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Developmental aspects : Metacognition and problem solving

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DEVELOPMENTAL ASPECTS: METACOGNITION AND PROBLEM SOLVING

by

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ABSTRACT

Mathematical problem solving has been the focus of recent curriculum reform. Researchers have investigated factors that appear to influence mathematical problem solving; one of these factors is metacognition. This study identified metacognitive aspects and investigated the relationship of metacognition and age in the context of mathematical problem solving. Twenty four children were randomly chosen; eight children from years two, four and six. The children were given the same non-routine problem to solve. A semi-structured interview and observation protocol were developed and used to determine students' metacognitive aspects. There was an extensive descriptive analysis of metacognitive aspects and a systematic quantification of metacognitive strategies in terms of their occurrence within the context of mathematical problem solving, and in relation to the subjects' age. A descriptive analysis of the data suggested a developmental trend to metacognitive awareness and strategies in relation to mathematical problem solving.

DECLARATION

I certify that this thesis does not incorporate, without acknowledgment, any material previously submitted for a degree or diploma in any institution of higher education and that, to the best of my knowledge and belief it does not contain any material previously published or written by another person except where due reference is made in text.

Signature:

Date: 29.11.96

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CHAPTER 1

INTRODUCTION

1.1

BACKGROUND OF THE STUDY

In recent years mathematical problem solving has been the focus of curriculum reform. It is widely acknowledged that our rapidly changing society requires individuals who are able to quickly adapt to repeated changes - to be able to problem solve. Researchers have investigated factors that appear to influence problem solving competence. One of these factors is metacognition (Garofalo and Lester, 1985; Schoenfeld, 1983, 1985a). Metacognition is defined by Garofalo and Lester (1985) as 'knowledge and beliefs about cognitive phenomena, and the regulation and control of cognitive actions' (p.163). Silver (1985) believes metacognition is the driving force in any intellectual activity. Schoenfeld (1985a) and Silver (1985) argue that problem solving analyses need to focus on metacognition:- strategy selection, cognitive monitoring and evaluation. Schoenfeld (1987) suggests analysis of a person's problem solving should include the person's: a) metacognitive knowledge and resources they bring to bear; b) metacognitive control, selection and implementation of resources; and c) beliefs and perceptions. Garofalo and Lester (1985) discuss the growing research and theory being generated about the benefits of metacognitive aspects and development, citing differences between experts and novices. Research has distinguished experts from

novices on the basis of heuristic application, knowledge representation, memory, and differences between high and low experience subjects (Cai, 1994). Siegler (1991) suggests strategies, metacognition and content knowledge developmentally advance and change with age. Flavell, Green and Flavell's (1995) research highlights important developmental differences in children's awareness and introspection of their thinking. This research builds on the above research to examine developmental aspects of metacognition in the context of mathematical problem solving.

1.2

STATEMENT OF THE PROBLEM

The purpose of this study is to identify and investigate metacognitive aspects in the context of mathematical problem solving. The goal of the study is to create a situation that allows for inferences to be made about the developmental aspects of children's metacognition in problem solving.

1.3

SIGNIFICANCE OF THE STUDY

Although it is important in a naturalistic study to realise the difficulty in generalising beyond the context of this study, results from this study should further add to the growing bank of theory and research that has focused on the relationship between metacognition and problem solving. Many studies have examined mathematical problem-solving processes and strategies.

Few studies have considered the nature of metacognitive development in relation to mathematical problem solving. Whilst many studies have alluded to the advantages to be gained from expert use of metacognitive processes and strategies in problem solving, there is little research on the nature of metacognitive differences between children of different ages when they are presented with a similar non-routine mathematical problem solving task. The outcome of this investigation should provide researchers with data on metacognitive progression and developmental differences in metacognition in relation to mathematical problem solving. This area of developmental research is important because it bears on whether children's metacognitive deficiencies are caused by lack of metacognitive development or failure to use metacognitive processes (Flavell, 1970). Perhaps the most significant benefit may lie with the classroom teacher, such information may enable teachers to link what they are teaching to what children need to be taught.

1.4

DEFINITION OF TERMS

Metacognition

> *Metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything related to them. . . And refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes. . .*
>
> (Flavell 1976, p. 232.)

Garafalo and Lester (1985) state that metacognition has two separate but related aspects: a) knowledge and beliefs about cognitive phenomena; and b) the regulation and control of cognitive actions. These are defined in this proposal collectively as metacognitive aspects and separately as metacognitive awareness and metacognitive strategies.

a) Metacognitive Awareness

This term defines a person's metacognitive knowledge and beliefs about cognitive processes and states (Wellman, 1985). Flavell (1979) suggests metacognitive awareness comprises three interacting categories of awareness:

i	<u>person</u> :-	beliefs about one's self and others' cognitions;
ii	<u>task</u> :-	information about aspects of the cognitive enterprise and;
iii	<u>strategy</u> :-	knowledge of ways to approach a particular cognitive enterprise.

b) Metacognitive Strategies

This term describes any strategies that monitor and control cognitive actions (Herrington, 1992). Cognitive theories on problem solving identify four key processes or strategies involved in metacognitive regulation, these correspond to Polya's (1957) phases and involve:-

i	<u>Orientation</u>:	assessing, understanding and analysing the problem;
ii	<u>Organisation:</u>	devising, planning, choosing and organising behaviour and actions;
iii	<u>Execution:</u>	executing and monitoring plans;
iv	<u>Verification:</u>	evaluating, verifying and reflecting on decisions and outcomes.

(Garafalo and Lester, 1985, p.171)

CHAPTER 2
REVIEW OF LITERATURE

The aim of this section is to review research findings and theories on metacognition, the relationship of metacognition to problem solving, developmental theories on metacognition, and to provide a summary of those findings and research.

2.1
METACOGNITION

Different theorists have emphasised different components and aspects of metacognition. In order to gain a coherent view of what metacognition is, it is first necessary to discuss different theorists' views.

Metacognition's core meaning is 'cognition about cognition' (Flavell, 1985). Flavell (1976) focused on two aspects of metacognition, a person's knowledge of their thinking and their control and monitoring of cognitive strategies.

Schoenfeld (1987) summarised metacognitive concepts as distinct related categories of intellectual behaviour:-

1. Knowledge about your thinking.
2. Control or self regulation.
3. Beliefs and intuitions.

Marzano, Brandt, Hughes, Presseisen, Rankin and Suhor (1988) suggest metacognition consists of two concepts; knowledge and control of self, and knowledge and control of process.

In sum, the common underlying similarity of these theories is that metacognition is thinking about, regulating, monitoring and controlling thinking, and the knowledge and control one has of one's thinking.

2.2
THE RELATIONSHIP OF METACOGNITION TO PROBLEM SOLVING

Brown and Palincsar (1982), Lester (1983, 1988) and Schoenfeld (1983) have argued the importance of metacognition in problem solving, suggesting metacognition may account for a significant part of good problem solving and understanding. Researchers have found that successful problem solvers exhibit higher degrees of metacognitive awareness and strategies, suggesting metacognitive aspects are tied to successful problem solving ability (Garofalo and Lester, 1985; Schoenfeld, 1985a, 1985b, 1987; Swanson, 1990). Swanson (1990) found high metacognitive students performed better on problem-solving tasks than low metacognitive students suggesting metacognitive ability might be a factor in performance.

Schoenfeld's (1987, 1983) research suggests that more able students have a higher degree of metacognitive awareness in problem solving tasks. Campione, Brown and Connell (1988) also suggest successful learners reflect, oversee and regulate thinking and strategies.

Schoenfeld's (1983) research suggests metacognitive managerial skills are an essential part of successful problem solving. In studies that compared expert and novice problem solvers Schoenfeld (1985a) found that metacognitive planning, monitoring and evaluating was exhibited by experts in contrast to lack of planning, regulation and monitoring exhibited by less successful problem solvers. Hembree's (1992) meta-analysis found the best problem solvers used reasoning skills suggesting students need higher order processes and metacognitive skills. Garofalo and Lester's (1985) research focused on providing a theoretical framework for conceptualising the relationship between metacognition and problem solving. They suggested a metacognitive framework for studying mathematical problem-solving performance. An instrument based on Garofalo and Lester's (1985) framework (Appendix 2) is used in the present research. Garofalo and Lester's (1985) framework specifies the key points where metacognitive decisions are likely to influence problem-solving cognitive actions. They suggest four main categories involved in mathematical problem solving. These are:

Orientation:	strategic behaviour to assess and understand a problem;
Organisation:	planning of behaviour and choice of actions;
Execution:	regulation of behaviour;
Verification:	evaluation of decisions made and of outcomes of executed plans.

There are different metacognitive strategies associated with each category, a summarised version of these strategies can be seen in Appendix 2.

Problem solving is influenced by students' beliefs. For example, Schoenfeld (1983) stressed a student's problem solving is influenced by three factors: the student's skills, control and belief systems. Similarly, Garofalo and Lester's (1985) research suggested that students' metacognitive awareness and beliefs about self, task and strategy played an important role in mathematical problem solving. Flavell (1979) suggests that metacognitive awareness is comprised of the interaction of person, task and strategy awareness and these three categories also interact to affect cognitive actions. Flavell (1979) suggests that these variables should be considered when examining metacognitive awareness.

