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Real Interest Rates and Inflation in Norway

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Abstract

We analyse the effectiveness of the monetary stance adopted by the Norwegian central bank in implementing monetary policy assessing how it relates to the neutral rate of interest. The neutral rate is frequently defined as the level of real interest rate consistent with both stable inflation and a level of production equal to potential production; providing no stimulus or restraint to the economy. We attempt to calculate this rate and to examine the long run link between real interest rates and inflation in Norway. We apply cointegration analysis to explore whether the gap between the real and neutral rate of interest determines the growth rate of inflation in the long run. The prevailing real interest rate is the price that equalises saving and investment and this may tend to deviate from the neutral rate due to the existence of an output gap and inflation gap in the short run. Thus we seek to find equilibrium between inflation growth and the real rate gap. Our estimate of the neutral rate lies in the range 2.7 to 3.2 per cent.

Keywords: Real interest rate; Inflation; Monetary policy; Cointegration analysis
1. Introduction.

The neutral real interest rate is frequently defined as the level of real interest rates consistent with stable inflation and a level of national production equal to potential national production; providing neither stimulus nor restraint to the economy. An accurate measure of the neutral rate might assist central bank monetary policy. In this paper we attempt to calculate the neutral rate for the Norwegian economy. We assess the real interest rate gap and the long run relationship between this gap and inflation. Norway has experienced four periods of very high inflation over the past hundred years: during the two world wars; the Korean War and a fifteen year period from the first half of the 1970s to the second half of the 1980s. Norway has recently had an effective inflation containment policy and has adopted an inflation targeting regime.

The paper examines whether there is a long run path set for inflation in Norway by an analysis of the relationship between the interest rate gap and actual inflation. We ignore any short run dynamics, and the influence of external shocks. We examine if the real interest rate gap: the difference between the real interest rate and its neutral level, determines the inflation growth rate by means of cointegration analysis.

Our results suggest: in the absence of structural breaks, a stable long-run equation links the growth rate of inflation to the interest rate gap; ex ante, the Norges Bank could make use of the neutral rate in deciding its policy accommodation; ex-post the neutral rate can give an indication of the current stance and objectives of monetary policy; finally the neutral rate for Norway appears to fluctuate between 2.5 and 3.5 percent.

Our paper is divided into five sections: we follow this introductory section in section 2 with a description of the Norwegian monetary policy regime, its evolution and its objectives and review the theory relating to real interest rates, the interest rate gap and its relationship with inflation. Section three sets out our models and hypotheses to be tested plus introduces our data set. In section four we present our empirical results and provide a general discussion and conclusion in section five.

2. Norwegian Monetary policy

In Norway, monetary policy is implemented by the central bank; the Norges bank, which conducts monetary policy by setting the interest rates on banks deposits and
overnight loans in the central bank and not by changing the price of liquidity supplied through market operations. The shortest money market rates (day-to-day money), indicate the price of available liquidity and will not normally fall below Norges Bank’s deposit rate or exceed it’s overnight lending rate (Gjegrem, 1999). Hence the deposit rate and the overnight lending rate form a corridor for the shortest money market rates.

The difference between these two key rates is maintained at two percentage points on an annual basis. Thus the sight deposit rate is the banking system’s marginal rate and the key policy rate in the central bank’s conduct of monetary policy. The banks use of an interest rate corridor is distinctive in that market operations take place at rates near the floor of the corridor. The effect of the asymmetrical corridor is to reduce the incentive to redistribute liquidity in the interbank market, because the deposit rate is normally so close to the market rate that banks earn little by investing surplus liquidity in the money market. (Kran, 2001)

After abandoning a fixed exchange rate regime in 1992, the Norges Bank conducted a managed float of the Krone with no explicit stipulation of a central rate and fluctuation margins. The Norwegian inflation target at 2.5 percent is slightly higher than that in Canada (2 percent), New Zealand (between 0 and 2 percent), Sweden (2 percent) and the Euro area (between 1.5 and 2 percent). However it is inline with the target in the United Kingdom and Australia. Therefore, most of Norway’s major trading partners have inflation targets below 2.5 percent. According to Andreassen et al (2001), Norway’s higher inflation target seems to imply an expectation of a real appreciation of the Norwegian exchange rate through prices instead of nominal exchange rate adjustments.2

The Norges Bank implements a flexible inflation target regime which gives weight to other macro objectives such as the output gap and unemployment.3 The output gap is the difference between the Real GDP and Trend Real GDP. Unemployment, the percentage of the labour force involuntarily not employed is currently around three percent (Statistics Norway, 2005). It has been show by Svensson (1997) that a strategy consisting only targeting inflation but allowing for a gradual adjustment of observed

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1 This is because banks can automatically borrow liquidity from Norges bank and deposit any available liquidity with the bank.
2 In comparison with other regimes, the Norwegian inflation target would appear to have half a percent (or somewhat above) implied real appreciation into its mandate.
3 A flexible inflation target is opposed to a strict inflation target which is associated with frequent and marked interest rate changes which keep inflation under control but could lead to wide variations in output and employment.
inflation towards its target is equivalent to a strategy in which the central bank explicitly targets the output gap. Norges Bank sets its interest rate instrument with a view of achieving the inflation target over a two year horizon, and will normally tolerate deviations of actual inflation from the target not exceeding one percentage point. This means that interest rates are set with a view of achieving an inflation rate of 2.5 percent two years ahead.

Svensson (2002) describes three channels in which interest rates affect inflation and the level of economic activity in Norway. Higher interest rates make it more attractive to take Krone positions and borrow in foreign currency. As a result, higher interest rates normally lead to appreciation of the Krone and this leads to a reduction in the prices of imported goods and thus domestic inflation. Interest rates also influence inflation indirectly via domestic demand; the demand channel. In addition, changes in inflation expectations influence price and wage inflation. Enterprises do not want to change prices too often hence they take inflation expectation into account when they set wages; the expectations channel. If market participants are forward looking they will normally set interest rates on money market securities factoring in their expectations of future movements in Norges Bank’s key rates.

