1995

Robust decentralised variable structure control for rigid robotic manipulators

Thasapalan Kuhan
Edith Cowan University

ROBUST DECENTRALISED VARIABLE STRUCTURE CONTROL FOR RIGID ROBOTIC MANIPULATORS

KUHAN.T

A Thesis Submitted in Partial Fulfilment of the Requirements for the Award of Honours in Bachelor of Engineering (Computer Systems)

at the Faculty of Science, Technology and Engineering
Edith Cowan University

Department of Computer and Communication Engineering
The School of Mathematics, Information Technology and Engineering
Edith Cowan University

OCTOBER, 1995

USE OF THESIS

I certify that this thesis does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any institution of higher educatio: and that to the best of my knowledge and belief it does not contain any material published or written by another person except where due reffernce is made in the text.

Signature ....

Date ....November 1995.......

Acknowledgment

Firstly, I would like to thank my supervisor and friend Dr. Zhihong Man for his guidance, support, encouragement and friendship throughout the last 2 years of this course.

My grateful thanks also go to all the members in the Department of Computer and Communication Engineering for their support given to me during my study of Engineering at Edith Cowan University.

Finally, I would like to thank my parents, my guardians Dr and Mrs S.Krishnan and my friends for their love, understanding and support during this period.

Abstract

In this thesis, the problem of robust variable structure control for non-linear rigid robotic manipulators is investigated. Robustness and convergence results are presented for variable structure control systems of robotic manipulators with bounded unknown disturbances, nonlinearities, dynamical couplings and parameter uncertainties. The major outcomes of the work described in this thesis are summarised as given below.

The basic variable structure theory is surveyed, and some basic ideas such as sliding mode designs, robustness analysis and controller design methods for linear or non-linear systems are reviewed. Three recent variable structure control schemes for robotic manipulators are discussed and compared to highlight the research developments in this area.

A decentralised variable structure model reference adaptive control scheme is proposed for a class of large scale systems. It is shown that, unlike previous decentralised variable structure control schemes, the local variable structure controller design in this scheme requires only three bounds of the subsystem matrices and dynamical interactions instead of the upper and the lower bounds of all unknown subsystem parameters. Using this scheme, not only asymptotic convergence of the output tracking error can be guaranteed, but also the controller design is greatly simplified. In order to eliminate chattering caused by the variable structure technique, local boundary layer controllers are presented. Furthermore, the scheme is applied to the tracking control of robotic manipulators with the result that strong robustness and asymptotic convergence of the output tracking error are obtained.

TABLE OF CONTENTS

Chapter 1

Introduction

1.1 Background

First computer-controlled robotic manipulator was designed thirty-four years ago, and still, it is a very active research field in both theory and applications. A number of books and survey papers have been published (Craig, 1986, 1988; Warwick and Pugh, 1988; Rehg, 1985; Ortega and Spong, 1988; Abdallah, et al., 1991). This thesis contributes to a further study on the control of robotic manipulators.

The control of robotic manipulators, in general, is concerned with the efficient managements of robotic manipulator systems. Due to the fact that robotic manipulators have high nonlinearity, large system uncertainties, strong dynamical couplings and external disturbances, it is generally difficult to design a simple controller which can guarantee high quality performance for robotic manipulators.

In the early days, low-level control of industrial robots was accomplished through the simple servo control of individual joints (Horn and Railbert, 1978; Van Brussel and Vastman, 1984; Silva, 1984, 1989). This approach has several disadvantages. Primarily, since servo parameters are set at constant values during each cycle of robot operation, these control parameters cannot adapt to compensate for robot

nonlinearities and parameter variations. Furthermore, an effective compensation for dynamical coupling among the joints of a robotic manipulator is impossible through the simple servo control.

To deal with the above difficulties, some linearization control techniques were developed. For example, in the work of Desa and Roth (1985), Whitehead et al. (1985) and Luh (1983), a Taylor series expansion is used to linearize the nonlinear dynamic equation for a general robotic manipulator. A feedback controller is then designed to compensate the nonlinearities and dynamical couplings so that good system performance can be achieved. Later, however, the physics and the special structure of the robotic manipulator equation, coupled with the fact that the generalised torque input vector provides an independent input for each degree of freedom, led to the global feedback linearization for robotic manipulators (Kreutz, 1989). However, it came to be realised that these linearization control schemes are based on some very restrictive assumptions. For example, it is assumed that symmetric positive-definite inertia matrix and the vector containing coriolis, centrifugal forces and gravity torques in the robotic dynamical equation are exactly known. Unfortunately, these assumptions are rarely satisfied in real robotic systems, and the violation of the ideal conditions can lead to failure of the linearization control schemes.

Later, several modified linearization control schemes were proposed by a number of researchers (Spong and Vidyasagar, 1987; Abdallah and Jordan, 1990; Anderson et al., 1989; Tarn et al., 1984; Shoureshi, 1990; Craig, 1988). In these control schemes, the known system dynamics is used to build up a nominal system model, and a nominal feedback controller is then designed. In order to deal with system uncertainties and external disturbances, a feedback compensator is designed so that the poles of the closed loop system are placed sufficiently far in the left-half-plane. The advantages of these modified control schemes are that large system uncertainties can be considered and the wealth of linear feedback techniques can be used in the

linear outer loop. However, the output tracking error cannot converge to zero and the high-gain control law may be the outcome in order to achieve robustness by the use of these control schemes.

With the rapid developments of adaptive control theory, many adaptive control approaches were developed for robotic manipulators where some useful structural properties of robotic manipulators are exploited to devise a suitable adaptive controller which does not necessarily linearize the plants. In Craig et al.(1987), the dynamic equation of a robotic manipulator is expressed in a linear function of unknown parameters, and controller is then designed by the use of parameter estimates so that the output tracking error can asymptotically converge to zero with all signals remaining bounded. The main drawbacks of this scheme are that the estimates of inertia matrix need to remain uniformly positive-definite, and the measurement of the acceleration is needed in order to realise the adaptive update law.

In Spong and Ortega (1988), the requirement that the estimates of inertia matrix remains uniformly positive definite in Craig (1986) is removed. The estimates of inertia matrix and other unknown parameters, which have the fixed values, are used in the feedback control. An additive signal that compensates for the deviation of the estimates of inertia matrix and other parameters is then adaptively adjusted so that the output tracking error can asymptotically converge to zero with all signals remaining bounded. During this same period, many other adaptive control schemes were developed. For example, in Amestegui et al. (1987), the requirement on boundedness of the estimated inertia matrix is removed but a different parameter update law is used. In Middleton and Goodwin (1988), the measurement of the joint acceleration is not required but the boundedness of the inverse of the estimates inertia matrix is still needed.

However, some practical issues for the use of the above adaptive control schemes have been noted by many researchers. First, the transient error performance can not

be specified. Second, since asymptotic stability has not been proved to be uniform, small changes in the dynamics or small unmodeled bounded disturbances may result in loss of stability and cause unacceptably large deviations from the desired response (Rohrs et al., 1985; Ortega and Spong, 1988).

A remarkable development in robotic control field is the use of variable structure control technique. The variable structure control technique was first used to solve control problems in Soviet Union in the 1960s (Emelyanov, 1962, 1966), and has been largely investigated by many researchers (Utkin, 1971, 1977, 1978, 1983; Itks, 1976; Young, 1978, 1988; Slotine and Sastry, 1983) in both theoretical and applied aspects.

A variable structure control system is characterised by a control structure which is switched as the system states cross certain discontinuous surfaces in the state space. The intersection of these surfaces forms a sliding mode which is intended to constrain the dynamics of the system trajectories. When the sliding occurs, the trajectory is kept on the sliding mode resulting in the desired system dynamics that is insensitive to parameter variations, nonlinearities and disturbances. It is due to the above advantages that the theory of variable structure systems has been widely used in the control of robotic manipulators.

The first application of the variable structure control theory to robot control seems to be the work of Young (1978), where the variable structure controller eliminates the nonlinear coupling of joints by forcing the system into the sliding mode after which the output tracking error asymptotically converges to zero. Later modifications of the Young controller were presented by Morgan and Ozguner (1985) and Abbass and Ozguner (1985), in which decentralised variable structure control schemes are developed and the controller designs are simplified. Unfortunately, for most of the above variable structure control schemes, chattering occur in the control input, which may excite undesired high-frequency dynamics. To solve this problem, a modified

variable structure controller using boundary layer technique was developed by Slotine and Sastry (1983). Using the boundary layer technique, the control input signal can be smoothed inside a possibly boundary layer. This will achieve optimal trade-off between control bandwidth and tracking precision, and therefore eliminate chattering and sensitivity of the controller to unmodeled high frequency dynamics.

More recently, the work of Yeung and Chen (1988) presented a new approach which takes advantage of an important property of the inertia matrix, namely its symmetric positive-definiteness, and allows a development of the control law without having to take the inverse of the inertia matrix in the variable structure controller design.

In the above variable structure control schemes, the upper and the lower bounds of unknown system parameters are required in controller designs. However, in some situations, it is difficult to know the upper and lower bounds of all unknown system parameters because robotic manipulators have high nonlinearity and large system uncertainties. On the other hand, a physical robotic manipulator is a partially known system, and the known knowledge and some useful structural properties are not fully used for the variable structure controller designs in these schemes.

The recent work of Leung et al. (1991) has made a great progress for the robust variable structure control of robotic manipulators. In this control scheme, the controlled robotic manipulator is assumed to be completely unknown, and the controller is designed based only on several uncertain system matrix bounds. Theoretically, robustness and convergence can be obtained. However, still there are some problems needed to be further improved. For example, when the boundary layer controller is carried out and the sampling interval is nonzero, the controller parameters will tend to infinity due to the fact that the switching plane variables and output tracking error can not converge to zero.

This thesis will further investigate variable structure control systems for robotic manipulators and presents a new robust decentralised variable structure control scheme.

1.2 Contributions of this thesis

In this thesis we focus our attention on the decentralised variable structure model reference adaptive control and control following terminal sliding mode.

The main contents of this thesis are organised as follow.

Chapter 2 gives a brief survey for the basic variable structure theory. Some basic ideas such as sliding mode designs, robustness analysis and controller design methods for linear or nonlinear systems are reviewed.

Chapter 3 discusses and compares three recent variable structure control schemes for robotic manipulators to highlight the research developments in this area.

Chapter 4 follows the line of chapter 4, a decentralised variable structure model reference adaptive control scheme is proposed for a class of large scale systems. It is shown that, unlike previous decentralised variable structure control schemes in Abbass and Ozguner (1985), Ozguner et al. (1987), Xu et al. (1990) and Morgan and Ozguner (1985), a set of adaptive mechanisms are introduced to estimate the uncertainty bounds. The local variable structure controller can then be designed without prior information of the bounds of the subsystem matrices and dynamical interactions. Therefore by the use of this adaptive sliding mode control scheme, not only asymptotic convergence of the output tracking error can be guaranteed, but also the controller design is greatly simplified. In order to eliminate chattering caused by the variable structure technique, local boundary layer controllers are presented. Furthermore, the scheme is applied to the tracking control of robotic manipulators

with the result that strong robustness and asymptotic convergence of the output tracking error are obtained

Chapter 5 proposes a decentralised terminal sliding mode control for rigid robotic manipulators. It is shown that, by the use of the terminal sliding mode technique, the output tracking error can converge to zero in a finite time. A theoretical analysis on the finite error convergence and robustness with respect to uncertain dynamics is carried out tin detail..

Chapter 6 gives conclusions and further research.

Chapter 2

A Survey of The Variable Structure Control Theory and Its Applications to Robotic Manipulators

2.1 Introduction

We have mentioned in chapter one and two the basic ideas of the variable structure control theory and the recent developments in the variable structure control for robotic manipulators. As we know from the previous chapter, variable structure control can be considered to be an extension of conventional feedback control in the sense that the structure of a state feedback regulator is allowed to change as its states cross discontinuity surfaces, which results in discontinuous feedback control input on one or more manifolds in the state space. From the point of the conventional feedback control theory, a variable structure control system can be treated as a combination of subsystems. Each subsystem has a fixed structure and operates in a specified region of the state space. The combination of these subsystems according to some prescribed

rules results in a new system which is different from the individual subsystems and has the desired system response.

The main feature of a variable structure control system is the sliding motion. For the design of a variable structure controller, the first thing is to define a set of switching plane variables which are a function of the system states. The intersection of these switching planes forms a sliding mode. The purpose of the variable structure controller is to drive the system states into the sliding mode on which the sliding motion occurs and the motion of the system is thus formally equivalent to a system of low order, called as equivalent system. Actually, the sliding motion on the sliding mode is the convergence motion of the system states from arbitrary initial values to the origin. The convergence rate depends on the design of sliding mode parameters. It is due to this feature that the variable structure control is also called sliding mode control.

Another feature of a variable structure system is that the transient response can be divided into two parts. First, the motion in which the variable structure controller drives the switching plane variables to reach the sliding mode. Second, the sliding motion in which the system states constrained on the sliding mode asymptotically converge to the origin. Usually, the sliding motion is determined only by the sliding mode parameters. However, the convergence of the switching plane variables are affected by the sliding mode parameters because the sliding mode parameters are involved in the controller gain matrices.

In this chapter, we will first review the basic variable structure control theory that has

been useful in establishing robust variable structure control algorithms. In view of the focus of the thesis, we will then restrict our discussion to recent research results on the robust variable structure control for robotic manipulators with uncertain dynamics.

In section 2.2 of this chapter, the basic variable structure control theory is briefly reviewed. The basic ideas and definitions such as system model, the sliding mode, the condition for existence of sliding mode, robustness property and an overview of four variable structure controllers are discussed. In section 2.3, we deviate to address more complicated variable structure control for a class of nonlinear systems.

2.2 Basic variable structure control study

2.2.1 System model and the sliding mode

We are going to consider a linear time invariant system .

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2.1}$$

where $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ . \\ . \\ x_n(t) \end{bmatrix}$ and $u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ . \\ . \\ u_m(t) \end{bmatrix}$ represent the state and control vectors,

$A \in R^{nxn}$ and $B \in R^{nxm}$ are constant system matrices. It assumed that n > m, B is of full rank m and pair (A,B) is completely controllable.

Now we define a set of variables called the switching plane variables s_i ($i = 1...m$) passing through the state space origin. { $s_1 = C_1X$, $s_2 = C_2X$, ..., $s_m = C_mX$ } where $C_i \in R^n$ is a constant vector and X is the state vector.

$$C = [C_1^T \;........\; C_m^T]^T \text{ is a nxm constant matrix.} \tag{2.2}$$

Sliding mode is achieved when the state vector X reaches and remains on the intersection (s = 0) of the m switching plane variables.

$$S_o = \bigcap_{i=1}^{m} s_i = \{X: C_iX = 0, i = 1......m\} \tag{2.3}$$

Now considering the input vector u(t), it can be expanded and it is usually of the form

$$u(t) = KX + \psi X \tag{2.4}$$

In the equation above KX is the linear feedback and ψX determines the switching component. We already know what X is, wherelse K and ψ are controller gain matrices

The task of the control input u(t) is to drive the switching plane variables to reach sliding mode (2.3) by suitable design of matrices (gain matrices) K and ψ. By doing so, and after reaching the sliding mode, the system performance will be determined by the sliding motion on the sliding mode. The sliding mode is designed such that the system response is restricted on the sliding mode and has a desired behaviour such as asymptotic stability and prescribed linear transient response. This can be achieved by

designing the switching plane variables as linear functions of the system states. This is done as it is easier or convenient for the design and analysis of a variable structure control system.

The next objective is to design the controller parameters to guarantee that the switching plane variables will remain on the sliding mode.

Switching plane variables are $\{ s_1 = C_1X, s_2 = C_2X, ..., s_m = C_mX \}$.

Utkin and Young (1982) manage to prove that the time derivative of the switching plane variables always point toward the sliding mode surfaces, then the switching plane variables s_i ($i = 1...m$) asymptotically converges to zero and the system states can remain on the sliding mode. The second Lyapunov method is chosen because the problem is a convergence problem.

$$\upsilon = \frac{1}{2}S^TS \tag{2.5}$$

In eqution the constant is included as to cancel the constant generated by derivation of S^TS.

To reach sliding mode surface it must be

$$S^T\dot{S} < 0 \quad \text{or} \quad S_i\dot{S}_i < 0 \quad (i = 1....m)$$

(2.6)

Note equation (2.6) is the derivative of equation (2.5).

Most of structure control algorithms are designed based on the sufficient conditions in expression (2.6) (Utkin 1978 and Decarlo 1988).

2.2.2 Equivalent control

First we will define a general equation for the switching plane variables.

$$S = CX \tag{2.7}$$

Then using equation (2.1) and combining these two equations and differentiating S, we have

$$\dot{S} = C\dot{X} \tag{2.8}$$

Combining the equations we have

$$\dot{S} = CA\,X(t) + CBu_{eq} = 0 \tag{2.9}$$

where u_{eq} is called the equivalent control.

If the CB is non-singular then u_{eq} from (2.9) can be written as

$$\begin{aligned} u_{eq} &= -(CB)^{-1}CAX \\ &= -KX \end{aligned} \tag{2.10}$$

by equating both sides we can get the expression of X

$$\text{where} \quad K = (CB)^{-1}CA \tag{2.11}$$

Now using the equivalent control equation ie equation (2.10), substitute into the linear time-invariant system ie equation (2.1) we get

$$\dot{X} = AX(t) - B(CB)^{-1}CAX(t) \tag{2.12}$$

Factorising AX(t) we get

$$\dot{X} = AX(t)[\,I - B(CB)^{-1}C\,] \tag{2.13}$$

The system in equation (2.13) is called the equivalent system. This system has characteristics noted below.

- The dynamical behaviour of this system is independent of the control input and depends only on the choice of matrix C from the expression S = CX.
- The control input here is used to drive the system states into sliding mode and therefore maintain it on the sliding mode.
- The determination of matrix C may thus be completed with prior knowledge of the form of the control input.
- With CB being non-singular, the equivalent system has an independent motion from the control input.
- When sliding motion occurs on the sliding mode or within N(C), the behaviour of the equivalent system is unaffected by the control input. This happen due to the above reasons. If CB was to be singular Utkin(1977) said that the equivalent control is either not unique or does not exists and sliding mode cannot be reached.
- (2.13) is a (n-m)th order system. Darling and Zinober (1986) has shown that for the matrix B with full rank m, there exists an othorgonal nxn transformation matrix T such that :

$$TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \tag{2.14}$$

where B_2 mxm nonsingular matrix and $T^T = T^{-1}$ (T is an othorgonal matrix)

Lets define :

$$Y = TX \tag{2.15}$$

Using equation (2.1) and (2.15) we can have a relationship between X and Y

$$\dot{X} = T^T Y \qquad (2.16)$$

Solving we have

$$T T^{-1} \dot{Y} = TAT^{-1}Y + TBu \qquad (2.17)$$

$$\dot{Y} = TAT^{-1}Y + TBu$$

$$Y^T = \begin{bmatrix} Y_1^T & Y_2^T \end{bmatrix}$$

$$TAT^{-1} = TAT^T = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$$

$$CT^T = [C_1 \; C_2]$$

$$\begin{bmatrix} \dot{Y}_1 \\ \dot{Y}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u \qquad (2.18)$$

$$\dot{Y}_1 = A_{11} Y_1 + A_{12} Y_2 \qquad (2.19)$$

$$\dot{Y}_2 = A_{12} Y_1 + A_{22} Y_2 + B_2 u \qquad (2.20)$$

on the sliding mode , we have

$$C_1 Y_1 + C_2 Y_2 = 0$$

$$Y_2 = (-C_1 Y_1) C_2^{-1} \qquad (2.21)$$

$$\text{let } F = (-C_1) C_2^{-1}$$

The equivalent system can be written in the following form

$$\dot{Y} = (A_{11} - A_{12}F)Y_1 \tag{2.22}$$

From the above we can see from expression (2.22) that the equivalent system is (n-m)th order system, ie the system dynamics is simplified on the sliding mode.