2.3

DEVELOPMENTAL THEORIES ON METACOGNITION

Flavell (1984) suggests it is not easy to describe or explain metacognitive development.

> *Serious theorising about basic mechanisms of cognitive growth has actually never been a popular pastime, now or in the past. It is rare indeed to encounter a substantive treatment of the problem in the annual flood of articles, chapters and books on cognitive development. The reason is not hard to find: good theorising about mechanisms is very, very hard to do. (Flavell, 1984, p. 189).*

Flavell, Green and Flavell's (1993, 1995) research highlights important developmental differences in children's awareness and introspection of their thinking. Flavell et al's (1993) studies suggested young children lacked the ability to introspect. Building on this previous research Flavell et al (1995) examined children's introspective skills by presenting the children with problems that required them to think about certain objects. Children's responses were categorised according to whether they reported or denied thoughts. Their research results suggest younger children lacked introspective skills and were unlikely to be aware of and reflect on their own thinking. In comparison older children had significantly better introspective skills and were more aware and reflective of their own thinking. Flavell et al (1995) believed this increase in

performance suggests children's introspective abilities and knowledge about thinking may undergo age-related developments. Schneider (1985) reviewed literature on developmental aspects of metacognition; results of studies on metamemory suggest young children are quite limited in monitoring and awareness of their cognitions. Similarly Brown (1987) suggests reflecting and awareness of one's cognitive operations appears to be more difficult for pre-school children than for children around twelve. Brown (1987) suggests that young children lack experience and are less aware about how they learn and are unable to verbalise their thinking.

Brown (1987) summarised Piaget's developmental progression of consciousness and suggested that metacognitive self regulation and reflective awareness develop over the same period as Piaget's formal operations stage. It is virtually agreed upon by contemporary developmentalists that cognitive development is not as stage like as Piaget suggested. Recent research suggests infants and young children are more competent than was thought by Piaget and developmentalists (Flavell, 1992; Siegler, 1991). Neo Piagetians have suggested that there is a regular, probably maturation-based, increase with age in some aspects of the child's cognitive abilities and capacities; as the child's capacity increases with increasing age new and more complex forms of cognition become possible (Flavell, 1992). Vygotsky (1986) suggests that the ability to use metacognitive processes is a developmental process and lags behind the ability to learn and use a strategy.

Siegler (1991) suggests strategies, metacognition and content knowledge developmentally advance and change with age.

2.4

SUMMARY

In summary, metacognition is managing thinking and thinking about thinking, it refers to the control and knowledge one has of one's cognitive processes.

Research in metacognition and problem solving suggests metacognitive aspects are related to successful problem-solving performance and mathematical ability. Higher degrees of metacognitive awareness and strategies appear to assist and influence successful problem solving. Garofalo and Lester's (1985) model and Flavell's (1979) metacognitive awareness categories can provide a framework to examine these metacognitive aspects in relation to mathematical problem solving.

Research suggests young children have less control and awareness of their thinking. It is possible that age does appear to act as an enabler of metacognitive growth (Flavell, 1992).

If age does act as an enabler of metacognitive growth and higher degrees of metacognitive awareness and strategies do assist

successful problem solving it is important that this relationship is examined. However, no research has been done to examine developmental aspects of metacognition in the context of mathematical problem solving. The following research objectives build on the need to examine these factors.

RESEARCH OBJECTIVES

AIM: To identify metacognitive aspects and investigate the relationship of metacognition and age in the context of solving a mathematical problem.

Research Questions

- Is there a developmental trend in metacognitive awareness evident in children between Years 2 - 6 in relation to mathematical problem solving?
- Is there a developmental trend in metacognitive strategies evident in children between Years 2 - 6 in relation to mathematical problem solving?
- What aspects of metacognition undergo development during Years 2 - 6?

Orientation Statement

It is possible that there is a developmental sequence to metacognitive awareness and strategies used for mathematical problem solving. It is also possible that this metacognitive development affects the implementation of successful problem solving strategies.

CHAPTER 3

METHODOLOGY

This study was designed to identify metacognitive aspects and to investigate the developmental aspects of metacognition in the context of mathematical problem solving.

3.1

SUBJECTS

The sample for this study was selected from an inner city Perth Primary School. The school was chosen because it has a range of children from mixed backgrounds and different socio-economic levels. This study focused on a total of twenty four children from Years two, four and six; there were eight children from each year level. Ten names (this included two reserves) from each class were randomly chosen.

3.2

PROCEDURE

Each individual child was given the same non-routine mathematical problem to solve independently (Appendix 1). It was acknowledged that metacognitive differences cited as developmental were possibly attributable to different degrees of learnt declarative and procedural knowledge (Garofalo & Lester, 1985). Cai (1994) found that non-routine problems appear to function well for examining metacognitive aspects because children do not respond to them with over-learned solution procedures and because children are unable to solve them quickly with a factual or rule-governed response. Therefore the non-routine problem was chosen to minimise the degree of expertise, domain specific and content knowledge brought to the problem solving process. This was crucial to the examination of developmental aspects of metacognition. Students were observed and requested to think aloud and write down their steps in solving the problem. All students had access to counters to use as concrete aids. There was a post activity interview to measure metacognitive awareness and to clarify think-aloud problem-solving comments. Think aloud problem solving and interviews were recorded and transcripts made.

3.3

INSTRUMENTS

The principal aim of the research instruments was to identify students' metacognitive awareness and strategies (in relation to mathematical problem solving) in order to investigate which aspects undergo development and whether any developmental trends were evident. This study used a semi-structured interview protocol, observation checklists, tapes of students' thinking aloud, description and analysis of students' written steps and activity. These focused on identifying Flavell's (1979) metacognitive awareness categories and, Garofalo and Lester's (1985) framework of metacognitive strategies (Appendices 2 - 4). These are explained in more detail on the following page.

The current study used Garofalo and Lester's (1985) model as a framework for the observation checklist in order to identify students' metacognitive strategies. Garofalo and Lester's (1985) framework of metacognitive strategies specifies four key points where metacognitive decisions are likely to influence mathematical problem solving. There are different metacognitive behaviours associated with each category. (Table 1).

TABLE 1

METACOGNITIVE STRATEGIES
ORIENTATION Comprehension Analysis of problem Selection of strategy to understand the problem
ORGANISATION Identifies goal Plans a course of action
EXECUTION Selection and performance of appropriate strategy Monitoring progress and execution
VERIFICATION Evaluation of orientation Evaluation of organisation Evaluation of execution Reflect on, revise and abandon nonproductive strategies

Flavell's (1979) three categories of metacognitive awareness were used as a framework for the interview questions in order to identify students' metacognitive awareness, namely:

Person awareness:	beliefs about self in relation to problem solving;
Task awareness:	information about the task in relation to problem solving;
Strategy awareness:	knowledge of ways to approach the task in relation to problem solving.

VALIDITY OF INSTRUMENTS

Think-aloud methods

Goos (1994), Nisbett and Wilson (1977) have questioned the validity of think-aloud methods suggesting: subjects may not be able to accurately explain and report their behaviour and metacognitive processing; subjects may not have the language to explain their behaviour and the demands of the situation may distort cognitive processing. However, Genest and Turk (1981) and Goos (1994) suggest think-aloud methods can provide useful information if treated as data for the researcher to infer explanations. Flavell et al (1993) acknowledges researchers need to be extremely sensitive to the way children talk about and conceptualise their thinking.

Observation checklist

The observation checklist used Garofalo and Lester's (1985) metacognitive framework as a model. This framework specifies four key points where metacognitive behaviours are likely to occur. The usefulness of this model for examining metacognitive aspects of problem solving has support from Artzt and Armour-Thomas (1992) and Cai (1994). Cai's (1994) and Artzt and Armour-Thomas's research have both used the model to examine metacognitive strategies in relation to mathematical problem solving. They suggest Garofalo and Lester's (1985) metacognitive

framework provides an effective model for examining metacognitive strategies in relation to mathematical problem solving.

Interviews

This research used a flexible interview approach. While a semi-structured interview format was used, the interviewer also used open, non-specific and probing questioning to encourage verbalisation. The researcher was aware of and responsive to the students' language and vocabulary. Questions were phrased: in language the student could understand; to ensure the student was given no cues as to how to respond and; to test and assess the validity of any developing hypotheses about the students' metacognitive development.

3.4

METHOD OF DATA ANALYSIS

A semi-structured interview and observation protocol were used to determine students' metacognitive aspects (Appendices 2 - 4). There was a descriptive analysis and a systematic quantification of metacognitive strategies. Think-aloud problem solving and interviews were recorded and transcripts made, these were analysed in conjunction with the observation checklists and clarification notes.

Presentation of data

The interview questions and a summary of students' responses were categorised in two sections:

Metacognitive strategies
- orientation
- organisation
- execution
- verification

Metacognitive awareness
- person
- task
- strategy

Data Analysis

The analysis emphasised descriptive profiles, trying to build 'portraits' of each year level, focusing on identification and description of metacognitive themes for each year level. Metacognitive developmental trends were identified and described.

3.5.

RESEARCH ASSUMPTIONS

This project was founded on a naturalistic paradigm with some quantitative features. This approach was based on the belief that: these are not competing paradigms, each can tell us something the other can't and the inferential nature of the research benefited from a combined approach. This combined approach was based on the following naturalistic beliefs and quantitative features.

