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## Determining the validity and reliability of an instrument designed to measure metacognitive behaviours

Anne L. Martin  
*Edith Cowan University*

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**DETERMINING THE VALIDITY AND RELIABILITY OF AN  
INSTRUMENT DESIGNED TO MEASURE METACOGNITIVE  
BEHAVIOURS**

**ANNE L. MARTIN, B. Ed.**

**A thesis submitted for the degree of Bachelor of Education with Honours  
at  
Edith Cowan University,  
Churchlands, W.A.**

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**Signature.**

**Date...12/3/94.....**

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# Abstract

This project was designed to study the role of metacognition in mathematical problem solving. More specifically, it was designed to determine the validity and reliability of an instrument proposed to identify metacognitive behaviours in Year 7 children solving problems.

The instrument was used to analyse audio tapes of pairs of students working on a non-routine problem (i.e., a problem that cannot be solved solely by the direct application of the basic operations). Analysis of the audio tapes involved categorizing metacognitive decisions as: *orientation*, *organization*, *execution*, and *verification* behaviours. A "cognitive-metacognitive" framework (Garofalo & Lester, 1985) was used as a basis for developing the instrument. The reliability of the instrument was determined by analysis of data gathered during its use by a group of experienced mathematics educators, rating the interactions of two Year 7 children using the problem *Taxi*, from the Microsmile package *The Next 17*. The instrument was found to have retest reliability of 0.57.



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**I dedicate this work to the memory of my sister, Patsy.**

# Chapter 1: Introduction

In the continuing search for ways to improve children's ability to learn and understand mathematics, much has been written about the role of problem-solving. Indeed, Schoenfeld (1992) calls problem solving the "theme of the 1980s" (p.334). The *Agenda for Action* (NCTM, 1980) began the decade with a clear call for problem solving to be extended a higher profile in the mathematics curriculum - a view expressed in Britain through the *Cockcroft Report* (DES, 1982); and it concluded with the appearance of *Everybody Counts* (NRC, 1989) and the *Curriculum and Evaluation Standards for School Mathematics*, (NCTM, 1989), both of which laid stress on problem solving.

One focus of the emphasis on problem solving has been on instructing children in a variety of strategies, with which to solve a given problem. However, there has been a general lack of success in simply teaching problem solving skills. (NRC, 1990; Silver, 1985; Campione, Brown & Connell, 1989; Resnick, 1989; Schoenfeld, 1992, 1985a; Shuard, 1986). Students who know mathematical content and problem solving strategies often fail to solve problems because they use their knowledge unwisely. The ability to solve problems requires learners to make managerial decisions as well as using problem solving skills (Schoenfeld, 1983a, 1992; Lester, Garofalo & Kroll, 1989). Hence, teachers' goals must be to "develop in students the ability to apply the subject matter they have studied with flexibility and resourcefulness," (Schoenfeld, 1992, p. 345), and to modify their

"beliefs and behaviour, not simply to record and store what they are told." (NRC, 1989, p. 59).

Research in the area of mathematical problem solving and managerial behaviours has been conducted.

For example, Schoenfeld (1983b) distinguished between two types of problem solving behaviours: tactical and managerial.

By the former, he meant "things to implement," (p. 20) such as algorithms and heuristics, by "managerial decisions" he meant:

Selecting perspectives and frameworks for a problem; deciding at branch points which direction a solution should take; deciding whether, in the light of new information, a path already taken should be abandoned; deciding what (if anything) should be salvaged from attempts that are abandoned or paths that are not taken; monitoring tactical implementation against a template of expectations for signs that intervention might be appropriate; and much, much more. (p. 20)

He believed the importance of tactical behaviours in these students' schooling had been emphasized, but that managerial behaviours were equally significant determinants of success in problem solving.

Garofalo and Lester (1985) indicated that many students have difficulty in mathematics because they lack these managerial skills.

It is particularly disturbing that they (students) are so deficient in the regulatory skills of monitoring and assessing. These skills are important in all mathematical performance, but especially so in problem solving. Problem solving is a complex activity involving a variety of cognitive operations, each of which needs to be managed and all of which need to be coordinated. (p. 169)

In another example, Garofalo and Lester (1985) found that primary students do not routinely analyze problem information, monitor progress or evaluate results when solving simple word

problems. Similarly, Schoenfeld (1983b) found that tertiary students lacked similar skills, when working on geometric proofs.

The significance of these findings is discussed later, in the chapter which focuses on literature relating to problem solving. Suffice it to say, at this point, that Garofalo and Lesier (1985) identified four stages during problem solving, within which to classify observable decision making behaviours.

The first of these observable decisions occurs at the *orientation* stage of any problem, and encompasses strategic behaviour to assess and understand the problem. Garofalo and Lester (1985) mention several instances where learners make judgements, governed by their own knowledge; for instance, they analyse the information in terms of their awareness of strategies which might assist them to solve the problem; they assess their familiarity with the task and assess the level of difficulty and chances of success.

Similarly, in formulating a plan to solve the problem, these authors suggest, under the heading of *organisation*, that several judgements are made. Final aspects of problem solving encompass *execution* and *evaluation* of the result achieved. At each of these four decision paths: *orientation*, *planning*, *execution* and *verification*, multiple metacognitive strategies are employed.

Some problems to be overcome in research on metacognition and in implementing its result in practice, are those of defining

*metacognition* and of finding valid and reliable ways of determining whether people are being metacognitive. (White, 1988). In order to distinguish between the terms cognition and metacognition, Garofalo and Lester (1985) point out:

A way of viewing the relationship between them is that cognition is involved in doing, whereas metacognition is involved in choosing and planning what to do and monitoring what is being done. (p. 164)

Flavell (1976) defined the construct of metacognition as follows:

I am engaging in metacognition . . . if I notice that I am having more trouble learning A than B; if it strikes me that I should double-check C before accepting it as a fact . . . metacognition refers, among other things, to the active monitoring and consequent regulation and organization of these processes to the cognitive objects on which they bear. (p. 232)

It is this regulation, or the "active monitoring and consequent regulation and organization of processes" (Flavell, p. 232) which provided the focus for this research..

Garofalo and Lester (1985) also identified two aspects of metacognition. ". . . metacognition has two separate but related aspects: (a) knowledge and beliefs about cognitive phenomena, and (b) the regulation and control of cognitive actions" (p. 163). Examples of behaviours that indicate these differences are shown in Figure 1.

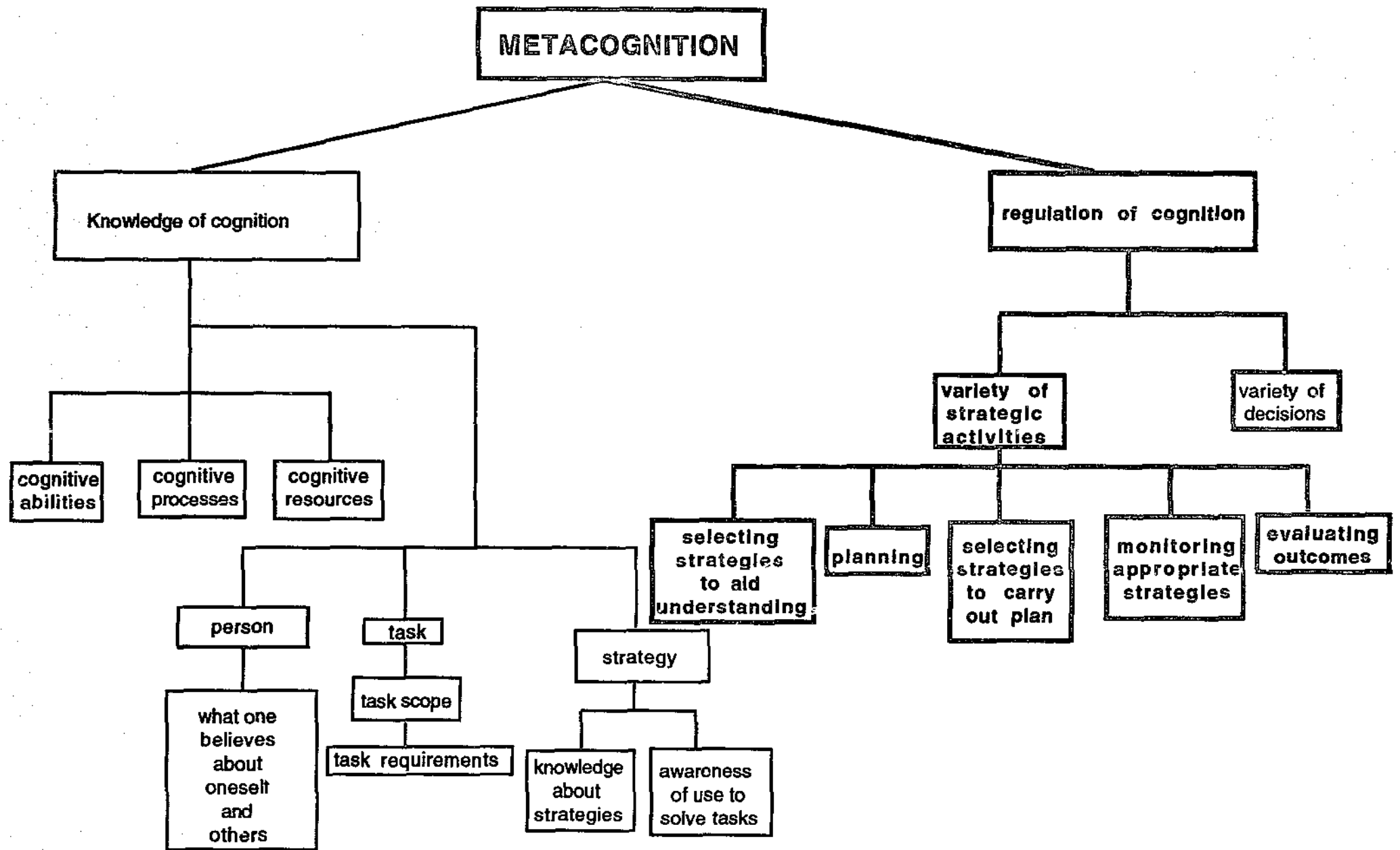


Figure 1: **Metacognition**, as described by Garofalo and Lester (1985). The bold type indicates area of research.

Within this aspect of metacognition the research concentrated on the development of a reliable instrument for measuring metacognitive strategies during problem solving.

A need clearly arises for valid and reliable instruments to measure and assess the aspects of managerial decision-making or metacognitive control, both from the viewpoint of the teacher and the researcher.

More research needs to be undertaken that explores these schemes and techniques for their utility and validity as assessment tools. . . In general, just as instruction and testing should be more closely integrated, so also should research and development efforts be merged and directed toward the creation and evaluation of innovative assessment tools that are appropriate for the important instructional goals associated with mathematical problem solving. (Silver & Kilpatrick, 1989, p. 181)

The aim of this project is to determine the reliability of an instrument using the framework suggested by Garofalo and Lester (1985), that categorizes children's metacognitive decisions during problem solving.

The research undertaken consisted of three parts: selection and pilot testing of a suitable problem-solving activity; audio taping and transcribing the dialogue of several pairs of children solving the problem, and determining the reliability of a questionnaire designed to categorize metacognitive strategies during problem solving.

The following chapter describes in more detail the current literature on aspects of the research.



# **Chapter 2: Metacognitive processes in problem solving**

## **2.1 Introduction**

In this chapter, the literature concerned with problem solving, in particular mathematical modelling, and simulations, is examined, followed by an analysis of aspects of metacognition.

Subsequently, comment is made upon literature which relates to the role of computer learning, as this is the context within which mathematical problem solving occurs in this project.

