_n
$
normal stress on the failure plane (N/m$^2$)
$
\tau
$
shear strength of the failure plane (kN/m$^2$)
$
\psi_f
$
angle of inclination of the slope face to the horizontal (degrees)
$
\psi_p
$
angle of inclination of the joint plane/failure plane to the horizontal (degrees)
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1.1 GENERAL

Stability analysis of soil and rock slopes has been a research problem for civil and mining engineers for several decades. In civil engineering applications, the slope stability is concerned with many projects in hilly as well as plain terrains, such as foundations of structures (buildings, bridges, power houses, dams, etc.), transportation routes (highways, railways, canals, pipelines, tunnels, etc.), and underground storages and basements. In mining engineering applications, projects related to open and underground excavations essentially need consideration of slope stability analysis in order to maintain the slope in stable condition during construction as well as operation. In large opencast mines, slope heights may be hundreds of meters; therefore, slope failures can cause severe losses in terms of productivity and safety, and even result in deaths of the workers.

Slopes can consist of soil, rock or a combination thereof. The analysis of rock slopes has always been a challenging task, mainly because of the presence of discontinuities in the rocks masses. The most common types of discontinuities in rock masses are fault, bedding, foliation, joint, cleavage and schistosity (Wyllie and Mah, 2004). A discontinuity along which there has been an observable amount of displacement is termed as fault. A plane parallel to the surface of deposition is known as bedding. Foliation is the form when orientation of platy minerals or mineral banding occurs in metamorphic rocks. A discontinuity in which there has been no observable relative movement is known as joint. Parallel discontinuities formed in incompetent layers in a series of beds of varying degrees of competency are known as cleavages. Foliation in schist or other coarse grained crystalline rock due to the parallel arrangement of mineral grains of the platy or prismatic type is termed as Schistosity.
No slopes made in rock can be regarded as fully guaranteed for their stability during their service over a period of many years (Jumikis, 1983). However, it is a general engineering practice to classify the rock slope failures in some idealised failure types, such as plane failure, wedge failure, circular failure, toppling failure and buckling failure, as shown in Fig. 1.1.

**(a) (b) (c)**

Fig. 1.1. Types of rock slope failure: (a) plane failure, (b) wedge failure, (c) circular failure, (d) toppling failure, and (e) buckling failure (adapted from Hoek and Bray, 1981; Goodman, 1989; Kliche, 1999; Goodman and Kieffer, 2000; Wyllie and Mah, 2004; Hoek, 2007; Ramamurthy, 2007).

A detailed review of different types of rock slope failures has been presented by Goodman and Kieffer (2000). Sliding of a rock mass on the joint/weak plane dipping away from the slope is termed as the *plane failure* (aka *block sliding*). It generally occurs in hard or soft rock slopes with well defined discontinuities and jointing, e.g., layered sedimentary rocks, volcanic flow rocks, block jointed granite,
and foliated metamorphic rocks. When two distinct planes of weaknesses, joints or fault planes exist, the rock mass between these planes can slide down; this mode of failure is known as the *wedge failure*. In a heavily fractured/weathered rock mass, failure takes place by movement along a cylindrical surface; this type of failure is called *circular failure*. *Toppling failure* takes place when a regularly spaced set of joints or bedding planes strike parallel, or nearly parallel, to the slope face and dip at a steep angle into the face. *Buckling failure* takes place when the excavation is carried out with its face parallel to the thin weakly bonded and steeply dipping layers, which may buckle and fracture near the toe, resulting in the sliding of the upper portions of the layers.

For the stability of rock slopes, Call and Savely (1990) stated the following three general principles of slope mechanics:

1. *Slope failures do not occur spontaneously.* One or more of the forces acting on a potentially unstable rock mass must change for making a part of the rock unstable.
2. *Most slope failures tend toward equilibrium.* A slope fails because it is unstable under the existing conditions. Failure tends to bring the slope to some sort of equilibrium. It normally involves a reduction in the driving forces and/or an increase in the resisting forces of the failed zones.
3. *A slope does not occur without warning.* Prior to failure, measurable movement and/or the development of tension cracks occur. These indicate imminent slope failure, and the slope may subside during a certain period of time to achieve stability.

For maintaining a stable slope in excavated or natural rock mass, stabilization is preferred. Common slope stabilization techniques can be divided into six general categories: *grading, controlled blasting, mechanical stabilization, structural stabilization, vegetative stabilization, and water control* (Kliche, 1999). An appropriate stabilization technique is used as per the project requirements, degree of urgency, availability of space, and specific site situation. Many of these techniques are routinely used simultaneously to achieve a stable rock slope with a better aesthetic look. *Structural stabilization* consists of structures that reinforce the rock at
slope face or support the slope at toe. Under specific field conditions, a number of reinforcement techniques are implied such as, shear keys, reaction wall, rock bolts, rock anchors, rock dowels, gunite/shortcrete, buttresses, retaining walls, etc (Kliche, 1999; Wyllie and Mah, 2004; Ramamurthy, 2007). Rock anchoring is the most common methods of rock slope stabilization. It requires a specialised technical skill for installations. Efforts were made earlier for developing appropriate installation steps for rock anchoring, and these steps are described in the literature (Littlejohn and Bruce, 1977; Federal Highway Administration (FHWA), 1982; British Standards Institute (BSI), 1989; Xanthakos, 1991; Post Tensioning Institute (PTI), 1996; Kliche, 1999; Wyllie, 1999; Ramamurthy, 2007). Basically, rock anchors are high tensile strength bars or strands pre-tensioned by anchoring at the end of the borehole within the unstable rock mass (Ramamurthy, 2007). Anchor force as well as anchor orientation both plays an active role in achieving the required slope stability. There is an optimum anchor orientation which minimizes the required anchor force. In practice, cement grouted anchors are installed at about 10°-15° to the horizontal in the downward direction to facilitate grouting, while resin grouted anchors may be installed in up-holes. It should be noted that bolts installed at an angle steeper than the normal to the sliding plane can be detrimental to stability because the shear component of the tension, acting down the plane, increases the magnitude of the driving force (Wyllie and Mah, 2004).

Rock slope can be stabilized using rock anchors in two main patterns of orientation as considered in the present work. In the first pattern, a rock slope can be stabilized by installing a set of anchors with a single orientation; in this case, the stabilized slope may be called single-directional anchored rock slope (SDARS). If the slope is stabilized with several sets of anchors with different orientations; it may be called multidirectional anchored rock slope (MDARS). Typical SDARS and MDARS under plane failure mode are shown in Figs. 1.2 and 1.3, respectively.

Various methods are utilized for the analysis of rock slope stability. These methods are: limit equilibrium analysis, sensitivity analysis, probabilistic analysis, and numerical analysis (Kliche, 1999; Wyllie and Mah, 2004). Though all these methods are available, the most frequently used methods are limit equilibrium analyses and numerical analyses. In limit equilibrium analysis, the factor of safety of rock slopes is calculated by developing analytical formulations for different
Fig. 1.2. A single-directional anchored rock slope (SDARS) (*Note: $A_1$, $A_2$ and $A_3$ form a set of anchors installed with a single orientation.*).

Fig. 1.3. A multi-directional anchored rock slope (MDARS) (*Note: $A_1$ and $A_2$ form one set of anchors, and $B_1$ and $B_2$ another set with different orientation.*)
failure modes of rock bounded by specified slide planes under consideration of field parameters. Numerical analyses are well recognised computer based software, which are commercially available and used for the analysis of rock slopes stability. In numerical analysis, finite-difference or finite-element based software are used for the simulation of rock mass behaviour. Computer programs attempt to represent the mechanical response of a rock mass subjected to a set of initial conditions, dividing the rock mass into zones with assigning specific material model/properties.

The analytical limit equilibrium approach for the estimation of factor of safety of the rock slope against plane failure is well accepted by the engineers, mainly because of simplicity in the development of explicit expressions and their frequent applications over a long period of time. Hoek and Bray (1981) presented most of the basic methods of limit equilibrium analysis for rock slope failures. Ling and Cheng (1997) presented an analytical expression for the factor of safety of the rock slope against plane failure induced by seismic force, ignoring the possibility of upward direction of vertical inertial seismic force, and without considering the surcharge and the anchoring force. Recently, Hoek (2007) described the idealisation of the rock slope failures in Hong Kong as plain failures and presented an analytical expression for estimating the factor of safety, considering many practical aspects including seismic loadings. This analytical model was improved by Shukla et al. (2009) to investigate the effect of surcharge on the stability of rock slopes, ignoring the seismic inertial forces applied by the surcharge on the slope. In the earlier works, the vertical seismic inertial force has also not been considered with their all possible directions for the generalized case. Therefore, an attempt is required to formulate a generalised analytical expression for the factor of safety of a rock slope against the plane failure, considering most of the factors that may be expected in field conditions under earthquakes and dynamic activities including the stabilizing forces for the stabilized slopes.

1.2 OBJECTIVES AND SCOPE OF THE PRESENT WORK

The methods of analysis based on limit equilibrium are widely used by practicing engineers, mainly because of their simplicity as explained in the previous section.
Moreover, because of wide applications over a long period of time, the results obtained from these methods have been well accepted by the engineers. Very recently, attempts have been made to present analytical expressions for the factor of safety of the rock slopes against plane failure, considering several field aspects in the analytical formulations. However, there are still some field aspects, which require special attention in the analytical formulations of the generalised expression for factor of safety of rock slopes against plane failure. So, further research is expected in this area.

This work aims at studying the stability analysis of anchored rock slopes against plane failure subjected to seismic loadings. Based on the research problem defined in detail at the end of literature review presented in Chapter 2, major objectives of this study are given below:

- Derivation of a generalised analytical expression for the factor of safety of a single-directional anchored rock slope (SDARS) against the plane failure, considering most of the factors that may be expected in field conditions.
- Study of the effects of various destabilizing parameters including the dynamic loads from earthquakes and other causes on the stability of slope.
- Presentation of analytical formulations for multi-directional anchored rock slope (MDARS) with an investigation of the effectiveness of rock anchoring stabilization method for rock slopes.
- Development of design charts for safe and economical design of rock slopes against the plane failure.

1.3 PUBLICATIONS BASED ON THE PRESENT WORK

During the progress of research work, attempts were made to prepare the manuscripts of research papers based on some parts of the thesis work for submission to journals and conference proceedings for their publications. The details of the published/accepted papers are as follows:
Chapter 1: Introduction

International Journals


International Conference Proceedings


1.4 ORGANISATION OF THE PRESENT WORK

In this chapter, the research area is introduced and basic information of the concerned subject is described. A critical review of the static and dynamic analyses of rock slopes are presented in Chapter 2, and subsequently the research problem is indentified. Chapter 3 describes the analytical formulation of the indentified problem as determining the analytical expression for the factor of safety of a single-directional anchored rock slope (SDARS), along with a discussion of its special cases in view of different practical situations. In Chapter 4 parametric studies and design charts for the stability of SDARS are presented, and an illustrative example is included to explain the calculation steps for the factor of safety. Chapter 5 describes the derivation of the analytical expression for a multi-directional anchored rock slope (MDARS) and presents some discussion. The work carried out in the thesis is summarized with specific conclusions and further research problems in Chapter 6.
2.1 GENERAL

The rock masses are generally heterogeneous and anisotropic because of presence of discontinuities; therefore, the stability analysis of rock slopes has been a challenging task for engineers. The stability of rock slopes is essentially governed by the joint sets, characteristics of joint materials, seepage pressure, and depth and steepness of the excavated slope face and its orientation with respect to the joint sets, as explained in chapter 1. Slope design is primarily concerned with the stability of unstable blocks of rock formed by discontinuities. Several types of slope failures such as plane failure, wedge failure, circular failure, toppling failure and buckling failure have been recognized in the past. The stability analysis becomes more complex when slopes are subjected to vibrations caused by earthquakes, blasting and other causes. This chapter presents an overview of stability analyses of rock slopes against plane and some other failure modes, categorizing the available methods of analysis in two sections as static analysis and pseudo-static analysis.

2.2 STATIC SLOPE STABILITY ANALYSES

Static slope stability analysis is specifically based on the static equilibrium of unstable rock mass. In static equilibrium, the sum of the forces, and moments, on each element of the system is zero. The unstable rock masses are defined on a categorized geometry of blocks isolated by discontinuity planes (Giani, 1992). The resisting and driving forces are calculated by solving equilibrium equations in order to determine the factor of safety ($FS$) defined as
Chapter 2: Literature review

\[
FS = \frac{\text{Resisting force}}{\text{Driving force}} \quad (2.1)
\]

The unstable rock block is in a condition of “limiting equilibrium” when the driving forces are exactly equal to the resisting forces and the factor of safety is equal to 1.0. For this reason, this method of slope stability analysis is termed as limit equilibrium analysis (Hoek and Bray, 1981; Wyllie and Mah, 2004; Hoek, 2007). This method is routinely used for the assessment of stability analysis of rock slopes in engineering practice.

The simplest expression for factor of safety of a rock slope (Fig. 2.1) against plain failure was presented by Hoek and Bray (1981) as

\[
FS = \frac{F_r}{F_l} = \frac{cA + W\cos\phi_p \tan\phi}{W \sin\phi_p} \quad (2.2)
\]

where \( F_r \) is the resisting sliding force, \( F_l \) is the inducing sliding force, \( W \) is the weight of the rock mass blocks \( A_1A_2A_3 \) with an inclination to the horizontal at an angle \( \phi_p \), and \( A \) is the area of the base \( A_2A_3 \). In the derivation of Eq. (2.2), it is assumed that the joint plane material is a \( c-\phi \) soil material with \( c \) and \( \phi \) as cohesion and angle of internal friction (also called angle of shearing resistance), respectively, that obeys the Mohr-Coulomb failure criterion.

![Fig. 2.1. Mechanism of rock slope failure under self weight only.](image-url)
For water forces acting on the sliding surface, the expression for factor of safety of a rock slope (Fig. 2.2) against plain failure was also presented by Hoek and Bray (1981) as

\[ FS = \frac{cA+(W\cos \psi_p-U_1\sin \psi_p-U_2)\tan \phi}{W\sin \psi_p+U_1\cos \psi_p} \]  

(2.3)

where, \( W \) is the weight of the rock mass block \( A_1A_2A_3A_4 \) with an inclination to the horizontal at an angle \( \psi_p \). \( U_1 \) is the horizontal force due to water pressure in the tension crack and, \( U_2 \) is the uplift force due to water pressure on the joint plane as given below:

\[ A = (H - z)\csc \psi_p \]  

(2.4a)

\[ W = \frac{1}{2} \gamma H^2 \left[ \left\{ 1 - \left( \frac{z}{H} \right)^2 \right\} \cot \psi_p - \cot \psi_f \right] \]  

(2.4b)
Chapter 2: Literature review

\[ U_1 = \frac{1}{2} \gamma_w z_w^2 \]  
\[ U_2 = \frac{1}{2} \gamma_w z_w (H - z) \csc \psi_p \] 

where \( \gamma_w \) is the unit weight of water and \( z_w \) is the depth of water in tension crack.

If the rock slope is anchored, as shown in Fig. 2.3, \( FS \) is given as (Hoek and Bray, 1981; Wyllie and Mah, 2004; Hoek, 2007)

\[ FS = \frac{cA + (W \cos \psi_p - U_1 \sin \psi_p - U_2 + T \sin \alpha) \tan \phi}{W \sin \psi_p + U_1 \cos \psi_p - T \cos \alpha} \]  

where \( T \) is the stabilizing tensile force and \( \alpha \) is the angle made by the stabilizing force to normal at the joint plane \( A_2A_3 \).

Fig. 2.3. Mechanism of rock slope failure under self weight, water forces and stabilizing force.
Aydan and Kawamoto (1992) proposed a limiting equilibrium approach for analysing the stability of rock slopes and underground openings against flexural toppling failure. The method also suggested the reinforcement effect of fully grouted rock bolts for the stabilisation of structures. The applicability and validity of the method was checked through model tests carried out in laboratory. It is found that the proposed method is valid for analysing the stability of model slopes and underground openings and can be used to predict the stability of the actual slopes and underground openings in layered rock masses.