2.2.3 Robustness Property

In this section, we are going to include uncertainty in matrix A and external disturbances.

This will result in a modified version of (2.1).

$$\dot{X} = (A_0 + \Delta A)X(t) + Bu + Df \tag{2.23}$$

where A_0 is the nominal system matrix, ΔA is the uncertainty, $f \in R^L$ is a bounded external disturbance vector and matrix D is compatibility dimensioned. Without loss of generality, it is assumed that matrices B and D are full rank and the uncertainty presented in the input distribution matrix B is incorporated in the system disturbance term. During the sliding motion, the state vector of the system satisfies the following equations.

form (2.7) $S = CX = 0$.

Using equation (2.23) we have

$$C(A + \Delta A)X + CBu_{eq} + CDf = 0 \tag{2.24}$$

Using the same steps as the previous section we can arrive at this equation

$$\dot{X} = [I - B(CB)^{-1}C](AX + \Delta AX + Df) \tag{2.25}$$

Using these conditions of rank relations

$$\text{rank}[B{:}D] = \text{rank}\,[B{:}\Delta AT] = \text{rank}\,[B] \tag{2.26}$$

From Spurgen (1991) it was said that with these conditions , the sliding mode system in (2.25) is insensitive to parameter variations and the external disturbances.

Expression (2.26) is called the invariance condition. Other researchers like Gutman (1979) and Bormish and Leitman(1983) have shown that if system uncertainties and disturbances satisfy the "matching conditions", then the system is completely insensitive on the sliding mode and the effect of disturbances and parameter variations can be minimised by minimising the time required to attain the sliding mode.

2.2.4 Methods of Sliding Mode Design

As we know now from (2.15) that the choice of parameter matrix C , can determine the behaviour of the system on the sliding mode. The asymptotic convergence and desired transient response is also determined by a suitable design of matrix C. There are 2 methods for the design of sliding mode . They are

- quadratic minimisation method
- the eigenstructure assignment method.

Quadratic Minimisation Method

This method for sliding mode design was first proposed by Utkin and Young (1978)

First lets define the cost function to be used

$$J(u) = \frac{1}{2}\int X(t)^T Q X(t)\, dt \tag{2.30}$$

where Q is a symmetric positive definite matrix and t_s denotes the time at which the sliding mode starts.

Partitioning the following matrix compatibility with Y

$$T^T Q T = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \tag{2.31}$$

where matrix T is defined earlier as an orthogonal matrix.

By substitution we have the cost function in the form

$$J(v) = \frac{1}{2}\int_{t_s}^{\infty} \{ Y_1^T Q^* Y_1 + v^T Q_{22} v\}\, dt \tag{2.32}$$

$$\text{where } Q^* = Q_{11} - Q_{12} Q_{22}^{-1} Q_{21} \tag{2.33}$$

$$A^* = A_{11} - A_{12} Q_{22}^{-1} Q_{21} \tag{2.34}$$

$$v(t) = Y_2 + Q_{22}^{-1} Q_{21} Y_1 \tag{2.35}$$

$$\dot{Y}_1 = A^* Y_1 + A_{12} v(t) \tag{2.36}$$

The expression (2.32) is in the form of standard linear quadratic optimal regulator problem.

By minimising expression (2.32), the optimal control v(t) is given by

$$v(t) = Q_{22}^{-1} A_{12}^T P Y_1$$

Using expression (2.36) in expression (2.34) we have

$$Y_2 = -Q_{22}^{-1}[Q_{21} + A_{12}^T P] Y_1 = -F Y_1 \tag{2.37}$$

where the matrix P satisfies the following Ricati equation

$$PQ^* + A^* P - P A_{12} Q_{22}^{-1} A_{12}^T P + Q^* = 0 \tag{2.38}$$

and matrix

$$F = Q_{22}^{-1}[Q_{21} + A_{12}^T P] \tag{2.39}$$

can be determined as required.

Eigenstructure assignment method

Utkin and Young(1978) used this method to design sliding mode. To begin with in this section lets recall (2.2) , which is the systems equation.

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (2.40-$$

a)

and determine the matrix K of the optimal control vector

$$u(t) = -Kx(t) \qquad (2.40\text{-b})$$

Solving the optimisation problem ie substituting (2.40) into (2.2)

$$\dot{X}(t) = (A - BK)X(t) \qquad (2.40-$$

c)

where matrix K is determined in expression (2.13)

During the sliding motion, the state variables must remain in N(C) so that

$$C[\,A - BK\,]\, v_i = \lambda_i C v_i = 0 \qquad (2.41)$$

Expression in the above shows that either

λ_i is zero or $v_i \in N(C)$. Since $A - BK = A_{eq}$ has m zero-valued eigenvalues, we can set $\{\lambda_i : i = 1,\dots,n-m\}$ to be the non-zero eigenvalue and therefore , specifying the corresponding eigenvalues $\{v_i : i = 1,\dots,n-m\}$ fix the null space of C(dim[N(C)] = n-m).

It is noted that C is not uniquely determined because of the equation

$$CV = 0, V = [v_1 \ldots\ldots v_{n-m}] \tag{2.42}$$

has m^2 degree of freedom, which may be easily seen if we define

$$W = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = Tv \tag{2.43}$$

Where the partitioning of $\dot{W}$ is compatible with that of Y, then the expression (2.42) becomes

$$0 = C^T T.Tv = [C_1\ C_2] \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = C_2 [F \quad I_m] \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \tag{2.44}$$

Therefore, F can be determined by the following equation

$$FW_1 = -W_2 \tag{2.45}$$

The work of Dorling and Zinober (1986) has shown that this approach has the drawback that the eigenvector may be assigned arbitrarily, after which the remaining n-m elements are fully determined by the assigned elements. Thus one approach to eigenvector assignment is to select m elements according to some scheme and accept the remaining elements as determined. This may allow a degree of adjustment to be carried out by inspection. Some other eigenvector assignment methods have also been proposed, and the details given can be found

in Moore (1976), Klein and Moore (1977) and Sinswat and Fallside (1977).

2.2.5 Controller designs

In most of the variable structure control schemes, the control law usually consists of a linear component u^L and a nonlinear component u^{NL} which are assumed to form control input u. The linear part is merely a state feedback

$$u^L = KX \tag{2.45}$$

While the nonlinear signal incorporates the discontinuous elements of the control. Some examples of possible types of nonlinearity are as given below.

(a) A nonlinear component with constant gains

$$u_i^{NL} = M_i \operatorname{sgn}(C_i X), \quad M_i > 0 \tag{2.46}$$

(b) A nonlinear component with state-dependent gains

$$u_i^{NL} = m_i(X) \operatorname{sgn}(C_i X) \quad m_i(.) > 0 \tag{2.47}$$

(c) A linear feedback with switching gains

$$u^{NL} = \Psi X \tag{2.48-a}$$

where $\Psi = [\psi_{ij}]$ and

$$\psi_{ij} = \begin{cases} \alpha_{ij} & s_i x_j > 0 \\ \beta_{ij} & s_i x_j < 0 \end{cases} \tag{2.48-b}$$

(d) A unit vector nonlinearity with scale factor

$$u^{NL} = \frac{NX}{\|MX\|} \tag{2.49}$$

where the null spaces of N, M and C are coincident.

The nonlinear control component is discontinuous on the individual hyperplane in cases (a) - (c). This may result in wasted control effort as the system state pierces one hyperplane, and is forced into another surface. In case (d), the individual controls are continuous, except on the intersection of the switching plane variables where all the nonlinear control elements become discontinuous together. The details of cases (a) - (d) are shown in Utkin (1978), Ryan (1983), Young (1977) and Dorling and Zinober (1983). Some special properties and behaviours of a system with control type (d) has been discussed in Surgeon (1991).

2.3 Variable structure control of nonlinear system

2.3.1 System model

In section (2.2) we have briefly reviewed the basic variable structure control theory of linear systems. Most of these ideas can be extended to the variable structure control of nonlinear systems. However, the complexity of the analysis and the controller designs may be increased due to the nonlinearity in the nonlinear system model. From the engineering point of view, the following nonlinear system is often considered (DeCarlo, et al., 1988).

$$\dot{X}(t) = f(t, X) + B(t, X)u(t) \tag{2.50}$$

where the state vector $X(t) \in R^n$, the control input vector $u(t) \in R^m$, $f(t, X) \in R^n$ and $B(t, X) \in R^{nxm}$. Further, each entry in f(t, X) and B(t, X) is assumed to be continuous with continuous bounded derivative with respect to X.

Each entry $u_i(t)$ of the control input vector has the following form

$$u_i(t, X) = \begin{cases} u_i^+(t, X) & \text{with } \sigma_i(X) > 0 \\ u_i^-(t, X) & \text{with } \sigma_i(X) < 0 \end{cases} \qquad i = 1 \ldots m \tag{2.51}$$

where $\sigma_i(x)$ is the ith switching surface associated with the (n-m) dimensional switching surfaces

$$\sigma(X) = \left[\sigma_1(X), \ldots, \sigma_m(X)\right]^T \tag{2.52}$$

2.3.2 Sliding mode and equivalent control

Following the sliding mode design for linear systems in section 2.2.4, the method of equivalent control is a way to determine the system motion restricted to the sliding mode $\sigma(X) = 0$. Suppose that there exists a time $t_0 > 0$, and the state of the system reaches the sliding mode after $t \geq t_0$. On the sliding mode, the following two equations are satisfied

$$\sigma(X(t)) = 0 \qquad t \geq t_0 \tag{2.53-a}$$

$$\dot{\sigma}(X(t)) = 0 \quad t \geq t_0 \tag{2.53-b}$$

Using system equation (2.50), expression (2.53-b) can be expressed as follows

$$\frac{\partial\sigma(X)}{\partial X}[\ f(t, X) + B(t, X)u_{eq}\ ] = 0 \tag{2.54}$$

where u_{eq} is the so called equivalent control which can be obtained from expression (2.54) as follows

$$u_{eq} = -\ [\frac{\partial\sigma(X)}{\partial X}B(t,\ X)\ ]^{-1}\frac{\partial\sigma(X)}{\partial X}f(t, X) \tag{2.55}$$

Using expression (2.55) in system model (2.50), the dynamics of the closed loop system on the sliding mode is given by

$$\dot{X} = [\ I - B(t, X)(\frac{\partial\sigma(X)}{\partial X}B(t, X))^{-1}\frac{\partial\sigma(X)}{\bullet X}\ ]f(t, X) \tag{2.56}$$

Therefore, the problem of the sliding mode design is to choose the parameters in $\sigma(X) = 0$ such that the equivalent system (2.56) is stable. In most of variable structure control schemes for nonlinear systems, the linear sliding modes are often used. Therefore, some methods of sliding mode design in sections 2.2.4 can also be used.

2.3.3 Controller design

In general, for nonlinear system equation (2.50), the control input is a m dimensional vector and each entry has the structure of the form

$$u_i = \begin{cases} u_i^+(t,\ X) & \text{for } \sigma_i(X) > 0 \\ u_i^-(t,\ X) & \text{for } \sigma_i(X) < 0 \end{cases} \tag{2.57}$$

To determine the switched feedback gains in control law (2.57), the following diagonalization method is often used (DeCarlo et al., 1988).

First, a new control vector is considered in terms of a nonsingular transformation

$$u^*(t) = Q^{-1}(t, X)\left[\frac{\partial \sigma(X)}{\partial X}\right]B(t, X)u(t) \tag{2.58}$$

where $Q^{-1}(t, X)(\partial\sigma/\partial X)B(t, X)$ is a nonsingular transformation, and $Q(t, X)$ is an arbitrary mxm diagonal matrix with elements $q_i(t, X)$ $(i = 1, \ldots, m)$ such that $\inf|q_i(t, X)| > 0$.

Using expression (2.58) in expression (2.50), the system dynamics becomes

$$\dot{X} = f(t, X) + B(t, X)\left[\frac{\partial \sigma(X)}{\partial X}B(t, X)\right]^{-1}Q(t, X)u^*(t) \tag{2.59}$$

If u_i^* is selected such that

$$q_i(t, X)\, u_i^{*+} < -\nabla \sigma_i(X)\, f(t, X) \qquad \sigma_i(X) > 0 \tag{2.60-a}$$

$$q_i(t, X)\, u_i^{*-} > -\nabla \sigma_i(X)\, f(t, X) \qquad \sigma_i(X) < 0 \tag{2.60-b}$$

then,

$$\sigma^T(X)\sigma(X) < 0 \tag{2.61}$$

Expression (2.61) is the reaching condition for the system states to reach the sliding mode surfaces $\sigma(X) = 0$. On the sliding mode, the desired system dynamics can be obtained. Also, the control input $u(t)$ can be obtained from equation (2.58).

In addition to the above diagnalization method, other methods which are similar to the ones in section 2.2.5 have been used by many researchers. The details can be found in DeCarlo et al. (1988).

2.3.4 Robust control of nonlinear systems

In practical situations, the system dynamics of a nonlinear system is different from its nominal system model due to parameter uncertainties. To represent parameter uncertainties in the plant, the following state equation is considered (DeCarlo, 1988).

$$\dot{X} = \left[f(t, X) + \Delta f(t, X, r(t)) \right] + \left[B(t, X) + \Delta B(t, X, r(t)) \right] u(t) \qquad (2.62\text{-a})$$

where r(t) is a vector function of uncertain parameters.

In most of researches (Corless and Leitmann, 1981; Gutman and Palmor, 1982; Peterson, 1985), the plant uncertainties Δf and ΔB are assumed to lie in the image of B(t, X) for all variables t and X (this is called "matching condition'). Then dynamic equation (2.62) can be expressed as follows

$$\dot{X} = f(t, X) + B(t, X)u + B(t, X)e(t, X, r, u) \qquad (2.62\text{-b})$$

where e(t, X, r, u) represents system uncertainties.

DeCarlo et al. (1988) shows that if e(t, X, r, u) is bounded by a positive function $\rho(t)$

$$\| e(t, X, r, u) \|_2 \leq \rho(t) \qquad (2.63)$$

and control input has the following form

$$u = u_{eq} + u_n \qquad (2.64\text{-a})$$

where

$$u_{eq} = - \left[\frac{\partial \sigma(X)}{\partial X} B(t, X) \right]^{-1} \left[\frac{\partial \sigma(X)}{\bullet t} + \frac{\partial \sigma(X)}{\partial (X)} f(t, X) \right] \qquad (2.64\text{-b})$$

$$u_n = -\frac{B^T(t, X)\,\nabla_x V(t, X)}{\|B^T(t, X)\,\nabla_x V(t, X)\|^2}\,\hat{\rho}(t, X) \tag{2.64-c}$$

$$\hat{\rho}(t, X) = \alpha + \rho(t, X) \tag{2.64-d}$$

$$\nabla_x V(t, X) = \left[\frac{\partial \sigma(t, X)}{\partial X}\right]^T \sigma(t, X) \tag{2.64-e}$$

then system state can reach the sliding mode surfaces $\sigma(X) = 0$, and the desired system dynamics can be obtained by the suitable choice of the sliding mode parameters.

The results discussed in this section forms the foundation of the variable structure control theory for nonlinear systems. Although there are many classes of nonlinear systems, robustness and convergence of variable structure control systems may be established based on the results in this section (Utkin, 1978; Young, 1978 and DeCarlo et al., 1988).

Chapter 3

Application of Variable Structure Control for Robotic Manipulators

3.1 Introduction

Research has been done over the past few decades to investigate the control algorithms of robotic manipulators and to improve the closed loop system performance. Generally, a robotic manipulator is a non-linear system. Control schemes such as feedback control and adaptive control have been modelled and have been the prime area of research in robotic manipulators. However they cannot deal with systems that have

- large uncertainties
- bounded disturbances
- non-linearities

With the Variable Structure technique, it is the most powerful and most importantly, deal with the above three effectively. The design of robust variable structure control laws for rigid robotic manipulators ensures robustness and asymptotic trajectory tracking. The results on the robustness and convergence have been obtained by many researchers namely Young (1978, 1988), Morgan and Ozguner(1985), Slotine and Sasatry (1983), Yeung (1988) and Leung et al (1991).

In the next section , the dynamics of the robotic manipulator and some recent variable structure control schemes will be briefly reviewed.

3.2 Dynamics of Robotic Manipulators

The robotic manipulator controls the movement of the robot. To move, the robot needs to know the position to move to and to control this we need to know the dynamic properties of the manipulator in order to know how much force is needed to move the robot to its desired position. Accuracy is vital, and deriving the dynamic equation is not simple, especially for large number of degrees of freedom and non-linearities present in the system.

In a perfect environment ie without friction and other disturbances, (Spong and Vidyasagar , 1988) derived the joint space dynamics of an n-link robotic manipulator using Langranian equations.

$$\sum_{j} d_{kj}(q)\,\ddot{q}_j + \sum_{i,j} f_{ijk}(q)\,\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k \quad k = 1, \ldots, n \tag{3.1}$$

d_{kj} are the coefficients of the inertia matrix D(q)

$\phi_k(q)$ are the gravitational forces

τ_k are the input torques

f_{ijk} is the coefficient of the coriolis and centrifugal terms

f_{ijk} can be defined as

$$f_{ijk} = \frac{1}{2}\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} + \frac{\partial d_{ij}}{\partial q_k} \} \tag{3.2}$$

The above equation can also be written as this

$$D(q)\,\ddot{q} + F(q, \dot{q})\,\dot{q} + G(q) = \tau \tag{3.3}$$

where the k,jth element of the matrix F is defined as

$$f_{kj} = \sum_{i=1}^{n} \frac{1}{2}(\frac{\partial d_{ki}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_i} - \frac{\partial d_{ij}}{\partial q_k})\,\dot{q}_i \tag{3.4}$$

and the component of G(q) is ϕ_k.

Equation (3.3) is very complex and non-linear for most robotic manipulators. This is not so for simple robotic manipulators. (Ortega and Spong 1989) found that there are several fundamental properties can be used to facilitate the design of control system. These are

1. The inertia matrix D(q) is symmetric, positive-definite , and both D(q) and $D(q)^{-1}$ are uniformly bounded as a function of q.

2. There is an independent control input for each degree of freedom.

3. The Euler-Lagrange equation for the robotic manipulator is linear in the unknown parameters. All the unknown parameters are constant (eg. link masses, link lengths, moments of inertia, etc.) and appear as coefficients of known functions of the generalised coordinates. By defining each coefficient or a linear combination of them as a separate parameter, a linear relationship results so that we may write equation (3.3) as

$$D(q)\ddot{q} + F(q, \dot{q})\dot{q} + G(q) = Y(q, \dot{q}, \ddot{q})\theta = \tau \qquad (3.5)$$

where Y is an nxr matrix of known functions, known as the regressor, and q is a n-dimensional vector of unknown parameters as shown in Spong and Vidyasagar (1989).

