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ABSTRACT

The Global Financial Crisis (GFC) highlighted the importance of measuring and understanding extreme credit risk. This paper applies Conditional Value at Risk (CVaR) techniques, traditionally used in the insurance industry to measure risk beyond a predetermined threshold, to four credit models. For each of the models we use both Historical and Monte Carlo Simulation methodology to create CVaR measurements. The four extreme models are derived from modifications to the Merton structural model (which we term Xtreme-S), the CreditMetrics Transition model (Xtreme-T), Quantile regression (Xtreme-Q), and the author's own unique *i*Transition model (Xtreme-*i*) which incorporates industry factors into transition matrices. For all models, CVaR is found to be significantly higher than VaR, and there are also found to be significant differences between the models in terms of correlation with actual bank losses and CDS spreads. The paper also shows how extreme measures can be used by banks to determine capital buffer requirements.

Keywords: credit risk, conditional value at risk, conditional probability of default, historical simulation, Monte Carlo simulation.

JEL Codes: G01, G21, G28

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1 INTRODUCTION

The Global Financial Crisis (GFC) has raised widespread spread concern about the ability of banks to accurately measure and provide for credit risk during extreme downturns.

Prevailing widely used credit models were generally designed to predict credit risk on the basis of 'average' credit risks over time, or in the case of Value at Risk (VaR) models on the basis of risks falling below a pre-determined threshold at a selected level of confidence, such as 95 percent or 99 percent. The problem with these models is that they are not designed to measure the most extreme losses, i.e. those in the tail of the credit loss distribution. It is precisely during these extreme circumstances when firms are most likely to fail, and it is exactly these situations that the models in this study are designed to capture.

Although the use of VaR (which measures potential losses over a given time period at a pre-determined confidence) is widespread, particularly since it's adaptation as a primary market risk measure in the Basel Accords, it is not without criticism. Critics include Standard and Poor's analysts (Samanta, Azarchs, & Hill, 2005) due to inconsistency of VaR application across institutions and lack of tail risk assessment. VaR has also been criticised by Artzner, Delbaen, Eber & Heath (1999; 1997) as it does not satisfy mathematical properties such as subadditivity.

Conditional Value at Risk (CVaR) is a measure initially used in the insurance industry for determining extreme returns (those beyond VaR). The metric has been shown by Pflug (2000) to be a coherent risk measure without the undesirable properties exhibited by VaR. CVaR has been applied to portfolio optimization problems by Uryasev and Rockafellar (2000), Rockafeller and Uryasev (2002), Andersson et.al (2000), Alexander et al (2003), Alexander and Baptista (2003), Rockafellar et al (2006), Birbil, Frenk, Kaynar, & Noyan (2009) and Menoncin (2009). CVaR has also been explored as a measure of sectoral market and credit risk by Allen and Powell (2009a, 2009b), but compared to VaR, CVaR studies in a credit context are still in their infancy.

Given the importance of understanding and measuring extreme credit risk, the first aim of this study is to show how CVaR techniques can be applied to prevailing models to measure tail risk, using a US dataset which includes 380 US companies, mixed between investment and speculative entities. Allied to this objective, this study investigates to what extent these CVaR measures are significantly differently from VaR measures.

Our second aim is to show how the CVaR measures can be used by banks to measure capital buffers required by banks to deal with volatility in credit risk. A link can be drawn between the volatility of the market asset values of banks and capital adequacy, as illustrated by the Bank of England (BOE, 2008). BOE report that in 2008 UK banks had equity ratios of around 3.3 percent, and assuming volatility in market value of assets of 1.5 percent, this gives a Probability of Default of around 1 percent (see equations 1 and 2). If volatility doubles, then PD increases substantially to 15 percent. As bank PDs increase with deteriorating market conditions, so too does the chance of the assets needing to be liquidated at market prices. Therefore as PDs rose during the GFC, market participants changed the way they assessed underlying bank assets, placing a greater weight on mark to market asset values, implying lower asset values and higher potential capital needs for banks. Thus BOE sees the mark to market approach of a bank's assets as providing a measure of how much capital needs to be raised to restore market confidence in the bank's capitalisation. In a similar fashion we will use the volatility metrics in this study to show what capital buffers are required to restore market confidence in volatile times.

To ensure a thorough examination of CVaR metrics we use a range of models (four in total), as well as apply two techniques (Historical and Monte Carlo Simulation) to each model. The Monte Carlo method generates multiple random scenarios, with the key advantage being that thousands of potential scenarios can be generated and considered, as opposed to just a few discrete observations. This is especially advantageous with CVaR, where historical observations are only limited to a small number of observations in the tail of the distribution. The third aim of this study is to ascertain which of the models most highly correlate with actual measures of credit risk, including Credit Default Swap spreads, delinquent loans and charge-offs.