## **2.2 Problem solving in mathematics**

*A National Statement on Mathematics for Australian Schools* (AEC, 1991) emphasized the importance of problem solving in today's curriculum. Similarly, *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) stressed the necessity for the development of problem solving skills in classrooms, arguing that it is the transferable nature of those skills which is vital in today's changing society. The processes employed in problem solving - classifying, generalizing, predicting, and so on, are known to be employed in successful mathematical activities (Kantowski, 1977). However, other processes are seen to be necessary: "Pupils need to think clearly, reflect on what an activity entails and consider what strategies are possible" (DES, 1989, p.4).

On the processes of mathematical thinking in the primary years, Shuard (1986) emphasized the need for teaching problem solving strategies, that is, the procedures which "problem solvers employ in addition to the mathematical knowledge they use" (p. 88). She also referred to the key importance detailed by Schoenfeld (1983b, 1985a,) of metacognition - knowing about and thinking about one's own thinking - in successful problem solving.

As a result of his investigations, Schoenfeld (1983b) maintained "purely cognitive" (p. 329) behaviour such as problem solving is complex. It is influenced by: a) the task in hand, b) the social environment and c) the problem solver's perceptions of self, and relationship to a) and b). He stressed that all learning must be analysed with these factors in mind and that there are three levels of behaviour to consider: resources (of the pupil) that is, the skills one brings to bear on the problem, control (by the pupil) and belief systems (of the pupil).

Beliefs, as well as managerial behaviours are seen as important determinants of problem solving success. For example, Garofalo and Lester (1985), describe strategy awareness that students may possess:

For instance, mathematical strategy knowledge naturally includes knowledge of algorithms and heuristics, but it also includes a person's awareness of strategies to aid in comprehending problem statements, organizing information of data, planning solution attempts, executing plans and checking results. (p. 168)

In the following section, attention is given to defining metacognition.

### 2.3 Defining metacognition

"Problem solving and metacognition . . . are perhaps the two most overworked and least understood buzzwords of the 1980s," states Schoenfeld (1992, p. 336).

Much discussion of metacognitive issues has a base in psychological literature, focusing on memory. Campione, Brown and Connell (1989) traced the development of the term *metacognition* to a paper by Tulving and Madigan (1969), which called for more research on memory.

From this beginning, Brown (1978) and Flavell (1976) began to investigate memory, and the awareness and control of cognition. Brown believed executive skills (e.g., monitoring, coordinating) were basic characteristics of thinking efficiently in a wide range of learning situations. Specifically, she described metacognitive behaviours as those of predicting, monitoring, checking, reality testing and co-ordination and control of deliberate attempts to solve problems.

Writing in collaboration with others, more recently, Brown (Campione, Brown & Connell, 1989) went on to note:

There are several aspects to the study of metacognition. One concerns students' conscious and stateable knowledge about cognition, about themselves as learners, about the resources they have available to them, and about the structure of knowledge in the domains in which they work. Another centres on self-regulation, students' monitoring and orchestration of their own cognitive skills. A further emphasis that cuts across the above is the ability to reflect upon both their knowledge and their management processes. . . . (these) taken together combine to paint a rich picture of how well students can learn independently in a domain. (p. 94)

The nexus between these two major aspects of metacognition is at the heart of difficulties with complete understanding of the term - students' beliefs about their own knowledge impinge upon their metacognitive strategies. (Schoenfeld, 1983b; Lester et al., 1989, Herrington, 1992). In this research, regulation is the key factor - the self-regulation involved in decisions made "concerning when, why and how one should explore a problem, plan a course of action, monitor one's actions, and evaluate one's progress, plans, actions, and results" (Lester et al., 1989, p. 1).

#### **2.4 Metacognition and mathematical problem solving.**

In categorizing approaches to teaching problem solving, Garofalo and Lester (1985), differentiated between three teaching methods which teachers employ to teach problem solving in classroom situations. The first is rote transmission of problem solving strategies, the second implies supporting instruction in strategies with a rationale for learning these techniques, and the third a conscious effort by the teacher to raise the level of awareness of students to their own use of the strategies: that is, to develop the ability to monitor and control their mental actions.

They emphasised that students must be assisted to monitor tasks they are performing, at the metacognitive level, and suggested that metacognitive beliefs, decisions and actions are "important, but frequently overlooked, determinants of success or failure in problem solving" (p. 163).

A case study supporting this conjecture, was carried out by Schoenfeld (1983b). The author selected two college students (mathematics' majors) and presented them with a non-routine problem. After a quick analysis of the problem the two decided upon a path of action, to find the solution. Their reasoning was at fault, but they spent the allotted twenty minutes pursuing the path they had chosen, without questioning if it was the most effective. They were not able to consciously question their plan of attack - the focal concept of the study.

Other approaches for developing metacognitive decision-making have been suggested. Schoenfeld (1992) recommends the following prompts as children are working on problems:

What (exactly) are you doing? (Can you describe it precisely?)

Why are you doing it? (How does it fit into the solution?)

How does it help you? (What will you do with the outcome when you obtain it?) (Schoenfeld, 1992, p. 356 )

These questions are believed to encourage the child to be aware of what they were doing. Another approach which has been applied to investigate students' abilities to modify beliefs and behaviours, as a self-regulatory tool to achieving success in problem solving, involved the students writing an account of their difficulties and successes during a period of instruction, (Bell & Bell, 1985) in an attempt to teach them metacognitive awareness.

In a post-instructional test, students who were in the group which wrote about their progress, were found to be more successful than those in a group who were only given instruction in problem solving skills, indicating the value of writing as a tool for the development of skills.

Two earlier studies which focused on the theory of this project are those of Schoenfeld (1983b) and Charles and Lester (1984), the latter a report on the Mathematical Problem Solving program (MPS).

The former involved pairs of students attempting to interpret a problem, their exchanges being taped and the dialogue analysed - the method used in the current work.

In the article describing this project, Schoenfeld (1983b) discussed tactical and strategic decision-making. It was in this work he observed that managerial skills are often overlooked, but essential, components of successful problem solving.

He analysed "protocols" of pairs of tertiary students working on a geometric problem, and developed a framework with which to analyse behaviours. Fundamentally, the framework was based upon the maxim that any problem contains "episodes" characterised as one of the following: reading, analysis, planning, implementation, exploration, verification or transition. For each episode, carefully thought-out managerial decisions are crucial to success. These "episodes" were coded and used to analyse the taped data. The framework was offered for analyzing problem-solving performance in control decision-making. At the same

time, Schoenfeld warned that great care must be taken in interpreting "verbal data" because of the influence of factors "beyond the purely cognitive" - beliefs held by the students (p. 350).

Charles and Lester (1984), conducted an evaluation of a process-oriented instructional program using the assessment instrument based on the one used in this study. With regard to assessment instruments the following comments were made: "Much work is needed on the development of valid and reliable problem-solving instruments both for research purposes and as informal classroom assessment aids" (p. 32).

Charles and Lester's experiment was of particular interest as it involved primary school children: Year 5s and Year 7s. It indicated that previous studies in the area of problem solving had little success, other than those that incorporated aspects of metacognition in the teaching.

Lester et al. (1989), reported on a study of two grade seven classes, one a regular class and one an advanced class, that looked into the role of metacognition in mathematical problem solving. One aspect of the report was to assess Year Sevens' metacognitive beliefs and processes and investigate how they affect problem-solving behaviours; the other aim was to explore the extent to which students could be taught to be more strategic and aware of their own problem-solving behaviours.

The instruction was presented over a period of twelve weeks for three days per week. Findings indicated instruction was most likely to be effective when it occurred over a prolonged period of time and within the context of regular day-to-day mathematics instruction. The framework shown in Table 1 was used as an assessment instrument. It was used for a number of tasks: to select research tasks, for the design of interview procedures, for the development of the instructional treatment and for organizing analyses and interpreting findings. Its validity or reliability were not at issue.

Among the four categories of the cognitive-metacognitive framework, which included *orientation*, *organization*, *execution* and *verification*, the *orientation* category was found to have the most important effect on students' problem-solving abilities. They identified *superficial* and *meaningful* attempts at orientation by the students, the former involving reading the problem and showing no understanding of the task, the latter involving the student endeavouring to understand the complete task before attempting to solve it.

The results of this study led to Lester et al. (1989), to conclude that "metacognitive decisions associated with problem solving can be identified as contributing to students' success or non-success" (p. 5). They also found that control processes and awareness of cognitive processes develop along with an understanding of mathematical concepts. Hence, to improve children's problem solving abilities, instruction must be on a regular basis and over a period of time. Metacognitive instruction, it was argued, is



most effective when provided in an organized manner under the direction of the teacher. They acknowledged, however, that it is difficult for the teacher to maintain the roles of monitor, facilitator, and model in the face of classroom reality, especially when the students are having trouble with basic subject matter (p. 90).

Students were evaluated by means of a self reporting questionnaire. On a pre- and post-instructional assessment students did not show any increase in metacognitive awareness. One important factor which the researchers attributed this to, was that the students' beliefs about their own ability were significant constraints upon regulation of behaviours.

In summary, the research shows that metacognitive decision-making behaviours are identifiable in students solving problems and that a measure of problem solving success can be achieved when instruction accounts for metacognitive strategies. The next section looks at an attempt at measuring metacognition.

## **2.5 Instruments to measure metacognition**

As a result of these previous theorists' positions, Garofalo and Lester (1985) concluded that researchers need more understanding of the effect of regulatory behaviour upon mathematical understanding and its assessment:

Before we can design effective instructional treatments, we must first begin a thorough and systematic research effort to understand the effect of metacognitive beliefs on mathematical activity and to learn more about the nature and development of regulatory behaviour. We should also begin to investigate the effect of

metacognitive beliefs and behaviour on attending to and processing the mathematical information presented in lectures and text. . . These are no simple tasks. . . new research methods need to be developed. (p. 174)

The assessment instrument being used for the current project was an outcome of the model of metacognition and mathematical performance suggest by Garofalo and Lester, (1985).

In this paper, the authors broached several issues, including a definition of metacognition and an attempt at dispersing confusion over the difference between *cognition* and *metacognition*. They then looked at metacognition and mathematical performance, and suggested a cognitive-metacognitive framework for studying mathematical performance.

The report detailed the evolution of the framework from the work of Polya (1957), two articles by Schoenfeld (1983a; 1983b) and of Sternberg (1980; 1982). Collectively, these authors focused on different elements of learning.

In *How To Solve It*, (Polya,1957) Polya developed the idea of an "heuristic", a trial and error approach to finding the solutions to problems.

He believed that for any problem, not necessarily mathematical, one could study the data given, formulate a plan to solve it, test out that plan and check the results to assess if they were reasonable and/or correct; if not, another plan was formulated, and the process repeated.

It should be pointed out that the four categories of the framework devised by Garofalo and Lester: *orientation, organization, execution* and *verification* are "related to, but are more broadly defined than, Polya's four phases" (Lester et al., 1989, p. 10).

We have already seen in Chapter 1 that Schoenfeld (1983b), discussed tactical and managerial decision-making, suggesting that the latter was undervalued and offering protocols to examine decisions made at crucial points in the solution of problems. Elements included reading, analysis, exploration, planning, implementation and evaluation; where these elements conjoined, metacognition occurred. Sternberg (1980) divided the elements of learning into five components: performance components, acquisition components, retention components, transfer components and metacognitive components. The latter, he maintained, are "higher order control processes for decision-making and executive planning" (Garofalo & Lester, 1985, p. 170).

A synthesis of the previous theorists' positions resulted in Garofalo and Lester suggesting the following model for assessing metacognitive behaviours (Table 1).