Adhikary et al. (1997) investigated the mechanism of flexural toppling failure of jointed rock slopes through a series of centrifuge experiments conducted on small scale manufactured models. The basal failure plane observed in the centrifuge models was found to emanate from the toe of the slope, and orient at an angle of $12^\circ$ to $20^\circ$ upward from the normal to the discontinuities. To analyse the centrifuged test data, a theoretical model based on a limiting equilibrium approach proposed by Aydan and Kawamoto (1992) was adopted. On the basis of the experimental results, the theoretical model proposed by Aydan and Kawamoto was calibrated to yield accurate predictions of slope collapse. After calibration, the model was found to predict the failure load accurately for all the tests reported. Using this model, a set of charts has been prepared to assist with the analysis of slopes susceptible to flexural toppling.

Nawaril et al. (1997) presented a Direct Sliding Block Method (DSBM) for the solutions of stability of jointed rock slopes assuming kinematically admissible collapse mechanisms consisting of several rigid blocks sliding over plan surfaces. The behaviour of the individual rigid block was under the control of the static and kinematics conditions. The collapse mechanisms of rigid sliding bodies were solved using geometry and properties of rock mass supported by field evidence. Comparison to other methods of analysis, there is no solution of linear equations and no necessity for complicated optimization procedure in the present method.

Pariseau et al. (1997) examined the potentially destabilizing effect of water pressure on rock slope stability assuming coupled poroelastic/plastic behaviour. A coupled finite element for the simultaneous effects of rock mass deformation and transient fluid flow was used for this purpose. Rock mass behaviour was based on the concept of effective stress, Hooke's law, Darcy's law, associated plasticity and a
parabolic yield condition appropriate to rock masses. The major effects of plasticity, which limits the range of purely elastic behaviour by rock mass strength, were greater displacements and persistent yielding. Yielding anticipated in poroelastic analyses, where the ratio of strength to stress was less than one, was initially somewhat more extensive than in the poroelastic/plastic case, but diminished considerably with time. In the poroelastic/plastic case, yielding that occurred during a slope cut persisted in time and space despite depressurization. Applicability of poroelastic/plastic finite element analysis to actual open pit mine slopes was demonstrated.

Mauldon et al. (1998) presented an energy-based model for analysing the stability of rock blocks with any number of contact planes or a curved contact surface. When the sliding of a prismatic rock block with more than two contacts or a curved surface is a concern, the distribution of the forces among the contact faces is statically indeterminate. Energy principle was used for finding the distribution of the total normal forces among the contact planes. The limiting equilibrium methods were used to validate the model for a special case blocks, showing very similar results. It was shown that treating a block with a curved failure surface as a wedge was generally unconservative.

Bobet (1999) provided analytical solutions for toppling failure based on the limiting equilibrium approach. The toppling mechanism was considered for small block in 2D-plane conditions considering water seepage. The derivations were verified with both the analytical and numerical method proposed by Hoek and Bray (1981). The analytical solution was found to give accurate results, within 10% of the numerical solution, height to length ratios larger than 50.

Adhikary et al. (2001) formulated a model for describing the deformation of rock masses. The behaviour of the intact rock layer assumed linearly elastic and the joints were elastic-perfectly plastic. Conditions of slip at the interfaces were determined by a Mohr-Coulomb criterion with tension cut off at zero normal stress and valid for large deformations. The model was incorporated into the finite element program AFENA and validated against an analytical solution of elementary buckling problems of a layered medium under gravity loading. Design charts suitable for assessing the stability of slopes in foliated rock masses against flexural buckling
failure were developed. The design chart is easy to use and provides a quick estimate of critical loading factors for slopes in foliated rock masses.

Chen et al. (2001) proposed a three-dimensional (3D) slope stability analysis method and provided the numerical procedures that implemented the three-dimensional upper-bound slope stability. A three-dimensional failure surface was generated by elliptical lines based on the slip surface in the neutral plane and extended in the z-direction. That failure surface was mathematically represented by a series of variables including the coordinates of the nodal points that define the slip surface at the neutral plane, the inclinations of the row-to-row interfaces and the coefficients that define the ratio of the long axis over the low one of the elliptic. A method of optimisation was followed in order to found the variables that offer the minimum factor of safety. A computer program EMU-3D was coded to perform the calculation for practical problems. The method was explained with a case study of the Tianshenqiao Landslide.

Kemeny (2003) developed a model for the time-dependent degradation of rock joint cohesion. A fracture mechanics model was developed utilizing subcritical crack growth, which resulted in a closed-form solution for joint cohesion as a function of time. A rock block containing rock bridges subjected to plane sliding was analysed. The cohesion was found to continually decrease, at first slowly and then more rapidly. At a particular value of time the cohesion reduced to value those results in slope instability. A probabilistic slope analysis was conducted, and the probability of failure as a function of time was predicted. The probability of failure was found to increase with time, from an initial value of 5% to a value at 100 years of over 40%.

Kim et al. (2004) developed an approach for slope stability analysis of rock cuts using Geographical Information System (GIS) considering plane, wedge, and toppling failure modes. Various factors affecting the slope stability analysis, such as the structural domain, the orientation and dip angle of the cut slopes, and the friction angle of discontinuities, were considered as the input parameters for GIS. By overlaying input data layers and using the developed computer algorithm, the factor of safety ($FS$) values, as an index of slope stability were calculated for each failure mode, which considered for stereographic analysis and limit equilibrium analysis simultaneously. The factors of safety for each failure mode were evaluated and the
minimum factor of safety was also evaluated in the divided small area. In order to verify the developed analysis method, the results of the cut rock slope stability were compared with actual failure modes and locations in the study area. Finally, the stable and economically appropriate cut angle for the planned rock slopes were suggested by using the developed algorithm and applying allowable factor of safety value.

Zheng (2005) developed numerical solutions of rock slope stability analysis in two-dimension for computation of the factors of safety and location of the critical slide line (CSL). Poisson’s ratio was adjusted to satisfy a proper factor of safety with less calculation and a rational distribution of plastic zones in the critical equilibrium state. Kinematic solution from a hydraulic project in construction was analysed to demonstrate the effectiveness of the procedures.

Qin et al. (2006) presented a cusp catastrophe model based on the catastrophe theory and discussed the necessary and sufficient conditions leading to landslide instability. It was assumed that the sliding surface of the landslide was planar and was a combination of two media: one was elastic-brittle or strain-hardening and the other was strain-softening. The shear stress-strain constitutive model for the strain-softening medium was described by the Weibull’s distribution law. The conditions leading to a fast-moving landslide were derived. It was found that the instability of a slope relies mainly on the ratio of the stiffness of medium 1 to the post-peak stiffness of medium 2, and the homogeneity index of medium 2. The role of water was to enhance the material homogeneity or brittleness and hence to reduce the stiffness ratio of the system.

Rodriguez et al. (2006) presented a systematic quantitative methodology for the reliability analysis of stability of rock slopes. A sliding mass resting on an inclined plane and composed of two blocks separated by a vertical tension crack was considered. A disjoint cut-set formulation was used to compute the reliability of the system, within that framework, each cut-set was associated with a failure mode and the probability of failure of the system was obtained as the sum of the probabilities of each failure mode. The analysis results of two-block sliding system were produced, which will help the designer to establish priorities during design and decision making.
Yang and Zou (2006) applied the kinematical theorem of limit analysis to calculate the stability factors of rock slopes subjected to porewater using the nonlinear Hoek–Brown (HB) failure criterion. Porewater pressures, regarded as external forces, are calculated using a pore water pressure ratio. The generalized tangential technique proposed by Yang et al. (2004a, b) was employed to formulate the stability factor as an optimization problem. A linear Mohr–Coulomb (MC) failure criterion was employed by extending it using the HB failure criterion. The main result is a convenient expression to estimate stability of rock slope subjected to pore water pressures. Numerical results for five types of rocks were presented for practical use in rock engineering.

Yang et al. (2006) presented the concept of the degree of reinforcement demand (DRD) for rock slope projects and the quantitative procedures for the DRD assessment. The main influencing factors were determined, classified and assigned based on knowledge from theoretical analyses, practical experiences and monitoring. A rock slope assessed by divided it into a number of slope zones and the construction schedule into different periods. The factors and measurement that influence the DRD were analysed via matrices. The feasibility of the DRD assessment was demonstrated to the shiplock slope of the Wuqiangxi hydropower station in China.

Low (2007) probabilistically analysed a two-dimensional jointed rock slope in Hong Kong and a three-dimensional hypothetical tetrahedral wedge using an intuitive and transparent constrained optimization approach for the first-order reliability method (FORM). The effects of correlation coefficients on the computed reliability index were studied and discussed. The results were compared with Monte Carlo simulations. The difference between probabilities of failure inferred from reliability index and from Monte Carlo simulations were investigated via the response surface method. It was shown that the efficiency of reliability-based approach can be combined with the robustness of Monte Carlo simulation. It was found that reliability-based design can be done quickly and efficiently using the procedure presented.

Tonon and Asadollahi (2008) carried out wedge failure validation under gravity loading using an algorithm BS3D developed by Tonon (2007) for analysis of single rock blocks that can handle general failure modes under conservative and non-conservative forces. Sixty-four physical models and two case histories were analysed.
Using the method. For the wedge stability problem, physical modelling and BS3D give the same failure modes except for six cases in which sliding on one plane were observed in physical models while BS3D predicted sliding on two planes. In all cases, safety factors obtained using BS3D analyses were the same as obtained using Block Theory limiting analysis. The results of BS3D analyses for two case histories agreed well with the observations that the wedges had already failed.

Li et al. (2008) produced stability charts for rock slopes using numerical limit analysis. Those charts were produced based on the most recent version of the Hoek–Brown failure criterion, applicable for isotropic and homogeneous intact rock, or heavily jointed rock masses. The accuracy of using equivalent Mohr–Coulomb parameters for the rock mass in a traditional limit equilibrium method of slice analysis was investigated. It was found that limit equilibrium method of slice analysis could be used in conjunction with equivalent Mohr–Coulomb parameters to produce factor of safety estimates close to the limit analysis results, provided modifications were made to the underlying formulations. Such modifications were made in the software SLIDE, where a set of equivalent Mohr–Coulomb parameters were calculated at the base of each individual slice. This approach predicts factors of safety remarkably close to the limit analysis solutions that are based on the native form of the Hoek–Brown criterion.

Liu et al. (2008) developed some analytical expressions based on limit equilibrium approach, for analyzing the toppling stability of rock slopes. Those rocks were characterized by blocks whose thickness was significantly smaller than the height of the block at the crest that can be considered as continuum. The effect of the angle of the block base with the normal to the dip of the dominant discontinuities on the toppling stability was analyzed. The transition position from toppling to sliding wear determined. A spreadsheet procedure was presented for facilitating the method and by which several cases of toppling were analyzed. The results indicate that the proposed solution represents the asymptotic value of the support force necessary to stabilize the slope against toppling as the slenderness ratio tends to infinity and that, when the slenderness ratio is greater than approximately 15–25, the support force calculated by the proposed solution provides an accurate estimate of the actual value.
Amini et al. (2009) presented some analytical expressions for the determination of the factor of safety and the stabilisation of rock mass instability considering flexural toppling as the mode of failure. The developed expressions were based on the principle of compatibility equations by which the magnitudes and points of application of inter-column forces were determined. It has been claimed that the safety factors for each rock column can be computed independently. The results obtained by using the expressions were compared with the results of existing laboratory approaches (base friction and centrifuge experiments) and were found appropriate for evaluating both the layered strata and the jointed rock mass stability against flexural toppling failure for rock slopes and underground openings as well.

2.3 PSEUDO-STATIC SLOPE STABILITY ANALYSES

Analysis of seismic slope stability problems in which forces due to earthquake shaking are represented as horizontal and vertical forces, equal to weight of the potential sliding mass multiplied by a coefficient, are commonly known as pseudo-static analysis. It is an approach used in earthquake engineering to analyze the seismic response of soil and rock slopes. In earthquake prone areas, horizontal and vertical pseudo-static (seismic) coefficients, $k_h$ and $k_v$, respectively, are used to compute the horizontal and vertical seismic inertial forces caused by a potential earthquake. These forces are then added to the overall equilibrium computation for the individual slice/block composing the failure mass (Melo and Sharma, 2004). Analytical formulations were developed earlier for calculating the factor of safety on plane failure mode under seismic loading conditions using pseudo-static (seismic) coefficients.

Siad (2003) analysed the stability of fractured rock slope located in seismic area based upon the kinematic approach of the yield design theory and the pseudo-static method. Upper bound solutions were obtained from consideration of simple translational failure mechanism based on the movement of rigid block which slides on fracture planes and through rock material. The results were presented in the form of stability charts relating in the estimated upper bound solutions.
Ling and Cheng (1997) presented a formulation based on two-dimensional limit equilibrium analysis. It was valid for a rock mass with sufficiently large width, typically with a plane strain condition. The rock mass was considered a rigid body. The strength of the joint plane was assumed to be plastic, obeying the Coulomb failure criterion. The developed analytical formulation considers a potential sliding rock mass of height $H$ and unit weight $\gamma$. The tension crack extends from the crest to depth $z$ with water filled to a height $z_w$. The forces acting on the joint are shown in Fig. 2.4, where $C$ and $F$ are the total cohesion and frictional resisting force acting along the joint plane. To see the effect of seismic force on the slope stability, factor safety was derived as:

$$FS = \frac{W \cos(\psi + \theta) \tan\phi - (U_1 (\sin\psi + U_2) \tan\phi - C) / k}{W \sin(\psi + \theta) + (U_1 \cos\psi) / k}$$

(2.6)

![Fig.2.4. Mechanism of rock slope failure under self weight, water forces, and horizontal and vertical seismic forces (adapted from Ling and Cheng, 1997).](image-url)
The formulation was then used to calculate yield acceleration and permanent displacement of the rock mass subject to random seismic excitations. Extension of the procedure to wedge sliding analysis was also included. An “example slope” was included to illustrate the proposed formulation. The effects of seismic coefficient on stability and permanent displacement were addressed. The vertical acceleration in an upward direction was found to reduce the stability and yield acceleration of the rock mass.

Li (2007) developed Numerical solutions for a finite element slope stability analysis using nonlinear shear strength criteria of power-law. Stability numbers as well as factors of safety for rock slopes and soil mechanics were computed separately. Depending on the factors of safety the design charts were prepared. Earthquake effects were also analysed by pseudo-static considerations. Estimated failure mechanisms were compared to those obtained using limit analysis and limit equilibrium. Stability numbers for different seismic coefficients were also computed and compared with other existing solutions.

Yang (2007) derived an expression for the yield seismic coefficient for homogeneous and isotropic rock slopes under the condition of no porewater pressure, using the kinematical theorem of limit analysis with an associative flow rule. The upper bound theorem of limit analysis was used to determine yield seismic coefficient and its corresponding failure mechanism. Seismic displacement induced
by earthquake was calculated with the modified Hoek–Brown (HB) failure criterion that is nonlinear. A linear failure criterion, which was tangential to the actual modified HB failure criterion, was used to calculate the rate of external work and internal energy dissipation. Equating the work rate of external forces to the internal energy dissipation rate, an objective function was obtained. The yield seismic coefficients were obtained by minimizing the objective function. It was reported that the yield seismic coefficients increased as the parameters related to rock type and geological strength index increased with other relevant parameters as constant. Though the approach for analysis adopted is more realistic, the findings have limited applications in field projects because the pore water pressure is expected in most of the cases, which has not be considered in the analysis.