2.2 Interest Rates

2.2.1 The Neutral Rate

The neutral real interest rate is frequently defined as the level of real interest rate consistent with stable inflation and production equal to potential production; providing no stimulus or restraint to the economy. Given the assumption that other variables like the real exchange rate neither stimulate nor contract the economy; the neutral rate should be an effective benchmark for evaluating monetary policy. According to the Taylor rule, when the output gap is equal to zero and inflation is on target, the nominal interest rate should be set equal to the neutral rate (equivalent to real interest rate) plus inflation expectations, the latter equal to the inflation target (hence the Taylor Rule is reduced to The fisher Equation)

\[ r_n = r_f + \pi \]

By setting its target short term nominal interest rate

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4 Amoto (2005) adds that it is generally assumed that the central banks inflation target is zero.
5 The Fisher equation equates the nominal interest rate to the real interest rate plus expected inflation, that is \( r_n = r_f + \pi \)
equal to the neutral rate plus a (low positive or zero) target rate of inflation, a central bank is essentially trying to mimic the ideal conditions of an economy without nominal rigidities.

The real interest rate gap (the difference between the real interest rate and its neutral level \( r - r^* \)) provides a measure of monetary policy stance.\(^6\) The Norges Bank should set the real interest rate above the neutral rate in cyclical upturns; hence the real interest rate gap should be positive and similarly in recessions the real interest rate gap should be negative. The neutral rate is difficult to estimate and may change over time. Giammarioli and Valla (2004) provide a survey of the different estimation methods including: a simple average of historic interest rate series, stochastic dynamic general equilibrium models, or applications of the Kalman filter method.

The determinants of the neutral rate are mainly structural conditions in the economy and include: the marginal product of capital, the productivity growth rate\(^7\) the subjective discount rate of private agents, households’ saving and consumption preferences over time\(^8\), the size of public debt\(^9\), the risk premium linked to uncertainty surrounding future inflation and exchange rates\(^10\) and liquidity of the financial markets\(^11\). Bernhardson (2005) argues that some of these traditional closed economy determinants could be less relevant for some small economies. Amoto (2005) highlights three issues of practical significance: nominal wage stickiness, nominal rigidities and financial imperfections. These factors imply there might not be a clean link between the real rate gap and funding costs of households and firms to undertake investment. In light of nominal wage stickiness, Erceg, Henderson and Levin (2000) show it is desirable for monetary authorities to respond to both price inflation and wage inflation.

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\(^6\) where \( r \) represents the real interest rate and \( r^* \) represents the neutral rate of interest

\(^7\) Higher productivity growth increases demand for investments and funding. In order to raise additional savings to supply loans, savers must be offered more real return; hence the real interest rate must increase.

\(^8\) If household would like to consume less now and save more for the retirement period, they would have to accept a lower real return as lower funding costs would be the only way through which investors would be willing to increase investment.

\(^9\) The government may need to offer a higher return to entice savings from the private sector to fund its borrowing. However if the private sector decides to save more today to offset a likely increase in taxes in the future, the public deficit may be financed without an increase in real interest rates.

\(^10\) The more uncertain future inflation is, the more uncertain the ex post real return. To compensate for this inflation risk, savers may demand an additional expected return to be willing to postpone consumption. Hence the higher the inflation risk, the higher the neutral real interest rate.

\(^11\) The less liquid the bonds market, the higher the probability that savers will influence the price if they desire to sell their bonds prior to redemption. To offset this potential loss, investors may demand an additional expected real return.
The neutral rate is both a short term and long term concept. In the long run when the inflation gap and output gap are zero, the real interest rate will be equal to the neutral level. In the short run, the real interest rate will deviate from the neutral real interest rate to the extent that the inflation gap and output gap deviate from zero. Hence the real interest rate gap depends on the state of the business cycle that is, the size of the inflation gap and the output gap. Thus we will proceed to examine the long run relationship between the interest rate gap and inflation.

2.2.2 Nominal and Real Interest Rates

The real interest rate can be regarded as the price equalising saving and investment. Investment, or demand for funding, increases with a lower real rate, while saving, the supply of loans, increases with the price. Saving and investment conditions and hence the real interest rate are determined by both structural factors and monetary policy. Assuming monetary policy is neutral, we are left with only structural factors determining the real interest rate. Basically all structural changes which tend to increase investment or reduce saving, lead to higher real interest rates and vice versa.

Over the years both inflation and real interest rates have declined. Moreover, low and stable inflation is in itself an important contribution to the fall in real rates as the inflation risk premium declines. The inflation risk premium compensates investors for the risk that ex post real return may undershoot the ex ante expected one. If we focus on the last 25 years, the period from the 1980s is naturally divided into two sub-periods: a period of fighting inflation (the 1980s up to the mid 1990s) and a period of low and stable inflation (from the mid 1990s). In the former period, governments, including the Norwegian government, tightened monetary policy to fight inflation and it fell to relatively low levels, then nominal rates continued to decline, leading to lower real rates compared to the 1980s (Bernhardsen, 2005). Low and stable inflation expectations would reduce the inflation risk premium and therefore ex ante real returns. In Norway, real rates in the seventies were virtually zero, as high nominal rates just compensated for inflation. Subsequently, inflation fell gradually and stabilised at a relatively low level of 2.5 percent in the early nineties.

