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ABSTRACT

Math modeling is currently at the focus of educational methodologists' attention. However, little is known about the extent to which principles of the math modeling lead to methodological fallacies in educational research. The main purpose of this paper is to explore the nature and principles of math modeling and to examine its application in educational research according to transcendental realism theory. The conclusion of the article suggests some methodological fallacies in educational research. Finally, the implications of the fallacies in educational research are considered.

INTRODUCTION

The fact that quantitative methods are once again very evident in educational research may lead one to conclude that the quantitative/qualitative debate has been settled in favor of the former. To some extent, this is as a result of managerial pressures and the need to produce research findings that allow predictions to be made by managers at both institutional and systemic levels. It also reflects pressures on educational researchers to mirror the methods of the natural sciences and an assumption that predictive, deterministic, rational and impersonal knowledge is possible in the educational sciences.

This orientation supposes that our failure to solve many of the problems in society reflects a lack of finesse with the method rather than the application of a misguided epistemology and ontology. In fact, there is still a fundamental, enduring controversy over the nature of educational sciences. We want to take this occasion, then, to reflect on whether in fact the world can be known in this way. And whether a distortion occurs when educationalists utilize to math modeling?

There is a long-standing tension in the educational sciences between two quite distinct and opposed ideas about the nature of educational life and the possibilities of our knowledge of it. One of the views is commitment to mathematics and the other one is based on an interpretative approach in which mathematical methods are irrelevant in the study of educational phenomena.

Our purpose in this article is to consider the issues raised by this persistent division of opinion, not with the intention of championing one side or the other, but rather of showing that the issue itself is misconceived and leads to methodological fallacies. This paper focuses on investigating the dominance of math modeling in educational research through the lens of transcendental realism. We identify some challenges to the approach, the problems of math modeling, and the relation between the problems and methodological fallacies in educational research.

NATURE OF MATH- MODELING IN EDUCATIONAL RESEARCH

Mathematical modelling is a technique for understanding the dynamics of a system and for predicting future outcomes within the system. From a simplified perspective, any system is composed of two fundamental things: (1) elements that have certain qualities and properties (2) relationships and actions that explain how these elements interact and change (Norris, et al, 1997). Indeed, mathematical models are an abstraction of the system they represent. By using such models, the user can study and understand the relationships between the elements of the system without having to actually manipulate the system. Abstraction also allows for the simplification of the system because it is not necessary or even desirable for it to be exact or replicate the exact mechanisms. Modelling is representing the educational phenomenon in mathematics terms giving meaning to mathematical symbols, clarifying understanding of mathematics ideas, and doing mathematics. Mathematics can replace direct interaction with the object under study. The experience of the phenomenon is transposed into mental images in the memory of the learner and can become associated with abstract mathematic representations. One can respond to the symbols without a model but by using its mental images as referents of thinking. Math models, as conceptual models, consist of a collection of principles or rules that describe the behaviour of the system under consideration. For example, representing the shape of the earth as a sphere provides a mathematical model, since the principles involved are those of elementary geometry. The active process of devising a mathematical model is called mathematical modelling (Breithach and Maltas, 2003). Thus, mathematical modelling is a systematic process that draws on many skills and employs the higher cognitive activities of interpretation, analysis, and synthesis. The modelling process is composed of five main stages.

1. Observing a phenomenon, delineating the problem situation inherent in the phenomenon, and discerning the important factors (variables/parameters) that affect the problem.
2. Conjecturing the relationships among factors and interpreting them mathematically to obtain a model for the phenomenon.
3. Applying mathematical analysis to the model.
4. Obtaining results and reinterpreting them in the context of the phenomenon under study and drawing conclusions.
5. Testing and refinement of the model (Breithach and Maltas, 2003).

So, mathematical models are underpinned by a number of beliefs about the nature of reality and how we can know it, which immediately involves us in a contradiction, since one of those assertions is that such methods are not reflections of underlying belief systems (Scott, 2000). However, this approach provides an invaluable tool to scientists because they can be easily manipulated and changed when necessary.