Table 1

## A Metacognitive Framework for Studying Mathematical Performance

(Garofalo &amp; Lester, 1985)

<b>1. Orientation:</b> Strategic behaviour to assess and understand a problem
1-a. Comprehension strategies
1-b. Analysis of information and conditions
1-c. Assessment of familiarity with task
1-d. Initial and subsequent representation
1-e. Assessment of level of difficulty and chances of success
<b>2. Organization:</b> Planning of behaviour and choice of actions.
2-a. Identification of goals and subgoals
2-b. Global planning
2-c. Local planning (to implement global plans)
<b>3. Execution:</b> Regulation of behaviour to conform to plans
3-a. Performance of local actions
3-b. Monitoring of progress of local and global plans
3-c. Trade-off decisions (e.g., speed vs. accuracy, degree of elegance)
<b>4. Verification:</b> Evaluation of decisions made and of outcomes of executed plans
4-a. Evaluation of <i>orientation</i> and <i>organization</i>
4-a-1. Adequacy of representation
4-a-2. Adequacy of organizational decisions
4-a-3. Consistency of local plans with global plans
4-a-4. Consistency of global plans with goals
B. Evaluation of <i>execution</i>
4-b-1. Adequacy of performance of actions
4-b-2. Consistency of actions with plans
4-b-3. Consistency of local results with plans and problem conditions
4-b-4. Consistency of final results with problem conditions

Garofalo and Lester (1985) defined the framework as specifying key points where metacognitive decisions are likely to influence cognitive actions. They listed categories involved in mathematical problem solving tasks as *orientation*, *organization*, *execution* and *verification* strategies. The framework emphasises the fact that there are different metacognitive behaviours associated with each category.

It was expected that strategic decisions and behaviours (or their absence) associated with each of the four categories of the framework could be identified as contributing to students' success or non-success on a mathematical task. Lester et al. (1989) found that the *orientation* category stands out as being the most important, since much of what follows (or doesn't follow) in the *organization*, *execution* and *verification* categories was connected to, or dependent upon, a student's understanding of the scope of the task.

In the following section, some idea is given of the observable behaviours which were identified in relation to the categories suggested by the framework.

## **2.6 Examples of metacognitive behaviours**

In trials carried out to assess the suitability of the problem *Taxi*, for the present research, the following interactions between pairs of children give an example of the types of behaviours believed to conform to those listed in the framework. The problem involved a computer simulation of two taxi drivers - the two children - competing for fares to pick up and transport passengers to and from random destinations. The object was to be the most efficient driver, in terms of cost.

The following dialogues are between two students solving the problem. Statements that are marked with an asterisk indicate strategic decisions being made within the categories given in the

framework. It should be noted that students' metacognitive behaviours are assumed from their verbal discourse. (For full trial transcripts, see Appendix 3).

In the following transcript, we see evidence of *orientation* - in terms of the assessment of chances of success:

Anna and Jill:

- A: I own the Yellow Cabs. Jill, you own the Blue Cabs. A call is coming in. From the school to go to the pub.  
J: Yeah.  
A: I bid, you gotta look away, you can't look at my bid. (bids)  
J: \* The lowest bid, does it have to be?  
A: Yeah. You bid yours now. (Silence) The numbers don't come up on the screen.  
J: The lowest bid?  
A: Yeah. You don't know what my bid is.

In the following transcript, we see evidence of *verification* - in terms of the assessment of ongoing achievement.

Anna and Jill:

- A: Um. . .from the docks to go to the stadium. (bids)  
J: What did you bid?  
A: Fifteen. What did you bid?  
J: Fifteen  
A: (Reads) Equal bids-the cost was nine; yes, I made six dollars.  
J: Did I made anything?  
A: No, you're still loss. If you make money, it comes off your loss. (Reads) From the hotel to go to the bus station. That is going to be a big one. Okay, (bids)  
J: Where do we gotta go?  
A: Just remember we've got to go from here-dur, dur, dur, (pointing, to show Jill the track. She hasn't shared any strategy with her as yet.)  
J: \* That'll be quite a big one. (bids)  
A: Yeah, I made 2 dollars.

And again:

- D: \* So, where is it? From there to there. Um. . . (spots the score card) Is that what you've lost?  
A: Yeah.  
D: Is that minus?  
A: \$18  
D: I haven't won anything  
A: You haven't lost anything either.

Other indications of behaviours listed in the framework being exhibited are found in the following interactions. In the succeeding statement, we see evidence of *orientation* - in terms of the assessment of chances of success.

Jaymie and Peter:

- P (reading from the computer score screen) Peter Cabs bid \$15. The cost was \$17. A loss of \$2. What did you bid Jaymie?  
J: \* \$20. From the stadium to the docks. Peter, you're here, so you have to pick them up there and go down to there.  
P: bids  
J: bids.  
P: Peter cabs gets the ride I bid \$20 and the cost was \$9 - a profit of \$11.  
J: only \$1  
P: But on this one it was \$11.  
J: We both put \$20.  
J: I did.

Again, with a different pair of students, *orientational* behaviours are apparent:

David and Anna:

- A: \* (Anna) So we have to go right around there (ignoring the present position of her car, points to mark a line from original position near Airport, although her car is now parked at the Pub). What is your bid, David?

Later, David behaved in much the same way:

D: \* I put \$8. Oh, (watching taxi move) is it (there) and back? Oh.

In the ensuing statement, we see evidence of *verification* - in terms of the assessment of ongoing achievement.

Jaymie and Peter:

- J: (Reads from computer screen). A call from the Nursery to go to the Cinema. (bids)  
P: bids. . . Who got it?  
P: I knew she would get it.  
T: Why did you know, Peter?  
P: Um  
J: \* Because he was up here and he had to come down this far.  
(Reads) Jaymie Cabs bid \$17. The cost was \$8. A profit of \$9.  
I'm on \$18 and Peter's on (-)\$11.  
From the Bus Station to the Shops.  
P: (Bids)  
J: (bids) (Reads) Jaymie Cabs gets the ride. \$9, the cost was \$5, a profit of \$4. \$22.

Close examination of the dialogue revealed behaviours indicating *verification* were exhibited frequently. This was put down to be the nature of the problem, *Taxi*, during which continual re-assessment was vital, in order to plan, and execute calculations under changing situations.

In the first example, when Peter and Jaymie worked through the problem, Peter was seen to be *organizing* his actions in the sense of planning his behaviour and choice of actions.:

P \*A call is coming in. From the hotel to the pub. That's a long way!  
(Types a bid.)



In the next interaction, Jaymie is *verifying* the decisions made and the outcomes of *executed* plans - in this case, Peter's - which are going horribly wrong, and Jaymie is going to take advantage of those mistakes:

J: (Types a bid, pause, reads) Peter Cabs bid \$20 the cost was \$24. That was a loss of \$4. Oh, man!

Further into the program, the children's *execution* and *performance of local actions* is commented upon:

J: (bids). . I put in \$28, and it's only saying 18!

In this statement, Jaymie is questioning the computer in response to her performance. It is to be assumed that she accidentally pressed 18, rather than 28. However, she did not interpret the data in this way. She let it lapse. By commenting, albeit indignantly, we may assume she modified her behaviour accordingly.

Finally, well into the program, we find an example which shows that the behaviours are not sequential. In the following statement, we see evidence of *orientation* - in terms of the assessment of chances of success:

J: A call is coming in from the Railway Station to the Bus station. I know who's going to win this bid (looking at position of cars).

These examples illustrate the decision-making processes believed to be evident in children's dialogue, as categorized in Table 1.

## **2.7 The role of computers**

This program, *Taxi*, and its use in this research was set in the context of children solving modelling problems on a computer.

The use of a computer is suggested by the literature (NRC, 1990; Herrington, 1988; Bell, Costello & Kuchemann, 1983) as a useful vehicle for developing processes or problem solving skills:

Developing mathematical processes is a long-term proposition (Bell, Costello & Kuchemann, 1983). To enable the novice learner to achieve the abilities of the expert, educators need to employ a range of suitable mathematical activities. These activities can include non-routine problem-solving, modelling real-world problems, investigations and strategy games. Each of these activities can be assisted with the use of a computer and appropriate software. (Herrington, 1988 p.7)

Bradbeer, DeBono and Laurie (1982) also discussed the use of the computer in developing metacognitive strategies:

Simulation or modelling programs deal with real-world events which can be mimicked on the screen. It is not necessary for the user physically to encounter the actual problem because data collected for the real world is often entered into the programs; the main aim is to develop decision-making skills as well as understanding. (p. 153)

Undeniably, the changing role of computers in schools is having a "compelling and inevitable impact" on mathematics (NRC, 1990, p.xi). They affect not only what mathematics is important, but also how mathematics is done (Rheinboldt, 1985). In this context of problem solving using a computer the framework for analysing metacognitive strategies was investigated.

The types of problems used in the study can be described as modelling problems. As defined by Burkhardt (1981), modelling is the representation of reality. The problem chosen was a modelling problem. It is a method of tackling practical problems from everyday life and is seen as requiring the use of a wider range of skills than is traditionally taught in schools.

The emphasis is upon the application of those skills to solve problems which are of real interest to children - problems relating to sport, money, music, clothes, school work and leisure. Use of the instrument was in the context of modelling problems as defined by Burkhardt.

## **2.8 The research question**

As a result of the previous review of the literature a study was designed to answer the following research question:

Does the framework suggested by Garofalo and Lester (1985) form a useful basis for developing a reliable instrument for measuring metacognitive strategies displayed by children solving modelling problems on a computer?

The following chapter describes the methods used to investigate this question.

## **2.9 Conclusion**

To summarize the results of the literature, it is clear that the study of metacognition is complex, but that successful projects will contribute to our understanding of how children learn. The importance of the idea of metacognitive control has been analyzed. Research indicates that students employ metacognitive decisions in problem solving tasks. The computer is a valid means of offering children the opportunity to simulate real-life problems, it is stimulating, and provides a small-group situation where the non-participant observer may carry out valid and useful analysis of children's metacognitive functions. The need for a reliable instrument to assess metacognitive strategies has been shown to be vital.

# CHAPTER 3: Method

## 3.1 Introduction

This section of the study consisted of three parts. Firstly, selection and pilot testing of a suitable problem. This was followed by audio taping and transcribing the dialogue of several pairs of children solving the problem. The final stage involved the development of a reliable questionnaire, to assess metacognitive strategies used by children as they solved the problems.

These three parts were completed in a school setting described below.

## 3.2 Description of the school and the students

The current mathematics syllabus *Learning Mathematics* (1989), focuses on developing problem solving skills. A group of Year 7 children were given specific instruction in problem solving skills for two terms with two weeks per strategy, in an effort to offer them a range of heuristics with which to solve non-routine problems.

On the premise that explicit teaching of cognitive and metacognitive strategies can enhance students' learning (Schoenfeld, 1987; Lester et al., 1989), as can small group cooperative learning (NRC, 1990; Shavelson, Webb, Stasz, & McArthur, 1988; Peterson & Carpenter,

1989), these lessons were conducted on a regular basis, once a week for forty minutes, with two teachers moving amongst small groups to offer suggestions, encouragement or answer questions. Children worked in groups of four to six, because, as Resnick (1987) points out: "something about performing in social settings seems to be crucial to acquiring problem solving habits and skills" (p. 40). Furthermore,

It seems possible that engaging in problem solving with others may teach students that they have the ability, the permission, and even the obligation to engage in a kind of independent interpretation that does not automatically accept problem formulations as presented. (p. 40)

This notion of socialization, also called enculturation by Schoenfeld, (1989; 1992) has been acknowledged as an important factor in learning.

