Sex-related differences in autonomous learning behaviours and mathematics achievement

Laura Beahan

Edith Cowan University

Follow this and additional works at: https://ro.ecu.edu.au/theses_hons

Part of the Educational Psychology Commons, and the Science and Mathematics Education Commons

Recommended Citation


This Thesis is posted at Research Online.
https://ro.ecu.edu.au/theses_hons/427
You may print or download ONE copy of this document for the purpose of your own research or study.

The University does not authorize you to copy, communicate or otherwise make available electronically to any other person any copyright material contained on this site.

You are reminded of the following:

- Copyright owners are entitled to take legal action against persons who infringe their copyright.

- A reproduction of material that is protected by copyright may be a copyright infringement. Where the reproduction of such material is done without attribution of authorship, with false attribution of authorship or the authorship is treated in a derogatory manner, this may be a breach of the author’s moral rights contained in Part IX of the Copyright Act 1968 (Cth).

- Courts have the power to impose a wide range of civil and criminal sanctions for infringement of copyright, infringement of moral rights and other offences under the Copyright Act 1968 (Cth). Higher penalties may apply, and higher damages may be awarded, for offences and infringements involving the conversion of material into digital or electronic form.
SEX-RELATED DIFFERENCES IN AUTONOMOUS LEARNING BEHAVIOURS
AND MATHEMATICS ACHIEVEMENT

BY

Laura Beahan BA

A Thesis Submitted in Partial Fulfilment of the
Requirements for the Award of
Bachelor of Education (Honours)

Faculty of Education
Edith Cowan University

Submitted 30th November 1992
USE OF THESIS

The Use of Thesis statement is not included in this version of the thesis.
The autonomous learning behaviour model proposed by Fennema and Peterson (1985a, 1985b) hypothesises that sex-related differences in mathematics are a result of sex-related differences in autonomous learning behaviours. Autonomous learning behaviours include choosing to engage in high-level tasks, preferring to work independently on such tasks and persisting at them. The purpose of this study was to investigate sex-related differences in autonomous learning behaviours and to determine any relationship between the presence of these behaviours and achievement in mathematics.

Twelve students studying the Year 11 unit "Foundations of Mathematics" were selected for the study, including two males and two females from each of the achievement levels; low, medium and high. They were given a number of mathematics problems and asked to think aloud while solving them. Scales were developed to identify the extent to which the students exhibited each of the autonomous learning behaviours while working on the mathematics problems. The students were also interviewed about their usual behaviours and preferences regarding mathematics.

It was observed that the males in this study chose to engage in more high-level tasks than the females. Sex-
related differences in independence were observed only between the medium-achieving males and females. No sex-related differences were found in the degree of persistence exhibited by the students. Differences between achievement levels were observed on the measure of persistence, but not on the other autonomous learning behaviours. The most autonomous students in this study were found to be medium-achieving males. The results of this study revealed some consistencies and some inconsistencies with both the autonomous learning behaviour model and previous research in the field.
DECLARATION

I certify that this thesis does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any institution of higher education; and that to the best of my knowledge and belief it does not contain any material previously published or written by another person except where due reference is made in the text.

Signature

Date ....30/11/92.............
ACKNOWLEDGEMENT

The author wishes to thank Mr Tony Edwards, Research Supervisor for this study, for his professional guidance and support throughout the study. The author also wishes to thank the teachers who allowed their students to participate in this study during class time.
TABLE OF CONTENTS

ABSTRACT ......................................................... ii
DECLARATION ....................................................... iv
ACKNOWLEDGEMENTS .............................................. v
LIST OF FIGURES .................................................. viii
LIST OF TABLES .................................................... ix

Chapter

1. INTRODUCTION .................................................. 1
   Background of the Study ........................................ 1
   Purpose of the Study .......................................... 2
   Statement of Research Questions .............................. 2
   Significance of the Study ..................................... 3

2. REVIEW OF THE LITERATURE ................................... 5
   Achievement of Girls in Mathematics ......................... 5
   Theoretical Models of Sex-Related Differences in Mathematics ........... 10
   Autonomous Learning Behaviours .............................. 11
   Conclusion .................................................... 15

3. THEORETICAL FRAMEWORK ..................................... 17
   Theoretical Framework and Philosophical Assumptions ................. 17
   Operational Definitions ....................................... 20

4. METHOD OF INVESTIGATION .................................... 22
   Design ......................................................... 22
   Subjects ...................................................... 23
   Instruments .................................................. 24
   Data Collection Procedures .................................. 27
   Justification of Methodology ................................ 30
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The autonomous learning behaviour model</td>
<td>18</td>
</tr>
<tr>
<td>2. Score for each student on the choosing to engage in high-level tasks scale</td>
<td>35</td>
</tr>
<tr>
<td>3. Score for each student on the Independence 1 scale: Nonreliance on an algorithm</td>
<td>38</td>
</tr>
<tr>
<td>4. Score for each student on the Independence 2 scale: Checking for the reasonableness of a result</td>
<td>40</td>
</tr>
<tr>
<td>5. Score for each student on the persistence scale</td>
<td>44</td>
</tr>
<tr>
<td>6. The average time spent on nonroutine problems by each student</td>
<td>45</td>
</tr>
<tr>
<td>7. Rank for each female on the four autonomous learning behaviours</td>
<td>47</td>
</tr>
<tr>
<td>8. Rank for each male on the four autonomous learning behaviours</td>
<td>48</td>
</tr>
</tbody>
</table>
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Subject Sex and Achievement Level</td>
<td>33</td>
</tr>
</tbody>
</table>

ix
CHAPTER ONE

INTRODUCTION

Background of the Study

The education of girls, particularly in science and mathematics, is an issue that was raised by the Commonwealth Schools Commission in their reports, Girls, School and Society (1975) and Girls and Tomorrow: The Challenge for Schools (1984). This led to the development of the National Policy for the Education of Girls in Australian Schools in 1987. This policy has recognised the need to educate girls for a technological society in which mathematics is an integral part.

Although girls are achieving as well as, if not better, than boys in mathematics at the primary and early secondary years, it is evident that boys are participating in more mathematics and excelling at the higher levels (Leder, 1990, p.13). The greatest differences appear to be on complex mathematical tasks requiring a high level of cognitive ability.

It has been suggested that sex-related differences in autonomous learning behaviours may be the cause of these differences in mathematics achievement (Fennema & Peterson, 1985a, 1985b). The autonomous learning
behaviour model proposed by Fennema and Peterson is the
basis of this research. Autonomous learning behaviours
include: choosing to engage in high-level mathematics
tasks; preferring to work independently on such tasks; and
persisting at them.

The achievement and participation of girls in mathematics
is an issue that has received much attention from
mathematics educators and researchers. The Commonwealth
Schools Commission states, however, that "the problem is
not essentially girls' failure, but the failure of
mathematics educators to teach mathematics in a way which
ensures equality of outcomes" (1984, p.17).

Purpose of the Study
The purpose of this study was firstly to observe any sex-
related differences among a sample of students in the
autonomous learning behaviours exhibited during
mathematical problem solving. Secondly, to determine any
relationship between the presence of these behaviours and
achievement in mathematics. Finally, to compare the
extent to which each of the autonomous learning behaviours
were exhibited by individual students.

Statement of Research Questions
(1) Can sex-related differences be observed in the
autonomous learning behaviours exhibited by the
students in this study during mathematical problem solving?

(2) Is there a relationship between the autonomous learning behaviours exhibited by the students in this study and their achievement in mathematics?

(3) Is there a relationship between the levels of autonomous learning behaviours displayed by individual students?

Significance of the Study

There are three points to be made regarding the significance of this study. Firstly, the autonomous learning behaviour model proposed by Fennema and Peterson is supported by only a limited amount of research. The results of this study may or may not provide further support for the model.

Secondly, if girls do not participate in autonomous learning behaviours to the extent that boys do, and if this results in their lower achievement on high-level mathematics tasks, then educational practices would need to be reassessed as to their contribution to this situation. It may be necessary to consider ways of teaching mathematics that encourage autonomous learning behaviours among girls in an effort to maintain educational equity.
Thirdly, the results of this study may reveal that other theoretical models explaining sex-related differences in mathematics (e.g. Eccles, 1985) should include autonomous learning behaviours as a mediator.

The following chapter reviews the literature on sex-related differences in mathematics achievement and autonomous learning behaviours.
CHAPTER TWO
REVIEW OF THE LITERATURE

The literature has been reviewed in three sections. Firstly, research into the achievement of girls in mathematics is reviewed. Secondly, theoretical models explaining sex-related differences in mathematics are discussed. Thirdly, research into autonomous learning behaviours and mathematics is reviewed.

Achievement of Girls in Mathematics

There has been an abundance of conflicting research in the area of gender and mathematics achievement. Many studies prior to 1978 found sex-related differences in mathematics achievement in favour of boys; however, they did not take into account differing levels of participation (Willis, 1989, p.5). More recently, a number of studies which account for differing levels of participation still report sex-related differences in achievement, although others do not (Battista, 1990; Ethington, 1990; Moore & Smith, 1987). These differences tend to depend on the age and level of the students and the particular type of mathematical tasks that they are required to perform.

