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Automatic Recall of Multiplication Facts and Number Sense

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AUTOMATIC RECALL OF
MULTIPLICATION FACTS
AND NUMBER SENSE

by

Maxine D. Jolly

A Thesis submitted in partial fulfillment of the requirements for the degree of Bachelor of Education (Primary) Honours

Edith Cowan University
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ABSTRACT

The development of students’ Number Sense has become a recent focus in primary mathematics education. Students also often learn the multiplication tables by rote in order to develop automatic recall of multiplication facts. One view of mathematics learning suggests that automatic recall of number facts is an important step to developing number sense, while another view suggests that rote learning to develop automatic recall of multiplication facts may interfere with the constructivist learning environment that is required to develop number sense.

This study examined whether automatic recall was associated with good number sense or not, and explored factors associated with automatic recall which may affect students’ development of number sense. Students from a local school were tested with a timed mental mathematics test and students were asked to reflect on this experience to identify themselves as mostly users of automatic recall or not mostly users of automatic recall. A number sense test on the same multiplication facts was then administered. A chi square analysis was performed on this data, and comparisons made.

Interviews were conducted with 13 students from different groups. Analysis of data from these interviews suggests that rote learning of the multiplication tables to develop automatic recall may have had a negative affect on the development of number sense for a small number of students who did not use any strategies other than automatic recall. However, for a large number of students who use other strategies as well as automatic recall, the development of automatic recall had no significant impact on their development of number sense.
DECLARATION

I certify that this thesis does not, to the best of my knowledge and belief:

3 incorporate without acknowledgment any material previously submitted for a degree or diploma in any institution of higher education;

4 contain any material previously published or written by another person except where due reference is made in the text; or

5 contain any defamatory material.

Signature ........................................

Date ............................................
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1. INTRODUCTION ............................................................... 11
   1.1 Overview ........................................................................ 11
   1.2 Research Questions .................................................. 12
   1.3 Operational Definitions ............................................. 12
   1.4 Review of Literature ................................................... 13
      1.4.1 Rote learning of multiplication facts .................... 13
      1.4.2 Number sense ................................................... 14
      1.4.3 View 1: Rote learning of multiplication facts facilitates
            students' development of number sense .................. 18
      1.4.4 View 2: Rote learning of multiplication facts does not
            facilitate development of number sense ................. 20
      1.4.5 Rote learning of multiplication facts in View 2 ....... 23
      1.4.6 Other variables affecting development
            of number sense .................................................. 27
   1.5 Hypotheses ............................................................ 30
2. METHODOLOGY ............................................................... 31

2.1 Overview ........................................................................ 31

2.1.1 Design of the study ............................................... 31

2.1.2 Participants ............................................................... 33

2.1.3 Instruments ............................................................... 34

2.1.4 Testing procedures ............................................... 35

2.2 Mental Mathematics Test ............................................. 36

2.3 Self-Identification of use of automatic recall .................... 37

2.3.1 Matched groups ..................................................... 38

2.4 Test for Number Sense ............................................... 40

2.4.1 Presentation of number sense questions ..................... 42

2.4.2 Marking the number sense test .................................. 43

2.4.3 Turn around facts .................................................... 43

2.4.4 Data analysis ......................................................... 44

2.5 Semi Structured Interviews .......................................... 45

2.5.1 Identified groups .................................................... 45

2.5.2 Interviews ............................................................. 45

2.5.3 Interview questions ............................................... 46

3. RESULTS ...................................................................... 49

3.1 Quantitative Data ........................................................... 49

3.1.1 Number of students who demonstrated good number sense ............................................. 49

3.1.2 Testing the null hypothesis ....................................... 54
3.1.3. Correlation between mental multiplication test score and number sense score.........................55
3.1.4 The effect of gender........................................55
3.1.5 Self-identification and demonstration of number sense within each class..........................56
3.1.6 Year level and demonstration of number sense........58
3.2 Qualitative Data........................................................59
3.2.1 Description of interviews.................................59
3.2.2 Accuracy of self-identification...............................60
3.2.3 Confidence in using own methods.........................64
3.2.4 Perception of the importance of remembering or thinking in doing mathematics...............66
3.2.5 Student response to “doing” mathematics...........70
3.2.6 Differences between AR\G and NAR\G students......71
3.2.7 Differences between AR\G and AR\NG students......71

4. DISCUSSION OF RESULTS..............................................73

4.1 Discussion of Quantitative Data .....................................73
4.1.1. Accuracy of self-identification...................................73
4.1.2. Rejecting the null hypothesis.................................73
4.1.3. Mental multiplication test scores and number sense scores........................................74
4.1.4. Affect of variation in dependent variables.................75

4.2 Discussion of Qualitative Data........................................75
4.2.1. Confidence in using own methods ..................... 76
4.2.2. Perception of the importance of remembering or
thinking in doing mathematics ................................. 77
4.2.3. Student response to “doing” mathematics ............ 77
4.2.4. Differences between groups of students .............. 78
4.2.5. Assessing the two views of developing number
sense ........................................................................ 80
4.3 Limitations ............................................................ 81
4.3.1 The sample population ........................................ 81
4.3.2 Number sense test ............................................ 82
4.3.3 Automatic recall ................................................ 82
4.3.4 Classroom practice ........................................... 83
4.3.5 Individual ability ............................................. 83
4.4 Conclusions Arising from the Results of the Study .... 84
4.4.1 Identifying use of automatic recall ...................... 84
4.4.2 ARG\NG students ............................................. 84
4.4.3 Implications for teaching for number sense .......... 85
4.5 Suggestions for further study .................................. 86
REFERENCES ............................................................. 88
LIST OF APPENDICES .............................................. 91
APPENDICES .......................................................... A-E
LIST OF TABLES

Table 2.1
Table of identified groups ............................................. 35

Table 2.2
Table of tested multiplication facts ................................... 37

Table 3.1
Number of students in whole population ......................... 50

Table 3.2
Mean mental multiplication test scores for females and males in whole population ................................. 56

Table 3.3
Number of students who identified themselves as mostly users of automatic recall in each class ......................... 57

Table 3.4
Number of students who demonstrated good number sense in each class ................................................. 57

Table 3.5
The number of students who demonstrate good number sense at each year level within the matched groups ........ 58
| Figure 1.1 | View 1: Automatic recall facilitates development of number sense ........................................................... 19 |
| Figure 1.2 | View 2: Ideal conditions for developing number sense ...................... 21 |
| Figure 1.3 | View 2: Rote learning of multiplication facts impedes development of number sense ........................................... 25 |
| Figure 2.1 | Flow chart of design of study, "Automatic Recall of Multiplication Facts and Number sense" ........................................... 32 |
| Figure 3.1 | Box plot of number sense test scores for the whole population (N=110) .............................................................................................................................................................. 51 |
| Figure 3.2 | Box plot of number sense test scores for the matched groups ...... 52 |
| Figure 3.3 | Number of students demonstrating good number sense within each of the matched groups ........................................... 53 |
CHAPTER 1

INTRODUCTION

1.1 OVERVIEW

Number Sense underlies major curriculum development in mathematics, such as the student outcomes outlined as goals by the Education Department of Western Australia (1998). In the report, “Everybody Counts: A Report to the Nation on the Future of Mathematics Education”, the National Research Council (1989, p. 46) stated that, “the major objective of elementary school mathematics should be to develop number sense.” This has been an issue of continuing concern to mathematics teachers who realize the import of mathematical understanding in our increasingly technological society (National Research Council, 1989). Number sense has been the focus of recent research (Bana & Korbosky, 1995; McIntosh, Reys, Reys, Bana & Farrell, 1997), but the processes by which number sense is developed are somewhat less clear than educators’ determination to develop it, as little research has been conducted in this area. In the primary years, many students expend time and effort in attempting to develop automatic recall of multiplication tables. Is this effort warranted in terms of development of number sense?

This thesis investigates whether the development of automatic recall of multiplication facts affects the development of students’ number sense with regard to those facts. It further attempts to identify some of the factors
associated with rote learning that may impact on students’ development of number sense. Literature relating to the development of number sense is reviewed. Two models of developing number sense are suggested and discussed, one in which rote learning to develop automatic recall aids the development of number sense, and one in which it impedes the development of number sense.

1.2 RESEARCH QUESTIONS

This study was designed to generate empirical information relative to the questions:

1. Is automatic recall associated with good number sense?

2. What factors associated with automatic recall affect the development of number sense?

1.3 OPERATIONAL DEFINITIONS

Number sense “refers to a person’s general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations.” (McIntosh, Reys, Reys, Bana & Farrell, 1997, p. 3)
Automatic recall of the multiplication number fact “means that the student can retrieve that fact from long term memory without any conscious mental processing.” (Bana & Korbosky, 1995, p. 6)

Rote learning refers to intentional memorisation of facts to develop automaticity of responding (Resnick & Ford, 1984).

Mental computation “any procedure that involves calculating ‘something in your head’ without the use of pencil and paper” (McChesney & Biddulph, 1994, p.10).

Multiplication number facts will be defined as the 121 multiplication facts from (0 x 0) to (10 x 10), commonly described as the ‘times tables’.

1.4 REVIEW OF LITERATURE

1.4.1 Rote learning of multiplication facts

Rote learning of multiplication facts was once routine in schools, based on Thorndike’s theory that “memories that are used repeatedly are strengthened” (cited in Ashcraft, 1994, p. 229). Suydam & Reys (1978, p. 17), comment that “Drill has long been recognized as an essential component of instruction in the basic facts. Practice is necessary to develop immediate recall.” The emphasis on rote learning of number facts and procedures has fluctuated throughout the history of education, and it is presently superceded by an interest in developing students’ number sense so that they will be able to operate effectively in an increasingly technological society (National Research Council, 1989), with an understanding of
mathematics concepts and an ability to use a variety of mathematical strategies.

1.4.2. Number sense

Current mathematics learning theories have evolved from Piaget’s developmental model (Biggs, E. & MacLean, J. R., 1969) and a constructivist paradigm which “implies adopting a style or process that facilitates the students in their construction of knowledge” (Malone & Ireland, 1996, p. 123). Working with the concept of students constructing their own meaning from mathematics experiences, several authors have coined phrases which describe the ability of students to understand numbers and their operations, to reason and to use them in a flexible and purposeful way. Among them are Skemp (1987), who describes “schematic learning” and “relational understanding” and Reys, Suydam and Lindquist (1992) who describe the development of many “learning bridges” between mathematical concepts. Sowder (1988, p. 183) defined number sense as “a well organized conceptual framework that enables a person to relate number and operation properties”, which suggests that number sense requires the kind of learning understanding that Skemp, and Reys, Suydam and Lindquist described. Sowder further describes a person who uses number sense as using “flexible and creative ways to solve problems involving numbers” (1988, p183).

Greeno (1991, p. 170) describes number sense as “several important but elusive capabilities, including flexible mental computation, numerical estimation, and quantitative judgement.” In his theoretical analysis of
number sense, he describes number sense as "an example of knowing in a conceptual domain, the domain of numbers and quantities" (p. 170). He also provides the most elaborate description of the way that concepts may be developed and connected by extensive activity to generate this cognitive expertise in the area of number.

Mcintosh, Reys and Reys (1992) defined number sense as a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to develop useful strategies for handling numbers and operations." They also identify six strands within their framework for examining number sense. These strands are:

1. Understanding of the meaning and size of numbers (number concepts)
2. Understanding and use of equivalent forms and representation of numbers (Multiple Representations)
3. Understanding the meaning and effect of operations (Effect of operations)
4. Understanding and use of equivalent expressions (Equivalent expressions)
5. Computing and counting strategies
6. Measurement benchmarks

These strands were used to formulate questions for the number sense test used by McIntosh, Reys, Reys, Bana and Farrell (1997) when testing the number sense of students in four countries. McIntosh, Reys, Reys, Bana & Farrell (1997, p. 3) extended and refined McIntosh, Reys and Rey's (1992)
definition to define number sense as “a person’s general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations.” This definition has been adopted for use in this study because, although it does not describe the conceptual framework, it does encompass the basic strands identified by McIntosh, Reys and Reys (1992), describing characteristics which are identifiable in a student. The definition suggests that the indicators of good number sense would be:

• an understanding of number and operations
• the ability to use this understanding in flexible ways to make mathematical judgements
• an inclination to use this understanding
• efficient strategies for managing numerical situations.