Hoek (2007) described the idealisation of the rock slope failures in Hong Kong as plain failures and presented an analytical expression for estimating the factor of safety, considering many practical aspects including horizontal seismic loadings in rock mass block as shown in Fig. 2.5. The slope stability was analysed as a two-dimensional problem, considering a slice of unit thickness, referring to a 1 metre thick slice through the slope and assuming negligible resistance to sliding at the lateral boundaries of the sliding block. The analysis considers only force equilibrium and assumes that all forces pass through the centroid of the rock block. In other words, moment equilibrium is not considered in this analysis. The analytical expression for the factor of safety of an anchored rock slope with horizontal seismic force was derived as

\[
FS = \frac{c \alpha + [W (\cos \psi_p - k_h \sin \psi_p) - U_1 \sin \psi_p - U_2 + T \cos \alpha] \tan \phi}{W (\sin \psi_p + k_h \cos \psi_p) + U_1 \cos \psi_p - T \sin \alpha}
\] (2.8)

Shukla et al. (2009) presented an analytical expression for the factor of safety of the rock slope incorporating most of the practically occurring destabilizing forces as well as the external stabilizing force through an anchoring system. The slope stability was analysed as a two-dimensional problem, considering a slice of unit thickness through the slope and assuming negligible resistance to sliding at the lateral boundaries of the sliding block. The analytical expression of the factor of safety of an anchored rock slope with surcharge \( q \) placed at the top of the slope
subjected to horizontal seismic force, as shown in Fig. 2.6. An analytical expression for the factor of safety was derived as

\[
FS = \frac{cA+[W(\cos \psi_p-k_h \sin \psi_p)-U_1 \sin \psi_p-U_2+T \cos \alpha+qB \cos \psi_p] \tan \phi}{W(\sin \psi_p+k_h \cos \psi_p)+U_1 \cos \psi_p-T \sin \alpha+qB \sin \psi_p}
\] (2.9)

Shukla et al. presented a detailed parametric study to investigate the effect of surcharge on the stability of the rock slope for practical ranges of governing parameters such as inclination of the slope face, inclination of the failure plane, depth of tension crack, depth of water in tension crack, shear strength parameters of the rock, and other influencing factors. The analysis was based on the mechanisms shown in Fig. 2.5.

**Fig.2.5.** Mechanism of rock slope failure under self weight, water forces, horizontal seismic force and stabilizing force.
material at the failure plane, unit weight of rock, stabilizing force and its inclination, and seismic load. It has been shown that the factor of safety of the rock slope decreases with an increase in surcharge; the rate of decrease being relatively higher for lower values of surcharge. It is also observed that for a specific surcharge, the factor of safety depends significantly on all other parameters, except for unit weight of rock and higher values of inclination of stabilizing force to the normal at the failure plane. For any combination of these variables, the surcharge plays a vital role in the stability. It was reported that a perfectly stable slope at relatively low surcharge could become unsafe with an increase in surcharge. The analysis presented can be used to carry out a quantitative assessment of the stability of the rock slopes.

Aydan and Kumsar (2010) presented some stability conditions for rock wedges under dynamic loading, and confirmed their validity through the laboratory experiments. A series of laboratory shaking table tests were carried out on wedge models under dynamic excitations for the assessment of the validity of the limiting equilibrium method as well as to evaluate their sliding responses during shaking. The shaking table experiments on the wedge models were performed under dry
conditions. The method of stability analysis for wedge failure of rock slopes proposed by Kovari and Firtz (1975) was extended to evaluate the dynamic sliding response of wedge blocks. The estimated sliding responses from the method presented were found to be in good agreements with the experimental results.

2.4 CONCLUSIONS

It is observed that the realistic assessment of stability of rock slopes under surcharge and seismic loading conditions has been a problem for engineers. The research work has been carried out until recently to present the analysis for different modes of failure under various practical site conditions. They can be classified mainly into two main groups as static slope stability analysis and dynamic slope stability analysis. The stability analyses have been made in various ways, including limit equilibrium and finite element approaches. Because of wide applications over a long period of time, the results obtained from limit equilibrium analyses have been well accepted by the engineers. Attempts have been made in earlier studies to present analytical expressions for the factor of safety of the rock slopes against plane failure, considering some field aspects; however, further effort is required to represent generalised expressions, considering most of the field parameters. The present research work is aimed at developing such an expression under surcharge and seismic loading conditions for practical applications.
CHAPTER 3

SINGLE-DIRECTIONAL ANCHORED ROCK SLOPE: ANALYTICAL FORMULATION

3.1 GENERAL

This chapter presents a derivation of an analytical expression for the factor of safety of a single-directional anchored rock slope (SDARS) against the plane failure as described earlier. The derivation considers most of the factors that may arise in field conditions under surcharge and seismic loadings. Additionally, the stabilization force caused by an anchoring system is also included in the analysis because rock anchors are generally installed as a ground improvement solution to increase the stability of the rock slopes.

3.2 GENERAL PLANE FAILURE CONDITIONS AND ASSUMPTIONS

The following conditions applicable to a rock slope plane failure as described by Hoek and Bray (1981) are considered in the present analysis:

1. The failure/sliding plane strikes parallel or nearly parallel (within approximately ± 20°) to the slope.
2. The sliding plane must “daylight” in the slope face, which means that the dip of the plane ($\psi_p$) must be less than the dip of the slope face ($\psi_f$), that is $\psi_p < \psi_f$ (Fig. 2.1(a)).
3. The dip of the sliding plane ($\psi_p$) must be greater than the angle of friction of this plane ($\phi$), that is, $\psi_p > \phi$ (Fig. 2.1(a)).
4. The upper end of the sliding surface either intersects the upper slope, or terminates in the tension crack.

5. Release surfaces that provide negligible resistance to sliding must be present in the rock mass to define the lateral boundaries of the slide (Fig. 2.1(b)).

![Fig. 3.1. Geometry of slope exhibiting plane failure: (a) cross-section of the rock slope with a plane failure situation; and (b) release surfaces at ends of plane failure (after Wyllie and Mah, 2004).](image)

In addition to above general conditions, the following assumptions are made in the present analysis:

1. The sliding rock mass is considered to be a rigid body.
2. Both sliding surface and tension crack strike parallel to the slope.
3. The slope stability is analysed as a two-dimensional plain strain problem, considering a slice of unit thickness through the slope.
4. The tension crack is vertical and is filled with water to a certain depth.
5. The joint plane material is assumed to be a $c$-$\phi$ soil material with $c$ and $\phi$ as cohesion and angle of internal friction (also called angle of shearing resistance), respectively, obeying the Mohr-Coulomb failure criterion.
6. Water enters the sliding surface along the base of the tension crack and seeps along the sliding surface, escaping at atmospheric pressure where the sliding surface daylights in the slope.
7. The porewater pressure along the joint plane is considered to distribute linearly with a zero-value at the toe of the slope.
8. All the forces act through the centroid of the sliding mass. There are no moments that would tend to cause rotation of the block, and hence failure is by sliding only.
9. The force equilibrium is considered without any resistance to sliding at the lateral boundaries of the sliding block.
10. The hydrodynamic force of the porewater is negligibly small, and it has been ignored in the analysis.
11. The analysis primarily considers horizontal and vertical seismic inertial forces, and the amplification aspect is not taken into account. In other words, a pseudo-static seismic analysis is considered.

3.3 DERIVATION

A rock slope of height $H$ with an inclination $\psi_f$ to the horizontal is shown in Fig. 3.1. The joint plane $A_2A_3$ inclined to the horizontal at an angle $\psi_p$ and a vertical tension crack $A_3A_4$ of depth $z$ separate a portion of the rock mass as the block $A_1A_2A_3A_4$ having a weight $W$. The tension crack is filled with water to a depth $z_w$. The stabilizing tensile force $T$ inclined at an angle $\alpha$ to normal at the joint plane $A_2A_3$ simulates the effect of a rock anchoring system such as rock bolts or cables, which are commonly used to stabilize the rock slopes. The horizontal and vertical seismic inertial forces, $k_h W$ and $k_v W$ with $k_h$ and $k_v$ as horizontal and vertical seismic coefficients, respectively, are shown to act on the sliding block. As reported in the literature, typically, $k_h$ varies from 0.0 to 0.5, and $k_v$ is considered as half of the $k_h$. A surcharge placed at the top of the slope $A_1A_4 (= B)$ applies a vertical pressure $q$ along with horizontal and vertical seismic inertial forces, $k_h q B$ and $k_v q B$, respectively. The horizontal force due to water pressure in the tension crack is $U_1$, and the uplift force due to water pressure on the joint plane is $U_2$. Under a critical combination of forces, the rock mass block $A_1A_2A_3A_4$ can slide along the joint plane $A_2A_3$ as a failure plane.
From Fig. 3.1, the volume of the sliding rock block $A_{1}A_{2}A_{3}A_{4}$ is

$$V = \frac{1}{2} (B + H \sin \psi_{f} + B)H - \frac{1}{2} (H - z)(H \cot \psi_{f} + B)$$

and

$$H \cot \psi_{f} + B = (H - z) \cot \psi_{f}$$

or

$$B = (H - z) \cot \psi_{f} - H \cot \psi_{f}$$
\[ H \left\{ \left( 1 - \frac{z}{H} \right) \cot \psi_f - \cot \psi_f \right\} \] (3.2)

By substituting \( B \) from Eq. (3.2) into Eq. (3.1),

\[
V = \frac{1}{2} \left\{ \left( 1 - \frac{z}{H} \right) \cot \psi_p - \cot \psi_f \right\} H - \frac{1}{2} H(H - z) \left\{ \cot \psi_f + \left( 1 - \frac{z}{H} \right) \cot \psi_p - \cot \psi_f \right\}
\]

\[
= \frac{1}{2} \left\{ 2 \left( (H - z) \cot \psi_p - H \cot \psi_f \right) H - \frac{1}{2} (H - z)^2 \cot \psi_p + (H - z) \cot \psi_p - H \cot \psi_f \right\}
\]

\[
= \frac{1}{2} \left\{ 2(H - z) \cot \psi_p - 2H \cot \psi_f + H \cot \psi_f \right\} H - \frac{1}{2} (H - z)^2 \cot \psi_p
\]

\[
= \frac{1}{2} \left\{ 2H \cot \psi_p - 2z \cot \psi_p - H \cot \psi_f \right\} H - \frac{1}{2} (H - z)^2 \cot \psi_p
\]

\[
= H^2 \cot \psi_p - H z \cot \psi_p - \frac{1}{2} H^2 \cot \psi_f - \frac{1}{2} H^2 \cot \psi_p + Hz \cot \psi_p - \frac{1}{2} z^2 \cot \psi_p
\]

\[
= \frac{1}{2} H^2 \cot \psi_p - \frac{1}{2} z^2 \cot \psi_p - \frac{1}{2} H^2 \cot \psi_f
\]

\[
= \frac{1}{2} H^2 \cot \psi_p \left( 1 - \frac{z^2}{H^2} \right) - \frac{1}{2} H^2 \cot \psi_f
\]

\[
= \frac{1}{2} H^2 \left\{ \left( 1 - \frac{z^2}{H^2} \right) \cot \psi_p - \cot \psi_f \right\}
\] (3.3)

The weight of the sliding rock mass block \( A_1A_2A_3A_4 \) is

\[ W = \gamma \times V \] (3.4)

where \( \gamma \) is the unit of rock mass constituting the sliding block.
Substituting $V$ from Eq. (3.3) into Eq. (3.4),

$$W = \frac{1}{2} \gamma H^2 \left[ \left\{ 1 - \left( \frac{z}{H} \right)^2 \right\} \cot \psi_p - \cot \psi_f \right]$$

(3.5)

The total force available to resist the sliding block is

$$F_r = \tau A$$

(3.6)

where $\tau$ is the shear strength of the sliding failure plane, and $A$ is the area of the base $A_2A_3$ of the sliding rock block given as

$$A = (H - z)cosec\psi_p$$

$$= H \left( 1 - \frac{z}{H} \right) cosec\psi_p$$

(3.7)

The Mohr-Coulomb failure criterion provides (Lambe and Whitman, 1969; Das, 2008)

$$\tau = c + \sigma_n \tan \phi$$

(3.8)

where $\sigma_n$ is the normal stress on the failure plane.

From Eq. (3.6) and Eq. (3.8),

$$F_r = (c + \sigma_n \tan \phi) A$$

$$= cA + \sigma_n A \tan \phi$$

$$= cA + F_n \tan \phi$$

(3.9)

where $F_n = \sigma_n A$ is the normal force on the failure plane, and it is given as
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\[ F_n = \sigma_n A \]

\[ = W \cos \psi_p - k_h W \sin \psi_p + k_v W \cos \psi_p - U_1 \sin \psi_p - U_2 + T \cos \alpha + qB \cos \psi_p - k_h qB \sin \psi_p + k_v qB \cos \psi_p \]

\[ = W \left( \cos \psi_p - k_h \sin \psi_p \pm k_v \cos \psi_p \right) - U_1 \sin \psi_p - U_2 + T \cos \alpha + qB \left( \cos \psi_p - \sin \psi_p \pm k_v \cos \psi_p \right) \]

\[ = W \left\{ \cos \psi_p \left(1 \pm k_v\right) - k_h \sin \psi_p \right\} - U_1 \sin \psi_p - U_2 + T \cos \alpha + qB \left\{ \cos \psi_p \left(1 \pm k_v\right) - k_h \sin \psi_p \right\} \]

\[ = (W + qB) \left\{ (1 \pm k_v) \cos \psi_p - k_h \sin \psi_p \right\} - U_1 \sin \psi_p - U_2 + T \cos \alpha \quad (3.10) \]

Horizontal force on the sliding block due to water pressure in the tension crack is

\[ U_1 = \frac{1}{2} \gamma_w z_w \times z_w = \frac{1}{2} \gamma_w z_w^2 \quad (3.11) \]

where \( \gamma_w \) is the unit weight of water.

Uplift force on the sliding block due to water pressure on failure plane is

\[ U_2 = \frac{1}{2} \left\{ (H - z) \csc \psi_p \right\} (\gamma_w z_w) \]

\[ = \frac{1}{2} \gamma_w z_w H \left( 1 - \frac{z}{H} \right) \csc \psi_p \quad (3.12) \]

By substituting \( B, W, U_1 \) and \( U_2 \) from Eqs. (3.2), (3.5), (3.11) and (3.12), respectively into Eq. (3.10),
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\[ F_n = \left[ \frac{1}{2} \gamma H^2 \left( 1 - \left( \frac{z}{H} \right)^2 \right) \cot \psi_p - \cot \psi_f \right] + qH \left[ \left( 1 - \frac{z}{H} \right) \cot \psi_p - \cot \psi_f \right] \{ \cos \psi_p (1 \pm k_v) - k_h \sin \psi_p \} - \frac{1}{2} \gamma_w z_w^2 \sin \psi_p - \frac{1}{2} \gamma_w z_w H(1 - \frac{z}{H}) \cosec \psi_p + T \cos \alpha \] (3.13)

Substituting \( A \) and \( F_n \) from Eqs. (3.7) and (3.13), respectively into Eq. (3.9),

\[ F_r = cH \left( 1 - \frac{z}{H} \right) \cosec \psi_p + \left[ \frac{1}{2} \gamma H^2 \left( 1 - \left( \frac{z}{H} \right)^2 \right) \cot \psi_p - \cot \psi_f \right] + qH \left[ \left( 1 - \frac{z}{H} \right) \cot \psi_p - \cot \psi_f \right] \{ \cos \psi_p (1 \pm k_v) - k_h \sin \psi_p \} - \frac{1}{2} \gamma_w z_w H(1 - \frac{z}{H}) \cosec \psi_p - \frac{1}{2} \gamma_w z_w^2 \sin \psi_p + T \cos \alpha \] \tan \phi

\[ = cH \left( 1 - \frac{z}{H} \right) \cosec \psi_p + \left[ \frac{1}{2} \gamma H^2 \left( 1 - \left( \frac{z}{H} \right)^2 \right) \cot \psi_p - \cot \psi_f \right] + qH \left[ \left( 1 - \frac{z}{H} \right) \cot \psi_p - \cot \psi_f \right] \{ \cos \psi_p (1 \pm k_v) \left[ \frac{k_h}{(1 \pm k_v)} \sin \psi_p \right] \} - \frac{1}{2} \gamma_w z_w H(1 - \frac{z}{H}) \cosec \psi_p - \frac{1}{2} \gamma_w z_w^2 \sin \psi_p + T \cos \alpha \] \tan \phi (3.14)