Nominal rates can be decomposed into three factors (1) an expected real return (or the expected real return required by investors), (2) a compensation for expected inflation, (3) an inflation risk premium due to that future inflation and hence real return are uncertain.
Bernhardsen, (2005) defines the real rate in light of risk adjusted real interest rate parity, where the domestic real interest rate is equal to the foreign real interest rate plus expected changes of real exchange rate plus a risk premium,

\[ r = r_g + (q^-q) + rp \]

where \( r \) is the domestic real interest rate, \( r_g \) is the foreign- or global real interest rate, \( q^\sim \) is the log of expected long term real exchange rate (the equilibrium real exchange rate), \( q \) is the log of the real exchange rate and \( rp \) is a risk premium. The expected change in the exchange rate is expressed by \( (q^\sim - q) \). If expected domestic real return deviates sufficiently from expected real return globally, capital movements would arise and equalise expected real return home and abroad. A risk premium arises due to uncertain exchange rates and price developments which make ex post real return uncertain at the time of investment. In the long run, where the economy has settled with inflation on target and the output gap equal to zero, it is reasonable to assume the exchange rate equals its long run value. Expected changes of the real exchange rate will be zero and the risk adjusted real interest parity is reduced to,

\[ r = r_g + rp \]

i.e., the domestic real interest rate is equal to the global real interest rate plus a risk premium. Moreover in the long run the real interest rate will equal the neutral rate \((r = r^*)\). Assuming similar long run conditions globally \((r_g = r_g^*)\), it follows that,

\[ r^* = r_g^* + rp \]

i.e., domestic neutral rate is equal to the global neutral real rate plus a risk premium. Hence the domestic neutral rate is determined by global structural factors plus country specific structural factors, the latter determining the risk premium.

Neiss and Nelson (2001) suggest the behaviour of the real interest rate is a reasonable approximation for the behaviour of the real interest rate gap, a finding supported by King and Watson (1996). Neiss and Nelson (2001) conclude that the variation in the real interest rate gap is dominated by variation in the observable component of the gap, the real interest rate, rather than the unobservable neutral rate.

\[ 13 \text{ Real interest parity is a real term extension of uncovered interest rate parity, the latter saying that nominal interest rate differentials compensate for expected nominal exchange rate changes.} \]
component.\textsuperscript{14} We proxy the changes in the interest rate gap with changes in the real interest rate in our empirical analysis.

### 2.2.3 Real interest rates and inflation

Wicksell (1898, 1901) was one of the first to describe the relationship between real interest rates and inflation. He envisaged the real rate gaps as causing secular changes in the price level. His cumulative process theory suggests changes in the price level are caused by non-zero real rate gaps similar to current New Keynesian models. Wicksell (1898, 1901) argued that if the loan and deposit rates set by banks were below the neutral rate, there would be excess demand for funds by firms to finance investment projects. Consequently the creation of liquidity by banks to absorb excess demand in the market for loadable funds would ultimately create excess money balances in the holding of households. This puts upward pressure on prices and this general price inflation ceases when, and only when, the market is brought into equality with the neutral rate. Basically, to re-establish reserves at the required rates, banks increase deposit rates to attract savings and, consequently, raise loan rates as well to maintain margins.

Amoto (2005) suggests that while banks play a crucial role in the Wicksell framework, one could suppose that it was the central bank that is determining the market rate of interest by injecting and withdrawing liquidity to manipulate the rates offered by central banks. However, because Norway’s central bank does not conduct monetary policy in this manner, this could be less evident.\textsuperscript{15}

Humphrey (1992) notes that Wicksell suggested the central bank could follow one of two different rules to contain the cumulative process. The first has the bank changing interest rates in response to inflation rising above or below (zero) target level (inflation rule). The second requires the central bank to change interest rates in response to deviations of the price level from a (constant) target path (“price-level rule). In 1995, Fuher and Moore (1995) argued that monetary policy makers had been roughly following Wicksell’s inflation rule for some time. Woodford (1999, 2000), revived the Wicksellian idea of the inflationary processes as being determined by the gap between

\textsuperscript{14} By contrast Neiss and Nelson (2001) find that the level of output is not a good indication of the behaviour of the output gap. The two series have an inverse relationship, with correlations ranging from -0.06 to -0.68, and the output gap has a standard deviation that is less than half that of output.

\textsuperscript{15} As mentioned earlier the Norges Bank uses the Deposit rate as its policy rate
the real and neutral rates of interest. Neiss and Nelson (2001) use a stochastic general equilibrium model to examine the properties of the interest rate gap as an inflation indicator. Humphrey (1997) suggests that the cumulative process put forward by Wicksell (1901) is squarely in the tradition of the quantity theory of money by saying the cumulative process was “nothing less than a full-scale extension of the [quantity] theory to account for bank deposits on the price level”. Prices rise due to the increase in money supply, which come about from loan-led deposit creation by banks to finance excess desired investment. Humphrey (1997) further suggests that even though it is the real rate gap that gets the ‘ball rolling’, price increases do not occur and would not occur without the expansion of deposits by banks. The pivot in the system is stable real money demand. As soon as the money supply changes, households attempt to change their cash holdings; price movements occur entirely through real balance effects.

In New Keynesian models, as well as in earlier neutral rate theories, price level changes are directly associated with the presence of positive or negative real rate gaps. This has lead to suggestions that inflation itself could serve as a good proxy variable for the real rate gap. (Amoto, 2005). Thus we shall now proceed to present our models which will test whether or not the growth rate of inflation is cointegrated with real interest rate gaps.

3. Theory, Data, Models and Hypotheses

3.1 Theory

In line with Brozoza-Brzezina (2001), we adopt a model that relates the interest rate gap to the inflation growth rate. Our model postulates that the gap between the real and neutral rates of interest after all lags have worked themselves out, determines the change in the growth rate of inflation rate. The long run relationship can be described by the following equation:

\[ \Delta \pi_t = \psi(r^* - r_{t-1}) \]

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16 Woodford suggests that this neo-Wicksellian analysis of price level determination also avoids the cumbersome task of estimating the implied money supply function.

17 This kind of relationship has been advocated among others by Fuhrer and Moore (1995), Henckel, Ize and Kovanen (1999) aswell as Andres, Mestre and Valles (1997).
where $\pi$ is the inflation rate, $r^*$, is the real rate of interest, $r^*$, is the neutral rate of interest. The properties of this model can be summarized in the following table:

**Insert Table 1 about here**

According to the model, when the interest rate gap is closed, inflation growth is zero and inflation is stable. Loose monetary policy ($r^* > r$) will start the process of inflation acceleration, whilst restrictive monetary policy ($r^* < r$) will reduce inflation. Brozoza-Brzezina (2001) suggests that under this model, expectations are the (implicit) driving force behind inflation persistence. If the neutral rate is quite stable as has been postulated by Neiss and Nelson (2001), permanently higher rates of inflation are related to permanently lower real rates of interest.