Naturalistic beliefs

This research accepted the view that it is impossible to separate a thing from its environment, activities are embedded in a context and the nature of the processes used for determining a student's metacognitive aspects will affect the results and conclusions.

Quantitative Features

There are contradictory viewpoints and considerable confusion about what metacognition is and claims that the concept is too ill-defined to be the object of inquiry and hence is unmeasurable (Campione et al, 1988). Therefore, while naturalistic inquiry generally rejects the use of a priori theory, this project assumed: a general theory of metacognition; that metacognition is definable and divisible into separate aspects and; it is possible to quantify metacognitive aspects in terms of their occurrence within a given context (Flavell, 1979; Garofalo and Lester, 1985; Schoenfeld, 1983).

CHAPTER 4

RESULTS AND ANALYSIS

This chapter includes the following sections.

4.1

PRESENTATION OF DATA

This section categorises students' responses in terms of their person, task and strategy awareness in relation to mathematical problem solving. Secondly, students metacognitive strategies in relation to mathematical problem solving, are identified. Thirdly, students' responses during think aloud problem solving are summarised and finally observation notes are summarised.

4.2

DATA ANALYSIS

In this section metacognitive themes for each year level are identified and described. This analysis emphasises a descriptive analysis:- the building of 'portraits' of each year level rather than a focus on problem-solving task performance. Secondly, metacognitive developmental trends are identified and described.

4.1
PRESENTATION OF DATA

4.1.1

CATEGORISATION OF STUDENTS' METACOGNITIVE AWARENESS

Data was collected through students' responses to interview questions (Appendix 4) in conjunction with observation notes. Any discrepancies and/or anomalies from think-aloud problem solving and interview responses were also clarified during the interviews.

The interview questions and individual students' responses are shown on the following pages, these are categorised in three sections relating to the students' person, task and strategy awareness in relation to mathematical problem solving. In each category there is a response from each of the twenty four students, these are grouped in their year levels with eight children in each year level group.

PERSON AWARENESS

Did you like solving the problem? Why?

YEAR 2
Yes, I don't know why.
Yes, It was fun because you had to count.
Yes, I like maths it was a bit hard.
Yes, it was hard and I had to think.
Yes, I like hard but I like more the hard work in class, 30 is a lot.
No, I just didn't.
Not really, I didn't understand it, 30 is a big number.
Yes, I liked how I had to think and count all those legs and how much eyes.
YEAR 4
Yes, because I sorted them into 2 and 4's. I used my head.
Yes, maths is my favourite subject and I'm usually good at it.
Kind of, didn't really, it was hard.
Yes, I like maths.
No, it was too hard.
Yes, it wasn't really easy or hard.
Yes, it was fun.
Yes, I don't know why, I like maths.
YEAR 6
No, it was hard.
Yes, it made me think.
Yes, very easy.
Yes, was fun and interesting.
Yes, because it was a challenge interesting.
Yes, I like maths.
Yes, it was easy.
Yes, needed to think it was a bit of a challenge.

Do you think some children were better or worse at solving the problem? Why?

YEAR 2
Better, I can't read numbers and I don't understand. Big children could be.
I don't know, some people are better at doing things.
Sort of the same, older children will find it easier.
Some better some worse, big kids will find it easier.
Worse, mum says I am much better.
Better, they have been at school longer than me.
Don't know.
Not sure.
YEAR 4
Both, younger might not understand, older would.
The same, I might have been better and faster.
Don't know.
Better, some are better.
Better.
Better, most people are smarter than me.
Better, I'm not good at maths.
Both, better probably.
YEAR 6
Both, some could do it quicker.
Both, because they do it differently.
About the same.
Better, I don't know.
Both, because some children don't have problem solving ability like others.
Both, some might have difficulties.
Better, they are smarter than me.
Both, some have different ways, processes.

Did you find it easy or hard to think about the problem?

YEAR 2
Hard because 30 is a big number.
Easy I was thinking the numbers in my head.
Bit hard, the eyes were the easy bit the legs were the hard bit.
Hard, don't know the way to do it.
Quite hard, because it took a long time, I don't know why it was really hard.
Well for how many chickens and pigs that was easy, the legs was hard.
Hard, I didn't really understand it, I did understand some of it.
The eyes were a bit easy and the legs were a bit hard.
YEAR 4
Pretty hard because you have uneven groups into 4's.
Pretty easy, there is one kid in my class who can think much faster but he doesn't always get them right.
Hard, I had to read it again.
Hard, I got confused, I don't know why.
Hard, the problem was hard, the numbers didn't go into even groups.
Hard, I didn't really get it.
Easy, if you write it down and don't have to think.
Hard at first, easier later.
YEAR 6
Hard, because there wasn't much information.
Hard, lots of things to think about, legs was the hard bit.
Easy, because I like maths.
Easy, because you imagine in your head.
Easy and a bit hard, forgot chickens had 2 legs, easy after you have all the information.
Easy, you think it through.
Easy, 30 eyes equals half the animals, I had trouble with the legs, then it was easy.
Easy, only has 2 actual questions in it both relevant to each other.

Did you find it easy or hard to solve the problem? Why?

YEAR 2
No.
Hard, because you had to count in 4's.
Hard, I didn't really understand, know how to work it out.
Hard.
Easy, my mum says I'm a clever girl and I think so too.
Hard, I don't know.
Hard.
A bit hard, a bit easy, I liked how I had to count.
YEAR 4
Pretty easy, 30 eyes, so I thought 2 eyes for each animal.
Easy, except when I lost track of my counters.
Easy, the legs on chicks.
Not sure.
Hard, it was hard because of halving groups.
Hard, I didn't get it.
It gets harder, you haven't got a math sign you need to think.
Hard, mainly because of the odd number of animals.
YEAR 6
Easy to solve, you just have to do times tables.
Easy to solve, do first part then second.
Easy, because I knew what to do.
Easy, because you can go wrong but you can work it out again.
Easy, when you get the information sorted.
Easy, read problem, then read it in bits, one question at a time, break it into bits.
Easy, with counters keep track of counting.
Easy, to work out in my mind.

TASK AWARENESS

Are problems like this important to do?

YEAR 2
You have to learn by doing them, you learn more and more.
So that you can get better.
Yes or other people won't know what you are talking about.
Yes you need to know how.
Only if you really have to.
Yes if you give some money for a bet you can count it.
Yes you need to know maths so that you can do things.
Because you learn if you have to do the same thing again you learn
YEAR 4
Yes, if you want to count some animals you would have to do it the proper way.
Some of them, to help me on my journey to finish school.
Yes, it helps you solve other problems.
If you're a farmer and it teaches tables for school.
I don't know, probably not.
Yes, so you can get a job.
Yes, if there was a test.
Not sure.
YEAR 6
No, you don't need to know how many legs chickens and pigs have.
Yes, there are things in life to work out.
Yes, you may have to find out how many... or do a problem like this.
Yes, in life you have lots of problems like this.
Yes, you need to work out problems when older.
Yes, you need to be able to work out tax etc.
Yes, when you buy something or need to solve something.
Depends on what you do in the future.

What would be a similar problem?

YEAR 2
Can't think of one.
Don't know.
Don't know.
How many dogs and geese, eyes and legs.
How many dinosaurs eggs and dinosaurs.
Don't know.
One that's the same.
Don't know.
YEAR 4
1 saw 30 chickens and pigs how many eyes?
15 people and 16 things to share.
Don't know.
1 saw 30 eyes, how many sheep?
If I had 20 bats, how many eyes?
Sheep and cows, 30 eyes, how many legs?
Changing it to cows and roosters.
Don't know.
YEAR 6
Just like the one we did.
Different animals.
The same, different animals.
Jenny and Joe were delivering 350 milk bottles and had to do half each.
If earn $2,000 a month and rent is $39 a week, how much money left to spend on clothes?
At a market saw 30 pieces of fruit in 5 bowls, how many in each bowl?
An easy one.
There are 80 eyes, different animals, different situation.

What would be a different problem?

YEAR 2
Don't know.
Don't know.
Don't know.
Don't know.
A lot of 2's and take away 10.
Don't know.
30 animals and each animal has 2 babies.
How many ears instead of legs.
YEAR 4
1 saw 15 pigs and 2 chickens, how many eyes.
Algebra.
Don't know.
There are 80 ears how many noses?
347 take away.
90 ÷ 30.
14 cows x 14.
a plus sum.
YEAR 6
Maths, adding etc.
How many leaves on a tree?
How much change if spend $2.50 out of $5?
Some tins and boxes, how many boxes?
How many litres in a ton?
A boat was travelling 20 km/h, tide was 5 km/h against it. How fast was boat going?
A hard one.
Long mathematics (e.g. calculus).

STRATEGY AWARENESS

What do you have to do to solve these sorts of problems?