Assuming

\[ \frac{k_h}{(1 \pm k_v)} = \tan \theta \] (3.15)

Eq. (3.14) reduces to

\[ F_r = cH \left( 1 - \frac{z}{H} \right) \cosec \psi_p + \left[ \frac{1}{2} \gamma H^2 \left( 1 - \left( \frac{z}{H} \right)^2 \right) \cot \psi_p - \cot \psi_f \right] + qH \left[ \left( 1 - \frac{z}{H} \right) \cot \psi_p - \cot \psi_f \right] \{ (1 \pm k_v) (\cos \psi_p - \tan \theta \sin \psi_p) \} - \frac{1}{2} \gamma_w z_w^2 \sin \psi_p - \frac{1}{2} \gamma_w z_w H(1 - \frac{z}{H}) \cosec \psi_p + T \cos \alpha \] \tan \phi
From Fig. 1, the total force tending to induce sliding is calculated as

\[ F_i = W \sin \psi_p + k_h W \cos \psi_p \pm k_v W \sin \psi_p + U_1 \cos \psi_p - T \sin \alpha + QB \sin \psi_p + k_h QB \cos \psi_p \pm k_v QB \sin \psi_p \]

\[ = W \left( \sin \psi_p + k_h \cos \psi_p \pm k_v \sin \psi_p \right) + U_1 \cos \psi_p - T \sin \alpha + QB \left( \sin \psi_p + k_h \cos \psi_p \pm k_v \sin \psi_p \right) \]

\[ = W \left[ \sin \psi_p \left( 1 \pm k_v \right) + k_h \cos \psi_p \right] + U_1 \cos \psi_p - T \sin \alpha + QB \left[ \sin \psi_p \left( 1 \pm k_v \right) + k_h \cos \psi_p \right] \]

\[ = (W + QB) \left[ \sin \psi_p \left( 1 \pm k_v \right) + k_h \cos \psi_p \right] + U_1 \cos \psi_p - T \sin \alpha \quad (3.17) \]
The factor of safety $FS$ of the rock slope is defined as (Hoek and Bray, 1981; Wyllie and Mah, 2004; Hoek, 2007):

$$FS = \frac{F_r}{F_i}$$  \hspace{1cm} (3.20)
Substituting \( F_r \) and \( F_i \) from Eqs. (3.16) and (3.19), respectively into Eq. (3.20),

\[
FS = \frac{cH \left( \frac{1 - z^*}{H} \right) \text{cosec} \psi_p + \left(1 + k_v\right) \frac{1}{2} \gamma H^2 \left[1 - \left(\frac{z^*}{H}\right)^2\right] \cot \psi_p - \cot \psi_f \right] + qH \left(1 - \frac{z^*}{H}\right) \cot \psi_p}{\cos \theta} - \cot \psi_f \right) + qH \left(1 - \frac{z^*}{H}\right) \cot \psi_p \]

(3.21)

Dividing denominator and numerator of Eq. (3.21) by \( \gamma H^2 \)

\[
FS = \frac{cH \left( \frac{1 - z^*}{H} \right) \text{cosec} \psi_p + \left(1 + k_v\right) \frac{1}{2} \gamma H^2 \left[1 - \left(\frac{z^*}{H}\right)^2\right] \cot \psi_p - \cot \psi_f \right] + \frac{1}{\gamma H^2} \gamma w \text{cosec} \psi_p - \frac{1}{\gamma H^2} \gamma w \text{cosec} \psi_p - \frac{1}{\gamma H^2} \gamma w \text{cosec} \psi_p - \frac{1}{\gamma H^2} \gamma w \text{cosec} \psi_p \}

(3.22)

Eq. (3.22) can be simplified further as

\[
FS = \frac{c^* \left(1 - z^*\right) \text{cosec} \psi_p + \left(1 + k_v\right) \frac{1}{2} \left(1 - z^*\right) \cot \psi_p - \cot \psi_f \right] + q^* \left(1 - z^*\right) \cot \psi_p - \cot \psi_f \right] + \frac{1}{\gamma H^2} \gamma w \text{cosec} \psi_p - \frac{1}{\gamma H^2} \gamma w \text{cosec} \psi_p - \frac{1}{\gamma H^2} \gamma w \text{cosec} \psi_p - \frac{1}{\gamma H^2} \gamma w \text{cosec} \psi_p \]

(3.23)

where \( c^* = \frac{c}{\gamma H} \), \( z^* = \frac{z}{H} \), \( z_w = \frac{z_w}{H} \), \( \gamma^* = \frac{\gamma}{\gamma w} \), \( q^* = \frac{q}{\gamma H} \) and \( T^* = \frac{T}{\gamma H^2} \) are nondimensional forms of \( c, z, z_w, \gamma, q \) and \( T \), respectively.

Eq. (3.23) can be simplified further as

\[
FS = \frac{2c^* + \left(1 + k_v\right) \left(1 + q^* R\right) \frac{\cos (\theta + \psi_p)}{\cos \theta} - \frac{z_w^2}{\gamma w} \sin \psi_p - \frac{z_w^2}{\gamma w} \sin \psi_p - 2T^* \cos \alpha \tan \phi}{\left(1 + k_v\right) \left(1 + q^* R\right) \frac{\sin (\theta + \psi_p)}{\cos \theta} + \frac{z_w^2}{\gamma w} \cos \psi_p - 2T^* \sin \alpha}
\]

(3.24)
where

\[ P = (1 - z^*) \cot \psi_p \]  
(3.25a)

\[ Q = (1 - z^2) \cot \psi_p - \cot \psi_f \]  
(3.25b)

\[ R = (1 - z^*) \cot \psi_p - \cot \psi_f \]  
(3.25c)

Eq. (3.24) is the general expression for the factor of safety of the rock slope against plane failure. This can be used to observe the effect of any individual parameter on the factor of safety of the rock slope and to carry out a detailed parametric study as required in a specific field situation.

3.4 SPECIAL CASES

The general equation [Eq. (3.24)] developed for the factor of safety of the rock slope against the plane failure can have several special cases as explained below:

**Case 1:** The joint material is cohesionless, and there are no surcharge, stabilizing force, seismic forces and water in the tension crack, that is, \( c^* = 0, \phi \neq 0, q^* = 0, T^* = 0, k_h = 0, k_v = 0, \theta = 0 \) and \( z_w^* = 0 \). Here, Eq. (3.24) reduces to

\[ FS = \frac{\tan \phi}{\tan \psi_p} \]  
(3.26)

**Case 2:** The joint material is cohesionless, and there are no surcharge, seismic forces and water in the tension crack, that is, \( c^* = 0, \phi \neq 0, q^* = 0, T^* \neq 0, k_h = 0, k_v = 0, \theta = 0 \) and \( z_w^* = 0 \). Eq. (3.24) becomes

\[ FS = \frac{(Q \cos \psi_p + 2T^* \cos \alpha \tan \phi)}{Q \sin \psi_p - 2T^* \sin \alpha} \]  
(3.27)
Case 3: The joint material is cohesionless, and there are no seismic forces and water in the tension crack, that is, \( c^* = 0, \phi \neq 0, q^* \neq 0, T^* \neq 0, k_h = 0, k_v = 0, \theta = 0 \) and \( Z_w^* = 0 \). Eq. (3.24) reduces to

\[
FS = \frac{((Q+2q^*R)\cos\psi_p + 2T^* \cos \phi) \tan \phi}{(Q+2q^*R) \sin \psi_p - 2T^* \sin \alpha}
\]

Case 4: The joint material is cohesive, and there are no seismic forces and water in the tension crack, that is, \( c^* \neq 0, \phi = 0, q^* \neq 0, T^* \neq 0, k_h = 0, k_v = 0, \theta = 0 \) and \( Z_w^* = 0 \). Here, Eq. (3.24) becomes

\[
FS = \frac{2c^* P}{(Q+2q^*R) \sin \psi_p - 2T^* \sin \alpha}
\]

Case 5: The joint material is \( c - \phi \) material, and there are no seismic forces and water in the tension crack, that is, \( c^* \neq 0, \phi \neq 0, q^* \neq 0, T^* \neq 0, k_h = 0, k_v = 0, \theta = 0, Z_w^* = 0 \). Eq. (3.24) becomes

\[
FS = \frac{2c^* P + ((Q+2q^*R) \cos \psi_p + 2T^* \cos \phi) \tan \phi}{(Q+2q^*R) \sin \psi_p - 2T^* \sin \alpha}
\]

Case 6: The joint material is \( c - \phi \) material, and there are no seismic forces, that is, \( c^* \neq 0, \phi \neq 0, q^* \neq 0, T^* \neq 0, k_h = 0, k_v = 0, \theta = 0, Z_w^* \neq 0 \). Here, Eq. (3.24) becomes

\[
FS = \frac{2c^* P + ((Q+2q^*R) \cos \psi_p - \frac{Z_w^2}{r} \sin \psi_p - \frac{Z_w^2}{r} \cos \psi_p + 2T^* \cos \phi) \tan \phi}{(Q+2q^*R) \sin \psi_p + \frac{Z_w^2}{r} \cos \psi_p - 2T^* \sin \alpha}
\]

Case 7: The joint material is \( c - \phi \) material, and there is only horizontal seismic force, that is, \( c^* \neq 0, \phi \neq 0, q^* \neq 0, T^* \neq 0, k_h = 0, k_v = 0, \theta = \tan^{-1}(k_h), Z_w^* \neq 0 \). Here, Eq. (3.24) becomes

\[
FS = \frac{2c^* P + ((Q+2q^*R) \cos(\theta + \psi_p) - \frac{Z_w^2}{r} \sin(\theta + \psi_p) - \frac{Z_w^2}{r} \cos \psi_p + 2T^* \cos \phi) \tan \phi}{(Q+2q^*R) \sin(\theta + \psi_p) + \frac{Z_w^2}{r} \cos \psi_p - 2T^* \sin \alpha}
\]
For a generalised case when the joint material is $c - \phi$ material, that is, $c^* \neq 0$, $\phi \neq 0$, $q^* \neq 0$, $T^* \neq 0$, $k_h \neq 0$, $k_v \neq 0$, $\theta = \tan^{-1}\left\{\frac{k_h}{(1 \pm k_v)}\right\}$, $z_w^* \neq 0$, Eq. (3.24) will be applicable. It should be noted that some of the above special cases have been presented in similar forms in the literature (Hoek and Bray, 1981; Ling and Cheng, 1997; Hoek, 2007).

Fig. 3.3 shows the variation of the factor of safety (FS) with the angle of shearing resistance (\(\phi\)) of the joint material along the failure plane for several possible field situations as described above in the form of Eqs. (3.24), (3.26), (3.27), (3.28), (3.30), (3.31) and (3.32), considering a particular set of governing parameters in their nondimensional form as: $\psi_f = 50^\circ$, $\psi_p = 35^\circ$, $\phi = 35^\circ$, $q^* = 0.25$, $T^* = 0.05$, $z^* = 0.1$, $z_w^* = 0.05$, $\gamma^* = 2.5$, $k_h = 0.1$, $k_v = 0.05$ and $\alpha = 10^\circ$. It is observed that the factor of safety increases nonlinearly with an increase in $\phi$ and is greater than unity in all cases for $\phi$ larger than 35°; the rate of increase is higher for larger value of $\phi$. As expected, for any $\phi$, the cohesion in the joint material and the stabilizing force increase the factor of safety, whereas the surcharge and water in the tension crack decrease the factor of safety. It is also noted that with an upward vertical seismic inertial force, the factor of safety is always greater than that with the downward vertical seismic inertial force. It should be noted that the horizontal line at $FS = 1$ divides the figure into safe and unsafe regions.

Fig. 3.4 shows the variation of the factor of safety (FS) with nondimensional cohesion ($c^*$) of the joint material along the failure plane for some possible field situations as described above in the form of Eqs. (3.24), (3.29), (3.30), (3.31) and (3.32), considering specific values of governing parameters in their nondimensional form as: $\psi_f = 50^\circ$, $\psi_p = 35^\circ$, $\phi = 35^\circ$, $q^* = 0.25$, $T^* = 0.05$, $z^* = 0.1$, $z_w^* = 0.05$, $\gamma^* = 2.5$, $k_h = 0.1$, $k_v = 0.05$ and $\alpha = 10^\circ$. It is observed that the factor of safety increases linearly with an increase in cohesion and is greater than unity in all cases for $c^*$ larger than 0.15; the rate of increase is the highest for field situation described by Eq. (3.29). From Figures (3.3) and (3.4), it can be noticed that that shear strength parameters of the joint material, surcharge, water pressure and the stabilizing force contribute significantly to the factor of safety of the rock slopes against plane failure.
Fig. 3.3. Variation of factor of safety of the rock slope with angle of shearing resistance of the join material for several possible field situations.

Fig. 3.4. Variation of factor of safety of the rock slope with cohesion of the join material for several possible field situations.
3.5 CONCLUSIONS

Eq. (3.24) provides a general expression for the factor of safety of the anchored rock slope against plane failure, incorporating most of the practically destabilizing forces. Several special cases of this general expression based on possible field situations have been described. The graphical presentations for typical values of governing parameters indicate that the factor of safety of the rock slope increases with an increase in both angle of shearing resistance and cohesion of the joint material. The rate of increase in the factor of safety increases with an increase in angle of shearing resistance, whereas it remains constant for any increase in cohesion. The vertically upward direction of the inertial seismic force results in an increase in factor of safety, but the vertically downward direction of the inertial seismic force causes a decrease. For the higher factor of safety of the rock slope against plane failure under seismic loading conditions, greater values of shear strength parameters and/or the stabilizing force must be present. It is also realised that surcharge and water pressure in the tension crack decrease the factor of safety significantly. For the stability analysis of rock slopes, it is therefore essentially required to consider realistic values of all these parameters based on the actual field conditions.
Chapter 4: Parametric study and design charts for SDARS

CHAPTER 4

PARAMETRIC STUDY AND DESIGN CHARTS FOR SINGLE-DIRECTIONAL ANCHORED ROCK SLOPE

4.1 GENERAL

The generalised expression for the factor of safety of rock slopes against a plane failure derived in Chapter 3 considered both horizontal and vertical seismic coefficients and most of the parameters that may arise at any field site. In this chapter, a parametric study is made in order to investigate the effects of horizontal and vertical seismic coefficients on the factor of safety of the slope using the nondimensional form of the analytical expression derived in Chapter 3. To take quick decisions in real life projects, design charts play an important tool for the practicing engineers. Therefore, some design charts have been prepared here, and an illustrative example has been presented to explain their applications.