Despite the general acceptance of a constructivist view of mathematics learning and the emphasis on developing number sense, some mathematics teaching practice still relies on a transmission style of teaching. Monroe & Clark (1998, p. 27) comment that “in spite of some movement toward pedagogy intended to help students develop mathematical thinking, many students and their teachers continue to rely on memory rather than reason as they perform mathematical calculations, indicating that they define mathematics, and mathematics has been defined for them throughout their years of schooling, as algorithmic thinking rather than reasoning.”
This style of teaching is apparent when teachers encourage rote learning to develop students’ automatic recall of basic number facts, and in particular, of the ‘times tables’ or basic multiplication facts. This practice raises the question as to whether developing automatic recall of multiplication facts through rote learning will facilitate the students’ development of number sense.

The Student Outcome Statements for Mathematics (EDWA, 1998) state at level 3, in regard to the basic multiplication facts (to 10 x 10), students will “remember quite a few basic multiplication facts and use mental methods to work out those they don’t remember or which go beyond the basic facts: for example, knowing four sevens (4x7=28) they can double to find eight sevens (8x7=56) and can further say that ‘8x70 is 56 tens, which is 560’.” This statement is clearly recommending the development of number sense in the area of multiplication facts, but this document does not recommend the means by which students should develop their automatic recall of “quite a few” multiplication number facts, nor clarify the number of multiplication facts that are expected to be automatically recalled. This leaves open the question of whether automatic recall of number facts should be taught in a manner that encourages rote learning, or whether the automatic recall of some facts might be one outcome of teaching strategies which aim to develop number sense within a constructivist learning environment. A review of literature found conflicting views as to the usefulness of rote learning multiplication tables in facilitating the development of student’s number sense.
1.4.3. View 1: Rote learning of multiplication facts facilitates students’ development of number sense

In defence of rote learning of multiplication tables, Hamrick & McKillip (in Suydam & Reys, 1978, p. 4) state that memorization of number facts is a prerequisite to learning computational skills (algorithms) which in turn “facilitates meaningful learning of both concepts and more advanced skills”.

Hope & Sherrill (1987, p. 98) report a common belief that mental calculation is “one of the best means of developing and deepening a child’s understanding of numbers and their properties.” They describe recall of basic number facts as “the fundamental building blocks of most calculations,” which was vital in identifying skilled mental calculators. Recall of basic number facts is therefore seen as a prerequisite for skilled mental calculation, which is considered to be a primary means of developing a student’s understanding of number which is important in developing number sense. McIntosh, Reys, Reys, Bana and Farrell (1997, p. 5) also concluded from their study of number sense in four countries that “one way to develop number sense is to develop mental computation ability”.

Askew (1997, p. 7) also supports the idea that automatic recall of number facts is useful in developing students’ mental facility with number, describing two aspects of mental computation: “mental methods of computation that are based on either instant or rapid recall of number facts.”
and "the ability of children to figure out mentally number calculations that they cannot rapidly recall". He also suggests that "these two aspects of mental mathematics - knowledge of number facts and strategic methods - appear to be complementary."

These views suggest that automatic recall of number facts is associated with number sense in the manner illustrated in figure 1, "View 1. Automatic Recall Facilitates Development of Number Sense".

<table>
<thead>
<tr>
<th>Factors that Promote Number Sense</th>
<th>Indicators of Number Sense</th>
</tr>
</thead>
<tbody>
<tr>
<td>Automatic recall of many number facts</td>
<td>Inclination to use understandings</td>
</tr>
<tr>
<td>Efficient mental computation</td>
<td>Flexible use of number</td>
</tr>
<tr>
<td>Taught strategies or own strategies</td>
<td>Understanding of number</td>
</tr>
<tr>
<td>Learned algorithms and practice or many experiences with number</td>
<td>Efficient strategies</td>
</tr>
</tbody>
</table>

Figure 1.1

View 1: Automatic Recall Facilitates Development of Number Sense
1.4.4. View 2: Rote learning of multiplication facts does not facilitate development of number sense.

Another view is that number sense can be developed effectively without rote learning of multiplication facts. Studies conducted by Brownell (Brownell & Chazal cited in Payne, 1975, p. 57) resulted in their conclusion that “The type of thinking that is developed and the child’s facility with the process of thinking are of greater importance than mere recall. Drill in itself makes little contribution to growth on quantitative thinking, since it fails to supply more mature ways of dealing with numbers.” Brownell and Chazal (cited in Suydam & Reys, 1978, p. 17) also concluded that “drill on basic facts increased pupil’s speed and accuracy but did not change the thinking they used to solve fact problems.”

Skemp (1987, p. 122) separates rote learning from developing number sense when he describes problems associated with memorization of the multiplication tables as the “burden on memory” and the lack of ability to adapt to other related problems. He recommends that students learn mathematics schematically instead, as they do when they develop number sense from many experiences with number, because this is “both more adaptable and reduces the burden on the memory.”

Greeno (1991, p. 173) suggests “it may be more fruitful to view number sense as a by-product of other learning than as a goal of direct instruction.”

Automatic recall of some multiplication facts could also be viewed as a byproduct of well developed number sense, rather than a goal of direct instruction. Kamii (1994, p. 73) points out that automatic recall of some
multiplication facts occurs naturally within a constructivist learning environment, as "Third graders come to remember easy combinations such as $4 \times 6 = 24$ and $10 \times 6 = 60$ through frequent use and will use them to deduce harder ones." These views suggest that automatic recall of number facts is associated with number sense in the manner illustrated in figure 1.2, "View 2: Ideal conditions for developing number sense".

<table>
<thead>
<tr>
<th>Factors that Promote Number Sense</th>
</tr>
</thead>
<tbody>
<tr>
<td>Many Real Number Situations</td>
</tr>
<tr>
<td>Discussion and Problem Solving</td>
</tr>
<tr>
<td>Experimenting and Reflecting on own strategies</td>
</tr>
<tr>
<td>Concrete Experiences</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Indicators of Number Sense</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding of Number &amp; Operations</td>
</tr>
<tr>
<td>Flexible use of number</td>
</tr>
<tr>
<td>Own efficient strategies</td>
</tr>
<tr>
<td>Inclination to use understandings</td>
</tr>
<tr>
<td>Automatic Recall of Some Basic Facts</td>
</tr>
</tbody>
</table>

*Figure 1.2*

**View 2: Ideal Conditions for Developing Number Sense**

This model assumes that number sense is best developed within a learning environment where students are encouraged to think mathematically (Curriculum Framework for K-12 Education in Western Australia, 1998) and construct their own mathematical understandings (Skemp, 1987).

McChesney and Biddulph (1994, p. 10) state that "Number sense is not
something that can be taught directly. Rather it is something that emerges from mathematical activity and exploration."

Identifying specific routes to number sense was not the focus of this research, but many experiences with number in real situations, many concrete mathematical experiences, students' experimenting and reflecting on their own mathematics strategies, problem solving and discussion with peers and teachers, have been identified in the literature below as probable factors in developing number sense included in figure 1.2: View 2.

McIntosh (1996) recommends that children concentrate on how they do mental computations in order to develop number sense. Greenes, Schulman & Spungin (1993) suggest that number sense is enhanced when students associate numbers with objects, events and real situations. This is supported by Burns (1992, p. 24) who writes;

"Learning mathematics requires that children create and re-create mathematical relationships in their own minds. Therefore, when providing appropriate instruction, teachers cannot be seduced by the symbolism of mathematics. Children need direct and concrete interaction with mathematical ideas; ideas are not accessible solely from abstractions. Continuous interaction between a child's mind and concrete experiences with mathematics in the real world is necessary."

The Curriculum Framework for K-12 Education in Western Australia (1998, p. 198) states that "mental computation should be developed through
discussion, comparison and reflection on alternative strategies and varied
practice.” Problem solving is also identified as an aid to developing number
sense by the National Council of Teachers of Mathematics (1991).
Markovitz and Sowder (cited in Reys & Nohda, 1994) believed that
encouraging students to use their own methods of calculating was good
teaching practice, and that an increase in the use of non-standard methods
indicated “an increase in student’s number sense.”

1.4.5. Rote Learning of Multiplication Facts in View 2
The beliefs and attitudes associated with memorizing multiplication tables
are different to those associated with the constructivist view of learning that
promotes number sense, so teaching for automatic recall within this
environment may have a negative impact on the development of number
sense.

In discussing the role of memorized written algorithms in the curriculum,
several authors have drawn attention to the negative impact of early
introduction of written algorithms on students’ development of number
sense (Shuard, 1986; Kamii & Dominick, 1989; Reys, Suydam, Lindquist &
Smith, 1998). Students’ automatic recall of multiplication facts is often
developed by rote learning and this is similar to many students’ learning of
written algorithms. It therefore seems possible that the rote learning of
multiplication tables may also have a negative impact upon student’s
development of number sense.
Number sense may be obstructed by early acquisition of automatic recall of number facts because:

**Students are encouraged to remember rather than to think**

Monroe & Clark, (1998, p. 27) found that “in spite of some movement toward pedagogy intended to help students develop mathematical thinking, many students and their teachers continue to rely on memory rather than reason as they perform mathematical calculations.” Madell (cited in Kamii, Lewis & Livingston, 1993,) claims that “The early focus on memorization in the teaching of arithmetic thoroughly distorts in children’s minds the fact that mathematics is primarily reasoning.”

**Students are discouraged from developing and having confidence in their own ways of calculating**

Kamii (1994, p. 73) states that “As for multiplication tables, memorization of these tables is not an appropriate goal for third graders. Such memorization would crush children’s excitement about what Duckworth (1987) called “wonderful ideas”.” Kamii & Dominick (1989, p. 135) criticize the rote learning of algorithms in arithmetic because “they encourage children to give up their own thinking.” McIntosh, Reys and Reys (1992, p.3) point out that “although many young children exhibit creative and sometimes efficient strategies for operating with numbers, attention to formal algorithms may, in fact, deter use of informal methods.” Similar difficulty may arise when students rote learn multiplication facts instead of using their own strategies to calculate products from familiar understood facts or benchmarks.
Students may be affected by increasing anxiety that decreases students' enjoyment and interest in mental mathematics.

In regard to mathematics anxiety, Buxton (1981, p. 7) comments that "Tests of mental recall of facts (often wrongly referred to as mental arithmetic) have much to answer for." Reys, Suydam, Lindquist & Smith (1998, p. 28) suggest that to help students cope with anxiety teachers should "Emphasize meaning and understanding rather than memorization."

These views suggest that the impact of rote learning of multiplication facts could impede the development of number sense in the manner illustrated in Figure 1.3, "View 2: Rote learning of multiplication facts impedes development of number sense."

![Figure 1.3](attachment:image.png)

**Figure 1.3**

**View 2: Rote Learning of Multiplication Facts Impedes Development of Number Sense**
Research on the relationship between automatic recall and number sense is limited. Information from recent research into number sense, however, will be helpful in exploring the relationship between number sense and automatic recall.

McIntosh, Reys, Reys, Bana and Farrell (1997, p. 5) used the framework for examining number sense developed by McIntosh, Reys and Reys (1992) to develop a test to assess students' number sense at different ages. Their results showed a link between mental computation ability and number sense in the Australian and American studies which led to their conclusion that "one way to develop number sense is to develop mental computation ability". Their study did not attempt to identify the role of automatic recall in mental computation or number sense, and they state that "While agreement exists that the development of number sense is an important goal for all children, many questions remain unanswered about the routes to achieve this goal" (p. 5).