It can be seen later that the manipulator system (3.3) can also be expressed into the generalised form in expression (2.40). Therefore, the basic variable structure theory can be used to design robust controllers and the structural properties mentioned in the

above can then be used to simplify controller designs.

3.3 The Young Controller

In 1978, Young was a pioneer in using variable structure control theory to control robotic manipulators. He later, in 1988 modified and generalised the robust variable structure control scheme. In this section, the Young controller scheme is shown

Given that the state variable is defined as

$$X = [\, q^T \quad \dot{q}^T \,]^T \tag{3.6}$$

Using

$$D(q)\,\ddot{q} + F(q,\dot{q})\,\dot{q} + G(q) = \tau$$

we can obtain that

$$\ddot{q} = -D^{-1}(q)[\,F(q,\dot{q})\,\dot{q} + G(q)] + D^{-1}(q)\tau \tag{3.7}$$

and therefore

$$\dot{X} = \begin{bmatrix} \dot{q} \\ -D(q)^{-1}(F(q,\dot{q})\dot{q} + G(q)) \end{bmatrix} + \begin{bmatrix} 0 \\ D^{-1}(q) \end{bmatrix}\tau \tag{3.8}$$

Using the reference model we have

$$\dot{X}_m = A_m X_m + B_m r \tag{3.9}$$

where we know that $X_m = [\, q_r^T \quad \dot{q}_r^T \,]^T$

and $A_m = \begin{bmatrix} 0 & I \\ A_{m1} & A_{m2} \end{bmatrix}$ $B_m = \begin{bmatrix} 0 \\ B_{m2} \end{bmatrix}$

matrix A_m is stable.

In Young's revised paper in 1988, he defined the output tracking error to be the difference between the reference angle and the systems angle or actual angle. He defined the error vector to be

$$\varepsilon = q_r - q, \qquad e = \left[\, \varepsilon^T, \quad \dot{\varepsilon}^T \,\right]^T \tag{3.10}$$

in the above equation ε is the output tracking error variable. This also means that the difference between the reference state vector and the system state vector would give us the error vector.

The sliding mode surfaces are chosen as

$$\sigma(e) = G_p \varepsilon + G_v \dot{\varepsilon} = 0 \tag{3.11}$$

$$\text{with} \quad \mathrm{Re}\lambda(- G_v G_p) < 0 \tag{3.12}$$

and the control input is designed such that it takes into consideration the states(x), error(e) and the reference model (r).

$$\tau = \psi_x X + \psi_e e + \psi_r r \tag{3.13}$$

$$\text{where} \quad \psi_x = \bar{\psi}_x \mathrm{diag}(\mathrm{sgn}(x_1), \ldots, \mathrm{sgn}(x_{2n})) \tag{3.14}$$

$$\psi_e = \bar{\psi}_e \mathrm{diag}(\mathrm{sgn}(e_1), \ldots, \mathrm{sgn}(e_{2n})) \tag{3.15}$$

$$\psi_r = \bar{\psi}_r \mathrm{diag}(\mathrm{sgn}(r_1), \ldots, \mathrm{sgn}(r_p)) \tag{3.16}$$

$$\bar{\Psi}_j^T = (\psi_{j1}, \ldots, \psi_{jn}) \tag{3.17}$$

$$\psi_{ji} = \begin{cases} k_{ji} & \sigma_i(e) > 0 \\ -k_{ji} & \sigma_i(e) < 0 \end{cases} \tag{3.18}$$

for i=1,.....,m and j = x, e, r

This will force the switching plane variables be driven into the sliding mode surfaces $\sigma(e) = 0$ and the desired error dynamics can be obtained on the sliding mode as follows

$$\text{from} \quad \sigma(e) = G_p\varepsilon + G_v\dot{\varepsilon} = 0 \tag{3.19}$$

$$\dot{\varepsilon} = -G_v^{-1}G_p\varepsilon \tag{3.20}$$

The switched controller gains are designed based on the upper and lower bounds of the unknown system parameters. In this control scheme the exact knowledge of the system is not required, the controller forces the whole system into sliding mode and this allows for good tracking performance on the sliding mode.

Similar techniques are produced in papers by Nicosia and Tomei (1984), Morgan and Ozguner (1985), Bailey (1987) and finally Bartolini and Zolezzi (1985).

3.4 Suction Control

The drawback in these techniques were that the schemes had control torques that were excessive and this caused chattering along the switching line. These chatterings are bad as they cause high-frequency dynamics that are not considered in the modelling.

To overcome this problem, in 1983 Slotine and Sastry proposed a suction control. This control technique contains two parts. In the first step, the trajectory is forced towards the sliding surfaces. In the second step, the controller is restricted to a smaller region or layer that is bounded. This will achieve optimal trade-off between control bandwidth and tracking precision. It will also eliminate chatterings as the controller is trapped in this boundary. Due to this smaller region, the sensitivity of the controller is reduced and is not affected by the unmodelled high frequency dynamics.

3.5 The Leung Controller

It can be seen from the above discussion that most of variable structure control schemes are proposed based on the restrictive assumption that the upper and the lower bounds of all unknown system parameters are known. However, in some situations, it is difficult to know the upper and the lower bounds of all unknown system parameters due to large uncertainties, disturbances and nonlinearities in robotic manipulators. To overcome this difficulty, Leung et al. (1991) proposed a new adaptive variable structure model following control scheme in which only several uncertain bounds of system matrices are used in the controller design.

In Leung et al. (1991), the robotic manipulator (3.3) or state equation (3.8) and the reference model (3.9) are considered. The state equation (3.8) is written in the following form

$$\dot{X} = AX + B\tau \tag{3.21}$$

$$\text{where} \quad A = \begin{bmatrix} 0 & I \\ A_1 & A_2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ B_1 \end{bmatrix} \tag{3.22}$$

It is shown that if the sliding mode is defined as in expression (3.19) and the following matching conditions and uncertain bound conditions are satisfied

$$(I - BB^{+})B_m = 0 \tag{3.23}$$

$$(I - BB^{+})(A_m - A) = 0 \tag{3.24}$$

$$(I - BB^{+})(A_m + A_n) = 0$$

(3.25)

$$\text{where } B^{+} = \begin{bmatrix} 0 & B_1^{-1} \end{bmatrix} \tag{3.26}$$

$$An = \begin{bmatrix} 0 \\ G_v^{-1}G_p(I + G_v^{-1}G_p) \end{bmatrix} \tag{3.27}$$

$$0 < \beta_1 \leq \|G_v B_1\| \leq \beta_2 < \infty \tag{3.28}$$

$$\|B^{+}(A_m - A) - K_1\| < \sum_{i=1}^{3} \alpha_i \|e\|^{i-1} \tag{3.29}$$

$$\|B^{+}B_m - K_2\| < \alpha_4 \tag{3.30}$$

$$\|B^{+}(A_m + A_n) - K_3\| < \alpha_5 \tag{3.31}$$

where $\alpha_i > 0$, $\beta_i > 0$ are some positive numbers

and the control law is designed such that

$$\tau = K_1X + K_2r + K_3e + \psi_1X + \psi_2r + \psi_3e \tag{3.32}$$

where K_1, K_2 and K_3 are constant matrices, and ψ_1, ψ_2 and ψ_3 are discontinuous gain matrices given by

$$\psi 1 = \begin{cases} \sum_{i=1}^{3} \bar{c}_i \|e\|^{i-1} \dfrac{G_v^T \sigma}{\|\sigma\|} \operatorname{sgn}(X)^T & \|\sigma\| \neq 0 \\ 0 & \|\sigma\| = 0 \end{cases} \tag{3.33}$$

$$\psi 2 = \begin{cases} \bar{c}_4 \dfrac{G_v^T \sigma}{\|\sigma\|} \operatorname{sgn}(r)^T & \|\sigma\| \neq 0 \\ 0 & \|\sigma\| = 0 \end{cases} \tag{3.34}$$

$$\psi 3 = \begin{cases} \bar{c}_5 \dfrac{G_2^T \sigma}{\|\sigma\|} \operatorname{sgn}(e)^T & \|\sigma\| \neq 0 \\ 0 & \|\sigma\| = 0 \end{cases} \tag{3.35}$$

$$\frac{d\bar{c}_j}{dt} = g_j \|e\|^{j-1} \|\sigma\| \sum_{i=1}^{2n} |x_i| \qquad j = 1, 2, 3 \tag{3.36}$$

$$\frac{d\bar{c}_4}{dt} = g_4 \|\sigma\| \sum_{i=1}^{n} |r_i| \tag{3.37}$$

$$\frac{d\bar{c}_5}{dt} = g_5 \|\sigma\| \sum_{i=1}^{2n} |e_i| \tag{3.38}$$

$$g_i > 0 \qquad i = 1, \ldots, 5$$

then the output angular position vector asymptotically converges to the desired reference signal vector.

Theoretically, this scheme has many advantages. For example, the exact knowledge of the robotic manipulator are not required and only some uncertain system matrix bounds are used in the controller design. However, it can be easily seen that it is difficult to use this scheme in some situations where the boundary layer technique is utilised or the sampling interval is not zero. Because, in these cases, the output tracking error e(t) and the switching plane variables $\sigma(t)$ cannot converge to zero, and thereafter the adaptive parameters $\bar{c}_i$ ($i = 1, \ldots, 5$) in expressions (3.36) - (3.38) may tend to infinity as time tends to infinity.

However, an important feature of the variable structure control system for the robotic manipulator has been revealed in this scheme, ie, the system matrix bounds can

provide enough structural information for the variable structure controller design. Such a system matrix bounds-based variable structure control technique may not only simplify the controller design, but also further improve the robustness with respect to large parameter uncertainties and nonlinearities..

3.6 Man Controller

This model was included as a chapter of his thesis that was submitted for his Doctorate at the University of Melbourne in 1992.

He found that the variable Structure control had many good features. For instance, its ease of use in linear or non-linear systems. In this chapter, he modified the control scheme to deal with more practical control problems. He also included the nominal systems model and the systems uncertainties to simplify the variable structure controller design. Another feature of his paper was that the bounds of the structure uncertainties was used in the design of the robust variable structure controller.

In his paper a class of large scale time-varying system with n number of interconnected system was considered.

Each subsystem was represented as

$$\dot{x}_i(t) = A_i(x(t))\, x_i(t) + B_i(x(t))\, u_i(t) + \Phi_i(x_j(t),\ \dot{x}_j(t),\ t) \quad i = 1, \ldots, n. \qquad (3.39)$$

where

$$A_i(x(t)) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ & & \cdot & & \\ & & & \cdot & \\ & & & & 1 \\ a_{i1}(x(t)) & \cdots & & & a_{ini}(x(t)) \end{bmatrix}, \quad B_i(x(t)) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} b_i(x(t))$$

$$\Phi_i(x_j, \dot{x}_j, t) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \phi_i(x_j, \dot{x}_j, t)$$

Φ is the dynamical interaction term.

A reference model was also specified for purpose of obtaining the output tracking error and it was defined as

$$\dot{x}_{mi} = A_{mi}x_{mi} + B_{mi}r_i(t) \tag{3.40}$$

r is a piecewise uniformly bounded ith reference input .

Using these assumptions

The subsystem (1) and its reference model (2) structurally satisfy the following so called matching conditions (Xu, Wu and Huang, 1990)

$$(I_i - B_i B_i^{+}) B_{mi} = 0 \tag{3.41}$$

$$(I_i - B_i B_i^{+}) (A_{mi} - A_i) = 0 \tag{3.42}$$

$$(I_i - B_i B_i^{+}) \Phi_i = 0 \tag{3.43}$$

where $B_i^{+} = (B_i^T B_i)^{-1} B_i^T$.

The dynamical interaction term of each subsystem is upper bounded by an unknown positive number. This is treated as a bounded uncertainties

$$\| \Phi_i(x_j, \dot{x}_j, t) \| < k_{1i} \quad i = 1, ..., n. \tag{3.44}$$

$c_i B_i$ is lower bounded by a known positive number

$$c_i B_i > k_{2i}. \tag{3.45}$$

The norm of $A_{mi} - A_i$ is upper bounded by a known positive number

$$\| A_{mi} - A_i \| < k_{3i}. \tag{3.46}$$

The last three assumptions were used as the subsystem structural information in the local controller design.

The control law for each subsystem is designed as follows

$$u_i = k_{ei} e_i + k_{xi} x_i + k_{ri} r_i + \delta_i \tag{3.47}$$

where

$$k_{ei} = \begin{cases} \dfrac{\|c_i\| \, \|A_{mi}\|}{k_{2i} \, \|e_i \, \sigma_i\|} e_i^T \sigma_i & \|e_i \, \sigma_i\| \neq 0 \\ 0_{1xni} & \|e_i \, \sigma_i\| = 0 \end{cases} \tag{3.48}$$

$$k_{xi} = \begin{cases} \dfrac{k_{3i} \, \|c_i\|}{k_{2i} \, \|x_i \, \sigma_i\|} x_i^T \sigma_i & \|x_i \, \sigma_i\| \neq 0 \\ 0_{1xni} & \|x_i \, \sigma_i\| = 0 \end{cases} \tag{3.49}$$

$$k_{ri} = \begin{cases} \dfrac{\|c_i\| \, \|B_{mi}\|}{k_{2i} \, |r_i \, \sigma_i|} r_i \, \sigma_i & |r_i \, \sigma_i| \neq 0 \\ 0 & |r_i \, \sigma_i| = 0 \end{cases} \tag{3.50}$$

$$\delta_i = \begin{cases} \dfrac{\|c_i\| k_{1i}}{k_{2i} \|\sigma_i\|} \sigma_i & \|\sigma_i\| \neq 0 \\ 0 & \|\sigma_i\| = 0 \end{cases} \tag{3.51}$$

where k_{ri}, k_{xi}, k_{ei} are adaptive gain matrices and δ_i is a discontinuous compensator.

In this paper Man only needed three uncertain bounds of subsystem matrices and the dynamical interaction term to be used in the local controller design for each subsystem. This is unlike other decentralised variable structure schemes where there is a requirement to compute and obtain controller gain matrices. This increases the simplicity in design. Moreover there is strong robustness with respect to large system uncertainties and asymptotic convergence of output tracking error.

3.7 Concluding Remarks

In this chapter, the dynamics of the robotic manipulator and some variable structure control schemes were briefly reviewed to highlight the research developments in this area. Although many variable structure control algorithms have been developed for robotic manipulators, still there are many issues that need further investigation. For example, the dynamical interaction term in the control scheme of Man (1992) is taken as a constant. There is a need for an adaptive mechanism to estimate this term as the dynamical interaction term varies for different tracking problems. Some terminal sliding mode techniques can also be used in the robust variable structure controller design to further improve the transient response and robustness.

In the following chapter of the thesis, some problems mentioned in the above will be fully investigated. Two new robust decentralised variable structure control schemes for robotic manipulators will be proposed, and it will be shown to enhance the robust control of robotic manipulators.

Chapter 4

A Decentralised Variable Structure Model Reference Adaptive Control for Robotic Manipulators

4.1 Introduction

Decentralised variable structure control is a powerful method for the control of large scale systems. The general principle of this method is that the upper and the lower bounds of all unknown system parameters are assumed to be known, and a set of local sliding modes are selected for the controlled system to describe the desired system response. The local variable structure controllers are then designed which drive subsystems to move in their local sliding modes. In the sliding modes, the desired system dynamics can be achieved for the overall system, which is completely insensitive to system uncertainties, dynamical interactions and bounded external disturbances (Abbass and Ozgunner, 1984; Ozguner, Yurkovich and Abbass, 1987; Xu, Wu and Huang, 1990; Morgan and Ozguner, 1985). However, in many practical situations, where the controlled system has many unknown parameters, the designs of real time local variable structure controllers based on the upper and the lower bounds

of unknown parameters will be very complicated and time-consuming by using the above control schemes.

In this chapter, a robust decentralised variable structure model following control for a class of large scale systems is proposed based on Leung and Zhou (1991). It is shown that two uncertain matrices in the error dynamics are assumed to be upper bounded by two known constants according to the structural properties of each subsystem, and the dynamical interaction term is upper bounded by an unknown constant, which is adaptively estimated in Lyapunov sense. A local variable structure controller can then be designed for each subsystem. It is easily seen that the local controller design is greatly simplified in this paper due to the fact that only two uncertain matrix bounds and an estimated upper bound of the dynamical interaction term as the subsystem structural information are used in the local variable structure controller design for each subsystem, which are independent of the subsystem order and the number of the unknown parameters. Also, asymptotic error convergence and strong robustness with respect to large system uncertainties can be obtained for the overall system.

It is well known that, in practical situations, some uncertain bounds of subsystem matrices can be obtained from experiments according to the structural properties of the controlled system. However, the upper bound of the dynamical interaction term of each subsystem is hardly known because the maximum value of the norm of the dynamical interaction term of each subsystem is varying for different trajectory tracking problems. To avoid the requirement of the prior knowledge of the upper bound of the dynamical interaction term, an adaptive mechanism is introduced to estimate this uncertain bound in Lyapunov sense. The estimate is then used as a controller parameter in the sense that the effects of dynamical interactions can be eliminated and asymptotic error convergence can be guaranteed. Furthermore, the scheme is applied to the tracking control of rigid robotic manipulators with the result that good tracking performance is obtained.

This chapter is organised as follows: In section 4.2, the system model and control objectives are formulated and an adaptive mechanism to estimate the upper bound of the dynamical interaction term of each subsystem is introduced. In section 4.3, a robust decentralised variable structure model following control scheme is developed. The error convergence and robustness are discussed in detail. In section 4.4, the scheme is applied to the tracking control of rigid robotic manipulator systems. In section 4.5, a simulation example on a two-link robotic manipulator is given in support of the theoretical results. Section 4.6 gives conclusions.

4.2 Problem formulation

Consider a class of large scale multivariable systems consisting of n interconnected subsystems. Each subsystem can be represented as

$$\dot{x}_i(t) = A_i(x(t))\,x_i(t) + B_i(x(t))\,u_i(t) + \Phi_i(x_j(t),\ \dot{x}_j(t),\ t)\quad i = 1, \ldots, n. \tag{4.1}$$

$$A_i(x(t)) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ & & \cdot & & \\ & & & \cdot & \\ & & & & 1 \\ a_{i1}(x(t)) & \cdots & & & a_{ini}(x(t)) \end{bmatrix},\quad B_i(x(t)) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} b_i(x(t))$$

$$\Phi_i(x_j, \dot{x}_j, t) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \phi_i(x_j, \dot{x}_j, t)$$

where $x_i \in R^{ni}$ is the state vector of the ith subsystem, $u_i \in R^1$ is the local control input, and $x(t) = [\, x_1(t), \ \cdots \ x_n(t) \,]^T$ is the state vector of the overall system. $A_i(x(t)) \in R^{ni \times ni}$ and $B_i(x(t)) \in R^{ni \times 1}$ are unknown subsystem parameter matrices.