There are two main approaches to mathematical logic used by educational methodologists (Scott, 2000; Edling, 2002):

APPROACHES

Deterministic Modelling

In the deterministic process, it is deemed possible to determine a future state if we know the current state of the process through differential (or difference) equations (Edling, 2002). These modelers reduce aspects of human behavior to variables, which are independent and logically distinct. Educational life is portrayed as consisting of a number of instances of those variables, which are, for the purpose of description, equivalent. This equivalence operates across time and place. The relationship between these variables is causal and linear. Furthermore, all educational phenomena can be characterized in this way. So the intentions, beliefs and reasons for action by human beings are no different from activities of chemicals. Antecedent conditions are understood as efficient causes of human behaviour. This means that is possible to develop a science of human behaviour, which allows predictions to be made about what will happen (Blossfeld and Rohwer, 1995 and Scott, 2000).

STATISTICAL MANIPULATION/ STOCHASTIC MODELS

In the stochastic model the future state can only be predicted from the present with some probability (Edling, 2002). This approach works on the basis of fuzzy logic, and describes a system that it is not determinate as such because built into it is a notion of probability, which allows for the possibility of counter-factual cases. According to this approach, we may sometimes be wrong about the nature of the world, but this is caused by the bias of researcher or observer when they fail to bracket out their values, preconceptions and experiences of the world or inappropriately apply the method. Values or preferences or choices do not play a significant part in either the activity of human beings or their description by other human beings. Furthermore, within the limits of probability it is possible to provide policy makers, administrators and the like with information that they can then confidently use to further their ends. This argument, therefore, seems on the surface to have provided us with a solution to the problem of finding a means of obtaining useful knowledge, while at the same time accounting for a voluntaristic dimension to human relations (Blossfeld and Rohwer, 1995; Scott. 2000). Historically, stochastic models have been used more frequently than deterministic ones in educational sciences. There are several reasons for this:

> *One being educationalists in general regard deterministic models as suspect, another that Coleman's (1964) textbook dealt with stochastic process models. Third, with stochastic process models, change in discrete variables can be modeled directly.* (Edling, 2002, 204)

However, this may present too simple a view of explanation in educational sciences, according to transcendental realism.

MATH MODELING AND TRANSCENDENTAL REALISM

Since Galileo, it has been understood that there is an essential connection between natural sciences and mathematics. Indeed, the underlying concepts of natural sciences can be formulated by mathematics. Consequently, there is also an insistence that the fundamental propositions in educational sciences can be formulated by mathematical methods. Some educational methodologists even claim this is no essential methodological distinction between the natural and educational sciences (Wilson, 1987). The idea that the educational sciences should model themselves on the natural sciences circulated in a general way in the Enlightenment and was formulated as an explicit thesis by Auguste Comte and John S. Mill. Since that time it has been the orthodox methodological position in the humanities, particularly in educational sciences (Wilson, 1987). However, there has also been continuing disagreement about the conception of educational sciences as the natural sciences of education (Giddens, 1976, 1979). One important version of the opposition is transcendental realism. According to transcendental realism, we seek to establish that the natural science model can inappropriate and misleading in the educational sciences and mathematics cannot play the same fundamental role in the study of educational environments as in understanding natural phenomena. Nevertheless, a purely ideographic approach is also ineffective and mathematics has an indispensable role to play in unravelling the complexities of educational phenomena. A strong argument has been made by transcendental realists that perception is theory- laden. So, what an observer sees, and also what he or she does not see, and the form that the observation takes is influenced by the background knowledge of the observer, of theories, of hypotheses, of assumptions, or of conceptual schemes that the observe harbours (Phillips and Burbules, 2000). The relativity of the light of reason is the second of this issue. It means that what is obvious to one person may not be obvious to another. Consequently, it is so hardly a solid basis on which to build a whole edifice about knowledge. And the third reason we can identify is the problem of induction. This is the longest standing issue for empiricists, with a lively history of discussion going been about 250 years to the work of David Hume. Hume's sceptical question is; what observation have we made that enable us to be certain about as yet unobserved cases? This question has rung out through the years. So, philosophers of science and logicians have been much exercised to find a solution. Some, like Popper, have denied that inductive reasoning is important in science. In fact, Popper denied that it exists at all. Indeed, these are a part of the main issues that are extremely troublesome for foundationalism (rationalism and positivism)

SCIENTIFICIZABILITY OF EDUCATIONAL RESEARCH

Transcendental realists hold that much educational research can be, and ought to be, scientific. But we add the vital proviso that this position is reasonable only if the positivist account of the nature of science prevalent in earlier times is replaced by a more up-to-date postpositivist account. Arguments for the disjunction of natural science and social science have often rested on an unrealistic account of the nature of the natural sciences. If they are viewed according to the positivist model as based on fundamentalist assumptions about evidence, proof, and trust, social science does seem to be quite different. However, when science is viewed according to the post-positivist model, in which observations are theory-laden, facts underdetermine conclusions, value affect choice of problems, and communities of researchers must examine methods and conclusions for bias, then the perceived gap between social and nature sciences begins to disappear (Philips and Burbules, 2000).

