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Minimizing the Number of Constraints for Shared Backup Path Protection (SBPP) in Shared Risk Link Group (SRLG) Optical Mesh Networks

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Abstract—Path-arc Integer Linear Programming (ILP) models for Shared Backup Path Protection (SBPP) in optical mesh networks generally contain many redundant constraints for calculating shared backup capacity. This greatly increases the computational time of the ILP solvers. In this paper, we first identify the sharing relationship between working and backup capacities, which facilitates the development of two novel algorithms for minimizing the number of constraints in ILP models for SBPP in mesh networks. Next, we consider the more realistic case of Shared Risk Link Group (SRLG) networks, where some optical fibers have the same risk of a physical cut due to being bundled in the same conduit. We propose a path-arc ILP model for SBPP in SRLG networks and minimize the number of constraints in this ILP model using the proposed algorithms. Simulation results show a remarkable reduction of around 50% in the number of constraints, which significantly improves the computational complexity of the model.

I. INTRODUCTION

Protection against network failures has been recognized as a critical issue in today’s telecommunication networks, especially in optical Wavelength Division Multiplexing (WDM) mesh networks. In order to provide 100% protection against network failures, a sufficient amount of spare capacity is required. Furthermore, optimizing the capacity utilization (working + backup capacity) is an important objective due to economic consideration. Amongst protection mechanisms, research has shown that path protection schemes offer better capacity efficiency than link protection schemes, and shared protection schemes offer better capacity efficiency than dedicated protection schemes [1], [2], [3]. In addition, protection design in WDM networks has been proven to be NP-hard [4]. There is a tradeoff between the optimality of the solution and the computational time. Optimal solutions can be obtained from ILP models [1], [2], [5], but computational time can be unacceptably high. In contrast, heuristic approaches with polynomial computational time can only offer near-optimal solutions. In this paper, we investigate the problem of capacity design for protection in WDM networks, with particular attention to ILP models for SBPP against conduit failures.

An ILP model for a SBPP scheme can take two different approaches, i.e using link-flow or path-arc models. A link-flow model [6] is based on link indicator variables and can offer absolute optimal solutions, but the variable complexity and the constraint complexity is high. Path-arc approaches [1], [2], [6], on the other hand, are based on path indicator variables representing the traffic volume carried on path candidates. The optimality of solutions in path-arc models depends on the quality (the minimization and the diversity) of sets of candidate routes. Current approaches to path-arc ILP models for SBPP are based on the general formulation to calculate sufficient spare capacity for a given traffic pattern. This leads to a high redundancy of constraints, which slows down the speed of the ILP solvers. In addition, in realistic WDM networks, many fiber cables may be bundled in one conduit to reduce the construction cost. Such networks are called Shared Risk Link Group (SRLG) Networks and a set of bundled fibers is called a Shared Risk Link Group (SRLG). In this paper, we extend the SBPP for traditional graph networks in [5] to SRLG networks. We then propose a method to minimize the number of constraints, followed by a new ILP model based on that method.

The rest of this paper is organized as follows. Section II outlines the background of protection and reviews approaches to SBPP in mesh networks. A path-arc ILP model for joint optimization of SBPP under SRLG networks is developed in Section III. In that section we also develop a method to reduce the number of constraints in the path-arc ILP model, and then propose an ILP model with minimum number of constraints. Finally, Section IV presents and analyzes our simulation results.

II. BACKGROUND AND RELATED WORK

SBPP has been studied extensively in [1], [7], [2], [8], [5], [9], [10], [11]. Since this is an NP-complete problem, heuristic approaches are employed to obtain near-optimal solutions in polynomial times [5], [9], [10], [11]. However, in this paper, we focus on the ILP models which can offer optimal solutions. We analyse these models and identify the redundancies in them. From these, we propose a method to optimize the number of constraints. Generally, there are two forms of ILP models for SBPP, i.e link-flow and path-arc ILP models.
A. Link-Flow ILP Model for Protection