During the primary school years, there appear to be differences in achievement in favour of girls. By the end
of high school, however, a number of studies report greater differences in favour of boys (Armstrong, 1981; Fennema & Carpenter, 1981). In Western Australia, the ratio of girls to boys receiving Advanced awards in mathematics under the Achievement Certificate increased from 0.76:1 in 1972 to 1.11:1 in 1986 (Parker & Offer, 1987). The Secondary Education Authority (1991) statistics indicate that a higher percentage of females than males received a grade of A for all the Year 12 accredited mathematics courses and for three of the four Year 11 accredited mathematics courses.

In Western Australia, more boys than girls score extremely well on tests to select students for gifted programmes (Kissane, 1986). A number of studies have revealed that differences in proportion of males to females are most evident in the top levels of achievement (Armstrong, 1981; Fennema & Carpenter, 1981; Fox & Cohn, 1980; Joffe & Foxman, 1986). The Australian Mathematics Competition awards more prizes for the top achievers to males (Annice, Peterson, Pollard & Taylor, 1988; Edwards, 1985). In 1983, the ratio of boy to girl prizewinners in Year 7 was 3.8:1 and in year 12 was 17.3:1. Annice et al. (1988) found a difference in achievement in the Australian Mathematics Competition favouring boys even after statistically adjusting females' marks upwards to take account of their lower response rate.
It is apparent that studies in achievement cannot be compared when different types of achievement are being measured. Most studies in achievement measure one of two types of mathematics problems. These are: (a) routine mathematics problems that have been taught in the classroom; and (b) nonroutine mathematics problems. The Achievement Certificate and the Tertiary Entrance Examinations generally measure achievement in mathematics that has been taught. The Australian Mathematics Competition and the tests used to select gifted students generally measure achievement in solving nonroutine mathematics problems. Some studies have revealed that girls receive better grades than boys in mathematics classes throughout schooling (Benbow & Stanley, 1982; Stockard & Wood, 1984). It has been suggested by Meyer and Fennema (1988) that girls learn what is taught in the classroom better, whereas boys are better at transferring what they have learnt to high-level tasks that have not been taught.

A recent meta-analysis of 487 reports on mathematical problem solving (Hembree, 1992) found no differences between males and females in Grades 1 to 8; however, it found differences in favour of males at the high school and college levels. A meta-analysis by Hyde, Fennema & Lamon (1990) also found differences in favour of males on tests of problem solving among high school students, college students and high achieving students. Fennema and
Carpenter (1981) and Armstrong (1981) found that boys perform better on high-level cognitive tasks such as problem solving, whereas Galbraith (1986) found that girls outperformed boys on a test of mathematical strategies or processes at each Year level 8, 9 and 10. Bourke and Stacey (1988) found no sex-related differences in the problem solving processes of Year 4, 5 and 6 children. The processes that Bourke and Stacey assessed included, for example, correctness of solution, accuracy of computation and quality of explanation. Therefore, the processes being assessed in this study were mainly precision and communication skills.

It appears evident that particular types of questions yield greater differences in male and female achievement. Wood (1976) studied responses to two GCE Ordinary level mathematics examinations and found that boys outperformed girls on the vast majority of items particularly those involving scale, measurement, probability and space-time relationships. Bradberry (1989) conducted a similar study over 10 years later and found that the situation had not changed despite a greater awareness of the educational needs of girls during this time period. Both Wood and Bradberry found that girls tended to leave the answer to an intermediate step of the problem; that is, they did not complete the final step. They both suggested that girls may not check to see if their solution is reasonable.
Many variables have been researched as to the cause of sex-related differences in mathematics achievement. These can be classified as either external or internal influences. External influences include bias in textbooks (Northam, 1986), interactions between teachers and students (Becker, 1981; Leder, 1987), parental, teacher and societal expectations (Sherman, 1983), and single sex versus mixed classes (Rowe, 1988). Internal influences include confidence in mathematics (Armstrong & Price, 1982; Fennema & Sherman, 1977, 1978; Joffe & Foxman, 1986), attitude and anxiety towards mathematics (Perl, 1982; Sherman, 1983), perceptions of the usefulness of mathematics (Armstrong & Price, 1982; Joffe & Foxman, 1984; Kelly et al., 1986; Perl, 1982), sex-role congruency or perception of mathematics as a male domain (Armstrong & Price, 1982; Sherman, 1979), fear of success (Clarkson & Leder, 1984; Leder, 1977, 1980, 1982), and attributional styles (Tapask, 1990; Wolleat, Pedro, Becker & Fennema, 1980). Interactions between these internal and external influences are hypothesized by a number of theoretical models explaining sex-related differences in achievement and participation (Eccles, 1985; Reyes & Stanic, 1988).

Research in sex-related differences in mathematics achievement has revealed conflicting results. However, when achievement in mathematics is considered as achievement on nonroutine problems, or high-level tasks,
there appear to be sex-related differences in high school and college settings in favour of males (Hembree, 1992; Hyde et al., 1990).

Theoretical Models of Sex-related Differences in Mathematics

Research into sex-related differences in mathematics has for some time been atheoretical; that is, not based on any particular theoretical model (Fennema, Walberg & Marrett, 1985). Much of this research has instead been based on observation and intuition. Since the need for unifying this research through the development of theoretical models has been noted, two particular models of achievement and participation in mathematics have arisen. The autonomous learning behaviour model proposed by Fennema and Peterson (1985a, 1985b) hypothesises that sex-related differences on high-level mathematics tasks are caused by differences in autonomous learning behaviours, which are in turn caused by internal and external influences. This model is the basis of this study and so will be explained in more detail in Chapter 3.

The second model proposed by Eccles (1985), and modified by Ethington (1992), has certain similarities to and differences from the autonomous learning behaviour model. It hypothesises that there are many interrelated variables that affect a child's perception of the value of a task.
and their expectancies which are seen as the mediators to mathematics achievement. Some of these variables include students' perceptions of their own abilities and future goals, their causal attributions for success and failure, and their perception of role-appropriate behaviours. This model contains many elements similar to the internal and external influences suggested by Fennema and Peterson's model. The difference is that Eccles' model does not explicitly view autonomous learning behaviours as mediating between internal/external influences and mathematics achievement. Eccles includes behaviours such as persistence, choice and performance together in the final outcome of the model which is labelled "achievement behaviours". That is, achievement behaviour may be defined as performance in mathematics or as the intention to continue studying mathematics. Research into sex-related differences in autonomous learning behaviours may reveal that alternative models such as Eccles' should incorporate autonomous learning behaviours as the mediator between internal/external influences and mathematics achievement.

**Autonomous Learning Behaviours**

The autonomous learning behaviour model (Fennema & Peterson, 1985a, 1985b) is a possible explanation of sex-related differences on high-level mathematics tasks such as problem-solving. These are the types of mathematical tasks that reveal the greatest sex-related differences
(Armstrong, 1981; Fennema & Carpenter, 1981; Hembree, 1992). Autonomous learning behaviours include choosing to work on high-level mathematics tasks, preferring to work independently on them, persisting at such tasks and succeeding at them. Fennema and Peterson (1985a, 1985b) suggest that unless students engage in these behaviours, they will not succeed at high-level mathematics tasks. Autonomous learning behaviours are seen as developing over many years and their development is dependent on internal and external influences.

Much research has focused on the internal and external influences, and there seems to be relatively little research on sex-related differences in autonomous learning behaviours and how they affect mathematics achievement. Grieb and Easley (1984) have noted that males show more independence in their mathematical problem solving and suggest that an independent attitude towards learning mathematics is necessary to achieve in mathematics beyond high school. They conducted case studies of primary school children over a number of years and noted the environmental influences that developed independent thinking in boys at such an early age. There were two ways that independence was revealed in the problem solving of males. Firstly, boys exhibited less reliance on algorithms, procedures and memory, and more reliance on common sense. Secondly, boys showed a greater reliance on their own judgements of the correctness of their solutions.
rather than depending on external authorities such as the teacher. Grieb and Easley suggest that these independent behaviours are developed by boys at a young age through interactions with their teachers.

Maccoby and Jacklin (1974) found that females show more dependency, and that independence or autonomy is positively related to intellectual performance. McLeod (1985, p.275) discusses independence in problem solving and suggests that it is a very important variable because it is essential for creative mathematical problem solving. Good, Grouws and Ebmeier (1983) classified students as dependent or independent according to how much they relied on the teacher for direction and feedback. They found that independent students were stronger at mathematics but were not among the highest achievers. Independent students also showed less response to external motivation and were often perceived by teachers as behavioural problems in the classroom. Independence has been observed as a common and important trait among exceptional mathematicians (Helson, 1980).