Roughly, the idea is that by sitting on the fringe of a community, one gets a sense of the enterprise; as one interacts with members of the community and becomes more deeply embedded in it, one learns its language and picks up its perspectives as well. (p. 365)

The eventual outcome of the formal lessons was to be the opportunity to "choose your own strategy" from a combination of those experienced, in an attempt to solve a real-life problem, as exemplified in Burkhardt (1981). The problem was to be selected from software developed for the express purpose of using those skills, that is, *Micro-Smile: The Next 17, Taxi* (ILEA).

At the commencement of the series of lessons a period was taken to set up a system of recording. This introductory period involved discussion by the children and teacher to develop a systematic approach to problem solving. They recorded all activities in a prescribed format, as shown in Figure 2.

The class was structured with an introductory class activity focusing on the process which had been selected by class teacher and specialist. Discussion was encouraged and a range of problems provided with any necessary equipment already selected. Children worked in groups of four, reading the problem card within their group. The groups then rotated, reading their problem to the class and teachers. They were asked to restate the problem in their own words, and to suggest a strategy for solving that problem from the list displayed in the room.

In the initial stages, the children were motivated to produce a set of solutions to the school's problem solving boxes, thereby providing a service to their school. At the same time it was pointed out to them, that the skills they would develop would be valued the following year when they move to the adjoining High School, where the Unit Curriculum is in place, and where problem-solving skills are equally valued.

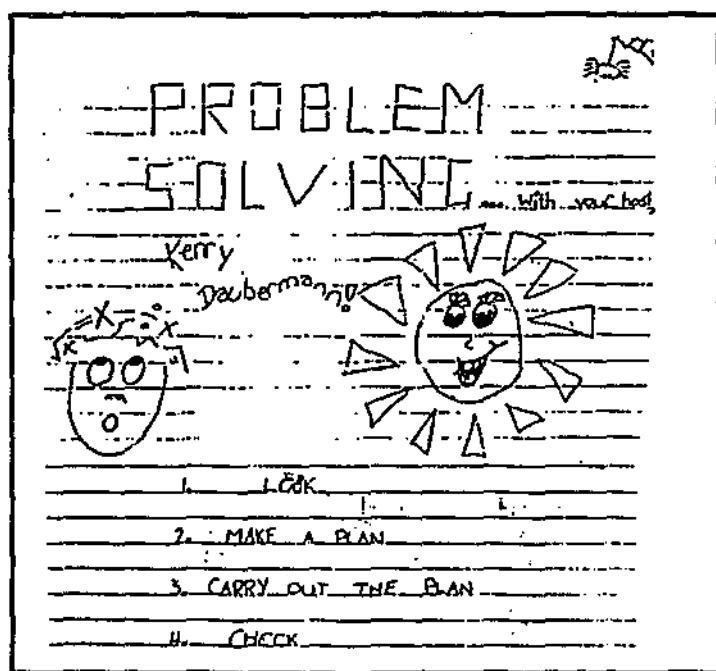


Figure 2: Child's problem solving planning page.

The first lesson included carrying out a problem as a group: the well known *Crossing the River*, which involves two men and two boys crossing a river in a small canoe which will carry only two children or one man at a time. Discussion following this activity revolved around the existence of a strategy which can be formalised, to solve this particular problem, and other strategies which may be used to solve other non-routine problems.

At this stage the attention of the class was drawn to the following list, derived from *Learning Mathematics* (1989):


Guess and check	Work backwards
Search for a pattern	Act it out
Solve a simpler problem first	Write an equation
Make a tally	Use logical reasoning
Make a table	Account for all possibilities.
Make an organized list.	(p. 7).

The list was displayed in the classroom, and after each lesson a child was called upon to note the strategies (always more than one) which had been useful to members of the class. Children then worked individually to write a short comment on the problem their group had solved, with an accompanying diagram, table or other form of model. They were asked to consider if their plan had been the most appropriate, and if not, to discuss their actions with their partner or group, to raise their consciousness of their actions.



Act it Out, continued →

We had to find how many ways boys could be so seated.



A	B	C	D
A B C D	B C A D	C A D B	D A B C
A B D C	B C D A	C A B D	D A C B
A C D B	B D C A	C D B A	D B A C
A C B D	B D A C	C D A B	D B C A
A D C B	B A D C	C B D A	D C B A
A D B C	B A C D	C B A D	D C A B

there are 24 different ways.

Figure 3: Child's problem solving solution page.

### 3.3 Problem selection

This phase involved selecting a problem that would provide evidence of the use of metacognitive strategies.

Selection of a problem involved searching through several books, including non-routine problems and computer based activities that acknowledged problem solving (e.g., Burkhardt, 1981; Coburn, Kelman, Roberts, Snyder, Watt & Weiner, 1985; Dessart, & Suydam, 1985; Shuard, 1986). Year 7 children were observed during regular problem solving classes, doing non-routine multi-step problems, as described in this chapter, and during computer lessons. Several problems were considered, until a decision was made to use the Microsmile program's *Taxi*, from *The Next 17*.

The problem chosen was a non-routine multi-step problem that could be solved by pairs of children. This was done because Schoenfeld (1983b) found in his study that children talk openly in pairs, and each is influenced by the others ideas. He noted that:

While the difficulties of using "verbal methods" to make sense of human problem solving should not be underestimated, I believe that those methods can be of tremendous assistance in helping us to understand our students' mathematical thinking. (p. 185)

His viewpoint is supported throughout the literature (NRC, 1990; Shavelson, Webb, Stasz, & McArthur, 1988; Peterson & Carpenter, 1989). The problem involved a simulation of two taxi drivers competing, by making bids to pick up and transport passengers to and from random destinations. The object was to be the most efficient driver, in terms of cost.

The children were asked to bid for taxi fares between given destinations, e.g. from the Factory to the Airport. The screen showed the layout of the district, with the destinations marked clearly (see Figure 4). The taxi fare was given to the child with the lowest bid. After each child had made a bid, a simulated taxi-cab moved between the two designated points.

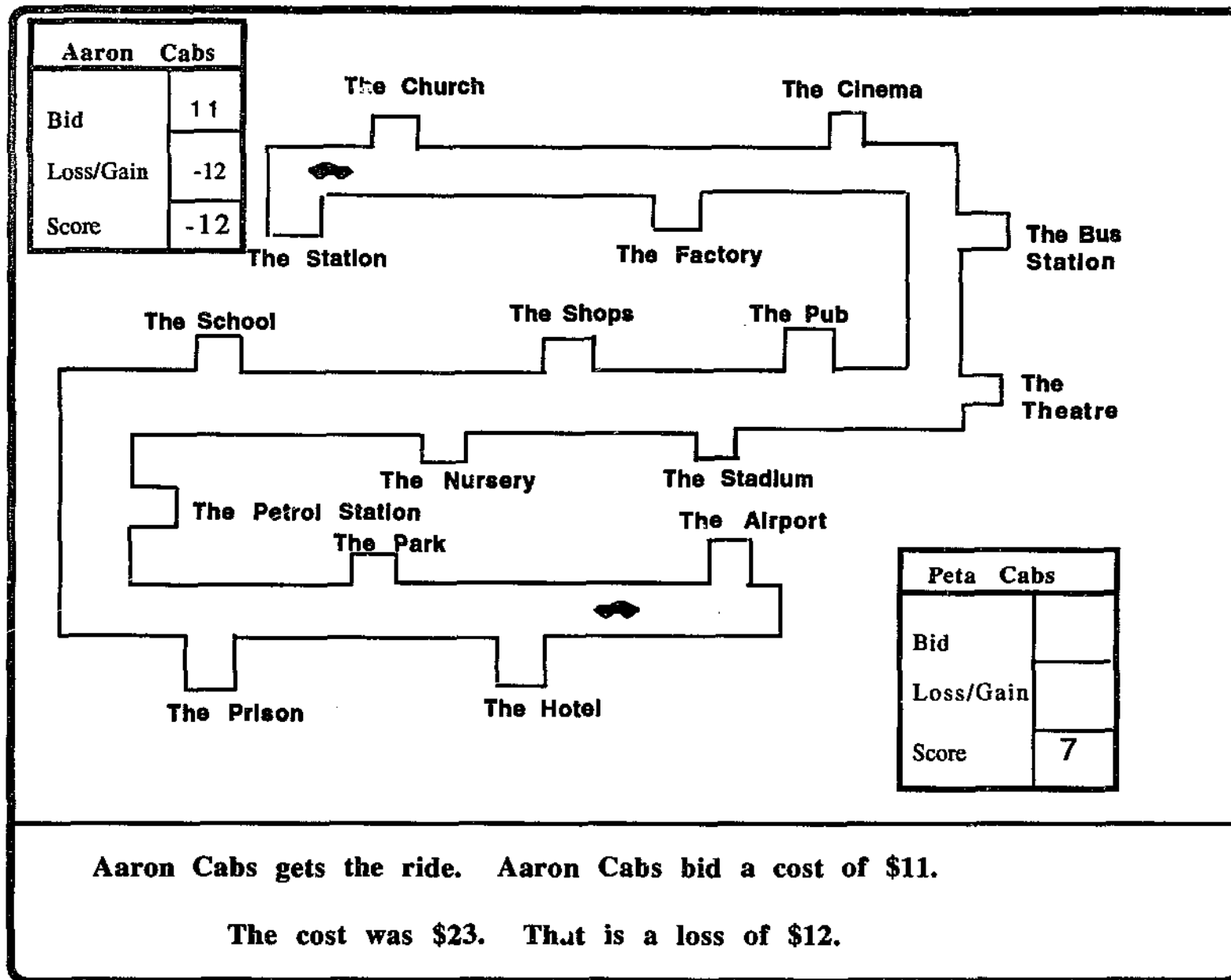


Figure 4: Copy of screen for "Taxi"

In addition, a score-board appeared in diagonally opposite corners of the screen. These gave progressive scores for each child. As the taxi moved, the scores were adjusted automatically, showing which child had bid the lowest, what the journey had cost, and the profit or loss made. A profit was made if the cost of the bid was greater than the cost incurred in making the journey, otherwise a loss was made.

The object of each player was to make a profit of twenty-five dollars (\$25). Alternatively, the game could be lost by losing twenty-five dollars (-\$25)

As already noted, children had solved problems using the following strategies in a whole class situation: guess and check, work backwards, look for a pattern, use logic, draw a picture, make an organized list, make a table, act it out, make a model, look for key words, and using resources (calculators, computers, books, etc.).

*Taxi* was primarily chosen as exhibiting possibilities for children to display decision-making behaviours during the solution, which related to those identified in the cognitive/metacognitive framework: *orientation, organization, execution and verification* decisions (Table 1).

Following the selection of *Taxi*, a trial was run as to the appropriateness of the problem for use with Year Seven children. To do this the consent of the school's principal and the

Year Seven class teacher was obtained to trial the problem. As a result of this pilot testing of the program it seemed to have promise for engaging students in a broad range of metacognitive behaviours.

### **3.4 Audio taping children solving the problem**

In order to pick up conversations between pairs of students and the instructor, a tape recorder was mounted on the computer monitor.

The standard practice followed was for the teacher to sit in on the activity, to provide assistance if necessary, and to have first hand experience of the conversation, in case of anomalies on the taped discussion. She never intervened during a lesson, but did discuss how the session had gone, and what might be done in a follow-up activity.

### **3.5 The sample**

Five pairs of year 7 children were selected, at random.

### **3.6 Development of the instrument**

The conversation of all pairs of students was transcribed, and one selected upon which to focus the study (Appendix 1). This was done by the researcher and supervisor studying the transcript carefully with the intention of producing a questionnaire that highlighted various metacognitive decision-

making behaviours that could be classified by raters for the purpose of determining validity and reliability.