4.2 PARAMETRIC STUDIES

The nondimensional generalised form of the analytical expression for the factor of safety of rock slopes against a plane failure derived in Chapter 3 is presented below for convenience in the parametric study.

\[
FS = \frac{2c^*P + ((1\pm k_v)(Q+2q^*R) \frac{\cos(\theta + \psi_p)}{\cos \theta}}{(1\pm k_v)(Q+2q^*R) \frac{\sin(\theta + \psi_p)}{\cos \theta} + \frac{\sin \psi_p}{\rho^2} - \frac{\cos \psi_p}{\rho^2} - 2T^*\cos \alpha + \tan \phi}
\]  

(3.24)
Chapter 4: Parametric study and design charts for SDARS

Where

\[
P = (1 - z^*) \csc \psi_p
\]  
(3.25a)

\[
Q = (1 - z^{*2}) \cot \psi_p - \cot \psi_f
\]  
(3.25b)

\[
R = (1 - z^*) \cot \psi_p - \cot \psi_f
\]  
(3.25c)

The following practical ranges of parameters in their nondimensional form are considered in the study:

- Angle of inclination of the slope face to the horizontal \( \psi_f \): 40° - 60°
- Angle of inclination of the failure plane to the horizontal \( \psi_p \): 35° - 45°
- Depth of tension crack \( z^* \): 0.05 - 0.25
- Depth of water in tension crack \( z_w \): 0 - 0.1
- Cohesion \( c^* \): 0 - 0.16
- Angle of shearing resistance \( \phi \): 20° - 40°
- Unit weight of rock \( \gamma^* \): 2 - 3
- Surcharge load \( q^* \): 0 - 1.5
- Stabilizing force \( T^* \): 0 - 0.2
- Angle of inclination of stabilizing force to the normal at the failure plane \( \alpha \): 0 - 80°
- Horizontal seismic coefficient \( k_h \): 0.05 - 0.3
- Vertical seismic coefficient \( k_v \): -0.15 - 0.15
4.2.1 Effect of Vertical Seismic Coefficient

Fig. 4.1 shows the variation of the factor of safety ($FS$) with vertical seismic coefficient ($k_v$) for different values of horizontal seismic coefficient, $k_h = 0.05, 0.1, 0.15, 0.2, 0.25$ and $0.3$, considering a particular set of governing parameters in their nondimensional form as: $\psi_f = 50^\circ$, $\psi_p = 35^\circ$, $z^* = 0.1$, $z_{n^*} = 0.05$, $c^* = 0.1$, $\phi = 25^\circ$, $q^* = 0.5$, $T^* = 0.1$, $\gamma^* = 2.5$ and $\alpha = 45^\circ$. It is observed that with an increase in $k_v$ in the downward direction, $FS$ decreases almost linearly, but it increases almost linearly as $k_v$ increases in the upward direction. It is also noted that $FS$ is greater than unity for any value of $k_h$ lower than $0.25$, and it is higher for smaller values of $k_h$, which is an expected observation. It should be noted that the curves have been plotted in view of the consideration that $k_v$ is generally equal to or smaller than half of $k_h$.

![Graph showing variation of factor of safety (FS) with vertical seismic coefficient (k_v) for different values of horizontal seismic coefficient (k_h).]
Fig. 4.2 shows the variation of the factor of safety (FS) with vertical seismic coefficient ($k_v$) for different values of angle of inclination of the slope face to the horizontal, $\psi_f = 40^\circ, 45^\circ, 50^\circ, 55^\circ$ and $60^\circ$, considering a particular set of governing parameters in their nondimensional form as: $\psi_p = 35^\circ$, $z^* = 0.1$, $c^* = 0.1$, $\phi = 25^\circ$, $q^* = 0.5$, $T^* = 0.1$, $\gamma^* = 2.5$, $\alpha = 45^\circ$ and $k_h = 0.2$. It is noted that for $\psi_f = 40^\circ$, which is closure to $\psi_p = 35^\circ$, $FS$ decreases nonlinearly with an increase in $k_v$ with its downward direction, but it increases with its upward direction; the rate of decrease/increase being higher. For $\psi_f$ greater than $55^\circ$, $FS$ is less than unity, making the slope unstable, and it does not vary significantly with an increase $k_v$.

![Graph showing variation of factor of safety (FS) with vertical seismic coefficient ($k_v$) for different values of angle of inclination of the slope face to the horizontal ($\psi_f$).]
Fig. 4.3 shows the variation of the factor of safety \( (FS) \) with vertical seismic coefficient \( (k_v) \) for different values of angle of inclination of failure plane to the horizontal, \( \psi_p = 35^\circ, 37.5^\circ, 40^\circ, 42.5^\circ \) and \( 45^\circ \), considering a particular set of governing parameters in their nondimensional form as: \( \psi_f = 50^\circ, \ z^* = 0.1, \ z_w^* = 0.05, \ c^* = 0.1, \ \phi = 25^\circ, \ q^* = 0.5, \ T^* = 0.1, \ \gamma^* = 2.5, \ \alpha = 45^\circ \) and \( k_h = 0.2. \) It is observed that for \( \psi_p = 45^\circ \), which is closer to \( \psi_f = 50^\circ \), \( FS \) decreases nonlinearly with an increase in \( k_v \) with its downward direction, but it increases with its upward direction; the rate of decrease/increase being higher. It is also noted that \( FS \) is greater than unity for any value of \( \psi_p \) and it does not vary significantly with an increase in \( k_v \) for smaller value of \( \psi_p. \)

\[
\begin{align*}
\psi_f &= 50^\circ, \ \phi = 25^\circ, \ q^* = 0.5 \\
T^* &= 0.1, \ z^* = 0.1, \ z_w^* = 0.05 \\
\gamma^* &= 2.5, \ k_h = 0.2, \ \alpha = 45^\circ, \ c^* = 0.1
\end{align*}
\]

**Fig. 4.3.** Variation of factor of safety \( (FS) \) with vertical seismic coefficient \( (k_v) \) for different values of angle of inclination of failure plane to the horizontal \( (\psi_p) \).
Fig. 4.4 shows the variation of the factor of safety \((FS)\) with vertical seismic coefficient \(k_v\) for different nondimensional values of depth of tension crack, \(z^* = 0.05, 0.1, 0.15, 0.2\) and 0.25, considering a particular set of governing parameters in their nondimensional form as: \(\psi_f = 50^\circ, \psi_p = 35^\circ, z_w^* = 0.05, c^* = 0.1, \phi = 25^\circ, q^* = 0.5, T^* = 0.1, \gamma^* = 2.5, \alpha = 45^\circ\) and \(k_h = 0.2\). It is observed that with an increase in \(k_v\) in the downward direction, \(FS\) decreases almost linearly, whereas it increases also almost linearly as \(k_v\) increases in the upward direction. It is also noticed that \(FS\) is higher for greater values of \(z^*\) and it is greater than unity for any value of \(z^*\).

\[
\begin{align*}
\psi_f &= 50^\circ, \quad \psi_p = 35^\circ, \\
\phi &= 25^\circ, \quad q^* = 0.5, \\
T^* &= 0.1, \quad z_w^* = 0.05, \\
\gamma^* &= 2.5, \quad \alpha = 45^\circ, \\
k_h &= 0.2
\end{align*}
\]

**Fig. 4.4.** Variation of factor of safety \((FS)\) with vertical seismic coefficient \((k_v)\) for different nondimensional values of depth of tension crack \((z^*)\).
Fig. 4.5 shows the variation of the factor of safety ($FS$) with vertical seismic coefficient ($k_v$) for different nondimensional values of depth of water in tension crack, $z_w^* = 0, 0.025, 0.05, 0.075, \text{ and } 0.1$, considering a particular set of governing parameters in their nondimensional form as: $\psi_f = 50^\circ$, $\psi_p = 35^\circ$, $z^* = 0.1$, $c^* = 0.1$, $\phi = 25^\circ$, $q^* = 0.5$, $T^* = 0.1$, $\gamma^* = 2.5$, $\alpha = 45^\circ$ and $k_h = 0.2$. It is noticed that with an increase in $k_v$ in the downward direction, $FS$ decreases almost linearly, but it increases almost linearly as $k_v$ increases in the upward direction. It is important to note that $FS$ is higher for smaller values of $z_w^*$. This shows that water present in tension cracks causes a significant reduction in $FS$.

**Fig. 4.5.** Variation of factor of safety ($FS$) with vertical seismic coefficient ($k_v$) for different nondimensional values of depth water in tension crack ($z_w^*$).
Fig. 4.6 shows the variation of the factor of safety \((FS)\) with vertical seismic coefficient \((k_v)\) for different nondimensional values of unit weight of rock, \(\gamma^* = 2, 2.25, 2.5, 2.75\) and 3, considering a particular set of governing parameters in their nondimensional form as: \(\psi_f = 50^\circ, \psi_p = 35^\circ, \phi = 25^\circ\), \(q^* = 0.5, T^* = 0.1, \alpha = 45^\circ\) and \(k_h = 0.2\). It is noted that with an increase in \(k_v\) in the downward direction, \(FS\) decreases almost linearly, but it increases with an increase in \(k_v\) in the upward direction. It is also noted that \(FS\) is greater than unity for any value of \(\gamma^*\) and the factor of safety is not much affected by variation in unit weight of rock, mainly because of almost similar effects on resisting and driving forces.

![Figure 4.6: Variation of factor of safety (FS) with vertical seismic coefficient \((k_v)\) for different nondimensional values of unit weight of rock \((\gamma^*)\).](image-url)
Fig. 4.7 shows the variation of the factor of safety ($FS$) with vertical seismic efficient ($k_v$) for different nondimensional values of surcharge, $q^* = 0, 0.25, 0.5, 1$ and $1.5$, considering a particular set of governing parameters in their nondimensional form as: $\psi_f = 50^\circ$, $\psi_p = 35^\circ$, $z^* = 0.1$, $z_w^* = 0.05$, $c^* = 0.1$, $\phi = 25^\circ$, $T^* = 0.1$, $\gamma^* = 2.5$, $\alpha = 45^\circ$ and $k_h = 0.2$. It is noted that with an increase in $k_v$ in the downward direction, $FS$ decreases almost linearly, whereas it increases almost linearly with an increase in $k_v$ in the upward direction. It is also noted that $FS$ is less than unity for any value of $q^*$ greater than 0.5, making the slope unstable, and it does not vary significantly with an increase $k_v$.

![Diagram showing variation of factor of safety ($FS$) with vertical seismic coefficient ($k_v$) for different nondimensional values of surcharge ($q^*$).]
Fig. 4.8 shows the variation of the factor of safety (FS) with vertical seismic coefficient \(k_v\) for different nondimensional values of stabilizing force, \(T^* = 0, 0.05, 0.1, 0.15,\) and 0.2, considering a particular set of governing parameters in their nondimensional form as: \(\psi_f = 50^\circ, \psi_p = 35^\circ, z^* = 0.1, z_w^* = 0.05, c^* = 0.1, \phi = 25^\circ, q^* = 0.5, \gamma^* = 2.5, \alpha = 45^\circ\) and \(k_h = 0.2\). It is observed that as \(k_v\) increases in the downward direction, \(FS\) decreases almost linearly, whereas it increases almost linearly with an increase of \(k_v\) in the upward direction. It is also noted that \(FS\) is greater than unity for any value of \(T^*\) greater than 0.05 and the slope is stable; it does not vary significantly for lower value of \(T^*\) with an increase in \(k_v\).

**Fig. 4.8.** Variation of factor of safety (FS) with vertical seismic coefficient \((k_v)\) for different nondimensional values of stabilizing force \((T^*)\).
Fig. 4.9 shows the variation of the factor of safety ($FS$) with vertical seismic coefficient ($k_v$) for different values of inclination of stabilizing force to the normal at the failure plane, $\alpha = 0^\circ$, $20^\circ$, $40^\circ$, $60^\circ$ and $80^\circ$, considering a particular set of governing parameters in their nondimensional form as: $\psi_f = 50^\circ$, $\psi_p = 35^\circ$, $z^* = 0.1$, $c^* = 0.05$, $\phi = 25^\circ$, $q^* = 0.5$, $T^* = 0.1$, $\gamma^* = 2.5$, and $k_h = 0.2$. It is observed that with an increase in $k_v$ in the downward direction, $FS$ decreases almost linearly, but it increases almost linearly as $k_v$ increases in the upward direction. It is also observed that the factor of safety is higher for greater values of $\alpha$ less than $60^\circ$; If $\alpha$ becomes greater than $60^\circ$, $FS$ decreases slightly. It should be noted that the horizontal line at $FS = 1$ divides the figure into safe and unsafe regions.

Fig. 4.9. Variation of factor of safety ($FS$) with vertical seismic coefficient ($k_v$) for different values of inclination of stabilizing force to the normal at the failure plane ($\alpha$).
Fig. 4.10 shows the variation of the factor of safety ($FS$) with vertical seismic coefficient ($kv$) for different values of angle of shearing resistance, $\phi = 20^\circ, 25^\circ, 30^\circ, 35^\circ$ and $40^\circ$, considering a particular set of governing parameters in their nondimensional form as: $\psi_f = 50^\circ, \psi_p = 35^\circ, z^* = 0.1, z_w^* = 0.05, c^* = 0.1, q^* = 0.5, T^* = 0.1, \gamma^* = 2.5, \alpha = 45^\circ$ and $k_h = 0.2$. It is observed that with an increase in $kv$ in the downward direction, $FS$ decreases almost linearly, whereas it increases almost linearly as $kv$ increases in the upward direction. It is also observed that the factor of safety is higher for greater value of $\phi$.

Fig. 4.10. Variation of factor of safety ($FS$) with vertical seismic coefficient ($kv$) for different values of angle of shearing resistance ($\phi$).
Fig. 4.11 shows the variation of the factor of safety (FS) with vertical seismic coefficient (kv) for different nondimensional values of cohesion, c* = 0, 0.04, 0.08, 0.12 and 0.16, considering a particular set of governing parameters in their nondimensional form as: \( \psi_f = 50^\circ \), \( \psi_p = 35^\circ \), \( z^* = 0.1 \), \( z_{wp} = 0.05 \), \( \phi = 25^\circ \), \( q^* = 0.5 \), \( T^* = 0.1 \), \( \gamma^* = 2.5 \), \( \alpha = 45^\circ \) and \( k_h = 0.2 \). It is observed that with an increase in \( k_v \) in the downward direction, \( FS \) decreases almost linearly, whereas it increases with \( k_v \) in the upward direction for nonzero values of \( c^* \). For \( c^* = 0 \), \( FS \) remains unaffected with variation in \( k_v \).

**Fig. 4.11.** Variation of factor of safety (FS) with vertical seismic coefficient (\( k_v \)) for different nondimensional values of cohesion of the joint material along the sliding surface (\( c^* \)).
4.2.2 Effect of Angle of Inclination of the Slope Face to the Horizontal

Fig. 4.12 shows the variation of the factor of safety (FS) with the angle of inclination of the slope face to the horizontal ($\psi_f$) for different sets of horizontal seismic coefficient ($k_h$) and vertical seismic coefficient ($k_v$) as: 0.0, 0.0; 0.1, 0.05↓; 0.2, 0.1↓; 0.3, 0.15↓; 0.4, 0.2↓; 0.1, 0.05↑; 0.2, 0.1↑; 0.3, 0.15↑ and 0.4, 0.2↑; considering a particular set of governing parameters in their nondimensional form as: $\psi_p = 35°$, $z^* = 0.1$, $z^*_w = 0.05$, $c^* = 0.1$, $\phi = 25°$, $q^* = 0.5$, $T^* = 0.1$, $\gamma^* = 2.5$ and $\alpha = 45°$. It is observed that $FS$ decreases nonlinearly with an increase in $\psi_f$, and the rate of decrease is very high for $\psi_f$ in the range of 40° to 45°. For $\psi_f$ greater than approximately 45°, $FS$ becomes less than 1 for higher values of seismic coefficients. Another important observation is that $FS$ does not vary significantly for higher values of seismic coefficients for $\psi_f$ greater than approximately 55°.

Fig. 4.12. Variation of factor of safety (FS) with angle of inclination of the slope face to the horizontal ($\psi_f$) for different values of horizontal ($k_h$) and vertical ($k_v$) seismic coefficients.
4.2.3 Effect of Angle of Inclination of Failure Plane to the Horizontal

Fig. 4.13 shows the variation of the factor of safety ($FS$) with the angle of inclination of failure plane to the horizontal ($\psi_p$) for different sets of horizontal seismic coefficient ($k_h$) and vertical seismic coefficient ($k_v$) as: 0.0, 0.0; 0.1, 0.05↓; 0.2, 0.1↓; 0.3, 0.15↓; 0.4, 0.2↓; 0.1, 0.05↑; 0.2, 0.1↑; 0.3, 0.15↑ and 0.4, 0.2↑; considering a particular set of governing parameters in their nondimensional form as: considering a particular set of governing parameters in their nondimensional form as: $\psi_f = 50^\circ$, $z^* = 0.1$, $z_w^* = 0.05$, $c^* = 0.1$, $\phi = 25^\circ$, $q^* = 0.5$, $T^* = 0.1$, $\gamma^* = 2.5$, and $\alpha = 45^\circ$. It is observed that $FS$ increase nonlinearly with an increase in $\psi_p$ and the rate of increase is very significant for $\psi_p$ in the range of $40^\circ$ to $45^\circ$. This is probably due to decrease in the weight of the sliding rock mass. For $\psi_p$ lower than approximately $40^\circ$, $FS$ becomes less than 1 for higher values of seismic coefficients. An important observation is that $FS$ does not vary significantly for $\psi_p$ lower than approximately $35^\circ$.