$a_{ik}(x(t))$ ($i = 1, ..., n$ and $k = 1, ..., n_i$) and $b_i(x(t))$ ($i = 1, ..., n$) are bounded parameters of subsystem matrices $A_i(x(t))$ and $B_i(x(t))$, respectively. Further, the sign of $b_i(x(t))$ is assumed to be known ($b_i > 0$). $\Phi_i(x_j, \dot{x}_j, t) \in R^{ni}$ and $\phi_i(x_j, \dot{x}_j, t) \in R^1$ ($j = 1, ..., n$ and $j \neq i$) are linear or nonlinear functions representing dynamical interactions of subsystems.

The desired performance of the ith subsystem (4.1) is embodied in the definition of a local reference model specified by the designer as

$$\dot{x}_{mi} = A_{mi}x_{mi} + B_{mi}r_i(t) \tag{4.2}$$

$$A_{mi} = \begin{bmatrix} 0 & 1 & 0 \cdots & 0 \\ & & \ddots & \\ & & & 1 \\ a_{mi1} & \cdots & & a_{mini} \end{bmatrix}, \quad B_{mi} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} b_{mi}$$

where $x_{mi} \in R^{ni}$ is the state vector of the ith local reference model, $r_i \in R^1$ is a piecewise continuous and uniformly bounded ith reference input, $A_{mi} \in R^{nixni}$ and $B_{mi} \in R^{nix1}$ are the known constant matrices and A_{mi} is stable.

The local output tracking error vector of each subsystem is defined as

$$e_i = x_{mi} - x_i = [\ e_{i1}, \ldots, e_{ini}\ ]^T = [\ e_{i1}, \dot{e}_{i1}, \ldots, e_{i1}^{(n_i - 1)}\ ]^T \tag{4.3}$$

and a set of local switching plane variables which are assumed to exist in the local error space passing through the origin are defined as

$$\sigma_i = c_i e_i(t) \qquad i = 1,..., n \tag{4.4}$$

where $c_i = [\, c_{i1}, \ldots, c_{ini} \,]$ is a constant vector to describe the desired error dynamics in the sliding mode

$$c_i e_i(t) = 0 \qquad i = 1, \ldots, n \tag{4.5}$$

or
$$c_{ini} e_{i1}^{(ni-1)} +, \ldots, + c_{i1} e_{i1} = 0 \tag{4.6}$$

If the constant parameter vector c_i are selected such that the eigenvalues of the differential equation (4.6) are negative, then, the output error e_i converges to zero asymptotically.

Expression (4.4) can also be expressed in the following form

$$\sigma(t) = [\, \sigma_1(t), \ldots, \sigma_n(t) \,]^T = [\, c_1 e_1(t), \ldots, c_n e_n(t) \,]^T \tag{4.7}$$

Expression (4.7) is called as the switching plane variable vector of the overall system. It is well known that the sufficient condition for the switching plane variable vector in expression (4.7) to be globally stable is given by (Abbass and Ozgunner, 1985; Ozguner, Yurkovich and Abbass, 1987; Xu, Wu and Huang, 1990; Morgan and Ozguner, 1985)

$$\sigma_i \dot{\sigma}_i < 0 \qquad i = 1, \ldots, n \tag{4.8}$$

For the further discussion, the following assumptions are made

(A4.1) The subsystem (4.1) and local reference model (4.2) are controllable (Abbass and Ozgunner, 1985; Ozguner, Yurkovich and Abbass, 1987).

(A4.2) The local state vectors x_i and x_{mi} are measurable for feedback to the ith input (Abbass and Ozgunner, 1985; Ozguner, Yurkovich and Abbass, 1987; Morgan and Ozguner, 1985).

(A4.3) The subsystem (1) and its reference model (2) structurally satisfy the following so called matching conditions (Xu, Wu and Huang, 1990)

$$(I_i - B_i B_i^+) B_{mi} = 0 \tag{4.9}$$

$$(I_i - B_i B_i^+)(A_{mi} - A_i) = 0 \tag{4.10}$$

$$(I_i - B_i B_i^+) \Phi_i = 0 . \tag{4.11}$$

where $B_i^+ = (B_i^T B_i)^{-1} B_i^T$.

(A4.4) The dynamical interaction term of each subsystem is upper bounded by an unknown positive number

$$\| \Phi_i(x_j, \dot{x}_j, t) \| < k_{1i} \quad i = 1, ..., n. \tag{4.12}$$

(A4.5) $c_i B_i$ is lower bounded by a known positive number

$$c_i B_i > k_{2i}. \tag{4.13}$$

(A4.6) The norm of $A_{mi} - A_i$ is upper bounded by a known positive number

$$\| A_{mi} - A_i \| < k_{3i}. \tag{4.14}$$

Remark 4.1: According to the structural properties of the control systems, k_{2i} and k_{3i} in **A4.5** and **A4.6** can be obtained in experiments. However, it is hard to know k_{1i}, the upper bound of the dynamical interactions of each subsystem in **A4.4**, because the maximum value of $\| \Phi_i(x_j, \dot{x}_j, t) \|$ is varying for different tracking problems.

In this paper, we avoid the requirement on the prior knowledge of k_{1i} and the following adaptive mechanisms used to estimate k_{1i}:

$$\dot{\hat{k}}_{1i} = \alpha_i \| c_i \| \, | \sigma_i | \tag{4.15}$$

where α_i is a positive number and $\hat{k}_{1i}$ is the estimate of k_{1i} with an arbitrary positive initial value.

It will be shown later that $\hat{k}_{1i}$ is the estimate of k_{1i} in Lyapunov sense. The detailed discussion of expression (4.15) is given in remark 6.

Remark 4.2: Since expression (4.5) multiplied by any arbitrary nonzero scaler does not change the position of the sliding mode, and the sign of $b_i(x(t))$ of matrix B_i is assumed to be known, assumption **A4.5** can always be valid (Khurana, Ahson and Lamba, 1986).

Remark 4.3: The general principle of the decentralised variable structure control for large scale systems has been investigated by (Abbass and Ozgunner, 1985; Xu, Wu and Huang, 1990). However, as mentioned in the introduction of this paper, if each subsystem has many unknown parameters, the local variable structure controller design in Abbass and Ozgunner (1985) and Xu, Wu and Huang (1990) based on the lower and the upper bounds of all unknown parameters will be very complicated and time-consuming. However, the objective of this paper is to design a local variable structure controller for each subsystem based on assumptions **A4.4** - **A4.6** and the adaptive mechanism in expression (4.15), which is independent of the subsystem order and the number of the unknown system parameters, so that the local controller design can be simplified and asymptotic error convergence and strong robustness with respect to large system uncertainties can be guaranteed for the overall system..

4.3 A decentralised variable structure control scheme

In this paper, the following control law, similar to the one in Abbass and Ozgunner (1985), Xu, Wu and Huang (1990), is used for each subsystem:

$$u_i = k_{ei}e_i + k_{xi}x_i + k_{ri}r_i + \delta_i \tag{4.16}$$

where $k_{ei} \in R^{1xni}$, $k_{xi} \in R^{1xni}$ and $k_{ri} \in R^1$ are adaptive gain matrices which are determined later. $\delta_i \in R^1$ is an adaptive compensator to eliminate the effects the dynamical interactions.

In order to design control law (4.16) based on assumptions **A4.4 - A4.6** and the adaptive mechanism (4.15) to guarantee the robustness and the asymptotic error convergence, we have the following main theorem.

Theorem : The motion of the switching plane variable vector of the overall system in expression (4.7) is globally stable and the output error in expression (4.3) asymptotically converges to zero if the gain matrices and the compensator in the control law (4.16) are chosen as given below

$$k_{ei} = \begin{cases} \dfrac{\|c_i\| \, \|A_{mi}\|}{k_{2i} \, \|e_i \sigma_i\|} e_i^T \sigma_i & \|e_i \sigma_i\| \neq 0 \\ 0_{1xni} & \|e_i \sigma_i\| = 0 \end{cases} \tag{4.17}$$

$$k_{xi} = \begin{cases} \dfrac{k_{3i} \, \|c_i\|}{k_{2i} \, \|x_i \sigma_i\|} x_i^T \sigma_i & \|x_i \sigma_i\| \neq 0 \\ 0_{1xni} & \|x_i \sigma_i\| = 0 \end{cases} \tag{4.18}$$

$$k_{ri} = \begin{cases} \dfrac{\|c_i\| \, \|B_{mi}\|}{k_{2i} \, |r_i \sigma_i|} r_i \sigma_i & |r_i \sigma_i| \neq 0 \\ 0 & |r_i \sigma_i| = 0 \end{cases} \tag{4.19}$$

$$\delta_i = \begin{cases} \dfrac{\hat{k}_{1i}\,\|c_i\|}{k_{2i}\,|\sigma_i|}\sigma_i & |\sigma_i| \neq 0 \\ 0 & |\sigma_i| = 0 \end{cases} \tag{4.20}$$

where $\hat{k}_{1i}$ is updated according to expression (4.15).

Proof: Using expressions (4.1), (4.2) and (4.3), we get the error dynamics of the ith subsystem in the following form

$$\dot{e}_i = A_{mi}e_i + (A_{mi} - A_i)x_i + B_{mi}r_i - B_iu_i - \Phi_i(x_j, \dot{x}_j, t) \tag{4.21}$$

Selecting a scalar positive-definite Lyapunov function

$$v_i = \frac{1}{2}(\sigma_i^2 + \alpha_i^{-1}\tilde{k}_{1i}^2) \tag{4.22}$$

with $$\tilde{k}_{1i} = k_{1i} - \hat{k}_{1i} \tag{4.23}$$

and $$\dot{\tilde{k}}_{1i} = -\dot{\hat{k}}_{1i} \tag{4.24}$$

and differentiating v_i with respect to time, and using expressions (4.4), (4.16) and (4.21), we have

$$\begin{aligned} \dot{v}_i &= \sigma_i\dot{\sigma}_i - \alpha_i^{-1}\tilde{k}_{1i}\dot{\hat{k}}_{1i} \\ &= c_i(A_{mi} - B_ik_{ei})e_i\sigma_i + c_i\left[(A_{mi} - A_i) - B_ik_{xi}\right]x_i\sigma_i \\ &\quad + c_i(B_{mi} - B_ik_{ri})r_i\sigma_i - \left[c_i(\Phi_i + B_i\delta_i)\sigma_i + \alpha_i^{-1}\tilde{k}_{1i}\dot{\hat{k}}_{1i}\right] \end{aligned} \tag{4.25}$$

Using expressions (4.17)-(4.20), four terms in expression (4.25) satisfy the following inequalities

$$\begin{aligned} & c_i(A_{mi} - B_ik_{ei})e_i\sigma_i \\ &= c_iA_{mi}e_i\sigma_i - \frac{c_iB_i}{k_{2i}}\|c_i\|\,\|A_{mi}\|\,\|e_i\sigma_i\| \\ &< c_iA_{mi}e_i\sigma_i - \|c_i\|\,\|A_{mi}\|\,\|e_i\sigma_i\| \le 0 \end{aligned} \tag{4.26}$$

$$c_i\left[(A_{mi}-A_i)-B_ik_{xi}\right]x_i\sigma_i$$

$$= c_i(A_{mi}-A_i)x_i\sigma_i - \frac{c_iB_i}{k_{2i}}k_{3i}\,\|c_i\|\,\|x_i\sigma_i\|$$

$$< c_i(A_{mi}-A_i)x_i\sigma_i - k_{3i}\|c_i\|\,\|x_i\,\sigma_i\| \le 0 \qquad (4.27)$$

$$c_i(B_{mi}-B_ik_{ri})r_i\sigma_i$$

$$= c_iB_{mi}r_i\sigma_i - \frac{c_iB_i}{k_{2i}}\,\|c_i\|\,\|B_{mi}\|\,\|r_i\,\sigma_i\|$$

$$< c_iB_{mi}r_i\sigma_i - \|c_i\|\,\|B_{mi}\|\,|r_i\,\sigma_i| \le 0 \qquad (4.28)$$

$$-\left[\,c_i(\Phi_i+B_i\delta_i)\sigma_i + \alpha_i^{-1}\tilde{k}_{1i}\,\dot{\hat{k}}_{1i}\right]$$

$$= -\,c_i\Phi_i\sigma_i - c_iB_i\frac{\hat{k}_{1i}\,\|c_i\|}{k_{2i}|\sigma_i|}\sigma_i^2 - \alpha_i^{-1}\tilde{k}_{1i}\,\dot{\hat{k}}_{1i}$$

$$\le -c_i\,\Phi_i\,\sigma_i - \hat{k}_{1i}\|c_i\|\,|\sigma_i| - (k_{1i}-\hat{k}_{1i})\,\|c_i\|\,|\sigma_i|$$

$$= -c_i\,\Phi_i\,\sigma_i - k_{1i}\|c_i\|\,|\sigma_i|$$

$$\le -\,(k_{1i}-\|\Phi_i\|)\,\|c_i\|\,|\sigma_i| < 0 \quad \sigma_i \ne 0 \qquad (4.29)$$

Then $\dot{v}_i = \sigma_i\,\dot{\sigma}_i < 0 \qquad \sigma_i \ne 0 \qquad (4.30)$

Expression (4.30) means that the global reaching condition in expression (4.8) is satisfied and therefore the motion of the switching plane variable vector of the overall system is globally stable.

On the sliding mode, expressions (4.5) or (4.6) is satisfied, then, the output error e_i converges to zero asymptotically.

Remark 4.4: One can show, from expressions (4.25) - (4.30), that the derivative of σ_i satisfies the following inequalities:

$$|\dot{\sigma}_i| > (k_{1i} - \|\Phi_i\|)\,\|c_i\| > 0 \qquad \sigma_i \neq 0 \tag{4.31}$$

This together with expression (4.30) means that σ_i goes to zero in a finite time, and then the sliding motion is started on the sliding mode surface $\sigma_i = 0$.

Remark 4.5: Expressions (4.17) - (4.20) show that, unlike the schemes in (Abbass and Ozgunner, 1985; Xu, Wu and Huang, 1990), the local variable structure controller design in this paper requires only two uncertain matrix bounds and an adaptive estimate of the norm of the dynamical interaction term of each subsystem, and the involved computations in Abbass and Ozgunner (1985), Ozguner, Yurkovich and Abbass (1987), Xu, Wu and Huang (1990) and Morgan and Ozguner (1985), to obtain the real time local controller gain matrices are not required here. Therefore, the local variable structure controller design is greatly simplified.

Remark 4.6: The adaptive mechanism in expression (4.15) can also written as the following form:

$$\hat{k}_{1i} = \hat{k}_{1i}(0) + \int_0^t \alpha_i \|c_i\|\,|\sigma_i|\,dt \tag{4.32}$$

with arbitrary positive initial value $\hat{k}_{1i}(0)$.

It can be seen from expression (4.32) that the upper bound of the norm of the dynamical interaction term in each subsystem is estimated in Lyapunov sense, and it is not necessary for the estimate to converge to the true upper bound of the norm of the dynamical interaction term of each subsystem because the value of the estimate in expression (4.32) is increased until the local sliding variable σ_i converges to zero. Therefore, how large the true upper bound of the norm of the dynamical interaction term in each subsystem is not required .

In addition, Assumption A4.4 is made for the local controller design using only the local information. If the states from other subsystems can be used in local controller design, the expression (4.12) in A4.4 can be modified into the following form:

$$\| \Phi_i(x_j, \dot{x}_j, t) \| < k_{0i} + k_{1i} f(x_j, \dot{x}_j, t) \tag{4.33}$$

where k_{0i} and k_{1i} are unknown positive numbers to be adaptively estimated and f(.) is a known positive function.

In this case, an adaptive mechanism, which is similar to expression (4.15), can be used to estimate k_{0i} and k_{1i} in expression (4.33) and the similar results for the controller design and the stability analysis can then be obtained.

Remark 4.7: The strong robustness property of the proposed control scheme is obvious. First, although the large scale system in expression (4.1) has high nonlinearities, dynamical couplings and uncertain dynamics, the proposed decentralised controller can make the switching plane variable vector in expression (4.7) converge to zero in a finite time (see remark 4.4). Second, in the sliding mode, the system is completely insensitive to nonlinearities, dynamical couplings and uncertain dynamics. The behaviour of the error dynamics is determined only by the sliding mode parameters in expression (4.6).

Remark 4.8: If the system in expression (4.1) has the bounded input disturbance, The dynamical couplings together with the input disturbance can be treated the bounded uncertainties (see expression (4.12)). Then the decentralised controller for each subsystem has the same structure as in expressions (4.16) - (4.20).

Remark 4.9: While the local control law u_i in expression (4.16) crosses the sliding mode $c_i\sigma_i = 0$, chattering occurs in the system and undesired system dynamics may be excited. To eliminate the problem of chattering, the controller gain matrices and the

compensator in expressions (4.17) - (4.20) can be modified using boundary layer technique as:

$$k_{ei} = \begin{cases} \dfrac{\|c_i\| \, \|A_{mi}\|}{k_{2i} \, \|e_i \, \sigma_i\|} e_i^T \sigma_i & \|e_i \, \sigma_i\| \geq \delta_{1i} \\[2ex] \dfrac{\|c_i\| \, \|A_{mi}\|}{k_{2i} \, \delta_{1i}} e_i^T \sigma_i & \|e_i \, \sigma_i\| < \delta_{1i} \end{cases} \tag{4.34}$$

$$k_{xi} = \begin{cases} \dfrac{k_{3i} \, \|c_i\|}{k_{2i} \, \|x_i \, \sigma_i\|} x_i^T \sigma_i & \|x_i \, \sigma_i\| \geq \delta_{2i} \\[2ex] \dfrac{k_{3i} \, \|c_i\|}{k_{2i} \, \delta_{2i}} x_i^T \sigma_i & \|x_i \, \sigma_i\| < \delta_{2i} \end{cases} \tag{4.35}$$

$$k_{ri} = \begin{cases} \dfrac{\|c_i\| \, \|B_{mi}\|}{k_{2i} \, |r_i \, \sigma_i|} r_i \, \sigma_i & |r_i \, \sigma_i| \geq \delta_{3i} \\[2ex] \dfrac{\|c_i\| \, \|B_{mi}\|}{k_{2i} \, \delta_{3i}} r_i \, \sigma_i & |r_i \, \sigma_i| < \delta_{3i} \end{cases} \tag{4.36}$$

$$\delta_i = \begin{cases} \dfrac{\hat{k}_{1i} \, \|c_i\|}{k_{2i} \, |\sigma_i|} \sigma_i & |\sigma_i| \geq \delta_{4i} \\[2ex] \dfrac{\hat{k}_{1i} \, \|c_i\|}{k_{2i} \, \delta_{4i}} \sigma_i & |\sigma_i| < \delta_{4i} \end{cases} \tag{4.37}$$

where δ_{1i}, δ_{2i}, δ_{3i} and δ_{4i} are positive numbers

The above local boundary layer control law offers a continuous approximation to the discontinuous local control law inside the local boundary layer and guarantees attractiveness to the boundary layer and ultimate boundedness of the output tracking error to within a neighbourhood of the origin. This will achieve optimal trade-off between the control bandwidth and tracking precision. Therefore, the chattering and sensitivity of the local controller to parameter uncertainties and dynamical interactions can be eliminated. But the drawback is that nonzero error exists. The detailed discussion on the boundary layer technique can be found in Corless and Leitman (1981).