A senior lecturer in mathematics was then invited to do a trial rating of the selected items, and asked for feedback into difficulties with the transcription or use of the framework with which it was to be analyzed.

By this stage a dilemma was emerging with the program, in that the nature of *Taxi* is such that most of the children's decisions were classified as *assessment* as a predominant behaviour. However, other behaviours *were* identified, and the decision was made to proceed with the study.

### **3.7 Description of the raters**

The questionnaire (Appendix 1) consisted of the transcript with statements highlighted with an asterisk. This was issued on a test/retest basis to raters. This phase commenced during the summer months of 1992 and continued on into the May holiday break. It was during this phase that the transcript was offered to twenty raters for the purpose of classifying metacognitive behaviours according to the Garofalo and Lester (1985) framework and hence providing an indication of validity. On a retest basis the transcripts were sent to the same raters after a period of several months with the intention of determining reliability.

A group of twenty people with an interest in mathematics and computing was asked for assistance, and obliged with a rating workshop. This group comprised principals, lecturers, and classroom teachers who were meeting for an out-of-term workshop in mathematics education.

Each one was provided with a copy of the questionnaire and a copy of the framework and were given an explanation of the different categories of the framework.

They were then asked to rate items already marked with an asterisk as showing evidence of one of the four behaviours: *orientation, planning, execution or verification* of outcomes. They were instructed to follow the broad categorisation of the framework (Table 1), referring to the sub-categories for clarification, if necessary. After three months the same group was asked to once again rate the items as before. The following chapter describes the results and analysis of this procedure.

### **3.8 Conclusion**

This chapter has explained the methodology used for the research. It involved the selection and testing of a suitable problem, the recording of several pairs of children solving the problem on a computer, the transcription of their dialogue, and the selection of one of those transcripts for analysis for evidence of metacognitive decision-making behaviours as elucidated by Garofalo and Lester's (1985) framework. The categorisations were *orientation, organization, execution and verification*. The

chosen dialogue was scrutinised for evidence of those behaviours. After identification of Items which were perceived to indicate any of the above categories, the transcripts were issued on a test/retest basis to a group of raters. The objective was to determine agreement about the nature of the behaviours (validity) and a measure of the reliability of the items chosen. The results and analysis of their categorisations follow in the next chapter of this study.



# **Chapter 4: Analysis and results**

## **4.1 Introduction**

The purpose of this study was to determine the validity and reliability of an instrument designed to measure metacognitive behaviours in children, specifically, Year 7 children solving a problem simulated on a computer. This chapter discusses the results and analyses of the data gathered from expert raters' categorisation of students' metacognitive decisions during problem solving.

A questionnaire was developed, based on a pair of children thinking aloud as they solved a computer simulated problem. Initially, the data from several pairs of children were recorded and transcribed and one pair's dialogue was chosen as a suitable discourse upon which to develop the questionnaire. The choice was influenced by the fact that the particular pair were willing to, and capable of, verbalising their approach to the problem. They offered more insights into what they were doing and why they had chosen to act in a particular way, than any of the other pairs.

From the chosen dialogue, a number of statements were identified by the author as statements that might be classified as exhibiting metacognitive behaviours. These were marked with an asterisk and a group of mathematics' educators were asked to

classify each statement in terms of four behaviours: *orientation*, *planning*, *execution* or *verification*.

The items chosen as representative of the four behaviours are to be found in Appendix 1. For purposes of clarity these are listed according to the researcher's classification. The numbers refer to the Line Numbers in the transcript.

**Table 2: Items belonging to each category of metacognitive behaviours, taken from the transcript of children solving the problem, *Tari*.**

Category (Code No)	Item number
Orientation (1)	1, 10, 19, 49, 106, 127.
Organisation (2)	42, 48, 51, 54, 75, 88, 106.
Execution (3)	37, 62, 67, 83.
Verification (4)	50, 59, 70, 84, 102, 105, 114, 135, 145, 150.

After a period of several months the same raters were asked to categorize these same statements as before. In the first rating, twenty raters returned the questionnaire. In the second rating, fifteen raters returned the questionnaire.

The group of mathematics "experts" who were asked to classify metacognitive behaviours were not experts in doing this activity. Many had never seen the instrument before, and may even have had no understanding of what metacognitive behaviours consisted of, prior to assisting in this project. At the same time, a good deal of time was taken in giving them a background into the research problem and the thesis proposed. The categories of the

instrument were explained, then they were given time to familiarise themselves with it, and invited to discuss any problems of interpretation.

## **4.2 Students' metacognitive behaviours during the problem solving task**

### **4.2.1 Orientation**

Lester et al. (1989) identified one approach to *orientation* which can be described as *superficial*, (p. 48) which involves no re-reading of the problem after the student has read it aloud. It was apparent that the boy in the study did not have a good understanding of the problem after reading aloud the instructions. The reading aloud was not followed by any re-reading. His explanation for not re-reading the problem was "I understood it."

The girl, however, approached the problem in what may be termed a more meaningful manner. She concentrated fully in a reflective manner during instructions, and achieved a *partial meaning* but not necessarily a complete understanding, of the problem as a whole. She tried to do some analysis of the problem conditions. For example, at line Number 1 she answered "Yes, we may as well," when asked if she would like to see the rules.

### **4.2.2 Organization.**

In terms of organisation, Lester et al. (1989) identified three levels of *organization* : *guessing*, *partially meaningful* and *fully meaningful* (p. 54). The guessing approach is exactly what its name suggests. A student taking this approach tries something

without much of a rationale. For example, after having obvious difficulty reading the problem aloud, neither student re-read it. The girl stated confidently that she did not need to re-read it; however, the boy was obviously unsure of the requirements of the problem from the beginning (Line 10). He commenced the game with a type of guess and check strategy. By the time he was at Line 75, he was saying "So, you've gotta go all the way back and I've got to go from there. I know what I'm going to do. . . So, it has to be the closest bid to it (the cost) does it?" At the same time, the girl could only be said to have been *partially organized* in her plan to attack the problem. She tried to identify some plan which was familiar, in Line 48: "This is problem solving. There's gotta be something to find this out. Look for a pattern or something."

#### 4.2.3 Execution

The *execution* category involved students assessing the progress of their approach as they worked through the problem. When students recognize a problem, their *organization* and execution of plans are aided (Lester et al., 1989, p. 57). The student who took a more meaningful approach to *orientation*, the girl, appeared to have better organizational strategies than the other, the boy. She almost always knew what was happening on the screen, and with the score. By Line 50 she says "Oh, I know how to do it now. I've worked out how to do this game". The boy seemed unable to initiate a challenge to her. He was unable to develop a plan which would see himself making a trade-off in

terms of moving for any reason other than as a short term measure.

#### **4.2.4 Verification**

Both students appeared comfortable working the mathematical calculations associated with the solution of the problem mentally. Neither asked for pen and paper to keep a running tally of the figures.

By the very nature of the problem, the children needed to make many assessments. However, neither student did sufficient *verification* in terms of planning. They did not check the reasonableness of what they did or what they were doing, or even where they were heading. However, they did undertake some strategies for evaluation, even if they were not based on a criteria of meaningfulness.

The evaluation strategy used by both students can be called *assessing by number considerations*. This strategy judged the appropriateness of a plan by the numbers which resulted from calculations made in carrying out the plan. As Lester et al. (1989) found: "This is a very reasonable strategy, because it can alert a student to an unreasonable result which might be due to the incorrect choice of a (bid) or to an incorrect calculation" (p. 61).

However, they pointed out that this evaluation strategy can mislead a student who uses it to replace an assessment based on the reasonableness of the plan.

Both students did go back to assess their understanding of the problem. The boy, in particular, made many attempts to evaluate, but always in terms of one criteria - the distance of the taxi from the pickup point, as illustrated at Line 145: "Well, if a cab is closer to the 'from' you are more likely to win. I had to go all the way from up here first." It should be pointed out that the random nature of the calls provided some justification for his assessment.

### **4.3 Validity of the categorisation of dialogue statements.**

In this section the analysis of the individual items from the questionnaire, identified as exhibiting metacognitive decisions, will be given. The analysis of the data considered the validity of each statement being classified and the overall reliability of the questionnaire. If a majority of the raters agreed on a particular classification for a statement, then this would be seen to indicate general agreement, and hence, validity.

In the first rating, twenty raters completed the questionnaire. If a majority (i.e.,  $\geq 10$ ) of the raters agreed on a category labelling when the test was given this would be seen to indicate general agreement of the nature of the item. Table 3 shows that in the majority of items this situation did not occur.

Table 3: No. of raters who placed items in each category (n=20)  
 \* = correct (researchers') categories.

ITEM	LINE	ORIENTATION	ORGANIZATION	EXECUTION	EVALUATION
1	1	*18	2	0	0
2	10	*10	9	0	1
3	19	*7	4	9	0
4	37	1	0	*8	11
5	42	5	*11	3	1
6	48	6	*7	2	5
7	49	*9	4	4	3
8	50	4	6	1	*9
9	51	4	*6	2	8
10	54	2	*11	5	2
11	59	1	1	3	*15
12	62	8	7	*3	2
13	67	0	5	*7	8
14	70	0	0	3	*17
15	75	3	*10	6	1
16	83	0	5	*9	6
17	84	2	1	3	*14
18	88	1	*9	7	3
19	102	1	1	6	*12
20	105	2	0	4	*14
21	106	*8	7	5	0
22	106	4	6	*10	0
23	114	0	2	4	*14
24	127	*12	5	0	3
25	135	1	2	3	*14
26	145	1	1	0	*18
27	150	0	0	0	*20



#### **4.3.1 Reliability of the categorisations**

Of the 27 items being classified, 15 of the items gained a majority agreement on their classification in accordance with the researcher's classification. Table 4 shows how each item was categorized on the first and second administrations. In this first rating, twenty raters returned the questionnaire, on the second administration 15 raters completed the questionnaire. If a majority (i.e.,  $\geq 10$ ) of the raters agreed on a category labelling when the test was given this would be seen to indicate general agreement of the item being reliably categorized. Table 4 shows that in the majority of items this situation did occur.

Table 4: Number of raters who placed items in the same categories (n=15)

ITEM	LINE NO	ORIENTATION		ORGANISATION		EXECUTION		EVALUATION	
		FIRST	SECOND	FIRST	SECOND	FIRST	SECOND	FIRST	SECOND
1	1	13	13						
2	10	7	10						
3	19	7	6						
4	37							7	8
5	42			9	6				
6	48			6	8				
7	49	9	4						
8	50							7	6
9	51			5	5				
10	54			6	7				
11	59							10	8
12	62			6	6				
13	67							5	5
14	70							12	10
15	75			7	6				
16	83					5	7	7	5
17	84							9	11
18	88			7	5	5	7		
19	102							9	11
20	105							13	9
21	106			7	7				
22	106					6	9		
23	114							9	10
24	127	10	10						
25	135							11	9
26	145							14	15
27	150							15	14

**In summary, for eighteen of the twenty-seven items, the majority (i.e.,  $\geq 10$ ) of raters agreed on the category in which items belonged - *orientation, organization, execution* or *evaluation*.**

## **4.4 Analysis of the reliability of the questionnaire**

### **4.4.1 Item reliability**

In the second rating, fifteen raters returned the questionnaire, so only 15 questionnaires were considered in this analysis. If a majority (i.e.,  $\geq 8$ ) of the raters agreed on the same categorisation of an item on both occasions a category labelling on both occasions when the test was given this would indicate that the items were reliably categorized. Table 4 shows that in the majority of items this situation did not occur.

Hence, only 10 items occurred where the raters (i.e.,  $\geq 8$ ) rated the item the same.