YEAR 2
Think so if you get it next time you should try to remember.
I don't know.
Solve it in your head.
I don't know, add them up in your head.
I don't know.
Count in your head, your fingers or counters, that's all really.
Ask a grown up person they help you because you don't know what to do yet.
Add/multiply.
YEAR 4
You are thinking about what kind of groups then about ways you are going to group them.
Use my head, I use counters to get them right.
Think about what to do, different ways.
Know times tables in head, start, work out bits.
Think, head swirling around.
Think.
Think about it.
Know times tables.
YEAR 6
Read then work out what you need to solve it.
Work out the numbers in your head.
Each bit at a time.
See whole problem as whole, break up if hard.
Read whole thing then break It up and get information down then put it together.
Break it up, solve bits at a time.
Think about it, break it up, then put it together again.
Need to think in a broad way, look at problem from different angles.

Is there only one way to solve maths problems?

YEAR 2
No, lots.
Yes, by counting the number.
No.
No, I think there are other ways.
No, I think there are quite a few ways.
No, I can't think of them.
No, You can count or look at maths charts.
Yes, multiplying and adding.
YEAR 4
No, different, adding or take away, or by reading and fixing it up, using my head.
Yes, use my head, think pretty fast, able to know division and times tables.
Not sure.
No, I could have done it in my head and not used counters, do it different if it was a different problem.
No, write it down, do it different ways.
No, different ways, writing down.
No, use calculators, computers.
No, different ways.
YEAR 6
No, you could visualise it, counting eyes and legs.
No, read or think the problem through another way.
No, try different ways.
Heaps of ways.
No.
No, can divide and multiply.
No, different ways.
No, different ways, opening the mind.

4.1.2

IDENTIFICATION OF STUDENTS' METACOGNITIVE STRATEGIES IN RELATION TO MATHEMATICAL PROBLEM SOLVING

Students' metacognitive strategies in relation to mathematical problem solving were identified through observations and audio recordings of the students while they were problem solving and thinking aloud. Following the problem solving task, interviews were used to clarify think-aloud problem solving and students were asked to elaborate on their comments and responses. Garofalo and Lester's (1985) four categories of metacognitive problem-solving strategies were used to categorise the metacognitive strategies employed and described by the students during problem solving. A summary of students' responses and a table of metacognitive strategies in terms of their occurrence within the problem-solving context was made. This information is shown in Table 3. Due to the methods used and small sample size, a statistical analysis was inappropriate. Children were grouped: yes, partial and no. 'Yes' meant the children exhibited problem solving behaviour and/or explained thinking that showed complete implementation of the strategy in that category. 'Partial' meant the children were developing and/or partially exhibited problem solving behaviour and /or explained thinking that showed partial implementation in that category. 'No' meant the children did not exhibit problem solving behaviour and/or thinking that category.

Examples of the decisions to group students into particular categories are shown in Table 2. These examples indicate metacognitive strategic decisions being made within the categories given in the framework by Garofalo and Lester (1985). It should be noted that students' metacognitive strategies were not assumed from isolated incidents and/or comments but from their full discourse during think-aloud problem-solving interviews and problem-solving behaviour including manipulation of counters and their working out on the provided paper.

TABLE 2

Examples of the statements and behaviours which were identified in relation to Garofalo and Lester's (1985) categories.

Category	Decision	Childrens' statements and behaviours
Comprehension Analysis of problem Selection of strategy to understand the problem Identifies goal Plans a course of action	Yes Yes Yes Yes Yes	I read the question about two times and then I think I need to find out how many animals and legs. Then I think half of thirty is fifteen animals, now how many pigs and chickens and legs, I'm thinking what kind of groups to put them in. But I need to know how many pigs and chickens. But I could have an odd number or half an animal - does it matter how many of each I have?
Analysis of problem	Partial	I'm thinking I can find out the animals because animals have two eyes but I don't know the legs because I just don't know.
Identifies goals Plans a course of action	Yes Yes	There were pigs and chickens and thirty eyes, each has two eyes, pigs have four legs and chickens have two legs, I could get fifteen then do twos and fours.
Selection and performance of appropriate strategy	Yes	Child draws eyes in groups of two then draws four goats' hooves and two chickens' legs on each group.
Monitoring progress and execution	Yes	What kind of sum is this, no that's not right - I'll start again (tries another way).
Evaluation of orientation	Yes	(After initial try) I'm thinking I should read it again, I'm not sure. I don't know the legs. Can I try again?
Evaluation of organisation Evaluation of execution	Yes Yes	(Finished task) I'm not happy with that. No, because one pig has three legs. How should I count them, you can't split fifteen to an even number. Pigs need four legs and chickens two.
Reflect on, revise and abandon non-productive strategies	Yes	(Previous attempt focused on four legs for each animal). Chickens can't have four legs. (Child then proceeds to place two counters then four counters for each animal).

Table 3 on the following page summarises the observed and described occurrence of metacognitive strategies of all the students during problem solving.

TABLE 3

OCCURRENCE OF METACOGNITIVE STRATEGIES DURING PROBLEM SOLVING

METACOGNITIVE STRATEGIES	YES			PARTIAL			NO		
	Yr 2	Yr 4	Yr 6	Yr 2	Yr 4	Yr 6	Yr 2	Yr 4	Yr 6
ORIENTATION: strategic behaviour to assess and understand the problem.									
Comprehension	3	6	7	3	1	1	2	1	
Analysis of problem	1	4	7	5	3	1	2	1	
Selection of strategy to understand the problem	1	1	7	5	6	1	2	1	
ORGANISATION: planning of behaviour and choice of actions									
Identifies goal	1	1	7	4	6	1	3	1	
Plans a course of action	1	1	7	4	6	1	3	1	
EXECUTION: regulation of behaviour and choice of actions.									
Selection and performance of appropriate strategy	1		7	4	7		3	1	1
Monitoring progress and execution	1	2	7	4	5		3	1	1
VERIFICATION: evaluation of decisions made and of outcomes of executed plans.									
Evaluation of orientation	1	2	7	4	5		3	1	1
Evaluation of organisation	1	1	7	4	6		3	1	1
Evaluation of execution	1	1	7	4	6		3	1	1
Reflect on, revise and abandon nonproductive strategies	1		7	3	7		4	1	1

Number of children (out of a total of 8 children for each year level)

(Garofalo and Lester, 1985)

The figures on the following pages show graphical representations of the above information.