Autonomous learning behaviours can be viewed as either those behaviours exhibited in learning mathematics or those behaviours exhibited while tackling mathematical problems. The first type has been studied in terms of classroom behaviours by Fennema and Peterson (1986), and Peterson and Fennema (1985). In studies of primary school
children, they found that teacher guidance and interaction with girls was negatively associated with girls' achievement on high-level mathematics tasks. They suggest that these findings support their autonomous learning behaviour model in that the dependent behaviours exhibited by the girls had restricted the development of independent behaviours that are essential for succeeding at high-level mathematics tasks.

Karp (1991) studied the relationship between teaching methods that fostered dependence or independence and teachers' attitudes towards mathematics. She found that elementary teachers with negative attitudes towards mathematics used teaching methods that fostered teacher dependence, whereas elementary teachers with positive attitudes towards mathematics used methods that encouraged independence.

Caporrimo (1990) studied autonomous learning behaviours that were exhibited during the problem solving of Year 8 students. Caporrimo's study investigated the relationship between gender, problem solving strategies and mathematics achievement, based on the autonomous learning behaviour model. A questionnaire was adapted from the Mathematics Assessment Questionnaire (Tittle & Hecht, 1988) which is designed to assess students' thoughts and feelings while engaging in and learning to solve mathematical word problems. This questionnaire draws on cognitive and
metacognitive studies in problem solving by Garofalo and Lester (1985) and Schoenfeld (1987). Caporrimo found no gender differences at the eighth grade level and suggested that future research should be aimed at the highest levels of mathematics. In this study, autonomous learning behaviours were considered to be synonymous with cognitive and metacognitive skills in problem solving. This relation is also evident in Peterson’s discussion (1988). These three aspects of problem solving are very closely related. For example, checking for the reasonableness of a result can be viewed as an independent behaviour, as an important step in successful problem solving, and as an act of self-regulation or metacognition. Similarly, persistence can be viewed in the light of all three behaviours. Caporrimo (1990) suggests that if boys engage in more autonomous learning behaviours and if this enables them to succeed at high-level mathematics tasks, then these behaviours would be evident in their problem solving, and one would expect to observe sex-related differences in problem solving. Caporrimo’s study, therefore, investigated sex-related differences in cognitive and metacognitive aspects of problem solving. The present study aimed to define autonomous learning behaviours independently of cognitive and metacognitive problem solving skills.
Conclusion

Sex-related differences in mathematics achievement appear to remain on tasks of high cognitive complexity such as problem solving, and the differences are most evident among the highest achievement levels and among older students. The autonomous learning behaviour model appears to be a possible explanation of these differences. Much research has concentrated on the part of this model dealing with internal and external influences. A limited amount of research has investigated sex-related differences in autonomous learning behaviours. These studies have tended to examine primary school students. This study differs from previous research in that it defines autonomous learning behaviours independently of other problem solving behaviours to observe sex-related differences among older students studying more advanced mathematics.

The following chapter outlines the theoretical framework and philosophical assumptions of the study, and presents operational definitions of the variables used in the study.
In this chapter, the autonomous learning behaviour model is described in detail as the theoretical framework of this study, and the philosophical assumptions of the study are outlined. The autonomous learning behaviours and other variables used in this study are operationally defined.

**Theoretical Framework and Philosophical Assumptions**

This study is based on the autonomous learning behaviour model (Fennema & Peterson, 1985a, 1985b) which stems from a social/psychological framework. This type of framework has been noted as appropriate for research in sex-related differences as it views societal and psychological influences as the ultimate causes of sex-related differences (Fennema, Walberg & Marrett, 1985). Biological variables are not considered in this study, nor are they considered in the autonomous learning behaviour model and other models explaining sex-related differences in mathematics achievement. This is owing to the fact that a consideration of biological variables would not be helpful in the pursuit of educational equity. The variables that are considered in this model are only those that can be changed, or that are alterable.
This study is based on the belief that mathematics is important for all people in society and is often a "critical filter" for many careers. Entry into such careers should not be restricted by sex, race or socioeconomic status. It is assumed that teachers and others involved in education can, and do, have an effect on the social forces that contribute to differences in learning and achievement.

The autonomous learning behaviour model (see Figure 1) hypothesises that sex-related differences in achievement on high-level mathematics tasks are a result of differing levels of participation in autonomous learning behaviours. Autonomous learning behaviours are gradually developed throughout life. The more one participates in autonomous learning behaviours, the more they are developed. Many variables, both internal and external, are believed to

![Diagram](image)

**Figure 1.** The autonomous learning behaviour model
influence the development of these behaviours. These may include students' beliefs about their ability to succeed in mathematics and the messages they receive about mathematics from their parents, teachers, friends and others.

Autonomous learning behaviours have been defined by Fennema and Peterson (1985a, p.309) as follows:

1. Choosing to engage in high-level mathematics tasks,
2. Preferring to work independently on high-level mathematics tasks,
3. Persisting at high-level mathematics tasks, and
4. Succeeding at high-level mathematics tasks.

A circular argument is evident here in that to succeed on high-level mathematics tasks, one needs to have had previous success on these tasks. This study has defined autonomous learning behaviours in terms of the first three dimensions listed above (excluding success) as well as the subcategories of independence in problem solving suggested by Grieb and Easley (1984, p.332). These are:

(a) Not relying on taught algorithms, procedures or memory,
(b) Not relying on external judgements of the correctness of a solution; that is, depending more on one's own judgement.
The definitions of high- and low-level mathematics tasks have been taken from studies by Peterson and Fennema (1985). The stages of problem solving have been identified from the work of Polya (1957). The following is a list of the operational definitions of these and other relevant variables.

**Operational Definitions**

**Choosing to engage in high-level mathematics tasks:**
Preferring to work on a high-level rather than a low-level mathematics task when given a choice.

**Nonreliance on an algorithm:** The degree to which a student is not dependent on rules, algorithms, procedures or memory.

**Checking for the reasonableness of a result:** The degree to which a student checks or judges the reasonableness of their solution to a mathematics problem in relation to the problem.

**Persisting at high-level tasks:**
(a) How far the student has proceeded into the problem solving process before giving up.
(b) The time spent actively working on the problem.
**High-level mathematics tasks:** Mathematical tasks involving application and understanding.

**Low-level mathematics tasks:** Mathematical tasks involving knowledge and skills.

**Mathematics achievement:** First semester result in the unit "Foundations of Mathematics" as determined by the school.
CHAPTER 4
METHOD OF INVESTIGATION

This chapter describes the design of the research, the subjects who participated in the research and the instruments that were used to collect the data. The data collection procedures are outlined, followed by a justification of the methodology of the study.

Design

The design of this research is causal-comparative because it attempts not only to describe a situation as it exists, but also to determine the cause or reason for it (Gay, 1990, p. 247). In causal-comparative research, two groups that differ on an independent variable are compared on a dependent variable. This form of research differs from an experimental design in that the independent variable is not, or cannot be, manipulated.

This research aimed to observe any differences in the autonomous learning behaviours of males and females, and of high-, medium- and low-achieving students. If differences were apparent, then a causal relationship was examined between sex and autonomous learning behaviours, and between the presence of autonomous learning behaviours and achievement in mathematics. Both sex and level of
achievement in mathematics were independent variables, and the observed presence of autonomous learning behaviours was the dependent variable.

Observation of students solving mathematics problems was the main source of data. The students were asked to think aloud while solving problems. A justification of this methodology is presented later in this chapter.

Subjects

Subjects were selected from the population of students studying the Year 11 unit "Foundations of Mathematics" at two metropolitan senior high schools. A list of all students enrolled in the unit "Foundations of Mathematics" at both schools was obtained, along with the students' first semester results. These lists were divided into high, medium and low achievement. Two males and two females with close to the same semester results were selected from each level of achievement. This made 12 students in all.

The subjects were Year 11 students, of 15 or 16 years of age. Two of the high-achieving students, a male and a female, were selected from the Year 10 Academic Extension Programme as they were also studying the unit "Foundations of Mathematics". The selection of these students was considered beneficial as it allowed the inclusion of high-achieving students. This benefit was assumed to outweigh
the possible effect of the one year difference of experience in mathematics.

Discussions with teachers of the unit "Foundations of Mathematics" led to the selection of this unit for a number of reasons. These were: (a) that there tends to be a great range of abilities among the students who elect it; (b) that there is a problem solving component of the course; and (c) that the mathematical topics covered are appropriate for the observation of autonomous learning behaviours.

**Instruments**

Due to the limited amount of research into autonomous learning behaviours in mathematics, no instruments were available that suited the purposes of this study. The instrument used to measure autonomous learning behaviours in problem solving was an observation schedule developed specifically for this research (see Appendices 1 and 2). The observation schedule was divided into sections corresponding to each of the autonomous learning behaviours defined. These behaviours had been taken from the literature on independence in problem solving and autonomous learning behaviours (as discussed in Chapter 3). A scale was developed for each autonomous learning behaviour to indicate the degree to which these behaviours were exhibited (see Appendix 3). These scales were developed prior to the research and presented at a
research proposal seminar where experts in the field of education had opportunity to comment.