In 1995, Bana & Korbosky published research that assessed the extent of students' automatic recall of basic number facts in the four operations and assessed their ability to apply their understanding of basic number facts to real life situations. Assessment of students' number sense was also made in relation to subtraction and division facts. Bana & Korbosky (1995, p. 40) report "The extent of understanding of the subtraction and division facts was not very different from performance on automatic response in these operations. However, as different items were used in this case, further study
is needed to determine whether or not there is a close relationship between knowledge and understanding of basic facts.” They also suggest (1995, p. 41), that further research is needed into the relationship between knowledge and understanding of basic number facts, stating that “This should be dealt with more systematically by using the same item for both assessments in each case.” Students tested for automatic recall and for number sense in this study were tested using the same multiplication facts.

1.4.6. Other Variables Affecting Development of Number Sense

Within the literature reviewed, several other factors are identified as affecting students’ development of number sense. These variables were controlled, as far as possible, in this study, and each of them are discussed in turn.

The year level at which the students are studying (maturity and curriculum content)

Reys, Suydam & Lindquist (1992, p. 4) explain that the school system has been geared to the belief that, “topics cannot be taught until the child is developmentally ready to learn them” so the age of the student is a factor in determining the curriculum. Students in different year levels would therefore be likely to have studied different content in mathematics and have had different mathematics experiences, which may affect their development of number sense.
The Curriculum Framework for Kindergarten to Year 12 Education in
Western Australia (1998, p.197) states that in middle childhood (typically
Years 3 - 7), "students are increasingly able to think of concepts such as
'multiplication'." If students are expected to be increasingly able as they
mature, older children may score higher than younger students in a test for
number sense. Bana and Korbosky (1995, p. 40) found that there was no
increase in performance on tests of automatic recall between years 5 and 6,
but there was a significant rise in scores between years 6 and 7. They
suggest that "it may also be the case that performance on basic facts levels
off over years 5-6 due to a lack of maturation over these age levels."

**Gender**

Barnes, Plaister and Thomas (1984, p. 23-24) point out that although the
mathematical performance of boys may not actually be superior to girls,
they do "significantly better on questions of a practical nature,... and
problems requiring multiple steps for their solution. Girls do better on
simple arithmetic and algebraic questions involving the application of a
memorized rule." Bana and Korbosky (1995) found a variation in strategies
used by girls and boys in their assessment of understanding of the number
facts. These variations may affect the results of tests for number sense.

**Teaching practice within the classroom**

Grouws (1992) describes classroom environments which "have students
interact (with each other and with the mathematics) in ways that promote
mathematical thinking." Reys, Suydam & Lindquist also state that "Helping
students to develop such number sense requires ... in general, creating a classroom environment that nurtures number sense." If some of the students involved in the study come from a classroom that 'nurture number sense' and some do not, the results of the tests may reflect this, rather than the students' use of automatic recall. A thorough inquiry was not made into the methods used to teach mathematics in each classroom, but students interviewed were asked to describe what happened when they “did maths” in their classroom. Students described chanting times tables to a tape, mental math speed tests, sheets of tables to write out, multiplication patterns to complete, math problems in text books, doing sums written on the board, solving story problems, measuring, and self paced and self marked assignments which included measurement and space. Mathematics games were not mentioned by the students, but were observed in some classes by the researcher. This evidence was not sufficient to make a reasonable comparison of classroom teaching strategies, as students from the same classroom often described different activities, but it does suggest a variety of approaches to mathematics, which may impact on student’s development of number sense.

**Individual Ability**

According to Stevenson (1975, p3), “Wide individual differences exist in the abilities of children to learn and to solve problems, and these differences are complex and difficult to determine. ... Whatever the group, whatever the task and its presentation, children tend to learn at different speeds.” These
differences may be attributed to differences in intelligence or previous experiences or other factors, but are nonetheless likely to affect the results of testing for number sense.

1.5 HYPOTHESES

1. Students who mostly use automatic recall of multiplication facts are less likely to demonstrate good number sense in regard to those facts.

2. Null Hypothesis
There will be no significant difference between the number of students who demonstrate good number sense in a group of students who mostly use automatic recall and the number of students who demonstrate good number sense in a group of students who mostly did not use automatic recall.
CHAPTER 2

METHODOLOGY

2.1 OVERVIEW

In this chapter, the design of the study will be described. The sample population will also be described and the instruments used will be described and discussed. The procedure followed for the collection of data will be outlined, along with the statistical treatment of the data.

2.1.1 Design of the Study

This study included both quantitative and qualitative aspects. A quasi-experimental method was used to test the research hypothesis that students who did not use automatic recall of multiplication facts are more likely to have good number sense in regard to those facts. The results of this testing were also used to identify groups of students with similar characteristics. Students from each of these groups were then interviewed to generate qualitative data regarding the students’ perceptions about mathematics. Below is a flow chart of the procedure.
134
year 5 - year 7
primary students
from one school

Mental
Multiplication Test
and Self Identification
of Users of
Automatic Recall

students who
mostly do not
use automatic
recall

students who
mostly do not
use automatic
recall

Students who
do not know
what they do

Matched Groups

30 students
mostly do not
use automatic
recall

30 students
mostly do not
use automatic
recall

Test for
Number Sense

Test for
Number Sense

End

ARNG Group
students who use
mostly Automatic
Recall and
demonstrate
Good number sense

4 students selected

Semi Structured
Interview

Figure 2.1

Flow chart of design of study “Automatic Recall of Multiplication Facts and Number Sense”
2.1.2 Participants

The population from which the original sample of students was drawn was upper primary students who attended a large South West primary school, chosen on the basis of expediency. Upper primary students were chosen because students at this level are assumed by the Curriculum Framework for Kindergarten to Year 12 Education in Western Australia (1998) to be developmentally able to understand and use multiplicative strategies. The first primary school approached agreed to facilitate this research and letters asking the class to participate in the research (see Appendix E) were sent to enough teachers of upper primary grades at this school to provide a sample of students larger than 100. The school has more than one class of each year group and has an experienced teaching staff. Years 5 and 6 were chosen to begin with, and as all of the teachers approached agreed to the testing, no further teachers were approached. The teachers were offered an overview of class results on the number sense test, which did not identify particular students but showed how many students demonstrated number sense in reply to each question. After the results had been analysed, interviews were sought with 15 students. Parental permission was received for 14 interviews, 13 of which were subsequently conducted.
2.1.3 Instruments

Three instruments were used in this study:


2. A test for number sense.

3. A semi structured interview.

The mental multiplication test and the number sense test were used for two purposes:

3  To provide data for analysis with regard to the number of cases where good number sense was demonstrated in the group of students who mostly used automatic recall compared to the number of cases where good number sense was demonstrated in the group of students who mostly did not use automatic recall in order to accept or reject the null hypothesis.

4  To identify students belonging to one of the four groups listed below:

   AR\G: Students who use automatic recall and demonstrate good number sense

   AR\NG: Students who use automatic recall and do not demonstrate good number sense

   NAR\G: Students who do not mostly use automatic recall and demonstrate good number sense

   NAR\NG: Students who do not mostly use automatic recall and do not demonstrate good number sense.
Table 2.1

Table of Identified Groups

<table>
<thead>
<tr>
<th>Mostly Use Automatic Recall</th>
<th>Demonstrate Good Number Sense</th>
<th>Do Not Demonstrate Good Number Sense</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR\G</td>
<td></td>
<td>AR\NG</td>
</tr>
<tr>
<td>NAR\G</td>
<td></td>
<td>NAR\NG</td>
</tr>
</tbody>
</table>

Four students were originally drawn from the NAR\G group and four students were drawn from the AR\NG group to participate in a semi-structured interview. Two students were also drawn from the groups AR\G and NAR\NG to participate in a semi-structured interview. When one of the AR\G students was found to belong in the NAR\G group (making 5 NAR\G students), an extra student was drawn from the AR\G group to replace him. These semi-structured interviews explored the students' perceptions about mathematics.

2.1.4. Testing Procedures

The mental multiplication test and the number sense test were administered to five whole classes on different days, over a two week period in July 1999. The mental multiplication test was administered prior to morning recess, and the number sense test was administered after morning recess.
The same instructions and explanations were given to each class before and during each test. The protocols used during testing are detailed in Appendix A.

2.2 MENTAL MATHEMATICS TEST

Participating classes of students were presented with a series of ten mental multiplication questions, using the same ten questions and a similar procedure to that outlined in the study by Bana and Korbosky (1995) for assessing students' automatic recall of number facts. In order to enable the whole class to be tested at once, students were given a sheet on which they would record their name, class and year level and their answers to the questions. The answer sheet can be found in Appendix A, along with instructions given to students at the time of the test. The students had three seconds in which to mentally recall or calculate the answer to each displayed and read question and correctly record their answers before the next question was read and displayed.

The multiplication facts tested in the study by Bana & Korbosky (1995), and in this study, are listed in Table 2.2.
Table 2.2

Table of Tested Multiplication Facts

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Multiplication Fact</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 \times 3</td>
</tr>
<tr>
<td>2</td>
<td>3 \times 4</td>
</tr>
<tr>
<td>3</td>
<td>5 \times 5</td>
</tr>
<tr>
<td>4</td>
<td>8 \times 2</td>
</tr>
<tr>
<td>5</td>
<td>4 \times 6</td>
</tr>
<tr>
<td>6</td>
<td>9 \times 0</td>
</tr>
<tr>
<td>7</td>
<td>7 \times 3</td>
</tr>
<tr>
<td>8</td>
<td>9 \times 4</td>
</tr>
<tr>
<td>9</td>
<td>6 \times 7</td>
</tr>
<tr>
<td>10</td>
<td>9 \times 8</td>
</tr>
</tbody>
</table>

2.3 SELF-IDENTIFICATION OF USE OF AUTOMATIC RECALL

In the study conducted by Bana and Korbosky (1995) it was assumed that a correct oral answer given within the three second time limit was an automatic response, that is, it was the result of recall from long term memory without conscious calculation. They comment, however, that "The three second response time ... did not necessarily prevent a student from using reconstructive processes ....Whether a student actually used such processes was not documented ... Hence for some facts the three second limit did not necessarily ensure automatic recall" (1995, p. 7). Anecdotal evidence also suggested that some students may not use automatic recall to answer multiplication questions within the three second limit. In order to increase the accuracy of identification of students who used automatic recall, students were asked to identify whether they had used automatic recall in the mental multiplication test or not. Before the test commenced,
students were primed by reviewing the question regarding automatic recall. The test procedure can be found in Appendix A. Immediately after they completed the mental mathematics test, the students were asked to answer the following question:

"Think about the answers you think you got right. How did you get these answers? Put a tick in the one box that is closest to your answer?

- I remembered these answers straight away
- I often worked them out quickly in my head
- I don’t know how I got them"

Using this method, students identified themselves as belonging to one of three groups.

3. Mostly use automatic recall
4. Use methods other than automatic recall.
3. Don’t know what they do

Asking the interviewed students why they ticked that box checked the validity of some of the students’ responses. The results of the mental multiplication test were also used to create two matching groups for analysis of the data from the number sense test.

2.3.1 Matched Groups

Matched groups were created in order to minimise the impact of the independent variables before a Pearson chi square analysis was performed on the data from the number sense test. This data was used to identify
whether students demonstrated good number sense (a number sense test score $\geq 10$). The results were used to test the null hypothesis.

To create two matched groups, thirty students were drawn from the group who self identified as mostly users of automatic recall (students who ticked "Mostly use automatic recall") and thirty students were drawn from the group who self identified as not using automatic recall, (students who ticked "Sometimes I worked them out quickly in my head"). These students were selected with the intent of creating two matching groups (Malhotra, Hall, Shaw & Crisp, 1996, p. 192) in terms of the identified independent variables;

*Competency in mental multiplication*

The mean score and standard deviation of the scores in the mental multiplication test of the two groups were comparable. The mean scores for both groups was 7.867, and the standard deviation from this score was 1.962 for the group that mostly used automatic recall, and 1.979 for the group that did not mostly use automatic recall. The differences in the students' mental multiplication test scores was minimized in order to reduce the impact on the number sense test score of the variable of individual ability of students to learn and calculate mathematically.