GRAPHICAL REPRESENTATIONS OF METACOGNITIVE PROBLEM SOLVING STRATEGIES

ORIENTATION

Figure 1: Comprehension

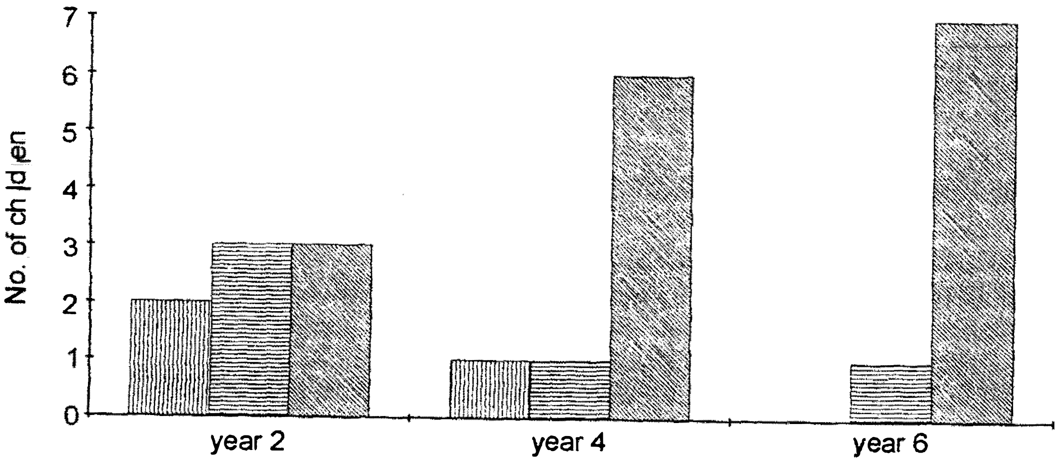


Figure 2: Analysis of the problem

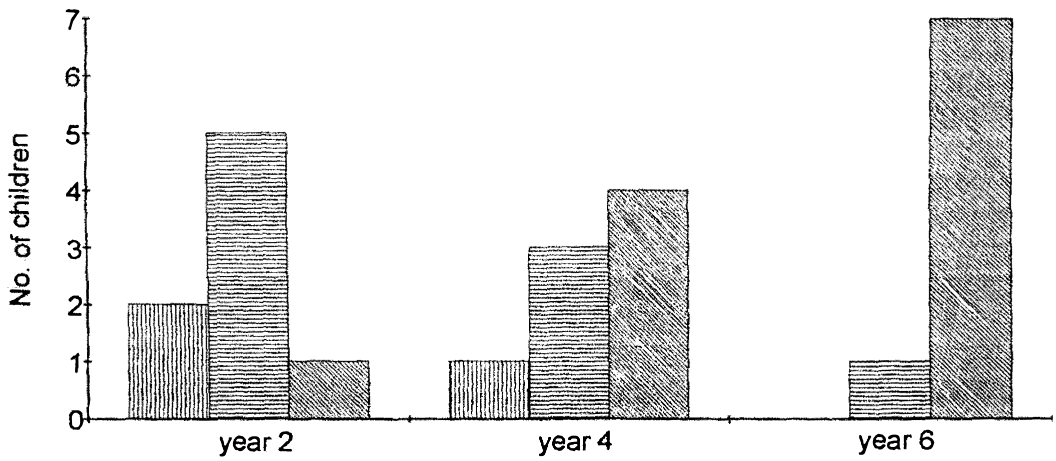
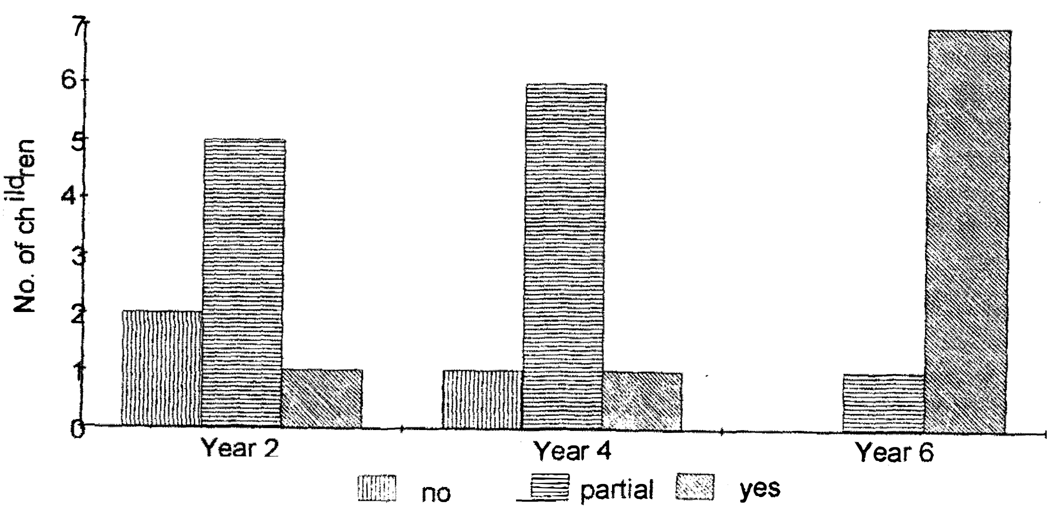


Figure 3: Selection of strategy to understand the problem



ORGANISATION

Figure 4: Identification of the goal

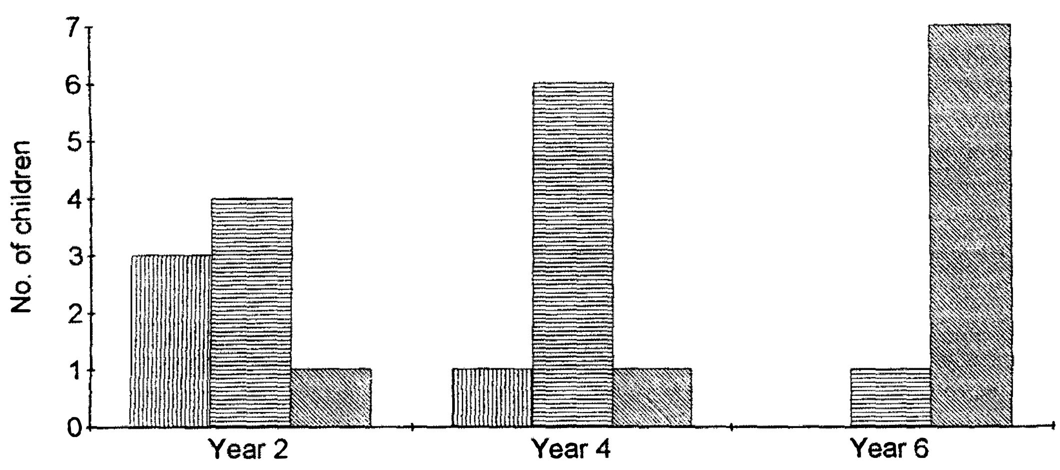
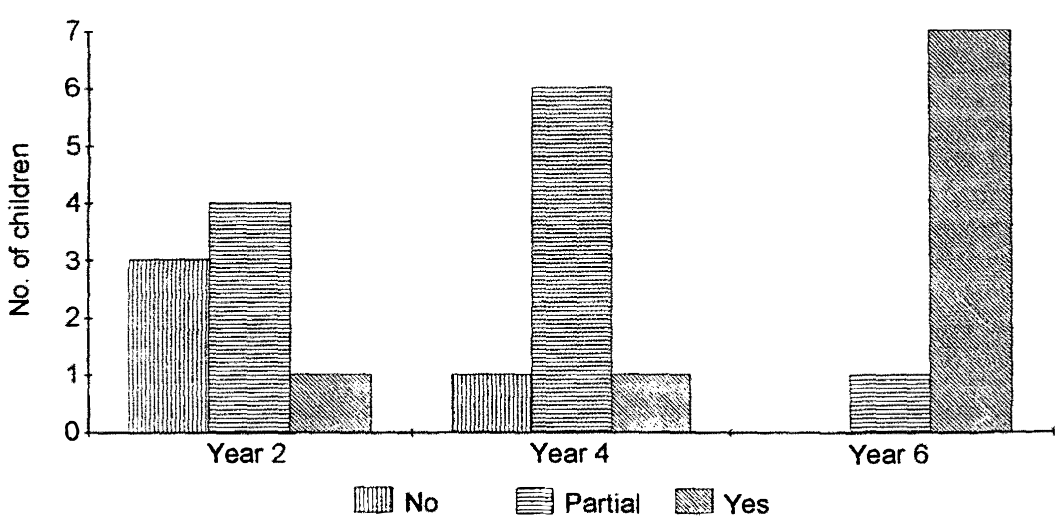


Figure 5: Planning of a course of action



EXECUTION

Figure 6: Selection and performance of an appropriate strategy

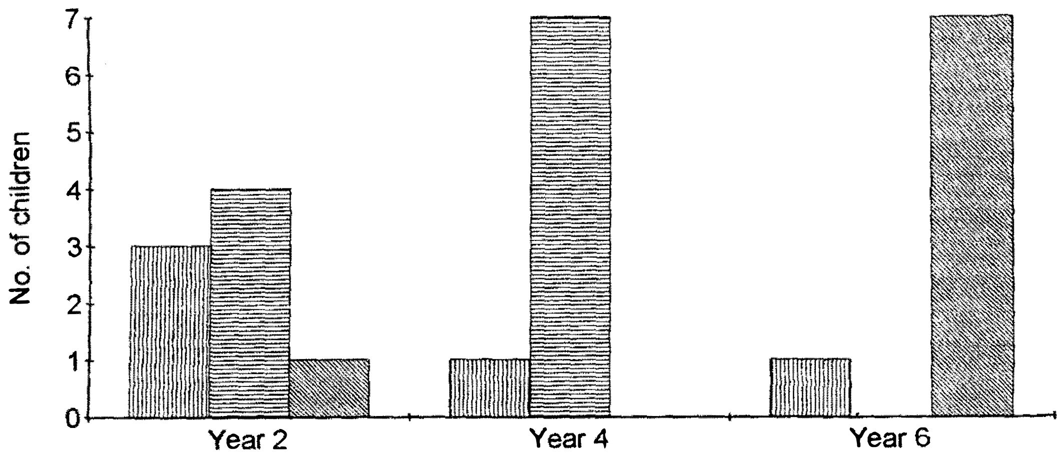
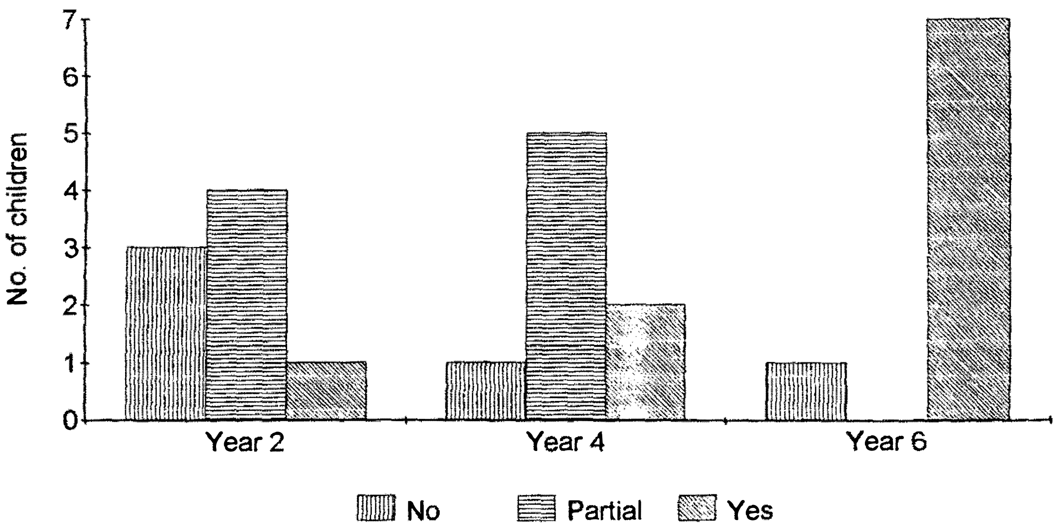