Nevertheless, there are some important points that the educational researcher must consider when he or she works as an educational scientist. These points are related to methodological fallacies associated with math modelling in educational research: (1) closed system versus open system (2) association versus causation and (3) intentionality versus extensionality.

PROBLEMS OF MATH MODELLING IN EDUCATIONAL RESEARCH

There are three important issues regarding math modeling in educational research that need to be examined (Scott, 2000). One of the most important problems regarding math modeling in educational research is relevance to the kind of systems within which the educational researcher actually operates. In other words, there is a main distinction between closed systems and open systems (Bhaskar, 1991, Archer, 1995, Sayer, 1992). Closed systems operate in two ways: (1) they operate consistently and (2) the external conditions of causal mechanism must remain constant to allow them to operate. Thus it is possible to suggest that, when both of these conditions hold, we can infer a causal relation. Of course, there is a third condition that lies in the realm of epistemology, and this knows that the causal relationship has not been contaminated by faulty, inconsistent or inadequate methods of data collection.

On the other hand, in open systems an object has powers and capabilities that are causally efficacious. These powers and liabilities do not just reside in individual human beings but also in the relation between them and even in the structural forms that they reproduce and change by their voluntaristic actions. Human beings do not operate in any deterministic way and are activated by individuals and groups of individuals creating the conditions for them to do so. In other words, educational researchers, in general, operate with open systems in which the two conditions of the closed system are violated. Objects do not operate consistently they change their essential nature, and the external conditions for the exercise of causality also change. Therefore, it is likely that over time and in different places, different manifestations of causal powers are at work. Because the constant conjunction of events that we think we have observed is not what it seems, educational researchers are clearly operating within open systems.

The second problem lies in the relationship between association and causation. Associations are identified as being between precisely defined variables. These variables have to be operationalised because they have to be understood and expressed as observable phenomena. They also cannot be singular because the defining operation of which they are a part involves the further identification of other identifiable items, which are similar in all essential respects. This is a necessary reductive act in the process of operationalising variables, which denies the need for any interpretive activity (Scott, 2000). Indeed, if we assume that the world consists of constant conjunctions of events, this inevitably conflates association and causal relations. For example, the relationship between two variables in educational research, such as poverty and academic achievement, is associational one. When researchers find a relationship between them, it tells us little about the causality, which produces the associations. As Bhaskar (1979) articulated in his discussion of transcendental realism, those regularities so produced do not relate in a straightforward way to the causal mechanism, which produced them. In deed, those causal mechanisms may really be in conflict with their appearances. Therefore, theorists who operate in open systems have to distinguish between associational properties and causal mechanisms. Those who operate in closed system do not have to cope with this problem.

The third issue is that because modelers operate extensionally, the intentional dimension of educational life is neglected. Extentionalism is a term that comes from the fact that standard logic satisfies the principle of extensionality. In standard logic, any expressions that are true of the same object, i.e. having the same extension can be substituted freely for one another without changing the truth of the larger context. Thus, standard logic deals with the reference of an expression, what it is true of, rather than with its meaning (Wilson, 1987, 390). Behaviorism, for example, seeks to eliminate any references to beliefs, purpose and meaning. But, educational sciences would embrace these human traits and suggests that educational researcher's descriptions of their experiences; project and desire are not purely epiphenomenal. Consequently, first, as Wilson (1987, 398-9) articulates:

“It is crucially important to note explicitly that use of a mathematical model does not imply that descriptions are untainted by intention. Rather, when we develop and apply such a model we arrange to package intentional idioms in such a way that, for the purpose at hand, we can proceed with formal calculations."

The second is that these educational actors and the relation between them are reduced to pale shadows of their real selves. Structural properties are reified and the voluntaristic dimension to educational life is inadequately accounted for (Scott, 2000).

PROBLEMS OF MATH MODELING AS A MAIN REFERENCE FOR RAISING METHODOLOGICAL FALLACIES

Logically, we have identified that each problem of application of the math modeling in educational research could result in some methodological fallacies as follows. Closed systems can result in the fallacy of homogeneity, reductive fallacy and deterministic fallacy. Association is the main reference of the causal fallacy and prospective fallacy. Extensionality is main reference of the fallacy of value-free knowledge.