4.4 Application of the scheme to robotic manipulators

In this section, the control scheme derived in section 4.3 is applied to the robust tracking control of rigid robotic manipulators.

The dynamics of an n-joint rigid robotic manipulator can be described by the following second-order nonlinear vector differential equation

$$M(q)\,\ddot{q} + F(q, \dot{q}) + G(q) = U(t) \tag{4.38}$$

where q is the nx1 vector of joint angular positions, U(t) is the nx1 vector of applied joint torques (control inputs), M(q) is the nxn symmetric positive-definite inertial matrix, $F(q, \dot{q})$ is the vector of coriolis and centrifugal forces, and G(q) is the vector of gravitational torques.

For the use of the decentralised control scheme proposed in section 4.3, it is convenient to treat each joint as a subsystem. The manipulator dynamic equation (4.38) is therefore represented by a collection of n second-order nonlinear scalar differential equations

$$m_{ii}(q)\,\ddot{q}_i + \Big[\sum_{\substack{j=1 \\ j\neq i}}^{n} m_{ij}(q)\,\ddot{q}_j(t)\Big] + f_i(q, \dot{q}) + g_i(q) = u_i \quad i = 1, \ldots, n$$

(4.39)

where the subscript "i" refers to the ith element, $m_{ii}(q)$ is the time varying effective inertia seen at the ith joint, and is always positive due to the positive-definiteness of M(q).

Defining $x_i = [\, q_i \quad \dot{q}_i \,]^T$, expression (4.40) can be written in terms of state variables:

$$\dot{x}_i = \begin{bmatrix} \dot{q}_i \\ -m_{ii}^{-1}(f_i + g_i) \end{bmatrix} + \begin{bmatrix} 0 \\ m_{ii}^{-1} \end{bmatrix} u_i - \begin{bmatrix} 0 \\ m_{ii}^{-1} \end{bmatrix} \sum_{\substack{j=1 \\ j\neq i}}^{n} m_{ij}\ddot{q}_j$$

$$= \begin{bmatrix} 0 & 1 \\ a_{i21} & a_{i22} \end{bmatrix} x_i + \begin{bmatrix} 0 \\ m_{ii}^{-1} \end{bmatrix} u_i + \begin{bmatrix} 0 \\ \phi_i \end{bmatrix}$$

$$= A_i x_i + B_i u_i + \Phi_i \qquad i = 1, \ldots n \tag{4.40}$$

and the ith local reference model is given in the following form:

$$\dot{x}_{mi} = A_{mi} x_{mi} + B_{mi} r_i \tag{4.41}$$

$$A_{mi} = \begin{bmatrix} 0 & 1 \\ a_{mi21} & a_{mi22} \end{bmatrix}$$

$$B_{mi} = \begin{bmatrix} 0 \\ b_{mi1} \end{bmatrix}$$

where a_{mi21}, a_{mi22} and b_{mi1} are known constant numbers determined from an engineering point of view.

The error dynamics is then given by

$$\dot{e}_i = A_{mi} e_i + (A_{mi} - A_i) x_i + B_{mi} r_i - B_i u_i - \Phi_i \tag{4.42}$$

where $e_i = [\varepsilon_i, \ \dot{\varepsilon}_i]^T$ and $\varepsilon_i = q_{mi} - q_i$.

In this case, a set of local sliding manifolds are defined as:

$$\sigma_i = c_i e_i \qquad i = 1, \ldots, n \tag{4.43}$$

where $c_i = [c_{i1} \quad c_{i2}]$, whose parameters are positive constant numbers

If the conditions in expressions (4.12), (4.13) and (4.14) are satisfied for all q and $\dot{q}$, the global reaching condition (4.8) can then be satisfied by the use of control law (4.16) and the controller gain matrices and the compensator in expressions (4.17) - (4.20).

On the sliding mode, the desired error dynamics is given by

$$\dot{\varepsilon}_i = - c_{i2}^{-1} c_{i1} \varepsilon_i \tag{4.44}$$

Therefore, the output tracking error ε_i (i = 1, ... n) converges to zero asymptotically.

4.5 A simulation example

In this section, a simple two-link robotic manipulator is simulated to test the decentralised variable structure model following control scheme derived in section 4.3. The full dynamic equations used in this simulation are given as follows.

$$m_{11}(q_2)\ddot{q}_1 + m_{12}(q_2)\ddot{q}_2 = \beta_{12}(q_2)\dot{q}_1^2 + 2\beta_{12}(q_2)\dot{q}_1\dot{q}_2 + \gamma_1(q_1, q_2)g + u_1$$

$$m_{12}(q_2)\ddot{q}_1 + m_{22}\ddot{q}_2 = -\beta_{12}(q_2)\dot{q}_2^2 + \gamma_2(q_1, q_2)g + u_2$$

where

$$m_{11}(q_2) = (m_1 + m_2)l_1^2 + m_2l_2^2 + 2m_2l_1\cos(q_2) + J_1$$

$$m_{22} = m_2l_2^2 + J_2$$

$$m_{12}(q_2) = m_2l_2^2 + m_2l_1l_2\cos(q_2) + m_2l_2\cos(q_1 + q_2)$$

$$\beta_{12}(q_2) = m_2l_1l_2\sin(q_2)$$

$$\gamma_1(q_1, q_2) = -((m_1 + m_2)l_1\cos(q_2) + m_2l_2\cos(q_1 + q_2))$$

$$\gamma_2(q_1, q_2) = -m_2l_2\cos(q_1 + q_2)$$

The parameter values are

$$l_1 = 1\text{m}, \quad l_2 = 0.8\text{m}$$

$$J_1 = 5\text{kg.m}, \quad J_2 = 5\text{kg.m}$$

$$m_1 = 0.5\text{kg}, \quad m_2 = 1.5\text{kg}$$

Now, each link is considered as a subsystem

$$S_1 \triangleq x_1 = [\, q_1, \ \dot{q}_1 \,]^T$$

$$S_2 \triangleq x_2 = [\, q_2, \ \dot{q}_2 \,]^T$$

The reference model used for each subsystem has the following form:

$$\begin{bmatrix} \dot{q}_{mi} \\ \ddot{q}_{mi} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} q_{mi} \\ \dot{q}_{mi} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r_i(t) \qquad i = 1, 2$$

where $r_i(t) = 5 \quad t > 0$

In this example, we let each subsystem and its local reference model have different initial values.

$$[\, q_{m1}(0),\ \dot{q}_{m1}(0) \,]^T = [\, 0.2,\ 0 \,]^T$$

$$[\, q_1(0),\ \dot{q}_1(0) \,]^T = [\, 0.4,\ 0 \,]^T$$

$$[\, q_{m2}(0),\ \dot{q}_{m2}(0) \,]^T = [\, 2,\ 0 \,]^T$$

$$[\, q_2(0),\ \dot{q}_2(0) \,]^T = [\, 1.8,\ 0 \,]^T$$

The initial values of the estimates of the upper bounds of dynamical interaction terms in expression (4.15) and two uncertain system matrix bounds in expressions (4.13) and (4.14) for subsystems S_1 and S_2 are chosen as

$$\hat{k}_{11}(0) = 1.5,\ k_{21} = 2,\ k_{31} = 2$$

$$\hat{k}_{12}(0) = 1.5,\ k_{22} = 2,\ k_{32} = 2$$

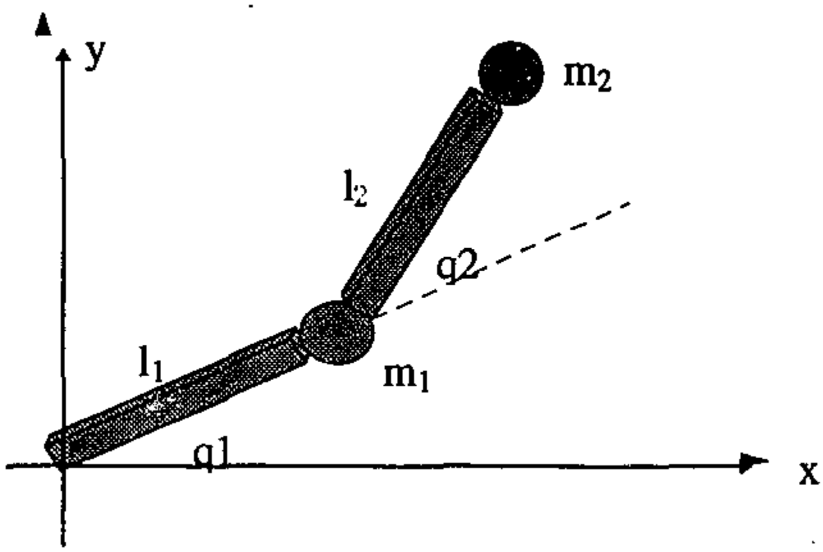
Switching plane variables are prescribed as

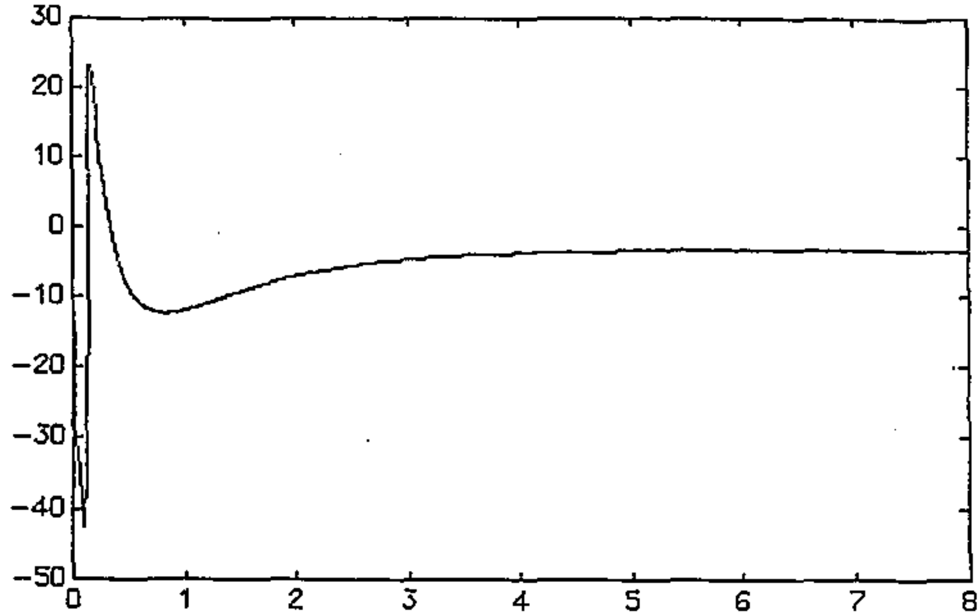
$$\sigma_1 = 5\,\varepsilon_1 + \dot{\varepsilon}_1$$

$$\sigma_2 = 5\,\varepsilon_2 + \dot{\varepsilon}_2$$

The computer simulation with a sampling interval $\Delta T = 0.01$s is performed. Fig.4.1 - Fig.4.3 show the output trackings, tracking errors and the control inputs by the use of control law (4.16) with the gain matrices and the compensator in expressions (4.17) - (4.20). It can be seen that good tracking performance has been achieved, but the control inputs have undesired chatterings. To eliminate the chatterings, the boundary layer scheme using control law (4.16) with the gain matrices and the compensator in expressions (4.34) - (4.37) is implemented. The simulation results are shown in Fig.4.4

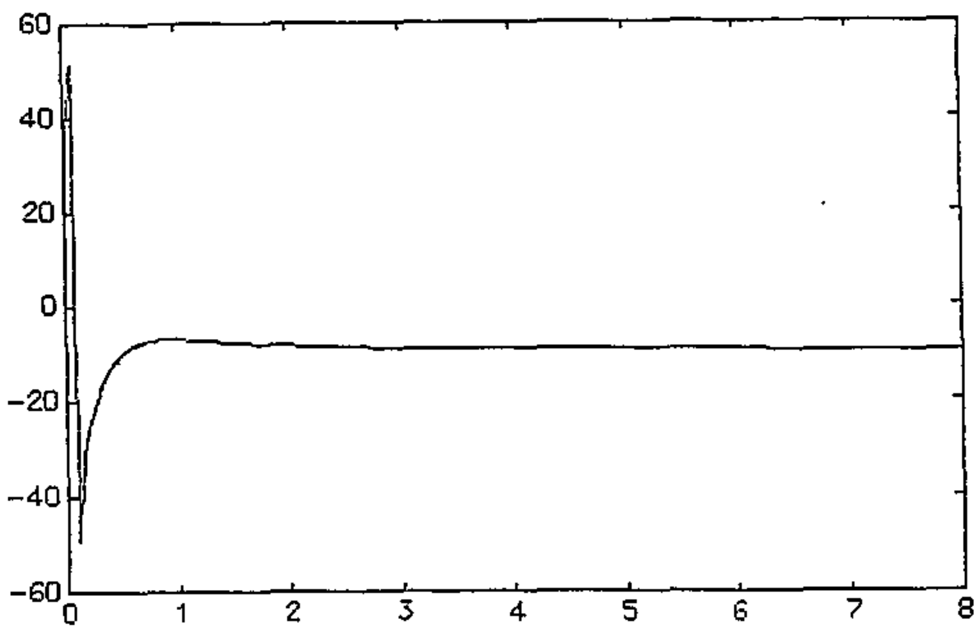
- Fig.4.6. It is shown that not only the problem of chattering is eliminated, but also the amplitude of the control inputs is greatly reduced.





Time t (sec)

Fig.5.7-(a) The control input of joint 1 with local boundary layer controller



Time t (sec)