To allow us to describe the long run equilibrium through cointegration analysis, some transformations have to be performed to our model. As the interest rate gap is expected to be stationary, equation (1) has to be transformed one level of integration “upwards”, to allow for order (one) integration of the variables. Equation (1) has been postulated for a stationary economy, with a constant level of potential output. Normally permanent growth of potential output will (ceteris paribus) lower the general price level. This fact is for example incorporated into the QTM equation through the presence of income $Y$. Hence, like Brozoza-Brzezina (2001), our model has to be enlarged by the potential output growth impact on prices. In our estimation income is proxied by the level of industrial production ($IP$) expanding our model to the following equation:

$$\pi_t = \pi_{o} + \psi \sum_{i=0}^{t-1} r_i - \Delta IP^*_t$$

where, $\pi_t$ is the inflation rate, $\pi_{o}$ is the time zero inflation rate, $\Delta IP^*_t$ is the change in income and $\psi \sum_{i=0}^{t-1} r_i$ is the matrix summing all previous real interest rates.

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18 Neiss and Nelson (2001) present a different equation $\pi_t = \alpha \pi_{t-1} + \psi (r^* - r_t)$, $0 < \alpha < 1$ but this yields the same steady state properties

19 Brozoza-Brzezina attributes the main reason for this as arbitrage between investment in financial instruments (return $r$) and physical capital (return $f'(k) = r^*$)

20 For simplicity we ignore the relationship between potential output (especially between the productivity growth rate) and the neutral rate of interest.
In line with Nelson and Neiss (2001), we also make another important assumption before empirically estimating the equation. As the neutral rate of interest is not observable, we will assume it is a stationary variable and its variance is small as related to the variance of the real interest rate. This allows us to keep $r^*$ constant which yields the equation:

$$
\pi_t = \pi_o + \psi \cdot r^* - \psi \sum_{i=0}^{t-1} r_i - \Delta IP^*_t
$$

where, $\pi_t$ is the inflation rate, $\pi_o$ is the time zero inflation rate, $\Delta IP^*_t$ is the change in income, $r^*$ is the constant neutral rate and $\psi \sum_{i=0}^{t-1} r_i$ is the matrix summing all previous real interest rates.

Thus equation (3) implies that the growth in the general price level will depend on the whole history of interest rate gaps. This equation will be subjected to cointegration analysis in subsequent chapters.

### 3.2 Data

The raw data collected for analysis is summarised in the following table:

**Insert Table 2 about here**

The data above needed to be transformed before we could apply it in our models. The adjusted data is summarised in the table below:

**Insert Table 3 about here**

The data series run between September 1886 and September 2005. Over this data period they were two significant policy changes which could potentially reflect structural breaks in our analysis. A structural break may have occurred at the end of 1992 when the fixed exchange rate regime was abolished. The second could have occurred in March 2001 when Inflation targeting was adopted. To verify if these policy breaks do indeed translate into structural breaks, exogenous “tests” shall be conducted in the next section.
4 Empirical Results

4.1 Structural breaks

In our study we will use exogenous approximation via observation of the behaviour of the stochastic series in our model. The logarithms of the series over the period 01/1986 to 08/2005 are plotted and presented in appendix 1. It appears like a break occurs around the time the fixed exchange rate regime was abolished in the early nineties. Several methodologies have been suggested that allow for the determination of the date of the structural break endogenously including Zivot and Andrews (1992), Banerjee, Lumsdaine and Stock (1992) and Perron (1990). However Perron (1989) conducts all his unit root tests using only a one time predetermined change. He argues that if the break date is known, like the Great Crash of 1929 or the oil price shock of 1973, then models with exogenous breaks are appropriate. The evidence from our graphs allow us to conclude that a structural break occurs at the end of 1992 when the fixed exchange rate system was abolished. Therefore, the subsequent cointegration tests will be estimated over two windows: a long sample (09/1986-08/2005) and a short sample (1/1993 -08/2005). Prior to our cointegration analysis we shall test the variables for their level of integration via Augmented Dicky Fuller tests (ADF).

4.2 Tests for level of Integration

The integration properties of inflation and interest rates are tested using ADF tests, reported below:

Insert Table 4 about here

From the table above it is evident that the hypothesis of a unit root can not be rejected for most of variables with or without a trend. Save for only CPI and LRFYBY with trend, the hypothesis of a unit root is not rejected. The uncertainty in the actual integration of the price level is disappointing as knowing with certainty the integration level of prices would allow us check for conformity with our model. For our model to

21 Zivot and Andrews (1992) present a procedure whereby a test statistic is estimated for each period while, simultaneously, allowing for the possibility of a structural break. The most negative value is compared with the critical values and is assumed as the date of the structural break.
be consistent with the data set, Inflation needs to be an $I(1)$ variable, and its growth rate stationary. Brzoza-Brzezina (2001) notes that the CPI is an imperfect approximation of the general price level and so ambiguity should rather be explained as a result of imperfection of the indices or low power of the integration tests.

The appearance of LRFYBY as a stationary variable is not abnormal as interest rates do tend to exhibit mean reversion. Furthermore, economic theory predicts the interest rate gap to be a stationary variable. To check if any of the variables are $I(2)$, we created their first-differences and applied the ADF command. In all cases we find that the hypothesis that the first differences of these variables have a unit root is strongly rejected. Hence we can conclude that LCPI, LRDR and LIP could be $I(1)$ variables.

### 4.3 Results of Cointegration tests

To test for cointegration we will make use of a Vector Error Correction Model built on three variables, the price level ($CPI$), the sum of all previous real interest rates ($\sum_{i=0}^{t-1} r_i$) and potential output ($IP$). Johansen tests are adopted in our estimation of the cointegrating relationships.