Our four models are based around some of the most widely used existing credit models. The Merton (1974) structural model (modified by KMV) uses a combination of asset value fluctuations and balance sheet characteristics to measure Probability of Default (PD), with Moody's KMV (2010) reporting use of their products by more than 2,000 leading financial institutions in over 80 countries, including most of the 100 largest financial institutions in the world. Our first model (Xtreme-S) applies CVaR techniques to this structural model, by measuring the tail asset value fluctuations (those beyond VaR). Our second model (Xtreme-Q) applies quantile regression to the Merton structural model, by dividing the dataset of asset value fluctuations into parts (quantiles), allowing the selected quantile (in our case based on tail observations) to be isolated and measured. Our third model (Xtreme-T) applies CVaR techniques to the CreditMetrics Transition model, which measures VaR and is the credit equivalent of the RiskMetrics model of JP Morgan who introduced and popularised VaR. The CreditMetrics model incorporates credit ratings and calculates VaR based on the probability of transitioning from one rating to another (including to a default rating). Our fourth model (Xtreme-*i*) applies CVaR techniques to our own *i*Transition model which is a transition model modified to incorporate market derived sectoral risk weightings.

The remainder of the paper is structured as follows: Section two describes data and the methodology (both Historical and Monte Carlo) used for each of the four models; Section three discusses results and implications for capital; Section four concludes.

2 DATA AND METHODOLOGY

2.1 Data

Data is divided into two periods: Pre-GFC and GFC. For each of the four models we generate separate measurements for each of these two periods. We also generate an annual measure for each model for each of the 10 years in the dataset. Our Pre-GFC period includes the 7 years from January 2000 to December 2006. This 7 year period aligns with Basel Accord advanced model credit risk requirements. Our GFC period includes January 2007 to June 2009.

For our Merton / KMV based models (Xtreme-S and Extreme Q) which require equity prices, we obtain daily prices from Datastream (approximately 250 observations x 10 years = 2500 observations per company). Required balance sheet data for the structural model, which includes asset and debt values, is also obtained from DataStream. To ensure a mix of investment and speculative entities, we obtain data from two sources: firstly entities listed on the New York Stock Exchange (NYSE) Standard & Poor's 500 index (S&P 500); secondly entities included in Moody's Speculative Grade Liquidity Ratings list (Moody's Investor Services, 2010a). In both cases we only include rated entities, for which equity prices and Worldscope balance sheet data are available in Datastream. Entities with less than 12 months data in either of the 2 periods are excluded. This results in 378 entities consisting of 208 S&P 500 companies and 170 speculative companies.

The transition based models (Xtreme-T and Xtreme-*i*) require credit ratings and transition probability matrices (as discussed in the methodology section) for each period. Credit ratings are obtained from Moody's (Moody's Investor Services, 2010b). We use Standard and Poor's (2009) US transition probability matrices which we obtain for each year in our study. For the Pre-GFC vs GFC periods, we average the matrices for the relevant years in the dataset.

Annual delinquent loans and charge-off rates were obtained from the U.S. Federal Reserve Bank (2010). Annual CDS figures for US Corporates were obtained from Datastream.

These CDS figures were extracted by credit rating, and weighted according to the dollar value of debt for each credit rating category in our data sample.

2.2 Methodology Model 1: Xtreme-S

We use the Merton / KMV approach to estimating default, and then modify this calculation to incorporate a CVaR component (which we term CPD as the model uses probability of default as opposed to VaR). The structural model point of default is where the firm's debt exceeds asset values. KMV (Crosbie & Bohn, 2003), in modeling defaults using their extensive worldwide database, which includes over 250,000 company-years of data and over 4,700 incidents of default, find that in general firms do not default when assets value reach total liability book values. Many continue to trade and service their debts at this point as the long-term nature of some of their liabilities provides some breathing space. KMV find that the default point, the asset value at which the firm will default, generally lies somewhere between total liabilities and current, or short-term, liabilities (modelling evidence from their extensive database shows approximately half way). Thus KMV use current debt plus half of long term debt as the default point. Distance to default (DD) and probability of default (PD) are measured as

$$DD = \frac{\ln(V/F) + (\mu - 0.5\sigma_V^2)T}{\sigma_V \sqrt{T}} \tag{1}$$

$$PD = N(-DD) \tag{2}$$

where

V = market value of firm's assets

F = face value of firm's debt (in line with KMV, this is defined as current liabilities plus one half of long term debt)

μ = an estimate of the annual return (drift) of the firm's assets

N = cumulative standard normal distribution function.