Another way to determine the reliability of the test was to consider the reliability in rating for each rater. Table 5 shows the correlation coefficient between the raters' first and second rating of the twenty-seven items. Analysis shows a medium positive correlation between the two ratings.

The correlation coefficient was calculated by tabulating each rater's response to the item number in terms of categories, numbered 1-4, for the two separate ratings.

**Table 5: Reliability of individual teachers' scoring**

<b>TEACHER</b>	<b>RELIABILITY CORRELATION COEFFICIENT (R)</b>
<b>1</b>	<b>0.87</b>
<b>2</b>	<b>0.57</b>
<b>3</b>	<b>0.49</b>
<b>4</b>	<b>0.65</b>
<b>5</b>	<b>0.57</b>
<b>6</b>	<b>0.67</b>
<b>7</b>	<b>0.63</b>
<b>8</b>	<b>0.58</b>
<b>9</b>	<b>0.55</b>
<b>10</b>	<b>0.4</b>
<b>11</b>	<b>0.33</b>
<b>12</b>	<b>0.65</b>
<b>13</b>	<b>0.57</b>
<b>14</b>	<b>0.55</b>
<b>15</b>	<b>0.42</b>
<b>AVERAGE</b>	<b>0.57</b>

# **Chapter 5: Conclusions and outcomes**

## **5.1 Introduction**

In this chapter, conclusions are drawn about the research and the limitations of the study are discussed.

The research question concerned the identification of metacognitive behaviours Year Seven students use during problem solving. In Chapter 1, metacognition was described as referring to self-regulation and awareness.

The object of the research was to use the framework provided by Garofalo and Lester, (1985) (Table 1), to determine the validity and reliability of a test for measuring four categories of metacognitive behaviours: *orientation, organization, execution* and *verification*. Of the 20 questionnaires completed a majority of raters agreed with 15/27 categorisations. This shows a reasonable indication of validity. For the fifteen questionnaires returned, analysis involved finding the average correlation coefficient for 15 raters of 27 items to be 0.57, a medium correlation. Based upon this figure and the results shown in various other analyses in Chapter 4, together with the acknowledged limitations in the following section, it is felt that the instrument has been shown to be both valid and reliable.

## **5.2 Limitations of the study**

Limitations of the study related in the first instance to methodology and also to content.

The use of a tape-recorder, and of an observer placed an unnatural barrier upon children's responses. It is difficult to interpret "out-loud" problem solving protocols (Schoenfeld, 1983b; 1985b). There is so much cognitive and metacognitive activity children are unaware of, or are unable to explain. Also, interpretations of their explanations are not necessarily correct. Judgements of this kind are, of their nature, subjective.

The problem chosen set the children up as opponents, rather than producing a collaborative atmosphere. This precluded their sharing ideas verbally, thus negating a valuable source of information. Expectations were not clearly elucidated. The children were presented with the problem with no expectations verbalised. They were aware that the problem was probably related to mathematics, or problem solving, as that was the only role for the researcher within their classroom. The children were considered to be novice problem solvers, or at least, not experts.

The problem-solving instruction might not have emphasized all that needed to be emphasized and/or it might not have emphasized aspects effectively. Since the children were still getting regular mathematics instruction between the short

periods of problem-solving instruction, the situation may have given the impression that problem solving and mathematics are somehow different from each other.

The problem chosen enabled the clear identification of metacognitive behaviours in the *orientation* and *verification* categories. However, the distinction between *organization* and *execution* categories was not nearly so well defined. This may, of course, be a property of the framework being evaluated as much as a deficiency in the problem.

There is a degree of variability in the potential for a problem to elicit behaviours associated with aspects of metacognition, as there is between individual's metacognitive behaviours, whilst solving that problem. Although some information was collected concerning the characteristics of the students, no effort was made to evaluate their beliefs and attitudes, nor the influence these may have had on their problem solving abilities.

As commented upon earlier, the group of mathematics "experts" who were asked to classify metacognitive behaviours were not experts in doing this activity. However, they were all committed to mathematics education, and had expertise in that field. Also, the researcher's classification of behaviours may not have been correct to begin with.

In addition, the sample size, both of children and of raters of the questionnaires, was minimal.



**In terms of the first twenty questionnaires it can be said that the instrument designed to measure metacognitive behaviours has a measure of validity. However, in terms of the test and retest analysis, the results concerning reliability are inconclusive.**

# **Chapter 6: Implications**

## **6.1 Introduction**

This study looked at the validity and reliability of an instrument designed to assess metacognitive behaviours. Data analyses reported here were performed on a very restricted base. The following paragraphs will review the analysis that were conducted and suggestions made for possible development of the study.

Analysis of data from pairs of students was restricted to interpretation of the transcription, with minimal teacher input. A full interview with the children could be conducted, the children being asked to go through the dialogue, or writing in their own words, stating their reasons for performing various calculations. The use of a video-recorder could be trialled to investigate its value.

Children could be asked to write a report of the interaction to allow for a different perspective in the in-depth analysis of students' strategic behaviour. Assumptions made concerning their self-regulation could then be investigated in terms of their explanations.

The original study could be repeated across several pairs of children, to search for more concrete evidence of the reliability of the framework in identifying metacognitive decision-making

behaviours. Alternatively, a single pair of children could be asked to perform several different problems, with an emphasis on different aspects of the framework.

For instance, problems might be chosen which involve a very obvious focus in the *execution* phase, or the *organization* phase, followed by the categorisation of behaviours using the framework, in an attempt to provide more systematic and thorough analysis of data.

## **6.2 Conclusion**

Metacognition is extremely complex. It is interesting to note the comment by Lester et al. (1989):

At present, what we believe about the role of metacognition and other noncognitive factors in mathematical problem solving is still based more on our reflections about our own experiences as teachers and learners of mathematics than on the results of carefully and systematically conducted research. (p.122)

The project discussed here has been an attempt to carefully and systematically add to the body of that research. The framework used represents a change from traditional methods of assessment, aimed at understanding metacognitive behaviours.

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## **Appendices**

## **Appendix 1: Problem solving dialogue.**

**Transcript**  
**Microsmile: Taxi-The Next 17**

Items to be assessed are marked with an asterisk.

**ITEM**

- |    |       |  |                          |                          |                          |                          |
|----|-------|--|--------------------------|--------------------------|--------------------------|--------------------------|
| 1. | Peta: | (reads from computer) Would you like to see the rules? | Orient                   | Org                      | Exec                     | Eval                     |
|    |       |  | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
- \* Yes, we may as well.
2. Aa: (reads) Customers will call in for rides. Each team makes a bid,
  3. Pe: . . .bid. . .
  4. Aa: . . .bid, and the lowest bid gets the ride. The journey from one building to the next will cost you one pound in fuel.
  5. T: one dollar
  6. Aa: the first team to make a profit of twenty-five dollars, wins. If a team makes twenty-five dollars loss, they lose.
  7. Both: Here are the names of the buildings: the hotel, the park, the prison, the docks, the petrol station, the school, the nursery, the shops, the stadium, the pub, the theatre, the bus station, the cinema, the factory, the church, the railway station.
  8. Tchr: (Reads) Would you like the rules again?
  9. Pe: m..m..m..nuh
  10. \* Aa: uh, uh, nuh.
- 
- |  |  |  |                          |                          |                          |                          |
|--|--|--|--------------------------|--------------------------|--------------------------|--------------------------|
|  |  |  | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
|--|--|--|--------------------------|--------------------------|--------------------------|--------------------------|
11. Pe: nuh
  12. Aa: nuh
  13. Pe: (reads) What is the name of the first team player? (Enters). (reads) Peta Cabs, you own the Yellow Taxi.  
Aaron, you own the Blue one.
  14. Aa: laughs, enters name.
  15. Pe: (reads) A call is coming in, press Space Bar to receive it. Call from the church
  16. Aa: . . .to, to. . .
  17. Pe: . . .to go to the theatre.
  18. Both: What is your bid?
  19. \* Pe: I think he knows.
- 
- |  |  |  |                          |                          |                          |                          |
|--|--|--|--------------------------|--------------------------|--------------------------|--------------------------|
|  |  |  | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
|--|--|--|--------------------------|--------------------------|--------------------------|--------------------------|
20. Tchr: Yes, you can stop him from looking if you want. Tell him. . .
  21. Pe: Turn away, Aaron. ( Enters a figure).
  22. Aa: OK, from the church to that, I bid. . . (enters a figure) You looked.
  23. Tchr: That's alright, because you are second. Press Return.
  24. Both: Peta Cabs gets the ride.

## ITEM

25. Both: giggles.

Orient Org Exec Eval

26. Aa How much did it cost you?  
 27. Pe: How much did it cost me?  
 28. Tchr: Check.  
 29. Pe (reads) Peta Cabs bid \$12 and



the cost was \$19. this is a loss of \$7.  
 Owww.

30. Aa: Oww.  
 31. T: (reads) Press Space bar to continue.  
 32. Aa: (reads) A call is coming in. Press space bar to receive.  
 33. Pe: (reads) From the Prison to go to the Cinema.  
 34. Aa: Oh, charge them lots!  
 35. What is your bid, Aaron Cabs? Oh... enters  
 36. Pe: (enters). Aaron Cabs gets the ride. Oh, I bet you...  
 37. \* Aa: Oh, oh, no way! I'm way off! I don't



believe this! (reads) Aaron Cabs bid a cost of 11.



- The cost was \$23. This is a loss of \$12.  
 38. Pe: I pressed \$34!  
 39. (reads) A call from the Stadium to go to the Airport.  
 40. Aa: What is your bid?  
 41. Pe: Giggles... Enters  
 42. Aa: OK From the Stadium, I start there, a call from the stadium to go to the airport. Where's the Stadium?  
 \* There to there to there. Mmm. ...



43. Pe: Seventy dollars? (?)  
 44. Aa: No, one. (enters)  
 45. Aaron Cabs get the ride. You bid \$9. Oh my God 7, 8, 9, (reads) Aaron Cabs gets the ride the cost was \$13. This is a loss of \$4. Press Space Bar to continue.  
 46. Pe: I'm \$16, (minus 16).  
 47. Aa: I'm \$7



48. \* Pe This is problem solving. There's



gotta be something to find this out.  
 Look for a pattern or something.

## ITEM

Orient Org Exec Eval

49. \* Aa: (points to score on screen). What does this say?

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50. \* Pe: Mine from there was \$12.  
Oh, I know how to do it now.

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51. \* Aa: I've worked out how to do this game.  
I lost \$4 . It cost me \$13 from there to there, so  
from there to there . . . OK. ITEM

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52. Pe: (reads) A call from the hotel to the church.  
53. Aa: from there that's that, then that. . . (enters)  
54. \* Pe: . . .I'm thinking. . . hang on I've got to. . .

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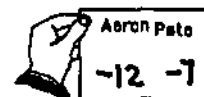
55. Tc: take your time. There is no hurry.  
56. Pe: Mmmm...mmmm...mmmm (enters a bid.)

## ITEM

57. Aa: (reads) Aaron Cabs gets the ride. I bid \$19  
58. Pe: I bid \$25.  
59.\* (watching score) ten, no you've lost

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60. Aa: this Aaron.  
(reads) Aaron Cabs bid \$19, the cost was \$15.



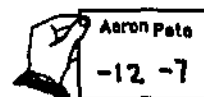
61. Pe: This is a profit of \$4 dollars!  
62. \* Pe: Oh! Well, you're still in debt.  
(reads) A call from the shop

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63. Aa: (I want the park)  
. . .to go to the prison.  
64. Pe: OK turn around again  
65. . . . (enters a bid.)  
66. Aa: . . . (enters a bid.)  
67. \* Pe: I'm going to get the ride!