During the pilot study, the researcher and an independent, suitably qualified secondary mathematics teacher, used the data obtained from the pilot study to rank one subject on the autonomous learning behaviour scales for each question. The ranks assigned for each question were compared and discussed, resulting in some refining of the scales. Inter-rater reliability was determined at the completion of the study and found to be an agreement on 15 out of the 18 items on the observation schedule. A difficulty arose in obtaining inter-rater reliability due to the fact that one of the main sources of data, particularly on the scale of checking for the reasonableness of a result, was observations made by the researcher while the students were working. Many of these observations were not apparent on the audiotape or written work of the students. This may explain the differences in agreement on the inter-rater reliability check. All the subjects in the main study were interviewed and rated by the researcher.

Triangulation of data was used to enhance the validity of the results in this study. This involves the use of alternative sources of data and methods of investigation (Miles & Huberman, 1984). In this study, autonomous learning behaviours were assessed in two ways.
Firstly, through the observation of students while working on mathematical problems. Secondly, by asking the students about their usual behaviours and preferences when engaging in mathematics. Therefore, any inconsistencies could be noted between the observation of students' behaviour, and the students' self-report of their usual behaviour. The behaviour of persistence was measured in two ways including the average time spent working on the problems, and on a scale to determine how far the student reached in the problem solving process before giving up.

The mathematics problems that the students were asked to solve can be seen in Appendix 4. These problems were chosen to highlight the specific autonomous learning behaviours. They were based on the topics that were covered by both schools during Semester 1 of the course. These topics included trigonometry, analytical geometry, algebra and problem solving. During the pilot study, the mathematics problems were assessed as to how well they allowed for the observation of autonomous learning behaviours. Any mathematics problems that were unsuitable were removed, replaced by other problems and tested on further subjects.

At the end of the interview sessions, the students were asked a number of questions about how they usually learn mathematics and solve problems. Due to time restrictions, this was a structured rather than open ended interview.
The interview schedule can be seen in Appendix 5. The students' responses were used to verify the data obtained through observation and to ascertain information that was not available through observation alone.

**Data Collection Procedures**

Prior to commencement of the research, a pilot study was undertaken to refine the scales used to measure autonomous learning behaviours, to determine the appropriateness of the mathematics problems being used, and to allow the researcher to gain experience in the techniques of interviewing. Four subjects were involved in the pilot study. After interviews with each of these subjects, the instruments described above were refined and trialled on the next subject. This resulted in a number of changes to both the scales and the mathematics problems.

During the first few weeks of Semester 2, the students were selected using the procedures previously described, and permission to participate in the study was obtained from the students and a parent or guardian through a consent form (see Appendix 6).

Due to the different lengths of school periods at each school, the interview session times ranged from 40 to 55 minutes. Each student required between two and three sessions to complete the interviews. The interviews were spread over no more than 2 weeks for each student.
The tasks that the students were asked to do were divided into three sections: A, B and C. The instructions for each section were different and can be seen in Appendix 1.

Section A assessed the degree to which students chose to engage in high-level tasks. The students were given cards containing one high-level and one low-level question (as shown in Appendix 4). They were instructed to read both questions and then choose to do one of them. They were reminded that it was not a test situation, and that they should choose the question that they would prefer to work on. The students were given four of these sets of questions. The students' choices were recorded, but not their solutions. Students were rated on a scale according to how often they chose high-level tasks (see Appendix 3). During this section, the students were encouraged to think aloud and become adapted to the interview situation.

Section B assessed the two aspects of independence described in Chapter 3. Problems were given in which an algorithm may or may not be used and in which checking for the reasonableness of a result is an appropriate process. There were five questions for each of these aspects of independence and they were given alternatively (as shown in Appendix 4). A statistical tables book, a copy of their text book and graph paper were made available to the students. The degree to which the students relied on an
algorithm (Independence 1) and the degree to which they checked their results (Independence 2) were rated on a scale for each question (as shown in Appendix 3). An average for both aspects of independence was then calculated. An audiotape was used to record this section and the data were obtained from the students' explanations as well as their written work.

Questions 6 and 10, measuring Independence 2, could not be assessed using observation alone. They both involved judgements about the size of objects. On completion of the interviews, the students were shown these two questions along with their solutions and asked whether or not they had thought about the reasonableness of their result in terms of the actual object. Their responses to this question were used to find a rating. All other ratings were taken from observations of the students working.

Section C assessed persistence during problem solving. Students were given two nonroutine problems (as shown in Appendix 4) that were more complicated, and in which the use of a specific procedure was not apparent. Persistence was measured according to the stage in the problem solving process during which the student gave up (see Appendix 3). These stages correspond to Polya's stages of problem solving (1957). Persistence was also measured in terms of the average time spent working on the problems. An
audiotape was used to record this section. A maximum of 20 minutes was allowed for each problem.

Finally, students were asked several questions regarding their usual way of solving mathematics problems and of learning mathematics. These questions can be seen in Appendix 5.

Justification of Methodology

The methodology of this study is one in which individual students are given a problem and asked to think aloud as they solve it. Verbal data of this nature has recently gained more popularity among researchers of problem solving (Schoenfeld, 1985). Prior to the recent interest in this method, it was considered to be unscientific and not reliable or replicable. Krutetskii's study of mathematical abilities (1976) made extensive use of this technique. Krutetskii argues that test scores, usually associated with the scientific methodology, only provide the end result or product of problem solving and do not reveal anything about the processes employed in reaching this end point. Giarelli (1988) notes that many researchers have begun to realise the limitations of the scientific methodology.

Artificial intelligence research used the "think-aloud" method of investigation to design problem solving programs...
for computers. Work in this field has enhanced the credibility of this methodology (Schoenfeld, 1985, p.172).

This study aimed to observe processes used during mathematical problem solving, and so the "think-aloud" method was the most appropriate. Grieb and Easley (1984) used a case study approach to note the development of independent behaviour in mathematics. This approach would also be appropriate; however, time restrictions made it an unsuitable method for this study. Capporimo's research into autonomous learning behaviours and problem solving (1990) used a self-report questionnaire. The present study assumed that actual observation of these behaviours would be more reliable and valuable data than students' self-reports of them.

The major criticism of the "think-aloud" method is that thinking aloud is not a normal experience for students, and so may affect their problem solving processes. On a complicated problem, some students may find thinking aloud difficult. This study aimed to overcome this potential problem in three ways. Firstly, the students became used to thinking aloud during Section A when only their choices were recorded, not their solutions. Secondly, during the difficult questions in Section C the students were asked to explain what they had been doing every few minutes rather than all the time. Finally, during the interview, the students were asked about their usual behaviours when
engaging in mathematics. This allowed for any inconsistencies to be noted between the observed behaviours while solving problems and the students' own conceptions of their usual behaviours.

The research was undertaken during the third term of 1992. The results of the research are discussed in the following chapter.
CHAPTER 5

RESULTS

In this chapter, the data analysis procedures are described, and the results of the investigation are presented. Throughout this section, the students will be referred to as subjects numbered from 1 to 12 according to Table 1.

Table 1

Subject Sex and Achievement Level

<table>
<thead>
<tr>
<th>Subject</th>
<th>Sex</th>
<th>Achievement Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>F</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>F</td>
<td>Low</td>
</tr>
<tr>
<td>3</td>
<td>M</td>
<td>Low</td>
</tr>
<tr>
<td>4</td>
<td>M</td>
<td>Low</td>
</tr>
<tr>
<td>5</td>
<td>F</td>
<td>Medium</td>
</tr>
<tr>
<td>6</td>
<td>F</td>
<td>Medium</td>
</tr>
<tr>
<td>7</td>
<td>M</td>
<td>Medium</td>
</tr>
<tr>
<td>8</td>
<td>M</td>
<td>Medium</td>
</tr>
<tr>
<td>9</td>
<td>F</td>
<td>High</td>
</tr>
<tr>
<td>10</td>
<td>F</td>
<td>High</td>
</tr>
<tr>
<td>11</td>
<td>M</td>
<td>High</td>
</tr>
<tr>
<td>12</td>
<td>M</td>
<td>High</td>
</tr>
</tbody>
</table>

The autonomous learning behaviours exhibited by the 12 students have been analysed in terms of the differences observed between groups of students and the differences observed within each individual. Differences observed between groups of students have been analysed in terms of
sex and achievement level for each of the four autonomous learning behaviours. These are: choosing to engage in high-level tasks, Independence 1 (nonreliance on an algorithm), Independence 2 (checking for the reasonableness of a result), and persistence.

Differences Between Groups of Students
The data obtained in this study included: (a) the written work of the students; (b) observations made by the researcher of students' behaviour while working on the problems; (c) audiotape recordings of students' thinking aloud while working on the problems; and (d) transcripts of the structured interview.