A link-flow ILP model [6], [9], [10] is based on the use of link indicator variables to determine working and backup routes between the end-nodes of a traffic connection request. A link indicator variable for a connection at a network link indicates whether the connection uses a wavelength channel on that link or not. An ILP model for traditional graph networks can be found in [6] and a model for SRLG networks can be found in [9], [10]. Generally, there are two indicator variables representing whether a working or a backup route of a connection traverses on a link. One variable represents the flow in the forward direction of the link and the other represents it for the reverse direction. In total, in a network of $|E|$ links, there are $4|E|$ indicator variables to represent the flow of only one required traffic connection. This number will be multiplied with the number of traffic connections required. The advantage of this model is its ability to provide the exact optimal solution. However, the number of variables required is very high, and hence it is only applicable to very small networks. In this paper, we shall investigate an alternative ILP model for protection, called the path-arc model.

B. Path-Arc ILP Model for Protection

Path-arc ILP models were introduced as an alternative to link-flow models in order to reduce the number of variables, while still maintaining the optimality of the solution. This model requires a number of pre-determined routes, being candidates of each traffic connection. Hence, the number of variables in this model depends on the number of candidate routes which also affects the optimality of the solutions. Theoretically, this model does not always offer the exact optimal solution, except when all distinct routes between the end-nodes of a connection are designated as candidates for that connection, in which case it will revert to the problem of large number of variables in the model. However, the number of candidate routes required for each connection in moderate size networks is not necessarily very high. For example in [12], each traffic demand requires on average 5 candidates to obtain 100% optimal solutions in networks from 20 to 50 nodes with nodal degrees of 3, 3.5, and 4.

Path-arc ILP formulation applied to dedicated protection schemes [1], [2] is simple. The selection constraint and the upper bound capacity constraint at each link are simple. The constraints for the working and the spare capacity are formulated using the same principle since working and spare capacities are allocated in the same manner. However, in SPBPP schemes, the model for calculating the spare capacity is much more complicated. This is due to the complexity of sharing methods. A general method for modeling the spare capacity can be found in [2], [5] using different classes of candidates. In [2], there are two independent sets of candidates for each connection, one is for working routes and another is for backup routes. On the other hand, [5] employs sets of disjoint route pairs as candidates for each connection. The complexity of modeling spare capacity, as analyzed in the next section, is $O(|E|^2)$, where $|E|$ is the number of network links. This is the main factor affecting the computational time of the Linear Programming (LP) solvers, and hence that of the ILP solvers. In fact, the actual number of constraints for spare capacity is $|E|\times(|E|-1)$ which does not depend on the number of traffic connections and the diversity of candidate routes. Consequently, it results in many redundant constraints in the model. In this paper we first propose a complete method to generate a minimum and sufficient set of constraints for spare capacity. This method removes all redundant constraints in the general method. Next, we propose another approach for modeling capacity utilization in SBPP schemes using sets of shareable/non-shareable capacity instead of a set of working/backup capacity to minimize the total number of constraints in the model. In addition, to our best knowledge, ILP models for SBPP in SRLG networks are only found as link-flow [9], [10]. In this paper, a path-arc model in SRLG networks is introduced and then the number of constraints in this model is minimized using the same method as used for traditional graph networks.

III. Minimum Number of Constraints for Shared Backup Path Protection

In this section, we first extend the SBPP ILP model in [6] proposed for traditional graph networks to SRLG networks. We point out the redundancy of constraints for calculating the shared backup capacity in this model. Consequently, we propose a method for obtaining the minimum number of constraints for spare capacity and introduce a new ILP model with a minimum number of constraints.