The equation for our VAR estimation is:

$$\Delta y_t = a_{y_0} + a_{y_1} t - \Pi y_{t-1} + \Psi w_t + u_{ny}$$

where, $z_t = \left( \begin{array}{c} y_t \\ x_t \end{array} \right)$, $y_t$ is an $m_y \times 1$ vector of jointly determined (or endogenous) $I(1)$ variables, $x_t$ is an $m_y \times 1$ vector of $I(1)$ exogenous variables, $w_t$ is a $q \times 1$ vector $I(0)$ exogenous variables, and $u_{ny}$ is the error term.

Results of the lag order of the VAR based on this estimation are presented in the table below:

**Insert Table 5 about here**

At this stage the cointegrating relationship of equation (3) will be found. This means finding the vector $[1, \psi, 1]$ with a constant such that the residuals $\epsilon$ from equation (3) are stationary. The econometric model that underlies the cointegrating VAR using
the Johansen maximum likelihood approach is given by the following general vector error correction model (VECM):

\[ \Delta x_t = \pi_0 + \pi_{t-1} \Delta x_{t-1} + \pi_2 \Delta x_{t-2} + \ldots + \pi_p \Delta x_{t-p} + \epsilon_t \]

where \( \pi_0 \) is an \((n \times 1)\) vector of intercept terms \( \pi_{t-1} \), \( \pi_i \) are \((n \times n)\) coefficient matrices with elements \( \pi_{jk}(i) \), \( \pi \) is a matrix with elements \( \pi_{jk} \) such that one or more of the \( \pi_{jk} \neq 0 \), and \( \epsilon_t \) is an \((n \times 1)\) vector with elements \( \epsilon_a \).

If all the variables in \( x_t \) above are \( \mathcal{I}(1) \), the error correction representation of these variables is necessarily a stationary linear combination giving rise to:

\[ \Delta x_{t-1} = \Delta x_t - \pi_0 - \sum \pi_i \Delta x_{t-1} + \epsilon_t \]

where \( \pi_0 \) is an \((n \times 1)\) vector of intercept terms \( \pi_{t-1} \), \( \pi_i \) are \((n \times n)\) coefficient matrices with elements \( \pi_{jk}(i) \), \( \pi \) is a matrix with elements \( \pi_{jk} \) such that one or more of the \( \pi_{jk} \neq 0 \), and \( \epsilon_t \) is an \((n \times 1)\) vector with elements \( \epsilon_a \). Incorporating this general equation above to equation (3) gives rise to equation (4) below:

\[ \pi = \pi_o + \psi \cdot r^* - \psi \sum \Delta y_i + \epsilon_i \]

where, \( \pi_i \) is the inflation rate, \( \pi_o \) is the time zero inflation rate, \( \Delta IP^* \) is the change in income, \( r^* \) is the constant neutral rate and \( \psi \sum \Delta y_i \) is the matrix summing all previous real interest rates and \( \epsilon_i \) is the error term.

From this equation, 4 different models will be tested based on:

- 1 price index; 2 data samples; 2 interest rates
  - Model 1: \textbf{IR, RDR, IP} -1993M1 - 2005M8
  - Model 4: \textbf{IR, LRFYBY, IP} -1993M1- 2005 M

The results of the Johansen test for cointegration applied to these four different models are presented below:

Insert Table 6 about here
For model 1 the trace statistics do not reject the hypothesis $H_0 = 1$ at a 90 percent significance level whilst the eigenvalue statistics do not reject this hypothesis at a 95 percent level. Thus both statistics seem to imply the existence of one cointegrating relationship (vector) between the variables.

For model 2 the trace statistics do not reject the hypothesis of $H_0 = 2$ at a 95 percent level whilst the eigenvalue statistics do not reject the hypothesis that $H_0 = 1$ at 90 percent significance level. The eigenvalue statistics also imply the existence of two cointegrating relationships at a 95 percent significant level. Thus both statistics seem to imply the existence of two cointegrating relationships (vectors).

For model 3 both the trace statistic and the eigenvalue statistics do not reject the hypothesis of $H_0 = 2$ at a 95 percent significance level. Hence the bias is towards the existence of two cointegrating relationships (vector).

For model 4 the trace statistic do not reject both the hypothesis $H_0 = 1$ and $H_0 = 2$ at a 95 percent significance level. The eigenvalue statistics do not reject the hypothesis of $H_0 = 1$ at a 95 percent significance level. Hence the bias is towards the existence of one cointegrating relationship (vector).

Next we estimate the cointegrating relationships having assumed the existence of one cointegrating vector in each model as suggested by most of the models and complimented by theory.

Insert Table 7 about here

As can be seen from the table, three out of the four relationships fulfil the criterion on coefficient signs. In the case of model two the interest rate and the industrial production coefficients are negative which implies a positive relationship between these variables and inflation, and thus contradicting the theoretical model. To this end no further analysis of model was carried out as this would be purposeless. This model seems to fail because of the existence of a structural break over the long period estimation. As noted in the preceding chapter a structural break seems to have occurred when Norway shifted from a fixed exchange rate regime to a floating currency at the end of 1992.
In the case of the other three models the signs on the industrial production coefficient and interest rate coefficient are positive. This implies a negative relationship between real rates and inflation and thus conforms to the theoretical model. Thus the error correction coefficients were estimated and in all three cases the standard errors were significantly different from zero. The error correction adjustment coefficients suggest causality is compliant with theory as the only significant error correction mechanism is obtained in the inflation equation. This implies a causal relationship of the type we would have expected, going from real interest rates to inflation.

*The error correction equations are given below:*

**Model 1**
\[ ecm1 = 1.0000 \times IR + 0.1037159 \times RDR + 0.69986 \times IP - 0.0031205 \]

**Model 3**
\[ ecm1 = 1.0000 \times IR + 0.17824 \times RFYBY + 0.6878 \times IP - 0.076100 \]

**Model 4**
\[ ecm1 = 1.0000 \times IR + 0.18563 \times RFYBY + 0.1322 \times IP - 0.0018857 \]

Whether the real interest rate gap should lead or lag inflation depends upon how forward looking private sector agents are in their decision making. If current spending decisions are largely based on expectations of future interest rates (approximately, long-term interest rates), and firms’ pricing decisions are largely based on their expectations of future excess demand, inflation should be negatively correlated with cumulated future real rate gaps. Conversely if private sector is mostly backward looking, or it is tightly constrained by past outcomes, then current inflation should be negatively correlated with past real rate gaps.