1. The fallacy of homogeneity: the characteristics given to a group of people are assumed to apply to individuals within that group.
2. The causal fallacy: observed patterns of behaviour are considered as caused configurations.
3. The fallacy of value-free knowledge: knowledge (educational science) is thought as value-free. Therefore, the researchers ignore the value-rich dimension of their activity.
4. The prospective fallacy: Retrospective viewpoints are frequently conflated with prospective viewpoints. We may be able to explain what has happened, but this dose not means that we know what will happen.
5. The reductive fallacy: Human characteristics are reduced to variable, which can not to be further reduced and which when combined capture the essence of either that human being or educational activities which they are engaged in.
6. The deterministic fallacy: Researchers neglect human intention and creativity in their description of educational activities (Scott, 2000).

CONCLUSION

The general purpose of this paper has been to examine the transcendental realist challenge to positivism in educational research. In the most general terms, positivism is an epistemology i.e. a theory of knowledge, which holds that reality exists and is driven by the law of cause and effect and can be discovered through empirical testing of hypotheses. Such inquiry is empirically objective and value free, as the laws or generalization exists independently of educational and historical context. Today, positivism as a concept, serves as much to fuel a polemic as it does to identify a distinct epistemological theory of movement. Transcendental realism offers an alternative view. It is grounded in the idea that reality exists, but can never be fully understood or explained, given both the multiplicity of causes and effects and the problem of social meaning. Objectivity can serve as an ideal, but requires a critical community of interpreters. Critical of empiricism, it emphasizes the social construction of theory and concepts (Guba, 1990).

The particular purpose of the present paper is to identify and to discuss major issues involved in attempting to employ mathematical models as tools in educational research. With respect to transcendental realism, we identified that mathematical modelling is the establishment of a simplified description of some aspect of the real world. We also note that each instance of modelling involves a goal: the solution of some real world problem. Nevertheless, as it was articulated by Abelson (1995, 1):

> *The field of statistics is misunderstood by students and nonstudents alike. The general public distrusts statistics because media manipulators often attempt to gull them with misleading statistical claims.*

In our view, the description need only be a sufficient approximation in context to provide an adequately accurate answer. This permits one to simplify the description, omitting aspects of irrelevant importance, so as to make the analysis and computation feasible.

Further, we have tried to illustrate a number of important points in relation to the role of complexity in knowing the educational world. Complexity denies the possibility of complete representations, and in this sense knowing the educational world must not only involve abstraction but it must also involve recognition that the processes of abstraction involve making selections. However, with complex situations, one can make numerous selections; that are always contingent.

We also identify that we, as educational researchers, can imagine the thrill of commanding new mathematical methodology that lead to novel perspectives. Math models are actually one of the power tools for understanding the intricacies of educational phenomena. However, they cannot play the same role as a vehicle for expressing fundamental concepts and propositions in educational sciences as they do in the natural sciences. The reason for this is that educational phenomena are inherently intentional and educational researchers work in open systems. Moreover, in educational research it is possible that association is not causality. Indeed, educational sciences cannot insist on extensional descriptions without abandoning their phenomena. However, this does not mean that mathematics has no place in educational research. Mathematics can play a heuristic rather than a fundamental role in research on educational phenomena. Several points are worth considering about the place of math modelling in educational research; (1) in using math model to represent dimensions of an educational situation, researchers gain the benefit of precise formulation. However, they must be sensitive when they are articulating the model of the historical and institutional context on which it is based. Otherwise they risk serious error in specifying the model and raising some methodological fallacies. (2) A well-specified model can identify the quantitative and categorical data required to address the questions that motivated the initial inquiry. However, if the model is viewed as heuristic, researchers are not so likely to miss the wood for the trees, as when they mistake the model for the whole story. (3) The line between math models in some pure sense and the use of math as an aid to data analysis becomes blurred once all math models in the educational sciences are recognized as having primarily a heuristic function. (4) Researchers must recognize more consistently than is now generally the case that the concepts and variables in terms of which a model is formulated are based directly or indirectly on the categories employed by the people being studied to organize their activities for themselves (Wilson, 1987 & Fischer, 1998). By considering these points and their theoretical foundations in educational research, the researchers are able to avoid the typical methodological fallacies which are associated with math modelling: the fallacy of homogeneity, the causal fallacy, the fallacy of value-free knowledge, the prospective fallacy, the reductive fallacy and the deterministic fallacy.

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