A. Notations:

- $G(V, E, R)$: The physical topology of a network, where $V$ is the sets of $|V|$ network nodes, index $v$, $E$ is the set of and $|E|$ network links, index $e$ and $R$ is the set of $|R|$ SRLGs in the network, index $r$.
- $T$: The set of $|T|$ traffic demands, index $t$.
- $d_t$: The volume of demand $t$.
- $p_t^k$: The $k$th candidate of demand $t$.
- $b_{r, e}^t$: The indicator constant, set to 1 if the working path of $p_t^k$ uses link $e$, or 0 otherwise.
- $b_{r, bac}^t$: The indicator constant, set to 1 if the backup path of $p_t^k$ uses link $e$, or 0 otherwise.
- $a_{r, e}^k$: The indicator constant, set to 1 if the working path of $p_t^k$ passes on SRLG $r$, or 0 otherwise.
- $\delta_t^k$: The decision variable indicating the volume of traffic demand $t$ carried on candidates $p_t^k$.
- $w_e$: The variable indicating the number of working channels on link $e$.
- $s_e$: The variable indicating the number of spare channels on link $e$.

B. An ILP Model for SBPP in Shared Risk Link Group Networks

We now present an ILP model for joint SBPP in SRLG networks. In fact, this is an extension of the typical ILP models from traditional graph networks [5]. Finding disjoint path-pairs
between two node in SRLG network has proven to be NP-complete [13]. However, this is outside the scope of this paper, and instead, the sets of disjoint path-pairs between end-nodes of traffic demands are predetermined and employed as eligible candidates.

- **Objective:**

  Minimize: \( \sum_{e \in E} (w_e + s_e) \) \hspace{1cm} (1)

- **Constraints:**

  \( \sum_{k=1}^{K_e} \delta^k_t = d_t \quad \forall t \in T \) \hspace{1cm} (2)

  \( w_e = \sum_{t \in T} \sum_{k=1}^{K_e} b_{t,r}^e \delta^k_t, \quad \forall e \in E \) \hspace{1cm} (3)

  \( s_e \geq \sum_{t \in T} \sum_{k=1}^{K_e} b_{t,r}^e \delta^k_t, \quad \forall r \in R, \forall e \in E: e \notin R \) \hspace{1cm} (4)

  \( w_e + s_e \geq W_e, \quad \forall e \in E \) \hspace{1cm} (5)

  \( \delta^k_t = \{0,1,\ldots,d_t\}, \quad \forall t \in T \) \hspace{1cm} (6)

Objective function (1) minimizes the total capacity utilization. The selection constraint (2) ensures sufficient routes and volume for each traffic demand. Constraints (3) and (4) ensure that sufficient capacity is available for working and backup routes. The capacity constraint in (5) enforces the upper limit on the number of wavelength channels used on each fiber link. We refer to this model as T-SBPP.

**C. An Improved ILP Model with Minimum Constraints**

We analyze circumstances which may occur in calculating backup capacity in constraints (4), shown in Fig. 1. For the sake of simplicity, we assume a SRLG group in the example contains only one link. We then generalize to SRLGs containing more than one link. Assume that we are modeling the spare capacity \( s_e \) at link \( e \) on which backup routes of candidates \( p_{i,t} \in [1,2,3] \) traverse, i.e. \( b_{t,bk}^e = 1 \).

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{backup_capacity_c1}
  \caption{Shared backup capacity calculating}
  \end{figure}

The constraint for each set of primary working routes traversing on link \( e_1, e_2, e_4 \) and \( e_5 \) are as follows:

\[
\begin{align*}
  s_3 & \geq \delta_1 + \delta_2 \\
  s_3 & \geq \delta_3 \\
  s_3 & \geq \delta_1 + \delta_2 + \delta_3 \\
  s_3 & \geq 0
\end{align*}
\] 

Clearly, constraints (7) and (8) can be removed from the model since they are contained by the constraint (9). In addition, constraint (10) can also be removed because of the spare capacity’s lower bound constraint. As a result, constraint (9) is sufficient for calculating the spare capacity at link \( e \):

- A backup route which can not be shared with any backup routes at a network link only belongs to the constraint having the maximum number of elements.
- A backup route sharing with all other backup routes at a network link becomes a constraint for spare capacity. We refer to such backup candidate routes as **fully-shareable backup candidates** at that link.

Based on these conditions, we propose an algorithm (Algorithm (1)) which provides the minimum and sufficient set of constraints for calculating the spare capacity at a network link.