Fig.5.7-(b) The control input of joint 2 with local boundary layer controller

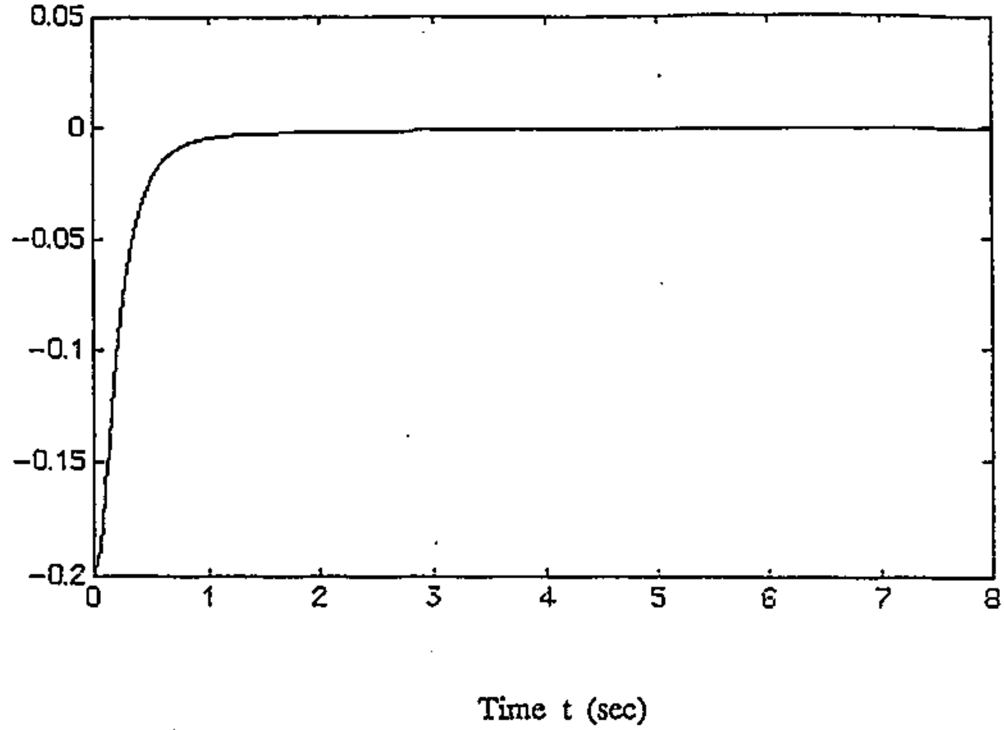


Fig.5.6-(a) The tracking error of joint 1 with local boundary layer controller

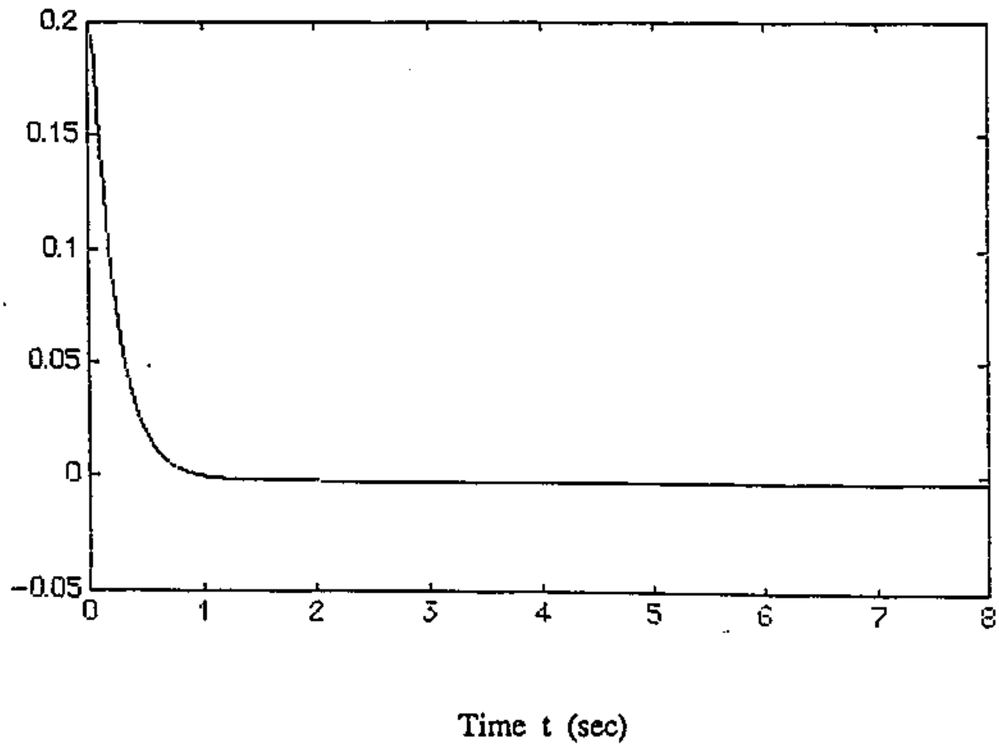


Fig.5.6-(b) The tracking error of joint 2 with local boundary layer controller

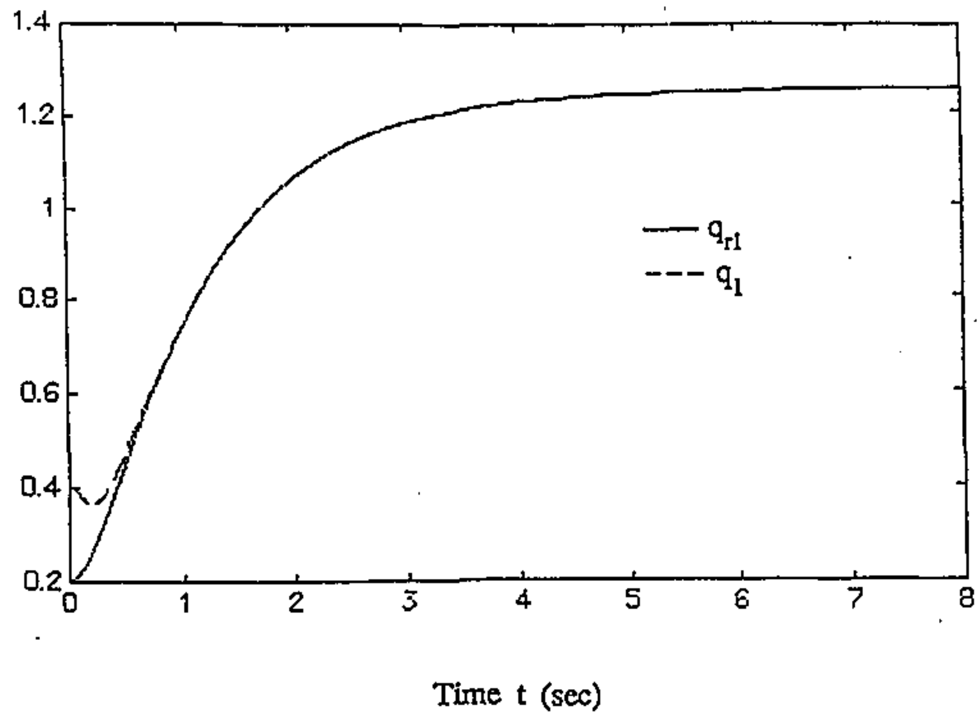


Fig.5.2-(a) The output tracking of joint 1

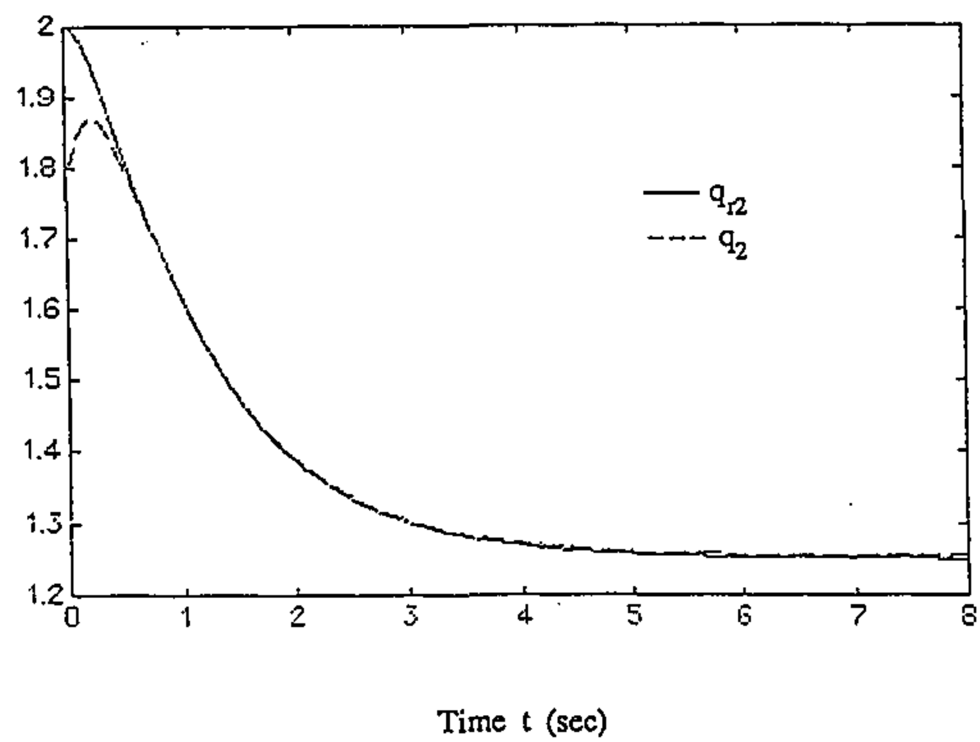


Fig .5.2-(b) The output tracking of joint 2

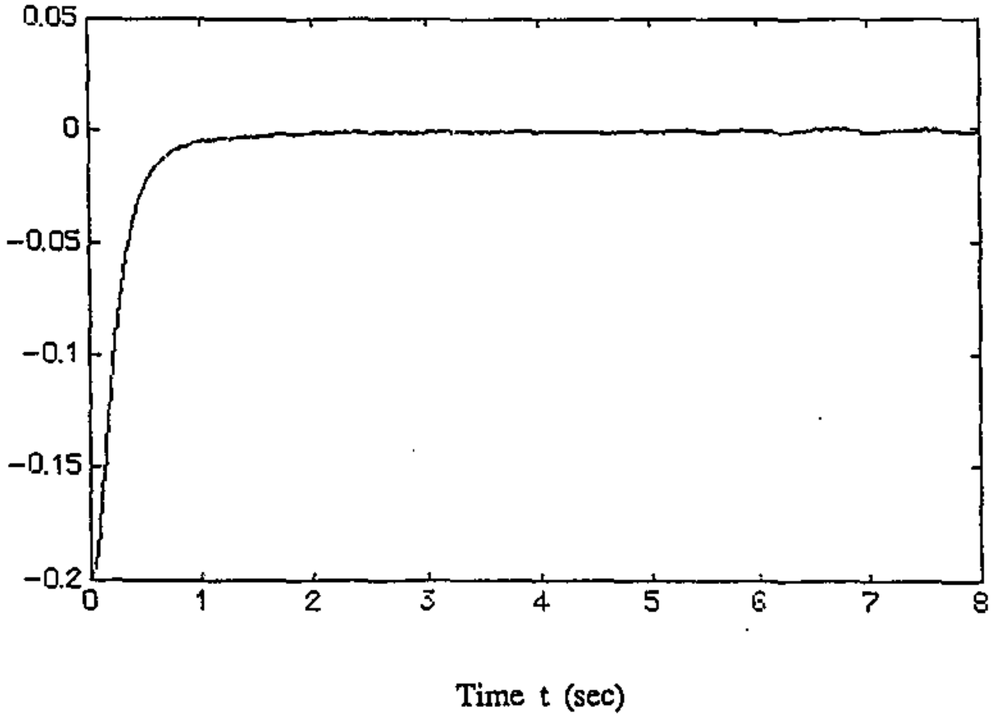


Fig.5.3-(a) The tracking error of joint 1

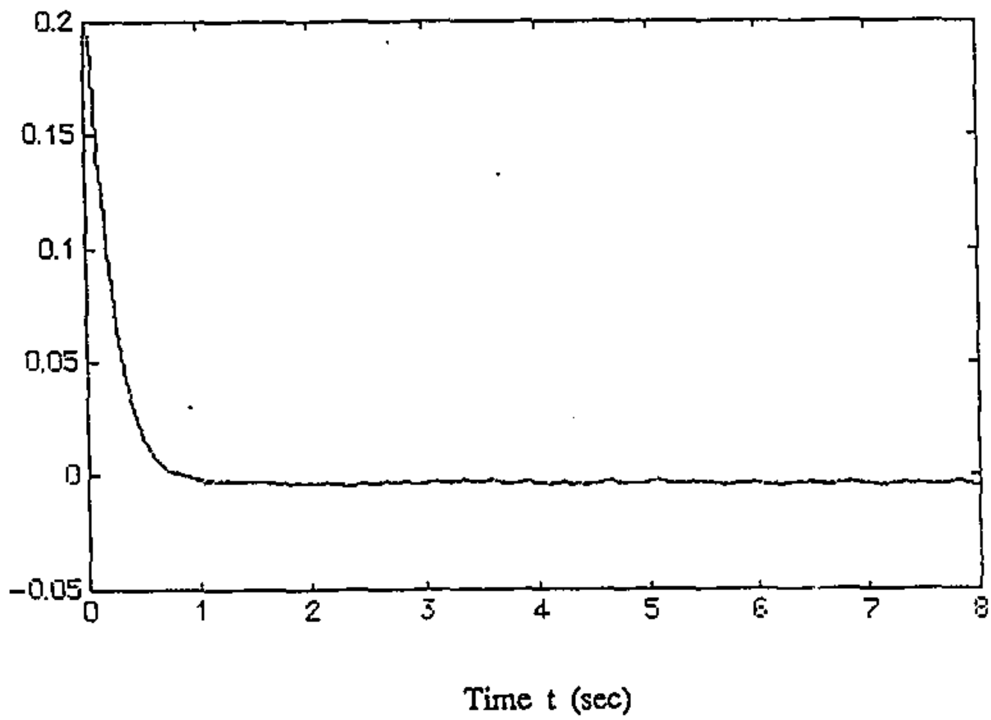


Fig.5.3-(b) The tracking error of joint 2

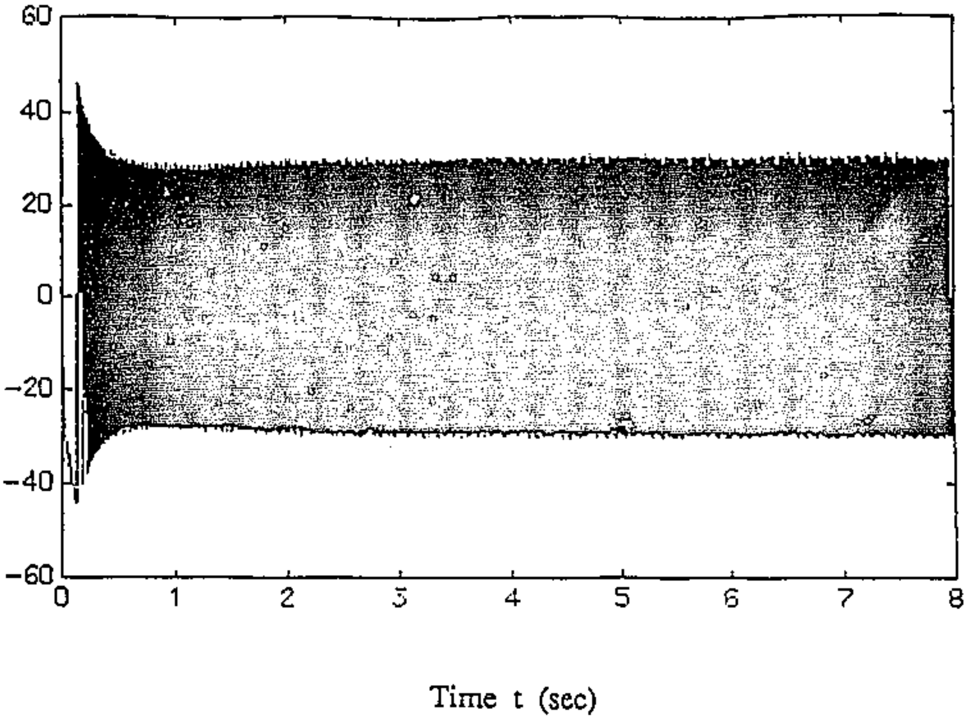


Fig.5.4-(a) The control input of joint 1

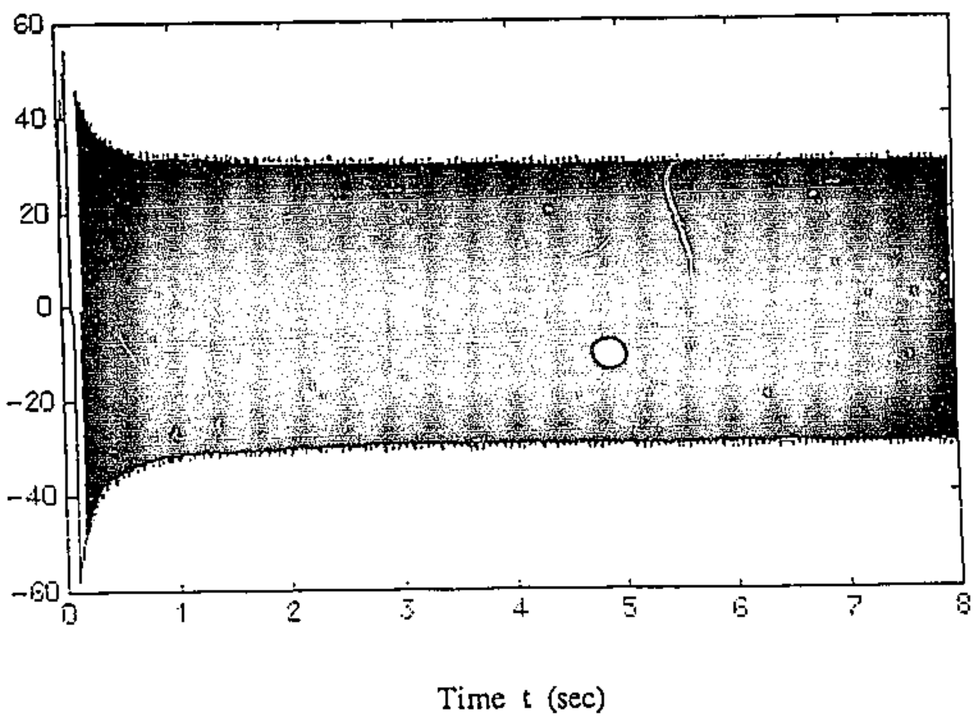
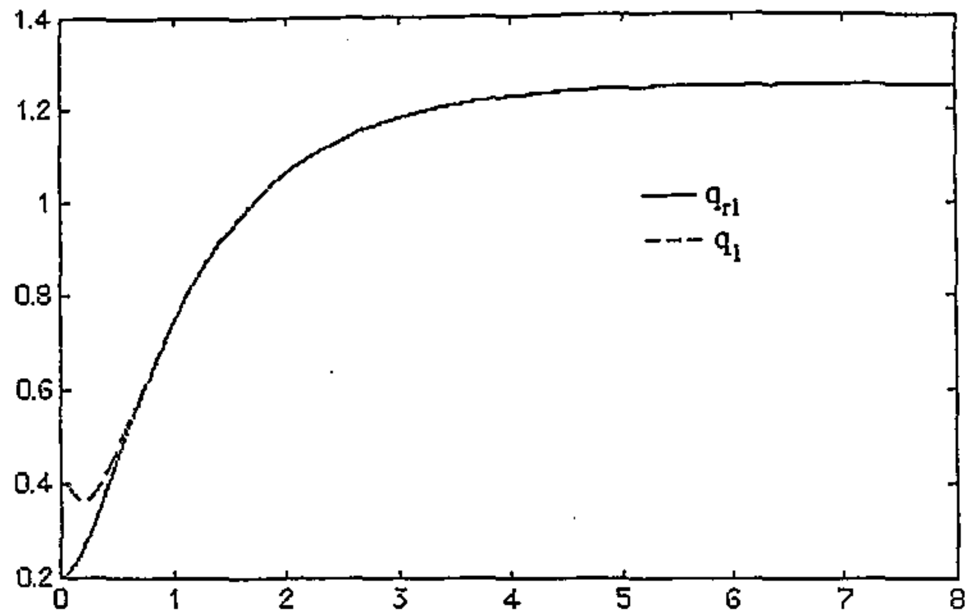
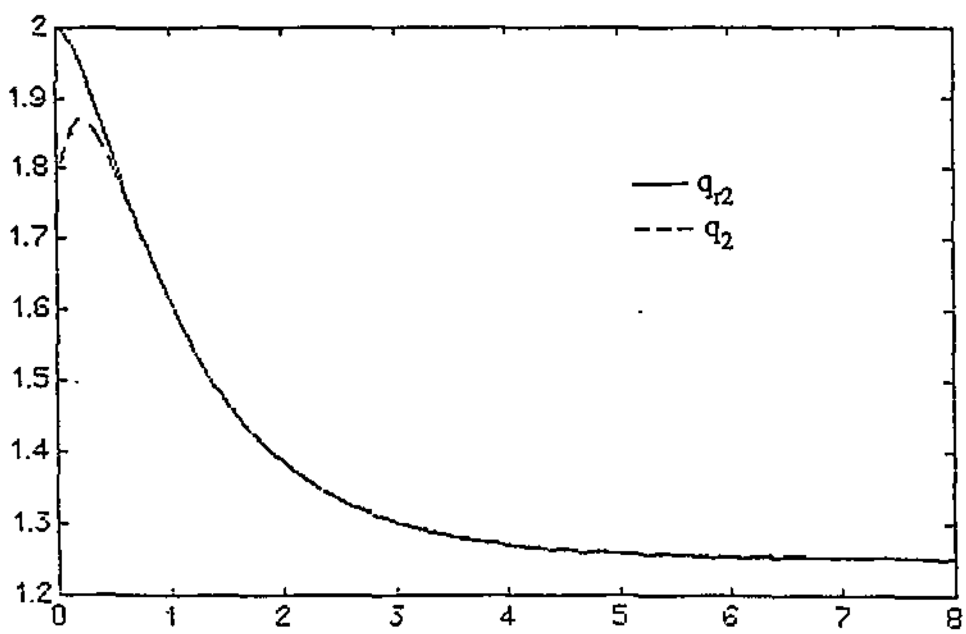


Fig.5.4-(b) The control input of joint 2



Time t (sec)

Fig 5.5-(a) The output tracking of joint 1 with local boundary layer controller



Time t (sec)

Fig 5.5-(b) The output tracking of joint 2 with local boundary layer controller

4.6 Concluding remarks

A robust decentralised model following control scheme using variable structure theory for a class of large scale systems is investigated in this chapter. The main contributions of this chapter are that only two uncertain bounds of the subsystem matrices and an Lyapunov estimate of the norm of the dynamical interaction term are required in the local controller design for each subsystem. The controller design is greatly simplified and robustness and asymptotic error convergence are guaranteed for the overall system. The scheme has been successfully applied to the tracking control of robotic manipulators.

Chapter 5

Decentralised Model Following Control Using Terminal Sliding Mode Technique

5.1 Introduction

In this chapter, we investigate a new terminal sliding mode technique to improve the error convergence developed by (Man. Z. et al, 1992). It is shown that a multi variable terminal sliding mode is first defined for the model following control of rigid robotic manipulators, and the relationship between the terminal sliding variable vector and the error dynamics of the closed loop system is established in order for the stability analysis of the error dynamics for each subsystem. The robust local terminal sliding controller can be designed based on a few structural properties of rigid robotic manipulators. Unlike the linear sliding mode control schemes, the terminal sliding variable vector has a non linear term of the velocity error. By suitably designing a controller, the local terminal sliding variable vector can converge to zero in a finite time, and the output tracking error can then converge to zero on the terminal sliding mode in a finite time.

Similar to the conventional linear sliding mode control schemes, the proposed terminal sliding mode control scheme can also provide the strong robustness with respect to large uncertain dynamics and bounded disturbances for the overall system. Further, the controller design is greatly simplified in the sense that only a few uncertain bounds of the controlled robot system are used as the controller parameter.

This chapter is organised as follows: In section 5.2, the system model and control objectives are formulated for each subsystem is introduced. In section 5.3, a robust decentralised variable structure model following control scheme is developed.. In section 5.4, the scheme is applied to the tracking control of rigid robotic manipulator .Section 5.5 gives conclusions.