Our empirical results suggest that investors are both forward looking and backward looking as significant cointegrating relationships exist with both past gaps (based on the real deposit rate) and future rate gaps (based on five year bond yields).

Finally we restrict the parameter \( IP \) to 1 (as in the theoretical model) and the validity of this restriction will be tested based on likelihood ratio statistics. The results are given in the table below.

*Insert Table 8 about here*
From the table above it can be seen that the IP parameter does not significantly differ from one only in the case of model 1. Since the p-values for this model is greater than 5 percent, we can not reject the null hypothesis that the data is consistent with the theory. However in the case of the other two models, model 3 and model 4, the IP parameter is significantly different from one which implies that the data is not supporting the theoretical model.

Summing up the results it can be concluded that only one out of the four models evaluated had a cointegrating relationship that fulfilled all the criteria imposed on the model.

**4.2 Neutral Rate Comparison**

If the neutral rate could be assumed to be constant over time, the real interest rate averaged over the whole business cycle, should approximate the neutral real interest rate because the positive and negative real interest rate gaps cancel out.\(^{22}\) Our estimation was based on looking at implied long term forward nominal rate given by RFYBY and the current rate proxied by RDR. Both series were deflated by the CPI inflation rate. In principle, the long term implied forward rates are determined by the market’s future interest rate expectations and should, at least in theory, be independent of the current business cycle. Hence the real rate deduced from them should be close to the markets estimate of the real interest rate. From the real rates we computed the average which provided our estimates of the neutral rate.

**Insert Table 9 about here**

Our estimates of the neutral rate based on the different model dates are not very reliable as the dates cross over different business cycles. In addition the results are skewed by the periods of high inflation before the turn of the century. To get a more accurate rate we proceed to estimate the neutral rate using data about real rates after the turn of the century. Our estimate lies in the range between 2.7 and 3.2 percent; consistent with estimates of the global neutral rate.

\(^{22}\)The problems related with this approach according to (Bernhardsen, 2005) are: inflation expectations and hence real interest rates are unobservable, it is not always clear when the business cycle starts and ends, and the neutral real interest s by nature not constant.
Giammarioli and Valla (2003), argue that the neutral rate for the Euro area has gradually declined from around 4 in the mid-1990s to around 3 percent in 2000. Cuaresma, Gnan and Ritzberger-Gruenwald (2003) suggest the neutral rate had settled to a level of about 2 percent at the end of 2002. Garnier and Wilhelmsen (2004) also find a drop in the neutral rate for both the Euro area and Germany. More recently Goldman Sachs (2005) estimates the level of the neutral rate to be close to 2 percent in October 2004. They further report that this rate has decreased over the past 15 years.

Laubach and Williams (2003) estimated the US neutral rate to be about 3 percent in 2001. The OECD (2004) argues that the neutral real interest rate in the US has varied between 2 and 5 percent since the 1960s and that it may now be slightly higher than 2 percent. Its temporary rise in the latter half of the 1990s is attributed to higher productivity growth. In the wake of the collapse of equity prices and the recession after the turn of the millennium, the estimates for the neutral real interest rate in the US have been lowered. Manrique and Marques (2004) also identify a similar pattern to Laubach and Williams (2005) that the neutral rate of interest in the US increased towards the millennium but then declined thereafter. The Financial Times (2005) puts the neutral real rate in the US around 2.75 percent, while Goldman Sachs (2005) puts it around 2.5 percent.

Hammerstrøm and Lønning (2000) suggest a neutral real interest rate in the range of 3-4 percent for Norway. The Norges Bank (2005) believes a more reasonable range for the neutral real interest rate is between 2.5-3.5 percent. Bernhardsen (2005) using implied long-term forward interest rates, concludes the real neutral interest rate varied between 3 and 4 percent in the period 1998-2003. However, from mid-2002 it has been trending downwards. This estimated interval for Norway is somewhat higher than the estimate for the global neutral real interest rate, implying a risk premium of about half a percent. Norges Bank (2005) attributes this difference to specific conditions such as exchange rate and liquidity risk in addition to large international investors’ preference for avoiding small markets.

5. Conclusion

The objective of our research was to test the validity of the Wicksellian theory in the context of Norway and simultaneously calculate the neutral rate of interest. The results of our empirical analysis supported this relationship in one out of the four models
estimated. Model 2 most likely failed because of the existence of a structural break over the estimation window. Models 3 and 4, although posting significant error correction equations, failed because the restrictions placed on the IP parameter (as with the theoretical model where \( y^* \) is set to one) were not significant. Thus only one out of the four models evaluated had a cointegrating relationship that fulfilled all the criteria imposed on the model. Therefore from this significant result we can conclude that, with respect to the inflationary process, that the economy can work like a space shuttle, which once accelerated will cruise at a certain speed without the use of engines. The central bank can open the interest rate gap to accelerate/decelerate inflation and once this has happened, the gap can be closed and inflation will remain at the higher/lower level.