Fig. 4.13. Variation of factor of safety ($FS$) with angle of inclination of failure plane to the horizontal ($\psi_p$) for different values of horizontal ($k_h$) and vertical ($k_v$) seismic coefficients.
4.2.4 Effect of the Depth of Tension Crack

Fig. 4.14 shows the variation of the factor of safety ($FS$) with the depth of tension crack ($z^*$) for different sets of horizontal seismic coefficient ($k_h$) and vertical seismic coefficient ($k_v$) as: $0.0, 0.0; 0.1, 0.05\downarrow; 0.2, 0.1\downarrow; 0.3, 0.15\downarrow; 0.4, 0.2\downarrow; 0.1, 0.05\uparrow; 0.2, 0.1\uparrow; 0.3, 0.15\uparrow$ and $0.4, 0.2\uparrow$; considering a particular set of governing parameters in their nondimensional form as: $\phi_f = 50^\circ$, $\phi_p = 35^\circ$, $z_w = 0.05$, $c^* = 0.1$, $\phi = 25^\circ$, $q^* = 0.5$, $T^* = 0.1$, $\gamma^* = 2.5$, and $\alpha = 45^\circ$. It is noticed that with an increase in $z^*$, $FS$ increases nonlinearly and the rate of increase is lower for higher value of seismic inertial force. $FS$ becomes less than 1 for higher values of seismic coefficients. It appears that increase in $FS$ is caused by a change in geometry of the sliding rock mass resulting in decrease in driving forces.

**Fig. 4.14.** Variation of factor of safety ($FS$) with depth of tension crack ($z^*$) for different values of horizontal ($k_h$) and vertical ($k_v$) seismic coefficients.
4.2.5 Effect of Depth Water in Tension Crack

Fig. 4.15 shows the variation of the factor of safety (FS) with the depth water in tension crack \( (z'_{uw}) \) for different sets of horizontal seismic coefficient \( (k_h) \) and vertical seismic coefficient \( (k_v) \) as: 0.0, 0.0; 0.1, 0.05↓; 0.2, 0.1↓; 0.3, 0.15↓; 0.4, 0.2↓; 0.1, 0.05↑; 0.2, 0.1↑; 0.3, 0.15↑ and 0.4, 0.2↑; considering a particular set of governing parameters in their nondimensional form as: \( \psi_f = 50^\circ, \psi_p = 35^\circ, z^* = 0.1, c^* = 0.1, \phi = 25^\circ, q^* = 0.5, T^* = 0.1, \gamma^* = 2.5 \) and \( \alpha = 45^\circ \). It is noticed that with an increase in \( z'^*_{uw} \), FS decreases almost linearly. It is observed that FS is higher for lower values of seismic coefficients.

Fig. 4.15. Variation of factor of safety (FS) with depth water in tension crack \( (z'_{uw}) \) for different values of horizontal \( (k_h) \) and vertical seismic coefficients \( (k_v) \).
4.2.6 Effect of Unit Weight of Rock

Fig. 4.16 shows the variation of the factor of safety \( (FS) \) with the unit weight of rock \( (\gamma^*) \) for different sets of horizontal seismic coefficient \( (k_h) \) and vertical seismic coefficient \( (k_v) \) as: 0.0, 0.0; 0.1, 0.05↓; 0.2, 0.1↓; 0.3, 0.15↓; 0.4, 0.2↓; 0.1, 0.05↑; 0.2, 0.1↑; 0.3, 0.15↑ and 0.4, 0.2↑; considering a particular set of governing parameters in their nondimensional form as: \( \psi_f = 50^\circ, \psi_p = 35^\circ, z^* = 0.1, z_{w} = 0.05, c^* = 0.1, \phi = 25^\circ, q^* = 0.5, T^* = 0.1 \) and \( \alpha = 45^\circ \). It is observed that with an increase in \( \gamma^* \), \( FS \) does not change significantly. This is mainly because of almost similar effects on resisting and driving forces.

\[ \text{Fig. 4.16. Variation of factor of safety (FS) with different nondimensional values of unit weight of rock (\gamma^*) for different values of horizontal (k_h) and vertical (k_v) seismic coefficients.} \]
4.2.7 Effect of Surcharge

Fig. 4.17 shows the variation of the factor of safety ($FS$) with the surcharge ($q^*$) for different sets of horizontal seismic coefficient ($k_h$) and vertical seismic coefficient ($k_v$) as: 0.0, 0.0; 0.1, 0.05↓; 0.2, 0.1↓; 0.3, 0.15↓; 0.4, 0.2↓; 0.1, 0.05↑; 0.2, 0.1↑; 0.3, 0.15↑ and 0.4, 0.2↑; considering a particular set of governing parameters in their nondimensional form as: $\psi_f = 50^\circ$, $\psi_p = 35^\circ$, $z^* = 0.1$, $z_{w^*} = 0.05$, $c^* = 0.1$, $\phi = 25^\circ$, $T^* = 0.1$, $\gamma^* = 2.5$ and $\alpha = 45^\circ$. It is observed that $FS$ decreases nonlinearly with an increase in $q^*$, and the rate of decrease is very high for $q^*$ in the range of 0 to 0.75. For $q^*$ greater than approximately 1, $FS$ becomes less than 1 for most combinations of seismic coefficients.

![Fig. 4.17. Variation of factor of safety ($FS$) with different nondimensional values of surcharge ($q^*$) for different values of horizontal ($k_h$) and vertical ($k_v$) seismic coefficients.](image-url)
4.2.8 Effect of Stabilizing Force

Fig. 4.18 shows the variation of the factor of safety ($FS$) with the stabilizing force ($T^*$) for different sets of horizontal seismic coefficient ($k_h$) and vertical seismic coefficient ($k_v$) as: 0.0, 0.0; 0.1, 0.05↓; 0.2, 0.1↓; 0.3, 0.15↓; 0.4, 0.2↓; 0.1, 0.05↑; 0.2, 0.1↑; 0.3, 0.15↑ and 0.4, 0.2↑; $\psi_f = 50^\circ$, $\psi_p = 35^\circ$, $z^* = 0.1$, $z_0^* = 0.05$, $c^* = 0.1$, $\phi = 25^\circ$, $q^* = 0.5$, $\gamma^* = 2.5$ and $\alpha = 45^\circ$. It is observed that $FS$ increases nonlinearly with an increase in $T^*$ and the rate of increase is higher for lower values of seismic coefficients.

Fig. 4.18. Variation of factor of safety ($FS$) with different nondimensional values of stabilizing force ($T^*$) for different values of horizontal ($k_h$) and vertical ($k_v$) seismic coefficient.
4.2.9 Effect of Inclination of Stabilizing Force to the Normal at the Failure Plane

Fig. 4.19 shows the variation of the factor of safety \((FS)\) with the inclination of the stabilizing force to the normal at the failure plane \((\alpha)\) for different sets of horizontal seismic coefficient \((k_h)\) and vertical seismic coefficient \((k_v)\) as: 0.0, 0.0; 0.1, 0.05↓; 0.2, 0.1↓; 0.3, 0.15↓; 0.4, 0.2↓; 0.1, 0.05↑; 0.2, 0.1↑; 0.3, 0.15↑ and 0.4, 0.2↑; considering a particular set of governing parameters in their nondimensional form as: \(\psi_f = 50^\circ\), \(\psi_p = 35^\circ\), \(z^* = 0.1\), \(z_w^* = 0.05\), \(c^* = 0.1\), \(\phi = 25^\circ\), \(q^* = 0.5\), \(T^* = 0.1\), \(\gamma^* = 2.5\) and \(\alpha = 45^\circ\). It is observed that \(FS\) increases nonlinearly with an increase in \(\alpha\), and it becomes maximum for \(\alpha \approx 70^\circ\) beyond which it decreases. It should be noted that the horizontal line at \(FS = 1\) divides the figure into safe and unsafe regions and it is seen that for high values of seismic coefficient, \(FS\) becomes less than 1; making the slope unsafe.

**Fig. 4.19.** Variation of factor of safety \((FS)\) with inclination of stabilizing force to the normal at the failure plane \((\alpha)\) for different values of horizontal \((k_h)\) and vertical \((k_v)\) seismic coefficients.
4.2.10 Effect of Cohesion of the Joint Material along the Sliding Surface

Fig. 4.20 shows the variation of the factor of safety ($FS$) with the cohesion of the joint material along the sliding surface ($c^*$) for different sets of horizontal seismic coefficient ($k_h$) and vertical seismic coefficient ($k_v$) as: $0.0, 0.0; 0.1, 0.05\downarrow; 0.2, 0.1\downarrow; 0.3, 0.15\downarrow; 0.4, 0.2\downarrow; 0.1, 0.05\uparrow; 0.2, 0.1\uparrow; 0.3, 0.15\uparrow$ and $0.4, 0.2\uparrow$; considering a particular set of governing parameters in their nondimensional form as: $\psi_f = 50^\circ$, $\psi_p = 35^\circ$, $z_w = 0.05$, $\phi = 25^\circ$, $q^* = 0.5$, $T^* = 0.1$, $\gamma^* = 2.5$ and $\alpha = 45^\circ$. It is observed that the factor of safety increases linearly with an increase in cohesion. This is probably due to linear relationship of cohesion with shear strength as defined by the Mohr-Coulomb failure criterion.

**Fig. 4.20.** Variation of factor of safety ($FS$) with cohesion of the joint material along the sliding surface ($c^*$) for different values of horizontal ($k_h$) and vertical ($k_v$) seismic coefficients.
4.2.11 Effect of Angle of Shearing Resistance of the Joint Material along the Sliding Surface

Fig. 4.21 shows the variation of the factor of safety ($FS$) with the angle of shearing resistance of the joint material along the sliding surface ($\phi$) for different sets of horizontal seismic coefficient ($k_h$) and vertical seismic coefficient ($k_v$) as: 0.0, 0.0; 0.1, 0.05↓; 0.2, 0.1↓; 0.3, 0.15↓; 0.4, 0.2↓; 0.1, 0.05↑; 0.2, 0.1↑; 0.3, 0.15↑ and 0.4, 0.2↑; considering a particular set of governing parameters in their nondimensional form as: $\psi_f = 50^\circ$, $\psi_p = 35^\circ$, $z^* = 0.1$, $c^* = 0.1$, $q^* = 0.5$, $T^* = 0.1$, $\gamma^* = 2.5$ and $\alpha = 45^\circ$. It is observed that the factor of safety increases almost linearly with an increase in $\phi$. This is also probably due to linear characteristics of the Mohr-Coulomb failure criterion used in the present study.

**Fig. 4.21.** Variation of factor of safety ($FS$) with angle of shearing resistance of the joint material along the sliding surface ($\phi$) for different values of horizontal ($k_h$) and vertical ($k_v$) seismic coefficients.
4.3 DEVELOPMENT OF DESIGN CHARTS

Based on the generalised analytical expression in Eq. (3.24) design charts have been developed as plots of factor of safety \((FS)\) of the slope against plane failure versus angle of shearing resistance \((\phi)\) of the joint material for different specific values of governing parameters in their nondimensional form. It is observed that Eq. (3.24) contains 13 parameters including \(FS\). It means \(FS\) is dependent on 12 field parameters. Figs. 4.22(a)-(i) have been developed for \(\psi_f = 40^\circ, \psi_p = 25^\circ, z^* = 0.25, z^*_w = 0.25, \gamma^* = 2\) and \(\alpha = 40^\circ\); Figs. 4.23 (a)-(i) have been developed for \(\psi_f = 50^\circ, \psi_p = 35^\circ, z^* = 0.25, z^*_w = 0.25, \gamma^* = 2\) and \(\alpha = 40^\circ\); Figs. 4.24 (a)-(i) have been developed with \(\psi_f = 60^\circ, \psi_p = 45^\circ, z^* = 0.25, z^*_w = 0.25, \gamma^* = 2\) and \(\alpha = 40^\circ\). These sets of parameters are typical values that can be expected in field situations. In all the design charts, horizontal seismic coefficient \(k_h\) varies from 0 to 0.4, and vertical seismic coefficient \(k_v\) has been kept as half of \(k_h\). Each design chart has been prepared under consideration of three different sets of nondimensional values of surcharge and stabilizing force, such as, \(q^* = 0, T^* = 0\); \(q^* = 0.5, T^* = 0.05\) and \(q^* = 1, T^* = 0.1\) for cohesion \(c^*\) = 0, 0.04, 0.08, 0.12, 0.16 and 0.2. In all the design charts, a horizontal line at \(FS = 1\) divides the chart into safe and unsafe regions. It is observed that with an increase in angle of shearing resistance \(\phi\), factor of safety \(FS\) increases almost linearly. It is also noticed that \(FS\) is higher for greater value of cohesion \(c^*\); however, its value is greatly controlled by the sets of values of stabilizing force \(T^*\) and surcharge \(q^*\) under seismic loading condition.

4.4 ILLUSTRATIVE EXAMPLE

Consider a rock slope with the following details and assume that the height of tension crack is one fourth of the height of the rock slope, and the tension crack is completely filled with water.
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Fig. 4.22(a).

Fig. 4.22(b).
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Fig. 4.22(c).

Fig. 4.22(d).
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Fig. 4.22(e).

Fig. 4.22(f).
Fig. 4.22(g).

Fig. 4.22(h).
Fig. 4.22(i).

Fig. 4.23(a).
Fig. 4.23(b).

Fig. 4.23(c).
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Fig. 4.23(d).

Fig. 4.23(e).
Fig. 4.23(f).

Fig. 4.23(g).
Fig. 4.23(h).

Fig. 4.23(i).
Fig. 4.24(a).

Fig. 4.24(b).
Fig. 4.24(c).

Fig. 4.24(d).
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Fig. 4.24(e).

Fig. 4.24(f).
Fig. 4.24(g).

Fig. 4.24(h).
Fig. 4.24(i).

Height of the rock slope, \( H = 10 \) m
Unit weight of rock, \( \gamma = 20 \) kN/m\(^3\)
Surcharge pressure, \( q = 100 \) kN/m\(^2\)
Stabilizing force, \( T = 100 \) kN/m
Depth of the tension crack, \( z = 2.5 \) m
Depth of water in the tension crack, \( z_w = 2.5 \) m
Angle of inclination of stabilizing force to the normal at the failure plane, \( \alpha = 40^\circ \)
Angle of shearing resistance of the joint material, \( \phi = 25^\circ \)
Angle of inclination of stabilizing force to the normal at the failure plane, \( \alpha = 40^\circ \)
Angle of shearing resistance of the joint material, \( \phi = 25^\circ \)
Cohesion of the joint plane material, \( c = 32 \) kN/m\(^2\)
Angle of inclination of the slope face to the horizontal, \( \psi_f = 50^\circ \)
Angle of inclination of the joint plane/failure plane to the horizontal, \( \psi_f = 35^\circ \)
Horizontal seismic coefficient, \( k_h = 0.2 \)
Vertical seismic coefficient, \( k_v = 0.1 \)
Nondimensional parameters can be calculated as:

Nondimensional unit weight of rock, $\gamma^* = \gamma / \gamma_w = \frac{20}{10} = 2$

Nondimensional cohesion, $c^* = \frac{c}{\gamma H} = \frac{32}{20 \times 10} = 0.16$

Nondimensional surcharge, $q^* = \frac{q}{\gamma H} = \frac{100}{20 \times 10} = 0.5$

Nondimensional stabilizing force, $T^* = \frac{T}{\gamma H^2} = \frac{100}{20 \times 10 \times 10} = 0.05$

Nondimensional depth of tension crack, $z^* = \frac{z}{H} = \frac{2.5}{10} = 0.25$

Nondimensional depth of water in tension crack, $z^* = \frac{z_w}{H} = \frac{2.5}{10} = 0.25$

Using the design chart as shown in Fig. 4.23(e) with the above values, factor of safety, $FS = 1.17$, which is also obtained directly from Eq. (3.24).