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68. Peta Cabs gets the ride. I bid \$8.  
69. Aa: (watching score) seven Oh, exactly! (reads)  
Peta Cabs bid \$8, the cost is \$8  
It's a profit of nothing.



## ITEM

Orient Org Exec Eval

70. \* Pe Why didn't I charge more? Oww!

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71. Aa: Yeah, why didn't you charge the highest? I know what I can do now.

72. Pe: I don't believe this!

73. Aa: I know what to do now. (reads) A call is coming in. From the airport.

74. Pe (locates on map) to the docks-there (pointing).

75. \* Aa: So you've gotta go all the way back

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and I've got to go from there.

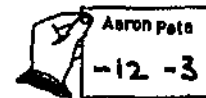
I know what I'm going to do. Turn around.

Turn around. So it has to be the closest bid

to it (the cost) does it? ... I'd say. ... (enters a bid.)

77. Pe: I'm going to get this right-I hope. ... (enters a bid)

78. Oh, no!, ... ten, eleven -you were closer. (reads) Peta Cabs bid \$11, the cost was \$7, that is a profit of \$4.



79. Aa: Oh, no! (reads) A call is coming in.  
A call from the Hotel, Mm-huhn to the ...  
I've gotta go all the way round. OK.

80. Pe: (bids)

81. Aa: ... (bids). Oh, no!

82. Pe: I bid \$16, what did you bid, Aaron? :

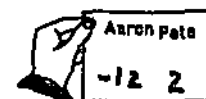
83. \* Aa: \$23 because I've gotta go all the

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84. \* Pe: way round there.  
Yes! (reads) Peta Cabs bid \$15 the

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cost was 11 This is a profit of \$5. At least I'm out of debt and I'm winning! Mmmm. Yes!



85. Aa: (reads) A call is coming in.

86. Both: A call from the Hotel to go to the Cinema.

87. Pe That's gonna cost a lotta money.

## ITEM

Orient Org Exec Eval

88. \*Aa: Oh, no! . . . Because it's a long distance

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it's much harder. 8, 9, (counting sections on the screen)... (bids)

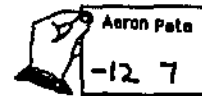
89. Pe: (enters a bid) Yes, yes yes.

90. Aa: Oh, no! What did you bid?

91. Pe: I went \$26

92. Aa: Oh, I went \$5. You were the closest one to it, too.

93. Pe: Yes, I made a profit, I did. (reads) Peta Cabs bid \$25. The cost was \$20. This is a profit of \$5.



94. Aa: (reads) A call is coming in, from the petrol station to the theatre.

95. Pe: . . . oh, no, yeah. (enters a bid)

96. Aa: Done it?

97. Pe: Yeah.

98. Aa: . . . (enters a bid).

99. Pe: (reads) Peta Cabs gets the ride. What did you bid Aaron?

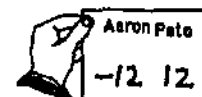
100. Aa: \$23

101. Pe: I bid \$19

102 \* Aa: \$19's . . . it couldn't be that mu. . . If I

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103. Pe: went down to \$20 I could have gotten the ride. (reads) Peta Cabs bid \$19, the cost was \$14. This is a profit of \$5.



104. Aa: Oh, no.

105. \* Pe: I'm beating Aaron, now. Aaron's

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loss is \$12 and mine's profit is \$12.

(reads) Press Space bar

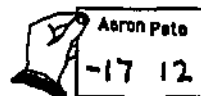
106. Aa: (reads) A call from the Park to go to the factory. OK.  
\* Go from there to there,

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\* so . . . I'll do (enters a bid).

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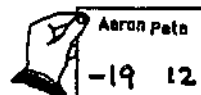
107. Pe: From the Park to the factory. Do you get it from. . . Oh . . . (bids).  
 108. Aa: Yes.  
 109. Pe: Oh, you got it Aaron. but you don't get a profit 18,19. .  
 110. Aa: Oh, no! I don't want this!  
 111. Tchr: Keep an eye on it and see if you can work out why that happened.  
 112. Aa: 18, 19., 20, 21, Aaron Cabs bid \$18. . .  
 113. Tchr: the cost was \$25, this is a loss of \$7



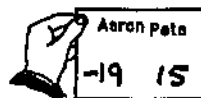
- 114 \* Aa: Oh, I know it now.



115. Pe: Aaron knows it now.  
 116. Aa: I think I know it.  
 117. Pe: You do!  
 118. T: (reads ) Minus \$19 and plus \$12.



119. Pe: (reads) From the Pub, yes; to there  
 120. T: (Yours is the) first bid Peta.  
 121. Pe: (enters a bid)  
 122. Aa: See if it will work this time. . . just wait, mmhuh. . .  
 123. You'll get a profit, yeah.  
 124. Pe: Peta Cabs bid \$11, the cost is \$8, a profit of \$3.



125. Pe A call is coming in from the hotel (there) to the petrol station.  
 126. Aa: Oh,no.. . OK. . . . .  
 127 \* Pe: is it the first to \$20?



128. Tchr: \$25.  
 129. Aa: Sorry . . . (enters a bid)  
 130. Pe: Now . . . (enters a bid).  
 131. Tchr (reads) Peta bid \$18, what did you bid Aaron?  
 132. Aa: I bid \$18  
 133. Pe: Peta Cabs bid \$13, the cost was \$6.  
 This is a profit of \$7 dollars, and I'm winning!  
 I've only got to get three more dollars.

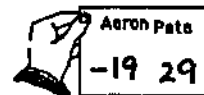




134. Tchr: Press the Space bar to continue. Have you worked it out yet Aaron?  
135 \*Aa: No, my idea keeps back-



- firing on me.  
136. Tchr: Don't worry, you are playing correctly,  
you have been unfortunate with the calls.  
137. Pe: (enters a bid)  
138. Aa: OK.  
139. Tchr: Tell me what you're thinking, Aaron  
140. Aa: Well, if Peta went from there to there for \$13,  
and she got a profit. . . (enters a bid)  
141. Tchr: (reads) Peta Cabs gets the ride. Why was that?  
142. Aa: . . . her play could be closer, to where its from. . .  
143. Tchr: (reads) Peta Cabs gets the ride. \$13 the cost was \$6,  
this is a profit of \$7.



144. That's the end of the game.  
If you were to play this again what would you do.  
What did you learn?

- 145.\* Aa: Well if the cab is closer to the



- "from" you are more likely to win.  
I had to go all the way from up here first.  
146. Tchr: So the call can affect the winner. Is there anything  
else Peta may know, that you didn't?  
147. Pe: Well, each of the sections is worth a dollar  
148. Aa: Yes, I knew that. I kept counting that, and adding  
a bit more, so that I could get a profit, but it didn't  
work. Peta always got the ride still.  
149. Tchr: Why did Peta always get the ride?  
150 \* Aa: Because she was closer.



151. Tchr: Why did you think you got the ride, Peta?  
152. Pe: Because my bid was lower.  
153. Aa: But I was always further away.

## **Appendix 2: Letter to colleagues**

# **Letter**

## **To colleagues**

**Mr. A. McIntosh**  
**Department of Mathematics and Computing**  
**Edith Cowan University**  
**Churchlands.**

**Dear Alistair,**

**I am writing to you to ask your assistance in some research I am conducting.**

**The research is involved with decisions that children make as they solve mathematical problems.**

**What I would like you to do (if you can spare the time!) is to:**

- 1. read the transcript of two children working through a problem on a computer; and**
- 2. classify the asterisked items according to the checklist described below.**

**The check-list is based on the work of Polya and tries to describe the types of decisions that students make when they solve a problem.**

**The framework has been adapted to a checklist.**

The categories are as follows:

<b>1. Orientation:</b> Strategic behaviour to assess and understand a problem
1-a. Comprehension strategies
1-b. Analysis of information and conditions
1-c. Assessment of familiarity with task
1-d. Initial and subsequent representation
1-e. Assessment of level of difficulty and chances of success
<b>2. Organization:</b> Planning of behaviour and choice of actions.
2-a. Identification of goals and subgoals
2-b. Global planning
2-c. Local planning (to implement global plans)
<b>3. Execution:</b> Regulation of behaviour to conform to plans
3-a. Performance of local actions
3-b. Monitoring of progress of local and global plans
3-c. Trade-off decisions (e.g., speed vs. accuracy, degree of elegance)
<b>4. Verification:</b> Evaluation of decisions made and of outcomes of executed plans
4-a. Evaluation of <i>orientation</i> and <i>organization</i>
4-a-1. Adequacy of representation
4-a-2. Adequacy of organizational decisions
4-a-3. Consistency of local plans with global plans
4-a-4. Consistency of global plans with goals
B. Evaluation of <i>execution</i>
4-b-1. Adequacy of performance of actions
4-b-2. Consistency of actions with plans
4-b-3. Consistency of local results with plans and problem conditions
4-b-4. Consistency of final results with problem conditions

The first category is called the *orientation* stage; it indicates a student attempting to assess and understand a problem. As well, within each category are listed more specific strategies (e.g. 1A-1E).

The *organization* category follows, during which planning behaviour and choice of actions are shown (2A-2C).

The *execution* category involves students assessing the progress of their approach as they work through the problem. (3A-3C)

Finally, comes the *verification* category, during which students evaluate both the decisions they have made and the final outcome. (4A4-4B4)

For the checklist to be useful it needs to be reliable. It is at this point that your co-operation is sought. Two Year 7 children were given the following problem to solve using a computer.

The problem involved a simulation of two taxi drivers competing by making bids to pick up and transport passengers to and from random destinations. The object was to be the most efficient driver, in terms of cost.

The children were asked to bid for taxi fares between given destinations, e.g. from the Factory to the Airport. The screen showed the layout of the district, with the destinations marked clearly. The taxi fare was given to the child with the lowest bid. After each child had made a bid, a simulated taxi-cab moved between the two designated points.

In addition, a score-board appeared in diagonally opposite corners of the screen. These gave progressive scores for each child. As the taxi moved, the scores were adjusted automatically, showing which child had bid the lowest, what the journey had cost, and the profit or loss made. A profit was made if the cost of the bid was greater than the cost incurred in making the journey, otherwise a loss was made. The object of each player was to make a profit of twenty-five dollars (\$25). Alternatively, the game could be lost by losing twenty-five dollars (-\$25)

The transcript of their problem solving is provided. Several selected items of the transcript have been marked with an asterisk. Would you read and consider which strategy, if any, each numbered line reflects, and place that number in the checklist provided? If possible, try to place the item in one category only.

I would appreciate your consideration of this matter and thank you in anticipation of your involvement.