It should be noted that KMV find the PD values arising from the normal distribution are very small, and hence use their own extensive database of defaulting entities to derive an Estimated Default Frequency (EDF) from DD values, which we do not have access to. For this reason, we will report DD values only as opposed to PD values. This has little impact on our study as we are concerned with changes from period to period rather than with absolute measures.

For our historical approach, we obtain daily equity returns for each entity, and calculate the standard deviation of the logarithm of price relatives. Following the estimation, iteration and convergence procedure outlined by KMV (2008), Bharath & Shumway (2009), and Allen and Powell (2009a), we obtain asset values and asset returns. These figures are then applied to the DD and PD calculations in equations 1 and 2. We measure μ as the mean of the change in lnV as per Vassalou & Xing (2004). Following KMV, debt is measured as current liabilities plus one half of long term liabilities.

We define conditional distance to default (CDD) as being DD on the condition that standard deviation of asset returns exceeds standard deviation at the 95 percent confidence level, i.e. the worst 5 percent of asset returns. We term the standard deviation of the worst 5 percent of returns for each period as CStdev, which we then substitute into equation 1 to obtain a conditional DD:

$$CDD = \frac{\ln(V/F) + (\mu - 0.5\sigma_V^2)T}{CStdev_V \sqrt{T}} \tag{3}$$

For our Monte Carlo approach we generate 20,000 simulated asset returns for every company in our dataset. This is done by generating 20,000 random numbers based on the standard deviation and mean obtained using the Historical approach. We then follow the same approach as for the Historical model, applying the standard deviation of all simulated returns to equation 1 to measure DD and the standard deviation of the worst 5 percent of simulated returns to equation 3 to measure CDD.

2.3 Methodology Model 2: Xtreme-Q

Quantile regression per Koenker & Basset (1978) and Koenker and Hallock (2001) is a technique for dividing a dataset into parts. Minimising the sum of symmetrically weighted absolute residuals yields the median where 50 percent of observations fall either side. Similarly, other quantile functions are yielded by minimising the sum of asymmetrically weighted residuals, where the weights are functions of the quantile in question per equation 3. This makes quantile regression robust to the presence of outliers.

$$\min_{\varepsilon \in R} \sum p_r (y_1 - \varepsilon) \tag{3}$$

where $p_{\tau}(.)$ is the absolute value function, providing the τth sample quantile with its solution.

Figure 1 Illustrative Quantile Regression Example

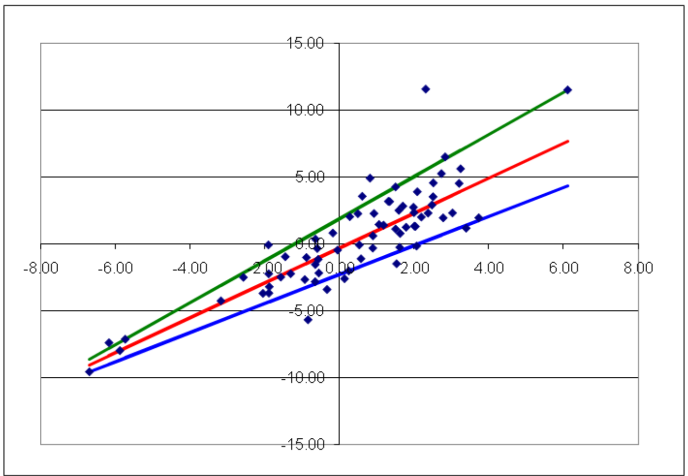


Figure 1 (Andreas Steiner, 2006) illustrates the quantile regression technique. The x and y axes represent any two variables being compared (such as age and height; or market returns and individual asset returns). The 50 percent quantile (middle line) is the median, where 50 percent of observations fall below the line and 50 percent above. Similarly, the 90 percent quantile (top line) is where 10 percent of observations lie above the line, and 10 percent quantile (bottom line) has 90 percent of observations above the line. The intercept and slope are obtained by minimising the sum of the asymmetrically weighted residuals for each line. Quantile Regression allows direct modelling of the tails of a distribution rather than 'average' based techniques such as ordinary least squares or credit models which focus on 'average' losses over a period of time. The

technique has enjoyed wide application such as investigations into wage structure (Buschinsky, 1994; Machado & Mata, 2005), production efficiency (Dimelis & Lowi, 2002), and educational attainment (Eide & Showalter, 1998). Financial applications include Engle & Manganelli (2004) and Taylor (2008) to the problem of VaR and Barnes and Hughes (2002) who use quantile regression analysis to study CAPM in their work on stock market returns.