### 4.5 CONCLUSIONS

A detailed parametric study has been carried out in order to investigate the effect of parameters that govern the stability of rock slope. The list of parameters include:

- angle of inclination of the slope face to the horizontal,
- angle of inclination of the failure plane to the horizontal,
- depth of tension crack,
- depth of water in tension crack,
- unit weight of rock,
- surcharge load,
- stabilizing force,
- angle of inclination of stabilizing force to the normal at the failure plane,
- angle of shearing resistance of the joint material along the sliding surface,
- cohesion of the joint material along the sliding surface,
- horizontal seismic coefficient,
- vertical seismic coefficient.

Many observations have been noted and discussed in details. Attempt has been made to explain the possible reasons of variation of factor safety of the rock slope when a specific parameter is varied. It is observed that with an increase in vertical seismic coefficient in the downward direction, factor of safety of the rock slope decreases almost linearly, whereas with an increase in vertical seismic coefficient in the upward direction, it increases almost linearly, and is greater than unity for any value of horizontal seismic coefficient lower than 0.25.

The generalised expression presented has been used to develop design charts for the estimation of factor of safety of the anchored rock slope against plane failure.
under surcharge and seismic loading conditions. An Illustrative example has been presented to explain the steps for the estimation of factor of safety. The design charts indicate that the factor of safety of the rock slope increases with an increase in both angle of shearing resistance and cohesion of the joint material. For the higher factor of safety of the rock slope against plane failure under surcharge and seismic loading conditions, greater values of shear strength parameters and/or the stabilizing force must be available.
CHAPTER 5

MULTI-DIRECTIONAL ANCHORED ROCK SLOPE:
ANALYTICAL FORMULATION

5.1 GENERAL

Civil and mining engineers often deal with projects involving excavated or natural slopes designed and built in rock masses. The stability analysis of the rock slope is generally an indeterminate problem, mainly because of the discontinuities present and their expected variation with time within the rock mass. Therefore, no slopes made in rock can be regarded as fully guaranteed for their stability during their service lives over a period of many years (Jumikis, 1983). However, it is a general engineering practice to classify the rock slope failures in some idealised failure types, such as plane failure, wedge failure, circular failure, toppling failure and buckling failure (Hoek and Bray, 1981; Goodman, 1989; Goodman and Kieffer, 2000; Wyllie and Mah, 2004; Hoek, 2007). The plane failure is observed in field situations when the discontinuity is in the form of joint planes inclined to the horizontal. There are several parameters that govern the stability of the rock slope against plane failure, such as inclination of the slope face, inclination of the joint or discontinuity plane, depth of tension crack, depth of water in tension crack, shear strength parameters of the joint material at the failure plane, unit weight of rock, stabilizing force and its inclination, seismic load, surcharge, etc (Hoek and Bray, 1981; Ling and Cheng, 1997; Wyllie and Mah, 2004; Hoek, 2007; Shukla et. al, 2009). For increasing the factor of safety of the slope, rock slopes are often stabilized by different methods. Rock anchoring is one of the common methods of rock slope stabilization (Kliche, 1999; Wyllie and Mah, 2004). The installation steps of rock anchors have been clearly described in the literature (Littlejohn and Bruce, 1977; Federal Highway Administration (FHWA), Washington DC, 1982; British Standards Institute (BSI), London, 1989; Xanthakos, 1991; Post Tensioning Institute (PTI),
1996; Wyllie, 1999; Wyllie and Mah, 2004). It is quite possible that there can be more than one set of anchors with different orientations to completely stabilize the slope in an economical way; such anchored slopes may be called multi-directional anchored rock slope (MDARS) as considered in the present chapter. There is currently no analytical expression available for the factor of safety against plane failure of a multi-directional anchored rock slope subjected to surcharge and seismic loads, which are often expected in field conditions. Therefore, in this chapter, an attempt is made to develop such an analytical expression considering most of the field parameters as mentioned above for its application by engineers in field projects.

5.2 ANALYTICAL FORMULATION

Fig. 5.1 shows a multi-anchored rock slope of height $H$ inclined to the horizontal at an angle $\psi_f$. The sliding rock mass block $A_1A_2A_3A_4$ is separated by a vertical tension crack $A_3A_4$ of depth $z$ and the joint/failure plane $A_2A_3$, which is inclined to the horizontal at an angle $\psi_p$. The tension crack is filled with water to a depth $z_w$. The slope is anchored with two sets of anchors, directed in different directions; $T_1$ and $T_2$ are the resultant anchor/stabilizing force for the first and second sets, inclined to the normal at the failure plane at angles $\alpha_1$ and $\alpha_2$, respectively. Though only two sets of anchors are shown in the figure, the analysis is carried out with $n$ number of multi-directional anchor sets. $B(=A_1A_4)$ is the top width of the slope and $W$ is the weight of the sliding rock mass block. The slope is subjected to surcharge $q$. The horizontal and vertical seismic loads ($k_h W$ and $k_v W$, and $k_h q B$ and $k_v q B$) are considered to act on the slope, where $k_h$ and $k_v$ are horizontal and vertical seismic coefficients, respectively.

Considering a slice of unit thickness through the slope, the stability of the rock block is analysed with destabilising and stabilising forces acting on it. Only the force equilibrium is considered without any resistance to sliding at the lateral boundaries of the sliding block. The total force available to resist the sliding block is

$$F_r = \tau A$$  \hspace{1cm} (5.1)
where $\tau$ is the shear strength of the sliding failure plane, and $A$ is the area of the base $A_2A_3$ of the sliding rock block given as

$$A = H \left(1 - \frac{z}{H}\right) \csc \psi_p$$  \hspace{1cm} (5.2)

**Fig. 5.1.** Multidirectional-anchored rock slope.

The top width $B$ is calculated as

$$B = H \left\{ \left(1 - \frac{z}{H}\right) \cot \psi_p - \cot \psi_f \right\}$$  \hspace{1cm} (5.3)

The Mohr-Coulomb failure criterion provides (Lambe and Whitman, 1979; Das, 2008)

$$\tau = c + \sigma_n \tan \phi$$  \hspace{1cm} (5.4)
where $\sigma_n$ is the normal stress on the failure plane, and $c$ and $\phi$ are, respectively, cohesion and angle of internal friction of the joint material. It is assumed that the joint plane material obeys the Mohr-Coulomb failure criterion. From Eqs. (5.1) and (5.4),

$$F_r = cA + F_n \tan \phi$$

(5.5)

where $F_n = \sigma_n A$ is the normal force on the failure plane. Considering equilibrium of forces acting on the rock block, $F_n$ is obtained as

$$F_n = W \cos \psi_p - k_h W \sin \psi_p \pm k_v W \cos \psi_p - U_1 \sin \psi_p - U_2 + \sum_{i=1}^{n} T_i \cos \alpha_i + qB \cos \psi_p - qB \sin \psi_p \pm k_v qB \cos \psi_p$$

$$= (W + qB) \{ (1 \pm k_v) \cos \psi_p - k_h \sin \psi_p \} - U_1 \sin \psi_p - U_2 + \sum_{i=1}^{n} T_i \cos \alpha_i$$

(5.6)

where $T_i$ ($i = 1, 2, 3, \ldots n$) is the stabilizing tensile force in the $i$th set of anchors inclined to the normal at the joint plane $A_1A_4$ at an angle $\alpha_i$ ($i = 1, 2, 3, \ldots n$), $U_1$ is the horizontal force due to water pressure in the tension crack, and $U_2$ is the uplift force due to water pressure on the joint plane.

The weight of the sliding rock block is

$$W = \frac{1}{2} \gamma H^2 \left[ \left( 1 - \left( \frac{z}{H} \right)^2 \right) \cot \psi_p - \cot \psi_f \right]$$

(5.7)

Horizontal force on the sliding block due to water pressure in the tension crack is

$$U_1 = \frac{1}{2} \gamma_w z_w \times z_w = \frac{1}{2} \gamma_w z_w^2$$

(5.8)

where $\gamma_w$ is the unit weight of water.
Uplift force on the sliding block due to water pressure on failure plane is

\[ U_2 = \frac{1}{2} \gamma_w z_w H (1 - \frac{z}{H}) \cosec \psi_p \]  

(5.9)

Substituting values from Eqs. (5.2), (5.3), (5.6), (5.7), (5.8), (5.9), into Eq. (5.5),

\[ F_r = cH \left(1 - \frac{z}{H}\right) \cosec \psi_p + \left[\frac{1}{2} \gamma H^2 \left[1 - (\frac{z}{H})^2\right] \cot \psi_p - \cot \psi_f\right] + qH \{(1 - \frac{z}{H}) \cot \psi_p - \cot \psi_f\} (1 + k_v) \left\{\cos \psi_p - \frac{k_h}{(1 + k_v)} \sin \psi_p\right\} - \frac{1}{2} \gamma_w z_w \sin \psi_p - \frac{1}{2} \gamma_w z_w H (1 - \frac{z}{H}) \cosec \psi_p + \sum_{i=1}^{n} T_i \cos \alpha_i \tan \phi \]

\[ = cH \left(1 - \frac{z}{H}\right) \cosec \psi_p + \left[\frac{1}{2} \gamma H^2 \left[1 - (\frac{z}{H})^2\right] \cot \psi_p - \cot \psi_f\right] + qH \{(1 - \frac{z}{H}) \cot \psi_p - \cot \psi_f\} (1 + k_v) \left\{\cos \psi_p - \tan \theta \sin \psi_p\right\} - \frac{1}{2} \gamma_w z_w \sin \psi_p - \frac{1}{2} \gamma_w z_w H (1 - \frac{z}{H}) \cosec \psi_p + \sum_{i=1}^{n} T_i \cos \alpha_i \tan \phi \]

(5.10)

Where \n
\[ \theta = \tan^{-1} \left(\frac{k_h}{(1 + k_v)}\right) \]  

(5.11)

From Fig. 1, the total force tending to induce sliding is calculated as
The factor of safety $FS$ of the rock slope is defined as failure \cite{Hoek, Bray, 1981; Wyllie and Mah, 2004; Hoek, 2007}:

$$FS = \frac{F_r}{F_i}$$ (5.13)

Substituting $F_r$ and $F_i$ from Eqs. (5.11) and (5.12), respectively into Eq. (3.13),

$$FS = \frac{2c^*P + (1 + k_v)(Q + 2q^*R) \frac{\cos(\theta + \psi_p)}{\cos \theta} - \frac{z^2_w}{\gamma_w \sin \psi_p} - \frac{z^2_w}{\gamma_w \cos \psi_p} + 2 \sum_{i=1}^{n} T_i \cos \alpha_i \tan \phi}{(1 + k_v)(Q + 2q^*R) \frac{\sin(\theta + \psi_p)}{\cos \theta} + \frac{z^2_w}{\gamma_w \cos \psi_p} - 2 \sum_{i=1}^{n} T_i \sin \alpha_i}$$ (5.14)

where $c^* = \frac{c}{\gamma H}$, $z^* = \frac{z}{H}$, $z^*_w = \frac{z_w}{H}$, $\gamma^* = \frac{\gamma}{\gamma_w}$, $q^* = \frac{q}{\gamma H}$ and $T^*_i = \frac{T_i}{\gamma H^2}$ are nondimensional forms of $c$, $z$, $z_w$, $\gamma$, $q$ and $T$, respectively, and

$$P = (1 - z^*) \csc \psi_p$$ (5.15a)
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\[ Q = (1 - z^2) \cot \psi_p - \cot \psi_f \]  \hspace{1cm} (5.15b)

\[ R = (1 - z^*) \cot \psi_p - \cot \psi_f \]  \hspace{1cm} (5.15c)

Eq. (5.14) is the general expression for the factor of safety of the rock slope against plane failure. This can be used to observe the effect of any individual parameter on the factor of safety of the rock slope and to carry out a detailed parametric study as required in a specific field situation.

5.3 SPECIAL CASES AND DISCUSSION

The general equation [Eq. (5.14)] developed for the factor of safety of the rock slope against the plane failure can have several special cases as explained below

**Case 1:** The joint material is cohesionless, and there is no surcharge loading, stabilizing force, seismic forces and water in the tension crack, that is, \( c^* = 0, \phi \neq 0, q^* = 0, T_i^* = 0, k_h = 0, k_v = 0, \theta = 0, z_w^* = 0 \). Here, Eq. (5.14) reduces to the expression given as

\[ FS = \frac{\tan \phi}{\tan \psi_p} \]  \hspace{1cm} (5.16)

**Case 2:** The joint material is cohesionless, and there is no surcharge loading, seismic forces and water in the tension crack, that is, \( c^* = 0, \phi \neq 0, q^* = 0, T_i^* \neq 0, k_h = 0, k_v = 0, \theta = 0, z_w^* = 0 \). Here, equation Eq. (5.14) becomes

\[ FS = \frac{(Q \cos \psi_p + 2 \sum_{i=1}^{n} T_i^* \cos \alpha_i) \tan \phi}{Q \sin \psi_p - 2 \sum_{i=1}^{n} T_i^* \sin \alpha_i} \]  \hspace{1cm} (5.17)

**Case 3:** The joint material is cohesionless, and there is no seismic forces and water in the tension crack, that is, \( c^* = 0, \phi \neq 0, q^* \neq 0, T_i^* \neq 0, k_h = 0, k_v = 0, \theta = 0, z_w^* = 0 \). Here, Eq. (5.14) becomes
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\[ FS = \frac{(Q+2q^*R)\cos\psi_p+2\sum_{i=1}^n T_i^*\cos\alpha_i)\tan\phi}{(Q+2q^*R)\sin\psi_p-2\sum_{i=1}^n T_i^*\sin\alpha_i} \] (5.18)

**Case 4:** The joint material is cohesive, and there is no seismic forces and water in the tension crack, that is, \( c^* \neq 0, \phi = 0, q^* \neq 0, T_i^* \neq 0, k_h = 0, k_v = 0, \theta = 0, z_w^* = 0 \). Here, Eq. (5.14) becomes

\[ FS = \frac{2c^*p}{(Q+2q^*R)\sin\psi_p-2\sum_{i=1}^n T_i^*\sin\alpha_i} \] (5.19)

**Case 5:** The joint material is \( c - \phi \) material, and there is no seismic forces and water in the tension crack, that is, \( c^* \neq 0, \phi \neq 0, q^* \neq 0, T_i^* \neq 0, k_h = 0, k_v = 0, \theta = 0, z_w^* = 0 \). Here, Eq. (5.14) becomes

\[ FS = \frac{2c^*p+(Q+2q^*R)\cos\psi_p+2\sum_{i=1}^n T_i^*\cos\alpha_i)\tan\phi}{(Q+2q^*R)\sin\psi_p-2\sum_{i=1}^n T_i^*\sin\alpha_i} \] (5.20)

**Case 6:** The joint material is \( c - \phi \) material, and there is no seismic forces, that is, \( c^* \neq 0, \phi \neq 0, q^* \neq 0, T_i^* \neq 0, k_h = 0, k_v = 0, \theta = 0, z_w^* \neq 0 \). Here, Eq. (5.14) becomes

\[ FS = \frac{2c^*p+(Q+2q^*R)\cos\psi_p-\frac{z_w^2}{p}\sin\psi_p-\frac{z_w^2}{p}\cos\psi_p+2\sum_{i=1}^n T_i^*\cos\alpha_i)\tan\phi}{(Q+2q^*R)\sin(\theta+\psi_p)+\frac{z_w^2}{p}\cos\psi_p-2\sum_{i=1}^n T_i^*\sin\alpha_i} \] (5.21)

**Case 7:** The joint material is \( c - \phi \) material, and there is only horizontal seismic force, that is, \( c^* \neq 0, \phi \neq 0, q^* \neq 0, T_i^* \neq 0, k_h \neq 0, k_v = 0, \theta = \tan^{-1}(k_h), z_w^* \neq 0 \). Here, Eq. (5.14) becomes

\[ FS = \frac{2c^*p+(Q+2q^*R)\cos(\theta+\psi_p)-\frac{z_w^2}{p}\sin(\theta+\psi_p)-\frac{z_w^2}{p}\cos\psi_p+2\sum_{i=1}^n T_i^*\cos\alpha_i)\tan\phi}{(Q+2q^*R)\sin(\theta+\psi_p)+\frac{z_w^2}{p}\cos\psi_p-2\sum_{i=1}^n T_i^*\sin\alpha_i} \] (5.22)

For a generalised case when the joint material is \( c - \phi \) material, that is, \( c^* \neq 0, \phi \neq 0, q^* \neq 0, T^* \neq 0, k_h \neq 0, k_v \neq 0, \theta = \tan^{-1}\left(\frac{k_h}{(1 \pm k_p)}\right) \), \( z_w^* \neq 0 \), Eq. (5.14) is
applicable. It should be noted that some of the above special cases have been presented in similar forms in the literature (Hoek and Bray, 1981; Ling and Cheng, 1997; Hoek, 2007; Shukla et al, 2009).