Figure 7: Monitoring progress and execution



VERIFICATION

Figure 8: Evaluation of orientation

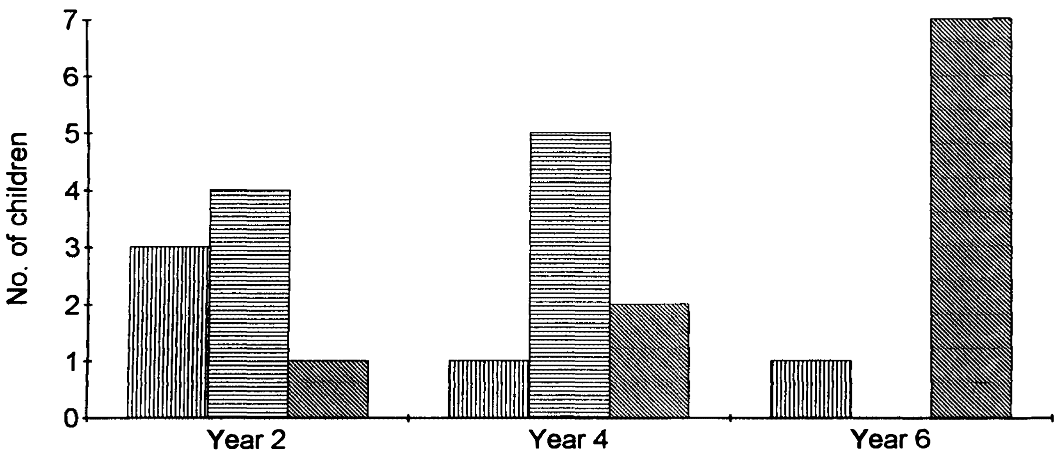
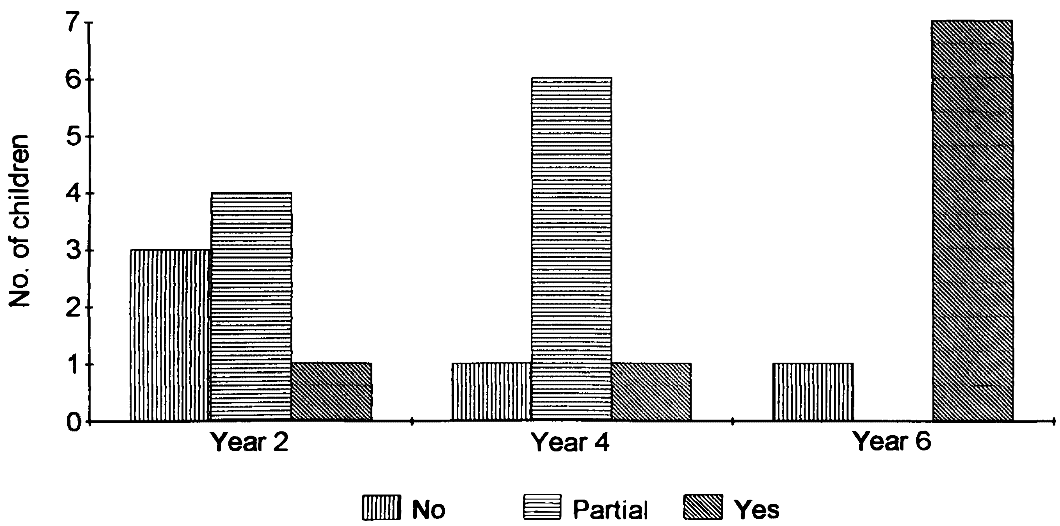


Figure 9: Evaluation of organisation



VERIFICATION

Figure 10: Evaluation of execution

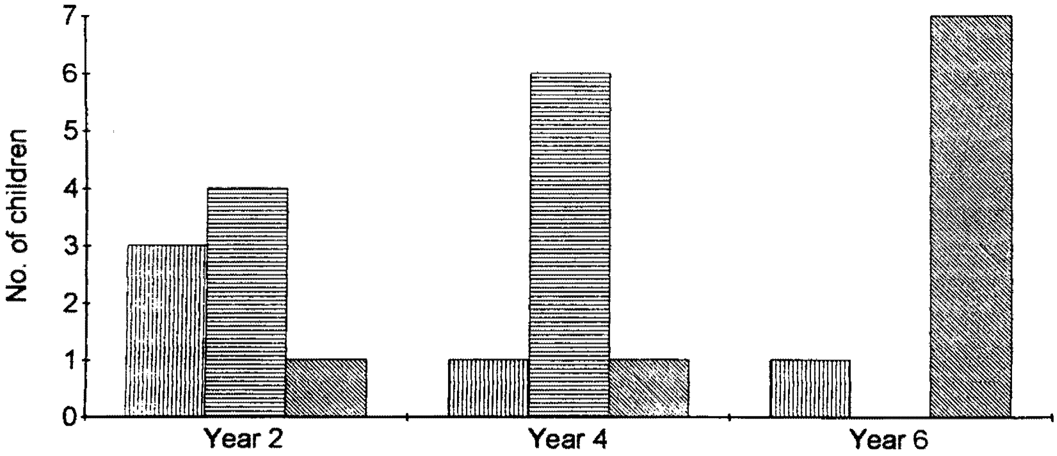
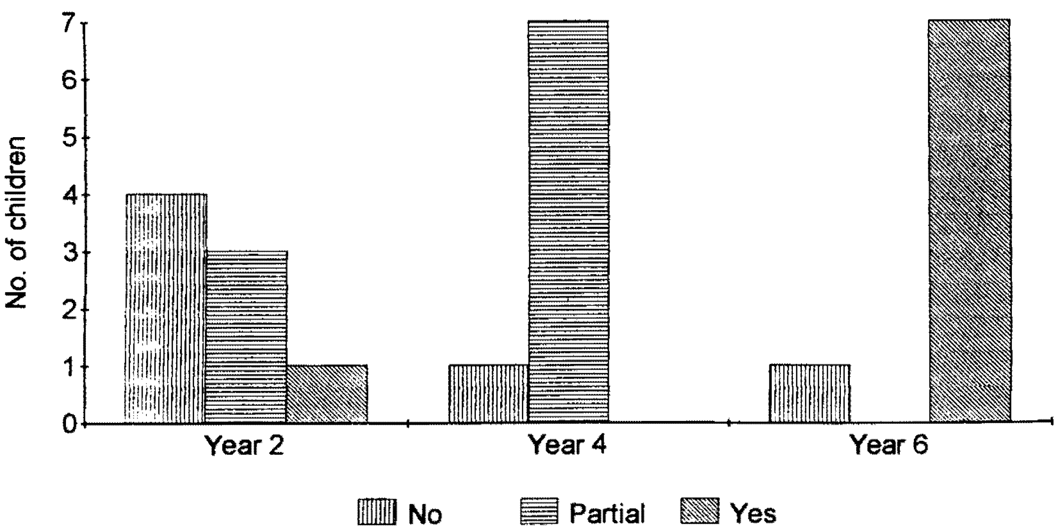


Figure 11: Reflection on, revision and abandoning unproductive strategies



4.1.3

STUDENTS' RESPONSES DURING THINK-ALOUD PROBLEM SOLVING

While the interview and observation checklist were designed to focus on specific metacognitive aspects, it was also possible to extract students' responses and comments in relation to metacognitive aspects from transcripts of think-aloud problem solving. These responses and comments provide another perspective on students' metacognitive aspects in relation to mathematical problem solving.

Year 2
I was thinking the numbers in my head. I'm using counters because I want to know. I'm thinking about it in my head. I was thinking about the numbers. I need to put the numbers down so my head won't get mixed up. It's a hard one to think about, get mixed up, can you tell me, can you give me a clue. I'm not thinking because there are none, the wolves have eaten them.

Year 4
Hard to keep information in my head. I read it about 2 times then I think. I'm thinking I have to split it in half. I'm thinking nothing, my heads swirling around. I'm thinking about counting in two's. I'm thinking about what kind of sum is this.

Year 6
I just have to think how to do it, there wasn't much information and that makes it a bit of a problem. The question should be in half, broken up a bit to make it easier. I just know what to do, each bit at a time. I'm just imagining in my head I can see the whole problem and break it up. Thinking about the whole thing, breaking it up and putting it together. Breaking it up and solving bits at a time. You need to think in a broad way changing the problem around to suit the answer, looking at it from different angles.

4.1.4

SUMMARY OF OBSERVATION NOTES

The students were observed during problem solving and this is a summary of observations made across the three year levels.

YEAR 2

All the children in year two used counters or drew lines or circles for eyes. Most stated they wanted to figure out how many chickens and pigs there were and quickly sorted the counters or lines into groups of two. Many children had difficulty and were confused with the legs and they did not have the metacognitive strategies needed to solve the problem, most decided they would all be 4 legged pigs or 2 legged chickens, or they drew hooves and chicken feet below the counters. Orientation metacognitive strategies were generally fairly competent.

YEAR 4

Most children in year four knew immediately there were 15 animals, immediately after most children went on to say there would be 60 legs, some of the children then evaluated and revised that decision , realising this was not possible and redid the problem. Many of the children described the legs as 'the tricky bit'. Evaluation of orientation strategies and reflection and revision strategies were generally partially implemented.

YEAR 6

All except one of the children in year six knew immediately there were 15 animals, they then realised there could be different answers for the legs depending on how the animals were divided. Generally extremely competent use of all metacognitive strategies.

4.2.

DATA ANALYSIS

This section uses data and children's responses presented in the previous section to identify themes and build portraits of each year level. A summary of metacognitive aspects is then presented followed by a descriptive analysis of trends.