In a stock market context Beta measures the systematic risk of an individual security with CAPM predicting what a particular asset or portfolio's expected return should be relative to its risk and the market return. The lower and upper extremes of the distribution are often not well fitted by OLS. Allen, Gerrans, Singh, & Powell (2009), using quantile regression, show large and sometimes significant differences between returns and beta, both across quantiles and through time. These extremes of a distribution are especially important to credit risk measurement as it at these times when failure is most likely. We therefore expand these quantile techniques to credit risk by measuring Betas for fluctuating assets across time and across quantiles, and the corresponding impact of these quantile measurements on DD. Our *x* axis depicts the asset returns for the quantile being measured (we measure the 50 percent quantile which corresponds roughly to the standard Merton model, and the 95 percent quantile to give us our CStdev). The *y* axis represents the returns for all the asset returns (all quantiles) in the dataset. The Historical approach is based on the actual historical asset fluctuations. The Monte Carlo approach uses 20,000 simulated asset returns generated in the same manner as for Xtreme-S.

2.4 Methodology Model 3: Xtreme-T

This model is based upon obtaining the probability (ρ) of a bank customer transitioning from one grade to another as shown for the following BBB example:

$$\text{BBB} \quad \rho_{AAA} \quad \rho_{AA} \quad \rho_{A} \quad \rho_{BBB} \quad \rho_{BB} \quad \rho_{B} \quad \rho_{CCC/C} \quad \rho_{D}$$

External raters such as Moody's and Standard & Poor's (S&P) provide transition probabilities for each grading and we use the S&P US transition probabilities. We exclude non-rated categories and adjust remaining categories on a pro-rata basis as is the practice of CreditMetrics (Gupton, Finger, & Bhatia, 1997). The sum of all probabilities must equal 1.

We follow CreditMetrics methodology as described in the following paragraphs. The model obtains forward zero curves for each rating category (based on risk free rates) expected to exist in a year's time. Using the zero curves, the model calculates the market value (V) of the loan, including the coupon, at the one year risk horizon. Effectively, this means estimating the change in credit spread that results from rating migration from one rating category to another, then calculating the present value of the loan at the new yield to estimate the new value. The following example values a 5 year loan, paying a coupon of 6 percent, where r = the risk free rate (the rate on government bonds) and s = the spread between a government bond and corporate bonds of a particular category, say AA (see CreditMetrics (Gupton et al., 1997)).

$$V = 6 + \frac{6}{(1+r_1+s_1)} + \frac{6}{(1+r_2+s_2)^2} + \frac{6}{(1+r_2+s_2)^3} + \frac{106}{(1+r_2+s_2)^4} \tag{4}$$

The above is calculated for each rating category (yields for government and corporate bonds are obtained from Datastream for each rating category for each year in the sample, and are weighted according to F for each entity in our data sample). Probabilities in the S&P (Standard and Poor's, 2009) transition tables are multiplied by F for each rating category to obtain a weighted probability. Based on the revised probability table, Historical VaR is obtained by calculating the probability weighted portfolio variance and standard deviation (σ), and then calculating Historical VaR using a normal distribution (for example 1.645σ for a 95 percent confidence level). It has become common practice for modellers of transition matrices to use the average historical transition probabilities over the time period being modeled (as opposed to varying the probabilities year by year). This approach is effective in isolating how changes in ratings affect VaR over time. However, credit ratings change only periodically and, especially over periods like the GFC, this will not be effective in predicting actual changes in VaR as it ignores the impact of volatility in the default probabilities associated with the ratings (these changed dramatically over the GFC). Standard and Poor's provide annual probability matrices as well as historical averages over various extended time periods. When correlating our VaR and CVaR outcomes to CDS spreads and bank defaults and charge-offs we examine both approaches – one which uses fluctuating probabilities and one which uses an average for the 10 year period.

We extend this VaR methodology (Gupton et al., 1997) to calculate Historical CVaR by using the lowest 5 percent of ratings for each industry.

CreditMetrics (see also Allen & Powell, 2009b) use Monte Carlo modelling as an alternate approach to estimating VaR, and we follow this approach for our Monte Carlo CVaR. transition probabilities and a normal distribution assumption are used to calculate asset thresholds (Z) for each rating category as follows:

$$Pr_{(Default)=}\ \Phi(Z_{Def}/\sigma)$$

$$Pr_{(CCC)\ =}\ \Phi(Z_{CCC}/\sigma) - \Phi(Z_{Def}/\sigma) \qquad (5)$$

and so on, where Φ denotes the cumulative normal distribution, and

$$Z_{Def\ =}\ \Phi^{-1}\sigma \qquad (6)$$

Scenarios of asset returns are generated using a normal distribution assumption. These returns are mapped to ratings using the asset thresholds, with a return falling between thresholds corresponding to the rating above it. In line with this methodology we generate 20,000 returns for each firm from which portfolio distribution and VaR are calculated. We extend this methodology to calculate Monte Carlo CVaR by obtaining the worst 5 percent of the 20,000 returns.