Using the students' written work, the observation notes and the audiotape recordings of thinking aloud, each student was rated on the autonomous learning behaviour scales for each of the problems (see Appendices 1, 2 and 3). An average was obtained for each student on the four autonomous learning behaviours. Bar graphs were constructed for each of the four autonomous learning behaviours, allowing comparison by sex and achievement level. Significant statements from the structured interview were extracted and added to the observation data where appropriate.
Choosing to Engage in High-Level Tasks

Figure 2 displays the frequency of choosing high-level tasks for each student. The higher the score obtained on this scale, the more high-level tasks the student chose to engage in (see Appendix 3). Figure 2 reveals that high-level tasks were more often chosen by males than females. Five of the females always chose low-level tasks, whereas four males chose at least one high-level task.

Note: Refer to Appendix 3 for an explanation of the choosing scale.

Figure 2. Score for each student on the choosing to engage in high-level tasks scale.

Responses to the interview questions revealed a similar pattern. Four males and two females said they preferred to work on challenging and different tasks, whereas two
males and four females said they preferred routine and familiar ones. Four males claimed sometimes to attempt mathematics or logic problems in their own time, whereas no females claimed to do this.

Low-level tasks were defined as those requiring only knowledge or skills and high-level tasks were defined as those requiring application or understanding. These definitions led to mathematics tasks of a qualitatively different nature. The high-level tasks involved problem solving whereas the low-level tasks involved the use of a simple algorithm or formula. Attitudes towards these two different types of mathematics tasks were revealed when students were asked how they felt about mathematics. Subject 6, a medium-achieving female, revealed that she would prefer to engage in low-level tasks when she explained, "I don't like problem solving. I only like maths that has like a formula and you've got the formula and you just do it." Subject 4, a low-achieving male, revealed that he would prefer to engage in high-level tasks when he said:

I don't see the point when you get a formula and you get a question that matches the formula and you use it and there's no point to it so you don't learn anything out of doing it. You're just putting numbers into a formula and getting out another number. But problem solving ... that's good maths.
A comparison of achievement levels reveals that the high-achieving students chose to engage in the least number of high-level tasks.

**Independence 1: Nonreliance on an Algorithm**

The Independence 1 scale (shown in Appendix 3) measured the students' degree of reliance on an algorithm or formula. Students scoring high on this measure were more likely to use their own methods and were less reliant on the memorisation of an algorithm or formula. These students tended to use methods that indicated an understanding of the concept. Students who were very dependent on a formula did not reveal an understanding of the concept because it is possible to use an algorithm or formula with little or no understanding.

Figure 3 displays each student's score for the Independence 1 measure. Sex-related differences in the degree to which a student relies on an algorithm were observed between the medium-achieving males and females, in favour of males. Slight differences in favour of males among the low-achieving students were found with no differences apparent among the high-achieving students.

During the interview, the students were asked whether they usually solve mathematics problems their own way or the way that they are shown. Subject 12, a high-achieving
Note: Refer to Appendix 3 for an explanation of the Independence 1 scale.

Figure 3. Score for each student on the Independence 1 scale: Nonreliance on an algorithm.

male, indicated that he liked to generalise what he learnt in mathematics, "I try and work out formulas for them. Just the general sort of thing. I normally do it a different way." Another student seemed to believe that there is only one correct way to do mathematics, which is the way that the teacher shows them. This student, a low-achieving female, said, "I try it my own way if I can't get it the proper way ... but ... I try and do it by the rules."

Therefore, students' beliefs about mathematics may influence the degree to which they rely on an algorithm.
Students who believe that there is only one correct way of doing mathematics will be more likely to study the example that the teacher gives and memorise the procedures. A student who believes that mathematics is more creative and that there are a number of ways to do mathematics problems is more likely to develop his or her own methods and be less reliant on an algorithm.

Figure 3 reveals that differences between achievement levels were not apparent. Low-, medium- and high-achieving students were equally likely to rely on an algorithm or use their own method.

Independence 2: Checking for the Reasonableness of a Result

The Independence 2 scale (shown in Appendix 3) measured the degree to which a student checked or judged the reasonableness of their result after completing a mathematics problem. A student scoring high on this measure would estimate and judge the reasonableness of his or her result in relation to the original question. A student scoring low on this measure would either leave an answer that could not be reasonable for the question, or would leave the correct answer without checking it.

Figure 4 indicates the score for each student on the Independence 2 measure. Sex-related differences were not observed between the low- or high-achieving males and
females; however, the medium-achieving males scored higher than the medium-achieving females.

![Bar chart showing score on the Independence 2 scale for low, medium, and high subjects.](chart)

**Note:** Refer to Appendix 3 for an explanation of the Independence 2 scale.

**Figure 4.** Score for each student on the Independence 2 scale: Checking for the reasonableness of a result.

When interviewed, all of the males responded that they usually think about whether their answer seems reasonable for the question, whereas two of the females indicated that they do not do this. One of these females, Subject 5, explained that this was because the mathematics problems that she had experienced in school were usually not related to real life situations. She suggested that because the problems were contrived, the results would not relate to real life situations, so there would be no point
in judging the reasonableness of her result. The practice of using contrived situations in mathematics problems may discourage students from developing this aspect of independence.

Another factor was seen to influence the degree to which a student checked for the reasonableness of a result. Subject 10, a high-achieving female, explained that she tried to estimate and check the reasonableness of her result for the problem involving the diameter of a truck tyre (see Appendix 4, question 10). However, she found that she had no idea of how big a truck tyre would be. Therefore, being unable to estimate sizes, or being unfamiliar with the object of the problem, can hinder one's ability to check for the reasonableness of a result. Another student drew on his experience with the object of the problem and on his good estimation skills to check his answer for the same problem. Subject 7, a medium-achieving male, said that he checks for the reasonableness of his solution "when it's an object, something you can measure". He said that he drew on his experience from living on a farm when checking his answer for the diameter of a truck tyre. This question led to one of the largest sex-related difference with the average score for males on the checking for the reasonableness of a result scale being 4 and the average score for females being 2.5.
Figure 4 indicates that differences between achievement levels in checking for the reasonableness of a result were not evident. It was noted that low-achieving students used the process of checking for the reasonableness of a result to determine which operations should be used to solve the problem. If unsure of the operations to use, they would choose one and see if the result seemed reasonable for the problem. Therefore, they were often deciding how to solve a problem by judging whether the result seemed reasonable. This may account for the high scores obtained by the low-achieving students on this scale. Alternatively, high-achieving students may have felt sufficiently confident in the method they used and in the accuracy of their calculations that they did not feel the need to check their result.

During the interview, the students were asked whether they felt confident in their own judgement of the correctness of their solution, or whether they relied on an external judgement such as the text book or the teacher. The responses indicated that four males felt confident that they were correct, whereas only two females felt this confidence. The other students were more reliant on an external judgement of the correctness of their result. All four low-achieving students did not feel confident in their own results in mathematics.
Other aspects of independence

The students were interviewed in regard to two other aspects of independence that could not be observed during problem solving. Firstly, students were asked whether they preferred to work on challenging mathematics problems on their own or with other people. Overall, the males indicated a preference for working with others. Four of the males preferred to work with other people whereas two males preferred to work on their own. Two females preferred to work with other people, three preferred to work on their own, and one said that it depended on the type of problem.

Secondly, the students were asked whether they preferred to work through a difficult problem on their own, or ask for help from others. The low-achieving students preferred to work through difficult problems on their own. The medium-achieving females preferred to ask for help whereas the medium-achieving males preferred to work through it on their own. Of the high-achieving students, one male and one female preferred to ask for help, one female preferred to work through problems on her own, and one male said that he did not have a preference and would either work through it on his own or ask for help.

Persistence

Persistence was measured both in terms of the number of minutes spent on the problem, and on a scale (shown in
Appendix 3) to indicate the stage of problem solving during which the student gave up. Figure 5 displays each student's score on the persistence scale. A student who scored high on this measure was more likely to try two or more alternative methods, or to obtain the solution. A student who scored low on this measure was more likely to give up before attempting to understand the problem or plan a solution.

![Figure 5](image)

**Note:** Refer to Appendix 3 for an explanation of the persistence scale.

**Figure 5.** Score for each student on the persistence scale.

Figure 5 indicates that there were no sex-related differences in the degree of persistence exhibited during nonroutine problem solving. Differences between high- and
low-achieving students are evident, with the high-achieving students exhibiting greater persistence. There was more variation among the medium-achieving students in the degree of persistence they exhibited.

The average number of minutes spent on the problem solving tasks by each student are displayed in Figure 6. This measurement of persistence reflects similar results to those on the persistence scale (see Figure 5). It indicates that the low-achieving students spent relatively little time on the problems. The use of time as a measurement of persistence is not appropriate for the student who solves the problem in a short amount of time.

![Figure 6. The average time spent on nonroutine problems by each student.](image-url)
This can be seen in the two persistence scores (Figures 5 and 6) for subject 10, a high-achieving female, and subject 11, a high-achieving male.