5.2 Problem Formulation

Consider a class of large scale multivariable systems consisting of n interconnected subsystems. Each subsystem can be represented as

$$\dot{x}_i(t) = A_i(x(t))\, x_i(t) + B_i(x(t))\, u_i(t) + \Phi_i(x_j(t), \dot{x}_j(t), t) \quad i = 1, \ldots, n. \tag{5.1}$$

$$A_i(x(t)) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & 1 \\ a_{i1}(x(t)) & \cdots & & & a_{ini}(x(t)) \end{bmatrix}, \quad B_i(x(t)) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} b_i(x(t))$$

$$\Phi_i(x_j, \dot{x}_j, t) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \phi_i(x_j, \dot{x}_j, t)$$

where $x_i \in R^{ni}$ is the state vector of the ith subsystem, $u_i \in R^1$ is the local control input, and $x(t) = [\ x_1(t), \ \cdots \ x_n(t)\ ]^T$ is the state vector of the overall system. $A_i(x(t)) \in R^{nixni}$ and $B_i(x(t)) \in R^{nix1}$ are unknown subsystem parameter matrices. $a_{ik}(x(t))$ ($i = 1, ..., n$ and $k = 1, ..., n_i$) and $b_i(x(t))$ ($i = 1, ..., n$) are bounded parameters of subsystem matrices $A_i(x(t))$ and $B_i(x(t))$, respectively. Further, the sign of $b_i(x(t))$ is assumed to be known ($b_i > 0$). $\Phi_i(x_j, \dot{x}_j, t) \in R^{ni}$ and $\phi_i(x_j, \dot{x}_j, t) \in R^1$ ($j = 1, ..., n$ and $j \neq i$) are linear or nonlinear functions representing dynamical interactions of subsystems.

The desired performance of the ith subsystem (1) is embodied in the definition of a local reference model specified by the designer as

$$\dot{x}_{mi} = A_{mi}x_{mi} + B_{mi}r_i(t) \tag{5.2}$$

$$A_{mi} = \begin{bmatrix} 0 & 1 & 0 \cdots & 0 \\ & & \cdot & \\ & & & \cdot \\ & & & \cdot \quad 1 \\ a_{mi1} & \cdots & & a_{mini} \end{bmatrix}, \qquad B_{mi} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} b_{mi}$$

where $x_{mi} \in R^{ni}$ is the state vector of the ith local reference model, $r_i \in R^1$ is a piecewise continuous and uniformly bounded ith reference input, $A_{mi} \in R^{nixni}$ and $B_{mi} \in R^{nix1}$ are the known constant matrices and A_{mi} is stable.

The local output tracking error vector of each subsystem is defined as

$$e_i = x_{mi} - x_i = [\ \varepsilon_{i1}, \ldots, \varepsilon_{ini}\ ]^T \tag{5.3}$$

and a set of local switching plane variables which are assumed to exist in the local error space passing through the origin are defined as

$$\sigma_i = c_i e_i + c_{i1}(\varepsilon_{i1}{}^p - \varepsilon_{i1}) \qquad i = 1,\dots, n \qquad (5.4)$$

where $c_i = \left[c_{i1}, 1, 0, ..0 \right]^T$ is a constant vector to describe the desired error dynamics in the sliding mode

$$c_i e_i + c_{i1}(\varepsilon_{i1}{}^p - \varepsilon_{i1}) = 0 \qquad (5.5)$$

or

$$c_{i1}\varepsilon_{i1}{}^p + \dot{\varepsilon}_{i1} = 0 \qquad (5.6)$$

If the constant parameter vector c_i are selected such that the eigenvalues of the differential equation (5.6) are negative, then, the output error e_i converges to zero in afinite time .

Expression (5.4) can also be expressed in the following form

$$\sigma(t) = \left[\sigma_1(t), \dots, \sigma_n(t) \right]^T = \left[c_1 e_1(t), \dots, c_n e_n(t) \right]^T \qquad (5.7)$$

Remark 5.1 : In expression 5.4 - 5.6, where $c_{i1} > 0$, $p = p_1 / p_2$ and p_1 and p_2 which are positive integers which are selected such that

$$p_1 = 2m - 1, \ m = 1,2,\dots\dots \qquad (5.8\text{-a})$$

$$p_2 = 2m + 1 \ m = 1,2,\dots. \qquad (5.8\text{-b})$$

Remark 5.2: It has been shown in (Zak 1988 & 1989) that $\varepsilon_i = 0$ is the terminal attractor of the system (5.6). Let the initial value of ε_i at time $t = 0$ be $\varepsilon_i(0)$ and parameter p be chosen as shown in ***remark 5.1***, then the relaxation time t_i for a solution of the system (5.5) is given as follow:

$$t_i = c_{ii}^{-1} \int_{\varepsilon_i(0)}^{\varepsilon_i \to 0} \frac{d\varepsilon_i}{\varepsilon_i^p} = \frac{\varepsilon_i(0)^{1-p}}{c_{ii}(1-p)} \tag{5.9}$$

Expression (5.9) also means that, on the terminal sliding mode in expression (5.6), the output tracking error converges to zero in a finite time. The details on the terminal attractor and its applications can be found in(Zak 1988 & 1989).

Expression (5.7) is called as the switching plane variable vector of the overall system. It is well known that the sufficient condition for the switching plane variable vector in expression (5.7) to be globally stable is given by (Abbass and Ozgunner, 1985; Ozguner, Yurkovich and Abbass, 1987; Xu, Wu and Huang, 1990; Morgan and Ozguner, 1985)

$$\sigma_i \dot{\sigma}_i < 0 \qquad i = 1,\ldots\ldots,n \tag{5.10}$$

For further discussion, the following asumptions are made.

A.6.1 The subsystem and local reference model are controllable

A.6.2 The local state vectors x_i and x_{mi} are measurable for feedback for the ith input

A.6.3 The subsystem and its reference model structurally satisfy the following matching co(Xu et al., 1990; leung et al., 1991).

$$(I_i - B_iB_i^+)B_{mi} = 0 \tag{5.11-a}$$

$$(I_i - B_iB_i^+)(A_{mi} - A_i) = 0 \tag{5.11-b}$$

$$(I_i - B_iB_i^+)\Phi_i = 0 \tag{5.11-c}$$

A.6.4 The dynamical interaction term in each subsystem is upper bounded

$$\|\Phi_i\| < k_{1i} \tag{5.12}$$

A.6.5 c_iB_i is lower bounded

$$c_iB_i > k_{2i} \tag{5.13}$$

A.6.6 The norm of $A_{mi} - A_i$ is upper bounded

$$\|A_{mi} - A_i\| < k_{3i} \tag{5.14}$$

A.5.7 The non-linearity term for each subsystem is upper bounded

$$\left\|(1 - c_{1i}p\varepsilon_{1i}^{p-1})\dot{\varepsilon}_{1i}\right\| < k_{4i} \tag{5.15}$$

where k_{1i} , k_{2i} , k_{3i} , k_{4i} are constant positive numbers.

Remark 5.3 : Assumption A.5.4 means that the dynamical interactions between subsystems and assumption A.5.5 which is the non-linearities in each subsystem are treated as bounded uncertainties.

Remark 5.4 : Since expression (5.9) multiplied by any arbitrary nonzero scalar does not change the position of the sliding mode, and the element $b_i(t)$ of matrix B_i is a bounded positive (or negative) time varying parameters, assumption A.5.5. can always be assumed (Khurana et al., 1985).

Remark 5.5 : Assumptions A.5.5. and A.5.6 show that two uncertain subsystem matrix bounds, together with the upper bound of dynamical interaction in assumption A.5.4, will be used as the subsystem structural information in the local controller design.

5.3. A decentralised variable structure control

In this part, the following control law, similar to similar to chapter 5, is used for each subsystem

$$u_i = k_{ei}e_i + k_{xi}x_i + k_{ri}r_i + \delta_i + \gamma_i \qquad (5.16)$$

where $k_{ei} \in R^{1xni}$, $k_{xi} \in R^{1xni}$ and $k_{ri} \in R^1$ are determined later, $\delta_i \in R^1$ is a discontinuous compensator picked according to the bound of the dynamical interactions and finally $\gamma_i \in R^1$ is non - linearity compensator picked according to the bound of the non - linearity for each subsystem

In order to design control law (5.16) by using three uncertain bounds in A.5.4 - A.5.7, and guarantee finite time convergence of the output tracking error, we have the following new results.

Theorem 5.1 : The motion of the switching plane variable vector of the composite system in expression (5.9) is globally stable, and the output tracking error in expression (5.3) converges in a finite time to zero if the gain matrices and the compensator in the control law (5.15) are designed as

$$k_{ei} = \begin{cases} \dfrac{\|c_i\|\|A_{mi}\|}{k_{2i}\|e_i\sigma_i\|} e_i^T \sigma_i & \|e_i\sigma_i\| \neq 0 \\ \\ 0_{1xni} & \|e_i\sigma_i\| \neq 0 \end{cases} \tag{5.17}$$

$$k_{xi} = \begin{cases} \dfrac{k_{3i}\|c_i\|}{k_{2i}\|x_i\sigma_i\|} x_i^T \sigma_i & \|x_i\sigma_i\| \neq 0 \\ \\ 0_{1xni} & \|x_i\sigma_i\| \neq 0 \end{cases} \tag{5.18}$$

$$k_{ri} = \begin{cases} \dfrac{\|c_i\|\|B_{mi}\|}{k_{2i}\|r_i\sigma_i\|} r_i \, \sigma_i & \|r_i\sigma_i\| \neq 0 \\ \\ O_{1xni} & \|r_i\sigma_i\| \neq 0 \end{cases} \tag{5.19}$$

$$\delta_i = \begin{cases} \dfrac{\|c_i\|k_{1i}}{k_{2i}\|\sigma_i\|}\sigma_i & \|\sigma_i\| \neq 0 \\ \\ O_{\,1xni} & \|\sigma_i\| = 0 \end{cases} \tag{5.20}$$

$$\gamma_i = \begin{cases} \dfrac{\|c_i\|k_{4i}}{k_{2i}\|\sigma_i\|}\sigma_i & \|\sigma_i\| \neq 0 \\ \\ O_{\,1xni} & \|\sigma_i\| = 0 \end{cases} \tag{5.21}$$

Proof : Using expressions (5.1), (5.2) and (5.3), we get the error dynamics of the ith subsystem in the following form

$$\dot{e}_i = (A_{mi} e_i + (A_{mi} - A_i) x_i + B_{mi} r_i - B_i u_i - \Phi_i \tag{5.22}$$

selecting the scalar positive definite Lyapunov function

$$v_i = \frac{1}{2}\sigma^2_i \tag{5.23}$$

and differentiating it with respect to time we have,

$$\dot{v}_i = \sigma_i \dot{\sigma}_i$$

$$= c_i(A_{mi} - B_i k_{ei})e_i\sigma_i + c_i[(A_{mi}A_i) - B_i k_{xi}]x_i\sigma_i + c_i(B_{mi} - B_i k_{ri})r_i\sigma_i$$
$$-c_i(\Phi_i + B_i\delta_i) - c_i\{(1 - c_1 p\varepsilon_i^{p-1})\dot{\varepsilon}_i + B_i\gamma_i)\sigma_i$$

(5.24)

$$c_i(A_{mi} - B_i k_{ei})e_i\sigma_i$$

$$= c_i A_{mi} e_i \sigma_i - \frac{c_i B_i \|c_i\| \|A_{mi}\|}{k_{2i}\|e_i\sigma_i\|} e_i^T\sigma_i e_i \sigma_i$$

$$= c_i A_{mi} e_i \sigma_i - \frac{c_i B_i \|c_i\| \|A_{mi}\|}{k_{2i}\|e_i\sigma_i\|} \|e_i\sigma_i\|^2$$

$$< c_i A_{mi} e_i \sigma_i - \frac{c_i B_i \|c_i\| \|A_{mi}\|}{c_i B_i\|e_i\sigma_i\|} \|e_i\sigma_i\|^2$$

$$< c_i A_{mi} e_i \sigma_i - \|c_i\| \|A_{mi}\| \|e_i\sigma_i\| \leq 0 \tag{5.24-a}$$

$$= c_i\left[\left(A_{mi} - A_i\right)\right]x_i\sigma_i - \frac{c_iB_ik_{3i}\left\|c_i\right\|}{k_{2i}\left\|x_i\sigma_i\right\|}x_i^T\sigma_ix_i\sigma_i$$

$$= c_i\left[\left(A_{mi} - A_i\right)\right]x_i\sigma_i - \frac{c_iB_ik_{3i}\left\|c_i\right\|}{k_{2i}\left\|x_i\sigma_i\right\|}\left\|x_i\sigma_i\right\|^2$$

$$< c_i\left[\left(A_{mi} - A_i\right)\right]x_i\sigma_i - \frac{c_iB_ik_{3i}\left\|c_i\right\|}{c_iB_i}\left\|x_i\sigma_i\right\|$$

$$< c_i\left[\left(A_{mi} - A_i\right)\right]x_i\sigma_i - k_{3i}\left\|c_i\right\|\left\|x_i\sigma_i\right\| \leq 0 \qquad (5.24\text{-b})$$

$$c_i(B_{mi} - B_ik_{ri})r_i\sigma_i$$

$$c_iB_{mi}r_i\sigma_i - c_iB_i\frac{\left\|c_i\right\|\left\|B_{mi}\right\|}{k_{2i}\left|r_i\sigma_i\right|}\left\|r_i\sigma_i\right\|^2$$

$$< c_iB_{mi}r_i\sigma_i - \left\|c_i\right\|\left\|B_{mi}\right\|\left|r_i\sigma_i\right| \leq 0$$

(5.24-c)

$$c_i(\Phi_i)\sigma_i + c_i(B_i\delta_i)\sigma_i$$

$$= c_i(\Phi_i)\sigma_i + \frac{c_iB_ik_{1i}\left\|c_i\right\|\left\|\sigma_i\right\|^2}{c_iB_i\left\|\sigma_i\right\|}$$

$$> c_i(\Phi_i)\sigma_i + \left\|c_i\right\|\left\|\Phi_i\right\|\left\|\sigma_i\right\| > 0 \qquad (5.24\text{-d})$$

$$c_i((1 - c_{11}p\varepsilon_{11}^{p-1})\dot{\varepsilon}_{11})\sigma_i + c_i(B_i\gamma_i)\sigma_i$$

$$= c_i((1 - c_{11}p\varepsilon_{11}^{p-1})\dot{\varepsilon}_{11})\sigma_i + \frac{c_iB_ik_{4i}\|c_i\|\|\sigma_i\|^2}{c_iB_i\|\sigma_i\|}$$

$$> c_i((1 - c_{11}p\varepsilon_{11}^{p-1})\dot{\varepsilon}_{11})\sigma_i + \|c_i\|\|(1 - c_{11}p\varepsilon_{11}^{p-1})\dot{\varepsilon}_{11}\|\|\sigma_i\| > 0 \qquad (5.24\text{-e})$$

Then $\dot{v}_i = \sigma_i\dot{\sigma}_i < 0 \qquad \sigma_i \neq 0$ (5.25)

Expression (5.25) means that the global reaching condition in expression (5.7) is satisfied, and therefore , the motion of the switching plane variable vector of the composite state is globally sable.

On the sliding mode, expression (5.6) is satisfied, the output error can the converge to zero in a finite time.

Remark 5.6: Expression (5.17)-(5.21) show that, unlike the decentralised variable structure schemes in Abbas and Ozgner (1985) and Khurana (1986), local variable structure controller design requires only four uncertain bounds of subsystem matrices and dynamical interactions. The involved computation in Abbass and Ozguner (1985), Ozgunner et al. (1987) and Xu et al. (1990) to obtain the controller gain matrices are not required here. Therefore, the local variable structure controller design is simplified and strong robustness with respect to large system uncertainties dynamical interactions and non-linearities can be obtained. I addition, the controller

gain matrices and the compensator in expression (5.17)-(5.21) can be calculated directly from the measurements according to assumption *A5.2* and the definition of σ_i.

Remark 5.7 From equation 5.6, on the terminal sliding mode, the term can be replaced by $(c_{11}{}^2 p\varepsilon_{11}^{2p-1} - c_{11}\varepsilon_{11}^{p})$. This is because on the sliding mode $\dot{\varepsilon} = -c_{11}\varepsilon_{11}^{p}$. Here we can show that finite time convergence can still be guaranteed because $0 < p < 1$.

Remark 5.8 while the local control law u_i in expression (5.16) crosses the local sliding mode $c_i\sigma_i = 0$, chattering occur in the system and undesired system dynamics may be excited. To eliminate the effects of chattering, the controller gain matrices and the compensatory expression (5.17)- (5.21) can be modified using boundary layer technique as follow.

$$k_{ei} = \begin{cases} \dfrac{\|c_i\|\|A_{mi}\|}{k_{2i}\|e_i\sigma_i\|} e_i^T \sigma_i & \|e_i\sigma_i\| \geq \delta_{1i} \\ \\ \dfrac{\|c_i\|\|A_{mi}\|}{k_{2i}\delta_{1i}} e_i^T \sigma_i & \|e_i\sigma_i\| < \delta_{1i} \end{cases} \tag{5.26}$$

$$k_{xi} = \begin{cases} \dfrac{k_{3i}\|c_i\|}{k_{2i}\|x_i\sigma_i\|} x_i^T \sigma_i & \|x_i\sigma_i\| \geq \delta_{2i} \\ \\ \dfrac{k_{3i}\|c_i\|}{k_{2i}\delta_{2i}} x_i^T \sigma_i & \|x_i\sigma_i\| < \delta_{2i} \end{cases} \tag{5.27}$$

$$k_{ri} = \begin{cases} \dfrac{\|c_i\|\|B_{mi}\|}{k_{2i}\|r_i\sigma_i\|} r_i \sigma_i & \|r_i\sigma_i\| \geq \delta_{3i} \\[2ex] \dfrac{\|c_i\|\|B_{mi}\|}{k_{2i}\delta_{3i}} r_i \sigma_i & \|r_i\sigma_i\| < \delta_{3i} \end{cases} \tag{5.28}$$

$$\delta_i = \begin{cases} \dfrac{\|c_i\|k_{1i}}{k_{2i}\|\sigma_i\|}\sigma_i & \|\sigma_i\| \geq \delta_{4i} \\[2ex] \dfrac{\|c_i\|k_{1i}}{k_{2i}\delta_{4i}}\sigma_i & \|\sigma_i\| < \delta_{4i} \end{cases} \tag{5.29}$$

$$\gamma_i = \begin{cases} \dfrac{\|c_i\|k_{4i}}{k_{2i}\|\sigma_i\|}\sigma_i & \|\sigma_i\| \geq \delta_{5i} \\[2ex] O_{1xni} & \|\sigma_i\| < \delta_{5i} \end{cases} \tag{5.30}$$

where $\delta_{1i}, \delta_{2i}, \delta_{3i}, \delta_{4i}, \delta_{5i}$ are positive numbers

using the above local boundary layer controller law, the local switching plane variables can be forced to move toward the local sliding mode surfaces and then the local control input can be smoothed in a boundary layer neighbouring the local sliding mode. This

will achieve optimal trade-off between control bandwidth and tracking precision. Therefore, dynamic interactions can be eliminated, but the drawback is that non-zero error exists (Slotine and Sastry, 1983; Slotine 1984)

5.4 Application of the scheme to robotic manipulators

In this section, the control scheme derived in section 5.3 is applied to the robust tracking control of rigid robotic manipulators.