2.5 Methodology Model 4: Xtreme-*i*

CreditPortfolioView (Wilson, 1998) is a variation to the transition model which incorporates an adjustment to transition probabilities based on industry and country factors calculated from macroeconomic variables. This model recognises that customers of equal credit rating may transition differently depending on their industry risk. Other studies have subsequently also linked macroeconomic / business cycle conditions to transition matrices (Belkin, Forest, & Suchower, 1998; Kim, 1999; Nickell, Perraudin, & Varotto, 2000; Trück, 2010; Wei, 2003). However, a study by APRA (1999) showed that banks did not favour using macroeconomic factors in their modelling due to complexities involved. Our own *i*Transition model (Allen & Powell, 2009b) uses the same framework as CreditPortfolioView, but (on the basis that differences in industry risk will be captured in share prices), incorporates market VaR (fluctuations in the share prices of industries) instead of macroeconomic variables to derive industry adjustments. This is done by calculating market VaR for each industry, then calculating the relationship between market VaR and credit risk for each industry, using the Merton model to calculate the credit risk component. We classify data into sectors using Global Industry Codes (GICS), which are Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Financials, Health Care, Retail, Information Technologies, Telecommunications and Utilities. These factors are used to adjust the Xtreme-T model as follows using a BBB rated loan example:

BBB	ρ_{AAAi}	ρ_{AAi}	ρ_{Ai}	ρ_{BBBi}	ρ_{BBi}	ρ_{Bi}	$\rho_{CCC/Ci}$	ρ_{Di}

Other than the industry adjustments, our Xtreme-*i* Historical and Monte Carlo VaR and CVaR calculations follow the same process as Xtreme-T.

3 RESULTS AND IMPLICATIONS FOR CAPITAL

Table 1. Results Summary

Historical

Model	Metric	Pre GFC	GFC
Xtreme-S	DD	8.64	4.07
	CDD	2.53	1.26
Xtreme-Q	DD	8.10	4.04
	CDD	2.31	1.96
Xtreme-T	VaR	0.0190	0.0453
	CVaR	0.0433	0.0908
Xtreme-i	VaR	0.0182	0.0570
	CVaR	0.0453	0.1052