**Algorithm 1 Finding constraints for SBPP in SRLG networks**

**Require:** A SRLG network \( G(V, E, R) \) and pairs of candidates \( P_t \).

**Ensure:** Set of constraints \( C_e \) for unshared backup capacity at link \( e \);

- \( S_e \leftarrow \text{Set of candidates } p_e^k \text{ of which } b_{t,bk}^e = 1 \); \( C_e \leftarrow \emptyset \);

  **for** (each group \( r \in R \) which does not contain \( e \)) **do**

  - \( S_r \leftarrow \text{Candidates } p_t^k \text{ whose backup routes on } r \);
  - \( S_{re} \leftarrow S_r \cap S_e \);
  - **if** \( (S_{re} \notin e) \text{ in } C_e \) **then**
  - Remove constraints being subset of \( S_{re} \text{in } C_e \);
  - Add \( S_{re} \text{in } C_e \text{as a new constraint};

  **end if**

**end for**

This algorithm only results in a set of constraints for which the number of elements is equivalent to or larger than 2, i.e these constraints contain candidates for which backup routes cannot be shared. Fully-shareable backup candidates at link \( e \) are determined as candidates not belonging to any constraints returning from Algorithm (1). The time complexity of Algorithm (1) is \( O(|R|) \). In order to determine the constraint for all links in the network, the time complexity will be \( O(|E| \times |R|) \). Let

- \( C_e = \{ c_{e,1}, \ldots, c_{e,D_e} \} \) be the set of constraints resulting from Algorithm (1), where \( D_e \) is the number of constraints at link \( e \).
- \( c_{e,m} \) presents candidate \( p_t^k \) in constraint \( e_{m} \).
- \( Q_e = \{ q_e^k \} \) is the set of fully-shareable candidates \( p_t^k \) at link \( e \). This contains the set of candidates for which backup routes traverse on link \( e \) and do not belong to any constraints \( C_e \) resulting from Algorithm (1).

The spare capacity constraint at each link \( e \) is now modeled as:

\[
\begin{align*}
  s_e & \geq \sum_{c_{e,m} \in C_e} \delta_{e,m}^k, \quad \forall c_{e,m} \in C_e \\
  s_e & \geq \sum_{q_{e,k}^k \in Q_e} ^k \delta_{e,k}^k, \quad \forall q_{e,k}^k \in Q_e
\end{align*}
\] 

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The number of constraints in this model depends on the relationship between the working/backup routes of the given traffic pattern. However, in the worst case, this equals to the number of constraints in T-SBPP. For convenience of discussion from here after, we refer to this proposed ILP model as WB-SBPP (Working/Backup - Shared Backup Path Protection).

D. Further Reducing the Number of Constraints for SBPP ILP Model

We observe in the WB-SBPP model that there are some backup candidates on a network link which are not necessary to be shared with the remaining backup candidates traversing that link. The bandwidth for such backup candidates must be dedicated without any sharing. In other words, the constraint for the capacity of these backup candidates is exactly the same as that for the working capacity. Hence, it is possible to combine the backup candidates into the working capacity constraint and thus reduce the number of constraints in the entire model. In this part, we propose an ILP model based on a different classification of capacity utilization, i.e., sharable and non-sharable capacity (rather than working and backup (spare) capacity). We refer to the model as SNS-SBPP (Sharable/Non-Sharable SBPP). The capacity utilization can be classified as follows:

Algorithm 2 Shared Group Algorithm (SGA)

Require: A SRLG network \( G(V,E,R) \), set of candidates;
Ensure: Set of shareable/non-shareable candidates \( (H_e, \hat{H}_e) \) at link \( e \);

\( H_e \leftarrow \emptyset; \hat{H}_e \leftarrow \emptyset; \)

\( S_e \leftarrow \) set of backup candidates using link \( e \);

\( n \leftarrow |S_e|; A \leftarrow \text{matrix}([(1) - [1])] \) of \((n \times n)\);

for (each group \( r \in R \) which does not contain \( e \)) do

\( S_r \leftarrow \) set of working candidates on group \( r \);