The dynamics of an n-joint rigid robotic manipulator can be described by the following second-order nonlinear vector differential equation

$$M(q)\,\ddot{q} + F(q,\dot{q}) + G(q) = U(t) \qquad (5.31)$$

where q is the nx1 vector of joint angular positions, U(t) is the nx1 vector of applied joint torques (control inputs), M(q) is the nxn symmetric positive-definite inertial matrix, $F(q,\dot{q})$ is the vector of coriolis and centrifugal forces, and G(q) is the vector of gravitational torques.

For the use of the decentralised control scheme proposed in section 5.3, it is convenient to treat each joint as a subsystem. The manipulator dynamic equation (5.31) is therefore represented by a collection of n second-order nonlinear scalar differential equations

$$m_{ii}(q)\,\ddot{q}_i + \Big[\sum_{\substack{j=1\\ j\neq i}}^{n} m_{ij}(q)\,\ddot{q}_j(t)\Big] + f_i(q,\dot{q}) + g_i(q) = u_i \qquad i = 1, \ldots, n \quad (5.32)$$

where the subscript "i" refers to the ith element, $m_{ii}(q)$ is the time varying effective inertia seen at the ith joint, and is always positive due to the positive-definiteness of M(q).

Defining $x_i = [\,q_i \quad \dot{q}_i\,]^T$, expression (5.32) can be written in terms of state variables:

$$\dot{x}_i = \begin{bmatrix} \dot{q}_i \\ -m_{ii}^{-1}(f_i + g_i) \end{bmatrix} + \begin{bmatrix} 0 \\ m_{ii}^{-1} \end{bmatrix} u_i - \begin{bmatrix} 0 \\ m_{ii}^{-1} \end{bmatrix} \sum_{\substack{j=1 \\ j\neq i}}^{n} m_{ij}\ddot{q}_j$$

$$= \begin{bmatrix} 0 & 1 \\ a_{i21} & a_{i22} \end{bmatrix} x_i + \begin{bmatrix} 0 \\ m_{ii}^{-1} \end{bmatrix} u_i + \begin{bmatrix} 0 \\ \phi_i \end{bmatrix}$$

$$= A_i x_i + B_i u_i + \Phi_i \qquad i = 1, \ldots n \tag{5.33}$$

and the ith local reference model is given in the following form:

$$\dot{x}_{mi} = A_{mi} x_{mi} + B_{mi} r_i \tag{5.34}$$

$$A_{mi} = \begin{bmatrix} 0 & 1 \\ a_{mi21} & a_{mi22} \end{bmatrix}$$

$$B_{mi} = \begin{bmatrix} 0 \\ b_{mi1} \end{bmatrix}$$

where a_{mi21}, a_{mi22} and b_{mi1} are known constant numbers determined from an engineering point of view.

The error dynamics is then given by

$$\dot{e}_i = A_{mi} e_i + (A_{mi} - A_i) x_i + B_{mi} r_i - B_i u_i - \Phi_i \tag{5.35}$$

where $e_i = [\varepsilon_i, \ \dot{\varepsilon}_i]^T$ and $\varepsilon_i = q_{mi} - q_i$.

In this case, a set of local sliding manifolds are defined as:

$$\sigma_i = c_i e_i + c_{i1}(\varepsilon_{i1}{}^p - \varepsilon_{i1}) \qquad i = 1, \ldots, n \tag{5.36}$$

where $c_i = [c_{i1} \quad 1]$, whose parameters are positive constant numbers

If the conditions in expressions (5.12), (5.13) (5.14) and (5.15) are satisfied for all q and $\dot{q}$, the global reaching condition (5.10) can then be satisfied by the use of control law (5.16) and the controller gain matrices and the compensator in expressions (5.17) - (5.21).

On the sliding mode, the desired error dynamics is given by

$$\dot{\varepsilon} = -c_{i1}\varepsilon_{i1}^{p} \qquad (5.37)$$

Therefore, the output tracking error ε_i (i = 1, ... n) converges to zero in a finite time.

5.6 Concluding remarks

A decentralised model reference terminal sliding mode control scheme using variable structure theory for a class of large scale systems is investigated in this chapter. The main contribution of this chapter is the usage of a nonlinear sliding mode and convergence of error faster than the ones of the linear sliding mode scheme . Similar to the discussion in Chapter 5, there are only 4 uncertain bounds, are required with local controller design and therefore the controller design is greatly simplified, strong robustness to large system uncertainties and strong dynamical interactions is obtained and finite time convergence of output tracking error is guaranteed. The chattering problem is eliminated by using the boundary layer technique.

Chapter 7

Conclusions

Variable structure technique is a powerful approach for the control of nonlinear robotic manipulators. It is advocated to solve complex control problems that are not within the scope of simple linear feedback controllers and adaptive controllers. A number of factors, such as nonlinearities, parameter uncertainties, nonlinear couplings and disturbances, are known to affect performance of robotic control systems. Therefore, this thesis has been mainly concerned with the study and improvements of robust control schemes for rigid robotic manipulators in the presence of these non-ideal conditions.

Chapter three and chapter four of this thesis has provided a survey for the basic variable structure control theory and recent significant results on variable structure control for robotic manipulators. The limitations of these results in non-ideal conditions have also been highlighted.

Chapter five have provided a robust decentralised variable structure control schemes. It has been shown that variable structure controllers can be designed based on several uncertain system matrix bounds, and the controller gain matrices are adaptively adjusted by input and output measurements so that strong robustness and asymptotic convergence of the output tracking error can be achieved.

In chapter 5, the linear sliding mode technique was replaced with a terminal sliding mode technique so that the output tracking error has a finite time convergence. It is seen that only theoretical analysis of the terminal sliding mode controller for rigid robotic manipulators are carried out. the simulation and further investigation need to be done.

In summary, the thesis has provided two new and improved robust variable structure control scheme aimed at achieving robustness and convergence against nonlinearities, parameter uncertainties, nonlinear couplings and external disturbances in the control of robotic manipulators. We believe that the result has potential to improve the performance of the robust control of robotic manipulator.

8. References

[1] Man Zhihong,"Robust Variable Structure Control For Rigid Robotic Manipulator - PhD. Thesis , The University of Melbourne , Feb 1993 .

[2] Man Zhihong and M. Palaniswami, "A robust tracking control for rigid robotic manipulators," IEEE Transactions on Automatic Control, Vol. 39, No.1, pp 154 - 159, 1994.

[3] Man Zhihong and M. Palaniswami, "A variable structure model reference adaptive control for nonlinear robotic manipulators,". Adaptive Control and Signal Processing, Vol. 7, pp 539 - 562, 1993.

[4] Man Zhihong and M. Palaniswami, "Decentralised three segment nonlinear sliding mode control for robotic manipulators," in Proc. of IEEE International Workshop on Emerging Technologies and factory Automation, Melbourne, pp. 607 - 612, 1992.

[5] Man Zhihong and M. Palaniswami, "A robust adaptive tracking control scheme for robotic manipulators with uncertain dynamics," in Proc. of IEEE International Conference on Systems Engineering, Japan, pp. 127 - 130, 1992

[6] Man Zhihong, A. P. Paplinski, and H. R. Wu, " Robust MIMO Terminal Sliding Mode Control Scheme for Rigid Robotic Manipulators" IEEE Trans Automatic Control Scheme for RigidRobotic ManipulatorsWorkshop on Emerging Technologies and factory Automation, Melbourne, pp. 607 - 612, 1992

[7] Abbass, A. F., and Ozguner, U. (1985). Decentralised model reference adaptive system using a variable structure control. Proc. 24th Conf. Decision & Control pp. 1473-1478.

[8] Abdallah, C., and Jordan, R. (1990). A positive-real design for robotic manipulators. Proc. IEEE Amer. Contr. Conf. pp. 991-992.

[9] Abdallah, C., Dawson, D., Daorato, P., and Jamshidi, M. (1991). Survey of robust control for rigid robotics. IEEE Control Systems, vol. AC - 11, pp. 24-30.

[10] Ambrosino, G., Celentano, G., and Garofalo, F. (1984). Variable structure model reference adaptive control systems. Int. J. Control, vol. 39, pp. 1339 - 1349.

[11] .Bahnasawi, A. A., Eid, S. Z., and Mahmoud, M. S. (1991) Adaptive model-following control based on variable structure systems. Int. J. Systems Sci., vol. 22, pp. 333 - 349.

[12] Bartolini, G., and Zolezzi, T. (1985). Variable structure nonlinear in the control law. IEEE Trans. Automat. Contr. vol. AC-30, pp. 681-684.

[13] Corless, M. J., and Leitmann, G. (1981). Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems. IEEE Trans. Automat. Contr., vol. AC-26, pp. 437 - 443.

[14] Crag, J. J., Hsu, P., and Sastry, S. S. (1986). Adaptive control of mechanical manipulators. IEEE Int. Conf. on Robotics and Automation, pp. 190-195.

[15] Crag, J. J., Hsu, P., and Sastry, S. S. (1987). Adaptive control of mechanical manipulators. Int. J. Robotics Research, vol. 6, pp. 16-28.

[16] Craig, J-J. (1986). Introduction to robotics: machine and control. Addison-Wesley Company Inc.

[17] Craig, J. J. (1988). Adaptive control mechanical manipulators. Addison-Westey, Reading, Massachusetts.

[18] DeCarlo, R. A., Zak, S. H. and Matthews, G. P. (1988). Variable structure control of nonlinear multivariable systems: A tutorial. Proc. of the IEEE, vol. 76, pp. 212 - 232.

[19] Desa, S., and Roth, B. (1985). Synthesis of control systems for manipulators using multivariable robust servo mechanism theory. Int. J. Robotic Res., vol. 4, pp. 18-34.

[20] Desoer, C., and Vidyasagar, M. (1975). Feedback Systems: Input - output Properties. New York: Academic.

[21] Dorling, C. M., and Zinober, A. S. I. (1986). Two approaches to hyperplane design in multivariable variable structure control systems. Int. J. Control, vol. 44, pp. 65 - 82.

[22] Drazenovic, B. (1969). The invariance conditions in variable structure system. Automatica, vol. 5, pp. 287 - 295.

[23] Emelyanov, S. V., Bermant, M. A., and Kostyleva, N. E. (1966). Design principles for variable structure control systems. In Proc. 3rd IFAC Congr., vol. 1, book 3, pp. 40c.1 - 40c.6.

[24] Gavel, D. T., and Hsia, T. C. (1987). Decentralised adaptive control of robotic manipulators. proc. IEEE Int. Conf. on Robotics and Automations, pp. 1230 - 1235.

[25] Gilbert, E. G., and Ha, I. J. (1984). An approach to nonlinear feedback control with applications to robotics. IEEE Trans. Sys., Man, and Cyber., vol. 14, pp. 879 - 884.

[26] Grimm, W. M. (1990). Robot non-linearity bounds evaluation techniques for robust control. Int. J. Adaptive Control and Signal Processing, vol. 4, pp. 501 - 522.

[27] Gutman, S. (1979). Uncertain dynamic system - A Lyapunov min-max approach. IEEE Trans. Automat. Contr., vol. AC-24, pp. 434 - 443.

[28] Gutman, S., and Polmor, Z. (1982). properties of min-max controllers in uncertain dynamical systems. SIAM. J. Contr. Optimization. vol. 20, pp. 850-861.

[29] Hsu, L., and Costa, R. R. (1989). Variable structure model reference adaptive control using only input and output measurements. Int. J. Control, vol. 49, pp. 399 - 416.

[30] Ioannou, P. A. (1986). Decentralised adaptive control of interconnected systems. IEEE Trans. Automat. Contr., vol. AC-31, pp. 291 - 298.

[31] Itkis, U. (1976). Control systems of variable structure. John Wiley, New York.

[32] Khurana, H., Ahson, S. I., and Lamba, S. S. (1986). On the stabilization of large-scale control systems using variable structure system theory. IEEE Trans . Automat. Contr. vol. AC - 31, pp. 176 - 178.

[33] Klein, G., and Moore, B. C., (1977). Eigenvalue-generalised eigenvector assignment with state feedback. IEEE Trans. Automat. Contr. vol.AC- 22, pp. 140-141.

[34] Kuo, C. Y., and Wang, S. P. T. (1989). Nonlinear robust industrial robot control. Trans. ASME J. Dyn. Syst. Means. Control, vol. 111, pp. 24 - 30.

[35] Leitmann, G. (1981). On the efficacy of nonlinear control in uncertain linear systems. Trans. ASME J. Dyn. Syst. Means. Control, vol. 102, pp. 95-102.

[36] Leung, T. P., Zhou, Q. J., and Su, C. Y. (1991). An adaptive variable structure model following control design for robotic manipulators. IEEE Trans. Automat. Contr. vol. AC-36, pp. 347-352.

[37] Middleton R. H. and Goodwin, G. C. (1988). Adaptive computed torque control for rigid link manipulators. Syst. Control Lett., vol. 10, pp.9 - 16.

[38] Miyasato, Y., and Oshima, Y. (1989). Non-linear adaptive control for robotic manipulators with continuous control input. Int. J. Control, vol. 49, pp. 545 - 559.

[39] Morgan, R. G., and Ozguner, U. (1985). A decentralised variable structure control algorithm for robotic manipulators. IEEE J. Robotics Automat., vol. RA - 1, pp. 57 - 65.

[40] Nicosia, S., and Tomei, P. (1984). Model reference adaptive control algorithms for industrial robots. Automatica, pp. 635-644.

[41] Ogata (1992). Modern Control Theory, Prentice Hall.

[42] Ortega, R., and Spong, M. W. (1989). Adaptive motion control of rigid robots: a tutorial," Proc. IEEE Conf. Dec. & Contr., pp. 1575-1584.

[43] Ozguner, U., Yurkovich, S., and Abbass, A. F. (1987). Decentralised variable structure control of a two-arm robotic system. Proc. IEEE Conf. Robotic & Automation, pp. 1248-1254.

[44] Rehg, J. A. (1985). Introduction to robotics: a system approach. Prentice-Hall, INC., Englewood Cliffs, New Jersey.

[45] Richter, S., Lefebvre, S., AND Decarlo, R. (1982). Control of a class of nonlinear systems by decentralised control. IEEE Trans. Automat. Contr. vol. AC-27, pp. 492 - 494.

[46] Rohrs, C., Athans, M., and Valavani, L. (1985). Robustness of continuous time adaptive control algorithms in the presence of unmodelled dynamics. IEEE Trans. Automat. Contr., vol. AC-30, pp. 881 - 889.

[47] Ryan, E. P. (1983). A variable structure approach to feedback regulation of uncertain dynamical systems. Int. J. Control, vol. 38, pp. 1121 - 1134.

[48] Samson, C. (1983). Robust nonlinear control of robotic manipulators. in Proc. 22nd IEEE Conf. Dec.Contr., pp. 1211 - 1216.

[49] Schmitedorf, W. E., and Barmash, B. R. (1986). Robust asymptotic tracking for linear system with unknown parameters. Automatica, vol. 22, pp. 353 - 360.

[50] Seraji, H. (1989). Decentralised adaptive control of manipulators: theory, simulation, and experimentation. IEEE Trans. Robotics Automat. vol. 5, pp. 183-201.

[51] Shoureshi, R., Momot, M. E., and Roesler, M. D. (1990). Robust control for manipulators with uncertain dynamics. Automatica, vol. 26, pp. 353 - 359.

[52] Shoureshi, R., Momot, M. E., and Roesler, M. D. (1990). Robust control for manipulators with uncertainties. Automatica, vol. 26, pp. 353-359.

[53] Silva, C. M. de. (1984). A motion control scheme for robotic manipulators in proc. Canadian CAD/CAM and Robotics Conf. vol. 13, pp. 131 - 137.

[54] Singh, S. N. (1985). Adaptive model following control of nonlinear robotic systems. IEEE Trans. Automat. Contr., vol. AC-30, pp. 1099 - 1100.

[55] Sinswat, V., and Fallside, F. (1977). Eigenvalue/eigenvector assignment by state-feedback. Int. J. Contr. vol. 26, pp. 389-403.

[56] Slotine, J-J. E. (1984). Sliding controller design for nonlinear systems. Int. J. Control, vol. 40, pp. 421 - 434.

[57] Slotine, J-J. E. (1985). The robust control of robotic manipulators. Int. J. Rob. Res., vol. 4, pp. 49 - 64.

[58] Slotine, J-J. E., and Sastry, S. S. (1983). Tracking control of nonlinear system using sliding surface with application to robotic manipulators. Int. T. Contr., vol. 38, pp. 465 - 492.

[59] Slotine, J. J. E., and Li, W. (1987). On the adaptive control of robotic manipulators Int. J. Robotics Research, vol. 6, pp. 49-59.

[60] Spong, M. W., and Ortega, R. (1990). On adaptive inverse dynamics control of rigid robots. IEEE Trans. Automat. Contr. vol. AC-35, pp. 92-95.

[61] Spong, M. W., and Vidyasagar, M. (1987). Robust linear compensator design for nonlinear robotic control. IEEE J. Rob. Automat., vol. 3, pp. 345-351.

[62] Spong, M. W., and Vidyasagar, M. (1989). Robot Dynamics and Control. Wiley, New York.

[63] Spurgeon, S. K., and Patton, R. J. (1990). Robust variable structure control of model reference systems. IEE Proceedings, vol. 137, pp. 341 - 348.

[64] Sundareshan, M. K., and Koeing, M. A. (1985). Decentralised model reference adaptive control of robotic manipulators. Proc. American Control Conf., pp. 44 - 49.

[65] Utkin, V. I. (1971). Equations of the sliding regime in the discontinuous systems. Automat. Remote Contr., vol, 32, pp. 1987 - 1907.

[66] Utkin, V. I. (1977). Variable structure systems with sliding mode. IEEE Trans. Automat. Contr., vol. AC-22, pp. 212 - 221.

[67] Utkin, V. I. (1983). Variable structure systems - present and future. Automat. remote Contr., vol. 44, pp. 1105 - 1120.

[68] Utkin, V. I., and Young, K-K. D. (1978). Methods for constructing discontinuous planes in multidimentional variable structure systems. Automation and Remote Control, vol. 31, pp. 1466 - 1470.

[69] Warwick, K., and Pugh, A. (1988). Robust control: theory and applications. Peter Peregrinus Ltd., London, U. K.

[70] Xu, X. H., Wu, Y. H., and Huang, W. H. (1990). Variable-structure control approach of decentralised model reference adaptive systems. IEE Proc. pt. D, vol. 137, pp. 302-306.

[71] Yeung, K. S., and Chen, Y, P. (1988). A new controller design for manipulators using the theory of variable structure systems. IEEE Trans. Automat. Contr., vol. AC - 33, pp. 200-206.

[72] Young, K-K. D. (1978). Controller design for a manipulator using theory of variable structure systems. IEEE Trans. on Systems, Man, and Cybernetics, vol. 8, pp. 101 - 109.

[73] Young, K-K. D. (1978). Design of variable structure model following control system. IEEE Trans. Automat. Contr. vol. AC - 23, pp. 1079 - 1085.

[74] Young, K-K. D. (1988). A variable structure model following control design for robotics applications," IEEE Trans. Automat. Contr. vol. 4, pp. 556-561.

[75] Young, K. K. D. (1988). A variable structure model following control design for robotics applications. IEEE Trans. Automat. Contr. vol. AC - 4, pp. 556-561.

A

B

C

D

E

F

G

H

I

J

K

L

M

N

O

P

Q

R

S

T

U

V