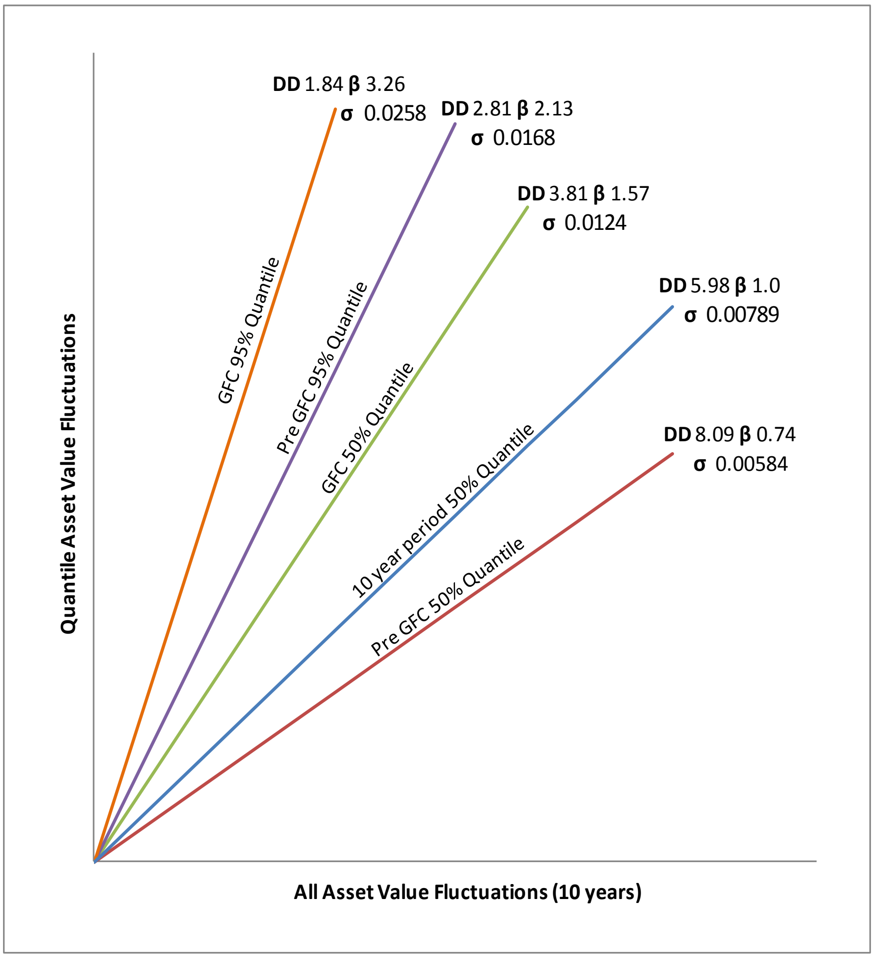
Monte Carlo

Model	Metric	Pre GFC	GFC
Xtreme-S	DD	8.63	4.06
	CDD	3.03	1.08
Xtreme-Q	DD	8.09	3.81
	CDD	2.81	1.84
Xtreme-T	VaR	0.0162	0.0507
	CVaR	0.0527	0.0865
Xtreme-i	VaR	0.0175	0.0543
	CVaR	0.0524	0.0859

DD (measured by number of standard deviations) is calculated using equation 1. CDD is based on the worst 5 percent of asset returns and is calculated using equation 3. VaR (95 percent confidence level) and CVaR (average of losses beyond VaR) are daily figures and can be annualised by multiplying by the square root of 250, being the approximate number of annual trading days. The pre-GFC period is the 7 years from 2000 – 2006 whereas the GFC period is the 3 years from 2007 – 2009.

Table 1 shows large differences between VaR and CVaR, or DD and CDD. For example, all Historical models show CDD being approximately 3 times higher than CVaR during the pre-GFC period, increasing to approximately 5 times higher for the transition based models (Xtreme-T and Xtreme-*i*) over the GFC period. The Monte Carlo models show similar trends to their corresponding Historical models, although Historical VaR and Monte Carlo VaR are slightly closer than Historical CVaR and Monte Carlo CVaR. The reason VaR is closer is because there are a large number of Historical VaR observations (95 percent of historical observations) to compare to the extremely large number of Monte Carlo VaR observations, whereas the Historical model generates only a smallnumber of CVaR observations (5 percent of historical observations) compared to the large number of Monte Carlo CVaR observations (5 percent of 20,000 observations), Although there are some differences between the models in the extent of the variation between the quantiles, the difference between VaR and CVaR (or DD and CDD) is nonetheless significant for all models at the 99 percent level using F tests for changes in volatility. This has significant implications for banks. Provisions and capital calculated on below the threshold measurements will clearly not be adequate during periods of extreme downturn. This is illustrated in Figure 2.

Figure 2. Illustration of Fluctuating Risk



The figure shows the results of the Quantile Regression (Xtreme-Q) Model for the 50 percent and 95 percent quantiles for pre- GFC and GFC periods. The pre-GFC period is the 7 years from 2000 – 2006 whereas the GFC period is the 3 years between 2007 – 2009. The *y* axis is calculated on the asset fluctuations (σ), using the Merton model, for the quantile in question. The *x* axis is the median σ for the entire 10 year period. Thus the Beta(β) for the 50 percent Quantile for the 10 year period is one. Where σ for a particular quantile is less (greater) than the median for the 10 year period, β<(>)1, and DD increases (reduces) accordingly.

The above graph shows that the 'median' DD (based on how the standard Merton structural model calculates DD) over the 10 year study period was 5.98 for US banks with an asset value standard deviation (σ) of 0.00789. As asset value σ is the denominator of the DD equation (equation 1), as σ increases (reduces) from one level to another (i.e from σ_1 *to* , σ_2) DD reduces (increases) by the same proportion. Thus the numerator of the equation (a measure of capital – the distance between as assets and liabilities) needs to increase to restore DD back to the same level (i.e., as per the BOE observation in Section 1, capital (K) will need to increase by the same proportion to restore market confidence in the banks' capital). Thus the required change in capital (K*) is;

$$K^* = \mathrm{K} \times \sigma_2 / \sigma_1 \tag{7}$$

Based on Figure 1, during the extreme fluctuations of the GFC (as measured by the 95 percent quantile) US banks needed in excess of 3 times more capital than during 'median' circumstances (as measured by the 10 year median). Whilst we have used the Xtreme-Q model to illustrate this, the same principle applies to all the models - a trebling of VaR or DD requires treble capital (100 percent buffer) to deal with it. Thus a bank with 5 percent capital during 'normal' times would need 15 percent during extreme times.