Fig. 5.2 shows the variation of the factor of safety ($FS$) of the slope with stabilizing force $T_1^*$ contributed by the first set of rock anchors for different values of stabilizing force $T_2^*$ from the second set of rock anchors, considering a particular set of governing parameters in their nondimensional form as: $\psi_f = 50^\circ$, $\psi_p = 35^\circ$, $\phi = 25^\circ$, $c^* = 0.1$, $q^* = 0.5$, $z^* = 0.1$, $z_w^* = 0.05$, $\gamma^* = 2.5$, $k_h = 0.1$, $k_v = 0.05$ and $\alpha_1 = 40^\circ$, $\alpha_2 = 20^\circ$. It is observed that the factor of safety increases nonlinearly with an increase in $T_1^*$ and is greater than unity in all cases for $T_1^*$ larger than 0.01; the rate of increase is higher for larger value of $T_1^*$.

Fig. 5.3 shows the variation of the factor of safety ($FS$) of the rock slope with angle of inclination $\alpha_1$ for the first set of rock anchors for different values of angle of inclination $\alpha_2$ of the second set of rock anchors, considering specific values of governing parameters in their nondimensional form as: $\psi_f = 50^\circ$, $\psi_p = 35^\circ$, $\phi = 25^\circ$, $q^* = 0.5$, $T_1^* = 0.05$, $T_2^* = 0.05$, $z^* = 0.1$, $z_w^* = 0.05$, $\gamma^* = 2.5$, $k_h = 0.1$, $k_v = 0.05$ and $c^* = 0.1$. It is observed that for any value of $\alpha_2$, $FS$ increases nonlinearly at it becomes the highest for $\alpha_1 \approx 70^\circ$. It is also noted that for any value of $\alpha_1$, an increase in the value of $\alpha_2$ beyond $60^\circ$ does not bring a significant change in the $FS$. This trend of variation of $FS$ is clearly noticeable from the concept of mechanics in the form of force components, which is experienced easily in Fig. 5.1. It indicates that with a fixed inclination of one set of anchors, the inclination of the second set is not essentially required to be fixed precisely for its values greater than approximately $60^\circ$.

**5.4 CONCLUSIONS**

Eq. (5.14) presents a general expression for the factor of safety of the multidirectional anchored rock slope against plane failure, incorporating most of the
Fig. 5.2. Variation of factor of safety (FS) of the rock slope with stabilizing force ($T_1^*$).

Fig. 5.3. Variation of factor of safety (FS) of the rock slope with angle of inclination ($\alpha_i$).
practically occurring destabilizing forces including surcharge and seismic loads. Several special cases of this general expression based on possible field situations have been described; some of which have been presented in similar forms in the literature. The graphical presentation for a particular set of specific values of parameters shows that factor of safety increases nonlinearly with an increase in inclination of the stabilizing force to the normal at the failure plane, and it becomes the highest for about 70°. It is also noticed that with a specific inclination of one set of anchors, the inclination of the second set of anchors is not essentially required to be fixed precisely for its values greater than about 60° because this task does not cause any significant change in the factor of safety of the slope. It is important to note that the numerical observations presented here are valid for typical values of parameters considered in the graphs. For investigating the variation of the factor of safety over a wide range of parameters, the developed analytical expression in the explicit form can be used conveniently.
6.1 SUMMARY

Rock masses including even intact rocks are generally heterogeneous and anisotropic mainly because of presence of discontinuities within them. Therefore, getting the realistic values of factor of safety of the excavated or natural rock slopes has been a challenging task for civil and mining engineers. It is a general engineering practice to classify the rock slope failures in some idealised failure types, such as plane failure, wedge failure, circular failure, toppling failure and buckling failure. There are several parameters that govern the stability of the rock slope against plane failure, such as inclination of the slope face, inclination of the joint or discontinuity plane, depth of tension crack, depth of water in tension crack, shear strength parameters of the joint material at the failure plane, unit weight of rock, stabilizing force and its inclination, surcharge, and seismic loads. For maintaining a stable slope in excavated or natural rock mass, stabilization is preferred. Rock anchoring is the most common methods of rock slope stabilization. Anchor force as well as anchor orientation both plays an active role in achieving the required slope stability. A rock slope can be stabilized by installing a set of anchors with a single orientation; in this case, the stabilized slope may be called single-directional anchored rock slope (SDARS). If the slope is stabilized with several sets of anchors with different orientations; it may be called multidirectional anchored rock slope (MDARS).

Various methods are utilized for the analysis of rock slope stability. The most frequently used methods are limit equilibrium analyses and numerical analyses. The analytical limit equilibrium approach for the estimation of factor of safety of the rock slope against plane failure is well accepted by the engineers, mainly because of simplicity in the development of explicit expressions and their frequent applications over a long period of time. Hoek and Bray (1981) presented
most of the basic methods of limit equilibrium analysis for rock slope failures. Ling and Cheng (1997) presented an analytical expression for the factor of safety of the rock slope against plane failure induced by seismic force, ignoring the possibility of upward direction of vertical inertial seismic force, and without considering the surcharge and the anchoring force. Recently, Hoek (2007) described the idealisation of the rock slope failures in Hong Kong as plain failures and presented an analytical expression for estimating the factor of safety, considering many practical aspects including seismic loadings. This analytical model was improved by Shukla et al. (2009) to investigate the effect of surcharge on the stability of rock slopes, ignoring the seismic inertial forces applied by the surcharge on the slope. In the earlier works, the vertical seismic inertial force has also not been considered with their all possible directions for the generalized case. Therefore, a derivation of an analytical expression for the factor of safety ($FS$) of a single-directional anchored rock slope (SDARS) against the plane failure is derived, considering most of the factors that may be expected in field conditions under earthquakes and dynamic activities including the stabilizing forces for the stabilized slopes. The expression for $FS$ is

$$FS = \frac{2c^*P + (1 \pm k_v)(Q + 2q^*R) \cos(\phi + \psi_p) - \frac{z^*_w - z^*_w}{\gamma} - \frac{z^*_w}{\gamma} \sin \psi_p - \frac{z^*_w}{\gamma} P + 2T^* \cos \alpha}{(1 \pm k_v)(Q + 2q^*R) \sin(\phi + \psi_p) - \frac{z^*_w}{\gamma} \cos \psi_p - 2T^* \sin \alpha}$$

(3.24)

where

$$P = (1 - z^*) \csc \psi_p$$

(3.25a)

$$Q = (1 - z^2) \cot \psi_p - \cot \psi_f$$

(3.25b)

$$R = (1 - z^*) \cot \psi_p - \cot \psi_f$$

(3.25c)

where $c^* = \frac{c}{\gamma H}$, $z^* = \frac{z}{H}$, $z^*_w = \frac{z_w}{H}$, $\gamma^* = \gamma$, $q^* = \frac{q}{\gamma H}$ and $T^* = \frac{T}{\gamma H^2}$ are nondimensional forms of $c$, $z$, $z_w$, $\gamma$, $q$ and $T$, respectively. $c$ is cohesion of the joint material along the sliding surface; $FS$ is factor of safety against sliding; $H$ is height of the rock slope; $k_v$ is horizontal seismic coefficient; $k_v$ is vertical seismic coefficient; $q$ is surcharge pressure; $T$ is stabilizing force; $W$ is weight of the sliding block; $z$ is depth of the tension crack; $z_w$ is depth of water in the tension crack; $\alpha$ is angle of inclination of stabilizing force to the normal at the failure plane; $\phi$ is angle of
shearing resistance of the joint material; \( \gamma \) is unit weight of rock; \( \gamma_w \) is unit weight of water; \( \theta \) is an angle equal to \( \tan^{-1}\left(\frac{k_h}{1 \pm k_w}\right) \); \( \psi_f \) is angle of inclination of the slope face to the horizontal; \( \psi_p \) is angle of inclination of the joint plane/failure plane to the horizontal. Several special cases of this general expression based on possible field situations have been described; some of which have been presented in similar forms in the literature.

The graphical presentations for typical values of governing parameters in the parametric study indicate that the factor of safety of the rock slope increases with an increase in both angle of shearing resistance and cohesion of the joint material. The rate of increase in the factor of safety increases with an increase in angle of shearing resistance, whereas it remains constant for any increase in cohesion. The vertically upward direction of the inertial seismic force results in an increase in factor of safety, but the vertically downward direction of the inertial seismic force causes a decrease. The factor of safety of the rock slope decreases with an increase in angle of inclination of the slope face to the horizontal whereas it increases with an increase in angle of inclination of the failure plane to the horizontal. For the higher factor of safety of the rock slope against plane failure under seismic loading conditions, greater values of shear strength parameters and/or the stabilizing force must be present. The orientation of stabilizing force to the normal at the failure plane plays a very important role on factor of safety of rock slope and it is higher for greater values of angle of inclination which becomes maximum for \( \alpha \approx 70^\circ \) beyond which it decrease. It is also realised that surcharge and water pressure in the tension crack decrease the factor of safety significantly.

A general expression has also been derived for the factor of safety of the multi-directional anchored rock slope against plane failure, incorporating most of the practically occurring destabilizing forces including surcharge and seismic loads.

\[
FS = \frac{2c'p + (1 \pm k_w)(Q + 2q^*R) \frac{\cos(\theta + \psi_p)}{\cos \theta}}{(1 \pm k_w)(Q + 2q^*R) \frac{\sin(\theta + \psi_p)}{\cos \theta} + \frac{\mu_w}{\gamma_w} \cos \psi_p + 2 \sum_{i=1}^{n} T_i \cdot \sin \alpha_i} \left( \frac{\sin(\theta + \psi_p)}{\cos \theta} - \frac{\mu_w}{\gamma_w} \cos \psi_p - \frac{\sum_{i=1}^{n} T_i \cdot \sin \alpha_i}{\gamma} \right) \tan \phi
\]

(5.14)
where \( T_i^* = \frac{T_i}{\gamma h^2} \) and \( T_i \) is stabilizing force in the \( i \)th set \((i = 1, 2, 3,...n)\); \( \alpha_i \) is angle of inclination of stabilizing force to the normal at the failure plane (degrees) in the \( i \)th set \((i = 1, 2, 3,...n)\).

Several special cases of this general expression based on possible field situations have been described some of which have been presented in similar forms in the literature. The graphical presentation for a particular set of specific values of parameters shows that the factor of safety increases nonlinearly with an increase in inclination of the stabilizing force to the normal at the failure plane, and it becomes the highest for about 70°. It is also noticed that with a fixed inclination of one set of anchors, the inclination of the second set of anchors is not essentially required to be fixed precisely for its very high values. For investigating the variation of the factor of safety over a wide range of parameters, the developed analytical expression in the explicit form can be used conveniently.

### 6.2 CONCLUSIONS

Based on the present study, the following general conclusions are made.

1. The derived analytical expression [Eq. (3.24)] can be used for the estimation of the factor of safety of a single-directional anchored rock slope (SDARS) against the plane failure, subjected to the most practically occurring destabilizing forces including surcharge and both horizontal and vertical seismic inertial forces.

2. Several special cases of Eq. (3.24) based on possible field situations have been described; some of which have been presented in similar forms in the literature.

3. The graphical presentations of Eq. (3.24) representing several field situations show that the factor of safety of the rock slope increases with an increase in both angle of shearing resistance and cohesion of the joint material. The rate of increase in the factor of safety increases with an increase in angle of shearing resistance, whereas it remains constant for any increase in cohesion.
4. The parametric study shows that the effect of vertical seismic coefficient on the stability of the rock slope for practical ranges of parameters governing the stability of the slope. With an increase in vertical seismic coefficient in the downward direction, the factor of safety of the rock slope decreases almost linearly, whereas with an increase in vertical seismic coefficient in the upward direction, it increases almost linearly.

5. The factor of safety of the rock slope decreases nonlinearly with an increase in angle of inclination of the slope face to the horizontal whereas it increases nonlinearly with an increase in angle of inclination of the failure plane to the horizontal.

6. Both stabilizing force and its orientation (i.e. angle of inclination of stabilizing force to the normal at the failure plane) play a significant role in achieving the required slope stability. The factor of safety of the rock slope increases nonlinearly with an increase in stabilizing force as well as with an increase in angle of inclination of stabilizing force to the normal at the failure plane and it becomes maximum for $\alpha \approx 70^\circ$ beyond which it decreases.

7. The factor of safety of the rock slope is not much affected by variation in unit weight of rock, and it decreases with an increase in horizontal seismic coefficient. It is also observed that surcharge and water pressure in the tension crack decrease the factor of safety significantly.

8. Developed design charts can be used for the estimation of factor of safety of the anchored rock slope against plane failure under surcharge and seismic loading conditions. The design charts indicate that the factor of safety of the rock slope increases with an increase in both angle of shearing resistance and cohesion of the joint material.

9. The numerical example illustrates the steps for the estimation of factor of safety using design charts.

10. The general expression [Eq. (5.14)] for the factor of safety of the multi-directional anchored rock slope (MDARS) against plane failure is derived to evaluate the effect of multi-directional anchors on the stability of rock slopes.

11. Several special cases for general expression Eq. (5.14) have been described, based on possible field situations, some of which have been presented in similar forms in the literature.
12. The graphical presentation of Eq. (5.14) for a particular set of specific values of parameters shows that the factor of safety increases nonlinearly with an increase in inclination of the stabilizing force to the normal at the failure plane, and it becomes the highest for about $70^\circ$, which is similar to the single-directional anchored rock slope system.

13. With a fixed inclination of one set of anchors, the inclination of the second set of anchors is not essentially required to be fixed precisely for its very high values (e.g. $65^\circ$, $70^\circ$, etc.).

14. For a higher factor of safety of the rock slope against plane failure under surcharge and seismic loading conditions, greater values of shear strength parameters and/or the stabilizing forces should be available.

15. The accepted values of safety factors for various special cases discussed in the thesis is 1.2-1.5 as reported in the research literature as well as in standards and codes of practice.

### 6.3 RECOMMENDATIONS FOR FUTURE WORK

For the success of many civil and mining engineering projects, especially in hilly and rocky terrains, and for mining excavations, it often becomes essential to know the realistic values of factor of safety of the rock slopes. Because of this fact, the subject area of the present work has been one of the important research topics in civil and mining engineering disciplines. The current research work can be extended further to consider the following:

- Development of a generalised expression for other failure modes including wedge failure under surcharge and seismic loading conditions.
- More design charts can be prepared for specific field situations following the approach described in the present work.
- Comparison of the analytical results with results obtained from finite element/numerical modelling.
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- Experimental model studies, though difficult to carry out, especially in laboratory environments, to compare the mathematical values of the factor of safety of safety.
- Effect of dynamic loads other than earthquakes on rock slope stability.
- Rock slope stability analysis using pseudo-dynamic approach.
- Effect of other reinforcing techniques on the stability of rock slopes.
REFERENCES