Differences Within Individuals

To illustrate the relationship between the presence of each of the autonomous learning behaviours within an individual, bar graphs were constructed for individual students displaying their result on each of the autonomous learning behaviour measures (see Figures 7 and 8). In order to compare the students, their scores on each of the autonomous learning behaviour measures were ranked from 1 to 12, with 12 being the highest and 1 the lowest. So, for example, on the Independence 1 measure, Subject 8 was ranked highest (12), Subject 7 was ranked next (11), followed by Subject 3 (10), and so on.

A number of students exhibited consistent levels of each of the autonomous learning behaviours, whereas others exhibited a high degree of some behaviours and a low degree of others. Of all the behaviours, persistence appeared to be the most variable.

Subject 2, a low-achieving female, showed a consistently low presence of all autonomous learning behaviours. Subject 11, a high-achieving male, exhibited an average presence of all autonomous learning behaviours. Subject 7, a medium-achieving male, and Subject 12, a high-
Figure 7. Rank for each female on the four autonomous learning behaviours.

Subjects

Females

Rank

Choosing  Independence 1
Independence 2  Persistence

0  2  4  6  8  10  12

1  2  5  6  9  10

47
Figure 8. Rank for each male on the four autonomous learning behaviours.

Males

Rank

Subjects

- Choosing
- Independence 1
- Independence 2
- Persistence

3 4 7 8 11 12
achieving male, exhibited a consistently high presence of all autonomous learning behaviours. Subject 8, a medium-achieving male, also exhibited very high levels of autonomous learning behaviours, but was lacking in persistence. Subject 5, a medium-achieving female, and subject 9, a high-achieving female, exhibited low to average levels of all autonomous learning behaviours, but were very persistent.

Subject 1, a low-achieving female, exhibited relatively high levels of Independence 2, checking for the reasonableness of a result, although exhibited very low levels of all other autonomous learning behaviours. Subject 3, a low-achieving male, exhibited high levels of Independence 1, nonreliance on an algorithm, although exhibiting low to average levels of all other autonomous learning behaviours. Subjects 4, 6 and 10 revealed great inconsistencies in the degree to which they exhibited each of the autonomous learning behaviours.

The graphs shown in Figures 7 and 8 represent individual profiles of the students' autonomous learning behaviours. The relationship between the autonomous learning behaviours is difficult to determine, with some students exhibiting consistent levels of all behaviours, and others exhibiting varying levels of the different behaviours. However: within individuals, persistence appears to vary the most from other autonomous learning behaviours.
An examination of the individual graphs reveals that the three students exhibiting the highest levels of autonomous learning behaviours, excluding persistence, (Subject 7, Subject 8 and Subject 12) were all males, whereas the three students exhibiting the lowest levels of autonomous learning behaviours (Subject 2, Subject 5 and Subject 9) were all females.

The results of this study are discussed in terms of the autonomous learning behaviour model and previous research in the following chapter.
The results of this research indicate certain consistencies and inconsistencies with both the autonomous learning behaviour model and other research in the field. This chapter compares the results of this study with previous research and discusses the findings in relation to the autonomous learning behaviour model. Each of the research questions will be examined, followed by a discussion of the limitations and then the conclusions and implications of the study. Given the small sample of students in this study, the results cannot be generalised to the population of students studying the unit "Foundations of Mathematics". The results of this research are discussed only in terms of the sample used. However, the results do indicate particular areas for future research into autonomous learning behaviours and mathematics. These are suggested throughout the discussion.

Sex-Related Differences in Autonomous Learning Behaviours

The autonomous learning behaviour model (Fennema & Peterson, 1985a, 1985b) hypothesises that sex-related differences in mathematics achievement are a result of
sex-related differences in autonomous learning behaviours. Therefore, this model suggests that there are sex-related differences in autonomous learning behaviours. It would predict that males: choose to engage in high-level tasks more than females; exhibit less reliance on algorithms than females; exhibit greater reliance on their own judgement of the reasonableness of their results than females; and persist at high-level tasks more than females.

This research found that the males in the study were more likely to engage in high-level tasks than the females. This finding supports Fennema and Peterson's theory that males choose to engage in high-level tasks more than females. There do not appear to be any other studies examining the choices students make regarding high- and low-level mathematics tasks with which these findings can be compared, so further research using a large sample seems warranted. The findings seem to parallel Grieb and Easley's assertion (1984) that boys are more likely to explore mathematics, rather than simply receive it. In particular, the finding that four of the six males, and no females, engage in high-level tasks in their own time seems to indicate a possible sex-related difference. Again, there does not appear to be any previous research in the degree to which students engage in mathematics outside of classroom time for pleasure or recreation.
This study found sex-related differences in the degree of reliance on an algorithm among the medium-achieving students, with the females exhibiting more reliance on algorithms than the males. Some differences were noted between the low-achieving students, with the females exhibiting more reliance than the males, and no sex-related differences were observed among the high-achieving students. The results for the low- and medium-achieving students are consistent with the autonomous learning behaviour model and the findings of Grieb and Easley (1984) that females show a greater reliance on taught procedures. These consistencies are not evident among the high-achieving students.

As suggested in Chapter 5, the relation between degree of reliance on an algorithm and beliefs about or conceptions of mathematics seems to be an important one. For example, one female considered mathematics to be a pre-defined set of rules and procedures of which there is only one correct method of use. Since these beliefs and conceptions influence the way students do mathematics and the degree to which they rely on an algorithm, it would be appropriate to examine any sex-related differences in conceptions of mathematics among students. Previous research on sex-related differences in beliefs about mathematics has been based around such beliefs as the perceived usefulness of mathematics and the perception of mathematics as a male domain (Fennema & Sherman, 1976).
These studies have not tended to examine students' conceptions of what mathematics is.

Sex-related differences in the degree to which students check for the reasonableness of their result were observed only between the medium-achieving males and females. No sex-related differences were apparent among the high-achieving students or low-achieving students. Overall, these results do not support the autonomous learning behaviour model, nor are they consistent with the suggestions of Bradberry (1989), Grieb and Easley (1984) and Wood (1976). These studies suggested that females tend to leave answers that are unreasonable, and are more dependent on external judgements of the reasonableness of their results than males. However, the differences found between the medium-achieving males and females in both the degree of reliance on an algorithm and the degree to which they check for the reasonableness of a result may indicate an interactional effect between sex and achievement level which could be further investigated.

The relation between estimation skills and the process of judging for the reasonableness of a result was highlighted by one female who explained that she could not estimate the size of an object in a problem. This may reflect the findings of Corle (1960), Rubenstein (1985) and Swan and Jones (1971, 1980) that boys performed better on estimation tasks than girls, particularly on the
estimation of measurements. Sowder (1992, p.382) notes that there is a considerable difference between estimating measurements and estimating computations, with the former being more contextually bound. Sex-related differences in the degree to which students check for the reasonableness of a result were noted in this study only between the medium-achieving males and females. However, if there are sex-related differences, as Grieb and Easley (1984) suggest, and if poor measurement estimation skills affect this process, then further research into sex-related differences in measurement estimation may contribute to an understanding of the differences in this aspect of independence.

The finding that more males felt confident in their own judgement of the correctness of a solution may reflect results of studies in confidence and mathematics (Armstrong & Price, 1982; Fennema & Sherman, 1977; Joffe & Foxman, 1986). These studies have revealed that females exhibit less confidence in mathematics than males.

This study found that more males than females preferred to work on mathematical tasks with other people rather than on their own. This does not support the assertions of Burton et al. (1986, p.74) that females prefer collaborative discussion-based learning.
No sex-related differences were observed in the degree of persistence exhibited during problem solving. This finding does not support the autonomous learning behaviour model which asserts that sex-related differences in behaviours such as persistence lead to sex-related differences in achievement.

Overall, sex-related differences were observed between all students on the scale of choosing to engage in high-level tasks, and between medium-achieving males and females on the measures of independence. Sex-related differences were not observed on the measure of persistence and between high-achieving males and females on measures of independence. These results reveal some consistencies and some inconsistencies with the autonomous learning behaviour model and previous research. The consistent differences observed between the medium-achieving males and females suggest the possibility that sex-related differences may operate differently at each level of achievement. The results of the measure of choosing to engage in high-level tasks, and the interview questions related to this, have indicated possible sex-related differences that could be further investigated. When the autonomous learning behaviours are viewed together (see Figures 7 and 8), sex-related differences become more apparent with the most autonomous students in this study being males and the least autonomous students being females.
Autonomous Learning Behaviours and Achievement in Mathematics

The autonomous learning behaviour model hypothesises that greater participation in autonomous learning behaviours leads to greater achievement in mathematics. This model suggests that high-achieving students would exhibit greater degrees of autonomous learning behaviours than low-achieving students. That is, high-achieving students would: choose to engage in more high-level tasks; exhibit less reliance on algorithms; exhibit greater reliance on their own judgement of the reasonableness of results; and persist at high-level tasks more than low-achieving students.