The second objective was to derive and compare the neutral rate. The adopted research method only allowed us to calculate the average level of the neutral rate and not the time series variations. Further research in this area could be explored through use of general equilibrium models or Kalman filter tests.
References


Bernhardsen, T. (2005) The Neutral Real Interest Rate, Monetary Policy Department Staff Memo, 2005/1


Financial Times (2005) ‘Fed thinking points to measured rate rises this year’, 19 January


Garnier, J., and J. Wilhemsen (2004) Estimating the Natural Rate of Interest for the Euro Area- The Laubach and Williams approach, European Central Bank, Memo


Hammerstrøm, G and I. Lønning, (2000) Can We Quantify the Neutral Real Interest Rate, Penger og Kredit 2/00, Norges Bank


Wicksell, K. (1898) *Interest and Prices*, Macmillan 1936
Table 1: Properties of Model

<table>
<thead>
<tr>
<th>The Model</th>
<th>( \Delta \pi_i = \psi(r^*-r_{i-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closed Interest Rate Gap</td>
<td>Impact on Inflation &amp; its growth rate</td>
</tr>
<tr>
<td>( P = ? )</td>
<td></td>
</tr>
<tr>
<td>( r = r^* )</td>
<td>( \pi = const. )</td>
</tr>
<tr>
<td></td>
<td>( \Delta \pi = 0 )</td>
</tr>
<tr>
<td>Positive Interest rate Gap</td>
<td>Impact on inflation &amp; its growth rate</td>
</tr>
<tr>
<td>( P = ? )</td>
<td></td>
</tr>
<tr>
<td>( r &gt; r^* )</td>
<td>( \pi \downarrow )</td>
</tr>
<tr>
<td></td>
<td>( \Delta \pi &lt; 0 )</td>
</tr>
<tr>
<td>Negative Interest Rate gap</td>
<td>Impact inflation &amp; its growth rate</td>
</tr>
<tr>
<td>( P = ? )</td>
<td></td>
</tr>
<tr>
<td>( r &lt; r^* )</td>
<td>( \pi \uparrow )</td>
</tr>
<tr>
<td></td>
<td>( \Delta \pi &gt; 0 )</td>
</tr>
</tbody>
</table>
Table 2. Raw Data used for Norwegian analysis

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI</td>
<td>Consumer Price Index&lt;sup&gt;23&lt;/sup&gt;</td>
</tr>
<tr>
<td>DR</td>
<td>Deposit Rate</td>
</tr>
<tr>
<td>FYBY</td>
<td>Yield on 5 Year Bonds</td>
</tr>
<tr>
<td>IR</td>
<td>Inflation rate&lt;sup&gt;24&lt;/sup&gt;</td>
</tr>
<tr>
<td>PI</td>
<td>Production index&lt;sup&gt;25&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

Notes: All the data were collected from the Primarc Thomson Data stream database with the exception of the industrial production index which was sourced from the Statistics Norway database.

<sup>23</sup>The CPI is the seasonally adjusted series.
<sup>24</sup>The inflation rate was derived from the CPI series as the periodic growth between the series. It was used to proxy expected inflation.
<sup>25</sup>The industrial production index measures seasonally adjusted index of oil and gas extraction, manufacturing, mining and quarrying and electricity, gas and steam supply. As these outputs would represent a large proportion of GDP, this index was used as a proxy for GDP.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCPI</td>
<td>$\ln(CPI)$</td>
</tr>
<tr>
<td>RDR</td>
<td>$(1 + DR)/(1 + IR) - 1$</td>
</tr>
<tr>
<td>RFYBY</td>
<td>$(1 + YFYB)/(1 + IR) - 1$</td>
</tr>
<tr>
<td>LPI</td>
<td>$\ln(PI)$</td>
</tr>
<tr>
<td>D (LCPI)</td>
<td>$LCPI - LCPI(-1)$</td>
</tr>
<tr>
<td>LRDR</td>
<td>$\ln(RDR)$</td>
</tr>
<tr>
<td>D (LRDR)</td>
<td>$LRDR - LRDR(-1)$</td>
</tr>
<tr>
<td>LRFYBY</td>
<td>$\ln(RFYBY)$</td>
</tr>
<tr>
<td>D (LRFYB)</td>
<td>$\ln(LFYBY - LRFYBY(-1))$</td>
</tr>
</tbody>
</table>

Notes: Transformation of the data, together with subsequent testing was carried out in Pesaran and Pesaran’s Microfit 4 software. Real interest rates were calculated by discounting the nominal rates with their respective inflation rates.
Table 4: Results of ADF Unit Root Tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lag Order in ADF Test</th>
<th>ADF Statistic without trend</th>
<th>Test Statistic</th>
<th>Lag Order in ADF Test</th>
<th>ADF Statistic with trend</th>
<th>Test Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCPI</td>
<td>7</td>
<td>-2.8745</td>
<td>-2.7382</td>
<td>1</td>
<td>-3.4311</td>
<td>-3.9675*</td>
</tr>
<tr>
<td>D(LCPI)</td>
<td>0</td>
<td>-2.8746</td>
<td>-10.5105</td>
<td>0</td>
<td>-3.4312</td>
<td>-11.2206</td>
</tr>
<tr>
<td>LRDR</td>
<td>4</td>
<td>-2.8822</td>
<td>-1.4252</td>
<td>6</td>
<td>-3.4426</td>
<td>-1.6249</td>
</tr>
<tr>
<td>D(LRD)</td>
<td>5</td>
<td>-2.8824</td>
<td>-8.3133</td>
<td>5</td>
<td>-3.4428</td>
<td>-8.3451</td>
</tr>
<tr>
<td>LRFYBY</td>
<td>6</td>
<td>-2.8745</td>
<td>-1.4976</td>
<td>6</td>
<td>-3.4311</td>
<td>-3.4850*</td>
</tr>
<tr>
<td>D(LRFYBY)</td>
<td>7</td>
<td>-2.8746</td>
<td>-9.3164</td>
<td>7</td>
<td>-3.4312</td>
<td>-9.3633</td>
</tr>
<tr>
<td>LIP</td>
<td>3</td>
<td>-2.8745</td>
<td>-1.9493</td>
<td>2</td>
<td>-3.4310</td>
<td>-7.0623</td>
</tr>
<tr>
<td>D(LIP)</td>
<td>2</td>
<td>-2.8745</td>
<td>-14.2935</td>
<td>2</td>
<td>-3.4311</td>
<td>-14.4903</td>
</tr>
</tbody>
</table>

Notes: Lag order in the VAR model was chosen based on information criteria (Akaike Information Criterion, Schwarz Bayseian Criterion, Maximised log likelihood and Hannan –Quinn Criterion).
* indicates null hypothesis of a unit root rejected a 95% Confidence Interval.