4.2.1.

IDENTIFICATION OF METACOGNITIVE THEMES

THEMES:

YEAR 2

Most students perceived themselves positively as mathematical problem solvers enjoying thinking and counting. They realised older children would find it easier to solve problems. They believed they had difficulty solving and thinking about the problem because thirty was a big number and the second part of the question was difficult. Observation suggests they all coped effectively with thirty as a number but did not have the strategies to solve the problem. They found it hard to separate 'thinking' and 'solving'.

The students believed problem solving was important because of their need to know how to do problems and learn. In general they were unable to separate or differentiate aspects of the problem to suggest similar or different problems.

The students' knowledge of ways to approach the problem stressed overall thinking, no segmenting and basic operations. They were aware of thinking about numbers in their heads. The most efficient metacognitive strategy they employed was orientation with most of the children either partially or completely comprehending, partially analysing the problem and partially selecting a strategy to understand the problem. Organisation, execution and verification strategies were either not or partially understood and/or implemented.

YEAR 4

Most students perceived themselves positively as mathematical problem solvers for a variety of reasons, such as liking maths, having fun, the task wasn't too easy or hard. In contradiction to this they thought other children were probably better. They found it hard to think and solve because they thought the problem was hard believing the difficulty lay in the numbers not their strategies.

The students believed problem solving was important because it can help you pass tests, learn tables, find jobs and learn the proper way. Suggestions of similar problems suggested an emphasis on the number and animals, while different problems focused on different symbols.

The students' knowledge of ways to approach the problem stressed thinking and thinking out what to do. They were developing understanding of breaking the problem into sections. Rather than an awareness of different strategies, they suggested using different tools such as calculators, computers and writing down. Different children used different strategies such as splitting the problem in half, counting in twos, analysing what kind of problem it was, but as a year level they were not yet aware of all the strategies involved in problem solving or the interaction of those strategies. The most efficient metacognitive strategies employed were orientation strategies with most of the children comprehending, nearly half analysing the problem and most children partially able to select a strategy to understand the problem. Organisation, execution and verification strategies were predominantly partially understood and implemented. Children in this year level were reflecting on their strategies but they did not abandon non-productive strategies.

YEAR 6

Most students perceived themselves positively as mathematical problem solvers enjoying the problem because it was fun, easy, interesting and a challenge. They realised some children would be better or worse depending on their problem solving abilities suggesting speed and different strategies or processes as reasons. They found the problem easy to think through and divide it into parts. They found solving the problem easy because they knew what to do.

The students believed problem solving was important because of the need to be able to solve problems. Suggestions of similar problems suggested an emphasis on problems of a similar problem solving nature. Suggestions of different problems emphasised different word problems that required a different form of problem solving.

The students' knowledge of ways to approach the problem stressed seeing the problem as a whole, breaking it up and working out bits at a time. They suggested trying other ways of solving, opening up the mind and visualising. Most of the children in this group (7 out of 8) successfully understood and implemented orientation, organisation, execution and verification strategies.

4.2.2

SUMMARY OF METACOGNITIVE ASPECTS

A summary identifying and describing metacognitive strategies and awareness is presented on the following page in Table 4 .
The table shows an overview of metacognitive aspects.

TABLE 4

A SUMMARY OF METACOGNITIVE ASPECTS

	Year 2	Year 4	Year 6
Person Awareness	Positive perception of self as a thinker and counter	Positive perception of self in relation to basic maths skills. Thinks other children are better	Positive perception of self as a problem solver. Realistic perception of others as better or worse depending on strategies used.
Task Awareness	Problem solving perceived as a way to know and learn	Problem solving perceived as a way to: pass tests, learn tables; learn the proper way and find jobs	Problem solving perceived as important in itself, i.e. the need to be able to solve problems
Strategy Awareness	Just 'thinking' and basic operations as ways to approach the task.	Thinking out what to do and use of different tools (calculators etc) rather than using different approaches. Developing understanding of breaking problem into parts.	
Orientation Strategies	Developing comprehension. Partial analysis of problem. Partial selection of strategy to understand	Most students comprehending the problem. Nearly half the group analysing the problem	
Organisation strategies	Children either partially able or unable to identify goal and plan a course of action.	Children predominantly partially able to identify goal and plan a course of action.	All students except one exhibited behaviour and/or explained understanding and implementation of orientation, organisation, execution and verification strategies
Execution Strategies	Children either partially able or unable to select a strategy and monitor progress	Children predominantly partially able to select a strategy and monitor progress (with 2 children implementing monitoring strategies)	
Verification Strategies	Children either partially able or unable to reflect on and evaluate, orientation, organisation and execution	Children predominantly partially able to reflect on and evaluate orientation, organisation and execution. While children did reflect, they did not abandon unproductive strategies	

4.2.3
DESCRIPTIVE ANALYSIS OF TRENDS

The preceding section identified metacognitive aspects. The research questions ask which aspects undergo development and whether there are developmental trends in metacognitive strategies and awareness. The preceeding data suggests a number of trends, this section will describe these trends.

Person awareness:

The children in year two perceived themselves as thinkers and counters, whereas the year four children perceived themselves in relation to basic mathematics skills, and by year six children had a realistic perception of self in comparison to others (some will be better, worse or use different ways) and a positive perception of self as a problem solver. This suggests a trend developing from a perception of self and others that is grounded in basic mathematical skills to a perception of self and others that is grounded in a belief of complex problem solving skills. The children progressed from an egocentric perception of oneself to a perception about oneself in relation to others.

Task awareness:

The children in year two perceived problem solving as a way to know and learn, whereas the year four children perceived it as a way to pass tests, learn tables and find jobs. By year six, children believed problem solving was important in itself. This suggests a

trend developing from a focus on problem solving for learning to problem solving for specific purposes to problem solving as worthwhile in itself.

Strategy awareness

The children in year two focused on just 'thinking' and basic operations as ways to approach the task, whereas by year four children were thinking out what to do emphasising different tools not different strategies, and by year six the children were emphasising different ways and strategies to approach the task and seeing the problem as a whole and in parts. This suggests a trend developing from strategies that focus on basic operations to a focus on using different tools to a focus on problem solving strategies.

Metacognitive strategies

From Table 3 and Figures 1 to 11 it can be seen that strategy use develops gradually. In general, year two children's comprehension strategies were ahead of their other strategies. Other strategies were either partially or not implemented and/or understood. The year four children were progressively developing metacognitive strategies with most children partially able to implement and/or explain understanding of the strategies.

By year six all except one child exhibited behaviour and/or explained thinking that showed complete understanding and implementation of orientation, organisation, execution and verification strategies.

Although there was a fairly even general strategy development there were some aspects that were predominant. Data suggests the younger children did not have the range of control, awareness of, or ability to implement many strategies, this seems to progressively develop. The year six children were able to look at the problem as a whole and differentiate parts of the problem, this was only beginning to develop in the year four children who generally saw only the two main parts to the problem. The children's awareness of their thinking and/or their ability to explain and differentiate aspects of their thinking progressively developed, with year two children explaining they just 'think', year four children explaining they think what to do with which method and year six children explaining their thinking using a wide perspective looking at the whole and differentiation between parts of the problem and strategies used.

Based on the above analysis Table 5 builds on Table 4 using arrows to indicate developmental trends of students' metacognitive aspects.

TABLE 5
INDICATION OF DEVELOPMENTAL TRENDS

<table>
<tr><th></th><th>Year 2</th><th>Year 4</th><th>Year 6</th></tr>
<tr><td>Person Awareness</td><td>Positive perception of self as a thinker and counter →</td><td>Positive perception of self in relation to basic maths skills. Thinks other children are better →</td><td>Positive perception of self as a problem solver. Realistic perception of others as better or worse depending on strategies used.</td></tr>
<tr><td>Task Awareness</td><td>Problem solving perceived as a way to know and learn →</td><td>Problem solving perceived as a way to: pass tests, learn tables; learn the proper way and find jobs →</td><td>Problem solving perceived as important in itself, i.e. the need to be able to solve problems</td></tr>
<tr><td>Strategy Awareness</td><td>Just 'thinking' and basic operations as ways to approach the task. →</td><td>Thinking out what to do and use of different tools (calculators etc) rather than using different approaches. Developing understanding of breaking problem into parts. →</td><td rowspan="5">All students except one exhibited behaviour and/or explained understanding and implementation of orientation, organisation, execution and verification strategies</td></tr>
<tr><td>Orientation Strategies</td><td>Developing comprehension. Partial analysis of problem. Partial selection of strategy to understand →</td><td>Most students comprehending the problem. Nearly half the group analysing the problem →</td></tr>
<tr><td>Organisation strategies</td><td>Children either partially able or unable to identify goal and plan a course of action. →</td><td>Children predominantly partially able to identify goal and plan a course of action. →</td></tr>
<tr><td>Execution Strategies</td><td>Children either partially able or unable to select a strategy and monitor progress →</td><td>Children predominantly partially able to select a strategy and monitor progress (with 2 children implementing monitoring strategies) →</td></tr>
<tr><td>Verification Strategies</td><td>Children either partially able or unable to reflect on and evaluate, orientation, organisation and execution →</td><td>Children predominantly partially able to reflect on and evaluate orientation, organisation and execution. While children did reflect, they did not abandon unproductive strategies →</td></tr>
</table>

CHAPTER 5
CONCLUSION AND DISCUSSION

The aim of this chapter is to firstly; draw conclusions, examine the research questions and provide a summary. Secondly, this chapter will discuss the findings of the present research and compare the results with previous research. Thirdly, this chapter discusses the limitations of the research and finally, this chapter discusses implications for the classroom and future research.