This study found no differences between achievement levels for choosing to engage in high-level tasks. This does not support the autonomous learning behaviour model. In particular, the finding that the high achieving students chose to engage in the least number of high-level tasks seems to be inconsistent with the model.

No differences were observed in this study between the achievement levels for degree of reliance on an algorithm. This does not support the autonomous learning behaviour model which asserts that greater independence in mathematics leads to greater achievement. This study found that the most independent students in regard to degree of reliance on an algorithm were medium-achieving
males. This result is consistent with that of Good, Grouws and Ebmeier (1983) who found that the most independent students were not the highest achievers, but had only a moderate prior achievement in mathematics. As Grieb and Easley (1984) note, independence becomes most important in advanced university mathematics; however, dependence on taught procedures is enough to succeed at most primary and secondary school mathematics. This may explain the findings, in both this study and that of Good, Grouws and Ebmeier, that high-achieving students at the secondary school level do not necessarily exhibit independent behaviours.

Differences between achievement levels in the degree to which students check for the reasonableness of a result were not found in this study. This does not support the autonomous learning behaviour model which asserts that greater independence will lead to greater achievement in mathematics. It was found that low-achieving students often used this process to determine the operations that should be used to solve the problem. The high results of the medium-achieving males are again consistent with the findings of Good, Grouws and Ebmeier (1983) that the most independent students are moderate achievers in mathematics.

Differences were observed between high- and low-achieving students in the degree of persistence exhibited during
nonroutine problem solving. This is consistent with the autonomous learning behaviour model as it indicates that persistence is related to greater achievement in mathematics.

Overall, differences in the degree of autonomous learning behaviours exhibited between the achievement levels were few, only being observed on the measure of persistence. It was noted that students within different levels of achievement used the processes and behaviours in different ways. For example, low-achieving students used the process of checking for the reasonableness of a result to determine how to solve a problem, while high-achieving students used it to check the reasonableness of their result. One may speculate that similar differences would occur in the degree of reliance on an algorithm. Students who have difficulty memorising all the steps of a procedure may not rely on an algorithm, while students who have an understanding of the concept may prefer not to use the algorithmic methods that they are shown. Therefore, although the observed behaviours are the same, they may have developed for different reasons.

**Relationship Between the Autonomous Learning Behaviours**

This study found that some students exhibit consistent degrees of all autonomous learning behaviours, whereas others exhibit varying degrees of each of the behaviours.
This suggests that studies of autonomous learning behaviours should examine the behaviours individually to take account of the fact that some students may exhibit high levels of one behaviour and low levels of others. The most variable behaviour appeared to be persistence. Students who exhibited a great reliance on taught procedures and external judgements of the reasonableness of results, and who tended not to engage in high-level tasks, did exhibit very persistent behaviours. Future studies should examine the relationship between individual aspects of autonomous learning behaviours to determine which ones best represent students with autonomy in learning mathematics.

The finding that the male medium achievers exhibited the highest levels of autonomous learning behaviours is consistent with Good, Grouws and Ebmeier's (1983) findings, but do not support the autonomous learning behaviour model which would expect that students exhibiting high levels of autonomous learning behaviours would be high achievers in mathematics. The results of this study and that of Good, Grouws and Ebmeier only indicate that the most independent or autonomous students are not the high achievers at school. Whether these students become the high achievers in university mathematics where independence is more important, is still unknown. It would seem that those students exhibiting
rule dependence during high school will not develop the understanding required for higher level mathematics.

Limitations
The limited size of the sample used in this study, and the fact that they were selected from two metropolitan secondary schools, does not allow for any generalisation of the results. The 12 students selected for this study came from five classes of the unit "Foundations of Mathematics" each with a different teacher. This study did not take account of the different methods of instruction of these teachers. For example, a teacher may stress the use of algorithms and strict procedures, or may encourage students to use their own methods. The use of the "think aloud" method has limitations, but steps were taken to minimise these. For example, the students were given time to adapt to thinking aloud while working on the problems, and they were asked to explain what they were doing every few minutes rather than all the time on the difficult problems (see Chapter 4).

Conclusions and Implications
Sex-related differences were observed in this study between males and females on the measure of choosing to engage in high-level tasks and between medium-achieving males and females in the degree of independence exhibited while solving mathematics problems. Sex-related differences were not observed on the measure of
persistence nor between high-achieving males and females in the degree of independence exhibited while solving mathematics problems. Differences between achievement levels were observed only on the measure of persistence. It was noted that the behaviours may have been used for different purposes by low- and high-achieving students.

The results of this study have implications for both future research in mathematics education and educational practices. Suggestions for future research have been made throughout the discussion. In addition to these, future research into autonomous learning behaviours in mathematics should concentrate on the development of reliable and valid instruments for measuring autonomous learning behaviours. The instruments used in this study appear to be appropriate for the identification of autonomous learning behaviours; however, further research would need to refine these instruments in terms of reliability and validity. The method used in this research is appropriate for the study of small samples of students. However, the method is time consuming and not suitable for large scale surveys. Future research should be concerned with the development of techniques to assess autonomous learning behaviours among large samples of students.

Behaviours need to be identified that constitute autonomy in learning and engaging in mathematics. Independence has
been noted as a common trait among exceptional mathematicians (Helson, 1980). It may be appropriate to determine how these behaviours are exhibited by these mathematicians. This would lead to observable behaviours that can be noted among students of mathematics. This study has revealed that persistence may not be a trait by which autonomous students can be recognised.

Autonomous learning behaviours should be studied in terms of the behaviours that are exhibited while learning mathematics, the behaviours that are exhibited while solving mathematics problems, and students' perceptions of their own behaviours. This data would need to be collected using a number of different techniques. This study found that it was important to ask students about their own behaviours both to clarify data obtained through observation and to acquire information about the students' learning styles that are not observable.

This study has suggested that certain beliefs about, and skills in mathematics may have a great affect on the presence of autonomous learning behaviours. Further research into this may reveal that they should be included as important components of the autonomous learning behaviour model. Research should continue to investigate the relationship between teaching styles, teacher beliefs and the development of autonomous learning behaviours in students.
Although this study found that there were negligible differences in the presence of independent behaviours between low-, medium- and high-achieving students, there is still a need to foster these behaviours in the classroom. These behaviours do not appear to be necessary to succeed in primary and secondary mathematics, however they are important in gaining a conceptual understanding of mathematics and in further mathematical studies. Teachers should not encourage dependence on rules and algorithms in mathematics. Assessment techniques should reflect this, with an emphasis on a relational understanding of why procedures work, rather than an instrumental understanding of how to use the procedures (Skemp, 1978).

A further implication of this study comes from the behaviour of checking for the reasonableness of a result. The continued use of obviously contrived situations in mathematics problems seems to discourage students from using the process of checking for the reasonableness of a result. Care should be taken in the construction of problems to ensure that their answers are not unreasonable.

Some sex-related differences were observed in this study, indicating that particular attention should be paid to the development of autonomous learning behaviours among females. However, regardless of whether sex-related
differences exist, educational practices should encourage the development of autonomous learning behaviours among all students.

Autonomous learning behaviours have been proposed as the mediators between internal and external influences and achievement in mathematics. However, there seems to be relatively little evidence in previous research to support this proposition. The results of this study indicate that there are differences in autonomous learning behaviours between students, some of which seem to be related to sex and achievement. The relationship between sex, autonomous learning behaviours and achievement in mathematics appears to be a complex one in which many factors are influential.
REFERENCES


Appendix 1
Observation Schedule Instructions

1. Relax the student by engaging them in conversation.
2. Inform the student that the sessions will be divided into three parts, with different instructions for each part.
3. Explain to the student that they are to think aloud while working through the problems.
4. Remind the student that this is not a test situation, that there are no time limits, and that they are to solve the problems any way they like.
5. Indicate to the student that the following are available for their use:
   - graph paper
   - statistical tables book
   - text book
   - math-o-mat

Section A

1. Inform the student that they will be given a card containing 2 questions. They are to read both questions and then choose to do one of them. Explain that there is no time limit and they are to choose the one that they would prefer to work on.
2. Present each of the sets of questions in Section A of the Mathematics Problems (Appendix 4). Enter the student's choice in the Observation Schedule (Appendix 2).
3. Determine the score for this section using the scale for choosing to engage in high-level tasks (Appendix 3), and enter it on the Observation Schedule.

**Section B**

1. Inform the student that for this section, they do not have a choice of questions. Remind the student that they may solve the problems any way they like, that they may use the materials available to them, and that they should continue to think aloud as they work on the problems.
2. Present each of the questions in Section B of the Mathematics Problems individually.
3. Circle a number for each question on the Observation Schedule according to the scales for Independence 1 and Independence 2.
4. Find the average score for Independence 1 and Independence 2 and enter it on the Observation Schedule.