*Gender*

Since Bana and Korbosky (1995) found a variation in strategies used by girls and boys in their assessment of understanding of the number facts, it
seemed prudent to draw a balance of boys and girls in the sample population. Fifteen boys and fifteen girls were selected in each group.

**Year of schooling**

It was important that the students be at a similar year levels to reduce the impact of the variable of maturity and of curriculum content on the results. In each group, eighteen year six and twelve year five students were selected.

**Teaching practice in the classrooms**

The impact that this variable may have on results was minimised as far as possible, by testing students from the same school where similar policies for the teaching of mathematics were implemented across the school. The same number of students from different classes was represented in each of the groups.

The number of students who identified as mostly users of automatic recall or not mostly users of automatic recall in each class varied dramatically in some classes and as the mental multiplication scores were generally higher for students using mostly automatic recall than for the other group, it was not possible to match pairs of subjects for each of the independent variables, so the characteristics of the two groups were matched for mean mental multiplication test score, gender, class and year. A list of the matched groups can be found in Appendix B.
2.4 TEST FOR NUMBER SENSE.

All students were tested for number sense using questions based on the number facts presented in the mental mathematics test. The number sense test was presented as a written test paper with 15 questions. Whole classes were tested together in order to minimize disruption to participating classes. The test was based on the definition of number sense by McIntosh, Reys, Reys, Bana & Farrell (1997, p. 3), on the six strands identified by McIntosh, Reys and Reys (1992) in their framework for number sense, questions used by McIntosh, Reys, Reys, Bana and Farrell (1997) to assess the number sense of students, questions in Bana & Korbosky's Test Section B - Application of Automatic Response (1995) and questions based on Haylock's think board, on which students represent mathematical ideas as symbols, real things, pictures and stories (Herrington, 1988). Advice was also received from a panel of experts with many years experience in the area of primary mathematics. The questions were constrained by the necessity to relate them to the multiplication questions in the mental multiplication test, and to reduce the likelihood of testing some other facet of mathematical understanding, such as understanding of place value. The marking of the number sense test was somewhat subjective, as it required the interpretation of several written answers, and the assessment of whether these answers demonstrated number sense in regard to multiplication. These decisions were based on the indicators of number sense previously discussed and upon insights gained from discussion with a panel of experts in the field of primary mathematics, and from students' comments. A copy of the number sense test can be found in Appendix C. Because the number sense test is
central to the validity of this research, each question has also been briefly discussed in Appendix D, along with issues that arose during the marking.

2.4.1 Presentation of Number Sense Questions

Questions 1 to 4 were multiple addition and arrays which were presented on an overhead projector for six seconds (see Appendix C) with the questions, "How much is that altogether?" and "How many dots are there?" asked by the researcher. Answers were written on the test paper.

Questions 5 to 15 were written questions presented on the test paper with space for written answers. Approximately 25 minutes was allowed for the students to complete the test.

A pilot test was conducted with four students who were not part of the test group, and alterations were made to the wording of some questions in order to make them easier to understand. In the trial, all the students completed the test in less than 20 minutes, so 25 minutes was set as the time for the test in order to allow ample time for its completion by most students. Of the 133 students tested, eleven students were still working at 25 minutes.

The fact that the number sense test was in written form may have had some impact upon the results of the test, as it might be expected that students with better literacy skills would perform better on the test. Teachers were asked about this aspect of the test and all teachers agreed that they would expect that the students in their classes would have no difficulty in reading the
questions, with the exception of one student. The questions that this student identified as difficult to understand in the test were read aloud to him.

2.4.2 Marking the Number Sense Test

Each answer that demonstrated number sense was awarded one mark, so that students who demonstrated number sense in all their answers could score fifteen marks altogether. Students who scored ten or more marks on the number sense test were classified as demonstrating good number sense. Students scoring nine marks or less were classified as not demonstrating good number sense. The number sense test was marked after the matched groups had been created using data from the mental multiplication test.

2.4.3 Turn around facts

The question of whether an array of items could be described by a multiplicative statement only or also by its reverse, (commonly called ‘the turn around fact’ by students at the school) impacted upon the marking of several questions.

During the trial of the number sense test, discussion with the students highlighted an inconsistency in the way they read multiplicative number sentences. Some students would read 2x3 as “two lots of three” or “two groups of three” which could be represented as while others read 2x3 as “two multiplied by three”, which could be represented as . All students agreed that 2x3 could be read as “two times
three". A survey of one class involved in the testing showed by a count of hands that the class was evenly divided over whether the representation could be written only as $2 \times 3$, or as either $2 \times 3$ or $3 \times 2$. Of eight students who were interviewed, four believed that represented only $2 \times 3$, and four believed that it represented $2 \times 3$ or $3 \times 2$.

Six teachers also failed to agree on the interpretation of the drawing. As a result of these inquiries it was decided that where questions in the number sense test required the interpretation of similar number sentences or representations, either the number sentence or its reverse would be accepted as an appropriate answer.

2.4.4 Data analysis

An arbitrary score of ten was used as the cut off point at which students were considered to demonstrate good number sense with regard to the tested multiplication facts. This was used to convert the scores into a nominal scale that identified the number of students within each group who demonstrated good number sense. A chi square test was then performed on the data from each of the matched groups, to determine whether there was a significant difference between the numbers of students who demonstrated good number sense within each group.
2.5 SEMI STRUCTURED INTERVIEWS

2.5.1 Identified Groups

Data from the number sense test which identified students as demonstrating good number sense or not demonstrating good number sense, and the students' self-identification as mostly users of automatic recall or not mostly users of automatic recall, was used to classify students as belonging to one of the four groups shown in Table 2.1 (p.35).

The researcher’s original intention was to interview only students from the AR\NG and NAR\G groups, but further consideration of the need for comparisons to be made between all of the groups led to the decision to interview four students from each of the groups AR\NG and NAR\G and two students from each of the groups AR\G and NAR\NG.

2.5.2 Interviews

A semi-structured interview was used in this study to provide additional information in regard to the students' perceptions of mathematics. The semi-structured interview was selected as a method in order to keep the interview focused on useful subject matter, and still allow the student being interviewed to express relevant ideas that the interviewer had not anticipated. The previous test experiences were used as a focus of questions regarding student perceptions of mathematics.
These interviews explored and compared the perceptions of students who mostly used automatic recall and the perceptions of students who mostly did not use automatic recall with particular reference to the students’:

5 perception of the importance of remembering and thinking in mathematics,

6 confidence in their own ability to develop strategies to solve harder problems, and

7 response to ‘doing’ mathematics, especially the presence of anxiety and whether they view mathematics as “useful”.

7.2.6 Interview questions

These questions were asked at the interviews.

- Who do you know who is really good at maths? Why are they so good at it?
- Do you think that you are really good at maths? Why is that?
- What would you need to do to become better at maths in your classroom?
- When doing maths is it more important to remember well or to think about things?
- (Showing a multiplication problem outside the range of the multiplication tables) Do you think you would be able to solve this? How might you do it? (Record strategies.)
- What did you think of the timed mental maths test? Why is that? Do you think it was a good way to measure students ability to use mathematics?
• What did you think of the Number Sense test? Why is that? Do you think it was a good way to measure student's ability to use mathematics?

• How do you usually do maths in your class? Is that useful to you?

• How do you usually do maths when you are not at school? Is that useful to you?

The question "What would you need to do to become better at mathematics in your classroom?" and "How do you usually do maths when you are not at school?" were expanded to contextualise the question in order to elicit a better student response to the question. A similar scenario was given to each student interviewed.

Questions regarding particular problems in the number sense test, and a question with regard to 'turn around facts' were added to gain insights into issues that arose during the marking of the number sense test. Students were also asked which box they ticked after the Mental Multiplication test, and why they chose to tick that box, in order for the researcher to assess whether the self-identification of users of automatic recall was accurate.

In addition, students were asked when they first began to learn the answers to the multiplication tables by heart, and when they thought they began to understand what multiplication meant or how you can use it. It was not expected that students would give accurate answers as to when they learned these things. This question enabled the researcher to establish whether
student had actually been taught automatic recall in school, and gave the
student the opportunity speak about the difference between “what
multiplication meant and how you can use it”, and automatic recall of
answers to the multiplication tables.

These interviews were audio taped and transcribed with permission from
the students’ caregivers. The request for permission is in Appendix E. The
resulting data was organised into comments about the importance of
remembering and the importance of thinking, anxiety, confidence and
competence, and using own methods, so that comparisons and contrasts
could be made and demonstrated.
CHAPTER 3

RESULTS

3.1 QUANTITATIVE DATA

The results of the study will be described in this chapter. Quantitative data from the testing will be described first, followed by a description of qualitative data obtained from the semi-structured interviews.

3.1.1. Number of students who demonstrated good number sense

After the mental multiplication test, students identified themselves as users of automatic recall (AR) or not users of automatic recall (NAR) by ticking boxes which described the manner in which they had found correct answers. More students identified themselves as not mostly users of automatic recall than mostly users of automatic recall. Students who demonstrated number sense in 10 or more of the 15 questions on the number sense test were judged to be demonstrating good number sense. The number of students who were identified as demonstrating good number sense in each group are presented in Table 3.1.
### Table 3.1

**Number of students in whole population**

<table>
<thead>
<tr>
<th></th>
<th>Demonstrate Good Number Sense</th>
<th>Do Not Demonstrate Good Number Sense</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mostly Use Automatic Recall</td>
<td>AR\G 34 students</td>
<td>AR\NG 10 students</td>
</tr>
<tr>
<td>Do Not Mostly Use Automatic Recall</td>
<td>NAR\G 32 students</td>
<td>NAR\NG 34 students</td>
</tr>
<tr>
<td>Don’t Know What They Did</td>
<td>11 students</td>
<td>7 students</td>
</tr>
</tbody>
</table>

The original population consisted of 134 Year 5 and Year 6 students. Forty-four students identified themselves as mostly users of automatic recall, sixty-six students identified themselves as not mostly users of automatic recall, eighteen identified themselves as "I don’t know what I did", and six students were unable to complete both tests. The 24 students who identified themselves as "I don’t know what I did" or were unable to complete both tests are excluded in further analysis of the results.

The larger number of students in the NAR\NG group raises the question of whether less able students are over represented in this group. It seems likely that if automatic recall of number facts is a learned strategy, then students who are better at learning might be more likely to learn automatic recall of the multiplication facts and therefore choose to use automatic recall. This highlights the importance of making an attempt to filter out the impact of
individual ability before testing the null hypothesis. This was done by matching the mean mental multiplication test scores for the two groups.

The actual scores on the number sense test for the groups identified as mostly users of automatic recall and not mostly users of automatic recall within the whole population are graphed in the box plot in Figure 3.1 which was generated from the data in SPSS.

![Box plot of number sense scores for the whole population (N=110)](image)

*Figure 3.1*

*Box plot of number sense scores for the whole population (N=110)*

The two boxes in this box plot depict the range of scores within the central 50% of each group and in this case the larger box for “does not use auto recall”, illustrates the wider range of scores in this larger group of students. The longer whiskers for the “does not use auto recall” group also describe the wider spread of scores in this group, from 4/15 to 15/15. The outlying
scores of the “uses auto recall group” (at six and fifteen) are designated by the dots. The position of the box and whiskers illustrate the tendency for students who identified themselves as mostly users of automatic recall to have slightly higher number sense scores, as the middle 50% of the “uses auto recall” group sits between ten and twelve marks, with a median (middle) test score of eleven, while the middle 50% of the “does not use auto recall” group sits between eight and eleven, with a median (middle) test score of nine. These results are affected by differences in year, class, ability and gender between the two groups.

The results of the number sense test for the groups matched for gender, class, year and score on the mental mathematics test are depicted in the box plot in Figure 3.2, which was generated in SPSS.