Yours sincerely

ANNE L. MARTIN

## **Appendix 3: Trial transcripts**

## **TAXI-THE NEXT 17 (SMILE)**

### **CHECKLIST TRIAL 1**

**Two children, Anna and David, Year 7. (Have commenced problem-solving classes, as documented.)**

- A: Do we want sound?  
D: Yes  
A: (reads) Two teams each have a taxi. Each customer will call in for rides. Each team makes a bid.  
D: The lower gets the ride. The journey from the building to the next costs you 1 in fuel. The first team to make \$25 wins. If a team makes \$25 loss, they lose. (They go through the diagram learning the names of each stop-teams register as David cabs and Anna Cabs)  
D: A call is coming in.  
A: A call from school.  
D: (Looking at score ) Anna Cabs has zero profit.  
A: . . .from the school to the Pub. (reads) What is your bid Anna? Do I just write in a number?  
T: Yes, but the idea is not to let David see it.  
A: (enters a figure).  
D: (reads) What is your bid, David? Long pause. (Enters a figure),  
A: (reads) Anna Cabs gets the ride. I bid \$5, how much did you bid?  
D: \$15. (Sound effects, giggles)  
T: What happened?  
A: (reads) Anna bid \$5. Her cost was \$10. So, how much do I get?  
D: We got the same. The difference was \$5. (?)  
D: (reads) Press space-bar to continue.  
A: (reads) A call is coming in.  
D: A call from the airport to the stadium.  
A: So we have to go right around there (ignoring the present position of her car, points to mark a line from original position near Airport, although her car is now parked at the Pub). What is your bid, David?  
D: Where's the stadium? Is that the stadium? No, that's the factory, Oh, that one there. Um. . . (enters a number)  
A: Don't look David. (enters number)  
D: (reads) Anna Cabs gets the ride. What did you put in?  
A: \$6, (reads) costs \$10.  
D: I put \$8. Oh, (watching taxi move) is it (there) and back? Oh.  
A: Oh.  
T: What happened?  
A: (reads) Anna Cabs bid \$6, the cost was \$13, this was a loss of \$19.  
A: David, there's a call from the Airport to go to school, which is from here round to here.  
D: Anna's to bid. (Anna enters a number.) Have you done it?  
A: Yes  
D: So, where is it? From there to there. Um. . . (spots the score card) Is that what you've lost?  
A: Yeah.  
D: Is that minus?  
A: \$18  
D: I haven't won anything  
A: You haven't lost anything either.

D: I don't know. I'll put that (enters a number). (David gets the ride)  
 T: What did you bid Anna?  
 A: \$10  
 D: I ran over her (as blue car goes over yellow). Back to the Stadium.  
 A: (reads) David Cabs bid \$9, the cost was \$22. This is a loss of \$13.  
 D: I think I've got it pretty well worked out now.  
 T: Good  
 A: What did you say?  
 D: The distance-I've got it pretty well worked out now.  
 A: Yeah.  
 A: A call from the petrol station to the hotel  
 A: No looking.  
 D: (Enters a figure)  
 A: From, from, from the Petrol Station to the Hotel. Where's the Petrol station? (long pause) (Enters a figure.)  
 D: (As Anna Cabs gets the fare) What did you bid?  
 A: \$14  
 D: I bid \$15  
 T: So Anna bid one less.  
 A: (reads) Anna Cabs bid \$14, the cost was \$8. This was a profit of \$6.  
 D: (pointing) Oh, I know what I did, I thought that was the first one that was \$19 around up to there. (he had ignored the position of his car, points to mark a line from current position of his which car is now parked at the School, but has now realised the implications)  
 T: Yes, you were really close there, you only needed to come around the corner. You have improved your loss very well Anna.  
 A: (reads) A call is coming in. A call from the Stadium to go to the church which is . . .  
 D: . . . a long way (beginning to understand the implication)  
 A: No looking. (Enters) Go, David.  
 D: Um. . . long pause (Enters)  
 T: Who gets it?  
 A: (reads) David, bids \$18  
 T: What did you bid Anna?  
 A: \$20  
 A: (reads) David bids \$18, the cost of the ride was \$9. This is a profit of \$9.  
 T: David finished with a loss of \$4, Anna \$12. David's the winner. Would you like to play that again another day?  
 (solution not complete)



## TAXI-THE NEXT 17 (SMILE)

### CHECKLIST TRIAL 2

**Two children, Anna and Jill, Year 7. (Have commenced problem-solving classes, as documented.)**

- A: Shall I show her the rules?  
T: You can tell them to her if you wish.  
A: No, no. (Reads) Two teams own a taxi firm. Customers will call you in for rides. Each team makes a bid and the lowest bid get-*and the lowest bid gets the ride!* (to teacher) *It's written in the rules!* (Reads) The journey from one building to the next costs you one dollar in fuel. (No comment about the six dollars it costs in the straight. Anna has worked out her clue by watching the digits which click over as the car travels, not by assessment of the rules.) The first team to make \$25 profit wins. If the team makes \$25 loss they lose. Do you get it?  
J: Mm.  
A: Here are the names of the buildings: Airport, like they'll say to the Airport. (They go through all the buildings). I'll go first.  
T: Do you understand the rules, Jill?  
J: Yeah.  
A: I own the Yellow Cabs. Jill, you own the Blue Cabs. (Reads) A call is coming in. From the school to go to the pub.  
J: Yeah.  
A: I bid, you gotta look away, you can't look at my bid. (bids)  
J: The lowest bid, does it have to be?  
A: Yeah. You bid yours now. (Silence) The numbers don't come up on the screen.  
J: The lowest bid?  
A: Yeah. You don't know what my bid is.  
J: How do you do it?  
A: Press the numbers  
J: Oh, those numbers! (bids) Is it return now?  
A: Yeah. Jill, you get the ride. You bid one!  
J: Giggles.  
A: You're going to lose about, you're going to lose thirteen dollars! You lost thirteen dollars.  
J: Oh.  
A: (Reads) From the docks to go to the stadium. Jill, what is your bid?  
J: (bids)  
A: Um. . .from the docks to go to the stadium. (bids)  
J: What did you bid?  
A: Fifteen. What did you bid?  
J: Fifteen  
A: (Reads) Equal bids-the cost was nine; yes, I made six dollars.  
J: Did I made anything?  
A: No, you're still loss. If you make money, it comes off your loss. (Reads) From the hotel to go to the bus station. That is going to be a big one. Okay, (bids)  
J: Where do we gotta go?  
A: Just remember we've got to go from here-dur, dur, dur, (pointing, to show Jill the track. She hasn't shared any strategy with her as yet.)  
J: That'll be quite a big one. (bids)  
A: Yeah, I made 2 dollars.

J: Did you press \*?  
 A: Yeah I've got 8 dollars. From the hotel to the nursery. (Reads) Jill, what's your bid?  
 J: (bids)  
 A: (bids) I'm going to lose something! A loss of 8 dollars. I'm back to zero. Call from the prison to there. (Reads) Anna, what is your bid? It's o.k. don't worry. . . (Makes counting motions, noises) Ah. (bids).  
 J: . . . (bids).  
 A: I made a profit of 2 dollars. Call from the school to go to the park. Jill, What is your bid?  
 J: Club \*  
 A: Call from the factory to go to the airport. Jill, What is your bid?  
 J: . . . (bids).  
 A: . . . (bids). Oh, I got it  
 J: Yes, because you bid lower.  
 A: I won't make any money, I don't think.  
 J: How much did it cost?  
 A: It cost 18 and I bid 25. So it's 7.  
 J: You made a profit. (She didn't)  
 A: Yeah, seven off my total. Call from the school to go to the park. Jill, What is your bid?  
 J: What did you bid?  
 A: Nine.  
 J: So did I. No, I lost some-two dollars. From the docks to go to the Stadium. (bids)  
 A: (bids)  
 J: You bid the lowest. Two dollars. From the factory to go to the park. Jill, What is your bid?  
 A: (bids)  
 J: (bids) What did you bid?  
 A: Twenty three  
 J: - - - eighty one. Oh, no!  
 A: - - -  
 J: You're going to lose three dollars. From the Pub to go to the Airport. Anna, what is your bid? Put me under pressure! (bids).  
 A: (bids)  
 J: What did you bid?  
 A: thirteen. You made zero dollars.  
 J: From the theatre to the school.  
 A: O.K. where am I?  
 J: You're up. . .  
 A: I know I'm not going to make anything..(bids).  
 v (bids)  
 A: I lost some. I lost four dollars.  
 T: How are you going? Have you worked it out?  
 Both: Yes.  
 T: Did you help Jill, Anna?  
 A: Yes.  
 T: Did the computer always take the lowest bid? Did you work out anything else to help you win? Did you tell Jill about using the length of the straight as a measure?  
 A: Oh, no. I forgot about it. But I did count it again to check, and it was right. See, Jill, it takes six to go from there to there, so if you want to go further it goes six, seven eight, and you can measure. Can we play this again?

## Microsmile Taxi The Next 17

### Jaymie and Peter Yr 7

from a once a week problem solving class focusing on strategies, over a period of six months ( a semester)

J: Reads rules. Reads names of places.  
J: (Reads) A call is coming in. Press space bar to receive it. From the petrol station to the fire station.  
P: (bids).  
J: I bid \$6. The cost was \$9. That's a profit of \$3.  
P: A call is coming in. From the hotel to the pub. That's a long way!  
J: types a bid. (pause)P Cabs bid \$20 the cost was \$24. That was a loss of \$4. Oh, man!  
P: From the Airport to the nursery. (bids)  
J: (bids)  
P: J Cabs gets the ride. J bid \$15. The cost was \$16. Loss of \$1. A call is coming in. From the motel to the factory.  
P: (bids).  
J: (bids). . . I put in \$28, and it's only saying 18!  
T: You must have pressed 18 by mistake.  
P: I bid \$35  
J: J Cabs bid \$18 and the cost was \$19, a loss of \$1. A call from the Railway Station to the Hotel. (bids)  
P: (bids) The customer does not accept either bid. Will you bid again, J?  
J: I put \$24 in.  
P: I bid \$34  
First, you've gotta move there, and then. . .  
T: It looks as though you have to bid again.  
Both bid.  
J: (reads) The cost was \$17 and you bid \$20. A profit of \$3.  
P: I bid \$25  
J: A call is coming in from the Railway Station to the Bus station. I know who's going to win this bid. (looking at position of cars) (both bid.)  
P: I bid \$6 and it cost \$10. I lost \$4. Mm.  
J: Cost \$8. From the Docks to the Cinema. (bids).  
P: (bids)  
J: You're going to lose more money.  
P: P Cabs bid \$15. The cost was \$17. A loss of \$2. What did you bid J?  
J: \$20. From the stadium to the docks. P, you're here, so you have to pick them up there and go down to there.  
P: bids  
J: bids.  
P: P cabs gets the ride I bid \$20 and the cost was \$9 - a profit of \$11.  
J: only \$1  
P: But on this one it was \$11.  
J: We both put \$20.  
J: From the Stadium to the Factory,

J: (bids).  
 P: (bids). (reads) Will you bid again?  
 J: (bids)  
 J: I put \$20  
 P: I put \$22  
 J: Ah, man. What did you put?  
 T: No, she doesn't tell you till you have bid.  
 J: Oh. (reads) Equal bids. P Cabs bid \$20. This is a profit of \$10.  
 It cost \$10. You now have \$11.  
 J: From the stadium to the petrol station.  
 P: bids  
 J: bids. (reads) J Cabs gets the ride. Oh, I'm going to make nil.  
 T: Did you forget something?  
 J: Yes, I forgot I have to come back. Reads J bid \$14, cost was \$12.  
 A profit of \$2.  
 P: I bid \$15.  
 J: (Reads). A call from the Nursery to go to the Cinema. (bids)  
 P: bids. . . Who got it?  
 J: I did, I made a profit of \$3.  
 P: I knew she would get it.  
 T: Why did you know, P?  
 P: Um  
 J: Because he was up here and he had to come down this far.  
 (Reads) J cabs bid \$17. The cost was \$8. A profit of \$9. I'm on  
 \$18 and Peter's on \$11.  
 From the Bus Station to the Shops.  
 P: (Bids)  
 J: (bids) (Reads) J gets the ride. \$9, the cost was \$5, a profit of \$4.  
 \$22.  
 J: A call is coming in. From the Stadium to the park.  
 (bids)  
 P: Um. . . bids  
 J: (reads) J gets the ride. I bid \$15. What did you bid, P?  
 P: \$20.  
 J: (Reads) J Cabs bid \$15 The cost was \$8, a profit of \$7. I've  
 won. I made a profit of \$29.