\( S_e \leftarrow S_e \cap S_r \;

for (each \( s_m \in S_e \)) do

\( A(s_m, S_e \setminus s_m) \);

end for

end for

for (each row \( m \) of matrix \( A \)) do

if \((A(m,k) = 0, \forall k = 1 \ldots n)\) then

\( H_e \leftarrow H_e \cup S_e(m); \)

end if

end for

\( H_e \leftarrow S_e \setminus \hat{H}_e; \)

- Any working capacity unit of a candidate is non-sharable.
- If the backup capacity of a candidate is able to share with the spare capacity of at least another candidate, then this capacity is sharable capacity, otherwise it is non-sharable capacity.

Algorithm (2) results in sets of shareable \((H_e)\) and non-shareable \((\hat{H}_e)\) candidates for backup capacity at link \( e \). The matrix \( A \) in the algorithm stores the shareable information for each backup candidate at link \( e \) to the remaining backup candidates at that link. A candidate is said to be sharable at a link if it is able to share with at least one backup candidate at that link. Otherwise, it is in the set of non-sharable candidates \( \hat{H}_e \). In implementation, Algorithm (1) and Algorithm (2) can be combined together. However, in this paper we keep them separate for clarity of discussion. The set of shareable candidates \( H_e \) is now considered as the set of backup candidates at link \( e \) in Algorithm (1) to determine the set of constraints \( C_e \). Some further notations are defined as follows:

- \( H_e \) is set of candidates \( p_{tk}^e \) that are sharable at link \( e \).
- \( \hat{H}_e \) is set of candidates \( p_{tk}^e \) for which backup routes are non-sharable at link \( e \).
- \( h_e \) denotes the sharable capacity at link \( e \).
- \( \hat{h}_e \) denotes the non-sharable capacity at link \( e \).

Given this information, the objective function of the model is presented as follows:

\[
\text{Minimize: } \sum_{e \in E} (h_e + \hat{h}_e) \tag{12}
\]

The selection constraint (2) and the integer constraint (6) are the same as the T-SBPP model. We introduce two constraint formulations for sharable and non-sharable capacities instead of working and backup capacities, given as:

- Non-sharable capacity constraint:

\[
\hat{h}_e = \sum_{p_{tk}^e \in \hat{H}_e} k_{t} \cdot \delta_{tk}^e + \sum_{p_{tk}^e \in H_e} k_{t} \cdot \delta_{tk}^e, \quad \forall e \in E \tag{13}
\]

- Sharable capacity constraint:

\[
h_e \geq \sum_{c_{e,m} \in C_e} k_{e,m} \cdot \delta_{tk}^e, \quad \forall e, m \in \sum_{C_e} \quad \forall e \in E \tag{14}
\]

It is worth noting that the set of constraints for sharable capacity is similar to those resulting from Algorithm (1), except that all non-sharable candidates are removed, thus reducing the number of constraints in the model.

The objective function (12) minimizes the total sharable and non-sharable capacities. Constraints (13) and (14) determine the non-sharable capacity and sharable capacity utilized at each network link respectively.

IV. SIMULATION RESULTS

In this section, we first examine the complexity of the proposed models, and the efficiency improvements of WB-SBPP and SNS-SBPP over T-SBPP in SRLG networks. We then analyze the capacity efficiency for SBPP in SRLG networks. We note that under the same network environment (network configuration and traffic pattern), all proposed ILP models result in the same optimal solutions i.e. optimal capacity utilization.

Two test case networks are adopted from [14], as shown in Fig. 2. The first network comprises of 24 nodes, 43 links...
and 11 shared risk link groups (Net1: USA-21N36L11G), and the second network has 21 nodes, 34 links and 11 shared risk link groups (Net2: USA-24N43L11G). We employ $K = 4$ shortest pairs of disjoint paths as the set of candidates for each connection requirement. The number of required connections is varied from 1 to 400 for each test case network. For each number of traffic demands, the number of constraints is calculated from the average of 100 random patterns of traffic demands.