Figure 3.2

Box plot of number sense test scores for the matched groups
Here the population sizes are the same, and the two boxes, which illustrate the range of scores within the central 50% of the population are also similar in size and position. The whiskers reflect slightly lower outlying scores for the students who do not mostly use automatic recall. The median score for the students mostly using automatic recall is eleven, which is higher than the median score of ten for the matched group who do not mostly use automatic recall.

The number of students who demonstrated good number sense (number sense test score $\geq 10$) in each of the matched groups is graphed below in Figure 3.3.

![Figure 3.3]

**Figure 3.3**

*Number of students demonstrating good number sense within each of the matched groups*
The mean score on the number sense test for students who mostly use automatic recall (AR) within the matched groups was 10.47. The mean score on the number sense test for students who mostly did not use automatic recall (NAR) within the matched groups was 10.03.

3.1.2 Testing the Null Hypothesis

The null hypothesis was tested by a chi square test for significant difference in nonparametric data. SPSS for Windows was used to calculate the $\chi^2$ probability that the difference in the frequency of demonstration of good number sense (scoring $\geq 10$ in the number sense test) in each of the matched groups (students who mostly use automatic recall and students who do not use mostly automatic recall) was due to chance. The chi square ($df=1, N=60$) probability was 0.18. Given that a probability of less than .05 would be significant, then the difference between the number of students who demonstrated good number sense in each of the matched groups was not significantly different to the difference one might expect to occur in a sample of this size if there actually were no difference between the demonstration of good number sense in students who mostly use automatic recall and students who do not mostly use automatic recall.

Further analysis of the data in regard to each of the independent variables (gender, year, class, mental math score) resulted in the following findings.
3.1.3  **Correlation between mental multiplication test score and number sense score.**

The correlation between the student scores on the mental multiplication test and on the number sense test was indicative of the similarity of the two tests. To test this, 50 students were randomly selected from the whole population and a *t* test was performed to ascertain the Pearson correlation coefficient with regard to the correlation between the students' mental multiplication scores and the students' number sense scores. The Pearson correlation coefficient was 0.43, which indicates a moderate positive correlation between the scores. The 2 tailed significance was 0.002 which indicates a significant result at 0.05 level of significance. The moderate positive correlation reflects the fact that the tests were based on the same multiplication facts, but supports the belief that the two tests actually tested something different in regard to those facts.

3.1.4  **The Effect of Gender on Number Sense Scores**

The mean scores for female and male students within the matched groups are displayed in the table below:
Table 3.2

Mean mental multiplication test scores for females and males in whole population

<table>
<thead>
<tr>
<th>WHOLE POPULATION N=110</th>
<th>Mean score in Mental Multiplication Test</th>
<th>Mean Score in Number Sense Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female: Mostly Use Automatic Recall (AR)</td>
<td>7.90</td>
<td>11.76</td>
</tr>
<tr>
<td>Female: Mostly Did not use automatic recall (NAR)</td>
<td>7.02</td>
<td>9.26</td>
</tr>
<tr>
<td>All Females</td>
<td>7.36</td>
<td>10.18</td>
</tr>
<tr>
<td>Male: Mostly Use Automatic Recall (AR)</td>
<td>8.78</td>
<td>10.43</td>
</tr>
<tr>
<td>Male: Mostly Did not use automatic recall (NAR)</td>
<td>7.03</td>
<td>9.87</td>
</tr>
<tr>
<td>All Males</td>
<td>7.78</td>
<td>10.11</td>
</tr>
</tbody>
</table>

The mean scores of females who mostly use automatic recall is higher than any other group, in the whole population (11.76). This also occurred within the matched groups, where the mean score for females who mostly used automatic recall was 11.46. A chi square test on the number of students demonstrating good number sense in samples of thirty females and thirty males randomly selected from the whole population gave a \( \chi^2 \) probability of 0.197 \((df=1, N = 60)\) which is not a significant result at a .05 level of significance.

3.1.5 Self-identification and demonstration of number sense within each class.

The number of students who identified themselves as mostly users of automatic recall differed considerably between classes.
Table 3.3

Number of students who identified themselves as mostly users of automatic recall in each class

<table>
<thead>
<tr>
<th>Class</th>
<th>No. students who use auto recall</th>
<th>No. students who do not use auto recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td>B</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>C</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>D</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>Total</td>
<td>44</td>
<td>66</td>
</tr>
</tbody>
</table>

The numbers of students who demonstrated number sense in each class also showed noticeable variations, which did not match with the differences in use of automatic recall.

Table 3.4

Number of students who demonstrated good number sense in each class

<table>
<thead>
<tr>
<th>Class</th>
<th>Year</th>
<th>No. students who do not demonstrate good number sense</th>
<th>No. of students who demonstrate good number sense</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>E</td>
<td>5/6</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>46</td>
<td>64</td>
</tr>
</tbody>
</table>
Differences of this type were expected as a result of observed differences in the characteristics of each class population and differences in the teaching methods used in each classroom. The matched groups, created for the testing of the null hypothesis, each contained the same number of students from the different classes.

3.1.6 Year Level and Demonstration of Number Sense

Table 3.5

The number of students who demonstrate good number sense at each year level within the matched groups

<table>
<thead>
<tr>
<th>Year Level</th>
<th>No. students who do not demonstrate good number sense</th>
<th>No. students who demonstrate good number sense</th>
<th>Total Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 5</td>
<td>11</td>
<td>13</td>
<td>24</td>
</tr>
<tr>
<td>Year 6</td>
<td>12</td>
<td>24</td>
<td>36</td>
</tr>
<tr>
<td>All</td>
<td>23</td>
<td>37</td>
<td>60</td>
</tr>
</tbody>
</table>

The number of students demonstrating number sense in year six was higher than the number of students demonstrating number sense in year five. This result was anticipated, as students in year six might reasonably be expected to be better at mathematics generally than students in year five since they are more mature and have received more tuition.
3.2 QUALITATIVE DATA

3.2.1 Description of Interviews

Interviews were requested with fifteen students, and written permission sought from their caregivers. Fourteen of these students, with their caregivers permission, agreed to the semi-structured interviews and thirteen interviews were conducted in a small room in the school office block, during normal class time. After the interview students were thanked for their help and offered a token for an ice-cream at the school canteen. One student, absent over several days, missed the interviews. Students were selected from each of the groups previously identified. Initially, students from the AR\NG and NAR\G groups were interviewed, and later the AR\G and NAR\NG students were interviewed.

Two students were interviewed from the AR\G group: students who mostly use automatic recall and demonstrate good number sense.

Four students were interviewed from the AR\NG group: students who mostly use automatic recall and do not demonstrate good number sense.

Five students were interviewed from the NAR\G group: students who do not mostly use automatic recall and demonstrate good number sense. This number includes one extra student who originally identified himself as AR\G.

Two students were interviewed from the NAR\NG group: students who do not mostly use automatic recall and do not demonstrate good number sense.
More students were interviewed from the AR\NG and NAR\G groups because the researcher was interested in the perceptions of these groups. If the development of automatic recall was a significant factor in the development of number sense then few students would be expected to fall into the AR\NG and NAR\G groups. The interviewer was therefore interested in these groups and in what factors related to automatic recall affected the AR\NG group who mostly used automatic recall, but did not develop good number sense, and what common perceptions might be found among students within the NAR\G group, who did not mostly use automatic recall, but nonetheless developed number sense.

3.2.2 Accuracy of self-identification

Students identified themselves as mostly users of automatic recall, or as not mostly users of automatic recall by ticking one of the boxes labeled “I remembered these answers straight away” or “I often worked them out quickly in my head” after their mental multiplication test. During the interviews, students were asked to recall which box they had ticked, and to describe why they chose to tick that box, so that the researcher could gauge the accuracy of the student’s self-identification. Of the thirteen students interviewed, one student appeared to have incorrectly identified himself. Ten students gave descriptions that confirmed that they had identified themselves correctly, while two students gave insufficient information for the researcher to determine the accuracy of their self-identification, so the student’s self-identification was deemed to be correct.
The student who was originally incorrectly identified as part of the AR\G group was moved to the NAR\G group when the interviews were analysed, and another student was interviewed as part of the AR\G group. As a result, one more student was interviewed in the NAR\G group than in the AR\G group. When asked why he ticked “I mostly remembered the answers straight away”, the incorrectly identified student replied, “Because I did. I find that since everybody knows that you are really good [at mental maths] I find that it’s a real pressure on me to do good. Say somebody beats me then I … it’s like I’m under pressure.” This student’s score in the mental mathematics test was 10/10, but the following transcript indicates that he actually worked the answers out quickly in his head.

*Interviewer:* Uh huh. So can you describe to me what it’s like when you remember the answer straight away? How do you know what the answer is going to be?

*Student:* Mmmm, because I just times the two sums or whatever the sum is.

*Interviewer:* Ah, Say four sixes?

*Student:* Ah, twenty four.

*Interviewer:* How did you know that?

*Student:* Because I’ve got an unusual way. I go two sixes are twelve, that’s half the first number, then I just double that number.

*Interviewer:* Oh right. So did you actually just do that? When I asked you that and I said four sixes, you actually did
that in your head. You didn’t just go four sixes ‘oh I remember that- twenty four”.

Student: Yep

Interviewer: You actually did that. Doubled it to twelve then doubled it. Is that what you did?

Student: Mmm Uh Huh. (speaks quietly)

Interviewer: That very interesting. Tell me about some of the other questions that you did. These are the questions.

(shows list of mental multiplication questions: 2x3, 3x4, 5x5, 8x2, 4x6, 9x0, 7x3, 9x4, 6x7, 9x8) Can you tell me how you knew the answers to them?

Student: (speaking immediately and quickly) Um I just knew just double three was six, I knew three fours are twelve. I knew five fives are twenty five. Because if you double that, its ten. And five tens are fifty, and if you halve its just the same as way. And eight twos. I wouldn’t go eight twos. ‘Cause I would go two eights are sixteen, so I just reverse the sums. I find it easier. Four sixes, I telled you about that I just go two sixes and then double it.

Interviewer: Yeah

Student: (continues speaking quickly) Nine zeros. I know that anything times by zero you get zero, so zero. I will go three sevens are 21. I used that. I reversed the sum. Same here (9x4) except that I used a cheat that Mrs ---- taught us. You put down your fourth finger then its tens, ones. So four,
three, that’s thirty six (shows on fingers). Six sevens, I would just go its easy to do six sixes are thirty six and then I just add on six which is 42 the answer there. The nine eights, I’ll just go eight eights are 64 then add on that to 72. Then also the other way, because it’s nine, you take away eight from ten, from ten times eight.”

Replies from other students who ticked “I mostly remembered the answers straight away”, included comments such as “Each question that I did just popped out of my head so fast!” and “ ‘Cause I remembered all my answers”.

When asked why they ticked “I often worked them out quickly in my head”, NAR students responded, “Um cause when I was doin’ it I just like, when you’d ask the question I’d just kind of work it out in my head as we went along.” and, “Some of them I worked out quickly and some I remembered.”

Common explanations for how students worked things out in their heads during the mental multiplication test included:

Reversing the question

“four times six and I think its like six times four so that’s 24.”
Adding

"With two times three I just put three and three and like it equals six, you just add." and, "I know that I just added 12 and 12 [for 6x4] together and that made 24."

Working from another known fact

"For 3 x 7, I knew 2x7 is 14 so I just added 7 to get 21."

Solving multiples of nine by using the "finger trick"

"I did it on my fingers [9x4] ... Well, you put down the fourth one (fourth finger put down, while holding up ten fingers) and that's thirty six."

An unusual response was this explanation for how a student quickly worked out 6x7, "And with 6 times 7 they got gypsy maths where you got thats 6, 7, 8, 9, 10. (counting on fingers) So you put the six with the sevens. That's thirty there - its the tens. And then you multiply the two top ones which is four times three equals twelve. So that equals 42. And then you add that on to the bottom one."