When comparing different volatility models, it is important to consider how well their relative outcomes compare to actual credit risk volatility experienced by US banks. Using 10 years of annual data, we correlate our measures for the four models to three measures three measures of actual credit risk. The first of these three measures is Credit Default Swap (CDS) spreads, which is measure of the premium the market is prepared to pay for increased credit risk. The second is Delinquent Loans as reported by the US Federal Reserve and are loans past thirty days or more and still accruing interest as well as those in non-accrual status, measured as a percentage of end-of period loans. The third is Charge-off rates, also reported by the US Federal Reserve which is the value of loans removed from the books and charged against loss reserves, measured net of recoveries as a percentage of average loans. These correlations are reported in Table 2. Various lags were tested, with most correlations being most significant with no lag and some correlations (the shaded areas of Table 2) being most significant with a 1 year lag (e.g. a 2009 measurement for actual risk compared to a 2008 measurement model). To avoid over-reporting of figures, we show only the results of the Histotrical model, but the Monte Carlo models produce very similar outcomes.

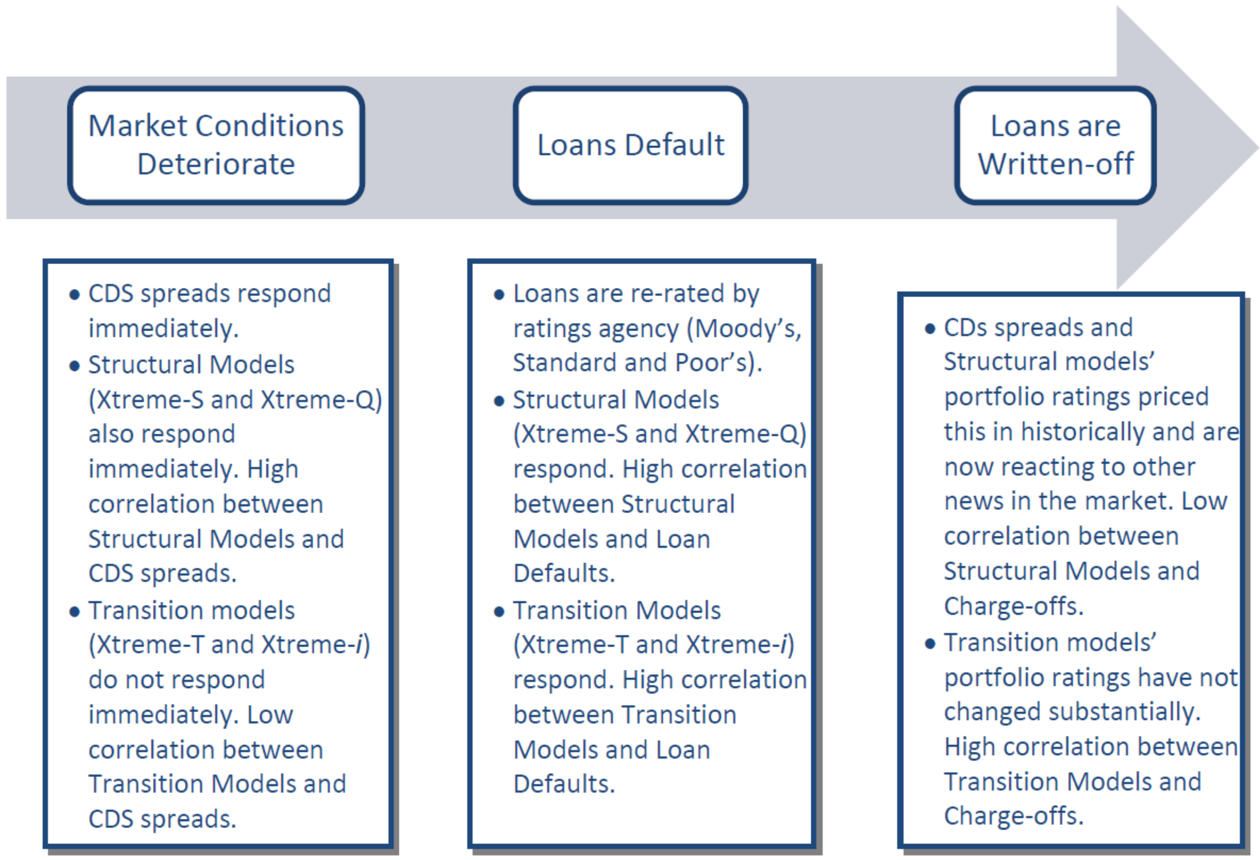
Table 2. Correlations

Model	Metric	CDS Spreads	Delinquent Loans	Charge-off rates
Xtreme-S	DD	0.915 **	0.786 **	0.606
	CDD	0.906 **	0.826 **	0.647 *
Xtreme-Q	DD	0.914 **	0.789 **	0.608
	CDD	0.885 **	0.865 **	0.683 *
Xtreme-T	VaR	0.573	0.929 **	0.936 **
	CVaR	0.741 *	0.893 **	0.908 **
Xtreme-i	VaR	0.831 **	0.952 **	0.926 **
	CVaR	0.768 **	0.920 **	0.920 **