A. Constraint Complexity Analysis

<table>
<thead>
<tr>
<th>Table I</th>
<th>CONSTRAINT EFFICIENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>USA-21N36L11G</td>
</tr>
<tr>
<td>$d$</td>
<td>3.43 (2.95)</td>
</tr>
<tr>
<td>WB-SBPP</td>
<td>50.4 %</td>
</tr>
<tr>
<td>SNS-SBPP</td>
<td>50.4 %</td>
</tr>
</tbody>
</table>

The results are shown in Fig. 3 where the number of constraints is the sum of constraints for the working and the spare capacities in T-SBPP (constraints (3) and (4)), WB-SBPP (constraints (3) and (11)), and SNS-SBPP (shareable constraint (13) and non-shareable constraint (14)).

The constraint complexity in the T-SBPP model is not dependant on the traffic connections required. However, it depends on the nature of SRLGs. Let us consider a network of $|V|$ nodes, $|E|$ links and $|E|$ SRLGs and assume that there are $|E|^*$ links being on at least one SRLG. The constraint complexity for modeling spare capacity is $O((|E| - |E|^* + |R|) \times |E|)$. WB-SBPP and SNS-SBPP can save around 50% on the number of constraints in the test networks, being 51.4% and 50.6% for USA-21N36L11G and USA-24N43L11G respectively (Table I).

The saving in the number of constraints in SNS-SBPP is a little bit better than WB-SBPP when the number of traffic connections is low, and it is equivalent when the number of traffic connections is high, say more than 50. However, SNS-SBPP always offers better constraint complexity in any case.

B. Capacity Efficiency of SBPP under SRLG Networks

Since ILP models for T-SBPP, WB-SBPP and SNS-SBPP in SRLG networks offer the same results in terms of capacity utilization, we only need to use one of these models to investigate the capacity efficiency of SBPP under SRLG networks. In this simulation, we employ the SNS-SBPP model due to the least number of constraints involved. We use the two test-case networks as shown in Fig. 2. Two metrics of network performance, i.e. the total capacity utilization of working and spare capacities, and the redundancy (the ratio of the spare capacity over the working capacity) are measured for different values of $K = \{3, 4, 5, 6\}$. The number of traffic connections is increased from 30 to 100 in steps of 5.

All simulations use the ILP solver developed under the MATLAB environment using the branch and bound technique. We limit the running time for each simulation to 12 hours over an IBM PC, Pentium 4, 3.0Ghz with 1Gb of memory.
Numerical results are shown in Fig. 4 in which Fig. 4(a) shows the redundancy and Fig. 4(b) shows the capacity utilization of our simulations over SRLG networks. First of all, we observe that all results are equivalent for \( K \geq 4 \), and are better than when \( K = 3 \). This is plausible because more candidates offer more choices to improve the quality of the solutions. When the number of candidates is adequate for a connection, the probability of the \( k^{th} \), \( k \geq 5 \) candidate being selected is insignificant. For example, in our simulations the probability of selecting the \( 5^{th} \) or the \( 6^{th} \) candidates is 0.

The objective in all proposed joint-optimization ILP models is to minimize the total working and spare capacities allocated in a network for a given pattern of traffic connections. Fig. 4(b) shows the capacity utilization against the number of traffic demands. The capacity utilization is increased when the number of traffic connections is increased. However, the redundancy in both test networks decreases and fluctuates around 72% and 75% respectively when the number of traffic connections is high.

V. CONCLUSION

In this paper, we have identified the redundancies in the constraints of ILP models for SBPP in optical mesh networks and have proposed algorithms for minimizing the number of constraints in SBPP models. We have shown that with a little effort in pre-processing the sharing relationship between working candidates and backup candidates, we can optimize the number of constraints in the ILP models for shared backup path protection in SRLG networks. Our proposed method attempts to generate the best possible ILP models for SBPP. Simulation results show that for SRLG networks we achieve a significant reduction of about 50% in the number of constraints. While the ILP models are not applicable for large scale networks, they offer optimal solutions, and can be used as benchmarks for assessing the efficiency of heuristic approaches developed for large scale networks.

REFERENCES