3.2.3 Confidence in using own methods

All ten students with a high score (9/10 or 10/10) in the mental multiplication test were confident that they could solve a problem outside the range of the multiplication tables mentally (32x4), and only one (AR) student was not successful. Two NAR students and one AR student, each with low scores on the mental multiplication test, initially declined to
attempt the problem with comments such as, “I think it is too hard ... because normally when we practice our times tables we normally go up to twelve. When we do questions its normally never over twelve.”

Students were given the opportunity to show their willingness to use their own methods to solve problems when they were presented with multiplication problems outside the range of the “times tables” and asked to find the answer without writing anything. If students described calculating by multiplying the ones, recording that, then multiplying the tens and adding that answer to the ones, then they were considered to be using the same method that they would use with a paper and pencil in school.

Among six AR students, one student used his own method to calculate an answer to the question “32x4”, explaining “Well I know 32 lots of two is 64 then I just add another 64 on then I’ve got the answer straight away.”

Among five NAR students who attempted the problems, four used their own methods to calculate an answer. For example:

**Student:** I don’t know. It’s nothing I’ve ever tried before - just doing a problem like that without writing it down.

**Interviewer:** OK do you want to have a go? You can talk if you want, while you’re doing it.

**Student:** Ok I’d probably like double 32 in my head which is 64. Then add 64 and 64 which would be a hundred and ... hang on...128.”
Students were also asked to describe how they used mathematics when they were not at school, to demonstrate whether they employed their own methods for calculating. The example of spending their pocket money at the shops was suggested.

Among the six AR students, four described using a piece of paper to work out the answer at the shops. Two of these students also suggested asking someone else. Of the other two students, one answered, “I don’t carry like anything with me so I have to work it out in my head” and the other AR\NG student gave no indication that he was able to transfer his mathematics skills to the situation of spending money at the shops.

Five of the NAR students said they would calculate in their heads when at the shops. Some were very confident, for example,

Oh, I work out what I want and then if I did a sum in my head if I worked out if I had enough money for what I wanted I’ll do it. And I don’t like getting like 10, like 5c and 10 c change, so if I say have ten cents to spare I just get something for ten cents so I don’t have to worry about having loose money.

Other students mentioned using their fingers as well as their heads, and one student said he would also use paper if the numbers were “big”.

66
3.2.4 Perception of the importance of remembering or thinking in doing mathematics

When asked whether they felt that in doing mathematics it was more important to remember well or to think well, all NAR students answered that thinking was more important. Reasons given for the importance of thinking were that if you relied on remembering, "you could be wrong", and because, "if you get a new sum you can't remember that because you haven't done it before".

Half of the AR students said that remembering was more important. Three of the AR students appeared to have difficulty grasping the difference between remembering and thinking. One AR\NG student had difficulty responding to the question appropriately while another AR\NG student answered that he felt that thinking was more important because, "Thinking I come up with the answers straight away, but remembering I have to like go through my brain and try to get it to work it out". During a discussion of question twelve in the number sense test this AR\NG student was unable to explain his knowledge of a multiplication fact as anything except remembering the right answer.

"Interviewer: Do you think you could explain to me why you think the answer is thirty six and not twenty seven?

Student: Well nine threes are twenty seven, and nine fours are thirty six.

Interviewer: How do you know that? How do you know that somebody wasn't tricking you when they told you that nine fours are thirty six, and you learned it?
Student: I just believe them (tiny voice).

Interviewer: You just believe them? What if they made a mistake on the chart? Say the teacher was writing up the multiplication chart and she made a mistake, how would you know?

Student: (confidently) Well I’d check through all the answers and I’d check through them all and tell her if any of them was wrong, but they’re mainly all right. Then if I saw any wrong I would just put my hand up and tell her.

Interviewer: And how would you know they were wrong?

Student: I’d just remember straight away. If they were in the once to twelve times tables I would know them all.”

When asked how they might get better at mathematics, most students responded that practice would improve their mathematics. One student responded that she would “practice more because practice makes perfect”. When asked what sort of things they would practice students gave responses similar to the following:

“Student AR\NG: Well my times every day and do maybe an hour or so of maths”

“Student AR\G: I would practice my maths at home. I would call them out to Mum.”

Students were also asked to speculate on what made some people really good at maths. “Practice” was again the most common response. Two students suggested practice on games was helpful; “She plays games with
maths at her house. She’s got games I know cause I’ve been there” and “I used to hang around the computer all the time and I used to have this game and it had the times and I’m really good at them”. Study was also mentioned on two occasions, and when the students were asked about what these really good maths students studied they suggested, “Multiplication and take aways and adds and that” and “The easier stuff and then he studied the harder stuff”. Unexpectedly, only one student mentioned innate ability. This may have been because the question was phrased “How do you think they got to be so good at maths?” which infers that a process was involved. Only two students, both from the NAR\G group, mentioned understanding in their answers.

“Interviewer:  Why do you think that you are so good at maths?
Student:       Because I understand what is going on between the two numbers - like what’s happening.
Interviewer:  How did you come to understand what’s going on?
Student:       If I saw something that I didn’t understand I kept on asking Mrs --, like ‘How do I do that?’ and she helped me out. So then I’m not just trying to work on something I don’t understand...”

“Interviewer: ....What could you do to get better at maths?
Student NAR\G: Learn the bits that I don’t really understand and just relearn the ones that I know”
All twelve students questioned about when they learnt their tables indicated that they had begun practicing for automatic recall of the multiplication tables in school several years ago.

Eight students spoke about when they thought they began to understand what multiplication meant or how you can use it. One NAR\G student explained how he first began to understand what multiplication meant, saying, "my year two [class] was starting to learn my two times table, and like at the start I didn’t really understand what it meant. But then in really early year three our teacher gave us a tape and we actually sang them. And if you could work it out in your head, you could see a pattern by singing it, you could see a pattern." Another NAR\G student explained that ".. when we first started to learn about multiplication was in year 2 and the teacher explained it very, very well so I knew what it was and then we got into and she asked us ‘what does this mean’ and we said that, and then we got into doing the answers."

Two AR students and one NAR student appeared to have difficulty grasping the difference between learning the answers to the multiplication tables by heart, and understanding what multiplication meant or how you can use it. The NAR student was unable to answer the question, while the one AR student responded with several more descriptions of automatic recall and the other answered, "I learned the meaning about the same time because the teacher wrote on the blackboard like 3 carrots times 2 apples and we had to draw lots of little apples you know?"
3.2.5 Student response to ‘doing’ mathematics.

Eight students described their timed mental multiplication test experience as putting them under time pressure, as a challenge, a bit quick, or described feeling “a bit nervous” or “relieved when it was over”. Four others described the mental multiplication test as easy or very easy. No pattern in these responses was found between NAR and AR students or the test results.

Students described the number sense test as a bit hard and a bit easy, harder, a bit weird, worried about getting some wrong, easier, better (than the mental test) or requiring more work. Students with lower scores in the number sense test made more negative comments. Students who did very well in the test (14 or 15/15) were the only students to describe it as enjoyable.

3.2.6 Differences between AR\G and NAR\G students

From the students’ responses to direct questions about the import of remembering or thinking, responses to questions about how students become ‘good at mathematics’ and descriptions of how students developed understanding of multiplication, it was noted that NAR students were more likely to value understanding of the multiplicative operation and related concepts and to use it than AR students.
No differences in confidence, or student responses to doing mathematics, were noted between the AR\G and AR\NG students. However, both of the AR\G students mentioned using the "finger trick" to work out multiples of nine, and one of these students described working out "3x7" as "I knew two times seven is fourteen so I just added seven to get twenty one", whereas no AR\NG student suggested that they had used any other strategy than remembering in their mental mathematics test. Two AR\NG students gave answers to some problems that did not demonstrate an understanding of the relative magnitude of numbers, or the effect of multiplication. \(32x4 = 21\), and \(21x55 = 15\). The AR\NG students either said remembering was more important than thinking in mathematics, or were unable to distinguish between the two. AR\NG students interviewed had more difficulty in expressing understanding of the operation of multiplication or of the process of reasoning than students in any other group.
CHAPTER 4

DISCUSSION OF RESULTS

4.1 DISCUSSION OF QUANTITATIVE DATA

4.1.1 Accuracy of self-identification

The results of the student interviews confirmed that most students were able to correctly identify themselves as mostly users of automatic recall or not mostly users of automatic recall. The one student who incorrectly identified himself indicated that he did this because remembering straight away was what was expected of him as a top mental mathematics student. Since the students who identified themselves as mostly using automatic recall were mostly students who were successful (score \( \geq 8/10 \)) in the mental multiplication test, it is possible that this perception had some influence on these students self-identification.

A longer preamble about choosing the right box may reduce the likelihood that students would choose a response because of peer pressure.

4.1.2 Rejecting the null hypothesis

The null hypothesis ‘that there will be no significant difference between the number of students who demonstrate good number sense in a group of students who mostly use automatic recall and the number of students who demonstrate good number sense in a group of students who mostly did not use automatic recall, was not rejected. The research question “Is automatic
recall associated with good number sense?” is therefore answered in the negative. Automatic recall does not appear to be associated with good number sense. This suggests that students’ development of automatic recall had little or no effect on the students’ development of number sense, or that students’ development of automatic recall had an effect on the development of number sense for only a few students. The results of the interviews suggest that the latter may be the case, as interviewed AR\NG students did not indicate that they used any strategy other than automatic recall in their mental multiplication test, whereas interviewed AR\G students indicated that they used a limited range of other strategies, but mostly automatic recall in completing the mental multiplication test.

4.1.3 Mental multiplication test scores and number sense scores

Correlation between scores on the mental multiplication test and scores on the number sense test was positive and moderate (0.43). This is not as strong as the correlation coefficient between mental computation tests and number sense tests conducted by McIntosh et al. (1997) which were reasonably strong (.65) for students aged 10 years in Australia. McIntosh et al. (1997) concluded that “mental computation may be a good indicator of number sense”, but this conclusion is not supported by the results of this study. Bana & Korbosky (1995, p. 40) report that in their study, “The extent of understanding of the subtraction and division facts was not very different from performance on automatic response in these operations.” They point out that different items were used in each test, so the results may not reflect a correlation between understanding and automatic recall. The results from
their study may also differ from these results because they assumed that if a response was given within the three second time limit, the student used automatic recall to make that response. The results of the students’ self-identification within this study suggest that this may not have been the case.

4.1.4 Affect of variation in dependent variables

McIntosh et al. (1997) note some differences between girls and boys performance in the number sense test and report a significant difference in the sample aged 10 years, where boys scored slightly better than girls. In this study, differences were also noted between the scores of boys and girls, but only when the sample was already separated into AR and NAR groups. However, in a chi square analysis of 30 girls and 30 boys randomly selected from the whole population, there was no significant difference between the scores of all girls and all boys or between the girls and boys scores within each year group.

4.2 DISCUSSION OF QUALITATIVE DATA

The second research question was answered by interviewing students about their perceptions of mathematics, with particular note being taken of factors which the literature review suggested may be associated with automatic recall and may negatively affect the development of number sense. No unexpected factors were identified in the interviews, but not all factors which were expected to have a detrimental affect on number sense were found to do so.
4.2.1 Confidence in using own methods

Confidence in ability to solve problems outside the range of the multiplication tables was strongly linked with student's score on the mental mathematics test.

Positive feedback for good automatic recall was noticed by the researcher in several of the classrooms (for example, publicly displayed charts, competitive class games and verbally expressed teacher approval), and it seems likely that the confidence of students is related to this positive feedback for good performance in mental mathematics in the classroom.

AR students were found to be much more likely to describe using pencil and paper to calculate with written algorithms, even when they were not at school, and calculating for their own purposes, than NAR students. NAR students were also more likely to use their own methods to mentally calculate the answer to a multiplication problem outside the range of the multiplication tables. This evidence supports the suggestion that students who rote learn multiplication facts may be more likely to rely on these well learned strategies instead of using their own strategies to calculate products from familiar understood facts or benchmarks. Reliance on learned strategies was also associated with automatic recall in View 2, based on Kamii & Dominick's (1989, p. 135) suggestion that in rote learning some students may give up "their own thinking". This may have occurred when some of the AR students rote learned their multiplication tables.