The table correlates the Historical model metrics produced by each of our four models for each of the ten years in our data sample with three measures of actual credit risk of US banks, being CDS Spreads, Delinquent Loans, and Charge-off rates. Level of significance is measured by a t-test, with * denoting 95 percent significance and ** denoting 99 percent significance. Non shaded areas are where highest correlation is experienced with no lag, and the shaded areas with a 1 year lag.

The structural based models (Xtreme-S and Xtreme Q) show a much higher correlation with CDS spreads than the other models. This is because CDS spreads change daily with market conditions, and so does the asset value component of the structural model. The transition based models (Xtreme-T and Xtreme-*i*) which largely depend on ratings (more sluggish than CDS spreads as ratings are often updated only annually) show no significant correlation in the same year, but a higher correlation when using a one year lag. All four models show highly significant (99 percent confidence) correlation with delinquent loans, meaning that the metrics of all the models are a good indicator of actual defaults. There is very high significance shown by the transition based models' correlations with charge-off rates. The timeline in Figure 3 shows how both CDS spreads and the structural model respond quickly to market events, resulting in high correlation between these two items, whereas ratings (and thus transition models) react slower to market events and thus have a higher correlation with actual write-offs which usually occur sometime after initial market deterioration.

Figure 3. Timeline and Correlations



Of note is that there is very little difference in the correlation significance levels for VaR (DD) as compared to CVaR (CDD). This means that, although CVaR (CDD) are at much higher levels than VaR as previously discussed, the trend (percentage increase or decrease from year to year) is similar for both VaR (DD) and CVaR (CDD) .

It should be noted that transition probabilities used in Table 2 to calculate VaR and CVaR for Xtreme-T and Xtreme-*i* have been updated each year according to the annual probability matrices provided by Standard and Poor's, as opposed to using a long term historical average (for example the probability of a B rated loan defaulting in 2008 is given as 3.82 percent compared to 0.64 percent in 2006, even though the underlying rating had not changed). As mentioned in Section 2.4 it is common practice to model transition matrices using long term probability averages, thus varying only the ratings. As an alternative to our approach of varying the probabilities, we used a 10 year probability average. We found no significant correlation at all between the VaRs and CvaRs produced using this averaging method with any of the 3 risk variables (CDS spreads, Defaults, or charge-offs). This means that ratings on their own (without the associated default probability) are a poor indicator of actual credit risk, as they change only periodically, whereas the actual credit risk may have increased substantially in the interim. This has major ramifications for banks in respect of capital. The Basel standard approach requires banks to calculate capital based on the rating alone, but this rating may have an entirely different probability of default from one period to the next. It should be noted that rating agents such as Standard and Poor's and Moody's stress that ratings are not absolute measures of default, but rather a relative ranking of one entity to another. Therefore ratings on their own, without the associated default probability are not a good predictor of default or a sound basis for determining capital adequacy.

4 CONCLUSIONS

This paper has shown how CVaR type metrics can be applied to credit risk models to measure extreme risk. A comprehensive study was undertaken by generating and comparing four Xtreme models and by applying Historical as well as Monte Carlo metrics to each. In addition the models were applied to pre-GFC as well as GFC data to capture different economic circumstances.

All four models showed highly significant differences between VaR (DD) and CVaR (CDD) measures. Increased volatility requires capital buffers to deal with the increased risk and the paper demonstrated how this volatility and buffer requirement can be measured.

There were no significant differences in outcomes between Xtreme-S and Xtreme-Q, nor between Xtreme-T and Xtreme-*i*. There were significant differences observed between the structural based models (Xtreme-S and Extreme-Q) as compared to the transition based models (Xtreme-T and Xtreme-*i*). The changes in risk as measured by the structural based models are more consistent with changes experienced in CDS Spreads than those shown by the transition based models, because both structural models and CDS spreads respond very rapidly to market conditions. The opposite is true of charge-offs where the transition based models show much greater correlation than the structural based models, as there is generally a delay between defaults and charge-offs, and credit ratings also often respond slower (often annually) to market conditions than the structural models. All models show a significant correlation with delinquent loans.

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