**Section C**

1. Inform the student that they will be given two problem solving questions. Instruct the student to explain every few minutes what they have been doing. If the student seems to have stopped working on the problem, ask them if they would prefer to leave it or keep going.
2. Circle a number for each question on the Observation Schedule according to the Persistence scale. Find an average and enter this.
3. Measure the time (in minutes) spent working on the problem and enter each time as well as their average on the Observation Schedule.

Interview

1. Ask the student each of the interview questions given in Appendix 5.

2. Ask the student if there is anything that they would like to add with regard to any of the questions asked.

3. Ask the student if they have any questions.
Appendix 2
Observation Schedule

**Section A**

<table>
<thead>
<tr>
<th>Low-level</th>
<th>High-level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Score for choosing to engage in high-level tasks:

---

**Section B**

<table>
<thead>
<tr>
<th>Question</th>
<th>Independence 1</th>
<th>Question</th>
<th>Independence 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 2 3 4</td>
<td>2</td>
<td>1 2 3 4</td>
</tr>
<tr>
<td>3</td>
<td>1 2 3 4</td>
<td>4</td>
<td>1 2 3 4</td>
</tr>
<tr>
<td>5</td>
<td>1 2 3 4</td>
<td>6</td>
<td>1 2 3 4</td>
</tr>
<tr>
<td>7</td>
<td>1 2 3 4</td>
<td>8</td>
<td>1 2 3 4</td>
</tr>
<tr>
<td>9</td>
<td>1 2 3 4</td>
<td>10</td>
<td>1 2 3 4</td>
</tr>
</tbody>
</table>

AVERAGE: 83
### Section C

<table>
<thead>
<tr>
<th>Problem</th>
<th>Persistence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 2 3 4</td>
</tr>
<tr>
<td>2</td>
<td>1 2 3 4</td>
</tr>
</tbody>
</table>

AVERAGE:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Average:
Appendix 3

Autonomous Learning Behaviour Scales

Choosing to Engage in High-Level Tasks

1: Always chose low-level tasks.
2: Chose one high-level task.
3: Chose two high-level tasks.
4: Chose three or four high-level tasks.

Independence 1: Non-reliance on an algorithm

1: - Does not complete question because cannot recall an appropriate rule, algorithm or procedure.
   - Uses an inappropriate rule or algorithm.
2: Relies on the memorisation of a formula or adheres to a strict routine procedure.
3: - Makes use of a rule but puts it into their own words thereby showing an understanding of the concept.
   - Uses a rule to begin with and resorts to a common sense method when this does not work.
4: Completes the question using common-sense only or a non-standard method. Does not rely on the knowledge of a formula.

Independence 2: Checking for the reasonableness of a result

1: Leaves an answer that is unreasonable for the question. Does not check to realise that it is wrong.
2: Obtains correct answer but does not check it, or checks calculations only.

3: - Re-reads the question to make sure they have answered it, or that correct units have been used.
   - Checks reasonableness in relation to the numbers only, not the real situation. (E.g. 9 divided by 4 should be just over 2).
   - Tries to estimate, but has no idea of actual sizes.

4: - Estimates and checks the answer, or judges the appropriateness of it in relation to the given question.
   - Checks and realises they must be wrong.
   - Method encorporates checking (e.g. guess and check).

**Persistence**

1: Gives up before attempting to understand the problem or plan a solution.

2: - Gives up during the understanding or planning stages.
   - Rushes in with an incorrect answer by guessing or without reasoning.

3: Tries one method of solution before giving up.

4: Tries more than one alternative method before giving up or obtains a solution to their satisfaction.
Appendix 4
Mathematics Problems

Section A

1. (a) On a balance scale 5 bricks exactly balances 2 bricks and a 9kg weight. How heavy is a brick?

OR

(b) Solve the following equation:

\[ 72(x - 5) = 63(5 - x) \]

2. (a) What can you say about the gradient of lines joining points \((-1,n)\) and \((3,n)\) for all values of \(n\)?

OR

(b) Calculate the gradient of the line joining the points \((1,2)\) and \((-2,4)\).

3. (a) Temperatures given in the Farenheit (F) temperature scale can be converted to the Celsius (C) scale via the formula \(C = \frac{5}{9}(F - 32)\). Thus, for example, \(50^\circ\) is equivalent to \(C = \frac{5}{9}(50 - 32) = 10^\circ\).

(i) Find the formula for converting Celsius temperatures to the Farenheit scale.

(ii) What temperature has the same numerical value in both the Farenheit and Celsius scales?

OR
(b) Rearrange this equation to find $x$ in terms of $y$.

$$y = x^2 - 25$$

4. (a) Find the surface area of a sphere of radius 2.5m.

OR

(b) A loop of rope fits right around the equator of the Earth. We shall assume the Earth to be a sphere of radius 6,400 km. In fact the rope is a little slack since it is exactly one metre too long. Suppose that you wished to take the slack out of the rope by raising it a fixed distance above the surface of the Earth all around the equator. What would the fixed distance be?

Section B

(1) Find the midpoint of the line segment joining the points $(-1,6)$ and $(1,6)$.

(2) If I add 6 to half a given number, my answer is twice the given number. Find the number.

(3) Without using a calculator or pen and pencil, can you tell me what $58 + 34$ is? How did you work that out?

(4) My French mark is 15 less than my Maths mark and the total of my two marks is 145. Find my two marks.
(5) Find a rule for the following pattern:

```
* * * * * * * * * * * * * * * * * * *
* * * * * * * * * * * * * * * * * * *
* * * * * * * * * * * * * * * * * * *
* * * * * * * * * * * * * * * * * * *
```

(6) The length of a factory to be built on level ground is 80 metres. What is its length, in centimetres, on the architect's site-plan which has a scale of 1:250?

(7) The point \((-2,4)\) is the mid-point of the line segment PQ where \(P\) is the point \((2,-2)\). Find the coordinates of Q.

(8) Susan's first 3 test results were 86, 75 and 91. How many marks must she score in the next test to have an average of 85?

(9) Find the distance between the points \((1,1)\) and \((5,1)\).

(10) The wheels of a truck travelling at 60 km/h make 4 revolutions per second. Find the diameter of each wheel.

(Or 10a) Given if question 10 is too difficult.

The wheels of a truck make 4 revolutions per second and travel a distance of 16 metres in one second. Find the diameter of each wheel.
Section C

(1) I have two watches with a 12 hour cycle. One gains a minute per day and the other loses $1\frac{1}{2}$ minutes per day. If I set them both on the correct time, how long will it be before they next tell the correct time together?

(2) One day adventurous Albert decided to find out how fast an escalator at a local shopping centre travels. He found that it took him 10 seconds to get half-way on the up-escalator, at which point he turned round and started walking down it at 2 metres per second. However he continued to move upwards and reached the top after a further 30 seconds. How fast was the escalator moving?
Appendix 5

Interview Schedule

1. Do you like maths?
   (a) Are there any particular parts of maths that you do or don't like?
   (b) What about problem solving?
2. Do you plan to study any maths when you leave school?
3. Do you like working on maths problems that are challenging and different or do you prefer to work on routine problems that you are more familiar with?
4. Do you try maths problems that your teacher has not specifically told you to do?
5. Do you ever attempt maths or logic problems in your own time such as these from the West Australian (show a sample of the Think section)?
6. Do you prefer to work on difficult or challenging maths problems on your own or do you prefer to work with other people?
7. When you are finding a problem difficult do you ask for help from your teacher or from other students in your class or do you prefer to work through it on your own?
8. Do you sometimes solve maths problems your own way or do you always follow the way you have been shown?
9. (a) When you did question 6 (show question and their solution) and you got an answer of _____, did you stop and think about whether that was a
reasonable size for the length of a factory on a site plan?

(b) When you did question 10 (show question and their solution) and you got an answer of _____, did you stop and think about whether that was a reasonable size for the diameter of a truck?

(c) Do you usually do this when you have an answer?

10. When you have found an answer to a maths problem, are you usually confident that you are correct or do you go straight to the back of the book to check it?

11. (a) When you are finding a maths problem difficult are you likely to give up, ask someone for help or keep persisting with it on your own?

(b) So would you say that you are persistent?
Appendix 6
Consent Form

I am currently undertaking research in mathematics education. The purpose of my study is to provide information about the way year eleven students solve mathematics problems. The study will involve approximately two hours of your time. You will be asked to attempt a number of mathematics problems and will be interviewed and tape recorded while you are working on them. The tape will be erased once the study is completed and your name will not be used when the study is published. Your participation in this study is voluntary. If you have any questions regarding any aspect of the study, you may contact me on 448 2916.

LAURA BEAHAN

I ____________________ have read the information above and any questions I have asked have been answered to my satisfaction. I agree to participate in this activity, realising I may withdraw at any time. I agree that the research data gathered for this study may be published provided my name is not used.

Participant's signature: _______________________

Parent/guardian signature: ___________________

Investigator: ________________ Date: __________