4.2.2 Perception of the importance of remembering or thinking in doing mathematics

NAR students were more likely to consider thinking more important than remembering in mathematics than AR students and only AR\NG students seemed to have difficulty grasping the difference between remembering and thinking. Most students felt that practice was a factor in students becoming 'really good at mathematics' and that practice would help them improve their own mathematics. Only NAR students described 'understanding' as a factor in these contexts. These results suggest that students who mostly use automatic recall value remembering over understanding of mathematics concepts and operations, and give some support to Madell's (cited in Kamii, Lewis & Livingston, 1993) claim that "The early focus on memorization in the teaching of arithmetic thoroughly distorts in children's minds the fact that mathematics is primarily reasoning".

4.2.3 Student response to "doing" mathematics

Eight of the interviewed students felt nervous or pressured in the mental multiplication test which only allowed three seconds for a response to each question, although most of these students performed well in the test. Buxton (1981, p. 7) suggested that tests such as the timed mental mathematics test may cause anxiety in students, and many students did report feeling nervous or pressured during this test. The suggestion that this anxiety is linked to student's use of automatic recall, made in View 2: Rote Learning of Multiplication Facts Impedes Development of Number Sense (Figure 1.3),
is not supported by these student responses, as similar numbers of AR and NAR students reported feelings of nervousness and being under pressure.

4.2.4 Differences between groups of students
Among students in the AR\NG group, all students had high mental multiplication test scores, and were confident in their ability to solve problems outside the range of the multiplication tables, but two gave answers to some problems that did not demonstrate an understanding of the relative magnitude of numbers, or the effect of multiplication. (32x4 =21, and 21x55=15). The AR\NG students were unlikely to use their own methods when calculating and either said remembering was more important than thinking in mathematics, or were unable to distinguish between the two. AR\NG students interviewed had more difficulty in using their own strategies when calculating, or expressing understanding of the operation of multiplication or of the process of reasoning than students in any other group. These results suggest that these students' reliance on automatic recall is having a detrimental effect on their development of number sense, as indicated in view 2.

Despite the fact that interviewed AR students were less likely to use their own methods and perceived remembering as more important than NAR students, for the larger group of students who mostly use automatic recall (AR\G) use of automatic recall has resulted in no apparent negative effect on development of number sense. This may be because these students also have a clearer understanding of the concept of multiplication than the
AR\NG students and greater ability to use other strategies to solve mathematics problems, although they use automatic recall almost exclusively when appropriate. The use of automatic recall was most appropriate in the timed mental multiplication test, when these students identified themselves as mostly users of automatic recall.

Despite the appropriateness of use of automatic recall in the mental multiplication test, thirty three of the sixty six students in the NAR group scored 8/10 or higher in this test without using mostly automatic recall. This finding does not support Hamrick & McKillip's (in Suydam & Reys, 1978,) belief that memorization of number facts is a "prerequisite" for meaningful learning of concepts and advanced skills, which was represented in View 1.

Most students at the school had been encouraged to automatically recall multiplication facts, so NAR students either preferred to often use strategies other than automatic recall, or they were unable to use automatic recall effectively as a strategy. The fact that the NAR students did not use mostly automatic recall, and seem more likely to use their own methods, however, was not a good indicator of their possession of other efficient strategies or understanding of multiplicative concepts which were suggested in View 2: Ideal Conditions for Developing Number Sense, as being related to the development of number sense. This was evidenced by the chi square test result, and the fact that more than half of these students did not demonstrate good number sense in the number sense test.
4.2.5 Assessing the two views of developing number sense.

Thirty-two of the students tested in this study demonstrated good number sense without mostly using automatic recall. This fact does not support the assumption made in View 1: Automatic Recall Facilitates Development of Number Sense (Figure 1.1), that automatic recall is a prerequisite to good number sense. The correlation between mental multiplication scores and number sense scores was only moderately positive ($r = 0.43$), despite the two tests being based on the same facts. If automatic recall was a good indicator of number sense, as suggested by View 1, then a stronger correlation might have been expected between the test scores.

It must also be noted, however, that some assumptions in View 2: Rote Learning of Multiplication Facts Impedes Development of Number Sense (Figure 1.3) have not been supported by the results of this study. Most importantly, the development of automatic recall does not appear to impede the development of all students, as 32 students who did use automatic recall also demonstrated good number sense, despite indications that AR students may be affected by the perception that mathematics is mostly remembering and a lack of inclination to use their own strategies. Secondly, the factor anxiety does not seem to be related to automatic recall.

There are, however, some indications that the two factors of perceiving mathematics as mostly remembering and a lack of inclination to use their own strategies have had a negative effect on the ARING group of students.
4.3 LIMITATIONS

4.3.1 The Sample Population

Results from the study will be unable to be generalised over a much larger population because they represent only a sample from one local school which was selected on the basis of expediency. The generally traditional approach to teaching mathematics in this school meant that all students had been encouraged to develop automatic recall of multiplication facts at some stage. This precluded any comparison with students who had not been taught automatic recall of multiplication facts, so the NAR population comprised only students who chose to use their own methods in preference to automatic recall of multiplication facts, or had been unsuccessful in developing automatic recall of multiplication facts. This may have had some impact on the results of the testing, and the rejection of the null hypothesis, so some possibility of a Type II error does exist.

Matching of the AR and NAR groups may have allowed some error due to sampling, because individual subjects were unable to be matched in pairs, so the independent variables were matched only for the whole of each group.

The number of students interviewed was small (N=13) compared to the larger population of identified students (N=110) so conclusions drawn from responses from this sample may not clearly reflect the perceptions of all students in the identified groups. This is particularly true for the ARG
groups (N=34) from which only 2 students were interviewed, and for the NARING group (N=34) from which only two students were interviewed.

4.3.2 Number Sense Test

Given that the test for number sense only covers multiplication basic facts, it cannot be assumed that it measures the students' general number sense, although it may be an indicator. The number sense test was designed to separate students into those who demonstrate good number sense and those who do not demonstrate good number sense. It would not be reasonable to assume that a student who scored 8 had twice as much number sense as a student who scored 4. For this reason, the researcher chose to test the null hypothesis using a chi square test for significant difference rather than using a parametric measure.

The number sense test was useful in this study but needs to be further refined. Marking of such a test will always require some subjective judgements and results will therefore be open to argument.

4.3.3 Automatic Recall

In regard to Research Question 1, findings may be distorted because students' development of automatic recall can be the result of rote learning, or of many meaningful experiences with these number facts. The multiplication facts tested replicated those in the test for automatic recall by Bana and Korbosky (1995). In order to increase the likelihood of automatic recall being the result of rote learning a much larger number of mental multiplication questions could be asked. This was not done because of
constraints on the amount of time the classroom tests would require. The likelihood that students had been taught automatic recall of multiplication number facts also reduces the likelihood of identifying students who developed automatic recall only as a result of many meaningful experiences with these number facts.

4.3.4 Classroom Practice

The study does not attempt any thorough record or analysis of classroom practice and only student comments or anecdotal evidence is cited. Nor does the study differentiate between the rote learning of multiplication tables by drill in the classroom or by other methods, although both these factors may be significant in developing students’ perceptions of mathematics.

4.3.5 Individual Ability

The study does not control the variable of individual ability, except by selecting students for the automatic recall and not automatic recall groups so that the mean score and standard deviation of scores on the mental mathematics tests are similar.
4.4 CONCLUSIONS ARISING FROM THE RESULTS OF THE STUDY

4.4.1 Identifying use of automatic recall
The interview results suggest that students at Year 5/6 level are able to make reasoned judgments about the strategies they use when answering mental mathematics questions, and that this may be a more accurate means of identifying users of automatic recall than only using the three second time limit. (4 of 7 NAR students interviewed scored 9/10 or 10/10 on the mental multiplication test).

4.4.2 AR\NG students

Analysis of the student responses in the interviews demonstrated that AR students were less likely to use their own methods and perceived remembering as more important than NAR students. These differences, however, appear only to have had a negative effect on the number sense of the students in the AR\NG group, who were very unlikely to apply strategies other than AR in mathematical situations, or to view mathematics as something other than remembering.

Only ten students from the whole population (N=110) were identified as belonging to the AR\NG group, and four of these students were interviewed. Use of automatic recall appears only to have had a negative
effect on these students because they rely heavily on it, and do not demonstrate the ability to use other strategies.

4.4.3 Implications for teaching for number sense.

All of the AR\NG students were good at automatic recall, so probably received positive feedback for this skill in the classroom, as was evidenced by their high confidence. This confidence may have reduced these students' inclination to attempt to use other strategies, or to seek to understand the meaning of multiplication. It seems likely that more opportunities to use mathematics in problem solving and real life situations (View 2) may help these students identify misunderstandings. Students might then be able to develop clearer understandings and learn new strategies, if the learning environment in the classroom was one in students felt able to share their difficulties and ideas with others.

The AR\NG are not alone in needing to develop further in the area of number sense, as the NAR\NG also fall into this category, but the AR\NG Group are most at risk of not being identified by their teachers, or by themselves, as being in need of tuition in this area because of their confidence and apparent classroom success.

Results of this study suggest that the view to be taken of the relationship between automatic recall and number sense is closer to Askew's (1997) view that automatic recall and development of other strategies for calculating mentally can be complementary. The development of automatic
recall taught as one strategy that students may choose to use, within a learning environment where students are also encouraged to think mathematically and construct and use their own mathematical understandings, appears to be harmless, and as useful as any other efficient strategy.

4.5 SUGGESTIONS FOR FURTHER STUDY

Clearer indications of the effect of automatic recall on number sense may have been identified if the students had been given the opportunity to identify themselves as “only users of automatic recall” by ticking “I remembered all of these answers straight away”. Only a small group of students may have identified themselves this way, (possibly most of those in the AR\NG group) but a comparison of their responses to a group who used “mostly methods other that automatic recall” would have made the effects much clearer. The self-identification of students was also affected by the perceived lack of confidentiality and peer pressure to identify oneself as “remembering straight away”. If the study was repeated under similar conditions, these affects could be minimised by a more detailed explanation before students ticked the box, and an instruction to immediately fold the test paper.

A useful further study would be to test and interview a sample of students from a non traditional school, where some students may not have been encouraged to develop automatic recall of the multiplication facts or
rewarded for doing so. Students in this situation may then clearly
demonstrate automatic recall of some facts that is developed as a result of
many experiences with number, which was not identified in this study.
REFERENCES


Education Department of Western Australia (1998). *Outcomes and standards framework student outcome statements: mathematics.* Western Australia: E.D.W.A.


## LIST OF APPENDICES

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>Answer Sheet for Mental Multiplication Test and Mental Multiplication Test Procedure</td>
</tr>
<tr>
<td>B</td>
<td>List of Matched Groups</td>
</tr>
<tr>
<td>C</td>
<td>Number sense test and number sense test procedure</td>
</tr>
<tr>
<td>D</td>
<td>Discussion of number sense test questions and issues that arose during marking</td>
</tr>
<tr>
<td>E</td>
<td>Requests for permission to test students and for permission to interview students</td>
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</table>
Appendix A: Answer sheet for Mental Multiplication Test and Mental Multiplication Test procedure.

Answer Sheet: Mental Multiplication
Student’s Name: ................................ Year ..........
Male / Female........ Class teacher’s name..................

Q 1. ........ Q 8. ........ Q 15. ........
Q 2. ........ Q 9. ........ Q 16. ........
Q 3. ........ Q 10. ........ Q 17. ........
Q 4. ........ Q 11. ........ Q 18. ........
Q 5. ........ Q 12. ........ Q 19. ........
Q 6. ........ Q 13. ........ Q 20. ........
Q 7. ........ Q 14. ........

Think about the answers you think you got right.