5.1
CONCLUSIONS

The aim of this research was to identify metacognitive aspects and investigate the relationship of metacognition and age in the context of solving a mathematical problem.

The orientation statement suggested the possibility of a developmental sequence to metacognitive awareness and strategies used for mathematical problem solving. The three 'year level' groups described in this research show clear differences in the development of metacognitive awareness and strategies. This research has substantiated this view through identifying metacognitive aspects and investigating the relationship of metacognition and age in the context of solving a mathematical problem. Descriptive analyses built up 'portraits' of each year level and identified and described metacognitive themes. Based on this

data and description, metacognitive trends were identified and discussed.

5.2 EXAMINATION OF RESEARCH QUESTIONS

The descriptive analysis provided an insight into the thought processes and metacognitive strategies children used. Table 5 indicates students' trends in the development of metacognitive aspects. Both inferred and verbal evidence of these were cited in the data.

Is there a developmental trend in metacognitive awareness evident in children between Years 2 - 6 in relation to mathematical problem solving?

This study suggests there is a developmental trend in person, task and strategy awareness.

Is there a developmental trend in metacognitive strategies evident in children between Years 4 -6 in relation to mathematical problem solving?

This study suggests there is a developmental trend in metacognitive strategies.

What aspects of metacognition undergo development during Years 2 - 6?

This study suggests all aspects of metacognition examined in this study underwent development, although some strategies such as comprehension strategies were already implemented and understood in Year 2. It is impossible to tell from this research whether this development is integrated and how this development occurs. It could be separate, combined, general, slow, in a specific order or in timed jumps.

5.3 SUMMARY

In sum, this research supports the view that there is a developmental trend of metacognitive aspects in relation to mathematical problem solving. In addition, results from this research suggest it is possible to describe and identify metacognitive aspects in relation to mathematical problem solving.

5.4
DISCUSSION OF FINDINGS AND COMPARISON WITH PREVIOUS RESEARCH

The descriptive data from this research did suggest there was a developmental trend to metacognitive awareness and strategies in relation to mathematical problem solving. This research distinguished older children from younger children on the basis of metacognitive development in relation to mathematical problem solving, suggesting metacognitive strategies and aspects developmentally advance and change with age.

Researchers who have investigated mathematical problem solving suggest metacognition influences problem solving competence (Garofalo and Lester, 1985; Schoenfeld, 1983, 1985a). The data from this study suggests that deficiencies in metacognitive aspects were caused by lack of metacognitive development, not failure to use metacognitive aspects. The descriptive analysis provided an insight into the thought processes, metacognitive awareness and metacognitive strategies that children of different year levels were able to understand and implement. The description of metacognitive development year level themes and trends provided a portrait of different 'year level' children.

Direct comparison between the results of this study and previous studies is difficult. Previous studies have focused on expert and novice differences and identification of experience related differences rather than age or developmental themes and trends of metacognition in relation to mathematical problem solving.

Perhaps the most significant patterns emerging from the present and previous research are from a developmental perspective. Whilst no studies have studied the developmental trends of metacognition in relation to mathematical problem solving, Flavell (1992) and Siegler (1991) have suggested cognitive and metacognitive abilities developmentally advance and change with age, and the present research also suggests metacognitive problem-solving strategies developmentally advance with age. Brown's (1987) and Flavell et al's (1993, 1995) research highlights developmental differences in children's awareness and introspection of their thinking suggesting the ability to be aware of and reflect on one's thinking and the ability to regulate and control thinking develops over the same time period as Piaget's stage of formal operations. A similar progression of consciousness and self regulation was shown in the present research with similarities and patterns emerging in relation to: the progression from an egocentric perception about oneself to a perception of oneself in relation to others; an awareness of just 'thinking' with a narrow frame of reference to an awareness of thinking about the whole problem and strategies using a wider frame of reference and; a progression from either partial and no reflection, control and monitoring to efficient implementation of these strategies.

In summary, there are similarities between the present and previous research and a number of developmental patterns and trends appear to be emerging.

5.5
LIMITATIONS OF THE RESEARCH

Reliability and validity

The classification of behaviours may be unreliable. Interater reliability measures for each of the instruments used in the study could be determined. Similarly, the validity of the classification scheme could be considered by comparing results across the range of instruments used to measure metacognitive aspects.

Complexity of factors

The relationship between age, metacognition and problem solving is a complex one in which many factors may influence the outcome. This research has investigated only a few of these factors, other factors that may influence this relationship are gender, culture, socio-economic background, ability and degree of problem-solving experience.

Metacognition diagnosis problem

Flavell (1992) discusses the formidable diagnosis problem and the difficulties in characterising, observing and inferring children's metacognition. Two significant limitations of 'think aloud' problem solving are: the task or stress may distort metacognitive aspects, and students may be unable to report or describe metacognitive aspects during problem solving (Goos, 1994). Pressley and Harris (1990) and Brown (1987) have suggested younger children are unable to verbalise their thinking.

Sample

The sample used in this research is not necessarily a representative sample of children from this age range. It would be difficult to generalise from the results of this research due to the limited size of the sample used and the fact that all students came from one inner city school.

5.6

IMPLICATIONS FOR THE CLASSROOM

The results of this research in combination with theories cited in the literature review suggest that teachers should consider metacognitive development when making decisions about problem-solving experiences and activities. Piaget stressed children's cognitive structures determine what they notice and how it is interpreted. If there is a developmental trend to metacognitive structures this may determine what children notice and interpret when presented with problem-solving activities. Silver (1985) suggests metacognition is the driving force in problem solving. If developmental trends are driving children's problem solving behaviour, they may be assisting or interfering with the children's ability to problem solve.

With the current focus on problem solving in curriculum documents, further study on the role of metacognition in problem solving may allow for more appropriate curriculum development.

Further research in this area may also enable teachers to link problem-solving instruction, activities and experiences to the developing needs of children. "Year level' portraits and identified themes and trends could assist teachers and parents to understand children's needs in this area and may provide realistic expectations in regard to assessment.

5.7
FUTURE RESEARCH

Perhaps future research could increase generalisability by using a larger group and broader range in order to examine the developmental trends identified here. Also, it would be beneficial to examine a range of classroom situations.

Chi and Glaser (1980) suggest children can function at higher developmental levels if they have acquired expertise in that field. Future research could look at ways to help students develop metacognitive aspects they are lacking and whether instruction, activities and/or learning experiences can assist this developmental sequence and increase problem-solving metacognitive aspects. Always remembering as Piaget suggested, children construct their own understandings, children's cognitive structures dictate what they accommodate to and how it is assimilated (Flavell, 1992). It is not the experiences or activities that influence children's metacognitive development but the way these are interpreted.

Currently there is no integrated view of metacognitive development in relation to problem solving or learning. Further research could

examine the timing and interaction of developmental aspects. With many teachers emphasising and encouraging problem solving, ‘learning how to learn’ and ‘thinking about thinking’ skills, it is important that future research should concentrate on developing or examining some coherent integrated view of metacognitive development to ensure a framework for assisting the development of these important skills.

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APPENDIX 1

Date Time: start finish

First Name..*Class*.................*Date of birth*........................*Boy/Girl*

....

PROBLEM

There are some chickens and sheep in a paddock.

There are 30 eyes.

How many sheep and chickens are there?

How many legs are there?

....

Please think aloud and show how you did it.

APPENDIX 2

OBSERVATION CHECKLIST

METACOGNITIVE STRATEGIES

	Yes	No	Partial
ORIENTATION			
Comprehension			
Analysis of problem			
Selection of strategy to understand the problem			
ORGANISATION			
Identifies goal			
Plans a course of action			
EXECUTION			
Selection and performance of appropriate strategy			
Monitoring progress and execution			
VERIFICATION			
Evaluation of orientation			
Evaluation of organisation			
Evaluation of execution			
Reflect on, revise and abandon nonproductive strategies			

(Garofalo and Lester, 1985)

NOTES

APPENDIX 3

OBSERVATION NOTES

METACOGNITIVE AWARENESS

Person: beliefs about self in relation to problem solving.

Task: information about the task in relation to problem solving.

Strategy: knowledge of ways to approach the task in relation to problem solving.

(Flavell, 1979)

STUDENTS' COMMENTS ON THEIR THINKING

CLARIFICATION NOTES ON 'THINK ALOUD'

APPENDIX 4

METACOGNITIVE AWARENESS

INTERVIEW QUESTIONS

PERSON

Did you like solving the problem? Why?

Do you think some children were better/worse at solving the problem? Why?

Did you find it easy or hard to think about the problem? Why?

Did you find it easy or hard to solve the problem? Why?

TASK

What would be a similar problem?

What would be a different problem?

STRATEGY

What do you have to do to solve these sorts of problems? How and why?

Is there only one way to solve problems?

(Herrington, 1992)