Table 5. Results of Unrestricted VAR Estimation

<table>
<thead>
<tr>
<th>Variables in the Model</th>
<th>Sample</th>
<th>LL</th>
<th>AIC</th>
<th>SC</th>
<th>LR</th>
<th>Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>DLCP; LRDR; LIP &amp; INPT</td>
<td>93M1-05M8</td>
<td>12</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>DLCPI; LRFYBY; LIP &amp; INPT</td>
<td>93M1-05M8</td>
<td>12</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>DLCPI; LRFYBY; LIP &amp; INPT</td>
<td>86M1-05M8</td>
<td>12</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

Notes: Given the monthly frequency of our data we set the maximum lag length in our unrestricted VAR estimation to 12.
Table 6. Results of The Johansen (1988) Tests for Cointegration

<table>
<thead>
<tr>
<th>Model 1 93M1-05M8</th>
<th>Hypothesis</th>
<th>Trace Test Statistic</th>
<th>Max Eigenvalue St</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ho</td>
<td>H1</td>
<td></td>
</tr>
<tr>
<td>IR</td>
<td>r = 0</td>
<td>r = 1 ( r ≥1)</td>
<td>(34.8700)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RDR</td>
<td>r = 1</td>
<td>r = 2 ( r ≥2 )</td>
<td>(34.8700)</td>
</tr>
<tr>
<td>IP</td>
<td>r = 2</td>
<td>r = 3</td>
<td>(34.8700)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode 1 2 86M1-05M8</th>
<th>Hypothesis</th>
<th>Trace Test Statistic</th>
<th>Max Eigenvalue St</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ho</td>
<td>H1</td>
<td></td>
</tr>
<tr>
<td>IR</td>
<td>r = 0</td>
<td>r = 1 ( r ≥1)</td>
<td>(34.8700)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RFYBR</td>
<td>r = 1</td>
<td>r = 2 ( r ≥2 )</td>
<td>(34.8700)</td>
</tr>
<tr>
<td>IP</td>
<td>r = 2</td>
<td>r = 3</td>
<td>(34.8700)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode 1 3 86M1-92M12</th>
<th>Hypothesis</th>
<th>Trace Test Statistic</th>
<th>Max Eigenvalue St</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ho</td>
<td>H1</td>
<td></td>
</tr>
<tr>
<td>IR</td>
<td>r = 0</td>
<td>r = 1 ( r ≥1)</td>
<td>(34.8700)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RFYBR</td>
<td>r = 1</td>
<td>r = 2 ( r ≥2 )</td>
<td>(34.8700)</td>
</tr>
<tr>
<td>IP</td>
<td>r = 2</td>
<td>r = 3</td>
<td>(34.8700)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode 1 4 93M1-05M8</th>
<th>Hypothesis</th>
<th>Trace Test Statistic</th>
<th>Max Eigenvalue St</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ho</td>
<td>H1</td>
<td></td>
</tr>
<tr>
<td>IR</td>
<td>r = 0</td>
<td>r = 1 ( r ≥1)</td>
<td>(34.8700)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RFYBR</td>
<td>r = 1</td>
<td>r = 2 ( r ≥2 )</td>
<td>(34.8700)</td>
</tr>
<tr>
<td>IP</td>
<td>r = 2</td>
<td>r = 3</td>
<td>(34.8700)</td>
</tr>
</tbody>
</table>

Notes: Critical values at 95% Confidence Interval in parenthesis, test statistics in bold.*denotes Ho rejected at 5%. **denotes Ho rejected at 10% but accepted at 5%. Blank denotes Ho accepted at both 10% and 5%.

26 If Ho is rejected at 5% it means it will automatically be rejected at 10% because the critical values will be getting smaller.
Table 7: Summary of Cointegrating Vectors of the 4 Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Inflation</th>
<th>Industrial Production</th>
<th>Interest Rate</th>
<th>Constant</th>
<th>Error Correction coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>1.0000</td>
<td>-0.0031</td>
<td>0.00372</td>
<td>-0.0031</td>
<td>-0.4599 (0.1261)</td>
</tr>
<tr>
<td>Model 2</td>
<td>1.0000</td>
<td>0.001</td>
<td>-0.333</td>
<td>-0.5286</td>
<td>****** (0.13224)</td>
</tr>
<tr>
<td>Model 3</td>
<td>1.0000</td>
<td>0.6878</td>
<td>0.1782</td>
<td>-0.761</td>
<td>-0.8981 (0.1500)</td>
</tr>
<tr>
<td>Model 4</td>
<td>1.0000</td>
<td>0.1856</td>
<td>0.1322</td>
<td>-0.001</td>
<td></td>
</tr>
</tbody>
</table>

Notes: ****** indicates no estimation was conducted because the industrial production and interest rate coefficient signs did not conform to theory. Tests are conducted with restricted intercepts and no trends in the VAR. Restrictions were specified on the IR variable in each case in order to identify the long run structural relationships.

Table 8: Results for tests of the Validity Restrictions on IP parameters

<table>
<thead>
<tr>
<th>Model</th>
<th>CHSQ( 1)</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>3.6407</td>
<td>(0.56)</td>
</tr>
<tr>
<td>Model 2</td>
<td>5.6314</td>
<td>(0.01)*</td>
</tr>
<tr>
<td>Model 3</td>
<td>41.2599</td>
<td>(0.00)**</td>
</tr>
</tbody>
</table>

Notes: * indicates that the IP parameter is significantly different from one at the 5 percent level Cointegration estimation set with restricted intercepts and no trends in the VAR, with over-identifying linear restrictions set on the IP variable in all three estimations.

Table 9. Neutral Rate Estimation

<table>
<thead>
<tr>
<th>Model</th>
<th>Neutral Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>4.1</td>
</tr>
<tr>
<td>Model 2</td>
<td>7.4</td>
</tr>
<tr>
<td>Model 3</td>
<td>4.5</td>
</tr>
<tr>
<td>Model 4</td>
<td>10.1</td>
</tr>
</tbody>
</table>

Notes: The estimation of the neutral rate was based on the computed average of the real interest rate over the respective estimation windows.
APPENDIX 1

Log CPI

Log IP