How did you get these answers?
Put a tick in the one box that is closest to your answer.

☐ I remembered these answers straight away
☐ I often worked them out quickly in my head
☐ I don’t know how I got them
Good morning, my name is Mrs Jolly.
Today I am going to give you two different maths tests. Both of them are on multiplication. When you do these tests you will be helping me with my research into the way students learn multiplication. Please fill in your name, year, and teachers name at the top of the answer sheet. For Male/Female, put M if you are a boy and F if you are a girl.

Look at the bottom of the answer sheet. When we finish the test, I would like you to think about the answers that you think you got right. Did you usually remember those answers straight away, or did you sometimes work them out quickly in your head? Don’t worry about that question now. We will concentrate on the test first.

For my research to be valid (right) you need to do your best to answer the questions. Don’t worry if you cannot answer some of the questions in the time allowed. Just skip to the next question. You will have only 3 seconds to answer each question before I go on to the next one. Do not put your hand up during the test, as I cannot stop the test once we have begun. If you don’t know an answer, just leave it and skip to the next question.

This is a sample question - do not write an answer for this question.

Q 0. 2 x 2
You can see that the question number is written here in blue. I will show you each question in turn and read it to you. You do not have to look at the question, you may just listen for the question if you prefer. If you forget which question we are up to, just look up here at the blue number next to the multiplication question.

I could read this question as two twos, as two lots of two, as two multiplied by two, or as two times two. Today I will read these questions as two times two. You only need to write your answer on the dotted line next to the question number.
Is everyone ready? Question 1:

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Please put down your pencils. Well done. Please do not add to or change your answers to the questions.
Would you please look at the bottom of your paper.
Think only about the answers that you are sure you got right. How did you get those correct answers?
Did you remember the answers straight away, or did you often work them out quickly in your head, or something like that?
Choose which of the answers on the paper are most like what you did and tick that box. If you really don’t know how you got those answers, tick the last box instead of one of the first two boxes.
Check that your name is written at the top of your page and turn it over on your desk. I will come and collect them. Thank you for your help. Your next test will be after recess.
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Av. Score: 7.867 10.032

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Av. Score: 7.867 10.4687
Appendix C: Number Sense Test and Number Sense Test procedure

Number Sense Test
Student’s Name: ........................................... Male /Female 
Year ........ Class teacher’s name ........................................

Please write your answers in the spaces provided on this sheet.

1. ........................................

2. ........................................

3. ........................................

4. ........................................

5. Which is larger, 6 x 4 or 5 x 5? ........................................

6. Compare the products of 6 x 7 and 7 x 6. What do you notice?
..................................................................................................................

7. I have five money boxes with $5 in each. How much money is that altogether? ........................................

8. Ice creams cost $2 each. I have $20, and I want to buy 8 ice creams. Do I have enough money?
..................................................................................................................

9. Write a number sentence for seven times three. (Write it in numbers.)
..................................................................................................................

10. Write a story problem for seven times three in words.
..................................................................................................................
..................................................................................................................
..................................................................................................................

11. Draw a simple picture that shows seven times three.
12. Kim thinks that $9 \times 4 = 27$

Is she right?  

Explain why you think she is right or wrong.

13. Explain how you could use multiplication facts to work out the number of squares that are shaded in this picture.

14. How much is $9 \times 0$?  

How do you know that is the right answer? Please explain in the space below.

15. What number sentence could describe this picture?
Over heads shown for questions 1 - 4:

Q1

[● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ●]

Q2

[● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ●]

Q3

\[6 + 6 + 6 + 6\]

Q4

\[7 + 7 + 7 + 7 + 7 + 7 + 7\]
This is the second maths test that is part of my research. This one is a written test. It is very different from the test we had before recess.

Please write your name, year and teacher’s name at the top of your paper. For Male/Female, write M if you are a boy and F if you are a girl.

You will have twenty-five minutes to complete the whole test. Please write your answers on the dotted lines or in the spaces provided on the sheet. If you run out of space for an answer you may continue on the back of the paper, but please remember to write the question number as well if you use the back of the paper. You may find some of the questions seem unusual. Just read the questions carefully and answer them the best you can. If, after reading the question and thinking, you are unsure how to answer the question, just have a go, then go on to the next question.

When you are completely finished you may raise your hand and your paper will be collected. You will be given some other work to go on with, but you must do this quietly as this is a test.

The first four questions will be shown to you on the overhead. Each picture will be shown for 6 seconds.

Question 1
How many dots are there?

Question 2
How many dots are there?

Question 3
How much is this altogether?

Question 4
How much is this altogether?

The rest of the questions are written on your paper. Please continue with the test.

(After 25 minutes)
Please put down your pencils. Leave your papers on your desk and they will be collected. Thank you for working so well on these tests. Your teacher will be sent a copy of the results of my research when it is finished, and if you ask, I am sure he/she will tell you what we were able to discover.
Appendix D: Discussion of Number Sense Test questions and issues that arose during marking

Questions

Questions 1 and 2 each presented an array of dots. The six second display did not allow enough time for the students to easily count the number of dots in the display in question 2, so a correct answer relied in part of the students’ ability to recognise the display as an expression of multiplication and to calculate the number of dots, or to recognise the display as an expression of a familiar number. A correct answer was considered to be an indicator of number sense.

Question 3 and 4 presented multiple addition. Students were expected to recognise multiple addition as one representation of multiplication, count the number of digits in the sum and then multiply to arrive at an answer. A correct answer was considered to be an indicator of number sense.

Questions 5 and 6 ask students to compare two multiplication facts, demonstrating an understanding of the effect of the operation by a correct answer (Q5) or a comment on the similarity of the product (Q6). Answers that stated “they have the same answer”, or “they are the same” were accepted as an expression of the similarity of the product.

Question 7 is taken from Bana & Korbosky’s Test Section B - Application of Automatic Response (1995) for multiplication. It used a real world word problem to test the ability of students to apply their knowledge to real life
numerical situations. An answer with the correct number, with or without $,$ was accepted as an indicator of number sense.

Question 8 was written in a similar style to question 7, but allowed students to use estimation to reach a correct answer. A ‘Yes’ answer was considered to be an indicator of number sense.

Discussing the uses of Haylock’s think board, on which students represent mathematical ideas as symbols, real things, pictures and stories, Herrington (1988) points out that “Being able to picture the algorithm in the “mind’s eye” displays another aspect of understanding that can be easily shown in the drawing of diagrams or pictures. ... Understanding can be seen as making connections between different representations of knowledge.” He asserts that “The think board can be used as an instrument to assess individual children’s understanding of a mathematical idea.” This form of assessment of understanding is used in questions 9, 10 and 11. These questions ask the student to represent a multiplication question as a number sentence, a word problem (story), and as a simple drawing. The term “word problem” was chosen because when the question was trialled, students responded readily to this request and provided the expected answers, whereas students asked for a “number story” or a “story using these numbers” often gave inappropriate responses or did not respond. When the test was administered, several students asked for clarification of the meaning of the phrase “word problem”. They were answered “I cannot explain the meaning of the question to you. Read the
question carefully and think about what it might mean, then have a go at answering the question."

Q9. Since all students read 2x3 as "two times three" (see Turn Around Facts, p. 43), only 7x3 or 7x3=21 was accepted as a correct number sentence, which demonstrated the student’s ability to represent the concept as numbers.

Q10. Any written story problem which described three groups of seven or seven groups of three was considered a correct answer, for example “If you had three boxes with seven glasses in them how many glasses would you have?” (Student no. 8) or “If there were 7 people and the people had 3 eyes each how many eye were there altogether?” (Student No. 63). Answers such as “If I had $7 and I gave 2 friends $3 each how much would I have left?” (Student No. 53) and “If you had to times three pigs and only three cows how many animals would you have altogether?” (Student No. 52) were not considered to demonstrate number sense.

Questions 12 and 14 use a similar format to the Division question in Bana and Korbosky’s Test Section C - Understanding Basic Facts (1995), but it has been adapted to apply to multiplication. The objective of this question was to allow children to demonstrate their understanding of the operation of multiplication, and the relationships between numbers by explaining their reasoning.

Q 12. Three responses were requested in this question, but only one mark was awarded, on the basis of whether, in any of the responses, the student had demonstrated an understanding of the numbers or operations as opposed to simple recall of the correct answer. The response “The right answer is 36”, or
"she should write out her 9 x tables", were not considered to demonstrate number sense. Examples of answers that were considered to demonstrate number sense are “Well you go 10 x 4 = 40 then take 4 which equals 9 x 4 = 36”, “If you added 9 + 9 + 9 + 9 what will it equal?” and “draw it: ..... ..... ..... ..... .....”.

Question 13 presented a partly shaded grid. The question tested the students ability to interpret the grid as an expression of multiplication and devise an efficient strategy using multiplication to calculate the number of squares that are shaded. Answers which demonstrated the ability to use multiplication to solve the problem were considered to demonstrate number sense. For example “Count the squares down one side of the shaded part. Then count across the top of the shaded side. multiply the numbers. After that add three.” or “6x8 +3 =51” or “Step 1) I would start by finding out how many squares there are altogether. 8x9=72 squares. Step 2) Now find out how many squares aren’t shaded. 3x7=21 Step 3) Now take away 21 from 72. 72-21= 51 squares. (student No. 117).

Question 14 asks the students to calculate 9 x 0, then explain how they know that answer is correct. In doing this the students demonstrated their understanding of the number 0, and of the multiplicative operation. For example, responses that were considered to demonstrate number sense included, “If there are nine piles with nothing in them the answer is zero”, and “because it is telling us 9 lots of 0. Zero is nothing so 9 nothings is 0”, “9x0=0
because the 0 is nothing, so you don’t have to x 9 by anything” and “I know because 0 x anything is nothing.”

Question 15 also related to multiple representations, (Herrington, 1988) but the problem was presented as a picture and the student was asked to represent it as a number sentence.

3x4, 4x3, 3x4=12, 4x3=12, 12 / 3 and 4+4+4=12 were accepted as responses that demonstrated number sense.
Appendix E: Requests for permission to test students and for permission to interview students

Dear Classroom Teacher,

I am undertaking research into the association between automatic recall of multiplication facts and number sense as an undergraduate honors student. This will entail my administering two mathematics tests to approximately 100 year 5, 6 or 7 students. Mr Lamb has given his approval for the research to take place in your school, and it has been given ethics clearance by the Faculty Research and Higher Degrees Committee at Edith Cowan University, but my research depends upon the kindness of teachers like yourself.

I would administer the mental mathematics test of twenty multiplication facts in approximately 15 minutes, and the number sense test on the same facts in approximately 30 minutes, during a morning that is convenient to you in week 1 or 2 of term 3.

I will be delighted to provide you with an overview of your class’s performance in each of the tests, but will not be able to inform you of any individual student’s results. A summary of the research findings will also be sent to you.

Individual, ten minute interviews regarding student’s perceptions of mathematics will also be requested with a maximum of ten students from the school, who have participated in the testing. These interviews will be dependent upon the student’s willingness, and upon permission for the interview being granted by their caregivers.

If you decide to participate (please do!), could you please fill in the attached form and either give it to me or leave it in Room 14’s pigeonhole? Thank you.

With kind regards
Maxine Jolly
Dear parent or caregiver,

As part of research being conducted at Eaton Primary School, your son / daughter ...................................................... has been selected to participate in an interview regarding their perceptions of mathematics. The research, into the association between automatic recall of multiplication facts and number sense, has been approved by the principal, Mr Kerry Lamb, and been given ethics clearance by Edith Cowan University, Bunbury. Student comments will be audio taped and transcribed. The interview conducted within the school, during class time.

I would greatly appreciate your son / daughter’s participation in this research. Please fill in the attached form, giving consent for your child to participate, and return it to the classroom teacher.

yours sincerely
Maxine Jolly, researcher

.................................... classroom teacher.

I give permission for my son/daughter ...................................................... to participate in the mathematics research interview.

Name of parent / caregiver.................................................................

Signature .................................................   Date .....................